Foundations of Market Power in Monetary Economies*

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This draft: January 2021

Abstract

We study the foundations of firms' market power in a continuous-time model where agents are price-makers who interact explicitly with each other. Market power arises from the existence of rents, the size of which depends on consumers' outside options, and firms' ability to appropriate these rents through rent seeking. We study how measures of market power (e.g., markups, concentration) are affected by search frictions, monetary policy, and self-fulfilling beliefs. As meeting speed becomes infinite, there exists a sequence of equilibria along which market power vanishes. But there can also exist equilibria where rents remain positive and firms behave as monopolists.

JEL Classification: D82, D83, E40, E50

Keywords: Search, bargaining, market power, rent seeking, money

*We thank Francisco Klapp for his excellent research assistance. We also thank for their comments Aleksander Berentsen, Jan Eeckhout, Paul Jackson, Thomas Philippon, Nicholas Trachter, Semih Uslu, and seminar participants at the University of Basel and the University of California, Irvine.

Market power is a key concept in economics. It measures the ability of a firm to raise its price and increase its profits at the expense of its customers. Clearly, that can happen only if the customers in question do not have readily available alternatives. If they did, they would react to any price increase by switching to another producer.

Thomas Phillipon, *The great reversal: How America gave up on free markets*, 2019

1 Introduction

We develop a theory of firms' market power in monetary economies from first principles. We adopt a game-theoretic approach to markets where agents interact with one another to form prices and allocations. We view firms' market power as stemming from two components: first, the existence of rents whereby the joint value of a trade between a consumer and a producer exceeds participants' opportunity costs which depend on their outside opportunities; second, the ability of the firm to capture a fraction of that surplus in a rent-seeking contest. We will show how market structure and monetary policy affects both components, the size of the surplus and the share accruing to firms, and standard measures of market power, e.g., markups, rent sizes, and market concentration. We will also study limits of economies when trading frictions vanish (e.g., due to the development of online trading) and ask whether market power disappears, and perfect competition prevails, when customers "*have readily available alternatives*", as suggested by the epigraph.

A theory of market power requires both a theory of markets and a theory of the power struggles between consumers and producers. Our approach to markets follows the game-theoretic literature on decentralized markets with bargaining pioneered by Rubinstein and Wolinsky (1985) and surveyed in Osborne and Rubinstein (1990). In this approach, agents are not passive price-takers trading in a non-strategic fashion against their budget constraint in a market operated by a fictitious auctioneer. Instead, as advocated by Makowski and Ostroy (2001), they are interacting explicitly with each other, making prices and allocations, and seeking actively for rents. In Makowski and Ostroy's words, ‹‹*Rather than dealing with an impersonal market, perfect competitors interact with one another in an environment involving intense rivalry. A perfect competitor will do whatever he can to increase his gain.*›› The second component of our theory captures the power struggles between consumers and producers in two ways: via bilateral negotiations, the outcome of which has both axiomatic and strategic foundations and depends on outside options, and via endogenous rent-seeking activities.[1] We study such a decentralized market within an economy with liquidity constraints and an explicit role for money, in the spirit of Lagos and Wright (2005), so that we can determine the effects of monetary policy on market power.[2]

Our benchmark setting is a New Monetarist economy in continuous time where pairwise meetings generate

[1] According to Makowski and Ostroy (2001), ‹‹*households and firms seek to maximize their rents (consumer and producer surpluses) directly by bargaining over terms of trade, innovating commodities, and doing whatever else that is not proscribed which increases their gains. Hence, (...) it would seem natural to call individual maximizing behavior "rent-seeking."*››

[2] The New Monetarist literature on decentralized markets in the context of monetary economies is surveyed by Lagos et al. (2017) and Rocheteau and Nosal (2017).

bilateral-monopoly rents (Choi and Rocheteau, 2020). Graphically, in Figure 1, these rents correspond to the area between the consumer's marginal utility and the seller's marginal cost. We measure the market power of producers by their profits, which are equal to a fraction of the green area in Figure 1, relative to the profits of a perfect monopoly in the absence of liquidity constraints.

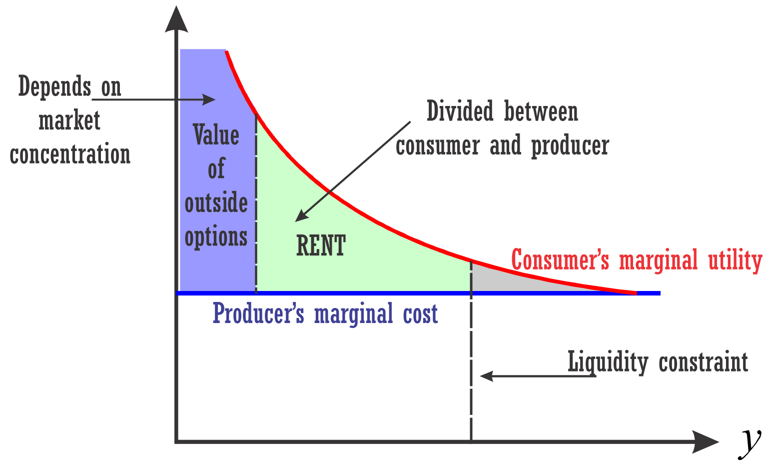

Figure 1: Determinants of market power

Our first contribution is to formalize the relative strengths of the consumer and producer in their negotiation to divide rents as a one-sided (main text) or two-sided (appendix) rent-seeking contest. Formally, prior to being matched, agents can exert costly rent-seeking efforts to increase their bargaining shares. In practice, this effort consists in investing in lobbying and marketing, or hiring pricing consultants to better price discriminate (Berry et al., 2019, p.58). We show that low trading frictions and low nominal interest rates exacerbate rent-seeking efforts and raise sellers' share in trade match surpluses. The logic is intuitive: changes that increase the size of the surplus in bilateral matches, or the frequency of those matches, gives incentives to sellers to spend more resources to raise their share in that larger surplus. This mechanism can explain why markups increase when interest rates decrease or when trading speed increases.

Maybe surprisingly, vanishing trading frictions fail to generate perfect competition as defined by the no-surplus condition of Ostroy (1980). On the contrary, high matching rates inflate rent sizes by reducing the effective cost of holding real balances, thereby relaxing consumers' liquidity constraints. A convincing theory of market power ought to account for the common wisdom captured by the epigraph according to which if consumers have readily available alternatives, then firms should have no market power. For this, one needs to recognize that market power is fundamentally about agents' outside options and their ability (or lack thereof) to leverage them to obtain better terms of trade.

We add outside options to our model by assuming that a consumer can search for alternative sellers until her desire for consumption is either fulfilled or vanishes exogenously. In this version, efforts to obtain more favorable terms of trade are two-sided in the following sense: consumers use their outside options to constrain the set of incentive-feasible agreements while producers actively seek rents to raise their share of surplus. Graphically, in Figure 1, we identify the value of the consumer's outside option with a fraction of the area (the purple area) between the consumer's marginal utility and the producer's marginal cost. Under perfect competition, the value of these outside options is maximum and covers the entire area between marginal utility and marginal cost, in which case market power is zero. At the opposite, if consumers have no outside option, face no liquidity constraint, and producers can appropriate the full rent, then market power is one.

We obtain two main insights by endogenizing consumers' outside options. The first insight is about the two channels through which monetary policy affects market power. The first channel operates via the value of consumers' outside options. An increase in the nominal interest rate reduces the value of consumers' outside options since in a monetary economy the cost of switching to another producer depends on the cost of holding real balances until that producer is found. In Figure 1, the purple area shrinks and rents (the green area) expand. The second channel operates through the tightening of liquidity constraints as the nominal interest rate rises. The overall effect on rent sizes and market power is ambiguous and can be non-monotone.

The second important insight of the model is the possibility of multiple steady-state equilibria across which rent-seeking efforts and market power differ. This multiplicity arises due to strategic complementarities between the rent seeking activities of producers that are channeled by consumers' outside options. If a small measure of (atomistic) producers invest more in rent seeking, then the value of consumers' outside falls, which inflates rents and gives incentives to other producers to ramp up their rent-seeking efforts. A necessary condition for this multiplicity of equilibria is that trading frictions are small. So, as the meeting technology between consumers and producers improves, market power is more likely to be explained by coordination and self-fulfilling beliefs.

Importantly, the multiplicity of equilibria prevails at the frictionless limit when consumers can switch to alternative producers almost instantly. In accordance with the common wisdom, there is an equilibrium that approaches a perfectly competitive outcome in which the value of consumers' outside options exhausts the match surplus and producers' rent seeking efforts go to zero. However, there can also exist equilibria that converge to an outcome where rents are positive and producers become monopolists, i.e., their bargaining share is one. This result illustrates how the reduction in trading frictions can exacerbate the rent-seeking efforts of producers, thereby inhibiting competitive forces. It can explain why the development of new trading technologies, such as online trading platforms, might increase market power, instead of reducing it. Also, in contrast with our epigraph, it shows that even if customers have readily available alternatives, firms might still be able to generate profits at customers' expense.

In a search-based environment, the concentration of the market in terms of the number of active producers per buyer affects the speed at which consumers can approach alternative producers, which makes it a key variable to determine consumers' outside options. But is market concentration positively or negatively correlated with consumers' outside option payoff and what is its relation to market power? We endogenize market concentration with an entry decision. In equilibrium, the value of outside options and entry are determined jointly and the co-movement between market power and concentration changes sign depending on the source of variation. For instance, an increase in entry costs raises concentration and worsens consumers' outside options. However, an increase in the nominal interest rate reduces the value of consumers' outside options and promotes firm entry. Hence, when monetary policy is the source of variation, the correlation between concentration and market power is negative.

We study the limit of the economy with firm entry as the speed of trade grows unbounded. We show that this frictionless limit depends critically on the size of entry costs. If entry costs are lower than some threshold, then rents vanish and social welfare approaches its constrained-efficient level. Even though firms still engage in rent seeking at the limit, aggregate rent-seeking costs vanish as market concentration goes to infinity. In contrast, if entry costs are above a threshold, then consumers' welfare is zero as firms act as perfect monopolists and match surpluses are dissipated in the form of entry and rent seeking costs.

We conclude by introducing firm heterogeneity according to which firms vary by the quality of their output. The subset of firms that are profitable in equilibrium (i.e., those who have a positive market share) is endogenous. As trading frictions are reduced, the value of consumers' outside options increases and only the highest quality firms can operate. Conversely, an increase in the nominal interest rate reduces the value of outside options, thereby allowing the less productive firms to remain in operation. The relationship between the concentration index and the nominal interest rate is U-shaped.

1.1 Literature

Our theory is motivated by a recent but fast-growing body of evidence on the evolution of market power in the US over the last four decades.[3] Influential studies in this literature include Eggertsson et al. (2018) who document an increase of the average price-over-marginal-cost markup calculated from aggregated data from 1.05 to over 1.2 between 1980 and 2015, De Loecker et al. (2020) who find an increase of markups from 1.2 to 1.5 using firm-level data, and Autor et al. (2020) who report that sales concentration, as measured by the Herfindahl-Hirschman Index (HHI), have increased during that time period.

The tradition that uses decentralized markets with pairwise meetings and strategic bargaining to discuss competitive forces dates back to Rubinstein and Wolinsky (1985) and Gale (1986a,b). In monetary economics, this class of models constitutes the core of the New Monetarist literature surveyed in Lagos et al. (2017). The continuous-time version used in this paper is developed in Choi and Rocheteau (2020). Among other things, continuous time allows us to study limits when meetings rates explode to infinity.

Our description of outside options whereby a consumer with a desire to consume can meet producers sequentially before its desire vanishes is absent from the workhorse New Monetarist model (e.g., Lagos and

[3] Other empirical studies on market power include Nekarda and Ramey (2013), Hall (2018), and Rossi-Hansberg et al. (2020). Some of the empirical facts that we mention are still controversial. For instance, Bond et al. (2020) raise concerns regarding the identification and estimation of firm-level markups in studies based on the production function approach. See also Berry et al. (2019) for a discussion of the problems with recent studies of market power.

Wright, 2005) but it is key to explain convergence to perfect competition at the frictionless limit. It has a long tradition in the literature on consumer sequential search (e.g., McCall, 1970). For instance, in the search-and-bargaining models of Wolinsky (1987), Bester (1993), and Chatterjee and Lee (1998), buyers bargain with one store, but can search in other stores for better deals during the process. A related description in the context of monetary economies can be found in Nosal (2011). A similar role for outside options emerges in models with noisy sequential search such as Burdett and Judd (1983) and their applications to monetary economies by Head and Kumar (2005) and Head, Liu, Menzio, and Wright (2012), among others.

In the New-Monetarist literature bargaining shares (Kalai solution) or bargaining powers (Nash solution) are set exogenously. The mechanism design approach to monetary theory endogenizes bargaining shares so as to achieve constrained-efficient allocations (see the survey in Wallace, 2010). Alternatively, an endogenous division of the match surpluses can be obtained with price posting and directed search (e.g., Rocheteau and Wright, 2005). Here we keep the assumption of ex post bargaining but endogenize bargaining strengths through ex ante investments.

The notion of "rent seeking" was introduced by Tullock (1967,1980) and Krueger (1974) to describe monopoly seeking activities, see Tollison (2012) for a review. Our formalization of rent seeking as a contest between agents to gain bargaining power is related to Farboodi, Jarosch and Menzio (2017) in a model of intermediation in over-the-counter markets à la Duffie et al. (2005). A difference is that we formalize rent seeking as a flow effort that needs to be maintained over time for the producer to keep its bargaining share instead of a one-time investment by the intermediaries in Farboodi et al. (2017). Moreover, our model incorporates an intensive margin (trade sizes) and liquidity constraints. Like our model, their environment also features multiple equilibria, but due to their specific setup the economy never converges to a competitive outcome at the frictionless limit. Our formalization also shares similarities with the description of search intensity in Lagos and Rocheteau (2005) and asset acceptability in Lester, Postlewaite, and Wright (2012). In all cases an investment is made ex ante that affects the expected discounted value of the next match. It is also closely related to the literature on the economics of conflicts, e.g., Hirshleifer (1995), Skaperdas (2006) and Garfinkel and Skaperdas (2007).[4]

We relate market concentration and market power by adding firm entry. Our description is analogous to the one in Pissarides (2000) in the context of a frictional labor market. Models with firm entry in monetary economies include Rocheteau and Wright (2005, 2013), Berentsen, Menzio, and Wright (2010), Rocheteau and Rodriguez (2014), among many others.

There is a long literature in Industrial Organization to study the impact of market structure (e.g., Cournot, Bertrand, and monopolistic competition) on market power and markups, see Tirole (1988) and Thisse and Ushchev (2018) for surveys of these models and Ritz (2018) for an estimation of the welfare cost of market power. In these models firms have market power usually because they sell differentiated products.

[4] As pointed out by Schelling (1980): "To study the strategy of conflict is to take the view that most conflict situations are essentially bargaining situations". See Anderton and Carter (2009) for a comprehension survey of this literature.

Relative to this literature, our model emphasizes search frictions as a source of rents and it is designed to analyze the impact of market structure and monetary policy on market power. Relatedly, Liu, Mian and Sufi (2019) study a dynamic oligopolistic competition model in which low real interest rates encourage market concentration by raising industry leaders' incentive to gain a strategic advantage over followers.

2 Environment

The benchmark environment is based on Choi and Rocheteau (2020).[5] Time is continuous and indexed by $t \in \mathbb{R}_+$. The economy is composed of two types of infinitely-lived agents: a unit measure of buyers and a unit measure of sellers. There are two types of perishable goods: a good $c \in \mathbb{R}$ that is traded in an on-going competitive market and that is taken as the numéraire, and a good $y \in \mathbb{R}_+$ exclusively produced and consumed in pairwise meetings. The labels *buyer* and *seller* refer to agents' role in pairwise meetings.

The lifetime expected discounted utility of buyers is

$$\mathcal{U}^b = \mathbb{E}\left\{\sum_{n=1}^{+\infty} e^{-\rho T_n} u\left[y(T_n)\right] + \int_0^{\infty} e^{-\rho t} dC(t)\right\}, \tag{1}$$

where $C(t)$ is a measure of the cumulative net consumption of the numéraire good.[6] Negative consumption of the numéraire good is interpreted as production. The first term between brackets on the right side of (1) accounts for the utility of consumption in pairwise meetings, while the second term accounts for the utility of consuming, or producing if $dC(t) < 0$, the numéraire good. The stochastic process, $\{T_n\}$, is Poisson with arrival rate $\alpha > 0$, and indicates the times at which the buyer is matched bilaterally with a seller.[7] The utility from consuming y units of goods in pairwise meetings is $u(y)$, where u is continuously differentiable, strictly increasing, strictly concave, $u(0) = 0$, $u'(0) = +\infty$, and $u'(\infty) = 0$.

The lifetime expected utility of a seller is

$$\mathcal{U}^s = \mathbb{E}\left\{-\sum_{n=1}^{+\infty} e^{-\rho T_n} y(T_n) + \int_0^{\infty} e^{-\rho t} dC(t)\right\}.$$

The first term corresponds to the disutility of producing y in pairwise meetings. The second term is the discounted linear utility from the consumption and production of the numéraire good.

The pairwise meetings are divided into two types. Conditional on being in a meeting, the buyer can access the technology to produce the numéraire good with probability $\chi_d \equiv \alpha^d/\alpha < 1$. Agents can then trade directly y against the numéraire. Equivalently, the buyer cannot produce the numéraire in the meeting but can promise to deliver it in the future. If $\chi_d = 1$ then the environment corresponds to a pure credit

[5] It can be interpreted as a continuous-time version of Lagos and Wright (2005) and Rocheteau and Wright (2005), except that centralized and decentralized markets do not alternate in discrete time but instead coexist in continuous time.

[6] A similar cumulative consumption process is assumed in the continuous-time models of OTC trades of Duffie et al. (2005). If consumption (or production) of the numéraire happens in flows, then $C(t)$ admits a density, $dC(t) = c(t)dt$. If the buyer consumes or produces a discrete quantity of the numéraire good at some instant t, then $C(t^+) - C(t^-) \neq 0$.

[7] In Rubinstein and Wolinsky (1985) agents only wish to trade once and exit the market afterwards. In contrast, in our model agents trade repeatedly and accepting a trade has no opportunity cost in terms of future opportunities. This assumption of no opportunity cost from accepting a trade is not innocuous and we will relax it later in order to endogenize outside options.

economy or, equivalently, an economy with transferable utility as in the literature surveyed in Osborne and Rubinstein (1990).[8] With probability $\chi_m \equiv \alpha^m/\alpha$ the buyer cannot produce the numéraire in the meeting and is not trusted to repay her debt in the future. In such meetings the buyer needs a means of payment.

There is an intrinsically useless object, called fiat money, that is perfectly storable and durable. The quantity of money at time t is denoted M_t. The constant money growth rate is $\pi \equiv \dot{M}_t/M_t$ and new money is injected into the economy through lump-sum transfers (or taxes if $\pi < 0$) to buyers. The price of money in terms of the numéraire is denoted ϕ_t and the lump-sum transfer is denoted $\tau_t = \phi_t \dot{M}_t$.

In pairwise meetings the quantities produced and consumed and payments are determined according to the proportional solution of Kalai (1977) where the share of the surplus received by sellers is $\mu \in [0, 1]$.[9] If $\chi_d = 1$ then the Kalai solution coincides with the generalized Nash solution. However, if $\chi_m > 0$, then the Kalai solution generates a different outcome from Nash in monetary matches whenever the liquidity constraint binds. The case for using the Kalai solution instead of the Nash solution in environments with liquidity constraints is made in Aruoba et al. (2007). Hu and Rocheteau (2020) provide strategic foundations based on a repeated Rubinstein game.

In the existing literature, the bargaining share, μ, is assumed to be exogenous.[10] We endogenize it by assuming that sellers can exert some costly effort to increase their share of the surplus. These efforts can correspond to marketing and lobbying expenses, investment in pricing strategies to better price discriminate, training costs for retail personnel, and so on. These rent-seeking activities involve a flow cost, $\upsilon(\mu)$, defined over $[0, 1]$, with $\upsilon' > 0$, $\upsilon'' \geq 0$, $\upsilon(0) = \upsilon'(0) = 0$. Our leading specification has $\upsilon'' > 0$, $\upsilon'(1) = +\infty$ but we will also consider $\upsilon'(1) < +\infty$. While we assume first that only sellers can engage in rent-seeking activities, in Appendix D we study an extension with a two-sided rent seeking contest.

3 Equilibria with rent seeking

Let $V^b(a)$ denote the value function of a buyer with $a \in \mathbb{R}^+$ real balances (expressed in terms of the numéraire). At any point in time between pairwise meetings, the buyer can readjust her asset holdings by consuming or producing the numéraire good. Given that preferences are linear in dC_t, $V^b(a) = a + V^b$ where $V^b \equiv -a^* + V^b(a^*)$ is independent of a while a^* denotes the buyer's optimal choice of real balances. In words, the buyer incurs a utility cost $a^* - a$ to readjust her real balances instantly from a to a target a^*.

Sellers have no transactional motive to hold real balances. In equilibrium they either do not want to hold money if $\dot{\phi}/\phi < \rho$ or they are indifferent between any quantity of money if $\dot{\phi}/\phi = \rho$. Hence, $V^s(a) = a + V^s$ where V^s is a term that will be determined later.

[8] If we set $\chi_d = 1$ we can abstract from the competitive market and assume that all meetings are pairwise as in, e.g., Rubinstein and Wolinsky (1985). The purpose of the competitive market is to allow consumers to make a choice of real balances when $\chi_d < 1$.

[9] We assume that the time of the negotiation is distinct from the time of the matching process. The extensive-form bargaining game that micro-found the choice of our bargaining solution in presence of liquidity constraints (Hu and Rocheteau, 2020) would be hard to solve in the presence of a termination risk generated by the matching process.

[10] The bargaining shares can be explained by the risk of termination of the negotiation in extensive-form games. See Hu and Rocheteau (2020). But it is typically invariant to market structure and policy.

Bargaining We now turn to the bargaining problem in a pairwise meeting between a buyer holding a units of real balances and a seller with bargaining share μ. A negotiation outcome is a pair, (y, p), that specifies a production of goods, y, by the seller in exchange for a payment expressed in the numéraire, p. In the fraction χ^m of the meetings where credit is not feasible, the negotiation is subject to the feasibility constraint, $p \leq a$, according to which the payment by the buyer cannot exceed her real balances at the time she enters the meeting, a. The surplus of the buyer is $u(y) + V^b(a - p) - V^b(a)$, which from the linearity of V^b reduces to $u(y) - p$. Similarly the surplus of the seller is equal to its profits, $p - y$. Under the proportional solution of Kalai (1977), the bargaining problem can be written as:

$$\max_{y,p} \{u(y) - p\} \quad \text{s.t.} \quad -y + p = \frac{\mu}{1-\mu}[u(y) - p] \quad \text{and } p \leq a. \tag{2}$$

According to (2) the Kalai solution is the Pareto efficient outcome that satisfies the proportionality constraint according to which the seller receives a fraction μ of the match surplus. The second constraint in (2) is a feasibility condition stating that the transfer of real balances cannot exceed the assets held by the buyer. In a credit match, this constraint is not operative. The last two constraints can be reduced to $p = \mu u(y) + (1-\mu)y \leq a$. In a monetary match, the solution is $y^m = y^*$ and $p^m = \mu u(y^*) + (1-\mu)y^*$, where $u'(y^*) = 1$, if $p \leq a$ is slack; otherwise, $p^m = a$ and $\mu u(y^m) + (1-\mu)y^m = a$. Let $y^m(a)$ be the bargaining solution for y in a monetary match. In a credit match, $y^d = y^*$ and $p^d = \mu u(y^*) + (1-\mu)y^*$.

Strategic foundations The strategic foundations for the proportional solution with quasi-linear payoffs and liquidity constraints have been established by Hu and Rocheteau (2020).[11] The game, called Rubinstein game with sliced bundles, is composed of $N \in \mathbb{N}$ stages that take place within an infinitesimal time interval. In each stage, the buyer and the seller play an alternating offer bargaining game à la Rubinstein (1982) with exogenous risks of break down. The maximum amount of goods that can be negotiated in each stage is y^*/N. The transfer of real balances is subject to a feasibility constraint according to which the buyer cannot transfer more money than what she holds in a given round taking into account the money spent in earlier rounds. The game ends when either the N^{th} round has been reached or the buyer's real balances have been depleted. In each stage, if an offer by the buyer is rejected by the seller, then the round is terminated with probability $1 - \xi^s$. If an offer by the seller is rejected by the buyer, then the round is terminated with probability $1 - \xi^b$. We assume $\left(\xi^b, \xi^s\right) = \left(e^{-(1-\mu)\varepsilon}, e^{-\mu\varepsilon}\right)$ for some $\mu \in [0, 1]$ and take the limit as $\varepsilon \to 0$. The outcome in each round corresponds to the generalized Nash solution with endogenous disagreement points where the bargaining power of the buyer is μ. We consider the limit of this game when N goes to infinity, which means that in each stage there is an infinitesimal amount of output up for negotiation, i.e., the

[11] There is a tradition in the literature on decentralized markets to describe bargaining with strategic foundations (Rubinstein and Wolinsky, 1984). Typically, the literature adopts a version of the alternating-offer bargaining game of Rubinstein (1982), which yields some version of the generalized solution with endogenous disagreement points depending on the details of the game. The use of the Nash solution in monetary models with divisible output generates a surplus for the consumer that is non-monotone in her real balances. As discussed in Aruoba et al. (2007) and Lebeau (2020), such non-monotonicity can be problematic, hence the use of the proportional solution.

players bargaining gradually over the output.[12] The bargaining share, μ, is obtained from the probabilities of termination within each round. We endogenize it later by assuming it can be controlled by sellers via some costly investment prior to match formation.

Measures of market power The average price of a unit of output is $p/y = 1 + \mu\left[u(y) - y\right]/y$ and the marginal cost of production is 1. Hence, the markup in pairwise meetings is defined as $MKUP \equiv (p/y) - 1$.[13] In credit matches, it is proportional to sellers' rent-seeking effort,

$$MKUP^d = \frac{\mu\left[u(y^*) - y^*\right]}{y^*}. \tag{3}$$

In monetary matches,

$$MKUP^m = \mu\left(\frac{u\left[y^m(a^*, \mu)\right]}{y^m(a^*, \mu)} - 1\right). \tag{4}$$

It is increasing with μ and decreasing with a^*. In both credit and monetary matches there is a one-to-one relationship between markups and sellers' surplus share. In monetary matches, the markup decreases with real balances because the unit price of output decreases in y due to the concavity of preferences. In the following we will provide microfoundations for the two determinants of markups, namely a^* and μ.

In Figure 2 we represent the surpluses from monetary and credit trades and their division between the buyer and the seller. The marginal utility of the buyer is represented by the red downward-sloping curve while the marginal cost of the seller is constant and normalized to one. In a credit trade (right panel), the joint surplus is maximum and corresponds to the entire area between the two curves. Even though the seller enjoys a positive surplus represented by the blue area, there is no loss in efficiency. In a monetary trade (left panel), the quantity is limited by the buyer's real balances, a. If $a < y^*$, then there is a loss of efficiency. If the seller has some bargaining power and can capture a fraction of the surplus from the trade, then the welfare loss is larger. The seller's surplus corresponds to either the green area, $p - y = a - y$, or the blue area. Hence, the gap between a and y widens as the seller's surplus increases. In the following definition we normalize the seller's surplus so that our measure of market power, denoted by MPS, is between 0 and 1.

Definition 1 *The market power of a seller when the terms of trade are (y, p) is defined as*

$$MPS \equiv \frac{p - y}{u(y^*) - y^*}. \tag{5}$$

The seller's market power is defined as the seller's profits, $p - y$, relative to the profits of a monopoly under perfect price discrimination, $u(y^*) - y^*$. In credit matches, $MPS^d = \mu$, i.e., the market power reduces to the bargaining share. In monetary matches, $MPS^m \equiv \mu\left[u(y^m) - y^m\right] / \left[u(y^*) - y^*\right]$. By the solution to the bargaining problem (2), it increases in a^* and μ.

[12] Hu and Rocheteau (2020) show that the agenda of the negotiation that consists in slicing the output in small bundles is optimal from the viewpoint of the producer.

[13] Lerner (1934) proposed an index of market power equal to $(price - MC)/price$ where MC is the marginal cost. In the context of our model this measure would be equal to $1 - (y/p)$.

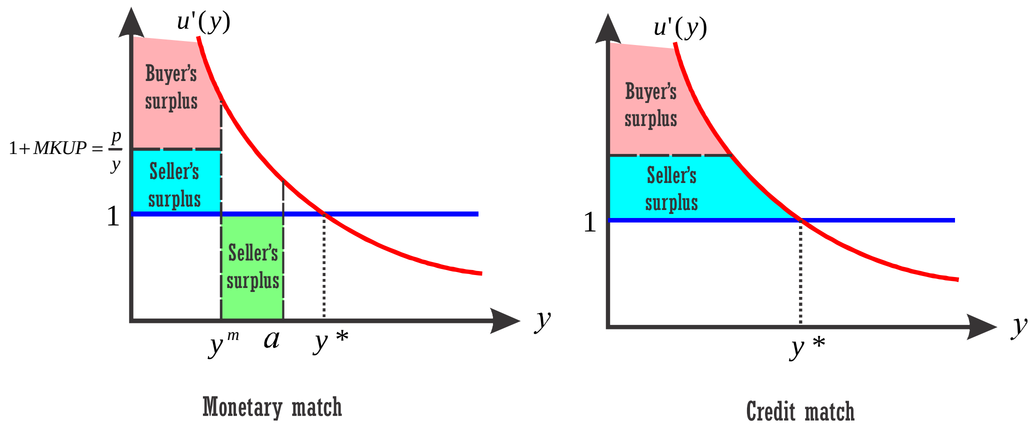

Figure 2: Size and division of trade surpluses in monetary and credit matches

Choice of real balances We now write the Hamilton-Jacobi-Bellman equation for the lifetime expected utility of a buyer holding a real balances, $V^b(a) = a + V^b$. As shown in Choi and Rocheteau (2020), V^b solves:

$$\rho V^b = \max_{a^* \geq 0} \left\{ -ia^* + \tau + \alpha^m (1-\mu) \left\{ u\left[y^m(a^*) \right] - y^m(a^*) \right\} + \alpha^d (1-\mu) \left[u\left(y^* \right) - y^* \right] + \dot{V}^b \right\}, \tag{6}$$

where $i \equiv \rho - r$ is the flow cost of holding a unit of money measured by the difference between the rate of time preference and the rate of return of money, $r = \dot{\phi}/\phi$. The second term, τ, is the lump-sum transfer from money creation. According to the third and fourth terms, the buyer receives an opportunity to consume at Poisson arrival rate α, in which case her match surplus is $u(y) - p(y)$. The meeting can be a monetary match (third term) or a credit match (fourth term). The last term is the change in the value function over time, $\dot{V}^b = \partial V_t^b / \partial t$. Using that the optimal choice of real balances maximizes the right side of (6) and assuming an interior solution, we obtain the following optimality condition:

$$i = \alpha^m (1-\mu) \frac{u'(y^m) - 1}{\mu u'(y^m) + 1 - \mu}. \tag{7}$$

The left side of (7) is the flow cost of holding real balances as measured by the difference between the buyer's rate of time preference and the rate of return of money. The right side is the marginal expected value of real balances which is the product of three terms: the frequency of trading opportunities, the buyer's share, and the marginal match surplus.

Rent seeking The value function of a seller solves the following HJB equation:

$$\rho V^s = \max_{\mu \in [0,1]} \left\{ -\upsilon(\mu) + \alpha^m \mu \left[u\left(y^m \right) - y^m \right] + \alpha^d \mu \left[u\left(y^* \right) - y^* \right] + \dot{V}^s \right\}. \tag{8}$$

The seller incurs a flow cost, $\upsilon(\mu)$, to be able to extract a share μ of the trade surplus when a match occurs at Poisson rate α. The size of the surplus is $u(y) - y$ where $y = y^m$ if the match is monetary and $y = y^*$

otherwise.[14] The FOC is

$$\upsilon'(\mu) = \alpha^m [u(y^m) - y^m] + \alpha^d [u(y^*) - y^*] + \alpha^m \mu [u'(y^m) - 1] \frac{\partial y^m}{\partial \mu}. \tag{9}$$

The left side is the marginal cost incurred by the seller to raise his share of match surplus. The first two terms on the right side are the match surpluses given the quantities traded in each match and the last term captures the effect of the bargaining share on the output in monetary matches. From the bargaining solution,

$$\frac{\partial y^m}{\partial \mu} = \frac{-[u(y^m) - y^m]}{\mu u'(y^m) + 1 - \mu} < 0.$$

In the left panel of Figure 2, an increase in μ expands the blue area which represents the fraction of the surplus received by the seller. The green area must expand as well, which means that y^m decreases. Substituting the expression for $\partial y^m / \partial \mu$ into (9), the optimal bargaining share solves

$$\upsilon'(\mu) = \alpha^m \frac{u(y^m) - y^m}{\mu u'(y^m) + 1 - \mu} + \alpha^d [u(y^*) - y^*]. \tag{10}$$

By (10), the convexity of υ and the concavity of u, one can check that μ increases with y^m. Intuitively, as buyers hold more liquid assets, the benefits from rent seeking increase.

3.1 Equilibrium rent seeking

We focus on steady states where real balances, $a^* = \phi M$, are constant. In such equilibria the rate of return of money is $r \equiv \dot{\phi}/\phi = -\pi$. A steady-state equilibrium can be reduced to a pair, (y^m, μ), that solves (7) and (10), i.e.,

$$i = \alpha^m (1 - \mu) \frac{u'(y^m) - 1}{\mu u'(y^m) + 1 - \mu} \tag{11}$$

$$\upsilon'(\mu) = \alpha^m [u(y^m) - y^m] \left[1 - \frac{\mu i}{\alpha^m (1 - \mu)}\right] + \alpha^d [u(y^*) - y^*], \tag{12}$$

where $i \equiv \rho + \pi$ can be interpreted as the nominal interest rate on an illiquid bond. Equation (11) gives a negative relationship between y^m and μ represented by the curve labelled YM in Figure 3. For all $\mu < \alpha^m / (i + \alpha^m)$, $y^m > 0$. Equation (12) gives a positive relationship between μ and y^m represented by the curve labelled MU. If $y^m = 0$ then $\mu = \upsilon'^{-1} \left[\alpha^d [u(y^*) - y^*]\right] \in (0, 1)$.

We measure social welfare by $\mathcal{W}$ defined as:

$$\rho \mathcal{W} = -\upsilon(\mu) + \alpha^m [u(y^m) - y^m] + \alpha^d [u(y^*) - y^*].$$

It is the sum of the surpluses in all pairwise meetings net of the rent-seeking costs by sellers. First-best allocations are such that $y^m = y^*$ and $\mu = 0$, i.e., sellers invest no resources in rent seeking and the surplus in monetary matches is maximum.

[14] Note from (8) that the choice of the rent-seeking effort seems formally equivalent to the choice of a search intensity. A key difference, however, is that μ has a direct effect on y^m given a^* through the bargaining solution, as illustrated below, and it enters into the definitions of market power given by (3)-(4) and (5) whereas search intensity would not.

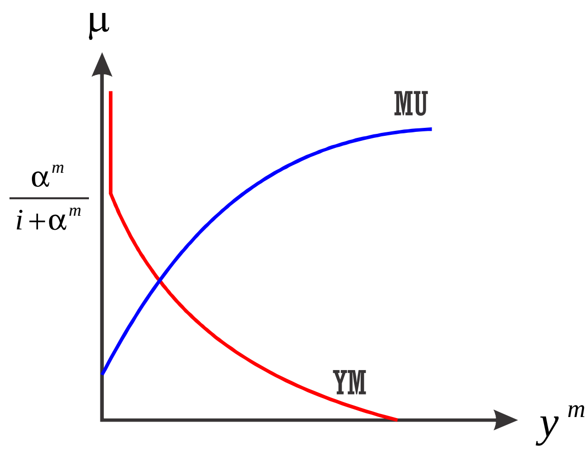

Figure 3: Steady state with endogenous rent seeking

Proposition 1 ***(Steady States with Endogenous Rent Seeking)***. *If*

$$\upsilon'\left(\frac{\alpha^m}{i+\alpha^m}\right) > \alpha^d \left[u\left(y^*\right) - y^*\right],$$

then there exists a unique steady-state monetary equilibrium.

1. *Market power and monetary policy.* $\partial y^m/\partial i < 0$, $\partial \mu/\partial i < 0$, $\partial MKUP/\partial i \lesseqgtr 0$, *and* $\partial MPS/\partial i < 0$. *The optimal monetary policy is such that* $i > 0$.

2. *Market power and trading frictions. For* i *close to 0,* $\partial y^m/\partial \alpha \approx 0$, $\partial \mu/\partial \alpha > 0$, $\partial MKUP/\partial \alpha > 0$, *and* $\partial MPS/\partial \alpha > 0$.

Low interest rates promote rent seeking and raise firms' market power by relaxing liquidity constraints, thereby inflating rents. As a result, the optimal nominal interest rate is positive, i.e., the Friedman rule is suboptimal as it generates too much wasteful rent seeking. The reduction of trading frictions contributes to an increase in sellers' market power by a similar mechanism, i.e., consumers hold larger real balances which increases rent sizes and promotes rent seeking.

We illustrate the results in Proposition 1 with a numerical example. The unit of time is a year and set $\rho = 0.03$ and $\alpha = 1$.[15] We adopt the following functional forms: $u(y) = y^{1-b}/(1-b)$ with $b = 0.12$, and $\upsilon(\mu) = \upsilon_0 \mu^{1+\eta}/(1+\eta)$ with $\upsilon_0 = 0.09$ and $\eta = 0.3$. The fraction of monetary matches is set to $\chi^m = 0.7$.[16] The top panels of Figure 4 show the effects of changes in the nominal interest rate. We distinguish the model with exogenous bargaining shares from the model with endogenous rent seeking. In the former, the markup,

[15] The frequency of meetings might seem low given that a unit of time corresponds to a year. Such low values are needed to match money demand in the data. For our purpose, a low value allows us to illustrate how comparative statics change when μ is endogenous. Note also that we chose a rent-seeking cost function such that $\upsilon'(1) < +\infty$. We discuss the importance of this assumption later in the paper. See the Appendix for further details on the numerical examples.

[16] In the Appendix we justify the choice of those values with a calibration aiming at matching simultaneously two empirical relationships in US data for the period 1985-2014: the aggregate money demand and the relationship between average markup and interest rate.

$\mu\,[u(y)/y-1]$, increases with i because output decreases, which raises the per unit gains from trade due to the concavity of $u(y)$. In the latter, monetary policy affects equilibrium allocations through two channels. There is a direct channel according to which an increase in i raises the cost of holding real balances, which tends to lower y^m. There is an indirect channel according to which lower real balances reduce gains from trades and sellers' incentives to invest in rent-seeking. The seller's bargaining share decreases, which mitigates the direct effect. For our numerical example, the markup decreases because of the reduced rent seeking effort of producers. Since sellers' bargaining share μ and output y^m fall, so does the MPS.

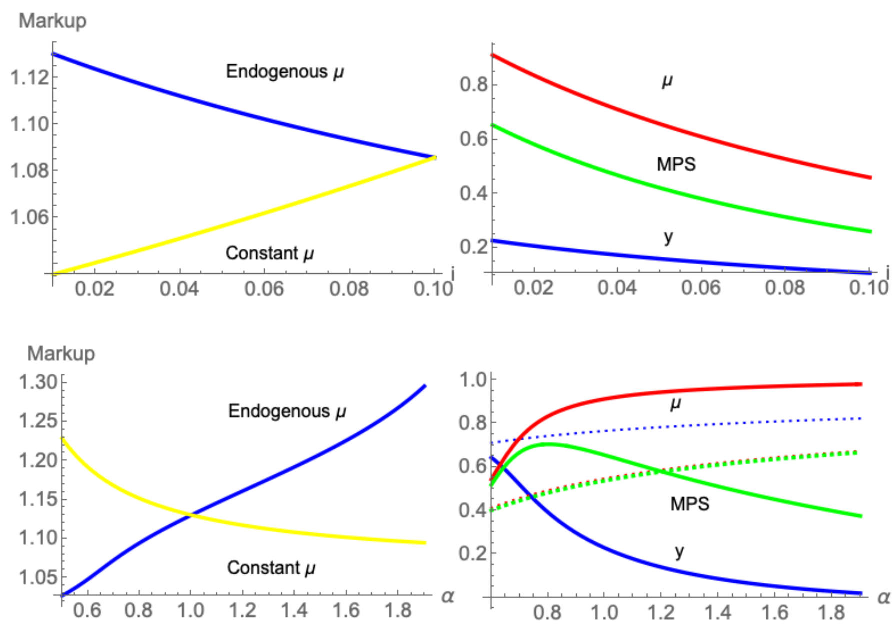

Figure 4: Numerical examples of changes in i and α.

The lower panels of Figure 4 illustrate the effects of a higher frequency of trading opportunities. If μ is constant, markup decreases with α because y^m increases due to the lower expected cost of holding real balances, i/α^m. If μ is endogenous, markup increases in α because producers ramp up their rent-seeking effort. While markup increases, our measure of market power, MPS, decreases. So changes in markups are not always a good indicator of changes in market power.

In the lower-right panel we illustrate the importance of the specification of $\upsilon(\mu)$ for equilibrium outcomes. When $\upsilon'(1) < +\infty$, an increase in α does not always lead to a higher y^m. In our numerical example, quantities traded in monetary matches shrink as the frequency of trade increases while producers acquire all the bargaining power at the limit. The specification with $\upsilon'(1) = +\infty$ (dashed lines) delivers opposite comparative statics for y^m and MPS, i.e., y^m and MPS increase in α.

3.2 High frequency trading

We now characterize the limit of decentralized equilibria as trading frictions vanish. In order to separate the meeting technology from the arrival rate of consumption opportunities we modify our environment as follows. Assume there is a flow $\omega > 0$ of new buyers per unit of time. Buyers meet sellers at Poisson arrival rate α and then exit the market permanently whether a trade occurs or not. In a steady state, the matching rate of sellers is ω. From (10),

$$\upsilon'(\mu) = \omega\chi^m \frac{u(y^m) - y^m}{\mu u'(y^m) + 1 - \mu} + \omega\chi^d \left[u(y^*) - y^*\right]. \tag{13}$$

The problem of each buyer is identical to the one described so far. Moreover, if $\omega = \alpha$ the economy is isomorphic to the one above.

The frictionless limit of our decentralized equilibrium is obtained when $\alpha \to +\infty$ taking ω constant. While the steady-state measure of buyers in the market, ω/α, goes to 0, the measure of trades remains constant and equal to ω. The next proposition describes the limit of the decentralized equilibrium under two assumptions: bargaining shares are exogenous, or they are determined endogenously by the rent-seeking activity of sellers. We measure the market power of sellers by the average of MPS across matches, i.e., $MPS = \chi^m MPS^m + \chi^d MPS^d$.

Proposition 2 ***(Rents and market power under high frequency trading)***

1. *Exogenous bargaining shares. Consider the limit of the decentralized equilibrium as α tends to $+\infty$. If $\mu < 1$, $y^m = y^*$ and $MPS = \mu$. If $\mu = 1$, then $y^m = 0$ and $MPS = \chi^d$.*

2. *Endogenous rent seeking. Let $\bar{\upsilon}' \equiv \omega[u(y^*) - y^*]$ and $\underline{\upsilon}' \equiv \omega\chi^d[u(y^*) - y^*]$. At the limit as $\alpha \to +\infty$:*

   (a) If $\upsilon'(1) \leq \underline{\upsilon}'$, then $y^m = 0$, $\mu = 1$, and $MPS = \chi^d$.

   (b) If $\upsilon'(1) \in (\underline{\upsilon}', \bar{\upsilon}')$, then $y^m \in (0, y^*)$, $\mu = 1$, and $MPS = \frac{\upsilon'(1)}{\omega[u(y^*)-y^*]} \in (\chi^d, 1)$.

   (c) If $\upsilon'(1) \geq \bar{\upsilon}'$, $y^m = y^*$ and $MPS = \mu = \upsilon'^{-1}\{\omega[u(y^*) - y^*]\} > 0$.

If bargaining shares are set exogenously, then the limit of the decentralized equilibrium implements y^* in all matches unless $\mu = 1$ in which case money is not valued. The rents in pairwise meetings – in Figure 2, the area between the marginal utility and the marginal cost over the interval $[0, y^m]$ – expand as trading frictions are reduced and, at the frictionless limit, they are maximum.[17] So, the limit of the decentralized equilibrium does not implement a perfect competition outcome with no rents. The reason is that the increase in the frequency of meetings only promotes competition if it intensifies the threat of a matched consumer to switch to another seller. In the canonical New-Monetarist model, if a consumer rejects a trade, she loses a trading opportunity that cannot be recovered. In other words, an increase in α raises the frequency of

[17] The idea that competition should be associated with no positive surpluses is consistent with the characterization by Ostroy (1980) of perfectly competitive equilibrium with a no-surplus condition.

trading opportunities but it does not improve the outside options of the matched consumers. However, the presence of rents does not generate allocative inefficiencies unless $\mu = 1$ because as trading frictions vanish the cost of holding real balances, i/α^m, also vanishes and the pricing protocol is pairwise Pareto efficient.

In the presence of costly rent seeking, the equilibrium in the frictionless limit implements y^* in all trades if the marginal cost of rent seeking when $\mu = 1$ is sufficiently large. Wasteful rent seeking persists even when meetings occur at infinite speed. If $v'(1)$ is in some intermediate range, then μ goes to one and producers act as monopolies but the output level falls below y^*. If $v'(1)$ is lower than a threshold, money is not valued and sellers capture all the surplus in credit matches.

4 Outside options and market power

Our benchmark model fails to generate perfect competition with no rent at the limit when trading frictions vanish. The reason is that there is no opportunity cost from agreeing on a trade as future trading opportunities exist irrespective of whether the current trade occurs or not. A theory of market power ought to be able to account for the common view that the market power of a producer vanishes if consumers can switch almost instantly from one producer to another. Therefore, in the following we amend our theory by adding endogenous outside options for consumers.

We now suppose that the rate at which buyers meet producers and the rate at which they receive consumption opportunities are distinct. Buyers can be in two states: idle or active. An idle buyer has no desire to consume. The desire to consume arrives at Poisson rate λ in which case the buyer becomes active. This desire is fulfilled after consumption of any quantity $y > 0$ or it disappears at Poisson rate $\gamma > 0$. In both events, the active buyer becomes idle.[18] One can interpret the λ and γ shocks as idiosyncratic preference shocks.[19] The model of Section 3 is a special case when $\lambda \to +\infty$, buyers are always active. We represent the transitions between the consumers' states in Figure 5. We adopt a simple matching function according to which $\alpha^b = \bar{\alpha}$ and $\alpha^s = \bar{\alpha} n_a$ where n_a is the measure of active buyers. In a steady state, $n_a = \lambda/(\lambda + \gamma + \bar{\alpha})$.

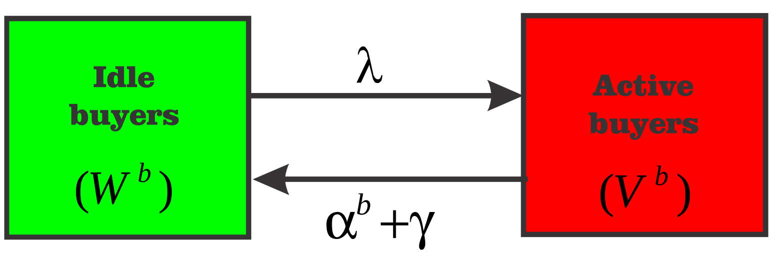

Figure 5: Preference shocks and consumers' states

[18] Alternatively, we could assume that it takes time for the buyer to rebuild her payment capacity following a trade, as in Rocheteau, Weill, and Wong (2018).

[19] In Duffie et al. (2005) similar preference shocks represent idiosyncratic valuations for an asset traded over the counter.

4.1 Rents

We denote $V^b(a)$ the value function of an active buyer with a real balances and $W^b(a)$ the value function of an idle buyer. Both value functions are linear in a and the difference, $Z^b \equiv V^b - W^b$, is the opportunity cost from accepting a trade. Hence, the buyer's surplus from trade is $u(y) - p - Z^b$. The seller's surplus is $p - y$ and the total surplus of the match is $u(y) - y - Z^b$. There exists gains from trade if $\max_{y \leq a} \{u(y) - y\} \geq Z^b$.

Definition 2 *We measure bilateral monopoly power (BMP) in a match between a consumer holding a^* real balances and a producer as:*

$$BMP \equiv \frac{\max_{y \leq a^*} \left\{u(y) - y - Z^b\right\}}{u(y^*) - y^*}. \tag{14}$$

The numerator of (14) corresponds to the maximum joint surplus that can be extracted given the buyer's outside option and the payment constraint. Our measure of bilateral monopoly power is equal to 0 if the gains from trade are exhausted by the buyer's outside options, i.e., $Z^b = \max_{y \leq a^*} \{u(y) - y\}$, and it is equal to 1 if buyers have no outside option and are not liquidity constrained. The market power of a seller, which is a fraction of BMP, is defined by (5).

We now turn to the determination of the terms of trade in pairwise meetings. We define the surplus of a monetary match as $S^m(a^*, Z^b, \mu) \equiv u\left[y^m\left(a^*, Z^b, \mu\right)\right] - y^m\left(a^*, Z^b, \mu\right) - Z^b$, where (y^m, p^m) is the outcome from proportional bargaining, i.e.,

$$p^m = \mu\left[u(y^m) - Z^b\right] + (1-\mu)y^m \leq a^*, \tag{15}$$

with an equality if $y^m < y^*$.[20] Similarly, we define the surplus in a credit trade as $S^d(Z^b) \equiv u(y^*) - y^* - Z^b$. The seller's market power, MPS, and markup, $MKUP = (p-y)/y$, in a trade are given by

$$MPS = \frac{\mu\left[u(y) - y - Z^b\right]}{u(y^*) - y^*}, \quad MKUP = \frac{\mu\left[u(y) - y - Z^b\right]}{y}.$$

For given μ and y, both quantities decrease with the buyer's outside option. These expressions also make clear that the two key determinants of market power are consumers' outside options (Z^b) and producers' rent seeking effort (μ).

In Figure 6 we represent the sizes of the rents and their division for monetary and credit matches. The value of the outside option is the purple area between the marginal cost of the seller and the marginal utility of the buyer for all $y \leq \underline{y}$ where $\underline{y}$ is the consumption-equivalent of the outside options, i.e., it solves $u\left(\underline{y}\right) - \underline{y} = Z^b$. The total rent from a credit match (right panel) is the area between the same curves for $y \in \left[\underline{y}, y^*\right]$. This rent is divided between the buyer and the seller (blue and red areas) without distorting pairwise efficiency. In a monetary match, the output is bounded above by the buyer's real balances. Graphically, the measure of bilateral monopoly power, BMP, is given by the ratio of the area between the blue (marginal cost) and red curves (marginal utility) over the interval $[\underline{y}, a]$ and the area between the same two curves over $[0, y^*]$. The following lemma characterizes the size of the rent in monetary matches.

[20] We provide strategic foundations for proportional bargaining with outside options in the Appendix based on the repeated Rubinstein game with sliced bundles of Hu and Rocheteau (2020).

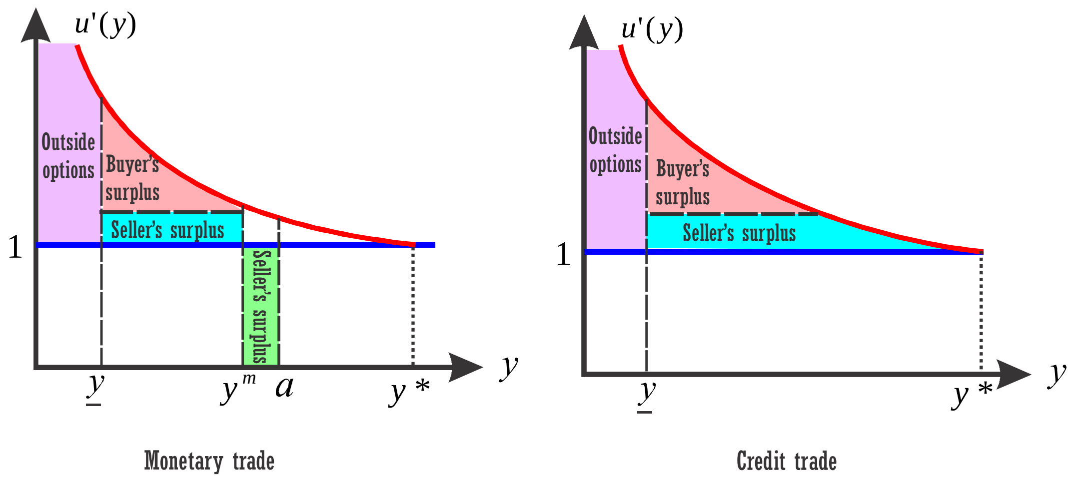

Figure 6: Size and division of surpluses with outside options.

Lemma 1 ***(Match surplus with outside options.)*** *The monetary match surplus, $S^m(a^*, Z^b, \mu)$, is positive if and only if $\max_{y \le a^*} \{u(y) - y\} > Z^b$. It reaches a maximum, $S^m = u(y^*) - y^* - Z^b$, whenever $a^* \ge \mu \left[u(y^*) - Z^b\right] + (1-\mu)y^*$. Moreover,*

$$\begin{aligned} \frac{\partial S^m(a^*, Z^b, \mu)}{\partial a^*} &= \frac{u'(y^m) - 1}{\mu u'(y^m) + 1 - \mu} \ge 0 \\ \frac{\partial S^m(a^*, Z^b, \mu)}{\partial Z^b} &= \frac{-1}{\mu u'(y^m) + 1 - \mu} < 0 \\ \frac{\partial S^m(a^*, Z^b, \mu)}{\partial \mu} &= \frac{-\left[u'(y^m) - 1\right] S^m}{\mu u'(y^m) + (1-\mu)} < 0. \end{aligned}$$

For all a^* such that $\max_{y \le a^*} \{u(y) - y\} > Z^b$ and $a^* < \mu \left[u(y^*) - Z^b\right] + (1-\mu)y^*$, $S^m(a^*, Z^b, \mu)$ is strictly concave in a^*.

According to Lemma 1, the match surplus is positive provided that the buyer holds enough real balances. It increases with the buyer's real balances, but decreases with her outside option and the seller's bargaining share. In the following we endogenize the three components of the match surplus, a^*, Z^b, and μ.

4.2 The value of outside options

We now endogenize the value of consumers' outside options and study how it is impacted by market structure, trading frictions, and monetary policy. The HJB equation for $V^b = V^b(0)$ in a steady state ($\dot{V}^b = 0$) is

$$\rho V^b = \max_{a^* \ge 0} \left\{-ia^* + \tau + \alpha^b \chi_m (1-\mu) S^m(a^*, Z^b, \mu) + \alpha^b \chi_d (1-\mu) S^d(Z^b) - \gamma Z^b\right\}. \tag{16}$$

An active buyer becomes idle if a match occurs, at Poisson rate α^b, or if the preference shock is reversed and the buyer no longer wants to consume, at Poisson rate γ. From (16) the optimal real balances are such that

$$a^* \in \arg\max \left\{-ia^* + \alpha^b \chi_m (1-\mu) S^m(a^*, Z^b, \mu)\right\}. \tag{17}$$

Assuming $Z^b \leq u(y^*) - y^*$, a buyer who holds less than $\underline{a}=\underline{y}$ incurs the cost of holding real balances but has no incentive to trade once a match is formed. There are only two candidate solutions for the buyer's optimal real balances: 0 or the solution to the first-order condition from (17).

Lemma 2 ***(Choice of real balances and outside options.)*** *The targeted real balances solve*

$$a^* = \mu \left[u\left(y^m\right) - y^m - Z^b\right] + y^m \text{ where } \frac{\alpha^b \chi_m (1-\mu)\left[u'(y^m) - 1\right]}{\mu u'(y^m) + 1 - \mu} = i, \tag{18}$$

if

$$\frac{\max_{y \geq 0}\left\{-iy + \left[\alpha^b \chi_m (1-\mu) - i\mu\right]\left[u(y) - y\right]\right\}}{\alpha^b \chi_m (1-\mu) - i\mu} \geq Z^b. \tag{19}$$

Otherwise, $a^* = 0$.

The HJB equation for the value function of an idle buyer, W^b, is:

$$\rho W^b = \tau + \lambda\left(V^b - W^b\right). \tag{20}$$

The idle buyer receives a preference shock with Poisson arrival rate λ, in which case she becomes active.

We start by assuming that bargaining shares are exogenous, $\mu = \bar{\mu}$, in order to highlight the role and determinants of consumers' outside options. An equilibrium is a list (V^b, W^b, a^*) solution to (16)-(17). The value of the buyer's outside option is obtained by taking the difference between (16) and (20) and is characterized in the following proposition.

Proposition 3 ***(Equilibrium with endogenous outside options.)*** *Assume* $\mu = \bar{\mu}$. *If either* $\chi_d > 0$ *or* $i < \alpha^b \chi_m (1-\mu)/\mu$, *then there exists a unique equilibrium. The buyer's equilibrium outside option is the unique* $Z^b > 0$ *solution to*

$$(\rho + \lambda + \gamma)\, Z^b = \max_{a^* \geq 0}\left\{-ia^* + \alpha^b (1-\mu)\left[\chi_m S^m(a^*, Z^b, \mu) + \chi_d S^d(Z^b)\right]\right\}. \tag{21}$$

It varies with exogenous variables as indicated below:

Exogenous variables	ρ	λ	γ	α^b	χ_d	μ	i
Outside option: Z^b	$-$	$-$	$-$	$+$	$+$	$-$	$-$

The buyer's outside option is determined by the HJB equation (21) where the effective discount rate is $\rho + \lambda + \gamma$. The flow value of the outside option is the product of the arrival rate of trading opportunities, the buyer's bargaining share, and the expected surplus of a match, net of the cost of holding real balances. If $\chi_d = 1$, then $a^* = 0$ and $(\rho + \lambda + \gamma)\, Z^b = \alpha^b (1-\mu) S^d(Z^b)$, which is the textbook determination of outside options in models of search and bargaining. If $\chi_m = 1$, then the model is analogous to a New Monetarist model with the addition of an endogenous outside option that solves $(\rho + \lambda + \gamma)\, Z^b = \max_{a^* \geq 0}\left\{-ia^* + \alpha^b (1-\mu) S^m(a^*, Z^b, \mu)\right\}$.

Comparative statics are intuitive. If agents are patient, if their desire of consumption is long-lasting, and if consumption shocks are infrequent, then the value of the outside option is high and producers' market power is low. Similarly, if the meeting rate increases, the value of the outside option goes up. This result formalizes the intuition that market power decreases when consumers can switch more easily between producers.

In terms of monetary policy, an increase in the nominal interest rate reduces the value of the buyer's outside option by making the search for alternative trading partners more costly. Indeed, in a monetary economy, a consumer who chooses not to trade with the producer she is currently matched with must carry her liquid assets until a new producer is found. As long as $i > 0$, holding liquid assets is costly. The mechanism at work is analogous to a 'hot potato' effect according to which inflation raises search costs, thereby raising buyers' incentives to spend their cash.[21] A consequence is that the rent to be shared between the consumer and the seller increases, which pushes markups up.

From (21) we obtain the following expression for market power in closed form for low interest rate environments.

Corollary 1 *When $i = 0^+$, seller's market power is equal to*

$$MPS = \mu \times BMP = \frac{\mu\left(\rho + \lambda + \gamma\right)}{\rho + \lambda + \gamma + \bar{\alpha}(1-\mu)}. \tag{22}$$

A small increase in i from $i = 0^+$ raises both MPS and BMP.

Seller's market power in the neighborhood of $i = 0$ is increasing in ρ, λ, γ, and μ but decreasing in $\bar{\alpha}$. The meeting speed, $\bar{\alpha}$, in the expression for MPS is multiplied by the consumer share, $1-\mu$. So, a reduction in the trading frictions, i.e., $\bar{\alpha}$ becomes large, might not reduce market power if it is accompanied by an increase in μ when μ is endogenous. We will see that this remark is of special importance when we consider frictionless limits.

4.3 More on market power and monetary policy

We now study the effects of monetary policy when both consumers' outside options and producers' rent-seeking efforts are endogenous. The HJB equation of a seller is:

$$\rho V^s = \max_{\mu \in [0,1]} \left\{ -\upsilon(\mu) + \alpha^s \mu \left[\chi_m S^m(a^*, Z^b, \mu) + \chi_d S^d(Z^b) \right] \right\}, \tag{23}$$

where $\alpha^s = \bar{\alpha} n_a$ is the Poisson rate at which a seller meets an active buyer.

Lemma 3 ***(Optimal rent seeking when buyers have outside options.)*** *Assume $\upsilon'(0) = 0$. There is a unique $\mu \in [0,1]$ solution to (23). It solves:*

$$\upsilon'(\mu) \leq \frac{\alpha^s \chi_m \left[u(y^m) - y^m - Z^b \right]^+}{\mu u'\left(y^m\right) + 1 - \mu} + \alpha^s \chi_d \left[u(y^*) - y^* - Z^b \right], \quad \text{“} = \text{”} \;\; \text{if } \mu < 1, \tag{24}$$

where $[x]^+ = \max\{x, 0\}$ and

$$y^m = u'^{-1}\left[1 + \frac{i}{\alpha^b \chi_m (1-\mu) - i\mu} \right].$$

[21] Such 'hot potato' effects of inflation have been formalized in Lagos and Rocheteau (2005), Ennis (2009), Liu, Wang, and Wright (2011), and Nosal (2011).

We study first monetary policy in the context of a pure monetary economy, $\chi_m = 1$.

Proposition 4 ***(Outside options and rent seeking in a pure monetary economy.)*** *Suppose $\chi_m = 1$. There exists at least one steady-state equilibrium with $Z^b > 0$ and $\mu > 0$. In the neighborhood of $i = 0^+$, as i increases, Z^b decreases, provided that $v'(\mu)(1-\mu)$ is increasing in μ for all $\mu \in (0,1)$, and the effect on μ is ambiguous.*

The effect of i on consumers' outside options is a priori ambiguous as it depends not only on the direct effect of i on Z^b (Z^b is decreasing in i) but also on the endogenous response of producers in terms of rent-seeking effort. Graphically, in Figure 6, as i increases, the purple area shrinks and rent sizes tend to increase. This effect gives producers incentives to raise their rent-seeking effort, which has the opposite effect on the purple area. Moreover, in monetary matches, an increase in i tightens liquidity constraints and has a first-order negative effect on rent seeking. If the condition on v imposed in Proposition 4 is satisfied, then the first effect dominates, i.e., an increase in i reduces Z^b. Otherwise, it is possible for Z^b to increase with i. By a similar logic, the overall effect of an increase of i on μ is ambiguous. Rent seeking decreases when λ is large because the role of the buyer's outside option vanishes.

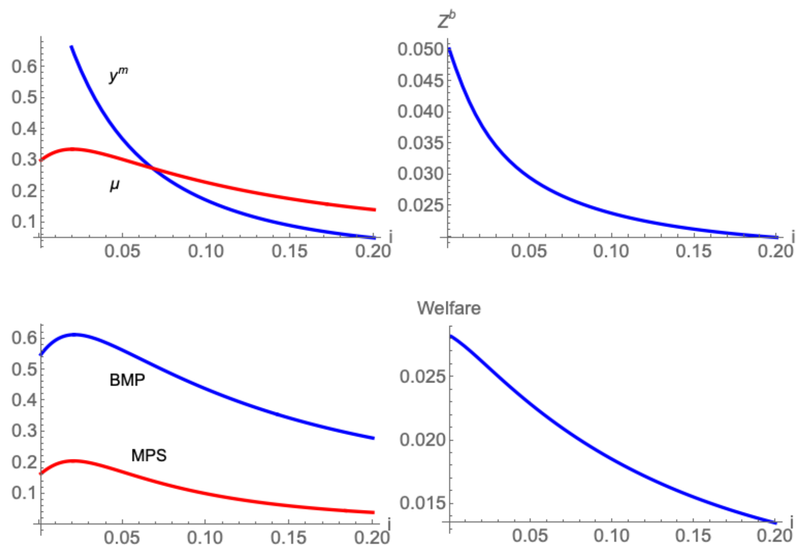

Figure 7: The impact of monetary policy.

We now turn to an economy with both money and credit trades to illustrate these effects with a numerical example. The parameter values are the same as in the previous section except $v_0 = 0.05$, $b = 0.1$, $\lambda = 0.5$ and $\gamma = 0.3$.[22] The key insight is that as i increases, rent seeking, our measures of bilateral monopoly power (BMP), and seller's market power (MPS) are all non-monotone (bottom left panel). Initially, rent seeking and market power increase due to the negative effect of i on consumers' outside options. Above a threshold, these measures decrease because the tightening of consumers' liquidity constraints reduces rent sizes. While

[22]The choice of the parameter values in Figure 7 is not meant to be realistic but to highlight some features of the equilibria.

in this numerical example the Friedman rule is optimal, there are other set of parameter values for which the Friedman rule is suboptimal.

4.4 Market power as a self-fulfilling prophecy

We now show that firms' market power can result from self-fulfilling beliefs. The key channel to explain the possibility of multiple equilibria comes from (24) according to which an increase in Z^b reduces firms' rent seeking by shrinking the size of match surpluses. In order to make the point that this multiplicity is not due to the presence of fiat money, we consider the case where all trades are pure credit trades, $\chi_d = 1$.

From (21) and (24), assuming an interior solution for μ, an equilibrium is a pair, (Z^b, μ), solution to:

$$Z^b = \frac{\bar{\alpha}(1-\mu)}{\rho+\lambda+\gamma+\bar{\alpha}(1-\mu)}\left[u(y^*)-y^*\right] \quad (25)$$

$$\upsilon'(\mu) = \alpha^s \frac{\rho+\lambda+\gamma}{\rho+\lambda+\gamma+\bar{\alpha}(1-\mu)}\left[u(y^*)-y^*\right]. \quad (26)$$

The equilibrium has a simple recursive structure. Equation (26) determines μ, and given μ (25) determines Z^b. Once again, note that the trading speed enters (25) and (26) through the term $\bar{\alpha}(1-\mu)$.

Proposition 5 ***(Outside options and rent seeking in a pure credit economy.)*** *Suppose* $\chi_d = 1$.

1. *If $\upsilon'(\mu)(1-\mu)$ is increasing in μ for all $\mu \in (0,1)$, then the equilibrium is unique. If $\bar{\alpha}$ is sufficiently large, then $\partial\mu/\partial\bar{\alpha} < 0$ and $\partial Z^b/\partial\bar{\alpha} > 0$.*

2. *Suppose $\upsilon(\mu) = \upsilon_0\mu^2/2$. If $\bar{\alpha} > \rho+\lambda+\gamma$, then there exists $0 < \underline{\upsilon}_0 < \bar{\upsilon}_0$ such that for all $\upsilon_0 \in (\underline{\upsilon}_0, \bar{\upsilon}_0)$, there exists two steady-state equilibria with $\mu \in (0,1)$ and one equilibrium with $\mu = 1$.*

The condition in the first part of Proposition 5 is satisfied, for example, for $\upsilon(\mu) = \left[\mu/(1-\mu)\right]^{1+a}/(1+a)$ with $a > 0$. In that case, the equilibrium is unique. If $\bar{\alpha}$ is sufficiently large, an increase in $\bar{\alpha}$ reduces rent seeking and raises consumers' outside option. Both effects contribute to a decline in sellers' market power.

The second part of Proposition 5 illustrates the possibility of multiple equilibria generated by the interaction between producers' rent seeking and consumers' outside options for a quadratic cost function for rent seeking. If $\upsilon'(1) = \upsilon_0$ is in some intermediate range, then there are two interior equilibria with $\mu \in (0,1)$ and a corner equilibrium with $\mu = 1$. The interior equilibria correspond to the two roots of the following quadratic equation:

$$\upsilon_0\mu\left[\rho+\lambda+\gamma+\bar{\alpha}(1-\mu)\right] = \alpha^s\left(\rho+\lambda+\gamma\right)\left[u(y^*)-y^*\right]. \quad (27)$$

The logic for multiple equilibria goes as follows. If consumers believe that all producers have a large bargaining share, then the value of their outside options is low. In that case, rents are large and it is optimal for producers to invest large efforts in rent seeking, which then confirms consumers' initial beliefs. By a similar logic, there can be an equilibrium where producers have a low bargaining share, the value of consumers' outside options is large, and rent seeking is low. A necessary condition for multiple equilibria is that trading frictions are not too large, i.e., $\bar{\alpha}$ is not too small.

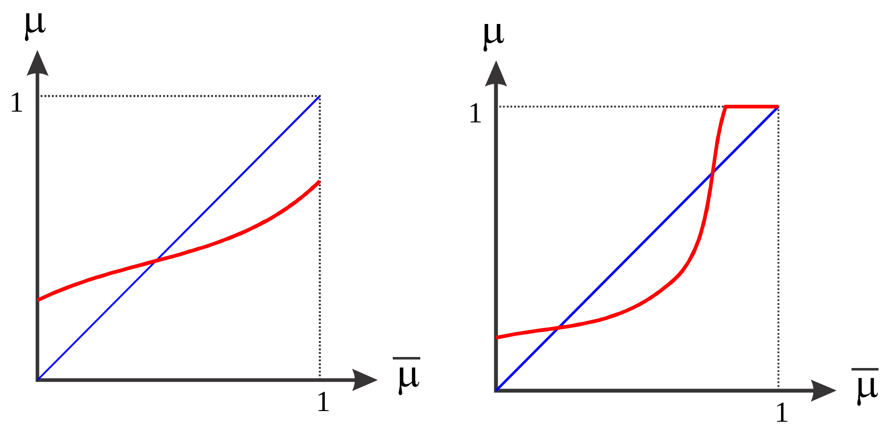

Figure 8: Best-response rent-seeking function. Left: unique Nash equilibrium; Right: multiple Nash equilibria.

Formally, we can show the existence of strategic complementarities by computing the effect of an increase in the rent-seeking effort of all producers except one, $\bar{\mu}$, on the rent-seeking effort of the remaining producer, μ. From (21) and (24),

$$\frac{\partial \mu}{\partial \bar{\mu}} = \frac{\partial \mu}{\partial Z^b} \times \frac{\partial Z^b}{\partial \bar{\mu}},$$

where

$$\frac{\partial \mu}{\partial Z^b} = \frac{-\alpha^s}{\upsilon''(\mu)} < 0 \text{ and } \frac{\partial Z^b}{\partial \bar{\mu}} = \frac{-\left[u(y^*) - y^* - Z^b\right]}{\rho + \lambda + \gamma + \bar{\alpha}(1-\bar{\mu})} < 0.$$

This calculation shows that the complementarities operate through endogenous outside options, $\partial Z^b/\partial \bar{\mu} < 0$, and the slope of the best response function is positive, $\partial \mu/\partial \bar{\mu} > 0$. The strategic complementarities are strong enough to generate multiple equilibria if $\partial \mu/\partial \bar{\mu} > 1$ over some interval. In the right panel of Figure 8, the best response function of the producer has a slope larger than one when $\bar{\mu}$ is sufficiently large, which leads to two interior Nash equilibria and one corner Nash equilibrium with $\mu = \bar{\mu} = 1$. If the strategic complementarities are not strong enough, then the best response is as described in the left panel of Figure 8, in which case there is a unique Nash equilibrium.

4.5 Frictionless limits

The next proposition studies the limit of the decentralized equilibrium when $\bar{\alpha}$ goes to $+\infty$. It illustrates two important insights of the model. First, there is an equilibrium that converges to perfect competition at the limit when trading frictions vanish. Second, endogenous rent seeking can prevent convergence to perfect competition in some other equilibria.

Proposition 6 ***(Frictionless limits with endogenous outside options.)***

1. ***Convergence to perfect competition***. *As $\bar{\alpha} \to +\infty$, there exists an equilibrium where rents vanish, $S^m \to 0$, $S^d \to 0$, $Z^b \to u(y^*) - y^*$, and $\mu \to 0$.*

2. ***Convergence to perfect monopolies***. *If $\upsilon'(1) \leq \lambda\chi^d[u(y^*) - y^*]$, then there exists a sequence of equilibria, $\{Z_n^b, \mu_n, \bar{\alpha}_n\}_{n=0}^{+\infty}$, such that $\bar{\alpha}_n \to \infty$, $\mu_n < 1$ for all n, $\mu_n \to 1$, $Z_n^b \to Z^b \in (0, u(y^*) - y^*)$, and $S_n^d \to S^d > 0$.*

The first part of the proposition shows that if trading frictions are reduced and buyers can find sellers almost instantly, then there always exists a sequence of equilibria where rents, S^m and S^d, vanish and Z^b approaches $u(y^*) - y^*$ as $\bar{\alpha}$ tends to $+\infty$. From (24), rent-seeking activities and markups disappear, $\mu = 0$. In accordance with the no-rent condition of Ostroy (1980), this allocation corresponds to a perfect competition outcome. Such equilibria formalize the intuition conveyed by the epigraph of this paper according to which the ability to quickly switch among potential trading partners erodes market power and promotes competition.

The second part of the proposition shows that there can be other equilibria that do not implement perfect competition at the limit. If $\upsilon'(1)$ is not too large, there exist limiting equilibria where producers have all the bargaining power, $\mu = 1$, and consumers' rents are positive, $Z^b < u(y^*) - y^*$. The failure of perfect competition is due to the response of rent seeking to reduced search frictions that overpowers the competitive effect on outside options.

We illustrate this result in the context of the pure credit economy with the quadratic cost of rent seeking studied in Proposition 5. From (27), when $\bar{\alpha}$ is large, μ approaches to one of the solutions to

$$\upsilon_0\mu\left[\rho + \lambda + \gamma + \bar{\alpha}(1 - \mu)\right] = \lambda\left(\rho + \lambda + \gamma\right)\left[u(y^*) - y^*\right].$$

So $\mu \to 0$ is a solution as $\bar{\alpha} \to \infty$. The other solution is $\mu \to 1$ with $\bar{\alpha}(1-\mu) \to (\rho + \lambda + \gamma)\left[\lambda\left[u(y^*) - y^*\right]/\upsilon_0 - 1\right]$. From (25),

$$\frac{Z^b}{u(y^*) - y^*} = \frac{\lambda\left[u(y^*) - y^*\right] - \upsilon_0}{\lambda\left[u(y^*) - y^*\right]} \in (0, 1).$$

Even though the producer's share approaches one asymptotically, the value of the outside option does not go to zero. Instead, it converges to a fraction of the overall gains from trade, $u(y^*) - y^*$.

We illustrate these results in the context of a monetary economy in Figure 9. Both panels assume $i = 0.01$ and use the same parameters as Figure 7, except $\upsilon_0 = 0.03$ and the left panel assumes an alternate rent seeking function with $\upsilon'(1) = \infty$ (see Table 1 in the Appendix for details). In the left panel, there is a single equilibrium at the intersection of the blue curve that specifies the optimal μ for given Z^b and the red curve that specifies Z^b for given μ. As shown by the green arrow, the equilibrium converges to a no-rent allocation (Z^b converges to $u(y^*) - y^*$) when $\bar{\alpha}$ becomes large.

In the right panel, there are multiple intersections between the blue and red curves corresponding to distinct equilibria. While the equilibrium with the highest Z^bconverges to perfect competition with $\mu = 0$ and $Z^b = u(y^*) - y^*$, a middle equilibrium converges to an equilibrium with $\mu = 1$ and $Z^b > 0$. The lowest equilibrium converges to a monopolistic outcome with $\mu = 1$ and $Z^b = 0$.

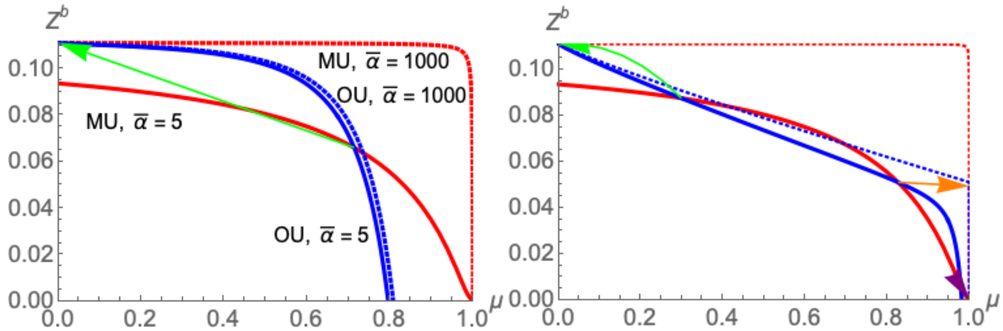

Figure 9: Equilibrium with outside option. (Left) $\upsilon'(1) > \lambda\chi^d[u(y^*) - y^*]$ (Right) $\upsilon'(1) \leq \lambda\chi^d[u(y^*) - y^*]$.

Figure 10 and 11 plot the equilibrium set as a function of $\bar{\alpha}$.[23] When $\upsilon'(1)$ and $\bar{\alpha}$ are large (Figure 10), there is a unique equilibrium represented by the green line. This equilibrium converges to the perfect competition outcome. If $\bar{\alpha}$ is small, then the unique equilibrium corresponds to the blue line, in which case an increase in $\bar{\alpha}$ leads to more rent seeking. This finding illustrates the role played by trading frictions for the emergence of different types of equilibria with different comparative statics.

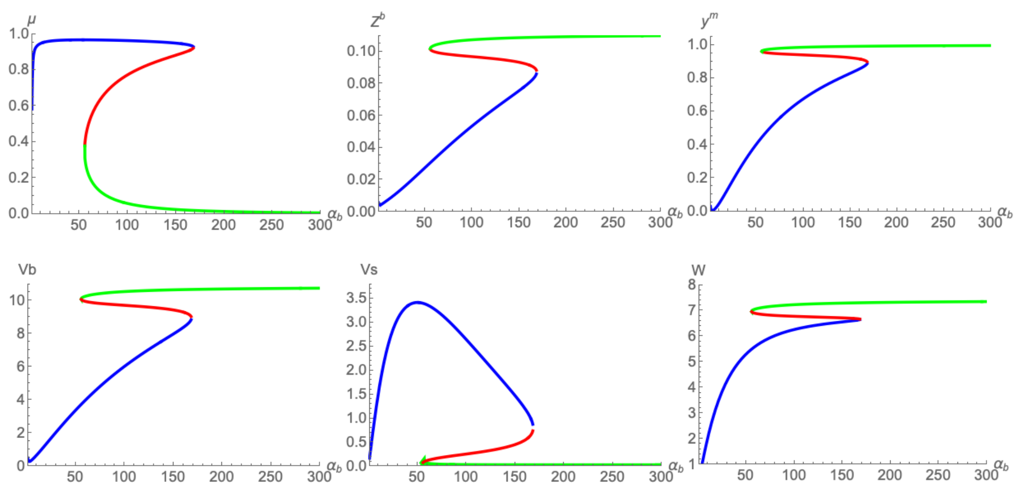

Figure 10: Equilibrium set when $\upsilon'(1)$ is large.

When $\upsilon'(1)$ is small (Figure 11), there are multiple equilibria at the limit when $\bar{\alpha} = +\infty$. For small $\bar{\alpha}$, there is a unique equilibrium (blue line) and it quickly converges to the monopoly outcome as $\bar{\alpha}$ rises. So if we consider a selection mechanism according to which the equilibrium is obtained by continuity by reducing trading frictions starting from an equilibrium with low $\bar{\alpha}$, then the selected equilibria do not converge to perfect competition. However, when $\bar{\alpha}$ is above a threshold, the equilibrium leading to perfect competition by continuity (the green line) exists. But then another equilibrium exists as well (the red line) where rent seeking increases with $\bar{\alpha}$ and positive rents persist at the limit.

[23] Figure 11 adopts the same parameters as the right panel of Figure 9. Figure 10 also uses the same parameters but with an alternate rent seeking function and $\lambda = 2$. See Table 1 in the Appendix for all parameter values.

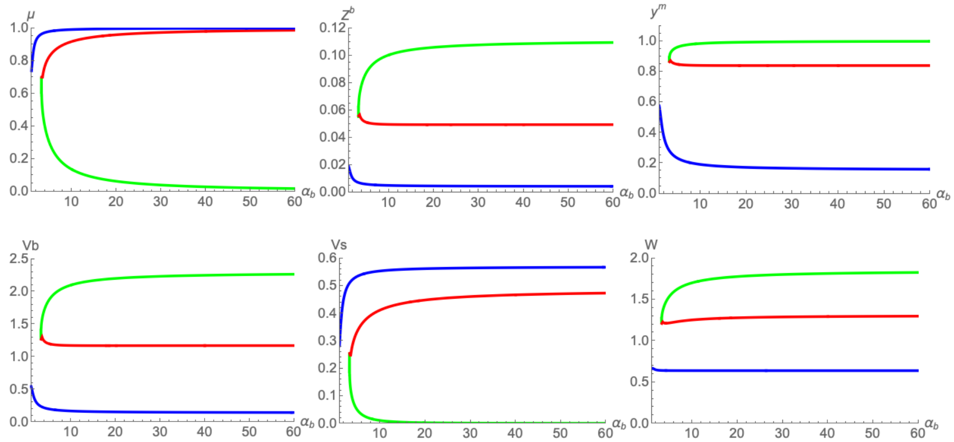

Figure 11: Equilibrium set when $\upsilon'(1)$ is small.

A lesson from these results is that even though an improvement in the matching technology between consumers and producers, such as the introduction of the Internet and online trading, can bring the economy closer to the perfect competition outcome, it can also bring the economy further away from it by exacerbating rent-seeking efforts and enhancing producers' market power.

5 Market concentration: Entry

We established that consumers' outside options play a central role to explain the size of rents in pairwise meetings. In Proposition 3 a key determinant of the value of these outside options is the rate at which consumers meet producers, α^b. We now endogenize this rate by introducing firm entry, which allows us to relate seller's market power to the market concentration of firms.

5.1 Market power and market concentration

The relationship between market power and market concentration is provided by (21) according to which the value of the consumer's outside option depends on the speed at which she can meet alternative producers, α^b. In order to establish a connection with market concentration, we define the measure of participating sellers per active buyer by θ. We think of θ as market dilution and $1/\theta$ as market concentration. The relationship between α^b and θ is provided by a constant-returns-to-scale matching technology between consumers and producers. Formally, the matching rates for buyers and sellers are $\alpha^b = \alpha(\theta)$ and $\alpha^s = \alpha(\theta)/\theta$, respectively, where $\alpha(0) = 0$, $\alpha'(0) = +\infty$, $\alpha'(\theta) > 0$, and $\alpha''(\theta) < 0$. The following lemma formalizes the relationship between outside option and market concentration.

Lemma 4 ***(Outside options and market concentration.)*** *In the neighborhood of $i = 0^+$, the elasticity* *of the consumers' outside options, Z^b, with respect to θ along the HJB equilibrium condition (21), is given by*

$$\left.\frac{\partial Z^b/Z^b}{\partial\theta/\theta}\right|_{Eq\ (21)} = \frac{\alpha'(\theta)\theta}{\alpha(\theta)}BMP. \tag{28}$$

The elasticity of Z^b with respect to θ is the product of the elasticity of the matching function and our measure of bilateral monopoly power. It is positive because a higher number of firms per consumer means higher α^b and hence higher Z^b.

Next, we endogenize θ by assuming free entry of producers. In order to participate in the market, sellers incur a flow cost $k > 0$.[24] In equilibrium, the expected utility of participating sellers is zero, which can be written as follows:

$$\max_{\mu\in[0,1]}\left\{-v(\mu) + \frac{\alpha(\theta)}{\theta}\mu\left[\chi_m S^m(a^*, Z^b, \mu) + \chi_d S^d(Z^b)\right]\right\} = k. \tag{29}$$

From (29), if consumers have better outside options, the surpluses in pairwise meetings shrink, and firms have less incentives to enter, which raises the concentration of firms. Note that the effective cost of entry is $k + v(\mu)$, which is endogenous.

Lemma 5 ***(Market concentration and outside options.)*** *In the neighborhood of $i = 0^+$, the elasticity of θ with respect to Z^b along the free-entry equilibrium condition (29), is given by*

$$\left.\frac{\partial\theta/\theta}{\partial Z^b/Z^b}\right|_{Eq.\ (29)} = \frac{-1}{1-\alpha'(\theta)\theta/\alpha(\theta)}\frac{1-BMP}{BMP} < 0. \tag{30}$$

The two elasticities given by (28) and (30), which originate from the two equilibrium conditions that determine jointly Z^b and θ, are of the opposite sign. Changes that affect the entry condition (e.g., entry costs) move the economy along the upward-sloping equation, (28), that determines Z^b, thereby leading to a positive correlation between concentration and sellers' market power.[25] On the contrary, changes that affect the condition that determines Z^b, e.g., the cost of holding real balances, move the economy along the downward-sloping free-entry condition, (29), thereby leading to a negative correlation between concentration and sellers' market power.

5.2 Definitions of concentration

We now turn to an empirical definitions of market concentration and relate it to θ. The market share of a firm is

$$s \equiv \frac{[\alpha(\theta)/\theta]\,p}{n_a\alpha(\theta)p} = \frac{1}{n_a\theta} = \frac{\lambda+\gamma+\alpha(\theta)}{\lambda\theta}, \tag{31}$$

where we used that $n_a = \lambda/\left[\lambda+\gamma+\alpha(\theta)\right]$. It is equal to the expected revenue of a firm, $[\alpha(\theta)/\theta]\,p$, divided by the total expenditure of active consumers, $n_a\alpha(\theta)p$, where p is the firm expected revenue in a match with

[24] Similar entry decisions can be found in the Pissarides (2000) model of unemployment and in the New Monetarist model of Rocheteau and Wright (2005, 2013).

[25] Covarrubias, Gutiérrez, and Philippon (2019) provide evidence that the rising concentration in the US is correlated with decreasing competition and productivity growth and increasing entry barriers and prices. Gutiérrez, and Philippon (2019) provide a novel measure of federal regulations and show that regulations and lobbying drive down the entry and growth of small firms relative to large ones.

a consumer. Hence, from (31), market share is equal to the inverse of the measure of sellers, $n_a\theta$, and it is decreasing in market tightness, θ. Following Philippon (2019, Box 2.1), we define market concentration according to the Herfindahl-Hirschman Index,

$$HHI \equiv \int_0^{n_a\theta} (s_j)^2 dj = \frac{\lambda + \gamma + \alpha(\theta)}{\lambda\theta}. \tag{32}$$

In the case of homogeneous firms, HHI is equal to s and it inversely related to the measure of firms per active consumer, θ.

5.3 Rent seeking and market concentration in a pure credit economy

Suppose first that all trades are credit trades, $\chi_d = 1$. From (21), (24), and (29), assuming an interior choice for rent seeking, an equilibrium is a triple, (μ, θ, Z^b), solution to:

$$k = \upsilon'(\mu)\mu - \upsilon(\mu) \tag{33}$$

$$\upsilon'(\mu) = \frac{\alpha(\theta)}{\theta}\frac{(\rho+\lambda+\gamma)}{\rho+\lambda+\gamma+\alpha(\theta)(1-\mu)}\left[u(y^*) - y^*\right] \tag{34}$$

$$Z^b = \frac{\alpha(\theta)(1-\mu)}{\rho+\lambda+\gamma+\alpha(\theta)(1-\mu)}\left[u(y^*) - y^*\right]. \tag{35}$$

The equilibrium is solved recursively, the free-entry condition, (33), determines μ. Given μ, the first-order condition for rent seeking, (34), determines uniquely θ. Finally, (35) determines Z^b given μ and θ.

Proposition 7 ***(Market concentration and rent seeking in a pure credit economy.)*** *Suppose there is free entry of sellers, endogenous rent seeking, and $\chi_d = 1$. Assuming $\upsilon'(0) = 0$ and $\upsilon'(1) = +\infty$, there is a unique equilibrium and it features $\mu \in (0,1)$, $\theta > 0$, and $0 < Z^b < u(y^*) - y^*$. Moreover, rent seeking (μ) and market concentration (HHI) increase with entry costs while the value of consumers' outside options (Z^b) decreases.*

In contrast to Proposition 5, if there is free entry of firms, the equilibrium with endogenous rent seeking and endogenous outside options is unique. The entry cost pins down firms' rent-seeking effort. An increase in entry costs raises market concentration, generates more rent-seeking by firms, and lowers the value of consumers' outside options.

5.4 Monetary policy and market concentration

Next, we study the effects of monetary policy on market concentration in a pure monetary economy, $\chi_m = 1$, where the producer's bargaining share is exogenous, $\mu = \bar{\mu}$.[26] The equilibrium is a pair (θ, y^m) solution to

$$\frac{\alpha(\theta)}{\theta}\bar{\mu}\left[u(y^m) - y^m - Z^b(\theta, i)\right] = k \tag{36}$$

$$\frac{\alpha(\theta)(1-\bar{\mu})\left[u'(y_m) - 1\right]}{\bar{\mu}u'(y_m) + 1 - \bar{\mu}} \leq i \quad \text{"="} \text{ if } y^m > 0, \tag{37}$$

[26] If λ goes to $+\infty$, this special case corresponds to a continuous-time version of Rocheteau and Wright (2005, 2013).

where $Z^b(\theta,i)$ is the solution to (21) with $\alpha^b = \alpha(\theta)$. It satisfies $\partial Z^b/\partial\theta > 0$, $Z^b(\theta,i) = 0$ for all $\theta < \underline{\theta}(i)$, and $Z^b(+\infty,i) = u(y^*) - y^*$. Equation (36) gives a positive relationship between θ and y^m with $\theta = 0$ if $y^m = 0$ and $\theta = \bar{\theta}$ if $y^m = y^*$. It is represented by the curve labelled FE in Figure 12. Equation (37) gives a positive relationship between y^m and θ where $y^m = 0$ if $\theta \leq \underline{\theta}$ and $y^m = y^*$ when $\theta = +\infty$. It is represented by the curve labelled YM in Figure 12. Therefore, the number of equilibria is generically even. We will focus on the equilibrium with the highest output and entry. In the left panel of Figure 12, an increase in k shifts the FE locus downward, in which case market concentration (HHI) increases and the value of buyer's outside options, Z^b, decreases. We now turn to monetary policy.

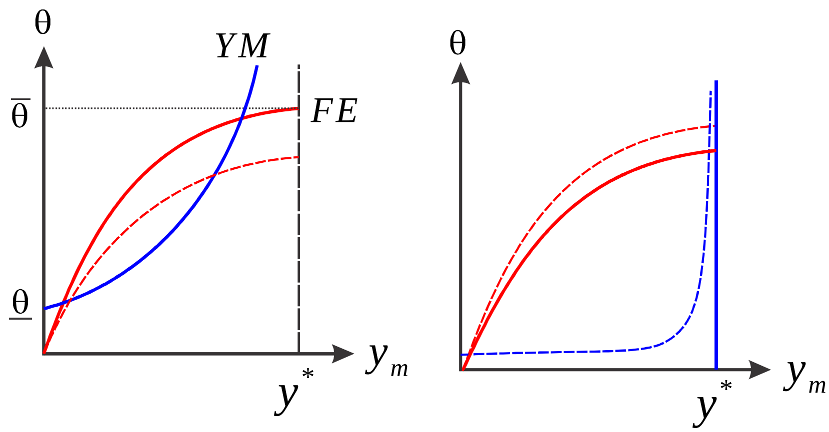

Figure 12: Equilibrium with endogenous outside option. Left: effect of a change in k; Right: Effect of a change in i.

Proposition 8 ***(Market concentration and monetary policy.)*** *Suppose there is free entry of sellers with $\mu = \bar{\mu}$ and $\chi_m = 1$. There exists $k_0 > 0$ such that for all $k < k_0$ there are an even number of equilibria with positive entry.*

1. *If $\lambda = +\infty$ then an increase in i raises firm concentration (HHI).*
2. *If $\lambda < +\infty$ and $i \approx 0$, then an increase in i reduces firm concentration (HHI) and buyers' outside options, Z^b.*

The effect of monetary policy on market concentration is a priori ambiguous. On the one hand, an increase in the nominal interest rate reduces the payment capacity of consumers, which reduces y^m for given θ. It is the standard money-demand effect in New Monetarist models. Graphically, the curve representing (37) moves to the left. On the other hand, it raises the cost of holding real balances while searching for alternative sellers, which is analogous to an increase in search costs. Graphically, as Z^b falls, the curve representing (36) moves upward, which leads to more entry by firms and lower market concentration. The overall effects on θ and y^m are a priori ambiguous. If $\lambda \to +\infty$, the second effect vanishes as there is no opportunity cost associated with a trade. In that case an increase in i reduces θ and y^m, and raises HHI. If $\lambda < +\infty$ but i is close to 0, then the first effect is only second order because y^m is close to y^* and the second effect dominates. In that case, an increase in i raises θ but reduces Z^b, y^m, and HHI. This case is illustrated in the right panel of Figure 12.

If we reintroduce endogenous rent seeking in this pure monetary economy and assume $\lambda \to \infty$ then the rent-seeking effort solves

$$\upsilon'(\mu)\mu = \frac{k}{\mu u'(y^m) + 1 - \mu}.$$

The rent-seeking effort is no longer pinned down by the entry cost alone. It is an increasing function of y^m which depends on monetary policy. It means that in a monetary economy, the endogenous part of the entry cost depends on all fundamentals and policy. We turn to such an economy next.

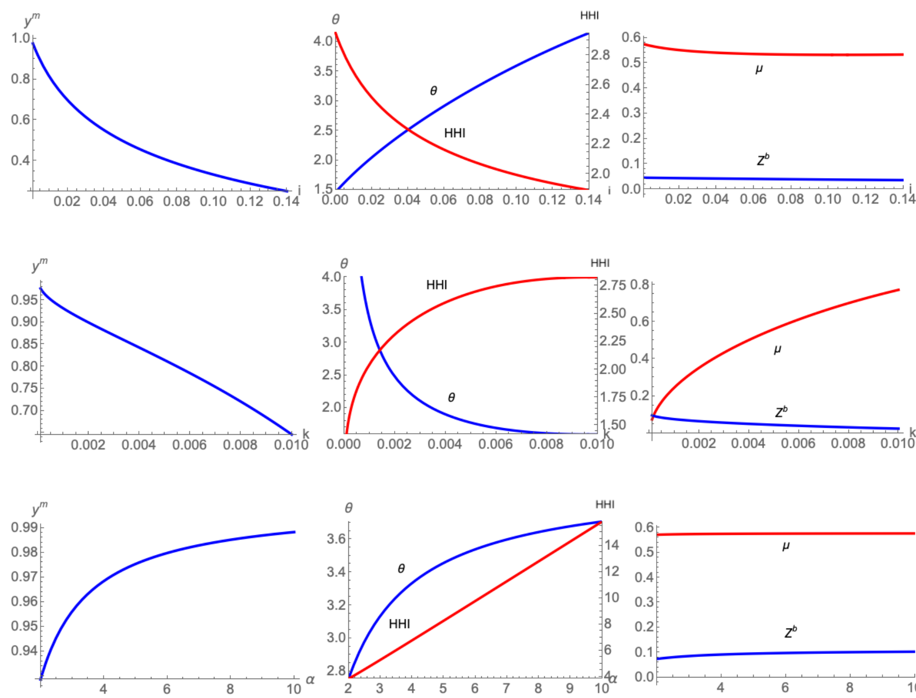

Figure 13: Numerical examples of changes in i, k and α.

We describe the economy with both monetary and credit trades with a numerical example in Figure 13. Equilibrium outcomes are consistent with the results described so far. The top panels show that an increase in the interest rate reduces both the value of consumers' outside options, Z^b, and market concentration. In the middle panels, an increase in k reduces the value of consumers' outside options but raises market concentration and rent seeking. Finally, in the bottom panels, an increase in the efficiency of the matching process raises both the value of consumers' outside options and market concentration.

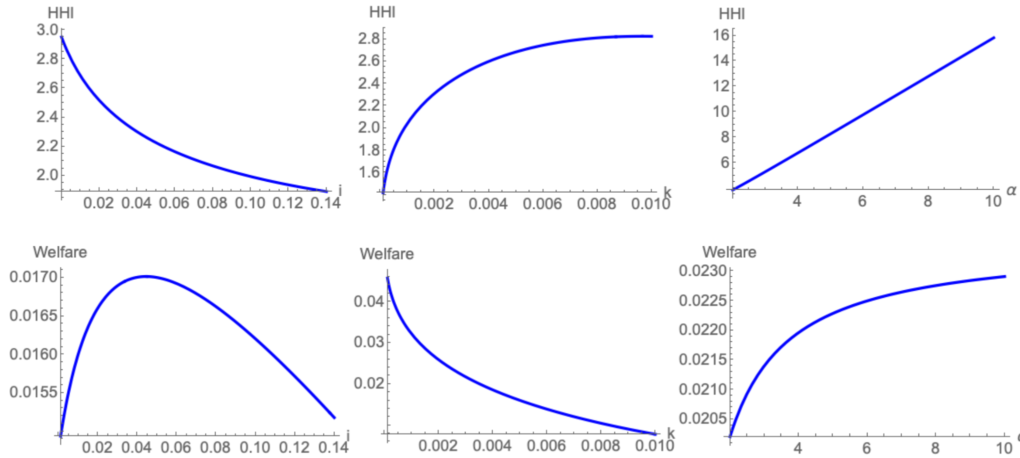

Figure 14: Correlation between HHI and Welfare.

Figure 14 presents the correlation between the index of concentration, HHI, and social welfare which is equal to consumers' surplus (since the sellers' profits cancel out with entry costs), namely the sum of $u(y)-p$ across meetings. The concentration index HHI drops as k falls and the economy becomes more competitive but it increases as α rises and the economy also becomes more competitive. Hence, HHI is not a measure of market power. It is not a good indicator of welfare either. Depending on the source of the exogenous variation, HHI and welfare can be positively or negatively correlated. The optimal i that maximizes welfare (the Friedman rule is suboptimal) does not minimize market concentration.

5.5 Vanishing barriers to entry

In the following, we describe the outcome as barriers to entry as represented by the entry cost, k, vanish.[27]

Proposition 9 ***(Frictionless limit with entry)*** *As k tends to 0, the decentralized equilibrium approaches a perfect competition outcome with $Z^b = u(y^*) - y^*$, $S^m = S^d = 0$, and $\mu = 0$. Market concentration goes to 0, i.e., $\theta = +\infty$ and $HHI = 0$.*

In order to understand this result, notice that from (29),

$$\max_{\mu \in [0,1]} \left\{ -\upsilon(\mu) + \frac{\alpha(\theta)}{\theta} \mu \mathbb{E}[S] \right\} = k,$$

where $\mathbb{E}[S]$ denotes the expected surplus of a pairwise meeting. As the cost of entry on the right side goes to 0, the left side of the equality must also go to 0, which implies that expected profits and rent-seeking efforts by firms vanish. The measure of firms per consumers goes to infinity, i.e., market concentration goes to 0, which allows consumers to switch instantly from one firm to another and drives the value of their outside options to its maximum, $u(y^*) - y^*$. As a result, rents disappear.

[27] One can also interpret entry costs in different ways. It can be a technological fixed cost that cannot be reduced by policy or some wasteful expenses due to various barriers to entry. Here we take the second interpretation.

5.6 Vanishing trading frictions

We now consider the limit as meeting rates go to infinity. We reexpress the meeting technology as $\alpha(\theta) \equiv A\bar{\alpha}(\theta)$ and take the limit as $A \to +\infty$. We restrict our attention to a pure credit economy where $\chi_d = 1$.

We start by characterizing the constrained-efficient allocations whereby the planner solves:

$$\max_{\theta, n_a} \int_0^{+\infty} e^{-\rho t} \left\{ A\bar{\alpha}(\theta_t) n_{a,t} \left[u(y^*) - y^* \right] - \theta_t n_{a,t} k \right\} dt \tag{38}$$

$$\text{s.t. } \dot{n}_{a,t} = \lambda(1 - n_{a,t}) - \left[A\bar{\alpha}(\theta_t) + \gamma \right] n_{a,t}, \tag{39}$$

and $n_{a,0}$ given. The planner maximizes the discounted sum of all utility flows subject to the law of motion of active buyers. We ignored rent-seeking efforts since they are socially wasteful and the planner would want to set $\mu = 0$. Let Υ_t denote the current-value costate variable associated with the law of motion for n_a.

Lemma 6 *Suppose $\chi_d = 1$ and $\theta\bar{\alpha}'(\theta)/\bar{\alpha}(\theta) < 1$ is non-increasing in θ. The constrained-efficient allocation is such that $(\theta_t, \Upsilon_t) = (\theta^*, \Upsilon^*)$ for all t where (θ, Υ) is the solution to*

$$k = A\bar{\alpha}'(\theta^*) \left[u(y^*) - y^* - \Upsilon^* \right] \tag{40}$$

$$(\rho + \lambda + \gamma) \Upsilon^* = \max_{\theta \geq 0} \left\{ A\bar{\alpha}(\theta^*) \left[u(y^*) - y^* - \Upsilon^* \right] - \theta^* k \right\}. \tag{41}$$

As $A \to +\infty$, $\Upsilon^* \to u(y^*) - y^*$, $A\bar{\alpha}(\theta^*) \to +\infty$, and the flow welfare tends to $\lambda \left[u(y^*) - y^* \right]$.

Condition (40) corresponds to the optimal entry of firms that equalizes the entry cost and the marginal benefit from having one more firm in the market. In accordance with Hosios (1990), market tightness is optimal when the entry cost is equal to a fraction $\theta^*\bar{\alpha}'(\theta^*)/\bar{\alpha}(\theta^*)$ of the expected match surplus, namely $A\left[\bar{\alpha}(\theta^*)/\theta^*\right]\left[u(y^*) - y^* - \Upsilon^*\right]$. Condition (41) defines the shadow value of an active buyer, Υ^*, which is the analog of Z for the social planner. The flow social value of an active buyer is equal to the expected match surplus, $A\bar{\alpha}(\theta^*)\left[u(y^*) - y^* - \Upsilon^*\right]$, net of the aggregate entry costs per buyer, $\theta^* k$. At the frictionless limit, Υ^* approaches $u(y^*) - y^*$. We now turn to the frictionless limit of the decentralized equilibrium.

Proposition 10 ***(Vanishing trading frictions under free entry.)*** *Suppose $\chi_d = 1$.*

1. *If $k < \upsilon'(1) - \upsilon(1)$, then $\mu \in (0, 1)$ for all $A \geq 0$. As $A \to +\infty$ market tightness converges to*

$$\theta_\infty = \frac{(\rho + \lambda + \gamma)}{\upsilon'(\mu)(1 - \mu)} \left[u(y^*) - y^* \right], \tag{42}$$

   $HHI \to +\infty$, and $Z^b \to u(y^*) - y^*$. Despite $\theta_\infty \neq \theta^*$ generically, social welfare at the frictionless limit approaches the constrained-efficient level.

2. *If $k \geq \upsilon'(1) - \upsilon(1)$, then $\mu = 1$ for all $A \geq 0$. As $A \to +\infty$, $\theta \to +\infty$, $\theta n_a \to \lambda[u(y^*) - y^*]/[k + \upsilon(1)]$, $HHI \to 0$, and $Z^b \to 0$. Social welfare at the limit is 0.*

If entry costs are not too large, $k < \upsilon'(1) - \upsilon(1)$, then the rent-seeking effort is interior. The expected profits of firms are commensurate to entry costs, i.e., they are small, and it is not optimal for firms to invest in maximum rent seeking so as to capture the whole match surpluses. As the efficiency of the matching technology goes to infinity, market tightness remains positive and finite but consumers' and producers' meeting rates go to infinity. The index of market concentration goes to infinity and rents disappear at the limit. So, the economy with small entry costs implements a perfect competition outcome at the frictionless limit. At the limit rents become vanishing small, as $Z^b \to u(y^*) - y^*$, but firms still seek rents (i.e. μ is interior) because firms get matched quickly and hence the effective rent-seeking cost is also small. Even though market tightness is not constrained-efficient in general – it is constrained-efficient if and only if $\upsilon'(\mu)(1-\mu) = \{\bar{\alpha}(\theta^*)/[\theta^*\bar{\alpha}'(\theta^*)] - 1\}\, k$ – the discrepancy is not welfare relevant at the limit as the measure of sellers is 0 and social welfare achieves its constrained-efficient level, $\lambda\,[u(y^*) - y^*]$.

If entry costs are large, $k \geq \upsilon'(1) - \upsilon(1)$, then it becomes profitable for firms to invest in maximum rent seeking so as to capture the full match surplus, $\mu = 1$. Irrespective of the efficiency of the meeting process, A, consumers have no valuable outside options, $Z^b = 0$, and rents remain maximum even as trading frictions are reduced. Market concentration, HHI, goes to 0 and the measure of firms is strictly positive. Even though consumers can switch almost instantly from one producer to another, producers' rent-seeking efforts prevent the economy from ever converging to a perfect competition outcome. In fact, the overall social welfare remains equal to zero for all A as consumers receive no surplus from trade and sellers' expected profit is zero due to free entry. In other words, the aggregate gains from trade, $\lambda\,[u(y^*) - y^*]$, are dissipated in the form of entry and rent-seeking costs. These results illustrate the important role played by entry costs to allow or prevent the convergence to perfect competition. If entry costs induce $\mu \in (0,1)$, then the economy becomes perfectly competitive at the limit; if the entry costs induce $\mu = 1$, then sellers become perfect monopolies.

6 Market concentration: Selection

In the following we introduce ex ante heterogeneity among firms in terms of the quality of their output. The production of a firm is valued according to $\varepsilon u(y)$ where the cumulative distribution of ε across firms is $F(\varepsilon)$ with support $[0, \bar{\varepsilon}] \subset \mathbb{R}_+$ for some $\bar{\varepsilon} > 0$. We denote y^*_ε the solution to $\varepsilon u'(y) = 1$. In contrast to the previous section, the measure of firms is constant and normalized to one but only a subset of firms will have a positive market share in equilibrium.

6.1 Selection and concentration

In a match where the seller is of type ε, there exists gains from trade if $\max_{y \leq a^*} \{\varepsilon u(y) - y\} > Z^b$. This inequality holds iff $\varepsilon > \varepsilon_R$ where the reservation value for output quality, $\varepsilon_R(a^*, Z^b)$, solves

$$\max_{y \leq a^*} \{\varepsilon_R u(y) - y\} = Z^b. \tag{43}$$

Lemma 7 ***(Quality threshold.)*** *The quality threshold above which gains from trade are positive obeys:*

$$\begin{aligned} \varepsilon_R(a^*, Z^b) &= \hat{\varepsilon}(Z^b) \quad \text{if } \hat{\varepsilon}(Z^b) \leq \tilde{\varepsilon}(a^*) \\ &= \frac{a^* + Z^b}{u(a^*)} \quad \text{otherwise,} \end{aligned}$$

where $\hat{\varepsilon}(Z^b)$ *is the solution to* $\varepsilon u(y^*_\varepsilon) - y^*_\varepsilon = Z^b$ *and* $\tilde{\varepsilon}(a^*)$ *is the solution to* $\tilde{\varepsilon} u'(a^*) = 1$.

The reservation value, $\varepsilon_R(a^*, Z^b)$, decreases with a^* and increases with Z^b. If buyers hold more real balances, then they are willing to buy from lower quality firms, in which case they compensate for the low quality by buying large quantities. If buyers' outside option improves, then they become pickier and raise their threshold for quality. Consumers are pickier in a monetary trade, $\varepsilon^d_R = \hat{\varepsilon}(Z^b) \leq \varepsilon^m_R = \varepsilon_R(a^*, Z^b)$, because they are constrained by the amount they can spend.

We assume that all agents are part of the matching process. The matching rate of a buyer is $\alpha^b = \bar{\alpha}$ and the matching rate of an active seller is $\alpha^s = \bar{\alpha} n_a$ where

$$n_a = \frac{\lambda}{\lambda + \gamma + \bar{\alpha}\left\{\chi_m\left[1 - F(\varepsilon^m_R)\right] + \chi_d\left[1 - F(\varepsilon^d_R)\right]\right\}}$$

is the steady-state measure of active buyers. The last term of the denominator is the rate at which an active buyer finds an acceptable opportunity to consume.

The terms of trade in pairwise meetings, $(y_\varepsilon, p_\varepsilon)$, are given by

$$p_\varepsilon = \mu_\varepsilon\left[\varepsilon u(y_\varepsilon) - Z^b\right] + (1 - \mu_\varepsilon) y_\varepsilon \leq a^*, \tag{44}$$

with an equality if $y_\varepsilon < y^*_\varepsilon$, where μ_ε is the bargaining share of a type-ε seller. We denote y^m_ε the solution to (44) and $S^m_\varepsilon(a^*, Z^b) \equiv \max\left\{\varepsilon u(y^m_\varepsilon) - y^m_\varepsilon - Z^b, 0\right\}$ the surplus in a monetary match. In a credit match, the surplus is $S^d_\varepsilon(Z^b) \equiv \max\left\{\varepsilon u(y^*_\varepsilon) - y^*_\varepsilon - Z^b, 0\right\}$.

The market share of firm ε is defined as

$$s_\varepsilon \equiv \frac{\bar{\alpha} n_a \bar{p}_\varepsilon}{\bar{\alpha} n_a \int_0^{\bar{\varepsilon}} \bar{p}_x dF(x)} = \frac{\bar{p}_\varepsilon}{\int_0^{\bar{\varepsilon}} \bar{p}_x dF(x)} \quad \text{for all } \varepsilon, \tag{45}$$

where $\bar{p}_\varepsilon = \chi_m p^m_\varepsilon + \chi_d p^d_\varepsilon$ is the average payment to a producer of type ε in monetary and credit matches. It is easy to check from (45) that $\mathbb{E}\left[s_\varepsilon\right] = 1$. Market concentration is defined by

$$HHI \equiv \int_0^{\bar{\varepsilon}} (s_\varepsilon)^2 dF(\varepsilon) = 1 + Var(s_\varepsilon), \tag{46}$$

where $Var(s_\varepsilon)$ is the variance of the distribution of market shares. The following lemma establishes a connection between the value of consumers' outside option and market concentration.

Lemma 8 ***(Concentration and competition.)*** *Consider a pure credit economy,* $\chi_d = 1$*, where all producers have the same bargaining share,* $\mu_\varepsilon \equiv \bar{\mu}$*. An increase in the value of consumers' outside options,* Z^b*, leads to higher market concentration, i.e.,* $\partial HHI / \partial Z^b > 0$.

As Z^b increases, the market share of the most productive firms increases while the market share of the least productive firms decreases. So a higher value of consumers' outside options generates a mean-preserving increase in the spread of the distribution of market shares, which then contributes to a higher market concentration.

6.2 Equilibrium selection

By a similar reasoning as in Section 4, $Z^b \equiv V^b - W^b$, is the unique solution to:

$$\begin{aligned}(\rho+\lambda+\gamma) Z^b &= \max_{a^* \geq 0}\left\{-ia^* + \bar{\alpha}\chi_m \int_{\varepsilon_R(a^*,Z^b)}^{\bar{\varepsilon}} (1-\mu_\varepsilon) S_\varepsilon^m(a^*, Z^b) dF(\varepsilon)\right\} \\ &\quad + \bar{\alpha}\chi_d \int_{\hat{\varepsilon}(Z^b)}^{\bar{\varepsilon}} (1-\mu_\varepsilon) S_\varepsilon^d(Z^b) dF(\varepsilon).\end{aligned} \tag{47}$$

The right side takes into account that consumers only purchase goods from firms whose quality is larger than ε_R, where $\varepsilon_R = \hat{\varepsilon}$ in credit meetings. Substituting $\partial S_\varepsilon^m / \partial a^*$ by its expression from Lemma 1 into the FOC of the buyer's maximization problem, a^* solves:

$$i = \bar{\alpha}\chi_m \int_{\varepsilon_R(a^*,Z^b)}^{\bar{\varepsilon}} (1-\mu_\varepsilon) \left\{\frac{\varepsilon u'\left[y_\varepsilon^m(a^* + \mu_\varepsilon Z^b)\right] - 1}{\mu_\varepsilon \varepsilon u'\left[y_\varepsilon^m(a^* + \mu_\varepsilon Z^b)\right] + 1 - \mu_\varepsilon}\right\}^+ dF(\varepsilon), \tag{48}$$

where $\{x\}^+ = \max\{0, x\}$. As i goes to 0, then the threshold, $\tilde{\varepsilon}$, above which the liquidity constraint binds approaches $\bar{\varepsilon}$, i.e., $y = y_\varepsilon^*$ in all active matches.

Finally, the rent-seeking effort of firms solves (24), i.e.,

$$\upsilon'(\mu_\varepsilon) = \frac{\alpha^s \chi_m S_\varepsilon^m(a^*, Z^b)}{\mu_\varepsilon \varepsilon u'(y_\varepsilon^m) + 1 - \mu_\varepsilon} + \alpha^s \chi_d S_\varepsilon^d(Z^b) \quad \text{for all } \varepsilon. \tag{49}$$

An equilibrium is a list, $\langle \varepsilon_R, Z^b, (\mu_\varepsilon), (y_\varepsilon^m), a^* \rangle$, where ε_R solves (43), Z^b solves (47), μ_ε solves (49) and y_ε^m solves (44) for each $\varepsilon > \varepsilon_R$ and a^* solves (48).

6.3 Market shares and search frictions

We start by studying the effects of reducing search frictions in a pure credit economy, $\chi_d = 1$, where the bargaining share is exogenous and equal to $\bar{\mu} < 1$ for all producers. From (47), an equilibrium reduces to a Z^b solution to

$$(\rho+\lambda+\gamma) Z^b = \bar{\alpha}(1-\bar{\mu}) \int_{\hat{\varepsilon}(Z^b)}^{\bar{\varepsilon}} \left[\varepsilon u(y_\varepsilon^*) - y_\varepsilon^* - Z^b\right] dF(\varepsilon). \tag{50}$$

The mathematical structure of this special case is isomorphic to that of the canonical job search model of McCall (1970). Indeed, if we reinterpret $w_\varepsilon = (1-\bar{\mu})\left[\varepsilon u(y_\varepsilon^*) - y_\varepsilon^*\right]$ as the wage drawn from a cumulative distribution $G(w) = \int_0^{\bar{\varepsilon}} \mathbb{I}_{\{w_\varepsilon \leq w\}} dF(\varepsilon)$, $\rho+\lambda+\gamma$ as workers' discount rate, and $\bar{\alpha}$ as the arrival rate of job offers, then Z^b is the reservation wage of the job seeker. Given this alternative interpretation, the following result should be intuitive:

Proposition 11 ***(Firm heterogeneity and search frictions)*** *Suppose* $\chi_d = 1$ *and* $\mu_\varepsilon = \bar{\mu} < 1$ *for all* ε. *There exists a unique equilibrium and it is such that* $Z^b \in (0, \bar{\varepsilon}u(y^*_{\bar{\varepsilon}}) - y^*_{\bar{\varepsilon}})$. *As* $\bar{\alpha}$ *increases, the value of consumers' outside options,* Z^b, *increases; the share of producers with positive market shares decreases, i.e.* $\hat{\varepsilon}$ *increases; market concentration,* HHI, *increases. A mean-preserving increase in the spread of the distribution* F *generates an increase in both* Z^b *and* $\hat{\epsilon}$.

An increase in the speed of the trading technology raises the value of consumers' outside options, which makes them pickier. As a result, rent sizes decrease and firms have less market power. The share of the producers that are profitable shrinks, which leads to higher market concentration. It is an example where market power and market concentration are negatively correlated. The last result of Proposition 11 shows that a technological shock that increases the variance of firms' productivity without changing its mean, is beneficial to consumers.

6.4 Market shares and monetary policy

Consider next a pure monetary economy, $\chi_m = 1$. We will distinguish the case where all producers have the same bargaining share, $\bar{\mu}$, and the case where μ_ε is endogenous and determined by rent seeking. In the former, from (47), an equilibrium is a Z^b solution to

$$(\rho + \lambda + \gamma) Z^b = \max_{a^* \geq 0} \left\{ -ia^* + \bar{\alpha}(1 - \bar{\mu}) \int_{\varepsilon_R(a^*, Z^b)}^{\bar{\varepsilon}} S^m_\varepsilon(a^*, Z^b) dF(\varepsilon) \right\}. \tag{51}$$

Proposition 12 ***(Firm heterogeneity and monetary policy)*** *All trades are monetary,* $\chi_m = 1$.

1. ***Exogenous bargaining shares:*** $\mu_\varepsilon = \bar{\mu} < 1$ *for all* ε. *For all* $i < \bar{\alpha}(1 - \bar{\mu})/\bar{\mu}$, *there exists a unique equilibrium and it is such that* $Z^b \in (0, \bar{\varepsilon}u(y^*_{\bar{\varepsilon}}) - y^*_{\bar{\varepsilon}})$. *As* i *increases,* Z^b *decreases. As* $i \to \bar{\alpha}(1-\bar{\mu})/\bar{\mu}$, $a^* \to 0$, $Z^b \to 0$, $\varepsilon_R \to 0$, *and* $HHI \to 1$.

2. ***Endogenous rent seeking.*** *Suppose* $i = 0^+$. *There exists an equilibrium and it is such that* $\varepsilon_R > 0$ *and* $Z^b > 0$. *A small increase in* i *generates a decrease in both* ε_R *and* Z^b. *For* ε *close to* ε_R, μ_ε *increases.*

An increase in the nominal interest rate raises the market power of producers by making it more costly for buyers to switch to alternative sellers, i.e., the value of consumers' outside option falls. At the limit when i approaches the upper bound for the existence of a monetary equilibrium, all firms have an equal market share. Even though concentration is minimum, so is Z^b and consumer welfare because high inflation allows the less productive firms to have a positive market share.

We obtain similar results when μ_ε is endogenous and i is close to 0, i.e., an increase in i allows less productive firms to become active and lowers the value of consumers' outside options. In addition, firms with lower ε increase their rent-seeking effort. These results are all consistent with the idea that inflation causes misallocation of resources by inducing consumers to spend their money on low-quality goods through a 'hot potato' effect.[28]

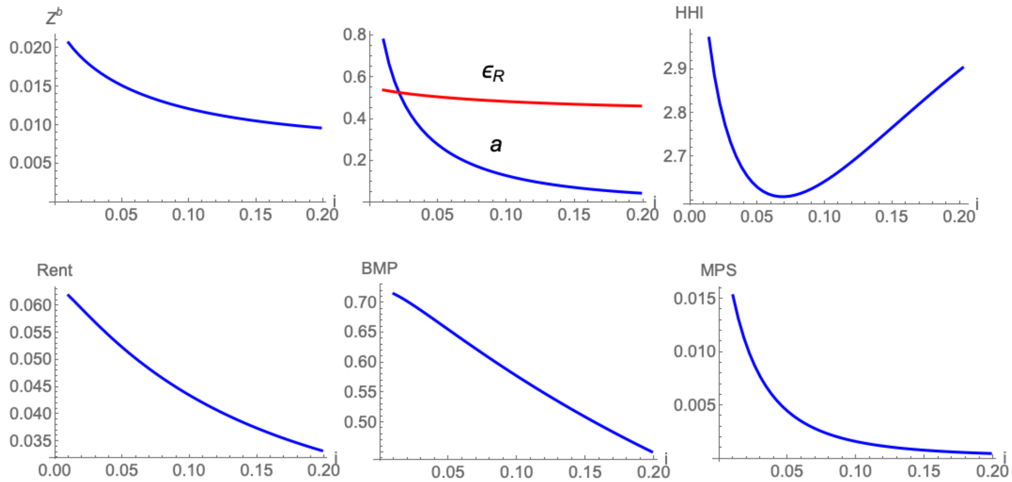

Figure 15: Impact of a change in monetary policy.

In Figure 15 we plot equilibrium outcomes for the general case with both monetary and credit trades. The results are consistent with Proposition 12: as i rises, the value of buyers' outside options falls and they carry less money. The quality threshold, ε_R, drops, i.e., firms of lower quality become profitable. The HHI measure of concentration is non-monotone in i. For low values of i, concentration decreases and it co-moves positively with the value of consumers' outside options. However, for large values of i, HHI increases and its correlation with Z^b becomes negative.

The bottom three panels plot different measures of market power: the sum of all of rents, $\varepsilon u(y_\varepsilon) - y_\varepsilon - Z^b$, in trade meetings, and the average BMP and MPS in meetings where trades take place. The total amount of rents falls in i because buyers carry fewer real balances. Since the amount of rents falls in i, firms have less market power and hence BMP and MPS drop.

Figure 16 plots equilibrium outcomes across heterogeneous firms, including rent-seeking effort (μ_ε), output levels (y_ε^m and y_ε^d), average rent size (S_ε), average markup, and market share (s_ε). The top-left panel shows that rent seeking increases with ε because firms of higher quality generate larger rents. The relationship between output and firm quality illustrated in the top-middle panel is more complex. For low ε, the buyer's liquidity constraint does not bind and $y_\varepsilon^m = y_\varepsilon^*$, which rises in ε. For large ε, the liquidity constraint binds and y_ε^m falls in ε for two reasons: firms with higher ε have a larger bargaining share, μ_ε, and each unit of their output is more valuable to the consumer. The output in a credit meeting $y_\varepsilon^d = y_\varepsilon^*$ rises in ε, provided that $\varepsilon > \hat{\varepsilon}(Z^b)$. The bottom panels show that rent sizes, markups and market share all rise in ε.

[28] A similar mechanism is formalized by Tommasi (1994, 1999).

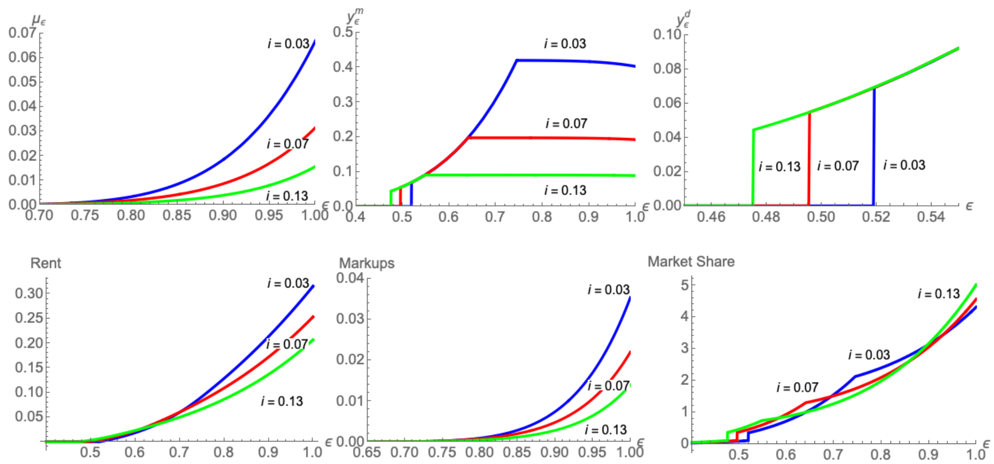

Figure 16: Terms of trade across firms.

Figure 16 also illustrates the effects of monetary policy on equilibrium outcomes. We consider three levels of $i \in \{0.03, 0.07, 0.13\}$ represented by curves of different color. As i increases, consumers reduce their real money balances, a, which tightens their liquidity constraint and reduces rent sizes. The value of consumers' outside options decreases which tends to increase rent sizes. The first effect dominates for firms with large ε and both their rent-seeking effort and production in monetary meetings decrease. The second effect dominates for low quality firms and both rent size and rent-seeking effort increase. The impact of i on markups is similar to that on firms' rent-seeking effort. The impact of i on market share is ambiguous.

6.5 Frictionless limits

We consider the limit of equilibria with heterogeneous firms as trading frictions vanish, $\bar{\alpha} \to +\infty$.

Proposition 13 ***(Frictionless limit with endogenous selection)*** *Suppose $F(\varepsilon)$ admits a positive density on $[0, \bar{\varepsilon}]$ and no mass point. Any limit of decentralized equilibria as $\bar{\alpha}$ tends to $+\infty$ is such that $\varepsilon_R = \bar{\varepsilon}$ and $Z^b = \bar{\varepsilon} u(y^*_{\bar{\varepsilon}}) - y^*_{\bar{\varepsilon}}$. Market concentration goes to $HHI = +\infty$.*

As trading frictions vanish, the consumer's outside option is equal to the full gain from trade in the match with the highest ε. Hence, only firms of the highest quality remain profitable, $\varepsilon_R = \bar{\varepsilon}$. As a result, market concentration goes to infinity. Because the meeting rate of active firms can become unbounded, we cannot conclude that $\mu_{\bar{\varepsilon}}$ converges to 0.

Figure 17 shows how market powers across firms, as measured by the average rent size, the average markup and the market share, are affected by the meeting technology in pure credit economies, $\chi_d = 1$. As $\bar{\alpha}$ rises, rents and markups decrease uniformly because the value of the consumers' outside option increases. The market shares become more disperse: it drops to zero for a larger set of low-quality firms and it rises for high-quality firms.

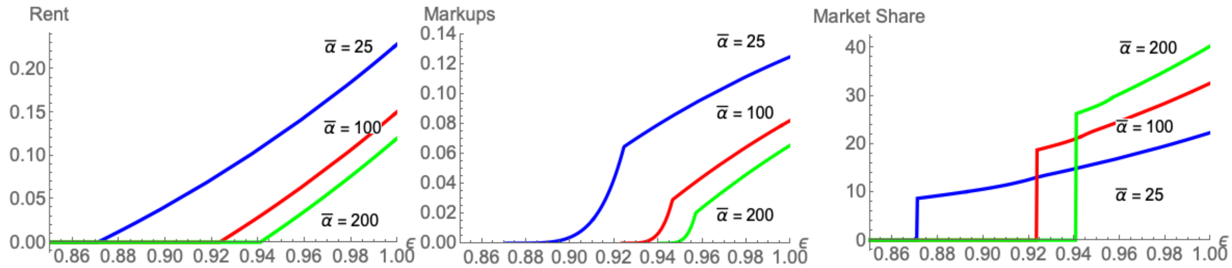

Figure 17: Rents, markups and market share across firms.

7 Conclusion

We described decentralized markets in monetary economies where agents interact explicitly and make prices and allocations in continuous time. We explained how agents gain power in markets through active rent seeking and by making use of their outside options. Among the key insights, we showed that monetary policy affects market power through two channels: a liquidity-constraint channel and a consumer-outside-option channel. According to the first channel, high nominal interest rates tighten liquidity constraints, reduce rent sizes, and discourage rent-seeking activities by producers. According to the second channel, high interest rates make the search for alternative producers more costly, which weakens consumers' outside options and raises rents, thereby promoting rent seeking. We explored the implications of these two channels for various measures of market power, including markups and market concentration.

A second insight is that consumers' endogenous outside options, which are a key determinant of firms' market power, provide a channel through which rent seeking activities by firms are strategic complements. As a result, if trading frictions are not too large, there can be multiple equilibria across which rent sizes and measures of market power differ. Hence, differences in market power can also be explained by self-fulfilling beliefs.

Finally, we showed that the role of consumers' outside options is critical for the convergence of the market equilibrium to a perfect competition outcome with no rent when meeting rates between consumers and producers become large. Such frictionless limit captures the common wisdom according to which firms' market power vanishes if consumers can easily switch from one producer to another. However, due to the multiplicity of equilibria described above, there can also exist other sequences of equilibria that do not converge to a perfectly competitive, no-rent outcome. At these alternative limits, firms have all the bargaining power and behave like perfect monopolists. Hence, an important lesson from our model is that improvements in trading technologies, such as the introduction of the Internet and e-commerce, is not enough to erode market power and eliminate rents.[29]

[29] This finding might help explain the puzzle proposed by Ellison and Ellison (2005) according to whom "evidence from the Internet... challenged the existing search models, because we did not see the tremendous decrease in prices and price dispersion that many had predicted."

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Appendix A. Proofs of propositions

Proof of Proposition 1. A monetary equilibrium exists and is unique provided that the value for μ solution to (12) at $y^m = 0$ is less than $\alpha^m/(i+\alpha^m)$, i.e.,

$$\mu = \upsilon'^{-1}\left[\alpha^d\left[u\left(y^*\right)-y^*\right]\right] < \frac{\alpha^m}{i+\alpha^m}.$$

(See Figure 3 for a graphical illustration of this argument.) The inequality can be rewritten as (1). In order to establish the comparative statics with respect to i we rewrite (11) as:

$$\frac{\mu i}{\alpha^m(1-\mu)} = \frac{\mu\left[u'(y^m)-1\right]}{\mu u'(y^m)+1-\mu}. \tag{52}$$

Substitute (52) into (12) to obtain:

$$\upsilon'(\mu) = \alpha^m\left[u\left(y^m\right)-y^m\right]\left\{1-\frac{\mu\left[u'(y^m)-1\right]}{\mu u'(y^m)+1-\mu}\right\} + \alpha^d\left[u\left(y^*\right)-y^*\right]. \tag{53}$$

Equation (53) gives a positive relationship between μ and y^m but the new MU curve is independent of i. As i increases, the curve labelled YM representing (11) shifts to the left. So μ and y_m decrease, i.e., $\partial y^m/\partial i < 0$, $\partial\mu/\partial i < 0$. It follows immediately that $\partial MPS/\partial i < 0$. From (11)-(12), when evaluated at $i = 0^+$,

$$\begin{aligned}\frac{\partial y^m}{\partial i} &= \frac{1}{\alpha^m(1-\mu^*)u''(y^*)}\\ \frac{\partial\mu}{\partial i} &= -\frac{\mu^*\left[u\left(y^*\right)-y^*\right]}{\upsilon''(\mu^*)(1-\mu^*)}.\end{aligned}$$

It follows that the change in the markup in a monetary match is

$$\frac{\partial MKUP}{\partial i} = \frac{-\mu^*}{1-\mu^*}\left[\frac{u(y^*)-y^*}{y^*}\right]\left\{\frac{\upsilon'(\mu^*)}{\left(\alpha^m+\alpha^d\right)\upsilon''(\mu^*)} + \frac{1}{\alpha^m y^* u''(y^*)}\right\}.$$

The last term between braces can take positive or negative values depending on the elasticities of υ and u'.

In order to show that the Friedman rule is suboptimal, we compute

$$\left.\frac{\partial\mathcal{W}}{\partial i}\right|_{i=0^+} = -\upsilon'(\mu)\left.\frac{\partial\mu}{\partial i}\right|_{i=0^+}.$$

By using the expressions for $\partial\mu/\partial i$ at $i = 0^+$ from above

$$\left.\frac{\partial\mathcal{W}}{\partial i}\right|_{i=0^+} = \frac{\mu^*\upsilon'(\mu^*)\left[u\left(y^*\right)-y^*\right]}{\upsilon''(\mu^*)(1-\mu^*)} > 0.$$

Finally, when $i \approx 0$, $\partial y^m/\partial\alpha \approx 0$ by (11). It follows that $\partial\mu/\partial\alpha > 0$ by (12). Then $\partial MKUP/\partial\alpha > 0$, and $\partial MPS/\partial\alpha > 0$ by using their definitions. ■

Proof of Proposition 2. Part 1. The allocation in monetary matches is determined by (11),

$$i = \alpha^m(1-\mu)\frac{u'(y^m)-1}{\mu u'(y^m)+1-\mu}.$$

If $\mu = 1$ then $y^m = 0$, in which case $MPS^m = 0$ and $MPS = \chi^d$. Assume next that $\mu < 1$. As $\alpha^m = \alpha\chi^m$ tends to infinity, y^m tends to y^* and MPS^m tends to μ. The average flow payment to sellers in terms of numéraire is

$$\omega p = \omega \left[\mu u(y^*) + (1-\mu)y^*\right].$$

In the Walrasian equilibrium it is ωy^*. Hence, the two coincide if and only if $\mu = 0$. If $\mu > 0$, the sellers enjoy a positive surplus but the quantities traded are not distorted and the gains from trade are fully exploited, $y = y^*$. Hence, the equilibrium is Pareto optimal.

Part 2. As $\alpha \to \infty$, by (11) either $\mu \to 1$ or $y^m \to y^*$. If $\upsilon'(1) \geq \bar{\upsilon}'$, then by (13) $\mu < 1$ regardless of the value of y^m. Hence as $\alpha \to \infty$ the limit of μ is strictly between 0 and 1 and $y^m \to y^*$. From (13), $\upsilon'(\mu) = \omega\left[u\left(y^*\right) - y^*\right]$. In that case, $MPS^m \to \mu$ and $MPS \to \mu$. If $\upsilon'(1) \leq \underline{\upsilon}'$, then for any finite value of α we have $\mu = 1$ by (13) and $y^m = 0$ by (11). Therefore $\mu = 1$ and $y^m = 0$ at the limit as $\alpha \to \infty$. In that case, $MPS^m = 0$ and $MPS = \chi^d$. If $\upsilon'(1) \in (\underline{\upsilon}', \bar{\upsilon}')$, then for any arbitrarily large α, there is a pair of (μ, y^m) that solves (11) and (13). By continuity, the limit of μ and y^m also satisfy these two equations as α explodes. If $\mu < 1$ at the limit, then $y^m \to y^*$. But since $\upsilon'(1) < \bar{\upsilon}'$, by (13) we must have $\mu = 1$, which is a contradiction. Hence $\mu = 1$ at the limit and y^m solves

$$\upsilon'(1) = \omega\chi^m\left[u\left(y^m\right) - y^m\right] + \omega\chi^d\left[u\left(y^*\right) - y^*\right].$$

In that case,

$$MPS^m \to \frac{\upsilon'(1)}{\omega\chi^m\left[u\left(y^*\right) - y^*\right]} - \frac{\chi^d}{\chi^m},$$

and

$$MPS \to \frac{\upsilon'(1)}{\omega\left[u\left(y^*\right) - y^*\right]}.$$

■

Proof of Lemma 1. There exists a positive surplus if and only if

$$\max_{y,p \leq a^*}\left\{u\left(y\right) - p - Z^b : p - y \geq 0\right\} > 0.$$

Using that an optimal solution to this problem is such that $p = y$, it can be rewritten as

$$\max_{y \leq a^*}\left\{u\left(y\right) - y - Z^b\right\} > 0.$$

The total surplus of the match is maximum when $y = y^*$, which from (15) requires $\mu\left[u\left(y^*\right) - Z^b\right] + (1 - \mu)y^* \leq a^*$. In order to compute the partial derivatives of $S(a^*, Z^b, \mu)$ we rewrite it as

$$S(a^*, Z^b, \mu) \equiv u\left[y\left(a^*, Z^b, \mu\right)\right] - y\left(a^*, Z^b, \mu\right) - Z^b$$

where $y\left(a^*, Z^b, \mu\right)$ is implicitly defined by

$$\mu u\left(y\right) + (1-\mu)y = a^* + \mu Z^b,$$

whenever $\mu\left[u\left(y^{*}\right)-Z^{b}\right]+(1-\mu)y^{*}>a^{*}$. ■

Proof of Lemma 2. Equation (18) is derived from (15), (17) and $i=\rho-r$. A positive solution of y_m exists provided that $\alpha^{b}\chi_m(1-\mu)-i\mu>0$. The candidate solution given by (18) is the actual solution provided that it dominates $a^{*}=0$. See Figure 18 for an illustration of the buyer's objective function and its two local maxima. This condition can be expressed as

$$-ia^{*}+\alpha^{b}\chi_m(1-\mu)S(a^{*},Z^{b},\mu)\geq 0.$$

Substituting $S(a^{*},Z^{b},\mu)$ by its expression and rearranging terms we obtain:

$$-ia^{*}+\alpha^{b}\chi_m(1-\mu)\left[u\left(y^{m}\right)-y^{m}\right]\geq\alpha^{b}\chi_m(1-\mu)Z^{b}.$$

Using that, from the bargaining, $a^{*}=\mu\left[u\left(y^{m}\right)-Z^{b}\right]+(1-\mu)y^{m}$, we can rewrite the inequality as:

$$-i\left\{\mu\left[u\left(y^{m}\right)-y^{m}\right]+y^{m}\right\}+\alpha^{b}\chi_m(1-\mu)\left[u\left(y^{m}\right)-y^{m}\right]\geq\left\{\alpha^{b}\chi_m(1-\mu)-i\mu\right\}Z^{b}.$$

One can easily rearrange this inequality to obtain (19).

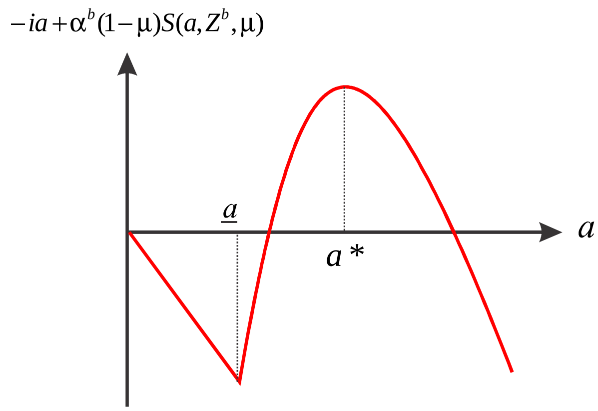


Figure 18: Choice of real balance with endogenous outside option: Buyer's objective

■

Proof of Proposition 3. Equation (21) is obtained from the bargaining solution (15), and the HJB equations for V^{b} and W^{b}, (16) and (20). The left side is increasing in Z^{b} while the right side is decreasing. In the neighborhood of $Z^{b}=0$, the right side is positive if either $\chi_d>0$ or $i<\alpha^{b}\chi_m(1-\mu)/\mu$. Hence, provided that one of these conditions holds, there is a unique $Z^{b}>0$ solution to (21). Given Z^{b}, allocations in pairwise meetings are determined uniquely.

From the Envelope theorem, the derivative of the right side with respect to i is equal to $-a^{*}$. Hence, provided that $a^{*}>0$, $\partial Z^{b}/\partial i<0$. Other comparative statics are obtained following similar arguments. ■

Proof of Corollary 1. Using (14) and (21), the measure of bilateral market power when $i = 0$ is

$$BMP = \frac{\rho + \lambda + \gamma}{\rho + \lambda + \gamma + \bar{\alpha}(1-\mu)}. \tag{54}$$

From (5) the seller's market power when $i = 0$ is (22). As i increases above 0, Z^b falls and hence BMP increases by (14). By a similar logic MPS rises in i at $i = 0$. ■

Proof of Lemma 3. From the first-order condition, the optimal rent-seeking activity solves:

$$\upsilon'(\mu) \leq \frac{\alpha^s \chi_m S(a^*, Z^b, \mu)}{\mu u'\left[y\left(a^*, Z^b, \mu\right)\right] + 1 - \mu} + \alpha^s \chi_d S^d(Z^b), \tag{55}$$

with an equality if $\mu < 1$. The left side is increasing in μ while the right side is decreasing in μ. Moreover, $\upsilon'(0) = 0 < \alpha^s \chi_m S(a^*, Z^b, 0) + \alpha^s \chi_d S^d(Z^b)$ for all (a^*, Z^b) such that $u(y) - y > Z^b$. Hence, there is a unique μ solution to (55). Using (18), (55) can be rewritten as (24). ■

Proof of Proposition 4. Suppose $\chi_d = 0$. From (21) and (24)

$$(\rho + \lambda + \gamma) Z^b = \max_{a^* \geq 0} \left\{-ia^* + \alpha^b(1-\mu) S^m(a^*, Z^b, \mu)\right\} \tag{56}$$

$$\upsilon'(\mu) = \frac{\alpha^s \left[u(y^m) - y^m - Z^b\right]}{\mu u'(y^m) + 1 - \mu}. \tag{57}$$

From (56), for all $\mu < \alpha^b / \left(i + \alpha^b\right)$, Z^b is a decreasing function of μ that is equal to Z_0^b when $\mu = 0$ and approaches 0 as μ tends to $\alpha^b / \left(i + \alpha^b\right)$. From (57), for all Z^b such that $Z^b < [u(y_0^m) - y_0^m]$ where $y^m = u'^{-1}\left(1 + i/\alpha^b\right)$, μ is a decreasing function of Z^b and it approaches 0 as Z^b tends to $[u(y_0^m) - y_0^m] > Z_0^b$ and it is equal to $\mu_0 < \alpha^b / \left(i + \alpha^b\right)$ when $Z^b = 0$. So there exists an equilibrium. Since we cannot guarantee uniqueness, we focus on equilibria where the curve representing (57) cuts (56) by above in the space (μ, Z^b).

An increase in i shifts (56) and (57) downward, so the effect on μ is ambiguous. Since (57) defines a negative relationship between Z^b and μ, we can define an implicit function $\mu(Z^b, i)$. Replace μ in (56) by $\mu(Z^b, i)$, then we have one equation with one endogenous variable Z^b. The solution of Z^b would fall in i if the right side of this equation falls in i for a given Z^b. By differentiating the right side of that equation with respect to i, and evaluate at $i = 0$, we have

$$-a^* + S^m(a^*, Z^b, \mu)\left(\frac{\upsilon'(\mu)\mu}{\upsilon''(\mu)(1-\mu)}\right) = -y^m + S^m(a^*, Z^b, \mu)\left(-1 + \frac{\upsilon'(\mu)}{\upsilon''(\mu)(1-\mu)}\right)\mu < 0,$$

where the equation uses $a^* = y^m + \mu S^m(a^*, Z^b, \mu)$ and the inequality uses $(1-\mu)\upsilon''(\mu)/\upsilon'(\mu) \geq 1$. ■

Proof of Proposition 5. Part 1. The equilibrium condition (26) can be rewritten as

$$\upsilon'(\mu)[\rho + \lambda + \gamma + \bar{\alpha}(1-\mu)]\frac{(\lambda + \gamma + \bar{\alpha})}{\bar{\alpha}\lambda} \leq (\rho + \lambda + \gamma)\left[u(y^*) - y^*\right],$$

with an equality if $\mu < 1$. If $\upsilon'(\mu)(1-\mu)$ is increasing in μ for all $\mu \in (0,1)$, then the left side is increasing in μ from 0 to some positive number and the equilibrium is unique, featuring $\mu \in (0,1)$. An increase in $\bar{\alpha}$ can raise or reduce the left side so that the sign of the effect of $\bar{\alpha}$ on μ depends on parameter values. As $\bar{\alpha} \to \infty$, fixing μ, the left side explodes to infinity and hence $\upsilon'(\mu)(1-\mu) \to 0$, which implies $\mu \to 0$. When $\mu \approx 0$, the left side of the displayed equation rises in $\bar{\alpha}$, provided that $\bar{\alpha}$ is sufficiently large. Therefore $\partial\mu/\partial\bar{\alpha} < 0$ and, from (25), $\partial Z^b/\partial\bar{\alpha} > 0$.

Part 2. Suppose the cost of rent-seeking is quadratic, $\upsilon(\mu) = \upsilon_0\mu^2/2$. Equation (26) can be rewritten as:

$$\upsilon_0\mu\left[\rho+\lambda+\gamma+\bar{\alpha}(1-\mu)\right] = \alpha^s\left(\rho+\lambda+\gamma\right)\left[u(y^*)-y^*\right].$$

The left side is a quadratic function of μ which is equal to 0 at $\mu = 0$, is strictly positive at $\mu = 1$, and whose vertex point is located at

$$\mu^{\text{vertex}} = \frac{\rho+\lambda+\gamma+\bar{\alpha}}{2\bar{\alpha}}.$$

A necessary condition for multiple equilibria is $\mu^{\text{vertex}} < 1$ so that the left side is hump-shaped, i.e., $\bar{\alpha} > \rho+\lambda+\gamma$. There are multiple steady states if the value of the right side is in-between the value of the left side evaluated at $\mu = \mu^{\text{vertex}}$ and $\mu = 1$, i.e.,

$$\frac{4\bar{\alpha}\left(\rho+\lambda+\gamma\right)}{\left(\rho+\lambda+\gamma+\bar{\alpha}\right)^2} < \frac{\upsilon_0}{\alpha^s\left[u(y^*)-y^*\right]} \leq 1.$$

Equivalently,

$$\begin{aligned}\underline{\upsilon}_0 &= \left(\frac{2\bar{\alpha}}{\rho+\lambda+\gamma+\bar{\alpha}}\right)^2\frac{\lambda\left(\rho+\lambda+\gamma\right)}{\lambda+\gamma+\bar{\alpha}}\left[u(y^*)-y^*\right]\\ \bar{\upsilon}_0 &= \alpha^s\left[u(y^*)-y^*\right].\end{aligned}$$

If the right inequality is strict, then there are two equilibria with $\mu \in (0,1)$. Moreover, if the condition above holds, $\mu = 1$ is also an equilibrium where sellers' rent-seeking effort is at a corner solution. ■

Proof of Proposition 6. Part 1. We focus on equilibria when $(1/\bar{\alpha}) \approx 0$. We rewrite (21) as follows:

$$\left(\frac{\rho+\lambda+\gamma}{\bar{\alpha}}\right)Z^b = \max_{a^*\geq 0}\left\{-\frac{i}{\bar{\alpha}}a^* + (1-\mu)\left[\chi_m S^m(a^*, Z^b, \mu) + \chi_d S^d(Z^b)\right]\right\}.$$

For all $\mu < 1$, the solution of Z^b to this equation in the neighborhood of $(Z^b, 1/\bar{\alpha}) \approx (u(y^*)-y^*, 0)$ can be approximated by:

$$Z^b \approx \left[u(y^*)-y^*\right]\left[1-\frac{\rho+\lambda+\gamma}{\bar{\alpha}(1-\mu)}\right].$$

By this approximation, as $\bar{\alpha}$ explodes, there is a one solution of μ in (24) such that $\mu \to 0$. It follows that there is an equilibrium (Z^b, μ) approaching $(u(y^*)-y^*, 0)$ as $\bar{\alpha}$ goes to $+\infty$.

Part 2. We now characterize limits of a sequence of equilibria, $\{Z^b_n, \mu_n, \bar{\alpha}_n\}$, such that $\mu_n \in (0,1)$ for all n, $\mu_n \to 1$, $\bar{\alpha}_n \to +\infty$, and $\bar{\alpha}_n(1-\mu_n) \to \tilde{\alpha} < +\infty$. From the definition of y^m in Lemma 3, we relate the limit $\tilde{\alpha}$ to y^m with the following correspondence:

$$\begin{aligned}\tilde{\alpha}\left(y^m\right) &= \frac{u'\left(y^m\right)i}{\left[u'\left(y^m\right)-1\right]\chi_m}\text{ if }y^m \in (0, y^*]\\ &= \left[0, \frac{i}{\chi_m}\right]\text{ if }y^m = 0.\end{aligned}$$

Note that at $y^m = 0$ the correspondence is equal to a nonempty interval because for all $\tilde{\alpha} \leq i/\chi_m$ the solution for y^m is 0. From (21) and (24), a limiting equilibrium is characterized by a pair, (Z^b, y^m), that solves:

$$(\rho + \lambda + \gamma) Z^b = \max_{a^* \geq 0} \left\{ -ia^* + \tilde{\alpha}(y^m) \left[\chi_m S^m(a^*, Z^b, 1) + \chi_d S^d(Z^b) \right] \right\} \quad (58)$$

$$\upsilon'(1) = \frac{\lambda \chi_m \left[u(y^m) - y^m - Z^b \right]^+}{u'(y^m)} + \lambda \chi_d \left[u(y^*) - y^* - Z^b \right]. \quad (59)$$

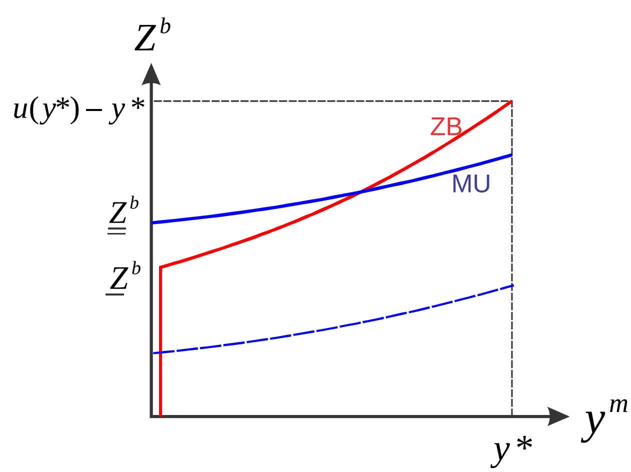
It should be noted that even if $y^m > 0$, the optimal solution for a^* given by the right side of (58) might be 0 if $u(y^m) - y^m \leq Z^b$. But if $a^* > 0$ then it satisfies (18). Equation (58) defines a positive relationship between Z^b and y^m. It is represented by a curve labelled ZB in the figure below. At $y^m = 0$, $Z^b \in [0, \underline{Z}^b]$ where $\underline{Z}^b$ solves

$$(\rho + \lambda + \gamma) \underline{Z}^b = \frac{i}{\chi_m} \chi_d S^d(\underline{Z}^b),$$

which, by the definition of $S^d(\cdot)$, can be solved in closed-form to obtain

$$\underline{Z}^b = \frac{i\chi_d}{\chi_m (\rho + \lambda + \gamma) + i\chi_d} \left[u(y^*) - y^* \right].$$

At $y^m = y^*$, $Z^b = u(y^*) - y^*$. Equation (59) also gives a positive relationship between Z^b and y^m. It is represented by a curve labelled MU in the figure below. If $y^m = y^*$ then $Z^b = [u(y^*) - y^*] - \upsilon'(1)/\lambda$. Hence the solution is positive provided that $\lambda [u(y^*) - y^*] > \upsilon'(1)$. At $y^m = 0$, $\underline{\underline{Z}}^b = u(y^*) - y^* - \upsilon'(1)/\lambda\chi_d$.



If $\underline{\underline{Z}}^b > \underline{Z}^b$, then (58)-(59) admits a solution such that y^m is interior. See the intersection between the two curves, ZB and MU, in the figure above. The condition $\underline{\underline{Z}}^b > \underline{Z}^b$ can be reexpressed as

$$\upsilon'(1) < \frac{\lambda \chi_d \chi_m (\rho + \lambda + \gamma)}{\chi_m (\rho + \lambda + \gamma) + i\chi_d} \left[u(y^*) - y^* \right].$$

In the neighborhood of a solution, $(\breve{Z}^b, \breve{y}^m)$, to (58)-(59), $\mu_n = 1 - \tilde{\alpha}(y_n^m)/\bar{\alpha}_n$ by the definition of $\tilde{\alpha}(y_n^m)$. An equilibrium is a sequence $\{Z_n^b, y_n^m\}_{n=0}^{+\infty}$ solving

$$\begin{aligned}(\rho+\lambda+\gamma) Z_n^b &= \max_{a^*\geq 0}\left\{-ia^* + \tilde{\alpha}(y_n^m)\left[\chi_m S^m\left(a^*, Z_n^b, 1-\frac{\tilde{\alpha}(y_n^m)}{\bar{\alpha}_n}\right) + \chi_d S^d(Z_n^b)\right]\right\} \\ \upsilon'\left(1-\frac{\tilde{\alpha}(y_n^m)}{\bar{\alpha}_n}\right) &= \frac{\lambda\bar{\alpha}_n}{\lambda+\gamma+\bar{\alpha}_n}\left\{\frac{\chi_m\left[u(y_n^m)-y_n^m-Z_n^b\right]^+}{u'(y^m)} + \chi_d\left[u(y^*)-y^*-Z_n^b\right]\right\},\end{aligned}$$

for some sequence $\{\bar{\alpha}_n\}_{n=0}^{+\infty}$ such that $\bar{\alpha}_n \to +\infty$. If $1/\bar{\alpha}_n$ is close to 0, the two equations that involve continuous functions admit a solution in the neighborhood of $(\breve{Z}^b, \breve{y}^m)$ and as $1/\bar{\alpha}_n$ tends to 0 this solution converges to $(\breve{Z}^b, \breve{y}^m)$.

If $\underline{\underline{Z}}^b \leq \underline{Z}^b$ but $\underline{\underline{Z}}^b > 0$, i.e., $\upsilon'(1) < \lambda\chi_d[u(y^*)-y^*]$, then (58)-(59) admits a solution such that $\breve{Z}^b > 0$ and $\breve{y}^m = 0$. See the intersection of the dashed curve and the ZB curve in the figure above. In the neighborhood of a solution, by (21), (24) and $\breve{y}^m = 0$, an equilibrium is a sequence $\{Z_n^b, \mu_n\}_{n=0}^{+\infty}$ solution to

$$\begin{aligned}Z_n^b &= \frac{\bar{\alpha}_n(1-\mu_n)\chi_d}{\rho+\lambda+\gamma+\bar{\alpha}_n(1-\mu_n)\chi_d}[u(y^*)-y^*] \\ \upsilon'(\mu_n) &= \frac{\lambda\bar{\alpha}_n}{\lambda+\gamma+\bar{\alpha}_n}\frac{\chi_d(\rho+\lambda+\gamma)}{\rho+\lambda+\gamma+\bar{\alpha}_n(1-\mu_n)\chi_d}[u(y^*)-y^*].\end{aligned}$$

This sequence is such that $(Z_n^b, \mu_n) \to (\breve{Z}^b, 1)$. ■

Proof of Lemma 4. Applying the Envelope Theorem to (21) in the neighborhood of $i = 0^+$,

$$[\rho+\lambda+\gamma+\alpha(\theta)(1-\mu)]\frac{\partial Z^b}{\partial\theta} = \alpha'(\theta)(1-\mu)\left[u(y^*)-y^*-Z^b\right],$$

where we used that $\alpha^b = \alpha(\theta)$. Using the expression from BMP in (54), we rewrite the equation above as:

$$\frac{\partial Z^b}{\partial\theta} = BMP \times \alpha'(\theta)(1-\mu)\frac{\left[u(y^*)-y^*-Z^b\right]}{\rho+\lambda+\gamma}.$$

Multiply both sides by θ/Z^b we obtain:

$$\frac{\partial Z^b/Z^b}{\partial\theta/\theta} = BMP \times \alpha'(\theta)\theta(1-\mu)\frac{\left[u(y^*)-y^*-Z^b\right]}{(\rho+\lambda+\gamma)Z^b}.$$

From (21) evaluated at $i = 0^+$,

$$(\rho+\lambda+\gamma)Z^b = \alpha(\theta)(1-\mu)\left[u(y^*)-y^*-Z^b\right],$$

which allows us to rewrite $\left(\partial Z^b/Z^b\right)/(\partial\theta/\theta)$ as in (28). ■

Proof of Lemma 5. If $i = 0^+$, $S^m(a^*, Z^b, \mu) = S^d(Z^b) = u(y^*)-y^*-Z^b$. From (29),

$$\frac{\partial\theta/\theta}{\partial Z^b/Z^b} = \frac{1}{\alpha'(\theta)\theta/\alpha(\theta)-1}\frac{Z^b}{u(y^*)-y^*-Z^b}.$$

Using that $BMP = \left[u(y^*)-y^*-Z^b\right]/[u(y^*)-y^*]$, we can rewrite this expression as (30). ■

Proof of Proposition 7. The right side of (33) is increasing in μ and, assuming $\upsilon'(0) = 0$ and $\upsilon'(1) = +\infty$, varies from 0 to $+\infty$ as μ increases from 0 to 1. Hence, (33) determines a unique $\mu \in (0,1)$. Given μ (34)-(35) determines a unique pair, $\theta > 0$ and $0 < Z^b < u(y^*)-y^*$.

From (33), an increase in k raises μ. Equation (34) can be rewritten as

$$\upsilon'(\mu)\left(\rho+\lambda+\gamma\right)+\alpha(\theta)\upsilon'(\mu)(1-\mu)=\frac{\alpha(\theta)}{\theta}\left(\rho+\lambda+\gamma\right)\left[u(y^*)-y^*\right].$$

If $\upsilon'(\mu)(1-\mu)$ is increasing in μ, then the left side is increasing in μ. It follows that θ is a decreasing function of μ, and hence a decreasing function of k. From (32), HHI increases in k. Finally, using that $\alpha(\theta)(1-\mu)$ is decreasing in k, it follows from (35) that Z^b decreases with k. ■

Proof of Proposition 8. From (36) we define a function $\theta(y^m;k)$ for all $y^m \in [0,y^*]$. This function is continuous and increasing with $\theta(0;k)=0$ and $\theta(y^*;k)=\bar{\theta}(k)$. Substitute $\theta(y^m;k)$ into (37) and rewrite the equilibrium condition as $\Gamma(y^m;k)\leq 0$ where

$$\Gamma(y^m;k)\equiv\frac{\alpha\left[\theta(y^m;k)\right]\chi_m(1-\bar{\mu})\left[u'(y^m)-1\right]}{\bar{\mu}u'(y^m)+1-\bar{\mu}}-i.$$

It can be checked that $\Gamma(0;k)=\Gamma(y^*;k)=-i$. For any $y^m\in(0,y^*)$, as k goes to 0, $\theta(y^m;k)$ and $\Gamma(y^m;k)$ go to infinity. Hence, $\sup_{y^m\in[0,y^*]}\Gamma(y^m;k)$ is decreasing in k from $+\infty$ when $k=0$ to $-i$ when $k=+\infty$. Define $k_0>0$ such that $\sup_{y^m\in[0,y^*]}\Gamma(y^m;k)=0$. For all $k<k_0$, $\sup_{y^m\in[0,y^*]}\Gamma(y^m;k)>0$ and hence $\Gamma(y^m;k)=0$ admits a positive and even number of solutions in $(0,y^*]$.

<u>Part 1</u>. From (21) if $\lambda=+\infty$ then $Z^b=0$. It follows that $\theta(y^m;k)$ is independent of i. An increase in i shifts $\Gamma(y^m;k)$ downward. Since Γ intersects the horizontal axis by above at the highest equilibrium, y^m and $\theta(y^m;k)$ decrease.

<u>Part 2</u>. We now turn to the case $\lambda<+\infty$. We total differentiate (36) in the neighborhood of $i=0^+$ to obtain:

$$\frac{\partial\theta}{\partial i}=\left\{\frac{1-\alpha'(\theta)\theta/\alpha(\theta)}{\theta}+\frac{\alpha'(\theta)(1-\bar{\mu})}{\rho+\lambda+\gamma+\alpha(\theta)(1-\bar{\mu})}\right\}^{-1}\frac{a^*}{S^m\left[\rho+\lambda+\gamma+\alpha(\theta)(1-\bar{\mu})\right]}>0,$$

where we have used, from (21), that

$$\begin{aligned}\frac{\partial Z^b}{\partial\theta} &= \frac{\alpha'(\theta)(1-\bar{\mu})S^m(a^*,Z^b,\bar{\mu})}{\rho+\lambda+\gamma+\alpha(n)(1-\bar{\mu})}\\ \frac{\partial Z^b}{\partial i} &= \frac{-a^*}{\rho+\lambda+\gamma+\alpha(\theta)(1-\bar{\mu})}.\end{aligned}$$

Moreover, from (36) and the fact that a change in y^m only has a second-order effect on θ, an increase in θ is associated with a decrease in Z^b. In order to determine the effects on y^m, rewrite (21) as

$$\left(\rho+\lambda+\gamma\right)Z^b=\alpha(\theta)\max_{a^*\geq 0}\left\{-\frac{i}{\alpha(\theta)}a^*+(1-\bar{\mu})S^m(a^*,Z^b,\mu)\right\}.$$

Given that Z^b decreases but $\alpha(\theta)$ increases, it has to be that $i/\alpha(\theta)$ increases. From (37) it follows that y^m decreases. ■

Proof of Proposition 9. From (29), as k goes to 0,

$$\max_{\mu\in[0,1]}\left\{-\upsilon(\mu)+\frac{\alpha(\theta)}{\theta}\mu\mathbb{E}S\right\}\to 0,$$

where $\mathbb{E}S = \left[\chi_m S^m(a^*, Z^b, \mu) + \chi_d S^d(Z^b)\right]$. Suppose $\mathbb{E}S > 0$ at the limit. Then, $\alpha(\theta)/\theta \to 0$ and $\theta \to +\infty$. From (21), $Z^b \to u(y^*) - y^*$ and $\mathbb{E}S \to 0$. A contradiction. So $\mathbb{E}S$ tends to 0 as k approaches 0. We can also rule out by a contradiction that $\alpha(\theta)/\theta$ approaches a positive value. Given that $[\alpha(\theta)/\theta]\,\mathbb{E}S \to 0$, it follows that $\mu \to 0$. Finally, $\theta \to +\infty$ implies from (32) that $HHI \to 0$. ■

Proof of Lemma 6.

The current-value Hamiltonian of the problem (38)-(39) is

$$H(\theta, n_a, \Upsilon) \equiv A\bar{\alpha}(\theta)n_a\left[u(y^*) - y^*\right] - \theta n_a k + \Upsilon\left\{\lambda(1-n_a) - \left[A\bar{\alpha}(\theta) + \gamma\right]n_a\right\}.$$

The necessary conditions from the Maximum Principle are:

$$\begin{aligned} k &= A\bar{\alpha}'(\theta_t)\left[u(y^*) - y^* - \Upsilon_t\right] \\ (\rho + \lambda + \gamma)\,\Upsilon_t &= A\bar{\alpha}(\theta_t)\left[u(y^*) - y^* - \Upsilon_t\right] - \theta_t k + \dot{\Upsilon}_t. \end{aligned}$$

We consider the solution of these two ODEs such that $\dot{\Upsilon} = 0$, which is given by (40)-(41). In order to establish that this solution is a solution to (38)-(39), we invoke Arrow's sufficiency condition. Let $\hat{H}(n_a, \Upsilon) \equiv \max_\theta H(\theta, n_a, \Upsilon)$. Then,

$$\hat{H}(n_a, \Upsilon) \equiv \max_\theta\left\{A\bar{\alpha}(\theta)\left[u(y^*) - y^* - \Upsilon\right] - \theta k - \Upsilon(\lambda + \gamma)\right\}n_a + \Upsilon\lambda.$$

So $\hat{H}(n_a, \Upsilon)$ is linear, hence concave, in n_a, which is the first requirement for sufficiency. The second requirement is

$$\lim_{t\to\infty} e^{-\rho t}\Upsilon_t n_{a,t} = \frac{\lambda\Upsilon^*}{\lambda + \gamma + A\bar{\alpha}(\theta^*)}\lim_{t\to\infty} e^{-\rho t} = 0$$

where we have replaced Υ_t by Υ^* and $n_{a,t}$ by its steady state value under $\theta_t = \theta^*$. Hence, the pair (θ^*, Υ^*) solving (40)-(41) corresponds to an optimum.

Next we consider the limit of (θ^*, Υ^*) as A explodes. It can be checked from (41) that $\Upsilon^* \to u(y^*) - y^*$ as A goes to infinity. To see this, rewrite (41) as

$$\left(\frac{\rho + \lambda + \gamma}{A}\right)\Upsilon^* = \max_{\theta \geq 0}\left\{\bar{\alpha}(\theta)\left[u(y^*) - y^* - \Upsilon^*\right] - \theta\frac{k}{A}\right\}.$$

Since $\Upsilon^* \in [0, u(y^*) - y^*]$ is bounded above, the left side goes to 0 as $A \to \infty$. Since $k/A \to 0$ in the right side, for the entire expression to go to 0, it must be that $\Upsilon^* \to u(y^*) - y^*$.

By (40)-(41) and $\Upsilon^* \to u(y^*) - y^*$, at the limit θ^* solves

$$\left[\frac{\bar{\alpha}(\theta^*)}{\theta^*\bar{\alpha}'(\theta^*)} - 1\right]\theta^* k = (\rho + \lambda + \gamma)\left[u(y^*) - y^*\right].$$

By assumption, $\bar{\alpha}(\theta)/[\theta\bar{\alpha}'(\theta)]$ is bounded above 1 and is increasing in θ. So, there is a unique solution for the limit of $\theta^* \in (0, +\infty)$ and thus $A\bar{\alpha}(\theta^*) \to +\infty$ as A tends to $+\infty$. Since active buyers get matched instantaneously, $n_a \to 0$. Then by (39) the flow of active buyers into trade meetings $A\bar{\alpha}(\theta^*)n_a \to \lambda$. Hence, the flow welfare, $A\bar{\alpha}(\theta^*)n_a\left[u(y^*) - y^*\right] - \theta^* n_a k$, tends to $\lambda\left[u(y^*) - y^*\right]$. ■

Proof of Proposition 10. Case 1: $k < \upsilon'(1) - \upsilon(1)$. The solution to (33) is interior, $\mu \in (0,1)$, and it is independent of A. The equilibrium condition, (34), can be rewritten as:

$$\upsilon'(\mu) = \frac{\bar{\alpha}(\theta)}{\theta} \frac{(\rho + \lambda + \gamma)}{(\rho + \lambda + \gamma)/A + \bar{\alpha}(\theta)(1-\mu)} \left[u(y^*) - y^*\right].$$

As A goes to $+\infty$ the term, $(\rho + \lambda + \gamma)/A$, vanishes and market tightness approaches to θ_∞ defined in (42). From (35),

$$Z^b = \frac{\bar{\alpha}(\theta)(1-\mu)}{(\rho + \lambda + \gamma)/A + \bar{\alpha}(\theta)(1-\mu)} \left[u(y^*) - y^*\right].$$

Hence, as $A \to +\infty$, $Z^b \to u(y^*) - y^*$. From (32)

$$HHI = \frac{\lambda + \gamma + A\bar{\alpha}(\theta)}{\lambda\theta} \to +\infty,$$

since $\theta \to \theta_\infty < +\infty$. Also since $\theta_\infty < +\infty$, $A\bar{\alpha}(\theta) \to +\infty$ and hence $n_a \to 0$. By (39) $A\bar{\alpha}(\theta_t)n_{a,t} = \lambda$ and the flow consumer payoff is $\lambda Z^b = \lambda\left[u(y^*) - y^*\right]$. Due to free entry, this flow is equal to the overall flow welfare and it is also equal to the flow welfare under the planner's solution by Lemma 6.

Case 2: $k \geq \upsilon'(1) - \upsilon(1)$. There is no interior solution to (33). The optimal rent-seeking effort is $\mu = 1$ for all A. The free-entry condition, (29), when A tends to $+\infty$ can be reexpressed as

$$k + \upsilon(1) = \frac{A\bar{\alpha}(\theta)}{\theta}\left[u(y^*) - y^*\right].$$

Hence,

$$\lim_{A \to +\infty} \frac{\bar{\alpha}(\theta)}{\theta} = \lim_{A \to +\infty} \frac{k + \upsilon(1)}{A\left[u(y^*) - y^*\right]} = 0.$$

So as $A \to +\infty$, $\theta \to +\infty$. The matching rate of producers, $A\bar{\alpha}(\theta)/\theta$, remains finite and equals to $[k + \upsilon(1)]/\left[u(y^*) - y^*\right]$. Since active buyers match instantly, $n_a \to 0$ and by (39) $A\bar{\alpha}(\theta_t)n_{a,t} = \lambda$ at the limit. Therefore $\theta n_a = \lambda\left[u(y^*) - y^*\right]/[k + \upsilon(1)]$. Finally, from (35), and using that for all A, $\mu = 1$, $Z^b \to 0$. From (32)

$$HHI = \frac{\lambda + \gamma + A\bar{\alpha}(\theta)}{\lambda\theta} \to 0,$$

since $A\bar{\alpha}(\theta)/\theta$ is finite and $\theta \to +\infty$. ■

Proof of Lemma 7. The proof is graphical. In Figure 19 the red curve plots $\varepsilon u(y) - y$ when $y = y^*_\varepsilon$ (the liquidity constraint, $y \leq a^*$, does not bind) while the blue curve plots the same function when $y = a^*$. The threshold, $\tilde{\varepsilon}$, above which the liquidity constraint binds is determined at the intersection of the two curves. The threshold, $\hat{\varepsilon}$, is determined at the intersection of the red curve and the outside options, Z^b. Above this threshold, there are gains from trade assuming the liquidity constraint does not bind. If $Z^b = Z^b_0$, then the intersection $\hat{\varepsilon}(Z^b_0)$ is less than $\tilde{\varepsilon}$, which means that the buyers have enough liquidity to generate gains from trade at $\hat{\varepsilon}(Z^b_0)$. Hence, $\varepsilon_{R,0} = \hat{\varepsilon}(Z^b_0)$. If $Z^b = Z^b_1$, then the intersection of the red curve and Z^b_1 is larger than $\tilde{\varepsilon}$, which means that the liquidity constraint does bind. In that case, from (43), $\varepsilon_R u(a^*) - a^* = Z^b$, i.e., $\varepsilon_R = \left(a^* + Z^b\right)/u(a^*)$.

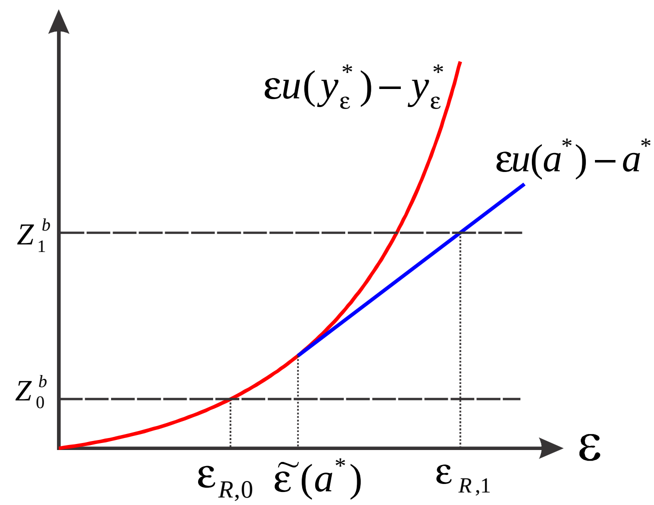


Figure 19: Thresholds for gains from trade and binding liquidity constraints

■

Proof of Lemma 8. If $\chi_d = 1$ and $\mu_\varepsilon = \bar{\mu}$, then

$$s_\varepsilon = \frac{\bar{\mu}\varepsilon u\left(y_\varepsilon^*\right) + (1-\bar{\mu})y_\varepsilon^* - \bar{\mu}Z^b}{\int_{\hat{\varepsilon}(Z^b)}^{\bar{\varepsilon}} \left[\bar{\mu}xu\left(y_x^*\right) + (1-\bar{\mu})y_x^* - \bar{\mu}Z^b\right] dF(x)} \quad \text{for all } \varepsilon \geq \hat{\varepsilon}(Z^b).$$

By differentiating s_ε with respect to Z^b it can be checked that $\partial s_\varepsilon / \partial Z^b$ is of the same sign as $-\{1 - [1 - F(\hat{\varepsilon})]\, s_\varepsilon\}$. If $\varepsilon = \hat{\varepsilon}$ then $s_\varepsilon = 0$ and $\partial s_\varepsilon / \partial Z^b < 0$. If $\varepsilon = \bar{\varepsilon}$ then $[1 - F(\hat{\varepsilon})]\, s_\varepsilon > 1$ and $\partial s_\varepsilon / \partial Z^b > 0$. Therefore, using that s_ε is increasing in ε, there is a $\varepsilon_0 \in (\hat{\varepsilon}, \bar{\varepsilon})$ such that $\partial s_\varepsilon / \partial Z^b > 0$ for all $\varepsilon > \varepsilon_0$ and $\partial s_\varepsilon / \partial Z^b \leq 0$ otherwise. Because $\int_0^{\bar{\varepsilon}} s_\varepsilon dF(\varepsilon) = 1$, an increase in Z^b is a mean-preserving increase in the spread of the distribution of market shares. By definition of the variance,

$$Var(s_\varepsilon) = \int_0^{\bar{\varepsilon}} (s_x - 1)^2 dF(x) = \int_0^{\bar{\varepsilon}} (s_x)^2 dF(x) - 1.$$

Hence,

$$HHI = 1 + Var(s_\varepsilon).$$

So an increase in Z^b raises the variance of the market shares (because it generates a mean-preserving increase in the spread of the distribution), which in turn raises the concentration index. ■

Proof of Proposition 11. The right side of (50) is decreasing in Z^b from a positive value when $Z^b = 0$ to 0 when $Z^b = \bar{\varepsilon}u\left(y_{\bar{\varepsilon}}^*\right) - y_{\bar{\varepsilon}}^*$. Hence, there is a unique solution in $(0, \bar{\varepsilon}u\left(y_{\bar{\varepsilon}}^*\right) - y_{\bar{\varepsilon}}^*)$. The right side is increasing in $\bar{\alpha}$, hence the comparative statics. The claim about HHI directly follows from Proposition 8.

Finally, the integral in the right side of (50) can be rewritten as

$$\int_0^{\bar{\varepsilon}} \max\{\varepsilon u\left(y_\varepsilon^*\right) - y_\varepsilon^* - Z^b, 0\} dF(\varepsilon)$$

and it is easy to check that the integrand is convex in ϵ. Therefore, fixing Z^b, the integral rises as F experiences a mean-preserving spread. Hence the value of Z^b that solves (50) rises and hence $\hat{\epsilon}(Z^b)$ rises. ■

Proof of Proposition 12. **Part 1: Exogenous bargaining shares**. The proof is analogous to that of Proposition 11. The key novelty is that the right side of (51) when $Z^b = 0$ is strictly positive (i.e. $a^* > 0$) if and only if $i < \bar{\alpha}(1-\bar{\mu})/\bar{\mu}$. Provided that this condition holds, the choice for a^* is interior and, by the Envelope Theorem, it decreases with i. It follows that Z^b falls in i by (51). As i approaches $\bar{\alpha}(1-\bar{\mu})/\bar{\mu}$, $a^* \to 0$ and $Z^b \to 0$. By Lemma 7, $\varepsilon_R \to 0$ because $\hat{\varepsilon} \to 0$ and $(a^* + Z^b)/u(a^*) \to 0$, where the latter limit is by L'Hospital's Rule (differentiating a^* and Z^b with respect to i). Finally, from (45), $s_\varepsilon \to 1$ for all ε and $HHI \to 1$.

Part 2. Endogenous rent seeking. From (48), if $i = 0$, then $y_\varepsilon = y^*_\varepsilon$ almost surely, i.e., $\tilde{\varepsilon} = \bar{\varepsilon}$. Hence, at the threshold ε_R the liquidity constraint is not binding, i.e., $\varepsilon_R = \hat{\varepsilon}(Z_b)$ is independent of a^*. Moreover, from (47), $Z^b = \varepsilon_R u\left(y^*_{\varepsilon_R}\right) - y^*_{\varepsilon_R} > 0$ implies $\varepsilon_R < \bar{\varepsilon}$. For $\varepsilon \in [\varepsilon_R, \bar{\varepsilon}]$, seller's rent-seeking effort is given by

$$v'(\mu_\varepsilon) = \frac{\bar{\alpha}\lambda}{\lambda + \gamma + \bar{\alpha}\left[1 - F(\varepsilon_R)\right]}\left\{\left[\varepsilon u\left(y^*_\varepsilon\right) - y^*_\varepsilon\right] - \left[\varepsilon_R u\left(y^*_{\varepsilon_R}\right) - y^*_{\varepsilon_R}\right]\right\}, \tag{60}$$

where μ_ε is a function of ε_R. The value of ε_R is given by

$$(\rho + \lambda + \gamma)\left[\varepsilon_R u\left(y^*_{\varepsilon_R}\right) - y^*_{\varepsilon_R}\right] = \bar{\alpha}\int_{\varepsilon_R}^{\bar{\varepsilon}}\left[1 - \mu_\varepsilon(\varepsilon_R)\right]\left\{\left[\varepsilon u\left(y^*_\varepsilon\right) - y^*_\varepsilon\right] - \left[\varepsilon_R u\left(y^*_{\varepsilon_R}\right) - y^*_{\varepsilon_R}\right]\right\} dF(\varepsilon). \tag{61}$$

The left side rises from 0 to ∞ as ε_R increases from 0. The right side is strictly positive at $\varepsilon_R = 0$ and vanishes as ε_R goes to $\bar{\varepsilon}$. Therefore, there is at least one solution for ε_R. We focus on the equilibrium with the highest ε_R, which is also the equilibrium with the highest Z^b.

From (47), an increase in i reduces the right side and hence leads to a decrease in Z^b. If i is close to 0, then $\varepsilon_R = \hat{\varepsilon}(Z_b)$ decreases. From (48) as i rises above 0, $\tilde{\varepsilon}$ falls below $\bar{\varepsilon}$ such that for all $\varepsilon \in (\tilde{\varepsilon}, \bar{\varepsilon}]$, $y_\varepsilon < y^*_\varepsilon$. In the neighborhood of $\varepsilon = \varepsilon_R^+$, by (60)

$$\frac{\partial \mu_\varepsilon}{\partial i} = \frac{-\bar{\alpha}\lambda u\left(y^*_{\varepsilon_R}\right)}{v''(\mu_{\varepsilon_R})\left\{\lambda + \gamma + \bar{\alpha}\left[1 - F(\varepsilon_R)\right]\right\}}\frac{\partial \varepsilon_R}{\partial i} > 0.$$

■

Proof of Proposition 13 . Consider a sequence of equilibria represented by $\{Z^b_n\}^{+\infty}_{n=0}$ and associated with an increasing sequence, $\{\bar{\alpha}_n\}^{+\infty}_{n=0}$, such that $\bar{\alpha}_n \to +\infty$. We establish that $\{Z^b_n\}^{+\infty}_{n=0}$ has a limit, Z^b_∞, and it is such that $Z^b_\infty = \bar{\varepsilon}u(y^*_{\bar{\varepsilon}}) - y^*_{\bar{\varepsilon}}$. The proof is by contradiction. Suppose $\underline{\lim}_{n\to\infty} Z^b_n = Z^b_\infty < \bar{\varepsilon}u(y^*_{\bar{\varepsilon}}) - y^*_{\bar{\varepsilon}}$, i.e., there exists positive rents in matches between consumers and firms with the highest quality. Consider a subsequence that converges to Z^b_∞. Define $\varepsilon^d_{R,\infty} < \bar{\varepsilon}$ as the ε solution to $\varepsilon u(y^*_\varepsilon) - y^*_\varepsilon = Z^b_\infty$. From (47), the associated subsequences, $\{(1-\mu_{\varepsilon,n})S^m_{\varepsilon,n}\}^{+\infty}_{n=0}$ and $\{(1-\mu_{\varepsilon,n})S^d_{\varepsilon,n}\}^{+\infty}_{n=0}$, must converge to 0 for almost all ε in the support of F since otherwise Z^b_∞ is unbounded. Since $\varepsilon^d_{R,\infty} < \bar{\varepsilon}$,

$$\alpha^s_\infty = \frac{\lambda}{\chi_m\left[1 - F(\varepsilon^m_{R,\infty})\right] + \chi_d\left[1 - F(\varepsilon^d_{R,\infty})\right]} < +\infty.$$

From (49), as $\varepsilon \searrow \varepsilon^d_{R,\infty}$, $\mu_{\varepsilon,\infty}$ tends to 0 in a continuous fashion, which is a contradiction provided that $\int_E dF(\varepsilon) > 0$ for any subset E of the support of F with positive measure. This proves that $Z^b_\infty = \bar{\varepsilon}u(y^*_{\bar{\varepsilon}}) - y^*_{\bar{\varepsilon}}$. Moreover, given that Z^b_n is bounded above by $\bar{\varepsilon}u(y^*_{\bar{\varepsilon}}) - y^*_{\bar{\varepsilon}}$, $\{Z^b_n\}^{+\infty}_{n=0}$ converges to $\bar{\varepsilon}u(y^*_{\bar{\varepsilon}}) - y^*_{\bar{\varepsilon}}$. It follows that $\varepsilon^d_{R,n}$ converges to $\bar{\varepsilon}$. From (46),

$$HHI \to \frac{(p_{\bar{\varepsilon}})^2 f(\bar{\varepsilon})d\bar{\varepsilon}}{\left[p_{\bar{\varepsilon}} f(\bar{\varepsilon})d\bar{\varepsilon}\right]^2} = \frac{1}{f(\bar{\varepsilon})d\bar{\varepsilon}} = +\infty,$$

because $d\bar{\varepsilon} \approx 0$. ■

Appendix B. Numerical examples and calibration

In this section we tabulate the parameter values for all numerical examples. We explain that the values in Section 3 are consistent with a calibration of the model with data.

Parameters for numerical examples

In all numerical examples we assume $\rho = 0.03$ and DM utility function $u(y) = By^{1-b}/(1-b)$. The cost of rent seeking is $\upsilon(\mu) = \upsilon_0\mu^{1+\eta}/(1+\eta)$ except in Figure 9 it is $\upsilon(\mu) = \upsilon_0[\mu/(1-\mu)]^{1+\eta}/(1+\eta)$ and in Figure 10 it is $\upsilon(\mu) = (1-w)\upsilon_0\mu^{1+\eta}/(1+\eta) + w[\mu^{10}/(1-\mu^{10})]$ where $w = 0.001$. In Figure 4 the dashed lines uses $\upsilon(\mu) = \upsilon_0\mu^5/(1-\mu^5)$. When there is free entry of sellers (i.e. Figure 13 and 14) we assume the matching function is $\alpha\theta$. For Figure 15, 16 and 17 we assume the firm's productivity $\varepsilon \sim U[0,1]$. The parameter values for each figure is listed below:

Figure	χ_m	χ_d	B	b	υ_0	η	α	λ	γ	i	k
4 (Top panels)	0.7	0.3	1	0.118	0.09	0.296	1	–	–	–	–
4 (Bottom panels)	0.7	0.3	1	0.118	0.09	0.296	–	–	–	0.01	–
7	0.7	0.3	1	0.1	0.05	0.9	1	0.5	0.3	–	–
9 (Right panel)	0.7	0.3	1	0.1	0.03	0.9	–	0.5	0.3	0.01	–
9 (Left panel)	0.7	0.3	1	0.1	0.001	0.5	–	0.5	0.3	0.01	–
10	0.7	0.3	1	0.1	0.03	0.5	–	2	0.3	0.01	–
11	0.7	0.3	1	0.1	0.03	0.9	–	0.5	0.3	0.01	–
13 and 14	0.7	0.3	1	0.1	0.03	0.9	1	2.13	0.33	0.01	0.005
15 and 16	0.7	0.3	1.129	0.2	0.5	0.295	–	2.13	0.33	–	–
17	0	1	1.129	0.2	0.5	0.295	–	2.13	0.33	–	–

Table 1: Parameter values for numerical examples

Calibration of the benchmark model

The parameter values in Section 3 can be obtained from the following calibration. The unit of time is a year and we set the rate of time preference to 0.03. We adopt the following functional forms: $u(y) = By^{1-b}/(1-b)$ and $\upsilon(\mu) = \upsilon_0\mu^{1+\eta}/(1+\eta)$ with $\eta > 0$. We set the meeting rate, $\alpha = 1$, as a normalization and calibrate the model by choosing parameters (B, b, υ_0, η) to simultaneously match two empirical relationships in US data for the period 1985-2014: the aggregate money demand and the relationship between average markup and interest rate. Aggregate GDP expressed in terms of the numéraire is $Y = 2\left[\alpha^m p(y^m) + \alpha^d p(y^*)\right]$ since after every expenditure in a pairwise meeting the buyer produces the numéraire to replenish her holdings of money. The aggregate demand for real balances is defined as

$$L \equiv \frac{M}{PY} = \frac{p(y^m)}{2\left[\alpha^m p(y^m) + \alpha^d p(y^*)\right]}.$$

where M is money supply, P is the price level and Y is real GDP. Using that $M/P = p(y^m) = \mu u(y^m) + (1-\mu)y^m$ and given that y^m is a function of i, L is also a function of i interpreted as aggregated money demand.

In the data, $L = M/(PY)$ where M is M1, PY is nominal GDP and i is interpreted as the nominal interest rate on T-bills. For $MKUP$ we use estimations from Eggertsson et al. (2018).[30] Finally, we rewrite $\alpha^j = \chi^j \alpha$, with $\chi^m + \chi^d = 1$, and we target (χ^m, χ^d) by using evidence on retail payments from the Survey of Consumer Payment Choice from the Federal Reserve Bank of Boston and the Methods-of-Payment Survey Report from the Bank of Canada. We set $\chi^d = 0.3$ which is in the middle of the range of numbers provided in those surveys.[31] The outcome of the calibration is given in Table 2:

Parameter	Description	Value
χ^m	Fraction of monetary meetings	0.700
α	Meeting rate	1.000
$b \equiv \frac{-u''(y)y}{u'(y)}$	Relative risk aversion of utility function	0.118
B	Scaling parameter of utility function	1
υ_0	Rent seeking cost scaling parameter	0.09
η	Rent seeking cost elasticity	0.296

Table 2: Calibration of benchmark model

Figure 20 (left) shows the money demand curve in the US over the period 1985-2014 and its calibrated counterpart from the model, while Figure 20 (right) presents Eggertsson et al. (2018) estimates for the markup and interest rate on T-bills.

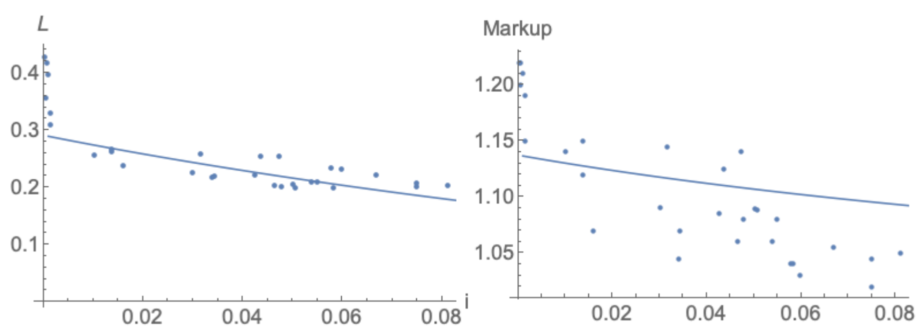


Figure 20: Markup and money demand: model and data

[30] The nominal rate on T-bills corresponds to the TB3MS series from the FRED database. Following Lucas and Nicolini (2015) we use M1J which is M1 plus money market deposit accounts (MMDA) due to their liquidity. This generates a more stable money demand curve after 1982. Nominal GDP is from the FRED database. Since Eggertsson's measure of markup is aggregated the corresponding model price markup is a weighted average the markup in pairwise meetings and in the centralized market, which is 1.

[31] Estimations for credit purchases range from below 20% in volume for the US to over 40% in Canadian data measuring value. See Bethune et al. (2020) and Wang et al. (2020) and references therein for more details on primary sources.

Appendix C: Strategic foundations for the proportional solution with outside options

We provide strategic foundations for the proportional solution with buyer's outside option. We showed in the main text that the outcome of the negotiation takes the form

$$p = \mu \left[u\left(y\right) - Z^b \right] + (1-\mu)y \leq a,$$

with an equality if $y < y^*$ and where a denotes the buyer's real balances. In order to provide game-theoretic foundations for this solution, we extend the Rubinstein game with sliced bundles proposed by Hu and Rocheteau (2020). The buyer's payoff is $u(y) - p$ if an agreement (y, p) is reached. The seller's payoff is $p - y$. The buyer has an outside option worth Z^b. The game is composed of $(N+1) \in \mathbb{N}$ stages. In each stage, the buyer and the seller play an alternating offer bargaining game à la Rubinstein (1982) with exogenous risks of break down. The maximum amount of goods that can be negotiated in stage n is $\Delta\bar{y}_n$ with $\sum_{n=1}^{N+1} \Delta\bar{y}_n = y^*$. The sequence, $\{\Delta\bar{y}_n\}_{n=1}^{N+1}$, is interpreted as the agenda of the negotiation. The transfer of real balances is subject to a feasibility constraint according to which the buyer cannot transfer more money than what she holds in a given round taking into account the money spent in earlier rounds. The overall agreement must be such that $p \leq a$. The game ends when either the $(N+1)^{\text{th}}$ round has been reached or the buyer's real balances have been depleted. In each stage, if an offer by the buyer is rejected by the seller, then the round is terminated with probability $1 - \xi^s$. If an offer by the seller is rejected by the buyer, then the round is terminated with probability $1 - \xi^b$. We consider $\left(\xi^b, \xi^s\right) = \left(e^{-(1-\mu)\varepsilon}, e^{-\mu\varepsilon}\right)$ for some $\mu \in [0,1]$ and consider the limit as $\varepsilon \to 0$. In the last stage, when the negotiation is terminated, the players can either honor the sum of all interim agreements or defect and take their outside option (Z^b for the buyer and 0 for the seller). The outcome in each round corresponds to the generalized Nash solution with endogenous disagreement points where the bargaining power of the buyer is μ.

Define $\underline{y}$ as the lowest solution to

$$u(\underline{y}) - \underline{y} = Z^b.$$

We assume $\underline{y} \in (0, a)$ exists since otherwise there are no gains from trade. We choose the agenda of the negotiation, $\{\Delta\bar{y}_n\}_{n=1}^{N+1}$, to have the following form:

$$\begin{aligned} \Delta\bar{y}_{N+1} &= \underline{y} \\ \Delta\bar{y}_n &= \frac{y^* - \underline{y}}{N} \text{ for all } n = 1, ..., N \end{aligned}$$

The bundle size in the last round implements the buyer's outside option. In the first N rounds, all good bundles have the same size equal to $(y^* - \underline{y})/N$ so that the total output that is up for negotiation over the $(N+1)$ rounds is equal to the first-best level, y^*. The game is solved by backward induction.

Consider round $N+1$ and suppose no agreement has been reached in earlier rounds. Given that the buyer can walk away with Z^b, the only offer that is acceptable and satisfies $u(y) - p \geq Z^b$ is $(y_{N+1}, p_{N+1}) = (\underline{y}, \underline{y})$.

We now move to round N and assume no agreement has been reached in the first $N-1$ rounds. The disagreement point is the outcome of the $(N+1)^{\text{th}}$ round, $(Z^b, 0)$. Hence, the final payoffs from the last two rounds correspond to the allocation given by

$$\max \left[u(y) - p - Z^b\right]^{1-\mu} [p - y]^{\mu}$$

with feasibity constraints $y \leq \Delta\bar{y}_N + \Delta\bar{y}_{N+1}$ and $p \leq z$. If the liquidity constraint does not bind,

$$\begin{aligned} y_2 &= \Delta\bar{y}_N + \Delta\bar{y}_{N+1} \\ p_2 &= (1-\mu)y_2 + \mu\left[u(y_2) - Z^b\right], \end{aligned}$$

and the players' payoffs are

$$\begin{aligned} \hat{u}_2^b &= Z^b + (1-\mu)\left[u(y_2) - y_2 - Z^b\right] \\ \hat{u}_2^s &= \mu\left[u(y_2) - y_2 - Z^b\right]. \end{aligned}$$

We now move to round $N-1$ and by the same logic the payoffs are determined by

$$\max \left[u(y) - p - \hat{u}_2^b\right]^{1-\mu} [p - y - \hat{u}_2^s]^{\mu}$$

with feasibility constraints $y \leq \Delta\bar{y}_{N-1} + \Delta\bar{y}_N + \Delta\bar{y}_{N+1}$ and $p \leq z$. If the liquidity constraint does not bind,

$$\begin{aligned} y_3 &= \Delta\bar{y}_{N-1} + \Delta\bar{y}_N + \Delta\bar{y}_{N+1} \\ p_3 &= (1-\mu)y_3 + \mu\left[u(y_3) - Z^b\right], \end{aligned}$$

and the players' payoffs are

$$\begin{aligned} \hat{u}_3^b &= Z^b + (1-\mu)\left[u(y_3) - y_3 - Z^b\right] \\ \hat{u}_3^s &= \mu\left[u(y_3) - y_3 - Z^b\right]. \end{aligned}$$

And we iterate until the very first round or until the liquidity constraint binds.

We consider the limit of this game when N goes to infinity, i.e., $\Delta\bar{y}_n \to 0$ for all $n \leq N$. If the liquidity constraint does not bind,

$$p = (1-\mu)y^* + \mu\left[u(y^*) - Z^b\right] \leq a.$$

If it does bind, we use that $|y_{n+1} - y_n| \to 0$ and the constraint is slack the iteration before it binds. Hence,

$$p = (1-\mu)y + \mu\left[u(y) - Z^b\right] = a.$$

The last round where the constraint binds is irrelevant because the amount of output negotiated is infinitesimal. This corresponds to the proportional solution described in the main text.

While we have assumed that only $y^* - \underline{y}$ was negotiated gradually, the result would go through if $\underline{y}$ is also negotiated gradually provided that at the very last round the buyer can opt out and take Z^b instead of the overall agreement.

Appendix D. Two-sided rent seeking

We extend the model of Section 3 by assuming that buyers and sellers can compete for the surplus of a bilateral match in a two-sided rent-seeking contest. We denote e^b and e^s the rent-seeking efforts of the buyer and the seller, respectively, and $\upsilon^b(e)$ and $\upsilon^s(e)$ the associated flow costs. Given those efforts, the seller's share in the match surplus is now

$$\mu(e^s, e^b) = \frac{e^s}{e^s + e^b}. \tag{62}$$

The value function of the seller satisfies an HJB equation analogous to (8) where the choice variable is now e^s taking as given e^b. The FOC is

$$\upsilon^{s\prime}(e^s) = \frac{e^b}{(e^s + e^b)^2} \left\{ \alpha^m \left[u\left(y^m\right) - y^m\right] \frac{e^b + e^s}{e^s u'(y^m) + e^b} + \alpha^d \left[u\left(y^*\right) - y^*\right] \right\}, \tag{63}$$

where y^m is the solution to

$$e^s u(y^m) + e^b y^m = \left(e^s + e^b\right) a. \tag{64}$$

Similarly, the optimal rent seeking activity of the buyer solves

$$\upsilon^{b\prime}(e^b) = \frac{e^s}{(e^s + e^b)^2} \left\{ \alpha^m \left[u\left(y^m\right) - y^m\right] \frac{\left(e^s + e^b\right) u'(y^m)}{e^s u'(y^m) + e^b} + \alpha^d \left[u\left(y^*\right) - y^*\right] \right\}. \tag{65}$$

The FOC for the choice of real balances is still given by (7), i.e.,

$$i = \alpha^m e^b \frac{u'(y^m) - 1}{e^s u'(y^m) + e^b}. \tag{66}$$

Note that a and e^b are complements in that if the buyer raises e^b then she will also find it optimal to raise a. A steady-state equilibrium is a list, (a^*, e^b, e^s), that solves (7), (63), and (65).

The next proposition characterizes the equilibrium set (including the possibility of multiple equilibria) and the impact of an increase in i on rent-seeking efforts and output at the equilibrium with the highest output. We say a utility function $u(y)$ is *not too concave* if its elasticity exceeds its relative risk aversion (RRA), namely $yu'(y)/u(y) > -yu''(y)/u'(y)$ for all $y \in [0, y^*]$.[32]

Proposition 14 ***(Two-sided rent seeking)*** *Suppose that $\upsilon^j(e) = \upsilon_0^j(e)^{1+\eta_j}/(1+\eta_j)$ and $\alpha^d = 0$.*

1. *If $\eta_s \leq \eta_b$, then there exists a unique steady state. As i rises, y^m, e^s/e^b and e^s fall. If u is not too concave, then e^b also falls in i.*

2. *If $\eta_s > \eta_b$, then there exist an even number of equilibria provided that η_s is sufficiently large, υ_0^b is sufficiently small, and u is not too concave. The equilibria are rank-ordered by $(y^m, e^b/e^s, e^b, e^s)$. Buyers' expected lifetime utility is lower in an equilibrium with higher y^m. At the highest equilibrium, as i increases, y^m, e^b/e^s, e^b and e^s fall.*

[32] For CRRA utility functions, this condition is satisfied if $RRA \leq 1/2$. For CARA utility functions, it is satisfied when the coefficient of absolute risk aversion is less than 2.

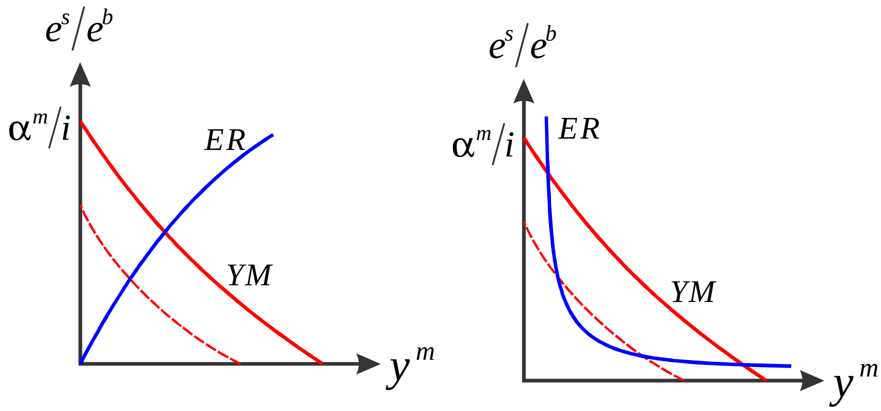


Figure 21: Steady state with two-sided rent seeking

Proof. The value function of the buyer with a units of real balances solves:

$$\rho\left[V^b(a)-a\right] = \tau+\max_{e^b,a^*}\left\{-\upsilon^b(e^b)-(\rho-r)a^*+\alpha^m\frac{e^b}{e^s+e^b}\left[u\left(y^m\right)-y^m\right]\right. \tag{67}$$
$$\left.+\alpha^d\frac{e^b}{e^s+e^b}\left[u\left(y^*\right)-y^*\right]+\dot{V}^b\right\}.$$

From (66),

$$u'(y^m)=\frac{i+\alpha^m}{\alpha^m-i\frac{e^s}{e^b}}. \tag{68}$$

The buyer's real balances decrease with e^s/e^b from a positive value when $e^s/e^b=0$ to 0 when $e^s/e^b>\alpha^m/i$. Suppose that $\upsilon^j(e)=\upsilon_0^j(e)^{1+\eta_j}/(1+\eta_j)$. From (63) and (65), the rent-seeking efforts solve

$$e^s=\left[\frac{e^be^s}{\upsilon_0^s\left(e^s+e^b\right)}\left\{\frac{\alpha^m\left[u\left(y^m\right)-y^m\right]}{e^su'(y^m)+e^b}\right\}\right]^{1/(1+\eta_s)}, \tag{69}$$
$$e^b=\left[\frac{e^se^b}{\upsilon_0^b\left(e^s+e^b\right)}\left\{\frac{\alpha^m\left[u\left(y^m\right)-y^m\right]u'(y^m)}{e^su'(y^m)+e^b}\right\}\right]^{1/(1+\eta_b)}. \tag{70}$$

Hence e^s and e^b only depend on the ratio e^s/e^b and y^m. Let $x=e^s/e^b$. Dividing (69) by (70) yields

$$x=\frac{(\upsilon_0^b)^{1/(1+\eta_b)}}{(\upsilon_0^s)^{1/(1+\eta_s)}u'(y^m)^{\frac{1}{1+\eta_b}}}\left\{\frac{\alpha^m\left[u\left(y^m\right)-y^m\right]}{(x+1)(u'(y^m)+1/x)}\right\}^{\frac{1}{1+\eta_s}-\frac{1}{1+\eta_b}}. \tag{71}$$

Part (a): If $\eta_s\leq\eta_b$, then $1/(1+\eta_s)-1/(1+\eta_b)\in[0,1)$. In this case (71) defines a positive relationship between x and y^m. This equation is shown in the left panel of Figure 3 as a blue curve ER. Since (68) defines a negative relationship between x and y^m (labelled YM in Figure 3), the steady state equilibrium is unique. As i rises, YM shifts downward (red dashed line in Figure 3) and hence y^m and e^s/e^b fall. By (69) and (70)

$$e^s(y^m,x)=\left[\frac{1}{\upsilon_0^s}\left\{\frac{\alpha^m\left[u\left(y^m\right)-y^m\right]}{(1+x)\left[u'(y^m)+1/x\right]}\right\}\right]^{1/(1+\eta_s)}, \tag{72}$$
$$e^b(y^m,x)=\left[\frac{1}{\upsilon_0^b}\left\{\frac{\alpha^m\left[u\left(y^m\right)-y^m\right]u'(y^m)}{(1+x)\left[u'(y^m)+1/x\right]}\right\}\right]^{1/(1+\eta_b)}. \tag{73}$$

Since y^m and x fall in i, $e^s(y^m, x)$ falls provided that $xu'(y^m)$ rises in i. If $xu'(y^m)$ falls in i, then e^s also falls because by (72) and (73)

$$e^s = \left\{ \frac{e^b(y^m, x)^{1+\eta_b}}{e^s(y^m, x)^{1+\eta_s}} x^{1+\eta_b} \right\}^{\frac{1}{\eta_b - \eta_s}} = \left\{ \frac{v_0^s}{v_0^b} [u'(y^m)) x^{1+\eta_b}] \right\}^{\frac{1}{\eta_b - \eta_s}}. \tag{74}$$

The right side falls as $xu'(y^m)$ and x fall. Altogether e^s falls in i. Next, if $xu'(y^m)$ rises in i, then by (73) $e^b(y^m, x)$ falls in i provided that $[u(y) - y]u'(y)$ rises in y, which happens when u is not too concave. If $xu'(y^m)$ falls in i, then e^b falls in i because by (72) and (73)

$$e^b = \left\{ \frac{e^b(y^m, x)^{1+\eta_b}}{e^s(y^m, x)^{1+\eta_s}} x^{1+\eta_s} \right\}^{\frac{1}{\eta_b - \eta_s}} = \left\{ \frac{v_0^s}{v_0^b} [u'(y^m) x^{1+\eta_s}] \right\}^{\frac{1}{\eta_b - \eta_s}}, \tag{75}$$

and the right side falls as $xu'(y^m)$ and x fall. Altogether, e^b falls in i if u is not too concave.

Part (b): Assume $\eta_s > \eta_b$. Rewrite (71) as

$$\left\{ x^{\frac{(1+\eta_s)(1+\eta_b)}{\eta_s - \eta_b}} \frac{\alpha^m [u(y^m) - y^m]}{(x+1)[u'(y^m) + 1/x]} u'(y^m)^{\frac{1+\eta_s}{\eta_s - \eta_b}} \right\}^{\frac{1}{1+\eta_b} - \frac{1}{1+\eta_s}} = \frac{(v_0^b)^{1/(1+\eta_b)}}{(v_0^s)^{1/(1+\eta_s)}}. \tag{76}$$

The left side rises in x because $\eta_s > \eta_b$. It rises in y^m if $[u(y^m) - y^m] u'(y^m)^{\frac{1+\eta_s}{\eta_s - \eta_b}}$ rises in y^m, which happens when u is not too concave and η_s is sufficiently large. If the left side rises in x and y^m, then (76) defines a negative relationship between x and y^m. As $y \downarrow 0$, x explodes to $+\infty$. As x falls, y^m explodes to $+\infty$. As v_0^b falls, the entire line shifts downward.

Equation (68) also defines a negative relationship between x and y^m. It is easy to check that it intersects with the x-axis at $x = \alpha/i$ and with the y-axis at some finite y^m. Therefore equation (68) and (76) intersect an even number of times provided that v_0^b is small, see the right panel of Figure 3. When multiple equilibria exist, they are rank ordered by $(y^m, e^b/e^s)$. By (74) and (75) e^s and e^b are larger when output y^m is higher. Buyers are worse off in an equilibrium with higher output because e^s is higher and thus the objective function in (67) falls for any given e^b and a^*. Therefore the buyers' equilibrium payoff is lower.

Finally, as i rises, YM in the right panel of Figure 3 shifts downward, hence y^m and e^b/e^s fall. By (74) and (75) e^s and e^b also fall in i. ■

In Figure 21 a steady-state equilibrium is an intersection between two curves, *ER* and *YM*, in the space $(y^m, e^s/e^b)$ where the former is obtained from (63) and (65) and the latter represents (68). The curve YM is downward sloping while *ER* can be increasing or decreasing.

When $\eta_s \leq \eta_b$, equilibrium is unique and an increase in i reduces output, y^m, and sellers' surplus share, $e^s/(e^b + e^s)$, fall (left panel of Figure 21). Intuitively, if i increases then buyers reduce their real balances and rent-seeking effort. The lower a reduces sellers' incentives to invest in rent seeking but the lower e^b has the opposite effect. The first effect dominates.

If $\eta_s > \eta_b$, there can exist multiple steady-state equilibria (right panel of Figure 21). In the equilibrium with high e^b/e^s ratio, it is optimal for the buyer to choose high levels of e^b and a^* and it is optimal for sellers to respond with a high e^s. As i rises, output y^m falls, all agents reduce their rent-seeking effort, but μ rises.

Buyers are worse off in an equilibrium with higher y^m because they have to exert a high level of effort to respond to sellers' rent seeking.