Discrimination against Ex-offenders in Frictional Labor Markets

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Abstract

We construct a model of discrimination against ex-offenders and workers' criminal behavior, based on a labor search and matching framework. Ex-offenders suffer from the difficulties in finding jobs generated by the discriminatory market segregation against them. It pushes ex-offenders to return to the crime market and to recidivate. To reduce crime and recidivism, the "Ban-the-Box" (BTB) policy is implemented statewise in 37 states in the US. The policy removes the market segregation such that the statistical discrimination against to ex-offenders within the same demographic group is eliminated. The formal labor market becomes attractive to the ex-offenders, but it hurts the employment of workers without criminal records. Workers without criminal records are more likely to commit crimes after the policy implementation. The overall crime rate increases by 26.23 offenses per 1000 individuals, while the recidivism rate decreases by 4.41 percentage points. Alternatively, the criminal record expungement is more effective for crime reduction. The overall crime rate increases by 8.50 to 11.45 offences per 1000 individuals, and the recidivism rate decreases by 4.15 to 7.42 percentage points.

Keywords: search and matching, unemployment, criminal records, crime, ban the box

JEL: E24, J64, K24

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1 Introduction

Discrimination in the labor market has been discussed for decades. Labor economists believe that discrimination on wages and employment comes from the tastes of employers (Becker, 1971) and the asymmetric information on the labor productivity of the minorities (Arrow, 1973; Phelps, 1972). In this paper, we focus on the discrimination against to ex-offenders within the same demographic group of the population. Discrimination against ex-offenders stops them from returning to the formal sector and rehabilitation. Therefore, ex-offenders prefer to return to the crime market and earn their lives by becoming involved in criminal activities. According to Durose and Antenangeli (2021), more than 60% of the released ex-offenders are arrested within 3 years. Ex-offenders are the main contributors to the overall crime rate.

Initiated in the late 1990s, the "Ban-the-Box" (BTB) movement garnered significant discourse. The movement aims to reduce discrimination against ex-offenders in the labor market. By deferring the background check until later in the hiring process, the BTB movement seeks to enhance fair employment opportunities and encourage employers to assess candidates on their merits rather than their criminal history. One of the earliest policy adoptions was in 2004, when the city of San Francisco passed a form of this policy. In a landmark move, former President Obama extended this policy to federal job applications in 2015. Since 2010, a majority of U.S. states have adopted legislation to eliminate queries about criminal convictions from job applications in the private sector. It is a significant policy initiative aimed at reducing employment barriers for individuals with criminal records. This policy, which has gained momentum across various states and municipalities in the United States, involves the removal of the checkbox on employment application forms that asks applicants if they have a criminal record. The core objective of the BTB is to enable individuals with past convictions to display their qualifications in the job market without being automatically disqualified at the initial application stage. Research on the effectiveness of BTB policies has yielded mixed results. Some studies suggest that these policies increase the likelihood of individuals with criminal records receiving callbacks and interviews. However, other studies indicate potential unintended consequences, such as employers who might statistically discriminate against demographic groups more likely to have a criminal record when they cannot directly inquire about criminal histories. Critics argue that BTB policies may lead to employer biases in hiring practices, particularly against minority groups (e.g. young black males), which discourages that minority workers without criminal records from staying in the formal sector. Moreover, minority ex-offenders are more likely to become involved in criminal activities again because it is difficult for them to find jobs.

This paper is the first to theoretically study the discrimination against ex-offenders in frictional labor markets theoretically. In the economy of this model, firms post vacancies and look for workers, while unemployed workers search for jobs. Both sides of the market meet randomly. When a match occurs, a matching quality is drawn from a random distribution. Firms and workers decide whether to accept the match or not given the matching quality. After they accept the match, workers start producing and the productivity is based on the matching quality. Firms are either prejudiced or unprejudiced. A prejudiced employer suffers disutility if he/she hires an ex-offender, while unprejudiced employers are indifferent to hiring an ex-offender or a worker without criminal records. Before the BTB policy is implemented, the labor market can be segmented because prejudiced employers avoid the disutility of hiring an ex-offender, by asking about the crime-related questions in job application forms. A market that excludes ex-offenders is a restricted market, and a market that welcomes all workers is a nonrestricted market. The restricted market consists of the firms that ask questions about criminal records in their job applications. These questions automatically exclude ex-offenders from the beginning of the hiring process. Ex-offenders know that even if they can hide their criminal record information in the application, they will be uncovered during the hiring process. They will not be hired eventually. Therefore, they do not apply for jobs in the restricted market. In the nonrestricted market, unprejudiced firms do not ask questions about criminal records. These firms hire workers that they randomly match with in the labor market, regardless of whether they are ex-offenders or not. All workers can search in a nonrestricted market, regardless of their criminal record status. Workers may encounter a criminal opportunity at some point in their lives. If a criminal opportunity pays well, workers will commit the crime. Criminals can be arrested at a certain probability and have to leave the labor force, i.e., employed workers have to leave their jobs, and unemployed workers are no longer able to search in the labor market. Workers' criminal behavior generates extra job separations. The extra job separations give rise to externalities for firms, and the traditional Nash bargaining share rule cannot internalize this externality. To internalize the externality, we design an employment contract in this economy, i.e., workers must pay a hiring fee upfront. At the equilibrium, this hiring fee is the firms' share of the surplus of this match, and workers have the production of the match. After that, firms do not care about the tenure of the match, and workers do not commit crimes more than they suppose to. Therefore, the economy achieves to the optimal equilibrium.

Since workers without criminal records can access both sub markets, they request a higher matching quality than ex-offenders. Ex-offenders would like to accept relatively worse matching quality, as it is challenging for them to find a job. This provides an explanation that some firms post vacancies in the restricted market: they do not want to match a worker with low matching quality. According to Arrow (1973) and Phelps (1972), the statistical discrimination between ex-offenders and workers without criminal records is caused by segmented markets and asymmetric information on the productivity, which is referred to as the matching quality in this paper. We only focus on the statistical discrimination against ex-offenders and assume that population is identical in terms of demographic characteristics. Quantitatively, we estimate empirical moments and calibrate the model with the Current Population Survey (CPS) and Uniform Crime Report (UCR) in the US in from 2010 – 2015. The model generates a wage gap between workers without criminal records and ex-offenders, in which the wage of ex-offenders is 92.5% of the wage of workers without criminal records. Rose (2021) estimates that the wage gap between workers without criminal records and ex-offenders is approximately 30%. Our model explains approximately 25% of the wage gap between ex-offenders and the workers without criminal records.

As the model sucessfully formulates the discrimination against ex-offenders, we adopt the model to shed light on the effects of the BTB policy. After the BTB policy implementation, prejudiced employers observe the matching quality before they discover the criminal history of the matched employee. This is because they are no longer allowed to ask questions about criminal records at the application stage. In the same labor market, unprejudiced employers have a greater expected value of posting a vacancy than prejudiced employers. At the equilibrium, prejudiced employers exit the labor market because they suffer a negative value of vacancies in this labor market. Therefore, only the nonrestricted market remains. The statistical discrimination against ex-offenders disappears because there are no prejudiced employers in the market. This policy increases the job finding rate of ex-offenders and helps the unemployed ex-offenders. However, it hurts the employment of workers without criminal records, as they lose their privilege in the labor market.

Quantitatively, after the BTB policy is implemented, the average wage of ex-offenders increases by 2.10%, while the wage of workers without criminal records decreases by 3.69%. The job finding rate of workers without criminal records decreases by 36%. A decrease in the job finding rate causes a reduction in the employment of workers without criminal records of 5.27 percentage points. The additional job opportunities for ex-offenders from the BTB policy increase the job finding rate of ex-offenders by 26%, such that the employment of ex-offenders increases by 3.25 percentage points.

The crime rate increases by 26.23 offenses per 1,000 individuals after the BTB policy implementation. The value in the labor market of workers without criminal records declines because of the reduced job finding rate. Moreover, the BTB policy reduces the cost of committing crime: ex-offenders have a higher job finding rate than before. Decreases in the value of the labor market and the cost of committing crimes make criminal opportunities more attractive to workers without criminal records. Hence, all workers except unemployed ex-offenders are more likely to commit crimes. Social welfare decreases by 6.26% after the BTB policy, including the loss from the increase in unemployment of workers without criminal records, and the increase in crime.

We also consider a policy alternative for counterfactual exercise. We adopt the policy of criminal records in Denmark: criminal record expungement. Under this policy, imprisonment records are expunged in five years after the ex-inmates are released from jail. The model predicts that this counterfactual policy has a smaller increase in the overall crime rate than the BTB policy. In the policy experiment, criminal records are expunged in 3 to 10 years after ex-offenders are released. The overall crime rate increases by 8.50 to 11.45 offenses per 1,000 individuals accordingly. It also reduces the recidivism rate, as the ex-offenders are rewarded by staying away from crime.

The relationship between labor market outcomes and crime has been discussed for decades. The literature has shown that labor market outcomes have strong effects on crime participation, especially on property crimes. Becker (1968) analyzes criminal issues via an economic approach. He discusses the cost of criminal punishment and social welfare. Since then, economic conditions have become one of the driving forces of crime. Danziger and Wheeler (1975) provide evidence of the relationship between income redistribution and crime. They compare the effects of income redistribution and punishment on crime reduction and conclude that criminal activities are associated with economic conditions, such as unemployment and wage disparities. Improving income redistribution could be more effective for crime reduction than punishment. Ehrlich (1975) finds that the labor market participation reduces the murder rate, while the murder rate increases with the unemployment rate. He suggests that the employment and the earning opportunities are correlated with the frequency of murder and other types of crime. Grogger (1998) further supports the relationship between youth unemployment and crime. This suggests that wages play a significant role in the participation in crime, such that higher wages lead to a reduction in crime participation. Grogger (1991) also discusses the impact of wages and employment across crime types. Higher employment may reduce felonies while it is associated with more misdemeanors. Wages are roughly the same across different types of crimes. Moreover, felony crimes are more sensitive to the duration of unemployment than to wages. Freeman (1999) notes that crime can be analyzed as a market with demand and supply. Unemployment is considered a legitimate opportunity that criminals have. A high unemployment rate is accompanied by a high crime rate, but the relationship is not strong. The decision to commit crimes is also highly related to the payoff of legal or illegal work. Fagan and Freeman (1999) show that income optimization affects the decision making of criminals. This income optimization makes the boundaries of legal and illegal work porous. Gould et al. (2002) show that the link between the wages of low skilled workers and crime is strong. Machin and Meghir (2004) study economic incentives and crime in England, Wales and the US, supporting the hypothesis that worse labor market opportunities for the bottom end of wage distribution may increase the crime rate. Bushway and Reuter (2011) summarize the literature on the relationship between labor market outcomes and crime. A one percentage point change in the unemployment rate at the state or county level has an approximately two percentage impact on property crime rate. Also, loss of high-paid manufacturing jobs is a key factor in increasing crime. However, the aggregate unemployment does not have significant effects on crime.

Moreover, researchers not only look at the link between labor market outcomes and crime but also examine how former prisoners perform in labor markets. This is an important issue to ask because the recidivism rate is high and most crimes are committed by ex-offenders.[1] Grogger (1992) suggests that the arrest records significantly affect the employment of young people, particularly the racial disparities between black and white youth. Pager (2003) audits a study on the effects of criminal records on employment opportunities. She finds that criminal records are the major barrier to employment. Workers without criminal records are 50% or more likely to receive callbacks. The gap in the callback rate between ex-offenders and workers without criminal records is bigger among black males. Only 44% of employers would like to hire ex-offenders. Pager et al. (2009) also investigate the effects of prison records on employment in New York City. They find that criminal records have more significant negative effects for black people, i.e., they receive fewer interview invitations than white people. Loucks et al. (1998) discuss the ex-offenders' situation in the European Union. In the EU, most countries ban ex-offenders from working in certain jobs, for example, jobs in the public sector. They summarize the rehabilitation policies for ex-offenders and state that a good rehabilitation policy can help prevent ex-offenders from getting involved in criminal activities again. Denver et al. (2018) provide evidence that most employers ask about potential employees' criminal records information in an early stage of job application. This means that employers worry about trust and safety issues of hiring an ex-offender. Agan and Starr (2018) carried out an experiment in New Jersey and New York City on effect of the "Ban-the-Box" policy on callbacks. They find that the BTB policy encourages statistical discrimination, i.e., employers may not hire a certain group of workers because they assume that group mainly consists of ex-offenders. They also showed that the criminal record is one of the main barriers to employment: Individuals are 63% more likely to receive a callback, if they have no criminal records. Doleac and Hansen (2020) show that the BTB policy reduces the employment probability of young, low-skilled black men by 3.4 percentage points. Rose (2021) shows that ex-offenders are penalized by unemployment and wage penalties when they return to the labor market. He investigates the effects of the BTB policy implemented in Seattle. He finds that the effects of the BTB policy on employment and earning are limited, and the BTB is unlikely to be an important tool for reducing the recidivism rate.

This is the first theoretical paper on crime and frictional labor markets that looks at both the statistical discrimination in frictional labor markets and criminal behavior, and analytically discusses the policy implementation. The majority of the theoretical literature focuses on the racial or gender discrimination. Lang and Lehmann (2012) discuss that the discrimination in the labor market comes from two channels: the preference of employers and the unobservable information of labor productivity. Becker (1971) discusses the

[1] According to Durose and Antenangeli (2021), about 78% of former prisoners are arrested again within 5 years after they are released from jail. We also provide empirical evidence in table 2 from the US Department of Justice Statistics.

discrimination that may be generated by the "taste" of employers. Black (1995) segments the labor market with unprejudiced and prejudiced employers. Unprejudiced employers hire both majority and minority workers, while prejudiced workers hire only majority workers. According to this market setup, minority workers match less-well jobs and are paid with lower wages than the majority workers. Black (1995) is close to the our article, as we show that the reservation matching productivity of ex-offenders is lower than that of the workers without criminal records. Mailath et al. (2000) also develop a model of the preference of employers. The asymmetric preference of employers develops an equilibrium in the frictional labor market, such that less-preferred workers suffer a higher unemployment rate and lower payoff given the same skill as preferred workers. Rosén (2003) adopts the Becker's theory on discrimination with the labor search frictions and Nash bargaining wage determination. Discriminationary employers pay lower wages to the minor groups and gain greater profits. They also suffer disutility from working with the less-liked groups. Bowlus and Eckstein (2002) analyze the wage differential between black and white workers and conclude that the productivity of black workers is only 3% less than that of white workers. The racial wage differential is mainly caused by the large number of prejudiced firms.

Phelps (1972) and Arrow (1973) introduce the discrimination caused by the asymmetric information, particularly in terms of labor productivity. This is called the statistical discrimination. Despite the average labor productivity among major and minor groups of workers, the minor groups may have more variance than the major groups. This variance difference and lack of information generate part of the discrimination in the labor market against to the minor groups, for instance, females, blacks, immigrants, or ex-offenders (Flabbi (2010);Larsen and Waisman (2016)). Rosén (1997) constructs a search model in which discriminationary employers only hire black workers when they do not have white applicants. The model generates two equilibria: discrimination equilibrium and nondiscrimination equilibrium. Only the discrimination equilibrium is stable. The gap in the probability of hiring generates a reservation productivity gap between black and white workers. Therefore, employers prefer to stay in the discrimination equilibrium and avoid matching workers with lower productivity. Moro and Norman (2004) also construct a search model with statistical discrimination, and conclude that the statistical discrimination affects the human capital investment in both the advantaged group and the discriminated groups. Lang et al. (2005) construct a model with wage posting and a direct search model. It leads to market segregation because black workers avoid applying for jobs that white workers would apply. Therefore, the market segments by worker search behavior and black workers only apply for jobs with lower wages. According to Lang and Manove (2011), all workers have the same payoff, but the discriminated group suffers a higher unemployment rate.

Becker (1968) provides a basic idea for theoretically linking the criminal behavior and labor market outcomes theoretically. He suggests that a worker becomes involved in a criminal activity because this criminal activity can provide a high economic value. Our theory follows Becker (1968) that the decision to commit crimes depends on the comparison of economic value. The closest work on crime and frictional labor markets is Engelhardt et al. (2008). In this model, workers consider both their flow wages and the expected value in the labor market for the future. The value in the labor market determines whether a worker would like to commit a crime when he/she meets a criminal opportunity. An optimal contract is designed to maximize the surplus of a match. This optimal contract internalizes the negative externality that firms have to suffer from employees' criminal behavior. Burdett et al. (2003), Burdett et al. (2004), Huang et al. (2004), and Engelhardt (2010) also construct models of frictional labor markets and criminal behavior. Differing from Nash bargaining with an optimal contract, Burdett et al. (2003) and Burdett et al. (2004) use wage posting to determine the employment contract. In these two models, criminal behavior is determined by workers' reservation wage, i.e., workers will not commit crimes if they are paid enough. Burdett et al. (2004) and Engelhardt (2010) extend the model with on-the-job search. Burdett et al. (2004) find that the model with on-the-job search can have two equilibria: i) low unemployment and a low crime rate, and ii) high unemployment and a high crime rate. They showed that a strong connection exists between crime and labor market outcomes in frictional labor markets. Huang et al. (2004) include educational and occupational choices in a labor search and matching model. They find multiple equilibria in which a high crime rate is correlated with low educational attainment, high unemployment, and poverty. Engelhardt (2010) constructs his model on an on-the-job search framework with heterogeneity of firms and workers. Firms with higher productivity pay higher wages, so workers working for high-productivity firms are less likely to commit crimes than workers working for low-productivity firms. Additionally, workers with high unemployment utility/leisure utility will never get involved in criminal activities. The model predicts that a policy helping workers find jobs can improve the labor market for ex-offenders and reduce the crime rates. In particular, the recidivism rate can be reduced by more than 5%.

The rest of the paper is organized as follows. Section 2 describes the economic environment and the model, including the value functions, wage determination, search behavior, and criminal behavior of workers. Section 3 shows the equilibrium of the model. The optimal contract with criminal behavior will be discussed. We calibrate the model in section 4. Section 5 discusses the policy effects of "Ban-the-Box". Section 6 extends the model with on-the-job search. In section 7, we extend the model to the case of criminal record expungement. Section 8 concludes.

2 Model

Before the BTB policy is implemented, there is a restricted (R) market and a nonrestricted (NR) market in this economy. Prejudiced firms (P) in the restricted market ask questions about criminal records in the application forms. Unprejudiced firms (NP) in the non-restricted market do not ask crime-related questions. Ex-offenders are excluded by the restricted market and only search for jobs in the nonrestricted market, while workers without criminal records search for jobs in both labor markets.

Time is continuous over an infinite horizon. New workers enter labor markets as unemployed workers without criminal records. All workers exit the labor markets at rate σ because of retirement or death. The labor markets are frictional. They are subject to the number of job seekers and the number of vacancies. The number of matchings follows a matching function $m(v_i, s_i)$, where s_i is the measure of job seekers in submarket $i \in \{R, NR\}$, and v_i is the number of vacancies in submarket i. In particular, the seeker measure in the nonrestricted market includes both job seekers with and without criminal records, while the measure of job seekers in the restricted market includes only job seekers without criminal records. This matching function $m(.,.)$ is continuous, strictly increasing, and concave. Moreover, $m(0,.) = m(.,0) = 0$ and $m(\infty,.) = m(.,\infty) = \infty$. According to Pissarides (2000), in the labor search and matching framework, the labor market tightness is defined as $\theta_i \equiv v_i/s_i$. The job finding rate for workers is $m(v_i, s_i)/s_i \equiv f(\theta_i)$, and the job filling rate for firms is $m(v_i, s_i)/v_i \equiv q(\theta_i)$. To simplify the model, we do not allow on-the-job search in the baseline model, such that the measure of seekers is the measure of unemployment in either submarket. We relax this restriction in section 6.

Both workers and firms are risk neutral, and they are subject to a discount rate r. Unemployed workers can obtain a flow utility b. When the worker matches the firm, a matching quality y is drawn from a certain distribution $F(y)$. According to the matching quality, the worker and the firm negotiate for an employment contract $(w_i^j(y), \phi_i^j(y))$. In this contract, he/she pays a lump sum hiring fee ϕ_i^j, where $j \in \{NC, C\}$, at the beginning of the employment.[2] The notation NC represents agents without criminal records and C represents ex-offenders. After the worker is hired, he/she receives a constant flow wage $w_i^j(y)$.

Prejudiced firms dislike ex-offenders such that they suffer an exogenous disutility from hiring ex-offenders, which is $\mathcal{U}_d$. The share of prejudiced employers in the market is φ and it is exogenous. Assume that the disutility $\mathcal{U}_d$ is high enough such that the prejudiced employers prefer to be in the restricted markets and exclude any applicants with criminal records. Therefore, the disutility does not play a role in the value

[2]Criminal behavior of employees brings externality to firms by adding extra job separation. According to Engelhardt et al. (2008), an employment contract with a constant wage and hiring fee paid by workers can internalize the externality of criminal behavior for workers. This contract is optimal in that to reduces the loss of firms from workers' criminal behavior and achieves optimal solution of the surplus maximization problem. We directly adopt this form of employment contract as we have the same externality issue with regards to firms.

functions of prejudiced employers in the restricted markets.[3] Firms enter one of the submarkets freely and post job vacancies by paying a flow cost c. Each firm has only one job, either vacant or filled. Firms enter the market freely. The match separates at rate δ exogenously.

All workers may confront criminal opportunity at rate μ_k, where $k \in \{E, U\}$ represents the employment status: E for employed workers, and U for unemployed workers. A criminal opportunity rewards the criminal with a value g. This crime value follows a certain random distribution $G(g)$. Workers observe the value of the criminal opportunities they encounter and decide whether to commit crimes or not. If the value of the criminal opportunity is greater than the reservation crime value, he/she commits the crime. We only consider property crimes in this economy. Criminals are arrested and are incarcerated at probability π, and they have to leave the labor market, either leave their jobs if they are employed or refrain from searching in the labor market if they are unemployed. Inmates leave jail at rate ρ and return to the labor market as unemployed workers with criminal records. Firms are able to exclude ex-offenders by asking crime-related questions in the job application form. The cost of committing crimes includes the capital loss of exiting the labor market and being incarcerated, and the capital loss when criminals return to the labor market as unemployed workers with criminal records after incarceration.

Workers can be victims of criminal activities. They take the expected loss from criminal activities into account when they calculate their value functions. The expected loss from criminal activities comprises by two parts. The first part is the wealth transfer. When criminals commit crimes and obtain the value of criminal opportunity, we consider the value of crime g as the wealth transfer from the victim to the criminal. Since workers commit crimes only when the crime value is higher than their reservation value, the expected wealth transfer from the victim to the criminal is conditional on the reservation crime value, considering the labor market status, $\mathbb{E}_{k_i^j}[g] = \int_{\bar{g}_{k,i}^j} g dG(g)$, where $k \in \{E, U\}$. The variable μ_k can be interpreted as the intensity of searching for criminal opportunities. Victims do not only suffer from the wealth transfer, but also suffer additional losses from criminal activities. The parameter m represents the extra loss of victims apart from the wealth transfer. Therefore, the expected criminal loss τ is the expectation of the crime value conditional on the reservation value of the crime for different groups of workers, i.e.,

$$\tau = (1+m) \sum_{i \in \{NR, R\}, j \in \{NC, C\}} \left\{ \mu_E e_i^j \mathbb{E}_{E_i^j}[g] + \mu_U u_i^j \mathbb{E}_{U_i^j}[g] \right\}. \quad (1)$$

The variable e_i^j and u_i^j represent the measure of employed (E) and unemployed (U) workers, respectively, with a criminal record status j in submarket i.

[3] The disutility will play a role in the implementation of the BTB policy because prejudiced employers must match an ex-offender before they discover his/her criminal history. See details in section 5.

2.1 Bellman equations

We are going to explain the value functions in this subsection according to the environment we described above. We start from the value function of the criminal activity. When criminals commit crimes, they receive a value g from the criminal activity. They still hold the value that they currently have in the labor market, until they are arrested and incarcerated at the probability π. Criminals suffer the capital loss from incarceration, $V_P - V^j_{k,i}$, where V_J is the value in jail. The value of the criminal opportunity g is randomly drawn from a certain distribution $G(g)$ with a support $[0, \infty)$. The value function of committing a crime $K^j_{k,i}$ is

$$\mathcal{K}^j_{k,i}(g) = g + V^j_{k,i} + \pi(V_J - V^j_{k,i}), \tag{2}$$

where $k \in \{E, U\}$ represents the employment status, $i \in \{NR, R\}$ represents the submarket, and $j \in \{NC, C\}$ represents the criminal record status. If the criminal is arrested, he/she will be incarcerated. The inmates receive a flow utility in jail x and also pay the "tax". This tax equals the expected loss by crime outside of jail and ensures that criminals do not commit crimes for the "safety" in jail. Inmates are released from jail at rate ρ. They return to the labor market as unemployed workers with criminal records. The value function of incarceration is

$$rV_P = x - \tau + \rho\left(V^C_U - V_P\right). \tag{3}$$

When workers confront a criminal opportunity, they commit crimes only when the value of committing a crime is strictly higher than the value in the labor market, which means that $K^j_{k,i} > V^j_{k,i}$. For employed workers, the value of employment also depends on the matching quality. As a result, the reservation crime value is at the equality of $K^j_{k,i}$ and $V^j_{k,i}$, which is

$$\bar{g}^j_U = \pi(V^j_U - V_J) \tag{4}$$

$$\bar{g}^j_E(y) = \pi(V^j_E(y) - V_J) \tag{5}$$

This reservation crime value shows that the reward from the criminal opportunity should be at least the expected loss of being caught. According to the environment, the value functions of ex-offenders V^C_U and unemployed workers without criminal records V_U^{NC} are

$$(r+\sigma)V_{U,NR}^{C} = b - \tau + \lambda_0^{C} \int \max\{V_{E,NR}^{C}(y) - V_{U,NR}^{C} - \phi_{NR}^{C}(y), 0\}dF(y) \tag{6}$$

$$+ \mu_U \int \max\{K_{U,NR}^{C}(g) - V_{U,NR}^{C}, 0\}dG(g)$$

$$(r+\sigma)V_{U,i}^{NC} = b - \tau + \lambda_0^{NC} \int \max\{V_{E,i}^{NC}(y) - V_{U,i}^{NC} - \phi_{i}^{NC}(y), 0\}dF(y) \tag{7}$$

$$+ \mu_U \int \max\{K_{U,i}^{NC}(g) - V_{U,i}^{NC}, 0\}dG(g).$$

Unemployed workers receive a flow unemployment benefit b and pay an expected loss from criminal activities τ. Workers without criminal records can search for both submarkets simultaneously. They accept the job offer when the offer provides an employment value higher than the unemployment value. The job finding rate of workers without criminal records is the sum of the job arrival rates in both submarkets, which is $\lambda_0^{NC} = f(\theta_{NR}) + f(\theta_R)$. Workers with criminal records only search in the nonrestricted market such that the job arrival rate is $\lambda_0^{C} = f(\theta_{NR})$. When a match occurs, workers decide whether to accept the job offer or not. If they accept the job match and become employed, they pay the lump-sum hiring fee $\phi_i^j(y)$. They also expect to obtain rewards from criminal activities. They encounter criminal opportunity at rate μ_U and decide whether to commit a crime or not.

When workers become employed, they receive the wage $w_i^j(y)$ according to the employment contract and the matching quality y. They also expect to lose τ from criminal activities. At an exogenous separation rate δ, employed workers lose their jobs and become unemployed again. Like unemployed workers, they meet a criminal opportunity at rate μ_E and decide whether to commit a crime or not. The value functions of employed workers with criminal records $V_{E,NR}^{C}(y)$ and employed workers without criminal records $V_{E,i}^{NC}(y)$ are as follows,

$$(r+\sigma)V_{E,NR}^{C}(y) = w_{NR}^{C}(y) - \tau - \delta(V_{E,NR}^{C}(y) - V_{U,NR}^{NC}) + \mu_E \int \max\{K_{E,NR}^{C}(g) - V_{E,NR}^{C}(y), 0\}dG(g). \tag{8}$$

$$(r+\sigma)V_{E,i}^{NC}(y) = w_{i}^{NC}(y) - \tau - \delta(V_{E,i}^{NC}(y) - V_{U,i}^{NC}) + \mu_E \int \max\{K_{E,i}^{NC}(g) - V_{E,i}^{NC}(y), 0\}dG(g) \tag{9}$$

Firms enter one of the labor markets by posting vacant jobs. They pay a flow cost k_i to establish a vacancy and randomly meet a worker in the labor market at rate $q(\theta_i)$. When the match is established, firms receive the lump-sum hiring fee paid by the worker. The value of a vacant job in the restricted market $\Pi_{V,R}$ is

$$r\Pi_{V,R} = -c + q(\theta_R) \int \max\{\Pi_{F,R}^{NC}(y) + \phi^{NC}(y), 0\}dF(y). \tag{10}$$

Since firms in the nonrestricted market do not have any information about the workers' criminal records, they have expectations regarding the distribution of unemployed workers with and without criminal records. The value of a vacant job in the nonrestricted market $\Pi_{V,NR}$ is

$$r\Pi_{V,NR} = -c + q(\theta_{NR})[\frac{u^{NC}}{u} \int \max\{\Pi^{NC}_{F,NR}(y) + \phi^{NC}_{NR}(y), 0\} dF(y) \tag{11}$$

$$+ \frac{u^{C}}{u} \int \max\{\Pi^{C}_{F,NR}(y) + \phi^{C}_{NR}(y), 0\} dF(y)] \tag{12}$$

where $u = u^{NC} + u^{C}$. Firms enter the labor market freely, such that $\Pi_{V,R} = \Pi_{V,NR} = 0$.

If a vacant job is filled by a worker, the firm receives the productivity y, which is drawn from the distribution of matching quality $F(y)$. Firms pay the worker with wage $w^j_i(y)$, according to the employment contract and the matching quality. The match separates at a Poisson rate δ or by the retirement of workers at rate σ, and the job becomes vacant. Additionally, the match can be destroyed by the criminal behavior of workers. If an employee meets a criminal opportunity with a value higher than the employee's reservation crime value, he/she commits the crime and may be arrested at probability π. The criminal has to leave the job when he/she is incarcerated. As a result, the value function of a filled job in the restricted market $\Pi^{NC}_{F,R}$ is as follows,

$$r\Pi^{NC}_{F,R}(y) = y - w^{NC}_{R}(y) - [\delta + \sigma + \mu_E \pi (1 - G(\bar{g}^{NC}_{E,R})](\Pi^{NC}_{F,R}(y) - \Pi_{V,R}). \tag{13}$$

and the value function of a filled job in the nonrestricted market $\Pi^j_{F,NR}$ is

$$r\Pi^{j}_{F,NR} = y - w^{j}_{NR}(y) - [\delta + \sigma + \mu_E \pi (1 - G(\bar{g}^{j}_{E,NR})](\Pi^{j}_{F,NR}(y) - \Pi_{V,NR}). \tag{14}$$

2.2 Wage determination

Wage determination aims to maximize the total surplus of a match following the Nash bargaining share rule. The bargaining power of workers is denoted as β. When firms and workers match and discover the matching quality, the employment contract $(w^j_i(y), \phi^j_i(y))$ is determined by the Nash bargaining share rule. In particular, firms that do not run background checks may hire a worker with or without criminal records. They do not know workers' background before hiring, but they will know it when they start bargaining on the employment contract with workers. We maximize the surplus of a match by Nash bargaining,

$$\max_{\phi^j_i, w^j_i} (V^j_{E,i}(y) - V^j_U - \phi^j_i(y))^{\beta} (\Pi^j_{F,i}(y) - \Pi_{V,i} + \phi^j_i(y))^{1-\beta}. \tag{15}$$

Lemma 1. *The optimal employment contract* $(w_i^j(y), \phi_i^j(y))$ *is the solution of the Nash bargaining surplus maximization problem, i.e.,*

$$w_i^j(y) = y,$$
$$\phi_i^j(y) = (1-\beta)(V_{E,i}^j(y) - V_{U,i}^j).$$

The proofs of all propositions and lemmas are in appendix B. The intuition of the optimal employment contract is as follows. We define the total surplus of a match with matching quality y as

$$\mathcal{S}_i^j(y) \equiv V_{E,i}^j(y) - V_{U,i}^j + \Pi_{F,i}^j(y).$$

From the value functions of employment, unemployment and filled jobs, we can conclude that the total surplus is

$$(r+\sigma)\mathcal{S}_i^j(y) = y - \tau - (r+\sigma)V_U^j + \mu_E \int_{\bar{g}_{E,i}^j(y)} \left[g + \pi(V_J - V_{U,i}^j) - \pi\mathcal{S}_i^j(y)\right] dG(g) - \delta\mathcal{S}_i^j(y).$$

Each match generates a flow value of the match, $y - \tau - (r+\sigma)V_U^j$, which includes the productivity of the match, an expected loss from the criminal activity, and the flow value of unemployed workers. The match separates either when the exogenous separation shock arrives at rate δ or when the employed worker commits a crime and is sent to jail. It is assumed that workers and firms can cooperate on the criminal behavior, meaning that they can jointly determine the reservation crime value together. The reservation crime value of employed workers is

$$\bar{g}_{E,i}^j(y) = \pi(V_{U,i}^j - V_J + \mathcal{S}_i^j(y)) = \pi(V_{E,i}^j(y) + \Pi_{F,i}^j(y) - V_J).$$

This value is different from the reservation crime value of employed workers in equation (5) and it is higher than (5) if $\Pi_{F,i}^j > 0$. This reveals that employed workers commit too many crimes when they do not determine the reservation crime value jointly with the firm. The surplus is not maximized because employed workers do not internalize the negative externality that they impose on firms through their criminal behavior.

When a worker commits a crime and is arrested, the capital loss of his/her criminal behavior is the employment surplus, i.e., $V_{E,i}^j(y) - V_{U,i}^j$. Meanwhile, the criminal behavior also breaks the match such that the match loss is the total surplus, $V_{E,i}^j(y) - V_{U,i}^j + \Pi_{F,i}^j(y)$. To minimize the loss from the worker's criminal behavior, we have $\Pi_{F,i}^j = 0$. Hence, the flow wage of workers is the total labor productivity given their type. Because $\Pi_{F,i}^j(y) = \Pi_V = 0$, the surplus of a match equals the surplus of employment $V_{E,i}^j(y) - V_{U,i}^j$. To maximize the matching surplus under the Nash bargaining share rule, the hiring fee equals the firms' share of the total surplus. This hiring fee ensures that firms obtain their surplus from the match, regardless of the tenure of the workers. They are no longer worried that workers will commit crimes and leave their jobs. As a result, this employment contract $(w_i^j(y), \phi_i^j(y))$ is Pareto-optimal for both sides of the match and it internalizes the externality of workers' criminal behavior.[4]

Given the optimal employment contract, workers without criminal records earn the same wage in both submarkets given the matching quality y. Therefore, workers without criminal records are indifferent to work in the two submarkets.

Proposition 1. *Workers without criminal records are indifferent to working in either the restricted or the nonrestricted market, i.e.,* $V_{E,NR}^{NC}(y) = V_{E,R}^{NC}(y) = V_E^{NC}(y)$.

2.3 Reservation matching quality

The matching quality is randomly drawn from a random distribution $F(y)$. Both agents, firms and workers only accept a match with quality that provides a positive surplus of the match, which is $V_E^j(y) - V_U^j > 0$ and $\Pi_{F,i}^j(y) \geq 0$. According to the optimal contract in section 2.2, the total surplus of a match is $V_E^j(y) - V_U^j$ and $\Pi_{F,i}^j(y) = 0$. In this case, firms accept any matches and the workers decide whether to accept the match or not.

Lemma 2. *For any type of worker, they are indifferent to accepting the match or remaining unemployed at the reservation matching quality* y_j^*. *The reservation matching quality is irrelevant to the submarket i.*

At the reservation matching quality y_j^*, $V_E^j(y^*) = V_U^j$. According to equations (6) to (9), we derive the reservation matching quality

$$y_j^* = b + \beta\lambda_0^j \int_{y_j^*} \frac{1 - F(y)}{r + \sigma + \delta + \pi\mu_E(1 - G(\bar{g}_E(y)))} dy + (\mu_U - \mu_E) \int_{\bar{g}_U^j} 1 - G(g)dg. \tag{16}$$

The reservation matching quality shows that the matching quality should at least cover the unemployment benefits b, the expected matching surplus, and the expected opportunity cost if the workers commit crimes.

Proposition 2. *The reservation matching quality of workers without criminal records is greater than that of ex-offenders, which is* $y_{NC}^* > y_C^*$.

Workers without criminal records are more likely to have a job when they search in both restricted and nonrestricted market, while ex-offenders only search in the nonrestricted markets. Therefore, workers

[4] I also provide the comparison of traditional Nash bargaining solution in table 3.

without criminal records are more likely to choose matching quality, which means that they require to have better matches as they have more job opportunities.

3 Equilibrium

From the free entry condition, we write the job creation conditions $\Pi_{V,i} = 0$ for both markets as follows:

$$\frac{c}{q(\theta_{NR})} = \mathbb{E}_j \left[\Pi^j_{F,NR} + \phi^j \right] \tag{17}$$

$$\frac{c}{q(\theta_R)} = \mathbb{E} \left[\Pi^{NC}_{F,R} + \phi^{NC} \right]. \tag{18}$$

The left hand side of equations (17) and (18) is the average cost of a match. Since $q(\theta_i) = m(v_i, u_i)/v_i$, the left hand side of equations (17) and (18) is $cv_i/m(v_i, u_i)$. The total vacancy cost in a market is cv_i so that $cv_i/m(v_i, u_i)$ represents the average cost of a match in market i.

The right hand side of equations (17) and (18) is the value of matching a worker. According to lemma 1, the expected value of matching a worker is

$$\mathbb{E}_j \left[\Pi^j_{F,NR} + \phi^j \right] = (1-\beta) \left[\frac{u^{NC}}{u} \int_{y^*_{NC}} V^{NC}_E(y) - V^{NC}_U dF(y) + \frac{u^C}{u} \int_{y^*_C} V^C_E(y) - V^C_U dF(y) \right]. \tag{19}$$

$$\mathbb{E} \left[\Pi^{NC}_{F,R} + \phi^{NC} \right] = (1-\beta) \int_{y^*_{NC}} V^{NC}_E(y) - V^{NC}_U dF(y) \tag{20}$$

According to the first order derivative of V^j_E,

$$(r+\sigma)\frac{\partial V^j_E}{\partial y} = 1 - \delta \frac{\partial V^j_E}{\partial y} + \mu_E \left(-\frac{\partial \bar{g}^j_E}{\partial V^j_E} \frac{\partial V^j_E}{\partial y} (1 - G(\bar{g}^j_E(y)))\right)$$

and

$$\frac{\partial \bar{g}^j_E}{\partial V^j_E} = \pi,$$

the expected surplus of type-j worker is

$$\int_{y^*_j} V^j_E(y) - V^j_U dF(y) = \int_{y^*_j} \frac{1 - F(y)}{r + \sigma + \delta + \mu_E \pi (1 - G(\bar{g}^j_E(y)))} dy$$

by applying the Newton-Leibniz formula and the partial derivation. The expected surplus decreases with an increase in the reservation productivity, such as

$$\frac{\partial}{\partial y_j^*} \int_{y_j^*} V_E^j(y) - V_U^j dF(y) = -\frac{1 - F(y_j^*)}{r + \sigma + \delta + \mu_E \pi (1 - G(\bar{g}_E^j(y)))} < 0.$$

Since the reservation matching quality of workers without criminal records is greater than that of ex-offenders, the expected matching surplus with ex-offenders is greater than the one with workers without criminal records. Intuitively, unprejudiced employers benefit from hiring ex-offenders. To satisfy the job creation condition and free entry condition, the average cost of a match in the nonrestricted market is greater than that in the restricted market. The job filling rate in the nonrestricted market is lower than that in the restricted market, i.e. $q(\theta_{NR}) < q(\theta_R)$.

Proposition 3. *The market tightness in the nonrestricted market is greater than that in the restricted market, which is*

$$\theta_{NR} > \theta_R.$$

Although the job finding rate in the nonrestricted market is higher, workers without criminal records still have their privileges in the overall labor market, since they can access to both the restricted and the nonrestricted market.

From equations (6) to (8) and the optimal contract, the surplus of a match in two submarkets are

$$V_E^{NC}(y) - V_U^{NC} = \frac{y - y_{NC}^* + \mu_E \int_{\bar{g}_E^{NC}(y)} [1 - G(g)] dg - \mu_U \int_{\bar{g}_U^{NC}} [1 - G(g)] dg}{r + \sigma + \delta} \tag{21}$$

$$V_E^C(y) - V_U^C = \frac{y - y_C^* + \mu_E \int_{\bar{g}_E^C(y)} [1 - G(g)] dg - \mu_U \int_{\bar{g}_U^C} [1 - G(g)] dg}{r + \sigma + \delta}. \tag{22}$$

The relationship between ϕ^{NC} and ϕ^C depends on the match surplus with workers. Given the same matching quality of ex-offenders and workers without criminal records, the surplus of matching with a worker without criminal records is lower than that of matching with an ex-offender. This is because the outside options of workers without criminal records are better than ex-offenders. Although ex-offenders may obtain greater value from the crime market, the reservation matching quality still dominates the impact on the relationship between the matching surplus of ex-offenders and workers without criminal records.

Lemma 3. *Given any matching quality, the hiring fee paid by workers without criminal records is lower than that paid by ex-offenders, which is*

$$\phi^{NC} < \phi^C.$$

3.1 Workers' flows

At the steady state equilibrium, the inflows and outflows of each group of workers remain the same. The number of new workers entering the labor market is denoted as n_U. They are considered as unemployed workers without criminal records. According to figure (1), the dynamics of each group of workers are as follows

$$\dot{u}^{NC} = n_U + \delta e^{NC} - [\sigma + \lambda_0^{NC}(1 - F(y^*_{NC})) + \mu_U \pi (1 - G(\bar{g}_U^{NC}))] u^{NC} \tag{23}$$

$$\dot{e}^{NC} = \lambda_0^{NC}(1 - F(y^*_{NC})) u^{NC} - [\sigma + \delta + \mu_E \pi \int_{y^*_{NC}} (1 - G(\bar{g}_E^{NC}(y))) dF(y)] e^{NC} \tag{24}$$

$$\dot{u}^{C} = \rho p + \delta e^{C} - [\sigma + \lambda_0^{NC}(1 - F(y^*_{C})) + \mu_U \pi (1 - G(\bar{g}_U^{C}))] u^{C} \tag{25}$$

$$\dot{e}^{C} = \lambda_0^{NC}(1 - F(y^*_{C})) u^{C} - [\sigma + \delta + \mu_E \pi \int_{y^*_{C}} (1 - G(\bar{g}_E^{C}(y))) dF(y)] e^{C} \tag{26}$$

$$1 = u^{NC} + e^{NC} + u^C + e^C + p, \tag{27}$$

where u^j and e^j represent the measure of unemployed and employed workers with criminal record status $j \in \{NC, C\}$, respectively, and p represents the measure of prisoners. In particular, the transition rate from employment to imprisonment depends on the distribution of matching quality.

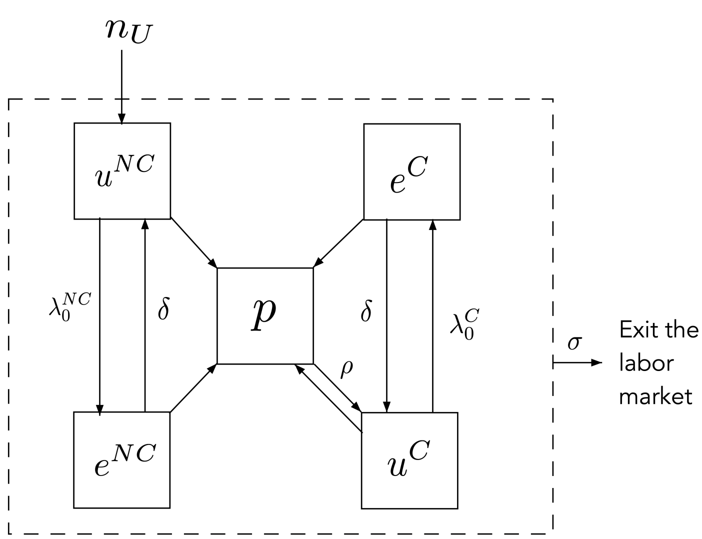

Figure 1: Workers' flows

To achieve the steady state equilibrium, $\dot{u}^j$ and $\dot{e}^j$ are equal to zero. The measure of new workers should satisfy the following

$$n_U = \sigma(u^{NC} + e^{NC}) + u^{NC} \mu_U \pi (1 - G(\bar{g}_U^{NC}) + e^{NC} \mu_E \pi \int_{y^*_{NC}} (1 - G(\bar{g}_E^{NC}(y))) dF(y), \tag{28}$$

which means that newcomers replace the outflows of workers without criminal records.

3.2 Existence and uniqueness of the equilibrium

According to the equilibrium conditions we have discussed above, the steady state equilibrium is defined as follows:

Definition 1. The steady state equilibrium is defined as a set of variables, $\{\theta_i, y_j^*, \bar{g}_{E,i}^j, \bar{g}_{U,i}^j, u^j, e^j, p, \tau\}$ for all $i \in \{R, NR\}$ and $j \in \{NC, C\}$, such that: θ_i satisfies equation (19) and (20); y_j^* satisfies equation (16); $\{\bar{g}_{E,i}^j, \bar{g}_{U,i}^j\}$ satisfy equation (4) and (5); $\{u^j, e^j, p\}$ satisfy equation (23) to (28) ; τ satisfies equation (1).

Figure (2) shows the equilibrium graphically.[5] To prove the existence and uniqueness of the equilibrium, we look at the first order derivatives with respect to both sides of (19) and (20). The left-hand side of the equations represents the average cost of a match in market i. It is represented by the upward sloping curve in figure (2). When the labor market tightness increases, i.e., there are more vacancies for each unemployed worker, firms match a worker more slowly. Therefore, the average cost of posting vacancies increases when θ_i increases.

The other side of (19) and (20) represents the expected surplus of a match, which is the downward sloping curve in figure (2). When θ_i increases, the surplus of employment decreases. Unemployed workers find jobs faster with an increase in the market tightness, so that the value of unemployment increases. Therefore, the reservation matching quality y_j^* increases and the surplus of employment decreases.[6] As a result, we can conclude that equilibrium exists and is unique in each market.

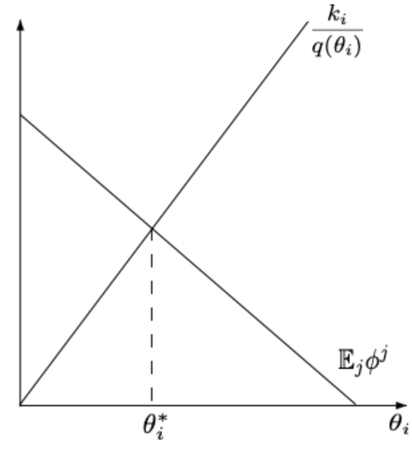

Figure 2: Equilibrium

[5]The concavity of the curves does not affect the determination of the equilibrium. The straight lines in figure (2) are drawn for simplification.

[6]The reservation matching quality of workers without criminal records are affected by both market tightness, while the reservation matching quality of ex-offenders is only influenced by the market tightness in the non-restricted market.

Proposition 4. *An equilibrium in submarket i exists with $\theta_i > 0$ and is unique.*

In the steady state equilibrium, we discuss the likelihood of committing crimes across all types of workers. According to (21) and (22), we have a positive employment surplus. It is straightforward to conclude that employees are less likely to commit crimes than unemployed workers, regardless of criminal record status and the submarket. Unemployed ex-offenders are obviously more likely to commit crimes than unemployed workers without criminal records because they have a lower job finding rate and employment surplus than workers without criminal records.

Proposition 5. *In the equilibrium, the reservation crime value of employed workers is greater than that of unemployed workers, i.e., $\bar{g}_E^j > \bar{g}_U^j$.*

We define the welfare function as follows:

$$\begin{aligned}\mathcal{W} =& e^{NC}\mathbb{E}_{NC}(y|y > y^*_{NC}) + e^{C}\mathbb{E}_{C}(y|y > y^*_{C}) + ub - c(\theta_{NR}u + \theta_R u^{NC}) \\ & - m\left[\sum_{j\in\{NC,C\}}\left(\mu_E e^j \int_{\bar{g}_E^j} gdF(g) + \mu_U u^j \int_{\bar{g}_U^j} gdF(g)\right)\right]\end{aligned}$$

Social welfare is defined as the total production in this economy ($e^{NC}\mathbb{E}_{NC}(y|y > y^*_{NC}) + c^{C}\mathbb{E}_{C}(y|y > y^*_{C})$), the unemployment benefits (ub), the cost of posting vacancies ($-c\theta_{NR}u - c\theta_R u^{NC}$), and the total loss from criminal activities ($-m\left[\sum_{j\in\{NC,C\}}\left(\mu_E e^j \int_{\bar{g}_E^j} gdF(g) + \mu_U u^j \int_{\bar{g}_U^j} gdF(g)\right)\right]$). [7]

4 Calibration

We estimate empirical moments and calibrate the model parameter values by using the US Current Population Survey (CPS) and FBI Uniform Crime Report (UCR) from 2010 to 2015. Table 1 shows the parameter values from the estimation and calibration.

The matching quality follows a standard uniform distribution with support $(0, 1]$.[8] According to Shimer (2005), the flow unemployment utility b equals to 0.4. The matching function is assumed to beCobb-Douglas, where $m(v_i, u_i) = Au_i^{\alpha}v_i^{1-\alpha}$. The elasticity of unemployment α is 0.5 according to the estimation of Petrongolo and Pissarides (2001). The bargaining power of workers follows the Hosios (1990) condition and equals the elasticity of the matching function, i.e., $\beta = \alpha$.[9] The matching efficiency A targets the annual

[7] We are not able to derive the first best condition or the solution of social planner's problem because of the criminal behavior. We calculate the welfare adapting the welfare function from Hosios (1990). Quantitatively, the employment contract with constant wage and lump-sum hiring fee provides a higher social welfare in our model.

[8] The assumption of the uniform distribution simplifies the calculation and interpretation. We also provide the results with Kumaraswamy distribution with support $(0, 1]$ and parameter $\epsilon = 2.3$, following the Pareto principle.

[9] The Hosios condition cannot guarantee the efficiency of the model because of the criminal behavior.

job finding rate from the estimation of Shimer (2005), which is 4.08. We convert the monthly job separation rate from Shimer (2005) to the average annual job separation rate δ, which is 0.408. The labor force exit rate σ is 0.025, following the estimation of Ortego-Marti (2016). The exit rate from jail follows Engelhardt et al. (2008), who estimate that the average jail period of property crimes is 16 months. The real interest rate r is 0.048 according to the Federal Reserve Bank of St. Louis.

Since the optimal contract requires that the wage equals the matching quality, the implied wage can be recovered as

$$\tilde{w}^j(y) = y - (r + \sigma + \delta + \mu_E \pi (1 - G(\bar{g}_E^j(y))))\phi^j(y). \tag{29}$$

The implied wage is the difference between the labor productivity and the flow hiring fee, which is the second term of equation (29). We can consider the lump-sum hiring fee as the present discounted value of a flow hiring fee with an effective discount rate, $r + \sigma + \delta + \mu_E \pi (1 - G(\bar{g}_E^i(y)))$. According to equation (29), the implied average wage of workers without criminal records is 0.8202, and that of ex-offenders is 0.7737.

We normalize the dollar figures in the data by the annualized earnings of workers aged 15 and above in the CPS from 2010 to 2015; the amount is \$41,374. In the crime sector, the average property crime rate from 2010 to 2015 is 34.6 per 1,000 individuals. According to the crime report data, the average property loss per offense is approximately \$1,721.86 from 2010 to 2015. Hence, we normalize the average property loss by the mean wage and calculate the mean crime loss as $\tau = \$1,721.86/\$41,374 = 0.0416$. It is assumed that the distribution of crime value follows an exponential distribution. We employ the estimation of m from Engelhardt et al. (2008) that $m = 0.105$ as the extra suffering from criminal activities. The mean reward for criminal activities equals the mean loss from crime, i.e., $g^e = \tau/(1 + m)$. The crime opportunity arrival rate $\mu = \mu_E = \mu_U$ targets the average overall property crime rate c and it is 0.1778.[10]

Quantitatively, we cannot estimate the market tightness of the restricted and the nonrestricted markets because of data limitations. We estimate the market tightness of submarkets by the overall labor market tightness. The overall labor market tightness is defined as

$$\theta \equiv \frac{v_R + v_{NR}}{u^C + u^{NC}},$$

which targets the market tightness θ that is estimated to be 0.72 by Pissarides (2009). The market tightness

[10]Because of the limitation of the data, we are not able to access the employment information of criminals before they committed a crime. Therefore, we assume the criminal opportunity arrival rate is the same for both employed and unemployed workers.

of each submarket is defined as follows:

$$\theta_R = \frac{v_R}{u^{NC}}$$
$$\theta_{NR} = \frac{v_{NR}}{u^{NC} + u^C}.$$

Hence, the overall market tightness can be rewritten as

$$\theta = \theta_{NR} + \frac{u^{NC} + u^C}{u^{NC}}\theta_R.$$

The job finding rate of workers with and without criminal records targets the estimation in Pager (2003). According to Pager (2003), only 44% of employers would like to hire workers with criminal records. Therefore, we assume that the job finding rate of ex-offenders is 44% of the total job finding rate of workers without criminal records, which is

$$f(\theta_{NR}) = 0.44[f(\theta_{NR}) + f(\theta_R)].$$

This relationship of job finding rates provides the link between θ_R and θ_{NR}. Quantitatively,

$$\theta_R = \left(\frac{1 - 0.44}{0.44}\right)^{\frac{1}{1-\alpha}} \theta_{NR}$$

Now we have two equations and three unknowns: θ_R and θ_{NR}, and the share of unemployed workers without criminal records, u^{NC}/u. We iterate the guess of the share of unemployed workers without criminal records to target the overall market tightness. By the iteration, the model produces moments that perfectly match the empirical moments. According to this calibration strategy, the market tightness of the restricted market is 0.4351, and that of the nonrestricted market is 0.2686. We assume that the posting cost c is the same in both markets. To calibrate the posting cost for the overall labor market, we apply the weighted average match surplus with the share of prejudiced employers, which is

$$\frac{c}{q(\theta)} = \varphi \mathbb{E}\left[\Pi_{F,R}^{NC} + \phi^{NC}\right] + (1-\varphi)\mathbb{E}_j\left[\Pi_{F,NR}^{j} + \phi^j\right],$$

where the share of prejudiced employers is $\varphi = 0.56$, targeting the evidence from Pager (2003).

4.1 Recidivism

As recidivism is one of the main concerns regarding the effects of criminal records, we calculate the recidivism rate to evaluate the model. The model calculates the probability of an ex-offender returning to jail within t

period. From the first period of release from jail, $t = 1$, an ex-offender can find a job or stay unemployed if he/she did not return to jail in previous periods. He/she may accept a criminal opportunity given his/her employment status. If he/she commits a crime, he/she may be caught at rate π. The estimated model provides the predictions of recidivism rates across different t periods, which lie within the estimation from Durose and Antenangeli (2021). The results are shown in table 2. The model prediction of the recidivism rate fits the data.

5 Policy effects: "Ban-the-Box" policy

Since 2010, most US states have implemented the "Ban-the-Box" (BTB) policy. The BTB policy does not allow employers to ask questions about criminal history in the application forms. At least after initial interviews, the criminal records are disclosed. This policy aims to give ex-offenders a fair chance, such that employers consider workers' qualifications before discovering their criminal history. However, this policy cannot stop employers from performing running background checks and making employment decisions according to background check reports. Even if potential employees are qualified, they may not be hired if they cannot pass the background check.

As prejudiced employers are no longer allowed to exclude ex-offenders from the application, the labor market is not segmented.[11] Prejudiced employers must review the matching quality before they discover the criminal history according to the policy. Therefore, prejudiced employers transfer their disutility to the ex-offenders they match. Prejudiced employers who are required to match ex-offenders should provide high-quality information that can compansate for their disutility. This means that the reservation matching quality of an ex-offender who matches a prejudiced employer is greater than that of an ex-offender who matches an unprejudiced employer. Similarly, as the discussed above 3, the expected matching surplus decreases with a higher reservation matching quality. Therefore, prejudiced employers have a lower expected matching surplus when they post a vacancy in the same labor market as unprejudiced employers. Formally, the value functions of a vacancy posted by prejudiced (P) and unprejudiced (NP) employers are

$$r\Pi_{V,P} = -c + q(\theta_{BTB}) \left[\frac{u^{NC}}{u} \int_{y^*_{NC}} \Pi^{NC}_{F,P} + \phi^{NC} dF(y) + \frac{u^C}{u} \int_{y^*_{C,P}} \Pi^C_{F,P} + \phi^C dF(y) \right]$$

$$r\Pi_{V,NP} = -c + q(\theta_{BTB}) \left[\frac{u^{NC}}{u} \int_{y^*_{NC}} \Pi^{NC}_{F,NP} + \phi^{NC} dF(y) + \frac{u^C}{u} \int_{y^*_{C,NP}} \Pi^C_{F,NP} + \phi^C dF(y) \right].$$

According to the optimal employment contract design, $\Pi^j_{F,P} = \Pi^j_{F,NP} = 0$. The value functions of the

[11] The details of the model after BTB policy are discussed in appendix D.

vacancies show that $\Pi_{V,P} < \Pi_{V,NP}$ as $y^*_{C,P} > y^*_{C,NP}$. Intuitively, to fulfill the free entry condition, suppose that $\Pi_{V,P} = 0$. Since $\Pi_{V,NP} > \Pi_{V,P}$, the market attracts more unprejudiced employers to enter the market and reduces the job filling rate, until $\Pi_{V,NP} = 0$. In these dynamics, the value of vacancies that posted by prejudiced employers becomes negative. Therefore, prejudiced employers exit the labor market market.

Proposition 6. *The BTB policy eliminates discrimination against ex-offenders.*

Without prejudiced employers in the market, the job finding rate of ex-offenders becomes the same as that of workers without criminal records. The reservation matching qualities of both types of workers are the same, so that the job creation condition is

$$\frac{c}{q(\theta_{BTB})} = \int_{y^*} [\Pi_F + \phi] dF(y).$$

Lemma 4. *With the BTB policy, the reservation matching quality of ex-offenders y^*_C increases, while that of workers without criminal records y^*_{NC} decreases.*

According to equation (16), the reservation matching quality depends on the job finding rate. Our analysis above shows that the job finding rate of ex-offenders increases such that they would like to match a firm with higher matching quality. However, the job finding rate of workers without criminal records decreases. Therefore, the researvation matching quality of workers without criminal records declines.

Proposition 7. *With the BTB policy,*

i) $\bar{g}^{NC}_k$ decreases given any $y \in [y^*_{NC}, 1]$;

ii) $\bar{g}^C_U$ increases;

iii) $\bar{g}^C_E$ increases if $\delta > \rho$, given any $y \in [y^*_C, 1]$.

Proposition 7 illustrates the effect of the BTB policy on criminal behavior. In particular, the reservation crime value of employed ex-offenders is ambiguous and depends on the relationship between the job separation rate and the exit rate from jail. The job separation rate represents the duration of employment, and the exit rate from jail represents the duration of incarceration. The matching surplus given any y decreases with the reservation matching quality. The higher the reservation matching quality is, the lower the matching surplus. Therefore, the matching surplus of ex-offenders given any matching quality y decreases with the BTB policy. Employed ex-offenders can leave their positions in two ways, either by exogenous job separation, or by committing crimes and being incarcerated. If $\delta > \rho$ and the value of unemployment V^C_U increases, the value from jail increases less than the loss from job separation. As a result, employed ex-offenders tend to commit fewer crimes when the duration of incarceration is longer than that of employment.

To discuss the effect of the BTB policy on the labor market outcomes, we first look at the expected matching surplus. Ex-offenders with lower reservation matching quality provide higher expected matching surplus. The policy affects the expected matching surplus in two channels: the matching surplus of ex-offenders and workers without criminal records, and the weight of unemployed ex-offenders on overall unemployed workers. After the BTB policy implementation, the market provides more job opportunities for ex-offenders, while workers without criminal records have to share to job opportunities with ex-offenders and only search in one market. The reservation matching quality of ex-offenders increases and shrinks the expected matching surplus, while the reservation matching quality of workers without criminal records decreases and their expected matching surplus increases.

Second, the weight of unemployed ex-offenders also matters. The weight of unemployed ex-offenders depends on the criminal behavior of unemployed workers without criminal records and the labor market conditions. Since the BTB policy provides more job opportunities to unemployed ex-offenders, they are less likely to commit crimes and stay in the labor market. To exit unemployment, ex-offenders could either commit crimes and be incarcerated again, or be hired. With more job opportunities, ex-offenders increases their reservation crime value, and the transition from unemployment to imprisonment slows down. Meanwhile, a higher job finding rate accelerates the outflows of unemployed ex-offenders. Quantitatively, the effect of the job finding rate dominates, and there are fewer unemployed ex-offenders in the overall unemployment. This means that the weight of unemployed ex-offenders decreases.

In summary, the weight and the expected matching surplus of ex-offenders decrease, while the expected matching surplus and the weight of unemployed workers without criminal records increase. It is straightforward to conclude that the overall expected matching surplus after the BTB policy increases, such that the average cost of a match increases. This makes it more difficult for firms to match a worker, which the job filling rate decreases, and the market tightness in the nonrestricted market increases.

Proposition 8. *The market tightness in the nonrestricted market θ_{NR} increases.*

Table 3 shows the numerical results of the changes after BTB policy implementation. The overall crime rate is a weighted average of the number of offenses that committed by employed and unemployed ex-offenders, and employed and unemployed workers without criminal records. Since the BTB policy increases the job finding rate of unemployed ex-offenders, the crime rate of unemployed ex-offenders decreases by 1.64 offenses per 1,000 individuals. The crime rate of employed ex-offenders also decreases, decreasing by 2.50 offenses per 1,000 individuals. Although the job separation rate is lower than the exit rate from jail, the crime committed by employed ex-offenders is also determined by the reservation matching quality, which is

$$c_{E,C} = \mu_E \int_{y_C^*} 1 - G(\bar{g}_E^C(y))dF(y).$$

Since the reservation matching quality of ex-offenders and the reservation crime value of unemployed ex-offenders increase, the population of employed ex-offenders who commit crimes declines. However, the policy hurts the employment of workers without criminal records and reduces the cost of committing crimes to the rest of the workers. It pushes workers without criminal records to the crime market and who commit more crimes. In particular, unemployed workers without criminal records face more challenges in the labor market post-BTB. It pushes them to the crime market and to commit more crimes.

The recidivism rate decreases with the BTB policy by 4.41 percentage points. According to the calculation of the recidivism rate in appendix C, the recidivism rate is determined by the job finding rate and the expected acceptance rate of criminal opportunity. An ex-offender is released from jail as an unemployed ex-offender. From the first month out of jail, he/she either stays unemployed or finds a job. He/she commits a crime accordingly when he/she confronts a criminal opportunity. Since both the expected acceptance rates of criminal opportunity when the ex-offender is employed and unemployed shrink according to the analysis above, it is straightforward to conclude that the recidivism rate decreases with the BTB policy.

The employment rate of ex-offenders increases by 3.25 percentage points, while the employment rate of workers without criminal records decreases by 5.27 percentage points. The overall employment rate decreases by 4.77%, which is consistent with the empirical finding in Doleac and Hansen (2020). The average implied wage of ex-offenders increases as they pay a lower hiring fee with a higher reservation matching quality. However, the average implied wage of workers without criminal records decreases with the reduction on the job finding rate and the reservation matching quality. The policy analysis also provides an analysis of the loss of social welfare. The expected matching quality of workers without criminal records may reduces since the reservation matching quality decreases. A reduction in the number of employed workers without criminal records also hurts the social welfare. Social welfare decreases by 6.26% due to the BTB policy in the baseline model. The reduction in social welfare comes from a reduction in the number of employed workers without criminal records and an increase in the overall crime rate.

We also provide the robustness checks with the assumption of Kumaraswamy-distributed matching quality and the text-book Nash bargaining wage determination without a lump-sum hiring fee in table 3.[12] The

[12] Kumaraswamy distribution is a distribution that similar as the Pareto distribution. Because we assume the matching quality is between 0 and 1, it does not satisfied the condition of Pareto distribution. The parameters of Kumaraswamy distribution are calculated by following the wage-distribution principle, which we $\epsilon = 2.3$ and the distribution is

$$F(y) = 1 - (1 - y)^{2.3}.$$

robustness checks show that with different assumptions and wage determinations, the model provides the same signs of the changes before and after the BTB policy.

6 Extension 1: On-the-job Search

Our baseline model is developed with a simple assumption that workers do not search for jobs. We relax this assumption in this section. On-the-job search behavior affects both the value of employment and the employment contract. We first rewrite the value function of employed workers as

$$
\begin{aligned}
(r+\sigma)V_E^j(y) = y - \tau - \delta(V_E^j(y) - V_U^j) + \lambda_1^j \int \max\{V_E^j(p) - V_E^j(y) - \phi_{OTJ}^j(p), 0\} dF(p) \\
+ \mu \int \max\{K(g) - V_E^j(y), 0\} dG(g).
\end{aligned}
$$

where λ_1^j represents the job finding rate of on-the-job search.

When the employed worker matches a new firm, he/she also bargains on the employment contract as unemployed workers, and the employed worker also needs to pay the lump-sum hiring fee to the new employer for contracting the unobservable criminal behavior and on-the-search behavior. According to the intuition of the employment contract, workers and firms bargain with their surplus of the match. When an employed worker bargains with the new offer, his/her outside option is the current employment value, $V_E^j(y)$.[13] The employment contract for on-the-job seekers satisfies

$$
\max_{w,\phi} \left[V_E^j(p) - V_E^j(y) - \phi_{OTJ,i}^j(p)\right]^{\beta} \left[\Pi_{F,i}^j(p) - \Pi_{V,i} + \phi_{OTJ,i}^j(p)\right]^{1-\beta}
$$

where p is the new matching quality and y is the matching quality of the current job. The solution of Nash bargaining with a hiring fee is

$$
\begin{aligned}
w_i^j(p) &= p \\
\phi_{OTJ,i}^j &= (1-\beta)\left[V_{E,i}^j(p) - V_{E,i}^j(y)\right].
\end{aligned}
$$

Similarly as the baseline model, we take the Newton-Leibniz formula and obtain the partial derivative of the

[13] At the footnote 22 in Ortego-Marti (2016), on-the-job search is unobservable and worker must quit the current job before negotiating the new offer. In the employment contract with hiring fee, employers and workers contract with both unobservable criminal behavior and on-the-job search. As a result, the employee does not quit the current job to negotiate the new offer and his/her outside options is to stay at the old job.

value function of employment as

$$\frac{\partial V_E^j(y)}{\partial y} = \frac{1}{r + \sigma + \delta + \beta\lambda_1^j(1 - F(y)) + \mu\pi(1 - G(\bar{g}_E(y)))}.$$

The job creation condition follows section 3,

$$\frac{c}{(1-\beta)q(\theta_R)} = \int_{y_{NC}^*} V_E^{NC}(y) - V_U^{NC} dF(y)$$
$$\frac{c}{(1-\beta)q(\theta_{NR})} = \frac{u^{NC}}{u}\left[\int_{y_{NC}^*} V_E^{NC}(y) - V_U^{NC} dF(y)\right] + \frac{u^C}{u}\left[\int_{y_C^*} V_E^C(y) - V_U^C dF(y)\right]$$

where the expected matching surplus

$$\int_{y_j^*} V_E^j(y) - V_U^j dF(y) = \int_{y_j^*} \frac{1 - F(y)}{r + \sigma + \delta + \beta\lambda_1^j(1 - F(y)) + \mu_E\pi(1 - G(\bar{g}_E^j(y)))} dy$$

The job arrival rate still depends on the market tightness, such that the job finding rate of unemployed workers and employed workers is

$$\lambda^{NC} = \lambda_0^{NC} = \lambda_1^{NC} = f(\theta_{NR}) + f(\theta_R) \quad (30)$$
$$\lambda^C = \lambda_0^C = \lambda_1^C = f(\theta_{NR}), \quad (31)$$

where $\theta_i \equiv v_i/s^j$, and s^j is the measure of job seekers as $s^j = e^j + u^j$. The definition of the job finding rate for both types of workers indicates that the job finding rate of workers without criminal records is greater than the job finding rate of ex-offenders, which is $\lambda^{NC} > \lambda^C$. This leads to the same conclusion as lemma 3 and proposition 3. Since employed and unemployed workers randomly search in the same market such that their job finding rates are the same, the reservation matching quality is

$$y_j^* = b + \beta(\lambda_0^j - \lambda_1^j)\left[\int_{y_j^*} V_E^j(y) - V_U^j dF(y)\right] + (\mu_U - \mu_E)\int_{\bar{g}_U^j} 1 - G(g)dg$$
$$= b,$$

as $\mu_U = \mu_E$. Since the reservation matching quality of ex-offenders and workers without criminal records are the same, which is the unemployment benefit, the model focuses on the differential of the job finding rate between ex-offenders and workers without criminal records. The job finding rate of workers without criminal records is still higher than that of ex-offenders pre-BTB, because they can access to both markets. The expected matching surplus of workers without criminal records is smaller than that of ex-offenders because of the higher job finding rate. Therefore, proposition 3 is also satisfied in the case with on-the-job search.

The effect of the BTB policy is different from that of the baseline model. Theoretically, on-the-job search lowers the reservation matching quality and it is not affected by the BTB policy. Workers are more likely to accept a relatively "bad" match because they climb on the job ladder and seek for a better match. In table 3, we compare the results of the on-the-job search model with those of the baseline model. Quantitatively, the results of the on-the-job search do not deviate from the baseline results (column 1 in table 3). The BTB policy does not affect the reservation matching quality, and changes in the job finding rate are fully influence the flows of workers, while the policy affects the reservation matching quality in the baseline model and shrinks the impacts on the worker flows. The reduction in social welfare is smaller than that in the baseline model. The average wages of both workers with and without criminal records are unchanged. This is because the BTB policy does not affect the reservation matching quality, and the job finding rate does not play a role in the effective discount factor in the implied wage (equation (29)).

7 Extension 2: case in Denmark

In Denmark, criminal records are expunged after 3 to 10 years given certain conditions. Access to criminal records for private entities is more restricted in Denmark. Specifically, individuals or companies (with the individual's consent) can access these records for a more limited period. The criminal records of the incarcerated individuals become inaccessible to the general public or private usage after they have maintained a crime-free status for five years after their release. In this section, we extend the baseline model following the idea in Denmark, allowing ex-offenders to have noncheckable criminal records a number of years after being released from jail, with a counterfactual experiment.

Suppose the Poisson rate of expunging criminal records is ζ, whose inverse represents the number of years after ex-offenders exit from jail. If ex-offenders do not reoffend and are not arrested during these years, their records are expunged. The value functions of ex-offenders are

$$\begin{aligned}(r+\sigma)V_E^C(y) =& w^C(y) - \tau - \delta(V_E^C(y) - V_U^C) + \mu_E \int_0^{g^{max}} \max\{K - V_E^i(y), 0\} dG(g) \\ &+ \zeta\left\{\mu_E[G(\bar{g}_E^C(y)) + (1-\pi)(1 - G(\bar{g}_E^C(y)))]\right\}(V_E^{NC}(y) - V_E^C(y)), \quad (32) \\ (r+\sigma)V_U^C =& b - \tau + \lambda_0^C \int \max\{V_E^C(y) - V_U^C - \phi^C(y), 0\} dF(y) + \mu_U \int_0^{g^{max}} \max\{K - V_U^C, 0\} dG(g) \\ &+ \zeta\left\{\mu_U[G(\bar{g}_U^C) + (1-\pi)(1 - F(\bar{g}_U^C))]\right\}(V_U^{NC} - V_U^C). \quad (33)\end{aligned}$$

The last terms of equations (32) and (33) represent the capital value change when the ex-offenders' criminal records are removed. For an expected duration value of time of ζ^{-1} years, ex-offenders also encounter some criminal opportunities at rate μ_k. To have a clean background, they may not to get involved in the criminal opportunity they encounter, or they commit the crime but do not get caught. Once the criminal records are expunged, ex-offenders have the same job finding rate as workers without criminal records. It also changes the dynamics of workers' flows. The new dynamics of worker flows are as follows

$$\dot{u}^{NC} = n_U + \delta e^{NC} + \zeta_U^* u^C - [\sigma + \lambda_0^{NC}(1 - F(y_{NC}^*)) + \mu_U \pi(1 - G(\bar{g}_U^{NC}))]u^{NC} \tag{34}$$

$$\dot{e}^{NC} = \lambda_0^{NC}(1 - F(y_{NC}^*))u^{NC} + \zeta_E^* e^C - [\sigma + \delta + \mu_E \pi(1 - G(\bar{g}_E^{NC}))]e^{NC} \tag{35}$$

$$\dot{u}^C = \rho p + \delta e^C - [\sigma + \lambda_0^C(1 - F(y_C^*)) + \mu_U \pi(1 - G(\bar{g}_U^C)) + \zeta_U^* u^C \tag{36}$$

$$\dot{e}^C = \lambda_0^C(1 - F(y_C^*))u^C - [\sigma + \delta + \mu_E \pi(1 - G(\bar{g}_E^C)) + \zeta_E^*]e^C \tag{37}$$

$$1 = u^{NC} + e^{NC} + u^C + e^C + p. \tag{38}$$

where $\zeta_k^* = \zeta\mu_k[G(\bar{g}_k^C) + (1 - \pi)(1 - G(\bar{g}_k^C))]$, given any $k \in \{E, U\}$.

Table 4 shows the numerical results of the extended model. The policy directly increases the surplus of employment of ex-offenders. According to (32) and (33), it is straightforward to have an additional capital gain from criminal record expunging, i.e.,[14]

$$V_E^C(y) - V_U^C = \frac{y - y_C^* - \mu \int_{\bar{g}_U^C}^{\bar{g}_E^C}[1 - G(g)]dg + (\zeta_E^* V_E^{NC}(y) - \zeta_U^* V_U^{NC})}{r + \sigma + s + \lambda_0^C + \zeta_E^*}.$$

Ex-offenders are less likely to commit crimes because of the gain from this policy, since they have a chance to become "nonrecorded" workers. The crime rates of ex-offenders reduce as expected, while the crime rates of workers without criminal records increase. Therefore, the recidivism rate decreases as ex-offenders are less likely to commit crimes with the expungement. Expungement does not reduce the job finding rate of workers without criminal records, but it reduces the cost of committing crime: the reduction of value in the labor market after release from jail. Workers without criminal records are more likely to commit crimes after the record expungement. In conclusion, the overall crime rate increases, driven by the increase in the criminal offenses by workers without criminal records. Compared with the BTB policy, the criminal record expunging policy has a smaller policy effect on the overall crime rate. When the BTB policy increases the overall crime rate by 26.23 offenses per 1,000 individuals, the criminal record expunging policy increases the overall crime rate by 8.50 to 11.45 offenses per 1,000 individuals.

In the labor market, the market tightness in the nonrestricted market increases because of the increase

[14]Because the difference of the rate of criminal record removal ζ^* for employed/unemployed ex-offenders is quantitatively small, we assume they have the same rate to simplify the calculation. The rate of criminal record removal ζ^* is calculated by $\zeta(\mu F(\bar{g}_U^C) + \mu(1 - \pi)(1 - F(\bar{g}_U^C)))$ for the numerical exercise.

in the expected matching surplus. The employment rate of ex-offenders increases by 1.90 to 3.45 percentage points. It also slightly increases the job finding rate of workers without criminal records because they also search in the nonrestricted market as well. Therefore, the employment rate of workers without criminal records increases by 0.65 to 1.25 percentage points. It also reduces the social welfare by 0.63% to 1.34%. The reduction in social welfare mainly comes from the increase in the overall crime rate.

8 Conclusion

This paper studies discrimination in the labor market against ex-offenders and the effects of BTB policy by constructing a model with a search and matching framework. Before the BTB policy is implemented, workers without criminal records search in both the restricted and the non-restricted markets simultaneously. However, ex-offenders search only in the nonrestricted market where they are not asked questions about criminal records. After BTB policy implementation, no firms run background checks and only the nonrestricted market exists, because the policy generates additional search costs for making hiring decisions based on the criminal history. Ex-offenders experience fewer employment difficulties after the BTB policy implementation, and the recidivism rate decreases. However, the BTB policy does not reduce the overall crime rate. The reason for the increase in the overall crime rate is that the BTB policy hurts the employment of workers without criminal records and reduces the cost of committing crimes. This makes the crime market becomes attractive to workers without criminal records. The increase in criminal offences by workers without criminal records dominates such that the overall crime rate increases by 26.23 offences per 1,000 individuals.

We consider a policy in which criminal records are expunged after ex-offenders refrain from re-offending for a certain period. This policy encourages ex-offenders to stay in the labor market such that the recidivism rate decreases. The overall unemployment rate increases because there are more ex-offenders remain in the labor market. The employment of ex-offenders decreases, and it drives the overall employment down. In the crime sector, this policy helps ex-offenders to refrain from committing crimes, but workers without criminal records are more likely to commit crimes because the cost of committing crimes decreases. Unlike the BTB policy, which provides job opportunities to ex-offenders, the expungment policy provides a chance that ex-offenders could become workers without a criminal record. Therefore, they have fewer incentives to commit crimes under the expungement policy.

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A Tables

Table 1: Calibration Results

		description	sources/target
b	0.4	Unemployment benefit	Shimer (2005)
α	0.5	Elasticity of matching function	Petrongolo and Pissarides (2001)
β	0.5	Bargaining power	Hosios (1990)
m	0.105	extra suffering from criminal activities	Engelhardt et al. (2008)
			Estimated from data:
δ	0.408	Job separation rate	Shimer (2005)
σ	0.025	Labor force exit rate	Ortego-Marti (2016)
ρ	0.75	Rate of exit from jail	Engelhardt et al. (2008)
π	0.019	Apprehension probability	Engelhardt et al. (2008)
τ	0.0416	Expected loss of victims	FBI Uniform Crime Report 2010-2015
r	0.048	Real interest rate	Fed. of Saint Louis
			Jointly calibrated to match
A	4.8083	Matching efficiency	Annual job finding rate: 4.08 from Shimer (2005)
k	0.4132	Cost of posting vacancy	$\theta = 0.72$ from Pissarides (2009)
μ	0.1026	Crime opportunity arrive rate	Job finding rate of workers without criminal records = 6.5160, job finding rate of ex-offenders = 1.8840 from Engelhardt (2010) Overall property crime rate: 0.0346

Table 2: Prediction of the Recidivism Rate

Duration	Model	Arrested[1]
12 months	24.01	36.8
24 months	47.45	52.9
36 months	70.32	61.5

Data resources: Durose and Antenangeli (2021)

Table 3: Effects of BTB policy

Changes in	Standard Uniform distribution	Kumaraswamy distribution	Nash Bargaining without ϕ	On-the-job search
Crime sector				
c	26.23	26.00	21.87	18.79
$c_{E,C}$	-2.50	-0.94	-0.97	-1.93
$c_{E,NC}$	18.80	15.24	13.70	13.44
$c_{U,C}$	-1.64	-0.65	-0.62	-1.28
$c_{U,NC}$	41.55	27.36	32.27	28.23
Rec. of 36 months	-4.41	-1.70	-2.24	-3.85
Labor distribution				
e	-4.77	-8.71	-5.35	-4.94
e^{NC}	-5.27	-9.32	-5.52	-5.23
e^{C}	3.25	3.77	1.27	4.20
Social Welfare	-6.26	-6.09	-5.93	-3.85
$\mathbb{E}\tilde{w}^{NC}$	-3.69	-3.67	-2.45	-
$\mathbb{E}\tilde{w}^{C}$	2.10	1.14	0.55	-

Note: 1. The variables c and c^i_j represent the overall crime rate and crime rates of type-ij workers in a population of 1,000, respectively. The changes in crime rates are the changes in the number of crimes in a population of 1,000. The change in the rate of recidivism after 36 months of release from jail (Rec. of 36 months) are in percentage points.
2. The variables e, and e^i represent the overall unemployment rates, overall employment rate, and employment rate of workers with and without criminal records respectively. The changes are in percentage points.
3. The variable $\mathbb{E}\tilde{w}^j$ represents the change in average implied wages for workers with or without criminal records. The changes are presented as percentage.

Table 4: Effects of Criminal Record Removal

Change in	3 years	5 years	10 years	BTB
Crime sector				
c	11.45	10.52	8.50	26.23
$c_{E,C}$	-5.33	-4.38	-3.11	-2.50
$c_{E,NC}$	9.13	8.30	6.62	18.80
$c_{U,C}$	-4.37	-3.86	-3.02	-1.64
$c_{U,NC}$	27.92	25.08	19.79	41.55
Rec. in 36 months	-7.42	-5.97	-4.15	-4.41
Labor distribution				
e	1.27	0.99	0.65	-4.77
e^{NC}	1.25	0.98	0.65	-5.27
e^{C}	3.45	2.78	1.90	3.25
Social Welfare	-1.34	-1.00	-0.63	-6.26
$\mathbb{E}\tilde{w}^{NC}$	1.25	0.97	0.64	-3.69
$\mathbb{E}\tilde{w}^{C}$	3.13	2.49	1.68	2.10

Note: Please see the note under table 3.

B Proofs of Lemmas and Propositions

Proof of Lemma 1

Proof. According to the Nash bargaining, the surplus must be maximized by the optimal employment contract. Compared with the expected capital loss of a match, $\pi(\Pi_F^j + V_E^j - V_P^j)$, and the employees' opportunity cost of committing a crime, $\pi(V_E^j - V_P^j)$, the surplus is maximized if and only if $\Pi_F^j = 0$. According to equation (13), the value of a filled job is

$$\Pi_F^j(y) = \frac{y^j - w^j}{r + s + \sigma + \pi\mu(1 - F(\bar{g}_E^j(y)))}.$$

Therefore, $\Pi_F^j(y) = 0$ requires

$$w^j = y^j.$$

By solving equation(15),

$$\phi^j(y) = (1 - \beta)(V_E^j(y) - V_U^j).$$

Proof of Proposition 1

Proof. Suppose that $V_{E,NR}^{NC} \neq V_{E,R}^{NC}$ given any matching quality. The subtraction of $V_{E,NR}^{NC}$ and $V_{E,R}^{NC}$ is

$$(r + \sigma + s)(V_{E,NR}^{NC} - V_{E,R}^{NC}) = \int_{\bar{g}_{E,NR}^{NC}}^{\bar{g}_{E,R}^{NC}} 1 - F(g)dg.$$

If $V_{E,NR}^{NC} > V_{E,R}^{NC}$, the left-hand side of the subtraction is positive, while the right-hand side of the subtraction is negative. Therefore, if $V_{E,NR}^{NC} < V_{E,R}^{NC}$. Only when $V_{E,NR}^{NC} = V_{E,R}^{NC}$, can the equality of the subtraction be satisfied. Therefore, $V_{E,NR}^{NC} = V_{E,R}^{NC} = V_E^{NC}$. □

Proof of Proposition 2

Proof. We assume that $\mu_E = \mu_U$, the subtraction of y_{NC}^* and y_C^* is

$$\begin{aligned} y_{NC}^* - y_C^* &= \beta\left[\lambda_0^{NC}\int_{y_{NC}^*}^1 \frac{1 - F(y)}{r + \sigma + \delta + \pi\mu_E(1 - G(\bar{g}_E(y)))}dy - \lambda_0^C\int_{y_C^*}^1 \frac{1 - F(y)}{r + \sigma + \delta + \pi\mu_E(1 - G(\bar{g}_E(y)))}dy\right] \\ &> \beta\lambda_0^C\int_{y_{NC}^*}^{y_C^*} \frac{1 - F(y)}{r + \sigma + \delta + \pi\mu_E(1 - G(\bar{g}_E(y)))}dy \end{aligned}$$

as $\lambda_0^{NC} > \lambda_0^C$. If $y_C^* > y_{NC}^*$, $y_{NC}^* - y_C^* < 0$ while $\beta\lambda_0^C \int_{y_{NC}^*}^{y_C^*} \frac{1-F(y)}{r+\sigma+\delta+\pi\mu_E(1-G(\bar{g}_E(y)))} dy > 0$. If $y_{NC}^* = y_C^*$, $y_{NC}^* - y_C^* = \beta\lambda_0^C \int_{y_{NC}^*}^{y_C^*} \frac{1-F(y)}{r+\sigma+\delta+\pi\mu_E(1-G(\bar{g}_E(y)))} dy = 0$. They contradict and $y_{NC}^* > y_C^*$. □

Proof of Proposition 3

Proof. According to equations (19) and and (20), the job filling rates are rewritten as

$$q(\theta_{NR}) = \frac{c}{\mathbb{E}\phi}$$
$$q(\theta_R) = \frac{c}{\int_{y_{NC}^*} \phi^{NC} dF(y)}.$$

and

$$\frac{q(\theta_{NR})}{q(\theta_R)} = \frac{\int_{y_{NC}^*} \phi^{NC} dF(y)}{\mathbb{E}\phi}$$
$$< 1.$$

Therefore, $\theta_{NR} > \theta_R$. □

Proof of Proposition 4

Proof. According to (19) and (20), the right hand side represents the average cost of a match in each market respectively, $kv_i/m(\theta_i)$. Take the first order derivative with respect to θ_i,

$$\frac{\partial}{\partial \theta_i}\left[\frac{c}{q(\theta_i)}\right] = -\frac{c}{(q(\theta_i))^2} q'(\theta_i)$$
$$> 0.$$

Since $q'(\theta_i) < 0$, the first order derivatives of the left hand side of the job creation conditions are positive.

The right hand side of the job creation conditions represents the surplus of a match. Take the first order derivative with respect to θ_i of the right hand side of the job creation conditions,

$$\frac{\partial}{\partial \theta_R}\left[\int_{y^*_{NC}}^{1} \frac{1-F(y)}{r+\sigma+\delta+\pi\mu_E(1-G(\bar{g}_E(y)))}dy\right] = -\frac{1-F(y^*_{NC})}{r+\sigma+\delta+\pi\mu_E(1-G(\bar{g}_E(y)))}\frac{\partial y^*_{NC}}{\partial \theta_R}$$

$$< 0$$

$$\frac{\partial}{\partial \theta_{NR}}\mathbb{E}_j\left[\int_{y^*_j}^{1} \frac{1-F(y)}{r+\sigma+\delta+\pi\mu_E(1-G(\bar{g}_E(y)))}dy\right] = -\frac{u^{NC}}{u}\frac{1-F(y^*_{NC})}{r+\sigma+\delta+\pi\mu_E(1-G(\bar{g}^{NC}_E(y)))}\frac{\partial y^*_{NC}}{\partial \theta_{NR}} - \frac{u^{C}}{u}\frac{1-F(y^*_{C})}{r+\sigma+\delta+\pi\mu_E(1-G(\bar{g}^{C}_E(y)))}\frac{\partial y^*_{C}}{\partial \theta_{NR}}$$

$$<0.$$

Since the left hand side (the right hand side) of job creation conditions are straightly increasing (decreasing) with the market tightness θ_i, there exists a unique equilibrium θ^*_i that satisfies the equality of the job creation conditions. □

Proof of Proposition 5

Proof. The surplus of employment for type-j workers is

$$V^{NC}_E(y) - V^{NC}_U = \frac{y - y^*_{NC} + \mu_E \int_{\bar{g}^{NC}_E(y)}[1-G(g)]dg - \mu_U \int_{\bar{g}^{NC}_U}[1-G(g)]dg}{r+\sigma+\delta}$$

$$>0$$

and

$$V^{C}_E(y) - V^{C}_U = \frac{y - y^*_{C} - \mu_E \int_{\bar{g}^{C}_E(y)}[1-G(g)]dg - \mu_U \int_{\bar{g}^{C}_U}[1-G(g)]dg}{r+\sigma+\delta}$$

$$> 0.$$

The reservation values of employed and unemployed workers are

$$\bar{g}^j_E = \pi(V^j_E - V_P)$$

$$\bar{g}^j_U = \pi(V^j_U - V_P).$$

The subtraction of them is

$$\bar{g}_E^j - \bar{g}_U^j = \pi(V_E^j - V_U^j)$$
$$> 0.$$

Therefore, we can conclude that the reservation crime value of employed workers is greater than that of unemployed workers. □

Proof of Proposition 7

Proof. According to equations (4) and (5),

$$\begin{aligned}\bar{g}_E^{NC} &= \pi\left(V_E^{NC}(y) - V_P\right) \\ &= \frac{\pi}{r+\sigma+\rho}\left[(r+\sigma)V_E^{NC}(y) + \rho(V_E^{NC}(y) - V_U^{NC}) + \tau - z\right]\end{aligned}$$

and

$$\bar{g}_U^{NC} = \frac{\pi}{r+\sigma+\rho}\left[(r+\sigma)V_U^{NC} + \rho(V_U^{NC} - V_U^{NC}) + \tau - z\right]$$

,where

$$V_J = \frac{z - \tau + \rho V_U^C}{r+\sigma}.$$

Since the BTB policy reduces the job finding rate of workers without criminal records, the value of the employment and unemployment of workers without criminal records decreases. However, the value of unemployment for ex-offenders increases as the job finding rate of ex-offenders increases. Given any matching quality, the reservation crime value of employed workers without criminal records decreases. The reservation crime value of unemployed ex-offenders also decreases.

The reservation crime values of unemployed ex-offenders is dependent only on the unemployment value, which is

$$\bar{g}_U^C = \frac{\pi}{r+\sigma+\rho}\left[(r+\sigma)V_U^C + \tau - z\right]$$

The reservation crime value of employed ex-offenders are

$$\begin{aligned}\bar{g}_E^C &= \frac{\pi}{r+\sigma+\rho}\left[(r+\sigma)V_E^C(y) + \rho(V_E^C(y) - V_U^C) + \tau - z\right] \\ &= \frac{\pi}{r+\sigma+\rho}\left[y + (\rho-\delta)(V_E^C(y) - V_U^C) + \mu_E\int_{\bar{g}_E^C(y)} 1 - G(g)dg - z\right].\end{aligned}$$

According to equation (22), the matching surplus given any y decreases with the reservation matching quality, y_C^*. The BTB policy provides more job opportunities to ex-offenders such that the reservation matching quality of ex-offenders rises. Therefore, the reservation crime value of employed ex-offenders increases if $\delta > \rho$. □

C Recidivism rate calculation

The recidivism rate is defined as the probability of ex-offenders committing crimes and returning to jail again within t period after being released from jail (Engelhardt, 2010). Criminals exit jail and return to the labor market as unemployed workers, such that they follow the Markov transition matrix between unemployment and employment,

$$P = \begin{pmatrix} (1 - f(\theta_{NR}))(1 - F(y_C^*) & f(\theta_{NR})(1 - F(y_C^*)) \\ s & 1 - s \end{pmatrix}.$$

Then the probability of returning to jail in the t-th period, $\psi(t)$, depends on the ex-offenders' employment status, the arrival rate and the acceptance rate of the criminal opportunity, and the apprehension probability. We also consider that he/she does not return to jail in the past periods. Therefore, the probability of returning to jail in the t-th period is

$$\psi(t) = \pi \left[\mu_E \int_{y_C^*} (1 - G(\bar{g}_E^C(y)))dF(y)f(\theta_{NR})(1 - F(y_C^*) + \mu_U(1 - G(\bar{g}_U^C))(1 - f(\theta_{NR}))(1 - F(y_C^*)) \right] (1 - Rec_t),$$

where the recidivism rate $Rec_t = \sum_{i=1}^{t} \psi(i)$ is the probability that the ex-offender returns to jail before the t-th period.

D Model after the BTB policy implementation

Compared with the model before the BTB policy, the model after the BTB policy implementation has only one market. Ex-offenders could meet either prejudiced or unprejudiced employers. Therefore, the bellman equations for workers after BTB policy implementation are as follows

$$
\begin{aligned}
(r+\sigma)V_E^{NC}(y) &= w^{NC}(y) - \tau - \delta(V_E^{NC}(y) - V_U^{NC}) + \mu_E \int \max\{K_E^{NC} - V_E^{NC}, 0\} dG(g) \\
(r+\sigma)V_U^{NC} &= b - \tau + \lambda_{BTB} \int_{y^*} V_E^{NC}(y) - V_U^{NC} - \phi^{NC}(y) dF(y) + \mu_U \int \max\{K_U^{NC} - V_U^{NC}, 0\} dG(g) \\
(r+\sigma)V_E^{C}(y) &= w^{C}(y) - \tau - \delta(V_E^{C}(y) - V_U^{C}) + \mu_E \int \max\{K_E^{C} - V_E^{C}, 0\} dG(g) \\
(r+\sigma)V_U^{C} &= b - \tau \\
&\quad + \lambda_{BTB} \left[(1-\varphi) \int \max\{V_{E,NP}^{C} - V_U^{C} - \phi_{NP}^{C}, 0\} dF(y) + \varphi \int \max\{V_{E,P}^{C} - V_U^{C} - \phi_{P}^{C}, 0\} dF(y) \right] \\
&\quad + \mu_U \int \max\{K_U^{C} - V_U^{C}, 0\} dG(g).
\end{aligned}
$$

According to proposition 5, firms match with any workers in the market. The Bellman equations for prejudiced firms are

$$
\begin{aligned}
r\Pi_{V,P} &= -c + q(\theta_{BTB})\mathbb{E}_j(\Pi_{F,P}^j + \phi_P^j) \\
r\Pi_{F,P}^{C}(y) &= y - w^C - \mathcal{U}_d - (\delta + \sigma + \mu_E \pi(1 - F(\bar{g}_E^C)))(\Pi_F^C(y) - \Pi_{V,P}) \\
r\Pi_{F,P}^{NC}(y) &= y - w^{NC} - (\delta + \sigma + \mu_E \pi(1 - F(\bar{g}_E^{NC})))(\Pi_F^{NC}(y) - \Pi_{V,P})
\end{aligned}
$$

where

$$
\mathbb{E}_j(\Pi_{F,P}^j + \phi_P^j) = \frac{u^{NC}}{u} \int_{y^*_{NC,P}} \Pi_{F,P}^{NC} + \phi_P^{NC} dF(y) + \frac{u^C}{u} \int_{y^*_{C,P}} \Pi_{F,P}^{C} + \phi_P^{C} dF(y).
$$

The employment contract between prejudiced employers and ex-offenders is also adjusted accordingly. Prejudiced employers now have to consider the disutility in their value functions and redesign the employment contracts when they match ex-offenders. Similarly, as in section 2.2, the optimal employment contract leads $\Pi_{V,P} = \Pi_{F,P}^C = \Pi_{F,P}^{NC} = 0$. Therefore, the employment contract of ex-offenders when they match prejudiced employers is

$$
\begin{aligned}
w_P^C &= y - \mathcal{U}_d \\
\phi_P^C &= (1-\beta)(V_{E,P}^C - V_U^C).
\end{aligned}
$$

We recalculate the reservation matching quality $y^*_{C,P}$ and the hiring fee ϕ^C_P when an ex-offender matches a prejudiced employer, and they are

$$y^*_{C,P} = b + \mathcal{U}_d + \beta\lambda_{BTB}\int_{y^*_{C,P}} \frac{1-F(y)}{r+\sigma+\delta+\pi\mu_E(1-G(\bar{g}_E(y)))}dy + (\mu_U - \mu_E)\int_{\bar{g}^C_U} 1 - G(g)dg$$

$$\phi^C_P(y) = \frac{y - y^*_{C,P} + \mu_E \int_{\bar{g}^C_{E,P}(y)}[1-G(g)]dg - \mu_U \int_{\bar{g}^C_U}[1-G(g)]dg}{r+\sigma+\delta}.$$

The expected hiring fee of prejudiced employers who match an ex-offender is

$$\int_{y^*_{C,P}} V^C_E(y) - V^C_U dF(y) = \int_{y^*_{C,P}} \frac{1-F(y)}{r+\sigma+\delta+\mu_E\pi(1-G(\bar{g}^C_E(y)))}dy$$

E Model without hiring fee (traditional Nash bargaining solution)

In this section, we are going to solve the traditional Nash bargaining problem for the wage determination, which is the solution without the lump-sum hiring fee. We rewrite the Nash bargaining problem as

$$\max_{w^j}(V^j_E(y) - V^j_U)^{\beta}(\Pi^j_F(y) - \Pi^j_V)^{1-\beta}.$$

The first-order condition with respect to wages given any matching quality $y \in (y^*_j, 1]$ is

$$(1-\beta)(V^j_E(y) - V^j_U) = \beta\Pi^j_F(y)$$

such that

$$(1-\beta)\int_{y^*_j}^1 V^j_E(y) - V^j_U dF(y) = \beta\int_{y^*_j}^1 \Pi^j_F(y)dF(y).$$

The wage solution for type-j workers is

$$w^j(y) = \frac{\beta(r+\sigma+\delta)y + (1-\beta)(r+\sigma+\delta+\mu_E\pi(1-G(\bar{g}^j_E))(y^*_j + \mu_E\int_{\bar{g}^j_E}^{\bar{g}^j_E}(1-G(g))dg - \mu_U\int_{\bar{g}^j_U}(1-G(g))dg)}{r+\sigma+\delta+(1-\beta)\mu_E\pi(1-G(\bar{g}^j_E))}.$$

Given the some matching quality y, the workers without criminal records have higher wages because their reservation matching quality is higher than that of ex-offenders.

In table 3, we compare the effect of the BTB policy with and without the hiring fee. The noticeable difference between two employment contracts is the crime rate of employed workers without criminal records. This causes a difference in the overall crime rate. To calculate the crime rate of employed workers without criminal records, we have

$$c_{E,NC} = \mu_E \int_{y^*_{NC}}^{1} 1 - G(\bar{g}_E^{NC}(y))dF(y).$$

This shows that the effects of the BTB policy on the crime rate of employed workers without criminal records depend on the reservation crime value of employed workers without criminal records when the matching quality is 1 and at the reservation y^*_{NC}, which relies on the gap between employment and unemployment value. The larger the gap is, the more crimes are committed by the employed workers without criminal records.

After the BTB policy, workers without criminal records are more likely to commit crimes because of the reduction in the job finding rate and the cost of committing crimes with both employment contracts. However, the employment surplus with the hiring fee increases, while that without the hiring fee remains almost the same before and after the BTB. This is because of the positive value of a filled job. When the BTB policy reduces the job finding rate of workers without criminal records, it reduces the reservation matching quality y^*_{NC}. Hence, the wages of workers without criminal records reduces, and the value of a filled job increases. The increase in the matching surplus mostly comes from the increase in the value of the filled job. Therefore, the employment surplus, $V_E^j - V_U^j$, remains the same almost before and after the BTB policy. In the case of the hiring fee, the increase in the matching surplus post-BTB comes from the gap between the employment and unemployment values, as the filled jobs have zero values. As a result, the employment surplus with hiring fee increases. Although both the values of employment and unemployment post-BTB decreases, the reduction in the value of employment is smaller than the reduction in the value of unemployment.