Heterogenous Human Capital Loss During Unemployment and Frictional Wage Dispersion

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Abstract

This paper studies frictional wage dispersion by extending Ortego-Marti (2016) with heterogeneous human capital depreciation rates. While baseline search models struggle to sufficiently explain empirical wage differences among observationally identical workers, the model with human capital loss during unemployment provides a significant improvement. Workers reduce their reservation wages because unemployment hurts their human capital. This paper shows that a model with heterogenous rates of human capital depreciation increases frictional wage dispersion even further. Workers with a lower depreciation rate increase the average wage, while workers with higher rates of depreciation decline the reservation wage in the labor market. Both changes imply that the model with heterogeneity yields more frictional wage dispersion than the model with homogenous workers and that the model can thus explain even more of observed frictional wage dispersion.

Key words: wage dispersion, search and matching, human capital

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1 Introduction

Frictional wage dispersion refers to the situation where observationally identical workers are paid different wages (Hornstein et al. 2011). Empirical evidence shows that such frictional wage dispersion plays an important role in explaining wage differentials across workers. Empirical literature tries to control as many variables as their Mincerian regressions can. However, the residual wage dispersion is still large. It implies that observable worker characteristics such as education and tenure can only explain a fraction of observed wage differentials. In particular, the 50-10 percentile ratio of the residuals lies between 1.7 and 1.9 (Hornstein et al. 2011), meaning that the data displays a 70 to 90 percentage differential between the 50th percentile (median) and 10th percentile residual wages.

Baseline search models of the labor market do not offer sufficient explanations of this frictional wage dispersion. McCall (1970) shows that even wages can be drawn from a random distribution. However, firms only post the reservation wages of workers because they realize workers accept job offers as long as the wage is equal or above their reservation wage. Pissarides (1985) shows that wage dispersion is generated in a related way as workers and firms draw a match-specific productivity from an exogenous distribution when they meet. Hornstein et al. (2011) show, however, that these baseline models do not generate sufficient wage dispersion among identical workers. They derive a theoretical measure of frictional wage dispersion for search models, the mean-min. wage ratio (Mm ratio) to which the 50-10 percentile ratio is a reasonable empirical counterpart (Ortego-Marti 2016). Hornstein et al. (2011) show that the Mm ratio for the two baseline search models is only 1.05 which is far less than the 50-10 percentile ratio of 1.7-1.9 that has been found empirically. Hence, baseline search models predict far too little frictional wage dispersion. Hornstein et al. (2011) also examine extensions to the baseline models, including endogenous job separation (Mortensen & Pissarides 1994), returns to experience, directed search (Moen 1997), and on-the-job (OTJ) search (Burdett & Mortensen 1998), but only find the latter extension to yield a satisfactory improvement in the magnitude of the Mm ratio.

To generate more frictional wage dispersion, Ortego-Marti (2016) proposes to add permanent loss of human capital during the unemployment spell to the model. In this model, unemployed workers gradually become less productive since their human capital depreciates during unemployment. Workers reduce their reservation wage because being unemployed hurts their human capital and affects their future income. As a result, the wage despersion in this market increases. The *Mm* ratio in Ortego-Marti (2016) increases to 1.21. It explains around 26% of the empirically observed residual wage dispersion, while the textbook model only accounts for around 6%.

In this paper, we adopt the model frame from Ortego-Marti (2016) with heterogeneous human capital depreciation rates among workers. We examine the *Mm* ratio to check whether the heterogeniety can explain observed frictional wage dispersion better. We consider workers in this model are identical on all observable characteristics. Differences in human capital depreciation rates are only due to some personal characteristics that cannot be observed in the data. For instance, workers who are more motivated to stay updated on relevant information and knowledge while unemployed could plausibly lose human capital less rapidly during unemployment.

The assumption that human capital depreciation happens during unemployment and that these losses are persistent is widely supported by the empirical literature. For instance, in the empirical part of his paper, Ortego-Marti (2016) shows that not only recent but also older accumulated unemployment history reduces wages significantly. This shows that human capital losses due to unemployment have persistent effects on wages. The part of the literature that focuses on displaced workers in the U.S. similarly finds that job displacement leads to persistent earnings losses (e.g., Neal 1995, Couch & Placzek 2010).[1]

Using Swedish data, Edin & Gustavsson (2008) investigate whether the negative empirical relationship between unemployment and subsequent earnings is indeed due to human capital loss. They find that cognitive skills decay significantly during time out of work, hence supporting the claim that unemployment does indeed lead to human capital losses. Kleinjans (2019) shows with German data that unemployment also hurts workers' non-cognitive skills. Finally, Ortego-Marti (2017) shows that the rate of wage (i.e., human capital) loss during unemployment varies across industries and occupations, and that the rate of decay is higher in industries and occupations where more skills and experience are required. This finding supports the assumption in this paper that workers have different human capital depreciation rates.

The literature provides some approaches to explain frictional wage dispersion. Ortego-Marti (2016) allows for human capital accumulation during employment and OTJ search which creates

[1] Displaced workers are defined as "workers who are fairly attached to their job and are involuntarily separated from it, with little chance of being recalled by their employer or finding a similar job within a reasonable span of time", Ortego-Marti (2016).

even more frictional wage dispersion. Burdett et al. (2016) follow a similar model but add differences in productivity among firms and find that this explains much of the wage differentials among equally skilled workers. Dolado et al. (2009) consider a model with OTJ search and distinguish between skilled and unskilled jobs as well as highly- and lowly-educated workers. Because of the possibility of OTJ search, highly-educated workers may accept unskilled jobs because they are allowed to keep searching while employed. This widens wage differences among highly-educated workers because some of them will accept a lower wage from unskilled jobs. Fu (2011) shows that firm-funded training of employees can enhance frictional wage dispersion, as differences in training among firms cause *ex ante* identical workers to differ in their productivities *ex post*.

The rest of the paper is structured as follows. Section 2 presents the search model with heterogenous workers. Section 3 discusses that the heterogeneity of human capital depreciation rate increases the overall Mm ratio analytically and quantitatively. Section 4 concludes.

2 The model

The model presented in this section is based on the stochastic random matching model by Ortego-Marti (2016). We consider an economy in continuous time. The labor market consists of two agents, firms and workers. Both are infinitely lived, risk neutral, and discount their future values at a constant rate r. Jobs end at the exogenous job separation rate s, and there is no OTJ search. Hence, only unemployed workers search in this labor market. Because of search frictions, it takes time for unemployed workers to get jobs. The total labor force is normalized to 1. During unemployment, workers gradually lose human capital at the rate δ_i. There are two types of workers $i \in \{H, L\}$ with different human capital depreciation rates. A fraction σ of the total labor force has a high human capital depreciation rate δ_H, while the remaining $1 - \sigma$ has a low depreciation rate $\delta_L < \delta_H$.

2.1 Matching function

As the two types of workers $i = \{H, L\}$ are similar in all observable characteristics, we assume that they search in the same labor market. Flows from unemployment to employment are determined by the number of matches between unemployed workers and open vacancies posted by firms. The matching technology in the labor market is represented by a matching function which determines the rate (equivalently, the number) of matches m as a function of the number of vacancies v and unemployed workers u:

$$m = m\left(v, u\right).$$

The matching function has the following properties. Firstly, it is increasing and concave in both of its arguments as more matches can be made when there are either more vacancies or more unemployed workers, but at a decreasing rate as more matches also imply that either less unemployed workers or vacancies are left to choose from. Secondly, the function exhibits constant returns to scale, that is, if the vacancy and unemployment rates increase by the same factor, matches will increase by the same factor.

Market tightness θ is defined as the ratio between vacancies and unemployed workers, $\theta = v/u$. By the properties of the matching function, the job arrival rate $\lambda(\theta)$ for unemployed workers can be calculated as

$$\lambda\left(\theta\right) = \frac{m\left(v, u\right)}{u} = m\left(\theta, 1\right),$$

and similarly for the job filling rate $q(\theta)$ for vacant jobs:

$$q\left(\theta\right) = \frac{m\left(v, u\right)}{v} = m\left(1, \frac{1}{\theta}\right).$$

According to the properties of the matching function, the job arrival rate increases in market tightness ($\partial\lambda/\partial\theta \geq 0$), while the job filling rate decreases ($\partial q/\partial\theta \leq 0$). That is, when market tightness is high, there are many vacancies per unemployed worker (or, few unemployed workers per vacancy) so unemployed workers find jobs more quickly while firms have more difficulty finding workers.

2.2 Workers

When workers initially enter the labor market, they are *ex ante* identical. However, workers lose and find jobs randomly because both job separations and matches happen at random. Over time, workers therefore accumulate different durations of time spent in unemployment. As human capital decays during unemployment, different unemployment histories also yield different human capital levels so workers become heterogenous *ex post*. Heterogenous human capital depreciation rates further add to this *ex post* heterogeneity, as two workers with similar unemployment histories but different depreciation rates will end up with different human capital levels.

A worker's unemployment history is summarized by the cumulative duration of unemployment spells γ. The distributions of unemployment histories γ among employed and unemployed workers are determined endogenously and are given by the cumulative density functions $G_i^E(\gamma)$ and $G_i^U(\gamma)$ for $i \in \{H, L\}$. These distributions are allowed to depend on worker types i, as different rates of human capital depreciation might affect unemployment durations. To ensure that these distributions are stationary, workers are assumed to leave the labor force at rate μ (e.g., due to retirement) and are replaced by new workers with zero unemployment history.

As we consider workers that are observationally identical, human capital is considered the net of such characteristics. Further, we assume for simplicity that workers do not accumulate skills during employment. Therefore, human capital only depends on unemployment history γ and the depreciation rate δ_i. The constant depreciation rate implies that human capital can be written as an exponentially decreasing function of unemployment history γ:

$$h_i(\gamma) = e^{-\delta_i \gamma}$$

for $i \in \{H, L\}$ and with the normalization $h_i(0) = 1$. Note that the assumption that workers do not accumulate human capital during employment implies that human capital losses during unemployment have a permanent impact. This simplifying assumption can be justified as empirical evidence shows that unemployment has long-term effects on wages (see Section 1). However, it would also be relevant to examine the more realistic case of skills accumulation during employment.

During unemployment, workers' valuations of non-market activities are assumed to be proportional to their human capital. Unemployed workers receive the utility flow $bh_i(\gamma)$ where b is the utility flow for an unemployed worker with no unemployment history (i.e., with $h_i = 1$). In general, this utility flow represents the return when the worker is not producing (Pissarides 1985) and will therefore include unemployment benefits, the value of leisure, and home production net of search costs and the disutility of being jobless (Hornstein et al. 2011). Ortego-Marti (2016) interprets $bh_i(\gamma)$ more narrowly as the flow payments that workers receive during unemployment and thus assumes that unemployment benefits are individual-specific and proportional to human capital. While this might seem as an unrealistic assumption, it will simplify the subsequent analysis significantly.

See a further discussion in Section F.

2.3 Firms

The productivity of a match (in terms of output) between a worker and a firm is determined by the product of a randomly drawn productivity parameter p and the individual-specific human capital $h_i(\gamma)$:

$$ph_i(\gamma).$$

As in the Pissarides (1985) model, not all matches start with the same productivity p. Instead, let the cumulative density function $F(p)$ represent a known distribution of match-specific productivities. When workers and firms meet, they make a random draw p from this distribution. This productivity represents the matching efficiency of the worker and the firm (Pissarides 1985, p. 679).

Firms post vacancies to hire workers for production. They pay a constant recruitment cost k for posting vacancies. The production starts once the firm matches with a worker. The match produces a flow productivity $ph_i(\gamma)$ that consists of the matching efficiency and human capital of the worker. The worker only stops producing if job separation occurs at the exogenous rate s.

2.4 Bellman equations

The above assumptions allow us to derive Bellman equations which represent the value functions of workers and firms. When workers and firms meet, they either accept the match and start producing or reject the job and keep searching in the next period. The threshold of productivity p that firms and workers accept the match is called the reservation productivity. Let $p^*_{i,\gamma}$ denote the reservation productivity for type-i workers. Matches with a productivity $p \geq p^*_{i,\gamma}$ are accepted, while matches with $p < p^*_{i,\gamma}$ are rejected.

Let $U_i(\gamma)$ denote the value function of an unemployed worker with unemployment history γ and human capital depreciation rate δ_i, and let $W_i(\gamma, p)$ denote the value function of an employed worker with the match-specific productivity p. Then, the Bellman equation for an unemployed worker is

$$(r+\mu)\,U_i(\gamma) = bh_i(\gamma) + \lambda(\theta)\int_{p^*_{i,\gamma}}^{p^{\max}} \left[W_i(\gamma, y) - U_i(\gamma)\right] dF(y) + \frac{\partial U_i(\gamma)}{\partial \gamma}. \tag{1}$$

This equation shows that the return to unemployment depends on the flow of unemployment benefits and on the probability of finding a job: unemployed workers receive job offers at the rate $\lambda(\theta)$ and accept them if $p \geq p^*_{i,\gamma}$. In this case, the transition into employment will be associated with a utility gain of $W_i(\gamma, p) - U_i(\gamma)$. Furthermore, the value of unemployment decreases with the worker's unemployment history due to human capital losses. Since workers leave the labor force at rate μ, the term $r + \mu$ on the left-hand side can be interpreted as the effective discount rate.

The value function of an employed worker similarly satisfies the Bellman equation

$$(r + \mu) W_i(\gamma, p) = w_i(\gamma, p) - s\left[W_i(\gamma, p) - U_i(\gamma)\right]. \tag{2}$$

Employed workers receive a utility flow from the wage $w_i(\gamma, p)$. They lose their jobs with capital loss $W_i(\gamma, p) - U_i(\gamma)$ at the rate s.

As explained previously, differences in workers' human capital depreciation rates are due to unobservable worker characteristics, so firms are not able to distinguish them before a match has been made. Similarly, firms receive applications across all unemployment histories as all workers search in the same labor market. Hence, firms posting a vacancy must consider the expected gain from hiring a worker across both worker types, all unemployment histories, and all potential match-specific productivities. Let V denote the value function of a vacancy, the Bellman equation for vacancies is

$$rV = -k + q(\theta)\left\{\sigma \mathbb{E}_H\left[J_H(\gamma, p) - V\right] + (1 - \sigma)\mathbb{E}_L\left[J_L(\gamma, p) - V\right]\right\}, \tag{3}$$

where

$$\mathbb{E}_i\left[J_i(\gamma, p) - V\right] = \int_0^\infty \left(\int_{p^*_{i,\gamma}}^{p^{\max}} \left[J_i(\gamma, y) - V\right] dF(y)\right) dG_i^U(\gamma)$$

is the expected capital gain if the firm matches with a worker of type $i \in \{H, L\}$ according to their unemployment history. Assume free entry in the market for vacancies such that firms keep posting vacancies until $V = 0$ which will hold in equilibrium.

In existing matches, the characteristics of the worker and the specific match are known to both agents. Therefore, $J_i(\gamma, p)$ denotes the value function of a filled job. The employee produces the flow output $h_i(\gamma)\, p$ and receives the wage $w_i(\gamma, p)$. This yields profits $h_i(\gamma)\, p - w_i(\gamma, p)$. At the rate s, the job is destroyed, and the firm will suffer a capital loss equal to $J_i(\gamma, p) - V$. The associated

Bellman equation thus reads

$$(r+\mu)\, J_i(\gamma,p) = h_i(\gamma)\, p - w_i(\gamma,p) - s\,(J_i(\gamma,p) - V)\,. \tag{4}$$

2.5 Wage determination and reservation productivities

When accepting the job contract, the worker gives up $U_i(\gamma)$ for $W_i(\gamma,p)$ and the firm gives up V for $J_i(\gamma,p)$. Hence, $U_i(\gamma)$ and V are the worker's and the firm's respective threat points or outside options. A successful match thus yields the net surplus $S_i(\gamma,p) = J_i(\gamma,p) - V + W_i(\gamma,p) - U_i(\gamma)\,.$ Wages are determined through Nash bargaining share rules. That is, the worker and the firm split the surplus from the match, weighted by their respective bargaining powers. Letting $\beta \in [0,1]$ denote the bargaining power of workers, the wage is the solution that satisfies

$$w_i(\gamma,p) = \arg\max_{w_i} \left[W_i(\gamma,p) - U_i(\gamma)\right]^{\beta} \left[J_i(\gamma,p) - V\right]^{1-\beta}. \tag{5}$$

The worker gets a share β of the surplus, and the firm a share $1-\beta$. The first order condition for this maximization problem is[2]

$$\beta J_i(\gamma,p) = (1-\beta)\,(W_i(\gamma,p) - U_i(\gamma))\,. \tag{6}$$

This condition shows that firms and workers agree on the same reservation productivity $p^*_{i,\gamma}$.[3] Further calculations allow us to arrive at an expression for the wage which links wages to reservation productivities:

$$w_i(\gamma,p) = h_i(\gamma)\left[\beta p + (1-\beta)\, p^*_{i,\gamma}\right]. \tag{7}$$

Rewriting yields $w_i(\gamma,p) = h_i(\gamma)\left[\beta\left(p - p^*_{i,\gamma}\right) + p^*_{i,\gamma}\right]$. At the reservation productivity, we have $W_i(\gamma,p^*_i) = U_i(\gamma)$ and $J_i(\gamma,p^*_i) = 0$. The wage at the reservation productivity is $w_i(\gamma,p^*_i) =$

[2] The derivations for this section can be found in Appendix B.

[3] Specifically, the productivity that makes firms indifferent between accepting and rejecting the match must satisfy the condition $J_i\left(\gamma,p^*_{i,\gamma}\right) = V$. With the free entry condition, $J_i\left(\gamma,p^*_{i,\gamma}\right) = 0$. Similarly, the productivity that makes workers indifferent satisfies $W_i\left(\gamma,p^*_{i,\gamma}\right) = U_i(\gamma)$. Inserting the $p^*_{i,\gamma}$ that satisfies the firm's condition into (6) yields

$$\begin{aligned}
\beta J_i\left(\gamma,p^*_{i,\gamma}\right) &= (1-\beta)\left(W_i\left(\gamma,p^*_{i,\gamma}\right) - U_i(\gamma)\right)\\
0 &= (1-\beta)\left(W_i\left(\gamma,p^*_{i,\gamma}\right) - U_i(\gamma)\right)\\
W_i\left(\gamma,p^*_{i,\gamma}\right) &= U_i(\gamma)\,.
\end{aligned}$$

Hence, the reservation productivity that satisfies the firm's condition also satisfies the worker's condition.

$h_i(\gamma)p^*_{i,\gamma}$, and the value of workers' outside options is $(r+\mu)U_i(\gamma) = w_i(\gamma, p^*_i)$. As a result, the reservation productivity $p^*_{i,\gamma}$ represents the worker's outside option. When the productivity p is higher than the reservation productivity, workers obtain their share of the additional productivity according to their bargaining power β. Hence, equation (7) shows that workers receive compensation for their outside options plus their share β of the exceeding productivity from the match.

Proposition 1. *The reservation productivity is independent of workers' unemployment history, i.e.,* $p^*_{i,\gamma} = p^*_i$.

All proofs of propositions and lemma are showed in appendix A. Following Ortego-Marti (2016), the result is proven by guessing this solution and then confirming that the guess is indeed correct. Intuitively, the output $h_i(\gamma)\,p$ that a worker can produce decreases with his unemployment history. Lower output reduces the value to the firm of hiring the worker, so the firm will offer a lower wage. This makes it less attractive for the worker to accept the job because the surplus of employment becomes lower. As a consequence, this increases the reservation productivity. However, the unemployment benefits $bh_i(\gamma)$ decrease with unemployment meanwhile. This lowers the worker's outside option, so the worker is willing to accept worse matches to leave unemployment by lowering the reservation productivity. Because both output and unemployment benefits are proportional to $h_i(\gamma)$, these two opposing effects exactly net out such that the reservation productivity is the same across unemployment histories.

Note that this simple result hinges on the assumption that unemployment benefits are proportional to human capital, as this causes the values of employment and unemployment to decrease proportionally. A more realistic assumption would be to assume constant benefits, as workers are generally compensated by similar amounts during unemployment. We relax this assumption in F.

Next we look at the relationship of the reservation productivity between two groups of workers. We derive the reservation productivity p^*_i as[4]

$$p^*_i = \frac{r+\mu}{r+\mu+\delta_i}\left[b + \frac{\lambda\beta}{r+\mu+s}\int_{p^*_i}^{p\max}\left(1-F(y)\right)dy\right], \tag{8}$$

According to (8), we are able to derive the first order derivative of p^*_i with respect to δ_i and the sign

[4] The derivation is in appendix C.

of the derivative is negative:

$$\frac{\partial p_i^*}{\partial \delta_i} < 0. \tag{9}$$

Hence, the reservation productivity decreases in the human capital depreciation rate. Workers with a faster human capital depreciation during unemployment thus have a lower reservation productivity than the ones with a lower depreciation rate:

$$p_H^* < p_L^*. \tag{10}$$

Lemma 1. *The reservation productivity is higher with lower human capital depreciation rate, i.e.,* $p_L^* > p_H^*$.

The reservation productivity is lower for workers with a high human capital depreciation because both their productivity and unemployment benefits decrease at a faster rate during unemployment. Hence, it is more costly for them to remain unemployed so they become less picky and are willing to accept worse (less productive) matches to leave unemployment faster.

These results above for the reservation productivity can be substituted into the wage in (7) to obtain a final expression for equilibrium wages:

$$w_i(\gamma, p) = h_i(\gamma)\left(\beta p + (1-\beta)\, p_i^*\right). \tag{11}$$

While the reservation productivity is independent of γ, wages still decrease in γ because workers with longer unemployment histories are less productive. For the same reason, wages increase in the productivity p of the match. Furthermore, wages increase in the reservation productivity p_i^* because workers with a better outside option require more compensation. Hence, when comparing two workers with the same unemployment history $\overline{\gamma}$ and productivity parameter $\overline{p}$ but different rates of human capital loss δ_i, wages are lower for workers with a high depreciation rate for two reasons. First, since their human capital depreciates faster, their human capital is lower for a given length of unemployment. Second, they require less compensation due to a lower outside option so they accept lower wages.

3 The *Mm* ratio

This section turns to the analysis of frictional wage dispersion in the model. Wage dispersion is measured by the *Mm* ratio, i.e., the ratio between the mean accepted and lowest accepted (reservation) wage that can be observed in the labor market:

$$Mm = \frac{\overline{w}}{w^*}. \tag{12}$$

A wide range of search models are able to derive closed form expressions of the *Mm* ratio without assuming any distribution of the offers that workers draw from. Instead, the *Mm* ratio in these models only depends on parameters of the model which can be estimated from the data (Hornstein et al. 2011). This is also the case in Ortego-Marti's (2016) model with homogenous workers. However, in our case with heterogenous workers, it is not straightforward to derive such a closed form expression. Therefore, we must instead rely on numerical methods to calculate the *Mm* ratio. It is, though, still possible to derive some analytical results regarding the size of the *Mm* ratio with heterogenous depreciation rates compared to the case with homogenous workers. The next subsection turns to this before moving on to the numerical calculations.

According to Ortego-Marti (2016), the *Mm* ratio should be calculated net of human capital as the aim is to show that even after controlling for observable worker characteristics, and in particular after controlling for unemployment history and human capital, the model still generates a more realistic wage dispersion than baseline models.[5] In this way, larger wage dispersion will not be driven by differences in human capital among workers, but only by the changes in search behavior that the model implies. Likewise, our goal is to examine whether the heterogeneity increases wage dispersion further. Therefore, human capital will similarly be disregarded from wages in the following calculations. This also allows us to compare the results to Ortego-Marti (2016) directly.

[5] From (11), it can be seen that wages are proportional to human capital $h_i(\gamma)$:

$$w_i(\gamma, p) = h_i(\gamma)(\beta p + (1-\beta)p_i^*) = e^{-\delta_i \gamma}(\beta p + (1-\beta)p_i^*).$$

Taking logarithm to this expression yields

$$\log w_i(\gamma, p) = -\delta_i \gamma + \log(\beta p + (1-\beta)p_i^*).$$

Hence, one can hold human capital fixed in a Mincerian regression (which also controls for other observable worker characteristics) by controlling for unemployment history γ. The logarithm of wages is theoretically equivalent to a Mincerian regression.

3.1 Analytical results

With two groups of workers in the labor market, the Mm ratio is given by

$$Mm = \frac{\overline{w}}{w^*} = \frac{\sigma \overline{w}_H + (1-\sigma)\,\overline{w}_L}{w^*}, \tag{13}$$

where $\bar{w}_i = \mathbb{E}\left(w_i\left(\gamma, p\right) \mid p \geq p_i^*\right) = \int_0^\infty \left(\int_{p_i^*}^{p^{max}} w_i(\gamma, p) \frac{dF(p)}{1-F(p_i^*)}\right) dG_i^E(\Gamma)$. The average wages can be calculated from (11) as[6]

$$\overline{w}_i = \bar{h}_i(\gamma)[\beta \overline{p}_i + (1-\beta)\, p_i^*]. \tag{14}$$

Since we are able to remove $h_i(\gamma)$ from wages by controlling unemployment history in a Mincerian wage regression, we consider the wages net of human capital, which is $\beta p + (1-\beta) p_i^*$.

Because $w_L^* = p_L^*$ and $w_H^* = p_H^*$ when wages are considered net of human capital, and according to (10), the lowest wage observed in the labor market is w_H^*. Therefore, (13) can be written as

$$Mm = \frac{\sigma \overline{w}_H + (1-\sigma)\,\overline{w}_L}{w_H^*}. \tag{15}$$

Lemma 2. *The average productivity conditional on the reservation productivity increases with the reservation productivty, i.e.,* $\bar{p}_L > \bar{p}_H$.

Since $\overline{p}_i = \mathbb{E}\left(p \mid p \geq p_i^*\right)$ and $p_L^* > p_H^*$, it must also be the case that the average observed match-specific productivity is higher among the group of workers with the low depreciation rate, as the two groups of workers draw p from the same distribution $F\left(p\right)$. This implies that $\overline{p}_L > \overline{p}_H$. Therefore, since both the reservation productivity and average productivity are higher among the group of workers with the low rate of depreciation, we have that $\overline{w}_L > \overline{w}_H$. These results allow us to compare the size of the Mm ratio with heterogeneity to the Mm ratio with homogenous workers. Rewrite equation (15), we get

$$Mm = \sigma \frac{\overline{w}_H}{w_H^*} + (1-\sigma)\frac{\overline{w}_L}{w_H^*} = \sigma Mm_H + (1-\sigma)\frac{\overline{w}_L}{w_H^*},$$

where Mm_H is the Mm ratio among workers with the high depreciation rate. Start by comparing the Mm ratio with heterogeneity to the case where workers only have the high depreciation rate.

[6] See this derivation in Appendix D.

Since $\overline{w}_L > \overline{w}_H$, it is also the case that $\overline{w}_L/w_H^* > \overline{w}_H/w_H^* = Mm_H$. Hence,

$$Mm = \sigma Mm_H + (1-\sigma)\,\frac{\overline{w}_L}{w_H^*} > Mm_H. \tag{16}$$

This shows that the model with heterogeneous depreciation rates yields a higher Mm ratio compared to the case where workers only have the high depreciation rate. Intuitively, because the group of workers with a lower depreciation rate has a higher average wage, this will increase the difference between the minimum and average wage, thus generating more wage dispersion.

We also consider whether the overall Mm ratio is greater than the Mm ratio when there are workers with low depreciation rate only. When $\sigma \to 0$, i.e. there are few workers with high depreciation rates, the overall Mm ratio converges to $\bar{w}_L/w_H^*$, which is greater than $Mm_L = \bar{w}_L/w_L^*$. When $\sigma \to 1$, the overall Mm ratio converges to Mm_H but still slightly greater than Mm_H. According to Ortego-Marti (2016), the model proves that higher human capital depreciation rate generates a greater Mm ratio when there is only one type of workers in the labor market. Therefore, we can conclude that $Mm > Mm_H > Mm_L$.

Hence, the model with heterogeneity will also yield a higher Mm ratio than the one with all workers having the low depreciation rate. This unambiguous result means that the positive effect on the Mm ratio with heterogeneity of a lower minimum wage will always outweigh the negative effect of a lower average wage.

Proposition 2. *The heterogeneity of human capital depreciation rate generates a greater overall Mm ratio.*

In summary, the analytical results show that heterogeneity always yields a higher Mm ratio than in the model with homogenous workers, irrespective of whether a higher or lower depreciation rate is introduced. Hence, we are able to increase the frictional wage dispersion implied by search models even further by adding heterogeneous rates of human capital decay.

3.2 Numerical solution

The following subsections turn to the numerical calculations of the Mm ratio net of human capital which is given by

$$Mm = \frac{\sigma\left[\beta\overline{p}_H + (1-\beta)\,p_H^*\right] + (1-\sigma)\left[\beta\overline{p}_L + (1-\beta)\,p_L^*\right]}{p_H^*}.$$

Hence, to compute the Mm ratio, we need to know p_i^* and $\overline{p}_i$ for $i \in \{H, L\}$ and the distribution of workers σ. The difference of depreciation rate is due to unobservable characteristics of workers from data. Therefore, the distribution of workers with different depreciation rates cannot be observed. We examine the Mm ratio with σ ranging from zero to one in this numerical exercise. From (8), the reservation productivity p_i^* is determined by the parameters of the model, the distribution of match-specific productivities $F(p)$, and the fixed part of unemployment benefits b. Since actual unemployment benefits vary across workers, we follow Ortego-Marti (2016) and define the replacement ratio ρ as the ratio of average unemployment benefits to the average wage.

Specifically, let $\overline{h}$ denote the average human capital across all workers. Then, $\rho = b\overline{h}/\overline{h}\overline{w} = b/\overline{w}$ where $\overline{w} = \sigma\overline{w}_H + (1-\sigma)\,\overline{w}_L$ is the average wage net of human capital. Hence, the fixed part of unemployment benefits will be given by $b = \rho\overline{w}$. The average wage $\overline{w}$ is determined by $\overline{w}_i$ and is, therefore, itself a function of p_i^* and $\overline{p}_i$ for $i \in \{H, L\}$. Likewise, p_i^* in (8) depends on both $\overline{w}_i$ and indirectly on $\overline{p}_i$. Furthermore, the average observed productivity $\overline{p}_i = \int_{p_i^*}^{p^{\max}} p/\left(1 - F\left(p_i^*\right)\right) dF(p)$ is itself determined by p_i^* and by the distribution $F(p)$.

In sum, this leaves us with six equations in six unknowns: the reservation productivities p_i^*, the average productivities $\overline{p}_i$, and the average wages $\overline{w}_i$. To solve these equations numerically, we need to assume a functional form for $F(p)$. Following Ortego-Marti (2016), we will assume a uniform distribution with support $[0, p^{\max}]$ where $p^{\max}$ is normalized to 1. This normalization will not affect the results as p_i^* is proportional to $p^{\max}$.[7] The uniform distribution with support $[0, 1]$ implies that the system of equations to be solved reads[8]

[7] See Ortego-Marti (2016, footnote 17).

[8] See the derivations of these expressions in Appendix E.

$$p_i^* = \frac{r+\mu}{r+\mu+\delta_i}\left[\rho\overline{w} + \frac{\lambda\beta}{r+\mu+s}\frac{(1-p_i^*)^2}{2}\right],$$
$$\overline{p}_i = \frac{1-(p_i^*)^2}{2(1-p_i^*)},$$
$$\overline{w}_i = \beta\overline{p}_i + (1-\beta)\,p_i^*,$$

for $i \in \{H, L\}$.

3.3 Parameter specification

The values attached to the parameters of the model follow Ortego-Marti (2016). Except from the human capital depreciation rates δ_i (which are elaborated below) and β, these values are taken from Hornstein et al. (2011) who calibrate the model to fit the US economy using CPS data. The assigned values are reported in Table 1 below. The parameter λ^* is the average job finding rate across worker types and can be observed from the data. In the model, it is determined by the product of the job arrival rate λ and the respective fractions of acceptable matches $1 - F(p_i^*)$. Hence, $\lambda^* = \lambda[\sigma(1-F(p_H^*)) + (1-\sigma)(1-F(p_L^*))]$. The reservation productivities in (8) depend on λ so λ is added as an unknown variable to the system of equations with the target to match the observed λ^*.[9]

Table 1: Parameter specification

r	μ	ρ	s	λ^*	β
0.0041	0.0021	0.4	0.03	0.43	0.5

All parameter values are monthly. The discount rate r of 0.0041 comes from the annual interest rate, which is 0.05. The average duration of working life is assumed to be 40 years, so that the monthly rate μ at which workers leave the labor force is 0.0021. Further, Hornstein et al. (2011) calibrate the monthly separation rate s as 0.03, and the monthly job finding rate λ^* as 0.43. The bargaining power of workers β is equated to 0.5 based on Petrongolo & Pissarides (2001) and following

[9] This approach follows Ortego-Marti (2016, p. 16).

Pissarides (2009) and Ortego-Marti (2016).

According to Ortego-Marti (2016), we are able to estimate the overall human capital depreciation rate by running a Mincerian regression on the unemployment history. However, the heterogeneity of the depreciation rate comes from the unobservable characteristics. We are not able to distinguish these two groups of workers and estimate their human capital depreciation rate respectively. Instead, we must rely on other existing estimates of the human capital depreciation rate although they will not exactly reflect our case. Using such estimates will ensure that realistic values are assigned to δ_L and δ_H.

In a Mincerian regression, Ortego-Marti (2016) estimates the monthly human capital depreciation rate to be 0.0122 and uses this in his calculations of the Mm ratio with homogenous workers. Further, Ortego-Marti (2017) finds that the monthly depreciation rate varies between 0.0040 and 0.0218 across occupations and between 0.0066 and 0.0181 across industries. Therefore, it seems reasonable to consider half of 0.0122 (corresponding to 0.0061) and the double of 0.0122 (corresponding to 0.0244) as further values of the human capital depreciation rate in the simulation below.

3.4 Numerical exercise

The Mm ratio is computed for three combinations of δ_L and δ_H. The first case assumes that the depreciation rate estimated by Ortego-Marti (2016) of 0.0122 corresponds to δ_H while δ_L is half its size. The second case reversely equates δ_L with Ortego-Marti's estimate and sets δ_H to twice its size. The last case considers the two "extremes" with δ_H twice the size of Ortego-Marti's estimate and δ_L half its size. In this latter case, Ortego-Marti's estimate can be viewed as an average between the two depreciation rates. Considering these different cases along with different values of σ allows us to examine whether the quantitative results seem robust to different specifications of δ_i and σ.

Figure 1 below plots the numerical values of the Mm ratio for each of the three cases and for values of σ ranging from zero to one. For $\sigma = 0$ or $\sigma = 1$, the model is reduced to the setting with homogenous workers as in Ortego-Marti (2016) with $\delta = \delta_L$ or $\delta = \delta_H$, respectively. With homogenous workers and $\delta = 0.0122$, the Mm ratio is therefore 1.209 as in Ortego-Marti (2016). For $\delta = 0.0061$, the Mm ratio is reduced to 1.129 while it increases to 1.369 for $\delta = 0.0244$. This conforms with the analytical finding for homogenous workers that a higher human capital depreciation rate unambiguously increases the Mm ratio because workers accept a wider range of wages.[10]

Firstly, we see that the numerical results confirm the analytical finding from above that heterogeneity will always yield a greater Mm ratio than if all workers had the high depreciation rate. This follows because the Mm ratio for $\sigma < 1$ is always higher than if $\sigma = 1$. As explained above, this happens because introducing workers with a lower depreciation rate will raise the average wage which generates more wage dispersion. Secondly, we also confirm the analytical finding that the Mm ratio with heterogenous workers is always greater than if all workers had the low depreciation rate.

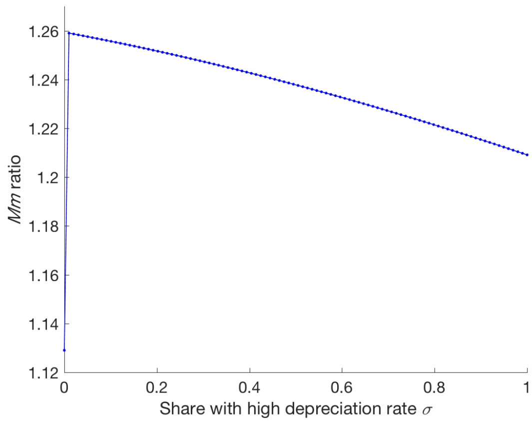
When we introduce heterogeneous workers into the model, we can observe a big jump of the Mm ratio from 1. We start from the labor market only with low-depreciation workers. Once the high-depreciation workers are introduced to the model, the reservation wage jumps to the high-depreciation one, while the effect on the average wage is not significant with a small share of high-depreciation workers. The jump in the reservation wage causes the jump of the Mm ratio. When σ increases, the average wage is affected by a higher share of high-depreciation workers. The more high-depreciation workers, the lower average wage. That is why we can observe that the Mm ratio decreases with σ.

Proposition 3. *The overall Mm ratio decreases with σ.*

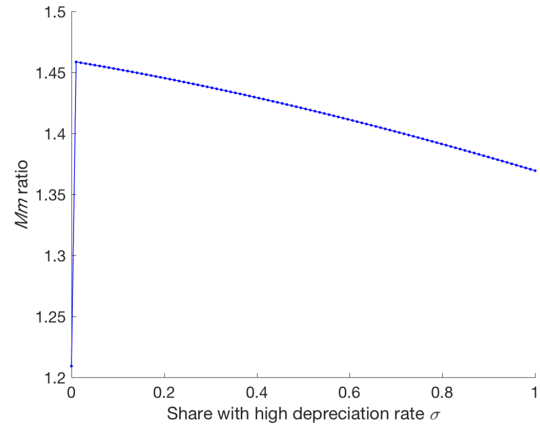
[10] This result has been proved by Ortego-Marti (2016).

Figure 1: Mm ratio for different combinations of δ_H and δ_L

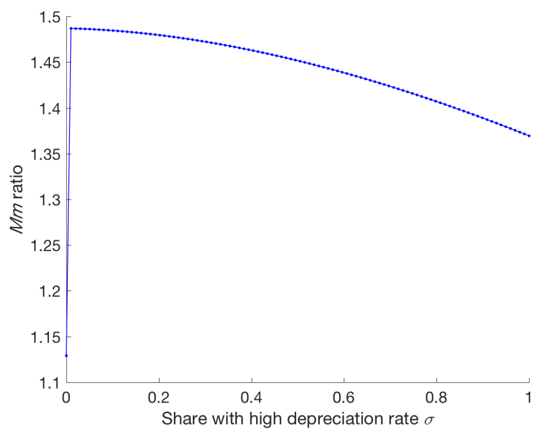
(a) Case I: $\delta_L = 0.0061$ and $\delta_H = 0.0122$



(b) Case II: $\delta_L = 0.0122$ and $\delta_H = 0.0244$



(c) Case III: $\delta_L = 0.0061$ and $\delta_H = 0.0244$



In sum, for all three cases considered, heterogeneity thus yields a higher Mm ratio than the value of 1.209 from Ortego-Marti (2016) with homogenous workers. Specifically, for case I, the ratio ranges from 1.210 to 1.259; for case II from 1.369 to 1.459; and for case III from 1.369 to 1.487. As explained in Section 1, a reasonable empirical counterpart to the Mm ratio is the 50-10 percentile ratio which is approximately equal to 1.8. Hence, the model with heterogeneity can explain between 26% and 61% of observed frictional wage dispersion.

4 Conclusion

This paper examines whether the heterogeneity of human capital depreciation generates more frictional wage dispersion. Extending the model in Ortego-Marti (2016) with heterogeneity, we find that the Mm ratio increases. Since workers are similar in all observable aspects, the heterogeneity that this model considers must be due to unobservable personal characteristics. Numerically, the overall Mm ratio increases from 1.209 in Ortego-Marti (2016) to between 1.210 and 1.487. Compared to the labor market with high-depreciation workers only, the share of low-depreciation workers increases the average wage so that the overall Mm ratio is higher with heterogeneity. Compared to the labor market with low-depreciation workers only, the share of high-depreciation workers reduces the minimum wage such that the overall Mm ratio also rises. As a result, we are able to conclude that the heterogeneity of human capital depreciation raises the Mm ratio, which represents the frictional wage dispersion.

The model has made use of simplifying assumptions such as no OTJ search, permanent human capital losses, and homogeneous firms. Obvious extensions of the model would thus be to relax these assumptions and examine how frictional wage dispersion is affected. OTJ search allows workers to not lose the option of searching for a better job if they accept a job offer. Adding returns to experience would ensure that human capital losses during unemployment are not permanent. Both extensions would raise the value of employment such that workers are willing to accept a wider range of jobs. This would make frictional wage dispersion increase. The literature also has shown that adding heterogeneous firms in terms of, for instance, differences in productivity or firm-funded training of employees can also raise frictional wage dispersion.

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5 Appendix

A Proofs of propositions and lemmas

Proof of proposition 1

Proof. This proof follows the method in Ortego-Marti (2016). The proof proceeds by guessing a solution to $U_i(\gamma)$ and then confirming that the guess is correct.

First, we guess that the value function for unemployment $U_i(\gamma)$ is given by $U_i(\gamma) = h_i(\gamma) U_i$ with U_i independent of unemployment history γ but dependent on worker types i. Assuming that the guess is correct, we obtain the following expression for the reservation productivity $p^*_{i,\gamma}$ by using the previous result that $(r+\mu) U_i(\gamma) = h_i(\gamma) p^*_{i,\gamma}$:

$$\begin{aligned}(r+\mu) h_i(\gamma) U &= h_i(\gamma) p^*_{i,\gamma} \\ p^*_{i,\gamma} &= (r+\mu) U_i.\end{aligned}$$

Since U_i is independent of γ, so is $p^*_{i,\gamma}$. Hence, we can write the reservation productivity as $p^*_{i,\gamma} = p^*_i$.

Second, the Bellman equation for $U_i(\gamma)$ in (1) has a unique solution for U_i because it is a contraction. To prove that our guess for $U_i(\gamma)$ is correct, we show that this guess yields a unique solution for U_i in (1). Start by inserting $U_i(\gamma) = h_i(\gamma) U_i$ and $p^*_{i,\gamma} = p^*_i$ into (1):

$$(r+\mu) h_i(\gamma) U_i = b h_i(\gamma) + \int_{p^*_i}^{p^{\max}} W_i(\gamma, p)\, dF(p) - \lambda \int_{p^*_i}^{p^{\max}} h_i(\gamma) U_i dF(p) + \frac{\partial [h_i(\gamma) U_i]}{\partial \gamma}.$$

Calculate the derivative:

$$\frac{\partial [h_i(\gamma) U_i]}{\partial \gamma} = -\delta_i e^{-\delta_i \gamma} U_i = -\delta_i h_i(\gamma) U_i.$$

Substitute back into the Bellman equation and calculate the second integral (using integration by parts):

$$\begin{aligned}(r+\mu) h_i(\gamma) U_i &= b h_i(\gamma) + \lambda \int_{p^*_i}^{p^{\max}} W_i(\gamma, p)\, dF(p) - \lambda \int_{p^*_i}^{p^{\max}} h_i(\gamma) U_i dF(p) - \delta_i h_i(\gamma) U_i \\ &= b h_i(\gamma) + \lambda \int_{p^*_i}^{p^{\max}} W_i(\gamma, p)\, dF(p) - \lambda h_i(\gamma) U_i [1 - F(p^*_i)] - \delta_i h_i(\gamma) U_i.\end{aligned}$$

Now, rewrite the remaining integral. Start by considering the Bellman equation for $W_i(\gamma, p)$ in (2):

$$(r + \mu + s) W_i(\gamma, p) = w_i(\gamma, p) + sh_i(\gamma) U_i.$$

Substitute in the wage $w_i(\gamma, p)$ from (7):

$$\begin{aligned}(r + \mu + s) W_i(\gamma, p) &= h_i(\gamma)[\beta p + (1-\beta) p_i^*] + sh_i(\gamma) U_i \\ &= h_i(\gamma)[\beta p + (1-\beta) p_i^* + sU_i].\end{aligned}$$

Substitute the rewritten expression for $W_i(\gamma, p)$ back into the integral to see that $h_i(\gamma)$ cancels out of the equation:

$$\begin{aligned}(r+\mu) h_i(\gamma) U_i &= bh_i(\gamma) + \lambda \int_{p_i^*}^{p^{\max}} \frac{h_i(\gamma)}{r+\mu+s}[\beta p + (1-\beta) p_i^* + sU_i]\, dF(p) - \lambda h_i(\gamma) U_i [1 - F(p_i^*)] - \delta_i h_i(\gamma) U_i \\ &= bh_i(\gamma) + \frac{\lambda}{r+\mu+s} h_i(\gamma) \int_{p_i^*}^{p^{\max}} [\beta p + (1-\beta) p_i^* + sU_i]\, dF(p) - \lambda h_i(\gamma) U_i [1 - F(p_i^*)] - \delta_i h_i(\gamma) U_i \\ (r+\mu) U_i &= b + \frac{\lambda}{r+\mu+s} \int_{p_i^*}^{p^{\max}} [\beta p + (1-\beta) p_i^* + sU_i]\, dF(p) - \lambda U_i [1 - F(p_i^*)] - \delta_i U_i.\end{aligned}$$

Now, calculate the integral on the right-hand side (using integration by parts):

$$\begin{aligned}\int_{p_i^*}^{p^{\max}} [\beta p + (1-\beta) p_i^* + sU_i]\, dF(p) &= \beta \int_{p_i^*}^{p^{\max}} p dF(p) + (1-\beta) p_i^* \int_{p_i^*}^{p^{\max}} dF(p) + sU_i \int_{p_i^*}^{p^{\max}} dF(p) \\ &= \beta \left(p^{\max} - \int_{p_i^*}^{p\max} F(p)\, dp \right) + p_i^* [1 - \beta - F(p_i^*)] - \beta p_i^* + sU_i [1 - F(p_i^*)]\end{aligned}$$

Substitute this back into the equation and gather the terms with U_i on the right-hand side:

$$\begin{aligned}(r+\mu) U_i &= b + \frac{\lambda}{r+\mu+s} \left[\beta \left(p^{\max} - \int_{p_i^*}^{p\max} F(p)\, dp \right) + p_i^* [1 - \beta - F(p_i^*)] + sU_i [1 - F(p_i^*)] \right] - \lambda U_i [1 - F(p_i^*)] - \delta_i U_i \\ &= b + \frac{\lambda}{r+\mu+s} \left[\beta \left(p^{\max} - \int_{p_i^*}^{p\max} F(p)\, dp \right) + p_i^* [1 - \beta - F(p_i^*)] \right] + \left[\left(\frac{s}{r+\mu+s} - 1 \right) \lambda [1 - F(p_i^*)] - \delta_i \right] U_i.\end{aligned}$$

Finally, we have the expression of U_i:

$$U_i = \frac{b + \frac{\lambda}{r+\mu+s} \left[\beta \left(p^{\max} - \int_{p_i^*}^{p\max} F(p)\, dp \right) + p_i^* [1 - \beta - F(p_i^*)] \right]}{r + \mu + \delta_i + \lambda \frac{r+\mu}{r+\mu+s} [1 - F(p_i^*)]}.$$

Simplify the expression,

$$U_i = \frac{b + \frac{\lambda}{r+\mu+s}\left[\beta \int_{p_i^*}^{p\max} (1 - F(p))\, dp + p_i^* [1 - F(p_i^*)]\right]}{r + \mu + \delta_i + \lambda \frac{r+\mu}{r+\mu+s} [1 - F(p_i^*)]}.$$

Since U_i is a function of p_i^* and it is independent on γ, it is proved that p_i^* is independent on γ. □

Proof of lemma 1

Proof. To determine the effect of the depreciation rate on the reservation productivity, insert the expression for U_i into $p_i^* = (r + \mu) U_i$:

$$p_i^* = (r + \mu) \frac{b + \frac{\lambda}{r+\mu+s}\left[\beta \int_{p_i^*}^{p\max} (1 - F(p))\, dp + p_i^* [1 - F(p_i^*)]\right]}{r + \mu + \delta_i + \lambda \frac{r+\mu}{r+\mu+s} [1 - F(p_i^*)]}.$$

Multiply with the denominator and rewrite the right-hand side:

$$\begin{aligned} \left[r + \mu + \delta_i + \lambda \frac{r + \mu}{r + \mu + s} [1 - F(p_i^*)]\right] p_i^* &= (r + \mu) \left(b + \frac{\lambda}{r + \mu + s} \left[\beta \int_{p_i^*}^{p\max} (1 - F(p))\, dp + p_i^* [1 - F(p_i^*)] \right] \right) \\ &= (r + \mu) \left(b + \frac{\lambda\beta}{r + \mu + s} \int_{p_i^*}^{p\max} (1 - F(p))\, dp \right) + p_i^* \frac{r + \mu}{r + \mu + s} \lambda [1 - F(p_i^*)]. \end{aligned}$$

Then, gather the terms with p_i^* on the left-hand side and simplify:

$$[r + \mu + \delta_i] p_i^* = (r + \mu) \left(b + \frac{\lambda\beta}{r + \mu + s} \int_{p_i^*}^{p\max} (1 - F(p))\, dp \right).$$

Isolate p_i^*:

$$p_i^* = \frac{r + \mu}{r + \mu + \delta_i} \left[b + \frac{\lambda\beta}{r + \mu + s} \int_{p_i^*}^{p\max} (1 - F(p))\, dp \right].$$

Take the derivative of the expression with respect to δ_i:

$$\begin{aligned} \frac{\partial p_i^*}{\partial \delta_i} = &- \frac{r + \mu}{(r + \mu + \delta_i)^2} \left[b + \frac{\lambda\beta}{r + \mu + s} \int_{p_i^*}^{p\max} (1 - F(p))\, dp \right] \\ &+ \frac{r + \mu}{r + \mu + \delta_i} \frac{\lambda\beta}{r + \mu + s} \frac{\partial \left(\int_{p_i^*}^{p\max} (1 - F(p))\, dp \right)}{\partial p_i^*} \frac{\partial p_i^*}{\partial \delta_i}. \end{aligned}$$

To calculate the derivative on the right-hand side, use the Leibniz rule:

$$\begin{aligned}\frac{\partial p_i^*}{\partial \delta_i} &= -\frac{r+\mu}{(r+\mu+\delta_i)^2}\left[b+\frac{\lambda\beta}{r+\mu+s}\int_{p_i^*}^{p\max}(1-F(p))\,dp\right] \\ &\quad + \frac{r+\mu}{r+\mu+\delta_i}\frac{\lambda\beta}{r+\mu+s}\left(-\left(1-F\left(p_i^*\right)\right)\right)\frac{\partial p_i^*}{\partial \delta_i}.\end{aligned}$$

Gather the terms with $\partial p_i^* / \partial \delta_i$:

$$\frac{\partial p_i^*}{\partial \delta_i}\left[1+\frac{r+\mu}{r+\mu+\delta_i}\frac{\lambda\beta}{r+\mu+s}\left(1-F\left(p_i^*\right)\right)\right] = -\frac{r+\mu}{(r+\mu+\delta_i)^2}\left[b+\frac{\lambda\beta}{r+\mu+s}\int_{p_i^*}^{p\max}(1-F(p))\,dp\right].$$

Isolate the derivative:

$$\frac{\partial p_i^*}{\partial \delta_i} = -\frac{\frac{r+\mu}{(r+\mu+\delta_i)^2}\left[b+\frac{\lambda\beta}{r+\mu+s}\int_{p_i^*}^{p\max}(1-F(p))\,dp\right]}{1+\frac{r+\mu}{r+\mu+\delta_i}\frac{\lambda\beta}{r+\mu+s}\left(1-F\left(p_i^*\right)\right)} < 0.$$

□

Proof of lemma 2

Proof. According to $\overline{p}_i = \mathbb{E}\left(p \mid p \geq p_i^*\right)$, we have

$$\bar{p}_i = \int_{p_i^*}^{p^{max}} \frac{p}{1-F(p_i^*)} dF(p).$$

Take the first order derivative with respect to p_i^*,

$$\begin{aligned}\frac{\partial \bar{p}_i}{\partial p_i^*} &= \frac{(1-F(p_i^*))(-p_i^* f(p_i^*)) - (-f(p_i^*))\int_{p_i^*}^{p^{max}} p dF(p)}{(1-F(p_i^*))^2} \\ &= \frac{f(p_i^*)[\int_{p_i^*}^{p^{max}} p dF(p) - p_i^*(1-F(p_i^*))]}{(1-F(p_i^*))^2} \\ &= \frac{f(p_i^*)\int_{p_i^*}^{p^{max}}(p-p_i^*)dF(p)}{(1-F(p_i^*))^2} \\ &> 0.\end{aligned}$$

Therefore, the average productivity conditionally on the reservation productivity, increases with the reservation productivity. □

Proof of proposition 2

Proof. According to the definition of Mm ratio, Mm_H and Mm_L are the Mm ratio when there are only high-depreciation/low-depreciation workers in the labor market, which are

$$Mm_H = \frac{\bar{w}_H}{w_H^*}$$
$$Mm_L = \frac{\bar{w}_L}{w_L^*}.$$

The overall Mm ratio is written as

$$\begin{aligned} Mm &= \frac{\sigma\bar{w}_H + (1-\sigma)\bar{w}_L}{w_H^*} \\ &= \sigma Mm_H + (1-\sigma)\frac{\bar{w}_L}{w_H^*}. \end{aligned}$$

According to lemma 1 and 2, we have $\bar{w}_L > \bar{w}_H$ and $w_L^* > w_H^*$. Therefore,

$$\begin{aligned} Mm &= \sigma Mm_H + (1-\sigma)\frac{\bar{w}_L}{w_H^*} \\ &> \sigma Mm_H + (1-\sigma)\frac{\bar{w}_H}{w_H^*} \\ &= Mm_H. \end{aligned}$$

To compare the overall Mm ratio and Mm_L, we consider the case when $\sigma \to 0$. When $\sigma = 0$, $Mm = Mm_L$. However, if σ is a positive number that approaches to zero, the reservation wage of high-depreciation workers is still taken into account. As a result, the overall Mm ratio is

$$\begin{aligned} Mm &\to \frac{\bar{w}_L}{w_H^*} \\ &> Mm_L. \end{aligned}$$

Therefore, we are able to conclude that the heterogeneity of human capital depreciation rate generates a higher Mm ratio. □

Proof of proposition 3

Proof. According to 16, take the first order derivative with respect to σ,

$$\frac{\partial Mm}{\partial \sigma} = \frac{\bar{w}_H - \bar{w}_L}{w_H^*}.$$

Given lemma 2,

$$\frac{\partial Mm}{\partial \sigma} < 0.$$

Therefore, the overall Mm ratio decreases with the share of high-depreciation workers. □

B Wage determination

This appendix derives the wage from Nash bargaining. The wage is the solution to the following maximization problem:

$$w_i(\gamma, p) = \arg\max_{w_i(\gamma,p)} [W_i(\gamma, p) - U_i(\gamma)]^{\beta} [J_i(\gamma, p) - V]^{1-\beta}.$$

Take logs to this expression:

$$\log w_i(\gamma, p) = \arg\max_{w_i} \beta \log [W_i(\gamma, p) - U_i(\gamma)] + (1-\beta) \log [J_i(\gamma, p) - V].$$

Calculate the first order condition:

$$\frac{\beta}{W_i(\gamma, p) - U_i(\gamma)} \frac{\partial [W_i(\gamma, p) - U_i(\gamma)]}{\partial w_i(\gamma, p)} + \frac{1-\beta}{J_i(\gamma, p) - V} \frac{\partial [J_i(\gamma, p) - V]}{\partial w_i(\gamma, p)} = 0.$$

Only $W_i(\gamma, p)$ and $J_i(\gamma, p)$ depend directly on the wage, so the partial derivates are calculated as $\partial [W_i(\gamma, p) - U_i(\gamma)] / \partial w_i(\gamma, p) = 1$ and $\partial [J_i(\gamma, p) - V] / \partial w_i(\gamma, p) = -1$:

$$\frac{\beta}{W_i(\gamma, p) - U_i(\gamma)} - \frac{1-\beta}{J_i(\gamma, p) - V} = 0.$$

Using $V = 0$, this yields the first order condition in (6):

$$\beta J_i(\gamma, p) = (1-\beta)\left[W_i(\gamma, p) - U_i(\gamma)\right].$$

Multiply both sides by $(r + \mu + s)$:

$$\beta(r+\mu+s)\, J_i(\gamma, p) = (1-\beta)(r+\mu+s)\left[W_i(\gamma, p) - U_i(\gamma)\right].$$

According to the Bellman equations and the first order condition, we obtain an expression for wages:

$$w_i(\gamma, p) = \beta h_i(\gamma)\, p + (1-\beta)(r+\mu)\, U_i(\gamma).$$

To obtain (7), the term $(r+\mu)\, U_i(\gamma)$ is now rewritten by evaluating the Bellman equations for an employed worker in (2) and for a filled job in (4) at the reservation productivity $p = p^*_{i,\gamma}$. For (2), it is used that the reservation productivity implies that $W_i\left(\gamma, p^*_{i,\gamma}\right) = U_i(\gamma)$:

$$(r+\mu)\, U_i(\gamma) = w_i\left(\gamma, p^*_{i,\gamma}\right).$$

Similarly for (4), it is used that $J_i\left(\gamma, p^*_{i,\gamma}\right) = 0$ as $V = 0$:

$$h_i(\gamma)\, p^*_{i,\gamma} = w_i\left(\gamma, p^*_{i,\gamma}\right).$$

Combining these two results implies

$$(r+\mu)\, U_i(\gamma) = h_i(\gamma)\, p^*_{i,\gamma}.$$

Inserting this into the current expression for wages yields (7):

$$\begin{aligned} w_i(\gamma, p) &= \beta h_i(\gamma)\, p + (1-\beta)(r+\mu)\, U_i(\gamma) \\ &= h_i(\gamma)\left(\beta p + (1-\beta)\, p^*_{i,\gamma}\right). \end{aligned}$$

C Reservation productivity

Following the guess in the proof of proposition 1, we can substitute U_i into $p_i^* = (r+\mu)U_i$ and get

$$p_i^* = (r+\mu)\frac{b + \frac{\lambda}{r+\mu+s}[\beta \int_{p_i^*}^{p^{max}} (1-F(p))dp + p_i^*(1-F(p_i^*))]}{r+\mu+\delta_i+\lambda\frac{r+\mu}{r+\mu+s}(1-F(p_i^*))}.$$

Simplify the expression, we have

$$p_i^* = \frac{r+\mu}{r+\mu+\delta_i}\left[b + \frac{\lambda\beta}{r+\mu+s}\int_{p_i^*}^{p^{max}} (1-F(p))dp\right]$$

D Mean wages

The mean accepted wage for a group of workers i is defined as

$$\begin{aligned}
\overline{w}_i &= E\left(w_i\left(\gamma,p\right) \mid p \geq p_i^*\right) \\
&= \int_0^\infty \left(\int_{p_i^*}^{p^{\max}} \frac{w_i\left(\gamma,p\right)}{1-F\left(p_i^*\right)} dF\left(p\right)\right) dG_i^E\left(\gamma\right) \\
&= \int_0^\infty \left(\int_{p_i^*}^{p^{\max}} \frac{h_i\left(\gamma\right)\left(\beta p + (1-\beta)\, p_i^*\right)}{1-F\left(p_i^*\right)} dF\left(p\right)\right) dG_i^E\left(\gamma\right),
\end{aligned}$$

where the second equality follows from the definition of a conditional expectation and the last equality follows from (11). Calculate the inner integral:

$$\begin{aligned}
\int_{p_i^*}^{p^{\max}} \frac{h_i\left(\gamma\right)\left(\beta y + (1-\beta)\, p_i^*\right)}{1-F\left(p_i^*\right)} dF\left(y\right) &= \int_{p_i^*}^{p^{\max}} \frac{h_i\left(\gamma\right)\beta y + h_i\left(\gamma\right)(1-\beta)\, p_i^*}{1-F\left(p_i^*\right)} dF\left(y\right) \\
&= h_i\left(\gamma\right)\beta \int_{p_i^*}^{p^{\max}} \frac{p}{1-F\left(p_i^*\right)} dF\left(p\right) + \frac{h_i\left(\gamma\right)(1-\beta)\, p_i^*}{1-F\left(p_i^*\right)} \int_{p_i^*}^{p^{\max}} dF\left(p\right) \\
&= h_i\left(\gamma\right)\left(\beta\overline{p}_i + (1-\beta)\, p_i^*\right).
\end{aligned}$$

Substitute back into the expression and calculate the outer integral:

$$\begin{aligned}\int_0^\infty h_i(\gamma)\left(\beta\overline{p}_i + (1-\beta)\,p_i^*\right) dG_i^E(\gamma) &= \int_0^\infty h_i(\gamma)\left(\beta\overline{p}_i + (1-\beta)\,p_i^*\right) dG_i^E(\gamma) \\ &= \left(\beta\overline{p}_i + (1-\beta)\,p_i^*\right)\int_0^\infty h_i(\gamma)\, dG_i^E(\gamma) \\ &= \overline{h}_i\left(\beta\overline{p}_i + (1-\beta)\,p_i^*\right),\end{aligned}$$

where $\overline{h}_i = \mathbb{E}[h_i(\gamma)]$.

E Expressions with uniform distribution

The uniform distribution with support $[a, b]$ is given by

$$\begin{aligned} f(y) &= \frac{1}{b-a}, \\ F(x) &= \int_a^x \frac{1}{b-a} dy = \frac{x-a}{b-a}. \end{aligned}$$

With $a = 0$ and $b = 1$,

$$F(x) = x.$$

Use this to calculate p_i^*:

$$\begin{aligned} p_i^* &= \frac{r+\mu}{r+\mu+\delta_i}\left[\rho\overline{w} + \frac{\lambda\beta}{r+\mu+s}\int_{p_i^*}^{p\max}(1-F(y))\,dy\right] \\ &= \frac{r+\mu}{r+\mu+\delta_i}\left[\rho\overline{w} + \frac{\lambda\beta}{r+\mu+s}\frac{(1-p_i^*)^2}{2}\right]. \end{aligned}$$

Similarly, calculate $\overline{p}_i$ using integration by parts:

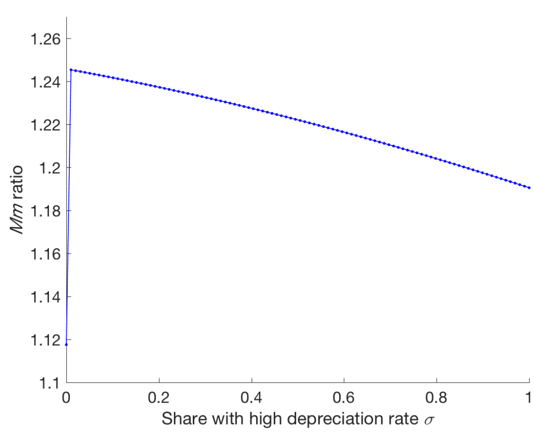
$$\begin{aligned} \overline{p}_i &= \int_{p_i^*}^{p^{\max}} \frac{y}{1-F(p_i^*)} dF(y) \\ &= \frac{1-(p_i^*)^2}{2(1-p_i^*)}. \end{aligned}$$

F Wage dispersion with constant unemployment benefits

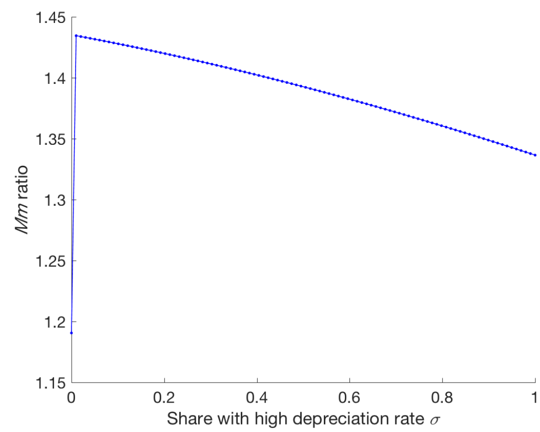
We also examine the Mm ratio with constant unemployment benefits. With constant unemployment benefits, unemployment hurts less than proportional benefits. It makes workers less worried about unemployment. Therefore, the overall Mm ratio is relatively lower than the baseline model, but the behavior with σ follows the same pattern. In this case, we set the unemployment benenfits as 0.4. Figure 2 presents the quantative results with constant unemployment benefits with the same experiments as the baseline model.

Figure 2: Mm ratio for different combinations of δ_H and δ_L

(a) Case I: $\delta_L = 0.0061$ and $\delta_H = 0.0122$



(b) Case II: $\delta_L = 0.0122$ and $\delta_H = 0.0244$



(c) Case III: $\delta_L = 0.0061$ and $\delta_H = 0.0244$