A Search and Matching Model of Housing and Rental Market Interactions

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Abstract

This study presents empirical evidence demonstrating a correlation between rental vacancy rates, frictions within the housing market, and housing prices. Decreased rental vacancies are correlated with higher price-to-rent ratios, increased sales, more housing vacancies, and shorter selling times. To examine these market interactions, I develop a search and matching model of the housing market that incorporates a rental market with search frictions, heterogeneous buyers, and free entry of sellers and landlords. I simulate the model using perfectly correlated demand and supply-side shocks to match the stylized facts. It illustrates that fluctuations in rental vacancies influence agents' decisions to enter the housing market, consequently altering market tightness in both markets. Additionally, the model replicates empirically observed elasticities and key literature findings.

Keywords: Housing market, Rental vacancy, Search and Matching, Housing dynamics

JEL Classification: E2, E32, R21, R32

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1 Introduction

Housing market has search frictions, which have been analyzed using the Diamond-Mortensen-Pissarides (DMP) framework commonly applied in labor markets. Similarly, searching for a rental home is costly—equivalent to at least one month's rent—and time-consuming, typically taking 1-2 months, indicating the presence of search frictions in the rental market. Bachmann and Cooper (2014) show that the pool of potential home-buyers includes both renters transitioning to buyers and former home-owners. Consequently, market tightness in the housing market, reflecting the number of potential buyers, is influenced by the influx of renters moving into home-ownership. However, the relationship between search frictions in both the rental and housing markets and this transition has not been studied before. This paper seeks to address two key questions: First, how are search frictions in the rental market related to those in the housing market? Second, what insights can this correlation provide regarding the movement agents within and between these markets, particularly the transition from renting to homeownership?

The first contribution of this paper is to provide new stylized facts linking the frictions between rental and the housing market. Using aggregated national-level time series data, I find a negative correlation between rental vacancy rates (properties for rent) and the price-to-rent ratio. Furthermore, rental vacancy rates display negative correlations with sales and housing vacancies (properties for sale) but a positive correlation with the time it takes to sell a property. Ngai and Tenreyro (2014) show that a "hot" housing market is characterized by higher prices, increased sales, more housing vacancies, and a shorter time to sell, while a "cold" market exhibits the opposite attributes. My findings complement this characterization by revealing that in a "hot" housing market, the price-to-rent ratio tends to be high and rental vacancies are low, whereas a "cold" market is associated with a lower price-to-rent ratio and higher rental vacancies.

The second contribution of this paper is to elucidate the mechanisms driving the movement of agents within and between housing and rental markets. I develop a model incorporating search frictions in both markets, with a key focus on the endogenous decision-making process behind agents' transitions from renting to homeownership. This approach is crucial for understanding how these transitions relate to market conditions. The model successfully replicates the upward-sloping Beveridge curve observed in both markets and matches stylized facts through demand

and supply shocks. Notably, this paper is the first to explore how such shocks can lead to divergent movements along the Beveridge curve in rental and housing markets. Specifically, the model shows that a "hot" housing market correlates with an increase in transitions from renting to buying, which leads to a rise in the number of home-buyers. Simultaneously, rental market tightness increases due to a greater decrease in rental seekers than vacancies.

The model's core elements include the incorporation of search frictions in the rental market, which accounts for rental vacancies and their correlation with the price-to-rent ratio. All households start in the rental market, matching with landlords to become tenants. Tenants then decide whether to move to the housing market or remain in rentals, based on the model's key aspect: the heterogeneity of home-buyers. This heterogeneity is introduced through an idiosyncratic utility draw that arises upon separating from rental properties, influencing whether households enter the housing market. Once households become home-buyers, they match with sellers and become home-owners. The model allows free entry for sellers and landlords, facilitating new construction in both markets. However, properties themselves remain static: rental properties stay as rentals, and housing market properties remain unchanged, aligning with Glaeser and Gyourko (2007), which shows no arbitrage between rental and housing properties. I relax this assumption in section A.2, but the results do not significantly change.

After calibrating the model, I conduct simulations of business cycles to match the observed stylized facts. A demand shock which makes home-ownership more valuable over being a tenant is used to match the correlations. While this shock successfully reproduces the cross-correlation between rental and housing market variables, it falls short in replicating the correlations within the housing market as reported in previous studies. However, research by Diaz and Jerez (2013), Head et al. (2014), and Gabrovski and Ortego-Marti (2019) indicate that multiple shocks are necessary to explain key features of the housing market. By incorporating supplementary supply shocks in both markets, the model is capable of matching with the correlations within the housing market, while also accurately replicating the cross-correlations between rental and housing market variables.

To illustrate the model's mechanism, consider a boom cycle. A demand shock elevates the utility of homeownership while diminishing the utility of renting, rendering entry into the housing market more appealing and increasing the number of buyers. Concurrently, rental market demand diminishes. With heightened demand for houses for sale, housing supply increases.

However, the rental market does not witness a parallel increase in supply, resulting in a decline in rental vacancies. Consequently, housing market activity intensifies while the rental market remains stagnant. These shocks generate a cycle of boom and bust, allowing me to capture the observed stylized facts.

Related literature This paper adds to the expanding body of literature employing the search and matching model to analyze the housing market. This housing search literature includes Arnott (1989), Wheaton (1990), Diaz and Jerez (2013), Head et al. (2014), Ngai and Tenreyro (2014), Anenberg (2016), Burnside et al. (2016), Gabrovski and Ortego-Marti (2019), Ngai and Sheedy (2020), Piazzesi et al. (2020), Gabrovski and Ortego-Marti (2022), and many more. An extensive survey is done by Han and Strange (2015). This paper extends this research by developing a model featuring interconnected search frictions in both housing and rental markets. While matching in each market operates independently, the transition of households from rentals to the housing market facilitates interaction between the two sectors.

Compared to existing real estate empirical studies, my research builds upon the contributions of Diaz and Jerez (2013), Head et al. (2014), and Ngai and Sheedy (2024) by utilizing aggregated time-series data to uncover the business cycle correlations between housing and rental market variables. Halket and di Custoza (2015) examine how scarcity in rental properties affects the housing market, echoing similar empirical findings to mine. They reveal that in regions where tenants have longer tenures, rental vacancy rates are negatively correlated with the price-to-rent ratio and the home-ownership rate. Confirming this correlation, my research expands upon their findings by incorporating additional insights on sales, time-to-sell, and housing vacancies.

In terms of the joint modeling of the rental and sales segment of the market, my paper is closest to Ioannides and Zabel (2017), Han et al. (2022), Bø (2022) and Badarinza et al. (2024). Ioannides and Zabel (2017) investigate the relationship between the labor and housing markets, while my focus lies in exploring the dynamics between the housing and rental markets. Han et al. (2022) analyze how taxes impact both rental and housing markets within a steady-state framework. Bø (2022) also incorporate a rental market into their search and matching model of the housing market, aiming to study the rental market's effect on house price dynamics. In contrast, my paper examines the rental and housing markets within a business cycle framework, aiming to develop a model consistent with empirically observed correlations. Finally, Badarinza et al. (2024) explore agents on both the supply and demand sides of the market, along with intermediary steps like online search, physical meetings, and price adjustments. In contrast, my paper delves into the dynamics of agents transitioning within the rental and housing markets, including movements from rental to housing, over the business cycle. Additionally, my paper contributes to this field by developing a tractable model that aligns with empirical data and enables the study of policies in both the housing and rental markets.

The subsequent sections of this paper are structured as follows: Section 2 outlines the data sources and highlights the observed stylized facts pertaining to the housing and rental market variables within the U.S. economy at the business cycle frequency. Section 3 describes the environment and establishes the steady state model. Section 4 entails solving the model with business cycle fluctuations. In Section 5, I discuss the calibration process and simulation of the model. Section 6 presents the results, while Section 7 concludes.

2 Data and Stylized Facts

For the data analysis, aggregated national level data is used. The empirical analysis spans from Q1 1991 to Q4 2019 to ensure consistency. This time frame was selected because preceding periods may exhibit different housing market characteristics and this period would make the empirical results comparable to Ngai and Sheedy (2024). Nevertheless, the findings remain robust across alternative time periods. The data on rental vacancy rate and housing vacancies is from Housing Vacancy Survey which is made available by the United States Census Bureau. As a measure of time on market (time to sell), I use the series labelled median number of months on sales market for newly completed homes from Federal Reserve Economic Data (FRED). The original source mentioned for this series is the US Census Bureau. Time to sell is also the measure of liquidity in the housing market. As an index for house prices, I use the series named all transactions house price index from FRED. The original source has been mentioned as the US Federal Housing Finance Agency. For rents, I use the series named consumer price index for all urban consumers: rent of primary residence in U.S. City Average from FRED. The source of this data is mentioned as the US Bureau of Labor Statistics. The data on housing sales is also taken from FRED with the original source mentioned as US Census Bureau. The frequency of the data is quarterly. The time period for all variables includes the Global Financial Crisis, however, the results are qualitatively similar even if the data between the period 2008 and 2021 is not included.

Table 1: Business cycle facts

	Price-to-Rent	R Vacancy	Sales	Time-to-Sell	H Vacancy
Elasticity with respect to Rental Vacancy					
	-0.15	1	-0.66	1.48	-0.82
Standard Deviation					
	0.01	0.03	0.10	0.14	0.09
Correlation Matrix					
Price-to-Rent	1				
R Vacancy	-0.39	1			
Sales	0.58	-0.22	1		
Time-to-Sell	-0.43	0.33	-0.44	1	
H Vacancy	0.75	-0.27	0.22	-0.23	1

Previous literature has noted the cyclical movements seen within the housing market. More specifically, Diaz and Jerez (2013) and Ngai and Sheedy (2024) show the correlation between housing market variables such as prices, sales, time to sell and housing vacancies. The main aim of this empirical exercise is to study the cyclical co-movements of the housing and rental market variables. These variables are, price to rent ratio, rental vacancy rate, housing vacancies, sales and time to sell. Following the approach used by Diaz and Jerez (2013), I calculate the business cycle correlations between these variables. All variables are transformed into natural logarithms to make their magnitudes comparable. Each variable is detrended using the Hodrick-Prescott filter. Standard deviations for each variable and the correlations between the variables are reported in table 1. The target for my model is going to be to match the elasticity of the variables mentioned above with respect to rental vacancy rate. Elasticity is a better way of capturing the effect of change in rental vacancies on other variables. This is based on the work of Mortensen and Nagypal (2007) who suggest that elasticities correspond to a regression coefficient. In this case, I am trying to study the interactions between the rental and housing market variables, and thus, the response of these variables to a change in rental vacancies. For this purpose, looking at a regression coefficient or elasticity makes more sense. Using the correlation coefficient and the standard deviations, I am able to get the elasticity for the variables mentioned above with respect to rental vacancy rate.

The first stylized fact is the significant negative correlation between the rental vacancy rate

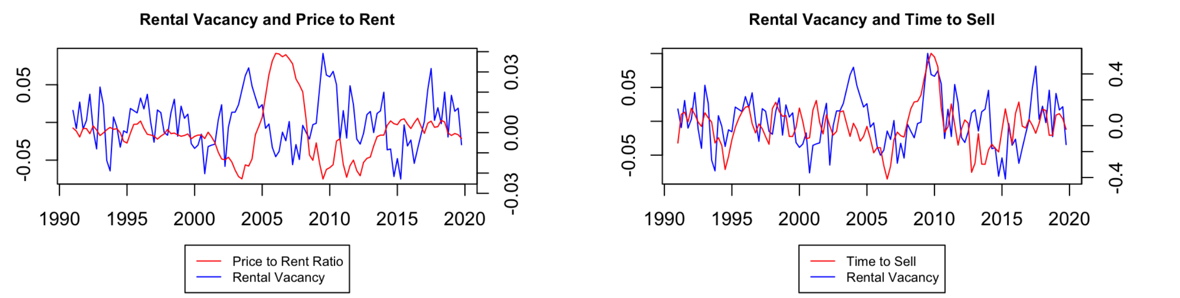

Figure 1: The elasticity of price to rent ratio with respect to rental vacancy rate is -0.15 while the elasticity of time to sell (time on market) with respect to rental vacancy rate is 1.48

and the price to rent ratio as shown in figure 1. This result is consistent with the findings of Halket and di Custoza (2015), although they use Craigslist data for their results. The price to rent ratio that I have used is similar to the one used by Han et al. (2022). It is simply the ratio of the time series value of price and rent in each quarter. This ratio does not take into consideration the fact that properties underlying average rent and average sales price are often very different from each other. Rental vacancy rate is significantly positively correlated with time on market as shown in figure 1. However, rental vacancy rate is significantly negatively correlated with both sales and housing vacancies as shown in figure 2. Data suggests that a decrease in rental vacancy rate is associated with an increase in the price to rent ratio, sales and vacancies while the time to sell decreases. This suggests that a decrease in rental vacancies can be associated with a "hot market" where the housing market activity increases. An increase in rental vacancies can be associated with a "cold market" where the housing market activity dies down a bit.

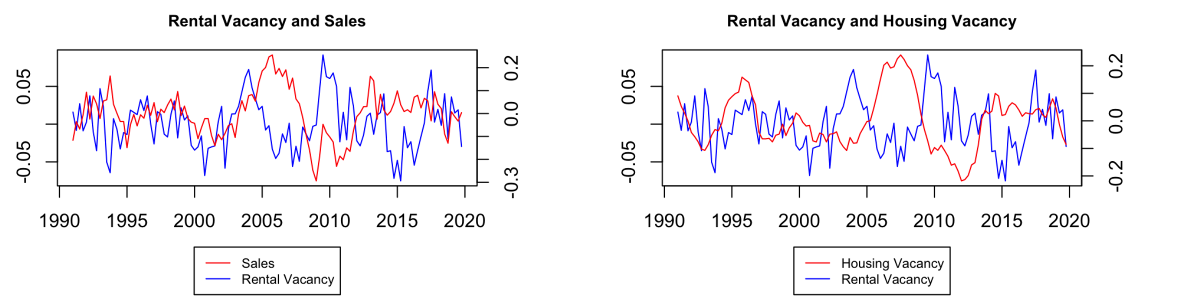

Figure 2: The elasticity of sales with respect to rental vacancy rate is -0.43 while the elasticity of housing vacancies with rental vacancy rate is -0.66

The correlations observed among housing market variables align with existing literature. Both sales and housing vacancies exhibit positive correlations with prices, while prices and time on the market display a negative correlation, approximately at -0.26. Sales are negatively correlated with the time to sell but positively correlated with housing vacancies. Finally, housing vacancies and time to sell are also negatively correlated. These findings are consistent with Ngai and Sheedy (2024) qualitatively, however there might be quantitative differences because of the different sources being used.

3 Model in Steady State

The model incorporates several essential components. Firstly, it comprises of a rental and housing market, where both finding and selling properties in the housing market, as well as locating rental properties and tenants in the rental market, entail time. This is captured through the inclusion of search and matching frictions in both markets, with a matching function akin to Pissarides (2000). Secondly, there is endogenous entry of rental seekers, resembling the approach taken by Gabrovski and Ortego-Marti (2019), wherein search costs increase as more rental seekers enter the market, contributing to the upward-sloping Beveridge curve in the rental market. Thirdly, the model accommodates new construction in both markets, facilitated by the free entry of landlords and sellers in each. Lastly, agents transition from rental to housing based on match-specific productivity, mirroring the framework of Mortensen and Pissarides (1994). This productivity encapsulates a spectrum of reasons driving agents' moves from the rental to housing market.

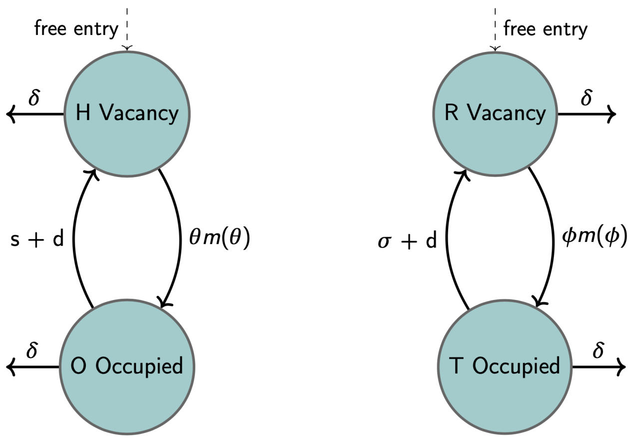

Figure 3: Property stocks and flows in the model.
There are no movements between ownership and rental market properties. T Occupied refers to those rental properties that are tenant occupied. O Occupied refers to those housing properties that are homeowner occupied. H Vacancy is the housing vacancy and R Vacancy is the rental vacancy.

The construction cost of a house in the rental market is denoted as k^R, while in the housing market, it is k^H. Once this cost is covered, the house is immediately built and can be listed as either a rental or housing vacancy. Newly constructed houses and existing ones are indistinguishable. Consequently, the economy features two property types: rental vacancies and housing vacancies. There is no movement between these properties as has been depicted in figure 3. This static separation is supported by Glaeser and Gyourko (2007), who argue that owned homes substantially differ from rental units and demonstrate the absence of arbitrage between rental and owner-occupied homes. This assumption is relaxed in section A.2, however, the results do not change.

The increasing search cost for rental seekers serves to pin the endogenous number of rental seekers. Without such a cost, the equilibrium would be characterized by either every agent seeking a rental property or none at all. Matching occurs within the rental market, where rental seekers transition into tenants. Upon separation, these tenants receive a draw of idiosyncratic utility, rendering them heterogeneous in their valuation of homeownership. This heterogeneity among home-buyers allows for a range of prices across different buyers, resulting in a better mapping with the data. Guided by their idiosyncratic utility of homeownership, individuals decide whether to remain in the rental market or enter the housing market.

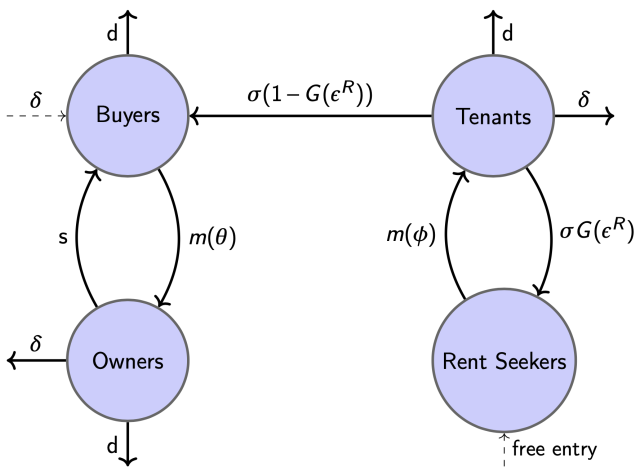

Figure 4: Households stocks and flows in the model.
There is free entry of rental seekers. There is no movement of agents from housing back to the rental market.

3.1 The Environment

Time is discrete. Agents are risk neutral and finitely lived. Households die with probability d in each time period. They discount the future at the discount rate β. At any point, a household will either be in the housing market, rental market or choose not to participate in any market. Within the rental market, they can either be rental seekers or be tenants. Tenants separate at the rate σ and get a draw of idiosyncratic utility ϵ from a known distribution $G(\epsilon)$. If this idiosyncratic utility is greater than ϵ^R, then the tenants enter the housing market. Within the housing market, the households are either looking for a house to buy (buyers) or they own a home (home-owners). Home-owners separate at the rate s to become home-buyers. The movement of households within the model are shown in figure 4. Idiosyncratic utility is a way of capturing the heterogeneity of buyers in the housing market like Head et al. (2014), Ngai and Sheedy (2020), Gabrovski and Ortego-Marti (2022), and others have done before. χ^H and χ^T denote the utility an agent gets from home-ownership and being a tenant respectively. This utility is common to each household. Overall, a home-owner will derive a utility $\chi^H \epsilon$ from owning a home.

The matching function in the housing market is conventional and has been widely utilized in previous literature. It is denoted as $M_h(b, h_v)$, where b is the measure of buyers and h_v is the measure of vacancies in the housing market. Market tightness in the housing market is denoted by $\theta \equiv b/h_v$. Buyers find homes at the rate of $m(\theta) \equiv M_h(b, h_v)/b = M_h(1, \theta^{-1})$ and sellers find buyers at the rate of $\theta m(\theta) \equiv M_h(b, h_v)/h_v = M_h(\theta, 1)$. The matching function adheres to standard properties: it increases with both b and h_v, is concave, and exhibits constant returns to scale. An increase in market tightness means an increase in the number of buyers relative to housing vacancies which implies that it has become more difficult for a buyer to match with a house and thus, as θ increase, $m(\theta)$ decreases. However, under this scenario, it is much easier to sell a house and finding rate for seller increases. Hence, an increase in θ leads to an increase in $\theta m(\theta)$. Homeowners separate at the rate s. After separation, homeowners enter the ownership market as buyers and list their house for sale, creating a housing vacancy. Buyers incur a flow cost denoted by c^B when searching for a property to purchase, while sellers incur a flow cost denoted by c^S for listing a vacancy and facilitating the sale of a house.

Similarly, the rental market tightness is denoted by $\phi \equiv n/r_v$ where n is the number of rental seekers and r_v is the measure of vacancies in the rental market. The matching function is given by $M_R(n, r_v)$. This matching function also follows the standard properties of being increasing in both n and r_v, concave and being a constant returns to scale function. Rental seekers find a place at the rate of $m(\phi) \equiv M_R(n, r_v)/n = M_R(1, \phi^{-1})$ and landlords find tenants at the rate of $\phi m(\phi) \equiv M_R(n, r_v)/r_v = M_R(\phi, 1)$. Like the ownership market, an increase in the rental market tightness leads to a decrease in $m(\phi)$ and an increase in $\phi m(\phi)$ because a relative increase in rental seekers would make it more difficult for them to find a place to rent but would make it easier for the landlord to find a tenant. The landlord has to pay a flow cost of c^R to fill a rental vacancy. I adopt the assumption of free entry for rental seekers, inspired by the framework outlined in Gabrovski and Ortego-Marti (2019). These agents face a flow cost $c(n)$, which increases with the number of seekers, n. This cost encompasses various expenses such as arranging viewings and visiting properties. Additionally, it reflects how scarcity in rental vacancies can drive up demand. With free entry, households continue to enter the market until the value of becoming a rental seeker equals zero, representing their outside option.

This movement of agents within markets is based on the work of Bachmann and Cooper (2014) who show that households will always rent a place first. Their idea is that during better economic times, households move into the rental market to try out of a new job or to live in a better neighborhood. Subsequently, they transition from the rental to the housing market. During economic booms, agents may move within the housing market to upgrade to better properties or neighborhoods. Importantly, they note that movement from the housing to the rental market is rare and tends to be non-cyclical, contrary to the notion that households downsize during economic downturns. I integrate these dynamics into my model, whereby once households enter the housing market, they do not revert to the rental market.

3.2 Housing Market

$B(\epsilon)$ and $H(\epsilon)$ denotes the value function of being a buyer and home-owner with idiosyncratic utility ϵ. The price that the buyer pays the seller is denoted by $p(\epsilon)$. V^H denotes the value function of a vacancy in the housing market which is created when the home-owner separates from his house or a developer creates a newly built house.

$$H(\epsilon) = \epsilon\chi^H + (1-d)\Big((1-\delta)\Big((1-s)\beta H(\epsilon) + s\beta(B(\epsilon) + V^H)\Big) + \delta\beta B(\epsilon)\Big) + d\beta V^H(1-\delta), \quad (1)$$

$$B(\epsilon) = -c^B + (1-d)\Big(m(\theta)\Big(\beta H(\epsilon) - p(\epsilon)\Big) + (1-m(\theta))\beta B(\epsilon)\Big). \quad (2)$$

Equation (1) captures the value of being a home-owner. The home-owner gets the utility $\epsilon\chi^H$

which is a combination of their idiosyncratic utility ϵ and the aggregate utility χ^H. With probability $(1-d)$, the home-owner does not die. With probability $(1-\delta)$, the house is not destroyed. At the rate s, the home-owner separates from their home, become a home-buyer and also create a housing vacancy with value V^H. At the rate δ, the house is destroyed and the home-owner becomes a home-buyer. Finally, in case the home-owner dies, they create a housing vacancy which is utilized by his family and thus, $d\beta V^H(1-\delta)$ is the utility the home-owner gets in the form of a bequest. Equation (2) captures the value of being a buyer in the ownership market. The buyer has to pay a flow cost of c^B. Again, $(1-d)$ is the probability that the buyer does not die. At the rate $m(\theta)$, they gets matched and becomes a home-owner after paying the price $p(\epsilon)$. If the home-buyer is not matched, then they will stay as a home-buyer in the next time period. The value function for the seller is given in equation (3),

$$V^H = -c^S + (1-\delta)\Big(\theta m(\theta)\int_{\epsilon^R}^{\infty} p(\epsilon)\frac{dG(\epsilon)}{1-G(\epsilon^R)} + (1-\theta m(\theta))\beta V^H\Big). \tag{3}$$

The seller has to pay a flow cost of c^S. At the rate $\theta m(\theta)$, they match with the buyer from the distribution $G(\epsilon)/1-G(\epsilon^R)$ and gets the price $p(\epsilon)$. This distribution captures those households that have the idiosyncratic utility above the reservation utility ϵ^R. The vacancy gets destroyed at the rate δ.

3.3 Rental Market

T is the value function of being a tenant. The tenant gets a common utility χ^T but has to pay a flow rent which equals ρ. With probability $(1-d)(1-\delta)$, the tenant does not die and the house does not get destroyed. At the rate σ, the tenant gets separated and it is at this time that they get a random draw of utility from the distribution $G(\epsilon)$. If their idiosyncratic utility is greater than the reservation utility, then they enter the housing market as buyers. If not, they continue to stay in the rental market and thus, look for a different place to rent. When tenants separate, they create a rental vacancy. The match between the tenant and the rental property gets destroyed at the rate δ. The bellman equation for the tenant is given by equation (4)

$$T = \chi^T - \rho + (1-d)\Big((1-\delta)\Big((1-\sigma)\beta T + \sigma\beta(1-G(\epsilon^R))\int_{\epsilon^R}^{\infty} B(\epsilon)\frac{dG(\epsilon)}{1-G(\epsilon^R)}\Big)\Big). \tag{4}$$

$$R = -c(n) + m(\phi)\beta T + (1 - m(\phi))\beta R, \tag{5}$$

The bellman equation for the households rental seekers is given in equation (5). R represents the value of being a household that is rental seeker. While searching for a place to rent, these households pay a flow cost $c(n)$ which is increasing in n. There is free entry of rental seekers and thus, households keep entering the market until the value of R equals 0. Matches are formed at the rate $m(\phi)$ at which point the household becomes a tenant. If a match isn't found, then they stay as rental seekers in the next time period. Equation (6) captures the bellman equation for a vacancy in the rental market. The landlord who owns the property has the value function L. The potential landlord has to pay a certain flow cost given by c^R. At the rate $\phi m(\phi)$, a match is created and she becomes the landlord. Again, if match does not take place, then the potential landlord still has the vacancy and has to search in the next time period.

$$V^R = -c^R + (1 - \delta)\Big(\phi m(\phi)\beta L + (1 - \phi m(\phi))\beta V^R\Big), \tag{6}$$

$$L = \rho + (1 - d)\Big((1 - \delta)\Big((1 - \sigma)\beta L + \sigma\beta V^R\Big)\Big) + d\beta V^R(1 - \delta). \tag{7}$$

The value of being a landlord L is given in equation (7). The landlord gets the rental flow income given by ρ. The probability with which the tenant occupying the rental house does not die is given by $(1 - d)$. At the rate σ, there is a separation which creates a rental vacancy. The tenant dies at the rate d, in which case, the landlord gets a rental vacancy again. δ is the rate at which the house gets destroyed. In this case, we assume that the landlord lives forever. Landlords can be thought of like corporations who buy houses to rent and thus, they can live forever.

3.4 Bargaining

Prices and rents are going to be determined by Nash Bargaining as in Nash (1950) and Rubinstein (1982). Each match that occurs generates a surplus for both parties involved in the transaction. As long as both parties derive a positive surplus from the match—meaning they both benefit from it—they will proceed with the transaction. Let $S^B(\epsilon)$ and $S^S(\epsilon)$ denote the buyer and seller surplus in the ownership market. Here, ϵ is the utility which the buyer gets from owning a house. When a house is sold, the buyer gets a value from owning a house $H(\epsilon)$ but has to pay $p(\epsilon)$ which goes to the seller. Thus, the buyer and seller surplus is given by,

$$S^B(\epsilon) = \beta H(\epsilon) - p(\epsilon) - \beta B(\epsilon), \tag{8}$$

$$S^S(\epsilon) = p(\epsilon) - \beta V^H. \tag{9}$$

Let η be the buyers surplus in the housing market. Then, the Nash Bargaining problem can be written as,

$$p(\epsilon) = \underset{p(\epsilon)}{\text{argmax}}(S^B(\epsilon))^{\eta}(S^S(\epsilon))^{1-\eta}, \quad \forall \epsilon \geq \epsilon^R. \tag{10}$$

The first order condition gives the Nash sharing rule in the housing market,

$$(1-\eta)S^B(\epsilon) = \eta S^S(\epsilon), \quad \forall \epsilon \geq \epsilon^R. \tag{11}$$

In the rental market, the surplus is shared between the landlord and the tenant. Let S^T and S^L denote the surplus of the tenant and the landlord. When a household finds a property to rent, she gets a value from being a tenant T. The surplus for tenants and landlords is given in equation (12) and (13).

$$S^T = \beta(T - R), \tag{12}$$

$$S^L = \beta(L - V^R). \tag{13}$$

Let α be the surplus of the tenant in the rental market. The Nash Bargaining problem can be written as,

$$\rho = \underset{\rho}{\text{argmax}} \quad (S^T)^{\alpha}(S^L)^{1-\alpha}. \tag{14}$$

The first order condition gives the Nash sharing rule in the rental market,

$$(1-\alpha)S^T = \alpha S^S. \tag{15}$$

Intuitively, under Nash Bargaining, both agents obtain their outside option along with a portion of the surplus, determined by their bargaining strength.

3.5 Equilibrium in the Rental Market

There is free entry of rental seekers. This means that these households will keep on entering the market until $R = 0$. Similarly, there is free entry of landlords such that the value of a rental vacancy covers the construction costs k^R. Thus, two equilibrium conditions are,

$$R = 0 \quad and \quad V^R = k^R. \tag{16}$$

Simplifying equation (7) gives the value of being a landlord as,

$$L = \frac{\rho + (1-\delta)\beta V^R(\sigma(1-d)+d)}{1-\beta(1-d)(1-\sigma)(1-\delta)}. \tag{17}$$

Intuitively, this suggests that the landlord gets his value from the rent and the property that he owns. Using equation (5), the free entry of those looking to rent in the market given by equation (16) and the first order Nash sharing rule given in equation (15), the rental seekers entry (RE) condition is given by,

$$(RE): \quad \frac{c(n)}{m(\phi)} = \beta T = \frac{\beta\alpha}{1-\alpha}\frac{\rho - k^R(1-\beta(1-\delta))}{1-\beta(1-d)(1-\sigma)(1-\delta)}. \tag{18}$$

The (RE) condition tells us that rental seekers expected net cost of searching $c(n)/m(\phi)$ equals the surplus from being a tenant. The value of being a tenant can be substituted with the Nash sharing rule to get the last part of the equation.

Landlords enter the market until they recover their construction cost k^R and user cost c^R through the rental vacancy. Simplifying equation (6) using the value of being a landlord from equation (17) gives the landlord entry (LE) condition as,

$$(LE): \quad \frac{c^R + k^R(1-\beta(1-\delta))}{\beta(1-\delta)\phi m(\phi)} = \frac{\rho - k^R(1-\beta(1-\delta))}{1-\beta(1-d)(1-\sigma)(1-\delta)}. \tag{19}$$

Again, the (LE) condition tells us that the expected net cost of searching for a tenant equals the surplus received by the landlord.

Substitute the surplus of the tenant and the landlord given in equations (12) and (13) into the first order Nash sharing rule given in equation (15) to get the equilibrium value of rent. The

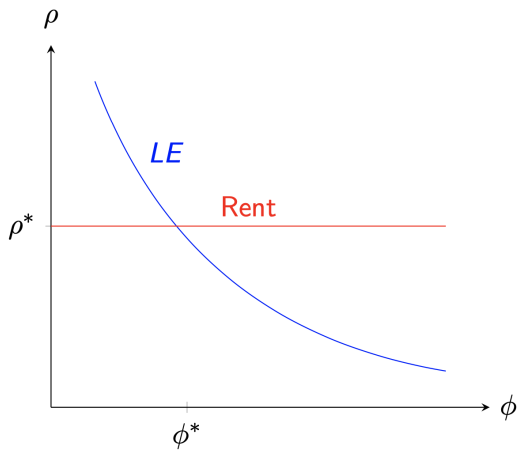

Figure 5: Equilibrium rent ρ^* and market tightness ϕ^*

value of T is simplified using equation (4) to get the final result which is given by,

$$(Rent): \quad \rho = (1-\alpha)\left[\chi^T + (1-d)(1-\delta)\sigma\beta(1-G(\epsilon^R))\overline{B(\epsilon)}\right] + \alpha[k^R(1-\beta(1-\delta))], \tag{20}$$

where, $\overline{B(\epsilon)} = \int_{\epsilon^R}^{\infty} B(\epsilon)\frac{dG(\epsilon)}{1-G(\epsilon^R)}$. Rent is a weighted average between the present discounted value of net flows from renting a home and the cost of construction of a home. The weights are the bargaining power of the tenant and the landlord.The inclusion of $\overline{B(\epsilon)}$ suggests that these markets are interdependent, influencing each other rather than being solved separately.

Combining equation (19) and (20) gives the equilibrium rents and the rental market tightness as shown in figure 5. Once these values are obtained, equation (18) can be used to pin down the value of n which is the number of rental seekers.

3.6 Equilibrium in the Housing Market

The housing market analysis is on the same lines as Gabrovski and Ortego-Marti (2022). In the housing market, there is no free entry of buyers. However, there is free entry of sellers which means that the value of a housing vacancy covers the construction costs k^H,

$$V^H = k^H. \tag{21}$$

The above condition along with equation (3) gives the Housing Entry (HE) condition as,

$$(HE): \quad \bar{p} - \beta k^H = \frac{k^H(1-\beta(1-\delta)) + c^S}{\theta m(\theta)(1-\delta)}, \tag{22}$$

where $\bar{p} = \int_{\epsilon^R}^{\infty} p(\epsilon) \frac{dG(\epsilon)}{1-G(\epsilon^R)}$. Equation (22) shows that sellers keep entering the market until the surplus from selling a house which is given by the left hand side of the expression is equal to the expected cost of finding a buyer which is given by the right hand side. The expected cost of finding a buyer is given by the sum of flow cost c^S and the user cost $k^H(1-\beta(1-\delta))$ for the duration of the vacancy which is given by $1/\theta m(\theta)$.

The equilibrium price is calculated using the Nash sharing rule given in equation (11). The value of $H(\epsilon)$ and $B(\epsilon)$ are calculated from equations (1) and (2). Combining these results will give the equilibrium price as,

$$(price): p(\epsilon) - \beta k^H = \frac{\beta(1-\eta)(\epsilon\chi^H + c^B - k^H(1-\beta(1-\delta)))}{1-(1-s)(1-d)(1-\delta)\beta + \beta(1-d)\eta m(\theta)}. \tag{23}$$

Average price can be calculated using equation (23) since $\bar{p} = \int_{\epsilon^R}^{\infty} p(\epsilon) \frac{dG(\epsilon)}{1-G(\epsilon^R)}$. Then average price can be expressed as,

$$(PP): \bar{p} - \beta k^H = \frac{\beta(1-\eta)(\bar{\epsilon}\chi^H + c^B - k^H(1-\beta(1-\delta)))}{1-(1-s)(1-d)(1-\delta)\beta + \beta(1-d)\eta m(\theta)}, \tag{24}$$

where, $\bar{\epsilon} \equiv E(\epsilon|\epsilon > \epsilon^R) = \int_{\epsilon^R}^{\infty} \epsilon \frac{dG(\epsilon)}{1-G(\epsilon^R)}$. This equation tells us that if there is an increase in the housing market tightness, $m(\theta)$ would decrease and that would lead to an increase in prices. This result is consistent with the existing housing market literature.

3.7 Reservation Utility

The ownership market lacks a condition of free entry for buyers, complicating the determination of equilibrium price and market tightness. To address this challenge, I examine the equilibrium value of the reservation utility, which in turn aids in establishing the equilibrium price and market tightness. This approach relies on the concept that when a household's idiosyncratic utility ϵ equals the reservation utility ϵ^R, the household becomes indifferent between remaining a tenant or transitioning to a buyer in the housing market. At this juncture, the value associated with being a buyer must equate to the value associated with being a tenant. Therefore, the equilibrium condition must be,

$$T = B(\epsilon^R). \tag{25}$$

Using equation (25), I find a unique equilibrium value of the reservation utility and the housing market tightness. Similar to the rental market equilibrium conditions, there will be two equilibrium conditions which will depict the relationship between ϵ^R and θ and that would in turn give the value of equilibrium price. Substituting the simplified value of T from equation (4) in the above expression gives,

$$\frac{\chi^T - \rho + (1-d)(1-\delta)\sigma\beta(1 - G(\epsilon^R)B(\epsilon^R))}{1 - \beta(1-d)(1-\delta)(1-\sigma)} = B(\epsilon^R). \tag{26}$$

To find the first relationship between ϵ^R and θ, substitute $B(\epsilon^R)$ from equation (2) combined with (11) to equation (26) to get,

$$(RU): \quad m(\theta)\eta\beta(1-d) = \frac{\left(\frac{(\chi^T-\rho)(1-\beta(1-d))+c^B}{1-\beta(1-d)(1-\delta)(1-\sigma G(\epsilon^R))}\right)(1-\beta(1-d)(1-\delta)(1-s))}{\epsilon^R\chi^H - k^H(1-\beta(1-\delta)) - \frac{(\chi^T-\rho)(1-\beta(1-d))}{1-\beta(1-d)(1-\delta)(1-\sigma G(\epsilon^R))}}. \tag{27}$$

From the above equation, an increase in θ leads to an increase in ϵ^R. Thus, we have a positive relationship between the two. For the second equation which connects θ and ϵ^R, I combine equation (22) and (24) to get,

$$(HE+PP): \frac{k^H(1-\beta(1-\delta)) + c^S}{\theta m(\theta)(1-\delta)} = \frac{\beta(1-\eta)(\bar{\epsilon}\chi^H + c^B - k^H(1-\beta(1-\delta)))}{1-(1-s)(1-d)(1-\delta)\beta + \beta(1-d)\eta m(\theta)}. \tag{28}$$

I am still unable to find a relationship between θ and ϵ^R using the above equation. To proceed further and get a relationship, I need to be able to express $\bar{\epsilon}$ as a function of ϵ^R. For that purpose, I assume a distribution for $G(\epsilon)$. I assume that G(.) follows a Pareto distribution as previously done by Lagos (2006), Han et al. (2022) and Gabrovski and Ortego-Marti (2022). The distribution is defined as,

$$G(x) = \begin{cases} 0 & , \text{if } \epsilon < \epsilon^R \\ 1 - (\frac{\epsilon_l}{\epsilon})^\lambda & , \text{if } \epsilon \geq \epsilon^R \end{cases} \tag{29}$$

where ϵ_l is the lowest possible value of the idiosyncratic utility that any household can draw and $\lambda > 1$ is the Pareto shape parameter. This assumption helps find a value of $\bar{\epsilon}$ in terms of ϵ^R. Simplifying the integral $\bar{\epsilon} = \int_{\epsilon^R}^{\infty} \epsilon \frac{dG(\epsilon)}{1-G(\epsilon^R)}$ using the Pareto distribution gives,

$$\bar{\epsilon} = \frac{\lambda}{\lambda - 1}\epsilon^R. \tag{30}$$

Substituting this value in equation (28) gives the desired new (HE) equation,

$$(HE + PP): \frac{k^H(1-\beta(1-\delta)) + c^S}{\theta m(\theta)(1-\delta)} = \frac{\beta(1-\eta)(\frac{\lambda}{\lambda-1}\epsilon^R\chi^H + c^B - k^H(1-\beta(1-\delta)))}{1-(1-s)(1-d)(1-\delta)\beta + \beta(1-d)\eta m(\theta)}. \tag{31}$$

From equation (31), it is straightforward to check that as θ goes up, ϵ^R will decrease. Thus, there is a negative relationship between the two variables in this equation. This was feasible because the chosen distribution ensures that as ϵ^R increases, $\bar{\epsilon}$ also increases, and the Pareto distribution possesses this property. In practice, any distribution that meets this criterion could have been employed.

Equation (27) gives us a upward sloping relationship between θ and ϵ^R and equation (31) gives us a downward sloping relationship. Combining these two equations will pin down the equilibrium value of reservation utility and housing market tightness as shown in figure (6). Once I get these equilibrium values, I can use equation (24) to get the average price and thus, get all equilibrium steady state values.

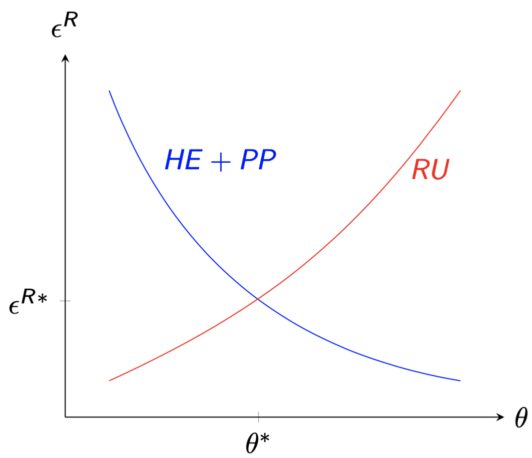

Figure 6: Equilibrium reservation utility ϵ^{R^*} and market tightness θ^*

The intuition for the above curves is based on how an increase in θ affects buyers and sellers differently. When θ goes up, $\theta m(\theta)$ also goes up, that is, it becomes easier for the seller to match with the buyer and thus, the seller will accept a lower price (HE condition) and hence, a lower value of the reservation utility will also get matched. In this case, as θ goes up, ϵ^R will go down and give me the HE+PP condition. However, for the buyers, when θ goes up, $m(\theta)$ goes down and thus, it is harder for them to find a match and thus, only if their draw of ϵ^R is really high, would they enter the housing market. Hence, in this case, as θ goes up, ϵ^R will also go up and give rise to the RU condition.

Definition 1. The equilibrium can be defined as a tuple consisting of $\{\phi, \theta, p(\epsilon), \bar{p}, \rho, \epsilon^R, n\}$ that satisfies: (1) the RE condition (18), (2) The LE condition (19), (3) The Rent equation (20), (4) The HE condition (22), (5) The Price equation (23), (6) The Average Price equation (24) and (7) The RU condition (27).

3.8 Laws of Motion

The quantities of home-buyers, home-owners, and tenants are determined using the laws of motion. These values are crucial for calculating the housing and rental vacancy rates. Additionally, it's essential to ensure that the Beveridge curve in both the housing and rental markets is upward-sloping, as demonstrated by Gabrovski et al. (2019) and Badarinza et al. (2024). As previously mentioned, n represents the number of rental seekers. In this context, t denotes the number of tenants, b represents the number of home-buyers, and h indicates the number of home-owners. Equations (32), (33), and (34) collectively provide the values of t, b, and h in terms of n. Once n is determined from the equilibrium, the equations below can be utilized to ascertain these values.

$$m(\phi)n = (\delta + d + \sigma(G(\epsilon^R) + (1 - G(\epsilon^R))))t, \tag{32}$$

$$(m(\theta) + d)b = (s + \delta)h + \sigma(1 - G(\epsilon^R))t, \tag{33}$$

$$m(\theta)b = (s + \delta + d)h. \tag{34}$$

Equation (32) signifies that the number of matched rental seekers must equate to the sum of tenants who separate, tenants who pass away, and tenants whose matches dissolve. Equation (33) illustrates that the outflow from home-buyers, comprising matched buyers or deceased individuals, must match the inflow into home-buyers, sourced from separated home-owners, home-owners whose matches dissolve, or tenants whose idiosyncratic utility exceeds the reservation value. Lastly, equation (34) demonstrates that the influx into the stock of home-owners equals the number of buyers who are matched and secure a house, while the efflux from this stock corresponds to home-owners experiencing separation, mortality, or a dissolution shock. Combining these three equations will give the values of t, b and h as,

$$t = \frac{m(\phi)n}{\sigma + d + \delta}, \tag{35}$$

$$h = \frac{m(\theta)\sigma(1 - G(\epsilon^R))t}{d(d + s + \delta + m(\theta))}, \tag{36}$$

$$b = \frac{(s + \delta + d)\sigma(1 - G(\epsilon^R))t}{d(d + s + \delta + m(\theta))}. \tag{37}$$

4 Model with Business Cycles

In this section, I extend the model to accommodate business cycle fluctuations. While the model demonstrates equilibrium at the steady state, the primary objective is to quantitatively align with the observed stylized facts. To achieve this goal, introducing business cycle fluctuations becomes imperative. Diaz and Jerez (2013), Head et al. (2014), Ngai and Sheedy (2020) and Gabrovski and Ortego-Marti (2019) have shown that multiple shocks are needed to match the key stylized facts of the housing market. In most cases, both demand and supply shocks are required.

In my model a demand shock indicates an increase in χ^H and a decrease in χ^T. Essentially, a positive demand shock boosts the appeal of homeownership relative to renting, making homeownership or entering the housing market more attractive. On the other hand, a supply shock entails an increase in k^H and k^R, indicating heightened construction costs in both the housing and rental markets. This reflects the notion that increased construction is linked with higher construction expenses—a straightforward and practical approach to capturing a supply shock as shown by Gabrovski and Ortego-Marti (2019). For example, more construction makes it harder to get construction permits, increases the price of materials, land, and labor required to build a house.

The shocks are perfectly correlated along the business cycle. This approach, utilized by Shimer (2005) in the labor market, entails a single underlying stochastic process governing both labor productivity and separations. Similarly, in the housing market, Diaz and Jerez (2013) and Gabrovski and Ortego-Marti (2019) employ this methodology. The rationale behind this approach is straightforward: during economic booms, there's heightened demand for homeownership, corresponding to a demand shock. The increased appeal of homeownership leads to augmented surplus and elevated prices during bargaining. Consequently, higher prices stimulate seller entry into the market. Moreover, an increase in construction is represented by elevated construction costs in both the rental and housing markets. By employing these correlated shocks, I'm able to align the elasticity of the price-to-rent ratio, sales, and housing vacancies with respect to rental vacancies.

The mechanism in the model with business cycle is identical to the model in steady state. Let i and i' denote the current state of the economy and the state of the economy in the next time period respectively. Further, X_i denotes the value function X at the current state i and $E_iX_{i'}$ denotes its expected value in the next time period i' conditional on state i.

The bellman equations for the housing market are given by

$$H(\epsilon_i) = \epsilon_i\chi_i^H + (1-d)\Big((1-\delta)\Big((1-s)\beta E_iH(\epsilon_{i'}) + s(\beta E_iB(\epsilon_{i'}) + \beta E_iV_{i'}^H)\Big) + \delta\beta E_iB(\epsilon_{i'})\Big) + d\beta E_iV_{i'}^H(1-\delta), \tag{38}$$

$$B(\epsilon_i) = -c^B + (1-d)\Big(m(\theta_i)\Big(\beta E_iH(\epsilon_{i'}) - p(\epsilon_i)\Big) + (1-m(\theta_i))\beta E_iB(\epsilon_{i'})\Big), \tag{39}$$

$$V_i^H = -c^S + (1-\delta)\Big(\theta_i m(\theta_i)\int_{\epsilon_i^R}^{\infty} p(\epsilon_i)\frac{dG(\epsilon_i)}{1-G(\epsilon_i^R)} + (1-\theta_i m(\theta_i))\beta E_iV_{i'}^H\Big). \tag{40}$$

The bellman equations for the rental market are given by

$$T_i = \chi_i^T - \rho_i + (1-d)\Big((1-\delta)\Big((1-\sigma)\beta E_i T_{i'} + \sigma\beta(1-G(\epsilon_i^R))\int_{\epsilon_i^R}^{\infty} E_i B(\epsilon_{i'})\frac{dG(\epsilon_i)}{1-G(\epsilon_i^R)}\Big)\Big), \tag{41}$$

$$R_i = -c(n_i) + m(\phi_i)\beta E_i T_{i'} + (1-m(\phi_i))\beta E_i R_{i'}, \tag{42}$$

$$V_i^R = -c^R + (1-\delta)\Big(\phi_i m(\phi_i)\beta E_i L_{i'} + (1-\phi_i m(\phi_i))\beta E_i V_{i'}^R\Big), \tag{43}$$

$$L_i = \rho_i + (1-d)\Big((1-\delta)\Big((1-\sigma)\beta E_i L_{i'} + \sigma\beta E_i V_{i'}^R\Big)\Big) + d\beta(1-\delta)E_i V_{i'}^R. \tag{44}$$

Prices and rents are determined by Nash bargaining as shown below,

$$p(\epsilon_i) = \underset{p(\epsilon_i)}{\text{argmax}} \Big(\beta E_i H(\epsilon_{i'}) - p(\epsilon_i) - \beta E_i B(\epsilon_{i'})\Big)^{\eta}\Big(p(\epsilon_i) - \beta E_i V_{i'}^H\Big)^{1-\eta}, \tag{45}$$

$$\rho_i = \underset{\rho_i}{\text{argmax}} \quad \beta\Big(E_i T_{i'} - E_i R_{i'}\Big)^{\alpha}\Big(E_i L_{i'} - E_i V_{i'}^R\Big)^{1-\alpha}. \tag{46}$$

Free entry of rental seekers implies $R_i = 0$ and $E_i R_{i'} = 0$. Free entry of sellers and landlords implies $V_i^H = k_i^H$ and $V_i^R = k_i^R$. In the next state i', $E_i V_{i'}^H = E_i k_{i'}^H$ and $E_i V_{i'}^R = E_i k_{i'}^R$. The reservation utility ϵ_i^R is calculated using

$$T_i = B(\epsilon_i^R). \tag{47}$$

Since I assume that G(.) follows a Pareto distribution, the relationship between ϵ_i^R and $\overline{\epsilon_i}$ is given by,

$$\overline{\epsilon_i} = \frac{\lambda}{\lambda-1}\epsilon_i^R. \tag{48}$$

The model is solved in the same way as the model in steady state in Section 3.

Definition 2. Let N denote the number of states of i. The equilibrium is characterized by a system of 11N unknowns $\{\theta_i, \phi_i, \overline{p_i}, \rho_i, \epsilon_i^R, \overline{\epsilon_i}, L_i, T_i, B_i, H_i, n_i\}$ and 11N equations. The 11N equations are given by equations (37-45). Following this, I can find the values for b_i, h_i and t_i

using the laws of motion equations (32), (33) and (34). This characterizes the equilibrium with business cycles and can be solved numerically.

5 Calibration

In this section, I quantify the impact of demand and supply shocks on my model. I start by calibrating the model to align with crucial moments of the housing and rental markets. Then, I analyze how these shocks influence key variables like the price-to-rent ratio, sales, time to sell, housing and rental vacancies, and buyer entry conditions.

The model is calibrated at quarterly frequency. The parameters and tagrets are mentioned in table 2. The value of β is selected as 0.987 to match the interest rate of 5%. The destruction rate δ is set to 0.004 which matches the annual housing depreciation rate of 1.6% as given by Van Nieuwerburgh and Weill (2010). The death rate d is set to 0.0044 which is used by Head et al. (2023) to match the population growth rate of 1% annually. In line with Diaz and Jerez (2013), the average tenure in a house is taken as 9 years which means the value of s is 0.0238. Following Gabrovski and Ortego-Marti (2019), the time required to sell a property is regarded as equivalent to the time taken to buy, resulting in a housing market tightness of 1. The time to sell is taken to be 1.4625 quarters which is the same as Gabrovski and Ortego-Marti (2021a). The functional form of $m(\theta)$ is the standard $\mu_H \theta^{-\psi}$. Since $\theta = 1$, μ_H equals 0.6838. I set ψ equal to 0.16 which is the estimate used in Genesove and Han (2012). The aggregate utility in the housing market denoted by χ^H, is normalized to 1 as is standard in the literature. The bargaining power of the buyer and seller in the housing and rental seeker and landlord in rental market is assumed to be equal. Thus, $\eta = \alpha = 0.5$. This assumption is frequently made in the literature, where it is common to attribute equal bargaining power to buyers and sellers. Following this convention, I extend the same principle to the rental market, granting both tenants and landlords equal bargaining strength.

Buyer flow costs, seller flow costs, and the shape parameter of the Pareto distribution are adjusted to align with vacancy rate and the price-to-rent ratio. Specifically, the buyer flow cost is set at 5.78% of the average price, while the seller flow cost is set at 5.40% of the average price. Ngai and Sheedy (2020) suggest that combined buyer and seller costs typically range from 6% to 12% of the price in the United States, with approximately 3% to 6% representing the realtor's fee paid by the seller. Considering these benchmarks and aiming to match the vacancy rate and price-to-rent ratio, I opt for a buyer cost of 5.78% and a seller cost of 5.40%. Using the above two targets and equation (22), I get the cost of construction in the ownership market k^H as 0.91 times the average price. The average price, as cited in Gabrovski and Ortego-Marti (2022) and Kotova and Zhang (2021), equals \$491,200. To establish equilibrium, I must target either the price, rent, or price-to-rent ratio. Following the approach of Gabrovski and Ortego-Marti (2022), I choose the average price as the target. I assume that the distribution G(.) follows a Pareto distribution. While the values of reservation and average utility are derived from the model, I manually input the Pareto shape parameter. This parameter serves as a scaling variable, and I set it at 2.8927 to align with a price-to-rent ratio of 3.5% as given by Halket and di Custoza (2015).

The main target in the rental market is the rental vacancy rate which is 7.76% in the data. To achieve that, the rental market tenure is taken as 2 years. This gives the value of separation rate σ in the rental market as 0.121. The functional form the matching function in the rental market is $m(\phi) = \mu_R \phi^{-\psi}$. The time to rent (TTR) a house is taken as 0.65 quarters which gives the value of μ_R as 1.667. This is based on Gabriel and Nothaft (2001) and Halket and di Custoza (2015), who observe the average rental vacancy duration to be between 1.5 to 2 months. Similar to the housing market, it is assumed that the time taken by a rental seeker to secure a rental property is equivalent to the time it takes for a landlord to find a tenant. Consequently, the rental market tightness is also set at 1. While in practice, it may take less time for a rental seeker to find an apartment, this assumption does not alter the outcomes.

Before a new tenant takes possession of the rental property, the landlord makes certain repairs and renovations to the house. That is the flow search cost of the landlord which is taken from Han et al. (2022). The landlord spends 16.3% of the rent on the maintenance every time a new match is made. I normalize the number of rental seekers to 2. This helps balance the flow search cost of rental seekers. I can also normalize this number to 1 and nothing apart from the flow search cost of rental seekers would change. Furthermore, the number of buyers, home-owners and tenants are determined relative to the number of rental seekers so this assumption does not have any affect on any of those. The functional form of $c(n)$ is taken as $\bar{c}n^{\gamma}$, consistent with Gabrovski and Ortego-Marti (2019), who apply it to buyers freely entering the market. The cost of construction in the rental market is determined by the model and amounts to 976.63. This

Table 2: Calibration Targets and Parameters

Preferences/Technology	Parameter	Value	Source/Target
Discount Factor	β	0.987	Interest rate = 5%
Utility Scale Housing	χ^H	1	Normalization
Utility Scale Rental	χ^T	18.56	Equilibrium
Elasticity of Matching Function	ψ	0.16	Genesove and Han (2012)
Destruction Rate	δ	0.004	Van Nieuwerburgh and Weill (2010)
Death Rate	d	0.0044	Head et al. (2023)
Separation Rate Housing	s	0.022	Tenure = 9 years
Separation Rate Rental	σ	0.131	Tenure = 2 years
Housing Matching Efficiency	μ_h	0.75	Time to buy = 1.4625 quarters
Rental Matching Efficiency	μ_r	1.667	Time to rent = 0.65 quarters
Bargaining Power	$\alpha = \eta$	0.5	
Seller Cost	c^S	26.54	Average seller cost = 5.4% of price
Buyer Cost	c^B	28.37	Average buyer cost = 5.7% of price
Rental Seeker Cost Elasticity	γ	2	Gabrovski and Ortego-Marti (2021a)
Rental Seeker Cost	$\bar{c}$	3.98	Equilibrium
Maintenance Cost Landlord	c^R	0.18	Han et al. (2022)
Construction Cost Housing	k^H	447.09	Equilibrium
Construction Cost Rental	k^R	976.63	Equilibrium
Pareto Shape	λ	2.8927	Price to Rent ratio
Average Utility	$\bar{\epsilon}$	16.91	Equilibrium
Reservation Utility	ϵ^R	11.06	Equilibrium
Average Price	$\bar{p}$	491.2	Gabrovski and Ortego-Marti (2022)

figure notably exceeds the cost of construction in the housing market. This discrepancy likely stems from the fact that landlords retain ownership of their properties indefinitely. As a result, the only scenario in which landlords part with their property is if it is destroyed, necessitating a significantly higher price to acquire property in the rental market.

I use the above values to compute the benchmark equilibrium as shown in table 3. The benchmark equilibrium has a rent to price ratio of 3.50% which is the same value given by Halket and di Custoza (2015). The value of housing vacancy rate is 4.49% compared to the empirically observed vacancy rate of 1.75%. The model is able to match the empirically observed rental vacancy rate of 7.76%. The value generated by the model is 7.75%. The housing vacancy rate

Table 3: Moments

Moment	Data	Value
Rent		17.20
Rent to Price	3.50%	3.50%
	Halket and di Custoza (2015)	
Housing vacancy	1.75%	4.49%
Rental vacancy	7.76%	7.75%

generated by the model does not perfectly match the one observed in the data. However, this is not a big problem since the stylized facts are established using the number of housing units that are vacant and not the housing vacancy rate.

Shocks. Firstly, I'll introduce the standard aggregate demand and supply shocks affecting both the rental and housing markets. As previously discussed, a demand shock entails an uptick in the appeal of homeownership coupled with a concurrent decline in the attractiveness of renting. A supply shock involves an increase in construction costs across both rental and housing sectors. Importantly, these shocks are assumed to be perfectly correlated. This approach is based on the work of Shimer (2005) in the labor market and Diaz and Jerez (2013) and Gabrovski and Ortego-Marti (2019) in the housing market.

I model χ^H, χ^T, k^H and k^R as AR(1) processes driven by the same underlying shock u_t. The AR(1) processes are given by,

$$ln(\chi_t^H) = \zeta_{\chi^H} + \nu ln(\chi_{t-1}^H) + u_t, \tag{49}$$

$$ln(\chi_t^T) = \zeta_{\chi^T} + \nu ln(\chi_{t-1}^T) + xu_t, \tag{50}$$

$$ln(k_t^H) = \zeta_{k^H} + \nu ln(k_{t-1}^H) + yu_t, \tag{51}$$

$$ln(k_t^R) = \zeta_{k^R} + \nu ln(k_{t-1}^R) + zu_t, \tag{52}$$

I simulate this model by discretizing the AR(1) processes following Rouwenhorst (1995). The processes are approximated with a discrete Markov Chain process with 5 grid points. The reason for using Rouwenhorst (1995) is based on the work of Kopecky and Suen (2010) who show that this method performs better than Tauchen (1986). To create the transition matrix, I use the auto-correlation coefficient ν as 0.956 to match the empirically observed auto-correlation value of price to rent ratio. With this, I use the Rouwenhorst (1995) approach to get the transition matrix which is given in the appendix A.2. I simulate the model 1000 times where each simulation consists of 160 quarters.

The standard deviation of the shock σ_u is 0.2121 to match the standard deviation of price to rent ratio observed in the data. The value of x is 0.08, y is 0.30 and z is 0.03. These values are used to match the elasticity of price to rent ratio, sales, time to sell and housing vacancies with respect to rental vacancy rate. The steady state value of χ^H is normalized to 1, that is, $log(\chi^H)$ equals 0 and so, $\zeta_{\chi^H} = 0$. The steady state value of χ^T is 18.56, $ln(\chi^T) = 2.92$ and so, $\zeta_{\chi^T} = 0.128$. Similarly, the steady state value of k_h and k_r gives the values of $\zeta_{k^H} = 0.268$ and $\zeta_{k^R} = 0.302$.

6 Results

In this section, I employ the framework to analyze the cyclic patterns within the rental and housing market. Initially, I employ the demand shock to match the elasticities presented in this paper. However, to reconcile with the elasticities previously observed in the housing market, I introduce additional supply shocks as is standard in the literature.

6.1 Demand Shock

Table 4 shows the business cycle properties of my benchmark model economy when only demand shocks are introduced. Column 1 reports the elasticities of individual variables with respect to the rental vacancy rate, as observed in the data and detailed in table 1. Meanwhile, Column 2 showcases these elasticities derived from my model. Notably, the model closely replicates the elasticity of the price-to-rent ratio, sales and housing vacancies. Although the model is able to match the elasticity for time to sell qualitatively, there is a quantitative difference.

Table 4: Results: Demand shocks in housing and rental market

Elasticity of X wrt to Rental Vacancy	Data	Model
Price to Rent	-0.15	-0.12
Time to Sell	1.48	0.03
Sales	-0.66	-0.60
Housing Vacancies	-0.82	-0.57

The dynamics of the model with a demand shock are illustrated in figure 7. The solid line represents the model at steady state and the dashed lines represent the movement that takes place post the demand shocks. A demand shock increases the utility of being a home-owner which makes it more lucrative for agents to enter the housing market. A housing demand shock increases χ^H in equation (27). This shifts the reservation utility (RU) curve to the right which decreases the barrier to entry into the housing market. This shift occurs because heightened utility motivates prospective home-buyers to engage in a more competitive market in pursuit of greater utility gains. Simultaneously, sellers respond to this increased housing demand by expanding housing vacancies, as reflected by an increase in χ^H in Equation 30. Consequently, the HE+PP curve shifts leftward, indicating increased market tightness due to the surge in vacancies posted by sellers. In the rental market, the utility from being a tenant diminishes, resulting in a decline in demand for rental properties which moves the Rent equation (20) downwards. There is an increase in rental market tightness because of a decrease in the supply in rental vacancies.

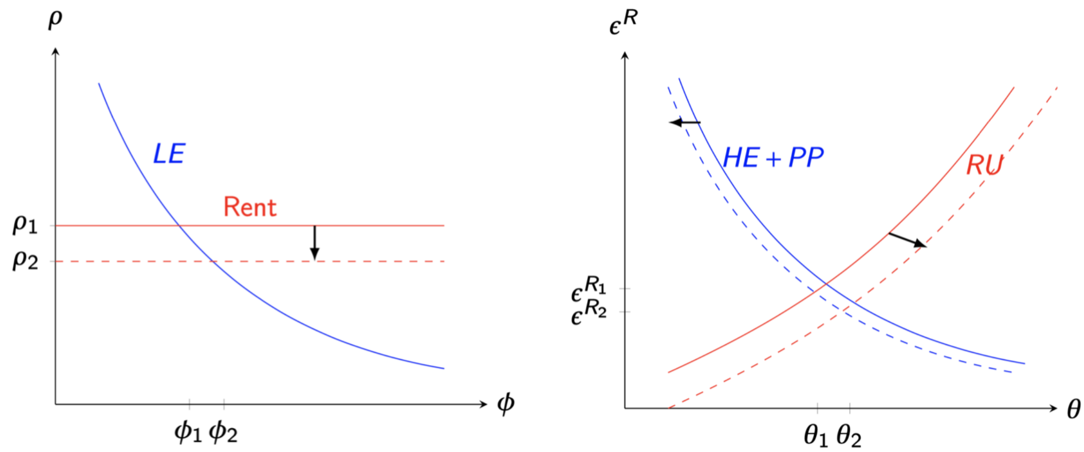

Figure 7: Demand Shock
The solid lines depict the equilibrium at steady state. The dashed lines represent the movement of curves after the shocks.

With my calibrated shocks, employing a demand shock alone leads to a decrease in prices. However, the price-to-rent ratio rises due to a more substantial decrease in rents. Market tightness increases in the housing market which decreases the time to sell. Moreover, as buyers increase, so do sales and housing vacancies. The rental market tightness increases because of the decrease in rental vacancies. Thus, while demand shocks can align with the cross-correlation between housing and rental market variables, they fail to match existing stylized facts in the housing market, where prices are negatively correlated with sales and housing vacancies. However, this discrepancy can be rectified by introducing a supply shock, a method previously utilized in this literature.

6.2 Supply and Demand Shock

Table 5 shows the business cycle properties of my benchmark model economy when supply and demand shocks are introduced.

Table 5: Results: Supply and Demand shocks in housing and rental market

Elasticity of X wrt to Rental Vacancy	Data	Model
Price to Rent	-0.15	-0.15
Time to Sell	1.48	0.09
Sales	-0.66	-0.65
Housing Vacancies	-0.82	-0.56

The dynamics of supply and demand shock are shown in figure 8. The difference here is that simultaneously, construction costs in the housing market rise, pushing the HE+PP curve upwards, consequently driving house prices higher. In the rental market, construction costs rise, exacerbating the reduction in supply and makes (LE) move towards the right. Consequently, rental vacancies decrease, tightening market conditions in the rental sector. However, the decrease in demand exerts downward pressure on rents. In this way, the demand and supply shock are still able to simulate a hot housing market where the rents and rental vacancies decrease.

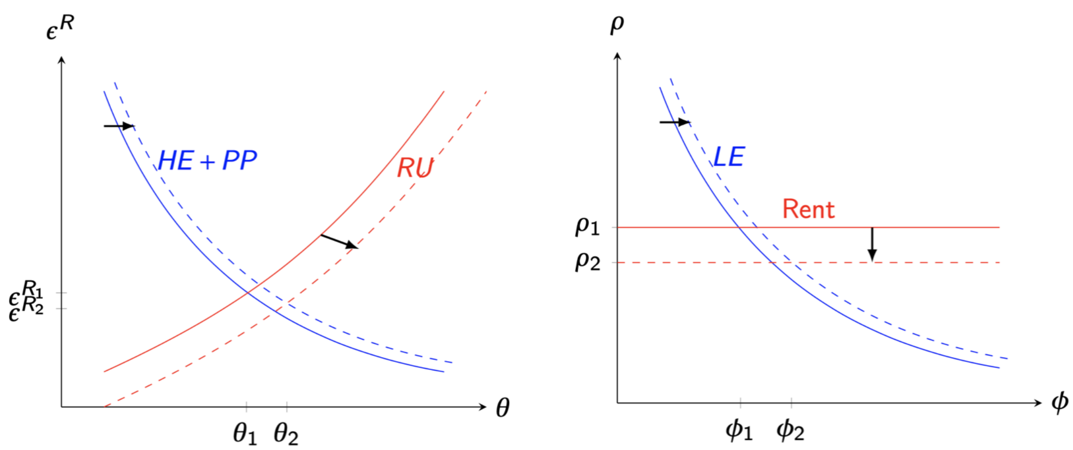

Figure 8: Supply and Demand Shock
The solid lines depict the equilibrium at steady state. The dashed lines represent the movement of curves after the shocks.

In this case, I am able to match the previous stylized facts where prices are positively correlated with sales and housing vacancies, and negatively correlated with time to sell. Furthermore, the Beveridge curve in the housing market in my model exhibits an upward slope, indicating a positive correlation between housing vacancies and the number of buyers. This consistency with prior studies, such as the work by Gabrovski and Ortego-Marti (2019), Gabrovski and Ortego-Marti (2021b), and Piazzesi et al. (2020), underscores the reliability of my model's framework. Unlike their model, which relies on the free entry of both buyers and sellers to achieve an upward-sloping Beveridge curve, my model endogenizes the free entry of buyers. Despite buyers entering through the rental market, the Beveridge curve maintains its upward slope. The reason is the use of an endogenous reservation utility. As illustrated in the calibration exercise, during a hot housing market an increase in housing vacancies coincides with a decrease in the reservation utility, thereby facilitating an influx of buyers.

The study also successfully replicates the upward-sloping Beveridge curve observed in the rental market, as documented by Badarinza et al. (2024). In my model, the demand shock encourages a shift from rental to homeownership, reducing the pool of rental seekers. Concurrently, there is a fall in the creation of new rental vacancies, further diminishing rental availability. Consequently, both rental vacancies and rental seekers decrease, indicating an upward-sloping Beveridge curve in the rental market.

This finding allows for insights into the relative movements along the Beveridge curves in both the housing and rental markets. While the model demonstrates that both curves slope upwards, there are contrasting movements as shown in figure 9: as there's an upward shift along the Beveridge curve in the housing market, resulting in increased buyers and housing vacancies, there's a simultaneous downward shift along the Beveridge curve in the rental market, leading to a reduction in rental seekers and vacancies. This is the first paper to talk about relative movements along the Beveridge curves in these two correlated markets.

Lastly, the model successfully reproduces the market crossover findings highlighted by Badarinza et al. (2024). Their research reveals that a tighter rental market correlates with an increased likelihood of transactions in the housing sales market. My quantitative analysis confirms these results. As depicted in figure 7, heightened market tightness in the rental sector correlates with greater housing sales. Moreover, Badarinza et al. (2024) also identify a relationship where a tighter housing market corresponds to more transactions in the rental market. However, they

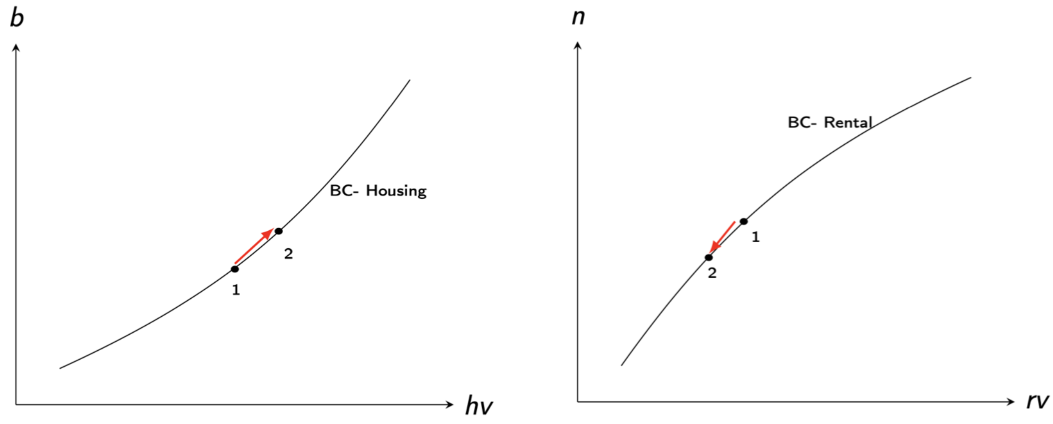

Figure 9: Beveridge Curves in the Housing and Rental Market.
In the housing model, there's a relative rise (from 1 to 2) in both buyers and housing vacancies due to the demand and supply shocks. Conversely, in the rental market, there's a concurrent decline (from 1 to 2) in both rental seekers and rental vacancies.

note this association to be economically marginal and statistically insignificant. Similarly, my model reflects this outcome: as market tightness in the housing market rises, the number of matches formed in the rental market ($nm(\phi)$) increases marginally. In summary, the model adeptly captures the established stylized facts of the housing market and contributes to the literature by documenting new insights resulting from interactions with the rental market.

7 Conclusion

This paper presents stylized facts regarding the business cycle correlations of rental vacancies, the price-to-rent ratio, time to sell, sales, and housing vacancies. The data suggests that a decrease in rental vacancies is linked to an increase in the price-to-rent ratio, sales, and housing vacancies, while the time to sell decreases.

The paper integrates a rental market into a standard search and matching model of the housing market, accounting for search frictions in both sectors. It incorporates endogenous entry of rental seekers, landlords, and sellers, with households entering the housing market through the rental market. This entry decision hinges on their individual utility of homeownership, thereby introducing heterogeneity among buyers.

Calibration and simulation of the model allow for comparison with empirical evidence on the elasticity of the price-to-rent ratio, sales, housing vacancies, and time to sell with respect to rental vacancies. Previous studies such as Diaz and Jerez (2013), Head et al. (2014), and

Gabrovski and Ortego-Marti (2019) indicate that both demand and supply shocks are necessary to match the stylized facts in the housing market. With these shocks, the model successfully matches the elasticity for the price-to-rent ratio, sales, and housing vacancies, while also qualitatively matching previously observed elasticities in the housing market. Additionally, the model generates an upward-sloping Beveridge curve for both the housing and rental markets, consistent with findings from Gabrovski and Ortego-Marti (2019) and Badarinza et al. (2024). The model can also shed light on the opposite relative movements along the Beveridge curve in both the markets.

The underlying mechanism of the model suggests that changes in rental vacancies affect endogenous reservation utility, consequently influencing household entry decisions into the housing market and leading to alterations in housing market variables. Although the model qualitatively matches the elasticity of time to sell, further investigation is required to understand other factors influencing time to sell that may not be accounted for in the model.

In conclusion, this paper underscores the pivotal role of the rental market in comprehending movements in the housing market.

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A Appendix

A.1 Model with Investors

In this section, I expand the model to incorporate investors into the market. Investors primarily buy housing vacancies, competing with regular buyers, and then convert these properties into rental vacancies. This buy-to-rent strategy enables the movement of properties from the housing market to the rental market. Similarly, investors with rental properties can move them back into the housing market in hopes of selling them. This mechanism facilitates the transition of properties from rental to the housing market.

The primary objective of this exercise is to determine whether the previously observed correlations remain consistent. Additionally, I aim to assess whether this approach increases the volatility in the time to sell, thereby aligning the model more closely with the stylized facts.

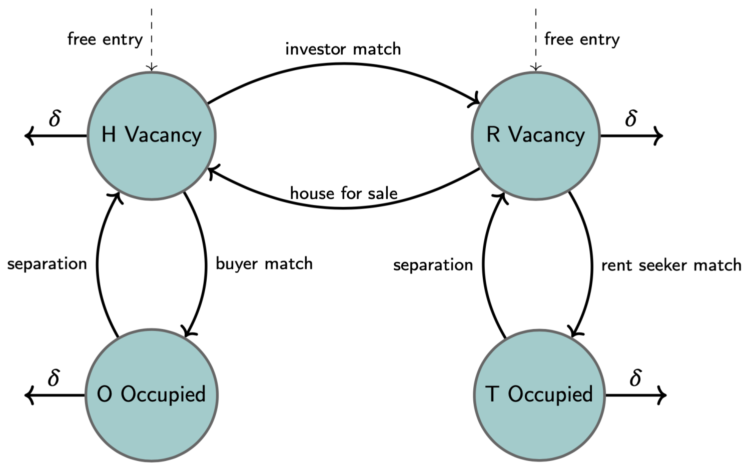

Figure 10: Property stocks and flows when investors are added to the model.
Investors can buy housing vacancies and convert them into rental vacancies which would provide them with rental utility. In the same way, investors can also move rental vacancies to housing vacancies which can be sold for profit.

Figure 10 illustrates the differences in this setup compared to the baseline model shown in Figure 3. Here, investors play a crucial role in facilitating the movement of properties between the rental and housing markets. Since the rest of the model remains similar to the previous version, the bellman equations in the steady state are the same, with some modifications as illustrated below.

The bellman equation for a home-owner is still given by equation (1), and for a home-buyer, it is given by equation (2). However, the value function for the seller changes because they can be matched with either a regular buyer or an investor. Here, I assume an exogenous matching rate τ for a seller matching with a regular household, receiving the price $\overline{p}$, and a rate of $(1-\tau)$ for matching with an investor, receiving the price p_i. Consequently, $(1-\tau)$ also represents the exogenous rate at which properties transition from the housing market to the rental market. The updated bellman equation is presented in equation (53),

$$V^H = -c^S + (1-\delta)\Big(\theta m(\theta)(\tau\overline{p} + (1-\tau)p_i) + (1-\theta m(\theta))\beta V^H\Big). \tag{53}$$

The bellman equations for a tenant, rental seeker, and landlord remain as described in equations (4), (5), and (7), respectively. However, the bellman equation for rental vacancies has been modified because investors now have the option to either rent out the property or sell it. I introduce an exogenous rate ω, representing the likelihood of an investor transferring a property from the rental market to the housing market, converting it into a housing vacancy. Consequently, at the rate $(1-\omega)$, the property remains in the rental market. The remainder of the Bellman equation remains unchanged, as described in equation (6). The updated bellman equation is presented in equation (54),

$$V^R = \Big(-c^R + (1-\delta)\Big(\phi m(\phi)\beta L + (1-\phi m(\phi))\beta V^R\Big)\Big)(1-\omega) + \omega\beta V^H. \tag{54}$$

Finally, equation (55) represents the bellman equation for an investor with utility I. Assuming free entry of investors into the market, the number of investors is determined by the search flow cost, which depends on the number of investors, i. This approach mirrors the modeling of free entry for rental seekers. At a rate of $m(\theta)$, investors match with sellers, convert housing vacancies into rental vacancies, and pay the price p_i. The bellman equation is given by:

$$I = -c^B(i) + m(\theta)(\beta V^R - p_i) + (1 - m(\theta)\beta I. \tag{55}$$

The free entry condition for investors, $I = 0$, pins down the number of investors i. The price paid by investors, p_i, is determined through Nash Bargaining between investors and sellers. The surplus for sellers remains the same and is given by, $S^S = p_i - \beta V^H$. The surplus for investors is the value they derive from the rental vacancy, expressed as $S^I = \beta V^R - p_i - \beta I$. Thus, the Nash Bargaining problem can be formulated as,

$$p_i = \underset{p_i}{\text{argmax}}(S^I)^\eta (S^S)^{1-\eta}. \tag{56}$$

The first order condition gives the Nash sharing rule in the investor market which can be used to determine the price as,

$$p_i = \beta(1-\eta)(V^R - I) + \beta\eta V^H. \tag{57}$$

This price represents the weighted average of the construction costs in the rental and housing markets. The remainder of the equilibrium is solved using the same method as previously described in Sections 3.5, 3.6, and 3.7. While the free entry condition for vacancies will change, the rest of the equations remain unchanged.

The model can once again be solved for a unique steady state, as previously demonstrated. After this, I do not re-calibrate the model; instead, I incorporate two additional parameter values for τ and ω, both sourced from Han et al. (2022). The value of $(1-\tau)$is set at 5.4% to align with the proportion of purchases made by buy-to-rent investors. Meanwhile, ω is set at 0.7% to reflect the exit rate of investors from the market, representing the rate at which investors sell their rental properties.

Once calibrated at the steady state, I introduce business cycle fluctuations as previously demonstrated. The process of converting this model into a dynamic setting remains unchanged. I apply the same demand shocks discussed in Section 6.1, followed by the demand and supply shocks outlined in Section 6.2. The results for the demand shock are presented in Table 6, and the results for the demand and supply shocks are shown in Table 7.

Table 6: Results: Demand shocks in housing and rental market

Elasticity of X wrt to Rental Vacancy	Data	Model
Price to Rent	-0.15	-0.12
Time to Sell	1.48	0.15
Sales	-0.66	-1.70
Housing Vacancies	-0.82	-1.97

The results above suggest that incorporating investors into the model and allowing for the movement of vacancies can be beneficial for two main reasons. First, we observe a slight am-

Table 7: Results: Demand and Supply shocks in housing and rental market

Elasticity of X wrt to Rental Vacancy	Data	Model
Price to Rent	-0.15	-0.15
Time to Sell	1.48	0.08
Sales	-0.66	-0.97
Housing Vacancies	-0.82	-1.60

plification in the elasticity of the time to sell when only the demand shock is introduced. This implies that the movement of properties could account for some of the volatility in the time to sell, as seen in the data. Second, there is a change in the ordering of the elasticities of sales and housing vacancies. Compared to Tables 4 and 5, these results show the correct ordering, where the elasticity for housing vacancies is lower than the elasticity of sales. This was not the case in the baseline results.

However, due to these movements, the series become even more volatile, making it challenging to match the results quantitatively, which was somewhat achievable previously. Thus, while the model with investors performs better in certain aspects, overall, it becomes more difficult to align with the data.

A.2 Transition Matrix

Table 8: Transition Matrix

		To State				
		1	2	3	4	5
From State	1	0.9149	0.0823	0.0028		
	2	0.0206	0.9163	0.0618	0.0014	
	3	0.0005	0.0412	0.9167	0.0412	0.0005
	4		0.0014	0.0618	0.9163	0.0206
	5			0.0028	0.0823	0.9149