Educational-Specific Decompositional Effects of Skill Loss on TFP

Ilsoo Han

University of California, Riverside

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Abstract

This paper presents a theoretical framework in which search and matching frictions serve as a transmission mechanism linking the composition of the unemployed workforce to fluctuations in Total Factor Productivity (TFP). It is well established that unemployment has a detrimental impact on individual human capital. This study demonstrates that unemployment can also adversely affect overall productivity. Specifically, the impact of unemployment on human capital varies by level of educational attainment, indicating that both the incidence and composition of unemployment can significantly influence economic productivity, particularly its fluctuations. The model proposed in this paper extends the framework of Ortego-Marti (2017a) by incorporating heterogeneous human capital depreciation rates. By accounting for the share of highly educated individuals among the unemployed, who are more susceptible to substantial skill loss during periods of unemployment, the model enhances our understanding of observed productivity fluctuations. These findings highlight the critical role that the composition of the unemployed plays in explaining TFP fluctuations.

1.Introduction

Human capital depreciates when it remains unused, meaning that workers experiencing unemployment will face negative effects on their wages and productivity. Consequently, if unemployment becomes more frequent and prolonged across the economy, overall productivity will decline. This has been demonstrated by Ortego-Marti (2017a), who shows that labor market flows can be a key factor in explaining differences in Total Factor Productivity(TFP).

In this paper, I present a channel through which worker heterogeneity can significantly enhance our understanding of TFP fluctuations in a quantitatively important way. To understand why worker heterogeneity might be a key factor affecting TFP fluctuations, consider that in an economic situation with the same unemployment rate over different periods, the impact of skill loss on overall productivity can vary depending on the composition of workers. For example, highly educated individuals tend to experience relatively greater human capital depreciation during unemployment. As the share of these highly educated individuals among the unemployed increases, the overall productivity of the economy decreases more significantly, leading to greater TFP fluctuations.

The paper extends the Ortego-Marti (2017a) model, which incorporates skill loss during unemployment within the Diamond-Mortensen-Pissarides (DMP) search and matching framework. This extension introduces heterogeneous human capital depreciation rates based on educational attainment. The two types of workers differ in several ways: highly educated workers experience a higher rate of human capital depreciation, as shown in the empirical findings discussed in Section 2. Additionally, worker heterogeneity necessitates the consideration of distinct labor markets for each type of worker. This allows us to separately introduce variables, such as job finding rates and job separation rates, which represent the characteristics of each labor market into the model.

In the model, TFP is endogenous and depends on each worker's human capital depreciation rates, as well as parameters related to labor market flows. The model offers two main advantages for quantitatively explaining TFP volatility. First, by considering different human capital depreciation rates for distinct worker groups, it enhances our understanding of the impact of skill loss transmitted through changes in the workforce composition. This approach provides a more comprehensive explanation of volatility than focusing on unemployment alone. Additionally, the incorporation of two separate labor markets means that fluctuations in unemployment within each market contribute to increased variability in labor market characteristics, resulting in greater TFP fluctuations.

To quantify TFP fluctuations caused by changes in the composition of unemployed workers, this study utilizes U.S. data. First, I calibrate human capital depreciation rates based on educational attainment by estimating the Panel Study of Income Dynamics using the regression model from Ortego-Marti (2016). The estimation results indicate that highly educated workers lose 2.15% of their wages for each additional month of unemployment, which is more than double the wage loss experienced by less educated workers (1.00%). Additionally, I estimate job finding and separation rates for the U.S. using the methods outlined in Elsby et al. (2013).

The simulation results indicate that the model presented in this paper, which accounts for the share of highly educated individuals among the unemployed, can explain 37.5% of the fluctuations in the observed data. This represents an approximate 12% increase in explanatory power compared to the 33.5% explained when only unemployment is considered. However, this increase in explanatory power is primarily attributed to the consideration of fluctuations in each separate labor market. The contribution of heterogeneous human capital depreciation to the overall variability accounts for only 0.9 percentage points of the 4 percentage points increase.

Finally, this paper analyzes the effect of hiring subsidies on TFP. Hiring subsidies can stimulate job creation by firms, thereby increasing the overall employment level in the economy and reducing unemployment spells, which in turn enhances overall productivity. This increase in productivity is primarily driven by the rise in employment among less educated workers, whose net productivity is relatively lower compared to that of highly educated workers. Consequently, the hiring subsidies, provided in the form of productivity increases, lead to a larger percentage increase in firms' accounting profits and significantly boost the incentive for firms to open vacancies for less educated workers, consistent with the findings of Pries (2008).

The rest of the paper is organized as follows. Section 2 review key empirical observations associated with relationship between the composition of highly educated workers among unemployed and TFP. Section 3 describes the model and the equilibrium. Section 4 will cover the calibration and simulation results. Section 5 will address the effects of labor market policies, particularly employment subsidies, on TFP. Section 6 concludes.

Related Literature

There have been attempts to develop models for TFP in frictional markets, as represented by works such as Marimon and Zilibotti (1999) and Lagos (2006). This paper is particularly similar to the studies by Ortego-Marti (2017a) and Doppelt (2019). Ortego-Marti (2017a) introduced skill loss due to unemployment history into a model with search and matching friction to endogenously derive TFP, using this framework to compare TFP differences among OECD countries. Doppelt (2019) demonstrated that skill loss and learning-by-doing among workers can link labor market dynamics and long-run economic growth within search and matching frameworks.

Additionally, models in which the speed of human capital accumulation varies with schooling and subsequently affects economic growth can be found in Laing et al. (1995), which presented an endogenous growth model based on frictional labor markets, schooling, and human capital accumulation.

As in this paper, I focus on quantitatively explaining the time series data of TFP that has not been addressed in existing models, specifically using a model based on frictional markets. While there has been a substantial amount of literature related to development accounting that explains differences in TFP across countries, most studies either focus on cross-sectional comparisons or emphasize qualitative results. In contrast, this paper distinguishes itself by providing a quantitative explanation of historical TFP data.

This paper is also broadly related to the job displacement literature. Numerous empirical studies have examined the impact of unemployment on workers' wages and productivity, commonly finding significant and persistent earnings losses. In particular, this paper builds on Ortego-Marti (2016), which estimated the impact of unemployment history on workers' wages using the PSID (Panel Study of Income Dynamics) in its calibration approach.

2. Empirical evidence

In this section, key empirical observations that motivate and support the model are presented. First, it is demonstrated that the effect of unemployment on wages varies according to educational attainment, with a particularly significant impact on highly educated individuals. Consequently, highly educated workers experience a relatively greater loss of human capital during unemployment, implying that an increased share of such workers within the unemployed population can lead to more pronounced negative effects on the overall economy. In other words, there is a negative relationship between their share and TFP, which is confirmed using CPS data. The strong correlation between the share of highly educated unemployed and TFP suggests that this share could be a major factor driving TFP fluctuations. To verify this, the impact of the share on TFP is examined through simple regression analysis.

2.1 Human capital depreciation during unemployment based on educational attainment

Rosen (1976) finds that the rate of growth for human capital differs between college graduates and high school graduates once they are employed. This finding supports the assumption in Laing et al. (1995) that a worker's educational efforts influence their productivity after employment. Conversely, it implies that workers with different levels of education incur varying costs when facing unemployment, particularly that highly educated workers experience a greater loss of human capital relative to their less educated counterparts during unemployment. To verify this, this paper measures skill loss during unemployment across different educational levels based on the methodology of Ortego-Marti (2016). The impact of an additional month of unemployment on wage decreases was estimated using the PSID, controlling for individual unobservable characteristics with fixed effects. The depreciation rate of human capital by educational level was estimated using the same dataset through the following regression model:

$$\log w_{it}^{(g)} = \alpha_i^{(g)} - \delta^{(g)} Unhis_{it}^{(g)} + \beta^{(g)} X_{it}^{(g)} + \varepsilon_{it}^{(g)} \tag{1}$$

To estimate the above panel regression model, I utilize the 1968-1997 waves of the PSID. In this model, the primary independent variable is $Unhis_{it}^{(g)}$, representing each worker's unemployment history in months, while $X_{it}^{(g)}$ includes controls for worker characteristics such as potential experience, regional dummies, and one-digit occupational dummies. Thus, $\delta^{(g)}$ captures the percentage wage loss attributed to accumulated unemployment spells, reflecting the loss of skills during unemployment. In other words, it indicates the human capital depreciation rates for each educational group.

Here, g denotes the group of workers based on their educational attainment, distinguishing between high and low education groups. Specifically, the high education group is defined as those with educational attainment equal to or exceeding a four-year bachelor's degree. This classification is based on findings that individuals with less than four years of college exhibit labor market outcomes more similar to those who did not attend college at all. Similarly, Flinn and Mullins (2015) also categorized individuals into high and low schooling groups using this criterion.

For context, the regression estimate for the overall sample in Ortego-Marti (2016) is 0.0122. The regression estimates for $\delta^{(g)}$ in this paper are presented in Table 1, with values of 0.010 and 0.022 for the lower-educated and highly educated groups, respectively. An estimated coefficient of 0.010 for the less educated group indicates that their skill levels depreciate by 1.0% per month. In contrast, the coefficient for the highly educated group is -0.022, signifying a depreciation of 2.2% per month, indicating that they lose significantly more human capital relative to the less educated during unemployment.

The intuitive reason why highly educated workers are more significantly affected by unemployment is as follows: highly educated individuals are more likely to work in jobs or sectors that require specialized skills, whereas less educated workers are more likely to be employed in positions involving relatively simple tasks. As a result, highly educated workers experience greater skill loss due to unemployment. This outcome aligns with the findings of Ortego-Marti (2017b), which showed that the wage decrease caused by unemployment varies by occupation and sector, with more negative effects observed in jobs requiring higher levels of skills.

We found a significant difference between the depreciation rates for the highly educated and less educated groups. This result also reaffirms Rosen (1976)'s finding, but in the context of unemployment.

2.2 Relationship between TFP and Share of Highly Educated Workers Among the Unemployed

The previous estimates suggest a prediction: because highly educated workers lose more human capital during unemployment, even if the unemployment rate remains constant, an increase in the share of highly educated workers among the unemployed may lead to lower economic growth or productivity levels. This conjecture can be empirically demonstrated using data, as illustrated in Figure 1. Total Factor Productivity (TFP) data was obtained from PWT 10.0, while the share was calculated based on wage data from the Current Population

Survey (CPS) spanning 1964 to 2019.

Figure 1 shows the series of the share of highly educated unemployed workers and total factor productivity (TFP) in the United States. The first graph displays the raw data for both series, indicating that both exhibit an upward trend. This is expected as college enrollment rates have increased over the period, leading to a higher share of highly educated individuals among the unemployed, which contributes to the upward trend in the share. However, the increase in college enrollment also raises the share of highly educated individuals among the employed, which may positively affect productivity and contribute to the upward trend in TFP. In particular, the share of highly educated individuals among the employed has increased almost linearly over the period, which could be closely related to the upward trend.

Since this paper focuses on analyzing the impact of unemployment on productivity, it is crucial to examine short-term volatility rather than long-term trends. Over the long term, as educational attainment increases, the absolute number of highly educated unemployed workers is likely to rise continuously. This makes the level of highly educated unemployed workers less suitable for analyzing TFP variability. To understand fluctuations that deviate from the trend in TFP, it is essential to focus on the share of highly educated unemployed workers rather than their absolute number.

The share of highly educated unemployed workers can change over time, providing information related to relative changes within the economy. In other words, even if the absolute number of highly educated unemployed workers continues to increase, the impact on TFP can vary depending on whether that share increases or decreases. Notably, as the share of highly educated unemployed workers rises, the negative effects on TFP may become more pronounced. Therefore, to explain TFP fluctuations, it is crucial to consider the share rather than simply the absolute number of highly educated unemployed workers.

To address this, we analyze the cyclical components of both TFP and the share of highly educated unemployed workers, removing long-term trends to isolate short-term relationships.

The second graph displays the detrended series for TFP and the share, showing that their movements tend to be opposite over considerable periods. This supports our conjecture that an increase in the share of highly educated individuals experiencing significant skill loss due to unemployment leads to a decrease in overall economic productivity.

The third graph presents the same detrended series in a scatter plot, clearly illustrating the negative correlation between the two variables, with a correlation coefficient of -0.36. This result suggests that fluctuations in the share of highly educated workers can closely relate to fluctuations in TFP, indicating that share is a key driver of changes in TFP.

To verify this, I performed the following regression analysis and present the results in Table 2:

$$TFP_t = \beta share_t^{(g)} + \varepsilon_t^{(g)} \tag{2}$$

We estimated the impact of the share of each educational group among the unemployed on TFP. The results indicate that the share of highly educated unemployed workers has a significant impact on TFP, while the share of less educated unemployed workers does not. This supports our conjecture. This empirical relationship is a new finding that has not been previously noted. It demonstrates that the share of highly educated unemployed workers is a key driver of TFP volatility, highlighting its importance in explaining fluctuations in TFP.

3. Model

This chapter presents a model that highlights the role of fluctuations in the share of highly educated unemployed workers in explaining TFP volatility arising from worker heterogeneity. To identify the mechanism linking these two factors, this paper considers a model that incorporates heterogeneous human capital depreciation rates based on educational attainment, building upon the model proposed by Ortego-Marti (2017a).

The model is continuous time. There are four types of agents in the model: two types of workers and two types of firms. Both workers and firms are infinitely lived, risk-neutral, and discount future income at a constant rate $r > 0$. We assume that the four types of agents meet in two separate labor markets. This assumption is based on the observation that primitive parameters across submarkets often differ significantly, as emphasized by Flinn and Mullins (2015). Specifically, it is commonly observed that unemployment rates vary across educational attainment levels, with individuals having lower completed schooling experiencing longer and more frequent unemployment spells. Due to these characteristics, we consider two separate labor markets based on educational achievement.

The total population size is L, which is normalized to one. Workers are categorized into two types $i \in \{1, 2\}$, representing different educational achievements, such as less educated and highly educated. Correspondingly, firms can also be classified into two types based on the type of workers they employ or the sector they operate in, aligning with the skills required by different types of workers. The proportion of highly educated workers in the population is ϕ, meaning the number of less educated workers is $L_1 = (1 - \phi)L$ and the number of highly educated workers is $L_2 = \phi L$. The share of highly educated unemployed workers among the total unemployed and the share of highly educated employed workers among the total employed can be defined respectively as follows:

$$\pi^u = \frac{u_2 L_2}{u_1 L_1 + u_2 L_2} = \frac{u_2 \phi L}{u_1(1-\phi)L + u_2 \phi L} = \frac{u_2 \phi}{u_1(1-\phi) + u_2 \phi} \tag{3}$$

$$\pi^e = \frac{(1-u_2)L_2}{(1-u_1)L_1 + (1-u_2)L_2} = \frac{(1-u_2)\phi \pi^u}{\pi^u - u_2 \phi} \tag{4}$$

u_i is the unemployment rate for group i, and we can see that the share π^u depends on the unemployment rate of each group and the weight of highly educated workers among the labor force. Additionally, π^e is a function of π^u.

The number of job matches occurring per unit time is determined by a matching function for each type i, denoted $m(U_i, V_i)$, where U_i represents the number of unemployed individuals of type i, and V_i represents the number of vacancies for type i. The matching function has the standard properties: it is concave, increasing in both arguments, and exhibits constant returns to scale. Market tightness, θ_i, is defined as the ratio of vacancies to unemployed workers, thus $\theta_i = \frac{V_i}{U_i}$. Given the properties of the matching function, workers find jobs at a Poisson rate $f(\theta_i) = m(1, \theta_i)$, and firms fill vacancies at a Poisson rate $q(\theta_i) = m(\theta_i^{-1}, 1)$. Based on these properties, the job-finding rate increases with θ_i, while $q(\theta_i)$ decreases with θ_i. Additionally, job separations are assumed to occur exogenously at a Poisson rate s_i.

In this paper, we aim to improve our understanding of how unemployment affects overall productivity via human capital depreciation, while accounting for worker heterogeneity. To fully grasp these effects, it is essential to recognize how human capital depreciation differs based on educational achievement. Following Ortego-Marti (2016), we assume that human capital depreciation is determined by unemployment history. Specifically, workers in each group lose human capital during unemployment at constant rates, denoted δ_1 and δ_2, with $\delta_1 < \delta_2$. This reflects the fact that, as discussed in Section 2, workers with higher educational attainment experience greater human capital loss during periods of unemployment.

With these constant depreciation rates, human capital for each worker type can be represented as $h_i(\gamma) = \exp(-\delta_i \gamma)$, where $h_i(0)$ is normalized to 1. Furthermore, we assume that overall labor efficiency, represented by p_i, differs across worker groups. This assumption is based on the understanding that the productivity impact of new technologies varies depending on the level of technological skill. For instance, the development of PC software is highly beneficial for occupations requiring such technology, whereas its productivity-enhancing effects would be minimal in occupations where it is not used. Therefore, the productivity of a match, denoted as y_i, can be defined as follows:

$$y_i = h_i(\gamma) p_i. \tag{5}$$

When each group of workers matches with firms, they earn wages $w_i(\gamma)$, while unemployed workers receive flow payments b_i. Here, b_i represents the value of non-market activities, including unemployment benefits, the value of leisure, and home production.

Workers are identical within each group when they enter the labor market. However, due to search frictions, they find and lose jobs at random. As a result, even within the same group, workers accumulate different unemployment histories. This leads to endogenous distributions of unemployment histories, denoted as γ, among employed and unemployed workers in each group, represented by $G_i^E(\gamma)$ and $G_i^U(\gamma)$, respectively. To ensure these distributions are stationary, we assume that workers exit the labor force at a rate of ψ. When workers leave, they are replaced by new workers who have an unemployment history of $\gamma = 0$.

Let $U_i(\gamma)$ denote the value that a worker associates with being unemployed, and let $W_i(\gamma)$ denote the value function for an employed worker. The Bellman equations for workers are:

$$(r + \psi)U_i(\gamma) = b_i + f(\theta i)[\max Wi(\gamma), U_i(\gamma) - U_i(\gamma)] + \frac{\partial U_i(\gamma)}{\partial \gamma} \tag{6}$$

$$(r + \psi)W_i(\gamma) = w_i(\gamma) - s_i[W_i(\gamma) - U_i(\gamma)] \tag{7}$$

Equation (5) describes the valuation of assets for unemployed workers. This valuation, $U_i(\gamma)$, includes the yield b_i and the expected capital gain from transitioning to employment, represented by $f(\theta_i)(W_i(\gamma) - U_i(\gamma))$. Notably, the valuation also depends on the partial derivative $\frac{\partial U_i(\gamma)}{\partial \gamma}$. A key feature of this model is that if a worker remains unemployed, their unemployment history γ increases, which negatively affects the overall asset value.

Similarly, Equation (6) describes the value of assets $W_i(\gamma)$ for employed workers. This value is determined by the flow wage $w_i(\gamma)$ and the job separation rate s_i.

Assuming that firms can distinguish between workers before a match is made, we consider two distinct labor markets for each level of educational achievement, where each type of firm only hires workers with the corresponding educational attainment. Let $J_i(\gamma)$ and V_i denote the value functions of a filled job and an open vacancy, respectively, for each labor market.

The Bellman equations for these are:

$$(r+\psi)J_i(\gamma) = h_i(\gamma)p_i - w_i(\gamma) - s_i J_i(\gamma) \tag{8}$$

$$rV_i = -c_i + q(\theta_i)\int_0^\infty \left[\max\{J_i(\Gamma), V_i\} - V_i\right] dG_i^U(\Gamma) \tag{9}$$

The intuition behind these Bellman equations parallels that of the workers' equations. For firms, if a position is filled, they receive a profit flow of $h_i(\gamma)p_i - w_i(\gamma)$. However, the job also runs a risk of an adverse shock s_i, which incurs a loss of $J_i(\gamma)$. Similarly, Equation (8) represents the expected capital gain if the firm successfully matches with a worker. This includes the flow cost c_i of posting a vacancy and the rate $q(\theta_i)$ at which the firm draws a worker from the pool of unemployed, considering the distribution of job seekers' unemployment histories $G_i^U(\gamma)$. Additionally, we assume free entry into the market for vacancies, so firms continue posting vacancies until $V_i = 0$.

When a worker and a firm are matched, the wage $w_i(\gamma)$ in each group is determined by the following process. Due to search frictions, in equilibrium, an occupied job generates a total surplus that is larger than the sum of the expected returns for both the searching firm and the searching worker. We assume that the rents from such a match are divided between the firm and the worker through Nash bargaining, where β represents the worker's bargaining power, and $w_i(\gamma)$ will be the Nash bargaining solution that maximizes the following weighted product of the worker's and the firm's net return from the match.

$$w_i(\gamma) = \underset{w_i(\gamma)}{\arg\max}\left[(W_i(\gamma) - U_i(\gamma))^\beta (J_i(\gamma) - V_i)^{1-\beta}\right] \tag{10}$$

Using the free entry condition $V_i = 0$, bargaining over the continuing wage $w_i(\gamma)$ gives

$$(1-\beta)(W_i(\gamma) - U_i(\gamma)) = \beta J_i(\gamma) \tag{11}$$

In this paper, a key feature of the model is that worker productivity depreciates with unemployment duration. Therefore, if human capital continues to depreciate as unemployment duration accumulates, at some point, the surplus from a match will become zero. This implies that each group of workers will be indifferent between market and non-market activities. Specifically, we can find a unique $\overline{\gamma}_i$ for $i \in \{1, 2\}$ which satisfies:

$$(r+\psi)U_i(\overline{\gamma}_i) = b_i \tag{12}$$

and

$$h_i(\overline{\gamma}_i)p_i = w_i(\overline{\gamma}_i) \tag{13}$$

This implies that:

$$h_i(\overline{\gamma}_i)p_i = b_i \tag{14}$$

As a result, $\overline{\gamma}_i$ is determined by:

$$\overline{\gamma}_i = -\frac{\log(b_i/p_i)}{\delta_i} \tag{15}$$

We assume that when workers accumulate unemployment history beyond $\overline{\gamma}_i$, firms can assign them to a zero surplus position. Given this assumption, the Bellman equation for unemployment (6) becomes:

$$(r+\psi)U_i(\gamma) = b_i + f(\theta_i)(W_i(\gamma) - U_i(\gamma)) + \frac{\partial U_i(\gamma)}{\partial \gamma}, \quad \forall \gamma \leq \overline{\gamma}_i \tag{16}$$

$$(r+\psi)U_i(\gamma) = b_i, \quad \forall \gamma > \overline{\gamma}_i \tag{17}$$

Similarly, the Bellman equation for vacancies (9) becomes:

$$rV_i = -c_i + q(\theta_i) \int_0^{\overline{\gamma}_i} J_i(\Gamma)\, dG_i^U(\Gamma) \tag{18}$$

This is because, when matched with workers having unemployment histories $\gamma_i \geq \overline{\gamma}_i$, the surplus obtained from such matches is zero.

3.1 Endogenous unemployment history distributions

In this section, we derive the stationary distributions $G_i^E(\gamma)$ and $G_i^U(\gamma)$.

First, $G_i^U(\gamma)$, which represents the fraction of unemployed workers with unemployment history less than a given γ, must satisfy the steady-state condition where flows in and out of this group are equal. This implies the following flow equation:

$$g_i^U(\gamma)u_i + (f(\theta_i) + \psi)G_i^U(\gamma)u_i = s_i G_i^E(\gamma)(1 - u_i) + \psi \tag{19}$$

In this equation, the left-hand side represents the flow out of the group, while the right-hand side represents the flow into the group. The term ψ on the right-hand side accounts for the rate at which workers leave the labor force, but they are replaced by workers who have $\gamma = 0$, thus appearing on the right-hand side of the equation.

Next, consider the flow of unemployed workers in each group. The inflows and outflows for each i must be equal, which leads to the following flow equation:

$$(f(\theta_i) + \psi)u_i = s_i(1 - u_i) + \psi \tag{20}$$

The above equation gives us the unemployment rate for each group u_i:

$$u_i = \frac{s_i + \psi}{s_i + \psi + f(\theta_i)} \tag{21}$$

The overall unemployment rate u is:

$$u = (1-\phi)u_1 + \phi u_2 \tag{22}$$

Finally, consider the group of employed workers with unemployment history less than a given γ. In steady state, the following equation must hold:

$$f(\theta_i)G_i^U(\gamma)u_i = (s_i+\psi)G_i^E(\gamma)(1-u_i) \tag{23}$$

In this equation, the left-hand side represents the inflow into the group, while the right-hand side represents the outflow.

Substituting (21) into the above flow equation shows that $G_i^U(\gamma) = G_i^E(\gamma)$. Using this result with (21), then (19) implies the following differential equation:

$$g_i^U(\gamma) + \frac{\psi(f(\theta_i)+s_i+\psi)}{s_i+\psi}G_i^U(\gamma) = \frac{\psi(f(\theta_i)+s_i+\psi)}{s_i+\psi} \tag{24}$$

Define $\alpha_i = \frac{\psi(f(\theta_i)+s_i+\psi)}{s_i+\psi}$. The solution of the above differential equation gives the following endogenous distribution:

$$G_i^U(\gamma) = 1 - e^{-\alpha_i \gamma} \tag{25}$$

This implies that the distribution is exponential with parameter α_i.

3.2 Equilibrium

This section describes the model's equilibrium. Equilibrium is defined as a triple (u_i, θ_i, w_i) for each group i. Therefore, we will first determine the wage equation and derive the job creation condition to find θ_i. Using the obtained θ_i and the unemployment rate u_i from Section 3.1, we will subsequently determine the equilibrium vacancies and unemployment rates.

First, by utilizing the Bellman equation and the Nash bargaining rule (11), we can derive the following wage equation:

$$w_i(\gamma) = (1-\beta)(r+\psi)U_i(\gamma) + \beta h_i(\gamma)p_i \tag{26}$$

Next, using the Bellman equations, we can derive the following surplus:

$$S_i(\gamma) = \frac{h_i(\gamma)p_i - (r+\psi)U_i(\gamma)}{r+\psi+s_i} \tag{27}$$

Here, the wage $w_i(\gamma)$ is a function of $U_i(\gamma)$. By substituting the surplus expression and the Nash bargaining rule (11) into Equation (6) and solving the resulting differential equation, we obtain the following results:

$$\begin{aligned}(r+\psi)U_i(\gamma) = &\left[e^{-\rho_i(\overline{\gamma}_i-\gamma)}\left(\frac{r+\psi+s_i+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}{r+\psi+s_i+\beta f(\theta_i)+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}\right)\right.\\ &\left.+\frac{(r+\psi+s_i)(1-e^{-\rho_i(\overline{\gamma}_i-\gamma)})}{r+\psi+s_i+\beta f(\theta_i)}\right]b_i\\ &+\frac{\beta f(\theta_i)}{r+\psi+s_i+\beta f(\theta_i)+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}h_i(\gamma)p_i\end{aligned} \tag{28}$$

where $\rho_i = \frac{(r+\psi+s_i+\beta f(\theta_i))(r+\psi)}{r+\psi+s_i}$.

We can then use Nash bargaining rule (11) and the free entry condition ($V_i = 0$) to derive the following job creation condition.

$$\frac{c_i}{q(\theta_i)} = \frac{(1-\beta)}{r+\psi+s_i}\Phi(f(\theta_i)) \tag{29}$$

where

$$
\begin{aligned}
\Phi(f(\theta_i)) = & \left(\frac{r+\psi+s_i+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}{r+\psi+s_i+\beta f(\theta_i)+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i} \right) \frac{\alpha_i}{\alpha_i+\delta_i}\left(1-e^{-(\delta_i+\alpha_i)\overline{\gamma}_i}\right) p_i \\
& - \left(\frac{r+\psi+s_i+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}{r+\psi+s_i+\beta f(\theta_i)+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i} \right) \frac{\alpha_i}{\rho_i-\alpha_i}\left(e^{-\alpha_i\overline{\gamma}_i}-e^{-\rho_i\overline{\gamma}_i}\right) b_i \\
& - \left(\frac{r+\psi+s_i}{r+\psi+s_i+\beta f(\theta_i)} \right) \left[1-e^{-\alpha_i\overline{\gamma}_i}-\frac{\alpha_i}{\rho_i-\alpha_i}\left(e^{-\alpha_i\overline{\gamma}_i}-e^{-\rho_i\overline{\gamma}_i}\right)\right] b_i
\end{aligned} \tag{30}
$$

The equilibrium labor market tightness θ_i for $i = 1, 2$ will be determined by the above conditions. Additionally, the left-hand side of the equation represents the expected cost of posting a vacancy, while the right-hand side represents the expected return from hiring workers. This can be interpreted similarly to the general intuition that employment is decided at the point where marginal cost equals marginal revenue.

3.3 Total factor productivity

In this paper, we assume that the economy's TFP is endogenous and depends on the average human capital. When $\gamma \leq \overline{\gamma}_i$, each group worker's productivity is given by $y_i = h_i(\gamma)p_i$, and when $\gamma > \overline{\gamma}_i$, $h_i(\gamma)p_i = h_i(\overline{\gamma}_i)p_i = b_i$ for $i \in \{1, 2\}$. Then, the economy's TFP ($\overline{y}$) can be defined as follows:

$$
\begin{aligned}
\overline{y} = & (1-\pi^e)\left\{\int_0^{\overline{\gamma}_1} h_1(\gamma)p_1\, dG_1^E(\Gamma) + \int_{\overline{\gamma}_1}^{\infty} b_1\, dG_1^E(\Gamma)\right\} \\
& + \pi^e\left\{\int_0^{\overline{\gamma}_2} h_2(\gamma)p_2\, dG_2^E(\Gamma) + \int_{\overline{\gamma}_2}^{\infty} b_2\, dG_2^E(\Gamma)\right\}
\end{aligned} \tag{31}
$$

From the result in Section 3.1 where $G_i^E(\gamma) = G_i^U(\gamma)$ and the fact that $G_i^U(\gamma)$ follows an exponential distribution, we obtain:

$$
\overline{y} = (1-\pi^e)\left[\frac{p_1\alpha_1}{\alpha_1+\delta_1}\{1-e^{-(\alpha_1+\delta_1)\overline{\gamma}_1}\}+b_1e^{-\alpha_1\overline{\gamma}_1}\right]+\pi^e\left[\frac{p_2\alpha_2}{\alpha_2+\delta_2}\{1-e^{-(\alpha_2+\delta_2)\overline{\gamma}_2}\}+b_2e^{-\alpha_2\overline{\gamma}_2}\right] \tag{32}
$$

As we can see from the above equations, the economy's TFP ($\overline{y}$) depends on the share of highly educated workers among the employed, denoted as π^e. This is because, similar to Lagos (2006), aggregate productivity is derived by aggregating across active production units. Moreover, as previously noted, π^e is a function of the share of highly educated workers among the unemployed, π^u. Therefore, fluctuations in π^u lead to fluctuations in π^e. Furthermore, changes in the share imply changes in the unemployment rates of each group, which means that labor market characteristics, such as the job finding rate and job separation rate for each group, are affected. Specifically, the parameter α_i in the unemployment history distribution is influenced by the job finding rate and the job separation rate. As α_i is dependent on these rates, fluctuations in the share of highly educated workers among unemployed directly imply variations in both the job finding rate and the job separation rate. Consequently, changes in the share lead to adjustments in α_i, which in turn affect the distribution of unemployment histories and, ultimately, the overall productivity in the economy.

As a result, in the endogenous TFP function derived in this paper, all components other than p_i and b_i will vary with changes in the share. This implies that changes in the share are the primary drivers of fluctuations in TFP.

4. Quantitative Results

This section reports various simulations of the model to highlight and distinguish the ways in which the share of highly educated workers among the unemployed affects TFP fluctuations.

The analysis and discussion are divided into several subsections. First, the chosen parameter values are discussed. The calibration generally follows Ortego-Marti (2017a) to measure the gain in explaining TFP from considering the share of highly educated workers rather than just unemployment. However, since the primary purpose of this paper is to analyze time series fluctuations, which were not the focus of previous studies, the parameter values of the model were also selected to reflect time series characteristics.

Second, the role of the share of highly educated workers among the unemployed is examined. Considering this share provides two main advantages over examining unemployment alone. First, the impact of skill loss on TFP varies depending on the composition of unemployment, leading to greater fluctuations. Second, it allows us to account for separate labor markets due to worker heterogeneity. When only unemployment is considered, there exists only one labor market, and the fluctuating variables represent characteristics arising from that single market. However, by considering two distinct markets, we must now account for the volatility arising from each market, which leads to greater variability compared to considering just one market. Consequently, when considering a model where productivity is endogenously determined, this additional volatility can induce larger fluctuations in productivity.

4.1. Calibration

To derive quantitative results, the calibration requires the following parameters: δ_i, f_i, s_i, ψ, b_i, and p_i. The parameter δ_i is set according to the empirical evidence presented in Section 2 and remains constant over time. The labor market flow parameters, f_i and s_i, are based on estimates calculated from 1964 to 2007 using the methodology proposed by Elsby et al. (2013). The parameter ψ, which represents the rate at which workers leave the labor force, is assumed to be identical for both groups, with a monthly value of 0.0021.

Finally, the parameters b_i and p_i are calibrated as follows:

- The overall productivity p_1 for the less educated worker group is normalized to 1. Using this, we calculate the average wage $\overline{w}_{1t}$ for each time period (the specific form of $\overline{w}_{it}$ can be found in the Appendix A). By multiplying the calculated $\overline{w}_{1t}$ by the UI replacement ratio of 0.73, as suggested by Hall and Milgrom (2008), we derive b_{1t} for each time period ($b_{1t}/\overline{w}_{1t} = 0.73$).

- The average wage $\overline{w}_{2t}$ for highly educated workers at each time period is calculated by multiplying $\overline{w}_{1t}$, obtained in the previous stage, by the wage ratio between the two groups as derived from the CPS data. Multiplying the calculated $\overline{w}_{2t}$ by the UI replacement ratio of 0.73 yields b_{2t}. Using the calculated $\overline{w}_{2t}$ and b_{2t}, we numerically compute p_{2t}.

4.2. The Impact of the Share of Highly Educated Unemployed Workers on TFP Fluctuations

The economy's TFP $\overline{y}$ in (32) is determined by the overall efficiency p_1 and p_2 of each group, as well as the labor market flows of each group and the average unemployment history of workers, which in turn influences the average human capital of the economy. This framework is similar in broad terms to the TFP determinants considered in Ortego-Marti (2017a), which only focused on unemployment. However, there are key differences:

First, the impact of the average unemployment history on economic productivity differs. Unlike the previous approach, which only considered unemployment levels, the composition of unemployment now becomes a significant channel affecting productivity. For instance, consider two periods with the same unemployment rate of 10%. Even if the unemployment rate remains constant, differences in the composition of the unemployed can lead to varying impacts on average productivity due to the differing degrees of skill loss associated with educational attainment. Specifically, if one period has a higher share of highly educated unemployed workers, this higher share results in relatively greater human capital loss, leading to a more pronounced decrease in productivity. While previous analyses assumed that identical unemployment rates would yield the same impact on productivity, incorporating the share reveals that differing compositions can significantly alter the effects of skill loss on productivity, even when the unemployment rates are the same.

Second, this framework allows us to account for separate labor markets due to worker heterogeneity. When only unemployment is considered, there exists only one labor market, and the fluctuating variables represent characteristics arising from that single market. However, by considering two distinct markets, we must now address the volatility arising from each, leading to greater variability compared to analyzing just one market. Consequently, in a model where productivity is endogenously determined, this additional volatility can induce larger fluctuations in productivity.

The results are reported in Table 4. We focus exclusively on the period before the financial crisis and the U.S. economy. Data on observed TFP is drawn from the Penn World Table 10.0 (PWT 10.0), with detailed information on its construction and TFP estimation provided by Feenstra et al. (2015).

Figure 2 shows the results from the benchmark model that considers only unemployment, as in Ortego-Marti (2017a). The left graph displays the observed TFP from PWT 10.0, represented by the blue line, while the red line shows the TFP computed from the benchmark model. Although the observed TFP exhibits an upward trend, the benchmark model's TFP does not, as it considers only unemployment.

However, our main focus is on explaining TFP fluctuations, so it is crucial to compare the detrended series from each model. This comparison is shown in the right graph, where the detrended series from both the model and observed TFP exhibit fairly similar movements, with a correlation of 0.579. To measure the explanatory power of the model TFP regarding the observed TFP, we performed a regression of observed TFP on model TFP, obtaining an R^2 value of 0.335. This indicates that the model TFP explains 33.5% of the movement in observed TFP.

Figure 3 compares the model TFP with the observed TFP for the model that considers the share of highly educated workers among unemployed. Unlike the benchmark model, this model's TFP also exhibits an upward trend. This trend can be attributed to the increase in the share of highly educated workers among employed during the period, which likely led to an increase in average human capital. The right graph compares the detrended series of both TFP measures, showing very similar movements with a correlation of 0.612. This correlation is higher than in the benchmark model, demonstrating the effectiveness of incorporating the share in the model. Similarly, the R^2 value from regressing model TFP on observed TFP is 0.375, indicating a 4%p or 12% increase in explanatory power compared to the benchmark. This shows that considering the share is crucial for explaining TFP fluctuations.

These results are due to the complex effects resulting from the introduction of the share of highly educated workers unemployed. To distinguish between the effects of composition and variations in the composition of unemployment, the following additional simulations were conducted. The third column of Table 4 presents the results of the model TFP calculation assuming that the human capital depreciation rates for the two groups are different, while assuming all other aspects are the same as in the benchmark model, as one method to exclude composition effects. In this case, unlike the baseline model, the analysis removes labor market fluctuations specific to each market, focusing only on overall labor market fluctuations similar to the benchmark model. This adjustment eliminates the effects of variation in composition. Consequently, the impact of unemployment history on average human capital is assumed to differ only for both groups, leaving only the pure composition effect. Now, the model TFP can explain 34.4% of the observed TFP movements. Thus, the 0.9%p difference in explanatory power between the benchmark case (33.5%) and the restricted case (34.4%) can be attributed to considering the composition. Consequently, the increase in the explanatory power of TFP due to the introduction of the share is primarily driven by the variation in the composition of unemployment resulting from considering separate labor markets.

5. The Impact of Hiring Subsidies on TFP

This section examines the effect of labor market policy on TFP, with a particular focus on hiring subsidies. As discussed in Pissarides (2000), Lagos (2006), and Ortego-Marti (2017a), hiring subsidies for firms in each group work to increase productivity by providing firms with a government subsidy when a job is created. The size of this subsidy is proportional to the job's productivity.

These subsidies enhance the incentives for job creation, leading to an increase in the number of active production units in the economy. As a result, overall economic productivity—particularly total factor productivity (TFP)—is positively impacted.

Therefore, the Bellman equation (9) for the value function of an open vacancy, when considering the hiring subsidy, is modified as follows:

$$rV_i = -c_i + q(\theta_i) \int_0^{\infty} [\max\{J_i(\Gamma) + \tau_{h_i} h_i(\Gamma) p_i, V_i\} - V_i] dG_i^U(\Gamma) \tag{33}$$

By incorporating the hiring subsidy, another aspect of the model changes compared to its previous version. Since the subsidy is only applied at the time of hiring, the rent generated by search frictions at this point differs from the rent when no subsidy is applied. As a result, there are now two distinct types of wages based on the timing of hiring.

- w_{0i}: The wage determined at the time of hiring through the bargaining process.
- w_i: The continuing wage, which is the wage after the worker has accepted the job offer.

The wage at the time of hiring, w_{0i}, is determined by solving the following surplus division problem:

$$w_{0i}(\gamma) = \arg\max (W_{0i}(\gamma) - U_i(\gamma))^{\beta} (J_{0i}(\gamma) + \tau_{h_i} h_i(\gamma) p_i - V_i)^{1-\beta} \tag{34}$$

where $W_{0i}(\gamma)$ and $J_{0i}(\gamma)$ represent the value functions at the time of hiring for group i. Specifically, $W_{0i}(\gamma)$ is the value function of the worker in group i at the time of employment, reflecting the expected utility based on the hiring wage w_{0i}. Similarly, $J_{0i}(\gamma)$ is the firm's value function at the hiring stage for group i, capturing the expected profit from the match, factoring in the hiring subsidy.

These value functions differ from the ongoing value functions, W_i and J_i, which apply after the initial hiring phase, when the hiring subsidy no longer has an effect. The distinction between these value functions emphasizes how the hiring subsidy creates a unique incentive at the time of job creation, influencing both productivity and TFP by increasing the number of active production units in the economy.

Using the free entry condition $V_i = 0$, we have the following relationship:

$$(1-\beta)(W_{0i}(\gamma) - U_i(\gamma)) = \beta \left(J_{0i}(\gamma) + \tau_{h_i} h_i(\gamma) p_i\right) \tag{35}$$

This equation can be solved for the hiring wage $w_{0i}(\gamma)$ as follows:

$$w_{0i}(\gamma) = (1-\beta)(r+\psi)U_i(\gamma) + \beta\left[1 + \tau_{h_i}(r+\psi+s_i)\right] h_i(\gamma) p_i \tag{36}$$

Combining the Bellman equations gives the surplus:

$$S_{0i}(\gamma) = \frac{h_i(\gamma) p_i \left[1 + \tau_{h_i}(r+\psi+s_i)\right] - (r+\psi)U_i(\gamma)}{r+\psi+s_i} \tag{37}$$

Solving the differential equation in the same manner as in the case without a subsidy, we obtain the following $U_i(\gamma)$:

$$\begin{aligned}(r+\psi)U_i(\gamma) = & \left[e^{-\rho_i(\overline{\gamma}_i - \gamma)} \left(\frac{r+\psi+s_i - \beta f(\theta_i) T_i + \left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}{r+\psi+s_i+\beta f(\theta_i) + \left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i} \right) \right. \\ & \left. + \frac{(r+\psi+s_i)(1-e^{-\rho_i(\overline{\gamma}_i-\gamma)})}{r+\psi+s_i+\beta f(\theta_i)} \right] b_i \\ & + \left[\frac{\beta f(\theta_i)(1+T_i)}{r+\psi+s_i+\beta f(\theta_i) + \left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i} \right] h_i(\gamma) p_i \end{aligned} \tag{38}$$

where $T_i = \tau_{h_i}(r+\psi+s_i)$. Finally, the job creation condition becomes

$$\frac{c_i}{q(\theta_i)} = \left(\frac{1-\beta}{r+\psi+s_i}\right) \Phi(f(\theta_i)) + \tau_{h_i} b_i e^{-\alpha_i \overline{\gamma}_i} \tag{39}$$

where

$$
\begin{aligned}
\Phi(f(\theta_i)) = & \frac{r+\psi+s_i+\frac{r+\psi+s_i}{r+\psi}\delta_i}{r+\psi+s_i+\beta f(\theta_i)+\frac{r+\psi+s_i}{r+\psi}\delta_i} \cdot \frac{a_i}{\alpha_i+\delta_i}\left(1-e^{-(\delta_i+\alpha_i)\overline{\gamma}_i}\right) p_i(1+T_i) \\
& - \frac{r+\psi+s_i-\beta f(\theta_i)T_i+\frac{r+\psi+s_i}{r+\psi}\delta_i}{r+\psi+s_i+\beta f(\theta_i)+\frac{r+\psi+s_i}{r+\psi}\delta_i} \cdot \frac{\alpha_i}{\rho_i-\alpha_i}\left(e^{-\alpha_i\overline{\gamma}_i}-e^{-\rho_i\overline{\gamma}_i}\right) b_i \\
& - \frac{r+\psi+s_i}{r+\psi+s_i+\beta f(\theta_i)}\left[1-e^{-\alpha_i\overline{\gamma}_i}-\frac{\alpha_i}{\rho_i-\alpha_i}\left(e^{-\alpha_i\overline{\gamma}_i}-e^{-\rho_i\overline{\gamma}_i}\right)\right] b_i
\end{aligned}
\tag{40}
$$

Although the equation appears complex, the intuition remains similar to the case without subsidies. The left-hand side of equation (39) represents the expected cost of vacancy posting, while the right-hand side represents the future benefits from hiring workers.

To quantify the impact of hiring subsidies on TFP, it is now necessary to additionally determine the market tightness, θ_i. In the case without subsidies, TFP depends solely on the job finding rate and does not require information about market tightness. However, with subsidies, which affect firms' job opening incentives, market tightness θ_i changes. Therefore, to measure the effect of subsidies, it is essential to calculate how the change in θ_i induced by the subsidy impacts overall productivity through the job finding rate.

We assume a specific functional form for the job finding rate: $f(\theta_i) = m_{0i}\theta_i^{1-\eta}$. To perform calibration, we need additional information on m_{0i}, c_i, η, and β. Firstly, following Pissarides (2009) and Ortego-Marti (2017a), we set $\eta = 0.5$. We also assume the Hosios condition, which is commonly assumed in the literature, $\eta = \beta$. The remaining parameters to determine are m_{0i} and c_i for each group. These are calculated through the following process:

- Assume $\theta_i = 1$, then $m_{0i} = f(\theta_i)$.
- Using the job creation condition, calculate c_i.

By completing this process, we obtain all the necessary variables to calculate the job finding rate. With the computed job finding rate function, we can then determine how the subsidy affects θ_i. Specifically, assuming $\tau_i = 0.5$, θ_i is calculated from the job creation condition using the formula:

$$\theta_i = \left\{ \frac{c_i}{\overline{\Phi}(f(\theta_i))m_{0i}} \right\}^{-1/\eta}. \tag{41}$$

Substituting the newly calculated θ_i into each group's job finding rate function allows us to derive the job finding rate after the subsidy and subsequently calculate the new TFP.

Table 5 reports the ratio of average TFP with a hiring subsidy to average TFP in the model without the subsidy across different periods, illustrating the impact of the subsidy on TFP. The parameter $\tau_{h_i} = 0.5$ represents a government policy where a one-off payment is given at the time of job creation, equal to half of the match's output. This subsidy is shown to increase average productivity in the economy by approximately 1%.

However, this effect is primarily driven by less-educated workers, with a productivity increase of 0.9 percentage points. This result aligns with Pries (2008), which found that firms with lower accounting profits are more responsive to productivity increases regarding job creation incentives. Intuitively, firms tend to respond to changes in the percentage of accounting profit. When accounting profits are low, a given absolute increase in productivity leads to a relatively larger change in accounting profit, prompting firms to react more strongly by opening more vacancies.

One way to enhance the impact of subsidies on TFP is by introducing fixed matching costs alongside the proportional posting cost, as discussed by Pissarides (2009). Fixed costs are interpreted as expenses incurred after a worker arrives but before wage negotiations, such as assessing the worker's qualifications, conducting interviews, and negotiating terms. These costs are sunk prior to the wage agreement can be regarded as part of the friction costs in search models.

When a worker arrives, the firm pays a fixed fee H_i before agreeing on the Nash wage. This fixed cost does not directly influence the wage bargaining process but increases the overall cost of hiring a worker in the vacancy equation. These fixed costs can enhance productivity for the following reasons. The hiring subsidy has an effect similar to a positive shock to productivity, leading firms to post more vacancies at a cost c. However, due to search externalities, the entry of more vacancies increases the average duration of vacancies ($1/q(\theta)$), which in turn raises the marginal cost of vacancy as represented on the left side of equation (39). Consequently, the incentive for firms to open vacancies due to the subsidy decreases, which indicates a response in tightness (θ), preventing the job finding rate from increasing as expected and thereby reducing the extent of productivity gains. HHowever, fixed costs do not depend on the average duration of vacancies and are not influenced by search externalities. As a result, when a positive productivity shock occurs, even if firms open more jobs, the duration does not increase proportionally. Consequently, the incentive for firms to open vacancies due to the subsidy decreases to a lesser extent, and the volatility of job creation increases.

Then, Bellman equation and job creation condition becomes

$$rV_i = -c_i + q(\theta_i)\left(\int_0^\infty (J_i(\Gamma) - V_i)\, dG_i^U(\Gamma) - H_i\right) \tag{42}$$

$$\frac{c_i}{q(\theta_i)} + H_i = \frac{1-\beta}{r+\psi+s_i}\Phi_i(f(\theta_i)) + \tau_h b_i e^{-\alpha_i \overline{\gamma}_i} \tag{43}$$

When using the value $H_i = 0.4$ as used by Pissarides(2009), the same size of subsidy now increases TFP by 1.23%. However, most of this increase still comes from less educated workers, contributing 1.12 percentage points.

6. Conclusion

This paper develops a theory of TFP fluctuations driven by skill loss due to unemployment. While traditional growth theory focuses on long-run relationships between variables, reality often involves frictions and persistent unemployment instead of full employment.

This paper aims to synthesize growth theory with short-run macroeconomic phenomena, particularly variations in unemployment rate. Specifically, we demonstrate that not only does unemployment itself impact productivity, but the composition of unemployment by educational attainment is also a crucial factor influencing TFP fluctuations. This approach serves as a guide for uncovering sources of long-run TFP growth, extending beyond merely explaining short-term fluctuations.

Appendix

A. Average Wage $\overline{w}_i$

In this section, we will derive the specific functional form for $\overline{w}_i$. First, the wage $w_i(\gamma)$ is determined as follows:

$$w_i(\gamma) = \beta h_i(\gamma) p_i + (1-\beta)(r+\psi)U_i(\gamma) \tag{44}$$

where $U_i(\gamma)$ is given as follows:

$$\begin{aligned}(r+\psi)U_i(\gamma) = &\left[e^{-\rho_i(\overline{\gamma}_i - \gamma)} \frac{r+\psi+s_i+\frac{r+\psi+s_i}{r+\psi}\delta_i}{r+\psi+s_i+\beta f(\theta_i)+\frac{r+\psi+s_i}{r+\psi}\delta_i} \right. \\ &\left. + \frac{(r+\psi+s_i)(1-e^{-\rho_i(\overline{\gamma}_i-\gamma)})}{r+\psi+s_i+\beta f(\theta_i)} \right] b_i \\ &+ \frac{\beta f(\theta_i)}{r+\psi+s_i+\beta f(\theta_i)+\frac{r+\psi+s_i}{r+\psi}\delta_i} h_i(\gamma) p_i\end{aligned} \tag{45}$$

Substituting equation (45) into equation (44), we obtain the following result:

$$\begin{aligned}w_i(\gamma) =& \beta h_i(\gamma) p_i \\ &+ (1-\beta) \left\{ \left[e^{-\rho_i(\overline{\gamma}_i - \gamma)} \frac{r+\psi+s_i+\frac{r+\psi+s_i}{r+\psi}\delta_i}{r+\psi+s_i+\beta f(\theta_i)+\frac{r+\psi+s_i}{r+\psi}\delta_i} \right. \right. \\ &+ \left. \left. \frac{(r+\psi+s_i)(1-e^{-\rho_i(\overline{\gamma}_i-\gamma)})}{r+\psi+s_i+\beta f(\theta_i)} \right] b_i + \frac{\beta f(\theta_i)}{r+\psi+s_i+\beta f(\theta_i)+\frac{r+\psi+s_i}{r+\psi}\delta_i} h_i(\gamma) p_i \right\}\end{aligned} \tag{46}$$

The average wage $\overline{w}_i$ for each group i, which is the average across all unemployment histories, can be calculated as follows:

$$\overline{w}_i = \int_0^\infty w_i(\Gamma)\, dG^E(\Gamma) \tag{47}$$

This is divided into the following two parts by $\bar{\gamma}_i$:

$$\overline{w}_i = \underbrace{\begin{array}{c}\int_0^{\overline{\gamma}_i}\left\{\beta h_i(\Gamma)p_i + (1-\beta)\left[e^{-\rho_i(\overline{\gamma}_i-\Gamma)}\cdot\frac{r+\psi+s_i+\frac{r+\psi+s_i}{r+\psi}\delta_i}{r+\psi+s_i+\beta f(\theta_i)+\frac{r+\psi+s_i}{r+\psi}\delta_i}\right.\right.\\ \left.\left.+\frac{(r+\psi+s_i)(1-e^{-\rho_i(\overline{\gamma}_i-\Gamma)})}{r+\psi+s_i+\beta f(\theta_i)}\right]b_i + \frac{\beta f(\theta_i)}{r+\psi+s_i+\beta f(\theta_i)+\frac{r+\psi+s_i}{r+\psi}\delta_i}h_i(\Gamma)p_i\right\}dG^E(\Gamma)\end{array}}_{\text{Part I}}$$

$$+\underbrace{\begin{array}{c}\int_{\overline{\gamma}_i}^{\infty}\left\{\beta b_i + (1-\beta)\left[e^{-\rho_i(\overline{\gamma}_i-\Gamma)}\cdot\frac{r+\psi+s_i+\frac{r+\psi+s_i}{r+\psi}\delta_i}{r+\psi+s_i+\beta f(\theta_i)+\frac{r+\psi+s_i}{r+\psi}\delta_i}\right.\right.\\ \left.\left.+\frac{(r+\psi+s_i)(1-e^{-\rho_i(\overline{\gamma}_i-\Gamma)})}{r+\psi+s_i+\beta f(\theta_i)}\right]b_i + \frac{\beta f(\theta_i)}{r+\psi+s_i+\beta f(\theta_i)+\frac{r+\psi+s_i}{r+\psi}\delta_i}b_i\right\}dG^E(\Gamma)\end{array}}_{\text{Part II}} \tag{48}$$

Then, Part I becomes

$$\begin{aligned}&\beta\frac{p_i a_i}{\alpha_i+\delta_i}(1-e^{-(\alpha_i+\delta_i)\overline{\gamma}_i})\\ &+(1-\beta)\left[\frac{r+\psi+s_i+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}{r+\psi+s_i+\beta f(\theta_i)+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}\right]\frac{\alpha_i}{\rho_i-\alpha_i}\left(e^{-\alpha_i\overline{\gamma}_i}-e^{-\rho_i\overline{\gamma}_i}\right)b_i\\ &+(1-\beta)\frac{r+\psi+s_i}{r+\psi+s_i+\beta f(\theta_i)}\left[1-e^{-\alpha_i\overline{\gamma}_i}-\frac{\alpha_i}{\rho_i-\alpha_i}\left(e^{-\alpha_i\overline{\gamma}_i}-e^{-\rho_i\overline{\gamma}_i}\right)\right]b_i\\ &+(1-\beta)\frac{\beta f(\theta_i)}{r+\psi+s_i+\beta f(\theta_i)+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}\frac{p_i a_i}{\alpha_i+\delta_i}(1-e^{-(\alpha_i+\delta_i)\overline{\gamma}_i})\end{aligned} \tag{49}$$

Part II becomes

$$\begin{aligned}&=b_i e^{-\alpha_i\overline{\gamma}_i}\left\{\beta+(1-\beta)\left[\frac{r+\psi+s_i}{r+\psi+s_i+\beta f(\theta_i)}\frac{\rho_i}{\rho_i-\alpha_i}\right.\right.\\ &\quad+\frac{\beta f_i(\theta_i)}{r+\psi+s_i+\beta f(\theta_i)+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}\\ &\quad\left.\left.-\left(\frac{r+\psi+s_i+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}{r+\psi+s_i+\beta f(\theta_i)+\left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}\right)\frac{\alpha_i}{\rho_i-\alpha_i}\right]\right\}\end{aligned} \tag{50}$$

Finally, average wage $\overline{w}_i$ is

$$
\begin{aligned}
\overline{w}_i =& \beta \frac{p_i a_i}{\alpha_i + \delta_i} \left(1 - e^{-(\alpha_i+\delta_i)\overline{\gamma}_i}\right) \\
& + (1-\beta) \Bigg\{ \left(\frac{r + \psi + s_i + \left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}{r + \psi + s_i + \beta f(\theta_i) + \left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i} \right) \frac{\alpha_i}{\rho_i - \alpha_i} \left(e^{-\alpha_i \overline{\gamma}_i} - e^{-\rho_i \overline{\gamma}_i}\right) b_i \\
& \quad + \frac{r+\psi+s_i}{r+\psi+s_i+\beta f(\theta_i)} \left[1 - e^{-\alpha_i\overline{\gamma}_i} - \frac{\alpha_i}{\rho_i - \alpha_i}\left(e^{-\alpha_i \overline{\gamma}_i} - e^{-\rho_i \overline{\gamma}_i}\right)\right] b_i \\
& \quad + \frac{\beta f(\theta_i)}{r + \psi + s + \beta f(\theta_i) + \left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i} \frac{p_i a_i}{\alpha_i + \delta_i} \left(1 - e^{-(\alpha_i+\delta_i)\overline{\gamma}_i}\right) \Bigg\} \\
& + b_i e^{-\alpha_i \overline{\gamma}_i} \Bigg\{ \beta + (1-\beta) \Bigg[\frac{r+\psi+s_i}{r+\psi+s_i+\beta f(\theta_i)} \frac{\rho_i}{\rho_i - \alpha_i} \\
& \quad + \frac{\beta f_i(\theta_i)}{r + \psi + s + \beta f(\theta_i) + \left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i} \\
& \quad - \left(\frac{r + \psi + s_i + \left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i}{r + \psi + s_i + \beta f(\theta_i) + \left(\frac{r+\psi+s_i}{r+\psi}\right)\delta_i} \right) \frac{\alpha_i}{\rho_i - \alpha_i} \Bigg] \Bigg\}
\end{aligned} \tag{51}
$$

B. Cost-Benefit Analysis for hiring Subsidy

In Section 6, we simplify the analysis by ignoring that the government's financing constraints. Requiring the government to run a balanced budget could be a natural extension, but instead, one way to evaluate the effect of the policy is by comparing the costs and benefits associated with hiring subsidies.

- **Cost of Subsidy**
  - The cost of the subsidy can be calculated as:
    - Subsidy amount $\times$ Number of jobs generated $=$ Subsidy amount $\times q(\theta) \times v$
    - Specifically,
      $(1-\pi^e) \times \tau_h \times h_1(\gamma) \times p_1 \times q(\theta_1) \times v_1 + \pi^e \times \tau_h \times h_2(\gamma) \times p_2 \times q(\theta_2) \times v_2$
    - Alternatively,
      $(1-\pi^e) \times \tau_h \times h_1(\gamma) \times p_1 \times f(\theta_1) \times u_1 + \pi^e \times \tau_h \times h_2(\gamma) \times p_2 \times f(\theta_2) \times u_2$
    - Note: Since $f(\theta_i) \times u_i = \theta_i \times q(\theta_i) \times u_i = q(\theta_i) \times v_i$, the above can be simplified.
- **Benefit of Subsidy**
  - The benefit is calculated as the difference in output due to the subsidy:
    - Benefit $=$ Number of workers employed $\times$ Output
    - Specifically,

$$(1-u_{\text{after}}) \times \overline{y}_{\text{after}} - (1-u_{\text{before}}) \times \overline{y}_{\text{before}}$$

- **Cost at t (integrate cost for all γ)**

$$\begin{aligned} &\int_0^\infty \left[(1-\pi^e) \times \tau_h \times h_1(\gamma) \times p_1 \times f(\theta_1) \times u_1\right] dG_1^U(\Gamma) \\ &+ \int_0^\infty \left[\pi^e \times \tau_h \times h_2(\gamma) \times p_2 \times f(\theta_2) \times u_2\right] dG_2^U(\Gamma) \end{aligned} \tag{52}$$

$$
\begin{aligned}
&= (1-\pi^e) \times f(\theta_1) \times u_1 \times \tau_h \left[\frac{p_1 \alpha_1}{\alpha_1 + \delta_1} \left(1 - e^{-(\alpha_1+\delta_1)\overline{\gamma}_1}\right) + b_1 e^{-\alpha_1 \overline{\gamma}_1} \right] \\
&\quad + \pi^e \times f(\theta_2) \times u_2 \times \tau_h \left[\frac{p_2 \alpha_2}{\alpha_2 + \delta_2} \left(1 - e^{-(\alpha_2+\delta_2)\overline{\gamma}_2}\right) + b_2 e^{-\alpha_2 \overline{\gamma}_2} \right]
\end{aligned} \tag{53}
$$

The magnitude of the subsidy, denoted by τ_i, affects both the cost and benefit. As τ_i increases, the ratio of benefit to cost decreases. Specifically, as the subsidy size increases, the relative cost becomes larger, and the ratio decreases from 0.914 to 0.755 as τ_i ranges from 0.05 to 1. This indicates that, at any level of subsidy, the cost always exceeds the benefit. This is because the job creation incentive provided by the subsidy is insufficient. One way to address this, as proposed by Pissarides (2009), is to introduce a fixed cost. Assuming a fixed cost of 0.4, as suggested in the paper, the ratio changes to a range of 1.183 to 0.907, meaning the benefit of the subsidy can exceed the cost.

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Table and Figures

Table 1. Effects of Unemployment History on Wages			
	(1)	(2)	(3)
Unhis	-0.012***	-0.010***	-0.022***
	(0.000)	(0.000)	(0.001)
N	34,542	23,065	11,477

Table 1: Effects of Unemployment History on Wages

Note. Column (1) shows the baseline regression model with all samples. Column (2) refers to the group of less educated individuals. Column (3) refers to the group of highly educated individuals.

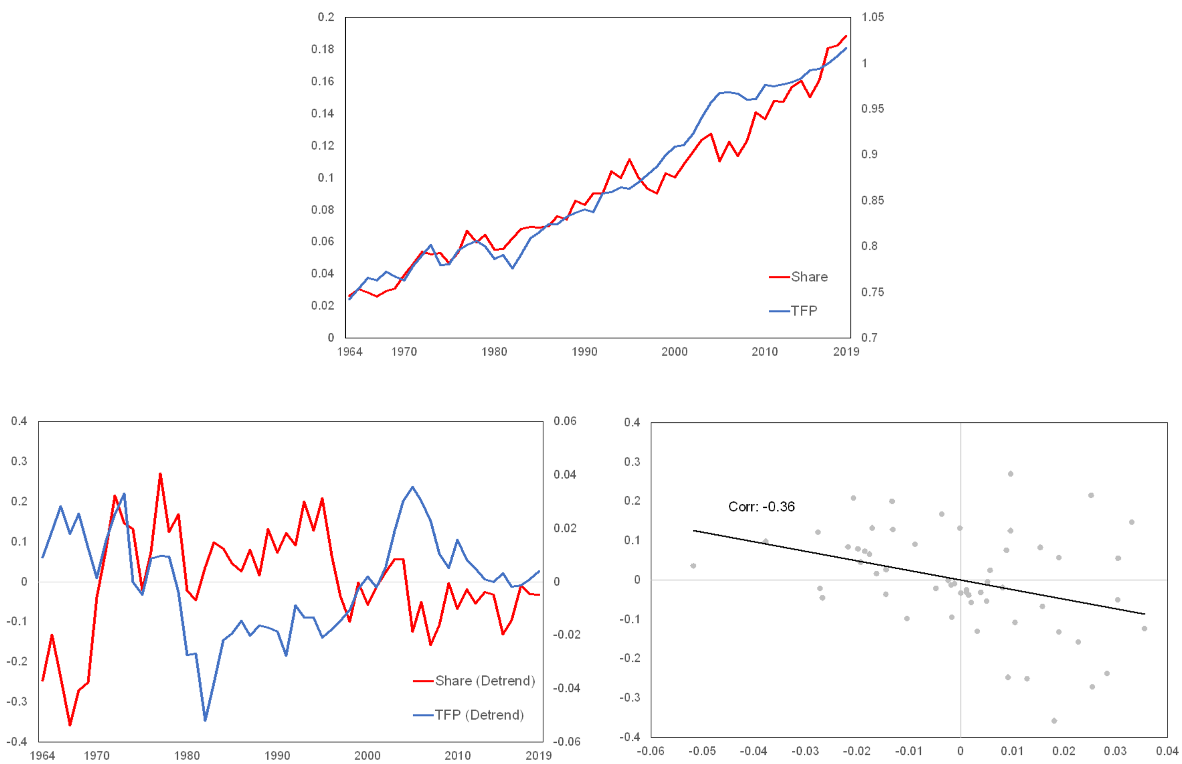

Figure 1: TFP and share of highly educated workers among unemployed.

Table 2. Effects of Share of Highly Educated Unemployed on TFP		
	(1)	(2)
Residuals of log(share)	-0.052**	-0.275
	(0.019)	(0.222)
T	56	56

Table 2: Effects of Share of Highly Educated Unemployed on TFP

Note. Column (1) corresponds to the case where the share of highly educated is used as the explanatory variable. Column (2) corresponds to the case where the share of less educated is used as the explanatory variable.

Table 3. Calibration of Parameters	
Parameter	**Value**
δ_1	0.010
δ_2	0.022
ψ	0.0021
p_1	1
p_2	Explanation Referenced
b_1	Explanation Referenced
b_2	Explanation Referenced
f_i	Elsby et al. (2013)
s_i	Elsby et al. (2013)

Table 3: Calibration of parameters

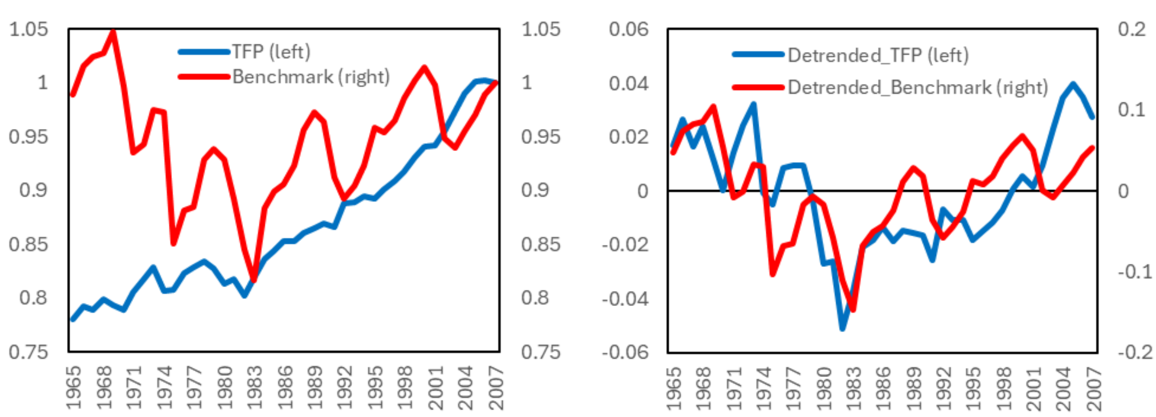

Figure 2: Benchmark vs Observed TFP

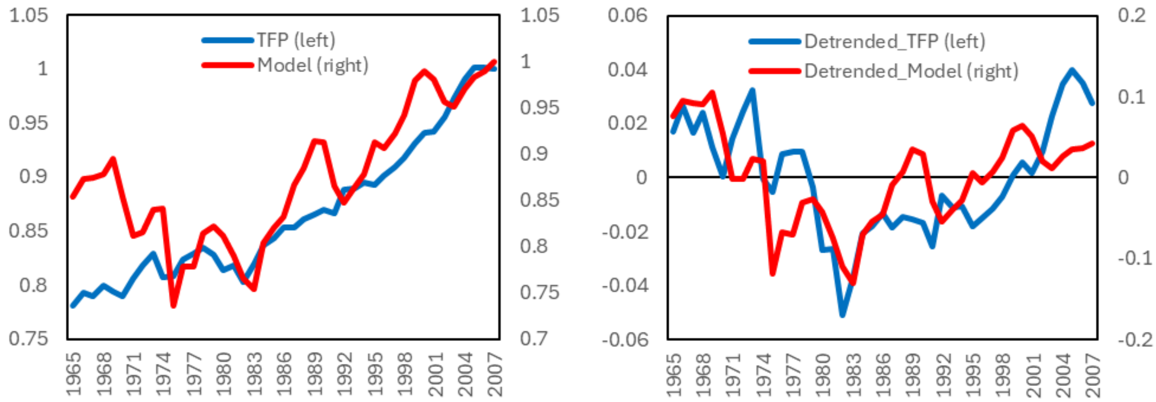

Figure 3: Model TFP vs Observed TFP

Table 4. Observed TFP vs Model TFP			
	Benchmark	**Model**	$\delta_1 \neq \delta_2$
Correlation	0.579	0.612	0.586
R^2	0.335	0.375	0.344

Table 4: Comparison of observed and model TFP across different assumptions.

Table 5. The Effects of Hiring Subsidy on TFP			
	Both	**Less Educated**	**Highly Educated**
Ratio	1.0099	1.0088	1.0010
with Fixed Cost	1.0123	1.0112	1.0011

Table 5: Effects of hiring subsidy on TFP for different groups.