

An Instrumental Variables Approach to Testing Firm Conduct*

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Abstract

Understanding firm conduct is crucial for industrial organization and antitrust policy. In this article, we develop a testing procedure based on the Rivers and Vuong non-nested model selection framework. Unlike existing methods that require estimating the demand and supply system, our approach compares the model fit of two first-stage price regressions. Through an extensive Monte Carlo study, we demonstrate that our test performs comparably to, or outperforms, existing methods in detecting collusion across various collusive scenarios. The results are robust to model misspecification, alternative functional forms for instruments, and data limitations. By simplifying the diagnosis of firm behavior, our method provides an efficient tool for researchers and regulators to assess industry conduct. Additionally, our approach offers a practical guideline for enhancing the strength of BLP-style instruments in demand estimation: once collusion is detected, researchers are advised to incorporate the product characteristics of colluding partners into own-firm instruments while excluding them from other-firm instruments.

Keywords: firm conduct, BLP-style instruments, Rivers and Vuong test, discrete choice models, demand estimation

JEL Classification Numbers: C36, L13, L21, L41

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1 Introduction

Understanding industry conduct – that is, the extent to which firms collude in a goods market – is one of the central issues in the study of industrial organization. This issue also carries practical importance, as antitrust policies and regulations based on incorrect conclusions about industry conduct could harm consumers and reduce market efficiency. For this reason, IO researchers and regulatory authorities have shown significant interest in accurately evaluating firm conduct. In response to this interest, various approaches to testing firm conduct have been developed.

Since existing tests on industry conduct (e.g., Duarte, Magnolfi, Sølvesten, and Sullivan (2024), Backus, Conlon, and Sinkinson (2021), Dearing, Magnolfi, Quint, Sullivan, and Waldfogel (2024)) require estimating the demand and supply system – a task that remains challenging despite advances in modeling and computing over the past three decades – their testing power can be reduced by model misspecification. Furthermore, the true firm conduct is often unknown, complicating the proper construction of BLP-style instruments – functions of exogenous product characteristics, such as the sums of characteristics of products within and across firms. For instance, if a researcher estimates demand in a differentiated product market and assumes that firms engage in price competition when they actually collude, the BLP-style instruments may not be constructed effectively, resulting in reduced identification power.

In this article, we propose a practical and powerful method for testing firm conduct that is also useful for designing more effective BLP-style instruments. The intuition behind our testing procedure is that if firms engage in price competition, the price effect of a change in own-firm product characteristics is not only larger in magnitude but also opposite in direction to the price effect of a change in other-firm product characteristics (Berry, Levinsohn, and Pakes, 1995; Gandhi and Nevo, 2021). In contrast, in the case of collusion, the two price effects are similar in magnitude (assuming the products are equally substitutable) and share the same direction. Therefore, treating the product attributes of other firms as own attributes when constructing BLP-style instruments would enhance their strength in the first-stage price regression in cases of collusion. On the other hand, it would weaken the instruments’ strength if the firms engage in price competition. Consequently, the strength of BLP-style instruments hinges on whether the assumption about firm conduct – unknown to the researcher – is correctly imposed. In other words, we can infer whether firms compete

or collude by comparing the effectiveness of instruments constructed under alternative firm conduct scenarios.

Based on the above idea, we develop a testing procedure by applying the non-nested model selection method proposed by Vuong (1989) and Rivers and Vuong (2002) (the RV test). The first step in our procedure is to construct two BLP-style instruments: one incorporating only own-firm product attributes (competition IVs) and the other incorporating both the characteristics of own products and those of *suspected* colluding partners (collusion IVs). The next step is to run two linear price regressions separately using each of the two instrument sets. Finally, the last stage involves constructing the RV test statistic based on the objective function values of the two regression models. A statistically significant positive (negative) value can be interpreted as evidence of collusive (competitive) behavior among the suspected firms. The primary advantage of our method over existing approaches built upon equilibrium markup conditions is that it does not require estimating the demand and supply system, as it simply compares the model fit of the two first-stage price regressions.

To evaluate the finite-sample performance of our proposed test statistic, we conduct an extensive Monte Carlo study. More specifically, we consider an indirect utility function with a normally distributed random coefficient for the price (as well as a simple logit demand model) and a linear marginal cost function. We then simulate 500 datasets for various Monte Carlo configurations, each representing a unique market condition (in terms of the degree of market concentration, the share of colluding firms among all firms, the extent to which a firm internalizes its colluding partners' profits). Using each simulated dataset, we calculate both our test statistic and the existing statistic. Finally, we compute the median values of the test statistics for each Monte Carlo configuration and examine how they change as industry conduct varies.

The results, which are robust to model misspecification, data limitations (e.g., the availability of only a few markets), and the use of alternative functional forms for BLP-style instruments, reveal that our testing procedure performs comparably to, or outperforms, existing approaches in detecting collusion under various market conditions. The high testing power of our method, combined with its ease of implementation – requiring only a comparison of the model fit of two first-stage price regressions without estimating the demand and supply system – and minimal data requirements, makes it a practical tool for the preliminary diagnosis of firm conduct. Once our test statistic

indicates the presence of collusion among *suspected* firms in the market, researchers and regulatory authorities can proceed with existing approaches to determine which specific conduct model best fits the observed data. In this way, the two methods complement each other and collectively create a more practical, efficient, and powerful framework for conduct testing.

Our Monte Carlo study also reveals that, as expected, the use of collusion IVs instead of competition IVs improves the estimation performance of demand parameters under various Bertrand-Nash collusive scenarios. This finding suggests that researchers can enhance the strength of BLP-style instruments by designing them to accurately reflect actual industry conduct. For instance, after finding suggestive evidence of collusion using our testing procedure, researchers are advised to incorporate the product characteristics of colluding partners into own-firm instruments while excluding them from other-firm instruments.

We apply our testing method to study firm conduct in two differentiated product markets in South Korea: the new passenger car market and the instant noodles market. Our test rejects the hypotheses of brand-level profit maximization and price collusion in favor of joint profit maximization at the parent company level in the car market. In addition, our results suggest that, despite suspicions from the Korean Fair Trade Commission, instant noodle manufacturers have not engaged in any price collusion. Our findings align with anecdotal evidence of firm conduct in each market, court rulings, and previous empirical findings that instant noodle prices remain significantly below the collusive level (Kim and Kim, 2024).

This article proceeds as follows. In the next section, we review previous literature and delineate the contributions of our study. In Section 3, we provide a conceptual framework for our testing procedure which we formally propose in Section 4. Based on an extensive Monte Carlo study whose setup is elaborated in Section 5, we examine the performance of our testing method and compare it with that of existing procedures in Section 6. We then conduct another simulation study in Section 7 to provide further intuition behind our approach. In Section 8, we apply our method to study industry conduct in two differentiated product markets. Finally, we conclude in Section 9.

2 Related literature

This article is closely related to the empirical literature on implementing the RV approach for testing firm conduct.¹ Bonnet and Dubois (2010) investigated vertical relationships between manufacturers and retailers in the French bottled water market, concluding that manufacturers use two-part tariff contracts with resale price maintenance. In the premium ice cream market, Sullivan (2017) found evidence that Ben & Jerry’s and Häagen-Dazs exhibited behaviors consistent with full coordination on both pricing and product choice decisions.² Hu, Xiao, and Zhou (2014) examined price collusion within or across large corporate groups in the Chinese passenger-vehicle industry and found no evidence of collusive pricing behavior. These previous studies backed out marginal costs from demand estimates under each assumed firm conduct model, constructed moment-based objective functions, and determined which model provided a better fit.

Recent studies on the RV test approach have built upon the falsifiable restrictions proposed by Berry and Haile (2014) to distinguish between alternative conduct models in differentiated goods markets. These studies have further examined the identification conditions required for testing, the selection of appropriate instruments, and the econometric properties of the RV approach. Dearing, Magnolfi, Quint, Sullivan, and Waldfogel (2024) identified cost pass-through as a key determinant for selecting instruments to falsify models of firm behavior. Duarte, Magnolfi, Sølvsten, and Sullivan (2024) analyzed the properties of the RV test statistic under weak instrument conditions and developed a robust inference framework for the RV approach in such scenarios. They also highlighted the advantages of the RV approach under model misspecification compared to other testing methods. Furthermore, Backus, Conlon, and Sinkinson (2021) refined the RV testing procedure by incorporating flexible nonparametric functional forms for marginal costs and proposing instrument functional forms that utilize scalar moment values to enhance the testing power of the RV framework. Our approach, while relying solely on first-stage price regressions, performs comparably to

¹As discussed in Duarte, Magnolfi, Sølvsten, and Sullivan (2024), alternative testing procedures include (i) the Estimation-Based (EB) test (Pakes, 2017), (ii) the Anderson-Rubin (AR) test (Anderson and Rubin, 1949), and (iii) the Cox test (Cox, 1961; Smith, 1992). Examples of empirical applications of these approaches include Miller and Weinberg (2017) (EB test), Bergquist and Dinerstein (2020) (AR test), and Villas-Boas (2007) (Cox test). An advantage of the RV test over these methods is that inference on conduct remains robust even under misspecification.

²To examine coordinated behavior in product choice decisions, Sullivan (2017) modeled endogenous product choices and estimated bounds for the fixed cost parameter using the partial identification approaches employed in Eizenberg (2014) and Wollmann (2018). He then implemented the RV test extended by Shi (2015) within a moment inequalities framework.

these methods, which are built upon equilibrium markup conditions, and thus offers a simple yet effective diagnostic for detecting collusive pricing behavior.

This article also contributes to the broader empirical industrial organization (IO) literature on assessing firm conduct in differentiated product markets, a longstanding and central topic of inquiry in the field and among antitrust authorities. Examples include airlines (Ciliberto and Williams, 2014), automobiles (Bresnahan, 1987; Verboven, 1996; Sudhir, 2001; Hu, Xiao, and Zhou, 2014), beer (Hausman, Leonard, and Zona, 1994; Slade, 2004; Rojas, 2008; Miller and Weinberg, 2017; Miller, Sheu, and Weinberg, 2021), instant noodles (Kim and Kim, 2024), ready-to-eat (RTE) cereal (Nevo, 2001; Backus, Conlon, and Sinkinson, 2021; Michel, y Mino, and Weiergraeber, 2024), soft drinks (Gasmi, Laffont, and Vuong, 1992), and ice cream (Sullivan, 2017). These studies relied on structural models of demand and supply to identify the firm behavior that best explains the observed market outcomes. In contrast, our proposed method requires only a pair of price regressions, making it straightforward to implement.

Finally, literature on demand estimation with aggregate data in the discrete choice framework also bears on this article. Prior works (e.g., Berry, Levinsohn, and Pakes (1999), Berry and Haile (2014), Reynaert and Verboven (2014), Armstrong (2016), Gandhi and Houde (2019), and Conlon and Gortmaker (2020)) have addressed challenges arising from weak instruments in non-linear GMM estimation. Our work contributes to this field by enhancing the understanding of instruments and improving estimation performance, demonstrating that the strength of BLP-style instruments depends on the correctness of the imposed assumption on industry conduct.

3 Conceptual framework

3.1 Discrete choice in differentiated product markets

There are $J_t + 1$ differentiated products in market $t = 1, 2, \dots, T$. For product $j = 0, 1, \dots, J_t$, there are observed attributes (x_{jt}, p_{jt}) and an unobserved component ξ_{jt} . The observed attributes are grouped into two parts: (i) the exogenous part, x_{jt} , which is uncorrelated with the unobservables $\boldsymbol{\xi}_t = (\xi_{0t}, \xi_{1t}, \dots, \xi_{J_t t})$ and (ii) the endogenous price, p_{jt} , which is correlated with $\boldsymbol{\xi}_t$. For each product, there are R exogenous attributes in $x_{jt} = (x_{jt}^{(1)}, \dots, x_{jt}^{(r)}, \dots, x_{jt}^{(R)})$. Market t is defined by the set of products offered, denoted by $\{0\} \cup \mathcal{J}_t$, the set of attributes of these products, denoted

by $\chi_t = \{(x_{0t}, p_{0t}, \xi_{0t}), \dots, (x_{Jt}, p_{Jt}, \xi_{Jt})\}$, and the set of characteristics of consumers, denoted by V_t .

A consumer i with characteristics $v_i \in V_t$ has the indirect utility from consuming product j given by $u_{ijt} = U(x_{jt}, p_{jt}, \xi_{jt}, v_i)$. A consumer purchases the product that maximizes his or her utility. The predicted market share of product j , denoted by s_{jt} , is the fraction of market consumers who prefer good j over all other J_t products:

$$s_{jt} = \Pr(v_i \in V_t : U(x_{jt}, p_{jt}, \xi_{jt}, v_i) \geq U(x_{kt}, p_{kt}, \xi_{kt}, v_i) \quad \forall k \in \{0\} \cup \mathcal{J}_t)$$

Consumer characteristics, v_i , are decomposed into a common component shared across all individuals and heterogeneous components: (i) random utility components, $\nu_i = (\nu_{ip}, \nu_{i1}, \dots, \nu_{iR})$, capturing heterogeneous valuations of (p_{jt}, x_{jt}) ; and (ii) an idiosyncratic taste shock, ε_{ijt} . The most commonly used utility specification in the literature is given by:

$$u_{ijt} = \delta_{jt} + \nu_{ip} p_{jt} + \sum_r \nu_{ir} x_{jt}^{(r)} + \varepsilon_{ijt}, \quad (1)$$

where $\delta_{jt} = x_{jt}\beta - \alpha p_{jt} + \xi_{jt}$ is the mean utility of the product, which also absorbs the common component of v_i . Since the heterogeneous components ν_i and ε_{ijt} are not directly observed in aggregated market-level data, we assume their distributions. Let ν_i follow a mean-zero multivariate normal distribution,³ while ε_{ijt} are i.i.d. (across both consumers and products) extreme value distributed.

We normalize the indirect utility from consuming product $j = 0$ by setting $u_{i0t} = \varepsilon_{i0t}$ for all consumers and refer to it as the *outside option*. Assuming that the two random components, ν and ε , are independent, the predicted market share of product j is given by

$$s_{jt}(\boldsymbol{\delta}_t, \mathbf{p}_t, \mathbf{x}_t; \theta_2) = \int \frac{\exp\left(\delta_{jt} + \nu_{ip} p_{jt} + \sum_r \nu_{ir} x_{jt}^{(r)}\right)}{1 + \sum_{k \in \mathcal{J}_t} \exp\left(\delta_{kt} + \nu_{ip} p_{kt} + \sum_r \nu_{ir} x_{kt}^{(r)}\right)} dF(\nu_i; \theta_2), \quad (2)$$

where $F(\nu_i; \theta_2)$ is the joint cumulative distribution function of ν_i , governed by a vector of non-linear

³Demographic-specific valuations of (x_{jt}, p_{jt}) can also be modeled either by drawing from an empirical distribution or by imposing distributional assumptions on the demographics of market t .

parameters θ_2 . Bold-faced letters indicate that the predicted market share depends on the vectors of mean utilities and attributes of all products in market t .

Berry (1994) demonstrated that, for a given non-linear parameter vector θ_2 , there exists a unique vector of mean utilities $\boldsymbol{\delta}_t$ that equates the observed market shares with the predicted market shares. Specifically, let $\mathbf{S}_t = (S_{0t}, S_{1t}, \dots, S_{Jt})$ denote the vector of *observed* market shares of products in market t . Then, $\mathbf{s}_t(\boldsymbol{\delta}_t, \mathbf{p}_t, \mathbf{x}_t; \theta_2) = \mathbf{S}_t$. This relationship allows us to invert the demand system in market t and solve for $\delta_{jt}(\mathbf{S}_t, \mathbf{p}_t, \mathbf{x}_t; \theta_2) = x_{jt}\beta - \alpha p_{jt} + \xi_{jt}$ for all $j \in \mathcal{J}_t$.⁴ Finally, for a given parameter vector $\theta = (\theta_1, \theta_2)$, where $\theta_1 = (\alpha, \beta)$ represents the linear parameters, we compute $\xi_{jt}(\theta) = \delta_{jt}(\mathbf{S}_t, \mathbf{p}_t, \mathbf{x}_t; \theta_2) - (x_{jt}\beta - \alpha p_{jt})$.

Utility parameters are estimated using a GMM framework based on the moment restrictions at the true parameter $\theta = \theta_0$ given by:

$$E[\xi_{jt}(\theta_0)\mathbf{Z}_{jt}] = 0,$$

where $\mathbf{Z}_{jt} = (x_{jt}, \mathbf{z}_{jt})$ is a vector of instruments, and \mathbf{z}_{jt} is a vector of *excluded* instruments. For the price of product p_{jt} , Berry, Levinsohn, and Pakes (1995) used the sum of attributes across own-firm products and the sum of attributes across other-firm products as excluded instruments.⁵ Specifically, $\mathbf{z}_{jt} = (\mathbf{z}_{jt}^{own}, \mathbf{z}_{jt}^{other})$ is given by

$$\mathbf{z}_{jt}^{own} = \left(\sum_{k \in \mathcal{J}_{ft} \setminus \{j\}} x_{kt}^{(1)}, \dots, \sum_{k \in \mathcal{J}_{ft} \setminus \{j\}} x_{kt}^{(R)} \right), \quad \mathbf{z}_{jt}^{other} = \left(\sum_{k \in \mathcal{J}_t \setminus \mathcal{J}_{ft}} x_{kt}^{(1)}, \dots, \sum_{k \in \mathcal{J}_t \setminus \mathcal{J}_{ft}} x_{kt}^{(R)} \right), \quad (3)$$

where $\mathcal{J}_{ft} \setminus \{j\}$ denotes the set of products, excluding j , offered by firm f in market t . These instruments, constructed as functions of all product attributes in the market, are referred to as BLP-style instruments throughout this article.

⁴This invertibility result also holds for more general specifications of $U(\mathbf{x}_t, \mathbf{p}_t, \boldsymbol{\xi}_t, v_i)$. Specifically, when the utility specification induces *connected substitutes* relationships among products in the market, invertibility is ensured. See Berry, Gandhi, and Haile (2013) for further details.

⁵Importantly, these instruments also help address endogeneity arising from observed market shares and prices of other products. Specifically, $\xi_{jt}(\theta)$ is a function of \mathbf{S}_t and \mathbf{p}_t . Therefore, estimating the inverted demand system with a flexible utility specification requires instruments that are exogenous to these variables as well. See Berry and Haile (2014) for identification results and instrument requirements in nonparametric settings.

3.2 Oligopoly model

There are F_t firms in market t , which has a size of M_t , measuring the number of consumers. Importantly, firms may engage in collusion, internalizing the profits of their colluding partners, while the true industry conduct is unknown to the researcher or regulatory authority. The variable profit of firm $f = 1, 2, \dots, F_t$ is given by:

$$\begin{aligned} \Pi_{ft}(\mathbf{p}_t; \mathbf{x}_t, \boldsymbol{\xi}_t, \phi) &= \sum_{k \in \mathcal{J}_{ft}} (p_k - mc_k) \cdot s_{kt}(\mathbf{p}_t; \mathbf{x}_t, \boldsymbol{\xi}_t) \cdot M_t \\ &+ \phi \cdot \sum_{k \in \mathcal{J}_{f_c t} \setminus \mathcal{J}_{ft}} (p_k - mc_k) \cdot s_{kt}(\mathbf{p}_t; \mathbf{x}_t, \boldsymbol{\xi}_t) \cdot M_t, \end{aligned} \quad (4)$$

where f_c is a firm index that treats colluding entities as a single entity. For instance, if firms 1 and 2 collude, then $f_c = 1$ for $f = 1$ and $f = 2$, resulting in $f_c = 1, 3, \dots, F_t$.

The degree of profit internalization is measured by $\phi \in [0, 1]$. As shown in equation (4), firm f partially internalizes the profits of its colluding partners, weighting them by ϕ as part of its own profit. Firms in market t set profit-maximizing prices simultaneously. The Bertrand-Nash equilibrium prices in market t , and consequently, the vector of equilibrium markups, are determined by the following First-Order Conditions (FOCs):

$$\mathbf{p}_t - \mathbf{mc}_t = - \left[\mathcal{H}_t \odot \frac{\partial \mathbf{s}_t(\mathbf{p}_t; \mathbf{x}_t, \boldsymbol{\xi}_t)}{\partial \mathbf{p}_t} \right]^{-1} \mathbf{s}_t(\mathbf{p}_t; \mathbf{x}_t, \boldsymbol{\xi}_t), \quad (5)$$

where \mathbf{mc}_t is a vector of marginal costs, \odot denotes component-wise multiplication, and \mathcal{H}_t is an unknown (to the researcher) $J_t \times J_t$ ownership matrix. For example, $\mathcal{H}_{t,jk} = 1$ if $j, k \in \mathcal{J}_{ft}$ and 0 otherwise in the case of competition ($\phi = 0$), while \mathcal{H}_t is a matrix of ones in the case of full collusion ($\phi = 1$ and $f_c = 1$ for all f). As shown in the markup equation (5) above, the endogeneity of prices arises from the fact that firms set these prices based on demand, making them functions of the unobservable components $\boldsymbol{\xi}$.

The markup equation (5) also demonstrates that in an oligopoly, pricing is influenced by the proximity of a product to its substitutes within the product characteristics space. Products facing close competition tend to have lower markups and prices, while those that are significantly differentiated can command higher markups and prices. More importantly, the equation highlights that

the impact of these characteristics on pricing depends on the unknown ownership structure.

3.3 Firm conduct and the strength of BLP-style instruments

Logit case

To clarify this point, assume a simple logit model where $v_{ip} = v_{ir} = 0$ in the utility specification (1). Under this assumption, the equilibrium markups for firm f in market t are derived as:

$$\mathbf{p}_{ft} - \mathbf{mc}_{ft} = \begin{cases} \frac{1}{\alpha} \left(\frac{1 + \sum_{k \in \text{market}} X_{kt}}{1 + \sum_{k \in \text{market}} X_{kt} - \sum_{k \in \text{own}} X_{kt}} \right) \cdot \mathbf{1}_{ft} & \text{under competition} \\ \frac{1}{\alpha} \left(\frac{1 + \sum_{k \in \text{market}} X_{kt}}{1 + \sum_{k \in \text{market}} X_{kt} - (\sum_{k \in \text{own}} X_{kt} + \sum_{k \in \text{colluding}} X_{kt})} \right) \cdot \mathbf{1}_{ft} & \text{under full collusion} \end{cases} \quad (6)$$

where $\mathbf{1}_{ft}$ is a $J_{ft} \times 1$ unit vector and $X_{kt} = \exp(\delta_{kt})$. Note that δ_{kt} is a function of the attributes of product k . One can see that the BLP-style instruments, \mathbf{z}^{own} and $\mathbf{z}^{\text{other}}$, affect the equilibrium markups and consequently, the equilibrium prices. Moreover, since the equilibrium markup is a function of \mathcal{H}_t , the identifying power of BLP-style instruments depends not only on the true market structure and competition level, but also on whether the industry conduct assumption upon which the BLP-style instruments are constructed is correct.⁶ If the assumption is incorrect, these instruments fail to properly capture markup variation and, as a result, have weaker identifying power. That is, when firms collude, the attributes of colluding firms' products should be included in \mathbf{z}^{own} rather than in $\mathbf{z}^{\text{other}}$.

As an illustration, consider an automobile market with two firms, Volkswagen and Toyota, and three SUVs: Atlas (A), Tiguan (T), and Highlander (H). Given that the first two SUVs are owned by Volkswagen and the third by Toyota, the ownership matrix under Bertrand-Nash competition, \mathcal{H}^B , and the ownership matrix under full collusion between the two firms, \mathcal{H}^C , are represented as follows:

$$\mathcal{H}^B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathcal{H}^C = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

⁶It also depends on variation in product attributes across products and markets as well as their distributional characteristics (e.g., right-skewness and symmetry).

As derived in Appendix A.1, assuming that $\alpha = \beta = 1$ and $X_A = X_T = X_H = 0.2$,⁷ we obtain

$$\begin{pmatrix} \frac{\partial p_A}{\partial x_A} & \frac{\partial p_A}{\partial x_T} & \frac{\partial p_A}{\partial x_H} \end{pmatrix} = \begin{cases} (0.124 & 0.124 & -0.037) & \text{if } \phi = 0 \\ (0.125 & 0.125 & 0.125) & \text{if } \phi = 1 \end{cases}$$

Two points are noteworthy. First, under Bertrand-Nash competition, the price of the Atlas is more significantly affected by a change in the attributes of the Tiguan than by a change in those of the Highlander, and the two price effects are opposite in direction (0.124 vs -0.037). For instance, after improving the fuel efficiency of the Tiguan, Volkswagen would increase the prices of both the Atlas and the Tiguan to capitalize on its strengthened competitive advantage. However, the firm would lower the price of the Atlas in response to an improvement in the fuel efficiency of Toyota's Highlander. Secondly, under full collusion, the price effect of an improvement in the Highlander's fuel efficiency is 0.125, which is more than three times larger in magnitude than its effect under price competition (-0.037) and aligns in direction with the effect on the Atlas's price due to a change in the fuel efficiency of the Tiguan. In fact, the three price effects under collusion are identical in magnitude because the three SUVs are equally substitutable for one another in this example.

These observations indicate that, under price competition, the sum of own-firm product attributes (competition IVs) has greater identifying power than the sum of both own- and other-firm product attributes (collusion IVs). In contrast, if the two firms collude, collusion IVs exhibit a stronger correlation with price than competition IVs. Therefore, comparing the strength of competition IVs and collusion IVs can provide insights into whether firms in a market are competitive or collusive.

Nested logit case

To assess whether the findings above hold in a more general setting, we assume that the idiosyncratic taste shock ε_{ijt} in the indirect utility function (1) follows a nested logit structure, parameterized by the nesting parameter $\sigma \in [0, 1)$.⁸ We then simulate equilibrium outcomes for five markets,

⁷The market share of the outside option is 62.5%. Also, results are qualitatively robust to different values of α , β , and X s.

⁸Specifically, $\varepsilon_{ijt} = \zeta_{igt} + (1 - \sigma)\bar{\varepsilon}_{ijt}$, where $\bar{\varepsilon}_{ijt}$ follows an i.i.d. extreme value distribution and ζ_{igt} has a unique distribution such that ε_{ijt} remains extreme value distributed (Cardell, 1997). A higher value of σ implies stronger substitution between products belonging to the same group, indexed by $g = 0, 1, \dots, G$. The outside option ($j = 0$) is the sole member of group $g = 0$.

each consisting of six firms ($F = 6$) and 60 inside products ($J = 60$), categorized into three groups ($G = 3$). In each market, the first three firms collude ($f_c = 1$ for $f = 1, 2, 3$) with varying degrees of profit internalization, $\phi \in \{0, 0.2, 0.5, 0.8, 1\}$; they compete in the first market and fully collude in the last market.

Our objective is to analyze the $J \times J$ Jacobian matrix, denoted by $\frac{\partial \mathbf{p}}{\partial \mathbf{x}}$, where the (j, k) entry corresponds to $\frac{\partial p_j}{\partial x_k}$ evaluated at equilibrium. Specifically, we first examine whether, under competition ($\phi = 0$), the following condition holds:

$$\left[\frac{\partial \mathbf{p}}{\partial \mathbf{x}} \right]_{jk} \cdot \left[\frac{\partial \mathbf{p}}{\partial \mathbf{x}} \right]_{jk'} < 0 \quad \text{for } k \in \mathcal{I}_{f(j)} \text{ and } k' \notin \mathcal{I}_{f(j)},$$

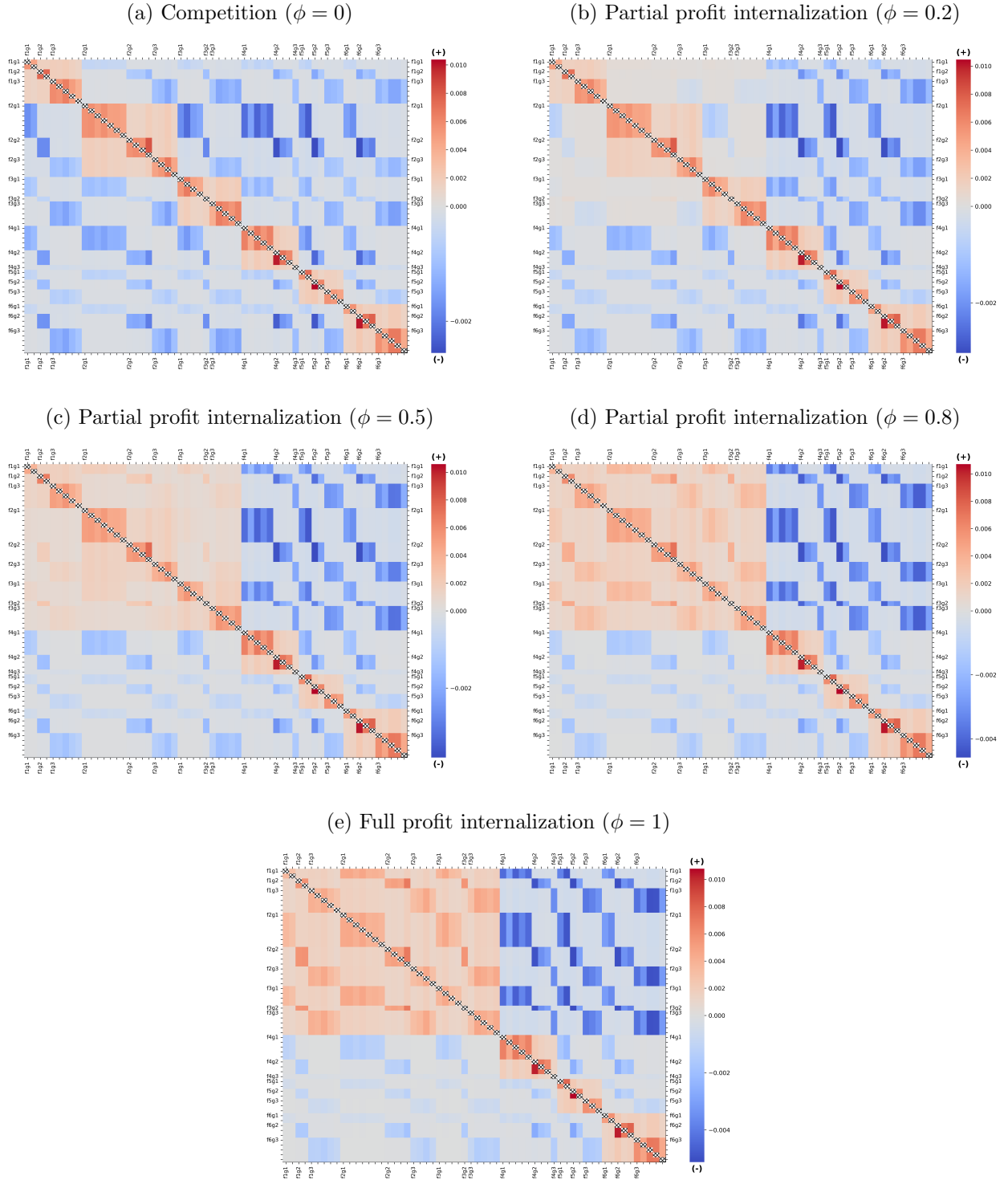
where $f(j)$ indexes the firm offering product j . Next, we investigate how varying the degree of profit internalization, ϕ , affects this relationship. Analytical and computational details for deriving the Jacobian matrix are provided in Appendix A.2.

Figure 1 presents heatmaps of the Jacobian matrix for the five markets. Each grid represents the (j, k) entry, where darker red indicates more positive values and darker blue indicates more negative values. Diagonal entries remain uncolored, as $\frac{\partial p_j}{\partial x_j}$ is significantly larger than $\frac{\partial p_j}{\partial x_k}$ for $j \neq k$. Products are indexed first by firm and then by group assignment. For instance, the first firm ($f = 1$) produces nine products: the first two belong to the first group ($g = 1$), the next two to the second group ($g = 2$), and the remaining five to the third group ($g = 3$). In each heatmap, rows and columns are marked to indicate transitions in ownership or product grouping when product j differs from product $j - 1$ in firm or group assignment.

The patterns of price effects in the figure align with the economic intuition derived from the simple logit case. First, in all markets, while a firm raises the prices of all its products after improving the attributes of one of its products, the direction of the price effect of other firms' product attributes depends on industry conduct.⁹ That is, under competition ($\phi = 0$, panel (a)), prices always decrease in response to improvements in other firms' product attributes. In contrast, under full profit internalization ($\phi = 1$, panel (e)), improvements in the attributes of products owned by colluding partners lead a firm to raise its prices with the magnitude of the price increase being larger for products in the same group.

⁹Additionally, products with larger market shares exert stronger effects, as indicated by darker red or blue colors in the figure.

Figure 1: Effects of own-firm and other-firm attributes on equilibrium prices



Notes: The figure presents heatmaps of the $J \times J$ Jacobian matrices, $\frac{\partial \mathbf{p}}{\partial \mathbf{x}}$, derived from the nested logit specification under various values of the profit internalization parameter, ϕ . See the main text for interpretation and Appendix A.2 for details on the data-generating process used to compute equilibrium outcomes and Jacobian matrices.

Second, the degree of profit internalization affects how closely the impact of colluding partners’ product attributes mirrors that of a firm’s own product attributes. When profit internalization is weak ($\phi = 0.2$, panel (b)), some product attributes of colluding partners have effects opposite in direction to those of the firm’s own product attributes. Specifically, a firm reduces the prices of its products that are in the same group as (and hence closer substitutes for) those of its colluding partners whose attributes are improved.¹⁰ For instance, improvements in the attributes of products in the first group ($g = 1$), offered by firms 1 and 3, lead to a price reduction for firm 2’s products within the same group, as shown in panel (b), where the corresponding grids are marked in blue.¹¹ As ϕ increases, the effects of colluding partners’ product attributes on prices of a firm converge to those of the firm’s own product attributes, both in direction and magnitude.

Our key findings – (i) the price effects of own-firm and other-firm attributes are opposite in direction, and (ii) the price effects of other-firm attributes reverse in direction after collusion – are consistently observed across various market structures with different values of F , J , and G , as well as different assignments of products to firms and groups, and varying utility and marginal cost parameters. Moreover, since these findings do not depend on imposing specific choice models, they are expected to remain valid when more flexible models are considered. Building on the analyses discussed thus far, we formalize a testing framework for detecting collusive behavior in the next section.

4 Testing framework

4.1 Proposed testing procedure

As demonstrated in Section 3 and Appendix A, properly accounting for strong collusive firm behavior enhances the power of BLP-style instruments in the first-stage price regression. More specifically, when firms collude, incorporating the attributes of colluding partners into own-firm instruments

¹⁰Under a random coefficient utility specification where ν_{ir} in the indirect utility function (1) is heterogeneous, products with similar values of $x_{jt}^{(r)}$ serve as closer substitutes. In this case, when a firm weakly internalizes the profits of its colluding partners, it is more likely to lower the prices of products that share similar characteristics with those of its colluding partners when their attributes improve.

¹¹Figure 1 also shows that the price effects are identical for products owned by the same firm and belonging to the same group, as illustrated by the long vertical strips in each heatmap. This uniformity stems from the uniform markup property under the nested logit model specification, as shown in (A1) in Appendix A.2; equilibrium markups are identical for products within the same group and offered by the same firm. Accordingly, in the logit case, the price effects are identical for all products owned by the same firm, as shown in Figure D1 in the Appendix.

strengthens the instruments. Conversely, if firms actually engage in price competition, this incorporation would weaken the instruments' strength in the first-stage price regression.

Based on this intuition, we propose assessing collusive behaviors among firms by comparing the performance of IVs in two first-stage price regressions. Specifically, we construct two distinct sets of BLP-style instruments: one based on the observed firm index (competition IVs), denoted by \mathbf{z}^{comp} , and the other based on the *suspected* colluding firm index (collusion IVs), denoted by \mathbf{z}^{coll} . We then estimate two separate first-stage price regressions using these two sets of instruments individually and compare their relative strengths using the non-nested model selection method proposed by Rivers and Vuong (2002).¹²

Our testing procedure is as follows. We first define the two BLP-style instruments for product $j \in \mathcal{J}_{ft}$ in market t , \mathbf{z}_{jt}^{comp} and \mathbf{z}_{jt}^{coll} . These instruments are specified as:

$$\begin{aligned} \mathbf{z}_{jt}^{comp} &= \left(\sum_{k \in \mathcal{J}_{ft} \setminus \{j\}} x_{kt}^{(1)}, \dots, \sum_{k \in \mathcal{J}_{ft} \setminus \{j\}} x_{kt}^{(R)}, \sum_{k \in \mathcal{J}_{ft} \setminus \{j\}} \left(x_{kt}^{(1)}\right)^2, \dots, \sum_{k \in \mathcal{J}_{ft} \setminus \{j\}} \left(x_{kt}^{(R)}\right)^2 \right) \\ \mathbf{z}_{jt}^{coll} &= \left(\sum_{k \in \mathcal{J}_{f_c t} \setminus \{j\}} x_{kt}^{(1)}, \dots, \sum_{k \in \mathcal{J}_{f_c t} \setminus \{j\}} x_{kt}^{(R)}, \sum_{k \in \mathcal{J}_{f_c t} \setminus \{j\}} \left(x_{kt}^{(1)}\right)^2, \dots, \sum_{k \in \mathcal{J}_{f_c t} \setminus \{j\}} \left(x_{kt}^{(R)}\right)^2 \right). \end{aligned} \quad (7)$$

Note that $\mathcal{J}_{f_c t}$ denotes the set of products owned by firm f and its *suspected* colluding partners, as f_c indexes these firms as a single entity. The choice of instruments and functional form in (7) is selected for brevity and clarity, with further discussion provided in Section 6.3 and Appendix B.

We emphasize that \mathbf{z}_{jt}^{comp} is constructed based on the *observed* competitive market ownership, whereas \mathbf{z}_{jt}^{coll} is constructed based on the *suspected* collusive market ownership. For example, suppose that the regulatory authority suspects that, among the four firms in a product market, indexed as A , B , C , and D , two firms, A and B , are engaging in price collusion. Based on observed price and product attribute data, the authority can construct two sets of BLP-type instruments as described above. In this case, \mathbf{z}_{jt}^{comp} treats firms A and B as two separate entities, whereas \mathbf{z}_{jt}^{coll} treats these two firms as a single entity, reflecting their *suspected* collusion.

¹²The implementation of Rivers and Vuong (2002)'s test in cross-sectional and panel regression settings can be inferred from Wooldridge (2010) (Section 13.11.2).

Next, we run the following two first-stage price regressions using \mathbf{z}_{jt}^{comp} and \mathbf{z}_{jt}^{coll} separately:

$$p_{jt} = \gamma^1 \cdot x_{jt} + \boldsymbol{\theta}^1 \mathbf{z}_{jt}^{comp} + e_{jt}^1 \quad (8)$$

$$p_{jt} = \gamma^2 \cdot x_{jt} + \boldsymbol{\theta}^2 \mathbf{z}_{jt}^{coll} + e_{jt}^2, \quad (9)$$

where x_{jt} represents the exogenous characteristics of product j , including an intercept. Continuing with the previous example, if firms A and B indeed collude and internalize the profit of their colluding partner, then \mathbf{z}_{jt}^{coll} should explain the equilibrium prices better than \mathbf{z}_{jt}^{comp} does. On the other hand, if the two firms do not engage in collusion and all four firms compete *à la* Bertrand, then \mathbf{z}_{jt}^{comp} should explain the equilibrium prices better.

Finally, we formalize our testing procedure by constructing the Rivers and Vuong (RV) model selection test statistic. Our RV test statistic, T_{IV}^{RV} , is given by:

$$T_{IV}^{RV} = \frac{\sqrt{n}(\hat{Q}_{comp} - \hat{Q}_{coll})}{\hat{\sigma}_{IV}} \xrightarrow{d} N(0, 1), \quad (10)$$

where \hat{Q}_{comp} and \hat{Q}_{coll} are the averages of the sum of squared residuals of the linear regressions in (8) and (9), respectively, and $\hat{\sigma}_{IV}$ is an estimator for the asymptotic variance of $\sqrt{n}(\hat{Q}_{comp} - \hat{Q}_{coll})$. In practice, we estimate the two linear regression models in (8) and (9) and obtain the two residuals, \hat{e}_{jt}^1 and \hat{e}_{jt}^2 . We then regress the difference of the two squared-residuals, $(\hat{e}_{jt}^1)^2 - (\hat{e}_{jt}^2)^2$, on a constant term only. The t -statistic for this constant term corresponds to the RV test statistic given above. Clustering is easily accommodated by specifying an appropriate cluster structure in the regression of the squared-residual difference on the constant term.

Our RV test statistic is asymptotically normal under the null hypothesis that the two models, (8) and (9), have the same fit: $Q_{comp} = Q_{coll}$. We may define two alternative hypotheses: (i) $H_1 : Q_{comp} < Q_{coll}$ and (ii) $H_2 : Q_{comp} > Q_{coll}$. In case of collusion, including product attributes of colluding partners in own-firm instruments is expected to enhance the strength of the instruments and improve the model fit in first-stage price regression. Therefore, a statistically significant positive test statistic (e.g., 1.65 at 0.05 significance level for a one-tailed test) can be interpreted as evidence of collusive behavior among *suspected* firms. In contrast, a statistically significant negative test statistic (e.g., -1.65 at the same significance level) suggests that firms engage in price competition.

4.2 Existing Testing Procedure

As elaborated in Section 2, there is a growing body of literature on firm conduct testing procedures that employ moment-based RV tests. These methods construct moments using the equilibrium markup conditions under the two alternative firm conduct models, and then compare the GMM objective functions of the two models to determine which one provides a better fit. This approach assumes that the researcher has access to a known demand system and a specified marginal cost function. Consequently, estimating demand parameters and specifying the functional form of marginal costs are necessary prerequisites. In this section, we briefly introduce the RV methodology in the existing literature. For expository purposes, we present a simplified form of this methodology, while extended discussions on this framework are provided in the footnotes accompanying the main text.

The marginal cost function is given by:

$$mc_{jt} = h(x_{jt}, w_{jt}) + \omega_{jt}, \quad (11)$$

where x_{jt} and w_{jt} represent observed product attributes and cost shifters excluded from the demand function, respectively, while ω_{jt} denotes an unobserved cost component. To simplify the discussion while conveying the main idea, we impose the following two assumptions: (i) the marginal cost function, $h(x_{jt}, w_{jt})$, which must be pre-specified by researchers to implement the testing procedure, is linear in x_{jt} and w_{jt} ; and (ii) marginal cost is constant with respect to the quantity produced, implying no economies of scale or scope.¹³

Let $D(\mathbf{x}_t)$ denote a known demand system, where \mathbf{x}_t represents a full set of exogenous product characteristics. Given demand estimates, the equilibrium markup for product j under firm conduct model m (e.g., competition, collusion, partial collusion, common ownership, etc.), η_{jt}^m , is also known from the profit-maximizing conditions. The marginal costs of products can then be recovered as follows:

$$mc_{jt} = p_{jt} - \eta_{jt}^m(\mathbf{S}_t, \mathbf{p}_t, D(\mathbf{x}_t)). \quad (12)$$

¹³Backus, Conlon, and Sinkinson (2021) proposed a testing procedure that incorporates a flexible specification of the cost function. Additionally, the marginal cost can be nonconstant with respect to quantities produced. In such cases, the instruments are required to satisfy additional conditions to distinguish between alternative firm conduct models. See Duarte, Magnolfi, Quint, S¸olvsten, and Sullivan (2025) for details and applications.

Under the *true* conduct model $m = 0$, assuming no specification error, the following moment condition holds:

$$E[\omega_{jt}^0 \mathbf{z}_{jt}] = 0,$$

where $\omega_{jt}^0 = p_{jt} - \eta_{jt}^0 - h(x_{jt}, w_{jt})$. A vector of excluded instruments, denoted by \mathbf{z}_{jt} , typically consists of BLP-style instruments (functions of exogenous product attributes) and cost shifters excluded from the demand specification, w_{jt} .¹⁴

The existing methods choose between two non-nested conduct models, $m = 1$ and $m = 2$. More specifically, the linear parameters in the marginal cost function $h(x_{jt}, w_{jt})$ are estimated by running an OLS regression of the marginal cost under conduct model m :

$$p_{jt} - \eta_{jt}^m = h(x_{jt}, w_{jt}) + \omega_{jt}^m,$$

resulting in different estimates across conduct models $m = 1$ and $m = 2$.¹⁵ After obtaining the residual $\widehat{\omega}_{jt}^m$, the GMM objective function, $\widehat{Q}^m(\eta^m)$, is constructed from the equilibrium markup condition under conduct model m as follows:

$$\widehat{Q}^m(\eta^m) = \widehat{g}'_m \widehat{W} \widehat{g}_m,$$

where, given the total number of observations n ,

$$\widehat{g}_m = n^{-1} \sum_t \sum_j \mathbf{z}'_{jt} \widehat{\omega}_{jt}^m \quad \text{and} \quad \widehat{W} = n \cdot \left(\sum_t \sum_j \mathbf{z}_{jt} \mathbf{z}'_{jt} \right)^{-1}.$$

¹⁴Backus, Conlon, and Sinkinson (2021) proposed a functional form for BLP-style instruments to enhance the power of the RV testing framework, drawing on the literature on optimal instruments in nonlinear GMM settings (Chamberlain, 1987). This functional form results in a scalar moment that is independent of the choice of the weighting matrix used to form the objective function. Additionally, the choice of instruments (beyond BLP-style instruments) to satisfy falsifiable conditions to distinguish between two alternative conduct models affects the validity of testing procedure. See Dearing, Magnolfi, Quint, Sullivan, and Waldfoegel (2024) for discussions on falsifiable conditions and empirical applications.

¹⁵In a more flexible framework, the marginal cost function can be estimated using non-parametric regression. For example, Backus, Conlon, and Sinkinson (2021) employed random forest regression (Breiman, 2001) to better capture nonlinear relationships.

Finally, the RV test statistic based on equilibrium markup conditions, T_{markup}^{RV} , is given by:

$$T_{markup}^{RV} = \frac{\sqrt{n}(\hat{Q}_1(\eta^1) - \hat{Q}_2(\eta^2))}{\hat{\sigma}_{markup}} \xrightarrow{d} N(0, 1), \quad (13)$$

where $\hat{\sigma}_{markup}$ is an estimator for the asymptotic variance of $\sqrt{n}(\hat{Q}_1(\eta^1) - \hat{Q}_2(\eta^2))$. The exact form of $\hat{\sigma}_{markup}$, along with a detailed discussion on adjustments for clustering and two-step estimation errors arising from demand estimation, is provided in Duarte, Magnolfi, Sølvsten, and Sullivan (2024). This RV statistic can be computed using the Python package `PyRVtest` (Duarte, Magnolfi, Sølvsten, Sullivan, and Tarascina, 2023).¹⁶

4.3 Summary

The basic idea behind our proposed conduct testing procedure aligns with that of existing methods: to select the firm conduct model that best fits the observed market outcomes. In our approach, model fit is evaluated using the first-stage price regression, whereas existing approaches rely on the GMM objective function. Unlike these methods, our testing procedure does not require demand estimation or marginal cost specification, making it straightforward to implement. Furthermore, our procedure is data-efficient, as it can be performed without market share data or additional instruments (beyond BLP-style instruments), including exogenous cost shifters. As demonstrated in the following section, the performance of our testing method is comparable to that of existing approaches. Consequently, our method complements these approaches by serving as a preliminary diagnostic tool for assessing industry conduct.

5 Monte Carlo set-up

5.1 Data-generating process

Our data-generating process (DGP) builds upon the framework in Armstrong (2016) and Conlon and Gortmaker (2020), with a focus on modeling various collusive behaviors among firms in a Bertrand-Nash setting. For each Monte Carlo configuration, the number of products (J) and the

¹⁶Additionally, diagnostics for weak instruments in this testing framework, as illustrated in (Duarte, Magnolfi, Sølvsten, and Sullivan, 2024), can be executed using this package.

number of firms (F) are fixed across markets ($t = 1, \dots, T$). Each firm produces J/F products.¹⁷ This allocation results in nearly symmetric market shares across firms and markets under the DGP described below.

The indirect utility of consumer i in market t from consuming product j , and the marginal cost of product j in market t , are specified as follows:

$$u_{ijt} = \beta_1 + \beta_2 x_{jt} - \alpha p_{jt} + \xi_{jt} + \sigma_x v_i x_{jt} + \varepsilon_{ijt}, \quad (14)$$

$$mc_{jt} = \gamma_1 + \gamma_2 x_{jt} + \gamma_3 w_{jt} + \omega_{jt}. \quad (15)$$

There are two exogenous product characteristics – a constant term and x_{jt} – and one exogenous cost shifter, w_{jt} , where both x_{jt} and w_{jt} are randomly drawn from a standard uniform distribution. Price, p_{jt} , is an endogenous product characteristic determined by equilibrium conditions in a differentiated goods market under a Bertrand-Nash framework. The true linear demand parameters are given by $\alpha = 1$, $\beta_1 = -4.5$, and $\beta_2 = 6$. When included, the heterogeneous component of demand is given by $\sigma_x v_i x_{jt}$, where $\sigma_x = 3$ and v_i follows a standard normal distribution. Cost parameters are given by $\gamma_1 = 2$, $\gamma_2 = 1$, and $\gamma_3 = 0.2$. The unobservable components of demand and cost, (ξ_{jt}, ω_{jt}) , are randomly drawn from a mean-zero bivariate normal distribution with standard deviations $\sigma_\xi = 0.2$, $\sigma_\omega = 0.2$, and covariance $\sigma_{\xi\omega} = 0.1$.

Firms may engage in collusion, internalizing the profits of their colluding partners as part of their own. The degree of collusion is measured by F/F_c , where $F_c \in \{1, \dots, F-2, F-1\}$ denotes the effective number of competitors – i.e., the number of distinct competitive entities after treating all colluding firms as a single entity. We assume that the first $F - F_c + 1$ firms either collude or are suspected of colluding by the regulatory authority. For example, suppose $F = 6$ and $F_c = 4$. In this case, the degree of collusion is 1.5, with the first three firms ($f = 1, 2, 3$) colluding or suspected of colluding to jointly set profit-maximizing prices, leaving the effective competitors indexed as $f_c = 1, 4, 5, 6$. The degree of profit internalization, represented by the conduct parameter $\phi \in [0, 1]$, quantifies the weight assigned to the profits of colluding partners, as shown in equation

¹⁷When J/F is not an integer, some firms are randomly assigned to produce $\lceil J/F \rceil$ products, while others produce $\lfloor J/F \rfloor$ products, ensuring that the total number of products remains $J = 36$. Here, $\lceil \cdot \rceil$ denotes the ceiling function, which rounds a number up to the nearest integer, and $\lfloor \cdot \rfloor$ denotes the floor function, which rounds a number down to the nearest integer.

(4). Continuing with the previous example, when $F_c = 4$ but $\phi = 0$, this configuration represents a scenario in which regulatory authorities falsely suspect three firms of collusion when, in fact, they are engaged in price competition.

Equilibrium prices in each market are endogenously determined by the Bertrand first-order conditions (5). These prices are solved using the fixed-point algorithm proposed by Morrow and Skerlos (2011) and implemented in PyBLP (Conlon and Gortmaker, 2020). We use nine Gauss-Hermite quadrature nodes to numerically integrate the choice probabilities when a random coefficient is included in the indirect utility function.

We also consider the case in which the product attribute is correlated with the unobservable demand component, ξ_{jt} , making it endogenous. We denote the endogenous attribute as x_{jt}^{endo} and derive it as follows. First, we compute $x_{jt}^{unscaled} = x_{jt} + \rho \cdot \xi_{jt}$, where $\rho \in \{-10, -5, -1, 0, 1, 5, 10\}$ determines the degree and direction of endogeneity; the case $\rho = 0$ corresponds to an exogenous product attribute. Next, given that the variation of $x_{jt}^{unscaled}$ depends on ρ , we apply min-max normalization to define $x_{jt}^{endo} = \frac{x_{jt}^{unscaled} - x_t^{\min}}{x_t^{\max} - x_t^{\min}}$, where x_t^{\min} and x_t^{\max} are the minimum and maximum values of $x_{jt}^{unscaled}$ in market t . In this way, we fix the support of the endogenous attribute: $x_{jt}^{endo} \in [0, 1]$ for any ρ .

Each Monte Carlo configuration is represented by a unique five-tuple (J, F, T, ϕ, F_c) . Specifically, $J = 36$, $F \in \{1, 2, \dots, 36\}$, $T \in \{10, 100\}$, $\phi \in \{0, 0.1, \dots, 1\}$, and $F_c \in \{1, 2, \dots, F - 1\}$.¹⁸ For each configuration, we generate 500 simulated datasets ($S = 500$) to evaluate the finite-sample performance of the proposed test statistic, T_{IV}^{RV} , and compare it with the performance of the test based on equilibrium markup conditions, T_{markup}^{RV} . We also investigate the performance of \mathbf{z}^{comp} and \mathbf{z}^{coll} as instruments for demand estimation. Furthermore, in Section 7, we focus on cases with varying F and fixed J but without collusive behavior ($\phi = 0$ and $F_c = F$) to evaluate the performance of BLP-type IVs, following the approach of Armstrong (2016).

¹⁸When the DGP includes an endogenous product attribute, ϕ is fixed at 1, and ρ is varied to generate unique five-tuples (J, F, T, ρ, F_c) .

5.2 Computation of test statistics

For each dataset with configuration (J, F, T, ϕ, F_c) , we construct \mathbf{z}^{comp} and \mathbf{z}^{coll} as outlined in Section 4.1.¹⁹ Note that even when $\phi = 0$, we still construct \mathbf{z}^{coll} based on a scenario in which the researcher falsely suspects collusion among the firms of interest. To assess the relevance of these two instruments under a given true conduct model, we compute two F-statistics associated with (i) $\boldsymbol{\theta}^1 = 0$ (Fstat₁) and (ii) $\boldsymbol{\theta}^2 = 0$ (Fstat₂) from equations (8) and (9), respectively. Our proposed test statistic, T_{IV}^{RV} , is then calculated using equation (10) and the description provided therein. Heteroskedasticity-robust standard errors are used to compute $\hat{\sigma}_{IV}$ in equation (10).

To compute T_{markup}^{RV} , we first estimate the demand parameters using the standard own- and other-firm IVs, defined as:

$$\mathbf{z}_{jt}^{own} = \left(\sum_{k \in \mathcal{J}_{ft} \setminus \{j\}} x_{kt}, \sum_{k \in \mathcal{J}_{ft} \setminus \{j\}} x_{kt}^2 \right), \quad \mathbf{z}_{jt}^{other} = \left(\sum_{k \in \mathcal{J}_t \setminus \mathcal{J}_{ft}} x_{kt}, \sum_{k \in \mathcal{J}_t \setminus \mathcal{J}_{ft}} x_{kt}^2 \right), \quad (16)$$

as excluded instruments $\mathbf{z}_{jt} = (\mathbf{z}_{jt}^{own}, \mathbf{z}_{jt}^{other})$. Note that these instruments are constructed based on *observed* ownership rather than *suspected* ownership. Next, we specify the marginal cost function as a linear function of the constant term and x_{jt} only, even though it is also linear in a cost shifter w_{jt} , as shown in equation (15).²⁰ Thus, we consider a scenario in which the cost function is potentially misspecified, either due to data limitations on cost shifters or the omission of a relevant cost shifter. This approach enables us to evaluate the performance of the two testing procedures without using additional data. We then follow the procedure outlined in Section 4.2 to construct T_{markup}^{RV} under two alternative firm conduct models: one with $\phi = 0$ (competition) and the other with $\phi = 1$ (full profit internalization under industry conduct consistent with the effective firm index used in the DGP).²¹ The standard error $\hat{\sigma}_{markup}$ in equation (13) is adjusted for two-step demand estimation errors and is heteroskedasticity-robust. The computation of T_{markup}^{RV} is done

¹⁹Note that the summation of the constant term, often referred to as the product counts IV, cannot be utilized in our DGP setting because it fails to generate cross-firm or cross-market variations in the number of products. Moreover, the inclusion of second-order polynomial IVs in (7) ensures that the rank condition for the GMM framework is satisfied in our DGP when a random coefficient is incorporated into the indirect utility function (14).

²⁰Accordingly, we do not include w_{jt} as an instrument when estimating the demand.

²¹Although the testing procedure built on equilibrium markup conditions allows us to compare conduct models with other values of ϕ , such as $\phi = 0.2$ versus $\phi = 0.8$, the primary interest of researchers and regulatory authorities is often to test full profit internalization ($\phi = 1$) against competition ($\phi = 0$). Therefore, we compute T_{markup}^{RV} under these two conduct models.

using `PyRVtest` (Duarte, Magnolfi, Solvsten, Sullivan, and Tarascina, 2023).

6 Monte Carlo evidence: testing performance of T_{IV}^{RV} vs T_{markup}^{RV}

6.1 Power of instruments and estimation performance

We begin by assessing the power of our proposed testing procedure outlined in Section 4.1. To do so, we consider a scenario with four firms, two of which collude ($F = 4, F_c = 3$). We generate Monte Carlo datasets, following the design in Section 5, for various values of $\phi \in [0, 1]$. The top panel of Table 1 presents the results when a random coefficient is excluded from the indirect utility function (14) in the DGP, while the bottom panel shows the results when it is included.

First, we examine and compare the two F-statistics: $Fstat_1$, associated with the null hypothesis $H_0 : \theta^1 = 0$ in equation (8), and $Fstat_2$, associated with $H_0 : \theta^2 = 0$ in equation (9). The results indicate that as the degree of profit internalization rises, $Fstat_1$ decreases slightly, whereas $Fstat_2$ rises substantially. This finding aligns with the price effect patterns illustrated in Figure 1 and our discussion on the strength of the two IVs in Section 3.3. As a result, for $\phi > 0.5$, \mathbf{z}^{coll} is significantly stronger than \mathbf{z}^{comp} . This pattern holds both with and without a random coefficient in the utility specification. Results from more extensive simulations with varying numbers of firms (F) and effective competitors (F_c) are illustrated in Figure D2 in the Appendix. The figure graphically depicts the share of cases where $Fstat_2 > Fstat_1$ in 500 simulation results for each Monte Carlo configuration. For a given degree of collusion (measured by the ratio F/F_c), the relative power of \mathbf{z}^{coll} always increases as the degree of profit internalization ϕ rises. Moreover, for a fixed ϕ , the more collusive the firms in the market are, the stronger the identifying power of \mathbf{z}^{coll} , except for the case in which all firms collude ($F_c = 1$).

Our simulation results presented in Table 1 also show that the median absolute error of the estimated price coefficient $\hat{\alpha}$ decreases significantly in response to a rise in ϕ when using \mathbf{z}^{coll} as instruments. As a result, \mathbf{z}^{coll} outperforms \mathbf{z}^{comp} in estimating α when firms collude, even with a low degree of profit internalization ($\phi \geq 0.4$ without a random coefficient and $\phi \geq 0.1$ with a random coefficient).

A similar pattern is observed for the median absolute error of the estimated non-linear coefficient $\hat{\sigma}_x$. Moreover, for any ϕ , the median absolute error obtained using \mathbf{z}^{coll} as instruments is smaller

Table 1: Comparison of IV performance: \mathbf{z}^{comp} vs \mathbf{z}^{coll} ($F = 4, F_c = 3, T = 100$)

ϕ	\mathbf{z}^{comp}			\mathbf{z}^{coll}			% of Fstat ₂ > Fstat ₁	Median of s_o
	$ \alpha - \hat{\alpha} $	$ \sigma_x - \hat{\sigma}_x $	Fstat ₁	$ \alpha - \hat{\alpha} $	$ \sigma_x - \hat{\sigma}_x $	Fstat ₂		
<i>Panel A: without a random coefficient</i>								
0 (competition)	0.143		10.377	0.190		6.123	0.060	0.659
0.1	0.143		10.329	0.171		7.215	0.132	0.660
0.2	0.142		10.308	0.155		8.663	0.266	0.662
0.3	0.143		10.296	0.144		10.377	0.472	0.663
0.4	0.144		10.269	0.134		12.557	0.694	0.664
0.5	0.144		10.209	0.121		15.164	0.866	0.665
0.6	0.144		10.155	0.108		18.336	0.958	0.666
0.7	0.146		10.213	0.097		21.690	0.990	0.667
0.8	0.146		10.177	0.090		25.504	1.000	0.668
0.9	0.146		10.199	0.083		29.670	1.000	0.669
1	0.148		10.095	0.077		34.193	1.000	0.669
<i>Panel B: with a random coefficient</i>								
0 (competition)	0.552	1.863	25.102	0.727	1.148	10.861	0.000	0.604
0.1	0.549	1.867	24.698	0.545	0.920	13.181	0.002	0.606
0.2	0.541	1.770	24.414	0.321	0.620	16.456	0.056	0.607
0.3	0.536	1.698	23.962	0.221	0.474	20.728	0.228	0.609
0.4	0.536	1.707	23.853	0.167	0.397	26.078	0.590	0.610
0.5	0.537	1.711	23.642	0.132	0.367	32.597	0.894	0.612
0.6	0.533	1.613	23.038	0.109	0.325	40.129	0.986	0.613
0.7	0.520	1.552	22.737	0.091	0.312	48.787	1.000	0.615
0.8	0.504	1.472	22.420	0.079	0.310	58.424	1.000	0.616
0.9	0.510	1.495	21.959	0.070	0.300	69.041	1.000	0.618
1	0.509	1.434	21.358	0.063	0.291	80.495	1.000	0.619

Notes: The table compares the median absolute errors of the estimated price and nonlinear coefficients, as well as the median F-statistics, across 500 simulated datasets for each Monte Carlo configuration ($J = 36, F = 4, T = 100, \phi, F_c = 3$), obtained using \mathbf{z}^{comp} and \mathbf{z}^{coll} as instruments individually. The top panel presents the results when a random coefficient is excluded from the indirect utility function (14) in the DGP, while the bottom panel presents the results when it is included.

than that obtained using \mathbf{z}^{comp} . This finding is not contradictory to our earlier observation that, when the degree of profit internalization is low ($\phi \leq 0.3$), using \mathbf{z}^{coll} instead of \mathbf{z}^{comp} leads to lower first-stage F-statistics. This is because identifying σ_x requires instruments that are correlated not only with price but also with market shares.²² We revisit this point in Section 7, where we discuss

²²As a simple illustration, consider a nested logit specification, a type of random coefficient model. After inverting a demand system (Berry, 1994), the resulting estimation equation using a standard linear instrumental variables approach is as follows:

$$\log \frac{s_{jt}}{s_{ot}} = \beta x_{jt} - \alpha p_{jt} + \sigma \log s_{j|g,t} + \xi_{jt},$$

where $s_{j|g,t}$ denotes the market share of product j within group g . Identifying this random coefficient model requires instruments for both the endogenous price and the endogenous market share $s_{j|g,t}$. See Berry and Haile (2014) for

the role of own- and other-firm instruments.

In sum, taking the true industry conduct into consideration when constructing instruments increases the strength of the instruments and enhances the estimation performance of the demand parameters.

6.2 Comparison of Testing Power

Now, we compute and compare the two test statistics, T_{IV}^{RV} and T_{markup}^{RV} . Specifically, we investigate how the degree of profit internalization, ϕ , as well as the degree of collusion, F/F_c , influence these two statistics. Recall that for both statistics, a statistically significant positive value (greater than 1.65 at the 0.05 significance level) indicates collusion, while a statistically significant negative value (less than -1.65 at the same significance level) indicates competition.

Table 2 reports these two statistics under various Monte Carlo configurations elaborated in Section 5.1. We first examine the results without a random coefficient presented in the upper panel. Overall, our test statistic, T_{IV}^{RV} , performs relatively well compared to the existing statistic, T_{markup}^{RV} , in detecting collusion. In contrast, the latter tends to outperform the former in rejecting collusion in favor of competition at lower values of ϕ . For example, when all six firms collude ($F_c = 1$) and fully internalize the profits of other firms ($\phi = 1$), both tests reject the competition hypothesis: $T_{IV}^{RV} = 3.309$ and $T_{markup}^{RV} = 3.564$. As the degree of profit internalization decreases, T_{IV}^{RV} also declines but remains statistically significant (and positive) for $\phi \geq 0.7$, while T_{markup}^{RV} remains statistically significant only for $\phi \geq 0.9$. For $\phi = 0.1$, only T_{markup}^{RV} has a statistically significant negative value (-6.088), allowing the rejection of collusion. When $F_c = 1$, there is no within-market variation in the collusion IVs, and their identification power relies solely on cross-market variations. While this suggests that the performance of our test may improve as the number of available markets increases, our testing procedure performs even better when $F_c \in \{2, 3, 4\}$ compared to the $F_c = 1$ scenario in detecting collusive behaviors among firms across a broader range of ϕ .

The relative testing power of our method is higher when a random coefficient is included in the consumer utility function.²³ For any degree of collusion (F/F_c), it effectively detects collusion

the instrument requirements under general nonparametric settings.

²³This higher testing power suggests that overall, the price effect of product attributes is more sensitive to industry conduct under the random coefficient specification than under the logit specification employed in our Monte Carlo study.

Table 2: (Exogenous) $F = 6$ and $T = 100$

ϕ	T_{IV}^{RV} (\mathbf{z}^{comp} vs \mathbf{z}^{coll})					T_{markup}^{RV} ($\phi = 0$ vs $\phi = 1$)				
	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$
<i>Panel A: without a random coefficient</i>										
0 (competition)	-1.657	-1.547	-1.483	-1.299	-0.925	-7.133	-3.872	-2.143	-1.033	-0.385
0.1	-1.458	-1.012	-1.030	-0.978	-0.758	-6.088	-3.360	-1.839	-0.880	-0.326
0.2	-1.112	-0.063	-0.129	-0.374	-0.451	-5.106	-2.819	-1.524	-0.719	-0.271
0.3	-0.604	0.992	0.845	0.328	-0.144	-4.083	-2.228	-1.175	-0.556	-0.227
0.4	0.011	1.986	1.782	1.004	0.211	-2.930	-1.614	-0.820	-0.393	-0.165
0.5	0.650	2.911	2.620	1.672	0.574	-1.758	-0.958	-0.453	-0.221	-0.131
0.6	1.272	3.787	3.405	2.294	0.886	-0.644	-0.300	-0.081	-0.055	-0.075
0.7	1.849	4.607	4.167	2.875	1.214	0.477	0.380	0.277	0.119	-0.011
0.8	2.372	5.373	4.880	3.404	1.522	1.553	1.010	0.593	0.305	0.043
0.9	2.851	6.111	5.599	3.925	1.825	2.597	1.622	0.944	0.469	0.097
1	3.309	6.824	6.277	4.414	2.105	3.564	2.208	1.283	0.643	0.162
<i>Panel B: with a random coefficient</i>										
0 (competition)	-2.737	-2.867	-2.905	-2.716	-2.039	-8.601	-2.536	-1.480	-0.754	-0.383
0.1	-2.636	-2.343	-2.344	-2.257	-1.683	-8.092	-2.353	-1.322	-0.640	-0.318
0.2	-2.549	-0.929	-0.963	-1.239	-1.198	-7.535	-2.126	-1.158	-0.512	-0.255
0.3	-2.460	0.845	0.676	-0.030	-0.612	-6.894	-1.844	-0.944	-0.354	-0.181
0.4	-2.320	2.644	2.302	1.168	0.019	-6.091	-1.504	-0.676	-0.200	-0.083
0.5	-2.109	4.343	3.826	2.293	0.639	-5.143	-1.090	-0.412	-0.037	0.004
0.6	-1.659	5.945	5.252	3.337	1.219	-4.004	-0.559	-0.036	0.110	0.071
0.7	-0.732	7.466	6.606	4.295	1.789	-2.480	0.009	0.276	0.280	0.149
0.8	0.721	8.896	7.892	5.220	2.307	-0.971	0.686	0.556	0.428	0.199
0.9	2.290	10.225	9.136	6.095	2.784	-0.833	1.299	0.818	0.548	0.271
1	3.620	11.442	10.300	6.911	3.227	-0.459	1.738	1.075	0.708	0.338

Notes: The table reports the median values of the two test statistics, T_{IV}^{RV} and T_{markup}^{RV} , across 500 simulated datasets for each Monte Carlo configuration ($J = 36$, $F = 6$, $T = 100$, ϕ , F_c). T_{markup}^{RV} is constructed under the two alternative firm conduct models: one with $\phi = 0$ (competition) and the other with $\phi = 1$ (full profit internalization under industry conduct consistent with the effective firm index). The top panel presents the results when a random coefficient is excluded from the indirect utility function (14) in the DGP, while the bottom panel presents the results when it is included.

at high values of ϕ while rejecting it in favor of competition at lower values of ϕ . Conversely, the inclusion of a random coefficient tends to reduce the testing power of T_{markup}^{RV} . These results highlight the advantage of our approach: it performs comparably to, or outperforms, the existing method under various Bertrand-Nash collusive scenarios, while its implementation does not require estimating the demand and supply system.

Figure D3 in the Appendix presents the median p-values under the alternative hypothesis, $H_2 : Q_{comp} > Q_{coll}$, derived from extensive additional simulations. Clearly, there is a positive monotonic relationship between testing power and the degree of profit internalization across all (F, F_c) configurations considered in our Monte Carlo study. The degree of collusion (F/F_c) also influences testing power; except in cases where all firms in the market collude, a higher degree corresponds to greater power in detecting collusive behavior among firms.

So far, we have treated the product attribute, x_{jt} , as an exogenous variable. However, this assumption may not hold in situations where firms determine product attributes and prices simultaneously with the contemporaneous unobservable components of demand, ξ_{jt} . In such cases, BLP-style instruments fail to satisfy the exclusion restriction, rendering them invalid and leading to inconsistent demand estimates. To evaluate the testing power of the two statistics under this model misspecification, we generate endogenous product attributes as described in Section 5.1. The true profit internalization parameter, ϕ , is fixed at 1, while the direction and degree of endogeneity are parameterized by $\rho \in \{-10, -5, -1, 0, 1, 5, 10\}$ as explained in Section 5.1.

The top panel of Table 3 reports the results obtained without a random coefficient in the indirect utility specification (14) in the DGP. While the power of both test statistics remains robust across various endogeneity scenarios, the direction of correlation between ξ_{jt} and x_{jt} affects the testing power. When the correlation is negative, the testing power declines; conversely, when the correlation is positive, the testing power improves. Results obtained with a random coefficient in the utility specification, presented in the bottom panel of the table, indicate that the testing power of both statistics tends to be higher when the model is misspecified.²⁴

In sum, our results are robust to model misspecification, which is one of the key advantages of the RV test, as highlighted by Duarte, Magnolfi, Sølvesten, and Sullivan (2024). Moreover, our test statistic, T_{IV}^{RV} , exhibits higher testing power than the existing statistic, T_{markup}^{RV} , across all specifications except in the case of $F_c = 1$, where the difference is only marginal.

Our simulation setting so far has assumed a scenario in which researchers have sufficient observations from many markets ($T = 100$). To evaluate the performance of the two testing procedures when only a few markets are available in the data, we repeat our analysis with $T = 10$. Results summarized in Tables D3 and D4 in the Appendix reveal that, as expected, both statistics experience a substantial loss in testing power under this data limitation. Interestingly, T_{IV}^{RV} continues to yield significantly positive statistics across various collusive scenarios, whereas T_{markup}^{RV} retains

²⁴We also consider an alternative DGP for the endogenous product attribute. Specifically, the triplet $(x_{jt}^{endo}, \xi_{jt}, \omega_{jt})$ is drawn from a mean-zero trivariate normal distribution with a covariance matrix:

$$\begin{bmatrix} 0.2 & \rho & 0 \\ \rho & 0.2 & 0.1 \\ 0 & 0.1 & 0.2 \end{bmatrix},$$

where ρ is in between -0.2 and 0.2. The results remain qualitatively unaffected.

Table 3: (Endogenous) $F = 6$ and $T = 100$

ρ	$T_{IV}^{RV} (\mathbf{z}^{comp} \text{ vs } \mathbf{z}^{coll})$					$T_{markup}^{RV} (\phi = 0 \text{ vs } \phi = 1)$				
	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$
<i>Panel A: without a random coefficient</i>										
Exogenous $\rho = 0$	3.546	7.240	6.630	4.679	2.268	3.905	2.469	1.489	0.739	0.243
Endogenous (-) $\rho = -1$	2.523	5.611	5.195	3.600	1.698	2.853	1.825	1.053	0.528	0.193
$\rho = -5$	2.446	5.644	5.231	3.650	1.663	2.782	1.755	1.011	0.484	0.139
$\rho = -10$	2.440	5.658	5.231	3.648	1.662	2.818	1.763	1.038	0.489	0.111
Endogenous (+) $\rho = 1$	4.494	7.743	7.065	4.984	2.431	4.858	3.060	1.853	0.967	0.366
$\rho = 5$	4.849	8.185	7.457	5.261	2.573	5.253	3.327	2.022	1.110	0.492
$\rho = 10$	4.857	8.193	7.488	5.271	2.606	5.130	3.195	1.913	0.979	0.359
<i>Panel B: with a random coefficient</i>										
Exogenous $\rho = 0$	3.650	11.492	10.361	6.948	3.248	0.265	1.712	1.033	0.720	0.326
Endogenous (-) $\rho = -1$	4.005	11.889	10.812	7.294	3.407	3.953	2.047	1.212	0.697	0.321
$\rho = -5$	4.057	12.188	11.114	7.466	3.480	4.839	2.061	1.145	0.677	0.297
$\rho = -10$	4.065	12.180	11.129	7.494	3.490	4.907	2.090	1.173	0.709	0.308
Endogenous (+) $\rho = 1$	5.126	12.991	11.749	7.835	3.709	5.299	2.426	1.319	0.894	0.421
$\rho = 5$	5.446	13.406	12.234	8.215	3.901	6.463	2.499	1.380	0.835	0.406
$\rho = 10$	5.441	13.434	12.277	8.232	3.906	6.502	2.424	1.348	0.847	0.358

Notes: The table reports the median values of the two test statistics, T_{IV}^{RV} and T_{markup}^{RV} , across 500 simulated datasets for each Monte Carlo configuration ($J = 36, F = 6, T = 100, \rho, F_c$). The product attribute x_{jt} is treated as an endogenous variable. The direction and degree of endogeneity are parameterized by $\rho \in \{-10, -5, -1, 0, 1, 5, 10\}$, while the true profit internalization parameter ϕ is fixed at 1. T_{markup}^{RV} is constructed under the two alternative firm conduct models: one with $\phi = 0$ (competition) and the other with $\phi = 1$ (full profit internalization under industry conduct consistent with the effective firm index). The top panel presents the results when a random coefficient is excluded from the indirect utility function (14) in the DGP, while the bottom panel presents the results when it is included.

significant power only under a few specific configurations: positive endogeneity of x_{jt} ($\rho \geq 5$), $F_c = 1$, and the inclusion of a random coefficient in the consumer utility specification, as shown in the bottom panel of Table D4. The results also reveal that T_{markup}^{RV} continues to outperform T_{IV}^{RV} in rejecting collusion when true industry conduct is closer to competition (low ϕ).

6.3 Alternative functional forms for BLP-style instruments

For expositional purposes, we have used only one functional form for BLP-style instruments: the summation of other product attributes (summation IVs), as originally proposed by Berry, Levinsohn, and Pakes (1995). To assess the robustness of our results to alternative functional forms, we incorporate Differentiation IVs (Gandhi and Houde, 2019) up to third-order polynomials in our Monte Carlo study.

Our results largely align with previous findings. Notably, our test statistic, T_{IV}^{RV} , whose testing power remains relatively unaffected by changes in functional forms, continues to outperform T_{markup}^{RV} in detecting collusive behavior among firms across various collusive scenarios. Interestingly, the testing power of T_{markup}^{RV} declines substantially when only *Local* and *Quadratic* IVs are used but is restored when the *third-order* analogue is incorporated within Differentiation IVs. We also observe similar effects of model misspecification arising from the endogenous product attribute x_{jt} to those presented in Table 3. Additionally, the use of Differentiation IVs enhances the accuracy of estimating the non-linear coefficient σ_x , consistent with the findings of Gandhi and Houde (2019). Further details on the alternative functional form design and extensive discussions of the results are provided in Appendix B.

The testing power of T_{IV}^{RV} depends on the strength of the instruments in the first-stage price regressions. We use summation IVs as the baseline instruments for our proposed testing procedure, primarily because, compared to other specifications, they provide stronger identification power in the first-stage price regression across most Monte Carlo configurations considered.²⁵ The core of our approach lies not in selecting the optimal functional forms for BLP-style instruments but in leveraging alternative firm indexes based on the *observed* and *suspected* ownership structures when constructing these instruments.

6.4 Discussion

The most notable difference between our proposed testing procedure and the existing one is that ours does not require researchers to estimate the demand system or impose a marginal cost specification. Therefore, T_{IV}^{RV} does not rely on the statistical properties of demand estimates, circumventing potential challenges associated with estimating the model by non-linear GMM and the inputs required for this task.²⁶ In contrast, reliable and powerful testing with T_{markup}^{IV} depends on obtaining consistent and efficient demand estimates. Furthermore, since our testing method simply compares the model fit of two linear price regressions, it can be implemented in data-limited settings where

²⁵Specifically, the median values of first-stage F-statistics obtained using summation IVs are higher than those from Differentiation IVs across various Monte Carlo specifications, including those presented in Section 7.

²⁶Moreover, our approach is expected to be less dependent on the functional form of choice probabilities – such as nested logit, multivariate probit, and multiple-choice models – compared to existing methods that require demand estimation as a prerequisite. This is because our testing procedure relies solely on firm indexing and the nature of non-nested tests.

researchers lack information on product market shares. Given these properties, our method can serve as a preliminary tool for researchers and regulatory authorities to diagnose collusive behavior among *suspected* firms in the market.

Once preliminary results indicate the presence of collusion, researchers can proceed with existing tests on industry conduct. The advantage of the existing statistic, T_{markup}^{RV} , is that researchers can test any model of conduct, provided the first-order profit-maximizing conditions can be derived from the model, and determine which model best fits the observed data based on the moment restrictions imposed on the supply-side model. On the other hand, our approach, while not assuming any particular supply model, aims to detect price collusion among firms grounded in Bertrand-Nash equilibrium conditions in differentiated goods markets. Therefore, these two approaches can complement each other, creating a more practical, efficient, and powerful framework for testing firm conduct.

In addition to its preliminary role in detecting collusive behavior, our proposed testing procedure can also be used to construct stronger BLP-style instruments. For instance, once researchers observe evidence of collusive behavior, they can construct \mathbf{z}^{coll} based on the colluding firm index and use it in demand estimation instead of \mathbf{z}^{comp} . Improved performance in the first-stage demand estimation, achieved through the use of instruments that properly capture true firm conduct behavior, would ultimately enhance performance in the second-stage estimation, where (feasible) optimal instruments are employed (e.g., Reynaert and Verboven (2014), Gandhi and Houde (2019), and Conlon and Gortmaker (2020)).

7 The role of own- and other-firm IVs in demand estimation

Our proposed testing method for firm conduct utilizes \mathbf{z}^{comp} and \mathbf{z}^{coll} , which are functions of product attributes produced by the firm and its *suspected* colluding partners (e.g., summation of attributes or squared attributes). Here, we set aside the issue of collusion and focus on the role of the standard own- and other-firm instruments, \mathbf{z}^{own} and \mathbf{z}^{other} , in demand estimation. The simulation study presented in this section provides additional intuition and aids in interpreting the previous simulation results related to our proposed testing procedure.

The Monte Carlo setup is as follows: We fix the number of products at $J = 36$ and vary the

number of firms $F \in \{1, 2, \dots, 36\}$. Since we do not consider any form of collusive behavior among firms, ϕ is fixed at 0. We generate instruments \mathbf{z}^{own} and \mathbf{z}^{other} as defined in equation (16). To investigate their roles in demand estimation, we report the median F-statistics from the first-stage price regression and the median absolute errors of the estimated utility parameters across 500 simulations under the following instrument configurations: (i) \mathbf{z}^{own} only, (ii) \mathbf{z}^{other} only, and (iii) $\mathbf{z}^{both} = (\mathbf{z}^{own}, \mathbf{z}^{other})$.

The results for the utility specification without (with) a random coefficient, illustrated in the upper panel (bottom panel, respectively) of Figure 2, reveal several interesting points.²⁷ First, own-firm instruments exhibit greater identification power than other-firm instruments in the first-stage price regression, as shown in Figures 2(a) and (c). Consequently, the median absolute error of the estimated price coefficient is lower when \mathbf{z}^{own} is used as the instrument, as shown in Figures 2(b) and (d). These observations are consistent with the argument in Section 3 and previous literature (Berry, Levinsohn, and Pakes, 1995; Gandhi and Nevo, 2021) that firms internalize the cross-price elasticities of their own products when setting profit-maximizing prices, resulting in greater explanatory power of own-firm product attributes in determining equilibrium prices. They also align with our simulation results presented in the previous section and echo the importance of correctly indexing firms based on the true conduct model.

In the previous sections, we designed our Monte Carlo study and conducted simulations using only own-firm instruments, \mathbf{z}^{comp} and \mathbf{z}^{coll} , as they exhibit greater identification power than other-firm instruments in the first-stage price regression. In fact, our testing procedure can be extended to incorporate other-firm instruments constructed in the same manner as the own-firm instruments, based on the original firm and the *suspected* colluding firm indices. Excluding the product attributes of colluding partners from other-firm instruments enhances their identification power because, as outlined in Section 3, the price effects of colluding and non-colluding firms' product attributes are opposite in direction.²⁸

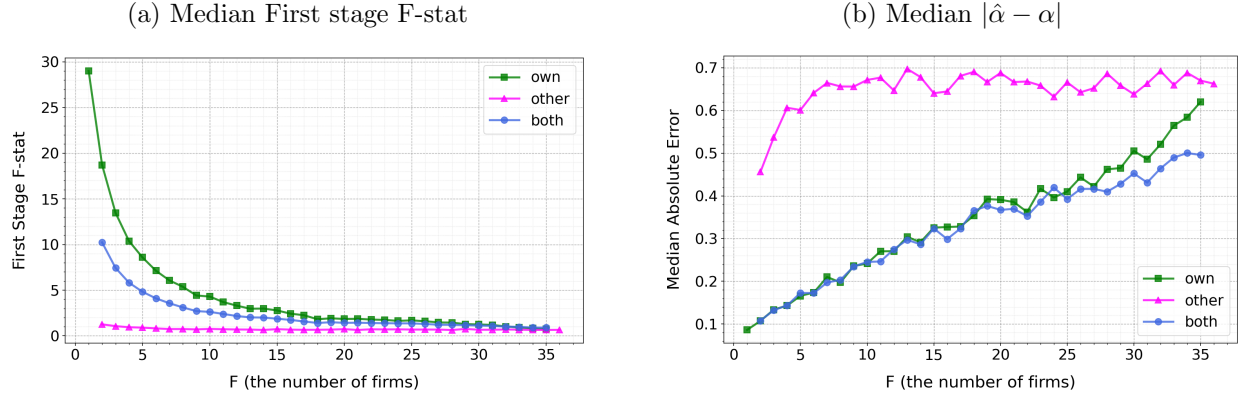
Second, the bottom panel of the figure shows that while own-firm instruments are more effective

²⁷Tables D1 and D2 in the Appendix provide the full numeric results, including the median root-mean-squared errors (RMSE) of the estimated coefficients.

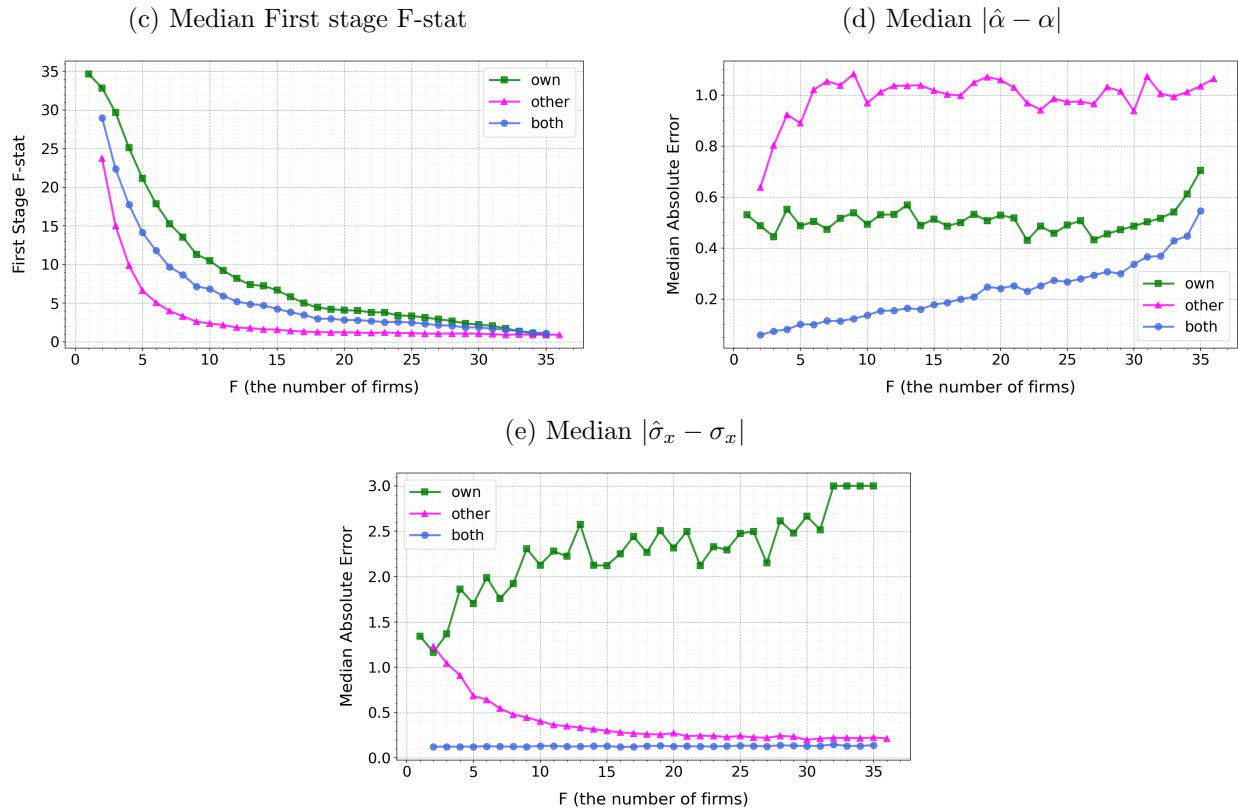
²⁸Simulation results available from the authors upon request are both quantitatively and qualitatively similar to those obtained using only the own-firm instruments. Additionally, it is worth noting that when testing whether all firms in the market collude ($F_c = 1$), constructing other-firm instruments under the *suspected* colluding firm index is impossible, making it more likely for our testing procedure to diagnose that the model of competition provides a better fit.

Figure 2: Own-firm vs other-firm instruments

Panel A: without a random coefficient



Panel B: with a random coefficient



Notes: The figure shows the median F-statistic from the first-stage price regression and the median absolute errors of the estimated utility parameters, $\hat{\alpha}$ and $\hat{\sigma}_x$, across 500 simulated datasets for each Monte Carlo configuration ($J = 36, F, T = 100$). The upper panel presents results for the utility specification without a random coefficient, while the bottom panel shows results with a random coefficient.

in the first-stage price regression and more powerful for identifying the price coefficient, other-firm instruments are more useful for identifying the non-linear coefficient σ_x .²⁹ These observations help explain the results in the bottom panel of Table 1, where using \mathbf{z}^{coll} leads to a smaller median absolute error for the estimated non-linear coefficient, even when the degree of profit internalization is low. Specifically, variations in product attributes of other firms unintentionally included in \mathbf{z}^{coll} may assist in identifying the non-linear coefficient.

Third, the less concentrated the market, the lower the explanatory power of the two BLP-style instruments. This occurs because, as competition among firms intensifies, equilibrium markups converge, which in turn weakens the correlation between markups and instruments. For instance, assuming a simple logit demand, when the market is competitive, the values inside the bracket in equation (6) are not significantly different across products in the market. In fact, this finding aligns with the observation made by Armstrong (2016) in the extreme case of $J \rightarrow \infty$.³⁰ On the other hand, in a more concentrated market, greater variation in markup sizes reduces the sensitivity of the correlation between markups and instruments to fluctuations from unobserved components.

8 Empirical application

In this section, we apply our testing method to study industry conduct in two differentiated product markets in South Korea: the new passenger car market and the instant noodles market. We then examine whether the results align with anecdotal and analytical evidence of industry conduct in each market. Namely, automobile manufacturers owned by the same parent company jointly determine the prices of their car models, while instant noodle prices remain significantly below the collusive level despite suspicions from the Korean Fair Trade Commission (KFTC).

²⁹Using \mathbf{z}^{other} instead of \mathbf{z}^{own} consistently results in a lower median absolute error and a lower median RMSE for the estimated non-linear coefficient, as shown in Table D2 in the Appendix.

³⁰Specifically, Armstrong (2016) examined cases where markets consist of many single-product firms ($J_t = F_t$). He demonstrated that as $J_t \rightarrow \infty$, the explanatory power of BLP-style instruments for equilibrium markups diminishes. Cross-firm or market-level variations in equilibrium markups can help maintain identification power in such cases. See Armstrong (2016) for further details.

8.1 The new passenger car market

South Korea is one of the largest car markets in the world, with approximately 1.6 million vehicles sold in 2022.³¹ The two largest firms, Hyundai and Kia, together accounted for nearly 70 percent of total vehicle sales over the past decade. Since both are owned by the Hyundai Motor Group, following Kia’s merger into the group in 1998, pricing is likely coordinated between the two brands.³² We test this joint profit maximization against own-profit maximization, in which each firm sets prices independently. We also consider three alternative firm conduct scenarios: joint profit maximization among German brands; collusion among domestic automakers and among foreign automakers; and full collusion. In February 2023, the KFTC imposed a total fine of 42.3 billion Won (approximately 33 million US dollars) on the four German automakers – Mercedes-Benz, BMW, Audi, and Volkswagen – for colluding on emission reduction technology applied to their diesel passenger cars.³³ Here, we investigate whether the German automakers also colluded on prices.

Drawing on province/year/product-level data on prices and attributes for 776 products from 13 brands between 2012 and 2023,³⁴ we conduct pairwise hypothesis tests by running the following first-stage price regressions:

$$p_{jt} = \gamma^h \cdot x_{jt} + \boldsymbol{\theta}^h \mathbf{z}_{jt}^h + \psi_j + \psi_y^{\text{fuel}} + \psi_r + e_{jt}^h, \quad (17)$$

where x_{jt} represents four exogenous characteristics of product j in market (year-province) t : fuel economy (km per 1,000 Won), acceleration (horsepower/weight), size (width×length×height), and a constant term. The regression model (17) also includes controls for product, year-fuel type (to account for differential trends in government policies regarding fuel types), and province, denoted as ψ_j , ψ_y^{fuel} , and ψ_r .

As for the exogenous instruments under conduct hypothesis $h \in \{H_0, H_1\}$, denoted by \mathbf{z}_{jt}^h , we use the four exogenous product attributes to construct four summation IVs (sums over own-firm products based on the effective firm index). In addition, we incorporate a two-level nesting

³¹<https://www.statista.com/statistics/265891/vehicles-sales-in-selected-countries/>

³²We use the terms “firm” and “brand” interchangeably.

³³The KFTC’s ruling is available at <https://www.ftc.go.kr/www/selectBbsNttView.do?pageUnit=10&pageIndex=92&searchCnd=all&key=12&bordCd=3&searchCtgr=01,02&nttSn=42758>.

³⁴We define a product as a unique combination of nameplate and fuel type. Appendix C provides details on the raw data and sample construction.

structure based on the car segment (small/compact, midsize, mid luxury, large/luxury, small SUV, standard SUV, large/luxury SUV) and fuel type (gasoline, diesel, LPG, EV, HEV),³⁵ and create 12 additional summation IVs: sums over own-firm products in the same segment, sums over own-firm products with the same fuel type, and sums over own-firm products with the same fuel type in the same segment.³⁶

We report the testing results (clustered by market) in the upper panel of Table 4 where a positive value greater than 1.65 (a negative value less than -1.65) constitutes evidence for the row (column) conduct model. The results indicate that, as expected, joint profit maximization by the parent company is more consistent with the data than own-profit maximization. Moreover, our test rejects all forms of price collusion against joint profit maximization by the parent company. These results, robust to alternative choices of fixed effects and clustering, align with the fact that no antitrust cases have been filed against the car manufacturers in South Korea in recent years, except for the non-price collusion case described above.

8.2 Instant noodles market

The South Korean instant noodles (*ramen*) market is highly concentrated, with four firms – Nongshim, Ottogi, Samyang, and Paldo – accounting for over 90 percent of total sales. In March 2012, the KFTC fined them 135 billion won (approximately 120 million US dollars) for price collusion. In particular, Nongshim, the market leader with more than a 50% sales share, was fined 100 billion won for leading the collusion. However, this ruling was overturned by the Supreme Court of Korea in December 2015.³⁷ Given that the KFTC remains suspicious of these firms and has been closely monitoring *ramen* prices,³⁸ we test various collusion hypotheses, with Nongshim as the collusion leader, against the hypothesis of own-profit maximization. Specifically, we consider full collusion and three cases in which Nongshim coordinates prices with two other firms.

³⁵Note that while the nesting order is important for demand estimation, it is irrelevant to our testing procedure.

³⁶We obtain testing results consistent with those reported in Table 4 when we proceed without any nesting structures and use just the four sums over own-firm products as IVs.

³⁷The rulings of the KFTC and the Supreme Court are available at <https://www.ftc.go.kr/www/selectBbsNttView.do?pageUnit=10&pageIndex=507&searchCnd=all&key=12&bordCd=3&searchCtgry=01,02&nttSn=37962> and <https://www.scourt.go.kr/supreme/news/NewsViewAction2.work?seqnum=5081&gubun=4&searchOption=&searchWord=>, respectively.

³⁸For example, the KFTC considered investigating collusion among the *ramen* manufacturers in 2023, citing that the *ramen* prices, which increased in the aftermath of the COVID-19 outbreak, did not decrease despite falling flour prices since late 2022: https://www.news1s.com/view/NISX20230623_0002349874.

Table 4: Testing firm conduct in the two differentiated product markets

Firm conduct	1.	2.	3.	4.	5.
<i>Panel A: new passenger car market</i>					
1. Brand ($F = 13$)		-3.718	1.013	0.335	4.940
2. Parent company ($F_c = 9$)	3.718		5.404	2.229	7.104
3. German automakers ($F_c = 7$)	-1.013	-5.404		-0.343	4.954
4. Domestic/Foreign ($F_c = 2$)	-0.335	-2.229	0.343		4.168
5. Full collusion ($F_c = 1$)	-4.940	-7.104	-4.954	-4.168	
<i>Panel B: instant noodles market</i>					
1. Competition ($F = 4$)		18.727	17.546	18.411	2.619
2. Full collusion ($F_c = 1$)	-18.727		-6.324	-5.444	-22.873
3. N-O-P ($F_c = 2$)	-17.546	6.324		3.747	18.316
4. N-O-S ($F_c = 2$)	-18.411	5.444	-3.747		21.808
5. N-P-S ($F_c = 2$)	-2.619	22.873	-18.316	-21.808	

Notes: The table reports the results of pairwise hypothesis tests using our testing framework. Results for the new passenger car market are in the upper panel, while those for the instant noodles market are in the bottom panel. The names of the four *ramen* producers – Nongshim, Ottogi, Paldo, and Samyang – are abbreviated using their initial letters: N, O, P, and S. A positive value greater than 1.65 (a negative value less than -1.65) constitutes evidence for the row (column, respectively) conduct model.

The data used for our test comprise region/year-month/product-level prices and product-level attributes of the 70 best-selling instant noodle products from 2010 to 2019.³⁹ For each of the seven product attributes – cholesterol, calorie, sugar, fat, protein, sodium contents, and a constant term – we calculate the sum over own-firm products (based on the effective firm index) and use them as exogenous instruments. In addition, given that the *ramen* market is differentiated along two dimensions (Hong, Kim, and Kim, 2023; Kim and Kim, 2024), that is, package (pouch vs. cup) and soup type (red soup, white soup, souplless),⁴⁰ we construct 21 additional IVs: sums over own-firm products that share the same package, sums over own-firm products that share the same soup type, and sums over own-firm products that share both the same package and soup type. Then, we regress the price on product, time (year-month), and region dummy variables, along with the 28 IVs, for each conduct scenario.⁴¹

According to the testing results (clustered by market) presented in the bottom panel of Table

³⁹A region is composed of two or more adjacent provinces. See Appendix C for data details.

⁴⁰We exclude cold noodle products from the analysis because their demand exhibits seasonality distinct from that of the products included in the analysis.

⁴¹Product attributes are absorbed by the product dummies.

4, our test rejects all collusion hypotheses considered in favor of the own-profit maximization.⁴² These results align with the 2015 Supreme Court ruling, as well as the findings of Kim and Kim (2024) concluding that the observed markups are too low to support any collusive behavior.

9 Conclusion

Correctly assessing industry conduct is essential for establishing antitrust policy and evaluating market efficiency. Existing approaches to testing firm conduct often suffer from reduced testing power due to model misspecification and challenges in demand estimation. In this article, we propose a practical and powerful testing procedure that circumvents these limitations. Our method, built upon the Rivers and Vuong (RV) non-nested model selection framework, simply compares the performance of two BLP-style instrument sets – competition IVs and collusion IVs – in first-stage price regressions and interprets statistically significant results as evidence of either competitive or collusive behavior among firms.

Through extensive Monte Carlo simulations, we evaluate the finite-sample performance of our test statistic under various market conditions, characterized by different levels of market concentration, collusion, and internalization of colluding partners' profits. The results show that our method is robust to model misspecification, alternative functional forms for instruments, and data limitations, performing comparably to, or better than, existing approaches in detecting collusion across various collusive scenarios. The simplicity of our approach despite its high testing power makes it a practical tool for the preliminary diagnosis of industry conduct. By complementing existing methods, our testing framework provides researchers and regulatory authorities with an efficient and effective way to assess firm behavior and guide antitrust interventions.

Moreover, our procedure can aid in designing more effective BLP-style instruments: once collusion is detected, researchers are advised to incorporate the product characteristics of colluding partners when building own-firm instruments while excluding them from other-firm instruments. This approach would strengthen the identification power of the instruments in demand estimation.

⁴²We also consider other collusion scenarios in which Nongshim, along with another firm, coordinate prices, and obtain results in favor of price competition.

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Appendix

A Partial derivatives of prices with respect to attributes

A.1 Logit case

We assume that there is only one exogenous attribute x without loss of generality. If the two firms compete *à la* Bertrand, then the three FOCs are:

$$p_A - mc_A = p_T - mc_T = \frac{1}{\alpha} \frac{1 + \sum_{j \in \{A, T, H\}} X_j}{1 + X_H}, \quad p_H - mc_H = \frac{1}{\alpha} \frac{1 + \sum_{j \in \{A, T, H\}} X_j}{1 + X_A + X_T},$$

where $X_j = \exp(\delta_j)$. By the Implicit Function Theorem, the partial derivatives of prices with respect to attributes are:⁴³

$$\begin{pmatrix} \frac{\partial p_A}{\partial x_A} & \frac{\partial p_A}{\partial x_T} & \frac{\partial p_A}{\partial x_H} \\ \frac{\partial p_T}{\partial x_A} & \frac{\partial p_T}{\partial x_T} & \frac{\partial p_T}{\partial x_H} \\ \frac{\partial p_H}{\partial x_A} & \frac{\partial p_H}{\partial x_T} & \frac{\partial p_H}{\partial x_H} \end{pmatrix} = - \begin{pmatrix} 1 + \frac{X_A}{1+X_H} & \frac{X_T}{1+X_H} & -\frac{(X_A+X_T)X_H}{(1+X_H)^2} \\ \frac{X_A}{1+X_H} & 1 + \frac{X_T}{1+X_H} & -\frac{(X_A+X_T)X_H}{(1+X_H)^2} \\ -\frac{X_A X_H}{(1+X_A+X_T)^2} & -\frac{X_T X_H}{(1+X_A+X_T)^2} & 1 + \frac{X_H}{1+X_A+X_T} \end{pmatrix}^{-1} \\ \times \frac{\beta}{\alpha} \begin{pmatrix} -\frac{X_A}{1+X_H} & -\frac{X_A}{1+X_H} & \frac{(X_A+X_T)X_H}{(1+X_H)^2} \\ -\frac{X_A}{1+X_H} & -\frac{X_A}{1+X_H} & \frac{(X_A+X_T)X_H}{(1+X_H)^2} \\ \frac{X_A X_H}{(1+X_A+X_T)^2} & \frac{X_T X_H}{(1+X_A+X_T)^2} & -\frac{X_H}{1+X_A+X_T} \end{pmatrix}.$$

In case of full collusion, the three FOCs are:

$$p_A - mc_A = p_T - mc_T = p_H - mc_H = \frac{1}{\alpha} \left(1 + \sum_{j \in \{A, T, H\}} X_j \right).$$

The partial derivatives of prices with respect to attributes are:

$$\begin{pmatrix} \frac{\partial p_A}{\partial x_A} & \frac{\partial p_A}{\partial x_T} & \frac{\partial p_A}{\partial x_H} \\ \frac{\partial p_T}{\partial x_A} & \frac{\partial p_T}{\partial x_T} & \frac{\partial p_T}{\partial x_H} \\ \frac{\partial p_H}{\partial x_A} & \frac{\partial p_H}{\partial x_T} & \frac{\partial p_H}{\partial x_H} \end{pmatrix} = -\frac{\beta}{\alpha} \begin{pmatrix} 1 + X_A & X_T & X_H \\ X_A & 1 + X_T & X_H \\ X_A & X_T & 1 + X_H \end{pmatrix}^{-1} \begin{pmatrix} -X_A & -X_T & -X_H \\ -X_A & -X_T & -X_H \\ -X_A & -X_T & -X_H \end{pmatrix}.$$

Therefore, with values of $\alpha = \beta = 1$ and $X_A = X_T = X_H = 0.2$, the partial derivatives matrices under competition and under competition are

$$\begin{pmatrix} 0.124 & 0.124 & -0.037 \\ 0.124 & 0.124 & -0.037 \\ -0.013 & -0.013 & 0.124 \end{pmatrix} \text{ and } \begin{pmatrix} 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 \end{pmatrix}, \text{ respectively.}$$

⁴³We also consider the case in which the marginal cost of a product is a linear function of its own attribute: $mc(x) = c + \gamma x$. Results remain qualitatively the same.

A.2 Nested logit case

First order conditions

Now, we extend the analysis to consider various collusive behaviors among firms and more flexible substitution patterns across products using a nested logit model. Note that

$$\frac{\partial s_k}{\partial p_j} = \begin{cases} \frac{\alpha}{1-\sigma} s_j ((1-\sigma)s_k + \sigma s_{k|g} - 1) & \text{if } j = k \\ \frac{\alpha}{1-\sigma} s_j ((1-\sigma)s_k + \sigma s_{k|g}) & \text{if } j \in \mathcal{I}_{g(k)}, j \neq k \\ \alpha s_j s_k & \text{if } j \notin \mathcal{I}_{g(k)} \end{cases}$$

where $s_{k|g}$ denotes the within-group market share for product k in group g . Given the profit function (4), the first order condition for profit maximization is given by

$$\begin{aligned} p_j - mc_j &= \frac{1-\sigma}{\alpha} + \sum_{k \in \mathcal{I}_{g(j)} \cap \mathcal{I}_{f(j)}} (p_k - mc_k) ((1-\sigma)s_k + \sigma s_{k|g(j)}) \\ &+ \sum_{k \in \mathcal{I}_{f(j)} \setminus \mathcal{I}_{g(j)}} (1-\sigma)(p_k - mc_k)s_k \\ &+ \phi \cdot \sum_{k \in ((\mathcal{I}_{g(j)} \cap \mathcal{I}_{f_c(j)}) \setminus (\mathcal{I}_{g(j)} \cap \mathcal{I}_{f(j)}))} (p_k - mc_k) ((1-\sigma)s_k + \sigma s_{k|g(j)}) \\ &+ \phi \cdot \sum_{k \in ((\mathcal{I}_{f_c(j)} \setminus \mathcal{I}_{g(j)}) \setminus (\mathcal{I}_{f(j)} \setminus \mathcal{I}_{g(j)}))} (1-\sigma)(p_k - mc_k)s_k \quad \forall j \in \mathcal{I}_{g(j)} \cap \mathcal{I}_{f(j)}, \end{aligned} \tag{A1}$$

where $g(j)$ indexes a group in which product j belongs. As shown in (A1), in the nested logit framework, equilibrium markups are identical for products within the same group produced by the same firm.

Implicit Function Theorem

Next, we define an implicit function for each j , denoted by $F_j(\tilde{\mathbf{x}}, \mathbf{p})$ as follows:

$$F_j := (\text{LHS of (A1)}) - (\text{RHS of (A1)}) = 0,$$

where $\tilde{\mathbf{x}} = (\mathbf{x}, \boldsymbol{\xi})$. We then have a vector of implicit function $\mathbf{F} : \mathbb{R}^{J \cdot (R+2)} \rightarrow \mathbf{R}^J$ given by

$$\mathbf{F}(\tilde{\mathbf{x}}, \mathbf{p}) = \begin{pmatrix} F_1(\tilde{\mathbf{x}}, \mathbf{p}) \\ F_2(\tilde{\mathbf{x}}, \mathbf{p}) \\ \vdots \\ \vdots \\ F_J(\tilde{\mathbf{x}}, \mathbf{p}) \end{pmatrix} = \mathbf{0}. \tag{A2}$$

A vector of prices, \mathbf{p} , is endogenously determined by firms. Without loss of generality, assume that there is one exogenous attribute for each product, denoted by x_j ($R = 1$). By the Implicit Function Theorem, we obtain

$$\begin{aligned} \frac{\partial \mathbf{p}}{\partial \mathbf{x}} &= \begin{pmatrix} \frac{\partial p_1}{\partial x_1} & \frac{\partial p_1}{\partial x_2} & \cdots & \cdots & \frac{\partial p_1}{\partial x_J} \\ \frac{\partial p_2}{\partial x_1} & \frac{\partial p_2}{\partial x_2} & \cdots & \cdots & \frac{\partial p_2}{\partial x_J} \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ \frac{\partial p_J}{\partial x_1} & \frac{\partial p_J}{\partial x_2} & \cdots & \cdots & \frac{\partial p_J}{\partial x_J} \end{pmatrix} \\ &= - \begin{pmatrix} \frac{\partial F_1}{\partial p_1} & \frac{\partial F_1}{\partial p_2} & \cdots & \cdots & \frac{\partial F_1}{\partial p_J} \\ \frac{\partial F_2}{\partial p_1} & \frac{\partial F_2}{\partial p_2} & \cdots & \cdots & \frac{\partial F_2}{\partial p_J} \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ \frac{\partial F_J}{\partial p_1} & \frac{\partial F_J}{\partial p_2} & \cdots & \cdots & \frac{\partial F_J}{\partial p_J} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots & \cdots & \frac{\partial F_1}{\partial x_J} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \cdots & \cdots & \frac{\partial F_2}{\partial x_J} \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ \frac{\partial F_J}{\partial x_1} & \frac{\partial F_J}{\partial x_2} & \cdots & \cdots & \frac{\partial F_J}{\partial x_J} \end{pmatrix} = -\mathbf{J}_{\mathbf{p}}^{-1} \mathbf{J}_{\mathbf{x}}. \end{aligned} \tag{A3}$$

Data setup and computational details

A consumer's indirect utility and marginal cost specification are given by

$$\begin{aligned} u_{ij} &= \beta_1 + \beta_2 x_j - \alpha p_j + \xi_j + \varepsilon_{ij} \\ mc_j &= \gamma_1 + \gamma_2 x_j \end{aligned}$$

where $\beta_1 = -3$, $\beta_2 = 7$, $\alpha = 1$, $\gamma_1 = 1$, $\gamma_2 = 6.5$, and ε_{ij} is i.i.d. extreme value distributed following the nested logit structure with $\sigma = 0.2$ ($\sigma = 0$ for the logit case shown in Figure D1). The exogenous attribute x_j is randomly drawn from a standard uniform distribution, while the unobserved attribute ξ_j follows a mean-zero normal distribution with a standard deviation of 0.2.

We then obtain the equilibrium price vector, \mathbf{p}^* , by numerically solving for the root of equation (A2), given the specified utility, marginal cost function, and other market primitives such as F , the colluding firm index, ϕ , J , G , and firm/group assignments for each product. Our data-generating setup produces reasonable equilibrium outcomes. For instance, the share of the outside option ranges from 75.4% at $\phi = 0$ to 76.8% at $\phi = 1$. Additionally, the share-weighted own-price elasticity varies from -6.76 at $\phi = 0$ to -6.86 at $\phi = 1$.

Next, we obtain analytical values of $\mathbf{J}_{\mathbf{p}}$ and $\mathbf{J}_{\mathbf{x}}$ evaluated at given $\tilde{\mathbf{x}}$ and \mathbf{p}^* . We utilize the automatic differentiation (AD) method to calculate these analytical values by using Python package `jax`.⁴⁴ Unlike numerical differentiation (ND), the AD method provides exact analytical derivatives at given points. In addition to its accuracy, AD eliminates the need to derive explicit analytical expressions for each Jacobian entry. The only requirement to compute $\frac{\partial \mathbf{p}}{\partial \mathbf{x}}$ is defining the implicit function as in (A2).

⁴⁴For API documentation on applying automatic differentiation to compute the Jacobian matrix using `jax`, see: <https://jax.readthedocs.io/en/latest/advanced-autodiff.html>

B Alternative functional forms for BLP-style instruments

IV construction

While \mathbf{z}^{comp} and \mathbf{z}^{coll} in the main text are constructed by summing the attributes of other products, as proposed by Berry, Levinsohn, and Pakes (1995), alternative functional forms for the instruments can also be used to test firm conduct. In this appendix, we extend the analysis presented in the main text by exploring Differentiation IVs (Gandhi and Houde, 2019) and instruments incorporating higher-order terms, such as the summation of cubed attributes of other products and the third-order polynomial analogue within Differentiation IVs, as detailed below.

We construct analogues of \mathbf{z}_{jt}^{comp} and \mathbf{z}_{jt}^{coll} in (7) as follows:

$$\begin{aligned}\mathbf{z}_{jt,diff}^{comp} &= \left(\sum_{k \in \mathcal{J}_{ft} \setminus \{j\}} \mathbf{1}(|x_{kt} - x_{jt}| < \sigma_{x,t}), \sum_{k \in \mathcal{J}_{ft} \setminus \{j\}} (x_{kt} - x_{jt})^2 \right), \\ \mathbf{z}_{jt,diff}^{coll} &= \left(\sum_{k \in \mathcal{J}_{fct} \setminus \{j\}} \mathbf{1}(|x_{kt} - x_{jt}| < \sigma_{x,t}), \sum_{k \in \mathcal{J}_{fct} \setminus \{j\}} (x_{kt} - x_{jt})^2 \right),\end{aligned}\tag{A4}$$

where $\mathbf{1}(\cdot)$ denotes the indicator function, and $\sigma_{x,t}$ is the standard deviation of the product attribute x_{jt} in market t . Recall that \mathcal{J}_{fct} represents the set of products owned by firm f and its suspected colluding partners. In the literature, the first component of each instrument in (A4), referred to as the *Local IV*, counts the number of *nearby* products within the characteristic dimension, while the second component, termed the *Quadratic IV*, measures the quadratic distance of product j within the characteristic dimension.

Using these two distinct sets of instruments, we construct T_{IV}^{RV} as outlined in Sections 4.1 and 5.2. To compute T_{markup}^{RV} , we estimate the demand parameters using the following excluded instruments:

$$\begin{aligned}\mathbf{z}_{jt,diff}^{own} &= \left(\sum_{k \in \mathcal{J}_{ft} \setminus \{j\}} \mathbf{1}(|x_{kt} - x_{jt}| < \sigma_{x,t}), \sum_{k \in \mathcal{J}_{ft} \setminus \{j\}} (x_{kt} - x_{jt})^2 \right), \\ \mathbf{z}_{jt,diff}^{other} &= \left(\sum_{k \in \mathcal{J}_t \setminus \mathcal{J}_{ft}} \mathbf{1}(|x_{kt} - x_{jt}| < \sigma_{x,t}), \sum_{k \in \mathcal{J}_t \setminus \mathcal{J}_{ft}} (x_{kt} - x_{jt})^2 \right).\end{aligned}\tag{A5}$$

We then construct $\mathbf{z}_{jt,diff} = (\mathbf{z}_{jt,diff}^{own}, \mathbf{z}_{jt,diff}^{other})$ as the vector of excluded instruments to compute T_{markup}^{RV} , following the procedures outlined in Sections 4.2 and 5.2.

Results: Fstat evidence and demand estimation performance

Table D5 presents the analogue of Table 1 in the main text, illustrating the strength of \mathbf{z}_{diff}^{comp} and \mathbf{z}_{diff}^{coll} in the first-stage price regression and demand estimation. Overall, the results are in line with those reported in Table 1, exhibiting similar patterns. When firms do not internalize the

profits of other firms ($\phi = 0$), instruments based on the observed firm index, \mathbf{z}_{diff}^{comp} , produce higher first-stage F-statistics ($Fstat_1$) and demonstrate better performance. However, instruments based on the suspected colluding firm index, \mathbf{z}_{diff}^{coll} , outperform \mathbf{z}_{diff}^{comp} even at a very low degree of profit internalization among colluding firms ($\phi \geq 0.1$), yielding higher first-stage F-statistics ($Fstat_2$) and a lower median absolute error of the estimated price coefficient, $\hat{\alpha}$.

Comparing the results in Table 1 and Table D5 also reveals that, consistent with the findings of Gandhi and Houde (2019), using Differentiation IVs improves the accuracy of estimating the non-linear coefficient σ_x . For instance, the range of the median absolute error of the coefficient is 1.435–1.863 for \mathbf{z}_{diff}^{comp} and 0.291–0.727 for \mathbf{z}_{diff}^{coll} , but it reduces to 0.271–0.289 for \mathbf{z}_{diff}^{comp} and 0.147–0.300 for \mathbf{z}_{diff}^{coll} . Additionally, \mathbf{z}_{diff}^{coll} outperforms \mathbf{z}_{diff}^{comp} in estimating the non-linear coefficient when firms collude, even at a low profit internalization degree ($\phi \geq 0.2$), yielding smaller median absolute errors. In summary, while Differentiation IVs exhibit stronger identification power than summation IVs, estimation performance can be further improved by designing Differentiation IVs to accurately reflect industry conduct.

Results: test performance

Next, we compare the two test statistics, T_{IV}^{RV} and T_{markup}^{RV} , constructed using the Differentiation IVs instead of summation IVs. The results presented in Table D7 show that the testing power of T_{IV}^{RV} in detecting collusion remains largely unaffected, except when $F_c = 1$. In contrast, the testing power of T_{markup}^{RV} declines significantly, failing to reject price competition in favor of collusion even when firms fully internalize the profits of colluding partners under any degree of collusion (F/F_c). Additionally, neither statistic yields statistically significant negative values (i.e., less than -1.65 at the 0.05 significance level), except in cases where $F_c = 1$, $\phi \leq 0.2$, and a random coefficient is included in the consumer utility, under which T_{markup}^{RV} rejects collusion.

We extend the Differentiation IVs in (A4) and (A5) by incorporating *third-order* polynomials.⁴⁵ Specifically, the instruments $\mathbf{z}_{jt,diff}^{comp}$, $\mathbf{z}_{jt,diff}^{coll}$, $\mathbf{z}_{jt,diff}^{own}$, and $\mathbf{z}_{jt,diff}^{other}$ now include the term $\sum_k (x_{kt} - x_{jt})^3$, where the indexing for the summands is consistent with the firm index used to construct each instrument. Unlike the *Local* and *Quadratic* IVs, the third-order polynomial can take both positive and negative values. A negative value suggests that other products owned by the firm (or by colluding firms) are relatively *inferior* to product j , assuming the coefficient for attribute x in the indirect utility function is positive. Table D8 reports the two statistics computed using these extended instruments and shows that the testing power of T_{markup}^{RV} improves. Under full collusion ($F_c = 1$ and $\phi = 1$), T_{markup}^{RV} now produces a statistically significant positive value, rejecting price competition in favor of collusion. T_{markup}^{RV} also generates statistically significant negative values when most firms in the market collude ($F/F_c \in 6, 3$) but the degree of profit internalization is low. Despite these changes, however, T_{IV}^{RV} – whose testing power remains relatively unaffected – continues to outperform T_{markup}^{RV} in detecting collusive behavior across various collusive scenarios.

⁴⁵We also incorporate $\sum_k x_{kt}^3$ into the instruments defined in (7) and (16). The results (available from the authors upon request) remain both quantitatively and qualitatively consistent with the findings presented in the main text.

Finally, we evaluate the performance of the two test statistics using Differentiation IVs in the presence of endogeneity for product attribute x_{jt} , as outlined in Section 5.1. The results presented in Table D9 (Differentiation IVs up to second-order polynomials) and Table D10 (Differentiation IVs up to third-order polynomials) align with those presented earlier. First, the inclusion of a random coefficient enhances the testing power of T_{IV}^{RV} across all Monte Carlo configurations considered. Second, although adding third-order polynomials improves its testing power, T_{markup}^{RV} continues to underperform T_{IV}^{RV} in detecting collusion, except in the case of $F_c = 1$.

In summary, the Monte Carlo study in this appendix suggests that the testing power of T_{IV}^{RV} in detecting collusion remains relatively strong compared to the power of T_{markup}^{RV} across different functional forms for the instruments. Specifically, T_{IV}^{RV} tends to produce more statistically significant positive values across the various Monte Carlo configurations that represent stronger collusive behavior among firms (i.e., higher ϕ and F/F_c except when $F_c = 1$).

C Appendix for empirical applications

In this section, we describe data used in Section 8 in detail and present some descriptive statistics of South Korean automobile and instant noodles industry.

C.1 Car data

Data description

Our raw data contain year/province-level information (12 years from 2012 to 2023 and 17 provinces) on the total number of new registrations, sales revenue, and various attributes – nameplate, model year, engine displacement, fuel type, fuel efficiency, size, and other miscellaneous physical characteristics that vary across trim levels or consumer-selected options – of passenger vehicles produced by 13 brands under nine parent companies. These brands, including six domestic ones, accounted for approximately 93% of all private passenger cars registered in South Korea during the sample period. We define a market as a unique combination of year and province and a product as a unique combination of nameplate and fuel type; for example, Toyota Camry Hybrid. Our sample includes 540 nameplates and five fuel types – gasoline, diesel, LPG, hybrid, and electric – resulting in 776 unique products.

Product attributes used in the empirical application include fuel economy (km per 1,000 Won), acceleration (horsepower/curb weight), and size (width \times length \times height). For each product sold in a given market, we calculate fuel economy by dividing fuel efficiency (measured in kilometers per liter or kilometers per kilowatt-hour (kWh) for electric vehicles (EVs)) by the per-liter fuel price (in 1,000 Won) of the corresponding fuel type (gasoline, diesel, or LPG) in that market, obtained from the Oil Price Information Network (Opinet).⁴⁶ We assume that hybrid electric vehicles (HEVs) are fueled by gasoline, as diesel HEVs are rare.⁴⁷ Additionally, we collect yearly per-kWh EV charging prices at quick-charging stations from the Ministry of Environment in South Korea to calculate fuel economy for electric vehicles.⁴⁸

We aggregate sales figures and attribute values at the market/product level, using the number of units registered in the market as weights. Consequently, the same product may exhibit different fuel economy, acceleration, and size across markets. Additionally, we calculate a product’s price in a market by dividing total sales revenue by the number of registrations in that market. This measure differs from the list price or the Manufacturer’s Suggested Retail Price (MSRP) commonly used in the literature. Specifically, sales in our data reflect acquisition prices, which account for manufacturer promotions at the point of sale as well as the costs of miscellaneous vehicle options

⁴⁶Opinet’s web address is <https://www.opinet.co.kr/user/main/mainView.do>.

⁴⁷Unlike HEVs, plug-in hybrid electric vehicles (PHEVs) can be recharged from an external power source (e.g., an EV charging station). Given that PHEVs have a very low market share (0.24%) in our sample, we exclude them from our empirical application to simplify fuel economy calculations for EVs. Similarly, we also exclude fuel-cell electric vehicles (FCEVs), which are powered by hydrogen and have a market share of approximately 0.24% as well.

⁴⁸Prices since 2016 are available at <https://me.go.kr/home/web/board/read.do?boardMasterId=1&boardId=1539980&menuId=10525>. We assume that prices prior to 2016 are identical to those in 2016.

selected by consumers. As a result, our price measure more accurately reflects market conditions than the MSRP. Our final data consist of 41,716 product-market-level observations in total.

Descriptive statistics

Table D11 presents the market share and average product price for each of the 13 brands and nine parent companies. Hyundai and Kia are the two leading brands in our sample, each accounting for approximately 32.5% of new passenger car sales during this period. Genesis, initially part of Hyundai, was established as an independent luxury brand in late 2015, specializing in high-end vehicles. The average price of a Genesis vehicle is 62 million Won, more than twice the average price of Hyundai (30 million Won) and Kia (28 million Won). Collectively, these three brands, owned by the Hyundai Motor Group, hold a 68.4% market share, solidifying the group’s position as the dominant market leader.

There are seven foreign brands in the sample, four of which are German, accounting for the majority (82.5%) of foreign brand sales. While Mercedes-Benz vehicles are the most expensive, with an average price of 74 million Won, Volkswagen offers the cheapest models, with an average price of 37.5 million Won. Additionally, there are two Japanese brands, Toyota and Lexus, both owned by the Toyota Group. Lexus targets the luxury segment alongside Mercedes-Benz, BMW, and Audi, whereas Toyota focuses on affordable vehicles. Tesla, the only U.S. brand in our sample, produces only EVs.

Figure D4 illustrates the yearly changes in market share composition at the brand level (upper panel) and parent company level (bottom panel). The market shares of German brands gradually increased in the early 2010s following the implementation of the Free Trade Agreement between South Korea and the European Union in 2011, which progressively reduced import tariffs on German automobiles. However, this upward trend slowed in the mid-2010s, particularly after the Dieselgate scandal in 2015. As shown in the right panel, the market share of Hyundai Motor Group declined gradually in the early 2010s but rebounded after Dieselgate and the launch of its luxury brand, Genesis. Meanwhile, the market shares of the other three domestic companies steadily contracted over the same period.

C.2 Instant noodles data

Data description

The data, acquired from NielsenIQ, contain monthly sales volume and prices for instant noodle products offered by four major firms – Nongshim, Ottogi, Samyang, and Paldo – across six regions of South Korea from January 2010 to December 2019. Each region comprises multiple adjacent provinces, with one exception: the capital, Seoul, which constitutes a region consisting of a single province. The remaining five regions geographically partition South Korea into (i) North, (ii) Mid-East, (iii) Mid-West, (iv) South-East, and (v) South-West, as illustrated in Figure D5.⁴⁹ Instant

⁴⁹The data cover 16 out of 17 provinces of South Korea; Jeju, an island located in the south, is not included.

noodles are available in two package types: (i) pouch and (ii) cup. For example, Nongshim’s Shin Ramyun, the most popular brand in South Korea, is available in both package types. We define a product as a unique combination of brand and package type, and a market as a unique combination of region and year-month pair.

We analyze the 70 best-selling products, which account for approximately 90% of total sales in the instant noodle market during the sample period. Cold noodle products are excluded from the analysis because their demand exhibits seasonality distinct from that of typical instant noodle products, which are consumed warm or hot. We obtain attributes of the 70 products from two databases: (i) the Ministry of Food and Drug Safety and (ii) FatSecret.⁵⁰ Specifically, we collect data on serving size, calorie content, and the quantities of key nutrients such as sugar, fat, protein, and sodium for each product. In our main analysis, nutrient quantities are divided by serving size. Our final dataset consists of a total of 44,670 product/market-level observations.

Descriptive statistics

Table D12 presents the number of products, market shares, and average product prices by firm (top panel), package type (middle panel), and soup type (bottom panel). Of the 70 products, 29 are owned by Nongshim, which holds a market share of nearly two-thirds (63.6%) during the sample period. The other three firms are distant followers, with market shares of 19% for Ottogi, 12.3% for Samyang, and 5.1% for Paldo. Additionally, Ottogi’s products are the cheapest (710 KRW), while Paldo’s are the most expensive (943 KRW), on average.

There are 44 products packaged in pouches and 26 in cups in our sample. The pouch type is the more common choice among consumers, accounting for 71.2% of total sales volume. On average, cup-type noodles are approximately 25% (or 200 KRW in absolute terms) more expensive than pouch-type noodles. Red-colored soup is the most popular type of instant noodle in South Korea, with 47 products in the sample categorized under this type. These products make up around four-fifths of the total sales volume, followed by souplless noodles (14%) and noodles served with white-colored soup (6%). On average, products in the latter two categories are approximately 200 KRW more expensive than those with red-colored soup.

Figure D6 depicts the trend in market share composition by firm (top panel), package type (middle panel), and soup type (bottom panel) during the sample period. Nongshim, while maintaining its position as the dominant firm, gradually lost sales throughout the 2010s, mostly to Ottogi, whose market share steadily expanded during the decade. Over time, cup-type noodles gained popularity, increasing their combined market share by eight percentage points during the sample period. While red-colored soup has consistently been the most preferred choice among consumers, white-colored soup experienced a temporary surge in popularity in the early 2010s. Additionally, sales of souplless products have increased by approximately 50 percent since the mid-2010s.

⁵⁰We primarily gather data from the Ministry of Food and Drug Safety database: <https://various.foodsafetykorea.go.kr/nutrient/>. When attributes are unavailable for some products in this database, we utilize FatSecret as an alternative: <https://platform.fatsecret.com/platform-api>. We also verify that the attributes of each product available on both websites are consistent with each other.

D Additional tables and figures

Table D1: Own-firm vs other-firm instruments without a random coefficient: full results

F	Median Fstat			Median $ \alpha - \hat{\alpha} $			Median RMSE ($\hat{\alpha}$)			s_o
	own	other	both	own	other	both	own	other	both	
1	29.00			0.09			0.16			0.72
2	18.71	1.26	10.23	0.11	0.46	0.11	0.20	0.84	0.19	0.68
3	13.46	1.06	7.43	0.13	0.54	0.13	0.24	0.95	0.24	0.67
4	10.38	0.94	5.82	0.14	0.61	0.14	0.28	1.02	0.26	0.66
5	8.59	0.90	4.82	0.17	0.60	0.17	0.30	1.06	0.30	0.66
6	7.13	0.81	4.10	0.17	0.64	0.17	0.33	1.13	0.31	0.65
7	6.07	0.74	3.57	0.21	0.66	0.20	0.37	1.12	0.36	0.65
8	5.37	0.75	3.11	0.20	0.66	0.20	0.38	1.15	0.36	0.65
9	4.42	0.70	2.71	0.24	0.66	0.23	0.43	1.17	0.40	0.65
10	4.30	0.76	2.62	0.24	0.67	0.25	0.44	1.15	0.40	0.65
11	3.70	0.72	2.40	0.27	0.68	0.25	0.47	1.13	0.43	0.65
12	3.31	0.70	2.17	0.27	0.65	0.28	0.50	1.16	0.45	0.65
13	2.97	0.68	2.02	0.30	0.70	0.30	0.53	1.21	0.48	0.65
14	2.99	0.65	2.00	0.29	0.68	0.29	0.52	1.21	0.48	0.65
15	2.78	0.73	1.88	0.33	0.64	0.32	0.55	1.14	0.51	0.64
16	2.43	0.68	1.74	0.33	0.64	0.30	0.57	1.13	0.50	0.64
17	2.26	0.66	1.57	0.33	0.68	0.32	0.60	1.18	0.52	0.64
18	1.82	0.66	1.41	0.35	0.69	0.37	0.67	1.18	0.57	0.64
19	1.92	0.68	1.49	0.39	0.67	0.38	0.68	1.17	0.60	0.64
20	1.85	0.73	1.42	0.39	0.69	0.37	0.67	1.18	0.58	0.64
21	1.86	0.65	1.44	0.39	0.67	0.37	0.68	1.20	0.58	0.64
22	1.78	0.72	1.41	0.36	0.67	0.35	0.69	1.14	0.58	0.64
23	1.75	0.70	1.38	0.42	0.66	0.39	0.71	1.15	0.60	0.64
24	1.64	0.71	1.37	0.40	0.63	0.42	0.76	1.17	0.62	0.64
25	1.69	0.69	1.36	0.41	0.67	0.39	0.73	1.20	0.62	0.64
26	1.60	0.69	1.30	0.44	0.64	0.42	0.76	1.21	0.64	0.64
27	1.47	0.68	1.21	0.42	0.65	0.42	0.76	1.15	0.63	0.64
28	1.45	0.64	1.20	0.46	0.69	0.41	0.79	1.19	0.64	0.64
29	1.28	0.76	1.17	0.47	0.66	0.43	0.84	1.12	0.64	0.64
30	1.25	0.67	1.13	0.51	0.64	0.45	0.86	1.13	0.69	0.64
31	1.20	0.67	1.09	0.49	0.66	0.43	0.86	1.18	0.68	0.64
32	1.02	0.70	1.01	0.52	0.69	0.46	0.95	1.18	0.70	0.64
33	0.92	0.67	0.98	0.56	0.66	0.49	0.98	1.15	0.73	0.64
34	0.82	0.64	0.91	0.58	0.69	0.50	1.08	1.14	0.78	0.64
35	0.76	0.65	0.90	0.62	0.67	0.50	1.12	1.16	0.78	0.64
36		0.66			0.66			1.14		0.64

Notes: The table reports the median values of the absolute error and RMSE of the estimated price coefficient, as well as the median F-statistics, when \mathbf{z}^{own} , \mathbf{z}^{other} , and $(\mathbf{z}^{own}, \mathbf{z}^{other})$ are used as instruments individually. A random coefficient is excluded from the indirect utility function (14) in the DGP. The median outside option share across 500 simulated datasets for each Monte Carlo configuration is denoted by s_o .

Table D2: Own-firm vs other-firm instruments with a random coefficient: full results

F	Median Fstat				Median $ \alpha - \hat{\alpha} $				Median RMSE($\hat{\alpha}$)				Median $ \sigma_x - \hat{\sigma}_x $				Median RMSE($\hat{\sigma}_x$)				s_0	
	own	other	both		own	other	both		own	other	both		own	other	both		own	other	both			
1	34.68				0.53	0.64	0.06		0.88	1.06	0.11		1.34	1.23	0.12		3.72	2.54	0.23		0.74	
2	32.85	23.75	29.00		0.49	0.64	0.06		0.86	1.06	0.11		1.16	1.23	0.12		2.14	2.54	0.23		0.64	
3	29.68	15.00	22.36		0.44	0.80	0.07		0.79	1.47	0.13		1.37	1.04	0.13		1.85	1.95	0.23		0.61	
4	25.10	9.88	17.75		0.55	0.92	0.08		0.83	1.70	0.15		1.86	0.91	0.12		1.85	1.63	0.23		0.60	
5	21.15	6.65	14.15		0.49	0.89	0.10		0.82	1.83	0.17		1.70	0.68	0.13		1.76	1.34	0.23		0.60	
6	17.89	5.10	11.80		0.50	1.02	0.10		0.84	2.04	0.18		1.99	0.64	0.13		1.71	1.25	0.24		0.59	
7	15.30	4.00	9.72		0.47	1.05	0.11		0.81	2.11	0.20		1.76	0.55	0.13		1.69	1.16	0.23		0.59	
8	13.56	3.32	8.68		0.52	1.04	0.11		0.86	2.20	0.22		1.92	0.48	0.13		1.76	1.06	0.24		0.59	
9	11.34	2.62	7.18		0.54	1.08	0.12		0.91	2.24	0.24		2.31	0.45	0.12		1.85	0.97	0.24		0.59	
10	10.51	2.38	6.85		0.49	0.97	0.14		0.81	2.18	0.25		2.13	0.41	0.13		1.68	0.89	0.24		0.59	
11	9.23	2.18	5.96		0.53	1.01	0.15		0.90	2.22	0.27		2.28	0.36	0.13		1.79	0.88	0.24		0.59	
12	8.20	1.87	5.22		0.53	1.04	0.15		0.94	2.35	0.29		2.22	0.35	0.13		1.85	0.77	0.24		0.59	
13	7.39	1.77	4.89		0.57	1.04	0.16		0.98	2.22	0.31		2.58	0.34	0.13		1.85	0.74	0.25		0.59	
14	7.23	1.64	4.70		0.49	1.04	0.16		0.84	2.34	0.31		2.12	0.32	0.13		1.76	0.76	0.25		0.59	
15	6.68	1.57	4.28		0.51	1.02	0.18		0.87	2.26	0.33		2.12	0.30	0.13		1.72	0.67	0.25		0.59	
16	5.83	1.43	3.84		0.49	1.00	0.19		0.88	2.29	0.34		2.25	0.28	0.12		1.72	0.68	0.25		0.58	
17	5.01	1.33	3.48		0.50	1.00	0.20		0.96	2.45	0.38		2.44	0.27	0.12		1.85	0.65	0.25		0.58	
18	4.44	1.26	2.98		0.53	1.05	0.21		1.02	2.39	0.40		2.27	0.26	0.13		2.00	0.66	0.25		0.58	
19	4.23	1.23	3.00		0.51	1.07	0.25		0.99	2.12	0.43		2.51	0.26	0.13		1.89	0.66	0.25		0.58	
20	4.10	1.24	2.82		0.53	1.06	0.24		0.98	2.32	0.42		2.31	0.27	0.13		1.91	0.61	0.25		0.58	
21	4.04	1.20	2.81		0.52	1.03	0.25		0.97	2.35	0.43		2.50	0.24	0.13		1.86	0.63	0.25		0.58	
22	3.83	1.14	2.70		0.43	0.97	0.23		0.85	2.26	0.43		2.12	0.25	0.13		1.75	0.63	0.25		0.58	
23	3.81	1.22	2.56		0.49	0.94	0.25		0.89	2.18	0.43		2.33	0.24	0.13		1.87	0.62	0.25		0.58	
24	3.42	1.14	2.59		0.46	0.99	0.27		0.93	2.25	0.46		2.30	0.23	0.13		1.76	0.60	0.25		0.58	
25	3.36	1.14	2.49		0.49	0.97	0.27		0.95	2.06	0.48		2.47	0.24	0.14		1.78	0.57	0.25		0.58	
26	3.14	1.06	2.33		0.51	0.97	0.28		0.91	2.25	0.49		2.50	0.23	0.13		1.76	0.53	0.25		0.58	
27	2.92	1.06	2.17		0.43	0.97	0.29		0.86	2.23	0.50		2.15	0.22	0.13		1.65	0.55	0.26		0.58	
28	2.70	1.07	2.09		0.46	1.03	0.31		0.87	2.22	0.51		2.61	0.24	0.14		1.68	0.58	0.26		0.58	
29	2.39	1.08	1.86		0.47	1.02	0.30		0.99	2.10	0.54		2.48	0.23	0.14		1.76	0.57	0.26		0.58	
30	2.22	1.06	1.87		0.49	0.94	0.34		0.94	2.15	0.59		2.66	0.20	0.13		1.74	0.55	0.26		0.58	
31	2.08	1.01	1.67		0.50	1.07	0.37		0.95	2.41	0.60		2.52	0.21	0.13		1.74	0.56	0.26		0.58	
32	1.73	0.90	1.59		0.52	1.01	0.37		1.05	2.37	0.64		3.00	0.22	0.15		1.88	0.56	0.27		0.58	
33	1.38	0.97	1.38		0.54	0.99	0.43		1.14	2.25	0.70		3.00	0.22	0.13		1.88	0.54	0.27		0.58	
34	1.10	0.90	1.22		0.61	1.01	0.45		1.22	2.26	0.78		3.00	0.22	0.13		1.93	0.52	0.27		0.58	
35	0.90	0.93	1.13		0.70	1.04	0.55		1.37	2.31	0.84		3.00	0.23	0.14		2.00	0.53	0.28		0.58	
36		0.93				1.06				2.32				0.21				0.52				0.58

Notes: The table reports the median values of the absolute errors and RMSEs of the estimated utility parameters, $\hat{\alpha}$ and $\hat{\sigma}_x$, as well as the median F-statistics when \mathbf{z}^{own} , \mathbf{z}^{other} , and $(\mathbf{z}^{own}, \mathbf{z}^{other})$ are used as instruments individually. A random coefficient is included in the indirect utility function (14) in the DGP. The median outside option share across 500 simulated datasets for each Monte Carlo configuration is denoted by s_0 .

Table D3: (Exogenous) $F = 6$ and $T = 10$

ϕ	T_{IV}^{RV} (\mathbf{z}^{comp} vs \mathbf{z}^{coll})					T_{markup}^{RV} ($\phi = 0$ vs $\phi = 1$)				
	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$
<i>Panel A: without a random coefficient</i>										
0 (competition)	-0.250	-0.285	-0.219	-0.179	-0.076	-3.341	-1.990	-1.197	-0.731	-0.396
0.1	-0.207	-0.228	-0.128	-0.132	-0.039	-3.036	-1.819	-1.088	-0.663	-0.360
0.2	-0.156	-0.042	0.007	-0.042	-0.004	-2.507	-1.609	-0.965	-0.583	-0.327
0.3	-0.087	0.175	0.162	0.053	0.030	-2.015	-1.374	-0.837	-0.488	-0.282
0.4	0.011	0.428	0.373	0.204	0.089	-1.549	-1.131	-0.671	-0.366	-0.222
0.5	0.112	0.707	0.621	0.352	0.142	-1.030	-0.844	-0.514	-0.278	-0.197
0.6	0.220	0.968	0.873	0.530	0.199	-0.613	-0.589	-0.378	-0.187	-0.158
0.7	0.353	1.225	1.117	0.695	0.276	-0.226	-0.320	-0.228	-0.101	-0.120
0.8	0.463	1.478	1.361	0.858	0.346	0.135	-0.061	-0.094	-0.032	-0.073
0.9	0.595	1.731	1.572	1.032	0.400	0.512	0.214	0.060	0.043	-0.046
1	0.712	1.967	1.795	1.184	0.468	0.803	0.458	0.226	0.120	0.005
<i>Panel B: with a random coefficient</i>										
0 (competition)	-0.575	-0.643	-0.634	-0.548	-0.331	-2.833	-1.905	-1.414	-0.956	-0.411
0.1	-0.525	-0.552	-0.470	-0.443	-0.288	-2.685	-1.822	-1.306	-0.859	-0.356
0.2	-0.495	-0.236	-0.194	-0.278	-0.198	-2.528	-1.683	-1.177	-0.740	-0.295
0.3	-0.445	0.207	0.226	-0.033	-0.085	-2.338	-1.557	-1.028	-0.603	-0.221
0.4	-0.397	0.738	0.700	0.306	0.007	-2.197	-1.385	-0.833	-0.442	-0.147
0.5	-0.355	1.254	1.131	0.604	0.149	-1.990	-1.173	-0.618	-0.291	-0.093
0.6	-0.271	1.762	1.569	0.901	0.283	-1.702	-0.861	-0.374	-0.154	-0.039
0.7	-0.113	2.256	2.003	1.201	0.441	-1.390	-0.480	-0.127	0.011	0.017
0.8	0.136	2.702	2.412	1.495	0.575	-1.159	0.039	0.234	0.182	0.102
0.9	0.460	3.131	2.766	1.764	0.715	-0.906	0.533	0.530	0.339	0.176
1	0.830	3.535	3.129	2.037	0.839	0.084	0.970	0.763	0.459	0.240

Notes: The table reports the median values of the two test statistics, T_{IV}^{RV} and T_{markup}^{RV} , across 500 simulated datasets for each Monte Carlo configuration ($J = 36, F = 6, T = 10, \phi, F_c$). T_{markup}^{RV} is constructed under the two alternative firm conduct models: one with $\phi = 0$ (competition) and the other with $\phi = 1$ (full internalization under industry conduct consistent with the effective firm index). The top panel presents the results when a random coefficient is excluded from the indirect utility function (14) in the DGP, while the bottom panel presents the results when it is included.

Table D4: (Endogenous) $F = 6$ and $T = 10$

ρ	$T_{IV}^{RV} (\mathbf{z}^{comp} \text{ vs } \mathbf{z}^{coll})$					$T_{markup}^{RV} (\phi = 0 \text{ vs } \phi = 1)$				
	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$
<i>Panel A: without a random coefficient</i>										
Exogenous $\rho = 0$	0.824	2.088	1.925	1.299	0.534	0.990	0.682	0.395	0.343	0.235
Endogenous (-) $\rho = -1$	0.412	1.541	1.418	0.908	0.328	0.635	0.462	0.259	0.062	-0.068
$\rho = -5$	0.386	1.548	1.428	0.889	0.235	0.685	0.496	0.343	0.202	0.068
$\rho = -10$	0.410	1.558	1.430	0.905	0.248	0.752	0.530	0.357	0.214	0.133
Endogenous (+) $\rho = 1$	1.113	2.220	2.033	1.390	0.645	1.307	0.920	0.596	0.349	0.168
$\rho = 5$	1.250	2.410	2.173	1.427	0.662	1.497	0.956	0.686	0.420	0.192
$\rho = 10$	1.248	2.435	2.164	1.434	0.663	1.457	0.979	0.657	0.436	0.218
<i>Panel B: with a random coefficient</i>										
Exogenous $\rho = 0$	0.878	3.574	3.166	2.050	0.845	0.301	1.153	0.881	0.646	0.385
Endogenous (-) $\rho = -1$	0.874	3.666	3.268	2.157	0.877	0.938	1.192	0.928	0.600	0.251
$\rho = -5$	0.907	3.730	3.341	2.199	0.886	1.474	1.259	0.922	0.654	0.261
$\rho = -10$	0.895	3.732	3.353	2.213	0.885	1.514	1.179	0.949	0.683	0.329
Endogenous (+) $\rho = 1$	1.343	4.007	3.551	2.368	1.102	1.478	1.358	1.026	0.683	0.361
$\rho = 5$	1.455	4.136	3.694	2.445	1.143	2.015	1.454	1.145	0.734	0.364
$\rho = 10$	1.462	4.183	3.688	2.466	1.126	1.997	1.352	1.013	0.753	0.463

Notes: The table reports the median values of the two test statistics, T_{IV}^{RV} and T_{markup}^{RV} , across 500 simulated datasets for each Monte Carlo configuration ($J = 36, F = 6, T = 10, \rho, F_c$). The product attribute x_{jt} is treated as an endogenous variable. The direction and degree of endogeneity are parameterized by $\rho \in \{-10, -5, -1, 0, 1, 5, 10\}$, while the true profit internalization parameter ϕ is fixed at 1. T_{markup}^{RV} is constructed under the two alternative firm conduct models: one with $\phi = 0$ (competition) and the other with $\phi = 1$ (full profit internalization under industry conduct consistent with the effective firm index). The top panel presents the results when a random coefficient is excluded from the indirect utility function (14) in the DGP, while the bottom panel presents the results when it is included.

Table D5: Comparison of IV performance: \mathbf{z}_{diff}^{comp} vs \mathbf{z}_{diff}^{coll} ($F = 4$, $F_c = 3$, $T = 100$)
 Differentiation IVs up to the second order

ϕ	\mathbf{z}_{diff}^{comp}			\mathbf{z}_{diff}^{coll}			% of Fstat ₂ > Fstat ₁	Median of s_o
	$ \alpha - \hat{\alpha} $	$ \sigma_x - \hat{\sigma}_x $	Fstat ₁	$ \alpha - \hat{\alpha} $	$ \sigma_x - \hat{\sigma}_x $	Fstat ₂		
<i>Panel A: without a random coefficient</i>								
0 (competition)	0.473		1.182	0.554		0.885	0.428	0.659
0.1	0.475		1.188	0.473		1.247	0.532	0.660
0.2	0.479		1.173	0.385		1.739	0.638	0.662
0.3	0.475		1.173	0.306		2.500	0.776	0.663
0.4	0.481		1.160	0.257		3.487	0.886	0.664
0.5	0.476		1.156	0.218		4.686	0.954	0.665
0.6	0.478		1.153	0.193		6.085	0.976	0.666
0.7	0.478		1.154	0.172		7.680	0.990	0.667
0.8	0.480		1.149	0.155		9.455	0.994	0.668
0.9	0.482		1.151	0.141		11.474	0.996	0.669
1	0.476		1.140	0.130		13.558	0.998	0.669
<i>Panel B: with a random coefficient</i>								
0 (competition)	0.853	0.276	2.725	0.962	0.300	2.275	0.432	0.604
0.1	0.850	0.277	2.662	0.804	0.298	3.408	0.626	0.606
0.2	0.850	0.277	2.684	0.541	0.257	5.089	0.810	0.607
0.3	0.856	0.273	2.734	0.361	0.207	7.146	0.952	0.609
0.4	0.861	0.271	2.720	0.283	0.178	9.975	0.992	0.610
0.5	0.862	0.283	2.666	0.223	0.167	13.329	1.000	0.612
0.6	0.868	0.289	2.690	0.190	0.162	17.157	1.000	0.613
0.7	0.893	0.284	2.656	0.162	0.157	21.654	1.000	0.615
0.8	0.905	0.279	2.597	0.142	0.153	26.572	1.000	0.616
0.9	0.888	0.281	2.597	0.127	0.149	32.005	1.000	0.618
1	0.867	0.278	2.625	0.115	0.147	37.722	1.000	0.619

Notes: The table compares the median absolute errors of the estimated price and nonlinear coefficients, as well as the median F-statistics, across 500 simulated datasets for each Monte Carlo configuration ($J = 36$, $F = 4$, $T = 100$, ϕ , $F_c = 3$), obtained using \mathbf{z}_{diff}^{comp} and \mathbf{z}_{diff}^{coll} as instruments individually. *Local* and *Quadratic* IVs are used as delineated in Appendix B. The top panel presents the results when a random coefficient is excluded from the indirect utility function (14) in the DGP, while the bottom panel presents the results when it is included.

Table D6: Comparison of IV performance: \mathbf{z}_{diff}^{comp} vs \mathbf{z}_{diff}^{coll} ($F = 4$, $F_c = 3$, $T = 100$)
Differentiation IVs up to the third order

ϕ	\mathbf{z}_{diff}^{comp}			\mathbf{z}_{diff}^{coll}			% of Fstat ₂ > Fstat ₁	Median of s_o
	$ \alpha - \hat{\alpha} $	$ \sigma_x - \hat{\sigma}_x $	Fstat ₁	$ \alpha - \hat{\alpha} $	$ \sigma_x - \hat{\sigma}_x $	Fstat ₂		
<i>Panel A: without a random coefficient</i>								
0 (competition)	0.212		3.917	0.334		1.642	0.086	0.659
0.1	0.212		3.903	0.288		1.942	0.140	0.660
0.2	0.215		3.915	0.261		2.367	0.212	0.662
0.3	0.214		3.926	0.241		2.995	0.336	0.663
0.4	0.214		3.911	0.215		3.804	0.492	0.664
0.5	0.215		3.921	0.194		4.727	0.636	0.665
0.6	0.217		3.901	0.173		5.814	0.764	0.666
0.7	0.216		3.868	0.157		7.100	0.862	0.667
0.8	0.216		3.846	0.143		8.327	0.930	0.668
0.9	0.216		3.811	0.131		9.848	0.974	0.669
1	0.216		3.834	0.122		11.416	0.988	0.669
<i>Panel B: with a random coefficient</i>								
0 (competition)	0.182	0.122	8.253	0.416	0.178	3.098	0.048	0.604
0.1	0.185	0.121	8.194	0.419	0.207	4.029	0.128	0.606
0.2	0.186	0.121	8.182	0.421	0.238	5.258	0.248	0.607
0.3	0.187	0.119	8.054	0.427	0.273	6.737	0.410	0.609
0.4	0.188	0.118	7.962	0.425	0.312	8.628	0.588	0.610
0.5	0.190	0.117	7.860	0.425	0.358	10.937	0.750	0.612
0.6	0.192	0.118	7.853	0.429	0.392	13.325	0.880	0.613
0.7	0.195	0.118	7.724	0.426	0.427	16.042	0.934	0.615
0.8	0.197	0.118	7.712	0.428	0.464	19.056	0.972	0.616
0.9	0.199	0.120	7.638	0.434	0.505	22.103	0.984	0.618
1	0.200	0.118	7.526	0.428	0.538	25.354	0.996	0.619

Notes: The table compares the median absolute errors of the estimated price and nonlinear coefficients, as well as the median F-statistics, across 500 simulated datasets for each Monte Carlo configuration ($J = 36$, $F = 4$, $T = 100$, ϕ , $F_c = 3$), obtained using \mathbf{z}_{diff}^{comp} and \mathbf{z}_{diff}^{coll} as instruments individually. *Local*, *Quadratic*, and *third-order* IVs are used as delineated in Appendix B. The top panel presents the results when a random coefficient is excluded from the indirect utility function (14) in the DGP, while the bottom panel presents the results when it is included.

Table D7: (Exogenous) $F = 6$ and $T = 100$ (Differentiation IVs up to the second order)

ϕ	$T_{IV}^{RV} (\mathbf{z}_{diff}^{comp} \text{ vs } \mathbf{z}_{diff}^{coll})$					$T_{markup}^{RV} (\phi = 0 \text{ vs } \phi = 1)$				
	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$
<i>Panel A: without a random coefficient</i>										
0 (competition)	-0.166	-0.150	-0.168	-0.205	-0.167	-1.166	-0.743	-0.525	-0.291	-0.190
0.1	-0.150	0.187	0.159	0.038	-0.061	-1.075	-0.650	-0.450	-0.239	-0.163
0.2	-0.113	0.801	0.767	0.460	0.086	-0.958	-0.578	-0.374	-0.219	-0.148
0.3	-0.087	1.404	1.410	0.879	0.281	-0.814	-0.468	-0.294	-0.153	-0.124
0.4	-0.041	1.974	1.985	1.325	0.482	-0.689	-0.360	-0.224	-0.119	-0.103
0.5	0.001	2.528	2.534	1.742	0.686	-0.556	-0.274	-0.152	-0.086	-0.079
0.6	0.030	3.068	3.065	2.137	0.883	-0.406	-0.172	-0.086	-0.046	-0.061
0.7	0.054	3.574	3.603	2.517	1.066	-0.246	-0.063	-0.010	-0.025	-0.042
0.8	0.084	4.071	4.120	2.897	1.236	-0.073	0.031	0.061	0.020	-0.010
0.9	0.135	4.554	4.635	3.279	1.411	0.058	0.155	0.152	0.076	0.013
1	0.154	5.030	5.153	3.646	1.573	0.224	0.259	0.215	0.105	0.024
<i>Panel B: with a random coefficient</i>										
0 (competition)	0.699	0.283	-0.013	-0.162	-0.270	-1.759	-1.130	-0.949	-0.633	-0.335
0.1	0.719	0.991	0.827	0.421	0.023	-1.786	-1.083	-0.896	-0.568	-0.279
0.2	0.720	1.901	1.762	1.141	0.400	-1.725	-1.026	-0.816	-0.499	-0.230
0.3	0.716	2.862	2.710	1.829	0.791	-1.647	-0.941	-0.716	-0.412	-0.184
0.4	0.711	3.798	3.627	2.505	1.115	-1.498	-0.835	-0.612	-0.333	-0.122
0.5	0.682	4.750	4.530	3.113	1.435	-1.330	-0.704	-0.503	-0.259	-0.073
0.6	0.654	5.698	5.433	3.727	1.726	-1.090	-0.547	-0.398	-0.136	-0.017
0.7	0.613	6.638	6.336	4.315	1.987	-0.717	-0.325	-0.234	-0.031	0.051
0.8	0.562	7.515	7.204	4.919	2.261	-0.245	-0.120	-0.020	0.076	0.108
0.9	0.508	8.322	8.052	5.502	2.535	0.281	0.166	0.181	0.169	0.181
1	0.470	9.071	8.878	6.076	2.773	0.842	0.425	0.336	0.241	0.238

Notes: The table reports the median values of the two test statistics, T_{IV}^{RV} and T_{markup}^{RV} , across 500 simulated datasets for each Monte Carlo configuration ($J = 36, F = 6, T = 100, \phi, F_c$). These two statistics are computed using *Local* and *Quadratic* IVs as delineated in Appendix B. T_{markup}^{RV} is constructed under the two alternative firm conduct models: one with $\phi = 0$ (competition) and the other with $\phi = 1$ (full profit internalization under industry conduct consistent with the effective firm index). The top panel presents the results when a random coefficient is excluded from the indirect utility function (14) in the DGP, while the bottom panel presents the results when it is included.

Table D8: (Exogenous) $F = 6$ and $T = 100$ (Differentiation IVs up to the third order)

ϕ	$T_{IV}^{RV} (\mathbf{z}_{diff}^{comp} \text{ vs } \mathbf{z}_{diff}^{coll})$					$T_{markup}^{RV} (\phi = 0 \text{ vs } \phi = 1)$				
	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$
<i>Panel A: without a random coefficient</i>										
0 (competition)	-1.007	-0.931	-0.959	-0.993	-0.847	-4.559	-2.466	-1.287	-0.568	-0.194
0.1	-0.959	-0.687	-0.682	-0.785	-0.743	-4.063	-2.193	-1.100	-0.456	-0.131
0.2	-0.810	-0.048	-0.073	-0.393	-0.586	-3.466	-1.898	-0.904	-0.361	-0.088
0.3	-0.606	0.678	0.613	0.069	-0.391	-2.792	-1.569	-0.694	-0.247	-0.055
0.4	-0.353	1.398	1.360	0.579	-0.174	-2.060	-1.162	-0.472	-0.143	-0.016
0.5	-0.047	2.076	2.022	1.091	0.050	-1.309	-0.744	-0.250	-0.015	0.038
0.6	0.301	2.705	2.651	1.542	0.300	-0.602	-0.341	-0.023	0.102	0.087
0.7	0.654	3.293	3.246	2.004	0.553	0.119	0.092	0.201	0.216	0.118
0.8	0.990	3.871	3.819	2.469	0.783	0.787	0.490	0.409	0.328	0.171
0.9	1.282	4.424	4.376	2.922	1.000	1.432	0.905	0.629	0.434	0.203
1	1.578	4.941	4.931	3.345	1.220	2.028	1.234	0.829	0.528	0.262
<i>Panel B: with a random coefficient</i>										
0 (competition)	-0.399	-0.821	-1.036	-1.209	-1.174	-5.427	-1.780	-1.150	-0.692	-0.345
0.1	-0.403	-0.133	-0.404	-0.776	-0.967	-5.135	-1.671	-1.027	-0.597	-0.287
0.2	-0.408	0.898	0.643	-0.050	-0.681	-4.783	-1.517	-0.901	-0.476	-0.230
0.3	-0.398	2.049	1.755	0.713	-0.368	-4.365	-1.339	-0.742	-0.361	-0.170
0.4	-0.385	3.163	2.855	1.494	-0.041	-3.835	-1.111	-0.531	-0.234	-0.100
0.5	-0.322	4.281	3.896	2.257	0.313	-3.196	-0.798	-0.312	-0.092	-0.037
0.6	-0.211	5.326	4.886	2.976	0.657	-2.305	-0.378	-0.070	0.057	0.027
0.7	-0.006	6.335	5.855	3.649	1.018	-1.217	0.092	0.204	0.197	0.107
0.8	0.470	7.271	6.796	4.304	1.345	0.099	0.487	0.438	0.356	0.179
0.9	1.063	8.120	7.721	4.947	1.655	1.526	0.854	0.707	0.530	0.235
1	1.707	8.883	8.589	5.558	1.974	2.949	1.199	0.877	0.650	0.292

Notes: The table reports the median values of the two test statistics, T_{IV}^{RV} and T_{markup}^{RV} , across 500 simulated datasets for each Monte Carlo configuration ($J = 36, F = 6, T = 100, \phi, F_c$). These two statistics are computed using *Local*, *Quadratic*, and *third-order* IVs as delineated in Appendix B. T_{markup}^{RV} is constructed under the two alternative firm conduct models: one with $\phi = 0$ (competition) and the other with $\phi = 1$ (full profit internalization under industry conduct consistent with the effective firm index). The top panel presents the results when a random coefficient is excluded from the indirect utility function (14) in the DGP, while the bottom panel presents the results when it is included.

Table D9: (Endogenous) $F = 6$ and $T = 100$ with Differentiation IVs up to the second order

ρ	$T_{IV}^{RV} (z_{diff}^{comp} \text{ vs } z_{diff}^{coll})$					$T_{markup}^{RV} (\phi = 0 \text{ vs } \phi = 1)$				
	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$
<i>Panel A: without a random coefficient</i>										
Exogenous $\rho = 0$	0.183	5.292	5.452	3.866	1.737	0.254	0.169	0.205	0.139	0.113
Endogenous (-) $\rho = -1$	0.019	4.362	4.325	3.038	1.238	0.213	0.226	0.117	0.055	0.006
$\rho = -5$	0.048	4.408	4.403	3.056	1.268	0.278	0.225	0.176	0.135	0.037
$\rho = -10$	0.074	4.424	4.384	3.080	1.302	0.219	0.208	0.161	0.126	0.056
Endogenous (+) $\rho = 1$	0.291	5.807	5.831	4.081	1.832	0.634	0.446	0.331	0.161	0.060
$\rho = 5$	0.370	6.146	6.131	4.331	1.967	0.720	0.494	0.325	0.219	0.130
$\rho = 10$	0.370	6.155	6.165	4.358	1.974	0.780	0.536	0.377	0.297	0.187
<i>Panel B: with a random coefficient</i>										
Exogenous $\rho = 0$	0.576	9.112	8.897	6.114	2.832	0.972	0.472	0.373	0.304	0.187
Endogenous (-) $\rho = -1$	0.668	9.599	9.317	6.307	2.860	0.341	0.552	0.456	0.365	0.207
$\rho = -5$	0.747	9.833	9.582	6.481	2.912	0.248	0.536	0.450	0.387	0.146
$\rho = -10$	0.757	9.851	9.591	6.491	2.938	0.310	0.522	0.427	0.364	0.129
Endogenous (+) $\rho = 1$	0.900	10.329	10.046	6.802	3.125	0.815	0.704	0.607	0.421	0.227
$\rho = 5$	0.950	10.706	10.469	6.999	3.203	0.798	0.620	0.484	0.453	0.284
$\rho = 10$	0.945	10.701	10.516	7.049	3.214	0.632	0.680	0.533	0.503	0.334

Notes: The table reports the median values of the two test statistics, T_{IV}^{RV} and T_{markup}^{RV} , across 500 simulated datasets for each Monte Carlo configuration ($J = 36, F = 6, T = 100, \rho, F_c$). These two statistics are computed using *Local* and *Quadratic* IVs as delineated in Appendix B. The product attribute x_{jt} is treated as an endogenous variable. The direction and degree of endogeneity are parameterized by $\rho \in \{-10, -5, -1, 0, 1, 5, 10\}$, while the true profit internalization parameter ϕ is fixed at 1. T_{markup}^{RV} is constructed under the two alternative firm conduct models: one with $\phi = 0$ (competition) and the other with $\phi = 1$ (full profit internalization under industry conduct consistent with the effective firm index). The top panel presents the results when a random coefficient is excluded from the indirect utility function (14) in the DGP, while the bottom panel presents the results when it is included.

Table D10: (Endogenous) $F = 6$ and $T = 100$ with Differentiation IVs up to the third order

ρ	$T_{IV}^{RV} (z_{diff}^{comp} \text{ vs } z_{diff}^{coll})$					$T_{markup}^{RV} (\phi = 0 \text{ vs } \phi = 1)$				
	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$	$F_c = 1$	$F_c = 2$	$F_c = 3$	$F_c = 4$	$F_c = 5$
<i>Panel A: without a random coefficient</i>										
Exogenous										
$\rho = 0$	1.785	5.143	5.167	3.514	1.281	2.212	1.434	0.944	0.515	0.307
Endogenous (-)										
$\rho = -1$	1.393	4.314	4.130	2.775	0.958	1.665	1.411	1.183	1.021	0.841
$\rho = -5$	1.137	4.371	4.225	2.859	0.991	1.611	1.259	0.918	0.766	0.401
$\rho = -10$	1.179	4.321	4.246	2.866	1.000	1.632	1.243	1.002	0.767	0.470
Endogenous (+)										
$\rho = 1$	1.112	5.746	5.690	3.920	1.621	1.890	0.874	0.311	-0.016	-0.114
$\rho = 5$	1.356	6.072	5.995	4.125	1.678	2.110	1.157	0.468	0.096	-0.073
$\rho = 10$	1.367	6.121	6.069	4.142	1.697	2.119	1.088	0.514	0.145	-0.055
<i>Panel B: with a random coefficient</i>										
Exogenous										
$\rho = 0$	1.860	8.886	8.633	5.587	1.973	3.105	1.283	0.860	0.575	0.187
Endogenous (-)										
$\rho = -1$	1.993	9.334	9.039	5.829	2.108	3.894	1.974	1.220	0.777	0.207
$\rho = -5$	1.836	9.592	9.287	6.032	2.226	3.503	1.692	0.962	0.600	0.146
$\rho = -10$	1.861	9.607	9.352	6.101	2.280	3.540	1.697	1.000	0.673	0.129
Endogenous (+)										
$\rho = 1$	1.425	10.169	9.899	6.560	2.699	2.568	1.216	0.898	0.617	0.227
$\rho = 5$	1.615	10.510	10.289	6.717	2.724	2.980	1.252	0.802	0.581	0.284
$\rho = 10$	1.685	10.560	10.336	6.758	2.702	2.930	1.255	0.874	0.636	0.334

Notes: The table reports the median values of the two test statistics, T_{IV}^{RV} and T_{markup}^{RV} , across 500 simulated datasets for each Monte Carlo configuration ($J = 36, F = 6, T = 100, \rho, F_c$). These two statistics are computed using *Local*, *Quadratic*, and *third-order* Differentiation IVs as delineated in Appendix B. The product attribute x_{jt} is treated as an endogenous variable. The direction and degree of endogeneity are parameterized by $\rho \in \{-10, -5, -1, 0, 1, 5, 10\}$, while the true profit internalization parameter ϕ is fixed at 1. T_{markup}^{RV} is constructed under the two alternative firm conduct models: one with $\phi = 0$ (competition) and the other with $\phi = 1$ (full profit internalization under industry conduct consistent with the effective firm index). The top panel presents the results when a random coefficient is excluded from the indirect utility function (14) in the DGP, while the bottom panel presents the results when it is included.

Table D11: Market shares and prices: the new passenger car market (2012–2023)

Parent Company	Brand	Market share		Average price (100 million Won)	
		by Brand	by Company	by Brand	by Company
<i>Domestic</i>					
Hyundai Motor Group	Hyundai	0.325	0.684	0.299	0.305
	Genesis	0.033		0.622	
	Kia	0.325		0.279	
KG Mobility	KG Mobility	0.069	0.069	0.280	0.280
GM Korea	GM Korea	0.088	0.088	0.208	0.208
Renault Korea	Renault Korea	0.062	0.062	0.265	0.265
<i>Foreign</i>					
Mercedes-Benz	Mercedes-Benz	0.028	0.028	0.740	0.740
BMW	BMW	0.029	0.029	0.632	0.632
Volkswagen Group	Volkswagen	0.013	0.023	0.375	0.470
	Audi	0.010		0.594	
Toyota Group	Toyota	0.007	0.013	0.387	0.486
	Lexus	0.006		0.606	
Tesla	Tesla	0.004	0.004	0.681	0.681

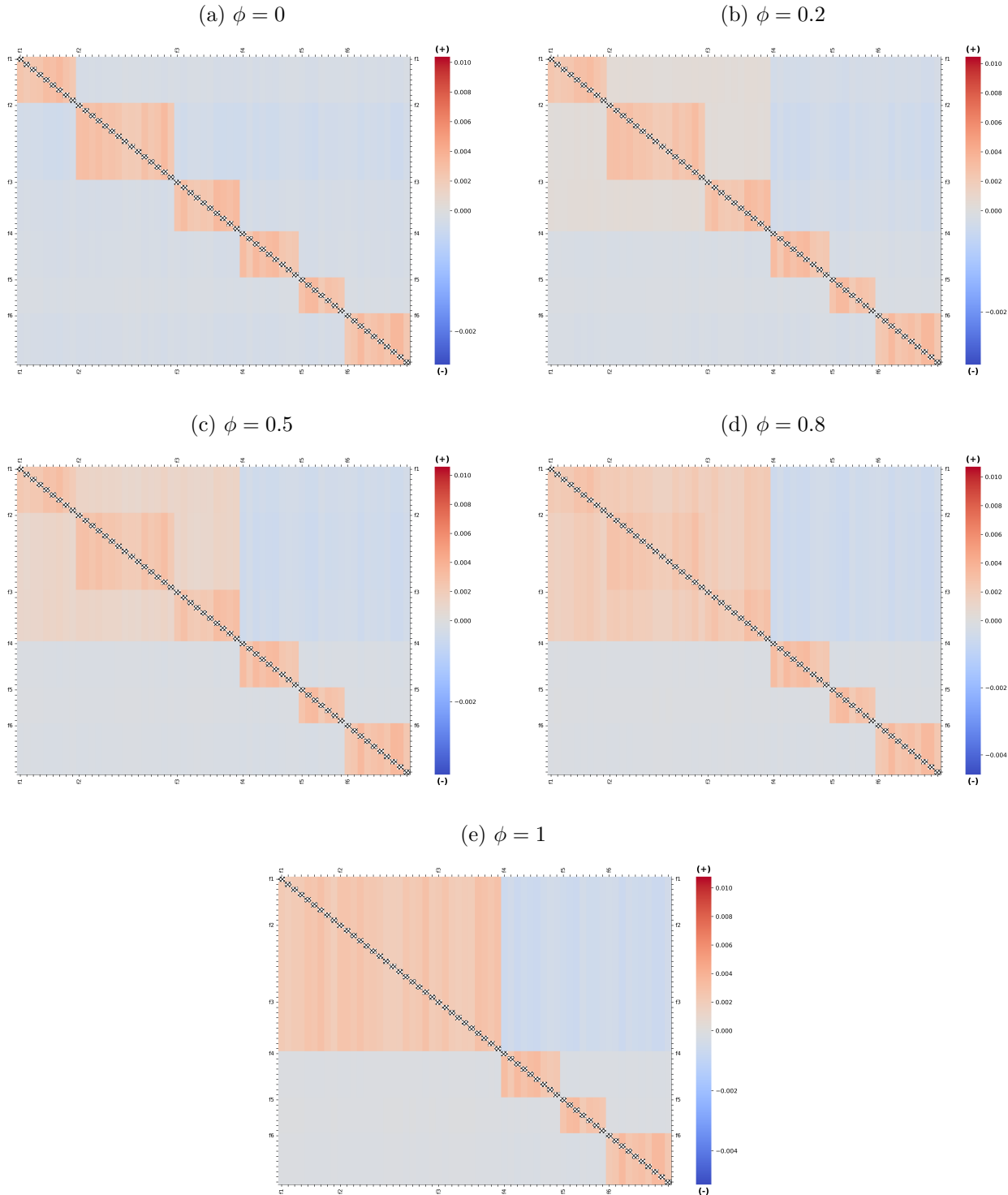
Notes: The table presents the market share and average product price for each of the 13 brands and nine parent companies. The average price per product is obtained by dividing the total sales revenue by the total number of registration by each firm and parent company during the sample period (2012-2023). Prices are adjusted for inflation using the 2020 Consumer Price Index (CPI) as the base year.

Table D12: Product counts, market shares, and prices: the instant noodle market (2010–2019)

	# products	Market share	Average price (1,000 KRW)
<i>Panel A: by firm</i>			
Nongshim	29	0.637	0.806
Ottogi	19	0.190	0.710
Paldo	10	0.051	0.943
Samyang	12	0.123	0.787
<i>Panel B: by package type</i>			
Pouch	44	0.712	0.736
Cup	26	0.288	0.930
<i>Panel C: by soup type</i>			
Red soup	47	0.799	0.754
White soup	11	0.062	0.954
Soupless	12	0.140	0.937

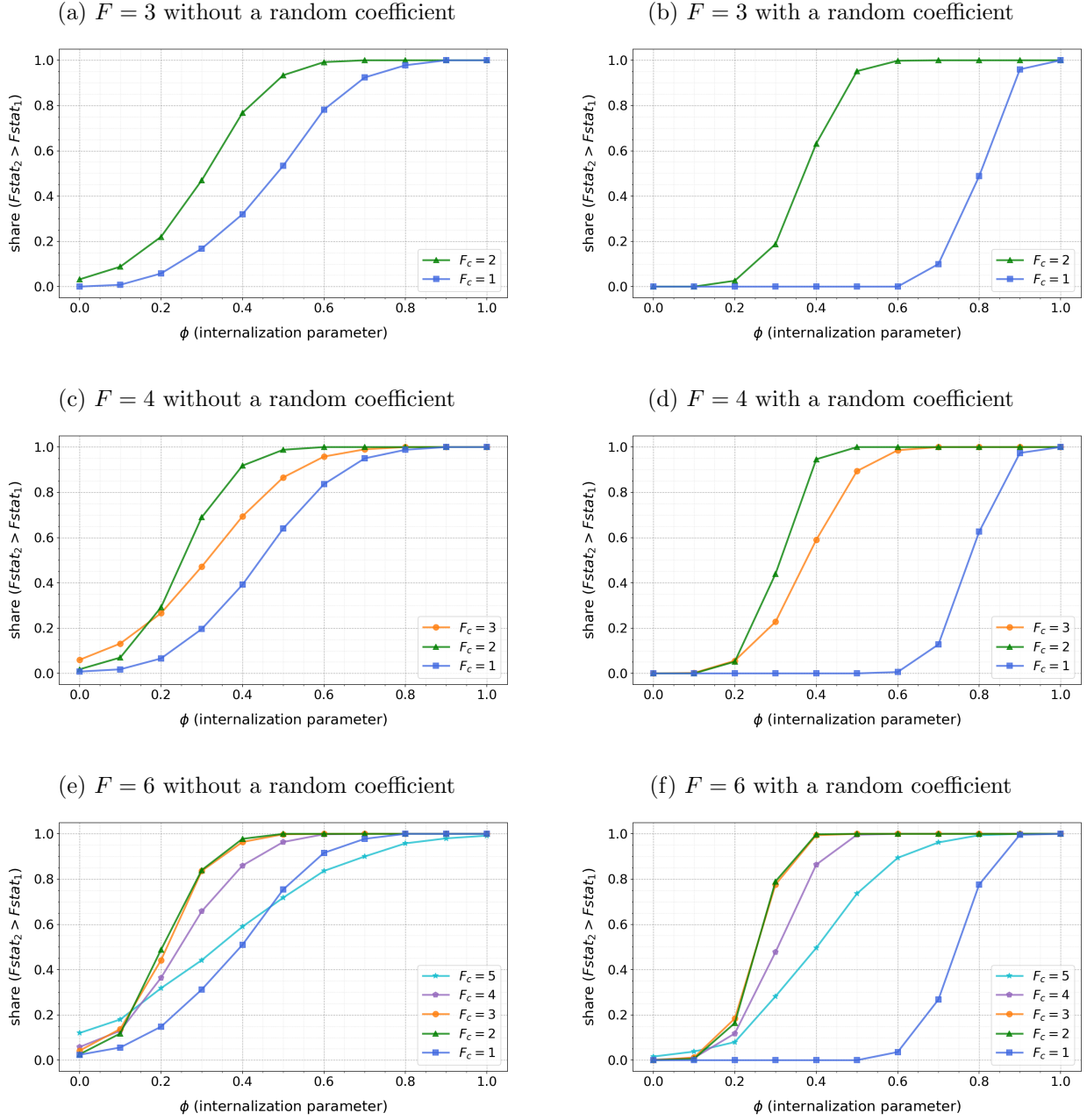
Notes: The table presents the number of products, market shares, and average product prices by firm (top panel), package type (middle panel), and soup type (bottom panel).

Figure D1: Effects of own-firm and other-firm attributes on equilibrium prices



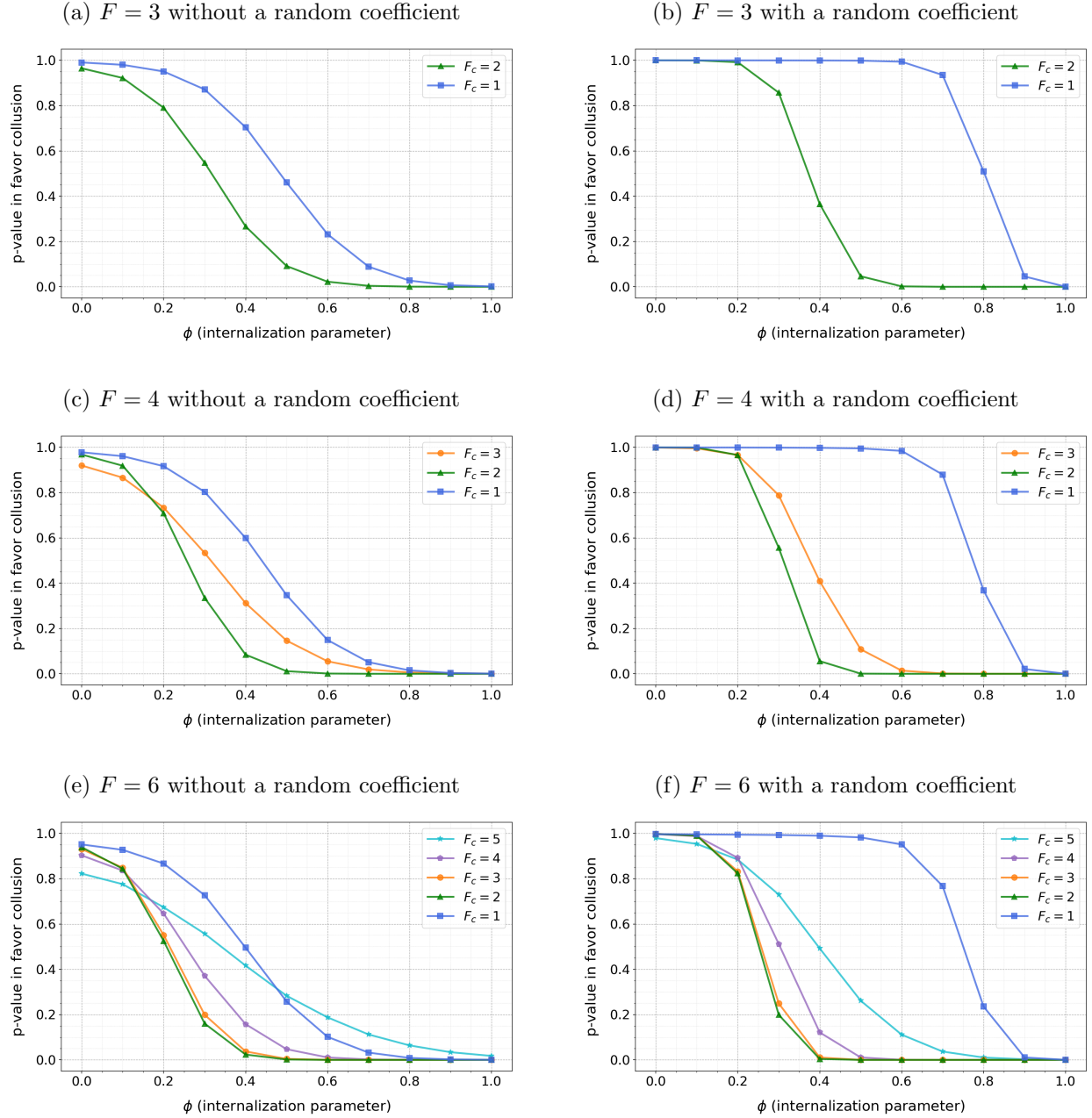
Notes: The figure presents heatmaps of the $J \times J$ Jacobian matrices, $\frac{\partial \mathbf{p}}{\partial \mathbf{x}}$, derived from the simple logit specification under various values of the profit internalization parameter, ϕ . The maximum and minimum ranges for the heatmaps are adjusted to match those corresponding to the subfigures in Figure 1. See the main text for interpretation and Appendix A.2 for details on the data-generating process used to compute equilibrium outcomes and Jacobian matrices.

Figure D2: F-stat evidence



Notes: The figure shows the share of cases where $Fstat_2 > Fstat_1$ in 500 simulation results for each Monte Carlo configuration ($J = 36, F, T = 100, \phi, F_c$). The left panel illustrates the results for the utility specification without a random coefficient, while the right panel presents the results with a random coefficient.

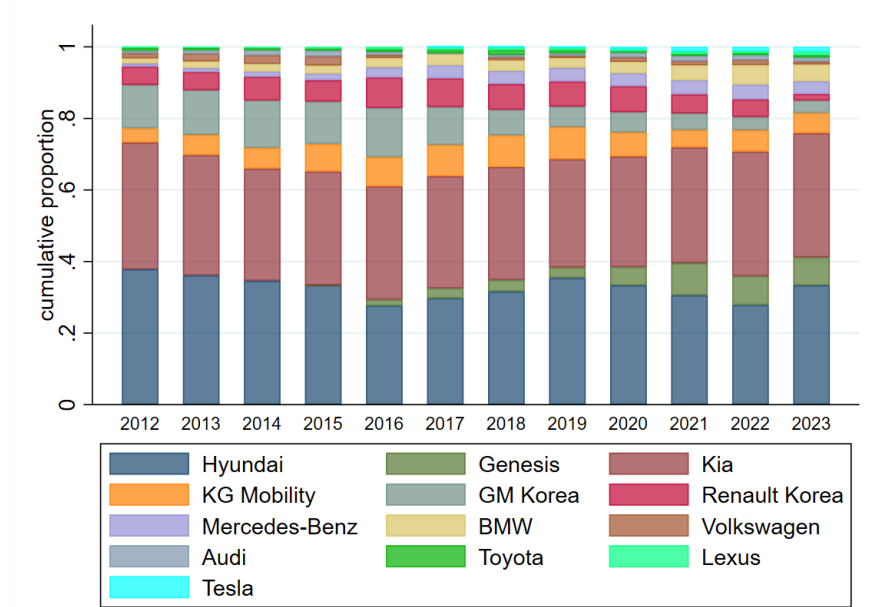
Figure D3: P-values of T_{IV}^{RV} under the alternative hypothesis $H_2 : Q_{comp} > Q_{coll}$



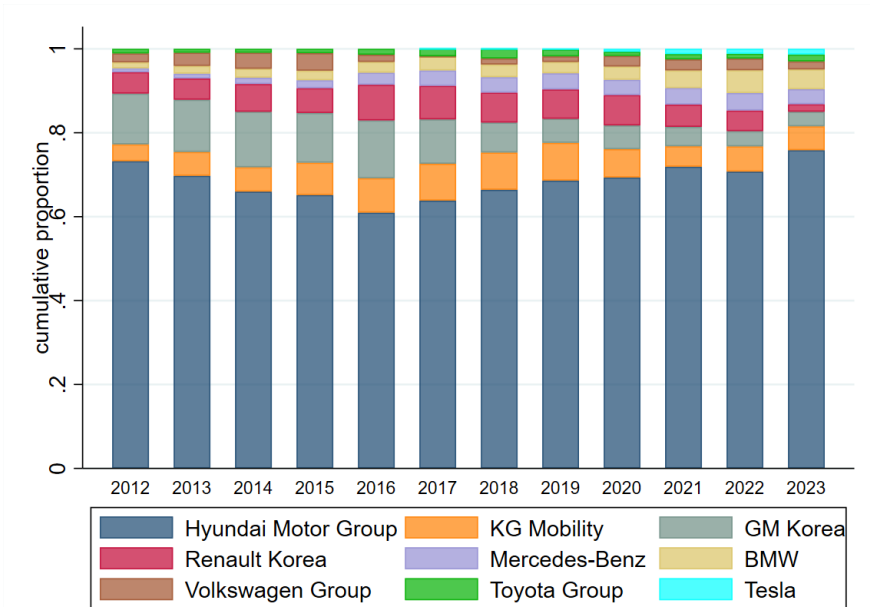
Notes: The figure shows the median p-value of T_{IV}^{RV} under the alternative hypothesis, $H_2 : Q_{comp} > Q_{coll}$, across 500 simulated datasets for each Monte Carlo configuration ($J = 36, F, T = 100, \phi, F_c$). The left panel illustrates the results for the utility specification without a random coefficient, while the right panel presents the results with a random coefficient.

Figure D4: Market share composition of new private passenger cars

(a) By brand

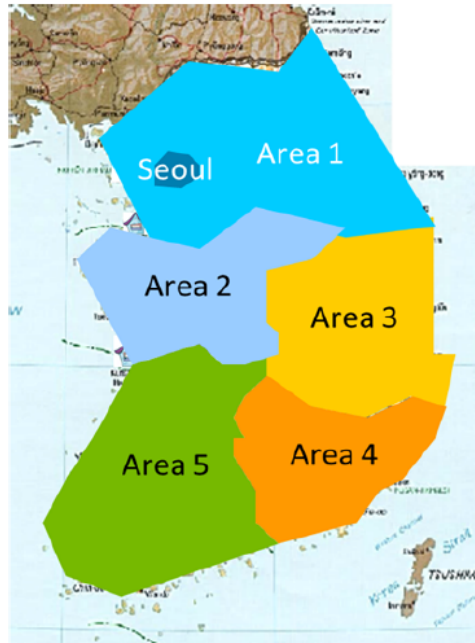


(b) By parent company



Notes: The upper and bottom panels of the figure depict the trend in market share composition by brand and parent company, respectively, from 2012 to 2023.

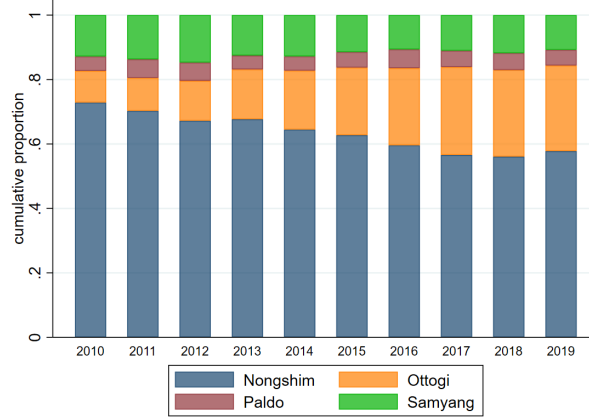
Figure D5: Six regions of South Korea



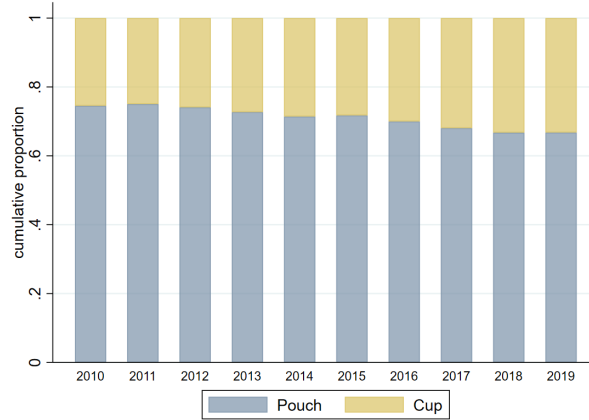
Notes: The figure shows the six geographical regions of South Korea classified by NielsenIQ.

Figure D6: Market share composition of instant noodles

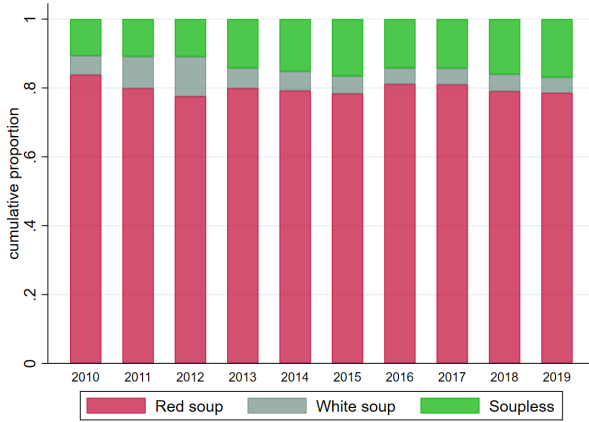
(a) By firm



(b) By package type



(c) By soup type



Notes: The figure depicts the trend in market share composition by firm (top panel), package type (middle panel), and soup type (bottom panel) from 2010 to 2019.