Online Supplement B: Monte Carlo Results
To
"Identifying and exploiting alpha in linear asset pricing models with strong, semi-strong, and latent factors"

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1 Introduction

This online supplement provides detailed Monte Carlo results for all experiments and risk factors. The summary tables below give the bias, RMSE and size (×100), for the DGP with one strong ($\alpha_M = 1$) and two semi-strong factors ($\alpha_H = 0.85,\ \alpha_S = 0.65$) for the two-step and the bias-corrected estimators of $\boldsymbol{\phi} = (\phi_M, \phi_H, \phi_S)'$, for different sample sizes. These are given for the twelve experimental designs listed in Table S-1 below. For experiments 8 and 9 we also report the results with larger values for the parameter of the pricing errors ($\alpha_\eta = 0.50$) and the spatial coefficients ($\rho_\varepsilon = 0.85$), denoted as Experiments 8a and 9a, respectively. Following each table the empirical power functions for the bias corrected estimator of $\boldsymbol{\phi}$ are displayed for different sample sizes. The threshold estimator of the covariance matrix described in Section 3.2 of the main paper is used in computing the standard errors of the tests.

2 List of Monte Carlo Experiments

The full list of Monte Carlo experimenst is provided in Table S-1. Six designs, the odd numbered ones, have errors in the return equations that are Gaussian, six, the even numbered ones, have errors that are t distributed with 5 degrees of freedom. Designs 3 and 4 add GARCH effects in the factor errors to designs 1 and 2, respectively. Designs 5 and 6, add the pricing error, η_i, to designs 3 and 4, and designs 7 and 8 further add the missing factor, g_t, to the error of the return equations. Designs 1-8 have a diagonal covariance matrix for the idiosyncratic errors, v_{it}. Designs 9 and 10 introduce spatial errors in the idiosyncratic errors, v_{it}, and continue to allow for GARCH effects, pricing errors, and a missing factor. Designs 11 and 12 generate v_{it} with a block covariance matrix structure, instead of the spatial pattern assumed in designs 9 and 10. All experiments are implemented using $R = 2,000$ replications.

Table S-1: List of experimental designs and their parameter values

	Error distribution	GARCH effects	Pricing errors	Missing factor	Error covariance
1	Gaussian	$b_k = 0, c_k = 0$	No	No	$\sigma_{ij} = 0,\ i \neq j$
2	$t(5)$	$b_k = 0, c_k = 0$	No	No	$\sigma_{ij} = 0,\ i \neq j$
3	Gaussian	$b_k = 0.8, c_k = 0.1$	No	No	$\sigma_{ij} = 0,\ i \neq j$
4	$t(5)$	$b_k = 0.8, c_k = 0.1$	No	No	$\sigma_{ij} = 0,\ i \neq j$
5	Gaussian	$b_k = 0.8, c_k = 0.1$	$\alpha_\eta = 0.3$	No	$\sigma_{ij} = 0,\ i \neq j$
6	$t(5)$	$b_k = 0.8, c_k = 0.1$	$\alpha_\eta = 0.3$	No	$\sigma_{ij} = 0,\ i \neq j$
7	Gaussian	$b_k = 0.8, c_k = 0.1$	$\alpha_\eta = 0.3$	$\alpha_\gamma = 0.5$	$\sigma_{ij} = 0,\ i \neq j$
8	$t(5)$	$b_k = 0.8, c_k = 0.1$	$\alpha_\eta = 0.3$	$\alpha_\gamma = 0.5$	$\sigma_{ij} = 0,\ i \neq j$
9	Gaussian	$b_k = 0.8, c_k = 0.1$	$\alpha_\eta = 0.3$	$\alpha_\gamma = 0.5$	$\rho_\varepsilon = 0.5$
10	$t(5)$	$b_k = 0.8, c_k = 0.1$	$\alpha_\eta = 0.3$	$\alpha_\gamma = 0.5$	$\rho_\varepsilon = 0.5$
11	Gaussian	$b_k = 0.8, c_k = 0.1$	$\alpha_\eta = 0.3$	$\alpha_\gamma = 0.5$	Block
12	$t(5)$	$b_k = 0.8, c_k = 0.1$	$\alpha_\eta = 0.3$	$\alpha_\gamma = 0.5$	Block

Notes: t-distributed errors are denoted by $t(5)$, b_k and c_k are the parameters of the GARCH(1,1), α_η is the strength of the pricing errors, α_γ refers to the strength of the missing factor, $\sigma_{ij} = 0,\ i \neq j$ means that the error covariance is diagonal, ρ_ε is the coefficient of spatial error process, and "Block" means that the error covariance matrix is block diagonal. See he online supplement A for further details.

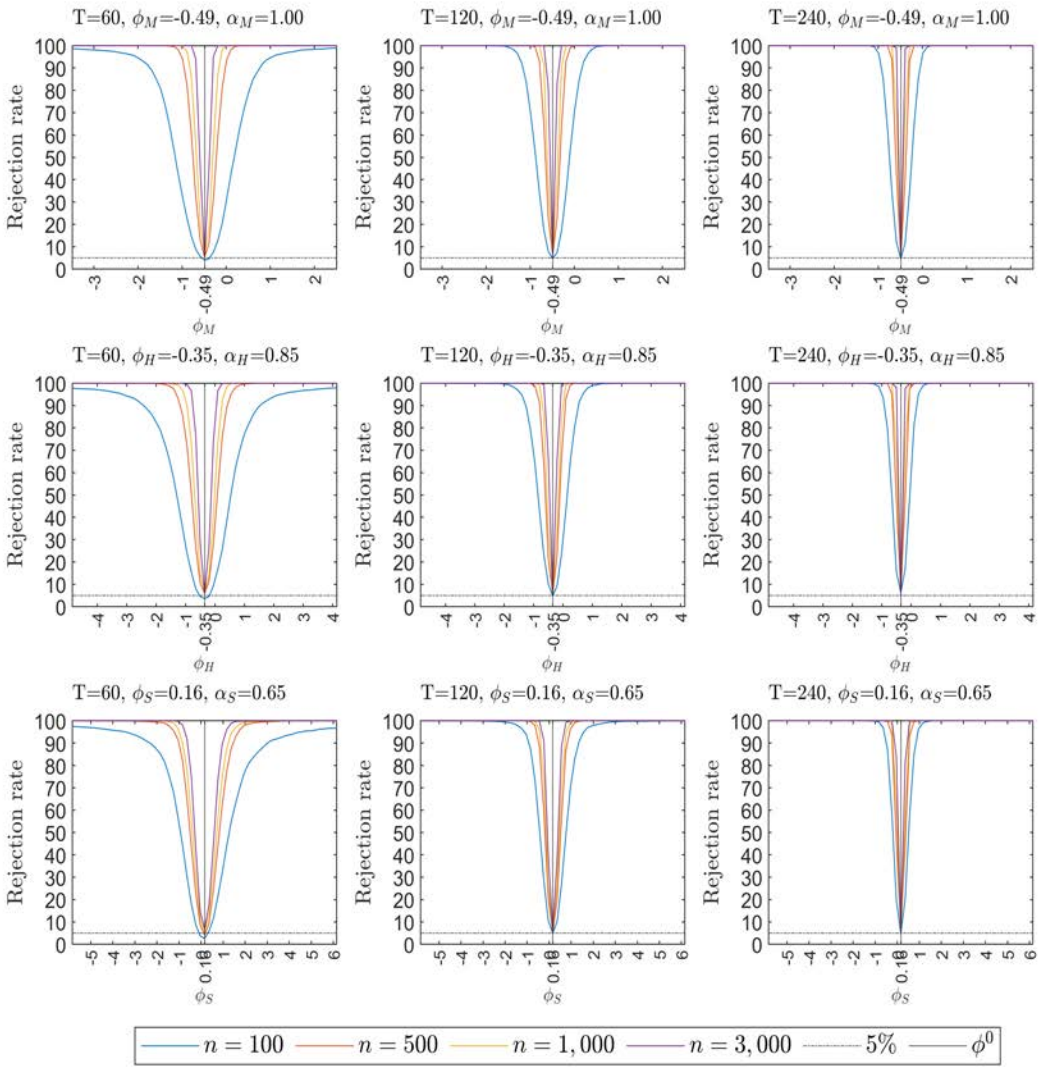
The simulation design, as presented in Table S-1, is aligned with the naming convention employed for tables and figures. Specifically, if a table is denoted as Table S-A-EX, it signifies that the table pertains to Experiment X when the GDP and the panel regressions correctly include one strong and two semi-strong factors. On the other hand, a table labeled as Table S-B-EX relates to Experiment X when the DGP includes one strong and two semi-strong factors, comparing the results when strong and semi-strong factors are included (correct specification) with the ones where only the strong factor is included (incorrect specification). Likewise, Table S-C-EX provides the summary results for Experiment X, when the DGP contains one strong and two weak factors, comparing the results to the case when strong and weak factors are included (correct specification) with the results obtained when the weak factors are excluded (incorrect specification). The aforementioned nomenclature also applies to the figures that present the empirical power functions.

Table S-A-E1: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ, for Experiment 1 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	0.51	-1.55	26.19	86.14	11.10	4.00
	500	-0.06	0.28	17.05	14.33	28.25	5.40
	1,000	-0.43	0.10	15.80	10.12	41.35	6.20
	3,000	-0.86	-0.01	14.77	5.56	60.90	5.70
$T = 120$	100	0.24	1.84	17.47	91.35	7.70	4.70
	500	0.18	0.15	9.34	8.99	13.95	6.15
	1,000	-0.04	0.01	7.57	6.22	19.15	5.65
	3,000	-0.32	-0.06	6.27	3.49	36.65	4.95
$T = 240$	100	0.24	-0.15	12.11	13.11	5.40	4.55
	500	0.31	0.09	5.75	5.75	8.00	4.85
	1,000	0.22	0.03	4.34	4.06	10.60	5.30
	3,000	0.07	-0.00	3.06	2.35	18.80	5.10
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	2.63	-0.89	29.26	140.65	17.55	3.65
	500	4.07	-1.16	25.10	24.87	45.65	5.90
	1,000	4.58	-0.12	25.19	18.25	57.25	6.30
	3,000	4.81	0.30	26.13	11.76	75.10	5.90
$T = 120$	100	2.51	3.60	19.56	190.94	9.95	4.75
	500	2.97	-0.47	13.83	13.46	25.65	5.55
	1,000	3.34	0.02	13.68	10.20	39.85	6.00
	3,000	3.57	0.15	13.85	6.48	61.55	5.50
$T = 240$	100	1.64	-0.50	13.44	16.67	7.75	6.15
	500	1.99	-0.39	7.87	8.01	12.95	5.55
	1,000	2.29	-0.15	7.28	6.04	23.05	5.40
	3,000	2.64	0.02	6.97	3.82	44.55	4.70
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-20.09	1.57	37.57	256.48	24.55	2.60
	500	-23.78	1.80	37.42	37.75	56.20	4.05
	1,000	-25.54	1.05	38.85	32.14	68.60	5.45
	3,000	-28.27	0.35	42.26	24.96	82.80	7.00
$T = 120$	100	-12.99	-6.16	25.50	378.75	16.25	4.90
	500	-16.99	0.65	23.97	19.49	46.85	4.80
	1,000	-18.80	0.56	25.41	16.40	62.60	5.55
	3,000	-22.08	0.44	28.91	12.73	80.55	5.75
$T = 240$	100	-8.52	0.16	17.30	19.66	10.80	4.55
	500	-11.19	0.19	15.08	11.66	36.15	4.90
	1,000	-12.71	0.34	15.72	9.27	55.90	4.60
	3,000	-15.93	0.18	18.70	7.02	81.90	5.60

Notes: The DGP for Experiment 1 allows for Gaussian errors, no GARCH effects, without pricing errors, no missing factors, and without spatial/block error cross dependence. For further details of the experiments, see Table S-1.

Figure S-A-E1: Power functions of the bias-corrected estimators of ϕ_M , ϕ_H and ϕ_S for Experiment 1



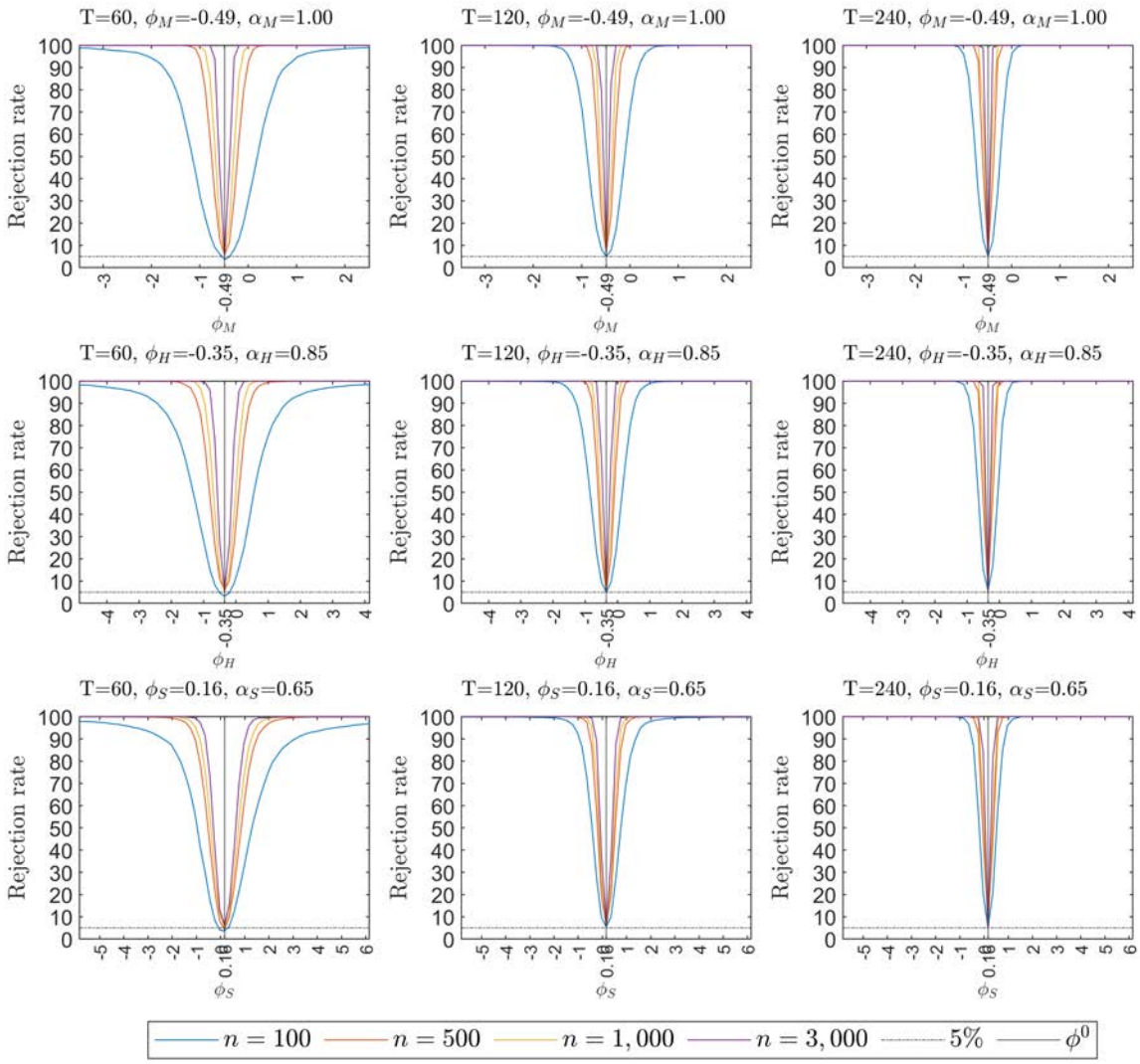
Note: See the notes to Table S-A-E1.

Table S-A-E2: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ, for Experiment 2 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	0.65	0.27	26.62	45.08	11.75	3.70
	500	-0.07	0.27	17.26	14.47	28.25	5.75
	1,000	-0.50	0.04	15.74	10.24	39.75	6.10
	3,000	-0.87	-0.05	14.80	5.72	60.70	5.90
$T = 120$	100	0.42	0.02	17.50	20.45	7.05	4.90
	500	0.25	0.23	9.47	9.16	14.80	6.60
	1,000	-0.03	0.03	7.46	6.12	19.20	5.55
	3,000	-0.36	-0.09	6.31	3.52	36.70	5.65
$T = 240$	100	0.27	-0.14	12.17	13.18	6.20	5.05
	500	0.34	0.11	5.84	5.81	8.15	5.05
	1,000	0.22	0.04	4.34	4.07	10.40	5.55
	3,000	0.04	-0.03	3.07	2.36	19.15	5.25
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	2.88	-2.77	29.60	81.53	17.70	3.25
	500	4.18	-0.95	25.36	25.93	44.85	5.80
	1,000	4.62	-0.02	25.27	18.42	57.40	5.00
	3,000	4.75	0.13	26.17	12.23	74.80	5.95
$T = 120$	100	2.49	-0.71	19.67	28.58	9.70	4.85
	500	3.17	-0.13	13.89	13.46	25.75	5.10
	1,000	3.30	-0.05	13.75	10.34	40.05	6.00
	3,000	3.52	0.08	13.84	6.61	62.40	6.05
$T = 240$	100	1.75	-0.39	13.41	16.61	7.70	5.65
	500	2.08	-0.27	7.96	8.01	13.75	5.10
	1,000	2.23	-0.23	7.22	6.08	22.20	5.15
	3,000	2.64	0.02	6.95	3.83	43.50	4.60
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-20.16	5.40	37.93	125.25	24.85	3.60
	500	-23.88	1.79	37.72	40.47	55.30	4.25
	1,000	-25.44	1.10	38.75	33.04	67.85	5.35
	3,000	-28.24	0.82	42.24	25.93	82.05	6.45
$T = 120$	100	-13.00	2.26	25.41	38.09	16.65	5.40
	500	-17.08	0.48	24.21	20.35	46.35	5.55
	1,000	-18.80	0.53	25.43	16.73	62.75	5.50
	3,000	-22.12	0.38	28.88	12.87	79.90	5.70
$T = 240$	100	-8.38	0.35	17.38	19.88	10.75	4.90
	500	-11.10	0.33	14.99	11.64	35.40	5.35
	1,000	-12.66	0.43	15.60	9.15	55.40	4.55
	3,000	-15.88	0.29	18.65	6.97	80.75	4.95

Notes: The DGP for Experiment 2 allows for t(5) distributed errors, no GARCH effects, without pricing errors, no missing factors, and without spatial/block error cross dependence. For further details of the experiments, see Table S-1.

Figure S-A-E2: Empirical Power Functions, experiment 2, for coefficient of the semi-strong factors



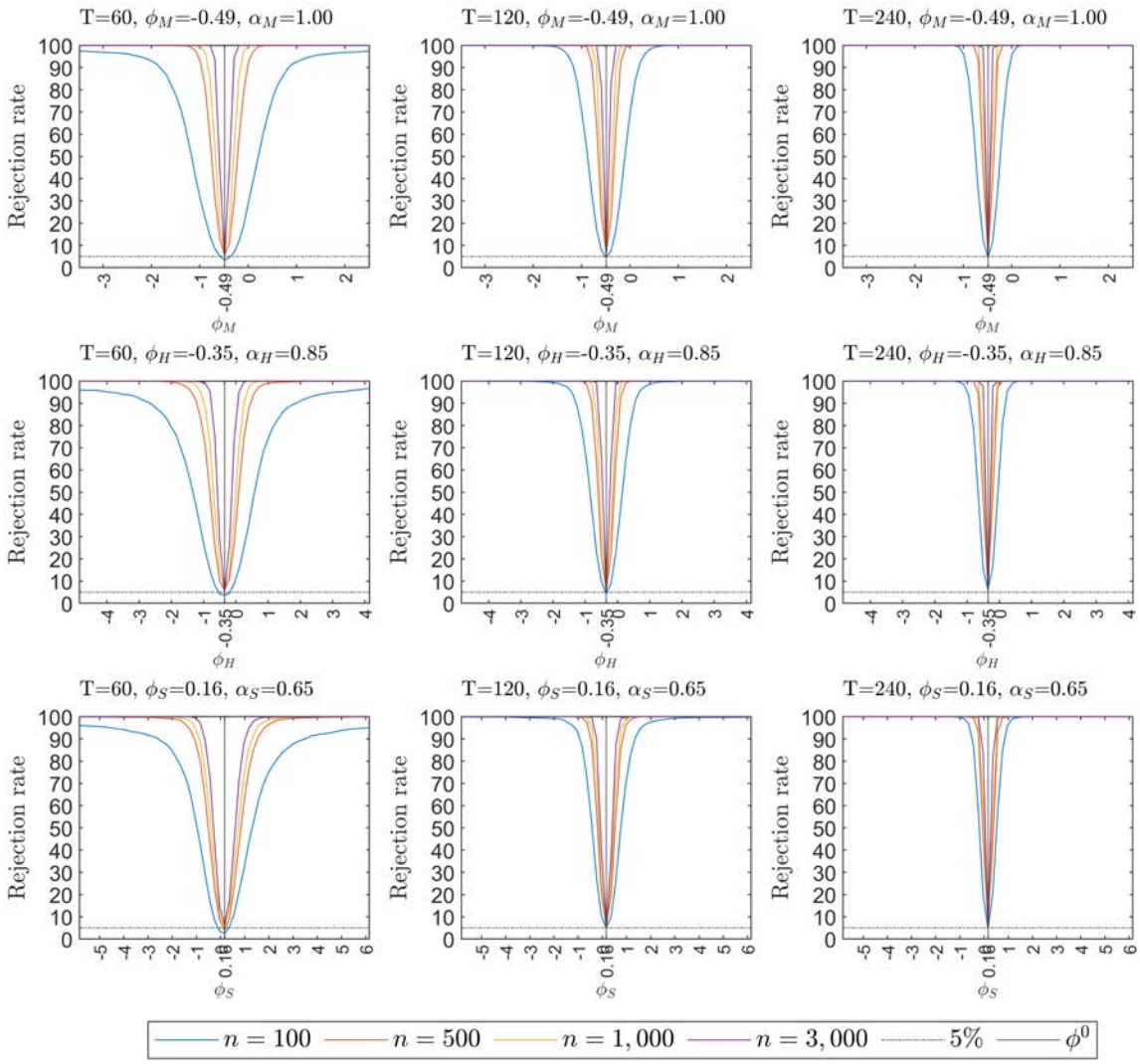
Note: See the notes to Table S-A-E2.

Table S-A-E3: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ, for Experiment 3 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	0.51	-6.00	26.18	227.89	11.05	3.40
	500	-0.14	0.29	17.03	14.71	28.20	5.65
	1,000	-0.51	0.09	15.81	10.33	40.30	6.35
	3,000	-0.92	0.00	14.83	5.65	61.15	5.65
$T = 120$	100	0.21	-0.19	17.45	20.63	7.85	4.50
	500	0.13	0.15	9.35	9.05	13.80	6.25
	1,000	-0.09	0.02	7.61	6.25	19.95	5.75
	3,000	-0.37	-0.05	6.31	3.50	36.65	4.90
$T = 240$	100	0.25	-0.13	12.15	13.17	5.35	4.45
	500	0.31	0.10	5.77	5.78	8.00	5.20
	1,000	0.21	0.05	4.36	4.07	11.15	5.25
	3,000	0.06	0.00	3.07	2.36	18.80	5.20
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	2.67	-7.85	28.98	155.63	17.70	3.80
	500	4.06	-1.08	24.84	26.10	45.75	6.05
	1,000	4.57	-0.15	24.93	18.93	56.85	6.35
	3,000	4.80	0.28	25.82	12.24	75.55	5.40
$T = 120$	100	2.51	-0.70	19.41	28.86	10.05	4.25
	500	2.95	-0.46	13.76	13.63	25.90	5.55
	1,000	3.31	0.05	13.56	10.28	39.70	5.70
	3,000	3.52	0.17	13.70	6.56	61.60	5.30
$T = 240$	100	1.64	-0.53	13.42	16.73	7.65	6.15
	500	2.02	-0.38	7.86	8.04	13.35	5.65
	1,000	2.32	-0.13	7.24	6.05	22.35	5.45
	3,000	2.65	0.03	6.93	3.82	44.45	4.35
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-20.59	25.62	37.71	892.51	25.80	2.50
	500	-24.12	1.65	37.45	40.44	57.75	3.90
	1,000	-25.89	1.16	38.88	34.07	68.95	5.40
	3,000	-28.50	0.41	42.22	26.13	82.65	6.70
$T = 120$	100	-13.31	2.54	25.72	37.94	16.80	4.95
	500	-17.21	0.73	24.19	19.85	47.55	5.25
	1,000	-19.04	0.61	25.61	16.70	63.75	5.65
	3,000	-22.27	0.51	29.04	12.99	80.65	5.95
$T = 240$	100	-8.63	0.19	17.37	19.69	10.70	4.55
	500	-11.31	0.21	15.22	11.72	36.85	5.15
	1,000	-12.87	0.33	15.91	9.31	56.35	4.75
	3,000	-16.07	0.17	18.87	7.04	81.75	5.60

Notes: The DGP for Experiment 3 allows for Gaussian errors, with GARCH effects, without pricing errors, no missing factors, and without spatial/block error cross dependence. For further details of the experiments, see Table S-1.

Figure S-A-E3: Empirical Power Functions, experiment 3, for coefficient of the semi-strong factors



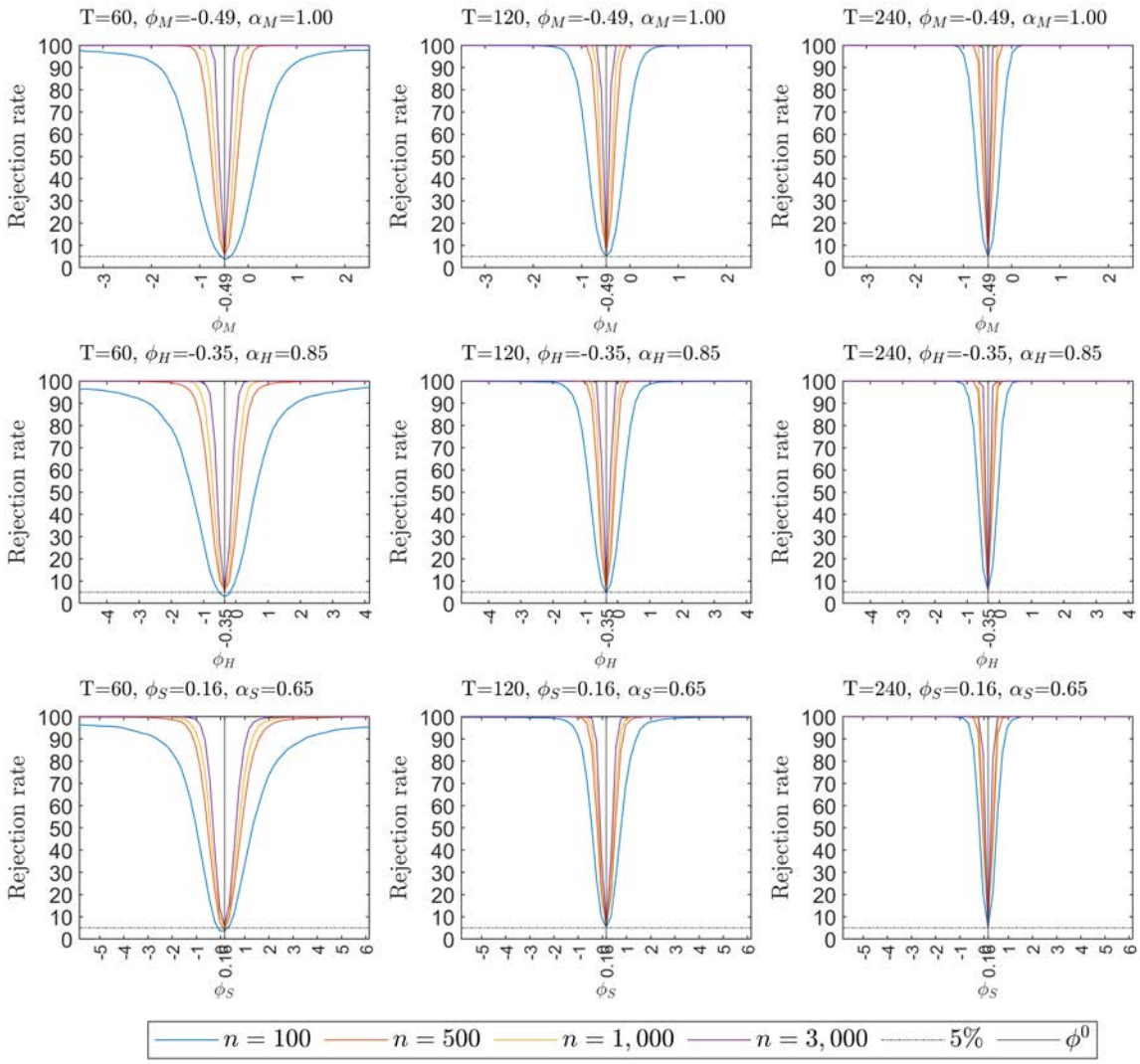
Note: See the notes to Table S-A-E3.

Table S-A-E4: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ, for Experiment 4 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	0.59	0.17	26.59	107.83	12.15	3.75
	500	-0.16	0.23	17.26	14.75	28.05	5.65
	1,000	-0.57	0.05	15.76	10.43	39.95	6.10
	3,000	-0.93	-0.04	14.85	5.82	60.70	5.95
$T = 120$	100	0.37	-0.86	17.48	44.37	7.05	4.95
	500	0.20	0.21	9.49	9.24	15.15	6.45
	1,000	-0.08	0.03	7.50	6.16	19.45	4.90
	3,000	-0.40	-0.08	6.35	3.53	36.70	5.85
$T = 240$	100	0.27	-0.13	12.20	13.22	6.35	4.85
	500	0.34	0.12	5.86	5.83	8.80	5.10
	1,000	0.22	0.05	4.36	4.08	10.60	5.60
	3,000	0.03	-0.02	3.08	2.36	19.15	5.25
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	2.92	-5.32	29.30	146.82	17.95	3.15
	500	4.18	-0.76	25.12	27.25	44.65	5.80
	1,000	4.61	-0.06	25.02	19.04	56.50	4.85
	3,000	4.73	0.12	25.86	12.64	74.85	5.60
$T = 120$	100	2.50	-3.13	19.53	113.15	9.75	4.55
	500	3.16	-0.11	13.84	13.64	25.65	5.25
	1,000	3.27	-0.02	13.64	10.41	40.10	6.05
	3,000	3.47	0.10	13.70	6.67	60.75	6.25
$T = 240$	100	1.75	-0.41	13.39	16.66	7.85	5.75
	500	2.12	-0.25	7.97	8.07	13.90	5.05
	1,000	2.25	-0.22	7.17	6.09	21.70	5.40
	3,000	2.65	0.02	6.92	3.85	43.35	4.70
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-20.62	12.61	38.03	267.24	25.90	3.60
	500	-24.23	1.81	37.80	44.22	56.50	3.85
	1,000	-25.79	1.07	38.78	34.41	68.90	5.45
	3,000	-28.50	0.96	42.21	27.28	81.45	6.20
$T = 120$	100	-13.32	7.33	25.66	214.62	16.60	5.25
	500	-17.29	0.59	24.43	20.72	46.95	5.50
	1,000	-19.05	0.56	25.63	16.96	62.30	5.80
	3,000	-22.32	0.41	29.03	13.12	80.50	5.85
$T = 240$	100	-8.49	0.40	17.46	19.99	10.75	5.10
	500	-11.22	0.35	15.14	11.73	36.10	5.20
	1,000	-12.80	0.44	15.77	9.19	55.95	4.40
	3,000	-16.02	0.29	18.80	6.99	81.40	5.00

Notes: The DGP for Experiment 4 allows for t(5) distributed errors, with GARCH effects, without pricing errors, no missing factors, and without spatial/block error cross dependence. For further details of the experiments, see Table S-1.

Figure S-A-E4: Empirical Power Functions, experiment 4, for coefficient of the semi-strong factors



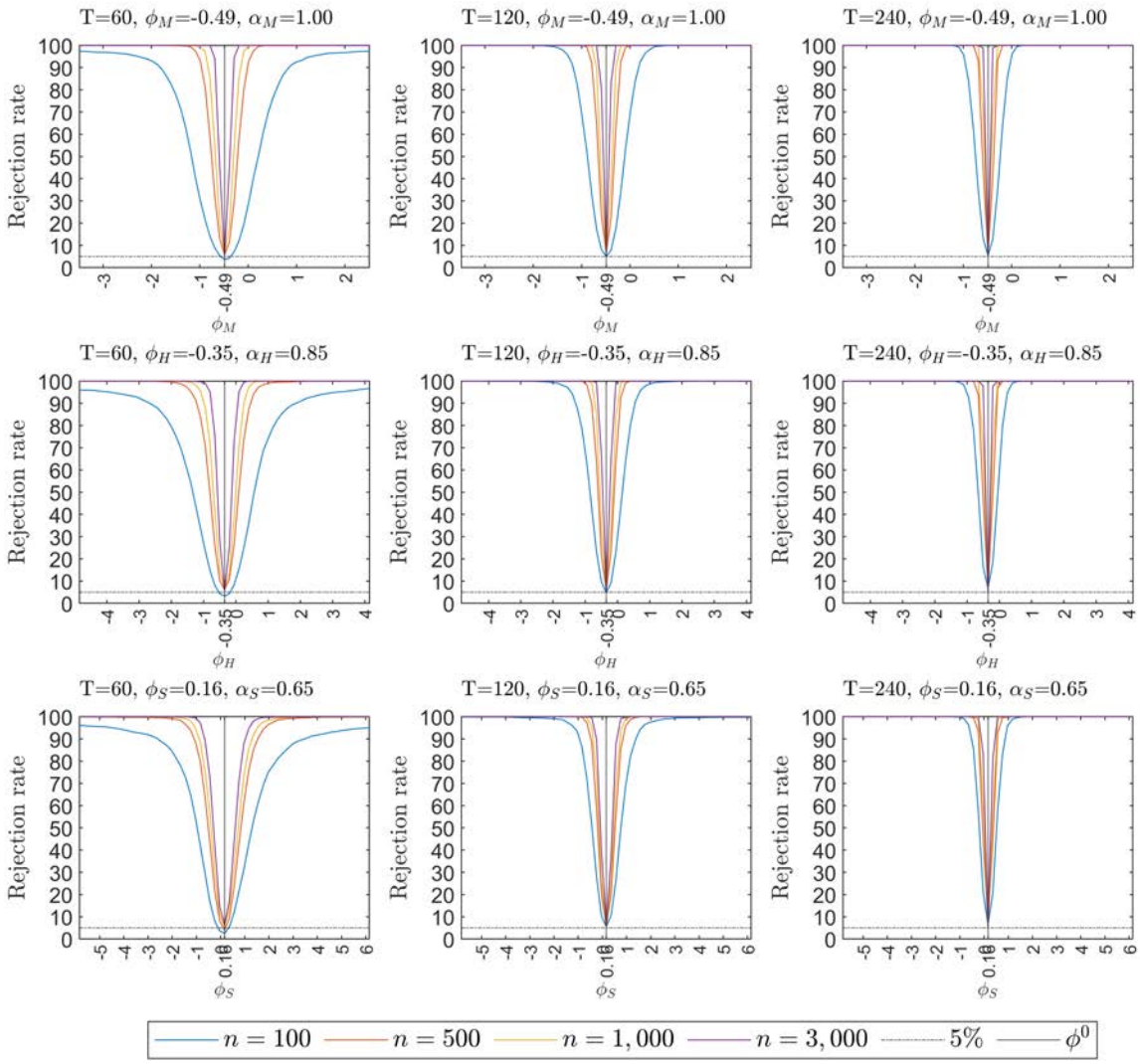
Note: See the notes to Table S-A-E4.

Table S-A-E5: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ, for Experiment 5 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	0.42	-7.58	26.27	285.21	11.35	3.75
	500	-0.15	0.28	17.05	14.79	28.10	5.80
	1,000	-0.49	0.10	15.81	10.35	40.35	6.25
	3,000	-0.92	-0.00	14.83	5.65	61.10	5.75
$T = 120$	100	0.06	-0.35	17.63	20.82	7.85	4.95
	500	0.13	0.14	9.40	9.15	14.85	6.45
	1,000	-0.07	0.04	7.60	6.28	20.00	5.90
	3,000	-0.37	-0.06	6.32	3.50	37.00	4.80
$T = 240$	100	0.08	-0.31	12.48	13.53	6.30	5.35
	500	0.31	0.10	5.83	5.84	8.35	5.35
	1,000	0.23	0.07	4.38	4.11	10.95	5.20
	3,000	0.06	0.00	3.08	2.36	18.75	5.25
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	2.58	-8.49	28.97	174.83	18.20	3.30
	500	4.10	-1.01	24.82	26.10	45.25	6.15
	1,000	4.58	-0.12	24.96	18.91	57.15	6.50
	3,000	4.80	0.28	25.82	12.25	75.60	5.60
$T = 120$	100	2.44	-0.79	19.68	28.94	10.35	4.70
	500	2.98	-0.43	13.78	13.79	25.75	5.60
	1,000	3.32	0.07	13.57	10.29	40.00	6.05
	3,000	3.52	0.17	13.70	6.56	61.85	5.45
$T = 240$	100	1.57	-0.63	13.97	17.37	8.25	6.75
	500	2.04	-0.35	7.92	8.17	13.85	5.75
	1,000	2.33	-0.12	7.28	6.09	22.70	5.50
	3,000	2.66	0.04	6.94	3.83	44.70	4.55
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-20.57	31.92	37.88	1112.05	25.45	2.55
	500	-24.15	1.61	37.49	40.53	57.90	3.95
	1,000	-25.88	1.16	38.88	34.12	68.80	5.70
	3,000	-28.51	0.40	42.23	26.13	82.65	6.60
$T = 120$	100	-13.21	2.55	26.09	37.89	16.70	5.50
	500	-17.24	0.70	24.27	20.01	47.90	5.40
	1,000	-19.03	0.62	25.61	16.77	63.50	5.85
	3,000	-22.26	0.53	29.04	13.01	80.55	6.05
$T = 240$	100	-8.55	0.29	18.03	20.67	11.85	6.25
	500	-11.35	0.16	15.31	11.89	37.35	5.85
	1,000	-12.87	0.33	15.91	9.37	56.40	4.75
	3,000	-16.07	0.18	18.86	7.07	81.75	5.65

Notes: The DGP for Experiment 5 allows for Gaussian errors, with GARCH effects, with pricing errors ($\alpha_\eta = 0.3$), no missing factors, and without spatial/block error cross dependence. For further details of the experiments, see Table S-1.

Figure S-A-E5: Empirical Power Functions, experiment 5, for coefficient of the semi-strong factors



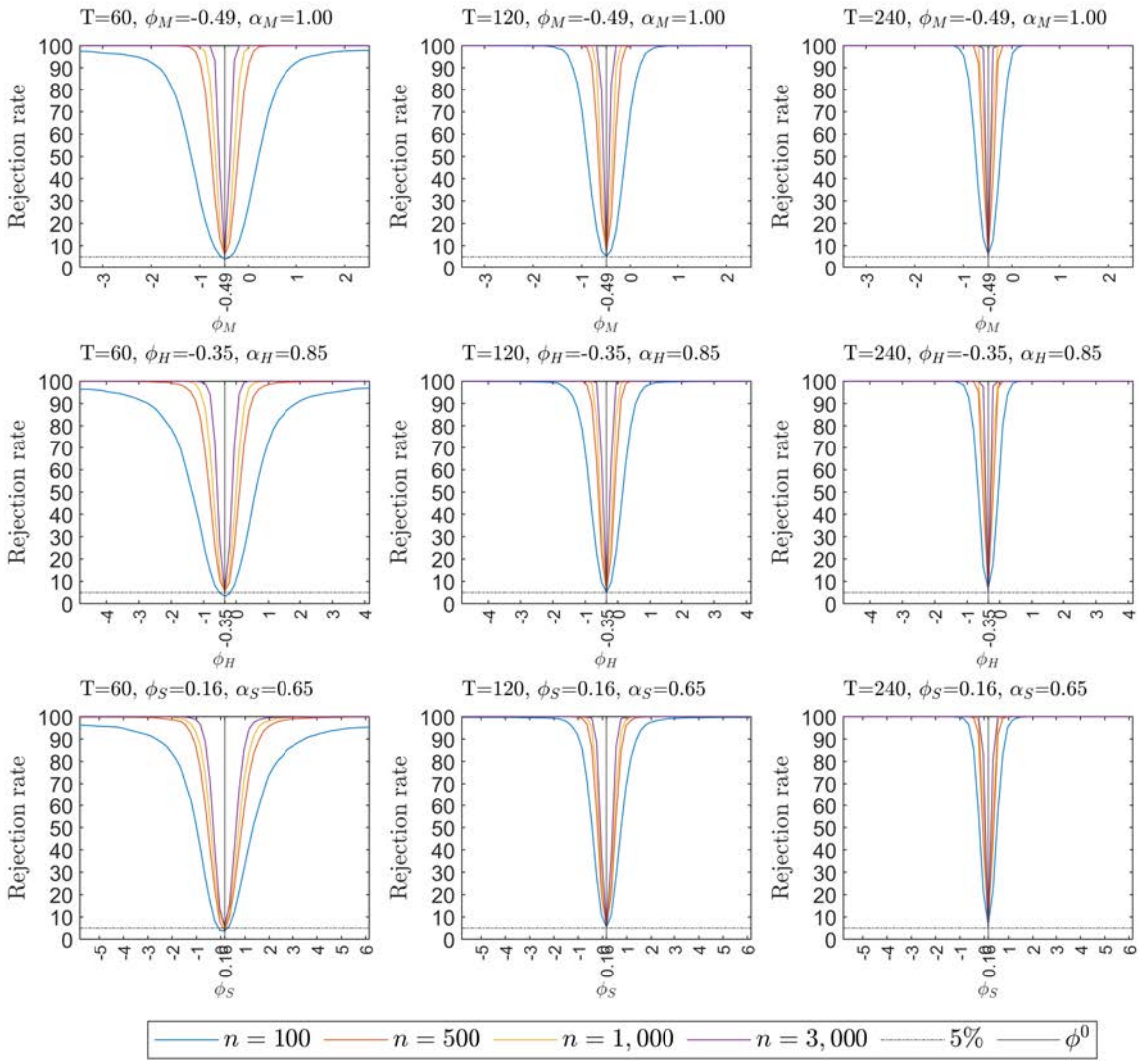
Note: See the notes to Table S-A-E5.

Table S-A-E6: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ, for Experiment 6 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	0.49	0.05	26.69	103.65	12.25	3.95
	500	-0.17	0.24	17.27	14.81	28.25	5.95
	1,000	-0.56	0.06	15.75	10.44	39.60	6.00
	3,000	-0.93	-0.05	14.85	5.82	60.75	5.80
$T = 120$	100	0.22	-0.76	17.65	33.47	7.45	5.15
	500	0.20	0.22	9.54	9.32	15.30	6.95
	1,000	-0.06	0.05	7.49	6.18	19.50	5.30
	3,000	-0.40	-0.09	6.36	3.54	37.05	5.70
$T = 240$	100	0.11	-0.30	12.57	13.63	7.55	6.10
	500	0.33	0.11	5.91	5.88	8.60	5.10
	1,000	0.24	0.07	4.38	4.13	10.30	5.80
	3,000	0.03	-0.03	3.09	2.37	19.25	5.45
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	2.83	-5.32	29.34	144.70	18.05	3.40
	500	4.22	-0.67	25.10	27.28	44.70	5.45
	1,000	4.62	-0.03	25.04	19.02	56.30	4.80
	3,000	4.73	0.12	25.86	12.64	74.70	5.65
$T = 120$	100	2.42	-2.45	19.82	78.69	11.40	4.90
	500	3.19	-0.06	13.87	13.81	25.85	5.45
	1,000	3.28	-0.00	13.65	10.44	40.80	6.35
	3,000	3.47	0.11	13.70	6.67	61.05	6.10
$T = 240$	100	1.68	-0.51	13.94	17.33	8.35	6.70
	500	2.15	-0.22	8.05	8.22	14.70	5.60
	1,000	2.26	-0.20	7.23	6.14	21.75	5.65
	3,000	2.65	0.03	6.93	3.86	43.85	5.00
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-20.59	12.67	38.25	257.83	25.55	3.70
	500	-24.27	1.73	37.83	44.57	56.60	3.95
	1,000	-25.78	1.10	38.79	34.52	68.85	5.30
	3,000	-28.50	0.95	42.22	27.27	81.40	6.05
$T = 120$	100	-13.22	5.85	26.06	146.46	16.95	5.40
	500	-17.32	0.55	24.50	20.89	47.05	5.55
	1,000	-19.04	0.58	25.64	17.05	62.35	5.95
	3,000	-22.32	0.42	29.03	13.16	80.25	5.90
$T = 240$	100	-8.41	0.50	18.09	20.93	12.15	6.40
	500	-11.25	0.31	15.25	11.95	36.35	5.70
	1,000	-12.80	0.44	15.77	9.26	55.35	5.05
	3,000	-16.02	0.29	18.80	7.02	81.00	5.10

Notes: The DGP for Experiment 6 allows for t(5) distributed errors, with GARCH effects, with pricing errors ($\alpha_\eta = 0.3$), no missing factors, and without spatial/block error cross dependence. For further details of the experiments, see Table S-1.

Figure S-A-E6: Empirical Power Functions, experiment 6, for coefficient of the semi-strong factors



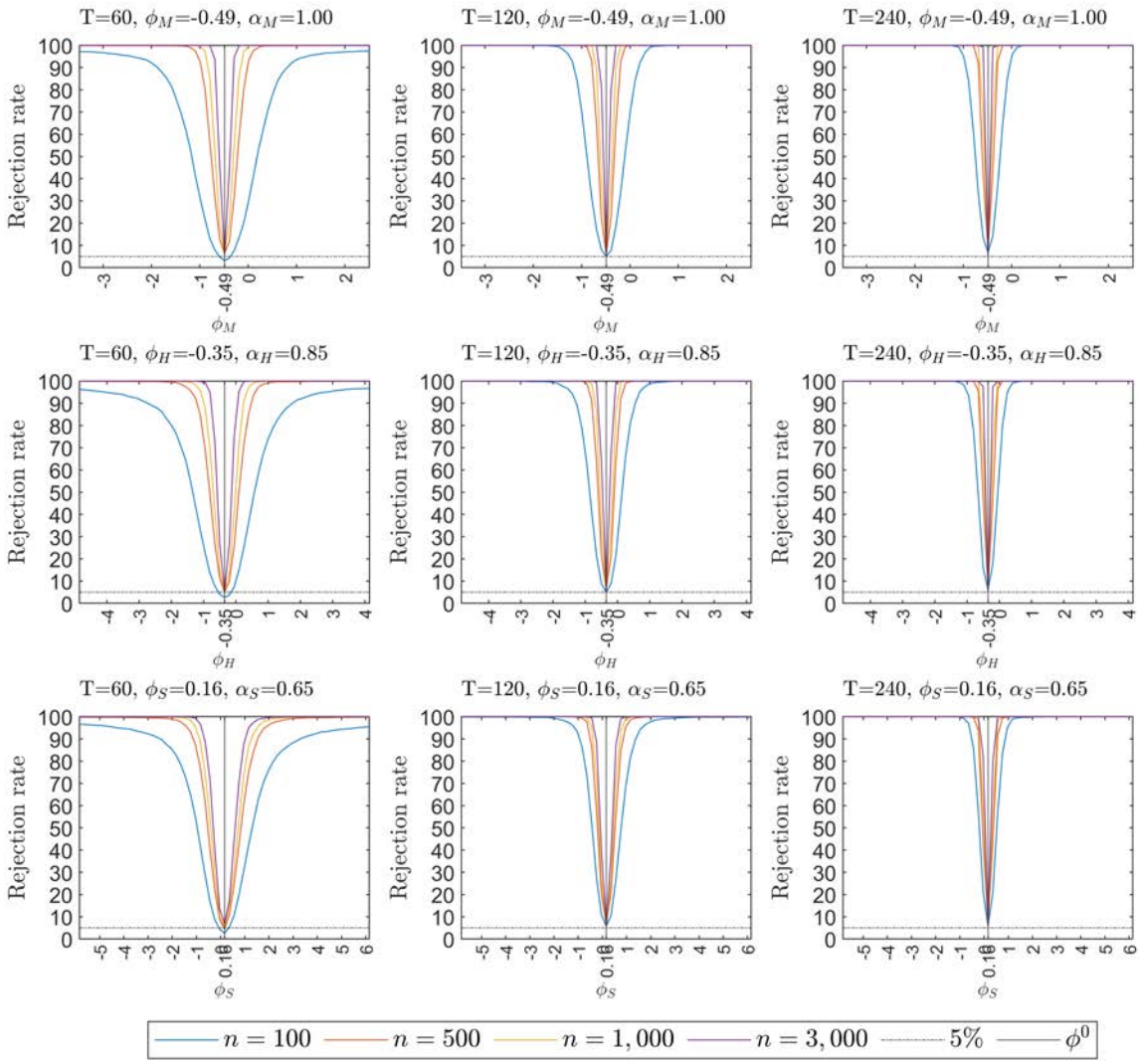
Note: See the notes to Table S-A-E6.

Table S-A-E7: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ, for Experiment 7 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	-0.33	-14.31	26.54	555.11	11.80	3.20
	500	-0.50	-0.18	17.67	15.13	29.80	6.40
	1,000	-0.73	-0.15	15.95	10.41	42.20	6.25
	3,000	-0.91	0.04	14.87	5.81	61.20	5.90
$T = 120$	100	-0.13	-0.49	17.44	20.30	7.55	5.05
	500	0.01	0.01	9.49	8.86	14.20	5.85
	1,000	-0.11	0.04	7.74	6.31	20.50	6.60
	3,000	-0.27	0.08	6.40	3.60	36.65	5.65
$T = 240$	100	-0.00	-0.39	12.82	14.02	7.30	6.75
	500	0.15	-0.08	6.14	6.13	9.85	6.95
	1,000	0.14	-0.03	4.52	4.22	11.90	5.80
	3,000	0.07	0.01	3.05	2.32	18.30	5.15
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	3.05	-222.60	29.40	9435.95	17.55	2.70
	500	4.42	-0.75	25.03	26.57	46.25	4.95
	1,000	4.43	-0.56	25.18	19.01	58.75	5.45
	3,000	4.63	-0.15	25.85	12.14	76.15	5.75
$T = 120$	100	2.45	-0.89	19.84	28.94	10.55	4.95
	500	3.24	-0.00	13.84	13.44	26.75	5.95
	1,000	3.30	0.05	13.47	9.91	41.30	5.20
	3,000	3.29	-0.25	13.67	6.56	62.45	5.75
$T = 240$	100	1.67	-0.55	13.68	16.80	7.45	6.10
	500	2.28	-0.04	8.00	8.00	13.35	5.50
	1,000	2.42	-0.02	7.22	6.00	22.60	5.10
	3,000	2.54	-0.13	6.85	3.82	44.00	5.25
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-20.23	62.14	37.82	2347.80	25.15	2.70
	500	-24.55	1.18	37.58	46.58	59.25	4.40
	1,000	-26.04	0.76	38.91	33.25	69.25	5.25
	3,000	-28.68	0.06	42.34	26.75	82.30	6.90
$T = 120$	100	-12.90	2.52	25.93	38.03	16.35	5.70
	500	-17.48	0.44	24.54	21.10	50.00	6.00
	1,000	-19.25	0.09	25.81	16.81	63.35	5.45
	3,000	-22.34	0.29	29.12	13.12	80.15	6.15
$T = 240$	100	-8.13	0.84	17.77	20.91	11.30	5.65
	500	-11.31	0.30	15.26	11.79	37.05	5.70
	1,000	-12.84	0.38	15.96	9.45	56.40	5.15
	3,000	-16.09	0.21	18.91	7.26	80.95	6.30

Notes: The DGP for Experiment 7 allows for Gaussian errors, with GARCH effects, with pricing errors ($\alpha_\eta = 0.3$), with one weak missing factor ($\alpha_\gamma = 0.5$), and without spatial/block error cross dependence. For further details of the experiments, see S-1.

Figure S-A-E7: Empirical Power Functions, experiment 7 for coefficient of the semi-strong factors



Note: See the notes to Table S-A-E7.

Table S-A-E8: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ, for Experiment 8 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	-0.27	9.06	27.12	251.34	11.45	3.75
	500	-0.74	-0.67	17.81	16.71	29.75	6.30
	1,000	-0.64	0.01	16.06	10.38	40.90	5.70
	3,000	-0.85	0.12	14.90	5.96	59.65	6.15
$T = 120$	100	0.04	-0.32	17.49	20.53	7.35	4.75
	500	-0.06	-0.04	9.49	8.89	13.95	5.80
	1,000	0.01	0.19	7.78	6.40	20.90	6.10
	3,000	-0.24	0.14	6.39	3.63	37.25	5.90
$T = 240$	100	0.09	-0.29	12.84	14.07	7.10	6.35
	500	0.20	-0.01	6.08	6.06	9.50	6.05
	1,000	0.17	0.01	4.52	4.25	11.55	6.30
	3,000	0.06	0.01	3.07	2.35	18.15	5.15
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	3.41	39.71	29.84	1913.29	17.65	3.15
	500	4.52	-1.03	25.21	30.79	44.70	4.95
	1,000	4.47	-0.47	25.14	19.34	57.65	5.85
	3,000	4.59	-0.21	25.88	12.19	75.20	5.45
$T = 120$	100	2.68	-0.54	20.19	28.91	11.20	5.05
	500	3.38	0.31	13.97	13.39	26.60	5.35
	1,000	3.36	0.14	13.38	9.90	40.55	4.90
	3,000	3.33	-0.18	13.64	6.52	61.95	5.20
$T = 240$	100	1.77	-0.40	13.95	17.05	7.45	6.00
	500	2.38	0.12	8.01	7.92	14.30	4.70
	1,000	2.58	0.20	7.25	5.99	23.15	4.75
	3,000	2.54	-0.13	6.81	3.84	42.85	4.95
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-20.01	-141.72	37.89	5932.49	25.40	3.15
	500	-24.52	3.22	37.63	77.87	57.05	4.45
	1,000	-26.14	0.57	39.14	34.26	69.30	5.10
	3,000	-28.54	0.84	42.33	26.81	81.00	5.85
$T = 120$	100	-13.01	2.45	26.01	36.96	16.80	5.60
	500	-17.52	0.34	24.66	20.94	48.35	6.55
	1,000	-19.32	-0.11	25.88	16.81	64.05	5.80
	3,000	-22.35	0.26	29.14	13.22	79.70	6.20
$T = 240$	100	-8.14	0.79	17.57	20.56	11.15	5.85
	500	-11.42	0.12	15.36	11.78	37.65	5.55
	1,000	-13.01	0.12	16.02	9.32	55.70	5.20
	3,000	-16.10	0.20	18.91	7.40	80.10	6.70

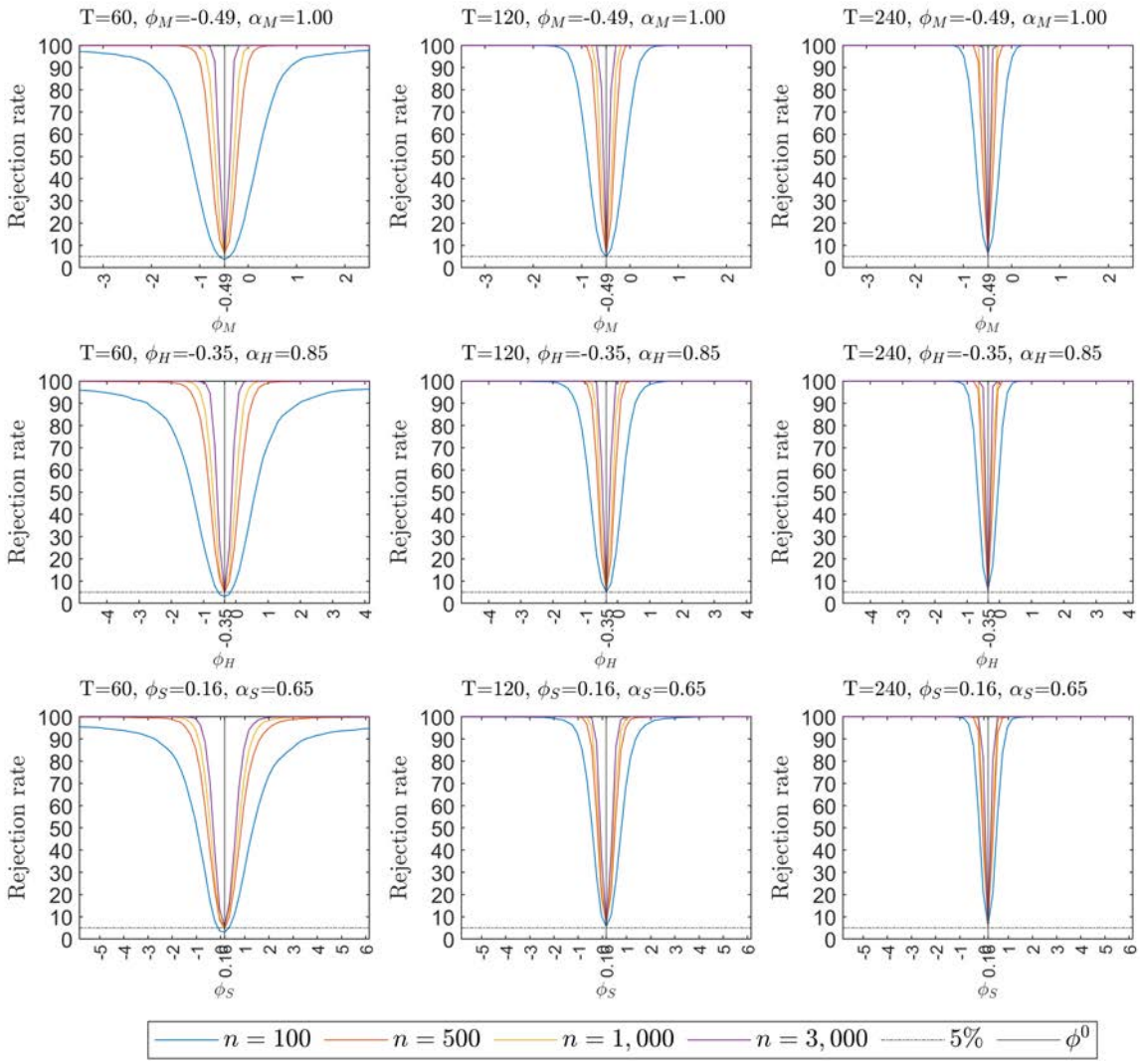
Notes: The DGP for Experiment 8 allows for t(5) distributed errors, with GARCH effects, with pricing errors ($\alpha_\eta = 0.3$), with one weak missing factor ($\alpha_\gamma = 0.5$), and without spatial/block error cross dependence. For further details of the experiments, see S-1.

Table S-A-E8a: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ for Experiment 8 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	-0.16	12.00	27.46	356.79	12.85	4.05
	500	-0.77	-0.71	17.89	16.74	29.55	6.50
	1,000	-0.66	-0.02	16.09	10.45	40.95	5.70
	3,000	-0.86	0.10	14.91	5.99	59.85	6.20
$T = 120$	100	0.19	-0.16	18.02	21.26	8.25	5.30
	500	-0.09	-0.08	9.66	9.09	14.80	6.00
	1,000	-0.01	0.16	7.84	6.51	20.95	6.70
	3,000	-0.25	0.12	6.40	3.65	38.10	6.35
$T = 240$	100	0.27	-0.10	13.79	15.14	8.85	8.70
	500	0.19	-0.03	6.34	6.36	11.05	7.05
	1,000	0.14	-0.03	4.63	4.40	12.05	7.50
	3,000	0.05	-0.00	3.09	2.39	18.85	5.70
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	3.10	55.07	29.92	2828.62	18.85	3.10
	500	4.53	-0.98	25.32	30.87	45.10	5.45
	1,000	4.46	-0.51	25.19	19.42	58.20	5.45
	3,000	4.59	-0.21	25.88	12.26	75.20	5.75
$T = 120$	100	2.39	-0.98	20.59	29.45	11.90	5.85
	500	3.39	0.32	14.17	13.75	27.35	5.45
	1,000	3.34	0.10	13.45	10.09	40.55	5.00
	3,000	3.33	-0.18	13.63	6.58	62.30	5.20
$T = 240$	100	1.52	-0.70	14.82	18.29	10.60	8.55
	500	2.38	0.12	8.27	8.27	15.65	6.05
	1,000	2.55	0.15	7.35	6.23	23.20	5.65
	3,000	2.54	-0.13	6.83	3.90	42.75	5.95
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-19.86	-210.04	38.24	8784.76	26.15	3.40
	500	-24.49	3.17	37.68	75.99	56.80	4.95
	1,000	-26.11	0.71	39.11	34.49	69.05	4.95
	3,000	-28.55	0.78	42.34	26.88	81.20	6.05
$T = 120$	100	-12.88	2.68	26.48	38.62	17.60	5.85
	500	-17.41	0.56	24.62	21.19	47.90	6.25
	1,000	-19.28	-0.02	25.86	17.01	63.85	6.00
	3,000	-22.36	0.23	29.18	13.36	79.65	6.75
$T = 240$	100	-8.03	0.96	18.42	22.23	13.25	7.35
	500	-11.34	0.24	15.47	12.20	36.80	6.50
	1,000	-12.95	0.21	16.03	9.63	56.70	6.20
	3,000	-16.10	0.19	18.96	7.59	80.25	7.25

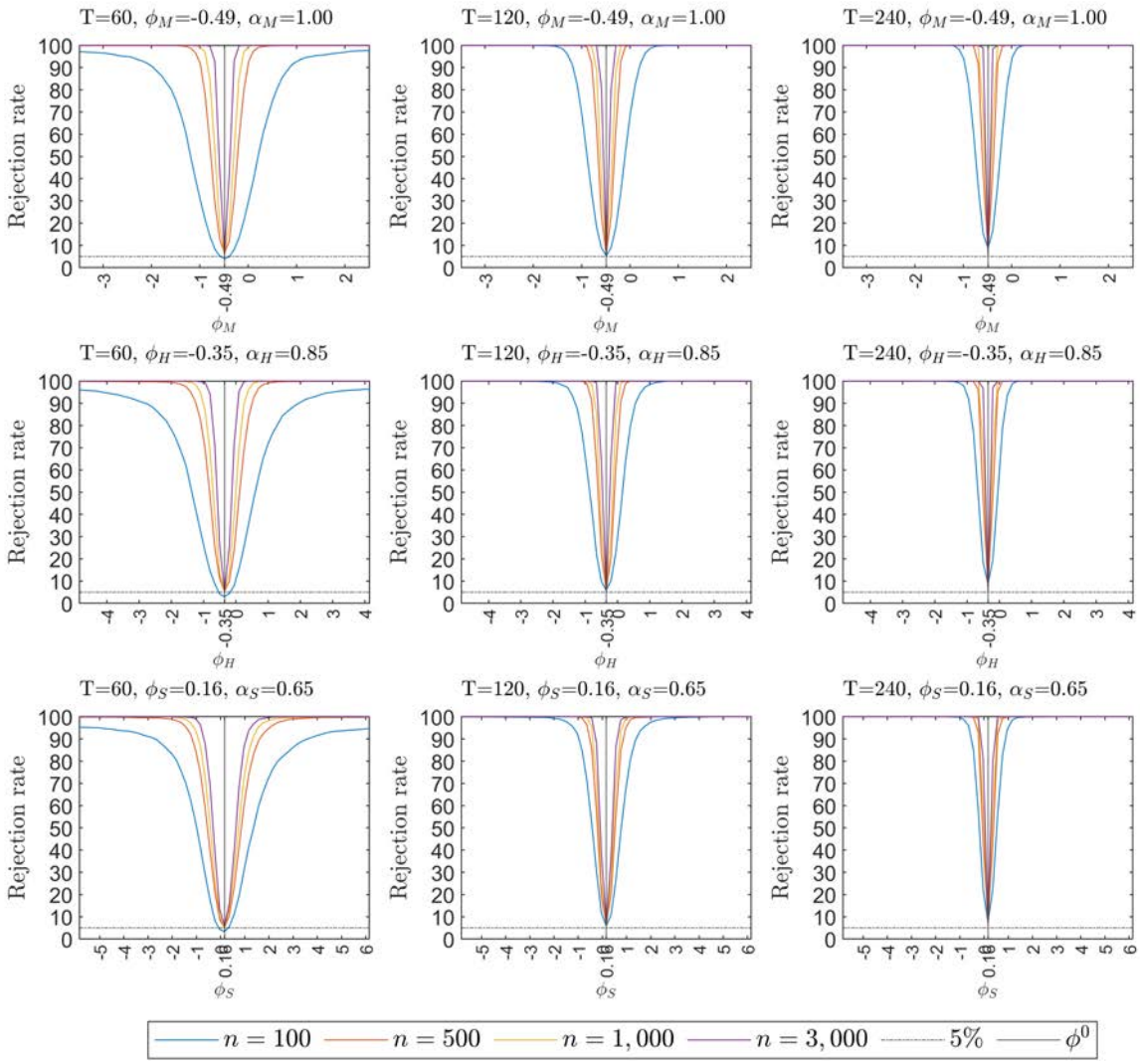
Notes: The DGP for Experiment 8 allows for t(5) distributed errors, with GARCH effects, with pricing errors ($\alpha_\eta = 0.5$), with one weak missing factor ($\alpha_\gamma = 0.5$), and without spatial/block error cross dependence. For further details of the experiments, see S-1.

Figure S-A-E8: Empirical Power Functions, experiment 8 for coefficient of the semi-strong factors



Note: See the notes to Table S-A-E8.

Figure S-A-E8a: Empirical Power Functions, experiment 8a for coefficient of the semi-strong factors



Note: See the notes to Table S-A-E8a.

Table S-A-E9: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ, for Experiment 9 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	-0.51	-15.47	27.16	662.68	11.80	3.40
	500	-0.55	1.29	18.02	84.39	31.25	7.75
	1,000	-0.85	-0.47	16.24	11.31	43.00	8.50
	3,000	-0.99	-0.16	14.97	6.25	61.30	7.75
$T = 120$	100	-0.17	-0.59	17.92	21.49	7.15	4.50
	500	0.03	-0.01	9.70	9.20	13.95	6.15
	1,000	-0.17	-0.07	7.88	6.53	20.35	6.30
	3,000	-0.33	-0.01	6.45	3.71	36.90	6.50
$T = 240$	100	-0.12	-0.54	12.73	14.02	6.80	6.85
	500	0.13	-0.12	6.18	6.19	9.65	6.55
	1,000	0.11	-0.08	4.57	4.30	11.75	6.60
	3,000	0.06	-0.00	3.09	2.37	17.90	5.45
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	2.80	3.36	30.96	683.86	19.20	3.90
	500	4.18	2.34	25.47	189.26	47.10	8.65
	1,000	4.41	-0.83	25.43	22.18	59.30	9.35
	3,000	4.63	-0.28	25.90	14.12	76.45	10.60
$T = 120$	100	2.27	-1.02	20.81	33.62	10.25	4.50
	500	3.04	-0.39	14.24	14.82	24.60	5.70
	1,000	3.17	-0.24	13.77	11.21	38.65	6.10
	3,000	3.24	-0.40	13.74	7.47	60.05	7.75
$T = 240$	100	1.62	-0.61	14.29	17.94	7.20	5.75
	500	2.25	-0.09	8.26	8.60	13.10	5.35
	1,000	2.36	-0.10	7.47	6.57	21.10	5.65
	3,000	2.57	-0.10	6.99	4.27	41.70	5.85
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-19.57	38.19	40.89	1634.08	26.25	4.75
	500	-24.60	-17.77	38.37	944.75	59.35	10.05
	1,000	-26.04	1.99	39.30	41.96	70.30	11.50
	3,000	-28.69	0.77	42.55	32.04	82.85	11.70
$T = 120$	100	-12.73	4.51	29.32	56.35	14.45	5.50
	500	-17.45	0.82	25.67	26.62	44.50	8.70
	1,000	-19.16	0.54	26.31	20.96	58.65	8.40
	3,000	-22.27	0.58	29.33	16.06	77.55	8.95
$T = 240$	100	-8.13	1.07	21.26	26.72	10.60	6.50
	500	-11.34	0.35	16.58	15.28	31.25	7.40
	1,000	-12.81	0.50	16.52	11.98	46.35	6.40
	3,000	-16.03	0.35	19.08	9.09	75.35	7.25

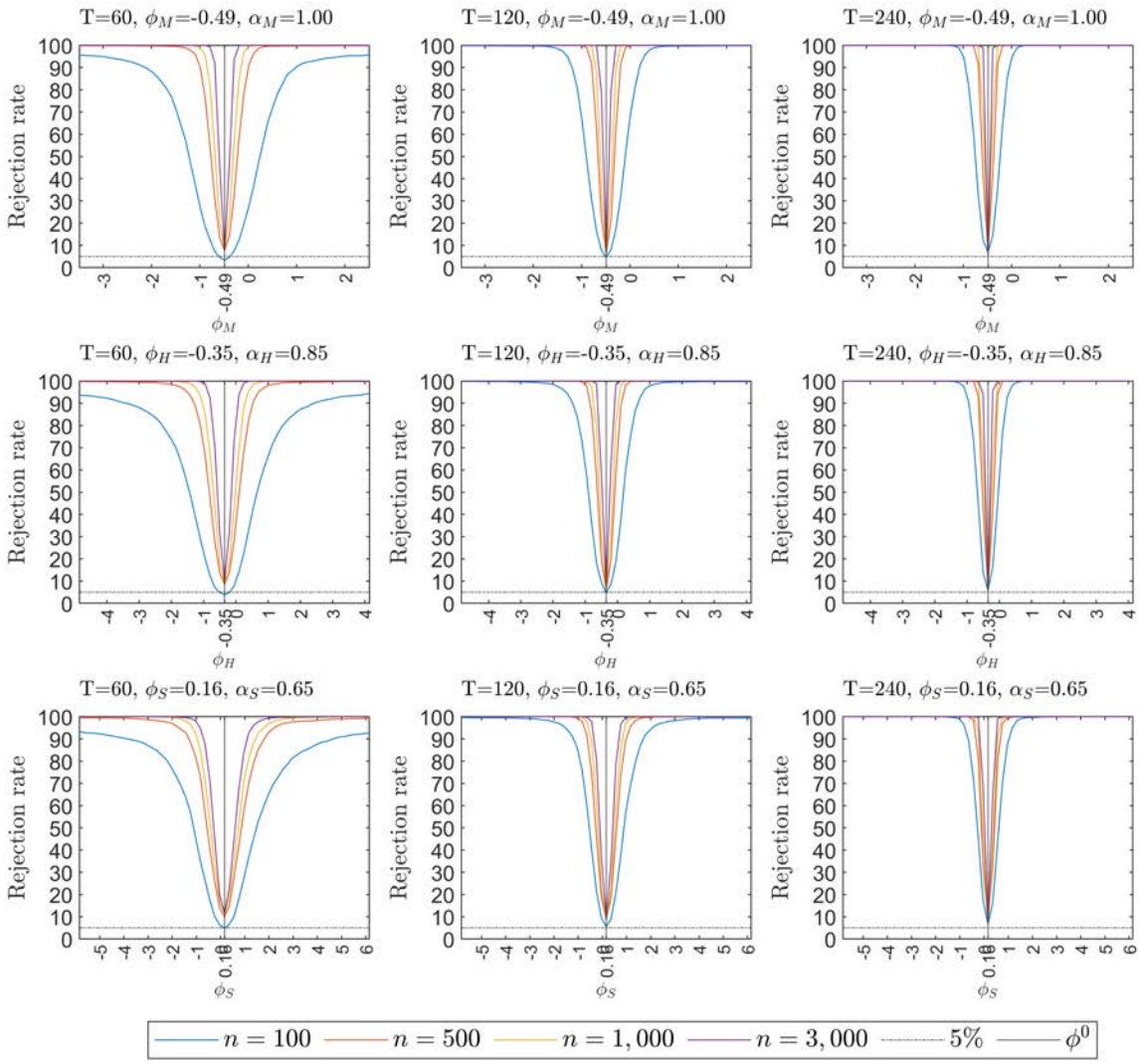
Notes: The DGP for Experiment 9 allows for Gaussian errors, with GARCH effects, with pricing errors ($\alpha_\eta = 0.3$), with one weak missing factor ($\alpha_\gamma = 0.5$), and with spatial error cross dependence ($\rho_\varepsilon = 0.5$). For further details of the experiments, see Table S-1.

Table S-A-E9a: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ for Experiment 9 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	-0.48	24.91	29.91	1802.71	10.25	3.05
	500	-0.58	-1.22	19.37	92.46	26.95	6.75
	1,000	-0.92	-0.95	17.00	17.30	37.20	8.60
	3,000	-1.03	-0.35	15.24	8.41	55.50	8.65
$T = 120$	100	-0.02	-1.97	19.95	35.08	8.60	6.20
	500	0.03	-0.10	10.58	10.55	13.75	6.10
	1,000	-0.24	-0.22	8.38	7.44	17.85	6.55
	3,000	-0.40	-0.12	6.67	4.26	33.75	6.90
$T = 240$	100	0.02	-0.44	13.81	15.39	8.65	7.40
	500	0.12	-0.14	6.62	6.70	10.25	6.35
	1,000	0.06	-0.15	4.86	4.72	10.85	7.45
	3,000	0.04	-0.03	3.26	2.62	16.95	6.45
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	1.63	29.05	36.11	1916.43	14.35	1.95
	500	3.76	1.48	27.36	303.36	36.10	7.25
	1,000	4.26	-1.84	26.21	39.83	48.45	8.20
	3,000	4.56	-0.62	26.15	22.78	69.25	11.05
$T = 120$	100	1.79	-3.12	24.84	50.47	9.30	4.50
	500	2.85	-0.87	16.01	19.66	18.60	5.60
	1,000	3.03	-0.57	14.77	15.08	28.80	6.90
	3,000	3.13	-0.68	14.02	10.07	49.45	8.40
$T = 240$	100	1.21	-1.21	17.34	22.52	8.30	6.75
	500	2.22	-0.15	9.58	10.83	11.00	5.95
	1,000	2.27	-0.24	8.46	8.56	18.20	6.15
	3,000	2.60	-0.08	7.42	5.55	33.25	7.10
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-18.49	110.60	50.00	7440.55	17.75	2.35
	500	-24.61	18.11	41.10	1183.88	44.00	6.75
	1,000	-26.08	6.93	40.81	118.48	58.05	8.75
	3,000	-28.71	2.78	43.19	66.65	75.45	11.30
$T = 120$	100	-12.19	13.59	37.47	192.49	10.25	4.45
	500	-17.28	2.13	28.78	40.29	30.50	8.05
	1,000	-19.06	1.36	27.98	31.28	44.05	8.90
	3,000	-22.25	1.02	30.06	23.69	66.95	8.90
$T = 240$	100	-8.15	1.74	28.35	38.76	8.85	5.65
	500	-11.28	0.68	19.74	22.51	19.25	7.65
	1,000	-12.72	0.82	18.22	17.60	31.20	7.15
	3,000	-15.99	0.51	19.76	13.26	62.25	8.00

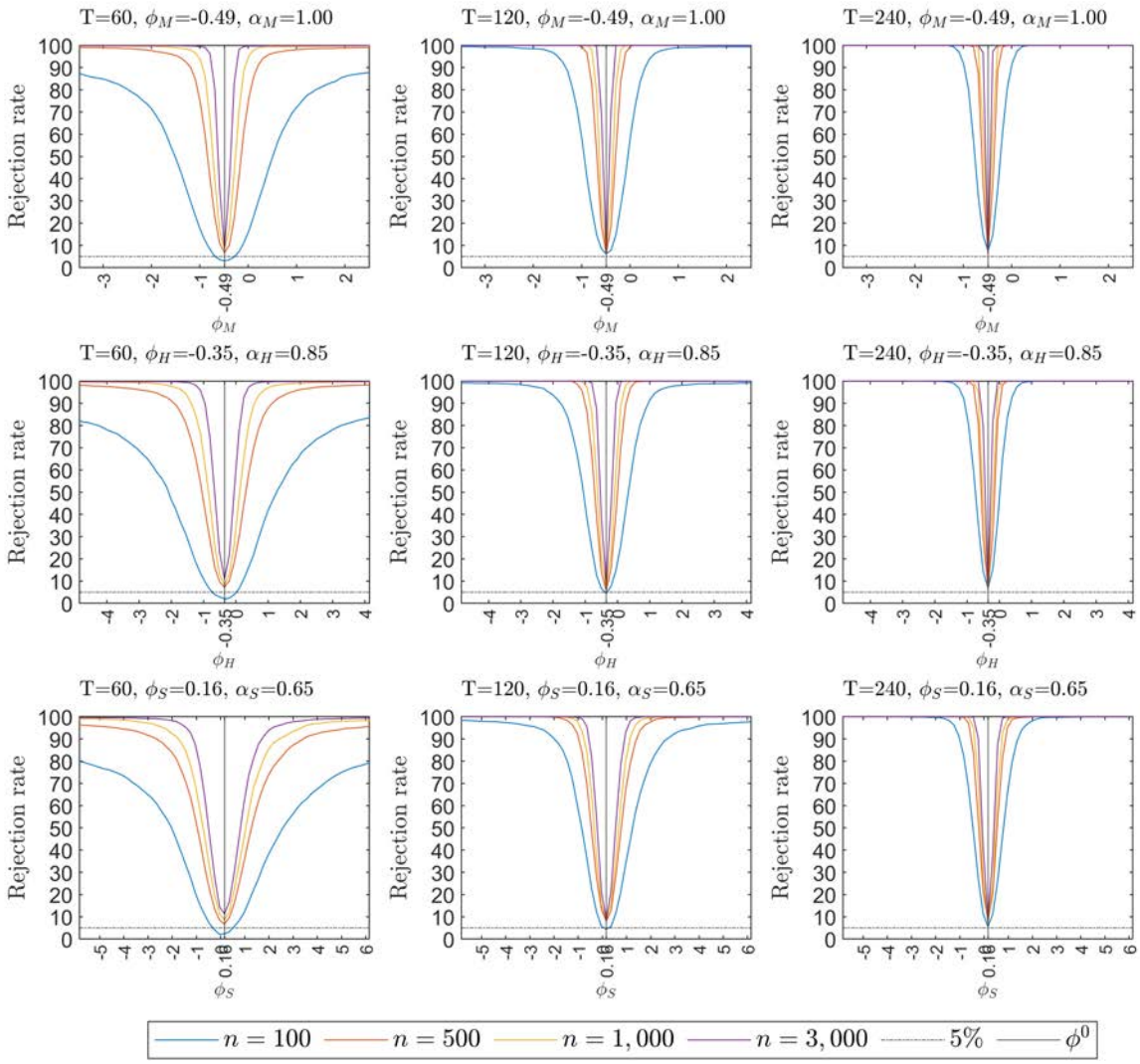
Notes: The DGP for Experiment 9 allows for Gaussian errors, with GARCH effects, with pricing errors ($\alpha_\eta = 0.5$), with one weak missing factor ($\alpha_\gamma = 0.5$), and with spatial error cross dependence ($\rho_\varepsilon = 0.85$). For further details of the experiments, see Table S-1.

Figure S-A-E9: Empirical Power Functions, experiment 9 for coefficient of the semi-strong factors



Note: See the notes to Table S-A-E9.

Figure S-A-E9a: Empirical Power Functions, experiment 9a for coefficient of the semi-strong factors



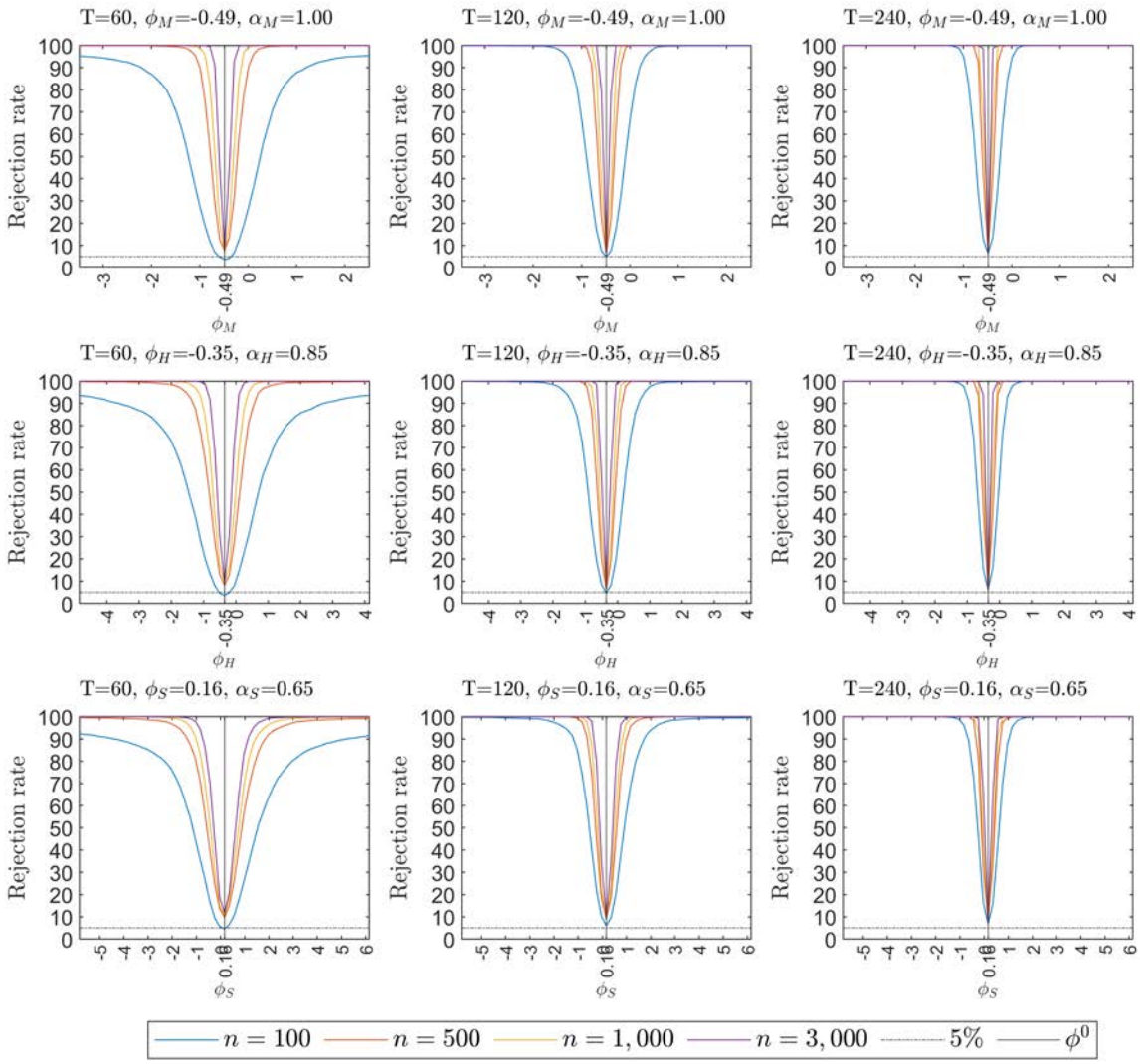
Note: See the notes to Table S-A-E9a.

Table S-A-E10: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ, for Experiment 10 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	-0.39	-1.61	27.55	151.91	11.70	3.75
	500	-0.82	-0.69	18.11	17.07	30.40	7.70
	1,000	-0.73	-0.31	16.33	11.30	42.50	7.05
	3,000	-0.92	-0.06	14.97	6.36	60.05	8.00
$T = 120$	100	0.02	-1.86	18.04	69.05	7.65	4.80
	500	-0.07	-0.09	9.78	9.29	15.05	6.10
	1,000	0.01	0.15	7.90	6.62	19.70	6.75
	3,000	-0.29	0.05	6.42	3.72	37.55	5.80
$T = 240$	100	0.01	-0.39	12.87	14.17	6.30	6.35
	500	0.17	-0.05	6.13	6.15	9.25	6.05
	1,000	0.17	0.00	4.55	4.30	11.30	6.40
	3,000	0.06	0.00	3.12	2.40	17.65	5.75
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	3.25	-8.09	31.16	504.30	19.35	3.65
	500	4.29	-1.32	25.64	33.61	46.05	8.10
	1,000	4.50	-0.68	25.36	22.95	58.25	9.70
	3,000	4.57	-0.37	25.89	14.12	76.05	9.50
$T = 120$	100	2.48	-1.13	21.16	32.42	11.15	4.50
	500	3.15	-0.09	14.31	14.78	25.55	5.70
	1,000	3.25	-0.07	13.66	11.19	38.65	5.55
	3,000	3.31	-0.26	13.70	7.36	60.50	6.70
$T = 240$	100	1.78	-0.40	14.58	18.18	8.25	6.05
	500	2.35	0.08	8.31	8.60	13.35	5.20
	1,000	2.53	0.12	7.51	6.58	21.05	5.00
	3,000	2.57	-0.10	6.96	4.28	41.35	6.00
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-19.35	25.35	40.86	579.34	25.25	4.60
	500	-24.56	2.56	38.40	68.65	57.50	9.85
	1,000	-26.17	1.78	39.59	43.66	69.65	10.70
	3,000	-28.53	1.55	42.55	32.38	81.95	11.90
$T = 120$	100	-12.71	10.19	29.68	304.30	14.70	6.05
	500	-17.42	0.86	25.76	26.78	43.95	8.75
	1,000	-19.30	0.15	26.41	20.74	59.65	8.50
	3,000	-22.31	0.47	29.35	16.12	77.35	8.95
$T = 240$	100	-8.09	1.05	21.11	26.50	10.55	6.65
	500	-11.52	0.08	16.67	15.20	30.35	7.15
	1,000	-12.98	0.22	16.56	11.84	47.50	6.80
	3,000	-16.05	0.32	19.11	9.25	75.50	8.00

Notes: The DGP for Experiment 10 allows for t(5) distributed errors, with GARCH effects, with pricing errors ($\alpha_\eta = 0.3$), with one weak missing factor ($\alpha_\gamma = 0.5$), and with spatial error cross dependence ($\rho_\varepsilon = 0.5$). For further details of the experiments, see Table S-1.

Figure S-A-E10: Empirical Power Functions, experiment 10 for coefficient of the semi-strong factors



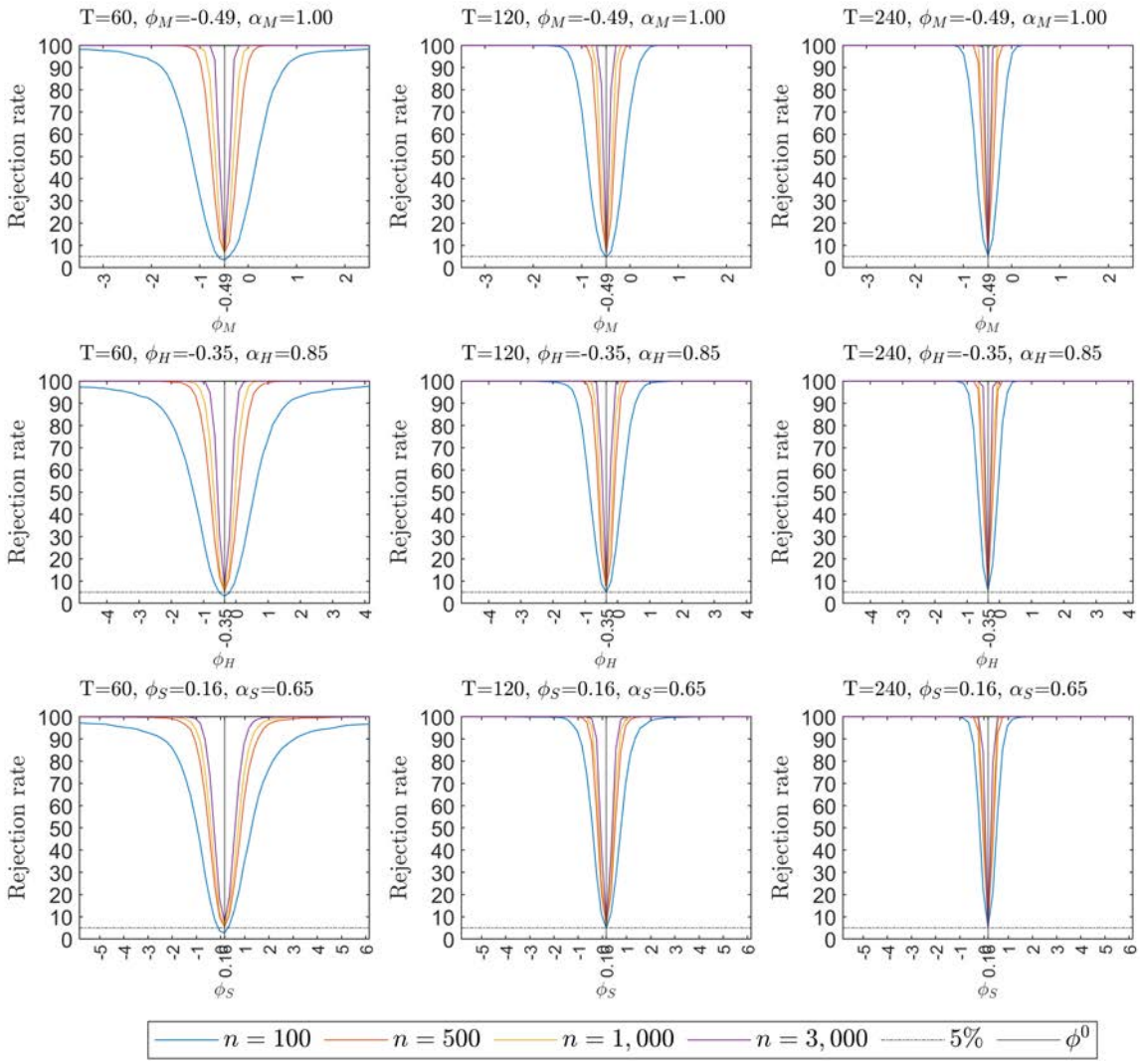
Note: See the notes to Table S-A-E10.

Table S-A-E11: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ, for Experiment 11 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	-0.05	-0.72	26.31	47.59	11.30	3.40
	500	-0.33	-0.07	17.63	15.12	29.40	6.70
	1,000	-0.71	-0.22	16.06	10.46	41.95	6.35
	3,000	-0.85	0.05	14.87	5.91	60.85	7.00
$T = 120$	100	0.10	-0.46	17.08	19.81	7.15	4.55
	500	0.13	0.10	9.51	8.93	13.90	6.40
	1,000	-0.11	-0.01	7.80	6.28	20.65	6.15
	3,000	-0.21	0.09	6.41	3.60	37.25	6.20
$T = 240$	100	0.20	-0.24	12.33	13.43	6.15	5.30
	500	0.24	0.01	6.12	6.11	10.05	6.75
	1,000	0.13	-0.07	4.52	4.19	11.75	5.85
	3,000	0.11	0.02	3.05	2.32	18.35	4.75
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	2.50	-2.25	29.66	103.01	18.75	3.35
	500	3.74	-0.07	24.90	25.61	45.55	5.45
	1,000	3.60	-0.52	24.93	18.83	59.20	6.05
	3,000	3.98	0.01	25.84	12.59	75.45	7.90
$T = 120$	100	1.81	-0.44	19.83	27.88	10.00	4.95
	500	2.49	0.09	13.69	13.12	26.00	6.15
	1,000	2.45	0.01	13.32	10.12	40.65	6.00
	3,000	2.63	0.01	13.61	6.86	62.05	7.20
$T = 240$	100	0.97	-0.57	13.36	16.38	6.90	5.55
	500	1.60	-0.05	7.76	8.01	13.50	5.40
	1,000	1.72	0.01	6.95	5.93	20.35	5.20
	3,000	1.85	-0.07	6.58	3.89	41.35	5.95
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-20.60	-0.57	38.15	212.41	27.80	2.75
	500	-23.77	2.07	36.92	42.46	58.25	5.55
	1,000	-25.88	1.00	39.00	34.14	69.55	6.15
	3,000	-28.31	0.81	41.99	28.50	82.20	7.90
$T = 120$	100	-12.85	2.34	26.13	35.00	17.60	4.80
	500	-16.99	0.79	23.96	20.38	47.50	5.65
	1,000	-19.15	0.29	25.77	16.81	64.20	5.85
	3,000	-21.97	0.81	28.73	13.50	80.80	7.00
$T = 240$	100	-7.60	1.18	17.18	20.04	10.50	5.15
	500	-10.88	0.56	14.80	11.65	35.40	5.55
	1,000	-12.61	0.61	15.89	9.46	55.65	5.80
	3,000	-15.72	0.52	18.57	7.12	80.50	5.35

Notes: The DGP for Experiment 11 allows for Gaussian errors, with GARCH effects, with pricing errors ($\alpha_\eta = 0.3$), with one weak missing factor ($\alpha_\gamma = 0.5$), and with block error cross dependence. For further details of the experiments, see Table S-1.

Figure S-A-E11: Empirical Power Functions, experiment 11 for coefficient of the semi-strong factors



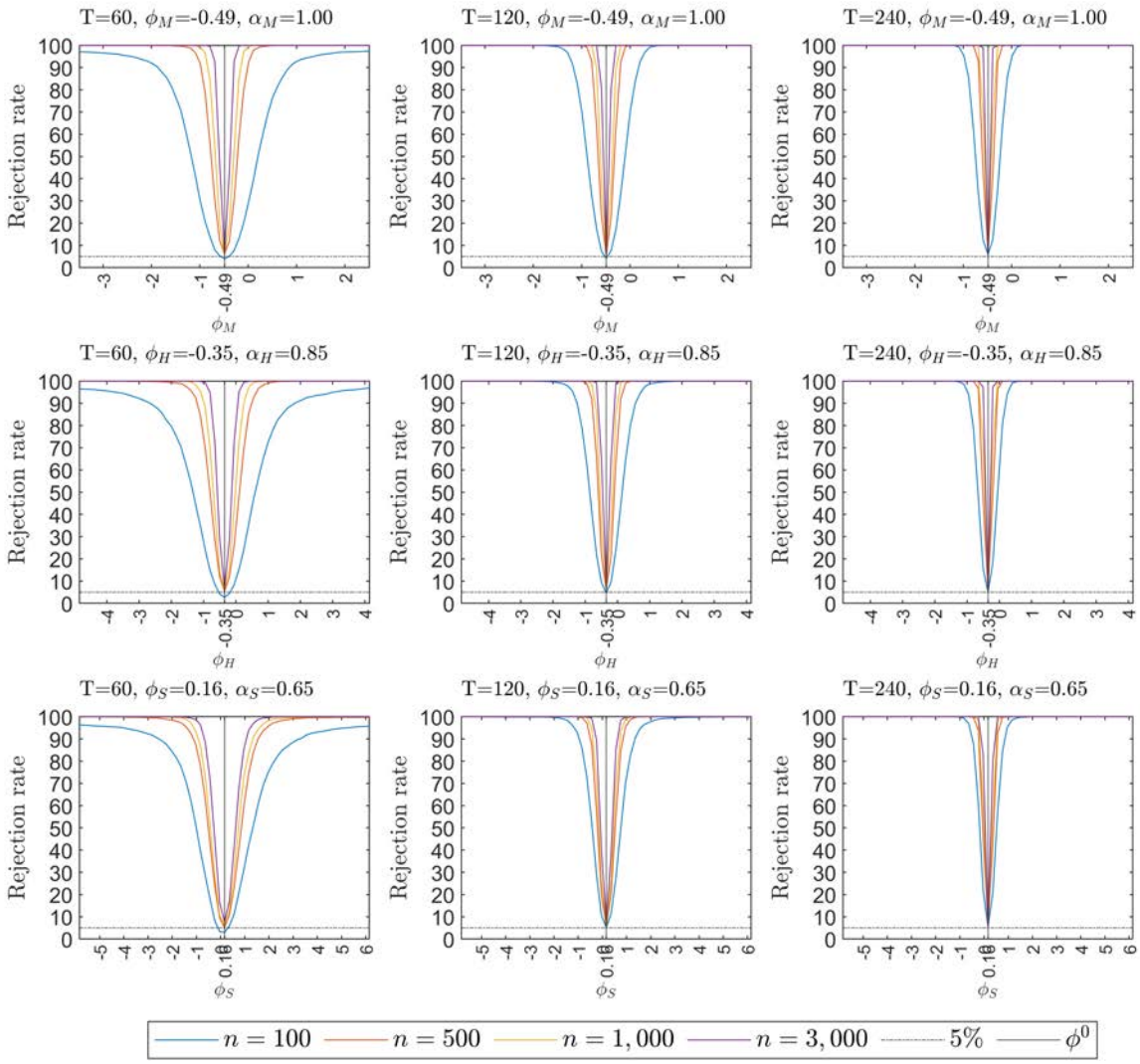
Note: See the notes to Table S-A-E11.

Table S-A-E12: Bias, RMSE and size for the two-step and bias-corrected (BC) estimators of ϕ, for Experiment 12 with one strong and two semi-strong factors

		Bias(x100)		RMSE(x100)		Size(x100)	
ϕ_M=−0.49, α_M=1	n	Two Step	BC	Two Step	BC	Two Step	BC
$T = 60$	100	0.09	-0.14	27.00	129.71	11.65	4.10
	500	-0.57	-0.25	17.72	15.76	29.85	6.10
	1,000	-0.60	-0.04	16.17	10.48	41.65	5.65
	3,000	-0.80	0.10	14.89	6.07	60.25	6.75
$T = 120$	100	0.22	-0.35	17.19	20.06	6.90	4.15
	500	0.07	0.06	9.47	8.91	13.70	5.40
	1,000	0.04	0.16	7.81	6.35	20.50	6.00
	3,000	-0.18	0.14	6.40	3.62	37.75	5.60
$T = 240$	100	0.30	-0.13	12.34	13.45	6.50	5.90
	500	0.30	0.07	6.02	6.01	8.90	6.30
	1,000	0.17	-0.02	4.50	4.21	11.40	5.80
	3,000	0.10	0.02	3.09	2.35	18.15	5.05
ϕ_H=−0.35, α_H=0.85							
$T = 60$	100	2.62	9.84	29.60	432.72	18.55	2.75
	500	3.90	0.37	25.08	28.76	45.25	5.35
	1,000	3.57	-0.62	24.92	19.22	58.20	5.95
	3,000	3.94	-0.10	25.92	12.64	74.50	6.90
$T = 120$	100	1.82	-0.38	19.87	27.99	9.25	4.65
	500	2.59	0.31	13.82	13.27	25.70	5.85
	1,000	2.45	0.02	13.31	10.26	39.70	5.95
	3,000	2.70	0.11	13.61	6.85	61.20	7.15
$T = 240$	100	0.92	-0.65	13.61	16.79	7.15	4.90
	500	1.65	0.02	7.79	8.06	12.50	5.05
	1,000	1.77	0.07	7.03	6.03	21.70	5.55
	3,000	1.87	-0.05	6.57	3.93	41.05	6.05
ϕ_S=0.16, α_S=0.65							
$T = 60$	100	-20.25	1.01	38.24	407.11	26.15	3.00
	500	-23.75	1.77	36.94	55.06	55.90	5.30
	1,000	-26.03	0.80	39.26	35.51	69.75	6.50
	3,000	-28.18	1.33	41.98	27.55	81.60	7.85
$T = 120$	100	-13.05	2.21	26.45	36.34	18.15	5.05
	500	-16.97	0.78	24.00	20.16	47.40	5.40
	1,000	-19.18	0.24	25.77	17.03	64.35	6.10
	3,000	-21.96	0.84	28.73	13.65	80.55	7.20
$T = 240$	100	-7.73	1.03	17.33	20.17	11.10	5.40
	500	-10.89	0.50	14.85	11.67	34.35	5.40
	1,000	-12.73	0.44	15.92	9.49	54.85	5.60
	3,000	-15.75	0.49	18.58	7.25	80.80	5.65

Notes: The DGP for Experiment 12 allows for t(5) distributed errors, with GARCH effects, with pricing errors ($\alpha_\eta = 0.3$), with one weak missing factor ($\alpha_\gamma = 0.5$), and with block error cross dependence. For further details of the experiments, see S-1.

Figure S-A-E12: Empirical Power Functions, experiment 12 for coefficient of the semi-strong factors



Note: See the notes to Table S-A-E12.

2.1 Estimators of ϕ for one strong and two semi-strong factors, threshold estimator of the covariance matrix with and without misspecification

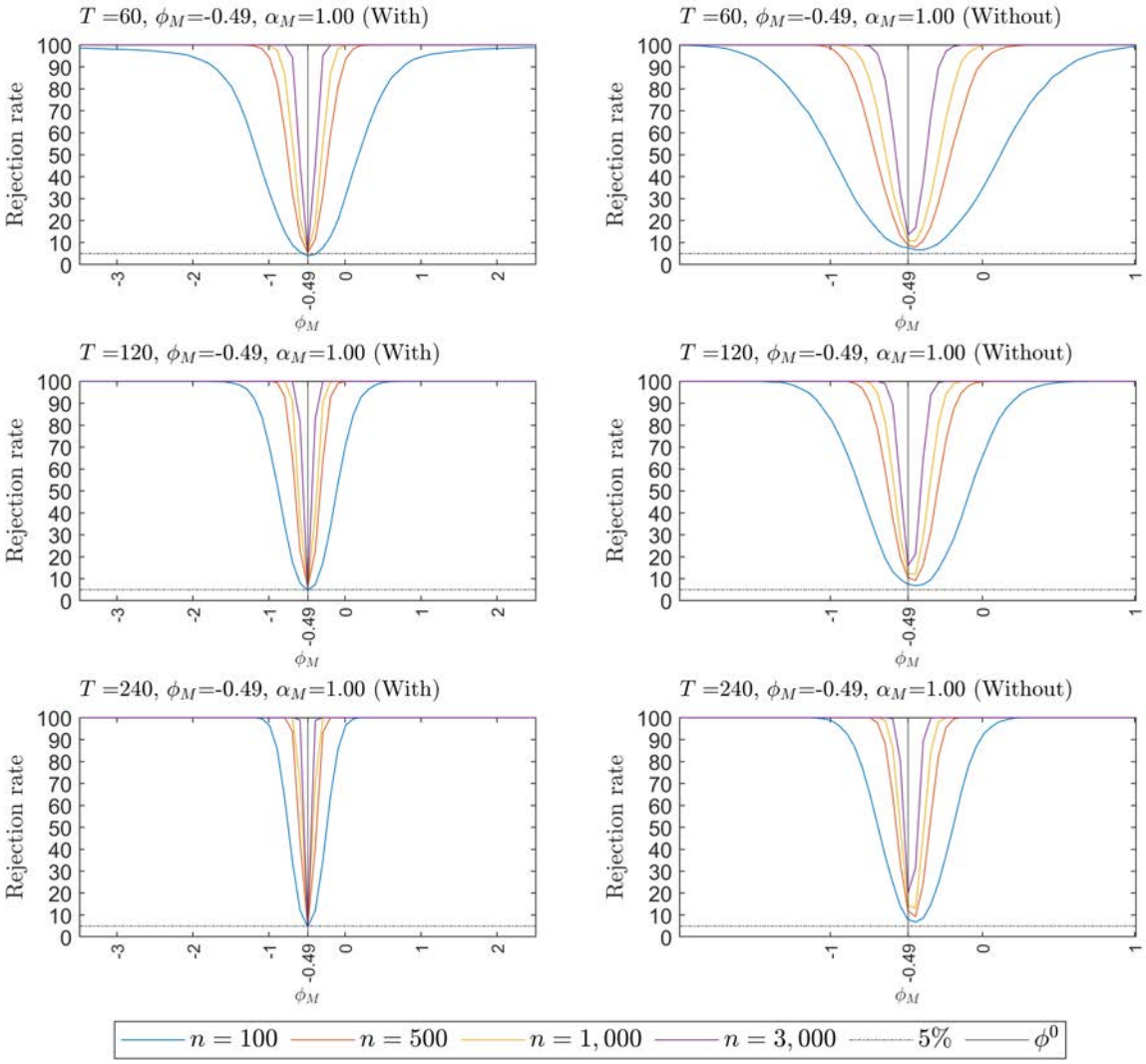
This subsection compares the estimators of ϕ for one strong and two semi-strong factors with and without misspecification. Under the misspecification, only strong estimators ϕ_M are considered in the estimation process. Each table contains the results of three of the 12 experiments described in Table S-1, and corresponding empirical power functions follow.

Table S-B-E1-3: Bias, RMSE and size for the bias-corrected estimators of ϕ_M=−0.49, α_M=1 with and without semi-strong factors ($\alpha_H = 0.85, \alpha_S = 0.65$) included in the regression for the cases of experiments 1, 2 and 3

		Bias(x100)		RMSE(x100)		Size(x100)	
Experiment 1	n	With	Without	With	Without	With	Without
		semi-strong factors		semi-strong factors		semi-strong factors	
$T = 60$	100	-1.55	5.63	86.14	31.56	4.00	7.60
	500	0.28	3.72	14.33	14.60	5.40	8.80
	1,000	0.10	2.96	10.12	11.02	6.20	10.85
	3,000	-0.01	1.96	5.56	6.98	5.70	13.50
$T = 120$	100	1.84	4.96	91.35	20.49	4.70	7.50
	500	0.15	3.36	8.99	9.93	6.15	10.50
	1,000	0.01	2.72	6.22	7.40	5.65	12.15
	3,000	-0.06	1.88	3.49	4.71	4.95	15.80
$T = 240$	100	-0.15	5.00	13.11	14.46	4.55	7.90
	500	0.09	3.33	5.75	7.03	4.85	11.90
	1,000	0.03	2.73	4.06	5.33	5.30	14.40
	3,000	-0.00	1.95	2.35	3.52	5.10	19.95
Experiment 2							
$T = 60$	100	0.27	6.04	45.08	32.20	3.70	7.25
	500	0.27	3.69	14.47	14.60	5.75	7.75
	1,000	0.04	2.85	10.24	11.07	6.10	11.10
	3,000	-0.05	1.94	5.72	7.03	5.90	14.45
$T = 120$	100	0.02	5.24	20.45	20.58	4.90	8.30
	500	0.23	3.46	9.16	10.04	6.60	10.35
	1,000	0.03	2.73	6.12	7.37	5.55	11.35
	3,000	-0.09	1.83	3.52	4.70	5.65	16.10
$T = 240$	100	-0.14	5.08	13.18	14.44	5.05	8.75
	500	0.11	3.39	5.81	7.06	5.05	11.65
	1,000	0.04	2.74	4.07	5.35	5.55	15.50
	3,000	-0.03	1.92	2.36	3.51	5.25	20.55
Experiment 3							
$T = 60$	100	-6.00	5.70	227.89	31.96	3.40	7.55
	500	0.29	3.68	14.71	14.81	5.65	9.10
	1,000	0.09	2.94	10.33	11.25	6.35	11.55
	3,000	0.00	1.96	5.65	7.18	5.65	14.55
$T = 120$	100	-0.19	4.89	20.63	20.51	4.50	7.45
	500	0.15	3.32	9.05	9.98	6.25	10.65
	1,000	0.02	2.68	6.25	7.46	5.75	12.15
	3,000	-0.05	1.85	3.50	4.75	4.90	15.20
$T = 240$	100	-0.13	4.96	13.17	14.48	4.45	7.90
	500	0.10	3.32	5.78	7.05	5.20	11.85
	1,000	0.05	2.72	4.07	5.34	5.25	14.10
	3,000	0.00	1.93	2.36	3.53	5.20	19.90

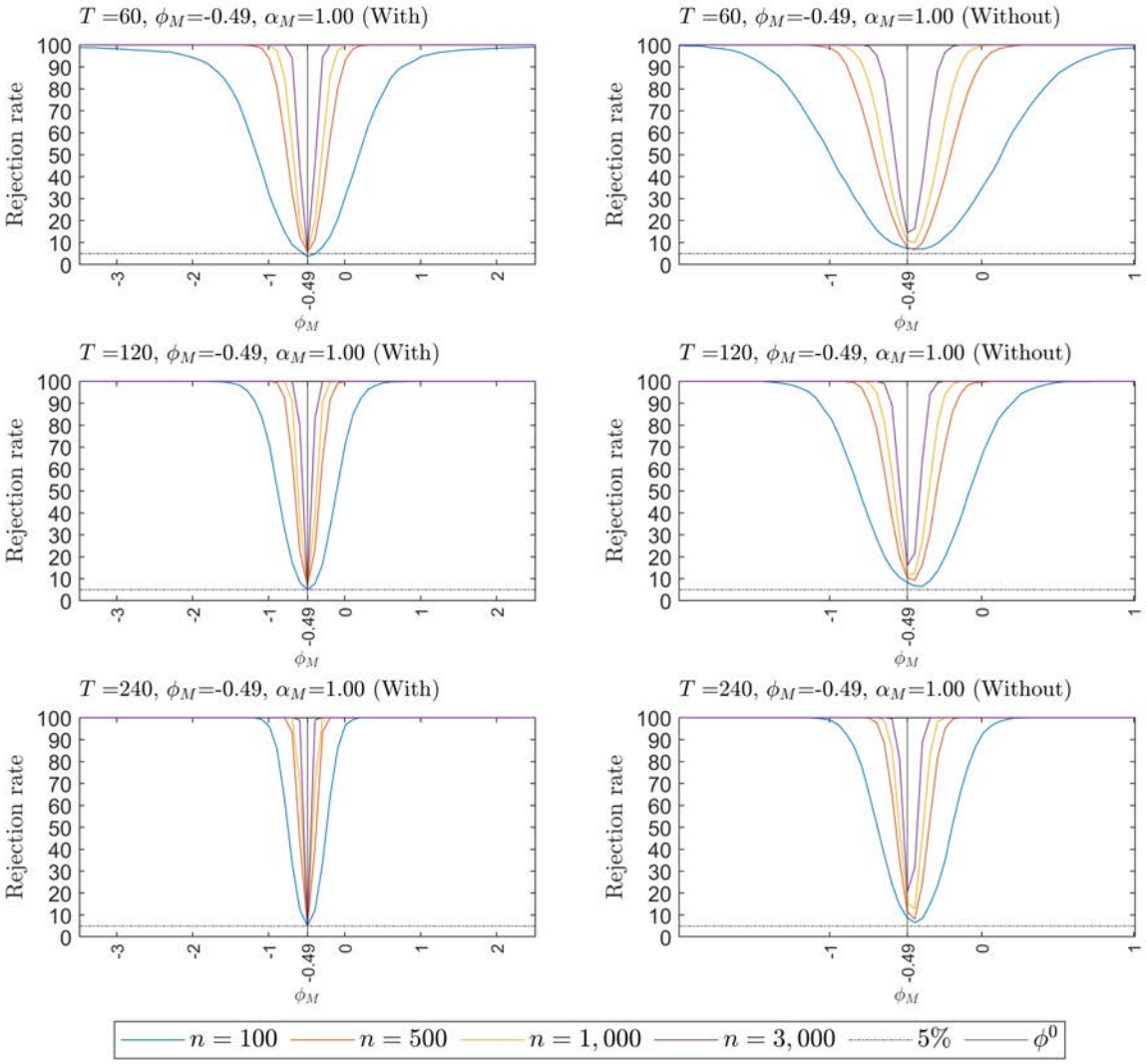
Notes: The DGP includes one strong $\alpha_M = 1$ and two semi-strong ($\alpha_H = 0.85$, $\alpha_S = 0.65$) factors, the regression with the two semi-strong factors includes them, the regression without excludes them. For further details of the experiments, see S-1.

Figure S-B-E1: Empirical Power Functions, experiment 1, for coefficient of the ϕ_M factor with and without misspecification



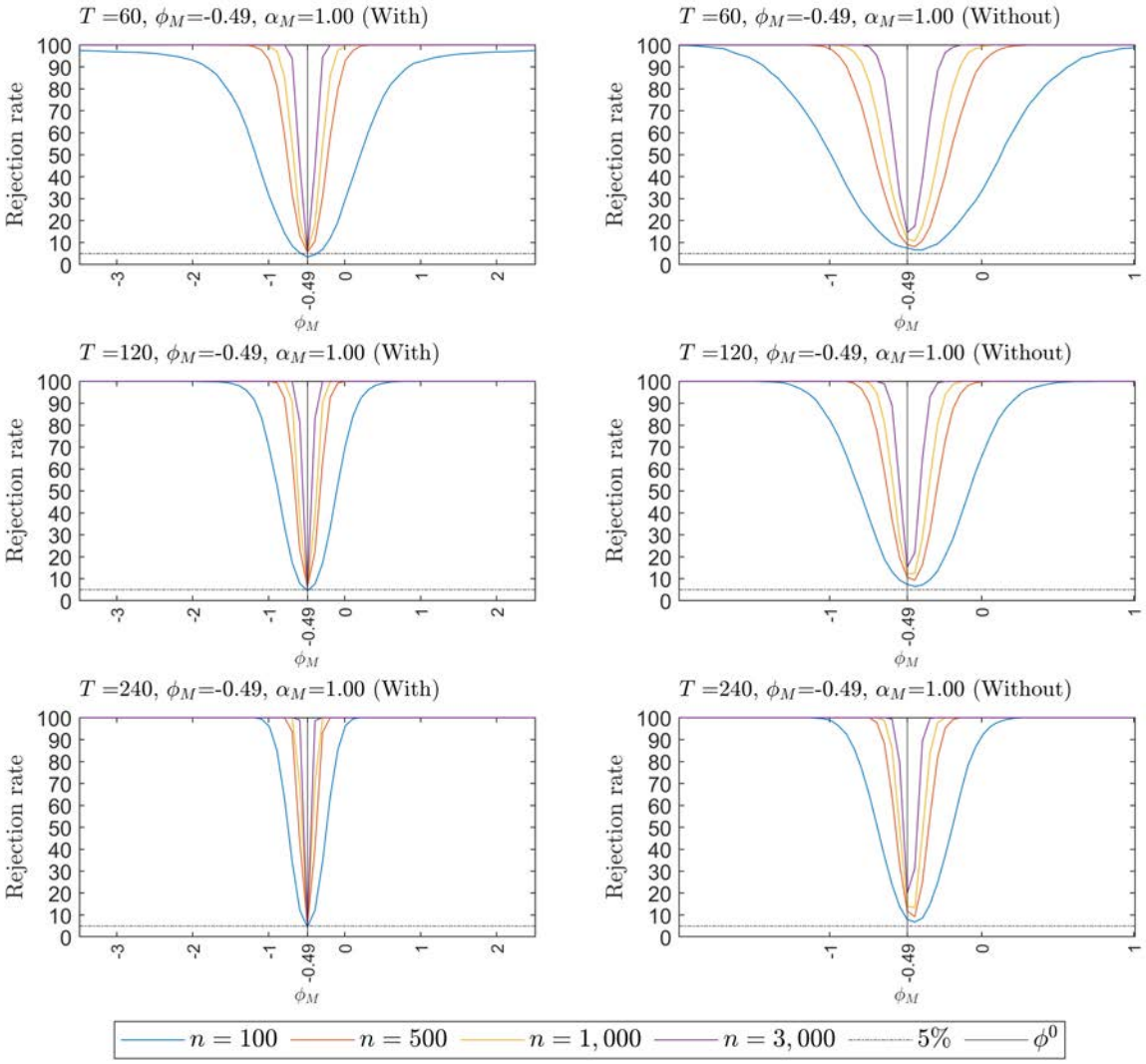
Note: See the notes to Table S-B-E1-3.

Figure S-B-E2: Empirical Power Functions, experiment 2, for coefficient of the ϕ_M factor with and without misspecification



Note: See the notes to Table S-B-E1-3.

Figure S-B-E3: Empirical Power Functions, experiment 3, for coefficient of the ϕ_M factor with and without misspecification



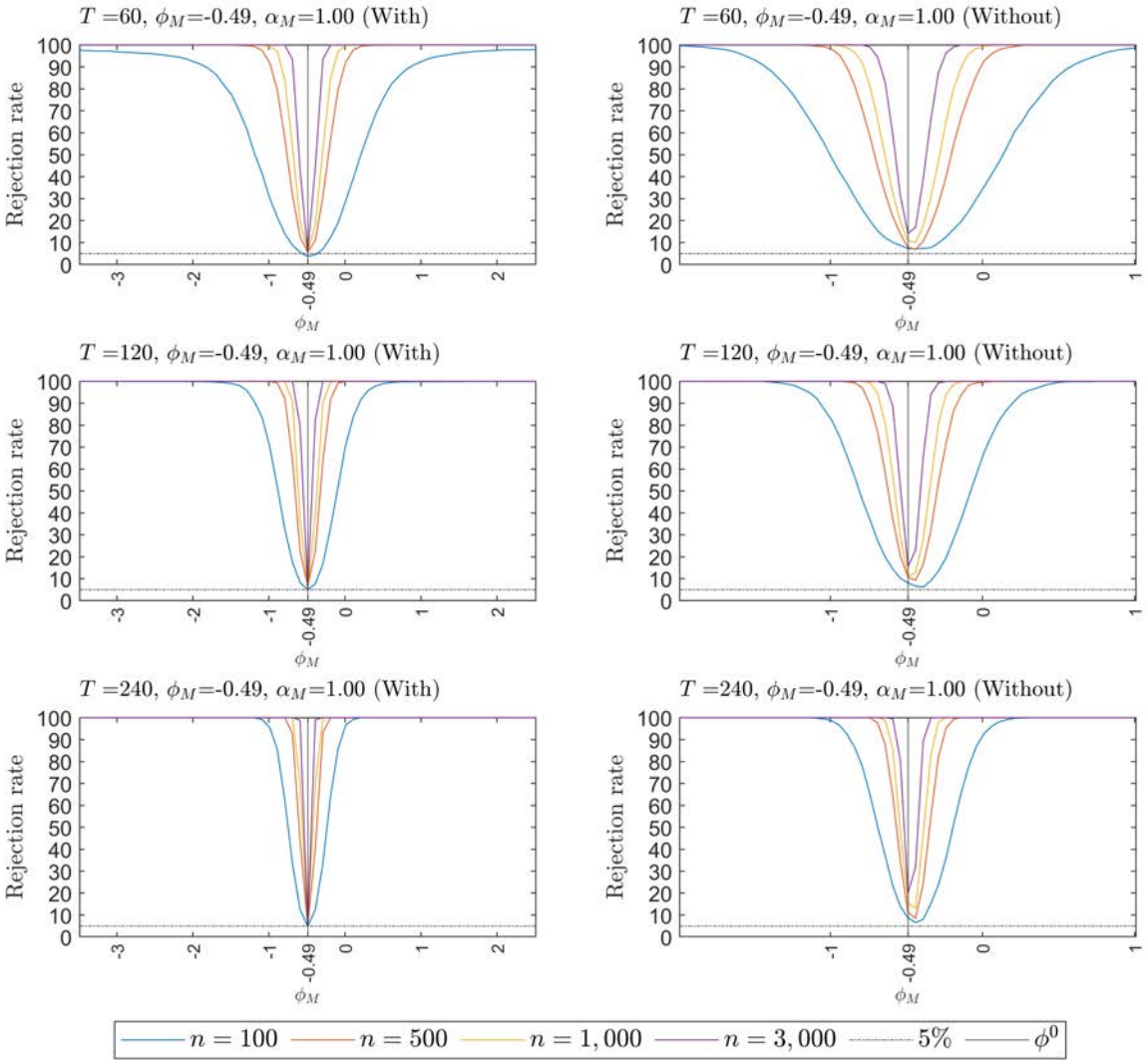
Note: See the notes to Table S-B-E1-3.

Table S-B-E4-6: Bias, RMSE and size for the bias-corrected estimators of $\phi_M = -0.49$, $\alpha_M = 1$ with and without semi-strong factors ($\alpha_H = 0.85, \alpha_S = 0.65$) included in the regression for the cases of experiments 4, 5 and 6

		Bias(x100)		RMSE(x100)		Size(x100)	
Experiment 4	n	With	Without	With	Without	With	Without
		semi-strong factors		semi-strong factors		semi-strong factors	
$T = 60$	100	0.17	6.08	107.83	32.68	3.75	7.25
	500	0.23	3.64	14.75	14.88	5.65	7.85
	1,000	0.05	2.84	10.43	11.29	6.10	10.85
	3,000	-0.04	1.94	5.82	7.21	5.95	14.15
$T = 120$	100	-0.86	5.15	44.37	20.62	4.95	8.10
	500	0.21	3.42	9.24	10.11	6.45	10.75
	1,000	0.03	2.69	6.16	7.41	4.90	11.05
	3,000	-0.08	1.81	3.53	4.75	5.85	15.55
$T = 240$	100	-0.13	5.03	13.22	14.46	4.85	8.70
	500	0.12	3.37	5.83	7.07	5.10	11.15
	1,000	0.05	2.72	4.08	5.36	5.60	15.50
	3,000	-0.02	1.90	2.36	3.52	5.25	20.00
Experiment 5							
$T = 60$	100	-7.58	5.70	285.21	31.96	3.75	7.55
	500	0.28	3.68	14.79	14.86	5.80	9.05
	1,000	0.10	2.92	10.35	11.25	6.25	11.80
	3,000	-0.00	1.97	5.65	7.19	5.75	14.70
$T = 120$	100	-0.35	4.89	20.82	20.51	4.95	7.45
	500	0.14	3.32	9.15	10.03	6.45	10.30
	1,000	0.04	2.67	6.28	7.48	5.90	12.20
	3,000	-0.06	1.86	3.50	4.76	4.80	15.45
$T = 240$	100	-0.31	4.96	13.53	14.48	5.35	7.90
	500	0.10	3.31	5.84	7.08	5.35	11.50
	1,000	0.07	2.71	4.11	5.36	5.20	14.10
	3,000	0.00	1.94	2.36	3.53	5.25	20.30
Experiment 6							
$T = 60$	100	0.05	6.08	103.65	32.68	3.95	7.25
	500	0.24	3.63	14.81	14.93	5.95	8.25
	1,000	0.06	2.82	10.44	11.30	6.00	11.00
	3,000	-0.05	1.95	5.82	7.22	5.80	14.40
$T = 120$	100	-0.76	5.15	33.47	20.62	5.15	8.10
	500	0.22	3.42	9.32	10.16	6.95	10.65
	1,000	0.05	2.68	6.18	7.43	5.30	11.25
	3,000	-0.09	1.82	3.54	4.75	5.70	15.40
$T = 240$	100	-0.30	5.03	13.63	14.46	6.10	8.70
	500	0.11	3.37	5.88	7.10	5.10	11.85
	1,000	0.07	2.71	4.13	5.38	5.80	15.50
	3,000	-0.03	1.91	2.37	3.52	5.45	20.40

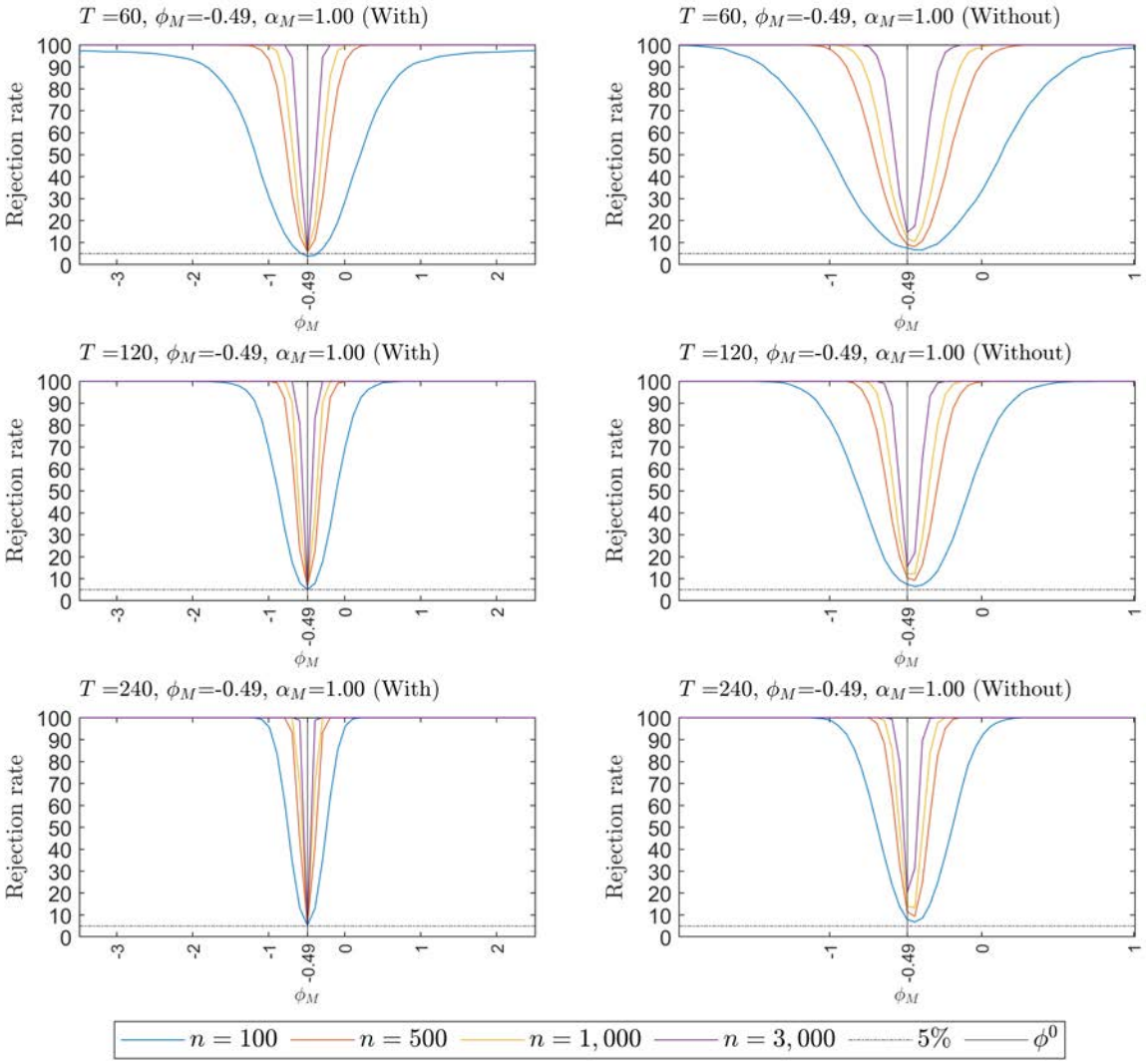
Notes: The DGP includes one strong $\alpha_M = 1$ and two semi-strong ($\alpha_H = 0.85$, $\alpha_S = 0.65$) factors, the regression with the two semi-strong factors includes them, the regression without excludes them. For further details of the experiments, see S-1.

Figure S-B-E4: Empirical Power Functions, experiment 4, for coefficient of the ϕ_M factor with and without misspecification



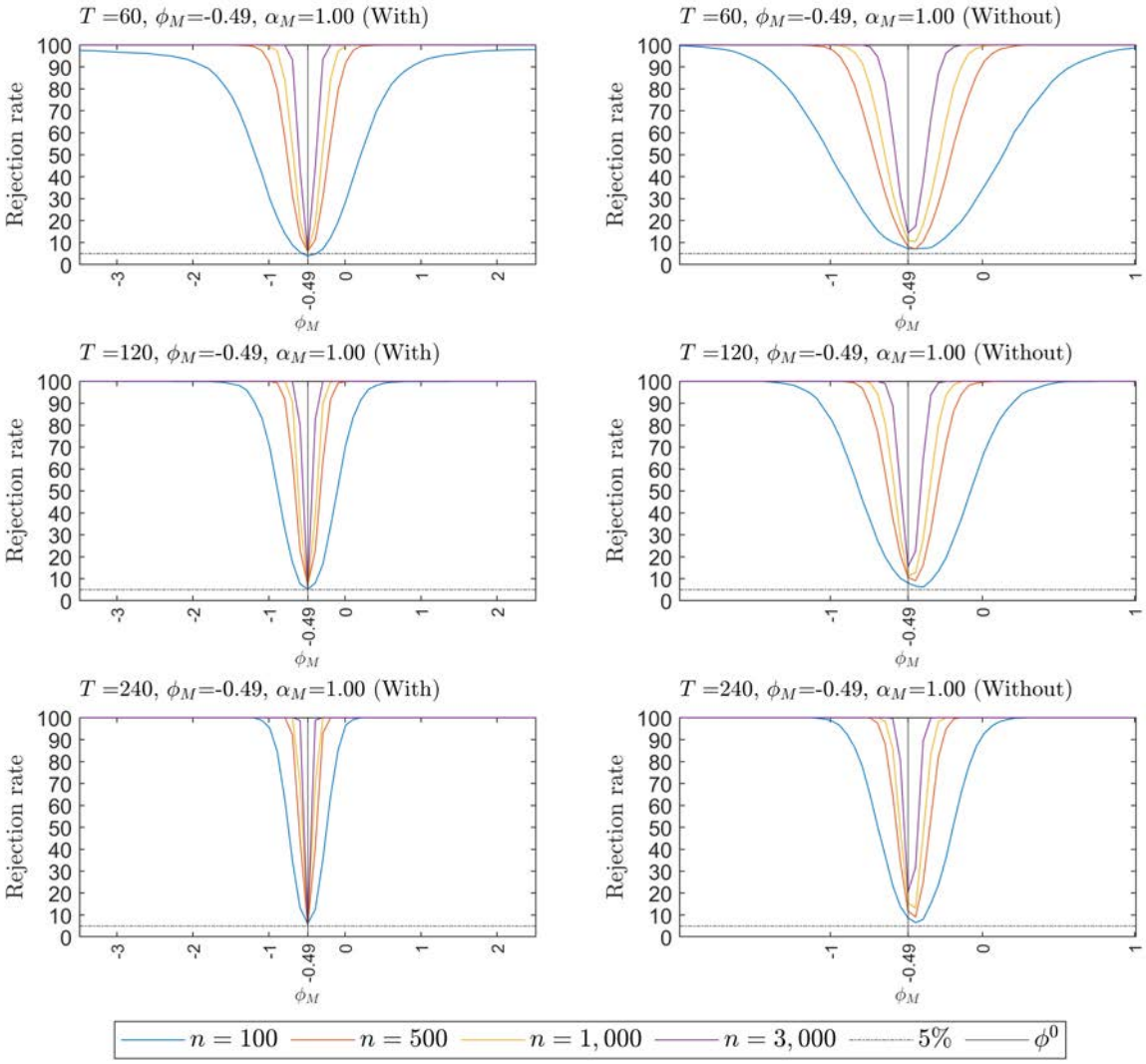
Note: See the notes to Table S-B-E4-6.

Figure S-B-E5: Empirical Power Functions, experiment 5, for coefficient of the ϕ_M factor with and without misspecification



Note: See the notes to Table S-B-E4-6.

Figure S-B-E6: Empirical Power Functions, experiment 6, for coefficient of the ϕ_M factor with and without misspecification



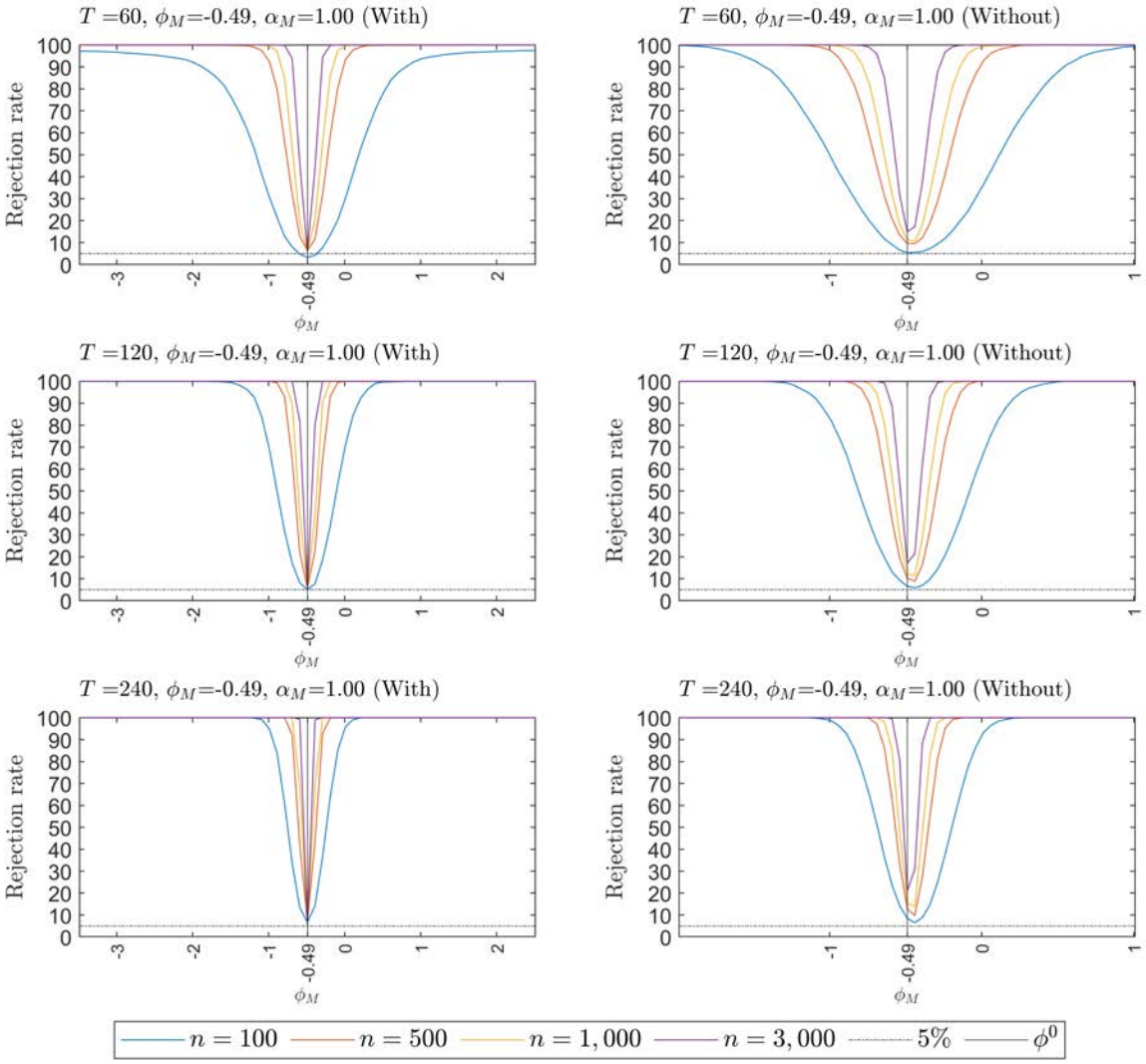
Note: See the notes to Table S-B-E4-6.

Table S-B-E7-9: Bias, RMSE and size for the bias-corrected estimators of $\phi_M = -0.49$, $\alpha_M = 1$ with and without semi-strong factors ($\alpha_H = 0.85, \alpha_S = 0.65$) included in the regression for the cases of experiments 7, 8 and 9

		Bias(x100)		RMSE(x100)		Size(x100)	
Experiment 7	n	With	Without	With	Without	With	Without
		semi-strong factors		semi-strong factors		semi-strong factors	
$T = 60$	100	-14.31	4.76	555.11	31.07	3.20	5.35
	500	-0.18	3.25	15.13	15.20	6.40	9.65
	1,000	-0.15	2.62	10.41	11.29	6.25	11.35
	3,000	0.04	1.99	5.81	7.17	5.90	14.95
$T = 120$	100	-0.49	4.70	20.30	19.98	5.05	6.55
	500	0.01	3.15	8.86	9.80	5.85	10.15
	1,000	0.04	2.62	6.31	7.45	6.60	11.85
	3,000	0.08	1.99	3.60	4.80	5.65	17.05
$T = 240$	100	-0.39	4.91	14.02	14.48	6.75	8.50
	500	-0.08	3.13	6.13	7.18	6.95	12.85
	1,000	-0.03	2.67	4.22	5.45	5.80	15.15
	3,000	0.01	1.97	2.32	3.50	5.15	20.90
Experiment 8							
$T = 60$	100	9.06	4.80	251.34	31.95	3.75	6.85
	500	-0.67	2.88	16.71	15.13	6.30	9.95
	1,000	0.01	2.75	10.38	11.32	5.70	10.25
	3,000	0.12	2.09	5.96	7.25	6.15	14.70
$T = 120$	100	-0.32	4.83	20.53	20.18	4.75	6.25
	500	-0.04	3.05	8.89	9.73	5.80	10.20
	1,000	0.19	2.76	6.40	7.60	6.10	12.70
	3,000	0.14	2.01	3.63	4.81	5.90	17.05
$T = 240$	100	-0.29	5.00	14.07	14.58	6.35	9.40
	500	-0.01	3.19	6.06	7.15	6.05	12.05
	1,000	0.01	2.68	4.25	5.46	6.30	15.90
	3,000	0.01	1.96	2.35	3.52	5.15	20.95
Experiment 9							
$T = 60$	100	-15.47	4.04	662.68	33.36	3.40	6.95
	500	1.29	2.84	84.39	15.74	7.75	11.40
	1,000	-0.47	2.26	11.31	11.50	8.50	12.25
	3,000	-0.16	1.69	6.25	7.20	7.75	15.35
$T = 120$	100	-0.59	4.09	21.49	21.20	4.50	7.00
	500	-0.01	2.95	9.20	9.95	6.15	9.15
	1,000	-0.07	2.40	6.53	7.37	6.30	10.80
	3,000	-0.01	1.74	3.71	4.68	6.50	14.30
$T = 240$	100	-0.54	4.06	14.02	14.60	6.85	8.50
	500	-0.12	2.85	6.19	7.11	6.55	10.80
	1,000	-0.08	2.42	4.30	5.27	6.60	13.35
	3,000	-0.00	1.75	2.37	3.37	5.45	17.50

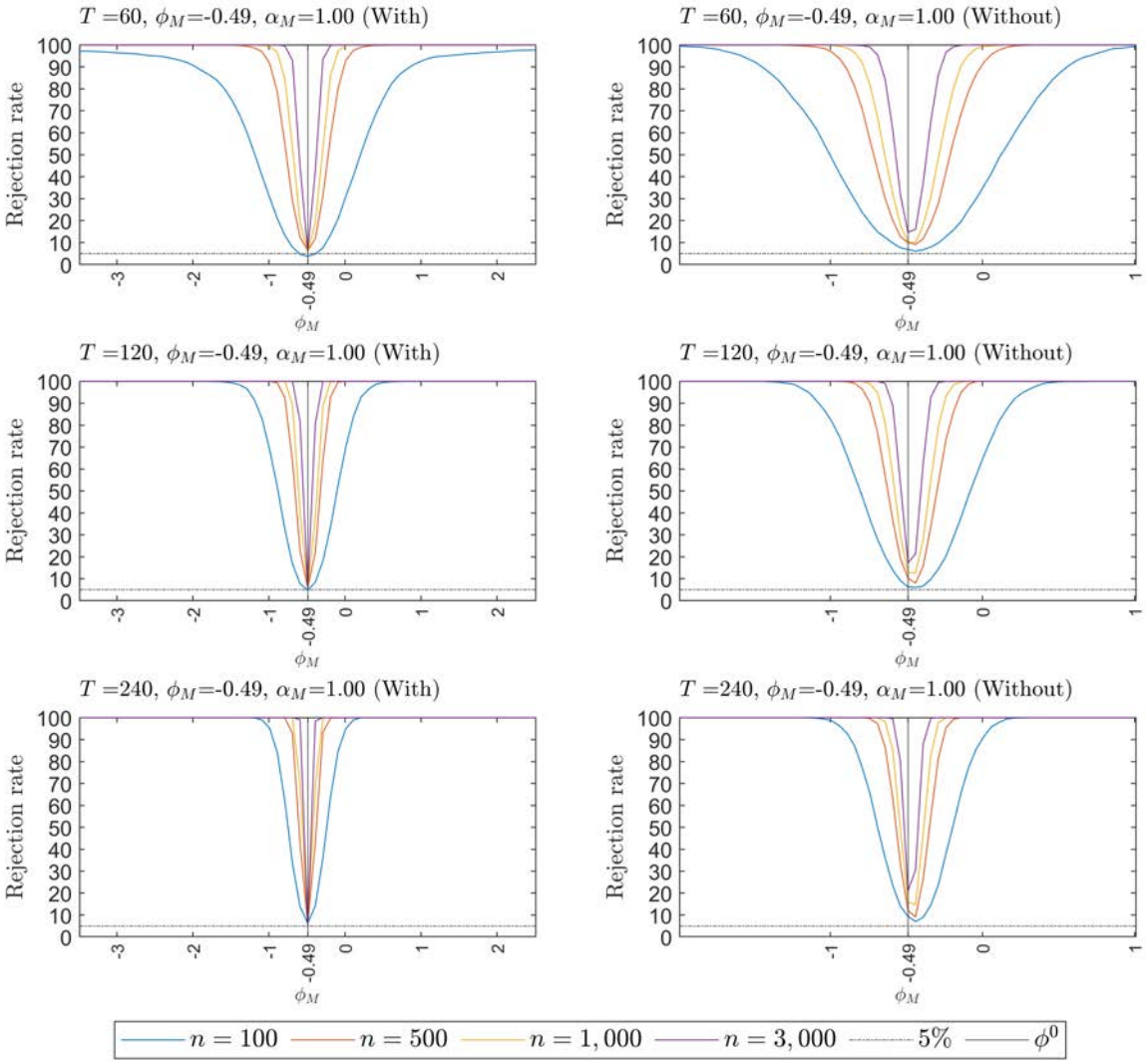
Notes: The DGP includes one strong $\alpha_M = 1$ and two semi-strong ($\alpha_H = 0.85$, $\alpha_S = 0.65$) factors, the regression with the two semi-strong factors includes them, the regression without excludes them. For further details of the experiments, see S-1.

Figure S-B-E7: Empirical Power Functions, experiment 7, for coefficient of the ϕ_M factor with and without misspecification



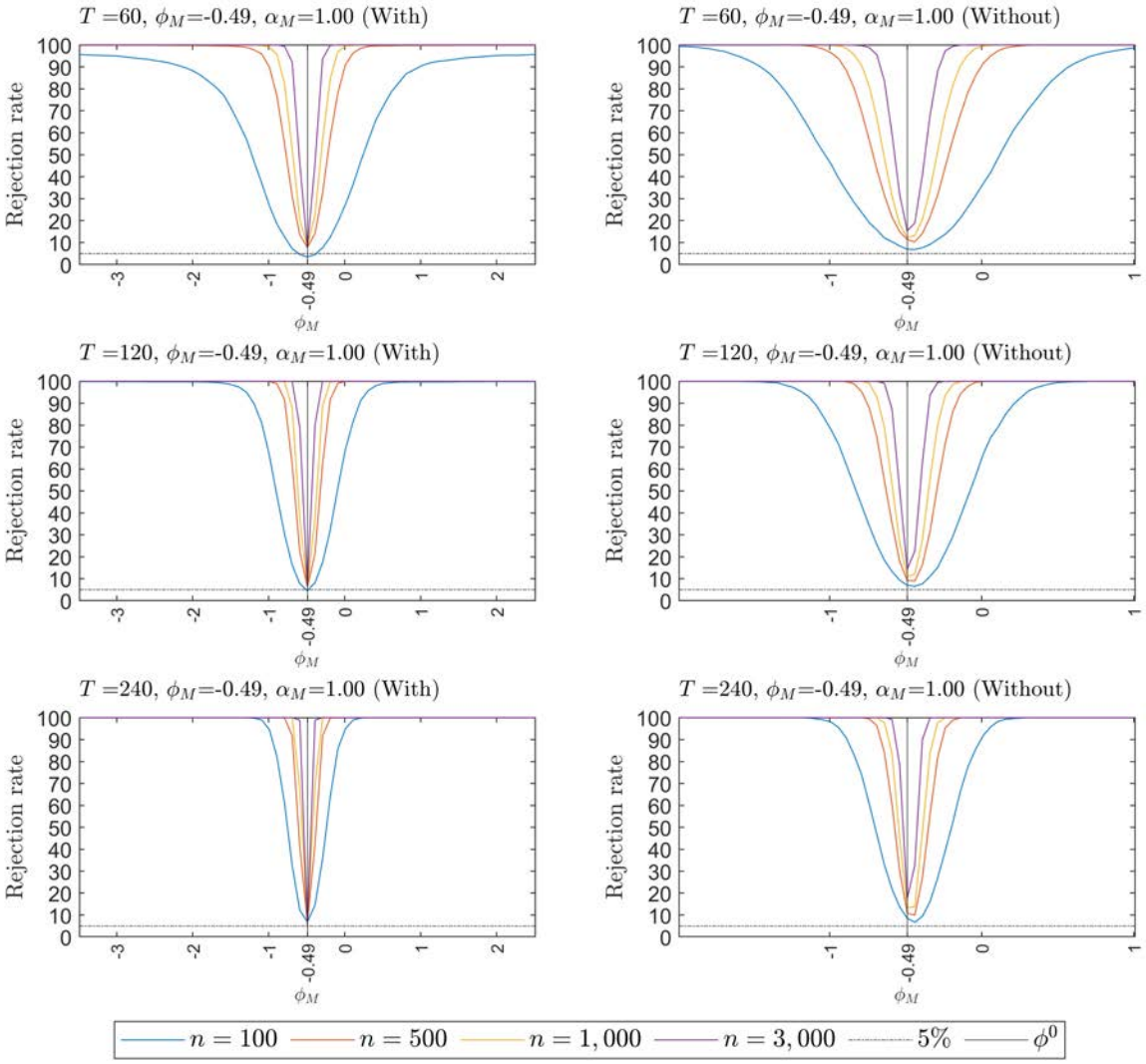
Note: See the notes to Table S-B-E7-9.

Figure S-B-E8: Empirical Power Functions, experiment 8, for coefficient of the ϕ_M factor with and without misspecification



Note: See the notes to Table S-B-E7-9.

Figure S-B-E9: Empirical Power Functions, experiment 9, for coefficient of the ϕ_M factor with and without misspecification



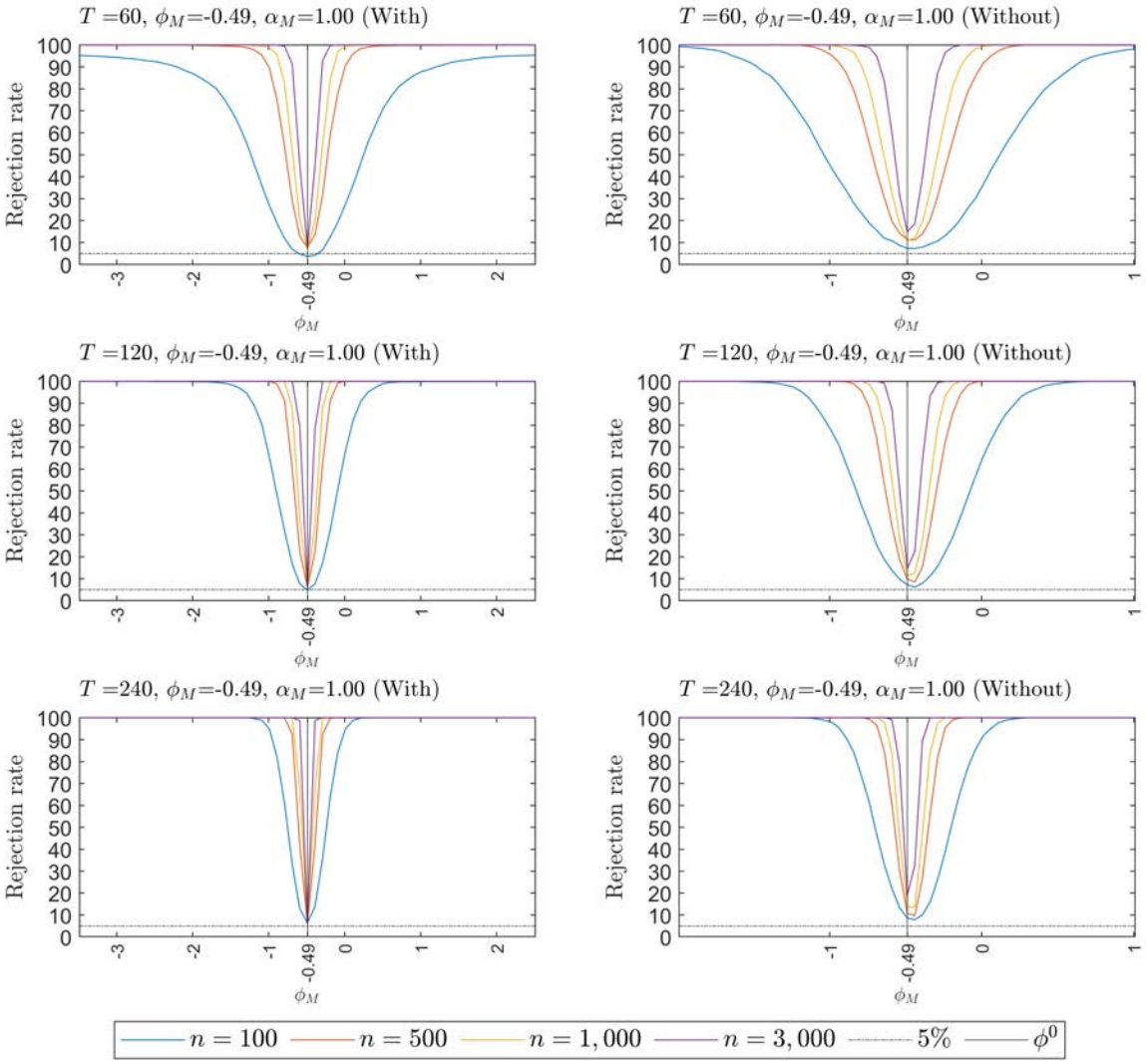
Note: See the notes to Table S-B-E7-9.

Table S-B-E10-12: Bias, RMSE and size for the bias-corrected estimators of $\phi_M = -0.49$, $\alpha_M = 1$ with and without semi-strong factors ($\alpha_H = 0.85, \alpha_S = 0.65$) included in the regression for the cases of experiments 10, 11 and 12

		Bias(x100)		RMSE(x100)		Size(x100)	
Experiment 10	n	With	Without	With	Without	With	Without
		semi-strong factors		semi-strong factors		semi-strong factors	
$T = 60$	100	-1.61	4.07	151.91	33.92	3.75	7.45
	500	-0.69	2.46	17.07	15.76	7.70	11.35
	1,000	-0.31	2.39	11.30	11.47	7.05	10.90
	3,000	-0.06	1.79	6.36	7.31	8.00	14.95
$T = 120$	100	-1.86	4.22	69.05	21.53	4.80	7.25
	500	-0.09	2.81	9.29	9.92	6.10	9.85
	1,000	0.15	2.59	6.62	7.51	6.75	11.40
	3,000	0.05	1.76	3.72	4.69	5.80	14.55
$T = 240$	100	-0.39	4.20	14.17	14.82	6.35	8.60
	500	-0.05	2.87	6.15	7.09	6.05	10.80
	1,000	0.00	2.46	4.30	5.28	6.40	13.65
	3,000	0.00	1.75	2.40	3.40	5.75	18.80
Experiment 11							
$T = 60$	100	-0.72	4.82	47.59	31.18	3.40	5.90
	500	-0.07	3.31	15.12	15.30	6.70	10.30
	1,000	-0.22	2.56	10.46	11.28	6.35	11.45
	3,000	0.05	1.96	5.91	7.26	7.00	16.00
$T = 120$	100	-0.46	4.75	19.81	20.00	4.55	6.60
	500	0.10	3.25	8.93	9.83	6.40	10.00
	1,000	-0.01	2.59	6.28	7.41	6.15	11.95
	3,000	0.09	1.99	3.60	4.83	6.20	16.70
$T = 240$	100	-0.24	4.94	13.43	14.51	5.30	8.45
	500	0.01	3.22	6.11	7.23	6.75	12.95
	1,000	-0.07	2.64	4.19	5.42	5.85	14.65
	3,000	0.02	1.97	2.32	3.50	4.75	20.55
Experiment 12							
$T = 60$	100	-0.14	4.87	129.71	32.05	4.10	6.75
	500	-0.25	2.94	15.76	15.19	6.10	9.45
	1,000	-0.04	2.71	10.48	11.34	5.65	10.40
	3,000	0.10	2.05	6.07	7.34	6.75	15.20
$T = 120$	100	-0.35	4.86	20.06	20.21	4.15	6.35
	500	0.06	3.16	8.91	9.76	5.40	10.35
	1,000	0.16	2.76	6.35	7.57	6.00	12.75
	3,000	0.14	2.01	3.62	4.82	5.60	16.95
$T = 240$	100	-0.13	5.03	13.45	14.61	5.90	9.25
	500	0.07	3.27	6.01	7.18	6.30	12.40
	1,000	-0.02	2.66	4.21	5.41	5.80	15.70
	3,000	0.02	1.96	2.35	3.52	5.05	20.65

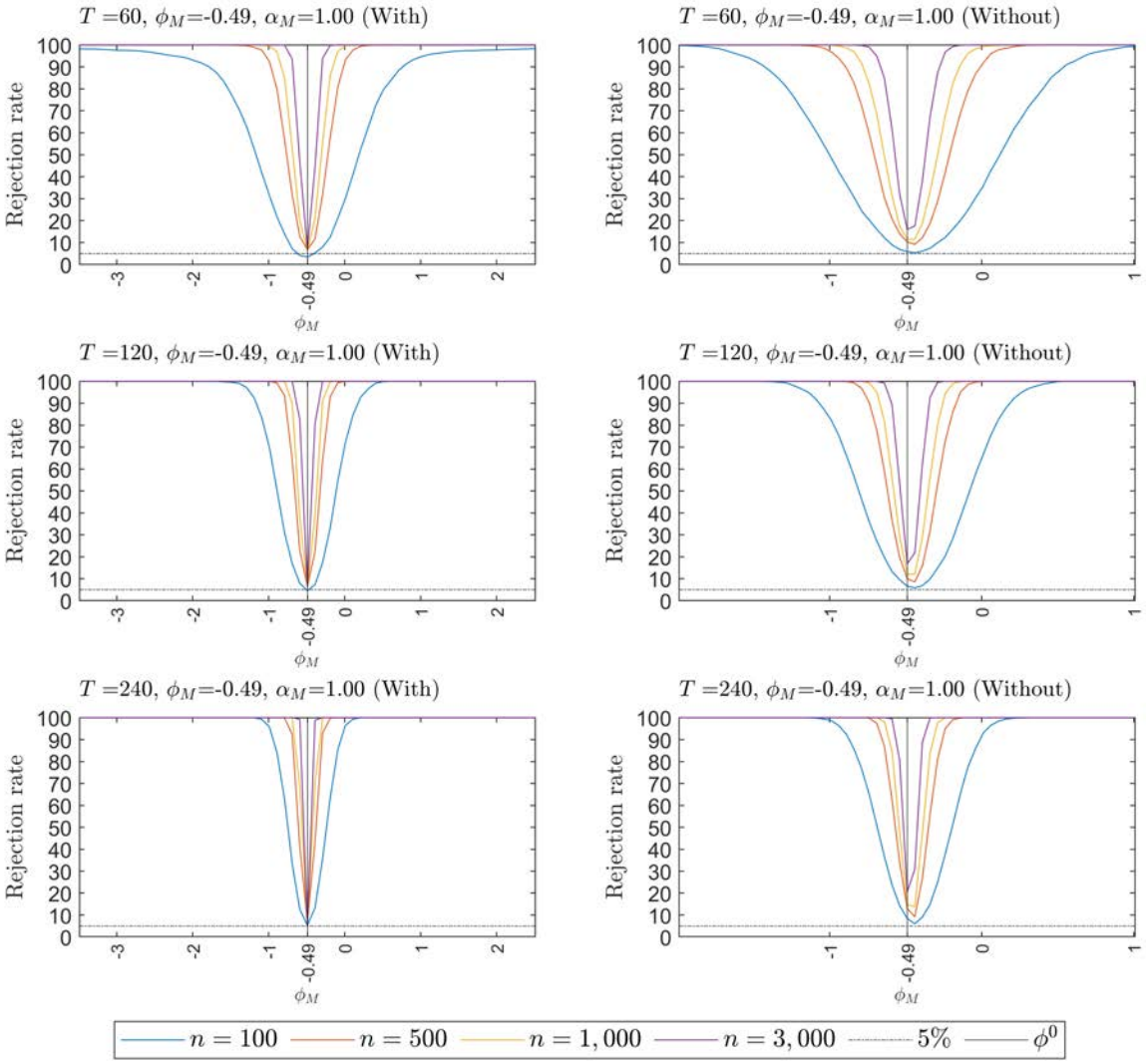
Notes: The DGP includes one strong $\alpha_M = 1$ and two semi-strong ($\alpha_H = 0.85$, $\alpha_S = 0.65$) factors, the regression with the two semi-strong factors includes them, the regression without excludes them. For further details of the experiments, see S-1.

Figure S-B-E10: Empirical Power Functions, experiment 10, for coefficient of the ϕ_M factor with and without misspecification



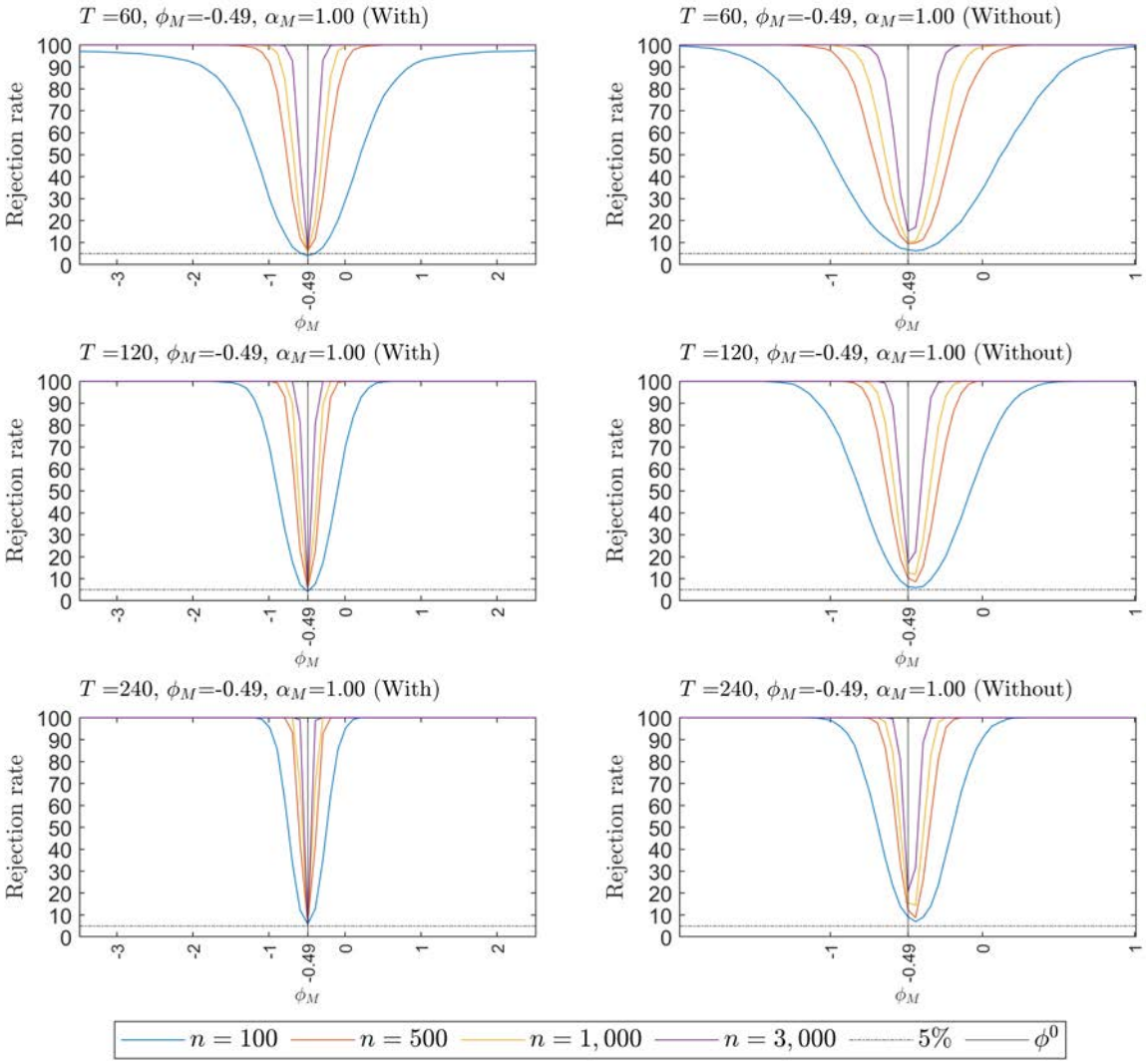
Note: See the notes to Table S-B-E10-12.

Figure S-B-E11: Empirical Power Functions, experiment 11, for coefficient of the ϕ_M factor with and without misspecification



Note: See the notes to Table S-B-E10-12.

Figure S-B-E12: Empirical Power Functions, experiment 12, for coefficient of the ϕ_M factor with and without misspecification



Note: See the notes to Table S-B-E10-12.

2.2 Estimators of ϕ for one strong and two weak factors, threshold estimator of the covariance matrix with and without misspecification

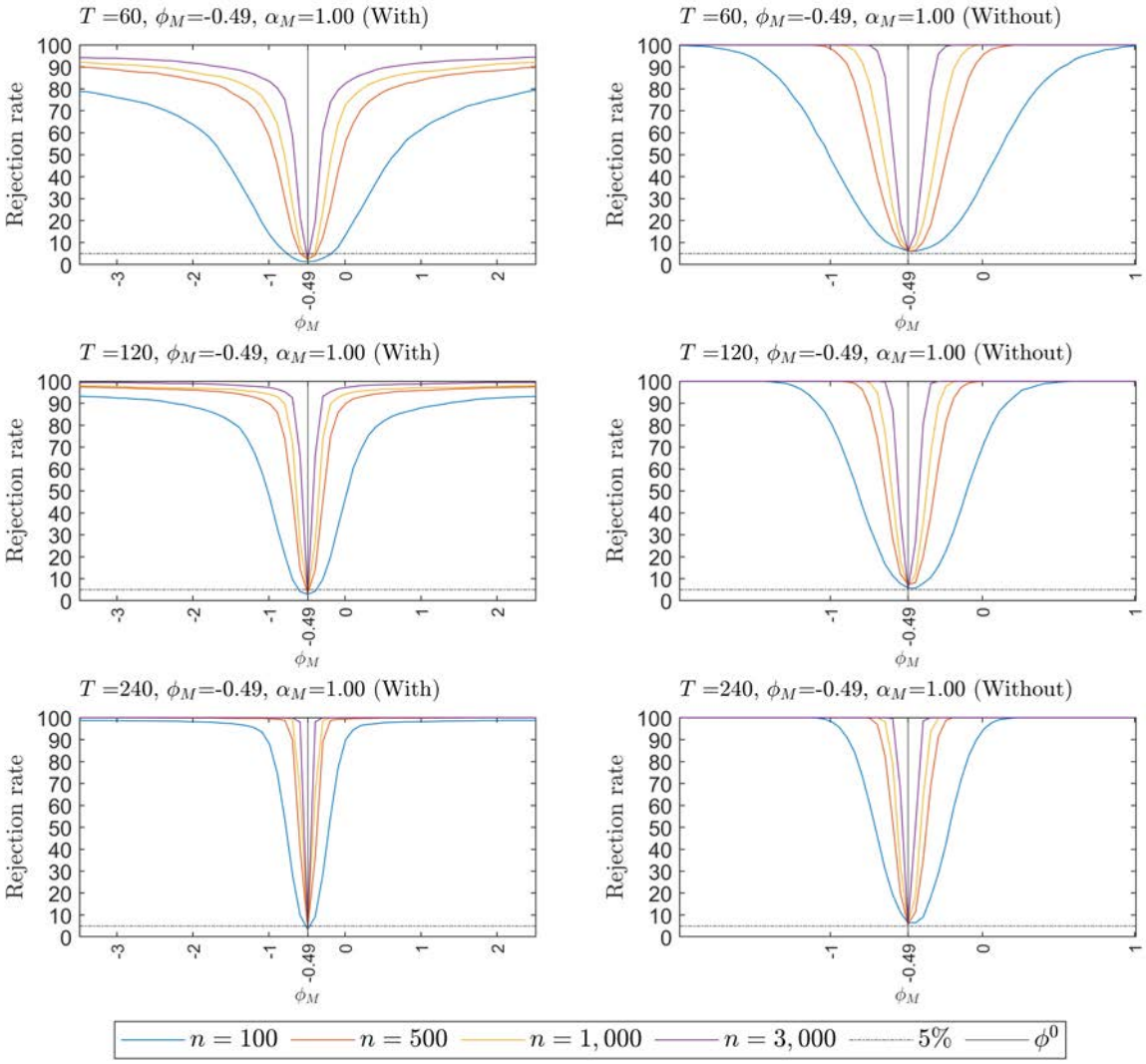
This subsection compares the estimators of ϕ for one strong and two weak factors with and without misspecification. Under the misspecification, only strong estimators ϕ_M are considered in the estimation process. Each table contains the results of three of the 12 experiments described in Table S-1, and corresponding empirical power functions follow.

Table S-C-E1-3: Bias, RMSE and size for the bias-corrected estimators of $\phi_M = -0.49$, α_M=1 with and without weak factors included in the regression for the cases of experiments 1,2 and 3

		Bias(x100)		RMSE(x100)		Size(x100)	
Experiment 1	n	With	Without	With	Without	With	Without
		weak factors		weak factors		weak factors	
$T = 60$	100	22.05	3.67	454.21	30.53	1.15	6.10
	500	-10.55	1.66	332.83	13.13	2.25	6.15
	1,000	-1.33	1.10	382.27	9.48	1.65	6.65
	3,000	-19.15	0.58	932.38	5.38	2.00	6.20
$T = 120$	100	6.52	3.04	237.70	19.49	2.90	5.80
	500	0.73	1.37	35.05	8.75	3.20	7.30
	1,000	4.61	0.94	152.17	6.12	3.60	6.50
	3,000	0.02	0.50	6.94	3.42	2.90	5.00
$T = 240$	100	8.56	2.91	385.16	13.50	3.45	6.45
	500	0.22	1.24	7.22	5.80	4.00	5.80
	1,000	0.06	0.90	4.23	4.14	4.00	6.50
	3,000	0.01	0.51	2.41	2.38	4.90	5.25
Experiment 2							
$T = 60$	100	-13.53	4.08	774.85	31.16	1.40	6.00
	500	-11.15	1.63	755.67	13.33	2.10	5.65
	1,000	10.51	1.01	256.15	9.61	2.00	6.55
	3,000	-0.10	0.56	71.71	5.50	2.25	6.60
$T = 120$	100	468.29	3.25	20351.06	19.57	2.25	6.60
	500	5.00	1.46	141.50	8.89	3.85	7.55
	1,000	-0.54	0.97	16.07	6.06	2.80	5.75
	3,000	0.00	0.45	9.71	3.45	3.35	5.45
$T = 240$	100	-1.38	2.96	67.20	13.49	4.15	7.50
	500	0.04	1.28	6.83	5.85	4.15	5.60
	1,000	0.11	0.91	4.59	4.15	4.65	7.15
	3,000	-0.03	0.48	2.46	2.39	5.15	6.10
Experiment 3							
$T = 60$	100	11.17	3.75	686.97	30.84	1.00	6.25
	500	-10.50	1.65	377.36	13.28	2.00	6.15
	1,000	-4.65	1.09	120.83	9.59	1.50	6.15
	3,000	-4.51	0.59	134.45	5.45	1.90	6.25
$T = 120$	100	5.69	3.01	267.53	19.52	2.65	5.65
	500	-5.01	1.36	140.66	8.79	3.15	7.45
	1,000	-0.91	0.94	23.35	6.14	3.35	6.45
	3,000	-0.35	0.50	20.92	3.43	2.70	4.75
$T = 240$	100	1.04	2.89	38.32	13.51	3.45	6.35
	500	0.32	1.24	8.13	5.82	3.95	5.90
	1,000	0.08	0.90	4.42	4.14	4.00	6.45
	3,000	0.04	0.51	2.59	2.39	4.85	5.45

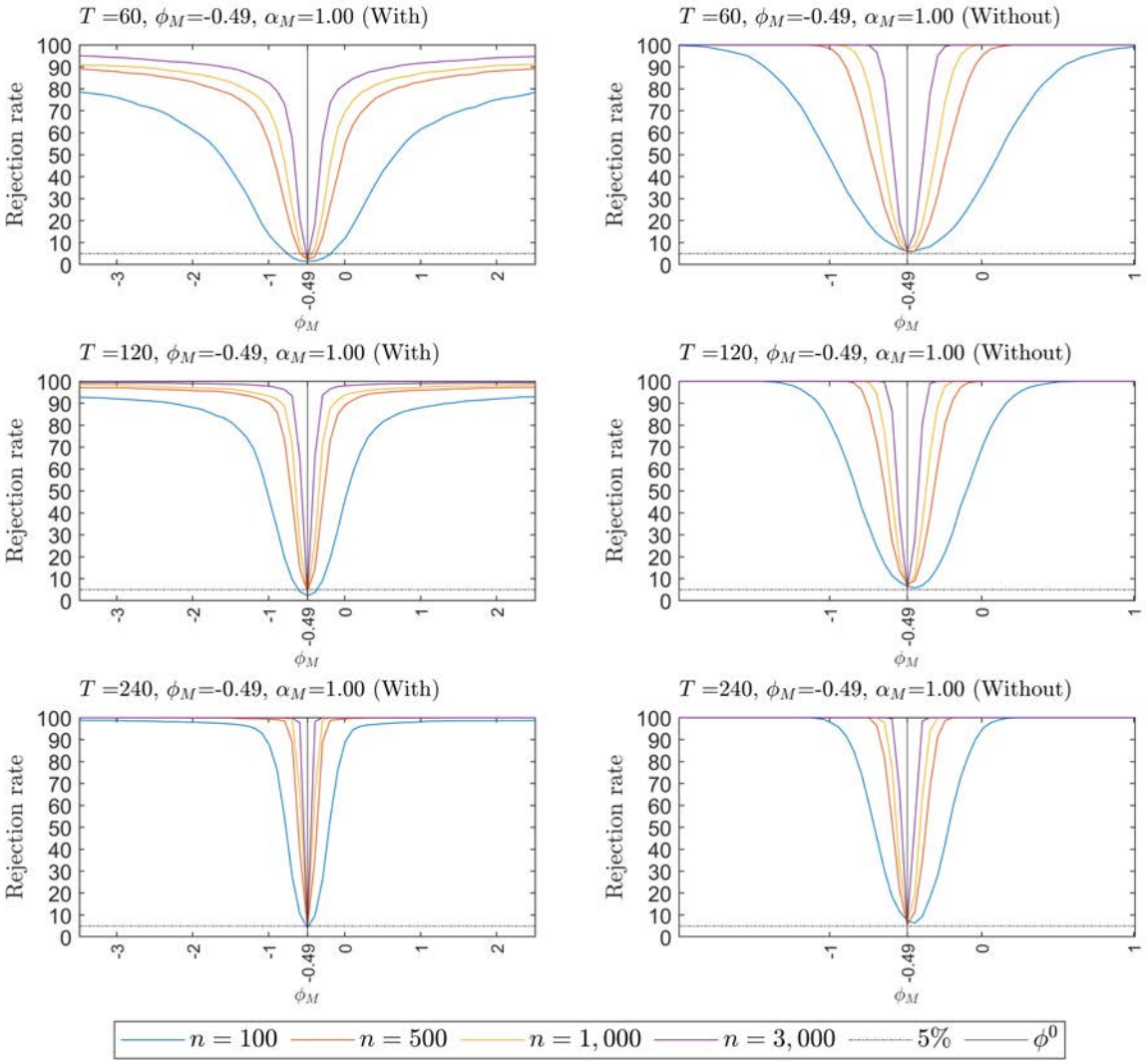
Notes: The DGP includes one strong $\alpha_M = 1$ and two weak ($\alpha_H = \alpha_S = 0.5$) factors, the regression with weak factors includes them, the regression without excludes them. For further details of the experiments, see Table S-1.

Figure S-C-E1: Empirical Power Functions, experiment 1, for coefficient of the ϕ_M factor with and without misspecification



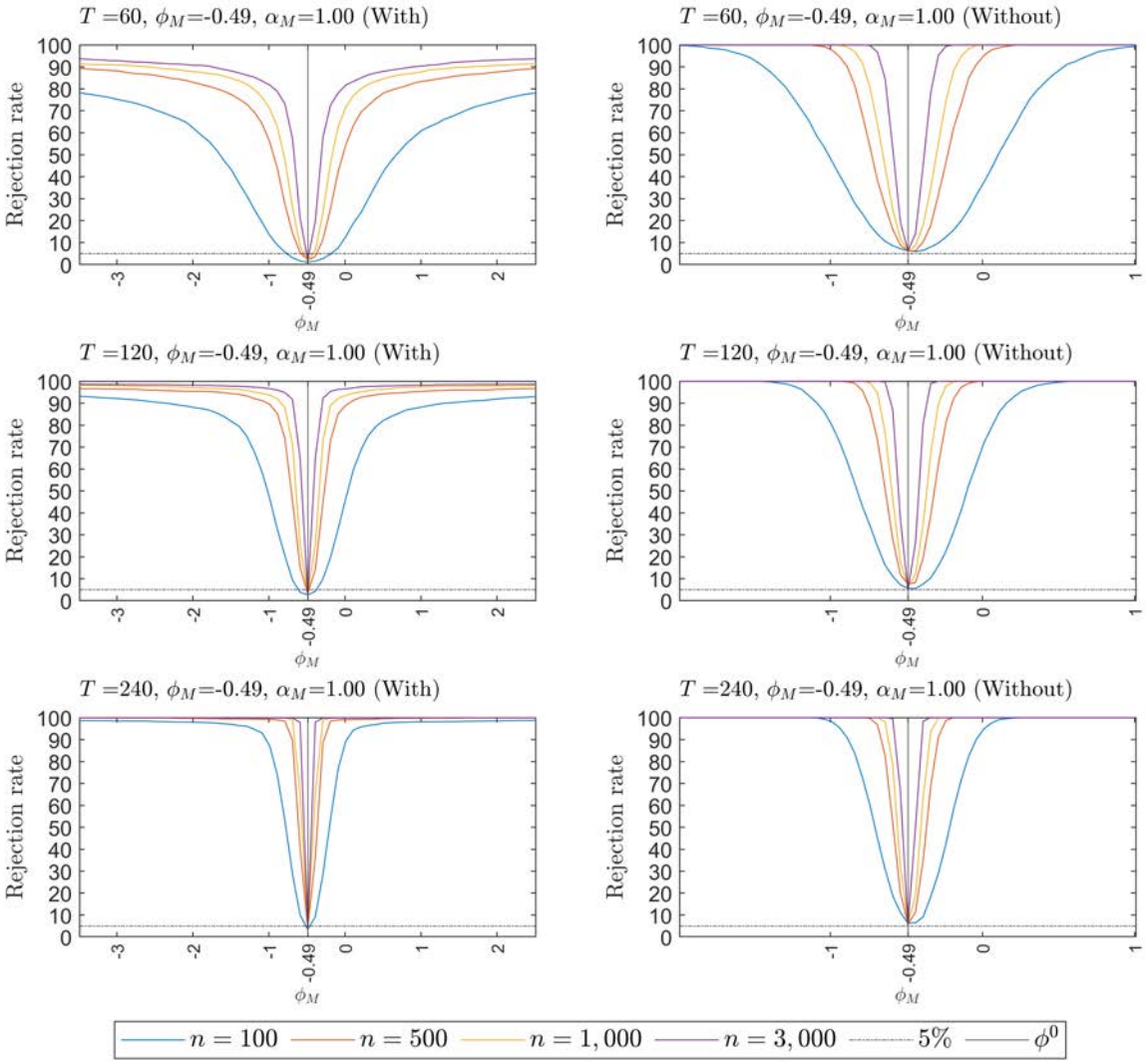
Note: See the notes to Table S-C-E1-3.

Figure S-C-E2: Empirical Power Functions, experiment 2, for coefficient of the ϕ_M factor with and without misspecification



Note: See the notes to Table S-C-E1-3.

Figure S-C-E3: Empirical Power Functions, experiment 3, for coefficient of the ϕ_M factor with and without misspecification



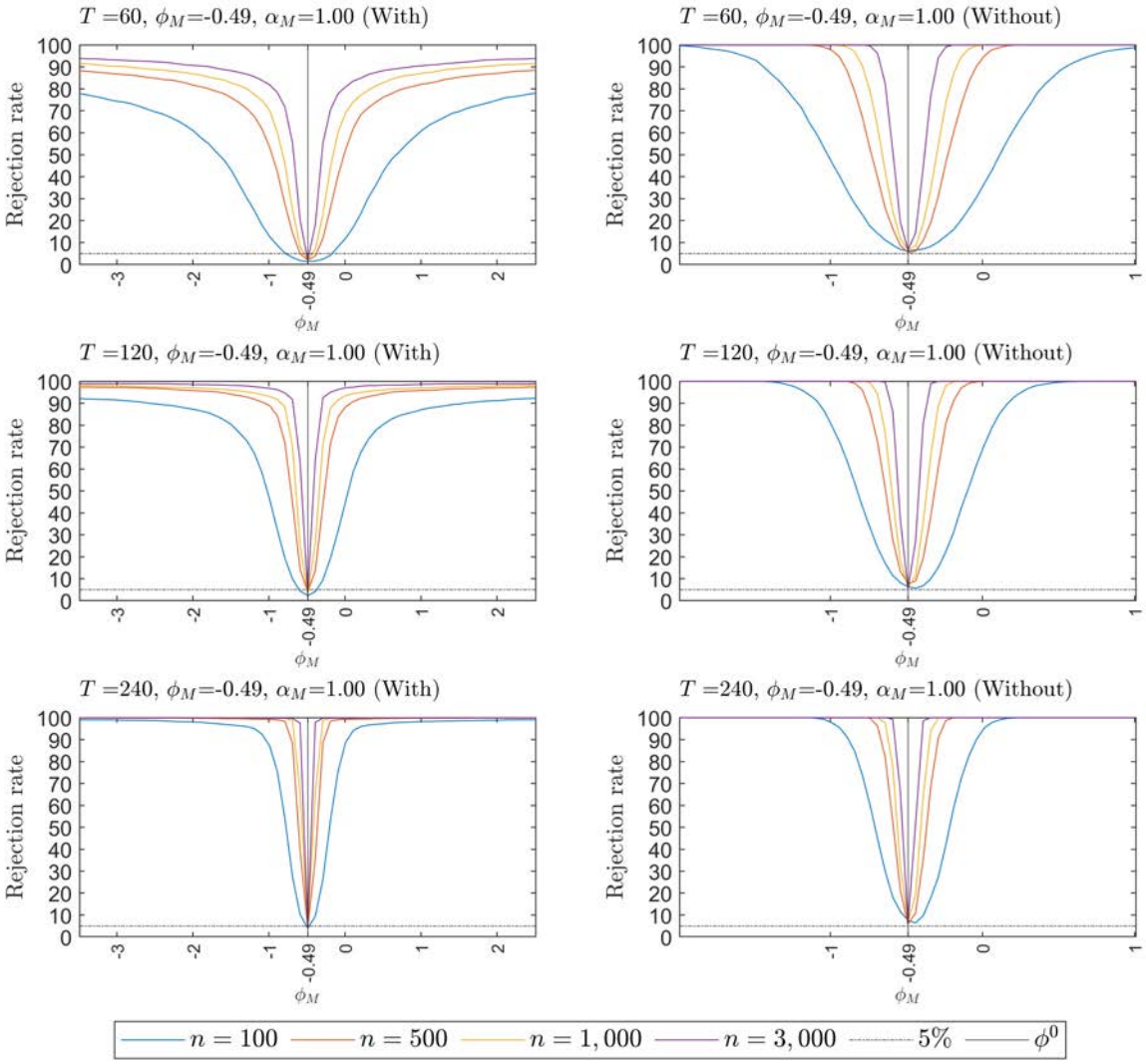
Note: See the notes to Table S-C-E1-3.

Table S-C-E4-6: Bias, RMSE and size for the bias-corrected estimators of $\phi_M = -0.49$, α_M=1 with and without weak factors included in the regression for the cases of experiments 4, 5 and 6

		Bias(x100)		RMSE(x100)		Size(x100)	
Experiment 4	n	With	Without	With	Without	With	Without
		weak factors		weak factors		weak factors	
$T = 60$	100	-24.42	4.12	1165.53	31.54	1.35	6.15
	500	-11.84	1.61	302.88	13.49	1.95	5.55
	1,000	5.16	1.01	229.86	9.72	1.90	6.75
	3,000	-8.17	0.57	450.22	5.57	2.05	6.60
$T = 120$	100	-4.28	3.22	452.39	19.63	2.30	6.30
	500	-0.54	1.44	26.55	8.94	3.80	7.85
	1,000	0.98	0.96	31.58	6.08	2.75	5.80
	3,000	-0.17	0.45	8.84	3.46	3.25	5.40
$T = 240$	100	-0.11	2.94	31.04	13.51	3.95	7.55
	500	-0.50	1.28	27.63	5.85	4.05	5.90
	1,000	-0.05	0.91	5.23	4.16	4.65	6.90
	3,000	0.02	0.48	3.36	2.39	4.85	6.05
Experiment 5							
$T = 60$	100	12.72	3.75	679.29	30.84	0.95	6.25
	500	-10.07	1.66	364.25	13.34	2.10	6.15
	1,000	-5.01	1.07	122.63	9.60	1.45	5.90
	3,000	-4.43	0.60	133.49	5.47	1.90	6.25
$T = 120$	100	6.87	3.01	277.10	19.52	2.55	5.65
	500	-4.67	1.36	127.57	8.85	3.65	7.60
	1,000	-0.85	0.92	24.13	6.16	3.60	6.40
	3,000	-0.35	0.51	20.50	3.44	2.65	5.20
$T = 240$	100	0.54	2.89	31.90	13.51	4.15	6.35
	500	0.29	1.25	7.91	5.86	3.85	6.00
	1,000	0.10	0.89	4.44	4.16	4.30	6.75
	3,000	0.04	0.52	2.58	2.40	5.20	5.45
Experiment 6							
$T = 60$	100	-27.78	4.12	905.91	31.54	1.45	6.15
	500	-11.62	1.60	305.30	13.55	2.05	5.95
	1,000	5.28	0.99	231.07	9.73	1.65	6.50
	3,000	-7.16	0.58	412.98	5.59	2.05	6.95
$T = 120$	100	-2.66	3.22	451.26	19.63	2.30	6.30
	500	-0.54	1.44	26.52	9.00	4.15	7.85
	1,000	0.92	0.94	30.26	6.10	2.90	6.30
	3,000	-0.18	0.46	8.91	3.47	3.25	5.55
$T = 240$	100	-0.18	2.94	29.74	13.51	5.10	7.55
	500	-0.49	1.29	27.08	5.89	4.40	6.30
	1,000	-0.02	0.90	5.12	4.17	5.20	6.80
	3,000	0.03	0.49	3.47	2.40	4.50	6.45

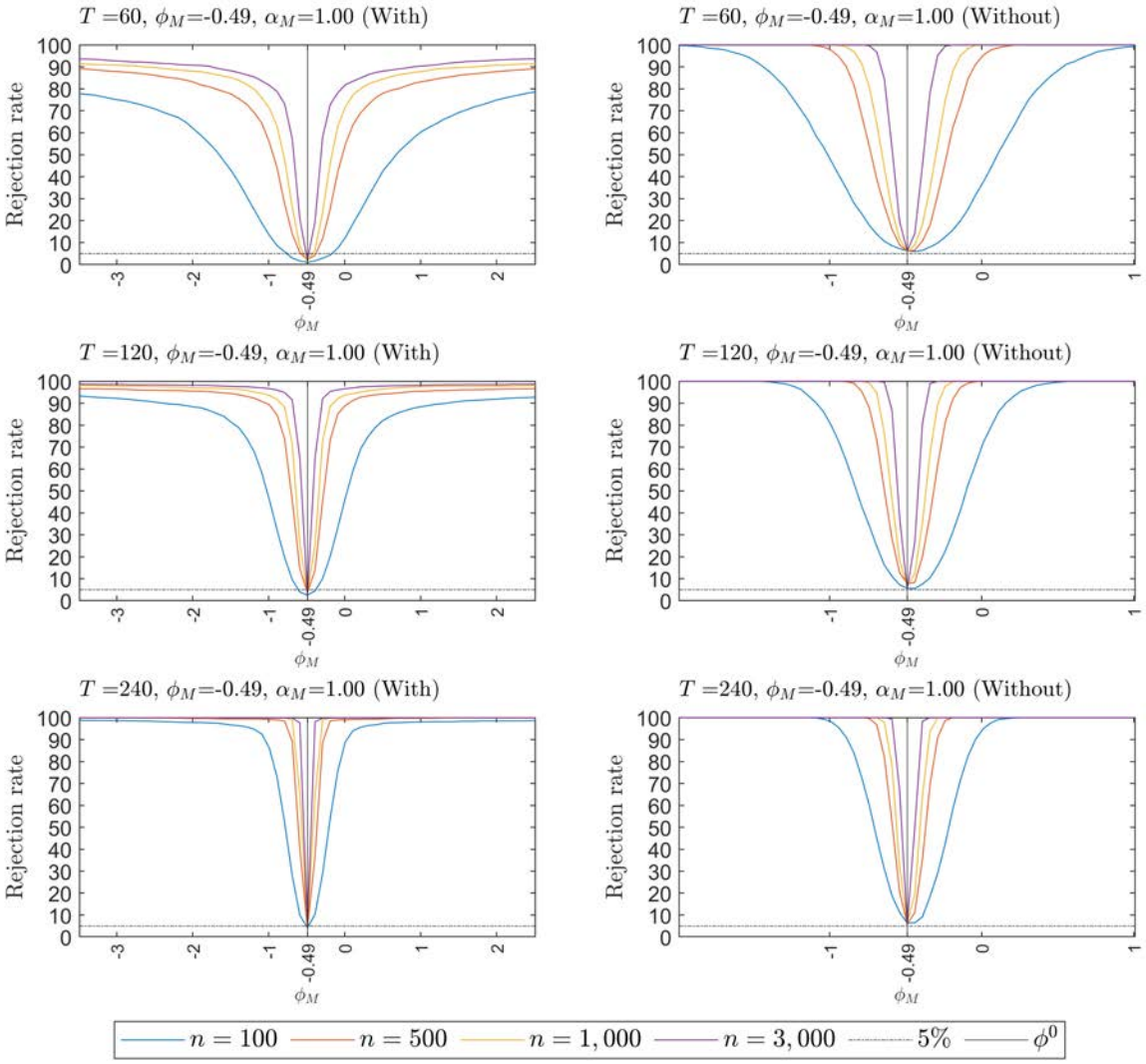
Notes: The DGP includes one strong $\alpha_M = 1$ and two weak ($\alpha_H = \alpha_S = 0.5$) factors, the regression with weak factors includes them, the regression without excludes them. For further details of the experiments, see Table S-1.

Figure S-C-E4: Empirical Power Functions, experiment 4, for coefficient of the ϕ_M factor with and without misspecification



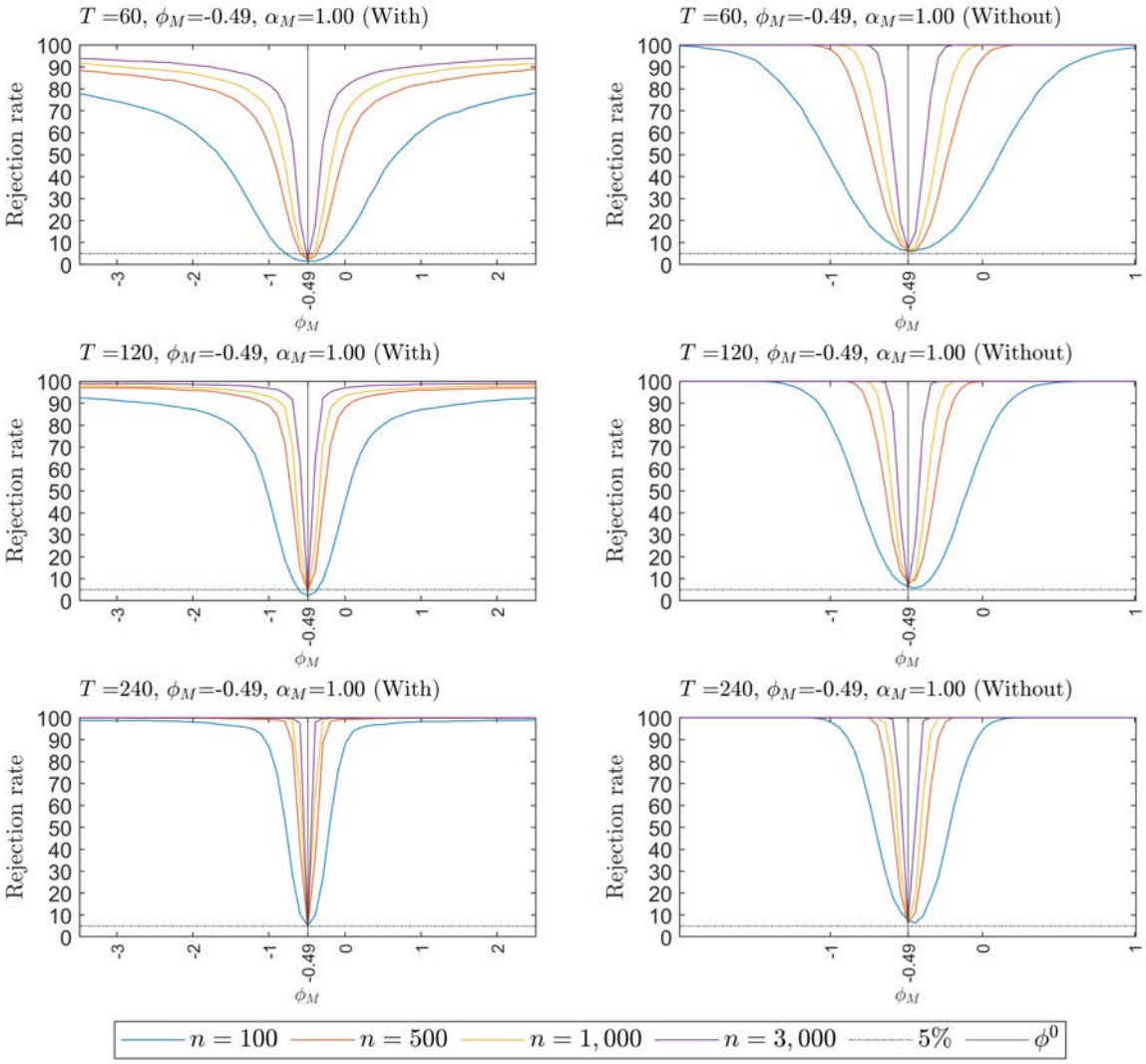
Note: See the notes to Table S-C-E4-6.

Figure S-C-E5: Empirical Power Functions, experiment 5, for coefficient of the ϕ_M factor with and without misspecification



Note: See the notes to Table S-C-E4-6.

Figure S-C-E6: Empirical Power Functions, experiment 6, for coefficient of the ϕ_M factor with and without misspecification



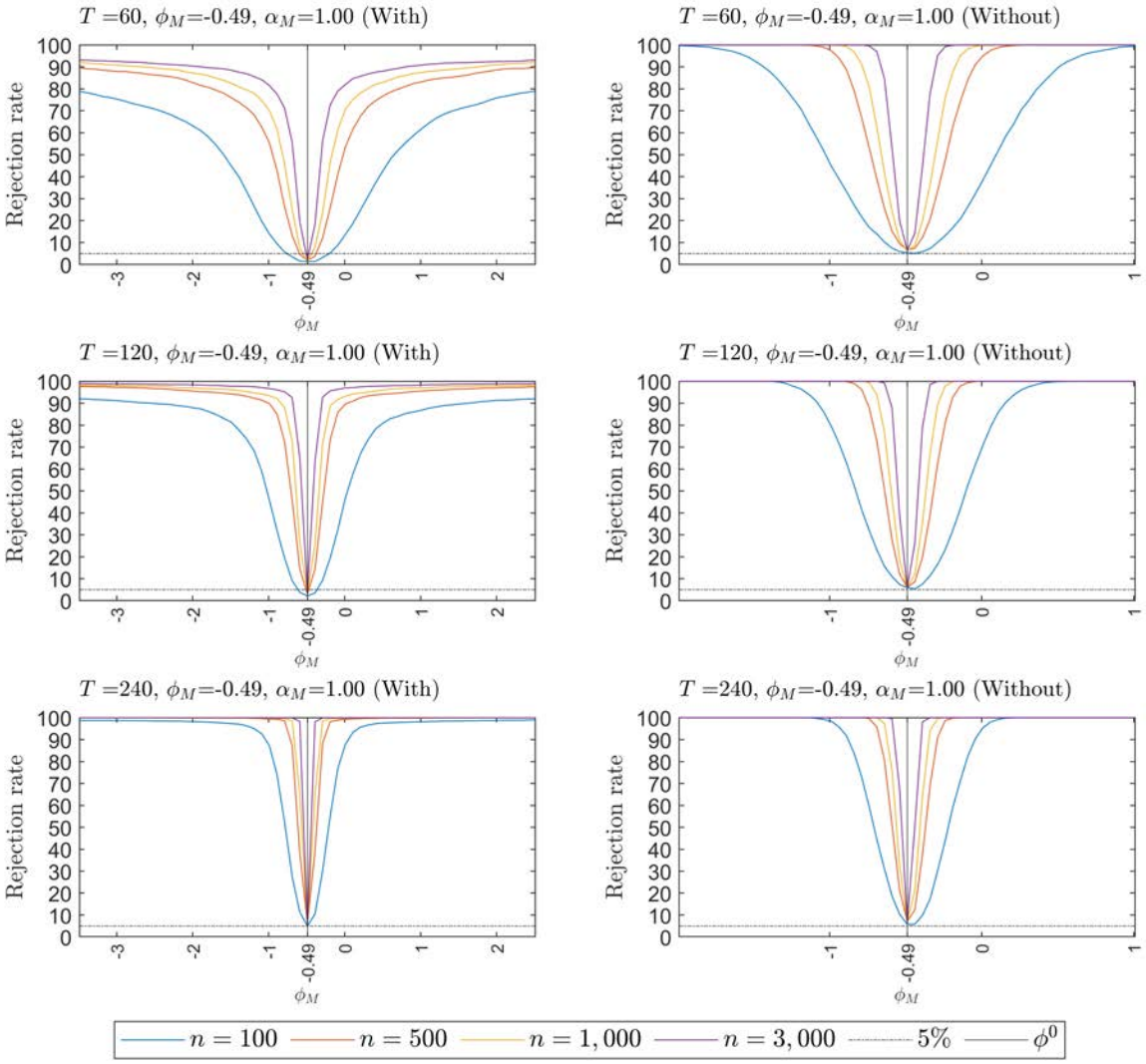
Note: See the notes to Table S-C-E4-6.

Table S-C-E7-9: Bias, RMSE and size for the bias-corrected estimators of $\phi_M = -0.49$, α_M=1 with and without weak factors included in the regression for the cases of experiments 7, 8 and 9

		Bias(x100)		RMSE(x100)		Size(x100)	
Experiment 7	n	With	Without	With	Without	With	Without
		weak factors		weak factors		weak factors	
$T = 60$	100	3.08	2.82	859.39	30.18	1.30	5.35
	500	24.88	1.20	840.21	13.62	1.85	6.65
	1,000	0.24	0.84	79.72	9.64	1.35	6.25
	3,000	-4.91	0.61	528.52	5.51	2.20	6.40
$T = 120$	100	4.42	2.67	216.43	19.15	2.10	6.00
	500	1.74	1.23	147.10	8.59	2.95	6.05
	1,000	-3.59	0.89	154.11	6.12	3.75	5.90
	3,000	-0.84	0.61	41.40	3.52	3.75	5.75
$T = 240$	100	-0.94	2.74	22.50	13.27	4.95	5.80
	500	-0.83	1.11	38.11	6.04	6.15	7.15
	1,000	-0.06	0.84	4.85	4.25	4.95	7.00
	3,000	-0.01	0.55	2.46	2.38	4.70	6.50
Experiment 8							
$T = 60$	100	-18.57	2.85	1090.60	31.04	1.30	5.70
	500	3.08	0.84	248.52	13.70	1.90	6.10
	1,000	11.56	0.98	511.88	9.66	1.60	5.60
	3,000	-0.24	0.71	45.00	5.66	2.05	6.50
$T = 120$	100	15.36	2.83	488.96	19.33	2.40	5.00
	500	-8.41	1.12	424.57	8.53	3.55	5.70
	1,000	-1.21	1.04	42.58	6.25	3.65	6.25
	3,000	-0.68	0.64	43.26	3.56	3.60	6.00
$T = 240$	100	-4.70	2.85	184.48	13.30	5.10	6.30
	500	0.07	1.17	7.15	5.98	5.45	6.30
	1,000	-0.07	0.85	4.44	4.27	5.10	6.60
	3,000	0.01	0.54	2.41	2.40	4.80	6.70
Experiment 9							
$T = 60$	100	16.90	1.71	1051.65	32.31	1.35	6.65
	500	-10.51	0.95	739.68	14.35	2.35	7.50
	1,000	-2.96	0.53	160.31	10.26	2.65	8.75
	3,000	-0.09	0.43	106.32	5.80	2.70	8.35
$T = 120$	100	61.02	1.74	2471.10	20.37	2.05	6.25
	500	-9.78	1.08	365.78	8.87	2.70	6.00
	1,000	0.35	0.73	77.55	6.36	3.00	7.30
	3,000	1.23	0.46	48.78	3.59	3.85	6.55
$T = 240$	100	0.51	1.73	52.46	13.89	4.55	6.90
	500	-0.22	0.91	8.62	6.08	5.75	6.40
	1,000	-0.20	0.71	7.44	4.27	5.50	7.15
	3,000	-0.04	0.43	2.52	2.37	4.30	5.60

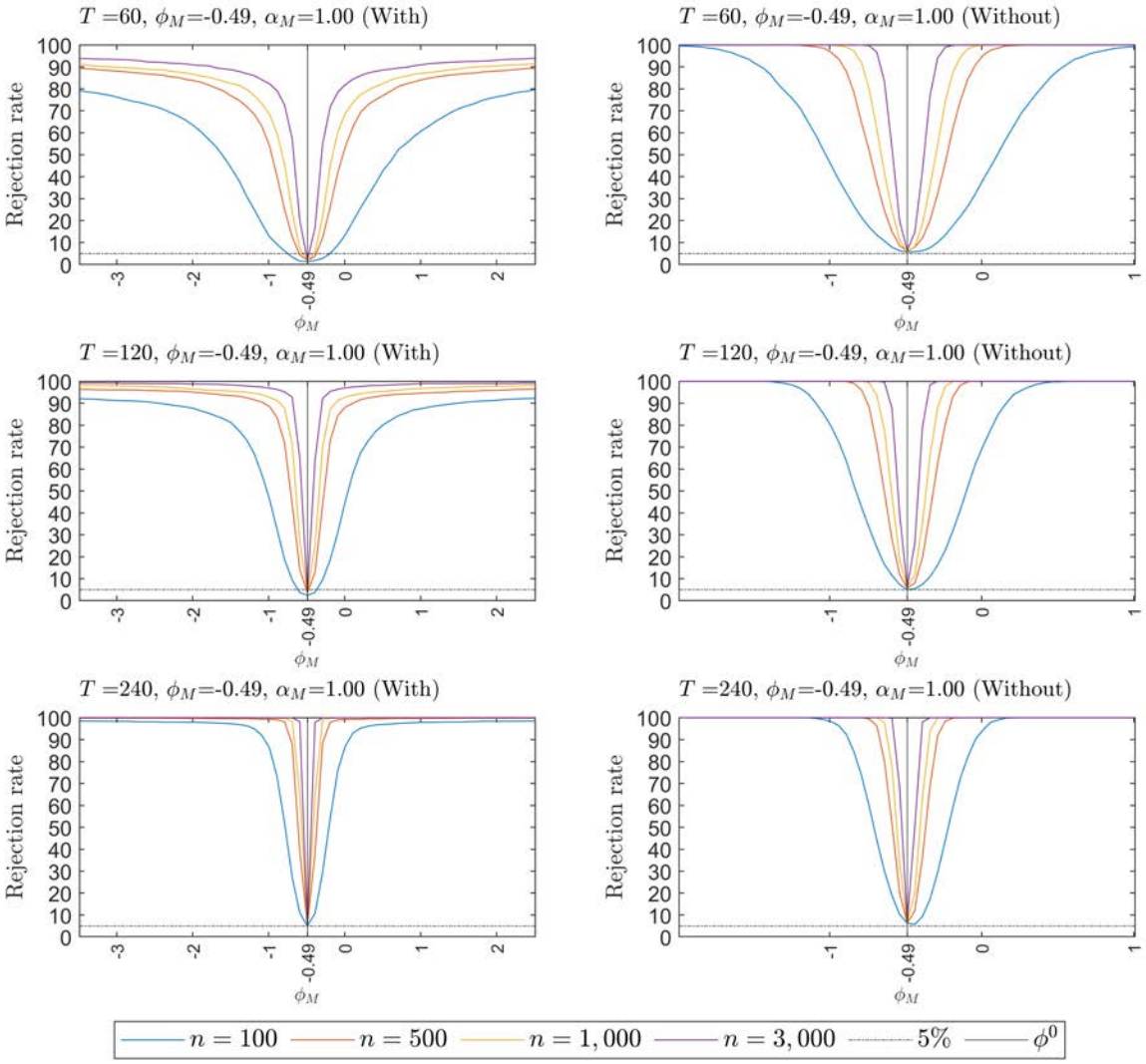
Notes: The DGP includes one strong $\alpha_M = 1$ and two weak ($\alpha_H = \alpha_S = 0.5$) factors, the regression with weak factors includes them, the regression without excludes them. For further details of the experiments, see Table S-1.

Figure S-C-E7: Empirical Power Functions, experiment 7, for coefficient of the ϕ_M factor with and without misspecification



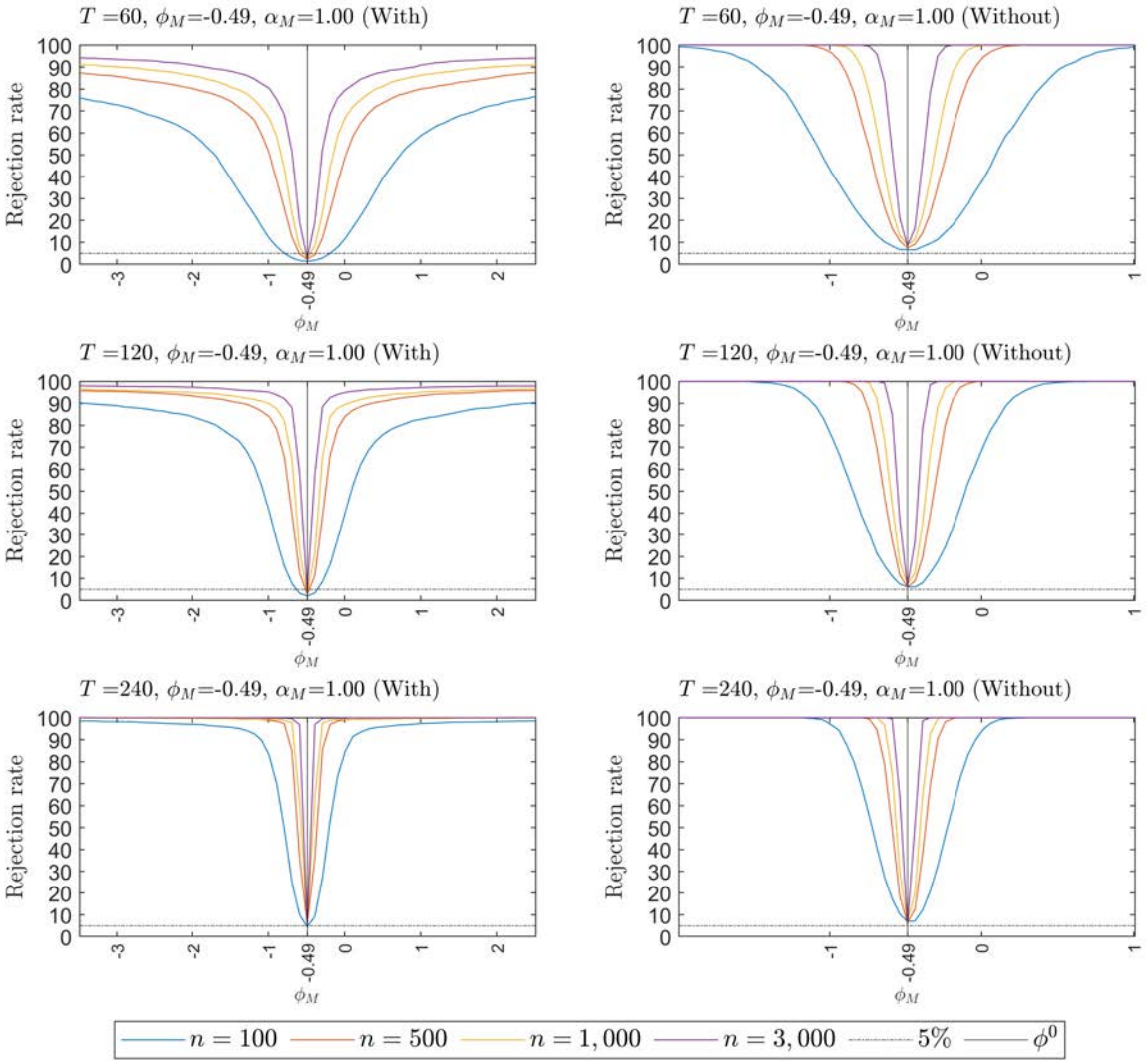
Note: See the notes to Table S-C-E7-9.

Figure S-C-E8: Empirical Power Functions, experiment 8, for coefficient of the ϕ_M factor with and without misspecification



Note: See the notes to Table S-C-E7-9.

Figure S-C-E9: Empirical Power Functions, experiment 9, for coefficient of the ϕ_M factor with and without misspecification



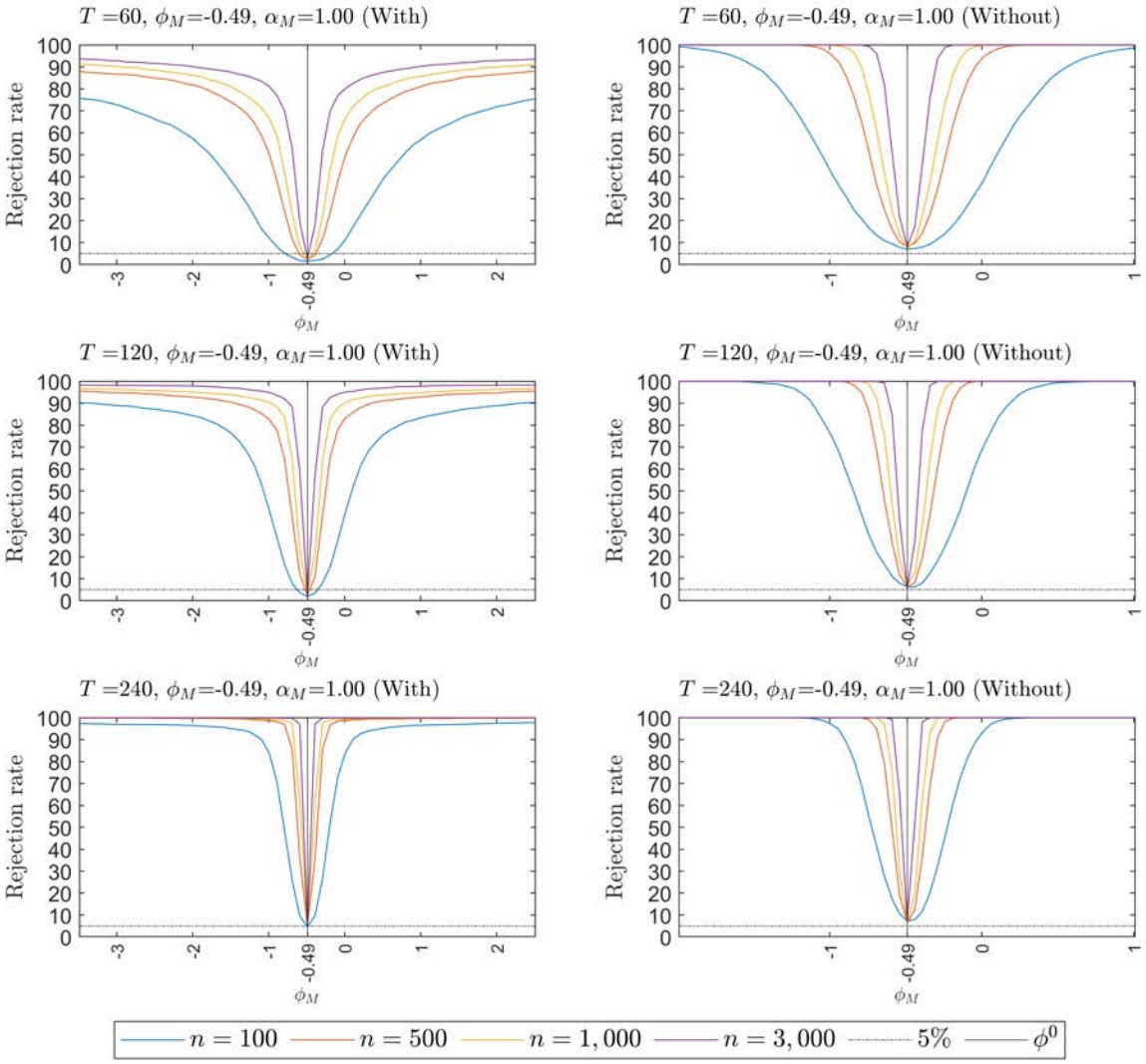
Note: See the notes to Table S-C-E7-9.

Table S-C-E10-12: Bias, RMSE and size for the bias-corrected estimators of $\phi_M = -0.49$, α_M=1 with and without weak factors included in the regression for the cases of experiments 10, 11 and 12

		Bias(x100)		RMSE(x100)		Size(x100)	
Experiment 10	n	With	Without	With	Without	With	Without
		weak factors		weak factors		weak factors	
$T = 60$	100	-1.30	1.80	581.15	33.08	1.55	6.95
	500	19.94	0.54	738.70	14.45	2.10	8.20
	1,000	1.31	0.69	76.46	10.23	1.85	7.35
	3,000	1.02	0.53	333.48	5.91	2.70	8.30
$T = 120$	100	27.62	1.91	810.64	20.78	1.90	6.30
	500	-4.30	0.95	195.46	8.90	2.40	6.25
	1,000	7.68	0.93	212.90	6.46	3.45	7.25
	3,000	-0.46	0.49	34.61	3.60	3.20	5.50
$T = 240$	100	0.01	1.87	109.19	14.08	4.60	7.30
	500	0.27	0.94	19.59	6.08	5.20	6.50
	1,000	0.41	0.76	21.39	4.28	5.50	6.55
	3,000	-0.04	0.76	2.57	2.40	4.70	7.10
Experiment 11							
$T = 60$	100	17.29	2.89	696.53	30.28	1.45	5.40
	500	-5.92	1.27	510.19	13.71	2.00	6.80
	1,000	45.15	0.78	2091.69	9.66	2.00	6.45
	3,000	2.77	0.59	75.60	5.61	2.65	7.05
$T = 120$	100	4.02	2.73	135.30	19.16	2.55	6.20
	500	0.44	1.34	24.35	8.61	3.50	5.60
	1,000	10.16	0.86	415.98	6.09	3.85	6.10
	3,000	0.10	0.62	7.88	3.54	4.65	6.70
$T = 240$	100	-0.31	2.77	14.31	13.28	4.10	6.15
	500	0.07	1.19	6.78	6.06	5.45	6.90
	1,000	-0.06	0.81	4.40	4.23	5.45	6.65
	3,000	0.01	0.55	2.37	2.38	4.65	6.50
Experiment 12							
$T = 60$	100	0.33	2.93	531.55	31.13	1.35	5.75
	500	-16.77	0.90	496.29	13.75	1.70	5.60
	1,000	17.22	0.93	766.48	9.68	1.65	6.15
	3,000	-0.91	0.68	162.80	5.75	2.40	7.00
$T = 120$	100	-11.47	2.88	529.74	19.36	2.55	5.15
	500	-16.06	1.23	555.75	8.55	3.55	5.80
	1,000	0.99	1.03	20.02	6.22	3.50	6.50
	3,000	-2.21	0.64	100.80	3.57	4.00	5.90
$T = 240$	100	-0.27	2.88	19.39	13.31	4.20	6.30
	500	0.10	1.24	6.30	5.99	5.35	6.25
	1,000	-0.05	0.83	4.37	4.23	5.10	6.70
	3,000	0.01	0.54	2.39	2.41	4.95	6.50

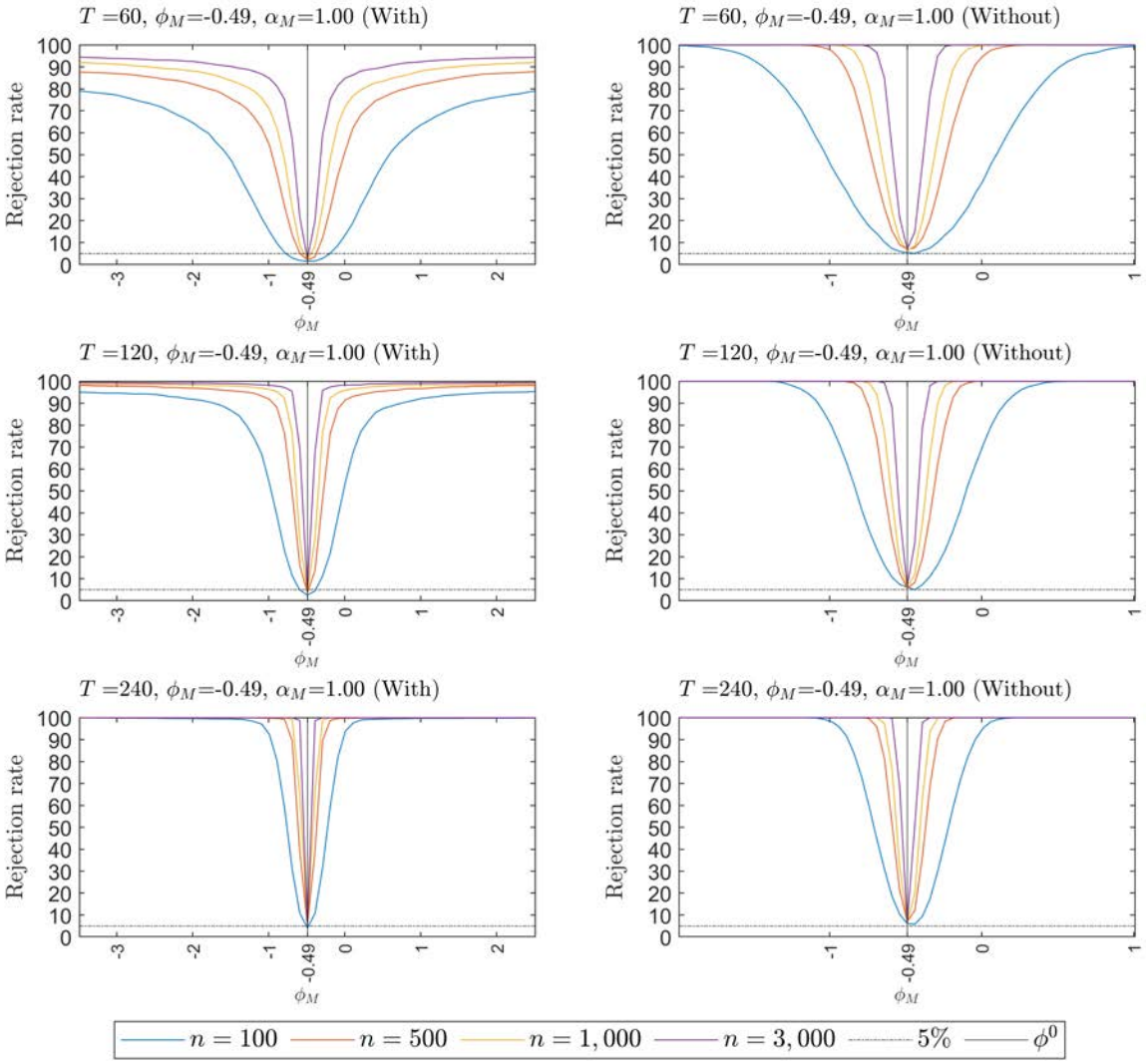
Notes: The DGP includes one strong $\alpha_M = 1$ and two weak ($\alpha_H = \alpha_S = 0.5$) factors, the regression with weak factors includes them, the regression without excludes them. For further details of the experiments, see Table S-1.

Figure S-C-E10: Empirical Power Functions, experiment 10, for coefficient of the ϕ_M factor with and without misspecification



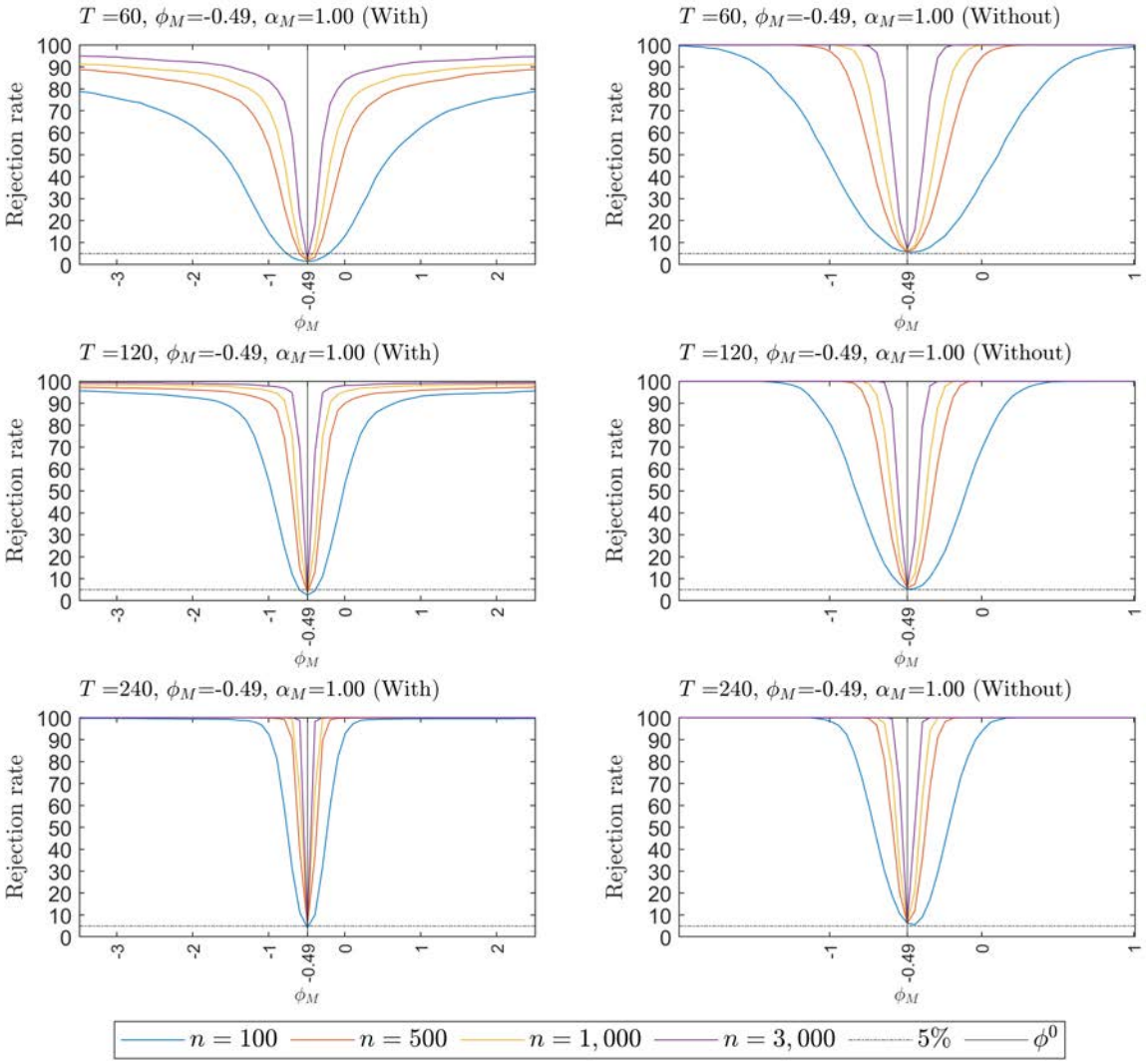
Note: See the notes to Table S-C-E10-12.

Figure S-C-E11: Empirical Power Functions, experiment 11, for coefficient of the ϕ_M factor with and without misspecification



Note: See the notes to Table S-C-E10-12.

Figure S-C-E12: Empirical Power Functions, experiment 12, for coefficient of the ϕ_M factor with and without misspecification



Note: See the notes to Table S-C-E10-12.