Strategic Exits in Stochastic Partnerships: The Curse of Profitability*

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We study dynamic partnerships where the output evolves stochastically, each player can exit at any time, and players who have exited continue to accrue some benefits if the remaining players keep contributing to the partnership. Players can strategically exit to free-ride on their partners' contributions, knowing that it may trigger subsequent exits of their partners. The unique Pareto optimal Markov perfect equilibrium may feature a *curse of profitability*: An increase in the partnership's output may strictly harm all the players by exacerbating free-riding. Another main finding is that a partnership's ability to sustain cooperation is non-monotonic in its group size.

Keywords: partnerships, strategic exits, ripple effect, group size, stochastic stopping games.
JEL Codes: C73, D62, L22.

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1 Introduction

In some partnerships, partners who have exited continue to accrue some benefits as long as the remaining partners keep contributing to the partnership. In a cartel, for instance, firms that have left can still benefit from the low quantities, or high prices, maintained by those that remain in the cartel. In a startup, co-founders who have ceased investment can still benefit from the startup's later success, including monetary returns (if they still hold some shares of the startup) and reputation gains. In a public protest, protesters who quit early still enjoy some achievements of those sticking to the end. In an environmental agreement, nations that have withdrawn still benefit from the reduction of greenhouse gases by those that stay.

These partnerships face the common problem of *strategic exiting* — partners may exit to save their private contribution costs while relying on the continued contributions of others. This phenomenon is well documented in the literature; for instance, Hellwig and Hüschelrath (2018) list strategic exiting as a common reason for cartels to shrink in size. Notice that a partner's exit makes it more difficult for the remaining partners to operate the partnership and thus may trigger them to exit as well. Such a *ripple effect*, in turn, determines whether a partner would like to strategically exit in the first place.

This paper builds a framework to investigate the dynamics of cooperation in partnerships where exited partners continue to benefit from the partnership's output. In particular, we focus on stochastic partnerships — the partnership's flow output, which we refer to as its level of *profitability*, changes stochastically over time. This reflects the fact that a partnership's output is affected by some uncertain exogenous factors, such as fluctuating market demand and evolving technology in the example of cartels.

The main finding of this paper is that the partnership may be subject to a *curse of profitability*, i.e., under some condition, a more profitable partnership leaves *all* the partners strictly worse off. Intuitively, higher profitability is a double-edged sword. On the one hand, it means the partnership generates more output. On the other, in case some partners exit, higher profitability makes the remaining partners more willing to keep operating the partnership, which stimulates strategic exiting in the first place. Moreover, partners have incentives to pre-empt each other since they prefer to be the free-riders (those who exit while others remain), and because of that, the free-riders exit "too early" in equilibrium. As a consequence, all the players — including the free-riders — may suffer from high profitability. This finding provides a novel and plausible explanation to the puzzle that some partnerships face more challenges in sustaining cooperation as they become more lucrative.[1]

As a starting point, Section 2 considers a baseline model where two players run a joint project.[2] Each player can exit at any time to save his contribution cost.[3] We refer to the player who exits first as the *first mover* and his partner as the *second mover*.[4] In the baseline model, the ripple effect is that the second mover, facing a higher cost after the first mover exits, may choose to exit as well and thus terminate the project.

Section 3 studies the Markov perfect equilibria (MPE) where each player decides whether to exit based on the project's current profitability level and whether his partner has already exited. Theorem 1 shows that in the *unique* Pareto optimal MPE, increasing the partnership's profitability level may strictly decrease both players' continuation value. This theorem formalizes the idea of the curse of profitability.

Section 4 generalizes the model to more than two players. In this setting, the ripple effect of a player's exit becomes more complicated, as an initial exit may trigger a second exit, which may further trigger a third, and so on.[5] Whether players will benefit from strategic exiting depends on the magnitude of such ripple effect, which itself is endogenously determined by the players' strategies in the equilibrium.

There are two main findings in the generalized model. First, under some condition, the curse of profitability continues to hold when there are more than two players (Theorem 2). Second, we utilize the setting to investigate how a partnership's ability to sustain cooperation is affected by its

[1]For instance, it is well documented that founders of a company may "share bitter but not sweet." As the company grows, some of the founders become less self-motivated and are more likely to become inactive or switch to other businesses. Also, in the context of cartels, Rotemberg and Saloner (1986) suggest that cartel members are less likely to collude during periods of high market demand.

[2]In the example of cartels, we can interpret the two players as two groups of firms. After one group exits the cartel, the other group may still collude among themselves, benefiting the group that has exited.

[3]In the baseline model, exits are assumed *irreversible* (i.e., exited partners cannot re-enter the partnership), capturing the idea that re-entry is either impossible or costly in many real-world partnerships. For legal cartels such as the U.S. Navel Orange Administrative Committee, re-entry is usually impossible because of the cartels' administrative power. For illegal cartels, re-entry is possible but costly; for instance, a firm that seeks to rejoin a cartel usually stands in a disadvantageous negotiation position, bears additional administrative cost, and suffers from stigma. Section 5.2 analyzes an extension where re-entry is possible but costly.

[4]The identities of the first and second movers are endogenously determined by the players' strategies. An alternative setting with designated identities is analyzed in Section 5.1.

[5]Such ripple effect is common for cartels. Hellwig and Hüschelrath (2018) document that a firm's exit usually triggers a "domino effect" until the cartel "re-stabilizes" again when a smaller group of firms continue to collude.

size, by which we mean the number of players. There are some sizes under which the players can sustain a *cooperative equilibrium* where no one exits. These are the sizes under which the rippling exits will halt, enabling a player (from a partnership of any size) to calculate the ripple effect if he exits. The calculated ripple effect, in turn, justifies that these sizes can sustain cooperation while others cannot. Theorem 3 formalizes this intuition and concludes that a partnership's ability to sustain cooperation is non-monotonic in its size.[6] This finding stands in contrast to the classic results in a static setting, which suggest that larger groups usually face more severe free-riding problems (Olson, 1965). In particular, the incentives for cooperation in our setting depend on the magnitude of the ripple effect triggered by an initial exit, which does not change monotonically as the partnership grows in size.

Section 5 considers three extensions of the model. First, some players in a partnership may act as leaders. A leader's various functions usually include the implicit commitment to stay to the end — to preserve their reputations, leaders do not exit the partnership before other players. Section 5.1 studies whether and how such a *last-exit commitment* mitigates the problem of strategic exiting. Departing from the baseline model, we analyze an alternative setting where one player (the *leader*) is designated as the second mover, while the other (the *follower*) is designated as the first mover. Proposition 2 shows that the leader's last-exit commitment can result in a Pareto improvement upon the baseline model where either player can exit at any time. For the leader, in particular, the benefit from avoiding pre-emption may outweigh the cost of abandoning the option to exit first.

Second, motivated by the fact that exited partners can sometimes re-enter a partnership, Section 5.2 analyzes a setting where re-entry is possible but costly. We show that the curse of profitability continues to hold if the re-entry cost is not too small. In other words, this paper's insight remains valid to some extent as long as partners cannot step in and out of the partnership for free.

Third, in some partnerships, the operation costs can be flexibly allocated among players, especially when players' contributions are homogeneous and substitutable (e.g., when they are in the form of money). Based on the generalized model, Section 5.3 introduces a designer to allocate the partnership's total cost of operation among the players who have not yet exited. Proposition 3 shows that flexibility of cost allocation enhances a partnership's ability to sustain cooperation.

[6]Theorem 3 shows that the set of sizes that sustain cooperation is generically "scattered" in $\mathbb{N}^+$. For instance, a numerical example in Section 4.3.1 yields the set $\{n^{(1)}, n^{(2)}, n^{(3)}, \ldots\} = \{3, 7, 16, \ldots\}$, where $n^{(k)} = \lceil 2.2 * n^{(k-1)} \rceil$.

Related Literature

First, this paper contributes to the study of dynamic incentives in cooperation (Abreu, Pearce, & Stacchetti, 1990). Our main departure from the literature is that players can exit and continue to free-ride on others' efforts, which, as we already described, is prevalent in real-world partnerships. This distinct feature leads to different implications from the literature regarding how to sustain cooperation — in our setting, the force to sustain cooperation is the rippling exits from a defection, while the majority of the literature focuses on designing the continuation equilibrium to punish a defector (Abreu et al., 1990). Rippling exits also appear in some papers, but not as the force to sustain cooperation. One such example is Cetemen, Urgun, and Yariv (2023), who study collective search within a team where each player can stop searching and implement the best discovery so far; the ripple effect occurs because one's cessation to search discourages others from continuing to search.[7] The main difference between our paper and Cetemen et al. (2023) is that we consider *two-way externalities*; namely, the players who exit early harm the remaining players, but are also harmed if the remaining players later exit. Hence, players in our setting are concerned about the ripple effect triggered by their exits, while such concerns are absent in Cetemen et al. (2023).

Second, this paper is also related to dynamic contribution games, where players individually contribute to a common stock of public goods over time (Admati & Perry, 1991; Fershtman & Nitzan, 1991; Marx & Matthews, 2000). Those papers focus on the ripple effect of one's irreversible investment, while our paper studies the ripple effect of exiting.[8] Such a difference gives rise to different conclusions in our paper. For instance, we find that a partnership's ability to sustain cooperation is non-monotonic in its size (Theorem 3), which does not hold for dynamic contribution games.[9]

Third, this paper is related to voluntary partnership games, where players repeatedly face the prisoner's dilemma and also have the options to opt-out of a partnership (Ghosh & Ray, 1996; Fujiwara-Greve & Okuno-Fujiwara, 2009; McAdams, 2011). Despite the similarity, the purpose

[7]Besides Cetemen et al. (2023), some other examples include models with information externalities where a player's (irreversible) decision reveals his private information that induces others to make the same decision. See, e.g., Rosenberg, Solan, and Vieille (2007), Moscarini and Squintani (2010), Murto and Välimäki (2011), Guo and Roesler (2018), Kirpalani and Madsen (2022), Margaria (2020), Awaya and Krishna (2021), etc.

[8]From a modeling perspective, those papers also concern repeated games with an endogenous state variable, which is the stock of public goods, while in our setting, the endogenous state variable is the number of remaining players.

[9]For instance, in Georgiadis (2015) a large group always outperforms a small one.

of exiting is somewhat opposite — players in our paper strategically exit to free-ride on others' efforts, while in the literature, the intention of an exit is to punish a free-rider.[10]

Fourth, this paper speaks to the discussion of how market demand (or other exogenous factors) affects dynamic cooperation, mostly in the context of cartels. Rotemberg and Saloner (1986) suggest that cartels are less likely to collude during periods of higher demands, because the one-period benefit from cheating is larger. Green and Porter (1984), by contrast, propose a theory where firms become less collusive when the demand is low, because they cannot distinguish low demand from each others' cheating behaviors while they want to deter cheating by being less collusive. Our paper provides a novel channel through which high demand dampens cooperation — it stimulates strategic exiting in the first place as the remaining partners are more willing to operate the project by themselves under high demand.

Finally, this paper contains some ideas related to the literature on farsightedness in cooperative games (Harsanyi, 1974; Chwe, 1994). In particular, Acemoglu, Egorov, and Sonin (2008) study dynamic organizations where members can eliminate some other members. In their paper, players intend to eliminate others in the hope that they themselves are not eliminated in the future. Our paper is conceptually different from Acemoglu et al. (2008), as players intend to exit themselves in the hope that others will not exit in the future.[11]

2 Baseline Model

2.1 Payoff

We consider a continuous-time model with an infinite horizon. Time is indexed by $t \in [0, \infty)$, and the discount rate for each player is $r > 0$. Two players ($i = 1, 2$) form a partnership to run a joint project. Player i's realized lifetime utility is the exponentially discounted sum of his flow payoff, $\Pi_i = \int_0^\infty e^{-rt}\pi_{it}dt$.

[10] Another paper where players can opt-out of a partnership is Chassang (2010), who analyzes a game in which players repeatedly receive noisy signals about a payoff-relevant state, based on which they choose when to exit the partnership. Roughly speaking, in our paper, players may exit to free-ride, whereas in Chassang (2010) they may exit for fear of mis-coordination.

[11] Besides the conceptual difference, we adopt a non-cooperative game approach and allow for a stochastic environment, which differ from Acemoglu et al. (2008).

	Stay	Exit
Stay	$X_t - c\ ,\ X_t - c$	$X_t - \beta c\ ,\ \alpha X_t$
Exit	$\alpha X_t\ ,\ X_t - \beta c$	$0\ ,\ 0$

Table 1: Flow payoff at time t in the baseline model

The flow payoff at time t, (π_{1t}, π_{2t}), is given in Table 1.[12] At each instant, staying in the partnership requires a flow contribution cost of $c > 0$. If both players stay, the project returns a flow output of $X_t > 0$ to each player. We interpret $X_t \in \mathcal{X} = \mathbb{R}^+$ as the project's *profitability* level. It is observable to both players and follows a geometric Brownian motion, $\frac{dX_t}{X_t} = \mu dt + \sigma dZ_t$, where $\mu < r$, $\sigma > 0$, and Z_t is a standard Wiener process.[13]

It is possible, however, for players to exit. If Player i ("he") exits while Player j ("she") stays, Player j suffers a negative externality: Her flow cost rises to βc with $\beta > 1$, since she has to run the entire project alone.[14] We refer to β as the *reliance parameter*, since it measures the extent to which players rely on each other's contributions to operate the project.

As long as Player j keeps operating the project, Player i enjoys some benefits from the partnership's output. His flow revenue becomes αX_t, where $\alpha \in (0, 1)$ is the *free-riding parameter*. We let $\alpha < 1$ to capture the idea that strategic exits are usually subject to some punishment.[15] Finally, when both players exit, the project ceases operation and returns no revenue.

2.2 Timeline

Players choose when to exit the partnership, and their past actions are perfectly observed. To allow players to instantaneously react to their partners' actions, we formulate the model as a two-stage dynamic game *à la* Murto and Välimäki (2013).

[12] This paper speaks to partnerships of all kinds and is not limited to cartels. Hence, we refrain from building a (less general) oligopoly model that only speaks to cartels.

[13] We use Brownian motion to capture the idea that the partnership's profitability is path-dependent. Also, Brownian motion provides some technical convenience that other diffusion processes do not.

[14] We specify the negative externality as an increase in flow cost; an alternative specification would be a decrease in flow revenue. In fact, these two specifications are equivalent up to a normalization. We choose the current one so that the corresponding parameter β is positively related to the extent of externalities. Also, although it would be natural to let $\beta = 2$, we allow for more generality. A value of $\beta < 2$ may arise from the economy of scale, while $\beta > 2$ could arise from additional frictions in the process of shifting responsibility.

[15] For instance, leaving a cartel means losing access to the cartel's resources, such as the marketing and technology support from agricultural associations. Appendix B.2 discusses the limit case where strategic exits are not directly punished, i.e., $\alpha = 1$.

In Stage 1, each player chooses when to exit, given that neither has exited yet. Player i's strategy in Stage 1 is an $\mathcal{H}_t$-adapted stopping time τ^i, where $\mathcal{H}_t$ contains all the information about the public history, including the history of the state variable during $[0,t]$ and the history of players' actions during $[0,t)$. Stage 1 ends at $\tau := \min\{\tau^1, \tau^2\}$. It is possible, however, that both players attempt to exit at the same time (i.e., $\tau^1 = \tau^2$). In case that happens, we make the following tie-breaking assumption: Only one player (selected at random through a coin flip or other fair randomization device) can successfully exit.[16] Given this assumption, whether or not tie-breaking is necessary, there is only one player exiting in Stage 1. We call this player the first mover ("he").

After the first mover exits, the game immediately proceeds to Stage 2, where the remaining player, whom we refer to as the second mover ("she"), chooses when to exit. Her strategy is an $\mathcal{H}_t$-adapted stopping time $\tau^s \geq \tau$. It is possible for the second mover to exit immediately after the first mover (i.e., $\tau^s = \tau$); we call this a *de facto joint exit*. Therefore, the loser of the coin flip (if any) in Stage 1 is essentially given a chance to take back her initial decision to exit; if she still chooses to exit, her exit is regarded as happening in Stage 2 for consistency.

3 Equilibrium

3.1 Stage 2

We use backward induction to solve for the equilibrium. In Stage 2, the second mover is facing an optimal stopping problem: She gets a flow payoff of $X_t - \beta c$ until she exits, at which point she collects a zero lump-sum payoff. As is standard for a time-homogeneous problem of this sort, the second mover's optimal strategy is a (stationary) Markovian decision rule, which can be represented as an *exit region* $\mathcal{X}^s \subseteq \mathcal{X}$ that specifies the value of x under which she exits at time t when $X_t = x$. Let $\gamma := \frac{\sigma^2 - 2\mu - \sqrt{(\sigma^2-2\mu)^2 + 8r\sigma^2}}{2\sigma^2}$ denote the negative root of $\Gamma(y) = \mu y + \frac{\sigma^2}{2} y(y-1) - r = 0$.

Claim 1. *In Stage 2, the second mover's optimal strategy is to adopt the exit region $\mathcal{X}^s = (0, x^*)$ where $x^* := \frac{(r-\mu)\gamma}{r(\gamma-1)}\beta c$. Her corresponding value function at time t when $X_t = x$ is*

[16] This tie-breaking assumption is common in stochastic stopping games (Dutta & Rustichini, 1993; Grenadier, 1996; Weeds, 2002; Murto, 2004). See Appendix B.3 for more discussion.

$$S(x) = \begin{cases} -\frac{\beta c}{r}\left[1-\left(\frac{x}{x^*}\right)^{\gamma}\right] + \frac{x}{r-\mu}\left[1-\left(\frac{x}{x^*}\right)^{\gamma-1}\right] & \textit{when } x > x^*, \\ 0 & \textit{when } x \le x^*. \end{cases}$$

Knowing the second mover's response in Stage 2, we can derive the first mover's lump-sum exit payoff. After exiting, he continues to receive a flow payoff of αX_t until the project is terminated by the second mover; i.e., until the state variable next hits x^* or lower.

Claim 2. *If the first mover exits at time t, his lump-sum payoff upon exit when $X_t = x$ is*

$$F(x) = \begin{cases} \frac{\alpha x}{r-\mu}\left[1-\left(\frac{x}{x^*}\right)^{\gamma-1}\right] & \textit{when } x > x^*, \\ 0 & \textit{when } x \le x^*. \end{cases} \tag{1}$$

Lemma 1. *There exists a unique $\tilde{x} \in (x^*, \infty)$ such that*

$$\begin{aligned} F(x) &= S(x) && \textit{for } x \in (0, x^*], \\ F(x) &> S(x) && \textit{for } x \in (x^*, \tilde{x}), \\ F(x) &= S(x) && \textit{for } x = \tilde{x}, \\ F(x) &< S(x) && \textit{for } x \in (\tilde{x}, \infty). \end{aligned}$$

Lemma 1 indicates that a *first-mover advantage* (i.e., $F(x) > S(x)$) exists in one and only one connected set of values of x, namely, the interval $(x^*, \tilde{x})$, as depicted in Figure 1. Notice that the first-mover advantage is a consequence of the model setup instead of an ad hoc assumption. Appendix A.3 uncovers the subtlety of this lemma.

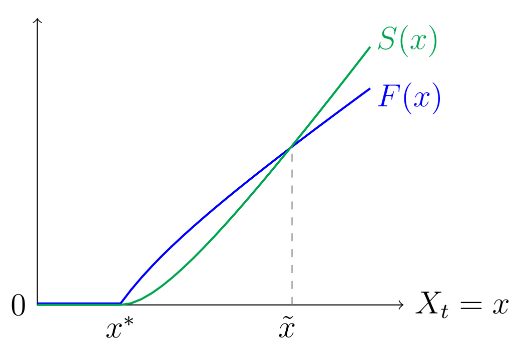


Figure 1: Value functions for the stopping game

3.2 Stage 1

Since the second mover's optimal strategy in Stage 2 is unique (up to the indeterminacy at the threshold x^*), we can induce backwards to Stage 1, where the players are facing the following stopping game. As long as no one has exited, each player receives a flow payoff of $X_t - c$. If one player chooses to exit at time t, he collects a lump-sum payoff of $F(X_t)$ as the first mover, while the remaining player receives a lump-sum payoff of $S(X_t)$ as the second mover.

While the solution concept of this paper is subgame-perfect Nash equilibrium (SPNE), we focus on pure-strategy MPE,[17] a subclass of SPNE where each player adopts a pure strategy that is (stationary) Markovian; i.e., the exit decision is based on the current state X_t alone and does not vary with time. Hence, Player i's strategy in Stage 1 can be represented by an exit region $\mathcal{X}^i \subseteq \mathcal{X}$ that identifies the value of x under which Player i exits at time t when $X_t = x$.

Lemma 2. *In any pure-strategy MPE, both players either always exit or always contribute for all the values of x in the interval $(x^*, \tilde{x})$. That is, the entire interval $(x^*, \tilde{x})$ is either included in or excluded from both players' exit regions.*

Lemma 2 is due to the effect of pre-emption. Notice that $(x^*, \tilde{x})$ is a connected set of values of x that features first-mover advantage. In the presence of first-mover advantage, once a player intends to exit, his partner will react by choosing to exit slightly earlier than he does; unraveling thus occurs as the pre-emption exercise diffuses to the entire connected set where first-mover advantage exists. With this lemma, any pure-strategy MPE must belong to one of the following two types.

Definition 1. *(a) A* ***cooperative equilibrium*** *is one where $(x^*, \tilde{x}) \cap \mathcal{X}^i = \emptyset$ for $i = 1, 2$.*
(b) A ***pre-emptive equilibrium*** *is one where $(x^*, \tilde{x}) \subseteq \mathcal{X}^i$ for $i = 1, 2$.*[18]

3.2.1 Cooperative Equilibria

We first analyze the cooperative equilibria. Notice that it is dominant for players to contribute to the joint project when $X_t \geq \tilde{x}$.[19] Hence, in any cooperative equilibrium, each player's exit

[17] Allowing mixed strategies does not change the main results, as we will discuss in Section 3.3.1.

[18] In a pre-emptive equilibrium, both players intend to exit in Stage 1 when $X_t \in (x^*, \tilde{x})$, but the one who (fails the coin flip and) proceeds to Stage 2 will still contribute until the next time x^* is reached.

[19] Notably, it is weakly dominant to contribute when $X_t = \tilde{x}$. A trivial cooperative equilibrium may exist if both players' exit regions include $\tilde{x}$ and exclude $(x^*, \tilde{x}) \cup (\tilde{x}, \infty)$, but such an equilibrium (if it exists) is always Pareto dominated. Hence, we do not consider this possibility.

region in Stage 1 must be a subset of $(0, x^*]$. This implies that both players will *de facto* jointly exit on the equilibrium path, since the second mover always immediately follows the first mover's exit when the current state $X_t \in (0, x^*]$. This observation enables us to determine the Pareto optimal equilibrium within the set of cooperative equilibria (if any exist). When the two players must jointly exit, they get the best possible outcome by adopting the exit region $(0, x^{**})$, where $x^{**} := \frac{(r-\mu)\gamma}{r(\gamma-1)}c$. This is derived by solving the single-agent optimal stopping problem where the flow payoff is $X_t - c$ and the lump-sum exit payoff is zero. We refer to this outcome as the *optimal cooperative outcome*. To implement it, we can let players choose $\mathcal{X}^1 = \mathcal{X}^2 = (0, x^{**})$.[20] Such a strategy profile gives each player the following value function at time t when $X_t = x$:[21]

$$V_c(x) = \begin{cases} -\frac{c}{r}\left[1 - \left(\frac{x}{x^{**}}\right)^{\gamma}\right] + \frac{x}{r-\mu}\left[1 - \left(\frac{x}{x^{**}}\right)^{\gamma-1}\right] & \text{when } x > x^{**}, \\ 0 & \text{when } x \leq x^{**}. \end{cases} \tag{2}$$

Lemma 3. *Let* $\beta^* := \left[\frac{1-(1-\alpha)^{\gamma}}{\alpha\gamma}\right]^{\frac{1}{1-\gamma}}$.

(a) *When $\beta \geq \beta^*$, there exists a cooperative equilibrium. Among all cooperative equilibria, $\mathcal{X}^1 = \mathcal{X}^2 = (0, x^{**})$ is uniquely Pareto optimal (up to payoff equivalence).*[22]

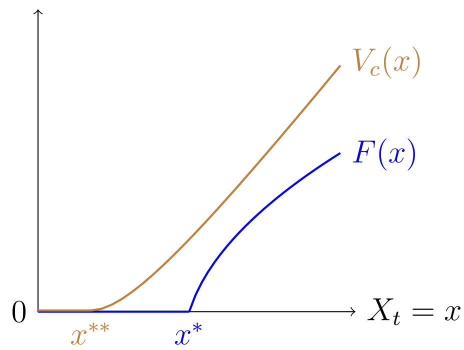
(b) *When $\beta < \beta^*$, no cooperative equilibrium exists.*

Since $V_c(x)$ is the highest possible value to jointly-exiting players at time t when $X_t = x$, the existence of a cooperative equilibrium boils down to whether $V_c(x) \geq F(x)$ holds for all $x \in (x^{**}, \infty)$, so that no one has an incentive to exit early. Lemma 3 shows that the above condition is equivalent to $\beta \geq \beta^*$. Intuitively, a larger β indicates that it is more difficult for the second mover to run the project alone, which, in turn, may deter players from strategic exiting in the first place. This is illustrated by Figure 2, where the deviation payoff is pointwise smaller than $V_c(x)$ if and only if $\beta \geq \beta^*$. We will refer to $\mathcal{X}^1 = \mathcal{X}^2 = (0, x^{**})$ as the *optimal cooperative equilibrium* when $\beta \geq \beta^*$.

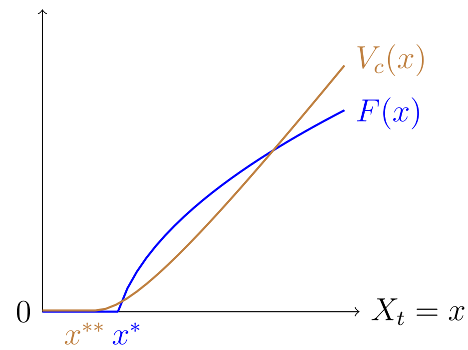
[20] This is not the only strategy profile to implement the optimal cooperative outcome; any strategy profile satisfying $\mathcal{X}^1 \cup \mathcal{X}^2 = (0, x^{**})$ will also do so.

[21] The derivation of x^{**} and $V_c(x)$ is omitted, as it is almost identical to that for Claim 1. Also, we use the subscript c to indicate the cooperative outcome.

[22] There are two sources of equilibrium multiplicity that do not affect players' payoffs. First, whether players should exit at the boundary of the exit region is indeterminate for optimality. Including x^* in the exit region also produces an equilibrium. Second, as shown by Footnote 20, the same equilibrium outcome can be implemented by different strategy profiles. The same argument on payoff equivalence also applies to Lemma 4 and Theorem 1.



(a) $\beta \geq \beta^*$



(b) $\beta < \beta^*$

Figure 2: Existence of cooperative equilibrium

3.2.2 Pre-emptive Equilibria

We now turn to the pre-emptive equilibria, where players always exit in the interval $(x^*, \tilde{x})$. Since it is still dominant for players to contribute when $X_t \geq \tilde{x}$, what remains undetermined in a pre-emptive equilibrium is the players' strategies when the current state $X_t \in (0, x^*]$.

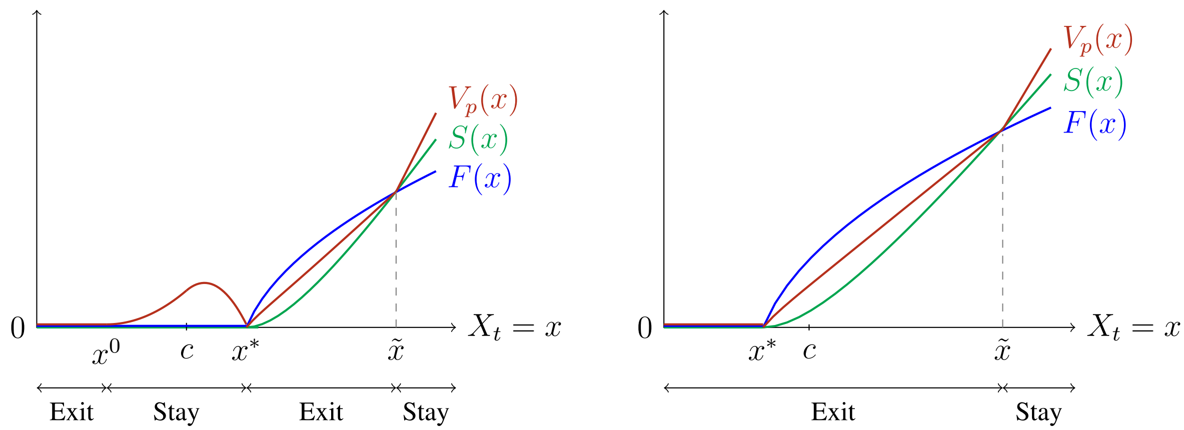
Lemma 4. *A pre-emptive equilibrium always exists. Let* $\beta^{**} := \frac{r}{r-\mu}\frac{\gamma-1}{\gamma}$.
(a) *If $\beta > \beta^{**}$, there are multiple pre-emptive equilibria. There exists $x^0 \in (0, x^*)$ such that $\mathcal{X}^1 = \mathcal{X}^2 = (0, x^0) \cup (x^*, \tilde{x})$ is uniquely Pareto optimal (up to payoff equivalence) among all pre-emptive equilibria.*
(b) *If $\beta \leq \beta^{**}$, the unique (up to payoff equivalence) pre-emptive equilibrium is $\mathcal{X}^1 = \mathcal{X}^2 = (0, \tilde{x})$.*

As Lemma 4 states, unlike cooperative equilibria, pre-emptive equilibria always exist. For instance, $\mathcal{X}^1 = \mathcal{X}^2 = (0, \tilde{x})$ is always an equilibrium.[23] However, this is not necessarily the best pre-emptive equilibrium for players. Notice that $x^* > c$ is possible, so players can gain positive payoffs by jointly contributing to the project when X_t is below x^* but above c.

If $\beta > \beta^{**}$, it follows that $x^* > c$. Similarly to the case of cooperative equilibria, any initial exit when $X_t \in (0, x^*]$ will trigger a *de facto* joint exit. Hence, the *optimal pre-emptive outcome* can be derived by solving the following single-agent stopping problem with $X_t \in (0, x^*)$: The flow payoff is $X_t - c$, the lump-sum exit payoff is zero, and additionally, there exists an exogenous exit point at x^* whose corresponding exit payoff is also zero. The solution to this single-agent problem is to

[23]When $X_t \in (0, x^*]$, each player is indifferent between initiating and not initiating an exit, as long as the other player chooses to trigger a joint exit. So this is an equilibrium.

adopt an exit threshold $x^0 \in (0, x^*)$, which is uniquely determined by value matching and smooth pasting conditions. To implement this outcome, we construct an *optimal pre-emptive equilibrium*, $\mathcal{X}^1 = \mathcal{X}^2 = (0, x^0) \cup (x^*, \tilde{x})$.[24] Figure 3(a) illustrates this equilibrium as well as the corresponding (expected) value function $V_p(x)$ when $X_t = x$.[25] A noteworthy feature of this equilibrium is that the exit strategy does not admit a threshold form, which rarely occurs for stopping games.



(a) $\beta > \beta^{**}$

(b) $\beta \leq \beta^{**}$

Figure 3: Optimal pre-emptive equilibrium

If $\beta \leq \beta^{**}$ (or equivalently, $x^* \leq c$), any joint contribution when the current state $X_t \in (0, x^*]$ is not worthwhile. Hence, the unique pre-emptive equilibrium is $\mathcal{X}^1 = \mathcal{X}^2 = (0, \tilde{x})$.[26] Figure 3(b) depicts this equilibrium and the corresponding value function $V_p(x)$.[27]

3.2.3 Pareto Optimal MPE

The previous analysis indicates three sources of multiplicity in the pure-strategy MPE of this stopping game: (1) When $\beta \geq \beta^*$, cooperative equilibria and pre-emptive equilibria coexist; (2) there

[24] It is worth pointing out the following difference between $(0, x^0)$ and $(x^*, \tilde{x})$. On the equilibrium path, the two players will *de facto* jointly exit when $X_t \in (0, x^0)$, whereas, when $X_t \in (x^*, \tilde{x})$, only one player will successfully exit, and the ex-post second mover will wait until x^* is reached again. Also, this strategy profile is not the only one to implement the optimal pre-emptive outcome as in Footnote 20.

[25] We use the subscript p to indicate pre-emption (cf. $V_c(x)$). The value function $V_p(x)$ can be derived as follows. From the tie-breaking assumption, $V_p(x) = \frac{1}{2}[F(x) + S(x)]$ when $x \in [x^*, \tilde{x}]$. When $x > \tilde{x}$, $V_p(x)$ can be determined using the Feynman–Kac equation plus two boundary conditions, for $x = \tilde{x}$ and $x \to \infty$. When $x < x^*$, $V_p(x)$ and x^0 can be jointly determined using the value matching and smooth pasting conditions at x^0 plus a value matching condition at x^*. Notice that smooth pasting is not required at x^*, as this threshold is not endogenously determined.

[26] Similarly to Footnote 24, $(0, x^*)$ is a region where the players *de facto* jointly exit, while $[x^*, \tilde{x})$ is a region where only one player successfully exits.

[27] The closed form of $V_p(x)$ can be derived using methods similar to those in Footnote 25.

can be multiple cooperative equilibria; (3) there can be multiple pre-emptive equilibria. In the presence of multiplicity, it is natural to adopt the Pareto criterion for equilibrium selection. The second and third sources of multiplicity are thus addressed by focusing on the *optimal cooperative equilibrium* and the *optimal pre-emptive equilibrium* according to Lemmas 3 and 4, respectively.

Lemma 5. *If the optimal cooperative equilibrium exists, it Pareto dominates the optimal pre-emptive equilibrium.*

Lemma 5 resolves the first source of multiplicity. Intuitively, a pre-emptive equilibrium resembles miscoordination whenever a cooperative equilibrium is sustainable. Now we are able to characterize the unique Pareto optimal MPE as below.

Theorem 1. *The Pareto optimal MPE is unique (up to payoff equivalence). If $\beta^{**} < \beta^*$, the equilibrium exit region for both players in Stage 1 is*

$$\mathcal{X}^1 = \mathcal{X}^2 = \begin{cases} (0, x^{**}) & \textit{when } \beta \in [\beta^*, \infty), \\ (0, x^0) \cup (x^*, \tilde{x}) & \textit{when } \beta \in (\beta^{**}, \beta^*), \\ (0, \tilde{x}) & \textit{when } \beta \in (1, \beta^{**}]. \end{cases}$$

If $\beta^{**} \geq \beta^*$, the equilibrium exit region for both players in Stage 1 is

$$\mathcal{X}^1 = \mathcal{X}^2 = \begin{cases} (0, x^{**}) & \textit{when } \beta \in [\beta^*, \infty), \\ (0, \tilde{x}) & \textit{when } \beta \in (1, \beta^*). \end{cases}$$

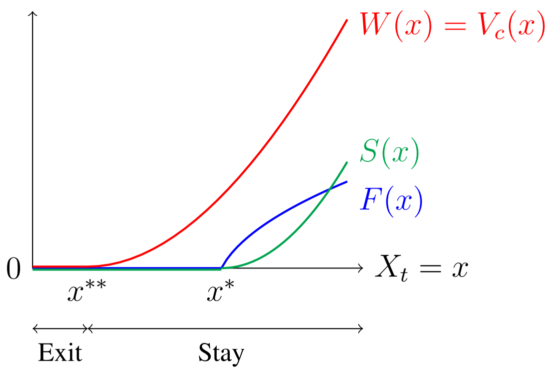
Proof. When $\beta \geq \beta^*$, there exists an optimal cooperative equilibrium (see Lemma 3), which also Pareto dominates the set of pre-emptive equilibria (see Lemma 5). When $\beta < \beta^*$, no cooperative equilibrium exists, so the optimal pre-emptive equilibrium will be Pareto optimal. Its particular form depends on whether $\beta \leq \beta^{**}$ or not (see Lemma 4). □

3.3 Interpretation of Theorem 1

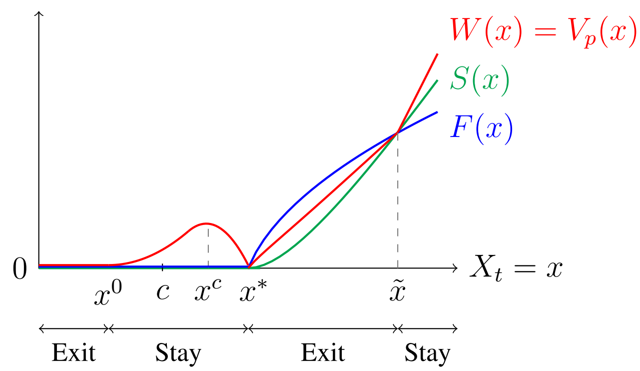
Theorem 1 provides a reasonable prediction for the baseline model. Figure 4 illustrates this result when $\beta^{**} < \beta^*$.[28] Let $W(x)$ denote each player's equilibrium value function at time t when

[28] We omit the case of $\beta^{**} \geq \beta^*$, since this is a degenerate case in which the intermediate scenario vanishes.

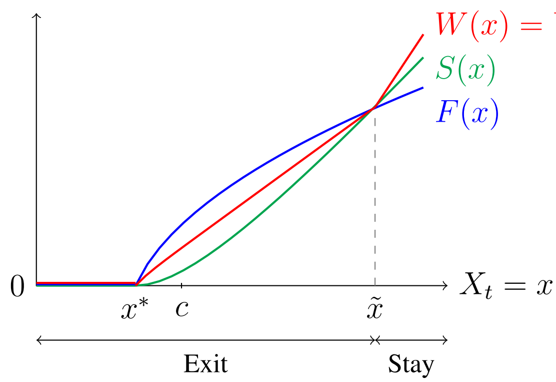
$X_t = x$. This function equals $V_c(x)$ if the optimal cooperative equilibrium exists and $V_p(x)$ if not.



(a) $\beta \geq \beta^*$



(b) $\beta^{**} < \beta < \beta^*$



(c) $1 < \beta \leq \beta^{**}$

Figure 4: Pareto optimal MPE (when $\beta^{**} < \beta^*$)

The most noteworthy property of this MPE is the possibility of a *curse of profitability*, by which we mean that increasing the project's level of profitability may render both players strictly worse off. To be more specific, as depicted in Figure 4(b), when $\beta^{**} < \beta < \beta^*$, $W(x)$ strictly decreases in x when $x \in [x^c, x^*]$ where $x^c := \arg \max_{x \in [x^0, x^*]} W(x)$. [29]

The intuition behind this property is as below. A larger X_t is a double-edged sword. While it means the project generates higher revenue, it also makes it less challenging for the second mover to run the project alone and thus stimulates strategic exiting in the first place. Furthermore, the harm of strategic exiting is amplified by players' pre-emption incentives, which disables players to cooperate when $X_t \in (x^*, \tilde{x})$. As a consequence, players can still cooperate when $X_t \in [x^c, x^*]$,

[29] Actually, the curse of profitability applies to any (possibly Pareto dominated) pre-emptive MPE satisfying $\mathcal{X}^1 = \mathcal{X}^2$ and $(0, x^*) \not\subseteq \mathcal{X}^i$ for $i = 1, 2$ (i.e., any pre-emptive MPE where players do not always exit when $X_t \in (0, x^*)$). See Figure 3(a).

but as X_t increases to approach x^*, players suddenly become enemies and try to pre-empt each other, letting go all the benefits from cooperation.

Indeed, because of pre-emption, even the ex-post first mover may suffer from the curse of profitability. This argument follows from the fact that the ex-post first mover's realized value function, $W(x)\mathbb{1}(x < x^*) + F(x)\mathbb{1}(x \geq x^*)$, also decreases in x when $x \in [x^c, x^*]$. Roughly speaking, the first mover exits "too early" because of pre-emption.

The curse of profitability provides a novel prediction that increasing the level of profitability can be harmful even when it is costless. This provides a plausible explanation to the phenomena that some partnerships are less likely to sustain cooperation as they become more lucrative. One such example is that the founders of a company may "share bitter but not sweet." Another example is that cartels find it harder to collude during periods of higher market demand (Rotemberg & Saloner, 1986).

Besides, Theorem 1 also explains when players can cooperate and avoid pre-emption. In particular, there is a *blessing of reliance* — players are more motivated to cooperate when they rely more heavily on each other's contributions.[30] This argument is captured by the fact that a cooperative equilibrium exists only when the reliance parameter β is sufficiently large. Intuitively, although a large β may exacerbate the second mover's burden upon the first mover's exit, it also enlarges the ripple effect, which decreases the first mover's benefits from strategic exiting. In other words, greater reliance generates a stronger threat against strategic exiting.

Corollary 1. *The value of β^* strictly increases with α and μ , and strictly decreases with σ.*

As we focus on the ripple effect, Theorem 1 regards β as the key parameter when discussing the equilibrium structure, since β is directly associated with the probability of the second mover's rippling exit. Corollary 1 complements the analysis by examining how other parameters relate to β^*, the threshold for β to sustain a cooperative equilibrium. This corollary conveys two messages. First, the cooperative outcome is more difficult to maintain if the joint project is more promising in the future (i.e., μ is larger) or less volatile in its profitability level (i.e., σ is smaller); intuitively, the second mover's high willingness to work alone stimulates strategic exiting. Second, maintaining

[30]The *blessing of reliance* and the *curse of profitability* have different implications. The former emphasizes that players' mutual reliance helps avoid pre-emption, while the latter indicates that, when pre-emption is unavoidable, higher profitability of the partnership may make everyone worse off in their welfare.

cooperation is more difficult if players have greater incentives to free-ride (i.e., α is larger).[31]

3.3.1 Comments on the Solution Concept

We conclude Section 3 by making several comments on the solution concept adopted in this paper. First, we focus on (stationary) MPE for the following reasons. In Stage 2, the second mover faces a single-agent time-homogeneous stopping problem, so the optimal decision rule is (stationary) Markovian. In Stage 1, no one has ever stopped; hence, there is not any variance in players' past actions that we can condition on to use non-Markovian strategies.[32] This stands in contrast to canonical repeated games where it is valuable to punish a player for their past actions.

Second, Section 3 restricts attention to pure strategies of the players. Appendix B.1 complements the analysis by considering mixed-strategy MPE. We show that allowing mixed strategies may bring new MPE, but cannot improve players' continuation values in equilibrium.

Third, we adopt the equilibrium selection criterion of Pareto optimality. In particular, for any value of X_t, any other pure-strategy MPE is Pareto dominated by the equilibrium characterized in Theorem 1 instead of a non-equilibrium strategy profile. Consequently, the equilibrium in Theorem 1 is also the unique *renegotiation-proof* MPE.

Finally, although all the equilibria reported in Section 3 are symmetric (i.e., $\mathcal{X}^1 = \mathcal{X}^2$), we do not rule out asymmetric MPE in the analysis. Examples of asymmetric MPE are shown in Footnotes 20 and 24. Notice that the asymmetry in these equilibria only lies in the region that will trigger *de facto* joint exits. Hence, for any asymmetric MPE, we can find another symmetric MPE that yields the same outcome (by which we mean players' exit time in equilibrium) and value functions for both players.

[31] Theorem 1 and Corollary 1 do not analyze how players' welfare is affected by the parameters β, α, μ, and σ. Our conjecture is that, players' value functions will jump up when each parameter crosses a threshold (e.g., when β crosses β^* upwards) such that the cooperative equilibrium can be sustained and pre-emption can be avoided.

[32] That being said, it is still possible to construct non-Markovian SPNE, in which players' strategies depend on the history of $(X_s)_{s\leq t}$ but not the history of players' actions. For instance, when the Pareto optimal coordinative equilibrium exists, we can let the players adopt this equilibrium (for the continuation game after $t = 0$) if $X_0 \neq x^{\#}$ and the Pareto optimal pre-emptive equilibrium (for the continuation game after $t = 0$) if $X_0 = x^{\#}$, where $x^{\#} > \hat{x}$. However, there is not much value in such construction, as the Pareto optimal MPE is unique.

4 More Than Two Players

In this section we generalize the model to $N \geq 2$ players. Section 4.2 shows that the partnership is, under some parameters, still subject to the curse of profitability. Section 4.3 utilizes the setting to analyze how the partnership's size affects its ability to sustain cooperation. Besides, Appendix B.5 includes some analysis about players' exit patterns in equilibrium.

4.1 Setup

As in the baseline model, each of the N players can choose when to exit the partnership, and those who have exited still enjoy some benefits as long as the project continues. Denote the number of contributors at time t by n_t.

A contributor's flow payoff at time t is $X_t - \beta_{n_t} c$. We assume $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_N > 0$; that is, her flow cost is smaller when there are more people sharing the responsibility.[33] A free-rider's (i.e., a player who has already exited) flow payoff at time t, on the other hand, is $\alpha_{n_t} X_t$. Besides the requirement that $\alpha_0 = 0$ and $\alpha_n \geq 0$ for $n \in \mathbb{N}^+$, we do not place any other assumption on these free-riding parameters.[34]

Finally, we maintain the following tie-breaking assumption: If multiple players attempt to exit at the same time, only one of them (selected at random) will succeed in doing so; the others then have the opportunity to retract their decisions to exit.

4.2 Curse of Profitability

The focus of this section is to show that the curse of profitability may also occur in the unique Pareto optimal MPE when there are more than two players. Some other findings in the baseline model continue to hold with more than two players, but we do not discuss them here.

Theorem 2. *For any group size N, there exist parametric values under which the curse of profitability occurs. That is, every player's value function in the unique Pareto optimal MPE is non-monotonic in the value of X_t.*

[33] A natural choice would be $\beta_n = \frac{N}{n}$, but here, as in the baseline model, we allow for more generality.

[34] In particular, we do not assume that α_n is monotonic in n for the following reason. Intuitively, the value of α_n is related to the total quantity of contribution $n\beta_n$, while $n\beta_n$ is generically non-monotonic in n.

We show Theorem 2 by considering the following parametric example.

Example 1. *Let* $\alpha_n = \alpha > 0$ *if* $n \geq 1$ *and* $\alpha_0 = 0$*; let* $\frac{\beta_{N-1}}{\beta_N} < \beta^*$, $\frac{\beta_{N-2}}{\beta_{N-1}} \geq \beta^*$, *and* $\beta_{N-1} > \beta^{**}$.[35]

In this parametric example, the free-riding parameter α_n is constant as long as the project is operating (i.e., $n \geq 1$). This is reasonable in contexts where the punishment for strategic exiting is based only on the misbehavior per se. It enables us to derive closed-form expressions for the following two value functions that are crucial for the analysis.

The first value function represents the n-player optimal cooperative outcome. Similarly to Section 3.2.1, we solve a single-agent optimal stopping problem for n players. The optimal jointly-exiting threshold is $x_n^* = \frac{(r-\mu)\gamma}{r(\gamma-1)}\beta_n c$, which gives each player the following value function when $X_t = x$:[36]

$$V_n(x) = \begin{cases} -\frac{\beta_n c}{r}\left[1 - \left(\frac{x}{x_n^*}\right)^{\gamma}\right] + \frac{x}{r-\mu}\left[1 - \left(\frac{x}{x_n^*}\right)^{\gamma-1}\right] & \text{when } x \geq x_n^*, \\ 0 & \text{when } x < x_n^*. \end{cases} \tag{3}$$

The second value function concerns any player who have already exited. Such a player will continue to receive αX_t until the project terminates (i.e., until $n_t = 0$). Denote $F_n(x)$ as his value function at time t when $X_t = x$, given that the project will shut down at the threshold x_n^* (which happens, for instance, when there are n remaining contributors and they agree to implement the n-player optimal cooperative outcome).[37] This function takes the following form:

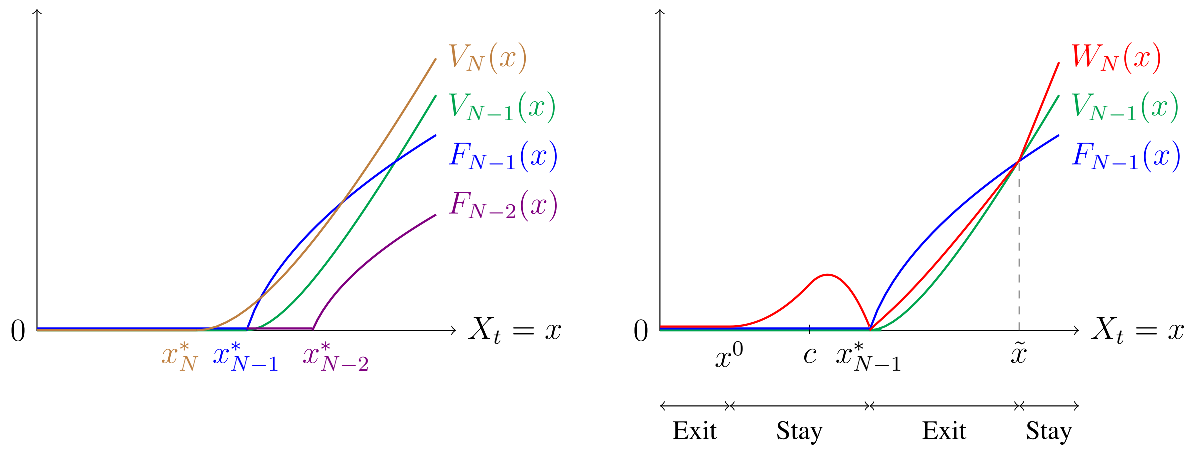
$$F_n(x) = \begin{cases} \frac{\alpha x}{r-\mu}\left[1 - \left(\frac{x}{x_n^*}\right)^{\gamma-1}\right] & \text{when } x \geq x_n^*, \\ 0 & \text{when } x < x_n^*. \end{cases}$$

Panel (a) of Figure 5 depicts the corresponding value functions for Example 1. As Lemma 6 will show (see Appendix A.8), $\frac{\beta_{N-2}}{\beta_{N-1}} \geq \beta^*$ implies that $V_{N-1}(x) \geq F_{N-2}(x)$ for $x \geq x_{N-1}^*$. This

[35] It is possible to construct $\{\beta_n\}_{n=1,\ldots,N}$ that satisfy the conditions specified in Example 1. Notice that β^{**} is a constant that is determined by r, μ, and σ, while the only requirement for β_{N-1} is that it is positive; hence, we can simply let β_{N-1} be larger than β^{**}. After that, we can set $\beta_N \in (\frac{\beta_{N-1}}{\beta^*}, \beta_{N-1})$ and $\beta_{N-2} \in [\beta_{N-1}\beta^*, \infty)$ to fulfill the requirements in the example.

[36] To save on notation, in Section 4 we omit the subscript c from the cooperative value functions.

[37] The baseline model corresponds to the following special case of Section 4: $N = 2$, $\beta_2 = 1$, $\beta_1 = \beta$, $V_2(x) = V_c(x)$, $V_1(x) = S(x)$, and $F_1(x) = F(x)$.



(a) Value functions

(b) Pareto optimal MPE

Figure 5: Curse of profitability when $N > 2$

indicates that the Pareto optimal MPE for $N-1$ players is a cooperative one: In case a player exits, his lump-sum exit payoff is at most $F_{N-2}(x)$ since the remaining players will, in any case, terminate the project when $X_t < x_{N-2}^*$.

Having anticipated the above situation, pre-emption is unavoidable when there are N players for the following reason. In the presence of N players, anyone who exits will gain a lump-sum payoff of $F_{N-1}(x)$. By Lemma 6, $\frac{\beta_{N-1}}{\beta_N} < \beta^*$ implies that $V_N(x) < F_{N-1}(x)$ for some $x > x_N^*$. Similarly to the baseline model, the pre-emptive motive will unravel to the entire interval with first-mover advantage, $(x_{N-1}^*, \tilde{x})$, as depicted in Panel (b) of Figure 5. Meanwhile, the condition $\beta_{N-1} > \beta^{**}$ indicates that $x_{N-1}^* > c$, and consequently, all N players are better off by contributing in the interval (x^0, x_{N-1}^*), giving rise to the curse of profitability in the unique Pareto optimal MPE. That is, the corresponding value function for each of the N players, $W_N(x)$, is non-monotonic in the value of x, as shown in Panel (b) of Figure 5.

4.3 Group Size and Cooperation

Besides the result on the curse of profitability, the setting in Section 4 also enables us to study how the group size of a partnership affects its ability to cooperate. With more than two players, the ripple effect takes the form that an initial exit triggers a second exit, which further triggers a third, and so on. A partnership is able to cooperate only when such a ripple effect deters all the players from strategic exiting in the first place.

To better illustrate the idea, we make two assumptions to simplify the analysis. First, we let $\alpha_n = \alpha > 0$ for any $n \geq 1$ as in Section 4.2. Second, we assume $\beta_1 \leq \beta^{**}$. Both assumptions are relaxed in Appendix B.5 where we show that the main insights of Section 4.3 remain valid in a more general model.

Before stating the characterization result, we analyze an example where a cooperative equilibrium exists with three players, but not with two players.

Example 2. *Let* $\frac{\beta_1}{\beta_2} < \beta^*$ *and* $\frac{\beta_1}{\beta_3} \geq \beta^*$.

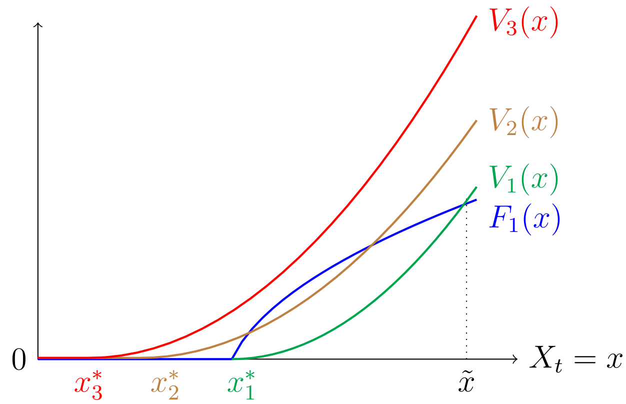


Figure 6: Value functions for Example 2

By Lemma 6 in Appendix A.9, $\frac{\beta_1}{\beta_2} < \beta^*$ implies that $V_2(x) < F_1(x)$ for some $x > x_2^*$. Hence, a two-player cooperative outcome is not an equilibrium. In fact, as we assume $\beta_1 \leq \beta^{**}$, Theorem 1 shows that for two players the unique MPE is that both adopt the exit region $(0, \tilde{x})$ when no one has exited yet.

What if there are three players? If one player deviates from the three-player cooperative outcome at time t, he should foresee the following ripple effect: If $X_t \leq \tilde{x}$, then a second exit will occur immediately; if $X_t > \tilde{x}$, then a second exit will occur later, when $\tilde{x}$ is reached again. In either situation, the third exit (and hence the shutdown of the project) will happen the next time the state variable hits x_1^*; until then, the deviating player continues to receive a flow payoff of αX_t. Hence, his lump-sum exit payoff is $F_1(X_t)$. By Lemma 6, $\frac{\beta_1}{\beta_3} \geq \beta^*$ implies that $V_3(x) \geq F_1(x)$ for all $x > x_3^*$, and thus a three-player cooperative equilibrium exists.

For this example, we can next analyze the case of $N = 4$. Since three players can sustain a cooperative equilibrium, one who deviates from a four-player cooperative outcome will receive αX_t until the state variable next hits x_3^*. Hence, his lump-sum exit payoff is $F_3(X_t)$.[38] Therefore, whether a four-player cooperative equilibrium exists boils down to whether $\frac{\beta_3}{\beta_4} \geq \beta^*$. This argument can be continued inductively to determine the entire set of group sizes where a cooperative equilibrium exists. Theorem 3 provides a general characterization result (that is not limited to Example 2).

Theorem 3. *Let $n^{(0)} = 1$ and $n^{(k)} = \min\left\{n : \frac{\beta_{n^{(k-1)}}}{\beta_n} \geq \beta^*\right\}$. An N-player cooperative equilibrium exists if and only if $N = n^{(k)}$ for some $k \in \mathbb{N}^+$.*

4.3.1 Interpretation of Theorem 3

The characterization result in Theorem 3 is fully determined by β^* and the sequence $\{\beta_n\}_n$. This provides a convenient way to find the set of group sizes that sustain cooperation. Consider the following numerical example: Let $\beta^* = 2.2$ and $\beta_n = \frac{N}{n}$. The set of cooperation-sustaining group sizes is then $\{n^{(1)}, n^{(2)}, n^{(3)}, \ldots\} = \{3, 7, 16, \ldots\}$, where $n^{(k)} = \lceil n^{(k-1)}\beta^* \rceil$. These are exactly the sizes where the ripple effect will halt. If the group size does not belong to this set (e.g., $N = 6$), players will exit one after another until the ripple effect halts at the next cooperation-sustaining size (3 in this example). Afterward, the remaining players will play the cooperative equilibrium.[39]

The exact numbers in the characterization result are unimportant. What is noteworthy is that the ripple effect will endogenously halt at some specific group sizes that are (generically) "scattered" in $\mathbb{N}^+$. Such a finding coincides with the observed pattern in real-world cartels. As documented by Hellwig and Hüschelrath (2018), in case a firm exits a cartel, a domino effect will be likely triggered as more firms gradually exit, and usually, the cartel becomes stabilized again in a smaller group size as some remaining firms continue to collude.

Based on the result above, we conclude that a partnership's ability to sustain cooperation is non-monotonic in its size. This finding stands in contrast to the traditional wisdom that expanding

[38]This argument assumes that the three remaining players will implement the optimal cooperative equilibrium, which is uniquely Pareto optimal for the remaining players (a direct extension of Lemma 5). This assumption is reasonable: When the Pareto optimal equilibrium is unique, Safronov and Strulovici (2018) show that players can always renegotiate to this equilibrium — any player can propose a switch to the unique Pareto optimal equilibrium and the others will approve the proposal. Appendix B.4 discusses the situation without renegotiation.

[39]Appendix B.5 includes more detailed analysis of the exit pattern.

a group will exacerbate the free-riding problem (Olson, 1965). We conclude this section with the following corollary. [40]

Corollary 2. *The set of cooperation-sustaining group sizes is bounded if and only if* $\lim_{n\to\infty} \beta_n > 0$.

5 Extensions

5.1 Last-Exit Commitment of Leaders

In many partnerships, some players play the role of leaders while others are followers. A leader's various functions usually include an implicit commitment to stay to the end — a leader does not exit before all the followers have done so. This section investigates whether such a *last-exit commitment* is rational. Based on the two-player setting, we designate one player (the leader, "she") as the second mover and the other (the follower, "he") as the first mover.[41] The game proceeds in a Stackelberg manner. In Stage 1, the follower chooses an $\tilde{\mathcal{H}}_t$-adapted stopping time $\tilde{\tau}^f$, where $\tilde{\mathcal{H}}_t$ contains information about the public history up to time t. After the follower exits, Stage 2 immediately starts and the leader chooses an $\tilde{\mathcal{H}}_t$-adapted stopping time $\tilde{\tau}^l \geq \tilde{\tau}^f$.

The leader's decision problem in Stage 2 is identical to that of the second mover in the baseline model: She adopts the (stationary) Markovian decision rule that can be represented by an exit region $\mathcal{X}^l = (0, x^*)$. We then induce backwards to the follower's optimal stopping problem in Stage 1: He receives a flow payoff of $X_t - c$ until he exits, then receives a lump-sum payoff of $F(X_t)$. Since this problem is time-homogeneous, it is without loss of generality to consider only a (stationary) Markovian strategy, which can be represented by an exit region $\mathcal{X}^f \subseteq \mathcal{X}$. Under the follower's (unique) optimal strategy, the leader's and follower's value functions in Stage 1 when $X_t = x$ are denoted by $U_l(x)$ and $U_f(x)$, respectively.

Proposition 1. *The follower's optimal exit strategy is unique (up to payoff equivalence).*[42]

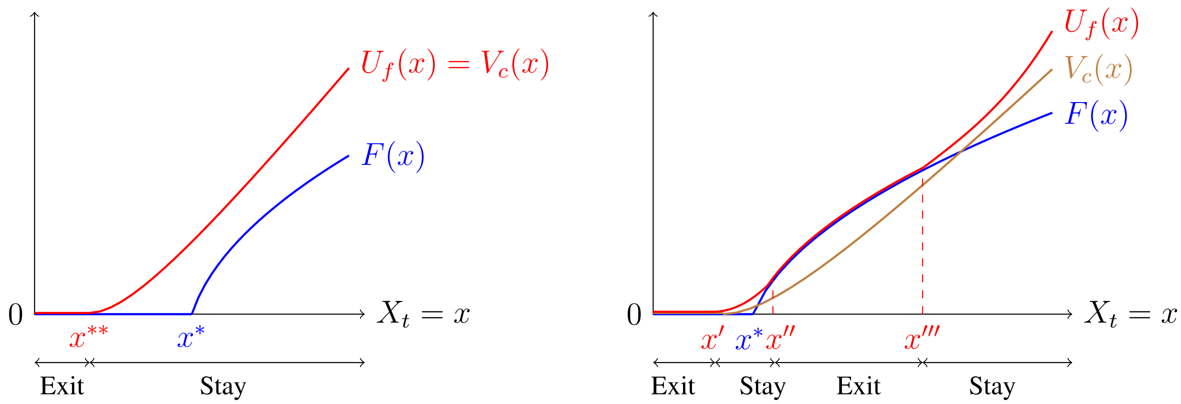
(a) *When* $\beta \geq \beta^*$*, the optimal exit region is* $\mathcal{X}^f = (0, x^{**})$.

[40] The limit $\lim_{n\to\infty} \beta_n$ exists since β_n is weakly decreasing and bounded by zero.

[41] It is worth noting that the follower moves first while the leader moves second. This is the opposite of the order in some classic papers on leadership, where the leader moves first and the follower moves second (Hermalin, 1998).

[42] The only source of payoff-equivalent multiplicity in this single-agent optimal stopping problem is the indeterminacy of whether to exit at the boundary of exit region. This differs from the stopping game in the baseline model.

(b) *When $\beta < \beta^*$, there exist three thresholds, $x' < x'' < x'''$, such that the optimal exit region is $\mathcal{X}^f = (0, x') \cup (x'', x''')$.*



(a) $\beta \geq \beta^*$ (b) $\beta < \beta^*$

Figure 7: The follower's optimal strategy

Proposition 1 shows that the follower's optimal strategy in Stage 1 depends on β. Figure 7(a) illustrates the strategy when $\beta \geq \beta^*$. Here, the follower finds it optimal to implement the optimal cooperative outcome, since $V_c(x) \geq F(x)$ for all $x \in [x^{**}, \infty)$: He will choose to trigger the *de facto* joint exit when X_t is below x^{**}.

When $\beta < \beta^*$, implementing the cooperative outcome is not optimal for the follower. As depicted in Figure 7(b), $U_f(x)$ coincides with $F(x)$ when $x \in \mathcal{X}^f$ and smoothly pastes with $F(x)$ at the three thresholds (x', x'', and x'''), which are pinned down by value matching and smooth pasting conditions (see Appendix A.11).

The noteworthy feature of this optimal strategy when $\beta < \beta^*$ is its non-monotonicity — it does not take the form of a threshold strategy. In particular, the follower stay in two disconnected intervals: $[x''', \infty)$, where strategic exiting is unnecessary, and $[x', x'']$, where strategic exiting is deterred. Strategic exiting occurs only when $X_t \in (x'', x''')$; in this interval, strategic exiting is sufficiently tempting while the concern of ripple effect is mild. To our knowledge, a non-monotonic strategy of this kind is novel to the literature on optimal stopping. It is mainly driven by the non-standard lump-sum exit payoff $F(x)$—in particular, by its kink at x^* (see Claim 3).

Proposition 2. *If $\beta < \beta^*$, there exists $x^l < \tilde{x}$ such that $U_l(x) > W(x)$ when $x > x^l$. In other words, when the current state X_t is large, the leader is better off by committing to exit last.*

The major purpose of the leader's last-exit commitment is to avoid pre-emption. It is natural that the follower should be better off in this situation than in the baseline model; somewhat surprisingly, Proposition 2 indicates that the leader may also be better off.[43] Intuitively, under such a commitment, the benefit to the leader from avoiding pre-emption may outweigh the cost of abandoning the option to exit first. In particular, the cost is less prominent when X_t is large. This may explain the implicit last-exit commitment of leaders in many real-world partnerships.

5.2 Re-entry

In some situations, players who have exited a partnership can return to it. We argue that, when the re-entry cost is large, the curse of profitability continues to hold.[44]

Consider the following variant of the baseline model. We refer to a player as being *active* if he is currently contributing and *inactive* if not. When there is only one active player, the inactive player can choose to re-enter the partnership and become active after paying a fixed cost K. When both players are inactive, the game ends and no one can re-enter.

A pre-emptive equilibrium of this extended game is similar to that of the baseline, except that now an inactive player will return to the partnership when X_t becomes very large (if the other player is active). When $\beta > \beta^{**}$ and K is sufficiently large, there exists a unique Pareto optimal pre-emptive equilibrium, which we depict in Figure 8.[45] It is also the unique Pareto optimal MPE in case a coordinative equilibrium does not exist.

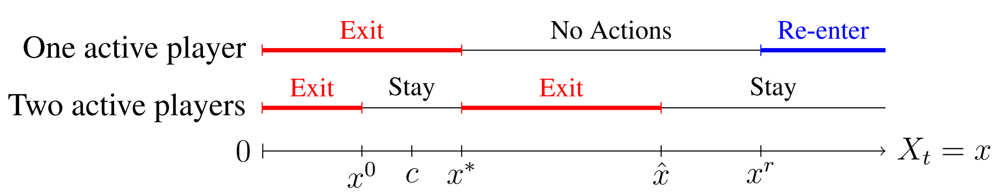


Figure 8: Pareto optimal pre-emptive equilibrium with re-entry ($\beta > \beta^{**}$)

In this equilibrium, when there is only one active player, the active player will exit and terminate the project when X_t falls below x^*, while the inactive player will re-enter the partnership

[43]This finding shares some similarities with earlier studies on how *sequentiality* of moves, compared with *simultaneity*, promotes cooperation in games with strategic complementarities (Zhou & Chen, 2015).

[44]The case of low re-entry cost is studied in Appendix B.6.

[45]Notice that the thresholds (x^0, x^*, and $\hat{x}$) take different values from the baseline. Their values correspond to the baseline when $K = \infty$.

when X_t is above a threshold x^r. When both players are active, their decisions are the same as the baseline: They both intend to pre-emptively exit when $X_t \in (x^*, \hat{x})$, while they terminate the partnership when $X_t < x^0$. In particular, when K is sufficiently large, $x^* > c$ holds, giving rise to the interval (x^0, x^*) where both players had better stay in the partnership together.[46] Because of this interval, the equilibrium exhibits the curse of profitability.[47]

5.3 Flexible Cost Allocation

In some partnerships, especially those that require monetary investments, players' contributed inputs are highly homogeneous. This makes it possible for the operation costs to be allocated flexibly among players. In this section, departing from the N-player benchmark in Section 4, we analyze whether and how flexibility of cost allocation benefits cooperation within a partnership.

We consider a designer who can choose how to allocate C, the total flow cost of running the project, across the players, given that their exit decisions are irreversible. Let $a_{it} = 1$ if Player i is still contributing at time t, and let $a_{it} = 0$ if he has already exited. At each moment t, the designer assigns to Player i a flow cost c_{it} such that (1) $\sum_{i=1}^{N} c_{it} = C$ if at least one player has not exited, and (2) $c_{it} = 0$ if $a_{it} = 0$. A complete plan of $\{c_{it}\}_{t \geq 0, i \in [1,N]}$ based on any information available to the designer is called a *cost allocation scheme*. Player i's flow revenue remains the same as in the benchmark: Given that the project is operating, Player i receives a flow revenue of X_t if $a_{it} = 1$ and αX_t if $a_{it} = 0$. Since the total cost required is now fixed regardless of how many people are contributing, inefficiency occurs when the project is still operating but some players have exited. Hence, the *socially optimal outcome* (subject to the irreversibility constraint) is a joint exit by all players at the threshold $x_{\text{opt}} = \frac{(r-\mu)\gamma}{r(\gamma-1)} \cdot \frac{C}{N}$, which is solved from the optimal stopping problem with flow payoff $NX_t - C$ and zero exit payoff. This outcome gives each player a value function $\tilde{V}(x) = \max\left\{ -\frac{C}{Nr}\left[1 - \left(\frac{x}{x_{\text{opt}}}\right)^{\gamma}\right] + \frac{x}{r-\mu}\left[1 - \left(\frac{x}{x_{\text{opt}}}\right)^{\gamma-1}\right], 0 \right\}$ when $X_t = x$.

Proposition 3. *There exists a cost allocation scheme to implement the socially optimal outcome if and only if $N \geq \beta^*$.*

[46] The threshold x^* increases in K while its value corresponds to that in the baseline model when K goes to infinity. Since $\beta > \beta^{**}$, there exists a threshold $\bar{K}$ such that $x^* > c$ if and only if $K > \bar{K}$.

[47] Notice that this equilibrium assumes players adopt Markovian strategies, i.e., their decisions only depend on the current value of X_t and the current activation status of the other player, but not the other player's past exit and entry history. This can be justified by the criterion of subgame Pareto perfection, which requires that the Pareto optimal equilibrium, if being unique, is adopted for any subgame regardless of the history.

One way to implement the socially optimal outcome is to evenly split the flow cost among the N players before anyone exits. This arrangement is not necessarily sustainable, since strategic exiting may occur. To minimize the players' incentives of strategic exiting, we consider the following cost allocation scheme in case one or more players exit: When $n_t < N$, we place the entire cost C on one of the remaining players. As shown in the proof of Proposition 3, this extreme scheme creates the worst possible punishment for a free-rider, as it maximizes the extent of ripple effect. Under this scheme, an initial exit will trigger the remaining players to exit one by one, and the project will shut down when $x_{\text{punish}} = \frac{(r-\mu)\gamma}{r(\gamma-1)}C$ is reached. Hence, a free-rider's lump-sum exit payoff is $\tilde{F}(x) = \max\left\{\frac{\alpha x}{r-\mu}\left[1-\left(\frac{x}{x_{\text{punish}}}\right)^{\gamma-1}\right], 0\right\}$ when $X_t = x$. To ensure that players do not deviate from the social optimum, we need $\tilde{V}(x) \geq \tilde{F}(x)$ when $x > x_{\text{opt}}$, which is equivalent to $N \geq \beta^*$.

This finding shows that when cost allocation is flexible, a larger group size is better for sustaining cooperation. Also, by comparing with Theorem 3, we see that flexibility of cost allocation enhances a partnership's ability to sustain cooperation. In particular, taking $\beta_n = \frac{N}{n}$ in Section 4, we get the comparable counterpart of this section; in this case, Theorem 3 predicts the set of cooperation-sustaining group sizes to be $\{n^{(1)}, n^{(2)}, n^{(3)}, \ldots\}$ where $n^{(1)} = \lceil\beta^*\rceil$ and $n^{(k)} = \lceil n^{(k-1)}\beta^*\rceil$. This is a strict subset of the prediction of Proposition 3, $\{\lceil\beta^*\rceil, \lceil\beta^*\rceil+1, \lceil\beta^*\rceil+2, \ldots\}$. In other words, flexible cost allocation enables more group sizes to sustain cooperation.

6 Concluding Remarks

In this paper, we study a class of dynamic partnerships where partners can exit and continue to free-ride on others' efforts. We analyze players' incentives of strategic exiting, focusing on how that can be deterred by the ripple effect of exits. We find a curse of profitability — players may actually suffer if the partnership generates high profit. We also show that a partnership's ability to sustain cooperation is non-monotonic in its size, standing in contrast to the traditional wisdom (in a static setting) that larger groups usually face a more severe free-riding problem (Olson, 1965).

Our framework is tractable and can be exploited to address other questions regarding the design of partnerships. In a companion paper, Xu (2023) studies a deterministic partnership where (1) partners are potentially asymmetric in their revenues and costs from the partnership and (2) partners can choose the level of contributed effort over time. We investigate the optimal way for the partners to monitor each other's efforts and show that noisy monitoring of efforts may turn out to facilitate cooperation. In particular, a well designed random auditing rule can help the partnership achieve the first-best outcome where partners exert socially optimal efforts and never exit.

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A Proofs

A.1 Proof of Claim 1

The Hamilton-Jacobian-Bellman equation for the optimal stopping problem is

$$0 = max\left\{0 - S(x), -rS(x) + x - \beta c + S'(x)\mu x + \frac{\sigma^2}{2}S''(x)x^2\right\}. \tag{A1}$$

The general solution for the homogenous ODE, $-rS(x) + x - \beta c + S'(x)\mu x + \frac{\sigma^2}{2}S''(x)x^2 = 0$, is $S(x) = -\frac{\beta c}{r} + \frac{x}{r-\mu} + k_1 x^\gamma + k_2 x^\eta$, where η and γ are the positive and the negative root of $\Gamma(y) = \mu y + \frac{\sigma^2}{2}y(y-1) - r = 0$.[48] The solution to Equation (A1) must also admit this form when $S(x) > 0$. From the boundary condition, $\lim_{x\to\infty}\left[S(x) - \frac{x}{r-\mu} + \frac{\beta c}{r}\right] = 0$, we infer that $k_2 = 0$.[49] The unique optimal exit threshold x^* must satisfy $S(x^*) = 0$ (value matching) and $S'(x^*) = 0$ (smooth pasting). Solving these two conditions yields $x^* = \frac{(r-\mu)\gamma}{r(\gamma-1)}\beta c$. We can also pin down the value of k_1, leading to the closed form in Claim 1.

A.2 Proof of Claim 2

Notice that the first mover is not doing any optimization in Stage 2. When $x < x^*$, his exit immediately triggers the second mover to exit and terminate the project, and thus $F(x) = 0$. When $x \geq x^*$, the Feynman–Kac formula is

$$0 = -rF(x) + \alpha x + F'(x)\mu x + \frac{\sigma^2}{2}F''(x)x^2,$$

whose general solution is in the form of $F(x) = \frac{\alpha x}{r-\mu} + k_3 x^\gamma + k_4 x^\eta$. Then we use two boundary conditions, $F(x^*) = 0$ and $\lim_{x\to\infty}\left[F(x) - \frac{\alpha x}{r-\mu}\right] = 0$, to pin down the coefficients,[50] yielding the closed form in Claim 2, where again $k_4 = 0$.

[48] $\Gamma(y)$ is a convex parabola with $\Gamma(0) = -r < 0$, so it must admit two roots of different signs. Specifically, $\gamma = \frac{\sigma^2-2\mu-\sqrt{(\sigma^2-2\mu)^2+8r\sigma^2}}{2\sigma^2} < 0$ and $\eta = \frac{\sigma^2-2\mu+\sqrt{(\sigma^2-2\mu)^2+8r\sigma^2}}{2\sigma^2} > 1$.

[49] This condition comes from the fact that the probability of the state variable being absorbed by the boundary x^* in finite time, when $X_t \to \infty$, approaches zero. In other words, when $X_t \to \infty$, the option value of exit is zero and thus $S(X_t)$ should be arbitrarily close to $\frac{X_t}{r-\mu} - \frac{\beta c}{r}$, the second mover's value if she never stops.

[50] The rationale of the second boundary condition is the same as Footnote 49.

A.3 Proof of Lemma 1

Claim 3. ***(a)*** *The function $F(x)$ is kinked at $x = x^*$ and strictly concave when $x \in [x^*, \infty)$;*
(b) *The function $S(x)$ is differentiable at $x = x^*$ and strictly convex when $x \in [x^*, \infty)$.*

Proof. The left derivative of $F(x)$ at x^* is 0, while the right derivative equals to $F'_+(x^*) = \frac{\alpha}{r-\mu} - \frac{\alpha\gamma}{(r-\mu)(x^*)^{\gamma-1}}(x^*)^{\gamma-1} = \frac{\alpha(1-\gamma)}{r-\mu} > 0$, so $F(x)$ has a kink at x^*. Also, when $x > x^*$, we have $F''(x) = -\frac{\alpha\gamma(\gamma-1)}{(r-\mu)(x^*)^{\gamma-1}}x^{\gamma-2} < 0$, justifying the strict concavity argument. Differentiability of $S(x)$ at x^* comes directly from the smooth pasting condition, $S'(x^*) = 0$. Strict convexity of $S(x)$ when $x > x^*$ comes from $S''(x) = -\frac{\beta c\gamma}{r(x^*)^\gamma}x^{\gamma-2} > 0$. □

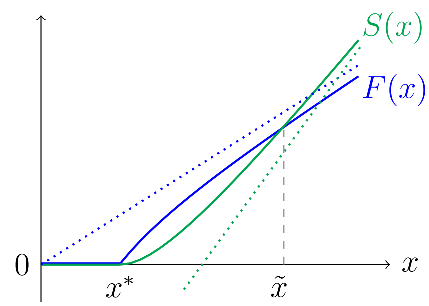

Figure A1: Value functions (solid) and their asymptotic lines (dotted)

As shown in Claim 3, the properties of $F(x)$ are unusual and somewhat subtle. The kink occurs since the exit threshold is not chosen by the first mover. To understand the concavity argument, we rewrite $S(x) = \frac{x}{r-\mu} - \frac{\beta c}{r} + \frac{\beta c}{r(1-\gamma)}\left(\frac{x}{x^*}\right)^\gamma$ and $F(x) = \frac{\alpha x}{r-\mu} - \frac{\alpha\gamma}{r(\gamma-1)}\left(\frac{x}{x^*}\right)^\gamma$ when $x \geq x^*$. Notice that $S(x)$ can be decomposed into a linear part (the first two terms that are linear in x), representing the value if she never exits, and a nonlinear part (the last term), reflecting the option value. In particular, the nonlinear part increases super-linearly as x decreases towards the exit threshold x^*, explaining the convexity of $S(x)$ when $x \geq x^*$. Similarly, $F(x)$ can be decomposed into a linear part (the first term) and a nonlinear part (the second term) where the latter accounts for the termination loss in case the second mover terminates the project. Like the case of $S(x)$, the nonlinear part in $F(x)$ also increases super-linearly as x decreases towards the exit threshold x^*, which accounts for the concavity of $F(x)$ when $x \geq x^*$. This is seen in Figure A1 where each value function's gap with its asymptotic line is exactly the value of the nonlinear part.

Denote $\Delta(x) = F(x) - S(x)$. It is bounded by the two asymptotic lines:

$$\Delta(x) < \frac{\alpha}{r-\mu}x - \frac{1}{r-\mu}x + \frac{\beta c}{r} = \frac{\alpha - 1}{r-\mu}x + \frac{\beta c}{r}. \tag{A2}$$

Therefore, when x is sufficiently large (i.e., $x > \frac{\beta c(r-\mu)}{r(1-\alpha)}$), the RHS of (A2) is negative and thus $\Delta(x) < 0$. Meanwhile, Claim 3 indicates that the right derivative of $\Delta(x)$ is positive at $x = x^*$, which implies that $\Delta(x^* + \epsilon) > 0$ with $\epsilon > 0$ arbitrarily small. By the continuity of $\Delta(x)$, there must be at least one zero of the function $\Delta(x)$ in the range of (x^*, ∞).

Such a zero point is also unique. Due to the convexity of $S(x)$ and the concavity of $F(x)$ as shown in Claim 3, we infer that $\Delta''(x) < 0$. A strictly concave function can at most admit two zero points, which are x^* and $\tilde{x}$ in our case. Strict concavity of $\Delta(x)$ also indicates that $\Delta(x) > 0$ for $x \in (x^*, \tilde{x})$ and $\Delta(x) < 0$ for $x \in (\tilde{x}, \infty)$.

A.4 Proof of Lemma 2

First of all, players' exit regions must be identical in the interval with first-mover advantage. Suppose, by contradiction, $x \in (x^*, \tilde{x})$ while x falls in $\mathcal{X}^i$ but not $\mathcal{X}^j$. Then Player j will be better off by deviating to exit when $X_t = x$, as $\frac{1}{2}(F(x) + S(x)) > S(x)$.

Hence, if Lemma 2 does not hold, for any $\epsilon > 0$, there must be x^+ and x^- in the interval $(x^*, \tilde{x})$ such that: (1) both players exit when $X_t = x^-$ and contribute when $X_t = x^+$; (2) x^+ and x^- are very close so that $|F(x^+) - F(x^-)| < \epsilon$, $|S(x^+) - S(x^-)| < \epsilon$, and $|V(x^+) - V(x^-)| < \epsilon$ where $V(\cdot)$ is each player's value function in the equilibrium. Notice that $V(\cdot)$ is continuous since the stochastic state variable X_t has a continuous path and can evolve in both directions.

Since $V(x^-) = \frac{1}{2}(F(x^-) + S(x^-))$, we infer that $V(x^+) < V(x^-) + \epsilon = \frac{1}{2}(F(x^-) + S(x^-)) + \epsilon < \frac{1}{2}(F(x^+) + S(x^+)) + 2\epsilon$. Together with the fact that $F(x^+)$ is strictly larger than $S(x^+)$ due to first-mover advantage, the above inequality indicates that $V(x^+) < F(x^+)$ when ϵ is sufficiently small. This contradicts the presumption that both players do not want to exit when $X_t = x^+$.

A.5 Proof of Lemma 3

Equation (1) indicates that $F(x)$ is non-increasing in β, while $F(x)$ is strictly concave and $V_c(x)$ is strictly convex when $x \in (x^*, \infty)$ (Claim 3). These properties imply the existence of a crucial value β^* under which $V_c(x)$ tangentially intersects with $F(x)$ when $x \in (x^*, \infty)$. Denote such a point of tangency by $\bar{x}$, so $F(\bar{x}) = V_c(\bar{x})$ and $F'(\bar{x}) = V_c'(\bar{x})$ must hold; i.e.,

$$-\frac{c}{r} + \frac{(1-\alpha)\bar{x}}{r-\mu} + \left(\frac{\bar{x}}{x^{**}}\right)^{\gamma} K = 0 \tag{A3}$$

$$\frac{(1-\alpha)\bar{x}}{r-\mu} + \gamma\left(\frac{\bar{x}}{x^{**}}\right)^{\gamma} K = 0, \tag{A4}$$

where $K = \frac{c}{r} - \frac{x^{**}}{r-\mu} + \frac{\alpha x^*}{r-\mu}\left(\frac{x^{**}}{x^*}\right)^{\gamma} = \frac{\left(1-\alpha\gamma(\beta^*)^{1-\gamma}\right)c}{r(1-\gamma)}$. We then subtract (A4) from (A3)$*\gamma$ and get

$$\bar{x} = \frac{1}{1-\alpha}\frac{r-\mu}{r}\frac{\gamma}{\gamma-1}c = \frac{1}{1-\alpha}x^{**}. \tag{A5}$$

Plugging (A5) into (A4), we get $1-(\beta^*)^{1-\gamma}\alpha\gamma = (1-\alpha)^{\gamma}$, which yields $\beta^* = \left[\frac{1-(1-\alpha)^{\gamma}}{\alpha\gamma}\right]^{\frac{1}{1-\gamma}}$. To complete the analysis, we also need to prove that $\beta^* > 1$. It suffices to show that $\frac{1-(1-\alpha)^{\gamma}}{\alpha\gamma} > 1$; i.e., $\alpha\gamma + (1-\alpha)^{\gamma} > 1$. Let $\Lambda(\alpha) = \alpha\gamma + (1-\alpha)^{\gamma}$. Since $\Lambda(0) = 1$ and $\Lambda'(\alpha) = \gamma[1-(1-\alpha)^{\gamma-1}] > 0$, it is verified that $\Lambda(\alpha) > 1$.

If $\beta \geq \beta^*$, we know from the construction of β^* that $V_c(x) \geq F(x)$ for any x. Thus there exists a cooperative equilibrium. If $\beta < \beta^*$, we need to show that players want to deviate when X_t falls in $(x^*, \tilde{x})$, the interval with first-mover advantage.

Claim 4. *When $\beta < \beta^*$, there exists $x \in (x^*, \tilde{x})$ such that $F(x) > V_c(x)$.*

Proof. According to the construction of β^*, $F(x)$ and $V_c(x)$ must intersect (non-tangentially) in the range of (x^*, ∞) when $\beta < \beta^*$. Moreover, the intersections must be in the range of $(x^*, \tilde{x})$, since $V_c(x) > S(x) \geq F(x)$ when $x \geq \tilde{x}$. □

A.6 Proof of Lemma 5

It suffices to show that when $\beta \geq \beta^*$, $V_c(x) \geq V_p(x)$ for all x. We break the proof for three different ranges of x. First, when $x \in [x^*, \tilde{x}]$, the existence of the optimal cooperative equilibrium implies that $V_c(x) \geq F(x)$, while $V_p(x) = \frac{1}{2}[S(x) + F(x)] < F(x)$. These two inequalities combined give $V_c(x) > V_p(x)$. Second, when $x > \tilde{x}$, we regard $V_c(x)$ (resp. $V_p(x)$) as the value to one who receives $X_t - c$ until exiting with a lump-sum payoff $V_c(\tilde{x})$ (resp. $V_p(\tilde{x})$) when $\tilde{x}$ is reached. Hence, the above argument holds when x falls in this range because $V_c(\tilde{x}) > V_p(\tilde{x})$. Third, when $x < x^*$, the argument still holds for a reason similar to the second case but now we exploit the fact that $V_c(x^*) > V_p(x^*)$.

A.7 Proof of Corollary 1

For α: It is straightforward from $\frac{\partial\left[\frac{1-(1-\alpha)^\gamma}{\alpha\gamma}\right]}{\partial\alpha} = \frac{(1-\alpha)^{\gamma-1}[\alpha(1+\gamma)-1]-1}{\gamma\alpha^2} > 0$.

For μ and σ: Since γ is the negative root of $\Gamma(y;\mu,\sigma) = \mu y + \frac{\sigma^2}{2}y(y-1) - r$, we know that $\frac{\partial\Gamma}{\partial y}|_{y=\gamma} < 0$, $\frac{\partial\Gamma}{\partial\mu}|_{y=\gamma} < 0$, and $\frac{\partial\Gamma}{\partial\sigma}|_{y=\gamma} > 0$. This implies that γ strictly decreases with μ and increases with σ. Hence, it suffices to show that β^* strictly decreases with γ.

We first replace $\frac{1}{1-\alpha}$ by z and let $\beta^* = f(\gamma) := \left[\frac{z^{1-\gamma}-z}{-(z-1)\gamma}\right]^{\frac{1}{1-\gamma}}$. We would like to show that $f'(\gamma) < 0$. Let $g(\gamma) = [f(\gamma)]^{1-\gamma}$ and $h(\gamma) = ln(g(\gamma))$. Since

$$
\begin{aligned}
f'(\gamma) &= \frac{1}{1-\gamma} g(\gamma)^{\frac{1}{1-\gamma}-1} g'(\gamma) + g(\gamma)^{\frac{1}{1-\gamma}} ln(g(\gamma)) \frac{1}{(1-\gamma)^2} \\
&= \frac{g(\gamma)^{\frac{1}{1-\gamma}}}{1-\gamma}\left[\frac{g'(\gamma)}{g(\gamma)} + \frac{ln(g(\gamma))}{1-\gamma}\right] \\
&= \frac{f(\gamma)}{1-\gamma}\left[h'(\gamma) - \frac{h(1)-h(\gamma)}{1-\gamma}\right],
\end{aligned}
$$

it suffices to show that

$$
h'(\gamma) - \frac{h(1)-h(\gamma)}{1-\gamma} < 0. \tag{A6}
$$

Notice that $\frac{h(1)-h(\gamma)}{1-\gamma}$ is the slope of the secant line between γ and 1 on the curve of $h(\cdot)$, one sufficient condition for (A6) to hold is that $h(\gamma)$ is convex; i.e., $g(\gamma)$ is log-convex.

To prove the log-convexity of $g(\gamma)$, we only need to show that $z^{\gamma y}g(\gamma)$ is convex for any

$y \in \mathbb{R}$.[51] We adopt Taylor Expansion w.r.t. γ on $z^{\gamma y} g(\gamma)$ as below

$$\begin{aligned} z^{\gamma y} g(\gamma) &= \frac{z}{(z-1)\gamma} \left(z^{\gamma y} - z^{\gamma y - \gamma} \right) \\ &= \frac{z}{(z-1)\gamma} \left[\sum_{n=0}^{\infty} (y ln(z))^n \gamma^n - \sum_{n=0}^{\infty} ((y-1) ln(z))^n \gamma^n \right] \\ &= \frac{z}{(z-1)} \sum_{n=1}^{\infty} (ln(z))^n \left[y^n - (y-1)^n \right] \gamma^{n-1}. \end{aligned}$$

Since $z > 1$, and $y^n - (y-1)^n > 0$ for all $n \geq 1$ and $y \in \mathbb{R}$, we conclude that $z^{\gamma y} g(\gamma)$ is convex.

A.8 Proof of Theorem 2

The only missing piece of this proof is the following lemma. It generalizes Lemma 3 and derives the crucial condition such that $V_n(x)$ and $F_{n'}(x)$ tangentially intersect for given $n > n'$.

Lemma 6. ***(a)*** *if* $\frac{\beta_{n'}}{\beta_n} \geq \beta^*$, $V_n(x) \geq F_{n'}(x)$ *for all* x*;*
(b) *if* $\frac{\beta_{n'}}{\beta_n} < \beta^*$, *there exist some values of* x *such that* $V_n(x) < F_{n'}(x)$. [52]

Proof. The proof is similar to Appendix A.5. The counterparts of Equations (A4) and (A5) are

$$\frac{(1-\alpha)\bar{x}}{r-\mu} + \gamma \left(\frac{\bar{x}}{x_n^*} \right)^{\gamma} K' = 0 \qquad \text{(A7)}$$

$$\bar{x} = \frac{1}{1-\alpha} \frac{r-\mu}{r} \frac{\gamma}{\gamma-1} \beta_n c = \frac{1}{1-\alpha} x_n^*, \qquad \text{(A8)}$$

where $K' = \frac{\beta_n c}{r} - \frac{x_n^*}{r-\mu} + \frac{\alpha x_{n'}^*}{r-\mu} \left(\frac{x_n^*}{x_{n'}^*} \right)^{\gamma} = \frac{\left(1 - \alpha\gamma \left(\frac{\beta_n}{\beta_{n'}}\right)^{-\gamma}\right) \beta_n c}{r(1-\gamma)}$. Substitute (A8) into (A7), we get

$$1 - \left(\frac{\beta_{n'}}{\beta_n} \right)^{1-\gamma} \alpha\gamma = (1-\alpha)^{\gamma}.$$

Hence, $\frac{\beta_{n'}}{\beta_n} = \beta^*$ is the crucial condition where $V_n(x)$ tangentially intersects with $F_{n'}(x)$. □

[51] See Page 70 of Niculescu and Persson (2006) for reference.
[52] Lemma 3 corresponds to the following special case: $n' = 1$, $n = 2$, $\beta_1 = \beta$, and $\beta_2 = 1$.

A.9 Proof of Theorem 3

We prove the theorem by induction. The cooperative outcome is not an equilibrium when the group size N is larger than 1 and smaller than $n^{(1)}$, since players' exit payoff is $F_1(X_t)$, which may outweigh the value from cooperating, $V_N(X_t)$, for some values of X_t. Meanwhile, a $n^{(1)}$-player cooperative equilibrium exists by the definition of $n^{(1)}$.

By induction, if n is larger than $n^{(i-1)}$ and smaller than $n^{(i)}+1$, an initial exit gives a lump-sum payoff of $F_{n^{(i-1)}}(X_t)$, since the ripple effect halts when $n^{(i-1)}$ players are remaining; i.e., the project will run until $x^*_{n^{(i-1)}}$ is reached. By Lemma 6, we conclude that a cooperative outcome is an equilibrium when $n = n^{(i)}$, but not an equilibrium when $n \in [n^{(i-1)}+1, n^{(i)}-1]$.

A.10 Proof of Corollary 2

We first prove "only if." Suppose M is the largest group size to maintain a cooperative equilibrium. By Lemma 6, $\beta_n \geq \frac{\beta_M}{\beta^*}$ for any $n > M$. Since $\frac{\beta_M}{\beta^*}$ is bounded away from 0, we conclude that β_n does not converge to zero.

We then prove "if." By Lemma 6, $\beta_{n^{(i)}} \leq \frac{\beta_1}{(\beta^*)^{i-1}}$. Hence, the maximal number of cooperation-sustainable group size $I = \left\lceil log_{\beta^*}\left(\frac{\beta_1}{\lim\limits_{n\to\infty}\beta_n}\right)\right\rceil$ is finite since $\beta^* > 1$ and $\lim\limits_{n\to\infty}\beta_n > 0$. This indicates that the corresponding set must be bounded.

A.11 Proof of Proposition 1

The associated Hamilton-Jacobian-Bellman equation for this optimal stopping problem is

$$0 = max\{F(x) - U_f(x), -rU_f(x) + x - c + U_f'(x)\mu x + \frac{\sigma^2}{2}U_f''(x)x^2\}.$$

According to Strulovici and Szydlowski (2015), to verify that a constructed $U_f(x)$ satisfies this HJB equation, it suffices to check three conditions: (1) $U_f(x) \geq F(x)$ for all x; (2) $U_f(x)$ is everywhere continuous and first-order differentiable; (3) $-rU_f(x)+x-c+U_f'(x)\mu x+\frac{\sigma^2}{2}U_f''(x)x^2 \leq 0$ when $U_f(x) = F(x)$. For Scenario (a), we already obtain the closed-form of $U_f(x)$ and it is not difficult to check that all three conditions are satisfied. For Scenario (b), it suffices to construct three thresholds, (x', x'', x'''), such that $0 < x' < x^* < x'' < x'''$ while the corresponding smooth pasting and value matching conditions are satisfied.

We start from x''' whose closed-form is exactly pinned down by Equations (A3) and (A4). Hence, $x''' = \frac{1}{1-\alpha}x^{**}$ as in (A5). For x' and x'', the analysis takes the following two steps.

Step 1: Existence. Let the general solution of $U_f(x)$ when $x \in [x', x'']$ be $U_f(x) = -\frac{c}{r} + \frac{x}{r-\mu} + k_5 x^\gamma + k_6 x^\eta$. Notice that k_6 is not necessarily zero as the boundary condition when $x \to \infty$ no longer holds. The value matching and smooth pasting conditions for these two thresholds are

$$\begin{aligned} -\frac{c}{r} + \frac{x'}{r-\mu} + k_5 \cdot (x')^\gamma + k_6 \cdot (x')^\eta &= 0 \quad &\text{(A9)} \\ \frac{x'}{r-\mu} + \gamma k_5 \cdot (x')^\gamma + \eta k_6 \cdot (x')^\eta &= 0 \quad &\text{(A10)} \\ -\frac{c}{r} + \frac{(1-\alpha)x''}{r-\mu} + (k_5 - k_3) \cdot (x'')^\gamma + k_6 \cdot (x'')^\eta &= 0 \quad &\text{(A11)} \\ \frac{(1-\alpha)x''}{r-\mu} + \gamma(k_5 - k_3) \cdot (x'')^\gamma + \eta k_6 \cdot (x'')^\eta &= 0. \quad &\text{(A12)} \end{aligned}$$

Claim 5. *$k_6 > 0$ and $k_5 > k_3$.*

Proof. Let (A12) − (A11)$*\gamma$ and (A12) − (A11)$*\eta$, we have

$$\frac{c\gamma}{r} + (1-\gamma)\frac{(1-\alpha)x''}{r-\mu} + k_6 \cdot (\eta - \gamma)(x'')^\eta = 0 \quad \text{(A13)}$$

$$\frac{c\eta}{r} + (1-\eta)\frac{(1-\alpha)x''}{r-\mu} + (k_5 - k_3) \cdot (\gamma - \eta)(x'')^\gamma = 0 \quad \text{(A14)}$$

By construction, we require that $x'' < x''' = \frac{x^{**}}{1-\alpha}$. Plugging it into (A13) yields $k_6 > 0$. Plugging it into (A14), we get $\frac{c\eta}{r} + (1-\eta)\frac{(1-\alpha)x''}{r-\mu} > \frac{c(\eta-\gamma)}{r(1-\gamma)} > 0$, which indicates that $k_5 > k_3$. □

From (A10) − (A9)$*\gamma$ and (A10) − (A9)$*\eta$, we have

$$\begin{aligned} \frac{c\gamma}{r} + (1-\gamma)\frac{x'}{r-\mu} + k_6 \cdot (\eta - \gamma)(x')^\eta &= 0 \\ \frac{c\eta}{r} + (1-\eta)\frac{x'}{r-\mu} + k_5 \cdot (\gamma - \eta)(x')^\gamma &= 0. \end{aligned}$$

We can thus express k_5 and k_6 as functions of x':

$$k_6(x') = \frac{c\gamma}{r(\gamma-\eta)}(x')^{-\eta} + \frac{1-\gamma}{(r-\mu)(\gamma-\eta)}(x')^{1-\eta} \tag{A15}$$

$$k_5(x') = \frac{c\eta}{r(\eta-\gamma)}(x')^{-\gamma} + \frac{1-\eta}{(r-\mu)(\eta-\gamma)}(x')^{1-\gamma}. \tag{A16}$$

We then construct the following function that takes z as a parameter,

$$\tilde{U}(x;z) = -\frac{c}{r} + \frac{x}{r-\mu} + k_5(z)x^{\gamma} + k_6(z)x^{\eta}. \tag{A17}$$

It suffices to find a value of z such that $\tilde{U}(x;z)$ tangentially intersects with $F(x)$ in the range of $x \in (x^*, \infty)$. Specifically, Equations (A9) to (A12) are satisfied by letting $x' = z$ and x'' be the point of tangency. Denote $\tilde{\Delta}(x;z) = \tilde{U}(x;z) - F(x)$.

Claim 6. *For any $z > 0$, $\tilde{\Delta}(x;z)$ is strictly convex in x.*

Proof. $\tilde{\Delta}''(x;z) = \gamma(\gamma-1)(k_5-k_3)x^{\gamma-2} + \eta(\eta-1)k_6x^{\eta-2} > 0$, as we already know from Claim 5 that $k_6 > 0$ and $k_5 > k_3$, together with $\gamma < 0$ and $\eta > 1$. □

On one hand, $\tilde{\Delta}(x;x^{**}) = V_c(x)$. Since $\beta < \beta^*$, we infer that $\inf_{x\in(x^*,\infty)} \tilde{\Delta}(x;x^{**}) < 0$ as $V_c(x)$ (non-tangentially) intersects with $F(x)$. On the other hand, for $\epsilon > 0$ sufficiently small, it is not difficult to see that $\inf_{x\in[x^*,\infty)} \tilde{\Delta}(x;\epsilon) > 0$. By continuity of $\tilde{\Delta}(x;z)$ w.r.t. z, there must exist $x' \in (0, x^{**})$ such that $\inf_{x\in[x^*,\infty)} \tilde{\Delta}(x;x') = 0$. According to the strict convexity of $\tilde{\Delta}(\cdot;x')$ (Claim 6), $\tilde{\Delta}(x^*;x') > 0$, and $\tilde{\Delta}(\infty;x') = \infty$, we know the infimum is uniquely attainable. Let this point of infimum be x''. We can verify that $\tilde{\Delta}(x'';x') = 0$, $\tilde{\Delta}'(x'';x') = 0$, and $\tilde{\Delta}(x;x') > 0$ for $x \in [x^*,\infty)/\{x''\}$. In other words, $\tilde{U}(x;x')$ smoothly pastes with $F(x)$ at x' and x'', while satisfying $\tilde{U}(x;x') > F(x)$ for $x \in (x', x'')$.

To conclude on the existence, we finally check that the constructed x'' is consistent with the presumption that $x'' < x'''$. Combining (A13) and (A15), we get

$$\frac{c\gamma}{r}\left[(x'')^{\eta} - (x')^{\eta}\right] + \frac{(1-\gamma)}{r-\mu}\left[x'(x'')^{\eta} - (1-\alpha)x''(x')^{\eta}\right] = 0,$$

which gives us $x'' < \frac{1}{1-\alpha}x' < \frac{1}{1-\alpha}x^{**} = x'''$.

Step 2: Uniqueness. To prove that there exists only one z satisfying $\inf_{x\in[x^*,\infty)} \tilde{\Delta}(x;z) = 0$, it suffices to show that $\inf_{x\in[x^*,\infty)} \tilde{\Delta}(x;z)$ is single-crossing w.r.t. z (i.e., crosses the horizontal axis only once) when $\inf_{x\in[x^*,\infty)} \tilde{\Delta}(x;z) = 0$. By the Envelope Theorem, we only need to show that $\frac{\partial\tilde{\Delta}}{\partial z}(x'', x')$ is either always positive or always negative.

$$\begin{aligned}
\frac{\partial\tilde{\Delta}}{\partial z}(x'', x')\cdot x' &= k_5'(x')x'\,(x'')^\gamma + k_6'(x')x'\,(x'')^\eta \\
&= (1-\gamma)k_5\cdot(x'')^\gamma + (1-\eta)k_6\cdot(x'')^\eta + \frac{c}{r(\eta-\gamma)}\left[\gamma\left(\frac{x''}{x'}\right)^\eta - \eta\left(\frac{x''}{x'}\right)^\gamma\right] \\
&= \frac{c}{r} + k_3\cdot(1-\gamma)\,(x'')^\gamma + \frac{c}{r(\eta-\gamma)}\left[\gamma\left(\frac{x''}{x'}\right)^\eta - \eta\left(\frac{x''}{x'}\right)^\gamma\right] \\
&= k_3\cdot(1-\gamma)\,(x'')^\gamma + \frac{c}{r(\eta-\gamma)}\left[\eta-\gamma+\gamma\left(\frac{x''}{x'}\right)^\eta - \eta\left(\frac{x''}{x'}\right)^\gamma\right] \\
&< k_3\cdot(1-\gamma)\,(x'')^\gamma + \frac{c}{r(\eta-\gamma)}\left[\eta-\gamma+\gamma-\eta\right] \\
&= k_3\cdot(1-\gamma)\,(x'')^\gamma < 0.
\end{aligned}$$

The first equality results from (A17). The second equality is obtained by plugging in the derivatives of $k_5(x)$ and $k_6(x)$ according to (A15) and (A16). The third equality makes use of (A11) and (A12). The fourth equality combines like terms. The first inequality holds because the function $\Phi(y) = \gamma y^\eta - \eta y^\gamma$, when $y \geq 1$, strictly decreases in y, while $\frac{x''}{x'} > 1$. The last inequality comes from $k_3 < 0$. We can eventually conclude that the single-crossing condition holds, so there exists a unique pair of (x', x'') satisfying Equations (A9) to (A12).

A.12 Proof of Proposition 2

First, we show that $x''' < \tilde{x}$. This is because x''' is smaller than the largest intersection of $V_c(x)$ and $F(x)$ according to the analysis in Appendix A.11, while $\tilde{x}$ must be larger than that intersection since $V_c(\tilde{x}) > S(\tilde{x}) = F(\tilde{x})$. Roughly speaking, the follower in Section 5.1 exits less aggressively than the (endogenous) first mover in the baseline model, due to the lack of pre-emption motives.

We then prove Proposition 2 for $x = \tilde{x}$. Notice that $S(\tilde{x})$ is equivalent to the value to a player (when $X_t = \tilde{x}$) who keeps receiving a flow payoff of $X_t - \beta c$ until exogenously exiting at x''' with a lump-sum payoff of $S(x''')$. Meanwhile, $U_l(\tilde{x})$ is equivalent to the value to a player (when $X_t = \tilde{x}$) who keeps receiving a flow payoff of $X_t - c$ until exogenously exiting at x''' with a lump-sum payoff of $S(x''')$. Comparing these two scenarios, we conclude that $U_l(\tilde{x}) > S(\tilde{x})$, as the flow payoff is larger. This further indicates that $U_l(\tilde{x}) > W(\tilde{x})$ since $W(\tilde{x}) = V_p(\tilde{x}) = S(\tilde{x})$ as shown by Panels (b) and (c) of Figure 4.

For $x > \tilde{x}$, $W(x)$ equals the value to a player (when $X_t = x$) who keeps receiving a flow payoff of $X_t - c$ until exogenously exiting at $\tilde{x}$ with a lump-sum payoff of $W(\tilde{x})$. Meanwhile, $U_l(x)$ equals the value to a player (when $X_t = x$) who keeps receiving a flow payoff of $X_t - c$ until exogenously exiting at $\tilde{x}$ with a lump-sum payoff of $U_l(\tilde{x})$. These two arguments, together with $U_l(\tilde{x}) > W(\tilde{x})$, indicate that $U_l(x) > W(x)$ when $x > \tilde{x}$.

The two paragraphs above show that $U_l(x)$ is strictly greater than $W(x)$ when $x \geq \tilde{x}$. Finally, according to the continuity of $U_l(x)$ and $W(x)$, there must exist $x^l < \tilde{x}$ such that $U_l(x) \geq W(x)$ when $x \geq x^l$.

A.13 Proof of Proposition 3

Step 1. We first show that, under the punishment scheme specified in Section 5.3, the project will be terminated when the state variable next reaches x_{punish}. Without loss of generality, let Player 1 be the initial free-rider (i.e., the first player to exit) and allocate the entire cost to Player i in Stage i (i.e., $i - 1$ players already exited) for $i \in \{2, 3, ..., N\}$.

In Stage N, Player N will adopt the exit threshold at x_{punish}. In Stage $(N-1)$, Player $(N-1)$ is facing a stopping problem with a flow payoff of $X_t - C$ and a lump-sum payoff of $\tilde{F}(X_t)$ if exiting at time t. We can show that the optimal strategy is to adopt the exit threshold at $\frac{x_{\text{punish}}}{1-\alpha}$. Then we induce backward to Stage $(N-2)$: After Player $(N-2)$ exits, the project will keep operating until the next time x_{punish} is reached. Hence, the lump-sum exit payoff for Player $(N-2)$ is also $\tilde{F}(X_t)$ and his exit threshold is also $\frac{x_{\text{punish}}}{1-\alpha}$. By induction, we infer that the lump-sum exit payoff for Player 1 is also $\tilde{F}(X_t)$ if he exits at time t.

Step 2. Substitute $\beta_{n'} = N$ and $\beta_n = 1$ into Lemma 6(a), we find that $N \geq \beta^*$ gives $\tilde{V}(x) \geq \tilde{F}(x)$ for any $x > x_{\text{opt}}$. Hence, the social optimum is implementable if $N \geq \beta^*$.

Meanwhile, the social optimum is not achievable if $N < \beta^*$. According to Lemma 6(b), there exists $x > x_{\text{opt}}$ such that $\tilde{V}(x) < \tilde{F}(x)$ due to $N < \beta^*$, while it is impossible for all players to achieve a higher value than $\tilde{V}(x)$. Therefore, strategic exiting must occur.

B Online Appendix

B.1 Mixed-Strategy MPE

This section analyzes the mixed-strategy MPE for Section 3. When $\beta \geq \beta^*$, the pure-strategy MPE specified in Theorem 1 already achieves the Pareto optimal outcome, so introducing mixed strategies cannot improve players' values. Therefore, we focus on the case with $\beta < \beta^*$, and will show that players cannot do better with mixed strategies as well.

We express Player i's (potentially mixed) Markovian strategy as $(\lambda_i(x), \psi_i(x))$ so that when $X_t = x$, Player i exits with probability

$$p_i(x) = \begin{cases} \lambda_i(x) & \text{, when } \lambda_i(x) > 0 \\ \psi_i(x)dt & \text{, when } \lambda_i(x) = 0. \end{cases}$$

For consistency, we let $\psi_i(x) = 0$ whenever $\lambda_i(x) > 0$.

Notice that the value functions have the following property: $(x^*, \tilde{x})$ is the only interval with *first-mover advantage*, while $(\tilde{x}, \infty)$ is the only interval with *second-mover advantage*. In our setting, the interval with second-mover advantage happens to be where players find it strategically dominant to contribute. Hence, in any Pareto optimal MPE, $\lambda_i(x) = \psi_i(x) = 0$ when $x \in [\tilde{x}, \infty)$.[53]

When X_t falls in $(x^*, \tilde{x})$, the interval with first-mover advantage, if players adopt mixed strategies, they must be indifferent between exiting or not. Formally, for $i \neq j$,

$$p_j(x) \cdot \frac{F(x) + S(x)}{2} + (1 - p_j(x)) \cdot F(x) = p_j(x) \cdot S(x) + (1 - p_j(x)) \cdot \left[(x - c)dt + (1 - rdt)\Omega_i(x)\right],$$

where $\Omega_i(x) := p_j(x) \cdot \frac{F(x)+S(x)}{2} + (1 - p_j(x)) \cdot F(x)$ is Player i's continuation value in case both players do not exit. The left-hand side represents Player i's value from exiting at time t, and the right-hand side corresponds to his value from staying. Rearranging the above equation, we get

$$p_j(x) = \frac{\Omega_i(x) - F(x) + (x - c - r\Omega_i(x))dt}{\Omega_i(x) - \frac{F(x)+S(x)}{2} + (x - c - r\Omega_i(x))dt}. \tag{B1}$$

[53] Players are weakly dominant to contribute when $X_t = \tilde{x}$. Like Footnote 19, we only consider the MPE where players contribute at $X_t = \tilde{x}$, due to the equilibrium selection of Pareto dominance.

According to the definition of $\Omega_i(x)$, it must satisfy $\frac{F(x)+S(x)}{2} \leq \Omega_i(x) \leq F(x)$. Meanwhile, if $\Omega_i(x) < F(x)$, from Equation (B1) we get $p_j(x) < 0$. Hence, $\Omega_i(x) = F(x)$ must hold, and thus $p_1(x) = p_2(x) = \frac{2(x-c-rF(x))}{F(x)-S(x)}dt$.

However, when $\beta < \beta^*$, there must exist an interval (x^1, x^2) such that $F(x) > V_c(x)$ when $x \in (x^1, x^2)$. These two thresholds are the intersections of $F(x)$ and $V_c(x)$ as shown in Figure 2(b). When $x \in (x^1, x^2)$, it follows that $x - c - rF(x) < x - c - rV_c(x) < 0$ where the last inequality comes from the HJB equation while deriving $V_c(x)$. This further indicates that $p_1(x) = p_2(x) < 0$, which obviously cannot hold. Hence, we infer that players will not play mixed strategies when $x \in (x^1, x^2)$ in any MPE.

Moreover, players' incentives to pre-empt each other still exist even when we allow for mixed strategies. In particular, due to the continuity of value function, there exists an interval $[x^2, x^2 + \epsilon)$ such that $\Omega_i(x) < F(x)$ when $x \in [x^2, x^2+\epsilon)$, contradicting our previous claim that $\Omega_i(x) = F(x)$ must hold if mixed strategies are employed. Therefore, players adopt pure strategies to exit when X_t falls in this interval, and such pre-emption incentives will expand to the entire interval of $(x^*, \tilde{x})$ as in the baseline model where only pure strategies are considered. We conclude that players always exit with probability one when $X_t \in (x^*, \tilde{x})$ in any (possibly mixed-strategy) MPE.

Finally, when $X_t \in (0, x^*]$, there is neither first-mover nor second-mover advantage, and thus players do not have strategic timing concerns when making the exit decisions. Intuitively, a *de facto* joint exit always occurs whenever anyone exits, so each player is as if making a single-agent decision problem. For that reason, mixed strategies are redundant.

The punchline of this section is as follow. Allowing for mixed strategies may introduce new MPE, but does not improve players' value in the equilibrium. Therefore, the equilibrium specified in Theorem 1 remains the unique Pareto optimal (possibly mixed-strategy) MPE up to payoff equivalence.

B.2 Strategic Exiting without Direct Punishment

This section considers the limit case for Theorem 1 where $\alpha = 1$, i.e., the first mover's exit does not decrease his flow revenue as long as the project is operating. To begin with, in Lemma 3, $\beta^* \to \infty$ as $\alpha \to 1$, indicating that the cooperative outcome cannot be achieved; in other words, pre-emption is unavoidable in any equilibrium. Moreover, in Lemma 1, $\tilde{x} \to \infty$ as $\alpha \to 1$. Intuitively, when the initial exit is not punished directly, the first mover's flow payoff is never lower than the second mover; hence, in any pre-emptive equilibrium, each player attempts to strategically exit when X_t is greater than x^*.

To formalize the idea, we rewrite Theorem 1 with $\alpha = 1$ as below: The unique Pareto optimal MPE is the following pre-emptive equilibrium

$$\mathcal{X}^1 = \mathcal{X}^2 = \begin{cases} (0, x^0) \cup (x^*, \infty) & \text{when } \beta \in (\beta^{**}, \infty), \\ (0, \infty) & \text{when } \beta \in (1, \beta^{**}], \end{cases}$$

where x^0 and x^* are defined as in Section 3. In particular, the only situation that the players both contribute is when β is large and the current state X_t is intermediate. Apparently, the curse of profitability still exists when $\beta \in [\beta^{**}, \beta^*]$.

B.3 Tie-Breaking Assumption

We provide three justifications for the tie-breaking assumption that commonly appears in the literature of stochastic timing games. First, a player's exit decision may need to be validated through an authorization procedure, while the authority in charge can only approve of one application at a time. Usually, the approval depends on who is first in line or other rationing methods (Grenadier, 1996).

Second, there can be a random delay between a player's exit decision (or the expression of intention) and the actual exercise of that decision. If the length of delay follows an continuous distribution that is identical and independent for both players, each player will exit first with 50% probability. The current setting can be regarded as a limit case when the random delay converges in probability to zero.[54] Actually, this is equivalent to another setting where players get identically distributed and independent random chances to act (so that they will not be asked to act at the same time), while we consider the limit case where the arrival rate of the random chances goes to infinity.

[54] See Weeds (2002) for an explicit formulation of this justification with a micro foundation, and Dutta and Rustichini (1993) for a reduced-form formulation.

Third, players may establish a communication scheme (or other coordination devices) to avoid miscoordination (i.e., an undesirable joint exit). One will prefer to exit only when his partner does not, so players will benefit from a coordination device that allows one of them to take back the exit decision in case both attempt to exit.

Given that both players intend to exit simultaneously, let Player i's value in the continuation game be $M_i(X_t)$. The tie-breaking assumption we use is essentially $M_1(X_t) = M_2(X_t) = \frac{1}{2}[S(X_t) + F(X_t)]$. One possible way to relax this assumption is, instead, to assume $\min\{F(X_t), S(X_t)\} \leq M_i(X_t) \leq \max\{F(X_t), S(X_t)\}$ for $i = 1, 2$. This alternative assumption retains the main results but does not provide new insights, so we stick to the original assumption for simplicity.

B.4 Absence of Renegotiation

Theorem 3 assumes that the Pareto optimal equilibrium is selected in any stage of the dynamic game. This equilibrium selection criterion is backed up by allowing renegotiation among the remaining players (see Footnote 38). A natural question to ask is, would it help cooperation if we disallow renegotiation so that a Pareto dominated equilibrium played by some remaining players can serve as a credible threat against a free-rider?

The answer is yes. As an extension to Lemma 4(b), the following strategy profile is a (subgame-perfect) equilibrium after one player exits: When $n_t \in [2, N-1]$, all remaining players adopt the exit region $(0, \tilde{x})$ where $\tilde{x}$ is the intersection of $V_1(x)$ and $F_1(x)$. Under such a "punishment," one who deviates from the N-player cooperative outcome will trigger a ripple effect so that the project will shut down when the state variable next hits x_1^* or lower. Hence, the lump-sum exit payoff for one who exits at time t is $F_1(X_t)$ regardless of N. According to Lemma 6, the N-player cooperative outcome can be sustained if and only if $\frac{\beta_1}{\beta_N} \geq \beta^*$; i.e., $N \geq n^{(1)}$.

This finding delivers two messages. First, with Pareto dominated (subgame-perfect) equilibrium utilized as a punishment against strategic exiting, a large group outperforms a small one in sustaining cooperation. Second, the set of cooperation-sustainable group sizes is strictly enlarged compared with Theorem 3. This is because players' ability to renegotiate may undermine the remaining players' commitment to punish a free-rider and thus make cooperation more difficult.

B.5 A General Model on Group Size and Exit Pattern

This section serves dual purposes. First, we relax the two assumptions in Section 4.3 that are introduced to simplify the analysis, but not necessary for delivering the main idea that a partnership's ability to sustain cooperation is non-monotonic in its group size. Specifically, we no longer require α_n to be identical for all $n \geq 1$ and $\beta_1 \leq \beta^{**}$. Second, we discuss how to determine players' equilibrium exit pattern in more detail. Instead of a characterization result, we develop the following algorithm to serve the above purposes.

We restrict attention to symmetric equilibrium. When there are n players remaining, denote the equilibrium exit region for all remaining players as $\tilde{\mathcal{X}}^n$ and the corresponding value function for each player as $\tilde{W}_n(x)$. When $n = 1$, the optimal exit region is $\tilde{\mathcal{X}}^1 = (0, x_1^*)$ and the corresponding value function equals $V_1(x)$. Inductively, suppose we know $\tilde{\mathcal{X}}^k$ and $\tilde{W}_k(x)$ for $k = 1, 2, ..., n-1$, we can examine whether a n-player cooperative equilibrium exists via the following steps.

Step 1 *Derive the optimal cooperative value function $\tilde{V}_{c,n}(x)$. This is identical to Equation (3). Here we relabel it as $\tilde{V}_{c,n}(x)$ for consistency with future steps.*

Step 2 *Derive a player's exit payoff, $\tilde{F}_n(x)$. The challenge of this step is to determine, based on the information of $\tilde{\mathcal{X}}^k$ for $k = 1, 2, ..., n-1$, how the remaining $(n-1)$ players gradually pull out following this player's initial exit. Then we can tell how α_{n_t} evolves stochastically and thus calculate the expected exit payoff for a free-rider, $\tilde{F}_n(x)$. Our remaining task is thus to derive the optimal symmetric MPE of the following stochastic stopping game: The flow payoff is $X_t - c$, the one who stops at time t collects $\tilde{F}_n(X_t)$ while each remaining player receives $\tilde{W}_{n-1}(X_t)$.*

Step 3 *If $\tilde{V}_{c,n}(x) \geq \tilde{F}_n(x)$ for any $x \geq x_n^*$, then a n-player cooperative equilibrium exists. Therefore, $\tilde{\mathcal{X}}^n = (0, x_n^*)$ and $\tilde{W}_n(x) = \tilde{V}_{c,n}(x)$.*

Step 4 *If $\tilde{V}_{c,n}(x) < \tilde{F}_n(x)$ for some $x \geq x_n^*$, then a n-player cooperative equilibrium fails to exist. Similarly to the baseline model, we derive the optimal n-player pre-emptive equilibrium. An exact algorithm to solve this problem is provided in Section 8 of Steg (2015). Here we provide a sketch of that algorithm, whose essential idea is to suppress pre-emption whenever possible.*

Step 4.1 *Determine the first-mover advantage region(s) by comparing $\tilde{F}_n(x)$ and $\tilde{W}_{n-1}(x)$, where each first-mover advantage region is an interval of values of x.*

Step 4.2 *Identify the regions that are subject to both first-mover advantage and pre-emption (i.e.,*

$\tilde{F}_n(x) \leq \tilde{V}_{c,n}(x)$ *does not always hold). Let players exit in these regions.*

Step 4.3 *Derive players' (unique) optimal exit strategy outside the regions that they will exit. In particular, solve the following optimal stopping problem: The flow payoff is $X_t - \beta_n c$, the endogenous exit payoff is zero, and the exogenous exit payoff is $\frac{1}{n}\tilde{F}_n(x) + \frac{n-1}{n}\tilde{W}_{n-1}(x)$ when X_t falls into the regions where players will exit. Let the corresponding value function from the optimal exit strategy be $\tilde{V}_{p,n}(x)$.*

Step 4.4 *If $\tilde{V}_{p,n}(x)$ is everywhere higher than $\tilde{F}_n(x)$ in the region(s) where players do not exit, we finish Step 4 with players' equilibrium exit region $\tilde{\mathcal{X}}^n$ and corresponding value function $\tilde{W}_n(x) = \tilde{V}_{p,n}(x)$. If not, identify more regions that are subject to both first-mover advantage and pre-emption (i.e., $\tilde{F}_n(x) \leq \tilde{V}_{p,n}(x)$ does not always hold). Let players exit in these regions and re-do Step 4.3.*

The equilibrium exit regions $\{\tilde{\mathcal{X}}^n\}_{n=1,...,N}$ provide sufficient information to determine players' exit pattern — the first exit occurs when X_t falls into $\tilde{\mathcal{X}}^N$, after which the second exit happens when X_t falls into $\tilde{\mathcal{X}}^{N-1}$, and so on. Notably, it is possible to have clustered exit waves as in Cetemen et al. (2023). For instance, if $X_t \in \tilde{\mathcal{X}}^N \cap \tilde{\mathcal{X}}^{N-1} \cap ... \cap \tilde{\mathcal{X}}^{N-k+1}$, $X_t \notin \tilde{\mathcal{X}}^{N-k}$, and $X_s \notin \tilde{\mathcal{X}}^N$ for $s < t$, then a wave of k clustered exits occur at time t.

B.6 Low Re-entry Cost

The paper's main result relies on the presumption the re-entry cost is high. This section complements the paper by considering the situation with low re-entry cost. For illustrative purpose, we focus on the extreme case where re-entry is costless.

The flow payoff is still determined by Table 1, but now players can freely switch between *Contribute* and *Defect*. We bypass the well-known difficulties[55] in defining and analyzing continuous-time stochastic games (in particular, in dealing with the complicated strategy space) by constructing a simple grim trigger strategy profile, which already yields the first-best outcome.

We define the *first-best outcome* as the strategy profile that maximizes the sum of the players' utilities.[56] Because of costless re-entry, the choice of strategy profile at each moment t does not depend on the players' past actions. Therefore, the first-best outcome can be determined by max-

[55] For details on these difficulties and potential solutions, see, e.g., Simon and Stinchcombe (1989).

[56] The first-best outcome is different from the constrained first-best outcome in the baseline model where re-entry is impossible.

imizing the players' aggregated flow payoff at each moment t. Specifically, this boils down to the question of whether to have two, one, or zero contributors for each X_t.

Proposition 4. *When re-entry is costless, the first-best outcome is attainable with a grim trigger strategy.*

As concretely specified in Appendix B.6.1, Proposition 4 implements the first-best outcome with the trigger being a simple *Nash reversion strategy profile* that requires the players to play a stage-game Nash equilibrium at every moment. Such a grim trigger strategy works because the one-period deviation benefit vanishes in a continuous-time setting, while the deviating player is worse off if the continuation game is the Nash reversion profile.

Proposition 4 is consistent with the classic conclusion in repeated games that, in a dynamic environment, the free-riding problem can be eliminated by employing a grim trigger strategy (McMillan, 1979). Comparing this result with the baseline model, we see that the irreversibility of defections is a possible "friction" to reintroduce the free-riding problem to partnerships that operate over time. Intuitively, it is more restrictive to punish a deviating player if he leaves the partnership forever. For instance, we cannot load him with more responsibility in the future as in Proposition 4.

B.6.1 Proof of Proposition 4

Step 1. We derive the *first-best outcome* by comparing $2X_t - 2c$ (two contributors), $(1+\alpha)X_t - \beta c$ (one contributor) and zero (no contributor) for any X_t. It is not difficult to verify that: When $\beta \geq 1+\alpha$, the first-best outcome is to have two contributors when $X_t \geq c$ and no contributor otherwise; when $\beta < 1+\alpha$, it is optimal to have two contributors when $X_t \geq \frac{2-\beta}{1-\alpha}c$, one contributor when $X_t \in \left[\frac{\beta}{1+\alpha}c, \frac{2-\beta}{1-\alpha}c\right)$, and zero contributor when $X_t < \frac{\beta}{1+\alpha}c$.

Step 2. We define a simple X_t-contingent *Nash reversion profile*, which requires the players to always play a stage-game Nash equilibrium at any moment t. Such a strategy profile is a subgame-perfect Nash equilibrium, and will be used as a credible punishment upon any deviation in the grim trigger strategy we construct. If $\beta \geq \frac{1}{1-\alpha}$, the stage-game Nash equilibria include

$$\begin{cases} (C,C) & \text{, when } X_t \in (\beta c, \infty) \\ (C,C) \text{ \& } (D,D) & \text{, when } X_t \in \left[\frac{c}{1-\alpha}, \beta c\right] \\ (D,D) & \text{, when } X_t \in \left(0, \frac{c}{1-\alpha}\right). \end{cases}$$

If $\beta < \frac{1}{1-\alpha}$, the stage-game Nash equilibria include

$$\begin{cases} (C,C) & \text{, when } X_t \in \left(\frac{1}{1-\alpha} c, \infty\right) \\ (C,D) \text{ \& } (D,C) & \text{, when } X_t \in \left[\beta c, \frac{1}{1-\alpha} c\right] \\ (D,D) & \text{, when } X_t \in (0, \beta c). \end{cases}$$

In particular, when $X_t \in \left[\beta c, \frac{1}{1-\alpha} c\right]$, (C,D) is unfavorable to Player 1 while (D,C) is unfavorable to Player 2.

Step 3. We construct the grim trigger strategy as follow. Let both players adopt the first-best outcome as we specify in Step 1. In case only one contributor is needed, we let the players take turns to contribute so as to ensure fairness. As the frequency of alternation gets sufficiently large, we can use $\frac{1}{2}\left[(1+\alpha)X_t - \beta c\right]$ to represent each player's flow payoff.

When $\beta \geq \frac{1}{1-\alpha}$, any deviation from the above outcome triggers a switch to the Nash reversion profile where both players play (D,D) if and only if $X_t \leq \beta c$. Such a grim trigger strategy is a subgame-perfect equilibrium. On one hand, there is no one-period deviation benefit for the deviating player in a continuous-time setting. On the other, (C,C) Pareto dominates (D,D) when $X_t \in [c, \beta c]$, the only interval of X_t where the first-best outcome does not coincide with the Nash reversion profile. In other words, the continuation value for both players is strictly worse in the Nash reversion profile compared with the first-best outcome.

When $\beta < \frac{1}{1-\alpha}$, we pick the Nash reversion profile that is most unfavorable to the deviating player. Similarly to the previous case, it is not difficult to check that a deviating player's continuation value will be worse off, while the one-period deviation payoff again vanishes in a continuous-time setting.