Due Diligence in Common Value Auctions*

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Job Market Paper

October 30, 2024

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Abstract

Multiple buyers compete to purchase an indivisible good in an informal common value auction. Buyers are symmetrically uninformed about the good's value but can privately learn it at a cost if the seller grants them access to confidential information. Should the seller grant such access? If so, when? We study the optimal timing of information acquisition that maximizes the seller's payoff guarantee across equilibria. Information acquisition before bidding is dominated by providing no information access. Instead, for high enough stakes, *due diligence* is optimal: The seller allows the auction winner to acquire information after bidding and possibly renege from the purchase thereafter.

*We are thankful to Dirk Bergemann, Tan Gan, Marina Halac, Ryota Iijima, Ravi Jagadeesan, Elliot Lipnowski, Song Ma, Doron Ravid, Larry Samuelson, Kelly Shue, Philipp Strack, Juuso Välimäki, and Kai Hao Yang for valuable comments and discussions.

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1 Introduction

Information acquisition is paramount in common value auctions: It can make the difference between submitting a winning or losing bid and between securing a great deal or overbidding. However, in many settings, buyers cannot access pivotal information about the seller's good without her approval. For example, in the context of mergers and acquisitions, the internal reports of a company for sale are not publicly available. Similarly, in the real estate market, potential buyers of a house cannot examine its structural integrity without the owner's permission. Should the seller grant access to her confidential information? If so, should she grant such access before or after buyers submit their bids?

In many high-stakes environments, like the sale of a company or a house, the seller grants access to information via *due diligence*, i.e., after a price has been agreed upon with the buyer. In turn, the buyer can use this information to decide whether to execute the transaction or renege from the purchase.[1] While allowing for due diligence is common business practice,[2] there is little theoretical foundation explaining the widespread adoption of this procedure. This paper fills some of this gap: We provide a revenue-based rationale for the use of due diligence in the canonical context of common value auctions.

In common value settings, buyers' information choices are strategic substitutes. The reason is that information benefits a buyer only if no other buyer has access to the same information. This creates significant strategic uncertainty if the seller allows buyers to conduct *research*, i.e., to acquire information before bidding. In particular, buyers find it optimal to remain uninformed if they believe that another buyer will conduct research, which may result in conservative bidding and a low expected revenue for the seller. As a result, if the seller deals conservatively with strategic uncertainty, we show that she prefers granting no access to information over granting such access via research.

Allowing only the auction winner to perform due diligence after the price is set avoids these issues, since buyers are symmetrically uninformed when bidding. However, two novel effects with opposing implications on the seller's revenue arise in this case. On the one hand, buyers may bid more aggressively, as the possibility of conducting due diligence carries option value. On the other hand, if the auction winner conducts due diligence, the seller's expected revenue is lower than the winning offer: The winner closes the deal only if the valuation exceeds his bid. Our main result shows that the size of the *stakes*, i.e., the size of the potential gains and losses from trade, determines which of these two effects dominates. In particular, granting access to information via due diligence maximizes the seller's revenue guarantee whenever the stakes are sufficiently high.

[1] In mergers and acquisitions, "due diligence" refers to the collection and processing of confidential information about the target company. Typically, this is done only after indicative bids have been submitted and at least a "Letter of Intent" outlining the main terms of the deal has been signed. While these terms could technically be renegotiated ex post, this is usually not done unless there is hard evidence about misrepresentations by the seller or objective changes in the market environment. This price rigidity can be justified by the softness of the information acquired by the buyer, which opens the door to renegotiations in bad faith. Our model captures this by assuming that due diligence can result only in a go or no-go decision, i.e., the price cannot be renegotiated.

[2] See Gole and Hilger (2009) for an extensive treatment of the role of due diligence in mergers and acquisitions.

Our model introduces the possibility of information acquisition into an informal common value auction. Multiple symmetric buyers submit bids to a seller, who subsequently selects the auction winner. Initially, all agents are symmetrically uninformed about the buyers' common valuation of the good for sale, which is distributed according to a regular distribution with positive and negative realizations net of the seller's reservation value.[3] The seller chooses whether and when to grant access to confidential information. This access allows the buyers to learn the valuation at a cost. We assume that this cost is not prohibitively large and that buyers' information choices are publicly observed. Whenever trade occurs, the price paid by the auction winner equals his bid.

To understand the scenarios our model aims to capture, consider the following motivating example: An owner-managed firm is up for sale. The current owner runs the business to the best of her capabilities. Multiple financial investors, such as private equity firms, are interested in acquiring the firm and believe that they might be able to increase its market value by improving management practices or the business strategy. This allows for the possibility of gains from trade. However, if such improvements are negligible, trade is inefficient due to the presence of transaction costs. To uncover whether and to what extent improvements are possible, the buyers need access to the firm's internal documents. The owner understands that the investors may be able to increase profits. Yet, she does not know whether this is the case or how to do so. (Otherwise, she could have implemented the improvements herself.)

This motivating example highlights some key features of our model. First, buyers have a common valuation: Due to their similarity in expertise, they would implement almost identical strategies to improve the profitability and, therefore, the market value of the firm. Second, the information about the firm is confidential, i.e., it cannot be accessed or acquired without the seller's permission. Finally, while the seller has access to the information, she cannot anticipate how buyers would react to it.[4]

Depending on the seller's choice of the timing of information acquisition, different dynamic games arise. If the seller does not allow for information acquisition, our model is outcome-equivalent to a first-price common value auction with symmetrically uninformed buyers. Thus, the winning bid and the seller's revenue are equal to the expected valuation of the good in every equilibrium.[5] Alternatively, the seller can allow buyers to become informed before bids are submitted, an action we call *research*, or after bids have been submitted and an offer has been accepted, namely via *due diligence*. In turn, if the auction winner conducts due diligence, he can use the information to decide whether to renege from the purchase, in which case the

[3]Thus, trade can be inefficient in our model. There are several reasons why this can be the case in practice. In mergers and acquisitions, for instance, transaction and integration costs can be substantial. In the real estate market, sellers can be sentimentally attached to their property. Furthermore, since real estate is often used as collateral, the sale of a house can result in the inefficient termination of an advantageous mortgage. More broadly, in various high-stakes situations – such as government auctions or the sale of subsidiaries by bankruptcy advisors – the buyers' use and evaluation of the good for sale may differ significantly from the seller's.

[4]This assumption distinguishes our model from informed seller problems. With an informed seller, the timing of information acquisition could also be used as a signaling device. To isolate the effect of the timing of information acquisition on equilibrium revenue, we therefore abstract away from such signaling considerations.

[5]Our solution concept is Perfect Bayesian Equilibrium under a standard "no signaling what you don't know" refinement, which we call *equilibrium* for short.

seller keeps the good.

Due to the existence of multiple equilibria, how the seller evaluates allowing for research depends on her attitude toward strategic uncertainty. In the main analysis, we focus on a seller who deals conservatively with this uncertainty, i.e., who maximizes her worst-case equilibrium revenue. We show that such a seller always prefers forbidding information acquisition over allowing for research. To see this, note that information is valuable to a buyer only when no other buyer conducts research: If two or more buyers become symmetrically informed, they compete away all their possible gains from trade. Because of this strong strategic externality, there always exists an equilibrium in which only one buyer conducts research. The resulting information asymmetry between the buyers leads to conservative bidding due to the winner's curse and, therefore, to a seller revenue strictly below the good's expected value. In other words, granting access to information before bidding reduces the seller's revenue guarantee across equilibria.

The above conclusion seems at odds with the seminal linkage principle. This principle, first shown by Milgrom and Weber (1982), states that the seller increases her expected revenue by always giving buyers access to her confidential information before bidding. However, this assumes that all the buyers automatically acquire and process the disclosed information. In contrast, we consider settings in which the buyers do so only when it is in their best interest, i.e., when the value of the information exceeds its cost. Since information about the common valuation benefits a buyer only if all others remain uninformed, this implies that no equilibrium exists where multiple buyers acquire information with certainty. In turn, the linkage principle fails: Granting access to information can reduce the seller's equilibrium revenue.

When the seller allows for due diligence, information acquisition occurs after, rather than before offers are submitted. Concretely, the auction winner has a choice: He can buy the good *sight unseen* or he can conduct due diligence and renege if the valuation is less than his bid. Thus, the winner trades off the cost of acquiring information against the benefit of avoiding overpaying for the good. Because this benefit is increasing in his bid, the winner conducts due diligence if and only if his bid is above some threshold $\hat{x}$. This decision creates a discontinuity in the seller's expected revenue. If the seller chooses a winner who offers $\hat{x}$ or less, the good is sold with certainty, and the revenue is equal to the winning bid. Instead, if she chooses a winner who offers more than $\hat{x}$, she sells the good only if the valuation is larger than the winning bid, resulting in an expected revenue smaller than this bid. Thus, the seller might no longer select the buyer making the highest offer as the winner.

We show that the seller's worst-case equilibrium outcome under due diligence can be of two forms. If the information cost is low, all equilibria feature the same winning bid and a winner who conducts due diligence. The winning bid maximizes the seller's revenue over the relevant range, i.e., over all bids such that the winner receives a weakly positive expected payoff. Instead, if the information cost is high, the seller's least preferred equilibrium outcome features sure-trade. In such an equilibrium, the winning bid is equal to $\hat{x}$, the largest bid that induces the winner to buy the good sight unseen. This can occur either because $\hat{x}$ is large enough to maximize the seller's revenue over the relevant range, or because multiple equilibrium outcomes exist, of which the one featuring sure-trade yields the lowest expected revenue. The multiplicity of equilibrium outcomes is possible since, to outbid $\hat{x}$, a buyer needs to submit an offer high enough to compensate the seller for the revenue losses due to the risk of reneging. Such a discrete deviation in bids may not be profitable if all buyers already win with a sufficiently high probability when bidding $\hat{x}$.

Our main result states that, whenever the stakes are sufficiently high, the seller maximizes her worst-case equilibrium revenue by allowing for due diligence. Intuitively, in high-stakes environments, the option value of conducting due diligence is large, so buyers have an incentive to bid aggressively in equilibrium. At the same time, high stakes also imply that the probability of the winner reneging on a substantial offer is relatively low, since the valuation is frequently even larger. Thus, the increase in buyers' bids when they can conduct due diligence more than offsets the potential revenue losses due to the winner reneging in high-stakes environments. Formally, we establish this result in two steps. First, we characterize conditions that are both necessary and sufficient for due diligence to be optimal. Then, we show that the economic force behind these conditions is the size of the stakes.

The seller prefers to allow for due diligence if and only if two conditions hold. The first condition requires that the information cost be low enough so that no sure-trade equilibrium exists. The second condition ensures that buyers are willing to submit bids that lead to an expected revenue larger than the good's expected valuation. This characterization enables us to derive comparative statics on the seller's preference for due diligence. In particular, we show that an increase in the number of buyers, a decrease in the information cost, a scaling up of the buyers' valuations, or an increase in the seller's reservation value, all make the seller more inclined to choose this timing of information acquisition.

Whether the conditions for the optimality of due diligence are satisfied depends on the size of the stakes. To show this, we consider the family of *mean-preserving scalings* of any given distribution. A mean-preserving scaling alters the stakes while holding fixed both the shape and mean of the distribution. Within such a family, we obtain a single-crossing result: Due diligence is preferred if and only if the stakes are sufficiently high.

Returning to our motivating example, our results imply that the seller of a firm should allow for due diligence whenever she is concerned about strategic uncertainty and the transaction can lead to large gains and losses. Similarly, our results can also help to explain why sellers often opt to allow for home inspections after an offer has been accepted, rather than before: Real estate transactions are not only some of the most important economic decisions of most households, but also incur high transaction costs, fees, and taxes. Thus, home sales fit our definition of high stakes. On the other hand, our results also provide an explanation for the lack of information acquisition in settings with comparatively low stakes, such as online car auctions.

In the Discussion, we analyze how our main predictions change as we vary our modeling assumptions. First, we consider the consequences of changing the seller's attitude toward strategic uncertainty. Concretely, we allow the seller to not only choose the timing of information acquisition but also the equilibrium that is selected in the resulting game. Such a seller aims to maximize her best-case equilibrium revenue. We show that which timing of information acquisition is best-case optimal depends on the size of the information cost: Research is preferred when the cost is low, while for high cost either no-information or due diligence is optimal. We then consider a variant of our model, where the seller can repeat the informal auction if the initial winner reneges, and show that such repeated due diligence is preferred to not allowing for information acquisition and to allowing for research, whenever the stakes are high enough. Next, we show that our results continue to hold if buyers observe a noisy signal of the common valuation, as long as the noise is small relative to the information cost. Finally, we show that the seller's preference for due diligence is strengthened if buyers can commit to buy the good sight-unseen. This is often the case in real-estate markets, where buyers can increase the perceived value of their offer by waiving their right to an inspection.

Related literature. Since the influential work of Milgrom (1981), information acquisition in auctions has become a prominent area of economic research. While much of this literature has focused on how different auction formats shape bidders' incentives to gather information about the item for sale, the role of the seller in guiding these information decisions has received less attention. Highlighting that in many instances the seller holds confidential information critical to the buyers to revise their valuation, our paper is among few that analyze the seller's incentives to provide buyers access to such information.[6] In particular, we are the first to explore the *timing* of information access, framing the problem as a design choice by the seller to maximize revenue.

So far, the literature on information acquisition in auctions (e.g., Milgrom, 1981; Lee, 1984; Hausch and L. Li, 1993; Persico, 2000; Shi, 2012; J. Kim and Koh, 2020; K. Kim and Koh, 2022; Bobkova, 2024; Gleyze and Pernoud, 2024) has mainly focused on investigating the impact of buyers' information acquisition before bids are submitted, a procedure we call *research*.[7] Our approach differs in at least two aspects. First, we allow for due diligence, i.e., the possibility that buyers can acquire information only after bids are submitted and a winner is selected. Second, we model buyers' research decision as an *overt*, rather than *covert*, action.[8] This assumption is motivated by the economic applications we wish to capture, namely, those where the information buyers seek is proprietary to the seller. In these settings, the seller can keep track of how many buyers decide to access her confidential information and has a strong ex post incentive to disclose this number to preserve the competitive pressure in the auction.[9]

Despite its prominent use in practice, due diligence has received little academic attention to date. To the best of our knowledge, the only theoretical work discussing due diligence is a recent paper by Daley, Geelen, and Green (2024). From a technical viewpoint, we depart from their work in how information acquisition is modeled: While we adopt a search-cost framework, they use a Brownian information revelation process. A more significant distinction lies in the objective of each paper. Daley, Geelen, and Green's (2024) primary goal is to provide a detailed equilibrium

[6]Among the few exceptions, see Lee (1984) and Hausch and L. Li (1993).

[7]This modeling restriction is not confined to auction theory. More broadly, the literature on information acquisition in mechanisms has mainly analyzed the cases where agents can acquire information only ex ante, i.e., before participating in the mechanism. See, e.g., Bergemann and Välimäki (2002) and Y. Li (2019).

[8]In this regard, our approach aligns with Milgrom (1981) and Lee (1982). In particular, our analysis of research generalizes Lee (1982) to a setting with a continuum of valuations and an arbitrary number of buyers.

[9]Lack of commitment power is not necessary for this disclosure result: As Lee (1984) shows, a seller may benefit from disclosing the number of informed buyers even if she could commit to remaining silent.

analysis of the dynamics of due diligence, offering comparative statics on the likelihood and expected time-to-completion of a deal. In contrast, we provide a revenue-based rationale for the widespread use of due diligence in common value auctions.[10] Due to a shortage of available structured data, due diligence is also relatively unexplored empirically. Among the few empirical papers discussing the topic, see Marquardt and Zur (2015) and Wangerin (2019).

The possibility of reneging on an accepted offer, inherent in the due diligence process, links our work to the literature on optimal return policies in retail markets (Courty and Hao, 2000; Matthews and Persico, 2007; Inderst and Ottaviani, 2013; Krähmer and Strausz, 2015; Haberman and Jagadeesan, 2024). However, our analysis of due diligence differs from this literature in two important ways. First, while buyers typically learn their valuation *for free* once the contract is signed and the good is purchased in return policy models, due diligence entails a significant cost of information processing in the settings we consider. Second, in return policy models, sellers directly set contract terms ex ante. Instead, in our model, the price of the good is determined in equilibrium through an informal auction.

Finally, the idea that information choices can be strategic substitutes when agents' equilibrium incentives in a continuation game are considered is not specific to our framework and known at least since Grossman and Stiglitz (1980). In fact, this finding connects our work to the large literature on information acquisition in coordination games stemming from Hellwig and Veldkamp (2009) (see, e.g., Myatt and Wallace, 2012; Colombo, Femminis, and Pavan, 2014; Herskovic and Ramos, 2020). Our analysis adds to these earlier studies by demonstrating that, in the context of common value auctions, the seller can adjust the timing of information acquisition to mitigate the issues originating from the strategic substitutability of information acquisition and avoid potentially unfavorable equilibrium outcomes.[11]

Outline. The remainder of the paper is organized as follows. Section 2 describes the model. In Section 3, we show that the seller prefers granting no access to information over allowing buyers to conduct research if she is concerned about strategic uncertainty. Section 4 analyzes the due diligence game and shows when due diligence is the optimal timing of information acquisition. Finally, in Section 5, we discuss how our predictions change if we allow for trade to be ex ante inefficient or consider a seller who aims to maximize her best-case equilibrium revenue, and consider several model extensions. All omitted proofs are in the Appendix.

2 Model

We consider a common value setting where $N \geq 2$ symmetric buyers (he) compete for the purchase of an indivisible good from a seller (she). The buyers, indexed by $i \in I := \{1, ..., N\}$,

[10] While Daley, Geelen, and Green (2024) state information acquisition and provision costs as the economic reasons for why due diligence is observed in practice, they do not explain this justification formally. Relatedly, Hansen (2001) explains why the seller might want to limit the number of bidders in an auction of a company because of informational externalities, but does not consider the timing of information acquisition. Thus, our work complements these papers by providing a formal justification that explains not only why due diligence can be the optimal timing of information acquisition, but also when.

[11] Relatedly, Liu, Ma, and Veldkamp (2023) show that a data seller can also alleviate the issues stemming from the strategic substitutability of information acquisition by changing the market structure, in their case by switching from the direct sale of data to the sale of data subscriptions.

participate in an informal auction, where they simultaneously submit bids (equivalently, offers) $x_i \in X := \{\varnothing\} \cup \mathbb{R}_+$ to the seller. We denote *abstaining* from bidding by $x_i = \varnothing$. A buyer is *inactive* if his bid satisfies $x_i = \varnothing$, and *active* otherwise. After observing the bids, the seller selects a *winner* among the active buyers. If trade occurs, the winner pays his bid. If no buyer is active, the seller keeps the good and the game is over. Model details are listed below.

Valuation. The buyers share a *common* valuation for the good, denoted by $\tilde{v}$. We assume that $\tilde{v}$ is distributed according to a cumulative distribution function (cdf) F with strictly positive and continuous density f over the compact support $V := [\underline{v}, \bar{v}] \subseteq \mathbb{R}$. We normalize the seller's reservation value to be equal to 0. Thus, $\tilde{v}$ represents the gains from trade that are available in the economy.

Information. Initially, neither the buyers nor the seller know the realization v of $\tilde{v}$. However, the seller can choose whether and when to grant buyers access to her confidential information. When they have access, each buyer can process this information at a cost $c > 0$ and (privately) learn the realization v. Buyers' information choices are publicly observed.[12] We formalize the seller's problem of choosing the optimal timing of information acquisition in Section 2.1.

Preferences. Let $\mathbb{1}^i_{\text{info}}$ be the indicator of buyer i's information choice, i.e., $\mathbb{1}^i_{\text{info}} = 1$ if and only if buyer i acquires information. All agents are risk-neutral expected utility maximizers with payoff functions specified as follows. Suppose buyer $i \in I$ won the auction with a bid of $x \in \mathbb{R}_+$. If trade occurs, the seller's payoff is $\Pi = x$, while the winner's payoff is $u_i = v - x - c \cdot \mathbb{1}^i_{\text{info}}$. If trade does not occur, the seller's payoff is $\Pi = 0$, while the winner's payoff is $u_i = -c \cdot \mathbb{1}^i_{\text{info}}$. Finally, all losing bidders $j \neq i$ always receive a payoff of $u_j = -c \cdot \mathbb{1}^j_{\text{info}}$.

Model assumptions. Throughout, we impose the following assumptions on the model's primitives.

Assumption 1. *Gains from trade are uncertain:* $\underline{v} < 0 < \bar{v}$.

Assumption 2. *F is regular, i.e.,* $v \mapsto v - \frac{1-F(v)}{f(v)}$ *is strictly increasing.*

Assumption 3. *Information costs are not too large:* $c < \bar{c} := \int_{\underline{v}}^{\mu} (\mu - v)\, dF(v)$.

Assumption 4. *Trade is ex ante efficient:* $\mu := \mathbb{E}_F[\tilde{v}] > 0$.

Assumption 1 states that the buyers' valuation can be negative. Given our normalization of the seller's opportunity cost, this assumption captures the fact that trade can be inefficient *ex post*. In Section 2.3, we show that the seller would never allow for any kind of information acquisition if this assumption were not satisfied. Assumption 2 is a standard technical condition generating some concavity in the seller's preferences. Assumption 3 guarantees that buyers

[12] In the applications that motivate our analysis, at least the seller observes how many buyers gained access to her confidential information. If she can verifiably disclose the number of informed buyers, she wants to do so unless only one buyer is informed. However, standard unraveling arguments imply that this is equivalent to perfect transparency. See also footnote 9.

might benefit from acquiring information, i.e., that information is not prohibitively expensive.[13] Finally, Assumption 4 streamlines our exposition as it allows us to focus on the most interesting parametric case.[14]

2.1 The Timing of Information Acquisition

Our goal is to understand how the timing of information acquisition affects equilibrium outcomes. Should the seller grant buyers access to her confidential information? If so, should this access be granted before or after the bidding phase? Of course, these choices change the timing of the model and, therefore, induce different games. In the *no-information benchmark*, buyers do not have access to the seller's confidential information. In the *research game*, the buyers can acquire information only before bidding. Finally, in the *due diligence game*, the buyers can acquire information only after bidding. We describe the details of these games below.

No-information benchmark. The no-information benchmark is outcome-equivalent to a common value first-price auction with symmetrically informed buyers. The following sequence of events illustrates the structure of this game (see also Figure 1):

0. Nature draws buyers' common valuation $v \in V$ according to F, but nobody observes it.

1. *Bidding phase:* Buyers simultaneously submit their bids or abstain.

2. *Winner selection phase:* The seller selects a winner if at least one buyer is active. Otherwise, the seller keeps the good and the game ends.

3. *Trading phase:* Trade occurs between the seller and the winner at a price equal to the winner's bid.

Figure 1: No-information Timeline.

Research game. In the research game, buyers can acquire information before submitting their bids, an action we call conducting *research*. The following sequence of events illustrates the structure of this game (see also Figure 2):

0. Nature draws buyers' common valuation $v \in V$ according to F, but nobody observes it.

1. *Research phase:* Buyers simultaneously and publicly choose whether to conduct research; those who conduct research privately observe v.

2. *Bidding phase:* Buyers simultaneously submit their bids or abstain.

[13] In particular, as we show in Section 4.1, the threshold $\bar{c}$ is the lowest information cost such that no buyer would ever prefer conducting due diligence over buying the good without acquiring information or abstaining.

[14] As we show in the Discussion, our main results still hold under the complementary assumption $\mu \leq 0$.

3. *Winner selection phase:* The seller selects a winner if at least one buyer is active. Otherwise, the seller keeps the good and the game ends.

4. *Trading phase:* Trade occurs between the seller and the winner at a price equal to the winner's bid.

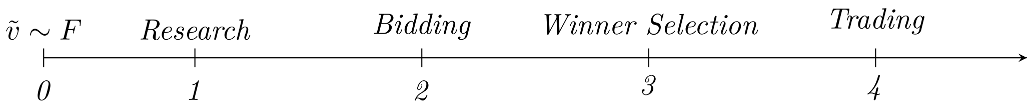

Figure 2: Research Timeline.

Due diligence game. In the due diligence game, only the auction winner can *conduct due diligence,* i.e., information acquisition is allowed only after bids are submitted and a winner is selected. The following sequence of events illustrates the structure of this game (see also Figure 3):

0. Nature draws buyers' common valuation $v \in V$ according to F, but nobody observes it.

1. *Bidding phase:* Buyers simultaneously submit their bids or abstain.

2. *Winner selection phase:* The seller selects a winner if at least one buyer is active. Otherwise, the seller keeps the good and the game ends.

3. *Due diligence phase:* The winner chooses whether to conduct due diligence or not.

4. *Trading phase:* If the winner conducts due diligence, he can choose whether to renege on his offer, in which case the seller keeps the good, or to execute the transaction. If the winner does not conduct due diligence, trade occurs immediately. If trade occurs, the price is equal to the winner's bid.

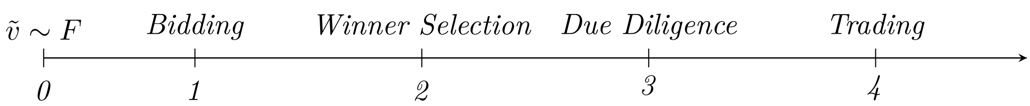

Figure 3: Due Diligence Timeline.

2.2 The Seller's Problem

We adopt Perfect Bayesian Equilibrium (PBE) under a "no signaling what you don't know" (NSWDK) refinement as our solution concept.[15] This refinement ensures that the buyers and

[15]See Fudenberg and Tirole (1991) for a formal definition of NSWDK.

the seller do not change their beliefs about $\tilde{v}$ in response to surprising moves by agents who have no private information.

We refer to any PBE satisfying NSWDK as an *equilibrium.* Note that under this refinement, agents' equilibrium beliefs about $\tilde{v}$ are pinned down uniquely both on- and off-path at all information sets where these beliefs influence best responses.[16] As a result, when characterizing equilibria, we do not specify agents' belief systems and describe only their equilibrium strategies.

We interpret the seller's task as a *design problem* in which she must decide whether and when to allow the buyers to acquire information about $\tilde{v}$. In our main analysis, we focus on the case in which the seller deals conservatively with strategic uncertainty, i.e., she evaluates her options as if the buyers were coordinating on her least preferred equilibrium.[17] Formally, for all $\tau \in \{re, dd, no\}$, let $\mathcal{E}^\tau$ be the set of equilibria in the research game, due diligence game and under the benchmark case of no information acquisition, respectively. It can be verified that these sets are non-empty. The seller's problem is given by

$$V_\Pi := \max_{\tau \in \{re,dd,no\}} \inf_{\sigma \in \mathcal{E}^\tau} \Pi(\sigma), \tag{\textbf{P}}$$

where $\Pi(\sigma)$ denotes the seller's payoff under the strategy profile σ. We say that the timing of information acquisition τ^* is optimal if it solves Problem (**P**), and that it is strictly optimal if it is the unique solution.

2.3 Benchmark: No Information

To better understand the trade-offs the seller faces when choosing the timing of information acquisition, it is useful to first establish a benchmark case in which information acquisition is not permitted. The following proposition characterizes the unique equilibrium revenue in this case.

Proposition 0. *All equilibria of the no-information benchmark are payoff equivalent and feature at least two buyers bidding μ. In particular, $\Pi(\sigma) = \mu$ for all $\sigma \in \mathcal{E}^{no}$.*

Without the possibility of acquiring information, buyers effectively compete for the expected valuation of the good. Standard Bertrand forces then drive buyers' expected payoffs to zero, implying that the seller extracts all the ex ante gains from trade in any equilibrium. Thus, allowing for information acquisition could not improve the seller's revenue if Assumption 1 were not satisfied. (Note that Proposition 0 would still apply in this case.) To see why, consider the

[16] For any measurable space Z, let $\delta_z \in \Delta(Z)$ denote the Dirac measure on $z \in Z$. Under NSWDK, an agent's equilibrium belief about $\tilde{v}$ always coincides with either F or δ_v, unless such agent observed the action of an informed buyer. Note that the only case where this is possible is in the winner selection phase of the research game if at least one of the buyers conducted research before bidding. However, the seller's best response in the research game is to accept the highest bid, irrespective of her belief about $\tilde{v}$.

[17] This selection criterion coincides with selecting the equilibrium that maximizes the buyers' joint payoff and, in the case of research, with considering only equilibria where the buyers use pure strategies in the research phase. In Section 5, we discuss the case in which the seller evaluates her choices according to the best-case equilibrium revenue they may induce.

maximal expected social surplus trade can generate

$$S := \mathbb{E}_F[\max\{\tilde{v}, 0\}] = \int_0^{\bar{v}} v\, dF(v). \tag{1}$$

If $\underline{v} \geq 0$, then $S = \mu$. As a result, the seller could already extract the full surplus by letting the buyers compete for their expected valuation of the good without allowing for information acquisition.

3 The Research Game

In this section, we analyze the *research game*, i.e., the setting where buyers can acquire information before bidding. The following proposition shows that a seller who is concerned about strategic uncertainty never wants to allow for research.

Proposition 1. *The seller strictly prefers granting no access to information over allowing buyers to conduct research, i.e.,*

$$\inf_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma) < \mu = \inf_{\sigma \in \mathcal{E}^{no}} \Pi(\sigma).$$

We prove Proposition 1 by showing that the research game admits an equilibrium inducing an expected revenue strictly below μ, i.e., strictly less than what the seller can guarantee by not allowing for information acquisition. Recall that the research game consists of the research phase followed by an informal auction. Consider now the strategic situations starting from the bidding phase. These are well-defined (Bayesian) continuation games for any given set of chosen actions in the research phase, since the buyers' information choices are public and their equilibrium beliefs about $\tilde{v}$ must be equal to the prior F for uninformed buyers and equal to the Dirac measure δ_v on the realized valuation $v \in V$ for informed buyers. Therefore, we can analyze the research game using a backward induction approach. First, we characterize the buyers' equilibrium payoffs in the continuation games starting from the bidding phase. Then, we use these payoffs to understand buyers' equilibrium incentives in the research phase.

Consider the continuation game where exactly the buyers in $J \subseteq I$ acquired information in the research phase, and let Π^J and U_i^J denote the seller's and buyer i's equilibrium payoff if this continuation game is reached. If $J = \emptyset$, i.e., no buyer conducted research, the situation is strategically equivalent to the no-information benchmark. Thus, the buyers compete à la Bertrand for the expected value of the good and receive an expected payoff of $U_i^{\emptyset} = 0$, while the seller receives a revenue of $\Pi^{\emptyset} = \mu$. The interaction also resembles a standard Bertrand competition game if $|J| \geq 2$, i.e., at least two buyers are informed. In this case, the equilibrium winning bid equals the realized valuation v whenever it is positive, while all buyers abstain from bidding if $v < 0$. Hence, the seller extracts the maximum available social surplus, i.e., her equilibrium revenue satisfies $\Pi^J = S$. In contrast, buyers do not realize any trade surplus. Thus, if $|J| \geq 2$, buyer i's overall payoff equals $U_i^J = 0$ if $i \notin J$ and $U_i^J = -c$ if $i \in J$, since information expenditure is sunk at this point.

It remains to analyze the continuation game where only one buyer conducted research, i.e.,

$J = \{i\}$ for some $i \in I$. In this case, buyer i can exploit his informational advantage. In particular, in equilibrium, he shades his bid while all uninformed buyers either abstain or submit a random bid. The uninformed buyers do not know the exact valuation and, therefore, are subject to the winner's curse: Since the informed buyer's bidding strategy is increasing in the observed valuation v, any buyer $j \neq i$ wins with a bid of $x \geq 0$ only if v is sufficiently low such that buyer i offers less than x. As a result, buyer j bids conservatively and never more than the expected valuation μ, since any higher bid would result in a negative payoff. In turn, buyer i has no incentive to bid above μ either, since a bid of μ would already win the auction with certainty. Thus, the winning bid is below μ almost surely, implying that the seller's expected revenue from reaching this continuation game, $\Pi^{\{i\}}$, satisfies $\Pi^{\{i\}} < \mu$. Furthermore, the uninformed buyers receive an expected payoff of $U_j^{\{i\}} = 0$, while his informational advantage allows buyer i to achieve a payoff of $U_i^{\{i\}} > 0$, even net of the information cost.[18]

To establish Proposition 1, we show that only buyer i becoming informed constitutes an equilibrium for any $i \in I$. By conducting research, the informed buyer achieves a payoff of $U_i^{\{i\}} > 0$, while his expected payoff would fall to $U_i^{\emptyset} = 0$, if he deviated and did not conduct research, instead. At the same time, no uninformed buyer has an incentive to conduct research: If they changed their decision, they would receive a payoff of $U_j^{\{i,j\}} = -c$, which is worse than their expected payoff from staying uninformed $U_j^{\{i\}} = 0$. Intuitively, buyers' research decisions are strategic substitutes, since research creates an informational advantage only if no other buyer becomes informed. Thus, an equilibrium where only one buyer acquires information always exists.[19] As discussed above, the resulting informational asymmetry leads to conservative bidding due to the winner's curse, and therefore to a low expected revenue for the seller.

4 The Due Diligence Game

In the due diligence game, the seller grants information access to the winner of the informal auction. The winner can then conduct due diligence and use the resulting information to decide whether to renege on his offer. This section analyzes the equilibrium implications of due diligence and characterizes when allowing for this timing of information acquisition is preferred by the seller.

4.1 Equilibrium Analysis of Due Diligence

Allowing for due diligence effectively changes the informal auction to one that allocates an option to buy the good at a strike price equal to the winning bid, instead of allocating the good directly. Therefore, when choosing which offer to accept, the seller must consider that the auction winner might renege on his offer ex post after a negative due diligence process. In other words, the seller must take into account the winner's incentives during the due diligence phase.

Due diligence phase. Suppose a buyer with bid $x \geq 0$ has been chosen as the winner of the auction. At this stage of the game, the winner faces the following two options: Either *(i)*

[18] See the proof of Proposition 1 in the Appendix for detailed derivations of Π^J and U_i^J.

[19] Note that this equilibrium exists regardless of how costly research is, as long as Assumption 3 holds. Moreover, the information cost c affects only the informed buyer's equilibrium payoff, but not the seller's revenue.

purchase the good "*sight unseen*", i.e., without conducting due diligence, for an expected payoff of

$$W^{su}(x) := \mu - x,$$

or *(ii)* conduct due diligence for a cost of $c > 0$, and purchase the good if and only if the realized valuation v is larger than the bid x. In this case, the expected payoff to the winner is given by

$$W^{dd}(x) := -c + \int_x^{\bar{v}} (v - x)\, dF(v).$$

In equilibrium, the winner conducts due diligence only when it is payoff maximizing. Therefore, the winner's continuation value at the beginning of the due diligence phase is given by

$$W(x) := \max\left\{W^{su}(x), W^{dd}(x)\right\}.$$

Observe that $W(x)$ is a strictly decreasing function of $x \in [0, \bar{v}]$: Irrespective of whether he decides to buy the good sight unseen or only after a positive due diligence process, the winner always strictly prefers to pay a lower price.

For all $c > 0$, let $\hat{x} = \hat{x}(c)$ be the (possibly negative) bid making the winner indifferent between buying the good sight unseen and conducting due diligence, i.e., $W^{su}(\hat{x}) = W^{dd}(\hat{x})$.[20] The following lemma uses $\hat{x}$ to characterize the best response of the winner during the due diligence phase.

Lemma 1. *During the due diligence phase, if the winning bid is $x \geq 0$, the winner buys the good sight unseen whenever $x < \hat{x}$, and conducts due diligence whenever $x > \hat{x}$.*

Figure 4 provides a graphical representation of Lemma 1: When the winning bid is low ($x < \hat{x}$), the winner is relatively confident that his ex post payoff conditional on trade, $v - x$, will turn out to be positive and therefore decides to buy the good sight unseen to economize on the cost of conducting due diligence. Conversely, when the winning bid is high ($x > \hat{x}$), the benefit of avoiding overpaying for the good more than compensates the winner for the due diligence cost. As a result, the winner prefers to conduct due diligence over buying the good sight unseen.

Whether the winner finds conducting due diligence optimal depends on the magnitude of the due diligence cost c. To see this, note that an increase in c leads to a one-to-one decrease in the payoff of conducting due diligence but does not affect the payoff of buying the good sight unseen. Therefore, as c increases, the *sight-unseen region* – i.e., the subset of non-negative bids inducing the winner to buy the good sight unseen – expands, while the *due-diligence region* – i.e., the subset of non-negative bids inducing due diligence – shrinks. Put differently, the

[20] The threshold bid $\hat{x} = \hat{x}(c)$ is implicitly defined by the equation

$$\int_{\underline{v}}^{\hat{x}} (\hat{x} - v)\, dF(v) = c. \tag{2}$$

Note that $\hat{x} = \hat{x}(c)$ is a well-defined continuously differentiable strictly increasing function of $c > 0$: The intermediate value theorem guarantees a solution $\hat{x} \in (\underline{v}, \infty)$ to (2) exists for all $c > 0$. Its uniqueness follows from the strict monotonicity of $x \mapsto \int_{\underline{v}}^{x} (x - v)\, dF(v)$. Finally, the Implicit Function Theorem ensures that $c \mapsto \hat{x}(c)$ is strictly monotone and continuously differentiable.

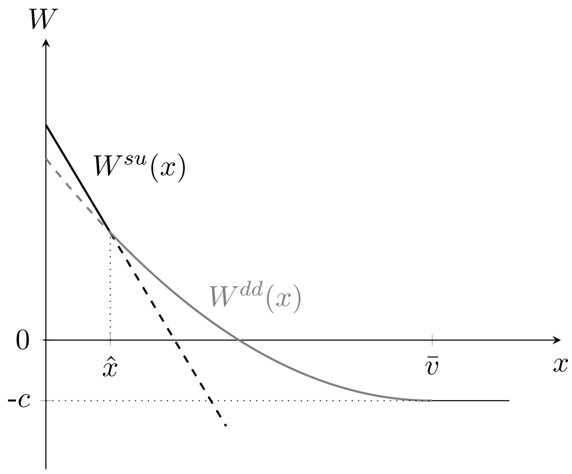

Figure 4: Winner's value $W(\cdot) = \max\left\{W^{su}(\cdot), W^{dd}(\cdot)\right\}$ when $\tilde{v} \sim \mathcal{U}\left(-\frac{1}{2}, \frac{3}{2}\right)$ and $c \approx 0.14$.

threshold bid $\hat{x}(c)$ is strictly increasing in $c > 0$. This monotonicity also implies that the sight-unseen region can be empty. This is the case whenever the due diligence cost is so small that $\hat{x}(c) < 0$.[21] Intuitively, since trade can be ex post inefficient, the winner always finds conducting due diligence optimal when c is small enough.

We are interested in studying the impact of due diligence on equilibrium outcomes. Of course, if the information cost $c > 0$ were so large that $W(\hat{x}) \leq 0$, due diligence would produce no equilibrium effect. In this case, any bid in the due-diligence region leads the winner to experience a strictly negative expected payoff, so no buyer would rationally submit a bid in this region if he thinks that it might win the auction. Assumption 3, which characterizes the conditions under which $W(\hat{x}) > 0$, rules out exactly this case.[22]

Winner selection phase. We now investigate the seller's selection of a winner. Let $p(x)$ be the equilibrium trade probability if the seller selects a winner whose bid is $x \geq 0$. Lemma 1 implies that this probability is given by:[23]

$$p(x) = \begin{cases} 1 & \text{if } x \leq \hat{x} \\ 1 - F(x) & \text{otherwise.} \end{cases}$$

For low bids ($x \leq \hat{x}$), the winner buys sight unseen, implying that his purchase probability is 1. For high bids ($x > \hat{x}$), the winner conducts due diligence, implying that he purchases the good if and only if the realized valuation v is larger than x.

[21] From equation (2) in footnote 20, one can verify that $\hat{x}(c) \to \underline{v} < 0$ as $c \downarrow 0$.

[22] Note that $W(\hat{x}) > 0$ is equivalent to $\hat{x} < \mu$. Thus, Assumption 3 guarantees that the cutoff bid $\hat{x}$ is lower than the expected valuation of the good μ.

[23] Technically, Lemma 1 does not pin down the winner's behavior when $x = \hat{x}$. However, to simplify the exposition, we assume that whenever indifferent between buying the good sight unseen and conducting due diligence, the winner chooses to buy the good sight unseen. This assumption is innocuous for the equilibrium analysis. If a continuation game where the winning bid is equal to $\hat{x}$ is reached, this tie-breaking rule must be satisfied in equilibrium: Otherwise, the auction winner could strictly benefit from submitting a slightly lower bid. Moreover, for any equilibrium where this tie-breaking rule is not satisfied in some continuation game that is not reached on-path, one can find an outcome-equivalent equilibrium that satisfies the tie-breaking rule both on- and off-path.

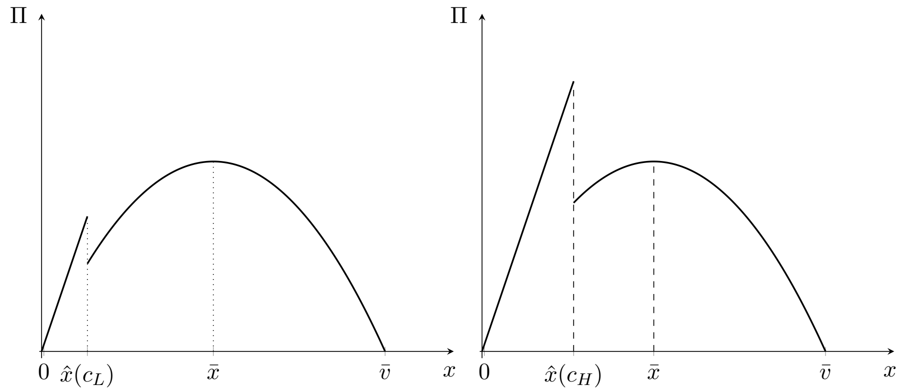

Figure 5: The seller's revenue curve $\Pi(\cdot)$ when $\tilde{v} \sim \mathcal{U}\left(-\frac{1}{2}, \frac{3}{2}\right)$ for two values of $c \in \{c_L, c_H\}$ such that $c_L < c_H < \bar{c}$.

In equilibrium, the seller anticipates the buyers' incentives during the due diligence phase. As a result, the equilibrium revenue to the seller associated with selecting any offer $x \geq 0$ as the winning bid is given by

$$\Pi(x) := x \cdot p(x).$$

Let $\beta_i(\vec{x}) \in [0, 1]$ denote the probability that buyer i is selected as the winner when the buyers' profile of bids is $\vec{x} \in X^N \setminus \{(\varnothing, ..., \varnothing)\}$, i.e., when at least one buyer is active. Sequential rationality requires that the seller selects a winner among buyers whose offers maximize her revenue. Therefore,

Lemma 2. *In equilibrium, $\beta_i(\vec{x}) > 0$ only if $x_i \geq 0$ maximizes $\Pi(\cdot)$ among all active bids in $\vec{x}$.*

Bidding phase. As Lemma 2 indicates, a buyer's offer is accepted in equilibrium only if it maximizes $\Pi(\cdot)$ among all submitted bids. Hence, to investigate the buyers' incentives during the bidding phase, it is useful to first understand the shape of the seller's revenue curve. If c is so small that $\hat{x} \leq 0$, then $\Pi(x) = x \cdot [1 - F(x)]$ for all $x \geq 0$. Since F is regular (Assumption 2), $x \mapsto x \cdot [1 - F(x)]$ is single-peaked around a strictly positive maximum point

$$\bar{x} := \arg\max_{x \geq 0} x \cdot [1 - F(x)].$$

If instead $\hat{x} > 0$, then $\Pi(x) = x$ for all $x \leq \hat{x}$, and $\Pi(x) = x \cdot [1 - F(x)]$ for all $x > \hat{x}$. In particular, the discontinuity of $\Pi(\cdot)$ at $\hat{x}$ implies that the seller's revenue curve may admit *two* local maximum points, $\hat{x}$ and $\bar{x}$, whenever c is not too small.

Figure 5 elucidates this fact by plotting the seller's revenue curve for the same distribution F under two distinct values of the due diligence cost. The figure shows that $\Pi(\cdot)$ can admit both $\hat{x}$ and $\bar{x}$ as local maximum points. It also shows that how the seller ranks these bids depends on c itself. In particular, since $\hat{x}$ strictly increases with c, one can have that $\Pi(\hat{x}) > \Pi(\bar{x})$ when the due diligence cost is large enough.

While the seller evaluates bids according to $\Pi(\cdot)$, buyers evaluate any bid $x \geq 0$ according to the value of winning with such a bid, $W(x)$, weighted by the associated winning probability,

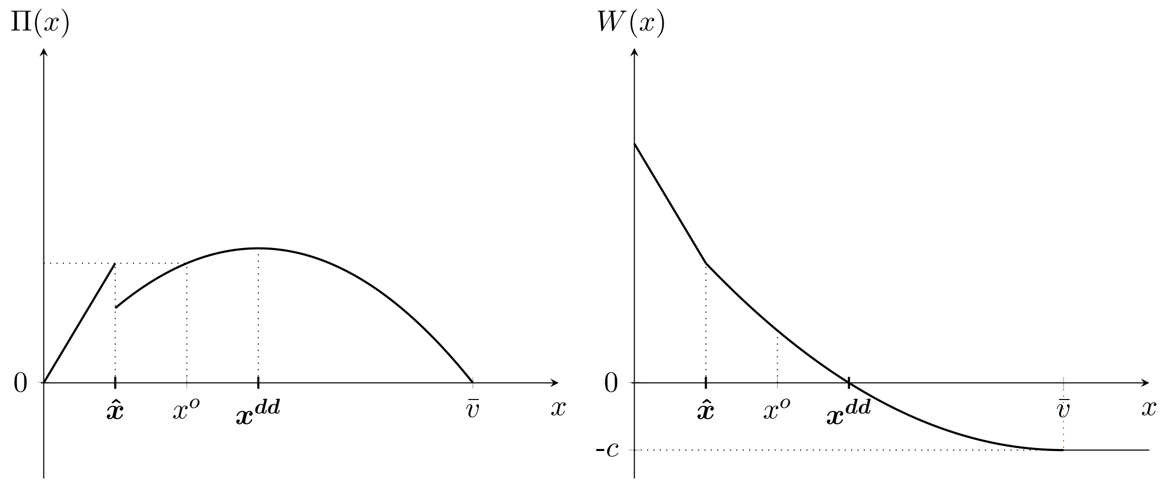

Figure 6: Seller's revenue $\Pi(\cdot)$ and Winner's value $W(\cdot)$ as a function of the winning bid x when $\tilde{v} \sim \mathcal{U}\left(-\frac{1}{2}, \frac{3}{2}\right)$, $c \approx 0.14$, and $N = 2$. In bold, we highlight offers that can be supported as winning bids in equilibrium.

which depends on the seller's winner selection rule β. Since due diligence is costly and reneging requires conducting due diligence, high bids may lead to a strictly negative payoff for the buyers if the seller accepts them. Of course, even if such bids result in a high revenue for the seller, they are not attractive to the buyers, as they prefer abstaining over winning with such a bid. Thus, the relevant bidding range is

$$R := \{x \geq 0 : W(x) \geq 0\}.$$

In the Appendix, Lemma 6 shows that R is a non-trivial compact interval, and that, over this range, $\Pi(\cdot)$ can be maximized only at two bids, either $\hat{x}$ or

$$x^{dd} := \arg\max_{x \in R} x \cdot [1 - F(x)]. \tag{3}$$

That is, it holds that

$$X^{\max} := \arg\max_{x \in R} \Pi(x) \subseteq \{\hat{x}, x^{dd}\}. \tag{4}$$

Equilibrium characterization. The set $X^{\max}$ plays an important role in the equilibrium analysis of due diligence. In particular, any $x^* \in X^{\max}$ can be supported as an equilibrium winning bid of the due diligence game: If at least one buyer submits a bid of x^*, it is a best response for the seller to reject all other bids in R. In turn, this implies that it is optimal for all other buyers to also bid x^*.

However, bids that do not maximize the seller's expected revenue can also be supported as winning bids in equilibrium. To see this, consider the setting with two buyers and a uniformly distributed valuation $\tilde{v} \sim \mathcal{U}\left(-\frac{1}{2}, \frac{3}{2}\right)$ depicted in Figure 6. At the information cost of $c \approx 0.14$, the expected revenue is maximized over the relevant range at x^{dd}. Nevertheless, both buyers bidding $\hat{x}$ can be supported as an equilibrium outcome. The reason lies in the discontinuity of

$\Pi(\cdot)$ at $\hat{x}$, which implies that to outbid $\hat{x}$, a buyer has to offer more than

$$x^o := \min\{x \geq 0 : x \cdot [1 - F(x)] = \hat{x}\} > \hat{x}.$$

Winning with a bid of $\hat{x}$ leads to a payoff that exceeds that of winning with a bid of x^o by a factor of

$$\rho := \frac{W(\hat{x})}{W(x^o)}. \tag{5}$$

As long as both buyers win with probability $\frac{1}{\rho}$ or higher by bidding $\hat{x}$, they prefer not to outbid their competitor. Of course, this is only possible if ρ is sufficiently large.

By extending the above logic to more than two buyers, the following proposition characterizes the seller's least preferred equilibrium of the due diligence game as a function of the information cost $c > 0$.

Proposition 2. *There exists a unique information cost threshold $c^* \in (0, \bar{c})$ such that:*

- *If $c < c^*$, the winning bid is x^{dd} and the winner conducts due diligence in every equilibrium.*
- *If $c \geq c^*$, an equilibrium where the winning bid is $\hat{x}$ and the winner buys the good sight unseen exists, and no equilibrium leads to a lower revenue than this sure-trade equilibrium.*

Moreover, as a function of the number of buyers N, $c^*(N)$ is weakly increasing in N and satisfies $\lim_{N\uparrow\infty} c^*(N) < \bar{c}$.

Whenever the due diligence cost c is sufficiently low, the winning bid is x^{dd} and the winner conducts due diligence in any equilibrium. Instead, if c is above the threshold c^*, there exists a sure-trade equilibrium where the winning bid is $\hat{x}$ and the winner buys the good sight unseen. Even though the winner does not conduct due diligence in this equilibrium, the possibility of acquiring information still influences the equilibrium outcome: The resulting winning bid is less than the expected valuation, i.e., $\hat{x} < \mu$.

A key step in the proof of Proposition 2 is to show, using regularity, that the ratio ρ, as defined in (5), is increasing in c. Thus, when $c < c^*$, $\hat{x}$ is negative or ρ is small. Therefore, all equilibria feature a winning bid of x^{dd}. Instead, a large information cost enables the existence of a sure-trade equilibrium where $\hat{x}$ is the winning bid – either because ρ is sufficiently large, or because $\hat{x} \in X^{\max}$. Whenever such an equilibrium exists, it is the seller's least preferred one: Since the expected revenue is continuous below $\hat{x}$, no bid in that range can win in equilibrium because of standard overcutting arguments. Similarly, no equilibrium can feature a winning bid above $\hat{x}$ that leads to a lower revenue, since then a buyer would rather offer $\hat{x}$, thus increasing both his winning probability and his expected payoff conditional on winning.

Proposition 2 also shows that the cost threshold c^* is weakly increasing in the number of buyers N. Recall that a sure-trade equilibrium can exist for two reasons: Either the seller's expected revenue is maximized at $\hat{x}$ ($\hat{x} \in X^{\max}$), or all buyers are content with bidding $\hat{x}$ as they all win with a probability of $\frac{1}{\rho}$ or higher when they do so.[24] Of course, the second scenario is only

[24] The seller's expected revenue $\Pi(\cdot)$, the winner's expected payoff $W(\cdot)$ and, therefore, also ρ, are independent of the number of buyers N.

possible if $\rho > N$. Thus, as N increases, it becomes harder to sustain a sure-trade equilibrium, implying that the cost threshold c^* is weakly increasing in the number of buyers. Nevertheless, regardless of the number of buyers, a sure-trade equilibrium always exists for sufficiently high information costs.

Comparative statics. We conclude the equilibrium analysis of due diligence by discussing the comparative statics in c and N, which are the main parameters of the model. To do so, for any $c > 0$, denote by $x^w(c)$, $\Pi^w(c)$, and $W^w(c)$ the winning bid, the seller's revenue, and the winner's payoff (which coincides with the buyers' joint payoff) under worst-case equilibrium selection, respectively.[25] Proposition 2 implies that

$$x^w(c) = \begin{cases} x^{dd}(c) & \text{if } c < c^* \\ \hat{x}(c) & \text{otherwise,} \end{cases}$$

$\Pi^w(c) = \Pi(x^w(c)|c)$, and $W^w(c) = W(x^w(c)|c)$.[26] Note that an increase in the information cost c weakly decreases the winner's payoff $W(x|c)$ for any bid x. Thus, the relevant range $R(c) = \{x : W(x|c) \geq 0\}$ shrinks, implying that $x^{dd}(c) := \arg\max_{x \in R(c)} x \cdot (1 - F(x))$ is non-increasing in c. Conversely, the highest bid with which the auction winner prefers buying the good sight unseen, $\hat{x}(c)$, is strictly increasing in $c > 0$ since higher information costs make due diligence less attractive. Jointly, these properties imply that the winning bid $x^w(c)$ is weakly decreasing for $c < c^*$ and strictly increasing otherwise. Furthermore, $x^w(c)$ discontinuously jumps downward at c^* since

$$x^{dd}(c^*) > \Pi(x^{dd}(c^*)|c^*) \geq \Pi(\hat{x}(c^*)|c^*) = \hat{x}(c^*).$$

The comparative statics of $\Pi^w(c)$ mimic those of $x^w(c)$ except that there need not be a downward jump at c^*. How $W^w(c)$ changes with $c > 0$ can also be derived from these observations. In particular, $W^w(c)$ is always weakly decreasing, except at c^*, where it jumps upward discontinuously. The following corollary summarize this discussion.

Corollary 1. *The comparative statics with respect to the information cost c in the seller's worst-case equilibrium of the due diligence game are as follows:*

- *For $c \in (0, c^*)$, the winning bid $x^w(c)$, the seller's revenue $\Pi^w(c)$, and the winner's payoff $W^w(c)$ are weakly decreasing in c;*
- *For $c \in (c^*, \bar{c})$, the winning bid $x^w(c)$ and the seller's revenue $\Pi^w(c)$ are strictly increasing*

[25] The proof of Proposition 2 shows that the winning bid must be equal to $\hat{x}$ in the worst-case equilibrium for almost all $c \geq c^*$. The only exception is at the cost parameter $c = c_1$, where it holds that $\Pi(\hat{x}) = \Pi(x^{dd})$. In this case, there exist equilibria where the winning bid is $\hat{x}$, x^{dd}, or a mixture of the two, which all result in the same equilibrium revenue to the seller. To facilitate the exposition of the comparative statics, we proceed with the analysis by selecting the worst-case equilibrium with a winning bid equal to $\hat{x}$ even when $c = c_1$.

[26] With the notation $\Pi(\cdot|c)$ and $W(\cdot|c)$, we highlight how the equilibrium objects depend on the cost c. In particular, the winner's expected payoff $W(\cdot)$ is affected by c not only through the resulting changes in the winning bid but also directly when due diligence is conducted. Similarly, $\Pi(\cdot)$ depends on the trade probability $p(\cdot)$, which in turn depends on the information cost through the cutoff $\hat{x}(c)$. Since we consider a fixed environment (and therefore, a fixed information cost) in most of the remaining analysis, we typically suppress such dependencies to simplify notation.

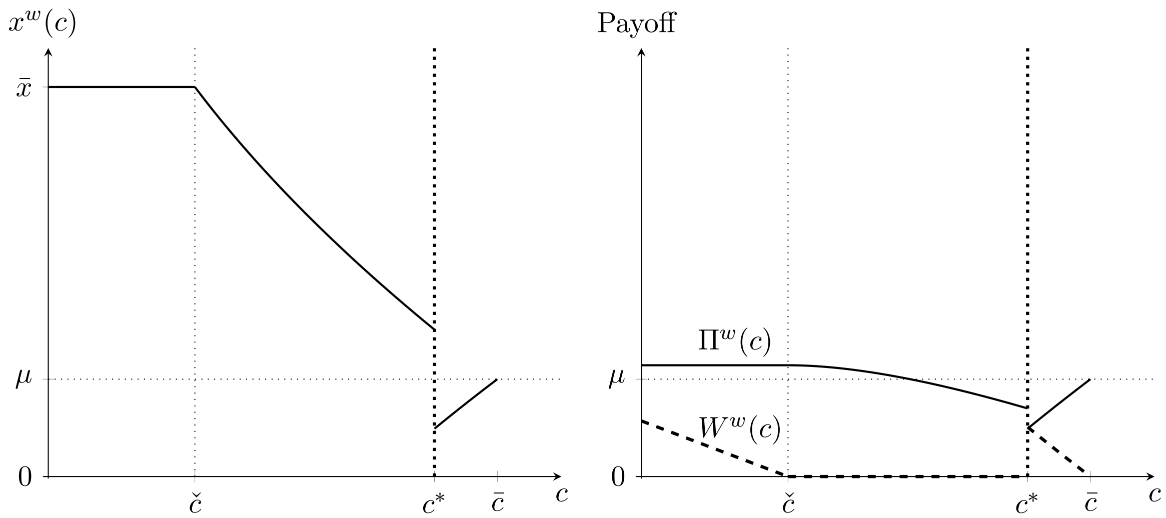

Figure 7: The winning bid $x^w(\cdot)$ (left panel), seller's revenue $\Pi^w(\cdot)$, and winner's payoff $W^w(\cdot)$ (right panel) in the seller's worst-case equilibrium as a function of the information cost c when $\tilde{v} \sim \mathcal{U}(-3, 4)$ and $N = 2$.

in c, while the winner's payoff $W^w(c)$ is strictly decreasing in c.

Furthermore,

$$\lim_{c \uparrow c^*} x^w(c) > x^w(c^*), \quad \lim_{c \uparrow c^*} \Pi^w(c) \geq \Pi^w(c^*), \text{ and } \quad \lim_{c \uparrow c^*} W^w(c) < W^w(c^*).$$

Figure 7 provides a graphical representation of Corollary 1 for a uniform distribution of the valuation. In particular, it shows that $x^w(\cdot)$, $\Pi^w(\cdot)$, and $W^w(\cdot)$ are all weakly decreasing in c for $c < c^*$. However, it also indicates that this region can be split up further: There exists an information cost threshold $\check{c} < c^*$, such that for information costs $c \leq \check{c}$, the worst-case winning bid and revenue for the seller are constant, while the winner's payoff is strictly decreasing. This is because, for such parameters, the relevant range $R(c)$ is large enough to contain the global maximum of the expected revenue curve $\bar{x} > 0$. Instead, when $c \in (\check{c}, c^*)$, bidding $\bar{x}$ is no longer individually rational as information is too expensive. Thus, the restriction to the relevant range is binding, implying that the winner receives an expected payoff of 0, and that the winning bid and seller's revenue are strictly decreasing in c.

Finally, since $c^* = c^*(N)$ is non-decreasing in the number of buyers N (Proposition 2), and N does not enter the equilibrium objects except through the cost threshold $c^*(N)$, Corollary 1 also implies that an increase in the number of buyers always (weakly) increases the winning bid and (weakly) benefits the seller at the expense of the buyers under worst-case equilibrium selection.

4.2 Optimality of Due Diligence

The results from the previous subsection allow us to analyze when due diligence is strictly optimal. Recall that the seller always prefers not providing any information, which results in an expected payoff of μ, over allowing the buyers to conduct research. Therefore, due diligence is strictly optimal if and only if it guarantees an equilibrium revenue strictly larger than μ.

From Proposition 2, we know that the seller's worst-case equilibrium can be of only two different types: Either the winning bid is $\hat{x}$ and trade occurs with certainty, or the winning bid is x^{dd} and the winner performs due diligence. If a sure-trade equilibrium exists, due diligence is not optimal since $\Pi(\hat{x}) = \hat{x} < \mu$. From Proposition 2, we know that this equilibrium exists whenever the information cost is high ($c \geq c^*$). Instead, if this cost is small enough ($c < c^*$), the winner conducts due diligence in every equilibrium. For due diligence to lead to an expected revenue larger than μ, it must therefore be the case that $\max_{x \geq 0} x \cdot [1 - F(x)] > \mu$. However, this condition ignores that the buyers are unwilling to win with bids that lead to a negative expected payoff. In particular, when $c < c^*$, the seller's equilibrium revenue is given by $\Pi(x^{dd}) = \max_{x \in R} x \cdot [1 - F(x)]$. Therefore, due diligence is strictly optimal whenever $\Pi(x^{dd})$ exceeds μ and the information cost is low enough. The following proposition summarizes this reasoning.

Proposition 3. *Due diligence is strictly optimal if and only if*

(i) $c < c^*$, *and*

(ii) $\max_{x \in R} x \cdot [1 - F(x)] > \mu$.

While Proposition 3 provides necessary and sufficient conditions for the optimality of due diligence that depend only on the model's primitives F, c, and N, it is difficult to understand the underlying economic reasons from these conditions alone. Therefore, the remainder of this section aims to clarify the implicit trade-offs. As a first step, the following proposition provides comparative statics on the optimality of due diligence.

Corollary 2. *Suppose due diligence is strictly optimal. Then, the same is true if*

- *The number of buyers increases, i.e,* $N' > N$*;*
- *The buyers' information cost decreases, i.e.,* $c' < c$*;*
- *The buyers' common valuation is scaled up, i.e.,* $\tilde{v}' = K \cdot \tilde{v}$ *for some* $K > 1$*;*
- *The buyers' common valuation is shifted to the left, i.e.,* $\tilde{v}' = \tilde{v} - \omega$ *for some* $\omega \in (0, \mu)$.

A change in the number of buyers affects only the cost threshold for the existence of the sure-trade equilibrium outcome, c^*. Since c^* is increasing in N, condition (i) of Proposition 3 is easier to satisfy the larger the number of buyers. Similarly, a decrease in the information cost c makes both conditions in Proposition 3 easier to satisfy. For condition (i), this is obvious. For condition (ii), recall that a decrease in c expands the relevant range R and therefore (weakly) increases the expected revenue at x^{dd}. Since the model is invariant to the units in which the valuations and costs are denominated, scaling up the valuations has the same effect as a decrease in the cost of information. Finally, a leftward shift in the distribution of the valuation $\tilde{v}$ makes due diligence (weakly) more attractive compared to trading the good sight unseen to *both* the winner and the seller.

Extending on these comparative statics, we can relate the optimality of due diligence to the size of the stakes, i.e., the size of the potential gains and losses from trade. Specifically, we consider changes to the size of the stakes that keep the shape and the mean of the distribution of valuations unchanged. We call such an operation a *mean-preserving scaling.*

Definition 1. *Fix a random variable X with finite expectation μ_X. For any constant $k > 0$, we say that Y is a mean-preserving k-scaling of X if $Y = \mu_X + k \cdot (X - \mu_X)$.*

Our main theorem provides a single-crossing result within any family of mean-preserving scalings.

Theorem 1. *For every distribution F and information cost $c > 0$, there exists a unique $k^* > 0$ such that, whenever $\tilde{v}$ is replaced by its mean-preserving k-scaling $\tilde{v}'_k := \mu + k \cdot (\tilde{v} - \mu)$, the following holds:*

- *No-information is strictly optimal if $k < k^*$;*
- *Due diligence is strictly optimal if $k > k^*$.*

Intuitively, in high-stakes environments, due diligence is attractive to buyers as it carries a high option value. Thus, it holds that $c < c^*$, and the unique equilibrium features due diligence. At the same time, in high-stakes environments, high bids lead to high expected revenue for the seller: Even though the winner might renege on high bids, the probability that he does so is not too large.[27] As a result, the upside potential of selling for a high price more than offsets the seller's revenue loss due to the winner reneging with positive probability.

A key step in proving Theorem 1 is to show that due diligence is strictly optimal whenever the stakes, measured as the expected *absolute value* of the buyers' valuation $\mathbb{E}_F[|\tilde{v}|]$, are high enough. Concretely, in the Appendix, we show that this is the case when the following inequality holds:

$$\mathbb{E}_F[|\tilde{v}|] > \mu \cdot \left[1 + 2\ln\left(\frac{\bar{v}}{\mu}\right)\right] + 2c. \tag{6}$$

To gain some intuition for inequality (6), suppose for a moment that information were costless, i.e., $c = 0$. Then, condition (i) of Proposition 3 would be satisfied trivially, and the relevant range R would coincide with all positive bids. Thus, due diligence would be strictly optimal unless $x \cdot [1 - F(x)] \leq \mu$ for all $x \geq 0$. This inequality implies a lower bound on the cdf of the common valuation: For all $x \geq 0$, it would have to be the case that $F(x) \geq \max\left\{1 - \frac{\mu}{x}, 0\right\}$. This restricts the thickness of the right tail of the distribution, and implies that due diligence is preferred unless the expected absolute value of the valuation is small enough, i.e., unless $\mathbb{E}_F[|\tilde{v}|] \leq \mu \cdot \left[1 + 2\ln\left(\frac{\bar{v}}{\mu}\right)\right]$.[28] In the Appendix, we show that this bound has to be replaced by inequality (6) to incorporate positive information costs.

A mean-preserving upward scaling affects both sides of inequality (6): The expected absolute valuation $\mathbb{E}[|\tilde{v}|]$ and the upper end of the distribution $\bar{v}$ both increase with the scaling parameter k. Nevertheless, since $\bar{v}$ enters logarithmically and $\mathbb{E}_F[|v|]$ is asymptotically linear in k, inequality

[27] Specifically, for any fixed bid $x > \mu$, the probability that a buyer reneges after winning with such a bid, i.e., that $\tilde{v}'_k < x$, is decreasing in the scaling parameter k.

[28] In particular, $1 - F(x) \leq \frac{\mu}{x}$ for all $x \geq \mu$ implies that

$$\int_\mu^{\bar{v}} x\, dF(x) = \mu \cdot (1 - F(\mu)) + \int_\mu^{\bar{v}} [1 - F(x)]\, dx \leq \mu \cdot \left[1 - F(\mu) + \ln\left(\frac{\bar{v}}{\mu}\right)\right],$$

where the equality follows from integration by parts.

(6) must hold for k arbitrarily high. This shows that a large enough mean-preserving scaling is sufficient for the optimality of due diligence. The single-crossing result in Theorem 1 then follows from Corollary 2. In particular, a mean-preserving scaling is a combination of a scaling and a shift in the buyers' valuation. Thus, Corollary 2 implies that if due diligence is strictly optimal under a scaling parameter of k, the same is true under any $k' > k$.

Theorem 1 has an important economic implication: In any high-stakes common value setting, the seller prefers allowing for due diligence over selling the good sight unseen or allowing for research if she is concerned about strategic uncertainty. Since both real estate transactions and mergers and acquisitions frequently exhibit high stakes, this provides a possible rationale for the widespread use of ex post information acquisition in these markets. Moreover, our analysis shows that when due diligence is strictly optimal, the winner always conducts due diligence. Thus, our results predict that we should observe due diligence and, possibly, trade failure in such settings.

5 Discussion

We conclude by discussing how our predictions change if we allow for trade to be ex ante inefficient and for the seller to evaluate her options as if she could select her preferred equilibrium. We then show how our results remain valid when considering several model extensions, such as allowing for repeated informal auctions, for noisy information processing, and for the buyers' commitment to buy the good sight unseen.

5.1 Negative Expected Gains from Trade

Assumption 4 requires that trade is a-priori efficient, i.e., $\mu > 0$. Even though this assumption streamlines the analysis, it is not necessary. In fact, our main takeaway is strengthened in the opposite case: If $\mu \leq 0$, the seller always finds due diligence weakly optimal, regardless of the information cost. Moreover, due diligence is strictly optimal if the information cost is small enough such that buyers might benefit from conducting due diligence. A necessary condition for this to be the case is that the information cost c is lower than the maximum available social trade surplus S. While this condition is implied by Assumption 3 when $\mu > 0$, this is no longer the case when $\mu < 0$. Therefore, we need to assume $c < S$ directly when considering settings where $\mu \leq 0$.[29]

Proposition 4. *If Assumptions 3 and 4 are replaced with $c < S$ and $\mu \leq 0$, respectively, due diligence is strictly optimal.*

When $\mu \leq 0$, an equilibrium resulting in a revenue of 0 exists in both the no information benchmark and the research game. If the buyers cannot acquire information, they choose to abstain, since winning with any positive bid leads to a negative payoff. In the research game, an equilibrium exists where only one buyer becomes informed and bids 0 whenever he learns that the valuation is positive, while all remaining buyers abstain. On the contrary, the expected revenue is strictly positive in all equilibria of the due diligence game as long as due diligence

[29] If $\mu = 0$, Assumption 3 and $c < S$ are equivalent.

matters: If $c < S$, the relevant range R is non-empty, and the unique equilibrium winning bid is $x^{dd} > 0$.[30] Thus, as Proposition 4 states, due diligence is strictly optimal if $\mu \leq 0$ and $c < S$.

5.2 Best-case Selection

Our main analysis focuses on a seller who is concerned about strategic uncertainty and wants to maximize her worst-case equilibrium payoff. The settings we aim to capture, such as the sale of a company or house, typically involve buyers who compete against each other frequently in the purchase of similar assets and a seller who seldom sells such goods.[31] Thus, it seems more plausible that the buyers coordinate on their preferred equilibrium, rather than the seller's. Since the buyers' and the seller's incentives are not aligned, this provides a motivation for our assumption that the seller deals conservatively with strategic uncertainty.[32,33]

For comparison, it is also interesting to ask: What is the optimal timing of information acquisition if the seller evaluates the alternatives using her best-case equilibrium revenue? Toward answering this question, we analyze the following alternative of the seller's problem in this subsection:

$$\max_{\tau \in \{re, dd, no\}} \left(\sup_{\sigma \in \mathcal{E}^\tau} \Pi(\sigma) \right). \qquad \textbf{(P')}$$

Problem (**P'**) is identical to Problem (**P**), except that the infimum over equilibrium revenues is replaced by the supremum. The following proposition characterizes the best-case optimal timing of information acquisition when the information cost is small or large, respectively.[34] In this proposition, we allow trade to be ex ante efficient, as in our main analysis, or ex ante inefficient, as in Section 5.1 above.

Proposition 5. *Suppose Assumptions 1 and 2 hold. Then,*

- *there exists an information cost threshold* $c' \in (0, \min\{\bar{c}, S\})$ *such that research is strictly best-case optimal whenever* $c < c'$.

Furthermore,

- *If* $\mu > 0$, *there exists a cost threshold* $c'' \in [c', S - \mu]$ *such that no-information is strictly best-case optimal whenever* $c \in (c'', \bar{c})$.
- *If* $\mu \leq 0$, *there exists a cost threshold* $c'' \in [c', S)$ *such that due diligence is strictly best-case*

[30] The assumption $c < S$ guarantees that $W^{dd}(0) > 0 > W^{su}(0)$ and therefore, that $\hat{x} < 0$. This implies that the expected revenue function $\Pi(\cdot)$ has a unique local maximum over the relevant range R at x^{dd}. Thus, the equilibrium winning bid is pinned down uniquely.

[31] In mergers and acquisitions, most buyers are institutional investors, who trade companies professionally. In the housing market, the buyers are mainly private individuals. Nevertheless, when they are in the market for a house, they typically submit bids on multiple properties before they are successful and leave the market. Moreover, similar houses in the same area typically attract the same group of potential buyers, leading to repeated competition between them.

[32] Our analysis shows that the seller's least preferred equilibrium coincides with the one that maximizes buyers' joint payoffs in all the games we study.

[33] There are several other reasons motivating a conservative approach toward strategic uncertainty. Perhaps most importantly, no uncontentious theory of equilibrium selection exists to date. Hence, we cannot rule out the possibility of the seller attaching a high probability to her worst-case equilibrium being played.

[34] We say that a timing of information acquisition is best-case optimal if it solves Problem (**P'**) and that it is strictly best-case optimal if it is the unique solution.

optimal whenever $c \in (c'', S)$.

Proposition 5 shows a major difference between best-case selection and our main analysis: Research can be optimal. In fact, for any distribution of valuations, this is the case as long as the information cost c is small enough. In the research game, there always exists an equilibrium in which all buyers randomize between conducting and not conducting research. As the information cost c vanishes, the probability that each buyer conducts research in such an equilibrium increases to 1. Thus, in the limit as $c \to 0$, at least two buyers become informed almost surely, implying that the seller extracts the entire available social surplus S in her preferred equilibrium. On the other hand, in the no-information benchmark and the due diligence game, equilibrium revenue is bounded away from S.

When information is sufficiently costly, the above conclusion is reversed: For high c, either no-information or due diligence maximizes the seller's best-case equilibrium revenue, depending on whether the expected valuation is positive or negative. When $\mu > 0$ and $c > S - \mu$, information acquisition is socially inefficient: In that case, the information cost is larger than the efficiency gains of avoiding trade when the valuation is negative. Accordingly, in such settings, not allowing for information acquisition is revenue maximizing. Instead, when $\mu \leq 0$, not granting information access leads to a revenue of 0, which makes selling the good sight unseen suboptimal irrespective of the information cost. At the same time, research leads to a positive revenue only if multiple buyers become informed. When c is large, the probability of this happening is so small in any equilibrium that the seller is better off allowing for due diligence rather than research.

A corollary of Proposition 5 is that due diligence is strictly best-case optimal when the buyers' valuation is shifted far enough to the left. Such a shift can be understood as an increase in the transaction cost or the seller's reservation value.

Corollary 3. *For every distribution F and information cost c, there exists a nonempty open interval $(\omega', \omega'') \subseteq \mathbb{R}_+$ such that, whenever the buyers' common valuation is shifted to the left by $\omega \in (\omega', \omega'')$, i.e., $\tilde{v}$ is replaced by $\tilde{v}' = \tilde{v} - \omega$, due diligence is strictly best-case optimal.*

Corollary 3 follows from Proposition 5 since a left-shift reduces the maximal available social trade surplus S, while keeping the information cost c unchanged. Thus, if the distribution of valuations is shifted far enough to the left such that S approaches c, due diligence must be preferred.

Proposition 5 might suggest that $\mu \leq 0$ is necessary for the best-case optimality of due diligence. However, this is not case: As Figure 8 shows, due diligence can be best-case optimal for a set of intermediate cost parameters also when $\mu > 0$. Whether this is the case depends on the shape of the distribution. In particular, in Appendix C we show that for any $c > 0$ and $\mu > 0$, one can find a distribution of valuations F consistent with μ such that the seller prefers to allow for due diligence, even if she aims to maximize her best-case equilibrium revenue. To understand this, note that there are two reasons why the seller cannot achieve a revenue equal to the available social surplus S. First, the final allocation of the good may be inefficient. Second, the seller cannot extract the entire social trade surplus whenever (there is a positive chance that) a

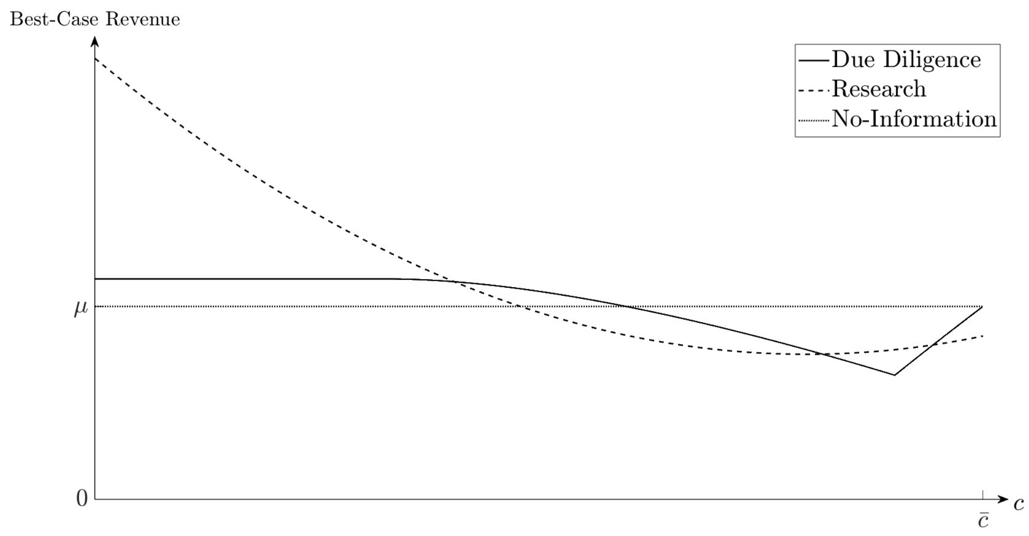

Figure 8: Seller's best-case equilibrium revenue for $c \in (0, \bar{c})$ when $\tilde{v} \sim \mathcal{U}(-3, 4)$ and $N = 2$.

buyer acquires information since the buyers have to be at least reimbursed for their information expenditure. Under due diligence there is at most one buyer incurring the information cost. Thus, due diligence results in a higher equilibrium revenue than any equilibrium of the research game whenever the distribution of valuations is such that the allocative inefficiency induced under due diligence is small.

5.3 Other Model Extensions

In this subsection, we show how our results remain valid when extending our model to allow for repeated informal auctions, for noisy information processing, and for the buyers' commitment to buy the good sight unseen.

Repeated Due Diligence. In the due diligence game, we assume that the seller keeps the good and the game ends if the auction winner reneges, even though gains from trade may still be possible.[35] Thus, we implicitly assume that the seller can commit not to auction off the good again after a negative due diligence. To show that this assumption is not driving our results, we now consider a model of repeated due diligence, where the seller can rerun the due diligence game with the remaining buyers in case the initial winner does not close the deal.

The analysis of repeated due diligence differs in two ways from our main model. For any given bid in the initial auction, the seller achieves a weakly higher expected revenue, since she may be able to sell the good for a positive price even if the first transaction fails. On the other hand, the buyers now may receive a positive payoff from losing the initial auction and being able to participate in the second round, which depresses their willingness to bid aggressively in the first period. While the combination of these two effects makes the revenue comparison of one-shot and repeated due diligence ambiguous, we show in Appendix D that the main insights from our

[35] Instead, in the no-information benchmark and the research game, trade always occurs in any equilibrium as long as it is efficient, i.e., as long as $v > 0$.

analysis continue to hold: The seller strictly prefers repeated due diligence over no-information and research as long as the stakes are sufficiently high.

Noisy Information Processing. The information technology in our main model is noiseless: Any two buyers who process information form identical posterior beliefs. In particular, they both learn the realized valuation v. We now discuss how our results change if we relax this assumption and instead allow for noisy information processing. In particular, we consider the case where a buyer who processes information observes a signal $s_i = v + \theta\varepsilon_i$, where $\varepsilon_i \overset{i.i.d.}{\sim} G_\varepsilon$ is a noise term whose distribution admits a strictly log-concave density,[36] and θ scales the size of the noise term.

For due diligence, only the distribution of the posterior expected valuation matters, which is now distributed according to F_θ, which depends on θ and G_ε. Our analysis of due diligence continues to hold for F_θ. Moreover, as the size of the noise θ decreases, F_θ converges uniformly to F. Therefore, the seller's worst-case equilibrium revenue also converges to that implied by Proposition 2 as $\theta \downarrow 0$.

Under research, adding relatively little noise may alter the equilibrium predictions. In the limit, as $c \downarrow 0$, any noise generates large enough information rents such that all buyers opt to become informed. However, as we show formally in Appendix E, for any fixed information cost, Proposition 1 remains valid as long as the resulting information rents are smaller than the cost of information acquisition.

Commitment to Buy Sight Unseen. So far, we considered buyers who do not have commitment power. In particular, in the due diligence game, buyers decide whether to conduct due diligence only after they have been selected as the winner. However, in some settings, buyers may be able to commit not to conduct due diligence. This is common, for example, in the housing market, where buyers often waive their right to a home inspection. Of course, they do so because it makes the offer more valuable to the seller: If she accepts such an offer, she is guaranteed to receive a revenue equal to the bid.

It turns out that the buyers' ability to commit to waiving due diligence benefits the seller at the expense of the buyers. Thus, due diligence becomes a more attractive option for the seller when the buyers can commit to buying the good sight unseen. Formally, in Appendix F, we show that condition (*ii*) of Proposition 3 alone is necessary and sufficient for due diligence to be strictly optimal given this alternative model specification. Consequently, the remainder of our baseline analysis also applies.

[36] Strict log-concavity of $g_\varepsilon := G'_\varepsilon$ ensures that the buyers' signals are strongly affiliated in the sense of Milgrom and Weber (1982).

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Appendix

The Appendix is organized as follows. Appendix A contains the omitted proofs from Sections 3 and 4.[37] Appendix B considers the case where the expected gains from trade are negative studied in Section 5.1. Appendix C analyzes the seller's problem under best-case equilibrium selection of Section 5.2. The extensions we consider in Section 5.3 are discussed formally in the Supplementary Appendix. Concretely, Appendix D discusses the possibility of repeated due diligence, Appendix E pertains to the case of noisy information processing, and in Appendix F we allow the buyers to commit to buying the good sight unseen in the due diligence game.

A Main Proofs

To simplify notation, throughout this Appendix we change the range of possible bids for the buyers from $X = \{\varnothing\} \cup \mathbb{R}_+$ to $X = \{-1\} \cup \mathbb{R}_+$, i.e., we replace the notation $x_i = \varnothing$ with $x_i = -1$, every time we need to specify a buyer's random strategy during the bidding phase. This change allows us to define a possibly mixed bidding strategy of a buyer by the cumulative distribution function $G_i : X \to [0, 1]$ it induces. Moreover, it implies that when a buyer abstains from bidding with strictly positive probability, his cdf G_i displays a probability mass at -1 given by $G_i(-1) > 0$. In light of this change of notation, we also extend the definitions of the functions $W(\cdot)$, $p(\cdot)$ and $\Pi(\cdot)$ to the whole domain X by setting $W(-1) := 0$, $p(-1) := 0$ and $\Pi(-1) := 0$.

Proof of Proposition 1

As we describe in Section 3, we prove Proposition 1 by showing that there exists an equilibrium of the research game where the seller's payoff is strictly less than $\mu > 0$. To do so, we use a backward induction approach. As a first step, note that the seller's equilibrium behavior in the winner selection phase of the research game is trivial: She always accepts one of the highest available offers. Given this observation, from now on we can focus on the buyers' equilibrium incentives in the previous phases of the game. Specifically, in Part 1 of this proof, we establish buyers' equilibrium behavior during the bidding phase taking their research-phase information choices as given. In Part 2, we use this equilibrium continuation behavior to study agents' incentives during the research phase and prove the result.

Proof of Proposition 1: Part 1

For every subset $J \subseteq I$, let Γ^J denote the continuation (bidding) game that follows a research phase where the buyers doing research are exactly those in J. As shown below, Γ^J always admits an equilibrium for every J. Given this, denote with U_i^J buyer i's payoff, and with Π^J the seller's revenue in a particular equilibrium of the continuation game Γ^J, respectively. The following sequence of lemmas characterizes these payoffs as a function of the cardinality of J.

Lemma 3. *Suppose $|J| = 0$, i.e., $J = \emptyset$. Then, in any equilibrium of the continuation game $\Gamma^{\emptyset}$, the payoffs are given by $U_i^{\emptyset} = 0$ and $\Pi^{\emptyset} = \mu$.*

[37] We omit the proof of Lemma 1 and 2 as they follow directly from arguments in the main text.

Proof of Lemma 3. If no buyer acquired information during the research phase, the continuation game $\Gamma^{\emptyset}$ is strategically equivalent to the no-information benchmark. Therefore, invoking Proposition 0 is sufficient to prove the result.

□

Lemma 4. *Suppose $|J| \geq 2$. Then, in any equilibrium of the continuation game Γ^J, the payoffs are given by $U_i^J = 0$ if $i \notin J$, $U_i^J = -c$ if $i \in J$, and $\Pi^J = S$.*

Proof of Lemma 4. By standard arguments, if at least two buyers learned the valuation v, none of them can achieve a positive equilibrium payoff, since one of the other buyers would have a strictly positive deviation otherwise. If $v < 0$, the informed buyers must submit a bid with which they never win. For any $v > 0$, at least two of the informed buyers must submit a bid of $x_i^*(v) = v$, unless an uninformed buyer bids more than v with certainty. However, this is impossible since if any uninformed buyer did so, at least one of them would receive a strictly negative payoff, which is impossible in equilibrium since buyers can abstain from bidding. To see this, fix any of these uninformed buyers, say buyer $j \in I$. At his lowest submitted bid $\underline{x}_j \geq v > 0$, he can only win if $\tilde{v} \leq \underline{x}_j$, which implies that he receives a strictly negative payoff conditional on winning. Since at least one of these uninformed buyers must win with strictly positive probability when submitting the bid $\underline{x}_j$, we reached a contradiction. Thus, for any $v > 0$, at least two informed buyers bid $x_i^*(v) = v$.

Therefore, all equilibria of this continuation game must be payoff equivalent to the outcome induced by the following strategies:

- After observing the valuation $v \in V$, any informed buyer $i \in J$ bids $x_i^*(v)$ given by

$$x_i^*(v) = \begin{cases} v & \text{if } v \geq 0, \\ \varnothing & \text{otherwise;} \end{cases}$$

- Any uninformed buyer $i \in I \setminus J$ abstains from bidding, i.e., $x_i^* = \varnothing$.

The above profile of strategies constitutes an equilibrium of Γ^J since no buyer has a strict incentive to deviate from his equilibrium bid. Moreover, it holds that $U_i^J = 0$ if $i \notin J$, $U_i^J = -c$ if $i \in J$, and $\Pi^J = S$, as required.

□

Lemma 5. *Suppose $|J| = 1$, i.e., $J = \{i\}$ for some $i \in I$. Then, in any equilibrium of the continuation game $\Gamma^{\{i\}}$ the payoffs are given by $U_i^{\{i\}} > 0$, $U_j^{\{i\}} = 0$ for $j \neq i$, and $\Pi^{\{i\}} < \mu$.*

Proof of Lemma 5. As a first step, we establish the following claim.

Claim 1. *A strategy profile constitutes an equilibrium of $\Gamma^{\{i\}}$ if and only if:*

- *After observing the valuation $v \in V$, the informed buyer bids $x_i^*(v)$ given by*

$$x_i^*(v) = \begin{cases} \max\left\{\mathbb{E}\left[\tilde{v} \,\middle|\, \tilde{v} < v\right], 0\right\} & \text{if } v > 0, \\ \varnothing & \text{if } v < 0; \end{cases}$$

- *The maximum bid of the uninformed buyers, $\max_{j \neq i} x_i^*$ is distributed according to the cdf G given by*

$$G(x) = \begin{cases} \exp\left(-\int_0^\mu \frac{1}{\hat{v}(t)-t}\, dt\right) & \text{if } x = -1, \\ \exp\left(-\int_x^\mu \frac{1}{\hat{v}(t)-t}\, dt\right) & \text{if } x \in [0, \mu), \\ 1 & \text{if } x \geq \mu, \end{cases}$$

where $\hat{v}(x) \in (0, \bar{v})$ is defined as the unique solution to

$$\mathbb{E}\left[\tilde{v} \,\middle|\, \tilde{v} < \hat{v}(x)\right] = x \tag{7}$$

for all $x \in (0, \mu)$.

- *No uninformed buyer $j \neq i$ has a strict incentive to deviate from x_j^*.*

Proof of Claim 1. Except for the presence of a reservation value, $\Gamma^{\{i\}}$ is equivalent to a special case of the model analyzed by Engelbrecht-Wiggans, Milgrom, and Weber (1983). As a result, a straightforward extension of the proof of Theorem 1 in Engelbrecht-Wiggans, Milgrom, and Weber (1983) suffices to prove Claim 1.

Here, for the sake of completeness, we illustrate the sufficiency part of Claim 1. In particular, we show that an equilibrium exists where buyer i behaves as described in the claim, some buyer $j \neq i$ randomizes according to the distribution G, and all buyers $m \in I \setminus \{i, j\}$ abstain from bidding with certainty. We need to show that no buyer has a strict incentive to deviate from this strategy profile given the behavior of the other agents. We start by investigating buyer j's equilibrium incentives.

Buyer j: From an inspection of the support of G, we deduce that either buyer j abstains from bidding ($G(-1) > 0$), or places an offer x_j such that $x_j \in [0, \mu]$. In what follows, we show that any bid $x_j \in [0, \mu] \cup \{\varnothing\}$ leads buyer j to earn an expected payoff of 0 given the other buyers' strategies. Instead, bidding any $x_j > \mu$ would induce a strictly negative payoff. This would prove that bidding according to G is indeed a best response for j.

Suppose first that buyer j bids $x_j > \mu$. Given the other buyers' strategies, buyer j would win the auction with certainty with this bid. (The informed buyer never bids above μ.) Therefore, buyer j's expected payoff would equal $\mu - x_j < 0$ under such a bid, as required.

If buyer j abstains from bidding, i.e., $x_j = \varnothing$, then his payoff would be 0. Thus, we are left to show that any bid $x_j \in [0, \mu]$ also leads to an expected payoff of 0. To do so, fix any $x_j \in [0, \mu]$. Notice that under this bid, buyer j wins the auction if and only if $x_i^*(\tilde{v}) < x_j$.[38] Therefore,

[38] This holds since $x_i^*(\tilde{v}) = x_j$ is a probability zero event, which implies that the seller's winner selection in

buyer j's expected payoff conditional on winning is

$$U_j^*(x_j) = \mathbb{E}\Big[\tilde{v} \,\Big|\, x_i^*(\tilde{v}) < x_j\Big] - x_j.$$

However, by construction it holds that $x_i^*(\tilde{v}) < x_j$ if and only if $\tilde{v} < \hat{v}(x_j)$, where $\hat{v}(x)$ is defined according to equation (7). This implies that $U_j^*(x_j) = 0$, as required.

Buyer $m \in I \setminus \{i, j\}$**:** Since buyer j's equilibrium behavior is statistically independent of buyer i's equilibrium behavior (and thus of his private information), each buyer $m \in I \setminus \{i, j\}$ faces the following equilibrium trade-off: Bidding any $x_m \in [0, \mu] \cup \{\varnothing\}$ would yield an expected payoff of 0, while bidding any $x_m > \mu$ would yield a strictly negative expected payoff. It follows that abstaining from bidding with probability 1 is a best response for all such buyers, as required.

Buyer i**:** Finally, consider buyer i's equilibrium incentives. Suppose first that buyer i observed a strictly negative valuation when he acquired information during the research phase, i.e., $v < 0$. Since buyer j abstains from bidding with strictly positive probability ($G(-1) > 0$), while each buyer $m \in I \setminus \{i, j\}$ abstains from bidding with certainty, any positive bid would induce buyer i to win the auction with strictly positive probability. However, this would result in a strictly negative payoff for him. Therefore, it must be that $x_i^*(v) = \varnothing$ for $v < 0$, as required.

We are left to show that $x_i^*(v) = \max\left\{\mathbb{E}\left[\tilde{v} \,\middle|\, \tilde{v} < v\right], 0\right\}$ is a best response for buyer i when v is strictly positive. To do so, fix $v \in (0, \bar{v}]$ arbitrarily. Given the other buyers' strategies, buyer i would never bid $x_i > v$ or abstain since he can guarantee a positive payoff by bidding $x_i = 0$. Instead, if buyer i bids $x_i \in [0, v]$, he gets a payoff equal to

$$U_i(x_i|v) = G(x_i) \cdot (v - x_i).$$

Note that $\frac{\partial}{\partial x_i} U(x_i|v) \geq 0$ if and only if $v \geq \hat{v}(x_i)$, whenever $x_i \in [0, v]$. In turn, $v \geq \hat{v}(x_i)$ is equivalent to $\mathbb{E}\left[\tilde{v} \,\middle|\, \tilde{v} < v\right] \geq x_i$. Therefore, we conclude that

$$x_i^*(v) = \max\left\{\mathbb{E}\left[\tilde{v} \,\middle|\, \tilde{v} < v\right], 0\right\}$$

is in fact a best response for buyer i when v is strictly positive, as required. This completes the proof of equilibrium existence in Claim 1.

□

To conclude the proof of Lemma 5, fix any equilibrium of $\Gamma^{\{i\}}$. We want to show that $U_i^{\{i\}} > 0$, $U_j^{\{i\}} = 0$ for $j \neq i$, and $\Pi^{\{i\}} < \mu$. That all uninformed buyers earn an equilibrium payoff of 0 follows immediately from the informed buyer's bidding function $x_i^*(v)$ described in Claim 1. That $\Pi^{\{i\}} < \mu$ instead follows from the fact that the buyers make offers strictly below μ almost surely. Finally, to see why buyer i earns a strictly positive overall equilibrium payoff $U_i^{\{i\}}$, notice that, by definition, his equilibrium strategy $v \mapsto x_i^*(v)$ must be better than always abstaining

case of a tie does not matter.

whenever $v < \mu$, and bidding exactly μ otherwise. Since the uniformed buyers never bid above μ according to G, this alternative strategy would yield buyer i an overall expected payoff of

$$\bar{U}_i = -c + \int_{\mu}^{\bar{v}} (v - \mu)\, dF(v).$$

Now, recall from Assumption 3 that $c < \bar{c}$, where

$$\bar{c} := \int_{\underline{v}}^{\mu} (\mu - v)\, dF(v).$$

Thus, $\bar{U}_i = \bar{c} - c > 0$. Since, $U_i^{\{i\}} \geq \bar{U}_i$, we conclude that $U_i^{\{i\}} > 0$, as required.

□

Proof of Proposition 1: Part 2

We now discuss the buyers' equilibrium incentives during the research phase. In particular, consider the game's outcome where only buyer i acquires information during the research phase, i.e., the play reaches the continuation game $\Gamma^{\{i\}}$, with certainty. Given Lemmas 3, 4, and 5, no buyer has a strict incentive to deviate from this outcome during the research phase. Thus, only one buyer becoming informed followed by any equilibrium behavior consistent with Claim 1 constitutes an equilibrium of the research game. Observe that Lemma 5 implies that the seller earns a revenue strictly less than μ in this equilibrium. Therefore,

$$\inf_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma) < \mu,$$

as required. This concludes the proof of Proposition 1.

Q.E.D.

Proof of Proposition 2

The proof of Proposition 2 is divided into three parts. In Part 1, we provide a characterization of the relevant bidding range $R = \{x \geq 0 : W(x) \geq 0\}$, and prove some useful properties that the set $X^{\max}$ defined in equation (4) satisfies. In Part 2, we analyze how $X^{\max}$ affects the equilibrium outcomes of the due diligence game. In Part 3, we use the results of Parts 1 and 2 to show how these equilibrium outcomes behave as a function of the buyers' information cost parameter $c > 0$, completing the proof.

Proof of Proposition 2: Part 1

Let $\check{x} = \check{x}(c)$ be the (possibly negative) bid such that $W^{dd}(\check{x}) = 0$, and recall the properties of $\hat{x} = \hat{x}(c)$ established in footnote 20. In this part of the proof, we state and prove the following preliminary lemma.

Lemma 6. *The following statements hold:*

(i) $\check{x} = \check{x}(c)$ is a well-defined continuously differentiable function of $c > 0$. Moreover, it is

strictly decreasing in $c > 0$*, and satisfies*

$$\lim_{c \downarrow 0} \check{x}(c) = \bar{v} \quad \text{and} \quad \lim_{c \uparrow \infty} \check{x}(c) = -\infty.$$

(ii) $\check{x}(c) > 0$ *for all* $c \in (0, \bar{c})$.

(iii) $\hat{x}(c) = \check{x}(c)$ *if and only if* $c = \bar{c}$*. Therefore,* $\hat{x}(c) < \check{x}(c)$ *for all* $c \in (0, \bar{c})$.

(iv) $R = [0, \check{x}]$.

(v) $X^{\max} \subseteq \{\hat{x}, x^{dd}\} \cap (0, \infty)$.

Proof of Lemma 6. By definition, $\check{x} = \check{x}(c)$ satisfies the equation

$$c = \int_{\check{x}}^{\bar{v}} (v - \check{x})\, dF(v). \tag{8}$$

Given this, all the assertions made in statement *(i)* hold: The intermediate value theorem guarantees that a solution $\check{x} \in (-\infty, \bar{v})$ to equation (8) exists for all $c > 0$. Uniqueness of this solution follows from the strict monotonicity of $x \mapsto \int_x^{\bar{v}} (v - x)\, dF(v)$. The Implicit Function Theorem ensures that $\check{x}(c)$ is continuously differentiable. Finally, that $\check{x}(c)$ is strictly decreasing in $c > 0$, and satisfies $\lim_{c \downarrow 0} \check{x}(c) = \bar{v}$ and $\lim_{c \uparrow \infty} \check{x}(c) = -\infty$ follows immediately from equation (8).

To prove statement *(ii)*, it is sufficient to show that $W^{dd}(0) = S - c > 0$ for all $c \in (0, \bar{c})$, where S is defined by equation (1). Recall that, $\bar{c} = \int_{\underline{v}}^{\mu} (\mu - v)\, dF(v)$ by definition (see Assumption 3). Hence, $\bar{c} < S$ if and only if

$$\int_0^{\mu} (-v)\, dF(v) < \mu \cdot (1 - F(\mu)).$$

Assumption 4 ensures that $\mu > 0$, which implies that the above strict inequality holds. Thus, $W^{dd}(0) > 0$ for all $c \in (0, \bar{c})$, as required.

To prove statement *(iii)*, note that $\hat{x}(c) = \mu = \check{x}(c)$ whenever $c = \bar{c}$. This proves the "*if*" part. The "*only if*" part and the fact that $\hat{x}(c) < \check{x}(c)$ for all $c \in (0, \bar{c})$ instead follow from the fact that $c \mapsto \hat{x}(c) - \check{x}(c)$ is a strictly increasing function of $c > 0$ (see footnote 20).

Statement *(iv)* follows from the definition of $\check{x}$, and the fact that $W(x) \geq 0$ if and only if $W^{dd}(x) \geq 0$.

Finally, $\Pi(\cdot)$ can admit at most two local maxima over R: One at $\hat{x}$ if $\hat{x} > 0$, and one at x^{dd} if $x^{dd} > \hat{x}$. To see why, note that $\Pi(\cdot)$ is strictly increasing on $[0, \hat{x}]$, and that by regularity of F, $x \mapsto x \cdot [1 - F(x)]$ is first strictly increasing, then strictly decreasing in x. Since, furthermore, $\Pi(0) = 0$ and $\Pi(x) > 0$ for all $x \in (0, \check{x})$, statement *(v)* follows.

This concludes the proof of Lemma 6.

□

Proof of Proposition 2: Part 2

The following lemma analyzes the equilibrium outcomes of the due diligence game in terms of the elements of $X^{\max}$.

Lemma 7. *Consider the due diligence game. The following statements hold:*

(a) If $X^{\max} = \{\hat{x}\}$ the winning bid is $\hat{x}$, and the winner buys the good sight unseen in every equilibrium. Thus, the seller's equilibrium revenue is unique and given by $\Pi^* = \Pi(\hat{x}) = \hat{x}$.

(b) If $X^{\max} = \{x^{dd}\}$, multiple equilibria may exist. In particular,

(i) There always exists an equilibrium where the winning bid is x^{dd} and the winner conducts due diligence. In this equilibrium, the seller's equilibrium revenue is given by $\Pi^* = \Pi(x^{dd}) = x^{dd} \cdot (1 - F(x^{dd}))$.

(ii) An equilibrium where the winning bid is $\hat{x}$ and the winner buys the good sight unseen exists if and only if $\hat{x} > 0$ and $W(\hat{x}) \geq N \cdot W(x)$ for all $x \neq \hat{x}$ such that $\Pi(x) \geq \Pi(\hat{x})$. In any such equilibrium, the seller's equilibrium revenue is given by $\Pi^* = \Pi(\hat{x}) = \hat{x}$.

(iii) An equilibrium inducing an outcome where the winning bid is non-random can only be of a type described in statements (b.i) or (b.ii) above.

(iv) An equilibrium inducing an outcome where the winning bid is random can only exist if $\hat{x} > 0$ and $W(\hat{x}) \geq N \cdot W(x)$ for all $x \geq 0$ such that $\Pi(x) > \Pi(\hat{x})$. In any such equilibrium, the seller's expected payoff lies in the range $[\Pi(\hat{x}), \Pi(x^{dd})]$.

(c) Otherwise, i.e., if $X^{\max} = \{\hat{x}, x^{dd}\}$, the seller's equilibrium revenue is given by $\Pi^* = \Pi(\hat{x}) = \Pi(x^{dd})$ in any equilibrium. Moreover, an equilibrium where the winning bid is $\hat{x}$ and the winner buys the good sight unseen exists.

Proof of Lemma 7. We first prove part *(a)*.

Proof of (*a*): Suppose $X^{\max} = \{\hat{x}\}$. Note that statement *(v)* of Lemma 6 implies that $\hat{x} > 0$. We first show that no bid $x > \hat{x}$ can win with positive probability. For $x \notin R$, this is the case since buyers can always abstain. To see why $x \in R$ is also impossible, note that any buyer strictly prefers winning the auction with a bid of $\hat{x}$ to winning with a bid of $x > \hat{x}$, and that the seller strictly prefers accepting $\hat{x}$ over accepting any $x \in R$ such that $x > \hat{x}$. Thus, if some bidder had a positive probability of winning with a bid of $x \in R$ such that $x > \hat{x}$, he could profitably deviate to bidding $\hat{x}$, which would weakly increase his winning probability while strictly increasing his expected payoff conditional on winning. On the other hand, standard Bertrand arguments show that no bid $x < \hat{x}$ can win with positive probability. Thus, the winning bid equals $\hat{x}$ in every equilibrium.[39] Given this, the fact that the winner buys the good sight unseen follows from Lemma 1 (see also footnote 23). This completes the proof of part *(a)*.

[39] In particular, at least two buyers bid exactly $\hat{x}$ in every equilibrium. Furthermore, all buyers bidding $\hat{x}$ can be supported as an equilibrium outcome.

Proof of (*b*): We now move to the proof of part *(b)*. To do so, for the remainder of this proof, assume that $X^{\max} = \{x^{dd}\}$, where x^{dd} is defined as in (3).

Proof of statement (*b.i*): Consider any winner selection rule $\beta : X^N \setminus \{(\varnothing, ..., \varnothing)\} \to \Delta(I)$ such that, whenever a profile of bids $\vec{x} \in R^N$ containing x^{dd} is submitted, $\beta(\vec{x})$ selects a winner among the buyers bidding x^{dd} uniformly at random. That is,

$$\beta_i(\vec{x}) = \frac{1}{|J^{dd}_{\vec{x}}|} \cdot \mathbb{1}_{(i \in J^{dd}_{\vec{x}})},$$

for all $\vec{x} \in R^N$ such that $x^{dd} \in \vec{x}$, where $J^{dd}_{\vec{x}} := \{j \in I : x_j = x^{dd}\}$ is the set of buyers bidding x^{dd} in $\vec{x}$. Note that since $\Pi(x^{dd}) > \Pi(x)$ for all $x \in R$, such a winner selection rule is consistent with Lemma 2 and, therefore, can be played in equilibrium by the seller. Given this rule, suppose all buyers respond by bidding x^{dd}. We show that $\vec{x} = (x^{dd}, ..., x^{dd})$ constitutes an equilibrium profile of bids. To see this, note that no buyer has a strict incentive to bid outside of the relevant bidding range R. Moreover, given β and $x_{-i} = \vec{x} \setminus \{x_i\}$, if buyer i bids any price $x_i \in R$ such that $x_i \neq x^{dd}$, he will surely lose the auction obtaining a payoff of 0, while if he offers $x_i = x^{dd}$, he will get an expected payoff of $\frac{1}{N} W(x^{dd}) \geq 0$. This proves that an equilibrium of the due diligence game where the winning bid is x^{dd} always exists. Finally, note that since $x^{dd} \cdot (1 - F(x^{dd})) > \hat{x}$, it holds that $\hat{x} < x^{dd}$. Therefore, in this equilibrium, the winner must conduct due diligence (Lemma 1). This concludes the proof of statement ($b.i$).

Proof of statement (*b.ii*): We first show that the "*if*" direction holds. To do so, suppose that $\hat{x} > 0$ and that $W(\hat{x}) \geq N \cdot W(x)$ for all $x \neq \hat{x}$ such that $\Pi(x) \geq \Pi(\hat{x})$. Consider the following strategic situation: Every buyer submits a bid $x_i = \hat{x}$ and buys the good sight unseen if selected as the winner. On the other hand, the seller responds to the profile of bids $\vec{x} = (\hat{x}, ..., \hat{x})$ by selecting a winner uniformly at random, i.e.,

$$\beta_i(\vec{x}) = \frac{1}{N}, \quad \forall i \in I,$$

while he responds to any other profile of bids $\vec{x}' \in X^N \setminus \{(\varnothing, ..., \varnothing)\}$ in a way that is consistent with Lemma 2. We show that this configuration of strategies constitutes an equilibrium of the due diligence game. First, during the due diligence phase, buyers behave as prescribed by Lemma 1. Second, the winner selection rule described above is consistent with Lemma 2. Finally, no buyer has a strict incentive to deviate from offering $x_i = \hat{x}$ during the bidding phase. To see this, observe that given $x_{-i} = \vec{x} \setminus \{x_i\}$ and β, if buyer i bids $x_i = \hat{x}$, he will receive a payoff of $\frac{1}{N} W(\hat{x})$, which is strictly positive by Assumption 3. If he bids $x_i \neq \hat{x}$ such that $\Pi(x_i) < \Pi(\hat{x})$, he will not be selected as the winner, implying that he will receive a payoff of 0. Finally, if he bids $x_i \neq \hat{x}$ such that $\Pi(x_i) \geq \Pi(\hat{x})$, he will be selected as the winner with probability of at most 1. However, by assumption, his payoff will satisfy $W(x_i) \leq \frac{1}{N} W(\hat{x})$. This completes the proof of the "*if*" direction.

We now prove the "*only if*" direction. As a first step, we show that it must be that $\hat{x} > 0$.

Lemma 8. *Suppose an equilibrium inducing with positive probability a winning bid different from x^{dd} exists. Then, it holds that $\hat{x} > 0$.*

Proof of Lemma 8. By contradiction, suppose that $\hat{x} \leq 0$. Then, $\Pi(x) = x \cdot [1 - F(x)]$, and $W(x) = W^{dd}(x)$ for all $x \geq 0$. Since both $x \mapsto x \cdot [1 - F(x)]$ and $x \mapsto W^{dd}(x)$ are continuous functions of $x \geq 0$, standard Bertrand arguments show that the winning bid is non-random and equals the offer $x^* \geq 0$ that maximizes the seller's revenue curve subject to the constraint $W^{dd}(x^*) \geq 0$ in every equilibrium. When $\hat{x} \leq 0$, this is the case if and only if $x^* = x^{dd}$. Thus, an equilibrium outcome inducing a winning bid different from x^{dd} can only exist if $\hat{x} > 0$, as required.

□

Given Lemma 8, let $\hat{x} > 0$, and suppose by contradiction that there exists $x^* \geq 0$ such that $\Pi(x^*) > \Pi(\hat{x})$, but $W(\hat{x}) < N \cdot W(x^*)$.[40] Also, suppose an equilibrium with $\hat{x}$ as the winning bid exists. According to Lemma 2, no buyer must be bidding any bid $x \neq \hat{x}$ such that $\Pi(\hat{x}) < \Pi(x)$ with positive probability in this equilibrium. Now, fix any buyer that is selected as the winner in this equilibrium with a probability weakly lower than $\frac{1}{N}$. (Such buyer necessarily exists.) Without loss of generality, suppose one such buyer is buyer $i \in I$. By assumption, buyer i cannot be earning an equilibrium payoff larger than $\frac{1}{N}W(\hat{x})$. However, if he deviates to $x_i = x^*$, his bid will be selected with certainty, implying that he will earn a payoff of $W(x^*) > \frac{1}{N}W(\hat{x})$. This contradicts the optimality of buyer i's equilibrium strategy. This proves the "*only if*" direction of statement (*b.ii*), as required.

Proof of statement (*b.iii*): Suppose by contradiction that an equilibrium where the winning bid $x^* \geq 0$ is non-random and such that $x^* \notin \{\hat{x}, x^{dd}\}$ exists. If $x^* \notin R$, any winning buyer would find it strictly optimal to deviate from their strategy and abstain with certainty. To see why $x^* \in R$ is also impossible, recall that only $\hat{x}$ and x^{dd} can be local maximum points of $\Pi(\cdot)$ in the relevant range R (Lemma 6). Therefore, because the winner's value function $x \mapsto W(x)$ is continuous in $x \geq 0$, standard Bertrand arguments imply that we can always find at least one buyer who has a strict incentive to make an offer x' such that $\Pi(x') = \Pi(x^*) + \varepsilon$ to be selected as the winner with certainty, as long as $\varepsilon > 0$ is small enough,.

Proof of statement (*b.iv*): Fix any equilibrium of the due diligence game inducing an outcome where the winning bid is random. For every $i \in I$, denote with G_i the cdf over bids induced by buyer i's equilibrium strategy, and with U_i^* buyer i's equilibrium payoff. Also, denote with $\mathcal{X}_i := \operatorname{supp} G_i$ the support of G_i, and with $J^* \subseteq I$ the subset of buyers that are selected by the seller as a winner with positive probability in equilibrium. From Lemma 8, we know that it must be the case that $\hat{x} > 0$. Given this, let

$$x^o := \min\{x \geq 0 : x \cdot [1 - F(x)] = \hat{x}\}.$$

[40] Note that by regularity of F, the set $\{x \neq \hat{x} : \Pi(x) \geq \Pi(\hat{x})\}$ is an interval strictly above $\hat{x}$. Thus, by the continuity of $W(\cdot)$, if there exists $x \neq \hat{x}$ such that $N \cdot W(x) > W(\hat{x})$ and $\Pi(x) \geq \Pi(\hat{x})$, there also exists $x' \neq \hat{x}$ such that $N \cdot W(x') > W(\hat{x})$ and $\Pi(x') > \Pi(\hat{x})$.

Note that x^o is a well-defined non-negative bid, and satisfies $\hat{x} < x^o \leq x^{dd}$ and $\Pi(x^o) = \Pi(\hat{x})$.[41] Since $x \mapsto W(x)$ is a decreasing function of $x \geq 0$, to prove statement $(b.iv)$, it is sufficient to show that $W(\hat{x}) \geq N \cdot W(x^o)$, and that the seller earns more than $\Pi(\hat{x})$ in equilibrium. We do so below, after first establishing a series of preliminary lemmas.

Lemma 9. *An equilibrium inducing an outcome where the seller's equilibrium revenue is strictly random must satisfy* $|J^*| \geq 2$.

Proof of Lemma 9. Suppose by contradiction that $|J^*| = 1$,[42] i.e., there is only one buyer that is selected by the seller as the winner of the auction in equilibrium. Without loss of generality, suppose such buyer is buyer 1. Since, by assumption, the winning bid is random, it must be that G_1 is non-degenerate, i.e., $\mathcal{X}_1$ is not a singleton. We show that this leads to a contradiction. To see this, note that if $\mathcal{X}_1$ is not a singleton, buyer 1 must be indifferent between winning with any bid in $\mathcal{X}_1$. This is only possible when $\mathcal{X}_1 \subseteq [\bar{v}, \infty)$, since $W(x)$ is constant for all $x \geq \bar{v}$ while it is strictly decreasing in $x \in [0, \bar{v})$. However, in this case, buyer 1 would strictly gain by abstaining from bidding. Hence, it must be the case that $|J^*| \geq 2$, as required.

□

Lemma 10. *An equilibrium inducing an outcome where the seller's equilibrium revenue is strictly random must satisfy* $U_j^* > 0$ *for all* $j \in J^*$.

Proof of Lemma 10. Fix $j \in J^*$ arbitrarily. Since buyers can always abstain from bidding, it must be that $U_i^* \geq 0$ for all $i \in I$. Now, suppose by contradiction that $U_j^* = 0$. Since no buyer would submit a bid $x > \check{x}$ in equilibrium if he believes such bid will be accepted with positive probability, it must be that buyer j wins the auction only when he offers $x_j = \check{x}$. There are two cases: Either $\check{x} = x^{dd}$ or $\check{x} \neq x^{dd}$. If $\check{x} \neq x^{dd}$, then $\check{x} > x^{dd}$ by Lemma 6, which implies that $x_j^\varepsilon := \min\{x \geq 0 : \Pi(x) = \Pi(\check{x}) + \varepsilon\}$ would be a strictly profitable deviation for buyer j for $\varepsilon > 0$ small enough.[43] Indeed, according to Lemma 2, x_j^ε has a weakly higher chance of being selected by the seller than $\check{x}$. Furthermore, it also satisfies $W(x_j^\varepsilon) > 0 = W(\check{x})$. This shows that $\check{x} \neq x^{dd}$ is impossible. We are left to show that $x^{dd} = \check{x}$ is also impossible. Toward a contradiction, suppose that $x^{dd} = \check{x}$ holds. Since the winning bid is random in the equilibrium under consideration, and $X^{\max} = \{x^{dd}\} = \{\check{x}\}$, it must be the case that no buyer submits the offer x^{dd} with certainty in equilibrium. But then, buyer j has a strict incentive to replace $\check{x}$ with $x_j^\varepsilon := \check{x} - \varepsilon$ for $\varepsilon > 0$ small enough. To see this, note that since $\Pi^* = \Pi(x^{dd})$ with probability strictly smaller than 1, there must exist $\varepsilon > 0$ small enough such that x_j^ε is selected by the seller with positive probability if the other buyers play according to their equilibrium strategies. However, $W(x_j^\varepsilon) > 0 = W(\check{x})$ for all $\varepsilon > 0$. This is a contradiction. We conclude that $U_j^* > 0$ for all $j \in J^*$, as required. □

[41] This follows from the continuity of $\Pi(x)$ for $x > \hat{x}$, and the fact that $\hat{x} > 0$ and $x^{dd} \in X^{\max}$.
[42] Clearly, $|J^*| \geq 1$, since J^* cannot be empty by definition.
[43] Since $X^{\max} = \{x^{dd}\}$, x_j^ε is well defined and satisfies $x_j^\varepsilon < x^{dd} < \check{x}$ whenever $\varepsilon > 0$ is small.

For every $i \in I$, denote with $S_i \in \Delta(\mathbb{R}_+)$ the cdf over the expected revenue that the seller would receive if she accepted buyer i's equilibrium offer with probability 1, i.e.,

$$S_i(\theta) := G_i\left(\left\{x \in \mathcal{X}_i : \Pi(x) \le \theta\right\}\right), \quad \forall \theta \ge 0.$$ [44]

Let $\underline{s}_i := \inf(\operatorname{supp} S_i) \ge 0$ for every $i \in I$.

Lemma 11. *An equilibrium inducing an outcome where the seller's equilibrium revenue is strictly random must satisfy*

$$\underline{s}_j = \underline{s}, \quad \forall j \in J^*. \tag{9}$$

Proof of Lemma 11. By contradiction, suppose that (9) does not hold. Then, there must exist $j_1, j_2 \in J^*$, with $j_1 \ne j_2$,[45] such that $\underline{s}_{j_1} < \underline{s}_{j_2}$. Note that this implies that buyer j_1 submits with positive probability a bid that the seller will reject with probability 1. Since buyer j_1 must be indifferent between all his equilibrium bids, it follows that $U^*_{j_1} = 0$. However, this is impossible by Lemma 10. Thus, equation (9) must hold, as required.

□

Lemma 12. *An equilibrium inducing an outcome where the winning bid is random must satisfy*

$$\underline{s} = \Pi(\hat{x}) = \hat{x}. \tag{10}$$

Proof of Lemma 12. We prove the statement by contradiction. Suppose first that $\underline{s} > \hat{x}$. Then, equilibrium winning bids must all be in the range $(\hat{x}, \check{x}]$. Note that, in this range, $\Pi(x) = x \cdot [1 - F(x)]$ and $W(x) = W^{dd}(x)$. Therefore, arguments similar to those employed in the proof of Lemma 8 imply that no equilibrium where the implied seller's revenue is strictly random can exist, a contradiction.

Alternatively, suppose that $\underline{s} < \hat{x}$. Fix $j \in J^*$ arbitrarily, and let $x_j \in \mathcal{X}_j$ be any bid in the support of G_j such that $\underline{s} \le \Pi(x_j) < \hat{x}$.[46] Buyer j must win the auction with strictly positive probability when playing this bid since, otherwise, he would earn a payoff of 0 in equilibrium, contradicting Lemma 10. But then, it must be the case that $x_j < \hat{x}$, i.e., x_j is located on the linear segment of the revenue curve $\Pi(\cdot)$. Otherwise, buyer j would find it strictly profitable to deviate and replace x_j with x'_j such that $x'_j = \Pi(x_j) + \varepsilon < x_j$ for $\varepsilon > 0$ arbitrarily small. Since this is true for all $j \in J^*$ and for all $x_j \in \mathcal{X}_j$ such that $\underline{s} \le \Pi(x_j) < \hat{x}$, we conclude that

$$\inf \mathcal{X}_j = \underline{s}, \quad \forall j \in J^*. \tag{11}$$

Given this, the following claim holds:

Claim 2. *If $\underline{s} < \hat{x}$ and $j \in J^*$, G_j places a probability mass on $\underline{s}$.*

[44] Throughout the proof of statement $(b.iv)$, for each measurable set $E \subseteq X$, we abuse notation and write $G_i(E)$ to denote the probability assigned by G_i to E.

[45] Recall from Lemma 9 that $J^* \subseteq I$ contains at least two elements.

[46] Such a bid exists by assumption.

Proof of Claim 2. Recall that $|J^*| \geq 2$ (Lemma 9) and, without loss of generality, assume that $\{1, 2\} \subseteq J^*$. Toward a contradiction, suppose that G_1 does not place probability mass on $\underline{s}$. In what follows, we show that buyer 2 earns an equilibrium expected payoff of 0, contradicting Lemma 10.

Since G_1 does not place probability mass on $\underline{s}$, it follows that for all $n \in \mathbb{N}$, there exists $\varepsilon_n > 0$ such that $G_1(\underline{s} + \varepsilon_n) < \frac{1}{n}$. Since $\limsup_n \varepsilon_n = 0$, it is without loss of generality to assume that $(\varepsilon_n)_n$ is a decreasing sequence. Fix n large enough so that $\underline{s}+\varepsilon_n < \hat{x}$, and take any $x_2^n \in \mathcal{X}_2$ such that $\underline{s} \leq x_2^n < \underline{s} + \varepsilon_n$. (Equation (11) implies that x_2^n must exist.) Since buyer 1 submits a bid leading to expected revenue of more than x_2^n with probability greater than $1 - \frac{1}{n}$, the expected payoff buyer 2 earns in equilibrium when offering x_2^n can be no greater than $\frac{1}{n} \cdot \mu > 0$.[47] Buyer 2 is indifferent between all bids in $\mathcal{X}_2$, which implies that buyer 2's equilibrium payoff is no greater than $\frac{1}{n} \cdot \mu > 0$. However, since this is true for all sufficiently large $n \in \mathbb{N}$, we conclude that buyer 2's equilibrium payoff U_2^* is equal to 0, as required.

□

Given Claim 2, it follows that a configuration of bids where all buyers in J^* submit the same bid $\underline{s}$ has a strictly positive chance to arise in equilibrium. Fix any buyer in J^* that (on average) is selected as a winner with a probability weakly smaller than $\frac{1}{|J^*|}$ under such configurations of bids. (Such a buyer necessarily exists.) We now show that such buyer $j \in J^*$ has a strict incentive to deviate from G_j. To see this, observe that by bidding $\underline{s}$, buyer j earns an expected payoff no greater than

$$\frac{1}{|J^*|} \cdot \left(\prod_{i \in J^* \setminus \{j\}} G_i(\underline{s}) \right) \cdot W(\underline{s}) \geq 0.$$

However, by replacing $\underline{s}$ with $x_j^\varepsilon := \underline{s} + \varepsilon$, buyer j would earn an expected payoff greater than or equal to

$$\left(\prod_{i \in J^* \setminus \{j\}} G_i(\underline{s}) \right) \cdot W(\underline{s} + \varepsilon) > \frac{1}{|J^*|} \cdot \left(\prod_{i \in J^* \setminus \{j\}} G_i(\underline{s}) \right) \cdot W(\underline{s}),$$

as long as $\varepsilon > 0$ is sufficiently small (Lemma 2). Since this contradicts the optimality of G_j, we conclude that $\underline{s} = \Pi(\hat{x})$, as required.

□

Lemma 13. *An equilibrium inducing an outcome where winning bid is random must satisfy* $\mathcal{X}_j \subseteq \{\hat{x}\} \cup [x^o, x^{dd}]$ *for all* $j \in J^*$.

Proof of Lemma 13. Suppose by contradiction that there exists $x_j \in \mathcal{X}_j$ such that $x_j < \hat{x}$, or $x_j > x^{dd}$, or $x_j \in (\hat{x}, x^o)$ for some $j \in J^*$. Because $\Pi(x) = x < \hat{x} = \Pi(\hat{x})$ for all $x < \hat{x}$, Lemma 12 implies that $x_j < \hat{x}$ is impossible. We now show that $x_j > x^{dd}$ is also impossible. There are two cases: Either $x^{dd} = \check{x}$ or $x^{dd} < \check{x}$. First, assume that $x^{dd} < \check{x}$. We show that there exists a strictly profitable deviation from x_j. Let $x_j^\varepsilon := \min\{y \geq 0 : \Pi(y) = \Pi(x_j) + \varepsilon\}$, for some $\varepsilon > 0$ sufficiently small. It is easy to see that since $x_j > x^{dd}$, x_j^ε exists and satisfies

[47] Since $x_2^n < \hat{x}$, $W(x_2^n) = \mu - x_2^n < \mu$ by Lemma 1.

$x_j^\varepsilon < \min\{x_j, \check{x}\}$. Furthermore, x_j^ε induces a strictly higher revenue than x_j and also implies a strictly higher expected payoff conditional on winning, because $W(x_j^\varepsilon) > W(x_j)$. Therefore, x_j^ε is a strictly profitable deviation from x_j, a contradiction. Now, suppose that $x^{dd} = \check{x}$. If $x_j > x^{dd}$ and x_j is accepted with positive probability, buyer j could profitably deviate to $x_j' = \varnothing$ which secures a payoff of 0, a contradiction. Instead, if x_j is accepted with probability 0, it follows that buyer j receives a payoff of $U_j^* = 0$ in equilibrium, contradicting Lemma 10. Finally, to see that $x_j \in (\hat{x}, x^o)$ is also impossible, note that x_j can be replaced by $x_j^\varepsilon := \Pi(x_j) + \varepsilon$. As long as $\varepsilon > 0$ is small enough, such deviation would be strictly profitable for buyer j, since it decreases the bid while increasing revenue. Thus, $\mathcal{X}_j \subseteq \{\hat{x}\} \cup [x^o, x^{dd}]$, as required.

□

Lemma 14. *An equilibrium inducing an outcome where the winning bid is random must satisfy that, for all $i \in J^*$, S_i admits a strictly positive probability mass on $\Pi(\hat{x})$. Furthermore, each such S_i admits no other mass point.*

Proof of Lemma 14. Fix $i \in J^*$ arbitrarily. To see why S_i must admit a strictly positive mass on $\Pi(\hat{x})$, suppose by contradiction that this is not the case. Then, for all $n \in \mathbb{N}$, there exists $\varepsilon_n > 0$ such that $S_i(\Pi(\hat{x}) + \varepsilon_n) < \frac{1}{n}$. By using steps analogous to those employed in the proof of Claim 2, one can verify that this implies that some buyer $j \in J^* \setminus \{i\}$ obtains a payoff of 0 in equilibrium. However, this would contradict Lemma 10.

Finally, standard Bertrand arguments imply that all buyers' winning probabilities have to be continuous for $x > x^0$ and, therefore, that S_i cannot admit another mass point in its support. For the sake of brevity, we omit the details. □

Lemma 15. *An equilibrium inducing an outcome where the winning bid is random must satisfy $|J^*| = N$, i.e., $J^* = I$.*

Proof of Lemma 15. Denote with $\mathcal{S}$ the cdf over the realized expected revenue accrued by the seller in equilibrium and induced by the profile of buyers' strategies $(G_i)_{i\in I}$. In other words, $\mathcal{S}$ is the cdf of Π^*. By Lemma 13, it holds that $\operatorname{supp}\mathcal{S} \subseteq [\Pi(\hat{x}), \Pi(x^{dd})]$. Moreover, since for every $i \in J^*$, S_i places a strictly positive probability mass on $\Pi(\hat{x})$ (Lemma 14), it follows that $\mathcal{S}$ does so as well. Given this, suppose by contradiction that there exists $i \in I \setminus J^*$. Clearly, $U_i^* = 0$. However, by submitting with certainty a bid x_i such that $\Pi(x_i) = \Pi(\hat{x}) + \varepsilon$, buyer i would win the auction with strictly positive probability and obtain a payoff conditional on winning equal to $W(x_i) > 0$, as long as $\varepsilon > 0$ is small enough.[48] Since this contradicts the optimality of G_i given G_{-i}, we conclude that $|J^*| = N$, as required. □

Lemma 16. *An equilibrium inducing an outcome where the winning bid is random must be such that G_i puts strictly positive probability mass on $\hat{x}$ for all $i \in I$.*

[48] The existence of such a small $\varepsilon > 0$ follows from the fact that at least one buyer $j \in J^*$ must win the auction with a bid $x_j \in [x^o, x^{dd}]$ (Lemma 13), and the fact that $U_j^* > 0$ (Lemma 10).

Proof of Lemma 16. From Lemmas 14 and 15, we know that each G_i induces a strictly positive probability mass on the revenue $\Pi(\hat{x})$. Observe that, by Lemma 13, there exist only two bids that induce such revenue and can be in $\mathcal{X}_i$: Namely, $\hat{x}$ and x^o. In what follows, we show that no G_i can place no probability mass on $\hat{x}$, proving the statement.

By contradiction, suppose buyer 1 only places probability mass on x^o, but not on $\hat{x}$, in equilibrium. Given this, let $\vec{x} \in R^N$ be any profile of bids that arises with strictly positive probability in equilibrium such that $x_1 = x^o$ and $\Pi(x_j) = \Pi(\hat{x})$ for all $j \neq 1$. The following claim holds:

Claim 3. *It holds that* $\beta_1(\vec{x}) = 1$.

Proof of Claim 3. If $\beta_1(\vec{x}) < 1$, buyer 1 would find it strictly profitable to deviate from x^o and bid $x_1 = x^o + \varepsilon$ for $\varepsilon > 0$ sufficiently small instead. In fact, in this way, buyer 1 would discretely increase his probability of winning the auction given G_{-i}, while only marginally reducing his payoff conditional on winning. Thus, $\beta(\vec{x}) = 1$, as required.

□

Claim 3 implies that any buyer $j \neq 1$ always loses against buyer 1 when submitting a bid x_j such that $\Pi(x_j) = \Pi(\hat{x})$. Thus, $U_j^* = 0$ for all $j \neq 1$. However, this is impossible since it contradicts Lemmas 10 and 15.

□

Using Lemmas 8,..., 16, we are ready to prove that

$$W(\hat{x}) \geq N \cdot W(x^o). \tag{12}$$

To see why (12) holds, let $i \in I$ be any buyer selected as a winner with probability weakly smaller than $\frac{1}{N}$ when the seller receives the profile of bids $\vec{x} = (\hat{x}, ..., \hat{x})$.[49] In equilibrium, buyer $i \in I$ must weakly prefer bidding $\hat{x}$ to any bid x_i' such that $\Pi(x_i') > \Pi(\hat{x})$. This can be the case only if $\frac{1}{N} W(\hat{x}) \geq W(x^o)$. Thus, inequality (12) holds, as required. Finally, that the seller's expected payoff lies in the range $[\Pi(\hat{x}), \Pi(x^{dd})]$. follows immediately from Lemma 13.

Proof of (c): Part (c) follows from arguments almost identical to those of $(b.iii)$ and $(b.iv)$. In particular, every equilibrium must feature either deterministic winning bids equal to $\hat{x}$, x^{dd}, or a mixture over these two bids. For the sake of brevity, we omit the details. This completes the proof of Lemma 7.

□

[49] By Lemma 16, this event occurs with strictly positive probability.

Proof of Proposition 2: Part 3

Recall from equation (3) the definition of x^{dd}. Observe that since $R = [0, \check{x}]$ and $\check{x} = \check{x}(c)$ (Lemma 6), also x^{dd} depends on $c > 0$, i.e., $x^{dd} = x^{dd}(c)$.[50] Given this, define $c_0 > 0$ as the unique solution to the equation $\hat{x}(c_0) = 0$, and $c_1 > 0$ as the unique solution to the equation $\hat{x}(c_1) = x^{dd}(c_1) \cdot \Big(1 - F(x^{dd}(c_1))\Big)$. Both c_0 and c_1 are well defined information cost thresholds and satisfy $0 < c_0 < c_1 < \bar{c}$.

Recall that $x^o = x^o(c)$ is given by

$$x^o(c) := \min\{x \geq 0 : x \cdot [1 - F(x)] = \hat{x}(c)\}, \quad \forall c \in (c_0, c_1).$$

Observe that the regularity of F implies that $0 < \hat{x}(c) < x^o(c) < x^{dd}(c)$ for all $c \in (c_0, c_1)$. Given this, define the real-valued function $\rho : (c_0, c_1) \to \mathbb{R}_+$ as follows:

$$\rho(c) := \frac{W^{su}(\hat{x}(c))}{W^{dd}(x^o(c))}.$$

The Implicit Function Theorem, together with the observation that $W^{su}(\hat{x}(c)) \in (0, \mu)$ and $W^{dd}(x^o(c)) \in (0, S - c)$ for all $c \in (c_0, c_1)$, implies that $\rho(c)$ is a well-defined continuously differentiable function of $c \in (c_0, c_1)$. Below, Claim 17 characterizes some key properties of this function.

Lemma 17. *The following holds:*

(i) $\lim_{c \downarrow c_0} \rho(c) = 1$*,*

(ii) $\lim_{c \downarrow c_0} \rho'(c) > 0$*,*

(iii) $\rho'(c) > 0$ *for all* $c \in (c_0, c_1)$*.*

Proof of Claim 17. To see why *(i)* holds, notice that as $c \downarrow c_0$, we have that $\hat{x}(c), x^o(c) \to 0$. This implies that $W^{su}(\hat{x}(c)) = \mu - \hat{x}(c) \to \mu$. Thus, we just need to show that also $W^{dd}(x^o(c))$ converges to μ as $c \downarrow c_0$. To see this, notice that, $c_0 = \int_{\underline{v}}^{0} (-v)\, dF(v)$ by construction. Therefore,

$$W^{dd}(x^o(c)) = -c + \int_{x^o(c)}^{\bar{v}} (v - x^o(c))\, dF(v) \to -c_0 + \int_0^{\bar{v}} v\, dF(v) = \int_0^{\bar{v}} v\, dF(v) + \int_{\underline{v}}^{0} v\, dF(v) = \mu,$$

as $c \downarrow c_0$, as required.

We now prove *(ii)*. Fix $c \in (c_0, c_1)$ arbitrarily. Note that $\rho'(c) > 0$ if and only if

$$\left(\frac{\partial}{\partial c} W^{su}(\hat{x}(c))\right) \cdot W^{dd}(x^o(c)) > \left(\frac{\partial}{\partial c} W^{dd}(x^o(c))\right) \cdot W^{su}(\hat{x}(c)). \tag{13}$$

Repeated applications of the Implicit Function Theorem show that

$$\frac{\partial}{\partial c} W^{su}(\hat{x}(c)) = -\frac{\partial}{\partial c}\hat{x}(c) = -\frac{1}{F(\hat{x}(c))} < 0, \tag{14}$$

[50] In particular, it is easy to see that $c \mapsto x^{dd}(c)$ is weakly decreasing in $c > 0$, and satisfies $x^{dd}(c) = \bar{x}$ if $\check{x}(c) \geq \bar{x}$, and $x^{dd}(c) = \check{x}(c)$ otherwise, where $\bar{x} > 0$ is the unique maximizer of $x \mapsto x \cdot [1 - F(x)]$.

and that

$$\frac{\partial}{\partial c} W^{dd}(x^o(c)) = -\left[1 + \Big(1 - F(x^o(c))\Big) \cdot \frac{\partial}{\partial c} x^o(c)\right] < 0. \tag{15}$$

Thus, inequality (13) is equivalent to

$$\frac{\frac{\partial}{\partial c} W^{su}(\hat{x}(c))}{\frac{\partial}{\partial c} W^{dd}(x^o(c))} < \frac{W^{su}(\hat{x}(c))}{W^{dd}(x^o(c))} = \rho(c). \tag{16}$$

In what follows, we show that inequality (16) is satisfied whenever $c \downarrow c_0$. To see this, observe that, by definition, $x^o(c) \cdot \Big(1 - F(x^o(c))\Big) = \hat{x}(c)$. Therefore:

$$\frac{\partial}{\partial c} x^o(c) = \frac{\partial}{\partial c} \hat{x}(c) \cdot \frac{1}{1 - F(x^o(c)) - x^o(c) \cdot f(x^o(c))} > 0, \tag{17}$$

where the strict inequality follows from the regularity of F and the fact that

$$x^o(c) < x^{dd}(c) \leq \bar{x} := \arg\max_{x \geq x} x \cdot [1 - F(x)]$$

for all $c \in (c_0, c_1)$.

Equation (15), together with equation (17), implies that:

$$\frac{\partial}{\partial c} W^{dd}(x^o(c)) = -\left[1 + \frac{\partial}{\partial c} \hat{x}(c) \cdot A(c)\right],$$

where

$$A(c) := \frac{1 - F(x^o(c))}{1 - F(x^o(c)) - x^o(c) \cdot f(x^o(c))} > 1, \quad \forall c \in (c_0, c_1).$$

Since, by equation (14), it holds that $\frac{\partial}{\partial c} \hat{x}(c) = \frac{1}{F(\hat{x}(c))} > 0$ for all $c \in (c_0, c_1)$, we conclude that

$$\frac{\frac{\partial}{\partial c} W^{su}(\hat{x}(c))}{\frac{\partial}{\partial c} W^{dd}(x^o(c))} = \frac{\frac{\partial}{\partial c} \hat{x}(c)}{1 + \frac{\partial}{\partial c} \hat{x}(c) \cdot A(c)} < 1,$$

for all $c \in (c_0, c_1)$. Because $\rho(c) \to 1$ as $c \downarrow c_0$, inequality (16) is satisfied eventually, i.e., for c close enough to c_0. This implies that $\lim_{c \downarrow c_0} \rho'(c) > 0$, as required.

Finally, to see why statement *(iii)* holds, observe that:

(a) Together, statements *(i)* and *(ii)* imply that $\rho(c) > 1$ for all c close enough to c_0.

(b) The arguments used to prove statement *(ii)* also show that $\rho'(c) > 0$ whenever $\rho(c) \geq 1$.

Statement *(iii)* follows from the above observations (a) and (b), and the fact that $c \mapsto \rho(c)$ is continuously differentiable.

□

To complete the proof of Proposition 2, define the $c^* > 0$ as follows:

$$c^* = \begin{cases} \rho^{-1}(N) & \text{if } \lim_{c\uparrow c_1} \rho(c) > N, \\ c_1 & \text{otherwise.} \end{cases}$$

From Lemma 17, it is easy to see that $c^* > 0$ is a well-defined information cost threshold. Moreover, it satisfies $c^* \leq c_1$ with $c^* < c_1$ if and only if $\lim_{c\uparrow c_1} \rho(c) > N$. In the remainder of this proof, we show that $c^* > 0$ satisfies all the properties mentioned in Proposition 2.

As a first step, fix any $c < c^*$. Since $c < c_1$, it holds that $\Pi(\hat{x}) = \hat{x} < \Pi(x^{dd})$, which implies that $X^{\max} = \{x^{dd}\}$. Moreover, it also holds that either $\rho(c) < N$ in case $c > c_0$, or $\hat{x} \leq 0$ in case $c \leq c_0$. In both cases, by part *(b)* of Lemma 7, we conclude that the winning bid is x^{dd} and the winner conducts due diligence in every equilibrium of the due diligence game.

Now, fix any $c \geq c^*$. If $c > c_1$, then $X^{\max} = \{\hat{x}\}$, and therefore the assertion that the winning bid is $\hat{x}$ and the winner buys the good sight unseen in the seller's least preferred equilibrium trivially holds by part *(a)* of Lemma 7. Thus, from now on suppose that $c \leq c_1$, so that $x^{dd} \in X^{\max}$. Because $c \geq c^*$, statements $(b.ii)$ and (c) of Lemma 7 imply that there exists an equilibrium where the winning bid is $\hat{x}$ and the winner buys the good sight unseen. That no other equilibrium leads to a lower revenue than that of the sure-trade equilibrium outcome follows directly from statements $(b.iii)$, $(b.iv)$, and (c) of Lemma 7.

Finally, since $\rho(\cdot)$ is strictly increasing in $c \in (c_0, c_1)$ by Lemma 17, $N \mapsto c^* = c^*(N)$ is a weakly increasing function of $N \geq 2$. Moreover, as $N \uparrow \infty$, we have that $c^*(N) \to c^*(\infty) = c_1 < \bar{c}$. This concludes the proof of Proposition 2.

Q.E.D.

Proof of Corollary 1

Proposition 2 implies that

$$x^w(c) = \begin{cases} x^{dd}(c) & \text{if } c < c^* \\ \hat{x}(c) & \text{otherwise,} \end{cases}$$

$\Pi^w(c) = \Pi(x^w(c)|c)$, and $W^w(c) = W(x^w(c)|c)$.

Note that $x^{dd}(c) = \min\{\check{x}(c), \bar{x}\}$, where $\bar{x} := \arg\max_{x\geq 0} x \cdot [1 - F(x)]$ and $W(\check{x}(c)|c) = 0$. Lemma 6 shows that $\check{x}(c)$ is strictly decreasing. Thus, $\frac{\partial}{\partial c} x^{dd}(c) \leq 0$. Since $x^w(c) = x^{dd}(c)$ for $c \in (0, c^*)$, this implies $\frac{\partial}{\partial c} x^w(c) \leq 0$ for such costs. To the contrary, when $c \in (c^*, \bar{c})$, $x^w(c) = \hat{x}(c)$, which is strictly increasing. Moreover, by Lemma 7 and Proposition 2, it must be that

$$\Pi(x^{dd}(c)|c) = x^{dd}(c) \cdot \Big(1 - F(x^{dd}(c))\Big) > \hat{x}(c)$$

for all $c < c^*$. Together with the continuity of $\hat{x}(c)$, this shows that

$$\lim_{c\uparrow c^*} x^w(c) \geq \frac{x^w(c^*)}{1 - F(\lim_{c\uparrow c^*} x^w(c))} > x^w(c^*).$$

Proposition 2 implies that $\Pi^w(c) = x^w(c) \cdot (1 - F(x^w(c)))$ for $c < c^*$ and $\Pi^w(c) = x^w(c)$ for $c \geq c^*$. Moreover, by regularity, $x \mapsto x \cdot [1 - F(x)]$ is increasing in x for $x < \bar{x}$. Thus, the comparative statics of $\Pi^w(c)$ follow from those of $x^w(c)$, continuity of $\hat{x}(c)$, and the fact that $x^{dd}(c) \cdot \Big(1 - F(x^{dd}(c))\Big) > \hat{x}(c)$ for all $c < c^*$.

Further, note that $W(\bar{x}|c)$ is decreasing in c, and that $W(\check{x}(c)|c) = 0$. Thus, $\frac{\partial}{\partial c} W^w(c) \leq 0$ for $c < c^*$. The remaining comparative statics of $W^w(c)$ follow from the fact that $W(x|c)$ is strictly decreasing in x and weakly decreasing in c.

Proof of Proposition 3

Because of Propositions 0 and 1, it is sufficient to show that $\inf_{\sigma \in \mathcal{E}^{dd}} \Pi(\sigma) > \mu$ if and only if *(i)* $c < c^*$ and *(ii)* $\max_{x \in R} x \cdot [1 - F(x)] > \mu$. To see why this is true, note that Proposition 2 implies that

$$\inf_{\sigma \in \mathcal{E}^{dd}} \Pi(\sigma) = \begin{cases} x^{dd} \cdot (1 - F(x^{dd})), & \text{if } c < c^*, \\ \hat{x}, & \text{if } c \geq c^*. \end{cases}$$

Given this, Proposition 3 follows from the definition of x^{dd} given by equation (3), and the fact that Assumption 3 implies that $\hat{x} < \mu$.

Q.E.D.

Proof of Corollary 2

As a first step, it is important to notice that, for any fixed profile of model primitives (F, c, N), none of the parameter changes listed in Corollary 2 can violate the assumptions of Section 2. Given this, fix any profile of primitives of the model (F, c, N) such that due diligence is strictly optimal. From Proposition 3, we know that this is equivalent to *(i)* $c < c^*$ and *(ii)* $\max_{x \in R} x \cdot [1 - F(x)] > \mu$. In what follows, we show that these conditions still hold once we apply each of the changes in model primitives listed in Corollary 2.

More competition: Consider an increase in the number of buyers, i.e., $N' > N$. This change does not affect condition *(ii)*. Instead, it relaxes condition *(i)* since, by Proposition 3, it holds that $c^*(N) \leq c^*(N')$. We conclude that conditions *(i)* and *(ii)* continue to hold after this parameter change, as required.

Cost decrease: Suppose the information cost decreases, i.e., $c' < c$. Condition *(i)* trivially still holds after this change. To see why the same is true for condition *(ii)*, recall from Lemma 3 in the proof of Proposition 2 that $R = [0, \check{x}]$ and that $\check{x} = \check{x}(c)$ is strictly decreasing in $c > 0$. Therefore, condition *(ii)* is effectively relaxed after this change.

Scale-up of valuation: Multiplying $\tilde{v}$ by a scalar $K > 1$ is equivalent to multiplying c by $\frac{1}{K}$. Thus, the arguments used above for a cost decrease suffice to prove this case.

Leftward shift of valuation. Finally, consider a leftward shift of the buyers' common valuation $\tilde{v}$, i.e., consider $\tilde{v}_{\text{new}} = \tilde{v} - \omega$ for some $\omega \in (0, \mu)$. Denote with $F_{\text{new}}(v) := F(v + \omega)$ the cdf of $\tilde{v}_{\text{new}}$, and let $\underline{v}_{\text{new}} := \underline{v} - \omega$ and $\bar{v}_{\text{new}} := \bar{v} - \omega$ be the corresponding endpoints of its support. The following lemma will be used extensively in the remainder of this proof.

Lemma 18. *Fix any $x \geq 0$. The following statements hold:*

(a) $W^{su}_{\text{new}}(x) = W^{su}(x+\omega)$,

(b) $W^{dd}_{\text{new}}(x) = W^{dd}(x+\omega)$,

(c) $x \cdot (1 - F_{\text{new}}(x)) > (x+\omega) \cdot (1 - F(x+\omega)) - \omega$.

Proof of Lemma 18. To see why statement *(a)* holds, observe that

$$W^{su}_{\text{new}}(x) := \mu_{\text{new}} - x = \mu - \omega - x = W^{su}(x+\omega).$$

Statement *(b)* follows because

$$\begin{aligned} W^{dd}_{\text{new}}(x) &:= \int_x^{\bar{v}_{\text{new}}} (v - x)\, dF_{\text{new}}(x) - c \\ &= \int_{x+\omega}^{\bar{v}} (v - \omega - x)\, dF(v) - c \\ &= W^{dd}(x+\omega). \end{aligned}$$

Finally, statement *(c)* holds since $F(x+\omega) > 0$ and

$$\begin{aligned} x \cdot (1 - F_{\text{new}}(x)) &= x \cdot (1 - F(x+\omega)) \\ &= (x+\omega) \cdot (1 - F(x+\omega)) - \omega \cdot (1 - F(x+\omega)) \\ &> (x+\omega) \cdot (1 - F(x+\omega)) - \omega. \end{aligned}$$

□

Now, let $\hat{x}_{\text{new}}$ and $\check{x}_{\text{new}}$ be defined implicitly by $W^{su}_{\text{new}}(\hat{x}_{\text{new}}) = W^{dd}_{\text{new}}(\hat{x}_{\text{new}})$ and $W^{dd}_{\text{new}}(\check{x}_{\text{new}}) = 0$. From Lemma 18, one can easily verify that $\hat{x}_{\text{new}} = \hat{x} - \omega$ and $\check{x}_{\text{new}} = \check{x} - \omega$.

Given this, consider the bid $x' = x^{dd} - \omega$, where $x^{dd} \cdot \left(1 - F(x^{dd})\right) = \max_{x \in R} x \cdot [1 - F(x)]$. Observe that x' is a relevant bid under $\tilde{v}_{\text{new}}$, i.e., it holds that $0 \leq x' \leq \check{x}_{\text{new}}$.[51] Moreover, by statement *(c)* of Lemma 18, it holds that

$$x' \cdot (1 - F_{\text{new}}(x')) > x^{dd} \cdot \left(1 - F(x^{dd})\right) - \omega > \mu - \omega = \mu_{\text{new}}, \tag{18}$$

where the second inequality holds by assumption. In turn, this implies that

$$x^{dd}_{\text{new}} \cdot \left(1 - F_{\text{new}}(x^{dd}_{\text{new}})\right) = \max_{x \in R_{\text{new}}} x \cdot (1 - F_{\text{new}(x)}) > \mu_{\text{new}}.$$

We now show that condition *(i)* holds under $\tilde{v}_{\text{new}}$ as well, i.e., $c < c^*_{\text{new}}$. To see why this is

[51] That $x' \leq \check{x}_{\text{new}}$ is obvious. To see why x' is positive, notice that $x^{dd} \cdot \left(1 - F(x^{dd})\right) \geq \mu$ implies that $x^{dd} > \mu$. Therefore, $x^{dd} - \omega > 0$ since $\mu_{\text{new}} = \mu - \omega > 0$ by assumption.

the case, recall from Proposition 2 that $c < c^*_{\text{new}}$ if and only if the due diligence game admits a unique equilibrium outcome where the winning bid is x^{dd}_{new} and the winner conducts due diligence. According to Lemma 7 in the proof of Proposition 2, this is the case if and only if $X^{\max}_{\text{new}} = \{x^{dd}_{\text{new}}\}$ and one of the following two conditions hold: Either $\hat{x}_{\text{new}} \leq 0$, or $\hat{x}_{\text{new}} > 0$ and

$$W^{su}_{\text{new}}(\hat{x}_{\text{new}}) < N \cdot W^{dd}_{\text{new}}(x^o_{\text{new}}), \tag{19}$$

where

$$x^o_{\text{new}} := \min\{x \geq 0 : x \cdot (1 - F_{\text{new}}(x)) = \hat{x}_{\text{new}}\}.$$

That $X^{\max}_{\text{new}} = \{x^{dd}_{\text{new}}\}$ holds follows from the fact that $x^{dd}_{\text{new}} \cdot \left(1 - F_{\text{new}}(x^{dd}_{\text{new}})\right) > \mu_{\text{new}} > \hat{x}_{\text{new}}$. Hence, if $\hat{x}_{\text{new}} \leq 0$, we are done, i.e., $c < c^*_{\text{new}}$.

Thus, for the remainder of this proof assume that $\hat{x}_{\text{new}} > 0$. To see why inequality (19) holds, observe that statement *(a)* of Lemma 18 implies that $W_{\text{new}}(\hat{x}_{\text{new}}) = W(\hat{x})$. Furthermore, note that since by assumption $c < c^*$ and $\hat{x} > \hat{x}_{\text{new}} > 0$, it must be the case that

$$W(\hat{x}) < N \cdot W(x^o).$$

Therefore, to prove that $c < c^*_{\text{new}}$, it is sufficient to show that $W_{\text{new}}(x^o_{\text{new}}) \geq W(x^o)$. However, that this is the case follows from statement *(c)* of Lemma 18 and the regularity of F, since they jointly imply that $x^o_{\text{new}} < x^o - \omega$.[52] This concludes the proof of Corollary 2.

Q.E.D.

Proof of Theorem 1

This proof proceeds in three parts. In Part 1, we characterize when due diligence is (weakly) optimal. In Part 2, we establish a sufficient condition for the strict optimality of due diligence which depends on the size of the stakes relative to a function of μ. In Part 3, we use this groundwork to prove our main result.

Proof of Theorem 1: Part 1

Observe that, together, Propositions 0 and 1 imply that research cannot be optimal. The following lemma characterizes when due diligence is (weakly) optimal. Moreover, it shows that when this is the case, any leftward shift in the buyers' valuation makes due diligence strictly optimal. Formally:

Lemma 19. *The following statements hold:*

(a) Due diligence is optimal if and only if $c < c^*$ *and* $\max_{x \in R} x \cdot [1 - F(x)] \geq \mu$.

(b) Suppose due diligence is optimal. Then,

- *If the buyers' valuation is scaled-up, i.e., it holds that* $\tilde{v}_{\text{new}} = K \cdot \tilde{v}$ *for some* $K > 1$*, due diligence remains optimal.*

[52] Recall that $x \mapsto W^{dd}_{\text{new}}(x)$ is a decreasing function.

– *If the buyers' valuation is shifted to the left, i.e., it holds that $\tilde{v}_{\text{new}} = \tilde{v} - \omega$ for some $\omega \in (0, \mu)$, due diligence becomes strictly optimal.*

Proof of Lemma 19. A straightforward modification of the arguments used in the proof of Proposition 3 suffices to prove the characterization in statement *(a)*. In turn, given this characterization, the same arguments used in the proof of Corollary 2 can be employed to show that statement *(b)* holds as well.[53] This completes the proof of the lemma.

□

Proof of Theorem 1: Part 2

In this part of the proof of Theorem 1, we prove the following lemma.

Lemma 20. *Due diligence is strictly optimal whenever*

$$\mathbb{E}[|\tilde{v}|] > \mu \cdot \left(1 + 2\ln\left(\frac{\bar{v}}{\mu}\right)\right) + 2c.$$

Proof of Lemma 20. By Proposition 1, we know that due diligence is strictly optimal if and only if it holds that $\inf_{\sigma\in\mathcal{E}^{dd}} \Pi(\sigma) > \mu$. Observe that

$$\mathbb{E}[|\tilde{v}|] = S - \int_{\underline{v}}^{0} v\, dF(v) \quad \text{and} \quad \int_{\underline{v}}^{0} v\, dF(v) = \mu - S.$$

This allows us to rewrite

$$\mathbb{E}[|\tilde{v}|] = 2S - \mu. \tag{20}$$

In what follows, we show that

$$\inf_{\sigma\in\mathcal{E}^{dd}} \Pi(\sigma) \le \mu \quad \Longrightarrow \quad S \le \mu \cdot \left(1 + \ln\left(\frac{\bar{v}}{\mu}\right)\right) + c. \tag{21}$$

Since by equation (20), the inequality on the right side of (21) is the opposite of inequality (6), proving the implication given in (21) is sufficient to show that Lemma 20 holds.

From now on, suppose throughout that $\inf_{\sigma\in\mathcal{E}^{dd}} \Pi(\sigma) \le \mu$. There are two possibilities:

C1: It holds that $\sup_{\sigma\in\mathcal{E}^{dd}} \Pi(\sigma) \le \mu$, i.e., no equilibrium resulting in expected revenue for the seller strictly higher than μ exists (i.e., condition (ii) of Proposition 3 is violated).

C2: It holds that $\sup_{\sigma\in\mathcal{E}^{dd}} \Pi(\sigma) > \mu$, which implies that $\inf_{\sigma\in\mathcal{E}^{dd}} \Pi(\sigma) < \sup_{\sigma\in\mathcal{E}^{dd}} \Pi(\sigma)$ (i.e., condition (i) of Proposition 3 is violated).

We proceed to show that both cases imply that $S \le \mu \cdot \left(1 + \ln\left(\frac{\bar{v}}{\mu}\right)\right) + c$.

Case C1: Assume $\sup_{\sigma\in\mathcal{E}^{dd}} \Pi(\sigma) \le \mu$. From Lemma 7 in the proof of Proposition 2, one can verify that this is the case only if either the buyers' payoff from bidding any $x \ge 0$ such that

[53] In particular, the conclusion regarding a left shift in the buyers' valuation follows from the fact that $\max_{x\in R_{\text{new}}} x \cdot (1 - F(x_{\text{new}})) > \mu_{\text{new}}$ still holds even if the second inequality in (18) is replaced by an equality.

$(1-F(x)) \cdot x > \mu$ is strictly negative, or no such bid x exists.

Suppose first that

$$x \cdot (1-F(x)) \leq \mu \text{ for all } x \geq 0. \tag{22}$$

Note that

$$\begin{aligned} S &= \int_0^{\bar{v}} v \, dF(v) \\ &= \int_0^{\bar{v}} (1-F(v)) \, dv \\ &\leq \int_0^{\frac{\mu}{1-F(0)}} (1-F(0)) \, dv + \int_{\frac{\mu}{1-F(0)}}^{\bar{v}} \left(\frac{\mu}{v}\right) dv \\ &= \mu \cdot \left[1 + \ln\left(\frac{\bar{v}}{\mu} \cdot (1-F(0))\right)\right] \\ &\leq \mu \cdot \left(1 + \ln\left(\frac{\bar{v}}{\mu}\right)\right), \end{aligned}$$

where the second equality follows from integration by parts and the first inequality from the monotonicity of F and inequality (22). We conclude that $S \leq \mu \cdot \left(1 + \ln\left(\frac{\bar{v}}{\mu}\right)\right) + c$, as required.

Next, suppose that $X_\mu := \{x \geq 0 : x \cdot (1-F(x)) > \mu\} \neq \emptyset$ but $W(x) < 0$ for all $x \in X_\mu$. Let $x_\mu := \inf X_\mu$. Observe that $x_\mu > \mu$, which implies that $W(x_\mu) = W^{dd}(x_\mu)$. By continuity of $x \mapsto x \cdot (1-F(x))$ and $x \mapsto W(x)$, it holds that $x_\mu \cdot (1-F(x_\mu)) = \mu$ and $W(x_\mu) \leq 0$. Furthermore, since $\mu = x_\mu \cdot (1-F(x_\mu)) \leq x_\mu \cdot (1-F(0))$, we have $0 < \frac{\mu}{1-F(0)} \leq x_\mu < \bar{v}$. This allows us to write the socially optimal trade surplus $S > 0$ as follows:

$$S = \int_0^{\bar{v}} v \, dF(v) = \int_0^{x_\mu} v \, dF(v) + \int_{x_\mu}^{\bar{v}} v \, dF(v).$$

We now separately bound both integrals from above. By definition, $x \cdot [1-F(x)] \leq \mu$ for all $x \in (0, x_\mu)$. Therefore,

$$\begin{aligned} \int_0^{x_\mu} v \, dF(v) &= -x_\mu \cdot (1-F(x_\mu)) + \int_0^{x_\mu} (1-F(v)) \, dv \\ &= -\mu + \int_0^{\frac{\mu}{1-F(0)}} (1-F(v)) \, dv + \int_{\frac{\mu}{1-F(0)}}^{x_\mu} (1-F(v)) \, dv \\ &\leq -\mu + \int_0^{\frac{\mu}{1-F(0)}} (1-F(0)) \, dv + \int_{\frac{\mu}{1-F(0)}}^{x_\mu} \left(\frac{\mu}{v}\right) dv \\ &= \mu \cdot \ln\left(\frac{x_\mu}{\mu} \cdot (1-F(0))\right) \\ &\leq \mu \cdot \ln\left(\frac{\bar{v}}{\mu}\right), \end{aligned}$$

where the first equality follows from integration by parts, the second equality follows from $x_\mu \cdot (1-F(x_\mu)) = \mu$, the first inequality follows from $x \cdot [1-F(x)] \leq \mu$ for all $x < x_\mu$ and the monotonicity of F, and the last inequality holds since $x_\mu \cdot (1-F(0)) < x_\mu \leq \bar{v}$.

At the same time, since $W(x_\mu) = -c + \int_{x_\mu}^{\bar{v}} (v - x_\mu)\, dF(v) \leq 0$, we know that

$$\int_{x_\mu}^{\bar{v}} v\, dF(v) \leq (1 - F(x_\mu)) \cdot x_\mu + c = \mu + c.$$

In combination, this shows that

$$S = \int_0^{x_\mu} v\, dF(v) + \int_{x_\mu}^{\bar{v}} v\, dF(v) \leq \mu \cdot \left(1 + \ln\left(\frac{\bar{v}}{\mu}\right)\right) + c,$$

as required.

Case C2: Assume $\sup_{\sigma\in\mathcal{E}^{dd}} \Pi(\sigma) > \mu$, which implies $\inf_{\sigma\in\mathcal{E}^{dd}} \Pi(\sigma) < \sup_{\sigma\in\mathcal{E}^{dd}} \Pi(\sigma)$. In this case, by Lemma 7 in the proof of Proposition 2, there exists an equilibrium of the due diligence game where the winning bid is $\hat{x}$ and the winner buys the good sight unseen. Moreover, it holds that $\hat{x} > 0$ and $W(\hat{x}) \geq N \cdot W(x)$ for all $x \geq 0$ such that $\Pi(x) > \Pi(\hat{x})$.

Observe that $\Pi(x) > \Pi(\hat{x})$ if and only if $x \cdot [1 - F(x)] > \hat{x}$. Thus, if $x \cdot [1 - F(x)] > \hat{x}$, it follows that $x > \hat{x}$ and, therefore, $W(x) = W^{dd}(x)$. Recall that $x^o := \min\{x \geq 0 : x \cdot [1 - F(x)] = \hat{x}\}$. By continuity of $x \mapsto W(x)$, we have that $W(x^o) \leq \frac{1}{N} W(\hat{x})$. Furthermore, since $\hat{x} = x^o \cdot (1 - F(x^o)) \leq x^o \cdot (1 - F(0))$, we have that $\frac{\hat{x}}{1-F(0)} \leq x^o < \bar{v}$.

By definition, for all $x \in (0, x^o)$, it holds that $[1 - F(x)] \cdot x \leq \hat{x}$. Therefore,

$$\begin{aligned}
\int_0^{x^o} v\, dF(v) &= -x^o \cdot (1 - F(x^o)) + \int_0^{x^o} (1 - F(v))\, dv \\
&\leq -\hat{x} + \int_0^{\frac{\hat{x}}{1-F(0)}} (1 - F(0))\, dv + \int_{\frac{\hat{x}}{1-F(0)}}^{x^o} \left(\frac{\hat{x}}{v}\right) dv \\
&= \hat{x} \cdot \left(\ln\left(\frac{x^o}{\hat{x}} \cdot (1 - F(0))\right)\right) \\
&\leq \hat{x} \cdot \ln\left(\frac{\bar{v}}{\hat{x}}\right) \\
&\leq \mu \cdot \ln\left(\frac{\bar{v}}{\mu}\right),
\end{aligned}$$

where the first equality follows from integration by parts, the first inequality follows from the fact that $x^o \cdot (1 - F(x^o)) = \hat{x}$ and that $x \cdot [1 - F(x)] \leq \hat{x}$ for all $x < x^o$, the second inequality follows from $x^o \cdot (1 - F(0)) < x^o \leq \bar{v}$, and the final inequality is implied by $\hat{x} \leq \mu$ and the fact that $x \mapsto x \cdot \left(\ln\left(\frac{\bar{v}}{x}\right)\right)$ is strictly increasing for all $x < \bar{v}$.

Next, since $W^{dd}(x^o) = -c + \int_{x^o}^{\bar{v}} (v - x^o)\, dF(v) \leq \frac{1}{N} W(\hat{x}) = \frac{1}{N}(\mu - \hat{x})$, it holds that

$$\int_{x^o}^{\bar{v}} v\, dF(v) \leq (1 - F(x^o)) \cdot x^o + \frac{1}{N}(\mu - \hat{x}) + c = \mu \cdot \frac{1}{N} + \frac{N-1}{N} \cdot \hat{x} + c \leq \mu + c.$$

Combining the above two inequalities, we obtain that

$$S = \int_0^{x^o} v\, dF(v) + \int_{x^o}^{\bar{v}} v\, dF(v) \leq \mu \cdot \left(1 + \ln\left(\frac{\bar{v}}{\mu}\right)\right) + c,$$

as required. This concludes the proof of Lemma 20.

□

Proof of Theorem 1: Part 3

We are now ready to prove our main result. Fix any distribution F and information cost $c > 0$ satisfying the Assumptions in Section 2. Let $\mathcal{E}_k^\tau$ be the set of equilibrium strategy profiles induced by $\tau \in \{no, re, dd\}$ whenever $\tilde{v}$ is replaced by its mean-preserving k-scaling $\tilde{v}'_k$, and define $V_\Pi^k \geq 0$ by

$$V_\Pi^k := \max_{\tau\in\{no,re,dd\}} \left(\inf_{\sigma\in\mathcal{E}_k^\tau} \Pi(\sigma) \right).$$

Since no-information always dominates research (Proposition 1), to prove Theorem 1 it is sufficient to show that there exists $k^* > 0$ such that

$$\{dd\} = \underset{\tau\in\{no,re,dd\}}{\arg\max} \left(\inf_{\sigma\in\mathcal{E}_k^\tau} \Pi(\sigma) \right), \quad \forall k > k^*,$$

and

$$\inf_{\sigma\in\mathcal{E}_k^{dd}} \Pi(\sigma) < V_\Pi^k, \quad \forall k \in (0, k^*).$$

As a first step, we show that there exists $\bar{k} > 0$ such that

$$\{dd\} = \underset{\tau\in\{no,re,dd\}}{\arg\max} \left(\inf_{\sigma\in\mathcal{E}_k^\tau} \Pi(\sigma) \right), \quad \forall k > \bar{k}. \tag{E}$$

To do so, recall from Lemma 20 that due diligence is strictly optimal whenever inequality (6) holds, which is equivalent to

$$S > \mu \cdot \left(1 + \ln\left(\frac{\bar{v}}{\mu}\right)\right) + c. \tag{23}$$

Thus, it is sufficient to prove that inequality (23) is satisfied eventually as $k \uparrow \infty$, once $\tilde{v}$ is replaced by its mean-preserving k-scaling $\tilde{v}'_k$.

When $\tilde{v}$ is replaced by $\tilde{v}'_k$, the RHS of (23) becomes

$$RHS(k) = \mu \cdot \left(1 + \ln(k) + \ln\left(\bar{v} - \frac{k-1}{k}\mu\right)\right) + c \leq \mu \cdot \ln(k) + M,$$

where $M > 0$ is some positive constant. On the other hand, the LHS of (23) becomes

$$LHS(k) = k \cdot \int_{\frac{k-1}{k}\mu}^{\bar{v}} \left(v - \frac{k-1}{k}\mu\right) dF(v) \geq k \cdot \int_{\mu}^{\bar{v}} (v - \mu)\, dF(v) = k \cdot \bar{c} > 0.$$

Since $\lim_{k\uparrow\infty} \frac{\ln(k)}{k} = 0$, the existence of a threshold $\bar{k} > 0$ satisfying property (E) follows.

Now, let k^* be defined by

$$k^* := \inf\left\{\bar{k} > 0 : \bar{k} \text{ satisfies property (E)}\right\}.$$

Observe that $k^* > 0$ since any $\bar{k}$ satisfying property (E) must be such that $\bar{k} \geq \frac{\mu}{\mu - \underline{v}} > 0$.[54] Furthermore, by construction, it holds that

$$\{dd\} = \underset{\tau \in \{no, re, dd\}}{\arg\max} \left(\inf_{\sigma \in \mathcal{E}_k^\tau} \Pi(\sigma) \right), \quad \forall k > k^*.$$

Thus, we are left to show that for all $\bar{k} \in (0, k^*)$, if we replace $\tilde{v}$ with its mean-preserving $\bar{k}$-scaling $\tilde{v}'_{\bar{k}}$, then due diligence is not weakly optimal, i.e., it holds that

$$\inf_{\sigma \in \mathcal{E}_{\bar{k}}^{dd}} \Pi(\sigma) < V_\Pi^{\bar{k}}.$$

By contradiction, suppose that this is not the case, i.e., there exists $\bar{k} \in (0, k^*)$ such that due diligence is weakly optimal whenever we replace $\tilde{v}$ with its mean-preserving $\bar{k}$-scaling $\tilde{v}'_{\bar{k}}$. Let $\varepsilon > 0$ be any strictly positive constant, and consider the mean-preserving $(\bar{k} + \varepsilon)$-scaling of $\tilde{v}$. Notice that $\tilde{v}'_{\bar{k}+\varepsilon}$ can be obtained from $\tilde{v}'_{\bar{k}}$ by applying a leftward shift and then scaling up the valuation to keep $\mu > 0$ constant. By Corollary 2 and statement *(b)* in Lemma 19, it follows that due diligence becomes strictly optimal under $\tilde{v}'_{\bar{k}+\varepsilon}$. Since this is true for all $\varepsilon > 0$, we conclude that $\bar{k} > 0$ satisfies property (E). However, this is implies that $k^* \leq \bar{k}$, a contradiction. This completes the proof of the theorem.

Q.E.D.

B Negative Expected Gains from Trade

In this Appendix, we prove Proposition 4.

Proof of Proposition 4

We want to show that due diligence continues to be strictly optimal when $\mu \leq 0$ and $c < S$. To do so, we proceed in order: First, we analyze the equilibrium implications of the no-information benchmark. Then, we investigate the equilibria of the research game. Finally, we consider the due diligence game.

No-information benchmark. Suppose information acquisition is not permitted, i.e., the buyers play the no-information benchmark. The following lemma characterizes the revenue the seller earns in every equilibrium of this game when $\mu \leq 0$. We omit the simple proof.

Lemma 21. *If $\mu \leq 0$, the buyers earn a payoff of 0 and the seller earns a revenue of 0 in every equilibrium of the no-information benchmark. Thus, it holds that $\Pi(\sigma) = 0$ for all $\sigma \in \mathcal{E}^{no}$.*

Research game. Suppose buyers can acquire information before bidding, i.e., they play the research game. The following lemma shows that if $\mu \leq 0$ and $c < S$, the revenue guarantee the seller accrues across all equilibria is equal to 0.

[54] If $\bar{k} < \mu/(\mu - \underline{v})$, the lower-end of the support of $\tilde{v}'_k$ becomes strictly positive, implying that information acquisition is socially inefficient. (See Section 2.3.) As a result, $\bar{k}$ could not satisfy property (E).

Lemma 22. *Suppose $\mu \leq 0$ and $c < S$. Then, it holds that*

$$\inf_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma) = 0.$$

Proof of Lemma 22. Let $\mu \leq 0$ and $c < S$. Our proof mirrors the proof of Proposition 1. In particular, once again we proceed via backward induction: Since the seller's equilibrium incentives are trivial, i.e., she always accepts one of the highest available offers, we first characterize buyers' equilibrium play during the bidding phase. Then, we study buyers' equilibrium incentives during the research phase.

For every subset $J \subseteq I$, let Γ^J denote the continuation (bidding) game that follows a research phase where the buyers doing research are exactly those in J. Given this, denote with U_i^J and Π^J buyer i's payoff and the seller's revenue in an equilibrium of the continuation game Γ^J, respectively. The following sequence of claims characterizes these utility levels as a function of the cardinality of J.

Claim 4. *Suppose $|J| = 0$, i.e., $J = \emptyset$. Then, it holds that $U_i^{\emptyset} = 0$ and $\Pi^{\emptyset} = 0$ in every equilibrium of the continuation game $\Gamma^{\emptyset}$.*

Proof of Claim 4. If no buyer acquired information during the research phase, the continuation game $\Gamma^{\emptyset}$ is strategically equivalent to the no-information benchmark. Therefore, invoking Lemma 21 is sufficient to prove the result.

□

Claim 5. *Suppose $|J| \geq 2$. Then, it holds that $U_i^J = 0$ if $i \notin J$, $U_i^J = -c$ if $i \in J$, and $\Pi^J = S$ in every equilibrium of the continuation game Γ^J.*

Proof of Claim 5. The proof of this result is identical to the proof of Lemma 4.

□

Claim 6. *Suppose $|J| = 1$, i.e., $J = \{i\}$ for some $i \in I$. Then, it holds that $U_i^{\{i\}} > 0$, $U_j^{\{i\}} = 0$ for $j \neq i$, and $\Pi^{\{i\}} = 0$ in every equilibrium of the continuation game $\Gamma^{\{i\}}$.*

Proof of Claim 6. Without loss of generality, assume $J = \{1\}$. We first show that buyer 1 must abstain when he observes a negative valuation $v < 0$. Since he cannot receive a positive payoff from winning the auction in this case, he must either abstain or submit a bid that loses with certainty. Towards a contradiction, suppose he does not always abstain after observing a negative valuation. For $i \neq 1$, let $G_i(x)$ denote the cdf describing i's bidding strategy, and let $\underline{x}_i = \inf\{x : G(x) > 0\}$. Let $\underline{x} = \max_{i \neq 1} \underline{x}_i$. We know that $\underline{x} \geq 0$ since there exists a bid with which buyer 1 loses with certainty. Note that since he can guarantee a positive payoff after observing $v > \underline{x}$, type v of buyer 1 never submits a bid of $x_1(v) < \underline{x}$, and never submits a bid of $x_1(v) = \underline{x}$ unless he is guaranteed to win the auction with such a bid. Now, without loss of generality, let buyer 2 be such that (i) $\underline{x}_2 = \underline{x}$ and (ii) he wins with positive probability if he submits a bid of $\underline{x}$, or just above $\underline{x}$ in case he does not bid $\underline{x}$ with positive probability. The above arguments imply that buyer 2's payoff of winning with a bid of $\underline{x}$ is at most $\int_{\underline{v}}^{\underline{x}}(v-\underline{x})\,dF(v) < 0$. Thus, it is suboptimal for buyer 2 to submit a bid of $\underline{x}$, a contradiction.[55] Thus, buyer 1 always abstains after observing a negative valuation, which also implies that all uninformed buyers abstain with probability 1.

Given that all uninformed buyers abstain, the unique best response for buyer 1 after observing a valuation $v > 0$ is to submit a bid of $x_1(v) = 0$. Thus, the equilibrium of $\Gamma^{\{1\}}$ is essentially unique[56] and given by the following profile of strategies:

- After observing the valuation $v \in V$, the informed buyer bids $x_1^*(v)$ given by

$$x_1^*(v) = \begin{cases} 0 & \text{if } v \geq 0, \\ \varnothing & \text{otherwise;} \end{cases}$$

- Every buyer $i \neq 1$ abstains from bidding, i.e., it holds that

$$x_i^* = \varnothing, \quad \forall \in I \setminus \{1\}.$$

To conclude the proof Claim 5, consider the equilibrium described in the bullet points above. We want to show that $U_1^{\{1\}} > 0$, $U_j^{\{1\}} = 0$ for $j > 1$, and $\Pi^{\{1\}} = 0$. That all uninformed buyers earn an equilibrium payoff equal to 0 follows immediately from the fact that they all abstain from bidding. That $\Pi^{\{1\}} = 0$ instead follows from the fact that the only active buyer, i.e., buyer 1, only makes an offer of 0 in equilibrium. Finally, to see why buyer 1 earns a strictly positive overall equilibrium payoff $U_1^{\{1\}}$, notice that by definition he earns a payoff of $U_1^{\{1\}} = S - c$, which is strictly positive by assumption.

□

To see why $\inf_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma) = 0$, suppose that buyers expect that only buyer 1 acquires information during the research phase. Because of Claims 4, 5, and 6, it is easy to verify that this outcome can be supported in equilibrium. But then, Claim 6 implies that $\inf_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma) = 0$, as required.

□

Due diligence game. Suppose $\mu \leq 0$ and $c < S$. Since $c < S$, it holds that $\hat{x} < 0$. Therefore, $\Pi(x) = x \cdot (1 - F(x))$ for all $x \geq 0$, which implies that x^{dd} is the unique equilibrium winning bid of the due diligence game (see Lemma 8, which does not require Assumption 4). Furthermore, since $c < S$, it holds that $\check{x} > 0$. Therefore, we have that

$$\inf_{\sigma \in \mathcal{E}^{dd}} \Pi(\sigma) = \Pi(x^{dd}) = \max_{x \in R} x \cdot [1 - F(x)] = \max_{x \in [0, \check{x}]} x \cdot [1 - F(x)] > 0,$$

[55] Technically, it could be that buyer 2 does not submit a bid of $\underline{x}$ with positive probability. However, then a similar argument shows that his payoff conditional on winning with any bid close enough to $\underline{x}$ is negative, contradicting the fact that $\underline{x}_2 = \underline{x}$.

[56] After observing a valuation of $v = 0$, buyer 1 could either abstain or submit a bid of $x_1(0) = 0$. However, this is a probability-zero event and does not affect the agents' payoffs.

which implies that due diligence is strictly optimal, as required. This concludes the proof of Proposition 4.

Q.E.D.

C Best-case Equilibrium Selection

In this Appendix, we consider the case where the seller's goal is to maximize her best-case equilibrium revenue, i.e., the case where her problem is given by (**P'**). Our analysis proceeds as follows: We first prove Proposition 5 and Corollary 3. Then, we show that for any $c > 0$ and $\mu > 0$, we can always find a distribution F of buyers' valuation consistent with μ such that due diligence is strictly best-case optimal.

Proof of Proposition 5

This proof is divided into three parts, corresponding to the three bullet points in the statement of the proposition. Throughout, we make extensive use of results in the proofs of Proposition 1, Proposition 2 and Proposition 4.

Part 1: Our goal is to show that there exists a $c' \in (0, \min\{S, \bar{c}\})$ such that research is strictly best-case optimal whenever $c < c'$. As a first step, we show that the information choice configuration where all buyers acquire information with the same probability

$$\alpha^* = 1 - \left[\frac{c}{U_i^{\{i\}} + c}\right]^{\frac{1}{N-1}} \tag{24}$$

constitutes a symmetric equilibrium outcome of the research phase, irrespective of whether $\mu > 0$ or $\mu \leq 0$.[57] To see this, fix any buyer $i \in I$. Given the other buyers' information choices, he faces the following trade-off:

- If he does not acquire information during the research phase, Lemmas 3, 4 and 5 in case $\mu > 0$ (or Claims 4, 5 and 6 in case $\mu \leq 0$) imply that his payoff will be given by

$$U_i^{no} = 0.$$

- Instead, if he acquires information during the research phase, Lemmas 3, 4 and 5 in case $\mu > 0$ (or Claims 4, 5 and 6 in case $\mu \leq 0$) imply that his payoff will be given by

$$U_i^{yes} = (1 - \alpha^*)^{N-1} \cdot U_i^{\{i\}} - c \cdot \left(1 - (1 - \alpha^*)^{N-1}\right).$$

Thus, buyer i is indifferent if and only if $U_i^{no} = U_i^{yes}$, which is equivalent to equation (24). This proves that the information choice configuration where all buyers acquire information with symmetric probability $\alpha^* \in (0, 1)$ indeed constitutes an equilibrium outcome of the research phase, irrespective of whether $\mu > 0$ or $\mu \leq 0$.

[57] See Lemma 5 or Claim 6 for a definition of $U_i^{\{i\}} > 0$.

Now, observe that as $c \downarrow 0$, we have $\alpha^* \to 1$. This implies that, holding fixed this equilibrium of the research game $\sigma^* \in \mathcal{E}^{re}$, as the information cost becomes smaller, the probability that at least two buyers become informed during the research phase converges to 1. In turn, because the continuation equilibrium revenue for the seller is given by $\Pi^J = S$ whenever $|J| \geq 2$, i.e., whenever at least two buyers are informed during the bidding phase (Lemma 4 or Claim 5), we conclude that

$$\lim_{c \downarrow 0} \sup_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma) \to S.$$

On the other hand, from Proposition 0, Proposition 2 and the proof of Proposition 4, we know that there exists $\varepsilon > 0$ such that

$$\sup_{\sigma \in \mathcal{E}^{no}} \Pi(\sigma) = \max\{0, \mu\} < S - \varepsilon$$

and

$$\sup_{\sigma \in \mathcal{E}^{dd}} \Pi(\sigma) \leq \max\Big\{\mu, \max_{x \geq 0} x \cdot (1 - F(x))\Big\} < S - \varepsilon$$

for all $c > 0$. This concludes the proof of Part 1.

Part 2: Suppose $\mu > 0$. We want to show that there exists a cost threshold $c'' \in [c', S - \mu]$ such that no-information is strictly best-case optimal whenever $c \in (c'', \bar{c})$. We first prove an auxiliary lemma.

Lemma 23. *Fix any $c \in (0, \bar{c})$. The following holds:*

$$\sup_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma) < \max\Big\{\mu,\, S - c - (c/S)^2\,(S - c - \mu)\Big\}.$$

Proof of Lemma 23. In the equilibrium $\sigma \in \mathcal{E}^{re}$ where only one buyer conducts research with positive probability, we have $\Pi(\sigma) < \mu$, by Lemma 5. By Lemmas 3 and 5, no equilibrium exists where no buyer conducts research with positive probability. Thus, it only remains to consider equilibria where at least two buyers conduct research with positive probability. Fix any such equilibrium $\sigma^* \in \mathcal{E}^{re}$. Because of Lemmas 3, 4 and 5, σ must feature a subset of buyers $J \subseteq I$ with $|J| \geq 2$ such that all buyers $i \in J$ conduct research with probability $\alpha_i \in (0, 1)$, while all buyers $i \notin J$ refrain from conducting research.

Any buyer $i \in J$ must be indifferent between conducting research and not:

$$U_i^{\{i\}} \prod_{j \in J \setminus \{i\}} (1 - \alpha_j) - c \cdot \left(1 - \prod_{j \in J \setminus \{i\}} (1 - \alpha_j)\right) = 0. \tag{25}$$

Since $U_i^{\{i\}} = U_j^{\{j\}}$ for all $i, j \in J$ (Lemma 5), this implies that all buyers in J must conduct research with the same probability, i.e., $\alpha_i = \alpha$ for all $i \in J$. In turn, this implies that $\alpha \in (0, 1)$ solves

$$(1 - \alpha)^{|J|-1} = \frac{c}{U_i^{\{i\}} + c}. \tag{26}$$

Now, observe that in the continuation game after only one buyer became informed during the research phase, the sum of the seller's and buyers' joint surplus generated by trade must be strictly smaller than the maximum socially available surplus S, since trade occurs with positive probability even if the valuation is negative (Lemma 5). Thus, it holds that $\Pi_i^{\{i\}} + U_i^{\{i\}} + c < S$, which implies:

$$\Pi_i^{\{i\}} < S - \left(U_i^{\{i\}} + c\right) = S - \frac{c}{(1-\alpha)^{|J|-1}}, \tag{27}$$

where the equality follows from equation (26).

The seller's revenue under σ^* is therefore given by:

$$\begin{aligned}
\Pi(\sigma^*) &= S - (1-\alpha)^{|J|}(S-\mu) + J\alpha(1-\alpha)^{|J|-1}\left(\Pi_i^{\{i\}} - S\right) \\
&< S - (1-\alpha)^{|J|}(S-\mu) - |J|\alpha c \\
&< S - c - (1-\alpha)^{|J|}(S-\mu-c) \\
&= S - c - \left(\frac{c}{U_i^{\{i\}} + c}\right)^{\frac{|J|}{|J|-1}} (S-\mu-c) \\
&< S - c - (c/S)^{\frac{|J|}{|J|-1}} (S-\mu-c) \\
&\leq S - c - (c/S)^2 (S-\mu-c),
\end{aligned}$$

where the first inequality follows from (27), the second inequality follows from the fact that $(1-\alpha)^{|J|} > 1 - |J|\alpha$, the third inequality follows from $U_i^{\{i\}} + c < S$, and the final inequality obtains from the fact that $\frac{|J|}{|J|-1} \leq 2$ and $c < \bar{c} < S$. We conclude that

$$\sup_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma) < \max\left\{\mu,\, S - c - (c/S)^2 (S-c-\mu)\right\},$$

as required.

□

Now, observe that

$$\mu \geq S - c \quad \Longrightarrow \quad \max\left\{\mu,\, S - c - (c/S)^2 (S-c-\mu)\right\} = \mu.$$

Thus, Lemma 23 shows that $\sup_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma) < \mu$ whenever $c \geq S - \mu$.

On the other hand, note that Lemma 7 implies that $\sup_{\sigma \in \mathcal{E}^{dd}} \Pi(\sigma) = \max\{\hat{x}, \Pi(x^{dd})\}$. Since $\mu > 0$, we have that $\hat{x} < \mu$. Moreover, since $W(x^{dd}) = -c + \int_{x^{dd}}^{\bar{v}} (v - x^{dd})\, dF(v) \geq 0$ and $\Pi(x^{dd}) = x^{dd} \cdot (1 - F(x^{dd}))$, it holds that

$$\Pi(x^{dd}) \leq \Pi(x^{dd}) + W(x^{dd}) = -c + \int_{x^{dd}}^{\bar{v}} v\, dF(v) < S - c.$$

Thus, whenever $c \geq S - \mu$, we have:

$$\sup_{\sigma \in \mathcal{E}^{dd}} \Pi(\sigma) < \max\{\mu, S - c\} = \mu.$$

Since

$$\sup_{\sigma \in \mathcal{E}^{no}} \Pi(\sigma) = \mu,$$

the proof of Part 2 is now complete.

Part 3: Finally, suppose that $\mu \leq 0$. Our goal is to show that there exists a cost threshold $c'' \in [c', S)$ such that due diligence is the strictly best-case optimal whenever $c \in (c'', S)$. Since no-information leads to an equilibrium revenue of 0, it cannot be optimal. We now prove preliminary claims about research and due diligence.

Claim 7. *When $\mu \leq 0$, the set of possible equilibrium revenues under research is given by*

$$\left\{\Pi(\sigma) : \sigma \in \mathcal{E}^{re}\right\} = \left\{\Pi(\sigma_J) : J \subseteq I,\, J \neq \emptyset\right\},$$

for some $\Pi(\sigma_J)$ that satisfy $\lim_{c\uparrow S} \Pi(\sigma_J) = 0$ and $\lim_{c\uparrow S} \frac{\partial}{\partial c}\Pi(\sigma_J) = 0$.

Proof of Claim 7. From Claims 4, 5, and 6, we know that no equilibrium exists where no buyer conducts research with positive probability. Similarly, these claims imply that in any equilibrium $\sigma_{\{i\}}$ where only one buyer (namely, buyer i) conducts research with positive probability, this buyer conducts research with probability 1, and the seller's revenue is $\Pi(\sigma_{\{i\}}) = 0$. Moreover, in any equilibrium where one buyer conducts research with probability 1, all other buyers have to conduct research with probability 0. Thus, in any remaining equilibrium candidate, a subset of buyers $J \subseteq I$ conducts research with positive probability $\alpha_i(J) > 0$, but not with certainty. In fact, for any given J such that $|J| \geq 2$, one can verify that unique equilibrium σ_J with these constraints exists. Fix any J with $|J| \geq 2$ and the corresponding equilibrium σ_J. In this equilibrium, all buyers $i \in J$ are indifferent between conducting research and not. Conducting research leads to a payoff of $S - c$ if no other buyer also conducts research and a payoff of $-c$ otherwise (Claims 5 and 6). On the other hand, remaining uninformed always leads to a payoff of 0 (Claims 4, 5 and 6). Therefore,

$$S \cdot \left(\prod_{j \in J\setminus\{i\}} (1 - \alpha_j(J))\right) - c = 0, \quad \forall i \in J,$$

which implies that $\alpha_i(J) = \alpha(J) := 1 - \left(\frac{c}{S}\right)^{\frac{1}{|J|-1}}$ for all $i \in J$. In turn, the corresponding equilibrium revenue is given by

$$\Pi(\sigma_J) = \left(1 - (1-\alpha(J))^{|J|} - |J|\alpha(J)(1-\alpha(J))^{|J|-1}\right) S = S \cdot \left(1 + (|J| - 1)\left(\frac{c}{S}\right)^{\frac{|J|}{|J|-1}}\right) - c|J|.$$

Note that $\lim_{c\uparrow S} \Pi(\sigma_J) = 0$. Furthermore,

$$\frac{\partial}{\partial c}\Pi(\sigma_J) = -|J| \cdot \left(1 - \left(\frac{c}{S}\right)^{\frac{1}{|J|-1}}\right) < 0,$$

which implies that $\lim_{c\uparrow S} \frac{\partial}{\partial c}\Pi(\sigma_J) = 0$. Hence, all possible equilibrium revenues under research satisfy the properties stated in the claim. □

Claim 8. *When $\mu \leq 0$, in any equilibrium of the due diligence game $\sigma \in \mathcal{E}^{dd}$, the seller's equilibrium revenue is given by $\Pi(\sigma) = \Pi(x^{dd}) = x^{dd} \cdot \Big(1 - F(x^{dd})\Big)$. Moreover, $\lim_{c \uparrow S} \Pi(x^{dd}) = 0$ and $\lim_{c \uparrow S} \frac{\partial}{\partial c} \Pi(x^{dd}) = -1$.*

Proof of Claim 8. Suppose $\mu \leq 0$, and let $\sigma \in \mathcal{E}^{dd}$. From the proof of Proposition 4, we know that $\Pi(\sigma) = \Pi(x^{dd}) = x^{dd} \cdot \Big(1 - F(x^{dd})\Big)$. Recall that

$$\bar{x} := \arg\max_{x \geq 0} x \cdot [1 - F(x)] > 0.$$

By Assumption 2, we have $x^{dd} = \min\{\check{x}, \bar{x}\}$, where $\check{x}$ is the unique solution to $W^{dd}(\check{x}) = 0$. Since $\check{x} = \check{x}(c)$ is continuously decreasing in c and satisfies $\lim_{c \uparrow S} \check{x}(c) = 0$, there exists a $c' < S$ such that $x^{dd} = \check{x}$ whenever $c \geq c'$. Note that

$$\frac{\partial}{\partial c} \check{x}(c) = -\frac{1}{1 - F(\check{x})}$$

and

$$\frac{\partial}{\partial c} \Pi(x^{dd}) = \frac{\partial}{\partial c} x^{dd} \cdot \Big(1 - F(x^{dd}) - x^{dd} f(x^{dd})\Big) < 0.$$

Thus, $\Pi(\sigma) \to 0$ as $c \uparrow S$ and

$$\lim_{c \uparrow S} \frac{\partial}{\partial c} \Pi(x^{dd}) = -\frac{1 - F(0) - 0 f(0)}{1 - F(0)} = -1.$$

This completes the proof of the claim. □

To conclude the proof of Proposition 5, note that by Claim 7, the set of possible equilibrium revenues under research can be described as a finite set given by

$$\Big\{\Pi(\sigma_J) : J \subseteq I,\ J \neq \emptyset\Big\}$$

where $\Pi(\sigma_J) = 0$ if $|J| = 1$, and

$$\Pi(\sigma_J) = S \cdot \left(1 + (|J| - 1)\left(\frac{c}{S}\right)^{\frac{|J|}{|J|-1}}\right) - c|J|,$$

otherwise. On the other hand, the seller's unique equilibrium revenue under due diligence is $\Pi(x^{dd})$ (Claim 8). Therefore, we need to show that $\Pi(\sigma_J) < \Pi(x^{dd})$ for all $J \neq \emptyset$ as $c \uparrow S$.

If $|J| = 1$, this is obvious. So, suppose $|J| \geq 2$. Then, since $\Pi(x^{dd}) = x^{dd} \cdot \Big(1 - F(x^{dd})\Big) > 0$ for $c \in (0, S)$, the ratio $\frac{\Pi(\sigma_J)}{\Pi(x^{dd})}$ is well-defined, non-negative, and continuous in c for $c \in (0, S)$. Moreover, by *L'Hopital's rule* implies:

$$\lim_{c \uparrow S} \frac{\Pi(\sigma_J)}{\Pi(x^{dd})} = \lim_{c \uparrow S} \frac{\frac{\partial}{\partial c} \Pi(\sigma_J)}{\frac{\partial}{\partial c} \Pi(x^{dd})} = 0.$$

Thus, there exists $c''(J) \in [0, S)$ such that $\frac{\Pi(\sigma_J)}{\Pi(x^{dd})} < 1$ for all $c \in (c''(J), S)$. Let $c'' := \max_{J \subseteq I,\, J \neq \emptyset} c''(J)$. Then

$$\sup_{\sigma \in \mathcal{E}^{dd}} \Pi(\sigma) = x^{dd} \cdot \left(1 - F(x^{dd})\right) > \max_{J \subseteq I,\, J \neq \emptyset} \Pi(\sigma_J) = \sup_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma)$$

for all $c \in (c'', S)$, as required.

Q.E.D.

Proof of Corollary 3

As $\omega \to \check{x}$, $S \downarrow c$. The result then follows from Proposition 5

Q.E.D.

We now state and proof Proposition 6, which shows that due diligence can be best-case optimal also when $\mu > 0$.

Proposition 6. *Under best-case selection, for any information cost $c > 0$ and expected valuation $\mu > 0$, there exists a distribution F consistent with μ such that due diligence is strictly best-case optimal.*

Proof of Proposition 6

By Lemma 23, we know that the equilibrium revenue under research is bounded above by a number strictly smaller than $S - c$. We now show that under due diligence, we can find a distribution F which achieves an equilibrium revenue arbitrarily close to $S - c$. Throughout, we focus on distributions for which information acquisition is socially beneficial, i.e., $S - c - \mu > 0$ since, otherwise, revenue would be maximized by not allowing for any information acquisition.

Lemma 24. *Fix any $\mu > 0$, $c > 0$, $S > \mu + c$, and $\delta \in \left(0, (S + c) - \sqrt{2c(S + c)}\right)$.*

Let

$$F(v) = \begin{cases} 0 & \text{if } v \leq -K_1 A, \\ \frac{2\delta}{K_1^2} \cdot (v + K_1 A) & \text{if } -K_1 A < v \leq K_1, \\ 1 + 2\frac{c}{K_2^2} \cdot (v - K_1 - K_2) & \text{if } K_1 < v \leq K_1 + K_2, \\ 1 & \text{otherwise,} \end{cases}$$

where $K_1 = S - c + \delta\,(1 + 2A) > 0$, $K_2 = \frac{2c}{1 - \frac{2\delta}{K_1}(1+A)} > 0$, and $A = \sqrt{\frac{S-\mu}{\delta}} > 0$.

Then, $\sup_{\sigma \in \mathcal{E}^{dd}} \Pi(\sigma) \geq S - c - \delta$.

Proof of Lemma 24. Note that, by construction, we have $\int_{\underline{v}}^{\bar{v}} v\, dF(v) = \mu$ and $\int_0^{\bar{v}} v\, dF(v) = S$. Furthermore, together with $c > 0$, F satisfies the assumptions in Section 2.[58]

[58] F does not admit a continuous density. However, you can approximate it arbitrarily closely by a distribution

In what follows, we show that a bid of $K_1 > 0$ leads to a non-negative expected payoff to the winner ($W(K_1) \geq 0$) and an expected revenue of $\Pi(K_1) = S - c - \delta$. Since all buyers submitting a bid equal to

$$\underset{x \geq 0 : W(x) \geq 0}{\arg\max} \; \Pi(x)$$

constitutes an equilibrium of the due diligence game, this would imply that

$$\sup_{\sigma \in \mathcal{E}^{dd}} \Pi(\sigma) \geq \Pi(K_1) = S - c - \delta,$$

thus proving the lemma.

Toward this goal, note that

$$W^{dd}(K_1) = -c + \int_{K_1}^{K_1+K_2} (v - K_1)\, dF(v) = -c + \int_0^{K_2} \frac{2c}{K_2^2} z\, dz = 0.$$

Thus, $W(x) \geq 0$ for all $x \in [0, K_1]$. Furthermore, a buyer who wins with a bid of K_1 conducts due diligence, since $K_1 > S - c > \mu$, and therefore $W^{dd}(K_1) = 0 > \mu - K_1 = W^{su}(K_1)$. Therefore,

$$\Pi(K_1) = K_1 \cdot (1 - F(K_1)) = K_1 \cdot \frac{2c}{K_2} = S - c - \delta,$$

as required.

□

To conclude the proof of Proposition 6, take any $S > \mu + c$, and consider the distribution F described in Lemma 24 for $0 < \delta \leq \min\left\{ \left(\frac{c}{S}\right)^2 (S - c - \mu), (S + c) - \sqrt{2c(S + c)} \right\}$. Then, it holds that

$$\sup_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma) < S - c - \delta \leq \sup_{\sigma \in \mathcal{E}^{dd}} \Pi(\sigma),$$

where the first inequality follows from Lemma 23 and $\mu < S - c$, and the second inequality from Lemma 24.

Q.E.D.

with a continuous denstity.

Supplementary Appendix

D Repeated Due Diligence

In this Appendix, we consider a two-period extension of our baseline due diligence model where the seller runs a second informal auction with the remaining buyers in case the winner of the initial auction withdraws after due diligence. We call this model the *repeated due-diligence model.* To streamline the exposition, we assume that there is no discounting, that buyers observe the winning bid of the first round, and restrict attention to equilibria where buyers submit deterministic bids.

The possibility of running a second auction produces two opposing effects on equilibrium revenue. On the one hand, a second auction increases the overall probability of a sale, pushing revenue up. On the other hand, since losing buyers can still hope to purchase the good in the second period in case the initial winner reneges, buyer competition in the initial auction is dampened. In what follows, we show that despite these equilibrium effects, our main result is unaffected: Due diligence remains the strictly optimal timing of information acquisition whenever stakes are sufficiently high. Formally:

Proposition 7. *Fix any distribution F and information cost $c > 0$ satisfying the assumptions of Section 2. There exists a threshold $k^* > 0$ such that, if $\tilde{v}$ is replaced by its mean-preserving k-scaling $\tilde{v}'_k$, repeated due diligence produces a revenue strictly larger than μ whenever $k > k^*$.*

Proof of Proposition 7

Fix any feasible distribution F and an information cost $c > 0$. From now on, we use k-subscripts to indicate pieces of notation corresponding to the mean-preserving k-scaling $\tilde{v}'_k$. For example, we let F_k denote the cdf associated with $\tilde{v}'_k$, while $\hat{x}_k$ denotes the threshold bid defined by equation (2). We begin by showing the following preliminary lemma.

Lemma 25. *For all sufficiently large $k > 0$, it holds that $\hat{x}_k < 0$.*

Proof of Lemma 25. By definition, it holds that

$$c = \int_{\underline{v}_k}^{\hat{x}_k} (\hat{x}_k - v)\, dF_k(v).$$

Changing the variable of integration to $y = \frac{1}{k}[v + (k-1)\mu]$, we obtain that

$$\frac{c}{k} = \int_{\underline{v}}^{a_k} (a_k - y)\, dF(y),$$

where $a_k := \frac{1}{k}[\hat{x}_k + (k-1)\mu]$. Now, notice that $a_k = \hat{x}(c/k)$, where $\hat{x}(\cdot)$ is the function corresponding to the cdf F defined implicitly by equation (2) (see footnote 20). Since $\lim_{c\downarrow 0} \hat{x}(c) = \underline{v}$, we obtain that

$$\frac{\hat{x}_k}{k} \to \underline{v} - \mu < 0, \qquad \text{as } k \uparrow \infty.$$

But then, it must be that $\lim_{k\uparrow\infty} \hat{x}_k = -\infty$, implying the result.

□

Lemma 25 implies that it is without loss to assume that $\hat{x} < 0$ for the purpose of this proof. Given Lemma 1, it follows that no buyer would ever buy the good sight unseen in equilibrium. The remainder of this proof proceeds in two steps. First, we prove the proposition under the assumption that there are only two buyers. We then show that, relative to this scenario, the seller's worst-case equilibrium revenue is larger when $N \geq 3$.

Proof of Proposition 7: Part 1

Suppose that $N = 2$, i.e., $I = \{1, 2\}$. For every $z \in [0, \bar{v}]$, let F^z denote the truncation of the distribution F at z, i.e., $F^z(x) = F(x)/F(z)$ for all $x \in [\underline{v}, z]$. Similarly, let $\hat{x}^z$, $\check{x}^z$, $\Pi^z(x)$ and $W^{dd,z}(x)$ be the mathematical objects corresponding to $\hat{x}$, $\check{x}$, $\Pi(x)$ and $W^{dd}(x)$, respectively. One can verify that $\frac{\partial}{\partial z}\hat{x}^z > 0$ and $\frac{\partial}{\partial z}\check{x}^z > 0$ for all $z \in (0, \bar{v})$. Thus, given that by assumption $\hat{x} < 0$, it holds that $\hat{x}^z < 0$ as well.

To analyze the two-period extension of our model, we use a backward induction approach. Suppose in period $t = 1$, buyer $i \in I$ won the initial auction with a bid of $z \geq 0$ but, after conducting due diligence, decided to renege on his offer. Because this buyer knew the seller would exclude him from the second informal auction of period $t = 2$, it must be the case that he reneged after observing a realized valuation of $v < z$ (Lemma 1). In turn, this implies that buyer $j \neq i$ (and the seller) perceives the good's valuation as distributed according to F_z when entering period $t = 2$. In other words, in period $t = 2$, buyer j is more pessimistic about the good's true valuation relative to the beginning of the game.

Now, observe that since $N = 2$, buyer j is the only bidder left in period $t = 2$. Therefore, in no equilibrium, he would buy the good for a price larger than 0 in this continuation game. However, it could be that buyer j has become so pessimistic about $\tilde{v}$ that he prefers to abstain from bidding over the prospect of purchasing the good at a price of 0. (For example, this would be the case if $z = 0$.) Thus, the remaining buyer always secures a continuation equilibrium payoff of

$$u^z := \max\left\{0, W^{dd,z}(0)\right\}$$

in period $t = 2$, while the seller always collects a revenue of 0 in the same period.

The equilibrium payoff $u^z \geq 0$ is crucial to determine buyers' bidding incentives in the initial auction. To see this, note that no buyer would submit a bid $x > 0$ such that $W^{dd}(x) < u^z$ in period $t = 1$, unless he expects to lose the auction with certainty. This implies that buyers' relevant range of initial bids is given by $R(z) := [0, \check{x}^*(z)]$,[59] where

$$W^{dd}(\check{x}^*(z)) = u^z.$$

At the same time, u^z depends on $z \geq 0$, which represents the winning bid in period $t = 1$. Since standard Bertrand arguments imply that buyers must submit the offer that maximizes the

[59]Note that $R(z) \subseteq R$, i.e., the relevant range of bids shrank relative to the baseline model. This follows from the possibility of purchasing the good in period $t = 2$, represented by $u^z \geq 0$.

seller's revenue $\Pi(x) = x \cdot [1 - F(x)]$ among the bids in $R(z)$ in any equilibrium, we conclude that any initial equilibrium winning bid z^* must be a solution to the following fixed-point problem:

$$z^* = \arg\max_{x \in R(z^*)} \Pi(x). \tag{28}$$

Standard arguments show that the fixed-point problem in (28) admits a unique solution $z^* > 0$. Thus:

Lemma 26. *In the repeated due diligence model, if $N = 2$ and $\hat{x} < 0$, the seller earns a revenue equal to $\Pi(z^*)$ in any equilibrium, where where z^* is the unique solution to (28).*

Given Lemma 26, our goal is to show that $\Pi(z^*) > \mu$ as $k \uparrow \infty$.

Toward this goal, consider the bid $x^a = a\mu$, where a is a positive scalar such that $a \cdot (1 - F(\mu)) > 1$. As a first step, we show that $\Pi_k(x^a) > \mu$ for all sufficiently large $k > 0$, where

$$\Pi_k(x) := x \cdot (1 - F_k(x)).$$

To see this, notice that for all $y \in \mathbb{R}$, we have that

$$F_k(y) = F\left(\frac{1}{k}(y + (k-1) \cdot \mu\right) \to F(\mu), \qquad \text{as } k \uparrow \infty.$$

Therefore, as $k \uparrow \infty$, it holds that

$$\Pi_k(x^a) = x^a \cdot (1 - F_k(x^a)) \to x^a \cdot (1 - F(\mu)) > \mu$$

since $a \cdot (1 - F(\mu)) > 1$ by assumption.

To complete the proof of Part 1, we are left to show that $x^a \in R_k(z_k^*)$ for all $k > 0$ sufficiently large, where z_k^* is the unique equilibrium winning bid solving the fixed-point problem (28) corresponding to F_k. To see why this is the case, suppose by contradiction that there exists a sequence $(k_n)_n$ with $k_n \uparrow \infty$ such that $x^a \notin R_k(z_k^*)$, i.e., $x^a > \check{x}_{k_n}^*(z_{k_n}^*)$ for all $n \in \mathbb{N}$. Observe that $W^{dd,z}(0) \leq W^{dd,z'}(0)$ whenever $z < z'$. Therefore, since by definition $z_{k_n}^* \leq \check{x}_{k_n}^*(z_{k_n}^*)$, we obtain that

$$W_{k_n}^{dd,z_{k_n}^*}(0) \leq W_{k_n}^{dd,x^a}(0).$$

Now, notice that

$$\begin{aligned}
W_{k_n}^{dd,x^a}(0) &= -c + \int_0^{x^a} v \, dF_{k_n}^{x^a}(v) \\
&= -c + \frac{1}{F_{k_n}(x^a)} \int_0^{x^a} v \cdot f_{k_n}(v) \, dv \\
&\leq -c + \bar{f}_{k_n} \cdot \frac{1}{F_{k_n}(x^a)} \int_0^{x^a} v \, dv \\
&= -c + \frac{\bar{f}}{k_n} \cdot \frac{1}{F_{k_n}(x^a)} \cdot \left(\frac{1}{2} (x^a)^2\right),
\end{aligned}$$

where $\bar{f}_k = \max_{v \in V_k} f_k(v)$ for all $k > 0$, and $\bar{f} = \max_{v \in V} f(v) < \infty$. Therefore,

$$\lim_{n \uparrow \infty} W_{k_n}^{dd,x^a}(0) = -c < 0.$$

In turn, this implies that $u_{k_n}^{z_{k_n}^*} = 0$ for all $n > 0$ sufficiently large. But then, $\check{x}_{k_n}^*(z_{k_n}^*) = \check{x}_{k_n}$ eventually. Since $\check{x}_k \to +\infty$ as $k \uparrow \infty$, we derived a contradiction.

This shows that, when $N = 2$, as long as stakes are sufficiently high, repeated due diligence leads to an expected revenue strictly larger than μ in every equilibrium, as required.

Proof of Proposition 7: Part 2

Now, suppose that $N \geq 3$. In period $t = 2$, after an initial winning bid of $z \geq 0$ is withdrawn, the remaining buyers (and the seller) perceive the good's valuation as drawn from the cdf F^z. Since F^z is regular[60] and $\hat{x}^z \leq \hat{x} < 0$, arguments similar to those used in the proof of Lemma 7 can be used to conclude that, in any continuation equilibrium of period $t = 2$, the following holds:

- If $\check{x}^z < 0$, all buyers abstain, and the seller earns a revenue of $\Pi_2(z) = 0$; or
- If $\check{x}^z \geq 0$, the winning bid is

$$x^{dd,z} := \arg\max_{x \in [0, \check{x}^z]} x \cdot (1 - F^z(x))$$

  and the seller earns a revenue of $\Pi_2(z) = \Pi^z(x^{dd,z}) = x^{dd,z} \cdot (1 - F^z(x^{dd,z})) \geq 0$.

The above result has the following two important implications: First, the seller's continuation revenue $\Pi_2(z)$ is weakly increasing in $z \geq 0$.[61] Second, the payoff any remaining buyer can obtain in any continuation equilibrium of period $t = 2$ is bounded above by

$$u^z := \begin{cases} 0 & \text{if } \check{x}^z < 0, \\ W^{dd,z}(x^{dd,z}) & \text{otherwise.} \end{cases}$$

Now, let us consider the buyers' bidding incentives in period $t = 1$. As a first step, note that the buyers know that the seller would evaluate each initial offer $z \geq 0$ according to the expected equilibrium revenue $\Pi_1(z)$ that offer would induce, which is given by

$$\Pi_1(z) := z \cdot (1 - F(z)) + \Pi_2(z).$$

Note that $\Pi_1(z) \geq z \cdot (1 - F(z))$. Furthermore, since $\Pi_2(z)$ is weakly increasing in $z \geq 0$, we have that $\Pi_1(z)$ is strictly increasing in z whenever the same is true for $z \mapsto z \cdot (1 - F(z))$.

In any equilibrium, at least two buyers submit the same bid z^* maximizing (at least locally)

[60] The regularity of F implies the regularity of F^z.
[61] This follows from $\frac{\partial}{\partial z}\check{x}^z \geq 0$ and the fact that $z \mapsto y \cdot (1 - F^z(y))$ increases with $z \geq 0$ for all $y \geq 0$.

$\Pi_1(z)$ over a relevant range $R(z^*)$ that includes the interval $[0, \check{x}^*(z^*)]$, where

$$W^{dd}(\check{x}^*(z^*)) = u^{z^*}.$$

Now, notice that, relative to the case where $N = 2$, when $N \geq 3$, the payoff u^z has increased. In turn, this implies that the relevant range of initial bids when $N \geq 3$ is a super-set of the corresponding set in the case where $N = 2$. Given this, suppose that when $N = 2$, repeated due diligence guarantees an equilibrium revenue $z^* \cdot (1 - F(z^*))$ strictly larger than μ to the seller. (By Part 1 of this proof, this is the case whenever the stakes are sufficiently high.) We now show that, when $N \geq 3$, the seller cannot earn a strictly lower expected revenue in any equilibrium. To see why this is the case, note that, when $N \geq 3$, the following statements hold:

- Because $\Pi_2(z)$ is weakly increasing in $z \geq 0$, the initial bid z^* is located to the left of any local maximum point of $z \mapsto \Pi_1(z)$.
- The initial bid z^* must be feasible in any equilibrium.
- We have that $\Pi_1(z^*) \geq z^* \cdot (1 - F(z^*)) > \mu$.

Together, the above statements imply the seller earns a revenue strictly larger than μ in any equilibrium when $N \geq 3$. Since this must be true whenever the stakes are sufficiently high, the proof of Proposition 7 is completed.

Q.E.D.

E Noisy Information Processing

In this Appendix, suppose that after acquiring information, any buyer i receives a signal about the common-valuation of the good given by $s_i = s_i^\theta = v + \theta\varepsilon_i$, where $\theta > 0$ is a scale parameter and $\varepsilon_i \overset{i.i.d.}{\sim} G_\varepsilon$ is a noise term. Throughout this Appendix, we assume that the distribution G_ε is symmetric around 0 and that $\log g_\varepsilon$ is strictly concave, where $g_\varepsilon > 0$ is the density of G_ε. This implies that buyers' signals are (strongly) *affiliated* in the sense of Milgrom and Weber (1982).[62] In what follows, we show that as long as θ is sufficiently small, *(i)* the equilibrium of the research game where only one buyer acquires information while the other buyers remain uninformed continues to exist, and *(ii)* the revenue the seller earns in this equilibrium is still bounded above by μ. In other words, we prove the following proposition.

Proposition 8. *When information processing is noisy, for every set of primitives* (c, F, G)*, there exists* $\theta^* > 0$ *such that the seller strictly prefers granting no access to information over allowing the buyers to conduct research, i.e.,*

$$\inf_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma) < \mu = \inf_{\sigma \in \mathcal{E}^{no}} \Pi(\sigma),$$

whenever $\theta < \theta^*$*.*

To prove Proposition 8, it is useful to generalize the notation used for the proof of Proposition 1 by making explicit the dependence of equilibrium variables on the scale parameter $\theta > 0$. For every subset $J \subseteq I$, let $\Gamma^J(\theta)$ denote the continuation (bidding) game that follows a research phase where the buyers doing research are exactly those in J. Given this, denote with $U_i^J(\theta)$ buyer i's payoff, and with $\Pi^J(\theta)$ the seller's revenue in a particular equilibrium of the continuation game $\Gamma^J(\theta)$, respectively. The following sequence of lemmas characterizes these payoffs as a function of $|J| \in \{0, 1, 2\}$ when θ is small.

Lemma 27. *Suppose* $|J| = 0$*, i.e.,* $J = \emptyset$*. Then, in any equilibrium of the continuation game* $\Gamma^\emptyset(\theta)$*, buyers' payoffs are given by* $U_i^\emptyset(\theta) = 0$*.*

The proof of Lemma 27 is identical to the one of Lemma 3. If no buyer acquired information during the research phase, buyers compete à la Bertrand for the expected valuation of the good μ during the bidding phase. Hence, noisy signals do not alter equilibrium predictions relative to the perfect-information benchmark when $|J| = 0$.

Lemma 28. *Suppose* $|J| = 2$*. Then, there exists* $\theta_0^{\text{two}} > 0$ *such that, for all* $\theta \in (0, \theta_0^{\text{two}})$*, there exists an equilibrium of the continuation game* $\Gamma^J(\theta)$ *where buyers' payoffs are given by* $U_i^J(\theta) = 0$ *if* $i \notin J$ *and* $U_i^J(\theta) < 0$ *if* $i \in J$*.*

Proof of Lemma 28. Fix some $\theta > 0$, and without loss of generality, suppose that $J = \{1, 2\}$. Denote by $H_\theta(\cdot|y)$ the cdf of s_1^θ given that $s_2^\theta = y$ (i.e., $H_\theta(x|y) := \mathbb{P}(s_1^\theta \le x | s_2^\theta = y)$ for all $x \in \mathbb{R}$), and let $h_\theta(\cdot|y)$ be the corresponding pdf of $H_\theta(\cdot|y)$. Also, let $w_\theta(x, y) := \mathbb{E}[\tilde{v} | s_1^\theta = x, s_2^\theta = y]$ be the conditional expectation of $\tilde{v}$ given $s_1^\theta = x$ and $s_2^\theta = y$. Note that $w_\theta(\cdot, \cdot)$ is strictly increasing

[62] For a proof of this statement, see, e.g., Lehmann, Romano, and Casella (1986).

in both its arguments.[63] In what follows, we show that as long as θ is sufficiently small, the following profile of strategies constitutes an equilibrium of the continuation game $\Gamma^J(\theta)$:

- All uninformed buyers abstain from bidding, i.e., $x_i^* = \varnothing$ for all $i \notin J$.
- Upon receiving signal $s_i^\theta = y$, buyer $i \in J = \{1, 2\}$ submits a bid $x_\theta^*(y)$ given by

$$x_\theta^*(y) := \begin{cases} \varnothing & \text{if} \quad y < s_\theta^* \\ \int_{s_\theta^*}^y w_\theta(z, z)\, dL_\theta(z|y) & \text{if} \quad y \geq s_\theta^* \end{cases}$$

  where s_θ^* is the unique solution to

$$\int_{-\infty}^{s_\theta^*} w_\theta(s_\theta^*, z)\, dH_\theta(z|s_\theta^*) = 0,$$

  and $L_\theta(z|y)$ is a cdf given by

$$L_\theta(z|y) := \exp\left(-\int_z^y \frac{h_\theta(t|t)}{H_\theta(t|t)}\, dt\right).$$

To prove that the above candidate equilibrium is indeed an equilibrium for θ sufficiently small, we need to show that no buyer has a strict incentive to deviate. From Chapter 6.4 of Krishna (2009), we know that given the candidate equilibrium strategies the other buyers play, the strategy each informed buyer $i \in J = \{1, 2\}$ plays is a best response. Therefore, it remains to show that no uninformed buyer would strictly benefit by participating in the auction rather than abstaining. To do so, we first prove the following two claims.

Claim 9. *It holds that*

$$\liminf_{\theta \downarrow 0} x_\theta^*(y) \geq y \quad \forall y \in (0, \bar{v}). \tag{29}$$

Proof of Claim 9. Let (θ_n) be a decreasing sequence such that $\theta_n \downarrow 0$, and fix any $y \in (0, \bar{v})$. Observe that since $s_\theta^* \to 0$ as $\theta \downarrow 0$, we can assume that $y > s_{\theta_n}^*$ for all $n \in \mathbb{N}$. Given this, fix any $\delta \in (0, y - s_{\theta_1}^*)$. Because $z \mapsto w_\theta(z, z)$ is increasing, we know that $w_\theta(z, z) \geq 0$ for all $z \geq s_\theta^*$. Thus, it holds that

$$\begin{aligned} x_{\theta_n}^*(y) &= \int_{s_{\theta_n}^*}^{y-\delta} w_{\theta_n}(z, z)\, dL_{\theta_n}(z|y) + \int_{y-\delta}^{y} w_{\theta_n}(z, z)\, dL_{\theta_n}(z|y) \\ &\geq w_{\theta_n}(y - \delta, y - \delta) \cdot (1 - L_{\theta_n}(y - \delta|y)) \end{aligned}$$

for all $n \in \mathbb{N}$. Therefore,

$$\liminf_n x_{\theta_n}^*(y) \geq \left(\liminf_n w_{\theta_n}(y - \delta, y - \delta)\right) \cdot \left(1 - \limsup_n L_{\theta_n}(y - \delta|y)\right).$$

Since $\liminf_n w_{\theta_n}(y - \delta, y - \delta) = y - \delta$, it is sufficient to show that $\limsup_n L_{\theta_n}(y - \delta|y) = 0$.

[63] This follows from the strict concavity of $\log g_\varepsilon$.

Toward this goal, observe that the fact that $z \mapsto e^z$ is increasing implies that

$$\limsup_n L_{\theta_n}(y-\delta|y) = \exp\left(-\liminf_n \int_{y-\delta}^{y} \frac{h_{\theta_n}(t|t)}{H_{\theta_n}(t|t)}\,dt\right).$$

Thus, to prove that $\limsup_n L_{\theta_n}(y-\delta|y) = 0$, it is sufficient to verify that

$$\liminf_n \int_{y-\delta}^{y} \frac{h_{\theta_n}(t|t)}{H_{\theta_n}(t|t)}\,dt = \infty. \tag{30}$$

Now, observe that

$$\liminf_n \int_{y-\delta}^{y} \frac{h_{\theta_n}(t|t)}{H_{\theta_n}(t|t)}\,dt \geq \int_{y-\delta}^{y} \liminf_n \frac{h_{\theta_n}(t|t)}{H_{\theta_n}(t|t)}\,dt \geq \int_{y-\delta}^{y} \liminf_n h_{\theta_n}(t|t)\,dt$$

where the first inequality follows from Fatou's lemma, and the second inequality comes from the fact that $H_\theta(t|t) \in [0,1]$ and $h_\theta(t|t) \geq 0$ for all $t \in (y-\delta, y)$. In what follows, we show that

$$\liminf_n h_{\theta_n}(t|t) = \infty$$

for all $t \in (y-\delta, y)$, thus proving equation (30). To do so, fix any $t \in (0, \bar{v})$. One can verify that

$$h_\theta(t|t) = \frac{1}{\theta} \cdot \frac{\int_{\underline{v}}^{\bar{v}} g_\varepsilon\left(\frac{t-v}{\theta}\right)^2 f(v)\,dv}{\int_{\underline{v}}^{\bar{v}} g_\varepsilon\left(\frac{t-v}{\theta}\right) f(v)\,dv}.$$

Now, let $m > 0$ be any positive number such that $g_\varepsilon(m) > 0$. Since $g_\varepsilon(\cdot)$ is unimodal and symmetric around 0, it holds that

$$\frac{\int_{\underline{v}}^{\bar{v}} g_\varepsilon\left(\frac{t-v}{\theta}\right)^2 f(v)\,dv}{\int_{\underline{v}}^{\bar{v}} g_\varepsilon\left(\frac{t-v}{\theta}\right) f(v)\,dv} \geq \frac{g_\varepsilon(m) \cdot \left(\min_{v\in[t-\theta m, t+\theta m]} f(v)\right) \cdot \left(G_\varepsilon(m) - G_\varepsilon(-m)\right)}{\left(\max_{v\in[\underline{v},\bar{v}]} f(v)\right) \cdot \left(G_\varepsilon\left(\frac{t-\underline{v}}{\theta}\right) - G_\varepsilon\left(\frac{t-\bar{v}}{\theta}\right)\right)},$$

where both the numerator and denominator are well-defined objects since $f > 0$ is continuous by assumption. Thus,

$$\begin{aligned}\liminf_n \frac{\int_{\underline{v}}^{\bar{v}} g_\varepsilon\left(\frac{t-v}{\theta_n}\right)^2 f(v)\,dv}{\int_{\underline{v}}^{\bar{v}} g_\varepsilon\left(\frac{t-v}{\theta_n}\right) f(v)\,dv} &\geq \liminf_n \frac{g_\varepsilon(m) \cdot \left(\min_{v\in[t-\theta_n m, t+\theta_n m]} f(v)\right) \cdot \left(G_\varepsilon(m) - G_\varepsilon(-m)\right)}{\left(\max_{v\in[\underline{v},\bar{v}]} f(v)\right) \cdot \left(G_\varepsilon\left(\frac{t-\underline{v}}{\theta_n}\right) - G_\varepsilon\left(\frac{t-\bar{v}}{\theta_n}\right)\right)} \\ &= \frac{g_\varepsilon(m) \cdot f(t) \cdot \left(G_\varepsilon(m) - G_\varepsilon(-m)\right)}{\max_{v\in[\underline{v},\bar{v}]} f(v)} > 0.\end{aligned}$$

Given this observation, we conclude that $\liminf_n h_{\theta_n}(t|t) = \infty$ for all $t \in (y-\delta, y)$, as required. This completes the proof of the claim.

□

Claim 10. *For every $\theta > 0$, let $\tilde{\sigma}_\theta := \max\left\{s_1^\theta, s_2^\theta\right\}$ be the maximum of the informed buyers' signals, and denote by $f_{\sigma_\theta}(s) > 0$ its corresponding density defined on the support $(\underline{\sigma}_\theta, \bar{\sigma}_\theta)$. Then,* *it holds that*

$$\lim_{\theta \downarrow 0} f_{\sigma_\theta}(s) \to f(s)$$

for all $s \in (\underline{v}, \bar{v})$.

Proof of Claim 10. For every $s \in (\underline{v}, \bar{v})$, we have that

$$f_{\sigma_\theta}(s) = \frac{1}{\theta} \int_{\underline{v}}^{\bar{v}} 2G_\varepsilon \left(\frac{s-v}{\theta}\right) \cdot g_\varepsilon \left(\frac{s-v}{\theta}\right) \cdot f(v)\, dv.$$

By changing the variable of integration to $\varepsilon = \frac{s-v}{\theta}$, we obtain that

$$f_{\sigma_\theta}(s) = \int_{\frac{s-\bar{v}}{\theta}}^{\frac{x-\underline{v}}{\theta}} 2G_\varepsilon(\varepsilon) \cdot g_\varepsilon(\varepsilon) \cdot f(s - \theta\varepsilon)\, d\varepsilon.$$

Note that

$$\int_{-\infty}^{\infty} 2G_\varepsilon(\varepsilon) \cdot g_\varepsilon(\varepsilon)\, d\varepsilon = 1.$$

Therefore, since $f > 0$ is continuous, we can apply the Dominated Convergence Theorem to conclude that $f_{\sigma_\theta}(s) \to f(s)$ as $\theta \downarrow 0$, as required. □

Now, fix $x > 0$ arbitrarily and suppose that an uninformed buyer, say buyer 3, offers such x. Since informed buyers abstain with strictly positive probability, bidding $x \geq \bar{v}$ is strictly suboptimal. Thus, assume that $x < \bar{v}$ from now on. Let $U_\theta(x)$ be the payoff buyer 3 would earn if the other buyers play according to the candidate equilibrium strategies described above. Fix any $\delta > 0$ small. Since the bidding function $s \mapsto x^*_\theta(s)$ of the informed buyers is increasing, Claim 9 implies that for θ sufficiently small, buyer 3 can outbid them only if they both received a private signal $s_i^\theta = y$ such that $y \leq x + \delta$. Therefore, for θ sufficiently small, it holds that

$$U_\theta(x) \leq \int_{\underline{\sigma}_\theta}^{x+\delta} \Big(\mathbb{E}[v|\sigma_\theta = s] - x\Big) \cdot f_{\sigma_\theta}(s)\, ds \leq \int_{\underline{v}}^{x+\delta} \Big(\mathbb{E}[v|\sigma_\theta = s] - x\Big) \cdot f_{\sigma_\theta}(s)\, ds.$$

Given this, the Dominated Convergence Theorem, together with Claim 10, implies that:[64]

$$\limsup_{\theta \downarrow 0} U_\theta(x) \leq \int_{\underline{v}}^{x+\delta} (v - x)\, dF(v) \leq \int_{\underline{v}}^{x} (v - x)\, dF(v) + \delta\big(F(x+\delta) - F(x)\big).$$

Because $\delta > 0$ can be taken arbitrarily small, we conclude that

$$\limsup_{\theta \downarrow 0} U_\theta(x) \leq \int_{\underline{v}}^{x} (v - x)\, dF(v) < 0.$$

Therefore, for $\theta > 0$ small enough, no uninformed buyer has a strict incentive to deviate from abstaining. This shows that there exists $\theta_0 > 0$ such that for all $\theta < \theta_0$, our candidate equilibrium is indeed an equilibrium, as required.

To complete the proof of the lemma, we need to verify the statements about the buyers' equilib-

[64] We can invoke this theorem since $\mathbb{E}[v|\sigma_\theta = s] \leq \bar{v}$ and $f_{\sigma_\theta}(s) \leq \max_{v \in [\underline{v}, \bar{v}]} f(v)$ for every $s \in (\underline{\sigma}_\theta, \bar{\sigma}_\theta)$. See also Claim 10.

rium payoff. Fix any $\theta < \theta_0$. Since the uninformed buyers abstain in the equilibrium described above, it holds that $U_i^J(\theta) = 0$ for all $i \notin J$. Finally, to see why the informed buyers $i \in J$ instead obtain a payoff $U_i^J(\theta) < 0$ for θ small enough, note that Claim 9 implies that

$$\limsup_{\theta \downarrow 0} \left(\mathbb{E}[\tilde{v}|s_i^\theta = x] - x_\theta^*(x) \right) \leq 0.$$

This shows that if θ is sufficiently small, no informed buyer can earn a trade surplus that is larger than the information cost in equilibrium, as required.

□

Lemma 29. *Suppose $|J| = 1$, i.e., $J = \{i\}$ for some $i \in I$. Then, there exists $\theta_0^{\text{one}} > 0$ such that, for all $\theta \in (0, \theta_0^{\text{one}})$, an equilibrium of the continuation game $\Gamma^{\{i\}}$ exists where the payoffs are given by $U_i^{\{i\}}(\theta) > 0$, $U_j^{\{i\}}(\theta) = 0$ for $j \neq i$. Furthermore, $\Pi^{\{i\}}(\theta) < \mu$.*

Proof of Lemma 29. The proof of this statement follows from the results in Engelbrecht-Wiggans, Milgrom, and Weber (1983),[65] and Lemma 5. We omit the details. □

We are now ready to prove Proposition 8.

Proof of Proposition 8

Let $\theta^* = \min\{\theta_0^{\text{one}}, \theta_0^{\text{two}}\}$, where $\theta_0^{\text{one}}, \theta_0^{\text{two}} > 0$ are the thresholds defined in the statements of Lemmas 29 and 28, respectively. Proposition 8 follows from Lemmas 27, 28 and 29, which jointly imply that when $\theta < \theta^*$, there exists an equilibrium of the overall research game where only one buyer acquires information while the other buyers remain uninformed. In particular, by Lemma 29, in this equilibrium the seller earns a revenue strictly smaller than μ. Thus, it holds that

$$\inf_{\sigma \in \mathcal{E}^{re}} \Pi(\sigma) < \mu = \inf_{\sigma \in \mathcal{E}^{no}} \Pi(\sigma),$$

whenever $\theta < \theta^*$, as required.

Q.E.D.

[65] In particular, Engelbrecht-Wiggans, Milgrom, and Weber (1983) prove that uninformed buyers always earn a payoff of 0 in equilibrium. In contrast, the informed buyer's equilibrium payoff and the seller's equilibrium revenue are continuous in θ. While their analysis does not incorporate a reservation value for the seller, their results do extend to such a setting directly. See also Hendricks, Porter, and Wilson (1994).

F Commitment to Buy Sight Unseen

In this Appendix, we analyze the due diligence game with buyer commitment. Specifically, a bid now consists of two elements: the price offer and whether the buyer buys the good sight unseen or conducts due diligence. Thus, a bid can be represented as a pair $(x, m) \in X \times \{su, dd\}$. Of course, in equilibrium, a buyer would never commit to conducting due diligence if she preferred to buy the good sight unseen following that bid. This is because conducting due diligence only lowers the seller's expected revenue and therefore her evaluation of the bid. Therefore, this variant of the model is outcome equivalent to only allowing the buyers to commit to buying the good sight unseen.

Let $W(x, m) := W^m(x)$, $R := \{(x, m) : W(x, m) \geq 0\}$, and

$$\Pi(x, m) = \begin{cases} x & \text{if } m = su, \\ x \cdot [1 - F(x)] & \text{if } m = dd. \end{cases}$$

We now prove the following proposition.

Proposition 9. *If the buyers can commit to buying the good sight unseen, due diligence is strictly optimal if and only if*

$$\max_{x \in R} x \cdot [1 - F(x)] > \mu.$$

Proof of Proposition 9

As a first step, note that an adjusted version of Lemma 2 continues to hold: A buyer's offer is accepted in equilibrium only if it maximizes $\Pi(\cdot, \cdot)$ among all submitted bids. Furthermore, any winning bid (x^*, m^*) must satisfy $W(x^*, m^*) \geq 0$. Given this, that $\max_{x \in R} x \cdot [1 - F(x)] > \mu$ is necessary follows since, otherwise, there exists no (x, m) such that $W(x, m) \geq 0$ and $\Pi(x, m) > \mu$.

It remains to prove sufficiency, i.e., that when $\max_{x \in R} x \cdot [1 - F(x)] > \mu$ any equilibrium of the due diligence game σ results in an equilibrium revenue of $\Pi(\sigma) > \mu$. We proceed to show that, in fact, when this is the case, the winning bid has to be equal to (x^{dd}, dd) in any equilibrium, and thus result in an equilibrium revenue of

$$x^{dd} \cdot \left[1 - F(x^{dd})\right] = \max_{x \in R} x \cdot [1 - F(x)] > \mu.$$

Claim 11. *If $x^{dd} \cdot \left[1 - F(x^{dd})\right] > \mu$, in any equilibrium of the due diligence game with buyer commitment where the winning bid is non-random, the winning bid is (x^{dd}, dd).*

Proof of Claim 11. Toward a contradiction, suppose an equilibrium σ with a non-random winning bid of $(x^*, m^*) \neq (x^{dd}, dd)$ exists. Note that $\Pi(x^*, m^*) < \Pi(x^{dd}, dd)$ since, otherwise, $W(x^*, m^*) < 0$. This implies that no buyer submits a bid $(x, m) \in X \times \{su, dd\}$ such that $\Pi(x, m) > \Pi(x^*, m^*)$. Now, let

$$(x', m') = \underset{(x,m):\Pi(x,m)=\Pi(x^*,m^*)}{\arg\max} W(x, m).$$

The bid (x', m') exists since, for fixed $m \in \{su, dd\}$, both $x \mapsto W(x, m)$ and $x \mapsto \Pi(x, m)$ are continuous functions. Furthermore, it holds that $W(x', m') > 0$.

In the equilibrium σ, at least one buyer, say buyer i, receives a payoff of less than $\frac{1}{N}W(x', m')$. This implies that buyer i has a strictly profitable deviation: He can bid $(x'+\varepsilon, m')$ for some small $\varepsilon > 0$ instead. Since $\Pi(x'+\varepsilon, m') > \Pi(x', m') = \Pi(x^*, m^*)$, this results in a payoff of $W(x'+\varepsilon, m')$ to buyer i. Since $W(\cdot, m)$ is continuous, and $W(x', m') > 0$, we know that for ε small enough, $W(x' + \varepsilon, m') > \frac{1}{N}W(x', m')$. This contradicts that σ is, in fact, an equilibrium. □

Fix any equilibrium of the due diligence game with buyer commitment inducing an outcome where the winning bid is random. For every $i \in I$, denote with $\mathcal{B}_i \in \Delta(X \times \{su, dd\})$ the buyer i's possibly random equilibrium bidding strategy, with U_i^* buyer i's equilibrium payoff, and with $J^* \subseteq I$ the subset of buyers that are selected by the seller as a winner with positive probability in equilibrium. Further, for every $i \in I$, denote with $S_i \in \Delta(\mathbb{R}_+)$ the cdf over the expected revenue that the seller would receive if she accepted buyer i's equilibrium offer with probability 1, i.e.,

$$S_i(\theta) := \mathcal{B}_i\left(\left\{(x, m) \in X \times \{su, dd\} : \Pi(x, m) \leq \theta\right\}\right), \quad \forall \theta \geq 0.$$

Let $\underline{s}_i := \inf(\operatorname{supp} S_i) \geq 0$ for every $i \in I$

Claim 12. *If $x^{dd} \cdot \left(1 - F(x^{dd})\right) > \mu$, any equilibrium of the due diligence game with buyer commitment where the winning bid is random must satisfy $|J^*| \geq 2$.*

Proof of Claim 12. Suppose by contradiction that $J^* = \{i\}$ for some $i \in I$. By definition, all buyers $j \neq i$ must satisfy $U_j^* = 0$. Furthermore, note that $\underline{s}_i < \Pi(x^{dd}, dd)$ since buyer i's bid must be random, he cannot submit offers outside of R, and there is only one bid in R that results in an expected revenue of $\Pi(x^{dd}, dd)$ by regularity. But then, there exists $x' > 0$ such that $\Pi(x', dd) > \underline{s}_i$ and $W(x', dd) > 0$.[66] However this is impossible since, otherwise, any buyer $j \neq i$ could profitably deviate to submitting (x', dd), which results in a strictly positive expected utility. □

Claim 13. *If $x^{dd} \cdot \left(1 - F(x^{dd})\right) > \mu$, any equilibrium of the due diligence game with buyer commitment where the winning bid is random must satisfy $U_j^* > 0$ for all $j \in J^*$.*

Proof of Claim 13. Fix $j \in J^*$ arbitrarily. Since buyers can always abstain from bidding, it must be that $U_i^* \geq 0$. Now, suppose by contradiction that $U_j^* = 0$. Since no buyer would submit a bid (x, m) with $W(x, m) < 0$ in equilibrium if he believes that such bid will be accepted with positive probability, it must be that buyer j wins the auction only when he offers a bid (x, m) such that $W(x, m) = 0$. There are two such bids: (μ, su) and $(\check{x}, dd)$. We first show that it cannot be that buyer j wins with positive probability after submitting (μ, su). Let $x' = \min\{x \geq 0 : \Pi(x, dd) = \mu\}$. Note that $x' < x^{dd}$ and, therefore, $W(x', dd) > W(x^{dd}, dd) \geq 0$. Thus, buyer j has a profitable deviation to bidding $(x' + \varepsilon, dd)$ for some small $\varepsilon > 0$ instead of (μ, su), since this bid guarantees a strictly positive expected utility as long as ε is small enough. Similarly, it can be shown that it cannot be that buyer j wins with positive probability after

[66] In particular, let $x' = x^{dd} - \varepsilon$ for some small $\varepsilon > 0$.

submitting $(\check{x}, dd)$ by arguments identical to those in Lemma 10. We conclude that $U_j^* > 0$ for all $j \in J^*$, as required.

□

Claim 14. *If $x^{dd} \cdot \left(1 - F(x^{dd})\right) > \mu$, any equilibrium of the due diligence game with buyer commitment where the winning bid is random must satisfy $\underline{s}_j = \underline{s}$ for all $j \in J^*$.*

The proof of Claim 14 is identical to the proof of Lemma 11.

Claim 15. *If $x^{dd} \cdot \left(1 - F(x^{dd})\right) > \mu$, an equilibrium of the due diligence game with buyer commitment where the winning bid is random does not exist.*

Proof of Claim 15. From Claims 12, 13, and 14, we can deduce that $S_i(\underline{s}) > 0$ for all $i \in J^*$ since, otherwise, some buyer $j \in J^* \setminus \{i\}$ would receive a payoff of zero.[67] Thus, all buyers $i \in J^*$ submit a bid that results in an expected revenue of $\underline{s}$ with positive probability. Let

$$(x', m') = \underset{(x,m):\Pi(x,m)=\underline{s}}{\arg\max} \; W(x, m).$$

Note that $\underline{s} < x^{dd} \cdot \left(1 - F(x^{dd})\right)$ and that no buyer ever submits a bid (x'', m'') such that $\Pi(x'', m'') = \underline{s}$ and $W(x'', m'') < W(x', m')$ since, otherwise, this buyer could deviate to bidding $(x' + \varepsilon, m')$ for $\varepsilon > 0$ small, which satisfies $\Pi(x' + \varepsilon, m') > \Pi(x'', m'')$ and $W(x' + \varepsilon, m') > W(x'', m'')$. Note that it is a positive-probability event that all buyers $i \in J^*$ submit a bid that leads to an expected revenue of $\underline{s}$. Fix any buyer in J^* that (on average) is selected as a winner with a probability weakly smaller than $\frac{1}{|J^*|}$ under such configurations of bids. (Such a buyer necessarily exists.) We now show that such buyer $j \in J^*$ has a strict incentive to deviate from $\mathcal{B}_j$. To see this, observe that by submitting bids resulting in an expected revenue of $\underline{s}$, buyer j earns, on average, an expected payoff no greater than

$$\frac{1}{|J^*|} \cdot \left(\prod_{i \in J^* \setminus \{j\}} S_i(\underline{s}) \right) \cdot W(x', m') \geq 0.$$

However, by replacing (x', m') with $(x' + \varepsilon, m')$, buyer j would earn an expected payoff greater than or equal to

$$\left(\prod_{i \in J^* \setminus \{j\}} S_i(\underline{s}) \right) \cdot W(x' + \varepsilon, m') > \frac{1}{|J^*|} \cdot \left(\prod_{i \in J^* \setminus \{j\}} S_i(\underline{s}) \right) \cdot W(x', m'),$$

as long as $\varepsilon > 0$ is sufficiently small. This contradicts the optimality of $\mathcal{B}_j$. We conclude that an equilibrium where the winning bid is random does not exist, as required. □

Claims 11 and 15 together show that, whenever $x^{dd} \cdot \left(1 - F(x^{dd})\right) > \mu$, the winning bid must be equal to (x^{dd}, dd) in any equilibrium of the due diligence game with commitment.

Q.E.D.

[67] This follows from arguments identical to those in Claim 2.