Sticky Inflation: Monetary Policy when Deficits Drag Inflation Expectations

Saki Bigio* — UCLA and NBER
Nicolas Caramp† — UC Davis
Dejanir Silva‡ — Purdue

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Abstract

We append the expectation of a possible monetary-fiscal reform to a standard new Keynesian model. If a reform occurs, monetary policy will temporarily aid debt sustainability through a temporary burst in inflation. Prior to a reform, rising public debt due to deficits pushes up inflation expectations. Monetary policy interest rates have two effects: they influence demand and affect expected inflation in opposite directions. The expectations effect is linked to the impact of interest rates on public debt. While lowering inflation in the short term is possible through demand control, inflation tends to rise again due to expectations (sticky inflation). In this context, optimal monetary policy may require negative real interest rates after fiscal shocks, temporarily breaking from the Taylor principle. This framework helps us assess whether the Federal Reserve was justified in ”staying behind the curve“ during the recent inflation surge.

*Bigio is at the Economics Department at UCLA. Email: sbigio@econ.ucla.edu.
†Caramp is at the Department of Economics at UC Davis. Email: ncaramp@ucdavis.edu
‡Silva is at the Mitchell E. Daniels, Jr. School of Business, Purdue University. Email: dejanir@purdue.edu

1. Introduction

In the wake of the Covid-19 pandemic, inflation became a persistent global issue. Central banks were slow to respond to calls for raising interest rates to curb inflation and were subsequently criticized for their delay. The reasoning behind this criticism assumes that a prompt rate hike would prevent inflation expectations from becoming unanchored, thus avoiding further inflationary pressures. This traditional view assumes that raising rates reduces demand and signals the central bank's resolve to control inflation. However, it overlooks the fact that higher rates also accelerate the growth of national debt, potentially pushing it to levels that cannot be sustained solely through taxes (see, for instance Zhengyang, Lustig, Nieuwerburgh and Xiaolan, 2022). This issue is especially concerning now, as inflation surged alongside national debt levels not seen in decades. Central banks face a further challenge not conceived by their critics: increasing rates may worsen the debt burden, leading people to expect future inflation as a means to stabilize that debt.

This paper examines inflation dynamics when agents anticipate scenarios in which public debt may need to be stabilized through inflationary finance. Our analysis is prompted by the sharp rise in medium-term inflationary disaster expectations following the Covid-19 pandemic, as reported by Hilscher, Raviv and Reis (2022). In the U.S., Hazell and Hobler (2024) link this surge in inflation expectations to fiscal deficits during the same period. We aim to provide a straightforward analysis of how the expectation of a future need to reduce government debt through inflation affects the current effectiveness of monetary policy.

To that end, we study a paper-and-pencil new- Keynesian model in which agents anticipate a possible monetary-fiscal reform. In the event of such a reform, the monetary authority temporarily allows higher inflation, resulting in negative real interest rates for a set period. After the reform, debt and inflation are stabilized. The key tension is that, before the reform, increases in real interest rates carry two effects with opposing effects on inflation: the first is a conventional decrease in aggregate demand, and the second, the novelty, is the increase in inflation expectations that respond to the greater debt burden. This increase in expected inflation enters as an endogenous cost-push and risk-premia shocks.

The paper makes two contributions. First, it introduces the concept of *sticky inflation*. Sticky inflation occurs when efforts to reduce inflation through interest rate hikes, while initially effective, fail in the medium term due to the increased debt burden, causing inflation to resurge. Under sticky inflation, attempts to control inflation with temporary rate hikes backfire.

The second contribution is to derive optimal policy prescriptions under sticky inflation. While the Taylor principle in the New Keynesian model suggests raising real interest rates aggressively to anchor expectations during inflation spikes, many central banks instead allowed very negative short-term real rates during recent inflation surges. This paper shows that, under sticky inflation, such underreaction is actually optimal.

The first part of the paper is devoted to characterizing inflationary dynamics for arbitrary paths of monetary policy rates. We perform three policy exercises. First, we characterize sticky inflation in the context of policy-rate paths that aim to close the output gap. This first exercise shows that stabilizing output generates a prolonged inflationary episode fueled by inflation expectations associated with higher debt levels. A second exercise shows that temporary increases in nominal rates aimed at controlling inflation may only be successful on impact. However, as long as the debt problems persist, inflation returns with greater force. Likewise, a policy that aims to stabilize debt permanently leads to an explosion in inflation and an undesirable overheating of the economy. The three exercises demonstrated that attempting to stabilize one outcome variable (inflation, the output gap, or debt) destabilizes other variables.

The policy exercises showcase that when debt levels add inflationary pressure through expectations, the effects of interest rates on debt financing affect inflation dynamics. The feedback from debt to inflation expectations breaks, in an endogenous way, the possibility of jointly stabilizing output and inflation (divine coincidence). This lack of divine coincidence leads to a non-trivial optimal monetary policy analysis, which we investigate in the context of commitment in the second part of the paper.

We show that before a reform, a monetary authority interested only in minimizing the square of inflation and the output gap should consider the squared deviation of debt relative to an inflation-neutral benchmark into its objective function.

We obtain an optimal real-interest path after a fiscal shock that maximizes this problem with fiscal considerations. After fiscal shocks, monetary policy allows for a period of negative rates, easing the debt burden. The period is characterized by a burst in inflation.

The key policy message is that unless fiscal effects are solved, the sole expectation of inflation-financed debt impairs current monetary policy and changes the standard prescriptions of Taylor rules, which are core to new Keynesian economics. Even a hawkish central bank that only cares about inflation stabilization, allows for temporary negative real interest rates to tame the inflationary pressures stemming from the expectation of a monetary-fiscal reform. This phenomenon may become particularly relevant in a world with greater public debt and more sensitive inflation expectations to these levels.

Literature Review. Since the surge in inflation after the COVID-19 pandemic, understanding the drivers of inflation has once again regained prominence in academic and policy debates. As a result, multiple papers have tried to explain inflationary dynamics from an analytical and quantitative standpoint. Research is divided into two camps: one that links a nominal anchor to fiscal factors and one that does not.

On the analytical front, for example, Lorenzoni and Werning (2023b) focus on the dynamic interaction between wage and price inflation.[1] Blanchard and Bernanke (2023) and Gagliardone and Gertler (2023) models that decomposes the drivers of inflation into labor-market shocks and energy shocks. This research heir leaves fiscal policy as either unexplained demand factors or implicitly detached from nominal variables. This stream of papers follows the new Keynesian tradition that typically abstracts away from how debt financing impairs monetary policy.

The interaction between monetary policy and fiscal solvency is vast and is part of core textbook material, (e.g., Ljungqvist and Sargent, 2018). The textbook approach abstracts away from nominal rigidities and assumes that deficits are financed by transfers of nominal money balances. Leeper (1991) studies the interaction between monetary policy and fiscal solvency in economies where monetary policy follows an interest-rate rule that violates the Taylor principle. A key feature of this theory is that government debt is nominal and, thus, associated with fiscal dominance and the Fiscal Theory of the Price Level (FTPL).[2] A sequence of papers in this FTPL tradition shows that in such environments, nominal policy rate increases lead to counter-factual inflation increases, following the logic of Fisherian effects. Woodford (2001); Cochrane (2001) show that with long-term debt, nominal policy rate increases lead to inflation decreases, overturning the original counterfactual Fisherian effect. However, Sims (2011) that the increase in nominal rates can reduce inflation only in the short run but eventually raises inflation.

This paper shares the emphasis on how fiscal solvency impacts inflation. However, our setting differs in important dimensions: In contrast to papers in the FTPL tradition, we focus on the dynamics of inflation where monetary policy is active and the Taylor principle

[1]On related work, Lorenzoni and Werning (2023a) shows how inflationary spirals can emerge from strategic interactions between different parties that set wages in a staggered fashion.

[2]Together with Leeper (1991), Woodford (1998); Cochrane (1998) show that under flexible-price, the price-level will jump in response to fiscal news, thus providing a fiscal-theory of the price level (FTPL). In these papers, the timing of taxes does not matter as Ricardian equivalence holds.

is satisfied. The difference is that monetary policy must live with a lurking expectation of possible inflationary finance. We find a similar “stepping-on-a-rake” as the one in Sims (2011), but here, the effect does not follow from the valuation of nominal long-term debt. Rather, the result follows from the confluence of two forces: a standard aggregate demand effect and an the effect on higher interest-rate burden.

A second generation studies the interaction between monetary policy and fiscal solvency that allows for sticky prices (e.g. Sims, 2011; Bianchi and Melosi, 2017; Leeper and Leith, 2016; Caramp and Silva, n.d.)). A common theme among these papers is that with nominal rigidities, interest-rate shocks, and fiscal shocks do not lead to price jumps but to persistent responses in inflation. Most analytical work in this area makes an assumption regarding fiscal or monetary dominance, although regime switching and expectations of policy changes are present in quantitative work.[3] Among these papers, the closest to ours is Bianchi, Faccini and Melosi (2023). In that paper, some fiscal shocks are adjusted with taxes and others are not, whereas a Taylor rule always guides monetary policy. The paper estimates that such unfunded fiscal shocks have been critical drivers of inflation in the US. Our approach is similar in that fiscal shocks are funded with a combination of inflation and taxes during and before reforms. Our contribution is to present a simple framework to characterize the sticky inflation phenomenon associated with expectations of refoms and what an optimal monetary policy should do about it.[4] This is not the case in our paper because we focus on the dynamics prior to a fiscal-monetary reform.

A key feature of our environment is that reforms happen in the future, so the expectation component of inflation is key in determining current inflation. This is a common feature in other papers such as Carvalho, Moench and Preston (forthcoming) and Eusepi and Preston (2012). In particular, Eusepi and Preston (2012) also shows that in similar environments, inflation will trend. On the empirical front, a number of papers have found that long-term forecasts are responsive to monetary shocks. In the US, Nakamura and Steinsson (n.d.) show that increases in policy rates reduce long-term forecasts of inflation. While this correlation is contrary, to our theory, it demonstrates that long-term inflation expectations

[3]See for example, Davig and Leeper (AER 2007); Chung, Davig, and Leeper (JMCB 2007), Bianchi (Restud 2013), Bianchi and Melosi (NBER Annual 2013, AER 2017) (Bianchi and Melosi, 2017).

[4]Our study of optimal policies also relates to some normative work in this area. Leeper, Leith, and Liu (JME 2021) and Leeper and Zhou (JME 2023) study optimal long-term debt policy. See also Leeper and Leith (2016). In these articles, the level and maturity of debt plays an important role, but the distinction between active and passive regimes disappear when considering optimal policy. But typical fiscal theory ingredients may play a larger or smaller role. The contrast between Schmitt-Grohe and Uribe (2004) - inflation plays a minor role with sticky prices - vs. Leeper and Zhou (2023).

are endogenous to policy. There is no reason why the relation between increases in policy rates reduce long-term forecasts of inflation should remain stable if economies enter unsustainable debt levels. Coibion, Gorodnichenko and Weber (2022) study a randomized-control trial and argue that news about future debt leads households to anticipate higher inflation, both in the short run and long run, and induces households to increase their spending. In a long sample covering multiple countries, Brandao-Marques, Casiraghi, Gelos, Harrison and Kamber (2023) shows that surprise increases in debt levels raise long-term inflation expectations predominantly in emerging markets. Moreover, consistent with our theory, they find that the effects are stronger when initial debt levels are already high. While developing countries who have traditionally held much higher debt levels, it is possible for developed economies to follow that path.[5]

2. Model

2.1 Environment

We cast the model in continuous time, $t \in [0, \infty)$. The economy is populated by households, firms, and a government. We next describe their behavior, relegating derivations to Appendix A.

Government. The government is comprised of fiscal and monetary authorities. The fiscal authority sends lump-sum transfers T_t (taxes if $T_t < 0$) to households and issues short-term real debt B_t. The monetary authority sets the nominal interest rate i_t.

The government's flow budget constraint is given by

$$\dot{B}_t = (i_t - \pi_t)B_t + T_t, \tag{1}$$

given $B_0 > 0$, where π_t denotes the inflation rate and i_t the nominal interest rate. Fiscal transfers satisfy the following rule:

$$T_t = -\rho B_t - \gamma(B_t - \overline{B}) + \Psi_t, \tag{2}$$

where ρ denotes the interest rate that prevails in a zero-inflation steady state, $\overline{B}$ is the steady-

[5] de Mendonca and Machado (2013) perform a similar study focusing on the case of Brazil.

state level of debt, and Ψ_t corresponds to a fiscal shock. Importantly, $\gamma \geq 0$ controls the strength of fiscal responses to the level of government debt. If $\gamma > 0$, debt is mean reverting; if $\gamma = 0$, transitory shocks fiscal shocks can permanently affect the debt level.

We study monetary policy while there are ongoing fiscal pressures, $\Psi_t > 0$, and there is uncertainty regarding the response of the monetary authority to the fiscal shock. The economy starts at the *fiscal-expansion phase*, where nominal rates satisfy a Taylor rule:

$$i_t = \rho + \phi \pi_t + u_t, \tag{3}$$

We focus on the case where the Taylor coefficient ϕ and the fiscal rule coefficient γ are such that the economy is always in an active monetary regime, following the Leeper (1991) terminology. The disturbance u_t allows the monetary authority to respond freely to the fiscal expansion. Below, we consider different monetary-policy objectives, corresponding to different choices of u_t.[6] These choices allow us to analyze an independent monetary authority that is free to choose interest rates during the fiscal-expansion phase, while agents form expectations of the likelihood of the arrival of a reform.

With Poisson intensity λ, the economy switches to the *inflationary-finance phase*, where debt is reduced through a mix of fiscal and monetary tools. In particular, the government sets $\Psi_t = 0$, provided that the monetary authority commits to set a constant real interest rate for T^* periods, as necessary to bring down debt to target level B^n. Once debt reaches B^n, monetary policy implements a zero inflation target, and the economy reaches its steady-state level. We assume that the inflationary-finance phase involves a period of length T^* of low interest rates to reduce debt to its agreed level. The period of future low rates is key.

Figure 1 summarizes the timeline of events. Over a small time interval Δt, the economy switches to the inflationary-finance phase with probability $\lambda \Delta t$, and stays in the fiscal-expansion phase with the remaining probability. Agents have views about the arrival rates of a reform that possibly differ from the objective arrival rate. Only the subjective arrival rates will play a role in equilibrium.

Discussion: Uncertainty about reforms. The possible reform to render debt sustainable highlights the uncertainty in the political process determining how debt sustainability will ultimately be achieved. The uncertainty arises from the possible negotiations, ability, and

[6] The disturbance u_t captures the *response* of the monetary authority to the fiscal expansion, so we refer to it as a disturbance to the policy rule instead of a shock.

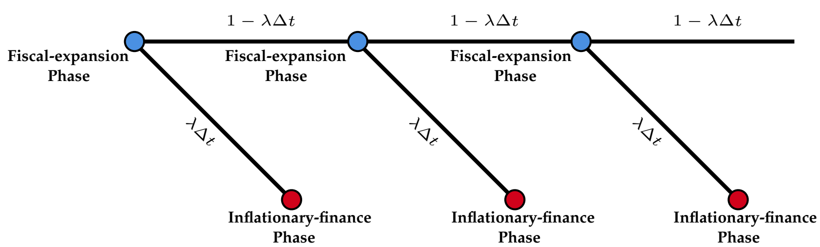


Figure 1: Timeline of events

decisions made by political actors, who either agree to raise taxes or seek a compromise with the monetary policy-makers to inflate the debt away. The scenarios reflect the complexity of real-world reforms. Our interest is on what this uncertainty means for monetary policy.

We abstract away from recurrent transitions between fiscal expansions and debt normalization processes. Furthermore, we do not consider potential signaling or reputational losses where policies during fiscal-expansion phase indicate the likelihood of inflationary-finance phase in the future. We do so to study monetary policy as transparently as possible.

Notation. We index variables in the inflationary-finance phase using an asterisk ($*$) superscript whereas variables during the fiscal-expansion phase do not carry that superscript. For example, π_t represents inflation at time t of fiscal-expansion phase whereas π_t^* is inflation at time t since the start of the inflationary-finance phase. Variables in the steady state are denoted by an upper bar. For example, consumption in a steady state is denoted by $\overline{C}$.

Households. In the fiscal-expansion phase, households form expectations of the arrival time of the inflationary-finance phase. The household's problem is given by

$$V_t(B_t) = \max_{[C_s, N_s]_{s \geq t}} \mathbb{E}_t^h \left[\int_t^{\tilde{t}} e^{-\rho(s-t)} \left(\log C_s - \frac{N_s^{1+\varphi}}{1+\varphi} \right) dt + e^{-\rho\tilde{t}} \tilde{V}_{\tilde{t}}(B_{\tilde{t}}) \right],$$

subject to

$$\dot{B}_t = r_t B_t + \frac{W_t}{P_t} N_t + D_t + T_t - C_t,$$

and a No-Ponzi condition $\lim_{T\to\infty} \mathbb{E}_t^h[\eta_T B_T] = 0$, where η_t denotes the stochastic discount factor (SDF) in this economy. B_t denotes the real value of bonds held by households, $r_t =$

$i_t - \pi_t$ is the real interest rate, W_t is the nominal wage, P_t is the price level, and D_t are firm dividends. The random time $\tilde{t}$ denotes the arrival time of the reform, and $\tilde{V}_t$ denotes the value function after the reform.

Households believe that the monetary-fiscal reform occurs with Poisson intensity λ_h. In the fiscal-expansion phase, the households' Euler equation is

$$\frac{\dot{C}_t}{C_t} = \underbrace{(i_t - \pi_t - \rho)}_{\text{standard term}} + \lambda_h \underbrace{\left[\frac{C_t}{C_t^J} - 1\right]}_{\text{m-f reform}}. \tag{4}$$

This Euler equation includes a standard term associated with the gap between real interest rates and the discount rate. The second term captures the jump in marginal utilities associated with the reform. It acts as a time-varying change in the discount rate.[7] Clearly, in the absence of policy uncertainty, $\lambda_h = 0$, we obtain the standard Euler equation. In turn, uncertainty provokes a change in the natural rates, a variable central to the New Keynesian model. The usual intra-temporal condition gives labor supply: $W_t/P_t = C_t N_t^{\varphi}$.

Firms. The production side follows the basic structure of the standard New Keynesian model. The economy has two types of firms: final-goods and intermediate-goods producers index by $i \in [0, 1]$. Final goods are produced by competitive firms using a constant-elasticity of substitution production function over intermediate inputs.[8] As usual, the demand for intermediate good i is given by $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$, where $P_{i,t}$ is the price of intermediate i, $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is the price level, and Y_t is the aggregate output.

Intermediate-goods producers operate the technology $Y_{i,t} = AN_{i,t}$, where $N_{i,t}$ denotes labor input. They compete monopolistically and they are subject to quadratic price-adjustment costs. The problem of intermediate-goods firm i is

$$Q_{i,t}(P_i) = \max_{[\pi_{i,s}]_{s \geq t}} \mathbb{E}_t^f \left[\int_t^{\tilde{t}} \frac{\eta_s}{\eta_t} \left(\frac{P_{i,s}}{P_{i,t}} Y_{i,s} - \frac{W_s}{P_s} \frac{Y_{i,s}}{A} - \frac{\varphi}{2} \pi_{i,s}^2 \right) ds + \frac{\eta_{\tilde{t}}}{\eta_t} \tilde{Q}_{i,\tilde{t}}(P_{i,\tilde{t}}) \right], \tag{5}$$

subject to their demand schedule, $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$, given $P_{i,t} = P_i$, where φ

[7]Recall that C_t is current consumption and C_t^J denotes consumption the instant of a fiscal-monetary reform. With log-utility, their ratio captures a jump in marginal utilities.

[8]That is, final goods are produced according to the production function $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon}{\epsilon-1}} di\right)^{\frac{\epsilon-1}{\epsilon}}$, where $Y_{i,t}$ denotes the output of intermediate $i \in [0, 1]$.

is a price adjustment cost parameter. As in the household's problem, $\tilde{t}$ is the random arrival time of a reform.

Firms believe that the monetary-fiscal reform occurs with Poisson intensity λ_f. The key object of this supply-side block is a modified New Keynesian Phillips curve (NKPC):

$$\dot{\pi}_t = \underbrace{(i_t - \pi_t)\,\pi_t + \epsilon\varphi^{-1}\left((1-\epsilon^{-1}) - \frac{W_t}{P_t}\right)Y_t}_{\text{standard term}} + \underbrace{\lambda_f \frac{\eta_t^J}{\eta_t}\left(\pi_t - \pi_t^J\right)}_{\text{m-f reform}} \tag{6}$$

Like the household's Euler equation, the firm's Phillips curve features standard terms associated with marginal costs and a modification associated with the beliefs about the possible reform. Firms anticipate that if a monetary-fiscal reform takes place, inflation will jump to π_t^J upon the announcement of the reform— π_t^J denotes the *jump inflation term*. This jump in inflation is adjusted by η_t^J, which captures the jump in the SDF.

2.2 A 4-equation log-linear representation

Part of the appeal of the standard new Keynesian model is its representation into a tractable 3-equation system. Here, we present a 4-equation representation that includes the feedback of fiscal variables on inflation expectations. We proceed as usual and produce a log-linear approximation of the model around the zero-inflation constant-debt steady state.

Steady State. The steady-state corresponds to the case $\Psi_t = 0$ and $u_t = 0$, so $B_t = \overline{B}$, $C_t = \overline{C}$, $i_t = \rho$, and $\pi_t = 0$, where $\overline{B}$ corresponds to the initial condition for government debt and $\overline{C}$ is the steady-state level of consumption.

Dynamics: inflationary-finance phase. When the inflationary-finance phase occurs, fiscal shocks are set to zero, $\psi_t^* = 0$, the parameter controlling the fiscal response to debt is set to zero, $\gamma = 0$, and the monetary authority implements a constant real interest rate r^* for T^* periods, as necessary to bring debt to a stable level. Hence, during the inflationary-finance phase, debt evolves according to $b_t^* = b_0^* + (r^* - \rho)t$ for $t \leq T^*$. To ensure that debt reaches the sustainable level after T^* periods, the real interest rate must satisfy the condition:

$$r^* = \rho - \frac{b_0^* - b^n}{T^*}, \tag{7}$$

where $b^n \equiv \frac{B^n - \overline{B}}{\overline{B}}$ denotes the natural level of debt, that is, the initial level of debt at the inflationary-finance phase such that the output and inflation would immediately jump to their steady-state level.

The monetary authority implements zero inflation when the target debt level is reached, that is, $\{x^*_{T^*}, \pi^*_{T^*}\} = \{0, 0\}$. Given this terminal condition, and the Euler equation $\dot{x}^*_t = r^* - \rho$, we obtain the output gap:

$$x^*_t = (r^* - \rho)(t - T^*) = (b^*_0 - b^n)\left(1 - \frac{t}{T^*}\right), t \in [0, T^*], \tag{8}$$

From the NKPC and the expression for the output gap, we obtain the inflation rate:

$$\pi^*_t = \kappa(r^* - \rho)\int_t^{T^*} \exp(-\rho(s - t))(s - T^*)ds. \tag{9}$$

For an economy that switches to the inflationary-finance phase after t periods, initial debt is given by $b^*_0 = b_t$. Using the expression for the real rate, we can then express the initial inflation and output gaps at the time of the fiscal-monetary reform in terms of a *debt gap* $b_t - b^n$:

$$\pi^*(b_t) \equiv \kappa\Phi(b_t - b^n) \quad \text{and} \quad x^*(b_t) \equiv (b_t - b^n), \tag{10}$$

where the coefficient Φ is defined as follows:

$$\Phi \equiv \int_0^{T^*} \exp(-\rho s)\left(1 - \frac{s}{T^*}\right) ds > 0.$$

Equation (10) shows that initial inflation and the output gap will jump at the start of the inflationary-finance phase. The size of the jump depends on the excess of the outstanding debt $b_t - b^n$, relative to the target level. The larger the initial level of government debt relative to its natural level, the lower the real rate must be, and the higher is the inflation rate. The distance, $b_t - b^n$ can be interpreted as the part of government debt that is not fully backed by future taxes at the start of the inflationary-finance phase. We assume throughout that $b_0 - b^n \geq 0$, so increases in debt above its initial level are not fully backed by future taxes.

The coefficient Φ corresponds to the pass-through from debt to inflation. It captures that a higher inflation rate is required to bring down a higher debt to its natural level during the inflationary-finance phase.

Dynamics: fiscal-expansion phase. The system of linearized Euler equation, NKPC, and government budget constraint is:

$$\dot{x}_t = i_t - \pi_t - \rho + \lambda_h x_t - \lambda_h (b_t - b^n) \tag{11}$$

$$\dot{\pi}_t = (\rho + \lambda_f)\pi_t - \kappa x_t - \lambda_f \kappa \Phi (b_t - b^n) \tag{12}$$

$$\dot{b}_t = i_t - \pi_t - \rho - \gamma b_t + \psi_t. \tag{13}$$

The Taylor rule (Eq. 3) completes the 4-equation system. In this system, lower-case variables denote log-linear deviations, $b_t \equiv \frac{B_t - \overline{B}}{\overline{B}}$, $x_t \equiv \frac{Y_t - \overline{Y}}{\overline{Y}}$. The variable $\psi_t \equiv \Psi_t / \overline{B}$ denotes the scaled fiscal shock. We assume the fiscal shock is exponentially decaying, so $\psi_t = e^{-\theta_\psi t}\psi_0$. In turn, $\kappa > 0$ is the slope of the Phillips curve.

This system has some noteworthy features. First, without the expectation of a reform, the debt dynamics are entirely decoupled from the inflation and the output gap. Moreover, the divine coincidence holds without the expectation of a fiscal-monetary reform, as $\{x_t, \pi_t, i_t - \rho\} = \{0, 0, 0\}$ is a solution to the New Keynesian block. Second, with the expectation of a fiscal-monetary reform, in contrast to the standard formulation of the New Keynesian model, the inflationary and output dynamics depend not only on the current policy rates but also on agents' expectations of inflation and output after the fiscal-monetary reform. The fiscal variable and the New Keynesian block are coupled through these expectation terms.

Determinacy, implementability, and monetary dominance. We provide next the conditions for local determinacy in our economy.

Proposition 1 (Determinacy and implementability). *Consider a given path of monetary disturbances u_t and fiscal shock ψ_t. Assume that $\gamma \in (0, \rho + \lambda_f + \lambda_h)$. Then,*

I. ***Determinacy.*** *There exists a unique bounded equilibrium if and only if*

$$[\gamma - \lambda_h (1 + \lambda_f \Phi)] (\phi - 1) > -\gamma \frac{\rho + \lambda_f}{\kappa} \lambda_h. \tag{14}$$

II. ***Implementability.*** *Let $\hat{i}_t$ denote a given path of nominal interest rates and $(\hat{x}_t, \hat{\pi}_t, \hat{b}_t)$ that satisfies the Euler equation (11), the NKPC (12), and the government's flow budget constraint (13). Suppose that $u_t = \hat{i}_t - \rho - \phi\hat{\pi}_t$, with ϕ satisfying condition (14), such that we can write the policy rule as*

$$i_t = \hat{i}_t + \phi(\pi_t - \hat{\pi}_t). \tag{15}$$

Then, the unique solution to the system (11)-(13) *and* (15) *is given by* $x_t = \hat{x}_t$*,* $\pi_t = \hat{\pi}_t$*, and* $b_t = \hat{b}_t$*.*

Condition (14) generalizes the Taylor principle to our economy with uncertainty about possible reforms.[9] We focus throughout on the case where condition (14) is satisfied, so monetary policy is active in the sense of Leeper (1991). Furthermore, we show that fiscal policy is passive when $\gamma \geq 0$—see Appendix C. Hence, we are never in a fiscally dominant regime, which shows our mechanism is different from the one prevalent under the fiscal theory of the price level.

Proposition 1 also shows how to implement any allocation satisfying conditions (11)-(13) by effectively adopting a time-varying inflation target.[10] A similar approach can be used to implement the equilibrium outcomes in the inflationary-finance phase by assuming the monetary authority follows the policy rule: $i_t^* = \rho + \phi\pi_t^* + u_t^*$, given the same coefficient ϕ.[11]

Integral Representation Given an arbitrary path for the real rate $r_t = i_t - \pi_t$, we can characterize the system in closed form. The path of debt satisfies:

$$b_t = e^{-\gamma t} b_0 + \int_0^t e^{-\gamma(t-s)}(\psi_s + r_s - \rho)ds. \tag{16}$$

Debt accumulates through two forces: fiscal pressures, ψ_s, and real interest rates that exceed the natural rate ρ. The parameter γ controls the mean reversion in government debt.

Policy uncertainty leads to a *discounted Euler equation* which takes the following form:[12]

$$x_t = -\int_t^\infty e^{-\lambda_h(s-t)}(r_s - \rho)ds + \lambda_h \int_t^\infty e^{-\lambda_h(s-t)} x^*(b_s)ds. \tag{17}$$

This equation states that changes in future interest rates are discounted by λ_h. The second term corresponds to an *expectation effect* related to consumption smoothing.[13] This term

[9] When $\lambda_h = 0$, we recover the standard Taylor principle: equilibrium determinacy requires $\phi > 1$. With $\lambda_h > 0$, determinacy can be achieved with a relaxed condition: $\phi \leq 1$. In particular, a real interest rate peg—$\phi = 1$—induces a unique equilibrium in this case.

[10] For models with a time-varying inflation target, see e.g. Ireland (2007) and Cogley and Sbordone (2008).

[11] Disturbances to the Taylor rule are regime-dependent, but the coefficients are fixed, in contrast to the literature on regime-dependent rules (see e.g. Davig and Leeper, 2007 and Farmer, Waggoner and Zha, 2009).

[12] This is similar to other forms of uncertainty, such as the uninsurable idiosyncratic income risk of McKay, Nakamura and Steinsson (2016) or the aggregate disaster risk in Caramp and Silva (2021).

[13] See e.g. Leeper and Zha (2003) for a definition and discussion of expectation-formation effects.

captures the impact on the current output gap x_t, of the expectation of a jump in the output gap from entering an inflationary-finance phase at different points in time.

Integrating the NKPC forward, we obtain the inflation rate

$$\pi_t = \kappa \int_t^{\infty} e^{-(\rho+\lambda_f)(s-t)} x_s ds + \lambda_f \int_t^{\infty} e^{-(\rho+\lambda_f)(s-t)} \pi^*(b_s) ds. \tag{18}$$

Inflation equals the expected present value of output gaps in the fiscal-expansion phase plus another expectation term, capturing the expected value of discounted inflation jumps. These expectation effects, which are a function of real rates, play an important role in shaping the trade-offs faced by the monetary authority when responding to a fiscal expansion.

The integral formulation of the NKPC, displayed in (18), shows that not only the output gap, as in the canonical new Keynesian model, but also the present value of expectations about future possible bursts in inflation can significantly shape current inflation. Hazell, Herreno, Nakamura and Steinsson (2022) empirically estimate a NKPC where inflation depends on the present discounted value of the temporary component on unemployment gaps and a term capturing long-term inflation expectations. That study finds a small Phillips-curve slope with expectation effects explaining the bulk inflation changes. The study interprets the evidence as indicating that most of the variation in inflation was related to expectations regarding *permanent* changes in the conduct of monetary policy—permanent changes in output gap targets.[14] Equation (18) shows that *temporary* fiscal shocks can rationalize that evidence since they will provoke movements in the expectation component.

Expectations, yields and break-even inflation. It is also possible to obtain an integral representation of the future inflation expectations—see Appendix B. Let $\pi_{t,\tau}$ be the date t expectation of inflation τ periods ahead. The are given by:

$$\pi_{t,\tau} = e^{-\lambda\tau} \pi_{t+\tau} + \int_t^{t+\tau} e^{-\lambda(s-t)} \lambda \pi^*_{t+\tau}(b_s)\, ds. \tag{19}$$

Thus, inflation expectations at any horizon are discount paths of inflation in the fiscal-expansion phase and inflationary-finance phaserespectively.

[14] Hazell et al. (2022) argue that the slope of the Phillips curve has been relatively small since the 1980s. They argue that (transitory) deviations of unemployment—and the output gap—from its natural level played a minor role during the Volcker disinflation, with expectation explaining the bulk of the change in inflation during that episode. See also Goodfriend and King (2005) and Bianchi and Ilut (2017) for a related view on the Volcker disinflation.

Likewise, we can also price a zero-coupon (real) bond at date t maturing τ periods ahead, promising one consumption unit and a zero-coupon (nominal) bond offering to pay one unit of account, nominal bond, $q_{t,\tau}$. These bond prices have a similar integral representation provided in the appendix. The nominal and real yield curves are respectively: $r_{t,\tau} \equiv -\frac{\ln(\phi_{t,\tau})}{\tau}$, and $i_{t,\tau} \equiv -\frac{\ln(q_{t,\tau})}{\tau}$. Importantly, we can construct a model analog of break-even inflation, $\tilde{\pi}_{t,\tau} \equiv i_{t,\tau} - r_{t,\tau}$. We return to this objects when we connect the model with the recent inflationary episode.

3. Three policy experiments

This section considers three policy experiments where the monetary authority attempts to stabilize the output gap, inflation, or government debt. We show that once the expectation of a monetary-fiscal reform lurks in the background, monetary policy can no longer jointly stabilize output and inflation. In other words, the expectation of a fiscal reform breaks divine coincidence.

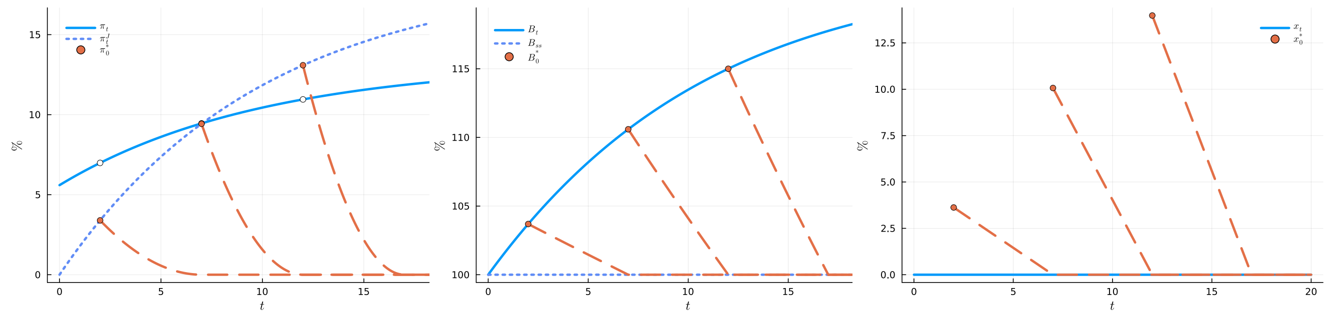
The impact of the fiscal shock depends crucially on firms' expectations. To isolate the role of firms' expectations, we temporarily make the simplifying assumption that $\lambda_h = 0$, so the Euler equation behaves as in the standard New Keynesian model. We further simplify the analysis by focusing on the case $\gamma = 0$. We revisit the role of households' expectation effects and of a more general fiscal rule in Section 3.4. Also, to spare notation, we assume that $\lambda_f = \lambda$.

3.1 Policy I: Output gap stabilization

In our first policy experiment, we assume that monetary policy is focused on stabilizing the output gap. That is, it sets $x_t = 0$ during the fiscal-expansion phase. We describe how to find the path of monetary disturbances u_t required to implement this outcome in Proposition 1.

To stabilize the output gap, the real rate must satisfy $r_t = \rho$. Due to the fiscal shock, government debt is increasing over time: $b_t = b^{lr} - \psi_t/\theta_\psi$, where $b^{lr} \equiv b_0 + \psi_0/\theta_\psi$ denotes the long-run level of debt in the fiscal-expansion phase. The proposition below shows that the expectation effects induced by the fiscal shock lead to an increasing path of inflation over time.

Proposition 2 (Inflation under output gap stabilization)**.** *Suppose $x_t = 0$. Then, inflation is*



(a) Inflation (b) Debt (c) Output Gap

Figure 2: Pre and Post Reform Equilibrium Objects

Note: Red dashed lines correspond to reform paths that occur at different points in time.

given by

$$\pi_t = \frac{\kappa\lambda\Phi}{\rho+\lambda}\left[b_t - b^n + \frac{\psi_t}{\rho+\lambda+\theta_\psi}\right]. \tag{20}$$

Moreover, inflation is increasing over time, $\dot{\pi}_t = \frac{\kappa\lambda\Phi}{\rho+\lambda+\theta_\psi}\psi_t > 0$*, and converges to a positive level,* $\lim_{t\to\infty}\pi_t = \frac{\kappa\lambda\Phi}{\rho+\lambda}(b^{lr} - b^n) > 0$*, in the fiscal-expansion phase.*

Proposition 2 shows that to stabilize the output gap, monetary policy must live with an ever-growing inflation rate. Before a reform, inflation grows proportionally to the size of deficits. Moreover, inflation is positive even after the deficits converge to zero. That is, *inflation is sticky*. Sticky inflation occurs because of its jump inflation component, $\pi_t^J = \kappa\Phi(b_t - b^n)$, which reflects the expected burst in inflation that trails the path of debt in a inflationary-finance phase. The role of expectations regarding the type of reform is clear from (20): if firms believe that switching to the inflationary-finance phase, the impact on inflation will be largely attenuated.

Proposition 2 tells us something potentially profound: an independent monetary policy focused on stabilizing output must live with inflation that trails debt, a fiscal variable. The sole belief, rational or not, of a future compromise to aid debt stabilization is enough to destabilize inflation in a monetary independent regime.

Figure 2 shows the typical paths of inflation, debt, and the output gap, during the fiscal-expansion phase and the inflationary-finance phase. In Panel (a), we find an example of a path of inflation which is plotted together with its corresponding jump inflation term (the blue dotted curve). The red dashed curves represent different inflation levels corresponding to different dates of the arrival of the inflationary-finance phase. Notice that when the inflationary-finance phase is initiated, inflation jumps to the jump inflation term. If the reform happens early, inflation may actually drop.[15] However, as the reform is postponed, inflation increases steadily and a reform is associated with an inflationary burst. Panel (b) shows the corresponding paths of debt. Prior to a reform, the debt grows over time, and then trends down to its target level in exactly T^* periods after the reform. The speed of the decline in debt after any inflationary-finance phase is faster the later the reform date. This reflects that since inflation must reduce debt to the same level in a fixed amount of time, reforms that begin with higher debt levels require greater bursts in inflation—lower negative real interest rates. Panel (c) shows the output gap. Again, the later the reform, the larger the response of the output gap and, hence, a greater undesirable labor wedge. A key lesson is that the later the reform, the greater its costs. Postponing reforms is costly.

3.2 Policy II: Inflation stabilization

In the previous section, we assumed that monetary policy focuses exclusively on the output gap. We now assume that the monetary authority attempts to contain inflation by temporarily raising rates, such that $r_t - \rho = e^{-\theta_r t}(r_0 - \rho)$, for a given initial rate $r_0 > \rho$ and persistence parameter $\theta_r > 0$.

We use a superscript og to denote the paths of variables corresponding to the output-gap stabilization in the previous section, where we set $r_t = \rho$. Instead, here the output gap depends on the evolution of r_t:

$$\dot{x}_t = r_t - \rho \Rightarrow x_t = -\frac{1}{\theta_r}(r_t - \rho), \tag{21}$$

where we used the terminal condition $\lim_{t\to\infty} x_t = 0$, a form of long-run neutrality. As $r_t > \rho$, the output gap is negative during the fiscal-expansion phase.

In turn, the path of debt is given by

$$b_t = b_t^{og} + \frac{1 - e^{-\theta_r t}}{\theta_r}(r_0 - \rho), \tag{22}$$

where $b_t^{og} = b_0 + \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\psi_0$ corresponds to the debt level under the output-gap stabilization

[15]This downward jump in inflation happens because Phase I inflation is the net present discounted of all future jump inflation, which increases over time. If the reform happens early, inflation jumps downward because the initial jump inflation is lower than the net present expected discounted value of future jump inflation.

policy.

We can solve for inflation using the NKPC (Equation 18). A policy that fights inflation deviates from the output-gap stabilization solution through the sum of two effects: a fight-inflation effect and a jump-inflation effect. Formally:

Lemma 1. *Suppose $r_0 > \rho$. With mean-reverting real interest rates $\dot{r}_t = -\theta_r(r_0 - \rho)$, inflation is given by:*

$$\pi_t - \pi_t^{og} = F_t^{\pi} + J_t^{\pi}.$$

where F_t^{π} and J_t^{π} are, correspondingly, fight and jump inflation components given by:

$$F_t^{\pi} = -\frac{\kappa}{\theta_r}\frac{e^{-\theta_r t}}{\rho + \lambda + \theta_r}(r_0 - \rho) < 0 \quad \textit{and} \quad J_t^{\pi} = \frac{\lambda\kappa\Phi}{\theta_r}\left[\frac{1}{\rho + \lambda} - \frac{e^{-\theta_r t}}{\rho + \lambda + \theta_r}\right](r_0 - \rho) > 0.$$

The first term, the fight-inflation term, captures the standard effect of contractionary policy through aggregate demand. The term is negative since $r_t > \rho$ and converges to zero as the contractionary stimulus vanishes. Thus, the increase in r_t above the natural rate ρ has a mitigating effect on inflation, as in standard versions of the New Keynesian model.

The second term, the jump inflation, is the expected present value of inflation surges after possible reforms, which depends on the path of debt, $\pi_t^J = \kappa\Phi(b_t - b^n)$. The jump inflation term is always positive and increasing over time. Jump inflation is related to the increase in the fiscal burden from an increase in the real interest rate. A greater fiscal burden accumulates over time: as interest payments on debt are not repaid immediately, they add to the stock of debt. Through debt accumulation, current rate hikes feedback into present inflation through the expectation of a greater burst in future inflation needed to support greater financing needs. In a nutshell, the debt accumulation resulting from higher real rates pressures prices upwards.

Clearly, the two effects, the fight or jump inflation, oppose each other. The fight-inflation term is negative whereas the jump inflation term is positive. Which effect dominates depends on the persistence of shocks and the horizon ahead of the stimulus we are looking at—not the size of the stimulus. Over time, the fight-inflation term vanishes, but the jump-inflation term increases. These observations lead to the following formal result:

Proposition 3 (Stepping on a Rake)**.** *Suppose $r_0 > \rho$. The path of monetary policy reduces inflation on impact, i.e., $\pi_0 < \pi_0^{og}$ if:*

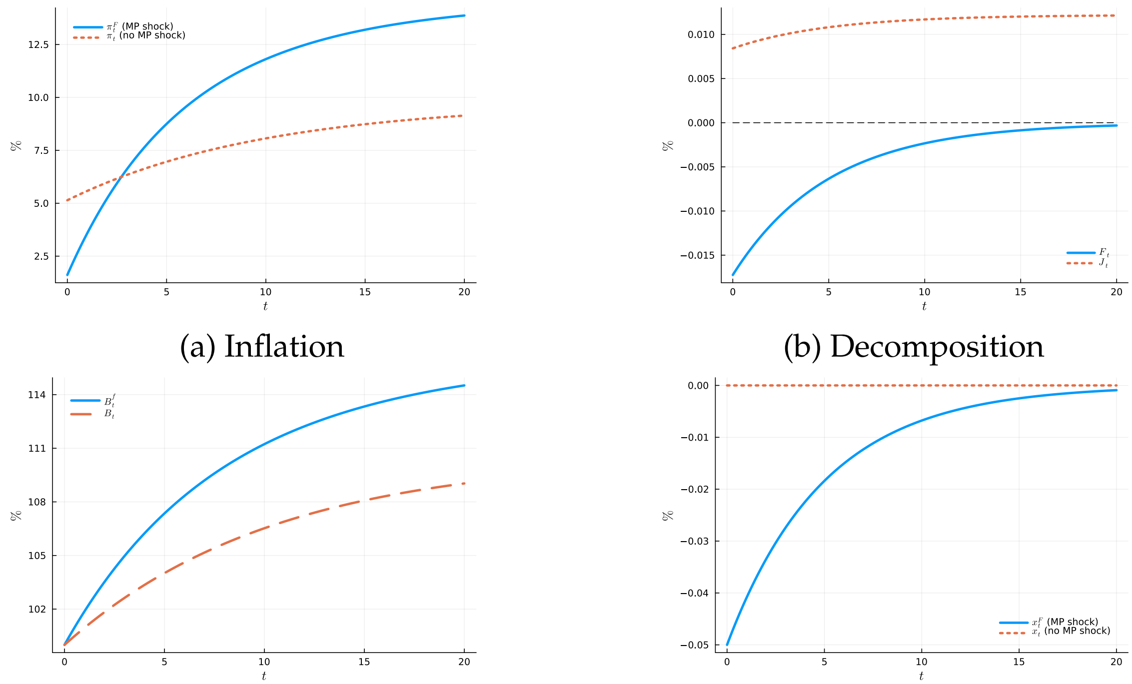
$$\theta_r < \frac{\rho + \lambda}{\lambda\Phi}.$$

However, there always exists a $\hat{T} > 0$ such that $\pi_t > \pi_t^{og}$ for $t > \hat{T}$.

The proposition shows that monetary policy can fight and reduce inflation at time zero, provided the policy is sufficiently persistent—a greater θ_r coefficient means faster mean reversion. The initial drop in the output becomes larger with a more persistent increase in real rates. For a sufficiently persistent increase in real rates, the reduction in the output gap dominates the increase in jump inflation caused by higher government debt. The lesson here is that if agents think that monetary policy may surrender to fiscal objectives in the future, to be successful in the present, it must have a sufficiently persistent contractionary policy stance.

Although a sufficiently persistent monetary policy can successfully fight inflation in the short run, it faces an unpleasant "stepping-on-a-rake" result. While policy can be successful at curtailing inflation temporarily, eventually, inflation will always come back stronger. Again, inflation is sticky! The reason is that the effect on the output gap eventually fades away, whereas the effect on government debt builds up over time as past interest rates add to the debt stock. As a result, the jump inflation term eventually dominates. There is no possibility of permanently controlling inflation prior to a debt-normalization phase. Sims (2011) calls that boomerang feature of the inflationary dynamics "stepping-on-a-rake" although here the result emerges for very different reasons. Sims (2011) finds stepping-on-a-rake effects in a model with long-term bonds and fiscal dominance—with a Taylor coefficient $\phi < 1$. Here, bonds are short-term, and there is monetary dominance—$\phi > 1$. As shown by Cochrane (2018), long-term bonds are strictly necessary to obtain these dynamics because debt revaluation of long-term bonds in the future requires inflationary finance. Here, a greater debt stock indicates a greater expectation of future inflation, and the revaluation effects in Sims (2011) play no role.

An Example of the Fight Inflation Policy. Figure 3 shows the paths of inflation, debt, and the output gap, considering an attempt to fight inflation in a fiscal-expansion phase. In Panel (a), we find an example of two paths of inflation: a baseline (dashed)—without a monetary policy disturbance—and a counterfactual (solid) corresponding to a temporary increase in policy rates. The example illustrates the essence of Proposition 3. While the strategy is successful at combating inflation earlier on, inflation returns a year into the policy. Panel (b) shows why: it plots the fight inflation (solid) and jump inflation (dashed) components. The fight inflation component is initially strong but eventually dies out, as the effect on aggregate demand vanishes. The jump inflation component is initially weaker, but continues to



(a) Inflation

(b) Decomposition

(c) Debt

(d) Output gap

Figure 3: Equilibrium Paths with and without Contractionary Monetary Shock

increase over time. Panel (c) shows the path of debt, which picks up with the higher real interest rates. Finally, Panel (d) shows the contractionary effect on the output gap.

The example shows that, with a lurking expected fiscal-monetary reform, attempts to curtail inflation have standard short-run effects, but unlike the canonical new Keynesian model, they lead to higher inflation in the long run. The example may be particularly pertinent to understand the many unsuccessful attempts to curb high inflation in countries such as Argentina, Brazil, or Turkey. These countries have recurrently appointed well–trained orthodox monetary policymakers who have attempted to raise real interest rates to combat inflation. While originally successful at curtailing inflation, inflation has consistently returned in those countries. We contend that temporary attempts cannot be successful at controlling inflation because the expectations of fiscal reforms are unanchored.

Full inflation stabilization. To obtain full inflation stabilization, the monetary authority must induce a persistent decline in the output gap. From the NKPC, inflation evolves according to

$$\dot{\pi}_t = (\rho + \lambda)\pi_t - \kappa x_t - \kappa\lambda\Phi(b_t - b^n). \tag{23}$$

Inflation stabilization, $\pi_t = 0$, requires $x_t = -\lambda\Phi(b_t - b^n)$, so the output gap offsets movements in the government debt. This condition implies that the real rate must satisfy:

$$r_t - \rho = -\frac{\lambda\Phi}{1 + \lambda\Phi}\psi_t. \tag{24}$$

In this case, government debt is given by $b_t = b_0 + \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\frac{\psi_0}{1+\lambda\Phi}$, so debt is increasing over time, but it is always below the level in the output-gap stabilization case.[16]

3.3 Policy III: Debt stabilization

Our third policy experiment investigates what happens when monetary policy attempts to stabilize debt. Stabilizing debt requires the real rate to neutralize the effects of deficits: $r_t - \rho = -\psi_t$, so $b_t = b_0$. Thus, the output gap is:

$$x_t = \frac{\psi_t}{\theta_\psi},$$

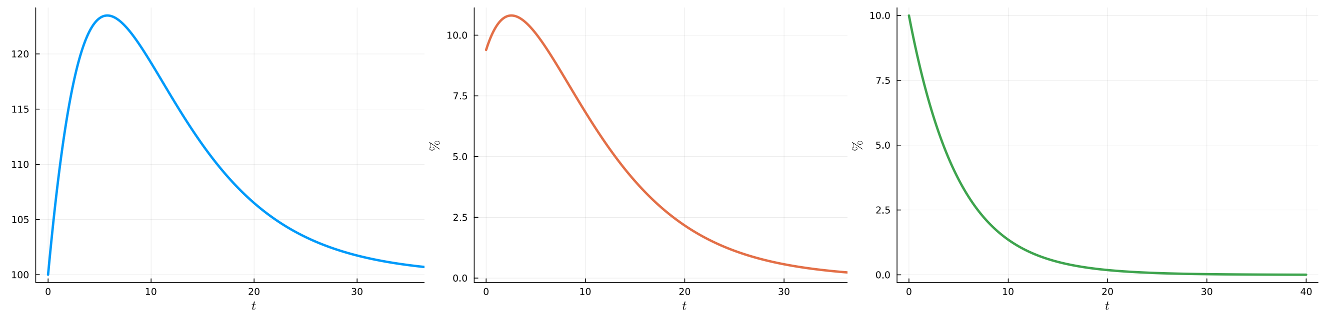
where again we used the terminal condition $\lim_{t\to\infty} x_t = 0$. Inflation is given by:

$$\pi_t = \frac{\kappa}{\theta_\psi}\frac{\psi_t}{\rho + \lambda + \theta_\psi}.$$

To stabilize the debt, the monetary authority overheats the economy generating inflation.

Discussion: policy trade-offs with fiscal cost-push shocks. We have seen that, in the presence of policy uncertainty, it is impossible to simultaneously stabilize output and inflation. Therefore, divine coincidence fails even in the absence of markup shocks. The reason is that expectation effects lead to an endogenous *fiscal cost-push shock*. A negative output gap is required to offset the inflationary effects of expectations of the fiscal-monetary reform, in the same way a negative output gap is required to offset standard cost-push shocks. It is the presence of this fiscal cost-push shock that breaks divine coincidence in the absence of

[16] Given the relationship between output gap and debt, full inflation stabilization requires the output gap to become ever more negative over time: $\lim_{t\to\infty} x_t = -\lambda\Phi(b^{lr} - b^n) < 0$, where $b^{lr} = b_0 + \frac{\psi_0}{\theta_\psi(1+\lambda\Phi)}$.



(a) Government debt (b) Inflation (c) Fiscal shock

Figure 4: Equilibrium dynamics in a soft landing

supply shocks. Stabilizing one of the three variables – output, inflation, or debt – causes the other two to continuously move away from its steady-state level.

3.4 Debt stabilizers, households' expectation effects, and soft landing

In the previous exercises, we abstract from households' expectation effects, $\lambda_h = 0$, and automatic debt stabilizers, $\gamma = 0$. Introducing these effects opens up an interesting new possibility: disinflation becomes possible even without negative output gaps, i.e., the economy can achieve a *soft landing*.

Consider the Euler equation with expectation effects:

$$\dot{x}_t = r_t - \rho + \lambda_h x_t - \lambda_h (b_t - b^n),$$

To ensure that disinflation is not due to changes in the output gap, suppose $x_t = 0$, which requires the following real rate: $r_t - \rho = \lambda_h (b_t - b^n)$. The law of motion of debt in this case is given by

$$\dot{b}_t = -(\gamma - \lambda_h) b_t + \psi_t \Rightarrow b_t = \frac{e^{-(\gamma - \lambda_h)t} - e^{-\theta_\psi t}}{\theta_\psi + \lambda_h - \gamma} \psi_0,$$

assuming $b_0 = b^n = 0$ for simplicity. Provided $\gamma > \lambda_h$, debt eventually reverts to its steady state level.

With no output gaps, inflation is given by the net present value of its jump inflation term:

$$\pi_t = \frac{\psi_0}{\theta_\psi + \lambda_h - \gamma} \left[\frac{e^{-(\gamma - \lambda_h)t}}{\rho + \lambda + \gamma - \lambda_h} - \frac{e^{-\theta_\psi t}}{\rho + \lambda + \theta_\psi} \right] > 0.$$

In the case $\gamma > \lambda_h$, inflation also reverts to its steady-state level. When $\gamma = \lambda_h$, we recover the case of Section 3.1, where output-gap stabilization leads to an increasing path of government debt and inflation. Full stabilization of the output gap becomes impossible when $\gamma < \lambda_h$, as the required feedback between debt and real rates would cause debt and inflation to spiral out of control.

Figure 4 shows the equilibrium dynamics in the case $\gamma > \lambda_h$. The fiscal shock causes a sharp increase in government debt and a bout of inflation. As the fiscal shock dissipates, the automatic debt stabilizer eventually starts to bring the debt level down. As this reduces the extent debt deviates from its natural level, inflation expectations start to recede bringing inflation down. Notice that the output gap is constant throughout this exercise, so the disinflation comes without a recession, it is entirely driven by expectation effects. Moreover, the disinflation happens well after the fiscal shock itself has nearly returned to its pre-shock level. The expectation effects depend on future deviations of debt from its natural level instead of contemporaneous fiscal deficits.

4. Optimal Policy

In this section, we study optimal monetary policy in the fiscal-expansion phase. As discussed in the previous section, uncertainty regarding the policy reform creates an endogenous fiscal cost-push shock, which breaks the divine coincidence. Therefore, the planner faces a simultaneous trade-off between stabilizing output, inflation, and debt.

4.1 The optimal policy problem

We consider a quadratic approximation to the planner's objective. The planner minimizes the expected present value of squared deviations of output and inflation from their steady-state values. The only policy instrument is the path of nominal interest rates during the fiscal-expansion phase. The planner affects the inflationary-finance phase indirectly through the level of government debt.

We focus again on the tractable case without automatic debt stabilizers, $\gamma = 0$, or households' expectation effects, $\lambda_h = 0$. The planner and firms share the same beliefs. We simplify notation and write $\lambda_f = \lambda$.

The planner's objective. During an inflationary-finance phase that starts with a debt level b_0^*, the value of the planner's objective is

$$\mathcal{P}^*(b_0^*) = \int_0^{T^*} e^{-\rho t}(\alpha x_t^{*2} + \beta \pi_t^{*2})dt.$$

Using that $x_t^* = (b_0^* - b^n)\left(1 - \frac{t}{T^*}\right)$ and $\pi_t^* = \kappa\Phi(b_0^* - b^n)\left(1 - \frac{t}{T^*}\right)$, we obtain $\mathcal{P}^*(b_0^*) = \Upsilon(b_0^* - b^n)^2$ where $\Upsilon \equiv [\alpha + \beta(\kappa\Phi)^2] \int_0^{T^*} e^{-\rho t}\left(1 - \frac{t}{T^*}\right)^2 dt$.

At the beginning of the fiscal-expansion phase, the planner's objective function can be written as

$$\mathcal{P} = -\frac{1}{2}\mathbb{E}\left[\int_0^{\tau} e^{-\rho t}\left(\alpha x_t^2 + \beta \pi_t^2\right)dt + e^{-\rho\tau}\mathcal{P}_\tau^*(b_\tau)\right],$$

where τ denotes the random time at which the economy switches to the fiscal-consolidation phase. Using the fact that the density of τ is $\lambda e^{-\lambda\tau}$, we can write welfare in the fiscal-expansion phase as:

$$\begin{aligned}\mathcal{P} &= -\frac{1}{2}\int_0^{\infty} \lambda e^{-\lambda\tau}\left[\int_0^{\tau} e^{-\rho t}\left(\alpha x_t^2 + \beta\pi_t^2\right)dt + e^{-\rho\tau}\mathcal{P}^*(b_\tau)\right]d\tau \\ &= -\frac{1}{2}\int_0^{\infty} e^{-(\rho+\lambda)t}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right]dt.\end{aligned}$$

Even though only output and inflation directly affect the planner's objective, the feature that government debt affects the economy's dynamics during the inflationary-finance phase creates an endogenous *debt-stabilization motive*, where the planner wants to minimize deviations of government debt from its natural level. Hence, the weight on debt does not come from the planner's concern about budgetary affairs, but from the fact that debt affects output and inflation after a reform.

The presence of a debt-stabilization motive distinguishes this problem from the classic analysis of Barro (1979), and its modern formulation by Aiyagari, Marcet, Sargent and Seppälä (2002), where the planner uses fluctuations in government debt to smooth variations in distortionary taxes. In our setting, deviations of the government debt from its natural level are themselves costly, given the expectation of an adjustment through inflationary financing. Moreover, given that the government issues short-term real debt, the planner does not have direct access to state-contingent debt, as in Lucas and Stokey (1983). In contrast, reductions in government debt come from a period of low real returns.

Competitive equilibria. The planner's problem consists of implementing the competitive equilibrium that maximizes the objective derived above. A competitive equilibrium corresponds to a bounded solution to the following conditions:

$$\dot{\pi}_t = (\rho + \lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n), \qquad \dot{b}_t = r_t - \rho + \psi_t, \qquad \dot{x}_t = r_t - \rho,$$

given b_0, the path of real interest rates r_t, and the path of fiscal shock ψ_t. For any given initial condition for the output gap, it can be shown that initial inflation in a bounded solution is given by:

$$\pi_0 = \kappa \frac{x_0 + \lambda\Phi(b_0 - b^n)}{\rho + \lambda} + \frac{\kappa}{\rho + \lambda} \int_0^\infty e^{-(\rho+\lambda)t} \left[(1 + \lambda\Phi)(r_t - \rho) + \lambda\Phi\psi_t\right] dt. \tag{25}$$

The set of competitive equilibria can be indexed by the path of real interest rates $\{r_t\}_0^\infty$ and an initial output gap x_0—the monetary authority can implement a particular equilibrium by appropriately choosing the monetary rule. While the planner can freely choose the initial condition for the output gap, it cannot independently choose the initial conditions for *both* the output gap and inflation.[17]

Incentives for debt expropriation and lack of a classical solution. As is often the case in optimal policy problems, the planner is incentivized to expropriate private agents in period zero. Debt is real, and prices are sticky, so expropriation does not take the form of a jump in the price level. Instead, the planner can effectively choose an arbitrarily negative return r_t on debt for an infinitesimal period, which leads to a downward jump in government debt in period zero. Therefore, the planner has de facto control over the initial debt level.

This implies that a classical solution to the optimal control problem, one where states follow a continuous path, does not exist in this environment. The possibility of an "expropriation-like" debt path occurs because the model does not penalize extremely low rates.[18] Thus, analogous to the approach in Marcet and Marimon (2019) and Dávila and Schaab (2023), we introduce a penalty to deal with this source of time inconsistency.

[17] It could do the converse and set an initial condition for inflation but then the output gap would be determined by (25).

[18] In discrete time, this condition appears as a restriction of positive definiteness of penalty functions in the controls. See, for example, the discussion in Quant Econ Website .

A planner's problem without debt expropriation. Following Dávila and Schaab (2023), we consider a penalized version of the problem in which the planner faces a penalty associated with the choice of the initial value for each one of the forward looking variables, namely x_0 and π_0. By appropriately choosing the penalties, we ensure there is no expropriation. The penalty otherwise does not affect the path of inflation and output. Its effect on the optimal solution is entirely mediated by the impact on the initial debt level. The planner's problem can be written as follows:

$$\max_{\{[\pi_t, b_t, x_t, r_t]_0^\infty\}} -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right] dt + \xi_x x_0 + \xi_\pi \pi_0, \tag{26}$$

subject to

$$\dot{\pi}_t = (\rho+\lambda)\pi_t - \kappa x_t - \kappa\lambda\Phi(b_t - b^n), \qquad \dot{b}_t = r_t - \rho + \psi_t, \qquad \dot{x}_t = r_t - \rho,$$

and the initial condition for inflation Eq. (25), given b_0 and the path of the fiscal shock ψ_t.

The integral above corresponds to the original objective. The last two terms capture the penalties on the initial output gap, ξ_x, and initial inflation, ξ_π. In the absence of these penalties, the initial value of the co-states for inflation, output gap, and debt are all equal to zero. In this case, there is a discontinuous jump in the value of debt at $t = 0$.[19] We choose the values of ξ_x and ξ_π such that $\lim_{t\to 0} b_t = b_0$, while the initial value of the co-states on output gap and inflation are still equal to zero.

Real rates. The following proposition characterizes real rates under the optimal policy.

Proposition 4 (Real interest rates.)**.** *The real interest rate under the optimal policy is given by*

$$r_t - \rho = -\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon + \alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t. \tag{27}$$

An important implication of Proposition 4 is that it is optimal to *reduce* the real interest rate in response to the fiscal shock, to the extent the shock is inflationary; i.e., if $\pi_t \geq 0$. Moreover, nominal rates move less than one-to-one with inflation along the equilibrium

[19]We show in the Appendix D.1 that a classical solution with smooth state variables does not exist when $\xi_x = \xi_\pi = 0$. Formally, it is optimal to have a Dirac mass on interest rates in period zero, and $\lim_{t\to 0} b_t \neq b_0$.

path:

$$i_t - \rho = \left[1 - \beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon + \alpha}\right]\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t.$$

This result is in sharp contrast with standard recommendations based on a Taylor rule, which emphasize the importance of nominal rates moving more than one-to-one with inflation.[20] In contrast, Proposition 4 shows that it is optimal to *underreact* to the shock. For example, when the planner only cares about output costs, $\beta = 0$, we have that $r_t - \rho = -\frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t$. In this case, the optimal response to the fiscal shock involves accommodation and a deviation from the standard Taylor principle.

Dynamics under optimal policy. To characterize the dynamics under the optimal policy, we use the optimal real interest rule, combined with the fact that $x_t = x_0 + b_t - b_0 - \hat{\psi}_t$, and collapse the solution into a bivariate system in π_t and b_t:

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{b}_t \end{bmatrix} = \begin{bmatrix} \rho+\lambda & -\kappa(1+\lambda\Phi) \\ -\hat{\beta} & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ b_t - b^n \end{bmatrix} + \begin{bmatrix} \kappa(\hat{\psi}_t + b_0 - b^n - x_0) \\ \frac{\alpha}{\lambda\Upsilon+\alpha}\psi_t \end{bmatrix}, \tag{28}$$

where $\hat{\beta} \equiv \frac{\beta\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}$ and $\hat{\psi}_t = \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\psi_0$. The eigenvalues of this system are given by

$$\overline{\omega} = \frac{\rho+\lambda+\sqrt{(\rho+\lambda)^2 + 4\kappa(1+\lambda\Phi)\hat{\beta}}}{2} > 0, \qquad \underline{\omega} = \frac{\rho+\lambda-\sqrt{(\rho+\lambda)^2 + 4\kappa(1+\lambda\Phi)\hat{\beta}}}{2} < 0.$$

As there is one positive and one negative eigenvalue, there is a unique bounded solution to the system above for any given x_0. The optimality condition for the initial output gap is given by:

$$\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t + \frac{\beta}{\rho+\pi}\pi_t\right] = 0. \tag{29}$$

This condition says that is not optimal to have output and inflation systematically above or below its steady-state level. The planner sets the average value of a combination of output and inflation to zero, depending on the relative weight of output and inflation on welfare. If $\alpha \gg \beta$, then the planner sets the average value of the output gap to zero. When $\beta \gg \alpha$, such

[20] Of course, the planner still uses the *threat* of reacting to movements in inflation more than one-to-one off the equilibrium to ensure the equilibrium is locally unique.

the planner cares primarily about inflation, the planner sets average inflation to its target.

4.2 Hawks vs. doves

While the optimal response is characterized for generic values of α and β, it is instructive to consider the extreme cases where the planner only cares about inflation, which we associate with a *hawkish* central bank, and the case where the central bank only cares about the output gap, which captures a more *dovish* central bank. In both cases, the planner (endogenously) assigns a positive weight on debt stabilization, as it depends on the weight on inflation and the output gap in a inflationary-finance phase. To simplify the message, we set $b_0 = b^n = 0$ and normalize the initial price level, $p_0 = 0$.

Doves. Consider first the case where the central bank does not give any weight to inflation and only cares about stabilizing output, so $\beta = 0$.

Proposition 5 (Optimal policy: Doves)**.** *Suppose $\beta = 0$. Then,*

(i) Inflation:

$$\pi_t = \frac{\kappa}{\overline{\omega}} \frac{\alpha\lambda\Phi}{\alpha + \lambda\Upsilon} \frac{\psi_0}{\overline{\omega} + \theta_\psi} + \frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha} \frac{1 - e^{-\theta_\psi t}}{\theta_\psi(\overline{\omega} + \theta_\psi)} \psi_0. \tag{30}$$

where $\pi_t > 0$ and $\dot{\pi}_t > 0$.

(ii) Output gap

$$x_t = \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha} \psi_t. \tag{31}$$

(iii) Government debt

$$b_t = \frac{\alpha}{\lambda\Upsilon + \alpha} \frac{\psi_0 - \psi_t}{\theta_\psi}. \tag{32}$$

Proposition 5 characterizes the equilibrium under the optimal policy for a dovish central bank. In this case, the planner faces a trade-off between stabilizing the output gap in the initial phase and stabilizing the output gap in the inflationary-finance phase, which ultimately requires influencing the government debt. The optimal response of the monetary authority involves reducing real rates to partially offset the effects of the fiscal shock on debt:

$$r_t - \rho = -\frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha} \psi_t.$$

Intuitively, starting from an equilibrium where $r_t = \rho$ and the output gap is constant, a reduction of real rates has a first-order benefit of reducing debt and only a second-order cost of distorting the output gap. The magnitude of the adjustment depends on the relative weight of debt stabilization on welfare. When λ is close to zero, such that is unlikely the economy will switch to the inflationary-finance phase, the planner barely reacts to the shock. In this case, the output gap is close to zero and government debt absorbs most of the fiscal shock. When λ is large, the planner offsets most of the fiscal shock, dampening the response of debt. Given the planner only cares about the output gap, there is no attempt to stabilize inflation, which ends up being positive and increasing over time.

Hawks. Consider next the case where the planner only cares about inflation, so $\alpha = 0$.

Proposition 6 (Optimal policy: Hawks)**.** *Suppose $\alpha = 0$. Then,*

(i) Inflation:

$$\pi_t = \kappa \frac{\psi_t - \frac{\overline{\omega}}{\rho+\lambda+\theta_\psi} e^{\underline{\omega} t}\psi_0}{(\overline{\omega}+\theta_\psi)(\underline{\omega}+\theta_\psi)}, \tag{33}$$

where $\pi_0 > 0$, $\dot{\pi}_0 < 0$, and $\pi_t < 0$ for t sufficiently large.

(ii) Output gap:

$$x_t = \frac{\psi_0}{\rho+\lambda+\theta_\psi}\left[\frac{\overline{\omega}}{\overline{\omega}+\theta_\psi} + \frac{\rho+\lambda+\theta_\psi}{\overline{\omega}+\theta_\psi}\right] - \beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon} p_t - \frac{\psi_0 - \psi_t}{\theta_\psi}, \tag{34}$$

where $p_t = \int_0^t \pi_s ds$ is the price level at date t.

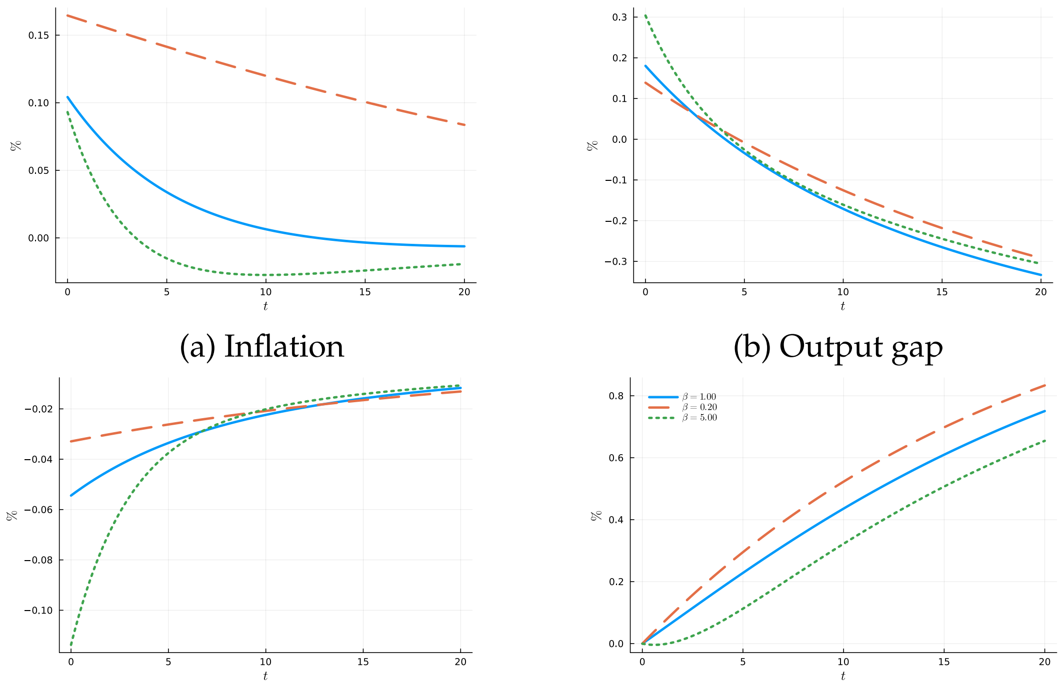
(iii) Government debt

$$b_t = -\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon} p_t, \tag{35}$$

where $\dot{b}_0 < 0$ and $\lim_{t\to\infty} b_t > 0$.

Proposition 6 characterizes the optimal policy for a hawkish central bank. The planner now faces a trade-off between stabilizing inflation in the fiscal-expansion phase and stabilizing inflation in the inflationary-finance phasethrough its impact on government debt. It is again optimal to reduce real rates on impact:

$$r_t - \rho = -\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon}\pi_t - \psi_t. \tag{36}$$



(a) Inflation

(b) Output gap

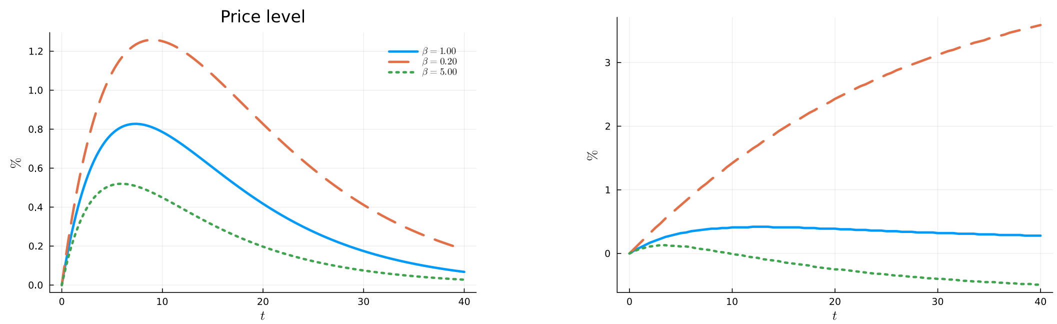
(c) Real rates

(d) Government debt

Figure 5: Equilibrium dynamics under optimal policy

Notice that $r_0 - \rho < 0$, as $\pi_0 > 0$. The case where $\lambda\Upsilon \gg \beta$ is particularly intuitive. As the planner cares mostly about the inflationary-finance phase, it offsets the fiscal shock almost fully, so $r_t - \rho \approx -\psi_t$. This way the planner stabilizes government debt. Interestingly, for a planner who cares relatively more about inflation in the initial phase, it is optimal to reduce rates even more aggressively, to the point where the government debt is initially decreasing over time, despite of the fiscal shock. Hence, a hawkish central bank reduces the real rate, at least initially, more than a dovish central bank.

Discussion: Hawks vs. doves. Figure 5 shows the optimal policy for different values of β, the welfare weight on inflation, for a fixed weight on the output gap, which we set to $\alpha = 1$. The case $\beta = 1$ corresponds to a planner who gives equal weight to inflation and the output gap, while the case $\beta > 1$ ($\beta < 1$) corresponds to a planner who gives more weight to inflation (output gap). A striking feature of this economy is that the optimal real interest is below its natural level in response to fiscal shocks, regardless of the value of β. Moreover, a hawkish central bank is able to reduce inflation more effectively despite having lower real



(a) Exogenous cost-push shock

(b) Fiscal cost-push shock

Figure 6: Price level: exogenous cost-push shock vs fiscal shock

rates, as this policy dampens the increase in government debt and, ultimately, its impact on inflation expectations.

Comparison with textbook model. The dynamics under the optimal policy with an endogenous fiscal cost-push shock differ in important ways from standard results on the optimal policy for the textbook model with exogenous cost-push shocks. The first main difference refers to the behavior of the output gap. In the textbook model, it is optimal to have a recession in response to a positive cost-push shock. In contrast, the planner engineers a boom in response to the fiscal shock, as shown in Panel (b) of Figure 5.

The second main difference corresponds to the behavior of the price level. In the textbook model, it is optimal to promise a sufficiently long period of deflation that brings the price level back to its pre-shock level, see Panel (a) of Figure 6. The fact that *price-level targeting* is optimal in the textbook model is one of the distinguishing features of optimal policy under commitment (see e.g. Woodford 2010 for a discussion). In contrast, price-level targeting is not optimal in the presence of a fiscal cost-push shock. As shown in Panel (b) of Figure 6, the price level does not return to its pre-shock level under the optimal policy.

4.3 Commitment versus discretion

So far, we have focused on the case where the planner commits to a time-zero plan. However, given the presence of forward-looking constraints, the optimal policy is not time-consistent. We then consider two alternatives to the time-zero commitment case. First, we consider the case of *discretion*, where the planner has commitment only over an arbitrarily small time frame. Second, we consider optimal policy under the *timeless perspective*, where the planner commits to a contingent plan decided in the distant past. In both cases, we find that it is optimal to reduce real rates in response to the fiscal shock, showing our results do not rely on assuming commitment to a time-zero plan.

Discretion. To capture the idea of discretion in our continuous-time setting, it is useful to assume the planner has commitment over a random period of time, and takes as given the actions of future planners.[21] Formally, assume that with Poisson intensity $\overline{\lambda}$ the control over monetary policy goes to a new planner. This implies that, in expectation, any planner has control over $\frac{1}{\overline{\lambda}}$ periods. We are interested in the limit as $\overline{\lambda} \to \infty$, so the planner has commitment over an infinitesimal period of time. This corresponds to the continuous-time analog of the case of discretion in discrete time, where the planner controls policy over a single period. The next proposition characterizes optimal policy under discretion.

Proposition 7 (Discretion)**.** *Consider an economy where a planner controls policy over a random period of time, and a new planner arrives with Poisson intensity $\overline{\lambda}$. In the limit as $\overline{\lambda} \to \infty$, the real interest rate under the optimal policy is given by*

$$r_t - \rho = -\psi_t.$$

Moreover, the output gap is given by $x_t = 0$.

Proposition 7 shows that the real rate is below its natural level under discretion. With an arbitrarily short planning horizon, the planner cannot directly control inflation, which depends on future decisions, and has no incentive to distort the output gap. Hence, the planner fully stabilizes the government debt, which is how the current planner affects future decisions. Notice that the planner sets the output gap to zero and promises that the output gap will decline over time, given the low interest rate. As a new planner arrives, the planner does not keep the promises of previous planners, and the output gap is again set to zero. In Appendix D.6, we consider the case of partial commitment, where the planner takes the initial value of the output gap as given. We show that optimal policy with partial commitment coincides with the case of full commitment for a dovish central bank, that is, $\beta = 0$. Therefore, in this case, it is also optimal for the real rate to be below the natural level.

[21] For a similar formulation of a problem with discretion in continuous time, see e.g. Harris and Laibson (2013) and Dávila and Schaab (2023).

Timeless perspective. We consider next the case of optimal policy under the timeless perspective, in the sense of Woodford (1999). When the planner commits to a time-zero plan, it sets the value of the co-states for the forward-looking variables equal to zero at $t = 0$. When considering the solution under the timeless perspective, the initial value of those co-states equals the corresponding value for a planner who started its planning in the distant past.[22] The next proposition shows that the two solutions actually coincide when $b_0 = b^n$.

Proposition 8 (Timeless perspective)**.** *Suppose that $b_0 = b^n$, such that government debt is at its natural level when the fiscal shock is announced. Then, the optimal policy when the planner commits to a time-zero plan coincides with the optimal policy under the timeless perspective.*

An implication of Proposition 8 is that the solution to the Ramsey problem satisfies an important *self-consistency* property. In particular, output and inflation can be described by time-invariant functions of the exogenous shock, ψ_t, a predetermined variable, b_t, and variables describing history-dependence, the co-states on the forward-looking variables. From the point of view of a planner who started planning in the distant past, there is no incentive to have output and inflation deviate from these time-invariant functions.

The assumption that $b_0 = b^n$ is important, as we would observe dynamics under the solution to the Ramsey problem even in the absence of shocks, so the optimal policy would be time-dependent and deviate from the solution under the timeless perspective. This observation motivates our focus on the case $b_0 = b^n$.

5. Staying behind the curve?

5.1 Motivation

In a final exercise, we compare optimal policies in the model with the responses following a Taylor rule. The exercise is motivated by the policy debates ongoing in the aftermath of the COVID-19 pandemic. In response to the COVID-19 crisis, the United States implemented an unprecedented fiscal expansion, resulting in the highest level of government debt (normalized by GDP) in the post-war era. The period also saw a burst in inflation that persisted for a while, unlike in more than 30 years.

[22] For a formal discussion of this procedure, see the discussion in Giannoni and Woodford (2017).

Policy debates. During the inflation surge, various commentators called for a more aggressive monetary policy stance, with nominal rates rates above the infiltration rate as dictated by the Taylor principle. [23] Meanwhile, long-run inflation expectations and market-implied probabilities of inflation spikes were taking off—see Hilscher et al. (2022).[24] In other words, the Fed was criticized for staying "behind the curve." Many commentators went even further stating that regaining control over inflation and inflation expectations would require a recession, i.e., that a "soft-landing" would be impossible?

It is clear that the Fed ignored the advice. Did it make a mistake by deviating from the Taylor rule? Did it risk triggering an inflationary spiral? The motivation behind the next exercise is to investigate whether following the Taylor rule would have been the correct, in the context of our model.

Inflation, Expectations, and debt financing during the post pandemic. To set the stage, we present some data patterns from the period. Figure 7 panel (a) displays the US debt-to-GDP ratio (un-adjusted for market values) from 2018 Q1 to 2024 Q3.[25] The figure illustrates a sharp increase in government debt in 2020 Q2, followed by a rapid decline. By 2023 debt plateaued and recovered almost have the increase. The figures in market-value terms are event more striking, with a decline of almost 40 percentage points in two and a half years.

Importantly, the decline in debt to GDP was not produced by real GDP growth nor fiscal surpluses. The primary deficit relative to GDP remained below its pre-pandemic average until 2022, as shown in panel (b). The decrease in government debt can be attributed to inflation exceeding the Federal Reserve's target and a negative real interest rate on government debt during much of the post-pandemic period, as indicated in panel (c). Furthermore, short-term and medium-term inflation expectations increased, as shown in the various panel measures (d).

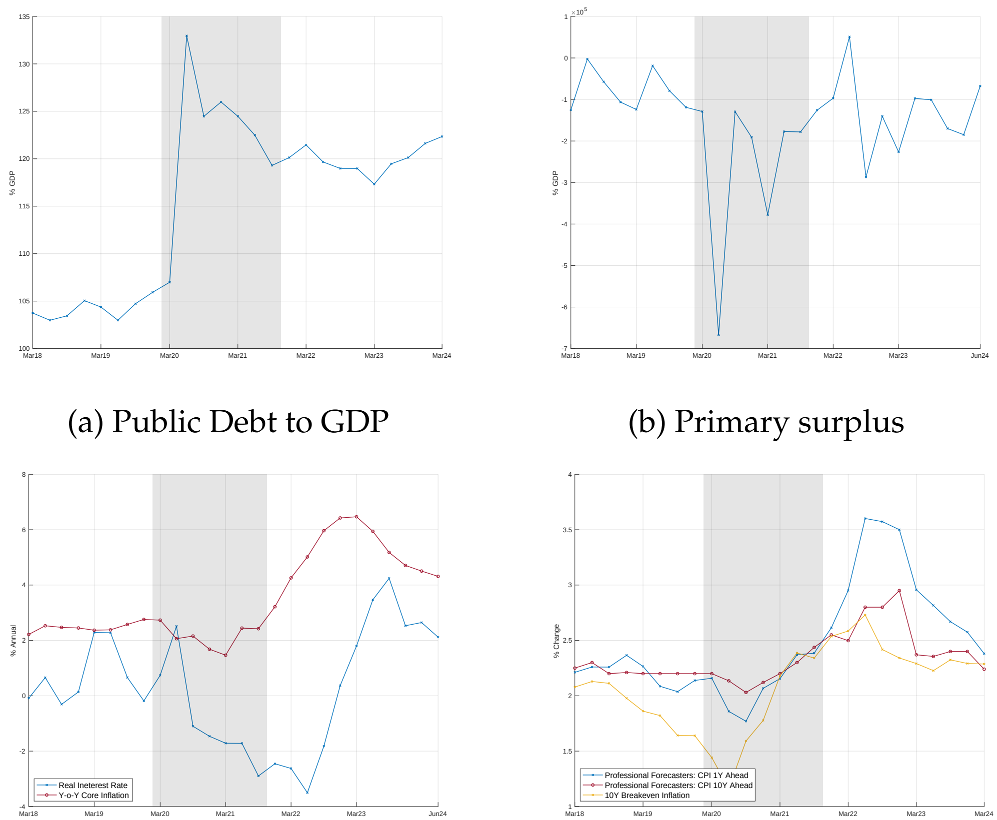
5.2 Comparing Taylor rules with the optimal policy

[TBA]

[23] See, for example, the following Wall Street Journal op ed by Thomas Sargent and William L. Silber or the following Brookings Institution discussion by Jon Steinsson.

[24] According to Hilscher et al. (2022) long-run inflation expectations deanchored in 2021-22, and reanchored as policy tightened, but remained higher.

[25] A similar dynamics is observed when plotting the debt held by the private sector instead.



(a) Public Debt to GDP

(b) Primary surplus

(c) Real rates and inflation

(d) Inflation Expectations

Figure 7: Pre- and Post-COVID-19 Dynamics

6. Conclusion

We have presented a few lessons immediately derived from the model in this paper. First, we demonstrated that in an environment where a monetary-fiscal reform is expected, we attempts to fight inflation backfire through the expectation of greater inflation when the reforms takes place. We called this phenomenon, sticky inflation. Second, we should that because inflation is sticky, optimal policy that cannot guarantee immunity against a forced reform, should balance inflation and debt objectives, often, keeping real interest rates low below their natural rate until a the reform takes place.

Several policy lessons follow indirectly. First, if a fiscal-monetary reform will happen, it is better to have the reform earlier than later. Second, we have not considered the possibility that earlier attempts to fight inflation are designed to signal that monetary policy will not finance deficits in the future. We have shown that these attempts are futile in bringing inflation back if the signalling effect is not present. Thus, it is important to understand how medium-term inflation expectations respond to the signalling effects.

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The parameter $\overline{\lambda}_h$ is used to represent the angle. The coefficient β is important in this equation. The constant γ is defined as...

Glossary

β Beta, a Greek letter. 40

γ Gamma, a Greek letter. 40

$\overline{\lambda}_h$ Poisson Intensity Fiscal Event. 40

A. Derivations for Section 2

Households. The household problem is given by

$$V_t(B_t) = \max_{[C_s, N_s]_{s \geq t}} \mathbb{E}_t \left[\int_t^{t^*} e^{-\rho(s-t)} \left[u(C_s) - h(N_s) \right] ds + e^{-\rho(t^*-t)} V^*_{t^*}(B^*_t) \right], \tag{37}$$

subject to

$$\dot{B}_t = (i_t - \pi_t) B_t + \frac{W_t}{P_t} N_t + D_t + T_t - C_t, \tag{38}$$

and a No-Ponzi condition, where t^* denotes the arrival time for a Poisson process with intensity $\theta \geq 0$, B_t denotes the real valued of bonds held by households, W_t is the nominal wage, P_t is the price level, D_t are dividends payed by firms, T_t denotes fiscal transfers.

The HJB equation for this problem is given by

$$\rho V = u(C) - h(N) + \dot{V} + V_B \left[(i - \pi) B + \frac{W}{P} N + T - C \right] + \theta [V^* - V]. \tag{39}$$

The first-order conditions are given by

$$u'(C) = V_B, \qquad h'(N) = V_B \frac{W}{P}. \tag{40}$$

The envelope condition is given by

$$\rho V_B = V_B (i - \pi) \dot{V}_B + V_{BB} \left[(i - \pi) B + \frac{W}{P} N + T - C \right] + \theta [V^*_B - V_B]. \tag{41}$$

Combining the envelope condition with the optimality condition for consumption, we obtain

$$0 = (i - \pi - \rho) + \frac{u''(C) C}{C} \frac{\dot{C}_t}{C_t} + \theta \left[\frac{u'(C^*)}{u'(C)} - 1 \right] \Rightarrow \frac{\dot{C}}{C} = \sigma^{-1} (i - \pi - \rho) + \frac{\theta}{\sigma} \left[\frac{u'(C^*)}{u'(C)} - 1 \right], \tag{42}$$

where $\sigma = -\frac{u''(C)C}{u'(C)}$.

The optimality condition for labor can be written as

$$\frac{h'(N)}{u'(C)} = \frac{W}{P}. \tag{43}$$

Firms. There are two types of firms in the economy: final-goods producers and intermediate-goods producers. Final goods are produced by competitive firms according to the production function $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon}{\epsilon-1}} di \right)^{\frac{\epsilon-1}{\epsilon}}$, where $Y_{i,t}$ denotes the output of intermediate $i \in [0, 1]$. The demand for intermediate i is given by $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$, where $P_{i,t}$ is the price of intermediate i, $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is the price level, and Y_t is the aggregate output.

Intermediate-goods producers have monopoly over their variety and operate the technology $Y_{i,t} = A_t N_{i,t}$, where $N_{i,t}$ denotes labor input. Firms are subject to quadratic adjustment costs on price changes, so the problem of intermediate i is given by

$$Q_{i,t}(P_i) = \max_{[\pi_{i,s}]_{s \geq t}} \mathbb{E}_t \left[\int_t^{t^*} \frac{\eta_s}{\eta_t} \left(\frac{P_{i,s}}{P_{i,t}} Y_{i,s} - \frac{W_s}{P_s} \frac{Y_{i,s}}{A_s} - \frac{\varphi}{2} \pi_{i,s}^2 \right) ds + \frac{\eta_{t^*}}{\eta_t} Q_{i,t}^*(P_{i,t}^*) \right], \tag{44}$$

subject to $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$, given $P_{i,t} = P_i$ and $\eta_t = e^{-\rho t} u'(C_t)$, where φ is the adjustment cost parameter.

The HJB equation for this problem is

$$0 = \max_{\pi_{i,t}} \eta_t \left(\frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_t}{P_t} \frac{Y_{i,t}}{A} - \frac{\varphi}{2} \pi_{i,t}^2 \right) dt + \mathbb{E}_t[d(\eta_t Q_{i,t})], \tag{45}$$

where $\frac{\mathbb{E}_t[d(\eta_t Q_{i,t})]}{\eta_t dt} = -(i_t - \pi_t) Q_{i,t} + \frac{\partial Q_{i,t}}{\partial P_{i,t}} \pi_{i,t} P_{i,t} + \frac{\partial Q_{i,t}}{\partial t} + \theta \frac{\eta_t^*}{\eta_t} \left[Q_{i,t}^* - Q_{i,t} \right]$.

The first-order condition is given by

$$\frac{\partial Q_{i,t}}{\partial P_i} P_{i,t} = \varphi \pi_{i,t}.$$

The change in π_t conditional on no switching in state is then given by

$$\left(\frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2} \pi_{i,t} P_{i,t} \right) P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} P_{i,t} = \varphi \dot{\pi}_{i,t}. \tag{46}$$

The envelope condition with respect to $P_{i,t}$ is given by

$$0 = \left((1-\epsilon) \frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t A} \right) \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_{i,t}} + \frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2} \pi_{i,t} P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} - (i_t - \pi_t) \frac{\partial Q_{i,t}}{\partial P_i} + \theta \frac{\eta_t^*}{\eta_t} \left(\frac{\partial Q_{i,t}^*}{\partial P_i} - \frac{\partial Q_{i,t}}{\partial P_i} \right). \tag{47}$$

Multiplying the expression above by $P_{i,t}$ and using Equation (46), we obtain

$$0 = \left((1-\epsilon) \frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t A} \right) \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t + \varphi \dot{\pi}_t - (i_t - \pi_t) \varphi \pi_{i,t} + \theta \varphi \frac{\eta_t^*}{\eta_t} \left(\pi_{i,t}^* - \pi_{i,t} \right).$$

Rearranging the expression above, we obtain the non-linear New Keynesian Phillips curve

$$\dot{\pi}_t = (i_t - \pi_t)\,\pi_t + \theta\frac{\eta^*}{\eta_t}\,(\pi_t - \pi_t^*) - \frac{\epsilon\varphi^{-1}}{A}\left(\frac{W_t}{P_t} - (1-\epsilon^{-1})A\right)Y_t.$$

Government and market clearing. The government flow budget constraint is given by

$$\dot{B}_t^g = (i_t - \pi_t)B_t^g + T_t, \tag{48}$$

where B_t^g denotes the real value of government debt. The government must also satisfy the No-Ponzi condition $\lim_{T\to\infty}\mathbb{E}_t[\eta_T B_T^g] = 0$.

The market clearing condition is given by

$$C_t = Y_t, \qquad N_t = \int_0^1 N_{i,t}di, \qquad B_t = B_t^g. \tag{49}$$

Government and market clearing.

B. Bond pricing and inflation expectations

B.1 Bonds, yield curve, and inflation forecasts.

While debt is short-term, we can price long-dated bonds, both real and nominal, and obtain break-even inflation measures and corresponding inflation expectations. By pricing these objects, we can provide a link to the ongoing literature on inflation expectations.

Consider a zero-coupon bond at date t maturing τ periods ahead, promising one consumption unit. Assuming household's price the bond, its price $\phi_{t,\tau}$ satisfies the following PDE:

$$r_t \cdot \phi_{t,\tau} = \dot{\phi}_{t,\tau} - \frac{\partial \phi_{t,\tau}}{\partial \tau} + \lambda_h \frac{C_t}{C_t^J} \underbrace{\left[\phi^*_{t,\tau} - \phi_{t,\tau}\right]}_{\text{m-f reform}} + \overline{\lambda}_h \frac{C_t}{\overline{C}} \underbrace{\left[\bar{\phi}_{t,\tau} - \phi_{t,\tau}\right]}_{\text{f-consolidation}}, \tag{50}$$

with terminal condition $\phi_{t,0} = 1$, the first-term is a standard term, the second and third terms capture the risk-adjusted revaluations of bonds if the reform events are realized. Likewise, a nominal bond, with price $q_{t,\tau}$ satisfies:

$$i_t \cdot q_{t,\tau} = \dot{q}_{t,\tau} - \frac{\partial q_{t,\tau}}{\partial \tau} + \lambda_h \frac{C_t}{C_t^J} \underbrace{\left[q^*_{t,\tau} - q_{t,\tau}\right]}_{\text{m-f reform}} + \overline{\lambda}_h \frac{C_t}{\overline{C}} \underbrace{\left[\bar{q}_{t,\tau} - q_{t,\tau}\right]}_{\text{f-consolidation}}, \tag{51}$$

with terminal condition $q_{t,0} = 1$. The interpretation is the same, expect that nominal bonds are discounted with the nominal rate.

The solution to the linearized version of equations (50) and (51), i.e. solutions to $\{\phi_{t,\tau}, q_{t,\tau}\}$, satisfy the same formulas:

$$\varphi_{t,\tau} = e^{-\int_t^{t+\tau}(\delta_s+\theta)ds} + \int_t^{t+\tau} e^{-\left(\int_t^s(\delta_z+\theta)dz\right)} \left(\theta^* e^{-\int_0^{\tau-(s-t)} \delta_z^*(b_s)dz} + \bar{\theta} e^{-\rho(\tau-(s-t))}\right) ds. \tag{52}$$

When $\varphi_{t,\tau} = \phi_{t,\tau}$ we substitute it: $\delta_t = r_t, \delta_t^* = r_t^*(b_s)$ and when $\varphi_{t,\tau} = q_{t,\tau}$ we substitute $\delta_t = r_t + \pi_t, \delta_t^* = r_t^*(b_s) + \pi_t^t(b_s)$.[26] The nominal and real yield curves are respectively:

$$r_{t,\tau} \equiv -\frac{\ln(\phi_{t,\tau})}{\tau}, \quad \text{and} \quad i_{t,\tau} \equiv -\frac{\ln(q_{t,\tau})}{\tau}. \tag{53}$$

From these expressions, we obtain a model analog of break-even inflation, $\tilde{\pi}_{t,\tau} \equiv i_{t,\tau} - r_{t,\tau}$.

Finally, let $\pi_{t,\tau}$ reflect the time t expectation about inflation after some time τ.[27] Inflation expec-

[26]Note that as required, for $\tau = 0$, prices equal one.

[27]The difference between expected and break-even inflation is that break-even inflation is an imperfect measure of inflation due to risk-premia. These terms are useful for inferring expectations of fiscal-monetary reforms from asset-market and expectation data.

tations satisfy an analogous PDE:

$$0=\frac{\partial \pi_{t,\tau}}{\partial t}-\frac{\partial \pi_{t,\tau}}{\partial \tau}+\theta^{*}\underbrace{\left[\pi_{t,\tau}^{*}-\pi_{t,\tau}\right]}_{\text{m-f reform}}+\underbrace{\bar{\theta}\left[\overline{\pi}_{t,\tau}-\pi_{t,\tau}\right]}_{\text{f-consolidation}}. \tag{54}$$

The solution to the log-linearized path of inflation expectations $\pi_{t,\tau}$ is given by:

$$\pi_{t,\tau}=e^{-\theta\tau}\pi_{t+\tau}+\int_{t}^{t+\tau}e^{-\theta(s-t)}\theta^{*}\pi_{t+\tau}^{*}\left(b_{s}\right)ds. \tag{55}$$

Thus, inflation expectations at any horizon are discount paths of inflation in the fiscal-expansion phase and inflationary-finance phaserespectively.

Bond pricing. Let $\eta_t = e^{-\rho t}C_t^{-1}$ denote the economy's real stochastic discount factor (SDF), which follows the process:

$$\frac{d\eta_t}{\eta_t}=-r_t dt+\frac{\eta_t'-\eta_t}{\eta}\left[d\mathcal{N}_t-\theta dt\right], \tag{56}$$

where $\mathcal{N}_t$ is a Poisson process with arrival rate $\theta=\theta^*+\bar{\theta}$. The SDF after the jump is given by η_t' and, conditional on the Poisson arrival, $\eta_t'=\eta_t^*$ with probability $\frac{\theta^*}{\theta^*+\bar{\theta}}$ and $\eta_t'=\overline{\eta}_t$ with probability $\frac{\bar{\theta}}{\theta^*+\bar{\theta}}$.

Denote the price of a real zero-coupon bond at date maturing τ periods ahead by $\phi_{t,\tau}$. The bond price after the jump is denoted by $\phi_{t,\tau}'$. Conditional on the Poisson arrival, $\phi_{t,\tau}'=\phi_{t,\tau}^*$ with probability $\frac{\theta^*}{\theta^*+\bar{\theta}}$ and $\phi_{t,\tau}'=\overline{\phi}_{t,\tau}$ with probability $\frac{\bar{\theta}}{\theta^*+\bar{\theta}}$.

The bond price satisfies the standard pricing condition:

$$0=\mathbb{E}_t\left[d(\eta_t\phi_{t,\tau})\right]\Rightarrow 0=\frac{\mathbb{E}_t[d\phi_{t,\tau}]}{\phi_{t,\tau}}+\frac{\mathbb{E}_t[d\eta_t]}{\eta_t}+\theta\mathbb{E}\left[\frac{\eta_t'-\eta_t}{\eta_t}\frac{\phi_{t,\tau}'-\phi_{t,\tau}}{\phi_{t,\tau}}\right], \tag{57}$$

where we used Ito's lemma for jump processes, and $\phi_{t,\tau}'$ denotes the bond price after the jump.

Rearranging the expression above, we obtain

$$r_t\phi_{t,\tau}=\frac{\partial \phi_{t,\tau}}{\partial t}-\frac{\partial \phi_{t,\tau}}{\partial \tau}+\theta^*\frac{\eta_t^*}{\eta_t}\left[\phi_{t,\tau}^*-\phi_{t,\tau}\right]+\bar{\theta}\frac{\overline{\eta}_t}{\eta_t}\left[\overline{\phi}_{t,\tau}-\phi_{t,\tau}\right]. \tag{58}$$

In a similar manner, let $\eta_t^{\$}$ denote the nominal SDF, which follows the process

$$\frac{d\eta_t^{\$}}{\eta_t^{\$}}=-i_t dt+\frac{\eta_t'-\eta_t}{\eta_t}\left[d\mathcal{N}_t-\theta dt\right], \tag{59}$$

where i_t denotes the nominal interest rate. As the price level does not jump with the Poisson arrival, the jump term of the nominal SDF coincides with the jump term of the real SDF.

Denote the time-t price of a nominal bond maturing τ periods ahead by $q_{t,\tau}$. The nominal bond price satisfies an analogous pricing condition

$$i_t q_{t,\tau} = \frac{\partial q_{t,\tau}}{\partial t} - \frac{\partial q_{t,\tau}}{\partial \tau} + \theta^* \frac{\eta_t^*}{\eta_t} \left[q_{t,\tau}^* - q_{t,\tau}\right] + \overline{\theta} \frac{\overline{\eta}_t}{\eta_t} \left[\overline{q}_{t,\tau} - q_{t,\tau}\right]. \tag{60}$$

Inflation expectations. Let $\pi_{t,\tau}$ denote the time-t expectation of inflation τ periods ahead. To derive a partial differential equation (PDE) for $\pi_{t,\tau}$, it is convenient to define the function $g_{t,T} = \mathbb{E}_t[\pi_T] = \pi_{t,T-t}$ for a fixed date $T > t$. From the law of iterated expectations, we obtain that $g_{t,T}$ is a martingale:

$$g_{t,T} = \mathbb{E}_t \left[\underbrace{\mathbb{E}_{t+\Delta}\left[\pi_T\right]}_{g_{t+\Delta,T}} \right]. \tag{61}$$

This implies that the drift of $g_{t,T}$, and then $\pi_{t,T-t}$, is equal to zero. Hence, $\pi_{t,T-t}$ satisfies the PDE:

$$0 = \frac{\partial \pi_{t,\tau}}{\partial t} - \frac{\partial \pi_{t,\tau}}{\partial \tau} + \theta^* \left[\pi_{t,\tau}^* - \pi_{t,\tau}\right] + \overline{\theta} \left[\overline{\pi}_{t,\tau} - \pi_{t,\tau}\right]. \tag{62}$$

B.2 Linearized conditions

The real bond price in steady state is given by $\overline{\phi}_{t,\tau} = e^{-\rho\tau}$. Let $\hat{\phi}_{t,\tau} = \phi_{t,\tau}/\overline{\phi}_{t,\tau}$. We can then write the PDE for $\hat{\phi}_{t,\tau}$ as follows:

$$(r_t - \rho)\hat{\phi}_{t,\tau} = \frac{\partial \hat{\phi}_{t,\tau}}{\partial t} - \frac{\partial \hat{\phi}_{t,\tau}}{\partial \tau} + \theta^* \frac{\eta_t^*}{\eta_t} \left[\hat{\phi}_{t,\tau}^* - \hat{\phi}_{t,\tau}\right] + \overline{\theta} \frac{\overline{\eta}_t}{\eta_t} \left[1 - \hat{\phi}_{t,\tau}\right]. \tag{63}$$

In steady state, we have $\hat{\phi}_{t,\tau} = \hat{\phi}_{t,\tau}^* = 1$ and $r_t = \rho$. Linearizing the expression above around the steady state, we obtain

$$(r_t + \overline{\theta} - \rho)\hat{\phi}_{t,\tau} = \frac{\partial \hat{\phi}_{t,\tau}}{\partial t} - \frac{\partial \hat{\phi}_{t,\tau}}{\partial \tau} + \theta^* \left[\hat{\phi}_{t,\tau}^* - \hat{\phi}_{t,\tau}\right], \tag{64}$$

using the fact that $\eta_t^* = \eta_t = \overline{\eta}_t$ in steady state.

C. Proofs

Proof of Proposition 1

Proof. We first show that fiscal policy is passive, that is, for any Lebesgue integrable path for (x_t, π_t, i_t), government debt is bounded if and only if $\gamma \geq 0$. Note that in the fiscal consolidation phase and the inflationary-finance phase, government debt is bounded by construction. In the fiscal-expansion phase, from equation (16) we get

$$\lim_{t\to\infty} b_t = \lim_{t\to\infty} e^{-\gamma t} b_0 + \lim_{t\to\infty} \int_0^t e^{-\gamma(t-s)} (i_s - \pi_s - \rho + \psi_s) ds.$$

Notice that since $\gamma \geq 0$, $e^{-\gamma t} \leq 1$ for all $t \geq 0$. Then

$$\lim_{t\to\infty} b_t = \lim_{t\to\infty} e^{-\gamma t} b_0 + \lim_{t\to\infty} \int_0^t e^{-\gamma(t-s)} (i_s - \pi_s - \rho + \psi_s) ds \leq \lim_{t\to\infty} b_0 + \lim_{t\to\infty} \int_0^t (i_s - \pi_s - \rho + \psi_s) ds < \infty,$$

where the last inequality follows from (i_t, π_t) being Lebesgue integrable.

For **I.**, notice that the dynamic system is given by

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{x}_t \\ \dot{b}_t \end{bmatrix} = \begin{bmatrix} (\rho + \theta_f) & -\kappa & -\theta_f^* \kappa \Phi \\ (\phi - 1) & \theta_h & -\theta_h^* \\ (\phi - 1) & 0 & -(\gamma - \rho) \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ b_t \end{bmatrix} + \begin{bmatrix} \theta_f^* \kappa \Phi b^n \\ u_t + \theta_h^* b^n \\ u_t + \psi_t \end{bmatrix}.$$

The equilibrium is uniquely determined if the matrix above has two eigenvalues with positive real components and an eigenvalue with a non-positive real component. The eigenvalues of the system above satisfies the characteristic equation:

$$f(\lambda) \equiv \lambda^3 + \underbrace{[\gamma - (\rho + \theta_f + \theta_h)]}_{\equiv a} \lambda^2 + \underbrace{\left[(\phi - 1)\kappa\left(1 + \theta_f^* \Phi\right) + \theta_h(\rho + \theta_f) - \gamma(\rho + \theta_f + \theta_h)\right]}_{\equiv b} \lambda +$$
$$\underbrace{\left[\gamma(\rho + \theta_f)\theta_h + (\phi - 1)\kappa\left[\gamma - \left(\theta_h^* + \theta_h \theta_f^* \Phi\right)\right]\right]}_{\equiv c} = 0.$$

Using Descartes' rule of signs, we get that $c > 0$ is a necessary condition for determinacy. To see this, suppose $c < 0$. Then, there are two options for the number of sign changes of $f(\lambda)$: one and three. This implies that there can be either 1 or 3 roots with a positive real part. Since we need two roots with positive real part for determinacy, we can rule out those cases.

Next, we show that $c > 0$ is a sufficient condition for determinacy. Because $\gamma < \rho + \theta_f + \theta_h$, $a < 0$. Then, we are guaranteed two sign changes. Using the Routh-Hurwitz criterion, not all roots of f are negative, completing the proof.

Part **II.** is immediately true by construction.

Proof of Lemma 2

Proof. From equation (18), given $x_t = 0$ and $\pi_t^J = \kappa\Phi(b_t - b^n)$, inflation is given by

$$\pi_t = \kappa\theta^*\Phi \int_t^\infty e^{-(\rho+\theta)(s-t)}(b_s - b^n)ds. \tag{65}$$

Debt is given by $b_s = b_0 + \frac{1-e^{-\theta_\psi s}}{\theta_\psi}\psi_0 = b_t + \frac{1-e^{-\theta_\psi(s-t)}}{\theta_\psi}\psi_t$. We can then write inflation as follows:

$$\pi_t = \frac{\kappa\theta^*\Phi}{\rho+\theta}\left[b_t - b^n + \frac{\psi_t}{\rho+\theta+\theta_\psi}\right]. \tag{66}$$

Differentiating the expression above with respect to time, we obtain

$$\dot{\pi}_t = \frac{\kappa\theta^*\Phi}{\rho+\theta}\left[\psi_t - \frac{\theta_\psi\psi_t}{\rho+\theta+\theta_\psi}\right] = \frac{\kappa\theta^*\Psi}{\rho+\theta+\theta_\psi}\psi_t. \tag{67}$$

□

Proof of Proposition 2.

Proof. From equation (18), inflation is given by

$$\pi_t = \kappa\int_t^\infty e^{-(\rho+\theta)(s-t)}x_s ds + \theta^*\kappa\Phi\int_t^\infty e^{-(\rho+\theta)(s-t)}(b_s - b^n)ds, \tag{68}$$

where $x_t = -\frac{1}{\theta_r}(r_t - \rho)$ and $b_t = b_t^{og} + \frac{1-e^{-\theta_r t}}{\theta_r}(r_0 - \rho)$.

We can then write inflation as follows:

$$\pi_t = \pi_t^{og} \underbrace{-\frac{\kappa(r_t-\rho)}{\theta_r(\rho+\theta+\theta_r)}}_{F_t} + \underbrace{\frac{\kappa\theta^*\Phi}{\theta_r}\left[\frac{1}{\rho+\theta} - \frac{e^{-\theta_r t}}{\rho+\theta+\theta_r}\right](r_0-\rho)}_{J_t}, \tag{69}$$

where $\pi_t^{og} = \kappa\theta^*\Phi\int_t^\infty e^{-(\rho+\theta)(s-t)}(b_s^{og} - b^n)ds$.

□

Proof of Proposition 3.

Proof. The fight-inflation strategy is successful at bringing inflation down at $t = 0$ if:

$$-F_0^\pi > J_0^\pi \iff \frac{\kappa(r_0 - \rho)}{\theta_r(\rho + \theta + \theta_r)} > \frac{\kappa\theta^*\Phi}{\theta_r}\left[\frac{1}{\rho + \theta} - \frac{1}{\rho + \theta + \theta_r}\right](r_0 - \rho).$$

We can write the inequality above as follows:

$$1 > \frac{\theta^*\Phi}{\rho + \theta}\theta_r \iff \theta_r < \frac{\rho + \theta}{\theta^*\Phi}. \tag{70}$$

Notice that $\lim_{t\to\infty} F_t^\pi = 0$ and $\lim_{t\to\infty} J_t^\pi = \frac{\kappa\theta^*\Phi}{\theta_r(\rho+\theta)}(r_0 - \rho) > 0$. Hence, there exits $\hat{T} > 0$ such that for $t > \hat{T}$ the following inequality holds:

$$-F_t^\pi < J_t^\pi. \tag{71}$$

Hence, $\pi_t > \pi_t^{og}$ for $t > \hat{T}$. □

Proof of Proposition 7.

Proof. □

□

D. Optimal policy

D.1 The planner's problem

Value of Phase II. The value of Phase II, conditional on starting with debt level b_0^*, is given by

$$\mathcal{P}^{II}(b_0^*) = \int_0^{T^*} e^{-\rho t}(\alpha x_t^{*2} + \beta \pi_t^{*2})dt. \tag{72}$$

Using the fact that $x_t^* = (b_0^* - b^n)\left(1 - \frac{t}{T^*}\right)$ and $\pi_t^* = \kappa\Phi(b_0^* - b^n)\left(1 - \frac{t}{T^*}\right)$, we obtain

$$\mathcal{P}^{II}(b_0^*) = \Upsilon(b_0^* - b^n)^2, \tag{73}$$

where $\Upsilon \equiv \left[\alpha + \beta(\kappa\Phi)^2\right] \int_0^{T^*} e^{-\rho t}\left(1 - \frac{t}{T^*}\right)^2 dt$.

Planner's objective. The planner's objective can be written as

$$\mathcal{P} = -\frac{1}{2}\mathbb{E}\left[\int_0^{\tau} e^{-\rho t}\left(\alpha x_t^2 + \beta \pi_t^2\right) dt + e^{-\rho\tau}\tilde{\mathcal{P}}_\tau\right],$$

where τ denotes the random time at which the economy switches to either Phase II or the steady state, and $\tilde{\mathcal{P}}_t$ denotes the value after the economy switches to either state. If the economy goes to steady state, then $\tilde{\mathcal{P}}_\tau = 0$, the value in steady state, and if the economy's go to Phase II, then $\tilde{\mathcal{P}}_\tau = \mathcal{P}^{II}(b_\tau)$. The density of τ is $\lambda e^{-\lambda\tau}$ and, conditional on τ, the economy switches to Phase II with probability $\frac{\lambda}{\theta}$ and to steady state with the remaining probability (see e.g. Cox and Miller (1977) for a derivation). We can then write the expression above as follows

$$\begin{aligned}\mathcal{P} &= -\frac{1}{2}\int_0^{\infty} \lambda e^{-\lambda\tau}\left[\int_0^{\tau} e^{-\rho t}\left(\alpha x_t^2 + \beta\pi_t^2\right) dt + e^{-\rho\tau}\frac{\lambda}{\theta}\mathcal{P}^{II}(b_\tau)\right] d\tau \\ &= -\frac{1}{2}\int_0^{\infty} e^{-(\rho+\lambda)t}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right] dt.\end{aligned}$$

Implementability conditions. The planner chooses the competitive equilibrium that maximizes the objective derived above. A competitive equilibrium corresponds to a bounded solution to the following system of equations:

$$\dot{\pi}_t = (\rho + \lambda_f)\pi_t - \kappa x_t - \lambda_f\kappa\Phi(b_t - b^n), \qquad \dot{x}_t = r_t - \rho + \lambda_h x_t - \lambda_h^*(b_t - b^n), \qquad \dot{b}_t = r_t - \rho + \psi_t, \tag{74}$$

given the initial condition b_0 and the path of real interest rate $[r_t]_0^\infty$. A bounded solution solution satisfies the transversality condition:

$$\lim_{t\to\infty} e^{-(\rho+\lambda_f)t}\pi_t = 0. \tag{75}$$

We can then solve the NKPC forward to obtain

$$\pi_0 = \kappa \int_0^\infty e^{-(\rho+\lambda_f)t}\left[x_t + \lambda_f \Phi(b_t - b^n)\right] dt. \tag{76}$$

We consider first the case without households' expectation effects: $\lambda_h = \lambda_h^* = 0$. We further assume that $\lambda = \lambda_f$ and $\lambda = \lambda_f$, so the planner's beliefs coincide with the firm's beliefs. In this case, the output gap is given by $x_t = x_0 + \hat{r}_t$ and debt is given by $b_t = b_0 + \hat{r}_t + \hat{\psi}_t$, where $\hat{r}_t \equiv \int_0^t (r_s - \rho)ds$ and $\hat{\psi}_t = \int_0^t \psi_s ds$. Plugging the value of x_t and b_t into the expression for π_0:

$$\pi_0 = \kappa \int_0^\infty e^{-(\rho+\lambda)t}\left[x_0 + (1+\lambda\Phi)\hat{r}_t + \lambda\Phi\hat{\psi}_t\right] dt. \tag{77}$$

Using the fact that $\int_0^\infty e^{-(\rho+\lambda)t}\hat{r}_t dt = \frac{1}{\rho+\lambda}\int_0^\infty e^{-(\rho+\lambda)t}(r_t - \rho)dt$, we obtain

$$\pi_0 = \frac{\kappa}{\rho+\lambda}\int_0^\infty e^{-(\rho+\lambda)t}\left[(\rho+\lambda)x_0 + (1+\lambda\Phi)(r_t - \rho) + \lambda\Phi\psi_t\right] dt. \tag{78}$$

We can then write the planner's problem as follows:

$$\max_{\{[\pi_t, b_t, x_t, r_t]_0^\infty\}} -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right] dt, \tag{79}$$

subject to

$$\dot{\pi}_t = (\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n) \tag{80}$$

$$\dot{b}_t = r_t - \rho + \psi_t \tag{81}$$

$$\dot{x}_t = r_t - \rho, \tag{82}$$

given b_0 and the initial value for inflation.

The lack of a classical solution. It turns out that a classical solution, where the states are continuous functions of time, does not exist. The issue of non-existence of a solution can be seen more clearly in the case $\beta = 0$, where inflation drops out of the problem. For simplicity, assume that

$b_0 = b_n = 0$. The optimality condition for r_t is given by

$$\alpha x_t + \lambda \Upsilon b_t = 0, \tag{83}$$

for all $t \geq 0$. The optimality condition for x_0 is given by

$$\mu_{x,0} = 0 \iff -\alpha \int_0^\infty e^{-(\rho+\lambda)t} x_t dt = 0. \tag{84}$$

Let (x_t^*, b_t^*) denote a candidate solution, where b_t^* is a differentiable function of time satisfying $b_0^* = 0$. Differentiating the optimality condition for r_t with respect to time, we obtain

$$r_t - \rho = -\frac{\lambda \Upsilon}{\alpha + \lambda \Upsilon} \psi_t \Rightarrow \hat{r}_t = -\frac{\lambda \Upsilon}{\alpha + \lambda \Upsilon} \hat{\psi}_t. \tag{85}$$

As $x_t = x_0 + \hat{r}_t$, the optimality condition for x_0 implies that the following condition must hold:

$$\frac{x_0}{\rho + \lambda} + \int_0^\infty e^{-(\rho+\lambda)t} \hat{r}_t dt = 0 \Rightarrow x_0 = \frac{\lambda \Upsilon}{\alpha + \lambda \Upsilon} \int_0^\infty e^{-(\rho+\lambda)t} \psi_t dt > 0. \tag{86}$$

However, from the optimality condition for the interest rate at $t = 0$, we obtain:[28]

$$\alpha x_0 + \lambda \Upsilon b_0 = 0 \Rightarrow x_0 = 0, \tag{87}$$

which contradicts the fact that $x_0 > 0$.

Incentive for expropriation. While a classical solution to this problem does not exist, a generalized solution with discontinuous states exists. In a classical solution, b_t is given by

$$b_t = \int_0^t (r_s - \rho + \psi_s) ds \tag{88}$$

The integral above is equal to zero at $t = 0$, so $b_0 = 0$. Following the approach in optimal impulsive control, consider the following generalization:[29]

$$b_t = \int_0^t (r_s - \rho + \psi_s) ds + \int_{[0,t]} \overline{r}_s d\mu, \tag{89}$$

[28] Notice that the optimality condition for the interest rate must hold at $t = 0$. From continuity of x_t and b_t, if $\alpha x_t + \lambda \Upsilon b_t > 0$ for $t = 0$, there exists $t_1 > 0$ such that this inequality holds for $t \in [0, t_1)$. By reducing interest rates in this interval, we can improve the planner's objective.

[29] See Arutyunov, Karamzin and Pereira (2019) for a discussion of optimal impulsive control theory.

where μ denotes a Borel measure on $\mathbb{R}_+$. For example, if μ is a Dirac measure with weight on zero, then b_t is given by

$$b_t = \int_0^t (r_s - \rho + \psi_s) ds + \overline{r}_0. \tag{90}$$

In this case, government debt can immediately jump at zero, provided $\overline{r}_0 \neq 0$.

Define $\hat{r}_t \equiv \int_0^\infty (r_s - \rho) ds + \int_{[0,t]} \overline{r}_s d\mu$, so $x_t = x_0 + \hat{r}_t$ and $b_t = \hat{r}_t + \hat{\psi}_t$. In a classical solution, $\hat{r}_t$ must be an absolutely continuous function satisfying $\hat{r}_0 = 0$, while it is a bounded variation function in the context of optimal impulsive control, where $\hat{r}_0$ can take any value. Without the constraint that $\hat{r}_0 = 0$, the planner's problem becomes particularly simple:

$$\max_{\{x_0, [\hat{r}_t]_0^\infty\}} -\frac{1}{2} \int_0^\infty e^{-(\rho+\lambda)t} \left[\alpha \left(x_0 + \hat{r}_t \right)^2 + \lambda \Upsilon \left(\hat{r}_t + \hat{\psi}_t \right)^2 \right] dt, \tag{91}$$

with optimality conditions

$$\alpha x_t + \lambda \Upsilon b_t = 0, \qquad -\alpha \int_0^\infty e^{-(\rho+\lambda)t} x_t dt = 0. \tag{92}$$

The solution in this case takes the form:

$$r_t - \rho = -\frac{\lambda \Upsilon}{\alpha + \lambda \Upsilon} \psi_t, \qquad x_0 = \frac{\lambda \Upsilon}{\alpha + \lambda \Upsilon} \int_0^\infty e^{-(\rho+\lambda)t} \psi_t dt, \qquad b_0 = -\frac{\alpha}{\lambda \Upsilon} x_0. \tag{93}$$

Hence, government debt jumps immediately down on impact, which requires $\overline{r}_0 = -\frac{\alpha}{\lambda \Upsilon} x_0$ and μ to be a Dirac measure with weight in zero. Intuitively, the planner has an incentive to expropriate part of the debt by having the real interest rate be very negative over a small period (the impulse from the Dirac measure).

D.2 Characterization of the optimal policy

The penalized planner's problem. To deal with the incentive to expropriate, we introduce a penalty associated with the initial value of each forward-looking variable:

$$\max_{\{[\pi_t, b_t, x_t, r_t]_0^\infty\}} -\frac{1}{2} \int_0^\infty e^{-(\rho+\lambda)t} \left[\alpha x_t^2 + \beta \pi_t^2 + \lambda \Upsilon (b_t - b^n)^2 \right] dt + \xi_x x_0 + \xi_\pi \pi_0, \tag{94}$$

subject to

$$\dot{\pi}_t = (\rho + \lambda) \pi_t - \kappa x_t - \lambda \kappa \Phi (b_t - b^n), \qquad \dot{b}_t = r_t - \rho + \psi_t, \qquad \dot{x}_t = r_t - \rho, \tag{95}$$

given b_0 and the initial value for inflation. We will choose the penalty ξ_x and ξ_π such that there is no discontinuity in b_t at $t = 0$, and the co-state for the output gap is equal to zero at $t = 0$.

Optimality conditions. The Hamiltonian to this problem is given by

$$\mathcal{H}_t = -\frac{1}{2}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right] + \mu_{\pi,t}\left[(\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n)\right] + \mu_{b,t}\left[r_t - \rho + \psi_t\right]$$
$$+ \mu_{x,t}[r_t - \rho] + (\mu_{x,0} + \xi_x)(\rho+\lambda)x_0 + (\mu_{\pi,0} + \xi_\pi)\left[\kappa x_0 + \frac{\kappa(1+\lambda\Phi)}{\rho+\lambda}(r_t - \rho)\right], \tag{96}$$

The dynamics of the co-states are given by

$$\dot{\mu}_{\pi,t} - (\rho+\lambda)\mu_{\pi,t} = \beta\pi_t - \mu_{\pi,t}(\rho+\lambda) \tag{97}$$
$$\dot{\mu}_{b,t} - (\rho+\lambda)\mu_{b,t} = \lambda\Upsilon(b_t - b^n) + \kappa\lambda\Phi\mu_{\pi,t} \tag{98}$$
$$\dot{\mu}_{x,t} - (\rho+\lambda)\mu_{x,t} = \alpha x_t + \kappa\mu_{\pi,t}. \tag{99}$$

The optimality condition for the real interest rate is

$$\mu_{b,t} + \mu_{x,t} = -\xi, \tag{100}$$

where $\xi \equiv \frac{\kappa(1+\lambda\Phi)}{\rho+\lambda}(\mu_{\pi,0} + \xi_\pi)$.

The optimality condition for the initial output gap:

$$(\rho+\lambda)(\mu_{x,0} + \xi_x) + \kappa(\mu_{\pi,0} + \xi_\pi) = 0. \tag{101}$$

We will choose $\xi_x = -\kappa\frac{\mu_{\pi,0}+\xi_\pi}{\rho+\lambda}$, such that $\mu_{x,0} = 0$. We show below that we can set $\mu_{\pi,0} = 0$ without loss of generality.

Real and nominal rates. The optimality condition for interest rates imply that $\dot{\mu}_{b,t} + \dot{\mu}_{x,t} = 0$. From the law of motion of the co-states, we obtain

$$\alpha x_t + \lambda\Upsilon(b_t - b^n) = \kappa(1+\lambda\Phi)\left(\mu_{\pi,0} - \mu_{\pi,t}\right) + (\rho+\lambda)\xi. \tag{102}$$

Differentiating the expression above with respect to time, we obtain

$$\alpha(r_t - \rho) + \lambda\Upsilon(r_t - \rho + \psi_t) = -\kappa(1+\lambda\Phi)\beta\pi_t. \tag{103}$$

Rearranging the expression above, we obtain the real interest rate

$$r_t - \rho = -\beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon + \alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t, \tag{104}$$

and the nominal interest rate is given by

$$i_t = \rho + \left[1 - \beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon + \alpha}\right]\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t. \tag{105}$$

Dynamics under the optimal policy. Using the expression for $x_t = x_0 + b_t - b_0 - \hat{\psi}_t$, we can write a dynamic system for π_t and b_t

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{b}_t \end{bmatrix} = \begin{bmatrix} \rho + \lambda & -\kappa(1+\lambda\Phi) \\ -\hat{\beta} & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ b_t - b^n \end{bmatrix} + \begin{bmatrix} \kappa(\hat{\psi}_t + b_0 - b^n - x_0) \\ \frac{\alpha}{\lambda\Upsilon + \alpha}\psi_t \end{bmatrix}, \tag{106}$$

where $\hat{\beta} \equiv \frac{\beta\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}$ and $\hat{\psi}_t = \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\psi_0$. As b_0 is given and π_0 can jump, there is a unique bounded solution to the system above if the system has a positive eigenvalue and a negative eigenvalue. The eigenvalues of the system satisfy the condition

$$(\rho + \lambda - \omega)(-\omega) - \hat{\beta}\kappa(1+\lambda\Phi) = 0 \Rightarrow \omega^2 - [\rho + \lambda]\,\omega - \kappa(1+\lambda\Phi)\hat{\beta} = 0.$$

Denote the eigenvalues of the system by $\overline{\omega} > 0$ and $\underline{\omega} < 0$, where

$$\overline{\omega} = \frac{\rho + \lambda + \sqrt{(\rho+\lambda)^2 + 4\kappa(1+\lambda\Phi)\hat{\beta}}}{2}, \qquad \underline{\omega} = \frac{\rho + \lambda - \sqrt{(\rho+\lambda)^2 + 4\kappa(1+\lambda\Phi)\hat{\beta}}}{2}. \tag{107}$$

The matrix of eigenvectors and its inverse are given by

$$V = \begin{bmatrix} \frac{\kappa(1+\lambda\Phi)}{\underline{\omega}} & \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}} \\ 1 & 1 \end{bmatrix}, \qquad V^{-1} = \frac{\overline{\omega}|\underline{\omega}|}{(\overline{\omega} - \underline{\omega})\kappa(1+\lambda\Phi)} \begin{bmatrix} -1 & \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}} \\ 1 & \frac{\kappa(1+\lambda\Phi)}{|\underline{\omega}|} \end{bmatrix}. \tag{108}$$

Let $Z_t = [\pi_t, b_t]'$ denote the vector of endogenous variables, A the matrix of coefficients, and U_t the vector of coefficients. We can then write the dynamic system as $\dot{Z}_t = AZ_t + U_t$. We can write the matrix of coefficients as $A = V\Lambda V^{-1}$, where Λ is a diagonal matrix with the eigenvalues. Using the matrix eigendecomposition, we can decouple the system using the transformation: $z_t \equiv V^{-1}Z_t$ and $u_t \equiv V^{-1}U_t$. This gives us the system of decoupled differential equations:

$$\dot{z}_{1,t} = \overline{\omega} z_{1,t} + u_{1,t}, \qquad \dot{z}_{2,t} = \underline{\omega} z_{2,t} + u_{2,t}. \tag{109}$$

Integrating the first equation forward and the second backwards, we obtain

$$z_{1,t} = -\int_t^\infty e^{-\overline{\omega}(s-t)} u_{1,s} ds, \qquad z_{2,t} = e^{\underline{\omega} t} z_{2,0} + \int_0^t e^{\underline{\omega}(t-s)} u_{2,s} ds. \tag{110}$$

Rotating the system back to its original coordinates, we obtain

$$\pi_t = \frac{\kappa(1+\lambda\Phi)}{|\underline{\omega}|} \int_t^\infty e^{-\overline{\omega}(s-t)} u_{1,s} ds + \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}} \left[e^{\underline{\omega} t} z_{2,0} + \int_0^t e^{\underline{\omega}(t-s)} u_{2,s} ds \right], \tag{111}$$

and

$$b_t - b^n = -\int_t^\infty e^{-\overline{\omega}(s-t)} u_{1,s} ds + e^{\underline{\omega} t} z_{2,0} + \int_0^t e^{\underline{\omega}(t-s)} u_{2,s} ds. \tag{112}$$

The disturbances $u_{1,t}$ and $u_{2,t}$ are given by

$$u_{1,t} = \frac{|\underline{\omega}|}{\overline{\omega} - \underline{\omega}} \left[\frac{\alpha}{\lambda\Upsilon + \alpha} \psi_t - \frac{\overline{\omega}}{(1+\lambda\Phi)} (\hat{\psi}_t + b_0 - b^n - x_0) \right] \tag{113}$$

$$u_{2,t} = \frac{\overline{\omega}}{\overline{\omega} - \underline{\omega}} \left[\frac{\alpha}{\lambda\Upsilon + \alpha} \psi_t + \frac{|\underline{\omega}|}{(1+\lambda\Phi)} (\hat{\psi}_t + b_0 - b^n - x_0) \right], \tag{114}$$

where $\hat{\psi}_t = \frac{1-e^{-\theta_\psi t}}{\theta_\psi} \psi_0$ if $\theta_\psi > 0$ and $\hat{\psi}_t = \psi_0 t$ if $\theta_\psi = 0$.

The forward integral of $u_{1,t}$ is given by

$$\int_t^\infty e^{-\overline{\omega}(s-t)} u_{1,s} ds = \frac{|\underline{\omega}|}{\overline{\omega} - \underline{\omega}} \left[\left(\frac{\alpha}{\lambda\Upsilon + \alpha} - \frac{\overline{\omega}}{(1+\lambda\Phi)} \frac{1}{\theta_\psi} \right) \frac{\psi_t}{\overline{\omega} + \theta_\psi} - \frac{\frac{\psi_0}{\theta_\psi} + b_0 - b^n - x_0}{1+\lambda\Phi} \right]. \tag{115}$$

The backward integral of $u_{2,t}$ is given by

$$\int_0^t e^{\underline{\omega}(t-s)} u_{2,s} ds = \frac{\overline{\omega}}{\overline{\omega} - \underline{\omega}} \left[\left(\frac{\alpha}{\lambda\Upsilon + \alpha} - \frac{|\underline{\omega}|}{(1+\lambda\Phi)} \frac{1}{\theta_\psi} \right) \frac{e^{\underline{\omega} t} - e^{-\theta_\psi t}}{\theta_\psi + \underline{\omega}} \psi_0 + \frac{|\underline{\omega}|}{(1+\lambda\Phi)} \left(\frac{\psi_0}{\theta_\psi} + b_0 - b^n - x_0\right) \frac{1 - e^{\underline{\omega} t}}{|\underline{\omega}|} \right] \tag{116}$$

From the expression for $z_{1,0}$, we obtain

$$\begin{aligned}\pi_0 &= \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}}\left[(b_0-b^n)+\frac{\overline{\omega}-\underline{\omega}}{|\underline{\omega}|}\int_0^\infty e^{-\overline{\omega}t}u_{1,t}dt\right]\\ &= \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}}\left[(b_0-b^n)+\left(\frac{\alpha}{\lambda\Upsilon+\alpha}+\frac{\overline{\omega}}{(1+\lambda\Phi)}\frac{1}{\theta_\psi}\right)\frac{\psi_0}{\overline{\omega}+\theta_\psi}-\frac{\frac{\psi_0}{\theta_\psi}+b_0-b^n-x_0}{1+\lambda\Phi}\right]\end{aligned} \tag{117}$$

We can then write initial inflation as follows:

$$\pi_0 = \frac{\kappa}{\overline{\omega}}\left[\lambda\Phi(b_0-b^n)+x_0+\frac{\alpha\Phi-\Upsilon}{\alpha+\lambda\Upsilon}\frac{\lambda\psi_0}{\overline{\omega}+\theta_\psi}\right].$$

The initial value for $z_{2,t}$ is given by

$$z_{2,0} = \frac{\overline{\omega}}{\overline{\omega}-\underline{\omega}}\left[\frac{|\underline{\omega}|}{\kappa(1+\lambda\Phi)}\pi_0+b_0-b^n\right].$$

Inflation is then given by

$$\pi_t = \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}-\underline{\omega}}\left[\left(\frac{\alpha}{\lambda\Upsilon+\alpha}+\frac{\overline{\omega}}{(1+\lambda\Phi)}\frac{1}{\theta_\psi}\right)\frac{\psi_t}{\overline{\omega}+\theta_\psi}-\frac{\frac{\psi_0}{\theta_\psi}+b_0-b^n-x_0}{1+\lambda\Phi}\right] \tag{118}$$

$$+\frac{\kappa(1+\lambda\Phi)}{\overline{\omega}-\underline{\omega}}\left[e^{\underline{\omega}t}\left[\frac{|\underline{\omega}|}{\kappa(1+\lambda\Phi)}\pi_0+b_0-b^n\right]+\left(\frac{\alpha}{\lambda\Upsilon+\alpha}-\frac{|\underline{\omega}|}{(1+\lambda\Phi)}\frac{1}{\theta_\psi}\right)\frac{e^{\underline{\omega}t}-e^{-\theta_\psi t}}{\theta_\psi+\underline{\omega}}\psi_0\right] \tag{119}$$

$$+\frac{\kappa(1+\lambda\Phi)}{\overline{\omega}-\underline{\omega}}\left[\frac{1-e^{\underline{\omega}t}}{(1+\lambda\Phi)}\left(\frac{\psi_0}{\theta_\psi}+b_0-b^n-x_0\right)\right]. \tag{120}$$

After some rearrangement, we obtain

$$\pi_t = \frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{e^{\underline{\omega}t}-e^{-\theta_\psi t}}{(\theta_\psi+\overline{\omega})(\theta_\psi+\underline{\omega})}\psi_0+e^{\underline{\omega}t}\pi_0. \tag{121}$$

Boundary conditions. The optimality condition for x_0 involves the co-states for x and π. Solving the equation for $\mu_{\pi,t}$ backward, we obtain

$$\mu_{\pi,t} = \mu_{\pi,0}+\beta\int_0^t \pi_s ds. \tag{122}$$

Solving the equation for $\mu_{x,t}$ forward, we obtain

$$\mu_{x,0} = -\int_0^\infty e^{-(\rho+\theta)t} \left[\kappa \mu_{\pi,t} + \alpha x_t\right] dt \tag{123}$$

$$= -\frac{\kappa}{\rho+\theta}\mu_{\pi,0} - \frac{\kappa\beta}{\rho+\theta}\int_0^\infty e^{-(\rho+\theta)t}\pi_t dt - \int_0^\infty e^{-(\rho+\theta)t}\alpha x_t dt. \tag{124}$$

The optimality condition for x_0 is given by

$$0 = \mu_{x,0} + \frac{\kappa}{\rho+\theta}\mu_{\pi,0} = -\int_0^\infty e^{-(\rho+\theta)t}\left[\frac{\beta}{\rho+\theta}\pi_t + \alpha x_t\right]. \tag{125}$$

Using the fact that $x_t = x_0 + \hat{r}_t$, we obtain

$$\int_0^\infty e^{-(\rho+\theta)t} x_t dt = \frac{x_0}{\rho+\theta} + \frac{1}{\rho+\theta}\int_0^\infty e^{-(\rho+\theta)t}(r_t - \rho)dt. \tag{126}$$

The optimality condition for x_0 can then be written as

$$0 = \frac{\alpha}{\rho+\theta}x_0 + \frac{1}{\rho+\theta}\int_0^\infty e^{-(\rho+\theta)t}\left[\kappa\beta\pi_t + \alpha\left(-\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t\right)\right]dt. \tag{127}$$

Rearraning the expression above, we obtain

$$\alpha x_0 = \beta\frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon+\alpha}\int_0^\infty e^{-(\rho+\theta)t}\pi_t dt + \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon+\alpha}\int_0^\infty e^{-(\rho+\theta)t}\psi_t dt. \tag{128}$$

The present discounted value of inflation is given by

$$\int_0^\infty e^{-(\rho+\theta)t}\pi_t dt = \frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{(\lambda\Upsilon+\alpha)(\theta_\psi+\overline{\omega})}\frac{\psi_0}{(\rho+\theta+|\lambda|)(\rho+\theta+\theta_\psi)} + \frac{\pi_0}{\rho+\theta+|\underline{\omega}|}. \tag{129}$$

Combining the previous two equations, we obtain

$$\alpha x_0 = \frac{\beta}{\theta_\psi+\overline{\omega}}\left(\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\right)^2\frac{\psi_0}{(\rho+\theta+|\lambda|)(\rho+\theta+\theta_\psi)} + \beta\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{\pi_0}{\rho+\theta+|\underline{\omega}|} \tag{130}$$

$$+ \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0}{\rho+\theta+\theta_\psi}. \tag{131}$$

Using the fact that $\pi_0 = \frac{\kappa}{\overline{\omega}}\left[\lambda\Phi(b_0 - b^n) + x_0 + \frac{\alpha\Phi - \Upsilon}{\alpha + \lambda\Upsilon}\frac{\lambda\psi_0}{\overline{\omega} + \theta_\psi}\right]$, we obtain

$$x_0 = \frac{\frac{\beta}{\theta_\psi + \overline{\omega}}\left(\frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha}\right)^2 \frac{\psi_0}{(\rho + \theta + |\underline{\omega}|)}\left[\frac{1}{\rho + \theta + \theta_\psi} + \frac{1}{\overline{\omega}}\right] + \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon + \alpha}\frac{\psi_0}{\rho + \theta + \theta_\psi} + \beta\frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha}\frac{\kappa\lambda\Phi(b_0 - b^n)}{\overline{\omega}(\rho + \theta + |\underline{\omega}|)}}{\alpha - \frac{\kappa\beta}{\overline{\omega}(\rho + \theta + |\underline{\omega}|)}\frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha}}. \tag{132}$$

Initial inflation is then given by

$$\pi_0 = \frac{\kappa}{\overline{\omega}}\left[\frac{\frac{\beta}{\theta_\psi + \overline{\omega}}\left(\frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha}\right)^2 \frac{\psi_0}{(\rho + \theta + |\underline{\omega}|)}\frac{1}{\rho + \theta + \theta_\psi} + \frac{\alpha^2\Phi}{\alpha + \lambda\Upsilon}\frac{\lambda\psi_0}{\overline{\omega} + \theta_\psi} + \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon + \alpha}\frac{|\underline{\omega}|\psi_0}{(\rho + \theta + \theta_\psi)(\overline{\omega} + \theta_\psi)} + \alpha\lambda\Phi(b_0 - b^n)}{\alpha - \frac{\kappa\beta}{\overline{\omega}(\rho + \theta + |\underline{\omega}|)}\frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha}}\right]. \tag{133}$$

Notice that the numerator is positive. The denominator is positive for α large or β large. In these cases, a fiscal shock leads to more inflation and higher output gap.

Real interest rate. The real interest rate is given by

$$r_t - \rho = -\beta\frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha}\left[\frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha}\frac{e^{\underline{\omega}t} - e^{-\theta_\psi t}}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})}\psi_0 + e^{\underline{\omega}t}\pi_0\right] - \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t. \tag{134}$$

We can write the interest rate as follows:

$$r_t - \rho = -\left[\frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha} + \beta\frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha}\frac{\kappa\lambda(\Upsilon - \alpha\Phi)}{\lambda\Upsilon + \alpha}\frac{1}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})}\right]\psi_t \tag{135}$$

$$- \beta\frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha}\left[\frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha}\frac{1}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})}\psi_0 + \pi_0\right]e^{\underline{\omega}t}\pi_0. \tag{136}$$

Output gap. Output gap is given by

$$x_t = x_0 + \hat{r}_t. \tag{137}$$

Plugging the expression for the interest rate, we obtain

$$x_t = x_0 - \left[\frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha} + \beta\frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha}\frac{\kappa\lambda(\Upsilon - \alpha\Phi)}{\lambda\Upsilon + \alpha}\frac{1}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})}\right]\hat{\psi}_t \tag{138}$$

$$- \beta\frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha}\left[\frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha}\frac{1}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})}\psi_0 + \pi_0\right]\frac{1 - e^{\underline{\omega}t}}{|\underline{\omega}|}\pi_0. \tag{139}$$

It can be shown that for $\beta = 0$, $x_0 > 0$ and $\lim_{t\to\infty} x_t < 0$.

Government debt. Government debt is given by

$$b_t - b_n = \frac{\overline{\omega}}{\overline{\omega} - \underline{\omega}}\pi_t - \left[\left(\frac{\alpha}{\lambda\Upsilon + \alpha} - \frac{\overline{\omega}}{(1+\lambda\Phi)} \frac{1}{\theta_\psi} \right) \frac{\psi_t}{\overline{\omega} + \theta_\psi} - \frac{\frac{\psi_0}{\theta_\psi} + b_0 - b^n - x_0}{1 + \lambda\Phi} \right]. \tag{140}$$

No weight on output gap. Suppose $\alpha = 0$. In this case, initial inflation is given by

$$\pi_0 = \frac{\kappa}{\theta_\psi + \overline{\omega}} \frac{\psi_0}{\rho + \theta + \theta_\psi} > 0. \tag{141}$$

Inflation at t is given by

$$\pi_t = -\kappa \frac{e^{\underline{\omega}t} - e^{-\theta_\psi t}}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})} \psi_0 + e^{\underline{\omega}t}\pi_0. \tag{142}$$

The derivative of inflation with respect to time is given by

$$\dot{\pi}_t = -\kappa \frac{\underline{\omega}e^{\underline{\omega}t} + \theta_\psi e^{-\theta_\psi t}}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})} \psi_0 + e^{\underline{\omega}t} \frac{\kappa}{\theta_\psi + \overline{\omega}} \frac{\underline{\omega}\psi_0}{\rho + \theta + \theta_\psi} \tag{143}$$

$$= -\frac{\kappa}{\theta_\psi + \overline{\omega}} \left[\frac{\theta_\psi}{\theta_\psi + \underline{\omega}} \psi_t - \frac{|\underline{\omega}|}{\theta_\psi + \underline{\omega}} \frac{\overline{\omega}e^{\underline{\omega}t}\psi_0}{\rho + \theta + \theta_\psi} \right]. \tag{144}$$

If $\theta_\psi > |\underline{\omega}|$, then inflation

Notice that inflation is decreasing at $t = 0$. If $\theta_\psi < |\underline{\omega}|$, so the fiscal shock is very persistent, then inflation is eventually increasing. If $\theta_\psi > |\underline{\omega}$, then inflation is decreasing even for large t.

No weight on inflation. Suppose $\beta = 0$. In this case, inflation is given by

$$\pi_0 = \frac{\kappa}{\overline{\omega}} \left[\frac{\alpha\Phi}{\alpha + \lambda\Upsilon} \frac{\lambda\psi_0}{\overline{\omega} + \theta_\psi} + \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha} \frac{|\underline{\omega}|\psi_0}{(\rho + \theta + \theta_\psi)(\overline{\omega} + \theta_\psi)} + \lambda\Phi(b_0 - b^n) \right], \tag{145}$$

which is positive when $b_0 - b^n = 0$. Inflation is given by

$$\pi_t = \frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha} \frac{e^{\underline{\omega}t} - e^{-\theta_\psi t}}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})} \psi_0 + e^{\underline{\omega}t}\pi_0. \tag{146}$$

D.3 A more general fiscal rule

Suppose transfers are given by $T_t = -\rho B_t - \gamma(B_t - \overline{B}) + \Psi_t$, such that the linearized government's flow budget constraint is given by

$$\dot{b}_t = r_t - \rho - \gamma b_t + \psi_t. \tag{147}$$

We have focused so far on the case $\gamma = 0$. If $\gamma > 0$, then the taxes react more strongly to debt than in our baseline case, while $\gamma < 0$ captures a weaker response of taxes to debt.

Suppose now that the Euler equation is given

$$\dot{x}_t = r_t - \rho + \theta_h x_t - \theta_h^* b_t, \tag{148}$$

assuming $b^n = 0$.

This generalizes the behavior of households and the fiscal authority. Consider next the following special case: $\lambda_f = 0$, so movements in government debt do not affect the firm's inflation expectation, and $\lambda = 0$, so the planner does not care directly about the level of government debt. In this case, we can solve for the real rate using the Euler equation to obtain

$$\dot{b}_t = \dot{x}_t - \theta_h x_t - (\gamma - \theta_h^*)\, b_t + \psi_t. \tag{149}$$

Notice that in this case the planner can implement $x_t = \pi_t = 0$. This requires that the real rate satisfies $r_t = \rho + \theta_h^* b_t$ and the government debt evolves according to

$$\dot{b}_t = -(\gamma - \theta_h^*) b_t + \psi_t. \tag{150}$$

Solving the differential equation above, we obtain

$$b_t = e^{-(\gamma - \theta_h^*)t} b_0 + \int_0^t e^{-(\gamma - \theta_h^*)(t-s)} \psi_s ds. \tag{151}$$

Using the fact that ψ_t is exponentially decaying, we obtain

$$b_t = e^{-(\gamma - \theta_h^*)t} b_0 + \frac{e^{-\theta_\psi t} - e^{-(\gamma - \theta_h^*)t}}{\gamma - \theta_h^* - \theta_\psi} \psi_0 \Rightarrow b_t = e^{-(\gamma - \theta_h^*)t} \left[b_0 - \frac{\psi_0}{\gamma - \theta_h^* - \theta_\psi} \right] + \frac{\psi_t}{\gamma - \theta_h^* - \theta_\psi}. \tag{152}$$

If $\gamma \geq \theta_h^*$, then debt and the real rate are bounded for any initial value of b_0. In contrast, if $\theta_h^* > \gamma$ then debt and the real rate are unbounded for $b_0 \neq \frac{\psi_0}{\gamma - \theta_h^* - \theta_\psi}$.

D.4 The general case

We consider next a generalized version of the optimal policy problem. We allow for subjective expectations to potentially differ among households, firms, and the planner. Moreover, we allow for a more general fiscal rule where taxes may react to the level debt. Finally, we allow for an arbitrary initial date for the planner's problem $t_0 \leq 0$. This enables us to nest the case of commitment, $t_0 = 0$, and the timeless perspective solution, $t_0 \to -\infty$. The planner's problem is then given by:

$$\max_{\{[\pi_t, b_t, x_t, r_t]_{t_0}^{\infty}\}} -\frac{1}{2}\int_{t_0}^{\infty} e^{-(\rho+\theta)(t-t_0)} \left[\alpha x_t^2 + \beta \pi_t^2 + \lambda \Upsilon (b_t - b^n)^2\right] dt, \tag{153}$$

subject to

$$\dot{\pi}_t = (\rho + \lambda_f)\pi_t - \kappa x_t - \lambda_f \kappa \Phi(b_t - b^n) \tag{154}$$

$$\dot{b}_t = r_t - \rho - \gamma(b_t - b^n) + \psi_t \tag{155}$$

$$\dot{x}_t = r_t - \rho + \theta_h x_t - \theta_h^*(b_t - b^n), \tag{156}$$

given b_0 and the process for the fiscal shock $\psi_t = e^{-\theta_\psi t}\psi_0$ for $t \geq 0$ and $\psi_t = 0$ for $t < 0$.

Optimality conditions. The Hamiltonian to this problem is given by

$$\mathcal{H} = -\frac{1}{2}\left[\alpha x_t^2 + \beta \pi_t^2 + \lambda \Upsilon (b_t - b^n)^2\right] + \mu_{\pi,t}\left[(\rho + \lambda_f)\pi_t - \kappa x_t - \lambda_f \kappa \Phi(b_t - b^n)\right] \tag{157}$$

$$+ \mu_{b,t}\left[r_t - \rho - \gamma b_t + \psi_t\right] + \mu_{x,t}[r_t - \rho + \theta_h x_t - \theta_h^*(b_t - b^n)], \tag{158}$$

The dynamics of the co-state on the output gap is given by

$$\dot{\mu}_{x,t} - (\rho + \theta)\mu_{x,t} = \alpha x_t + \mu_{\pi,t}\kappa - \mu_{x,t}\theta_h, \tag{159}$$

given the initial condition $\mu_{x,t_0} = 0$, as x_{t_0} is free to jump.

The dynamics of the co-state on inflation is given by

$$\dot{\mu}_{\pi,t} - (\rho + \theta)\mu_{\pi,t} = \beta \pi_t - \mu_{\pi,t}(\rho + \lambda_f), \tag{160}$$

given the initial condition $\mu_{\pi,t_0} = 0$, as π_{t_0} is free to jump.

The dynamics of the co-state on government debt is given by

$$\dot{\mu}_{b,t} - (\rho + \theta)\mu_{b,t} = \lambda\Upsilon(b_t - b^n) + \mu_{\pi,t}\lambda_f\kappa\Phi + \mu_{x,t}\theta_h^* + \mu_{b,t}\gamma. \tag{161}$$

The optimality condition for the real interest rate is

$$\mu_{b,t} + \mu_{x,t} = 0. \tag{162}$$

Dynamical system. Combining the optimality condition for debt and output gap, we obtain

$$\lambda\Upsilon(b_t - b^n) + \alpha x_t = -\mu_{\pi,t}\kappa(1 + \lambda_f\Phi) + \mu_{x,t}(\gamma + \overline{\theta}_h). \tag{163}$$

We can solve the equation above for x_t in terms of states and co-states:

$$x_t = -\frac{\kappa}{\alpha}(1 + \lambda_f\Phi)\mu_{\pi,t} + \frac{(\gamma + \overline{\theta}_h)}{\alpha}\mu_{x,t} - \frac{\lambda\Upsilon}{\alpha}(b_t - b^n). \tag{164}$$

Differentiating the expression above, we obtain

$$\begin{aligned} \dot{b}_t + (\gamma - \theta_h^*)(b_t - b^n) - \psi_t + \theta_h x_t = &-\frac{\kappa}{\alpha}(1 + \lambda_f\Phi)\left[\beta\pi_t + (\theta - \lambda_f)\mu_{\pi,t}\right] \\ &+ \frac{(\gamma + \overline{\theta}_h)}{\alpha}\left[(\rho + \theta - \theta_h)\mu_{x,t} + \alpha x_t + \kappa\mu_{\pi,t}\right] - \frac{\lambda\Upsilon}{\alpha}\dot{b}_t \end{aligned} \tag{165}$$

Rearranging the expression above, we obtain

$$\dot{b}_t = A_{b,\pi}\pi_t + A_{b,\mu_\pi}\mu_{\pi,t} + A_{b,\mu_x}\mu_{x,t} + A_{b,b}b_t + u_b\psi_t, \tag{166}$$

where

$$A_{b,\pi} \equiv -\frac{\kappa(1 + \lambda_f\Phi)}{\alpha + \lambda\Upsilon}\beta \tag{167}$$

$$A_{b,\mu_\pi} \equiv -\frac{\kappa(1 + \lambda_f\Phi)}{\alpha + \lambda\Upsilon}(\theta - \lambda_f) - (\gamma - \theta_h^*)\frac{\kappa\lambda_f\Phi}{\alpha + \lambda\Upsilon} + \frac{\kappa\theta_h}{\alpha + \lambda\Upsilon} \tag{168}$$

$$A_{b,\mu_x} \equiv \frac{\gamma + \overline{\theta}_h}{\alpha + \lambda\Upsilon}(\rho + \theta - \theta_h) + (\gamma - \theta_h^*)\frac{\gamma + \overline{\theta}_h}{\alpha + \lambda\Phi} \tag{169}$$

$$A_{b,b} \equiv -(\gamma - \theta_h^*) \tag{170}$$

$$u_b \equiv \frac{\alpha}{\alpha + \lambda\Upsilon}. \tag{171}$$

The law of motion of $\mu_{\pi,t}$ can be written as follows:

$$\dot{\mu}_{\pi,t} = A_{\mu_\pi,\pi}\pi_t + A_{\mu_\pi,\mu_\pi}\mu_{\pi,t} + A_{\mu_\pi,\mu_x}\mu_{x,t} + A_{\mu_\pi,b}b_t + u_{\mu_\pi}\psi_t, \tag{172}$$

where

$$A_{\mu_\pi,\pi} \equiv \beta, \qquad A_{\mu_\pi,\mu_\pi} \equiv \theta - \lambda_f \qquad A_{\mu_\pi,\mu_x} = A_{\mu_\pi,b} = u_{\mu_\pi} = 0. \tag{173}$$

The law of motion of $\mu_{x,t}$ is given by

$$\dot{\mu}_{x,t} = A_{\mu_x,\pi}\pi_t + A_{\mu_x,\mu_\pi}\mu_{\pi,t} + A_{\mu_x,\mu_x}\mu_{x,t} + A_{\mu_x,b}b_t + u_{\mu_x}\psi_t, \tag{174}$$

where

$$A_{\mu_x,\pi} \equiv 0 \tag{175}$$

$$A_{\mu_x,\mu_\pi} \equiv -\kappa\lambda_f\Phi \tag{176}$$

$$A_{\mu_x,\mu_x} \equiv \rho + \theta + \gamma - \theta_h^* \tag{177}$$

$$A_{\mu_x,b} \equiv -\lambda\Upsilon \tag{178}$$

$$u_{\mu_x} \equiv 0. \tag{179}$$

The law of motion of inflation is given by

$$\dot{\pi}_t = A_{\pi,\pi}\pi_t + A_{\pi,\mu_\pi}\mu_{\pi,t} + A_{\pi,\mu_x}\mu_{x,t} + A_{\pi,b}b_t + u_\pi\psi_t, \tag{180}$$

where

$$A_{\pi,\pi} \equiv \rho + \lambda_f, \tag{181}$$

$$A_{\pi,\mu_\pi} \equiv +\frac{\kappa^2}{\alpha}(1 + \lambda_f\Phi) \tag{182}$$

$$A_{\pi,\mu_x} \equiv -\frac{\kappa}{\alpha}(\gamma + \overline{\theta}_h) \tag{183}$$

$$A_{\pi,b} \equiv -\lambda_f\kappa\Phi + \frac{\kappa\lambda\Upsilon}{\alpha} \tag{184}$$

$$u_\pi \equiv 0. \tag{185}$$

Let $z_t = [\pi_t, \mu_{\pi,t}, \mu_{x,t}, b_t]$, so the dynamics of z_t is given by

$$\dot{z}_t = Az_t + u\psi_t. \tag{186}$$

Model solution and boundary conditions. Suppose that the matrix A is diagonalizable, so we can write $A = V\Lambda V^{-1}$, where Λ is a diagonal matrix. Define $Z_t \equiv V^{-1}z_t$ and $U \equiv V^{-1}u$, so we obtain the decoupled system

$$\dot{Z}_t = \Lambda Z_t + U\psi_t. \tag{187}$$

Solving the system backwards, for $t \geq 0$, we obtain

$$Z_{k,t} = e^{\lambda_k t}\left[e^{-\lambda_k t_0} Z_{k,t_0} + \frac{U_k}{\theta_\psi + \lambda_k}\psi_0\right] - \frac{U_k}{\theta_\psi + \lambda_k}\psi_t, \tag{188}$$

and the value at t_0 is given by $Z_{k,t_0} = e^{\lambda_k t_0} Z_{k,0}$.

In matrix form, we can write the dynamics for $t \geq 0$ as follows:

$$Z_t = \exp(\Lambda t)c - \tilde{U}\psi_t, \tag{189}$$

where c is a vector of constants and $\tilde{U}_k \equiv \frac{U_k}{\theta_\psi+\lambda_k}$. If $\lambda_k > 0$, then we must have $c_k = 0$ to obtain a bounded solution. Let $V = [\underline{V}\ \overline{V}]$, where $\underline{V}$ denote the matrix of eigenvalues associated with the non-positive eigenvalues and $\overline{V}$ the corresponding matrix associated with the positive ones. Similarly, let $\underline{\omega}$ denote the diagonal matrix with the non-negative eigenvalues and $\overline{\omega}$ the diagonal matrix with the positive eigenvalues. We can then write the solution as follows:

$$z_t = \underline{V}\exp\left(\underline{\Lambda}t\right)\underline{c} - V\tilde{U}\psi_t. \tag{190}$$

We can decompose the system into jump variables, z_t^J, co-states, z_t^C, and states, z_t^J, such that $z_t = [(z_t^J)', (z_t^C)', (z_t^J)']'$. The system satisfies the following initial conditions

$$z_0^S = \underline{V}^S\underline{c} - V^S\tilde{U}\psi_0, \qquad z_{t_0}^C = \underline{V}^C\exp\left(\underline{\Lambda}t_0\right)\underline{c} - V^C\exp\left(\Lambda t_0\right)\tilde{U}\psi_0, \tag{191}$$

where $z_{t_0}^C$ is a vector of zeros, and V^S denotes the rows of V associated with the state variables, and similar notation applies to the other matrices.

Commitment solution. To obtain the commitment solution, we set t_0 to obtain the system of equations

$$\begin{bmatrix} z_0^C \\ z_0^S \end{bmatrix} = \begin{bmatrix} \underline{V}^C \\ \underline{V}^S \end{bmatrix} \underline{c} - \begin{bmatrix} V^C \\ V^S \end{bmatrix} \tilde{U}\psi_0. \tag{192}$$

Let's assume that the matrix $\begin{bmatrix} \underline{V}^C \\ \underline{V}^S \end{bmatrix}$ is invertible, so the vector of coefficients $\underline{c}$ is given by

$$\underline{c} = \begin{bmatrix} \underline{V}^C \\ \underline{V}^S \end{bmatrix}^{-1} \left(\begin{bmatrix} z_0^C \\ z_0^S \end{bmatrix} + \begin{bmatrix} V^C \\ V^S \end{bmatrix} \tilde{U}\psi_0 \right). \tag{193}$$

Given the coefficients $\underline{c}$, we can use equation (190) to compute the dynamics of z_t.

D.5 Perturbation solution

Let $r = \{r_t \in \mathbb{R} : t \geq 0\}$ denote the path of real interest rates. Define $w_t = (\pi_t, x_t, b_t)$ as the vector of non-policy variables and $w = \{w_t \in \mathbb{R}^3 : t \geq 0\}$ as the path of w_t. We say that a path of non-policy variables w is feasible if there exists a path of real interest rates r such that w is a *bounded solution* to the system of differential equations:

$$\dot{\pi}_t = (\rho + \lambda_f)\pi_t - \kappa x_t - \lambda_f \kappa \Phi(b_t - b^n) \equiv g_\pi(w_t, r_t, \psi_t) \tag{194}$$

$$\dot{b}_t = r_t - \rho + \psi_t \equiv g_x(w_t, r_t, \psi_t) \tag{195}$$

$$\dot{x}_t = r_t - \rho \equiv g_b(w_t, r_t, \psi_t), \tag{196}$$

given the initial condition b_0 and the process for the fiscal shock $\psi_t = e^{-\theta_\psi t}\psi_0$.

Denote the set of feasible w by $\mathcal{F}$. The optimal policy problem is then given by:

$$\max_{w \in \mathcal{F}} \int_0^\infty e^{-(\rho+\theta)t} f(w_t) dt, \tag{197}$$

where $f(w_t) \equiv -\frac{1}{2}\left[\alpha x_t^2 + \beta \pi_t^2 + \lambda \Upsilon (b_t - b^n)^2\right]$.

Let w^* denote a candidate solution at the interior of the feasible set $\mathcal{F}$. Given a scalar

$\epsilon > 0$, consider the perturbation $\hat{w}_t = w_t^* + \epsilon\eta_t$. We say that the deviation $\eta = \{\eta_t \in \mathbb{R} : t \geq 0\}$ is feasible if the path of $w_t^* + \epsilon\eta_t$ belongs to the feasible set.

Fixing a given deviation η and a candidate solution w^*, the value of a perturbed solution is a function of ϵ:

$$\mathcal{W}(\epsilon) = \int_0^\infty e^{-(\rho+\theta)t} f(w_t^* + \epsilon\eta_t)dt. \tag{198}$$

Given functions $(\mu_{\pi,t}, \mu_{x,t}, \mu_{b,t})$, we can write the value of the perturbation as follows:

$$\mathcal{W}(\epsilon) = \int_0^\infty e^{-(\rho+\theta)t} \left[f(\hat{w}_t) + \sum_{z\in\{\pi,x,b\}} \mu_{z,t} \left(g_z(\hat{w}_t, \hat{r}_t, \psi_t) - \dot{\hat{z}}_t \right) \right] dt, \tag{199}$$

where $\hat{r}$ corresponds to the path of real interest rates associated with the perturbed solution $\hat{w}$. Notice that $g_z(\hat{w}_t, \hat{r}_t, \psi_t) - \dot{z}_t = 0$ for all $t \geq 0$, as $\hat{w}$ is feasible, so the value of $\mathcal{W}(\epsilon)$ is independent of the functions $(\mu_{\pi,t}, \mu_{x,t}, \mu_{b,t})$.

We can use integration by parts to express the following integral in a more convenient form:

$$\int_0^\infty e^{-(\rho+\theta)t} \mu_{z,t} \dot{\hat{z}}_t dt = \lim_{t\to\infty} e^{-(\rho+\theta)t} \mu_{z,t} \hat{z}_t - \mu_{z,0}\hat{z}_0 - \int_0^\infty e^{-(\rho+\theta)t} \left[\dot{\mu}_{z,t} - (\rho+\theta)\mu_{z,t} \right] \hat{z}_t dt, \tag{200}$$

for $z \in \{\pi, x, b\}$. Combining the previous two expressions, we obtain

$$\begin{aligned} \mathcal{W}(\epsilon) = \int_0^\infty e^{-(\rho+\theta)t} & \left[f(\hat{w}_t) + \sum_{z\in\{\pi,x,b\}} \mu_{z,t} g_z(\hat{w}_t, \hat{r}_t, \psi_t) + \sum_z \left(\dot{\mu}_{z,t} - (\rho+\theta)\mu_{z,t} \right) z_t \right] dt \\ & + \sum_z \left[\mu_{z,0} z_0 - \lim_{t\to\infty} e^{-(\rho+\theta)t} \mu_{z,t} z_t \right], \end{aligned} \tag{201}$$

Notice that b_0 is fixed, x_0 is free to be chosen by the planner, but π_0 is determined by the choice of r and x_0. It is useful to eliminate π_0 from the expression above. First, notice that π_0 can be written as

$$\pi_0 = \int_0^\infty e^{-(\rho+\theta)t} h_0(r_t, \psi_t; x_0, b_0) dt, \tag{202}$$

where $h_0(r_t, \psi_t; x_0, b_0) \equiv \kappa \left[x_0 + \lambda\Phi b_0 + \frac{1+\lambda\Phi}{\rho+\theta}(r_t - \rho) + \frac{\lambda\Phi}{\rho+\theta}\psi_t \right]$.

We can then write the $\mathcal{W}(\epsilon)$ in terms of a Hamiltonian, properly modified to incorporate the effect of the initial conditions:

$$\mathcal{W}(\epsilon) = \int_0^\infty e^{-(\rho+\theta)t} \left[H(\hat{w}_t, \hat{r}_t, \psi_t; b_0, \hat{x}_0) + \sum_{z\in\{\pi,x,b\}} \left(\dot{\mu}_{z,t} - (\rho+\theta)\mu_{z,t}\right) z_t \right] dt$$
$$- \sum_{z\in\{\pi,x,b\}} \lim_{t\to\infty} e^{-(\rho+\theta)t} \mu_{z,t} z_t, \tag{203}$$

where $H(\hat{w}_t, \hat{r}_t, \psi_t) \equiv f(\hat{w}_t) + \sum_{z\in\{\pi,x,b\}} \mu_{z,t} g_z(\hat{w}_t, \hat{r}_t, \psi_t) + (\rho+\theta)\left[\mu_{b,0} b_0 + \mu_{x,0} x_0\right] + \mu_{\pi,0} h_0(\hat{r}_t, \psi_t; b_0, \hat{x}_0)$.

A necessary condition for w^* to be an interior solution of the optimal policy problem is that

$$\mathcal{W}'(\epsilon) = 0. \tag{204}$$

Let $\hat{r} = r^* + \epsilon\eta_{r,t}$ and $\hat{x}_0 = x_0^* + \epsilon\eta_{x,0}$ denote the path of real interest rates and initial output gap associated with the perturbation $\hat{w}$. We can then write the derivative with respect to ϵ as follows:

$$\mathcal{W}'(0) = \int_0^\infty e^{-(\rho+\theta)t} \left[\sum_z \left(H_z(w_t^*, r_t^*, \psi_t) + \dot{\mu}_{z,t} - (\rho+\theta)\mu_{z,t}\right) \eta_{z,t} \right] dt - \sum_z \left[\lim_{t\to\infty} e^{-(\rho+\theta)t} \mu_{z,t} \eta_{z,t} \right]$$
$$+ \int_0^\infty e^{-(\rho+\theta)t} H_r(w_t^*, r_t^*, \psi_t) \eta_{r,t} dt. \tag{205}$$

The functions $\mu_{z,t}$ are arbitrary, so we can choose them to satisfy the condition:

$$\dot{\mu}_{z,t} - (\rho+\theta)\mu_{z,t} = -H_z(w_t^*, r_t^*, \psi_t), \tag{206}$$

subject to the boundary condition $\lim_{t\to\infty} e^{-(\rho+\theta)t}\mu_{z,t} = 0$. As any feasible perturbation is bounded, this ensures that the term $\lim_{t\to\infty} e^{-(\rho+\theta)t}\mu_{z,t}\eta_{z,t}$ is equal to zero.

As the perturbation $\eta_{r,t}$ is arbitrary, the following condition must be satisfied:

$$H_r(w_t^*, r_t^*, \psi_t) = 0. \tag{207}$$

Finally, the optimality condition for x_0 is given by

$$(\rho+\theta)\mu_{x,0} + \mu_{\pi,0}\kappa = 0. \tag{208}$$

The general case. Suppose that equilibrium variables evolve according to the more general dynamics:

$$\dot{\pi}_t = (\rho + \lambda_f)\pi_t - \kappa x_t - \lambda_f \kappa \Phi(b_t - b^n) \tag{209}$$

$$\dot{b}_t = r_t - \rho - \gamma(b_t - b^n) + \psi_t \tag{210}$$

$$\dot{x}_t = r_t - \rho + \theta_h x_t - \theta_h^*(b_t - b^n), \tag{211}$$

We are interested in the effect of the initial conditions, so let's set $r_t = \rho$ and $\psi_t = 0$. In this case, the evolution of b_t and x_t is given by

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{x}_t \\ \dot{b}_t \end{bmatrix} = \begin{bmatrix} \rho + \lambda_f & -\kappa & -\kappa\lambda_f\Phi \\ -1 & \theta_h & -\theta_h^* \\ -1 & 0 & -\gamma \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ b_t - b^n \end{bmatrix}. \tag{212}$$

The homogeneous solution is given by

$$b_t - b^n = e^{-\gamma t}(b_0 - b^n), \qquad x_t = \frac{\theta_h^*}{\gamma + \theta_h}(b_t - b^n), \qquad \pi_t = \left[\frac{\theta_h^*}{\gamma + \theta_h} + \lambda_f \Phi\right] \frac{\kappa}{\rho + \lambda_f + \gamma}(b_t - b^n). \tag{213}$$

D.6 Optimal policy with discretion

Optimal policy with finite planning horizon. Consider a planning with a finite planning horizon. We assume that a new planner takes over with a Poisson intensity $\overline{\lambda}$. The current planner takes the actions of future decision-makers as given. This ensures that the Euler equation is satisfied even after a new planner takes over. Let $\mathcal{P}_t(b_t)$ denote the value of a planner at period t with a given level of government debt, and $\mathcal{P}^*(b^*)$ denotes the value of a planner in the inflationary-finance phase. The planner's objective is given by

$$\mathcal{P}_0(b_0) = \mathbb{E}_0 \left[-\frac{1}{2} \int_0^\tau e^{-\rho t} \left[\alpha x_t^2 + \beta \pi_t^2\right] dt + e^{-\rho\tau} \tilde{P}_\tau(b_\tau) \right], \tag{214}$$

where τ denotes the random time the economy switches to either the inflationary-finance phase, so the planner's value becomes $\tilde{P}_\tau(b_\tau) = \mathcal{P}^*(b_\tau)$, or a new planner's take over, so the planner's value is $\tilde{\mathcal{P}}_\tau(b_\tau) = \mathcal{P}_\tau(b_\tau)$. The density of τ is given by $(\lambda + \overline{\lambda})e^{-(\lambda+\overline{\lambda})t}$ and, conditional on switching, the probability of moving to the inflationary-finance phase is $\frac{\lambda}{\lambda+\overline{\lambda}}$, while the probability of a new planner taking over is given by $\frac{\overline{\lambda}}{\lambda+\overline{\lambda}}$ (see e.g. Cox and Miller (1977) for a derivation).

Using the density of τ, we can then express $\mathcal{P}_0(x_0, b_0)$ as follows:

$$\mathcal{P}_0(b_0) = -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda+\overline{\lambda})t}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right]dt + \int_0^\infty e^{-(\rho+\lambda+\overline{\lambda})t}\overline{\lambda}\mathcal{P}_t(b_t)dt. \quad (215)$$

The planner's problem consists of maximizing the objective above subject to the constraints

$$\dot{\pi}_t = (\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n), \qquad \dot{b}_t = r_t - \rho + \psi_t, \qquad \dot{x}_t = r_t - \rho.$$

We also include a penalty on π_0 and x_0, as in the case with full commitment.

Optimality conditions The optimality conditions are given by

$$\dot{\mu}_{\pi,t} - (\rho+\lambda+\overline{\lambda})\mu_{\pi,t} = \beta\pi_t - (\rho+\lambda)\mu_{\pi,t} \quad (216)$$

$$\dot{\mu}_{b,t} - (\rho+\lambda+\overline{\lambda})\mu_{b,t} = \lambda\Upsilon(b_t - b^n) - \overline{\lambda}\mathcal{P}_{b,t}(b_t) + \lambda\kappa\Phi\mu_{\pi,t} \quad (217)$$

$$\dot{\mu}_{x,t} - (\rho+\lambda+\overline{\lambda})\mu_{x,t} = \alpha x_t + \kappa\mu_{\pi,t}, \quad (218)$$

where $\mathcal{P}_{b,t}(b_t)$ denotes the partial derivative of $\mathcal{P}_t(b_t)$ with respect to debt.

The optimality condition for the interest rate is given by

$$\mu_{x,t} + \mu_{b,t} = -\xi, \quad (219)$$

where $\xi \equiv \frac{\kappa(1+\lambda\Phi)}{\rho+\theta}\xi_\pi$.

The optimality condition for x_0 is given by

$$\mu_{x,0} = 0. \quad (220)$$

Standard envelope arguments imply that

$$\mu_{b,t} = \mathcal{P}_{b,t}(b_t). \quad (221)$$

The discretion limit. Consider the limit as $\overline{\lambda} \to \infty$, so each planner has commitment only over an infinitesimal amount of time. In the limit, the co-states on π_t and x_t are given by

$$\mu_{\pi,t} = 0, \qquad \mu_{x,t} = 0. \tag{222}$$

Integrating the expression for $\mu_{x,t}$ forward, we obtain

$$\mu_{x,t} = -\int_t^\infty e^{-(\rho+\lambda+\overline{\lambda})(s-t)} \left[\alpha x_s + \kappa \mu_{\pi,s}\right] ds \Rightarrow \lim_{\overline{\lambda}\to\infty} \overline{\lambda}\mu_{x,t} = -\alpha x_t, \tag{223}$$

using the fact that $\lim_{\overline{\lambda}\to\infty} \mu_{\pi,t} = 0$. Hence, from the optimality condition for x_0, we obtain $x_0 = 0$. Differentiating the optimality condition for the interest rate with respect to time, we obtain

$$(\rho + \lambda + \overline{\lambda})\xi = \alpha x_t + \lambda\Upsilon(b_t - b^n) - \overline{\lambda}\mu_{b,t} + \kappa(1 + \lambda\Phi)\mu_{\pi,t}, \tag{224}$$

where we used the envelope condition for b_t

Given $\mu_{b,t} = -\xi - \mu_{x,t}$, and combining the previous two expressions, we obtain

$$(\rho + \lambda)\xi = \lambda\Upsilon(b_t - b^n). \tag{225}$$

Therefore, the interest rate is given by

$$r_t - \rho = -\psi_t. \tag{226}$$

The case of partial commitment. In the case of discretion, planner's do not take into account promises made by prior planners. Hence, each planner sets a new value of x_t as they take control, and promise that output gap will evolve according to the Euler equation in the future. As we reduce the planning horizon to zero, each planner chooses the value of the output gap regardless of the path of interest rates. We consider next the case of partial commitment, where the planner has to respect past promises made about the output gap. In this case, the output gap must satisfy the Euler equation at every point in time, except at $t = 0$ when news about the shock arrives.

In this case, the planner's objective is given by

$$\mathcal{P}_0(x_0, b_0) = -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda+\overline{\lambda})t} \left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right] dt + \int_0^\infty e^{-(\rho+\lambda+\overline{\lambda})t}\overline{\lambda}\mathcal{P}_t(x_t, b_t)dt, \tag{227}$$

and we impose a penalty on π_0, but not on x_0, as the initial output gap is not free.

The optmality conditions are now given by

$$\dot{\mu}_{\pi,t} - (\rho + \lambda + \overline{\lambda})\mu_{\pi,t} = \beta\pi_t - (\rho + \lambda)\mu_{\pi,t} \tag{228}$$

$$\dot{\mu}_{b,t} - (\rho + \lambda + \overline{\lambda})\mu_{b,t} = \lambda\Upsilon(b_t - b^n) - \overline{\lambda}\mathcal{P}_{b,t}(x_t, b_t) + \lambda\kappa\Phi\mu_{\pi,t} \tag{229}$$

$$\dot{\mu}_{x,t} - (\rho + \lambda + \overline{\lambda})\mu_{x,t} = \alpha x_t + \kappa\mu_{\pi,t} - \overline{\lambda}\mathcal{P}_{x,t}(x_t, b_t). \tag{230}$$

The optimality condition for the interest rate is the same as under discretion, and the envelope conditions for output gap and debt are given by

$$\mu_{x,t} = \mathcal{P}_{x,t}(x_t, b_t), \qquad \mu_{b,t} = \mathcal{P}_{b,t}(x_t, b_t). \tag{231}$$

Differentiating the optimality condition for the interest rate with respect to time, we obtain

$$(\rho + \lambda + \overline{\lambda})\xi = \alpha x_t + \lambda\Upsilon(b_t - b^n) - \overline{\lambda}(\mu_{b,t} + \mu_{x,t}) + \kappa(1 + \lambda\Phi)\mu_{\pi,t}, \tag{232}$$

where we used the envelope conditions.

Taking the limit as $\overline{\lambda} \to \infty$, we obtain

$$(\rho + \lambda)\xi = \alpha x_t + \lambda\Upsilon(b_t - b^n) \Rightarrow r_t - \rho = -\frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t. \tag{233}$$

In period $t = 0$, the planner is allowed to choose x_0, which must satisfy the condition:

$$\mu_{x,0} = 0 \Rightarrow 0 = \int_0^\infty e^{-(\rho+\lambda)t}\alpha x_t dt = 0, \tag{234}$$

where we used the fact that $\mu_{\pi,t} = 0$ as $\overline{\lambda} \to \infty$. Therefore, optimal policy with partial commitment coincides with the optimal policy with commitment for a dovish central bank, that is, when $\beta = 0$.

Taking the limit of a discrete-time economy. Welfare is measured by

$$\sum_{t=0}^{\infty} \left(e^{-\rho\Delta t}\right)^t \left[\alpha x_t^2 + \beta\pi_t^2\right] \Delta t. \tag{235}$$

The NKPC is given by

$$\pi_t = e^{-\rho\Delta t}\mathbb{E}_t\left[\pi_{t+\Delta t}\right] + (\kappa x_t + u_t)\,\Delta t. \tag{236}$$

Under discretion, the planner's problem is given by

$$\max_{x_t,\pi_t} -\frac{1}{2}\left[\alpha x_t^2 + \beta \pi_t^2\right]\Delta t, \tag{237}$$

subject to

$$\pi_t = e^{-\rho\Delta t}\mathbb{E}_t\left[\pi_{t+\Delta t}\right] + \left(\kappa x_t + u_t\right)\Delta t, \tag{238}$$

taking as given $\mathbb{E}_t \pi_{t+\Delta t}$.

The optimal solution is given by

$$x_t = -\frac{\kappa\beta}{\alpha}\pi_t \Delta t. \tag{239}$$

D.7 Optimal policy under the timeless perspective

The dynamics under the optimal policy are characterized by the conditions:

$$\dot{\pi}_t = (\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n) \tag{240}$$

$$\dot{b}_t = r_t - \rho - \gamma(b_t - b^n) + \psi_t \tag{241}$$

$$\dot{x}_t = r_t - \rho + \theta_h x_t - \theta_h^*(b_t - b^n) \tag{242}$$

$$\dot{\mu}_{\pi,t} = \beta\pi_t \tag{243}$$

$$\dot{\mu}_{b,t} = (\rho+\lambda)\mu_{b,t} + \lambda\Upsilon(b_t - b^n) + \kappa\lambda\Phi\mu_{\pi,t} \tag{244}$$

$$\dot{\mu}_{x,t} = (\rho+\lambda)\mu_{x,t} + \alpha x_t + \kappa\mu_{\pi,t}, \tag{245}$$

where the real rate is given by

$$r_t - \rho = -\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t, \tag{246}$$

given the initial value of debt, b_0, and the boundary conditions $\mu_{x,0} = \mu_{\pi,0} = 0$.

Consider the case without a fiscal shock, $\psi_t = 0$, and denote the co-states in this case with no shocks by $\mu_{x,t}^{ns}$ and $\mu_{\pi,t}^{ns}$. The optimal policy under the timeless perspective corresponds to the solution to the system above when we replace the initial conditions by the long-run values of these multipliers: $\mu_{x,0} = \lim_{t\to\infty}\mu_{x,t}^{ns}$ and $\mu_{\pi,0} = \lim_{t\to\infty}\mu_{\pi,t}^{ns}$ (see Giannoni and Woodford (2017) for a discussion in the context a general model). This is equivalent to the problem of a planner who started its planning in a distant past, so the multipliers had time to converge to their long-run values.

Even without shocks, the limits $\lim_{t\to\infty} \mu_{x,t}^{ns}$ and $\lim_{t\to\infty} \mu_{\pi,t}^{ns}$ will not be equal to zero, provided that $b_0 \neq b^n$. However, in the case $b_0 = b^n$, the solution to the system above in the absence of shocks is simply $\pi_t = x_t = b_t = \mu_{\pi,t} = \mu_{x,t} = \mu_{b,t} = 0$. Hence, we have that $\lim_{t\to\infty} \mu_{x,t}^{ns} = 0$ and $\lim_{t\to\infty} \mu_{\pi,t}^{ns} = 0$, so the boundary conditions for the problem under the timeless perspective coincide with the time-zero commitment solution.