A Theory of Cultural Revivals*

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Abstract

Why do some societies have political institutions that support productively inefficient outcomes? And why does the political power of elites vested in these outcomes often increase over time, even when they are unable to block more efficient modes of production? We propose an explanation centered on the interplay between political and cultural change. We build a model in which cultural values are transmitted intergenerationally. The cultural composition of society, in turn, determines public good provision as well as the future political power of elites from different cultural groups. We characterize the equilibrium of the model and provide sufficient conditions for the emergence of *cultural revivals*. These are characterized as movements in which both the cultural composition of society as well as the political power of elites who are vested in productively inefficient outcomes grows over time. We reveal the usefulness of our framework by applying it to two case studies: the Jim Crow South and Turkey's Gülen Movement.

Keywords: institutions, cultural beliefs, cultural transmission, institutional change

JEL codes: D02, N40, N70, O33, O38, O43, Z10.

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1 Introduction

Why do some societies have political institutions that support productively inefficient outcomes? A common view in the literature is that vested interests block the adoption of new technologies (Acemoglu and Robinson 2006, 2012; Chaudhry and Garner 2006, 2007). This interpretation is useful, but its explanatory power is limited to cases in which vested interests actually have the power to block such changes. More often than not, the political power of vested interests is more limited than this. As we shall elaborate below, there are salient historical cases in which social groups vested in inefficient production modes and technologies were not able to directly block changes detrimental to their interests. Curiously, such groups were still able to maintain their social and political dominance over time, and in some cases even *came out ahead*.

In this paper, we develop a theory that explains how elites vested in inefficient economic production are able to gain political power despite the presence of more efficient modes of production which they are unable to block. We propose that when elites have limited power to directly block modes of production detrimental to their interests, they can instead influence a society's *culture*. We present a model focusing on interactions between elites and citizens: elites provide public goods, and their power to do so reflects the proportion of the citizens that share their cultural trait. The main mechanism through which public good provision operates is by affecting citizens' socialization decisions. Citizens care about the welfare of their children and thus invest more effort in transmitting a cultural trait that better aligns with public good provision. In turn, cultural changes strengthening or weakening a given cultural type lead to commensurate changes in the political power of the elites. That is, political power changes in response to cultural change.

Our model provides insight into the emergence of *cultural revivals*. We define cultural revivals as occurring when two conditions are satisfied. First, given the initial composition of the population, it is more efficient for elites to provide public goods complementary to only one sector of the economy. Second, in spite of the first feature, the political power of the elites who benefit from public good provision in the *inefficient sector* increases over time.

The primary insight underlying the existence of cultural revivals is that an economic disadvantage threatens the future political power of elites who benefit from public good provision in the inefficient sector. Hence, on the margin, these elites benefit significantly from the provisioning of public goods used by citizens that share their cultural trait, as this affects socialization decisions which in turn affect future political weights. Under some conditions, which we derive in the model, this consideration results in public good provision by the elites favoring the less efficient sector. This in turn triggers cultural evolution in the direction of that sector.

Two historical examples help motivate the model. First, how did American white planter elites maintain their political and economic power following Reconstruction? This is a puzzle: poor whites and freed blacks vastly outnumbered the white elite, and the former two groups were mired in poverty. Political changes favoring the vast majority of the (poor) population would certainly have improved the economic prospects of most Southerners. Conceivably, the white planter elites could have lost their political power to any number of groups who tried to unite poor whites and freed blacks into a voting bloc. Indeed, the Populists and Republicans attempted to create such an alliance. Yet, poor whites largely rejected such an alliance, aligning culturally and politically with the white economic elites. Public goods favoring whites (e.g., segregated schools and hospitals) were key to creating a more politically salient "white identity" that aligned much of the former Confederacy on racial, rather than economic, lines.[1] After Reconstruction, the salience of white identity enabled white elites to strengthen their grip on the Southern economy and politics. Jim Crow laws were a

[1]The type of public good investments we describe in these examples and in the model are different from conventional pork barrel spending. Whereas pork barrel spending occurs when politicians win spending concessions for their own constituents, we are describing a broader effort by political elites of the same "type" to provide targeted public goods to one part of the population. While both types of spending are targeted to keep the prevailing elites in power, the type of spending we focus on works through altering society's cultural composition in the longer run. On the other hand, pork barrel spending works through a more straight-forward "buying votes" mechanism.

manifestation of this outcome. In the parlance of our model, a cultural revival of racist values encouraged poor whites to align with white economic elites, which in turn facilitated political changes strengthening the old economic and political structures.

Turkey's Islamist Gülen Movement is another historical narrative that fits our theory well. After the Turkish Republic was founded in 1923 following the collapse of the Ottoman Empire, there followed a host of mainly top-down sociopolitical and economic reforms. These reforms were motivated by the fact that the returns to secular schooling and human capital had risen markedly following the Industrial Revolution, whereas the economic productivity of the more established conservative Ottoman culture had long been stagnant. These reforms were politically and economically empowering to a new group of secular elites. Thus, throughout its nearly seven decade nascent existence, the Turkish Republic adhered to its French-style "laicité" whereby the state hierarchy was fully under the control of secular elites. Nonetheless, this new regime did not tip the political balance of power completely in one group's favor given the culturally more conservative leanings of Turkish society. With a succession of elections starting in 1995, Turkish Islamists were able to firmly regain and consolidate their political power. This revival was a manifestation of deep-rooted cultural change spearheaded by investment in public goods. The seeds of this cultural revival were sown at the end of Turkey's single party era in 1950, when Islamist groups ratcheted up their social and political activism. At the forefront of this movement was Fethullah Gülen, a religious cleric who mainly focused on establishing K-12 schools which stressed the importance of quality education with an emphasis on science and math proficiency. In the half century starting in the mid-1960s, the growth in Gülenist schools was remarkable. Gülenist supporters and their associated Islamist culture became more prominent in Turkey after the 1990s. In line with the theory we present below, this subsequently culminated in a shift in the balance of political power from the seculars to the Islamists in Turkey.

Our model departs from—and adds to—the standard political economy explanation of institutional calcification, in which stagnation occurs when it is in the interest of the politically powerful for the status quo to prevail (Acemoglu 2003; Acemoglu and Robinson 2006, 2012). This view is rooted in North's (1990) idea that a society's *formal institutions*—those political, legal, social, and economic mechanisms that establish the formal "rules of the game" and the incentives faced by the players therein—are the key drivers of economic and political outcomes. The "formal institutions of political economy" view clearly explains many cases of economic and institutional stagnation. We complement this literature, as our theory implies two key insights that cannot be explained by the "formal institutions of political economy" view. First, we demonstrate that political changes which move society away from productively efficient outcomes can be triggered *despite* vested interests lacking the capacity to directly enact such changes. Second, we provide an explanation as to why cultural changes are so often linked to economic stagnation.

This paper is not the first in economics to suggest an interaction between cultural and political change.[2] In a closely related work, Bénabou, Ticchi and Vindigni (2021) develop a theory on the role that religion can play in preventing scientific progress. Religion often plays a role in cultural revivals, as we conceptualize them, because the grasp religious elites tend to have on "eternal truths" often means that new, more productive ways of doing things upset the status quo in which they are powerful. In this light, Bénabou, Ticchi and Vindigni (2021) focus on the threat that certain technologies pose to religious beliefs and how this interacts with the political structure. By contrast, we are interested in the interaction between the cultural composition of society and political power. Hence, the two views are highly complementary and help explain different, although related, phenomena. More broadly, our theory can explain why not just religious, but also secular values can be leveraged by elites in order to facilitate political change.

[2]For key insights and overviews of recent developments of various aspects of this literature, see Bisin and Verdier (2000*b*), Bénabou and Tirole (2006), Guiso, Sapienza and Zingales (2006), Nunn (2012), Spolaore and Wacziarg (2013), Ticchi, Verdier and Vindigni (2013), Algan and Cahuc (2014), Alesina and Giuliano (2015), Bénabou, Ticchi and Vindigni (2015), Bisin and Verdier (2017), and Bisin, Seror and Verdier (2019).

Our paper is related to the theoretical literature on religious and cultural leaders.[3] Hauk and Mueller (2015) present a model in which individuals transmit their cultural norms and elites seek to spread their culture. They do so by interpreting cultural aspects of their own and other cultures. Our model is close to Hauk and Mueller (2015), as we also advance a theory whereby culture is transmitted intergenerationally à la Bisin and Verdier (2001) and elites affect the incentive of parents to pass their cultural values to their offspring.[4] We complement the related literature in two ways. First, existing works consider elites that are not constrained in their ability to affect the cultural composition. By contrast, we incorporate our model of cultural transmission within a broader political economy theory where the elites' ability to affect the cultural composition reflects their political power. We further model how the political power of elites changes over time in response to cultural change. Second, the existing literature typically focuses on the effect of leaders on cultural diversity (Prummer 2019).[5] We study the effect of elites on economic outcomes, and thereby connect as well to the growing empirical literature on the effect of leaders on economic growth (Jones and Olken 2005; Yao and Zhang 2015; George and Ponattu 2020; Ferraz, Finan and Martinez-Bravo 2020).

The rest of our paper proceeds as follows. Section 2 lays out the model. Section 3 elaborates further on the formal analysis to provide an interpretation for the existence of cultural revivals in history, and Section 4 offers some concluding thoughts.

[3]See, among others, Hauk and Mueller (2015), Verdier and Zenou (2015), Carvalho, Koyama and Sacks (2017), Prummer and Siedlarek (2017), Verdier and Zenou (2018), Carvalho and Sacks (2021), Prummer (2019), and Almagro and Andrés-Cerezo (2020).

[4]For similar models, see Verdier and Zenou (2018) and Almagro and Andrés-Cerezo (2020).

[5]A notable exception is Almagro and Andrés-Cerezo (2020), who study the rise of national identities.

2 The Model

2.1 Setup

We describe the model here first before laying it out formally below. We consider a two-period model with two types of agents: elites and citizens. The productivity of (adult) citizens is a function of their cultural type and public good provision by the elites. The political power of the elites, which determines their capacity to provide their preferred public good, is a function of the share of the adult citizenry sharing their cultural trait. In the first period (hereafter, period 0), elites choose the level of public good provision. Then, adult citizens produce and socialize their offspring to their cultural type. The cultural types of the children are then realized. In the second period (hereafter, period 1), parents die and children become adults and the cultural profile of the latter determines the political power of elites. Elites then choose the level of public good provision in period 1, adult citizens produce, and the game ends. We employ the Subgame Perfect Nash Equilibrium concept and solve using backward induction.

2.1.1 The Citizens

There are a continuum of adult citizens in period zero, each with one child. Citizens belong to one of two cultural types, which we label type 1 and type 2. The cultural types complement the two types of public goods, $\{g_t^1, g_t^2\} \in [0,1]$, in the production process, where $t \in \{0,1\}$ represents the period.

The utility of adult citizens of type $i \in \{1,2\}$ in period t can expressed as:

$$U^i(g_t^i) = (\eta + \phi^i) g_t^i, \tag{1}$$

for $i \in \{1,2\}$. The utility of adult citizens of type i depends on the provision of public good g_t^i in two ways. First, adult citizens consume public good g_t^i and receive a linear utility ηg_t^i, with $\eta \in [0, \frac{1}{4}]$.[6] Second, adult citizens produce using public good g_t^i and get additional utility based on their production.[7] We take production of adult citizen of type i as $\phi^i g_t^i$.[8] The parameter $\phi^i \in [0, \frac{1}{2}]$ represents a fixed marginal productivity of public good i for adult citizens of type i.[9]

In period t, fraction $q_t \in [0, 1]$ of the adult citizens are of type 1 and fraction $1 - q_t$ are of type 2. q_0 is exogenous, but q_1 evolves according to dynamics we describe below. We assume that $q_0 \in (0, 1)$, so that both cultural groups are initially present.

Cultural Dynamics: The only decision adult citizens make is investment in socialization of their children in period 0. Following Bisin and Verdier (2001), we model the transmission of cultural values as a mechanism which interacts intergenerational socialization and socialization by society. Intergenerational socialization to type i occurs with probability τ^i, the effort of the parent. If direct intergenerational socialization fails, the child receives its cultural trait through horizontal or oblique transmission (i.e., via peers or other adults such as teachers). This occurs with probability equaling the trait's share in the population.

[6]The assumption $\eta \leq \frac{1}{4}$ ensures that there exists an interior solution for the parent's optimization problem, as explained below.

[7]The assumption of parent's deriving utility from their production is tantamount to parents earning a piece-rate wage or keeping their after-tax production. Since we are not concerned with wages or taxes in the present model, we have chosen for the sake of parsimony to ignore these considerations. For a theory of cultural evolution that accounts for production and taxation, see, for example, Bisin et al. (2021).

[8]This is a simplified version of a model in which citizens provide effort, and the cost of effort, $d(e_t^i)$, is a function of ϕ^i and g_t^i. For instance, setting utility such that $U^i(e_t^i) = \eta g_t^i + e_t^i - \frac{(e_t^i)^2}{2\phi^i g_t^i}$ would yield indirect utility (at optimization) of $(\frac{1}{2}\phi^i + \eta) g_t^i$ (for $g_t^i > 0$), which is similar to (1).

[9]As we explain further below, the assumption $\phi^i \leq \frac{1}{2}$ ensures an interior solution for the parent's optimization problem.

Let P^{ij} denote the probability that the child of a citizen of type $i \in \{1, 2\}$ is socialized to type $j \in \{1, 2\}$. We can express P^{ij} for any $i, j \in \{1, 2\}$ as:

$$\begin{aligned} P^{11} =& \tau^1 + (1 - \tau^1)q_0 \\ P^{12} =& (1 - \tau^1)(1 - q_0) \\ P^{22} =& \tau^2 + (1 - \tau^2)(1 - q_0) \\ P^{21} =& (1 - \tau^2)q_0. \end{aligned} \tag{2}$$

As an illustration, the probability that a child from a type 1 parent is socialized to type 1 is equal to the sum of the probability that direct socialization succeeds, τ^1, and of horizontal transmission by a peer of type 1, $(1 - \tau^1)q_0$. Assuming that transmission efforts are symmetric, we can express q_1 (the fraction of adult citizens of type 1 in period 1) as:[10]

$$q_1 = q_0 + q_0(1 - q_0)(\tau^1 - \tau^2). \tag{3}$$

The Citizens' Optimization Problem: Parents are forward-looking, and their time preference is set to 1 for simplicity. We assume that parents have *imperfect empathy* towards their offspring. This is a form of altruism where parents evaluate their children's utility using their own preferences.

Let U_t^{ij} denote the utility of a child of type j in period t, as perceived by a parent of type i, for $i, j \in \{1, 2\}$. In period 0, children consume the public goods but do not produce. Under the imperfect empathy assumption,

$$U_0^{ii} = \eta g_0^i \qquad \text{and} \qquad U_0^{ij} = 0 \text{ when } j \neq i. \tag{4}$$

[10]The symmetry assumption of the transmission efforts is a common feature of the studies in the related literature (Bisin and Verdier 2000*b*,*a*, 2001; Tabellini 2008; Hauk and Mueller 2015). Equation (3) follows from the fact that there are a proportion $(1 - q_0)P^{21}$ children of type 2 parents socialized by peers of type 1, and there are a proportion q_0P^{12} children of type 1 parents socialized by peers of type 2. We can therefore write $q_1 = q_0 + (1 - q_0)P^{21} - q_0P^{12}$. Substituting P^{21} and P^{12} from (2), we derive (3).

According to (4), if the child is socialized to cultural type i, a parent of type i perceives that the child gets utility ηg_0^i given that the child consumes the public good g_0^i but does not produce. By contrast, if the child is socialized to cultural type $j \neq i$, then a parent of type i perceives that the child gets no utility, given that the child does not consume public good g_0^i.[11]

In period 1, children become adults, produce and consume the public goods. Hence, as perceived by a parent of type i, the utility of her child is

$$U_1^{ii} = (\eta + \phi^i) g_0^i \qquad \text{and} \qquad U_1^{ij} = 0 \text{ when } j \neq i. \tag{5}$$

The inequality $U_0^{ii} + U_1^{ii} \geq U_0^{ij} + U_1^{ij} = 0$ is always satisfied, so parents have incentive to socialize their children to their own cultural trait.

Now, let $c(\tau^i)$ denote the socialization cost, where τ^i is the probability of direct socialization to type i. Since the value of parental socialization is orthogonal to the parent's own utility represented in (1),[12] the optimization problem faced by a parent of type i in period 0 can be written as:

$$\max_{\tau^i \in [0,1]} P^{ii}(U_0^{ii} + U_1^{ii}) + P^{ij}(U_0^{ij} + U_1^{ij}) - c(\tau^i), \tag{6}$$

with P^{ij} given by (2), U_0^{ij} by (4) and U_1^{ij} by (5) for $i, j \in \{1, 2\}$. We assume that $c(\tau^i) = \frac{1}{2}(\tau^i)^2$ for simplicity.

[11]The imperfect empathy assumption simplifies the model. Our results are robust to weaker assumptions, as long as the parents derive more utility from having a child belonging to their own type. For a related theoretical application of the imperfect empathy concept to public good consumption, see Bisin and Verdier (2000*b*). Bisin and Verdier (2011) provide a review of the related literature.

[12]Based on the formulation in (6), we abstract from the parent's own utility from production in period 0. One could easily incorporate this, however, by adding a parameter of "altrusim" which would gauge the weight of the child's utility relative to that of the parent. Doing so would not impact the qualitative nature of our key results. For a similar specification of the parent's optimization problem, see, for example, Bisin and Verdier (2000*b*, 2001) and Hauk and Mueller (2015).

2.1.2 The Elites

Elites also derive utility from public goods 1 and 2. We therefore denote them as either type 1 or type 2, depending on whether they derive utility from public good 1 or public good 2. The utility of elites of type $i \in \{1, 2\}$ in period t can be expressed as:

$$V^i(g_t^i) = \log(\phi^i g_t^i), \tag{7}$$

for $t \in \{0, 1\}$. The log specification is taken for simplicity and ensures the concavity of the utility function $V^i(.)$.[13]

The primary idea behind (7) is that elites have a vested interest in the provision of a particular type of public good. For instance, merchants desire protection of property rights as well as transport infrastructure (North 1981; Acemoglu and Robinson 2012), military elites desire spending on defense (Tilly 1990; Hoffman 2015), religious authorities advocate for spending on religious infrastructure and education (possibly to the detriment of spending on secular public education; see Gill (1998), Coşgel and Miceli (2009), Chaudhary and Rubin (2016), and Rubin (2017)), and elites in declining industries may push for subsidies or tariffs to revitalize their industry (e.g., coal mining in the United States).

Public Good Provision by the Elites: In each period t, elites choose the allocation of the two public goods g_t^1 and g_t^2. We normalize the resources available to the elite to 1 in both periods, so $g_t^1 + g_t^2 \leq 1$ for $t \in \{0, 1\}$.

We conceptualize the elites as having different weights when it comes to the allocation of resources for public provision. As a matter of simplicity, we assume a one-to-one mapping from the adult population shares to the weights of the elites in their intra-temporal utility. The weights of the elites therefore reflect the cultural composition, so provision decisions

[13] It is important that the utility function of the elites is *different* from that of the citizenry. This ensures that the elites make self-interested decisions that are not completely aligned with the interests of citizens of their own type.

represent adult citizens' preferences.[14] In reality, these weights are presumably influenced by the composition of the citizenry, the inclusiveness of political institutions, the likelihood of social unrest, and various other dimensions that mediate the effect of the cultural composition of the citizenry on public good provision.[15] We employ this allocation mechanism to "stack the deck" against frictions or non-representative political power being the root cause of public good provision and cultural change.

In other words, the political process results in an allocation (g_t^1, g_t^2), for $t \in \{0, 1\}$, that maximizes the weighted discounted utility of the elites under the constraint $g_t^1 + g_t^2 \leq 1$. In period 1, the allocation mechanism maximizes:

$$W_1(g_1^1, g_1^2) = q_1 V^1(g_1^1) + (1 - q_1) V^2(g_1^2). \tag{8}$$

In period 0, denoting $\beta \in [0, 1]$ the time preferences of the elites, the allocation mechanism maximizes:

$$W_0(g_0^1, g_0^2) = q_0 V^1(g_0^1) + (1 - q_0) V^2(g_0^2) + \beta \max_{g_1^1, g_1^2} W_1(g_1^1, g_1^2), \tag{9}$$

given that the constraints $g_t^1 + g_t^2 \leq 1$ are satisfied, for $t \in \{0, 1\}$, and the elites internalize the dynamics of cultural change (3).

2.1.3 Timeline and Solution Concept

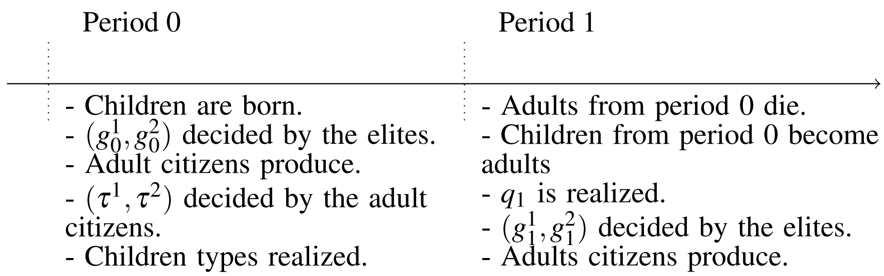
The timeline of the model is summarized in Figure 1. At the beginning of period 0, children are born. The provision of the two public goods g_0^1 and g_0^2 in period 0 is then decided via optimization of (9) and adults produce. Adults then choose their intergenerational socialization efforts τ^1 and τ^2 via optimization of (6). Socialization then occurs and the types of the children are realized. At the beginning of period 1, the children become adults,

[14] More broadly, as long as there is a positive relationship between the weights of the elites and the cultural composition, then the results established in this paper remain robust, as demonstrated in the working paper version Iyigun, Rubin and Seror (2019).

[15] On how social unrest affect policies, see, for instance, Passarelli and Tabellini (2017) and Almagro and Andrés-Cerezo (2020). The weights of the elites may also change so as to reflect social efficiency. See, for instance, Bisin and Verdier (2017) and Bisin et al. (2021).

the parents die, and q_1 is realized.[16] The provision of the public goods (g_1^1, g_1^2) in period 1 is then decided via the optimization of the allocation mechanism in (8). The adults then produce and the game concludes.

Figure 1: Timeline



Our solution concept is Subgame Perfect Nash Equilibrium (SPE). A SPE consists of the optimal provision scheme in both periods and the intergenerational cultural transmission efforts. Accordingly, the SPE will be denoted $\{(g_0^{1*}, g_0^{2*}); (g_1^{1*}, g_1^{2*}); (\tau^{1*}, \tau^{2*})\}$ in the remainder of the paper.

2.2 Solution

We solve the model via backward induction. First, we solve for the provision of public goods in period 1. We proceed to solve for the socialization efforts of the citizens as well as public good provision in period 0.

2.2.1 Period 1

The public good allocation mechanism chooses (g_1^1, g_1^2) to optimize (8) in period 1 under the constraint $g_1^1 + g_1^2 \leq 1$. Taking the first-order conditions, we find that:

$$g_1^{1*} = q_1 \text{ and } g_1^{2*} = 1 - q_1, \tag{10}$$

[16] We assume that the old generation is entirely replaced by the new one for simplicity, although such an assumption could be relaxed. For example, in a closely related model of cultural transmission, Hauk and Mueller (2015) assume an overlapping structure where a Poisson birth and death process keeps the population size constant.

with q_1 given by (3). The optimal provision scheme in period 1 perfectly reflects population shares. An increase in the fraction of individuals of type 1 implies a commensurate increase in the weight of type 1 elites and the optimal provision of public good 1.

2.2.2 Period 0

In period 0, the citizens solve (6) for the optimal socialization efforts. We find that

$$\tau^{1*} = (1 - q_0)(\eta g_0^1 + (\eta + \phi^1)g_1^{1*}) \qquad \text{and} \qquad \tau^{2*} = q_0(\eta g_0^2 + (\eta + \phi^2)g_1^{2*}), \tag{11}$$

with g_1^{i*} given by (10) for $i \in \{1, 2\}$. Since $\phi^i \leq 1/2$, $\eta \leq 1/4$ and $g_t^i \leq 1$, the optimization problem faced by adults always admits a solution $\tau^{i*} \leq 1$, $i \in \{1, 2\}$. This is consistent with our probabilistic interpretation of this parameter. Furthermore, as in Bisin and Verdier (2001), there is a substitution between vertical and horizontal socialization mechanisms. All else equal, when the initial fraction of individuals of type i increases, citizens of type i invest less effort in socializing their offspring. Likewise, when the initial fraction of individuals of type i increases, the likelihood that their offspring will switch their cultural affiliation through horizontal socialization decreases.

Using (3), we now establish the following result:

Remark 1 *q_1 is the unique solution of the fixed point equation*

$$q_1 = q_0 + q_0(1 - q_0)(\tau^{1*} - \tau^{2*}), \tag{12}$$

with τ^{i*} given by (11) for $i \in \{1, 2\}$.

Proof: See Appendix A.1.

Parents' socialization efforts affect the cultural composition in period 1. However, socialization decisions depend on the optimal provision of the public goods in period 1, which is a function of the cultural composition in that period. Hence, q_1 solves a fixed point equation. We find in the Appendix that this fixed point equation admits a unique interior solution.

As q_1 solves a fixed point equation, there is a multiplier effect in the socialization decision. All else equal, an increase in the provision of good 1 in the first period, g_0^1, leads to higher socialization efforts by citizens of type 1, given (11). In turn, adult citizens expect an increase in the fraction of citizens of type 1 in period 1. Therefore, they expect a higher provision of public good 1 and a lower provision of good 2 in period 1, as $g_1^{1*} = q_1$ and $g_1^{2*} = 1 - q_1$. Adult citizens of type 1 therefore invest even more in socializing their offspring, and adult citizens of type 2 invest less. Marginal changes in the provision of the public goods in the initial period can thus have substantial effects on socialization efforts and the evolution of the weights of the elites.

The optimal allocation of the public goods (g_0^{1*}, g_0^{2*}) maximizes (9) under the constraints $g_t^1 + g_t^2 \leq 1$ for $t \in \{0, 1\}$, and given the dynamics of cultural change (3). As demonstrated in the Appendix, the optimal allocation of the public goods in period 0 is characterized by the following first-order condition:

$$\frac{\partial W_0}{\partial g_0^1} = \frac{q_0}{g_0^1} - \frac{1 - q_0}{1 - g_0^1} + \beta \frac{\partial q_1}{\partial g_0^1} \log\left(\frac{\phi^1 q_1}{\phi^2 (1 - q_1)}\right) = 0. \tag{13}$$

Only one first-order condition is sufficient to find the optimal allocation of the public goods in period 0 because the budget constraint $g_0^1 + g_0^2 \leq 1$ is necessarily satisfied at equality in equilibrium. The first two terms on the RHS of equation (13) describe the trade-off in period 0 between allocating resources to either good 1 or good 2, given the initial weights of the elites. The third term gives the effect of a marginal increase in the provision of good 1 on the weighted utility of the elites in period 1. Since $\frac{\partial q_1}{\partial g_0^1} > 0$, an increase in the provision of public good 1 in period 0 shifts the citizens' socialization decisions, and it affects the cultural composition in period 1 and the period 1 weights of the elites. The elites thus internalize the effect of the initial provision of the public goods on the future cultural composition.

2.2.3 Characterization of Subgame Perfect Equilibria

We establish the following result in the Appendix:

Proposition 1 *There exists a threshold $\tilde{\beta} > 0$ such that:*

- *If $\beta \leq \tilde{\beta}$, there exists a unique stable SPE.*
- *If $\beta > \tilde{\beta}$, there exists two thresholds $\underline{q}$ and $\overline{q}$ in $(0,1)$ with $\underline{q} < \overline{q}$ such that:*
  - *If $q_0 \in [\underline{q}, \overline{q}]$, there exists two stable SPE.*
  - *If $q_0 \notin [\underline{q}, \overline{q}]$, there exists a unique stable SPE.*

Proof: See Appendix A.2.

The intuition behind Proposition 1 is that there can be increasing marginal returns to provisioning public goods. Incremental changes in the initial provision of public goods can lead to increasingly higher future utility levels for the elites by increasing their weights in provision decisions and shifting the cultural composition. This non-convexity can formally be observed in the first-order condition given in (13). The marginal benefit of increasing the provision of good 1 in period 0 is proportional to the period-1 relative utility of the elites of type 1, which necessarily increases with g_0^1, as $\frac{\partial}{\partial g_0^1} \log[\frac{\phi^1 q_1}{\phi^2(1-q_1)}] > 0$.

Since the non-convexity arises from the inter-temporal concerns of the elites, its magnitude is related to the time preferences of the elites. When β is lower than the threshold $\tilde{\beta}$, the elites' concern for the future is minimal. It follows that the non-convexity does not meaningfully affect the period-0 optimization problem, so there is a unique stable SPE.

By contrast, when the time preferences of the elites are such that $\beta > \tilde{\beta}$, then the elites care enough about the future that the non-convexity substantially affects their period-0 decision problem. When the initial fraction of individuals of type 1, q_0, has intermediate values, there are two solutions to the optimization problem faced by the elites in period 0. This is because the multiplier effect in socialization decisions is strong, so it is conceivable that *either* type of elite could increase its future weight in the provision decision of period 1 if enough individuals adopt their cultural type. The optimization problem faced by the elites in period 0 therefore admits two stable solutions. In both cases, the elites are able to affect citizens' socialization decisions in order to shift the cultural composition in their favor. Alternatively, when one cultural type has a clear initial majority (i.e., q_0 is above $\overline{q}$ or below $\underline{q}$), then the extent of cultural change is limited. In such cases, it is too costly for the elites from the minority group to affect the trajectory of culture in order to increase their future weight in provision decisions. Consequently, the optimization problem faced by the elites in period 0 admits a single stable solution.

2.3 Cultural Revivals

We now extend our model to account for the type of cultural revivals highlighted in the introduction. In the case of the postbellum South, white elites faced a serious threat to their political and economic power following emancipation. Poor whites and freedmen could have joined forces to improve their economic and political power, both of which were previously almost non-existent. In order to prevent this from happening, white elites funneled resources into public goods used by whites (e.g., white-only schools), which increased the returns to a racist cultural ideology. Ultimately, the equilibrium that was reached was one in which racist policies and cultural ideologies reinforced each other, and poor whites largely aligned with the wealthy elite. Importantly, this outcome arose *in spite of* the fact that a political and economic alliance with African-Americans would have likely improved labor market outcomes for poor whites (see Section 3.1). Likewise, the nascent Turkish Republic's strictly secular reforms and principles were not enough to block conservative Islamists from reasserting their influence in Turkish society. With a succession of elections starting in 1995, Islamists in Turkey were able to firmly consolidate their political power. This was a manifestation of deep-rooted cultural change spearheaded by investment in public goods (see Section 3.2).

One commonality unites these examples. Cultural change induced institutional change *despite* these changes entailing a movement away from what might have been more efficient outcomes.

How do we relate these insights to the model? In particular, what do we mean by "more efficient" outcomes? We address this issue by introducing the notion of *dynamic production efficiency* (DPE). Accordingly, we consider a public good allocation to be dynamic production efficient if, given the initial cultural profile of society (q_0), the period-0 allocation maximizes production across both periods and the period-1 allocation maximizes production in that period. This conceptualization is important for our understanding of cultural revivals, because we are seeking to understand the conditions under which cultural change triggers institutional change *despite* these changes being (dynamically) production inefficient.

We operationalize these insights in the context of our model by first formalizing dynamic production efficiency. To that end, we can express the aggregate production of the citizens in period $t \in \{0, 1\}$ as follows:

$$p_t(g_t^1, g_t^2) = q_t \phi^1 g_t^1 + (1 - q_t)\phi^2 g_t^2. \tag{14}$$

Denoting the DPE allocation of public goods as $\{(g_0^{1,DPE}, g_0^{2,DPE}), (g_1^{1,DPE}, g_1^{2,DPE})\}$, we can now formally introduce the concept of *Dynamic Production Efficiency (DPE):*

Definition 1 *The production of public goods* $\{(g_0^1, g_0^2), (g_1^1, g_1^2)\}$ *is* ***dynamically production efficient*** *when:*

$$(g_1^{1,DPE}, g_1^{2,DPE}) = \arg\max_{(g_1^1, g_1^2)} p_1(g_1^1, g_1^2) \tag{15}$$

under the constraint $g_1^1 + g_1^2 \leq 1$*, and*

$$(g_0^{1,DPE}, g_0^{2,DPE}) = \arg\max_{(g_0^1, g_0^2)} p_0(g_0^1, g_0^2) + \beta p_1(g_1^{1,DPE}, g_1^{2,DPE}), \tag{16}$$

given that $g_0^1 + g_0^2 \leq 1$, *the dynamics of cultural change (12) are internalized, and* $(g_1^{1,DPE}, g_1^{2,DPE})$ *maximizes production in period* 1.

Hence, if the elites were to maximize the citizens' production, they would choose a provision scheme $\{(g_0^{1,DPE}, g_0^{2,DPE}), (g_1^{1,DPE}, g_1^{2,DPE})\}$. We can now characterize dynamic efficient production as follows:

Proposition 2 *There exists a threshold* $\overline{\beta}^{DPE} > 0$ *and a threshold* $\tilde{q}^{DPE} \in [0,1]$ *such that if* $\beta > \overline{\beta}^{DPE}$, *there is a unique efficient dynamic production path such that*

- *if* $q_0 \geq q^{DPE}$, *it is dynamically efficient to produce good* 1,

$$g_0^{1,DPE} = g_1^{1,DPE} = 1 \text{ and } g_0^{2,DPE} = g_1^{2,DPE} = 0. \quad (17)$$

- *if* $q_0 < q^{DPE}$, *it is dynamically efficient to produce good* 2,

$$g_0^{1,DPE} = g_1^{1,DPE} = 0 \text{ and } g_0^{2,DPE} = g_1^{2,DPE} = 1. \quad (18)$$

- q^{DPE} *is non-decreasing in* β *when* $\phi^2 > \phi^1$, *and non-increasing in* β *otherwise.*

Proof: See Appendix A.3.

From Proposition 2, which good will be produced along the dynamic efficient path depends on the initial cultural composition. If type 1 adults are initially sufficiently numerous (i.e. $q_0 > \tilde{q}^{DPE}$), then only good 1 will be the efficient one to produce in the two periods. Conversely, when adults of type 1 are not initially numerous (i.e., $q_0 < \tilde{q}^{DPE}$), then only good 2 will be the efficient one to produce.[17]

While we derive the full characterization of the dynamic efficient production in the Appendix, we restrict our attention in Proposition 2 and in the rest of the paper to the parameter values such that $\beta > \overline{\beta}^{DPE}$.[18] By doing so, we abstract from the less interesting cases where

[17]In the proof of Proposition 2, we assume that when $\tilde{q}^{DPE} = q_0$, the default option is to produce good 2.

[18]A formal characterization of $\overline{\beta}^{DPE}$ is provided in the proof of Proposition 2.

production in period 0 and the ensuing evolution of cultural norms do not affect efficient production in period 1. When $\beta > \overline{\beta}^{DPE}$, the dynamic efficient production is *path dependent*. When only good 1 is produced in period 0, then the fraction of individuals of type 1 increases sufficiently between the two periods so that producing good 1 remains efficient in period 1. Conversely, if only good 2 is produced initially, then the fraction of type-2 individuals increases and it remains optimal to produce good 2 in period 1. Intuitively, when the time preferences are sufficiently large, the evolution of cultural norms and the period-1 allocation substantially affect the optimization problem in period 0.

With the definition of DPE and Proposition 2 in hand, we can now turn to cultural revivals. As we noted before, cultural revivals have two features: i) given the initial cultural composition, it is dynamically efficient to produce one public good; ii) the cultural type and the elites associated with the *productionally inefficient* sector becomes predominant. The first condition entails that the share in the population of one type and the political weight of the corresponding elite are initially low enough that it is dynamically inefficient to produce the public good favored by that elite. This is important, because we are not interested in the case in which cultural and institutional change is either efficient or driven by vested interests.

Moreover, we are not interested in the case in which the cultural and political profile moves in the direction of the type with higher marginal productivity. While our model can account for such movements, there are other explanations for such an outcome that we cannot rule out. These include external influence (i.e., a country will fall behind if it does not adopt the cutting-edge production method) and externalities (i.e., adopting a more productive 'type' gives a society access to other, unforeseeable windfalls). We do not deny the importance of such influences. However, this is not the phenomenon we are interested in, nor is it salient for our motivating examples. Rather, we are interested in the case in which the sector with *lower marginal productivity* becomes more predominant over time. For this reason, for the remainder of the paper we assume without loss of generality that $\phi^1 < \phi^2$.

We therefore focus on cultural revivals favoring type 1, since economic activities performed by this type have lower marginal productivity.

We can therefore define cultural revivals in the context of our model as follows:

Definition 2 *A* ***cultural revival*** *in favor of type* 1 *occurs in a SPE when the following conditions are satisfied:*

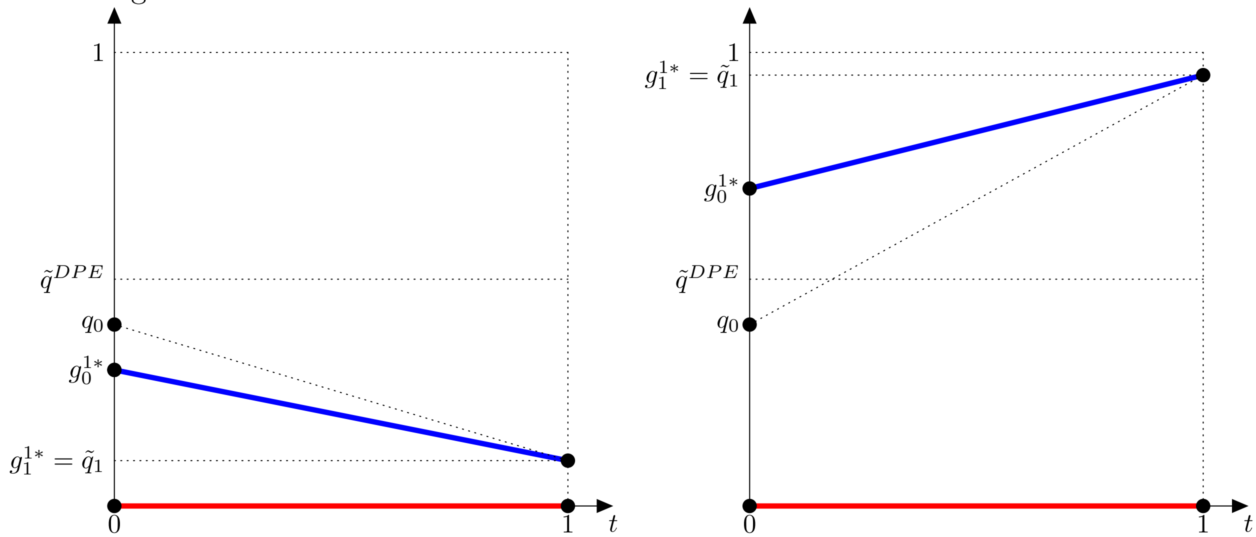
i) $q_0 < \tilde{q}^{DPE}$, *and*

ii) $q_1 > q_0$.

The first condition above, $q_0 < \tilde{q}^{DPE}$, implies (via Proposition 2) that it is dynamically efficient to produce good 2 in both periods of the game. However, the second condition, $q_1 > q_0$, implies that despite type 2 individuals being more marginally productive (since $\phi^1 < \phi^2$) and public good provision associated with them being more production efficient in the two periods, the fraction of type 1 individuals *increases* between periods. Furthermore, the weight of the elites of type 1 in provision decisions *increases* in a cultural revival, while it would have decreased if the provision of the public good was close to the DPE.

The effect of a cultural revival on the equilibrium of the game is illustrated in Figure 2. The red line represents the dynamically efficient production path. Since $q_0 < \tilde{q}^{DPE}$, it is dynamically efficient to provision good 2 in the two periods of the game (i.e., $g_0^{1*} = g_1^{1*} = 0$, by Proposition 2). The left panel illustrates a typical case where there is no revival in the SPE. The period-0 allocation $(g_0^{1*}, 1 - g_0^{1*})$ is close to the efficient provision in period zero, $(g_0^{1,DPE} = 0, g_0^{2,DPE} = 1)$ and the share of citizens of type 1 decreases over time. The equilibrium production represented by the blue line converges towards the dynamic efficient path. The right panel of the figure illustrates an equilibrium where there is a cultural revival. The period-0 allocation $(g_0^{1*}, 1 - g_0^{1*})$ is far from the dynamic efficient production. The high provision of good 1 in period 0 triggers significant cultural changes and the fraction of citizens of type 1, q_1, increases. Since $g_1^{1*} = q_1$, equilibrium production diverges from the dynamic efficient production path.

Figure 2: Period-0 Allocation in a SPE with and without a Cultural Revival



a) No Cultural Revival **b) Cultural Revival**

Note: the red line represents the DPE path and the blue line represents equilibrium production.

Given our characterization of the SPE of this model, we are thus able to derive sufficient conditions under which cultural revivals emerge in at least one SPE:

Proposition 3 ***Cultural Revivals:*** *Assuming that $\beta > \overline{\beta}^{DPE}$, there exists a threshold $\overline{\beta}$ and a threshold $\tilde{q}_0 < \tilde{q}^{DPE}$ such that there is a cultural revival favoring type 1 in at least one SPE if $\tilde{q}_0 < q_0 < \tilde{q}^{DPE}$ and $\beta > \overline{\beta}$.*

Proof: See Appendix A.4.

Although the result of Proposition 3 is related to cultural revivals favoring type 1, by symmetry, a similar result can be established for cultural revivals favoring type 2 for $\phi^1 > \phi^2$. The assumption that $\beta > \overline{\beta}^{DPE}$ ensures that the dynamic efficient production is characterized by Proposition 2.

The intuition associated with Proposition 3 is that when $\phi_1 < \phi_2$, elites of type 1 face a marginal productivity disadvantage that threatens their future weight in provision decisions. Hence, type 1 elites receive a particularly high marginal benefit, relative to type 2 elites, from citizens' socialization decisions when good 1 is over-provided. Formally, $\frac{\partial^2 q_1}{\partial \phi^2 \partial g_0^1} > 0$, meaning that when ϕ^2 is large, so is the marginal (positive) effect of provisioning good 1 on the fraction of citizens of type 1, $\frac{\partial q_1}{\partial g_0^1}$.

We find that type 1 elites can still thrive over time, despite good 1 being dynamically inefficient to produce (i.e. $q_0 < \tilde{q}^{DPE}$) and good 1 yielding lower marginal production (i.e. $\phi^2 > \phi^1$). In order for this to happen, two conditions must be met. First, there must be a sufficiently large fraction of individuals of type 1, (i.e. $q_0 > \tilde{q}_0$). It must be conceivable for the elites of type 1 to increase their future weight in provision decisions by affecting socialization decisions. This necessitates a sufficiently high population of type 1 individuals. Second, q_0 must be less than $\tilde{q}^{DPE}$ from the definition of cultural revivals.

Finally, the time preference parameter β must be sufficiently large (i.e. $\beta > \overline{\beta}$). If β is too small, the elites have a limited effect on the evolution of the cultural composition. The evolution of the cultural composition is rather driven by citizens' socialization decisions, which reflect the economic conditions of their children. Hence, if good 1 is less efficient to produce than good 2, the citizens of type 2 tend to invest higher socialization efforts than their peers of type 1. The fraction of citizens of type 2 increases, and the elites provisioning good 2 thrive. Conversely, if $\beta > \overline{\beta}$, the elites of type 1 care enough about the future to shift the cultural composition in their favor and can set in motion a cultural revival.

To summarize, despite the fact that elites cannot affect structural differences by force or directly alter politics to their benefit, they can still leverage the resources at their disposal to change the prevailing cultural norms and, ultimately, their future political power. We have demonstrated that even when providing public goods to a cultural type is inefficient and is associated with a productivity disadvantage, cultural and political change favoring that type is still possible. By influencing the course of cultural change, self-interested elites that provision inefficient public goods might still be able to thrive.

3 Historical Evidence of Cultural Revivals

3.1 "Poor Whites" and Jim Crow in the Postbellum South

Historians of the postbellum South have long been fascinated by the acquiescence of poor whites after the Civil War to racist policies that limited the rights of blacks and excluded them from many basic services (Frazier 1949; Woodward 1974). The large literature on this topic tends to frame this as a puzzle. While poor whites did face some degree of economic competition from blacks, the potential gains from cooperation—both in labor relations and at the ballot box—seemed to have been much greater. Indeed, for a brief period, many poor whites joined black men in the Republican Party. For instance, in North Carolina, a "biracial coalition of freedmen and disaffected lower-class whites, resentful of planter domination, channeled their frustrations into politics, ushering into state and local offices Republican administrations of a reformist bent, pursuing measures calculated to end aristocratic privilege and forge a more democratic society" (Forret 2006, p. 229). Some of these reforms were led by recently-freed slaves who were elected to office during Reconstruction.

Poor whites were *poor*, and they faced similar class and employment relations with wealthy whites as did blacks. An alliance between the two groups would have allowed them to dominate Southern politics and ultimately receive the associated economic benefits. Yet, such alliances, where they existed, did not last. The white economic elite recognized the potential threat to their political power, and they successfully prevented the alliance from happening. In the end, poor whites tended to align with the rich white elite on political and social issues. How can this be explained, given that (as a class) this alliance was to the economic detriment of poor whites?

The dominant theory in the literature focusing on economic issues stresses the role of economic competition between blacks and poor whites. With the freeing of slaves, blacks and poor whites were now in competition for the same jobs. Hence, racist laws were favored by poor whites because it limited economic competition (Marshall 1961; Wilson 1976). Yet, a purely economic explanation has a difficult time explaining why poor whites continued to align with the same elites that kept them in such a subjugated economic position. Alternatives did exist. Every southern state had some experience with interracial coalitions in the decades following the Civil War (Forret 2006). In the last two decades of the 19th century, the Populist party attempted to bring together poor whites and blacks. Yet, every time such a coalition was attempted it failed. Race, not economic relations, tended to draw poor whites back to the Democratic Party fold: "flagrant race-baiting, playing to whites' racial fears and anxieties, cemented loyalty to the party of white supremacy ... When poor white voters aided in the restoration of Democratic governments, they removed from power the very politicians most sympathetic to their plight" (Forret 2006, p. 231).

Theories focusing primarily on the role of economic competition are likely correct in many respects. It is not our intention to undermine the importance of such economic factors. Yet, these theories have shortcomings. Our theory of cultural revivals helps address these shortcomings. We can think of there being two types of elites in the postbellum South: the old planter elite and Republican or Populist mobilizers. Both offered the possibility of providing goods via the political process. Under Jim Crow, the planter elite offered goods that favored poor whites (white schools, white churches, white hospitals, white drinking fountains, and many "public" goods discriminated by race), while the Populists promised policies that would improve the plight of both the poorest whites and blacks. Such policies were briefly enacted during Reconstruction, when many recently-freed slaves were elected to office. Logan (2020) finds that counties with more black officials had greater tax revenue, which was spent on improving literacy (for both black and white children) and land redistribution. These were clearly policies favoring poor Southerners.

From the perspective of poor whites, the marginal productivity of public goods (ϕ^2) complementary to Populist politics was almost certainly greater than that of public goods complementary to the planter elite (ϕ^1). In the context of our model, Populist cultural values would have been those conducive to an alliance between poor whites and blacks, whereas the cultural values of the planter elite were racist and meant to undermine any such alliance. Given that poor whites and blacks made up a vast majority of the Southern population, it was almost certain that the provision of public goods complementary to Populist politics was dynamically production efficient (to employ the terminology of our model). Hence, multiple equilibria were possible. One (non-DPE) equilibrium consisted of the old white planter elite dominating the political process, while the other (DPE) equilibrium consisted of poor whites and blacks gaining a greater political voice.

Clearly, the cultural values of the planter elite won out, and this was manifested in Jim Crow laws. Our model helps explain this outcome. The planter elite, facing the prospect of losing their political and economic power following the Civil War, could not simply alter political institutions to their benefit, especially during Reconstruction. Widespread suffrage—for males, at least—meant that freed blacks and poor whites could unite to upend their political dominance. This outcome would have been dynamically production efficient. How could the white elites prevent this outcome from happening? That is, how could they trigger a cultural revival? The logic of our model, as formalized in Proposition 3, indicates one possible solution: *overinvestment* in public goods favoring poor whites, which in turn triggered a change in cultural beliefs conducive to their desired outcomes.

These were precisely the actions taken by white southern elites. It was a departure from the antebellum period, in which the relationship between the planter elite and poor whites was much more antagonistic. As recently explained by Merritt (2017), the planter elite limited educational opportunities for poor whites prior to the Civil War, because keeping them ignorant was best for limiting social unrest. More generally, poor whites in the antebellum South were marginalized. Yet, after the Civil War, and especially after Reconstruction, "formerly marginalized poor whites were welcomed into a fuller participation in the benefits of whiteness ... by the end of the nineteenth century, antebellum cooperation between slaves and poor whites was forgotten, replaced by the reality of racial hatred and Jim Crow segregation" (Forret 2006, p. 228, 231). As explained by Feldman (2004, p. 164), "race repeatedly exerted pressure on poor whites to ally themselves with their privileged white 'betters' against their own class interests and potential biracial alliance, and race produced a series of reforms that largely made life better for whites and worse or no better for blacks." Some of these benefits, which took the form of racially segregated public goods, are well-known. Segregated schools funneled resources to white communities—poor or not—at the expense of black communities. These are a prototypical example of public goods that are complementary to a certain cultural identity. Numerous other types of public goods complementary to a racist, white identity were provided by elites. One important example is parades and statues commemorating the Confederacy. This was the period when a new narrative around the "struggles of the Confederacy" emerged. In Alabama, for instance, this period was characterized by "attempts to memorialize the Lost Cause, disparage Reconstruction, glorify Redemption, romanticize the Reconstruction Klan, and paint a dark picture of [Reconstruction] as a tragic time of black rule, Yankee pillage, federal repression, corruption, and chaos" (Feldman 2004, p. 165). Memorials and public events were a common means of forging such values. In the terms of our model, this narrative was part of a larger "cultural revival" that increased the cultural imprint of white identity, which in turn allowed for the codification of racist laws that primarily benefited the white elite.

This glorification of the past was clearly intended to establish and cement a white cultural identity. Another mechanism through which the old white elite attempted to affect culture was propaganda. Ottinger and Winkler (2020) find that the emergence of the Populist party caused a rise in anti-Black propaganda in the media (i.e., the word "rape" in co-occurrence with the word "negro"). It seemingly worked. The Populist-desired alliance between poor whites and blacks never came to fruition, and the white elite were largely able to co-opt the former and suppress the latter in the century following the Civil War.

In short, the rise of a white supremacist culture among many poor southern whites in the decades following the Civil War was part of a broader "cultural revival." This is precisely what Proposition 3 predicts can happen when established elites face a threat to their political power. An elite-driven "overinvestment" in goods complementary to a white supremacist cultural identity helped the elite maintain their political power in the face of an alternative that promised greater potential returns to the masses of blacks and poor whites. Yet, most poor whites never gained the cultural capital to take advantage of this alternative. This would have required a cultural change in which an alliance with working class blacks would have been desirable. This cultural revival had numerous long-run, negative consequences, many of which are still with us today. While it is not within the scope of this paper to explore these long-run consequences, they are indicative of just how hard it is to escape from an equilibrium in which culture and political power reinforce each other.

3.2 The Gülen Movement in Turkey

The Turkish Republic was founded in 1923 after the Ottoman Empire collapsed following a longer than six century tenure. The modern republic was founded on the back of mainly top-down sociopolitical and economic reforms that were primarily inspired by Western Enlightenment principles and strictly secular social and political norms. The main impetus for these reforms was provided by the fact that the returns to secular schooling and human capital had markedly risen following the Industrial Revolution. Meanwhile, the economic productivity of the more established but conservative Ottoman culture had long been stagnant. As Kuran (2011) documents, this was a new reality that had been borne out by the upturn in economic fortunes of the better educated, non-Muslim citizens of the crumbling empire. These reforms were implemented fairly swiftly in a country whose population was more than 95 percent Muslim and most of them highly devout.

Religious groups have been a constituent element of Turkey's Ottoman legacy. They managed to persevere even the Ottoman modernization campaigns in the mid- to late-19th century, collectively known as the Tanzimat Era. Nevertheless, they were disbanded and outlawed by Kemal Ataturk in the early years of the Turkish Republic. The outlawing of religious education and the introduction of the Latin alphabet in 1927 further limited their influence and forced them to go underground (Tee 2016; Bozçağa and Christia 2020).

Throughout its first seven decades of existence, the Turkish Republic adhered to a French-style "laicité" whereby the state hierarchy was fully under the secular elites' control and all public goods and services—most notably, all three levels of education—were established and guided by secular norms. The upshot is that the demise of the Ottoman Empire and the fledgling new republic very clearly and swiftly upended the well-rooted Ottoman hierarchy and elite who derived their political legitimacy from Islam and the Muslim clerics (Rubin 2017).

By the early 1990s, however, the Turkish seculars' grip on political power and institutions began to wane. With a succession of elections starting in 1995, Turkish Islamists were able to firmly regain and consolidate their political power. This consolidation was evident in the wake of the 2018 Presidential elections, in which the century old Turkish Parliament disbanded and unprecedented powers were bestowed upon the president-elect Tayyip Erdoğan, who had been the Turkish Prime Minister since late 2002.

How was this complete reversal achieved? The model we outlined above provides some insight. In the parlance of the model, two types of elites existed in Turkish politics: secularists and Islamists. Even though the latter were forced underground for decades after the fall of the Ottoman Empire, as Proposition 3 dictates, the Islamist ideology had enough support and adherents in Turkey so as to enable an eventual revival. Moreover, and in line with our theory, such a reversal came on the back of deep-rooted cultural change spearheaded by investment in public goods.

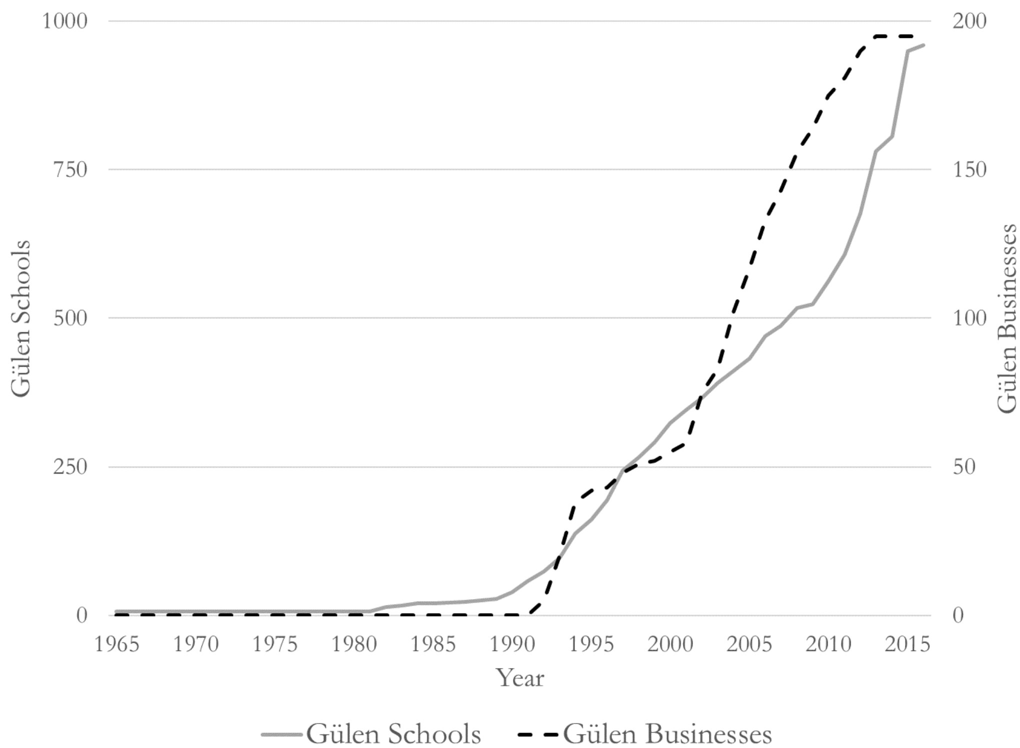
The seeds of this transformation were planted in the 1950s by a religious revival—the Gülen Movement—whose primary emphasis lay in public goods provision in the form of primary and secondary schooling. The seeds of this cultural revival were sown at the end of Turkey's single party era in 1950, when Islamist groups ratcheted up their social and political activism. At the forefront of this movement was Fethullah Gülen, a religious cleric in the Western coastal city of Izmir who mainly focused on establishing K-12 schools. Gülen purported to preach an inclusive brand of Sunni Islam that emphasized cooperation and tolerance, and he viewed Western capitalism and economic modernity as generally compatible with Islam (Matthews 2020).

The Gülen Movement stressed charity and public goods provision to the lower and middle-income classes, but by far the most important element in that drive was investment in public schools. As Bozçağa and Christia (2020) note, "Hizmet's primary emphasis was on education services and, similar to many Islamist movements that have viewed the school system as a way to yield control over the hearts and minds of students ... Gülenists used educational institutions as a way to spread their ideas, win over youth, and strengthen the movement."

In the half century between the mid-1960s and 2016 but especially after the 1970s, the growth in Gülenist "Hizmet" schools and other educational institutions was quite remarkable. Based on Bozçağa and Christia's data, while there were 7 Hizmet schools between 1965 and 1981, their numbers grew to 28 by the close of the latter decade; to 292 by the end of the 1990s; to 524 by the end of the aughts; and to 960 by 2016 (see Figure 3). In fact, "the proportion of Gülenist educational institutions as compared to all private ones varied from about 5 percent for tutoring centers, to 11 percent for schools and 18 percent for dorms" (Bozçağa and Christia 2020). More importantly, the growth of the Hizmet movement represented only a subset of the expanding weight and social influence of Islamism in Turkey.

Based on our model, the number of those who were sympathetic to the Hizmet movement ought to have increased more rapidly in Turkey after the 1990s, and their associated Islamist culture should have become more prominent and influential over time. Moreover, politics should have subsequently evolved in ways that were more amenable to the Islamists. This is precisely what happened starting in the late 1990s and the early 21st century. It culminated in a permanent shift in the balance of power from the Kemalist seculars to Islamists. This is the mechanism through which Proposition 3 predicts a cultural revival will occur (if it occurs at all).

Figure 3: Gülen Schools and Businesses



Source: Bozçağa and Christia (2020).

According to Bozçağa and Christia (2020), the proportion of Gülen-affiliated officials across different civil service sectors ranged between about 1.5 percent in healthcare to roughly 5 percent in the judiciary and 11.3 percent in the police. These estimates are indicative of the extent of the penetration of the Gülenist movement among the high-ranking officials within in the government bureaucracy, judiciary and the police. Relatedly, and as depicted in Figure 3, the number of Gülen affiliated businesses in Turkey began to rapidly increase starting in the 1990s, following more than a decade lag in the ascension of Gülenist schools.

The Gülenist Hizmet movement was part of a broader Islamist revival in Turkey. In fact, while the Islamists remain in power and their political control is further entrenched through institutional interventions, Gülenist political influence and control came to an abrupt end following the failed military coup attempt in the summer of 2016. The Islamist Tayyip Erdoğan government, which was in a tight and decades-old alliance with the Gülenists in their collective power struggle against the Kemalist seculars, ascribed the failed coup attempt to Fethullah Gülen and his followers and it began a sweeping purge of Gülenists from all levels of governmental, educational, and economic hierarchies which continues to this day.

In sum, the birth, spread and growth of the Gülenist Hizmet movement and Islamism in Turkey is a historical example which supports our model. It was achieved almost exclusively on the back of a focused emphasis on educational supply, followed by a subsequent and unambiguous political transformation of the country. This shift in the balance of political power came about only after the cultural dynamics of the country were altered in ways that slowly but steadily favored the Islamists.

3.3 Other Examples of Cultural Revivals

In this section, we briefly provide more examples of cultural revivals upon which our model provides insight. Squicciarini (2020) provides a prototypical example. She finds that the Catholic Church responded to the second wave of industrialization in the 19th century by imposing an anti-scientific curriculum in Catholic schools, which harmed the economic outcomes of students in highly Catholic regions of France. Much as in our model, one type of elites (the Church) altered cultural norms by investing in public goods provision (schools). This shifted the institutional and cultural paths against the headwinds of modernization in highly Catholic regions of France.[19]

Cultural revivals need not be associated solely with religious elites, however. Iyigun and Rubin (2017) study macro-level cultural revivals in the 17th-century Ottoman Empire, 19th-century Imperial China, and 18th–19th century Tokugawa Japan. Only in the first of these cases were religious elites important in facilitating the cultural revival. In each of these

[19] There are many other relevant historical examples. For instance, Chaney (2016) studies the decline of Islamic science, finding that the decline began in the 12th and 13th centuries and that scientific learning was replaced by more traditional modes of religious education in madrasas. Carvalho and Koyama (2016) and Carvalho, Koyama and Sacks (2017) find that ultra-Orthodox European Jews responded to emancipation in the 19th century by imposing unprecedented restrictions on secular education, further closing themselves off from society. Fouka (2020) provides an example of a cultural backlash from post-WWI US education policy. A prohibition of German in public schools, which was intended to promote assimilation, had the effect of heightening cultural identity among Germans.

cases, rulers and elites were confronted with Western institutions and technologies that had the potential to upend the economic and social order. In the context of our model, the old political, military, and economic elite had cultural values complementary to the production of "traditional" goods, such as *tımars* or *waqf* in the Ottoman Empire or Confucian education in Imperial China and Tokugawa Japan. Meanwhile, certain types of merchants, producers, and others with access to capital but not social prestige or political power had values consistent with more "non-traditional" goods. This latter group would have seen their returns rise immensely with the adoption of Western technologies, education, and institutions. Such adoption would have almost certainly been dynamically production efficient. Yet, in *each of these cases*, the reaction to the West was what we call a "cultural revival": cultural values favoring the established elites became *more predominant* in society, and institutions suited for the pre-industrial world became further entrenched. In many ways, these cultural revivals mimic the revival of white supremacist culture in the postbellum South discussed in greater detail in Section 3.1. In all of these cases, institutions ended up supporting the interests of the established elites all the more.

4 Conclusion

There have been many historical cases in which social groups vested in inefficient production modes and technologies were able to maintain their social influence and political dominance over time despite having no capacity to block modes of production detrimental to their interests. This paper develops a theory to explain this phenomenon. We propose that when elites have limited power to directly block economic activities detrimental to their interests, they can instead affect a society's *culture*. We call such outcomes "cultural revivals." The primary intuition for the existence of cultural revivals is that when an economic disadvantage threatens the future political weight of one type of elites, these elites have particularly high incentive to affect citizens' socialization decisions by provisioning public goods. If they are successful, they increase their political weight and, as a result, economic activities complementary to their interests also increase. This happens *despite* the fact that these economic activities are associated with less productively efficient outcomes.

The insights provided by the model offer an explanation for two historical case studies: the Jim Crow South and Turkey's Gülen Movement. In both of these cases, one group of elites—whose economic power and political influence were threatened by new economic realities—could not prevent institutional changes by simply altering political institutions. Yet, they were still successful in preventing changes that would have undermined their power. They did so by altering society's cultural composition via the provision of public goods. In both cases, cultural changes favoring more "traditional" values became predominant. This in turn allowed the prevailing elites to strengthen their grip on political and economic power.

Such a series of events has been shown time and again to be a potent means for established elites to maintain their power in the face of social and economic headwinds pushing to undermine their power. Our theory highlights why elites so often succeed in pushing against these headwinds even when they cannot directly alter political and economic institutions to their benefit.

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Appendices

A Proofs

A.1 Proof of Remark 1

Plugging in (11) into (12), we get:

$$q_1 = q_0 + q_0(1-q_0)\{(1-q_0)\{\eta g_0^1 + (\eta+\phi^1)q_1\} - q_0\{\eta g_0^2 + 1 - (\eta+\phi^2)(1-q_1)\}\} \quad \text{(A.1)}$$

The solution $q_1(g_0^1, g_0^2)$ is unique, as represented in Figure A.1. The RHS of (A.1) (the thick black line in Figure A.1) belongs to $(0,1)$ when $q_0 \in (0,1)$; it is linearly increasing in q_1 with a slope below 1.[20] Hence, the RHS of (A.1) only crosses the 45° line once. The solution $q_1(g_0^1, g_0^2)$ is necessarily stable: if $q_1 > q_1(g_0^1, g_0^2)$, then the socialization efforts are such that q_1 is too high to be an equilibrium. The inverse is true if $q_1 < q_1(g_0^1, g_0^2)$. We find that

$$q_1(g_0^1, g_0^2) = \frac{q_0(1-q_0(1-q_0)(\eta+\phi^2)) + \eta q_0(1-q_0)\{(1-q_0)g_0^1 - q_0 g_0^2\}}{1-q_0(1-q_0)(\eta+\overline{\phi})}, \quad \text{(A.2)}$$

with $\overline{\phi} = (1-q_0)\phi^1 + q_0\phi^2$.

A.2 Proof of Proposition 1

The optimal allocation in period 0, (g_0^1, g_0^2), solves:

$$\max_{(g_0^1, g_0^2)} W_0(g_0^1, g_0^2) = q_0 V^1(g_0^1) + (1-q_0)V^2(g_0^2) + \beta W_1(g_1^{1*}, g_1^{2*}), \quad \text{(A.3)}$$

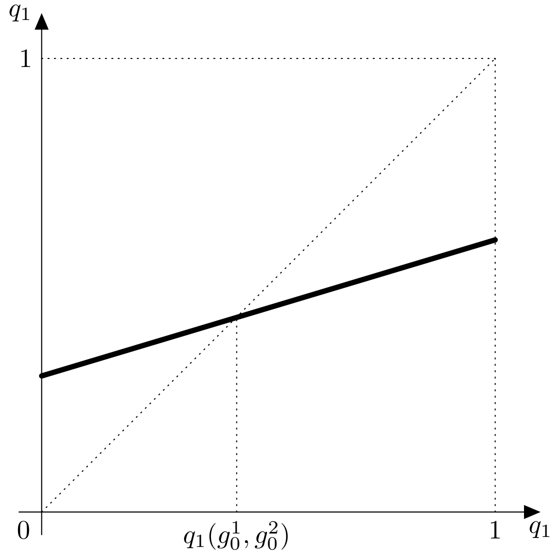
with W_1 given by (8) under the following constraints:

$$\begin{cases} g_0^1 + g_0^2 \leq 1, \\ g_1^{1*} = q_1, \\ g_1^{2*} = 1 - q_1, \text{ and} \\ q_1 = \frac{q_0(1-q_0(1-q_0)(\eta+\phi^2)) + \eta q_0(1-q_0)\{(1-q_0)g_0^1 - q_0 g_0^2\}}{1-q_0(1-q_0)(\eta+\overline{\phi})}, \end{cases} \quad \text{(A.4)}$$

with $\overline{\phi} = (1-q_0)\phi^1 + q_0\phi^2$.

[20] Formally, if we denote $Z(q_1)$ the RHS of (A.1), we find that $Z'(q_1) = q_0(1-q_0)\eta\overline{\phi} < 1$, with $\overline{\phi} = (1-q_0)\phi^1 + q_0\phi^2$. If $q_1 = 0$, $Z(0) = q_0\{1 + (1-q_0)[(1-q_0)\eta g_0^1 - q_0\eta g_0^2 - q_0(\eta+\phi^2)]\}$. A $g_0^2 = 1 - g_0^1$, $Z(0) \geq q_0\{1 - (1-q_0)q_0[2\eta + \phi^2]\} > 0$ for any $\phi^1, \phi^2 \in [0, 1/2]$ and $\eta \in [0, 1/4]$. By symmetry, $Z(1) < 1$.

Figure A.1: Determination of $q_1(g_0^1, g_0^2)$



Note: the thick black line represents the RHS of (A.1).

As the elites always benefit at the margin from more spending on their most preferred good, the budget constraint is necessarily satisfied at equality, $g_0^1 + g_0^2 = 1$. Hence, only one first-order condition can be written for public good g_0^1, while we substitute g_0^2 with $1 - g_0^1$. We denote $g_0^1 = g$, $g_0^2 = 1 - g$, $q_1(g_0^1, g_0^2) = q_1(g)$ and $W_0(g_0^1, g_0^2) = W_0(g)$ in the rest of the proof to simplify the notation.

Substituting g_0^2 by $1 - g$ and maximizing $W_0(g)$, we find the following FOC:

$$\frac{\partial W_0(g)}{\partial g} = \frac{q_0}{g} - \frac{1-q_0}{1-g} + \beta \frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})} \log(\frac{\phi^1 g_1^{1*}}{\phi^2 g_1^{2*}}) + \frac{\partial g_1^{1*}}{\partial g}\frac{\partial W_0(g)}{\partial g_1^{1*}} + \frac{\partial g_1^{2*}}{\partial g}\frac{\partial W_0(g)}{\partial g_1^{2*}} = 0. \tag{A.5}$$

In the previous FOC, and given our equilibrium concept, the envelope theorem can be applied. In period 0, the elites internalize their optimal choice of period 1, so

$$\frac{\partial g_1^{1*}}{\partial g}\frac{\partial W_0(g)}{\partial g_1^{1*}} + \frac{\partial g_1^{2*}}{\partial g}\frac{\partial W_0(g)}{\partial g_1^{2*}} = \beta \frac{\partial q_1}{\partial g}\{\frac{\partial w_1}{\partial g_1^{1*}} - \frac{\partial w_1}{\partial g_1^{2*}}\} = \beta \frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}[\frac{q_1}{g_1^{1*}} - \frac{1-q_1}{g_1^{2*}}] = 0, \tag{A.6}$$

given that $g_1^{1*} = q_1$ and $g_1^{2*} = 1 - q_1$. Hence, $\frac{\partial W_0(g)}{\partial g}$ can be written as:

$$\frac{\partial W_0(g)}{\partial g} = \frac{q_0}{g} - \frac{1-q_0}{1-g} + \beta \frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})} \log(\frac{\phi^1 q_1}{\phi^2(1-q_1)}) = 0, \tag{A.7}$$

with q_1 is given by (A.2).

The sign of $\frac{\partial W_0}{\partial g}$ is ambiguous. Writing the second-order derivative of W_0, we find that:

$$\frac{\partial^2 W_0(g)}{\partial g^2} = -\{\frac{q_0}{g^2} + \frac{1-q_0}{(1-g)^2}\} + \beta[\frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}]^2 \frac{1}{q_1(1-q_1)}. \quad (A.8)$$

The second term in the expression above is positive. Indeed, as long as $\beta > 0$ non-convexities may arise in the optimization problem of the elites for the following reason. For the elites of type 1, the marginal benefit of increasing their weight is equal to their period-1 utility, which is necessarily increasing in g, the provision of public good 1.

From (A.8), we deduce that the second-order derivative is equal to zero when:

$$\{\frac{q_0}{g^2} + \frac{1-q_0}{(1-g)^2}\} = \beta[\frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}]^2 \frac{1}{q_1(g)(1-q_1(g))}, \quad (A.9)$$

which rewrites

$$q_1(g)(1-q_1(g)) = \beta[\frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}]^2 \frac{g^2(1-g)^2}{(1-q_0)g^2+q_0(1-g)^2}, \quad (A.10)$$

with $q_1(g)$ given by (A.2), so

$$\frac{\partial^2 W_0(g)}{\partial g^2} < 0 \text{ when } q_1(g)(1-q_1(g)) > \beta[\frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}]^2 \frac{g^2(1-g)^2}{(1-q_0)g^2+q_0(1-g)^2}, \text{ and} \quad (A.11)$$

$$\frac{\partial^2 W_0(g)}{\partial g^2} \geq 0 \text{ when } q_1(g)(1-q_1(g)) \leq \beta[\frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}]^2 \frac{g^2(1-g)^2}{(1-q_0)g^2+q_0(1-g)^2}. \quad (A.12)$$

$q_1(g)(1-q_1(g))$ as a function of g is represented by the black curve in Figure A.2 in the case where there exists $g \in [0,1]$ such that $\frac{\partial^2 W_0(g)}{\partial g^2} \geq 0$ (this case will formally be characterized below). As the function $q_1(g)$ is linear and increasing, the function $q_1(g)(1-q_1(g))$ is concave in g, has an inverted U shape, and takes strictly positive values on $[0,1]$, as $q_1(g) \in (0,1)$.

$\beta[\frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}]^2 \frac{g^2(1-g)^2}{(1-q_0)g^2+q_0(1-g)^2}$ as a function of g is represented by the blue curve in figure A.2. The function is single peaked, and necessarily equal to zero in the corners.

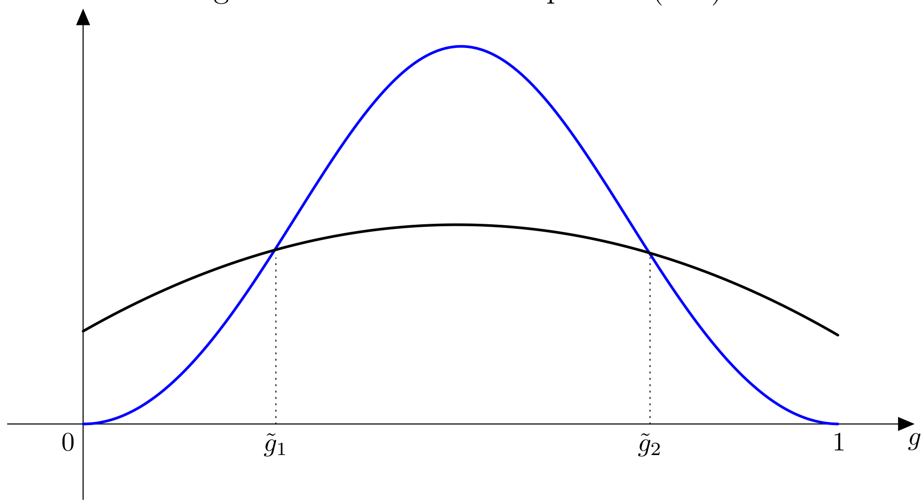
The following result follows from the previous discussion, as summarized in Figure A.2:

Lemma 1 *Let*

$$\tilde{\beta} = \frac{1}{[\frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}]^2} \min_{g\in[0,1]} \frac{q_1(g)(1-q_1(g))}{\frac{g^2(1-g)^2}{(1-q_0)g^2+q_0(1-g)^2}}. \quad (A.13)$$

- *If $\beta \leq \tilde{\beta}$, then $\frac{\partial^2 W_0(g)}{\partial g^2} \leq 0$ for any $g \in [0,1]$.*

Figure A.2: Solutions of equation (A.8)



.

Note: the blue curve is the RHS of (A.10), and the black curve is the LHS of (A.10).

- *If* $\beta > \tilde{\beta}$*, then the equation* $\frac{\partial^2 W_0}{\partial g^2} = 0$ *admits two solutions* $\tilde{g}_1, \tilde{g}_2 \in (0,1)$*,* $\tilde{g}_1 \neq \tilde{g}_2$ *such that:*
  - *If* $g < \tilde{g}_1$ *or* $g > \tilde{g}_2$ *then* $\frac{\partial^2 W_0}{\partial g^2} < 0$.
  - *If* $g \in [\tilde{g}_1, \tilde{g}_2]$*, then* $\frac{\partial^2 W_0}{\partial g^2} \geq 0$.

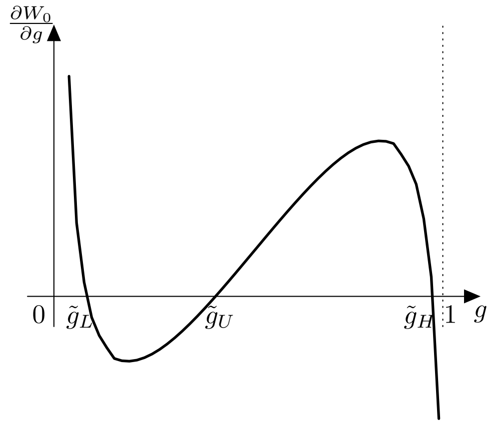
Since the non-convexity arises from the inter-temporal concerns of the elites, the magnitude of the non-convexity can be simply related to the magnitude of the time preference parameter β. In the case where $\beta \leq \tilde{\beta}$, the non-convexity in the optimization problem is weak, so that the function $W_0(.)$ is concave.

In the case where $\beta > \tilde{\beta}$, Figure A.2 depicts the determination of the solutions of the equation $\frac{\partial^2 W_0}{\partial g^2} = 0$.

We deduce that the convexity of the function $\frac{\partial W_0(.)}{\partial g}$ changes twice. The function is first decreasing, then increasing, and decreases again. Additionally, when $g \to 0$, then $\frac{\partial W_0(g)}{\partial g} \to \infty$. When $g \to 1$, then $\frac{\partial W_0(g)}{\partial g} \to -\infty$. Given these results, we have represented the function $\frac{\partial W_0(.)}{\partial g}$ in Figure A.3.

As represented in Figure A.3, the function $\frac{\partial W_0(g)}{\partial g}$ can at most cross the horizontal axis three times. When the function crosses the horizontal line and is decreasing, then the solution of the equation $\frac{\partial W_0(g)}{\partial g} = 0$ is stable. This is the case of the two extreme solutions $\tilde{g}_L$ and $\tilde{g}_H$, where the subscripts L and H stand for "low" and "high" respectively. When the function $\frac{\partial W_0(g)}{\partial g}$ crosses the horizontal axis and is increasing, then the solution of the equation $\frac{\partial W_0(g)}{\partial g} = 0$ is unstable. This is the case of the intermediate solution $\tilde{g}_U$, as represented in Figure A.3.

Figure A.3: $\frac{\partial W_0}{\partial g}$ as a function of g.



The second step of the proof consists in establishing the following Lemma:

Lemma 2 $\frac{\partial^2 W_0(g)}{\partial g \partial q_0} > 0$ *when g is such that* $\frac{\partial W_0(g)}{\partial g} = 0$

In order to establish this result, we need to prove the following Lemma first:

Lemma 3 $\frac{\partial q_1}{\partial q_0} > 0$*, with q_1 given by (A.2).*

From (A.1), q_1 solves the following fixed point equation:

$$q_1(g) = q_0 + q_0(1 - q_0)((1 - q_0)U^{11}(g) - q_0 U^{22}(g)), \tag{A.14}$$

with $U^{11}(g) = \eta g + (\eta + \phi^1)q_1(g)$ and $U^{22}(g) = \eta(1 - g) + (\eta + \phi^2)(1 - q_1(g))$.

In order to prove that $\frac{\partial q_1(g)}{\partial q_0} > 0$, we prove that the RHS of (A.14) is increasing in q_0. As represented in Figure A.4, this would shift the RHS of (A.14) upward and prove that $q_1(g)$ increases with q_0, as the intersection between the LHS and RHS of (A.14) is shifted to the right.

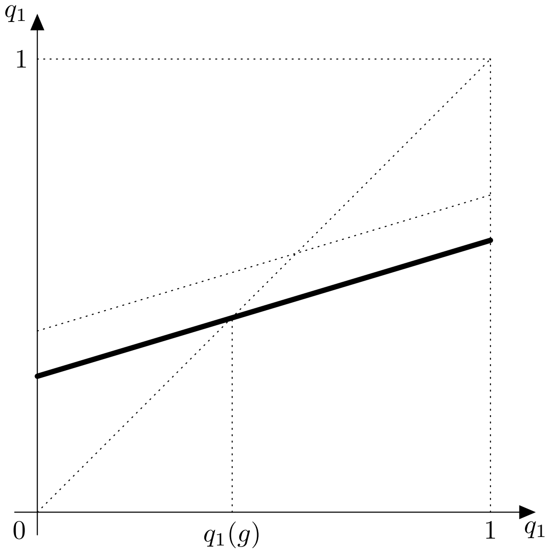
After denoting the RHS of (A.14) as $RHS(g)$ to ease the notation, we find that

$$\begin{cases} \frac{\partial RHS(g)}{\partial q_0} = 1 + (1 - q_0)(1 - 3q_0)U^{11}(g) - q_0(2 - 3q_0)U^{22}(g), \text{ and} \\ \frac{\partial^2 RHS(g)}{\partial q_0^2} = (-4 + 6q_0)U^{11}(g) + (-2 + 6q_0)U^{22}(g). \end{cases} \tag{A.15}$$

Hence, the second-order derivative of $RHS(g)$ is linearly increasing in q_0 and it equals zero at

$$q_0 = \tilde{q}_0 = \frac{2U^{11} + U^{22}}{3(U^{11} + U^{22})} \in (0, 1). \tag{A.16}$$

Figure A.4: Solutions of the fixed point equation (A.14)



Note: the black line is the RHS of (A.14).

Thus, $\frac{\partial RHS(g)}{\partial q_0}$ is U-shaped, and is minimum in $\tilde{q}_0$. Notwithstanding a few computations, we find that in $q_0 = \tilde{q}_0$,

$$\frac{\partial RHS(g)}{\partial q_0} = 1 - \frac{[U^{11}(g)]^2(U^{11}(g) + 2U^{22}(g))}{3(U^{11}(g) + U^{22}(g))^2} - \frac{[U^{22}(g)]^2(U^{22}(g) + 2U^{11}(g))}{3(U^{11}(g) + U^{22}(g))^2}, \tag{A.17}$$

which rewrites

$$\frac{\partial RHS(g)}{\partial q_0} = 1 - \frac{1}{3}(U^{11}(g) + U^{22}(g)) + \frac{1}{3}\frac{U^{11}(g)U^{22}(g)}{U^{11}(g) + U^{22}(g)}. \tag{A.18}$$

As

$$\frac{1}{3}(U^{11}(g) + U^{22}(g)) = \frac{1}{3}\{2\eta + \phi^1 q_1 + \phi^2(1 - q_2)\}, \tag{A.19}$$

$$\frac{1}{3}(U^{11}(g) + U^{22}(g)) < \frac{1}{3}\{2\eta + \max(\phi^1, \phi^2)\} < 1, \tag{A.20}$$

given that $\eta \in [0, 1/4]$ and $\max(\phi^1, \phi^2) \leq 1/2$. We deduce that

$$\frac{\partial RHS(g)}{\partial q_0} > 0. \tag{A.21}$$

This concludes the proof of Lemma 3. We can now turn to the proof of Lemma 2.

By differentiating $\frac{\partial W_0(g)}{\partial g}$ given in (A.7) with respect to q_0, we find:

$$\frac{\partial^2 W_0(g)}{\partial g \partial q_0} = \frac{1}{g} + \frac{1}{1-g} + \beta\eta \frac{(1-2q_0) + [q_0(1-q_0)]^2(\phi^2 - \phi^1)}{(1-q_0(1-q_0)(\eta+\overline{\phi}))^2} \log(\frac{\phi^1 q_1}{\phi^2(1-q_1)})$$
$$+ \beta \frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})} \frac{\partial q_1}{\partial q_0} \frac{1}{q_1(1-q_1)}. \quad \text{(A.22)}$$

When $\frac{\partial W_0(g)}{\partial g} = 0$,

$$\beta \log(\frac{\phi^1 q_1(g)}{\phi^2(1-q_1(g))}) = \frac{\frac{1-q_0}{1-g} - \frac{q_0}{g}}{\frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}}. \quad \text{(A.23)}$$

Rewriting the cross derivative of W_0 by substituting (A.23) in (A.22), we find

$$\frac{\partial^2 W_0(g)}{\partial g \partial q_0} = \frac{1}{g} + \frac{1}{1-g} + \frac{(1-2q_0) + [q_0(1-q_0)]^2(\phi^2 - \phi^1)}{q_0(1-q_0)(1-q_0(1-q_0)(\eta+\overline{\phi}))} \{\frac{1-q_0}{1-g} - \frac{q_0}{g}\}$$
$$+ \beta \frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})} \frac{\partial q_1(g)}{\partial q_0} \frac{1}{q_1(g)(1-q_1(g))}, \quad \text{(A.24)}$$

which can be rewritten:

$$\frac{\partial^2 W_0(g)}{\partial g \partial q_0} = \frac{q_0}{g}\{\frac{1-(1-q_0)^2(\eta+\overline{\phi}+q_0(\phi^2-\phi^1))}{(1-q_0)(1-q_0(1-q_0)(\eta+\overline{\phi}))}\} + \frac{1-q_0}{1-g}\{\frac{1-q_0^2(\eta+\overline{\phi}+(1-q_0)(\phi^2-\phi^1))}{q_0(1-q_0(1-q_0)(\eta+\overline{\phi}))}\} +$$
$$\beta \frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})} \frac{\partial q_1(g)}{\partial q_0} \frac{1}{q_1(g)(1-q_1(g))} > 0. \quad \text{(A.25)}$$

Take the case where $\phi^2 \geq \phi^1$ without loss of generality. The second term in the RHS of the previous equation is then necessarily positive. We find that the first term in the RHS of the previous equation is also positive when $\eta \in [0, 1/4]$ and $\phi^2 \in [0, 1/2]$.[21] Finally, the third term in the RHS is also positive, as $\frac{\partial q_1(g)}{\partial q_0} > 0$ from Lemma 3. We have proven that $\frac{\partial^2 W_0(g)}{\partial g \partial q_0} > 0$ when $\frac{\partial W_0(g)}{\partial g} = 0$. The result holds when $\phi^1 \geq \phi^2$ by symmetry of the problem.

Combining this last result with our previous analysis, we deduce our final intermediary result:

Lemma 4

- *If $\beta \leq \tilde{\beta}$, then there is a single stable solution $\tilde{g}_a \in (0, 1)$ that solves* $\frac{\partial W_0(g)}{\partial g} = 0$.

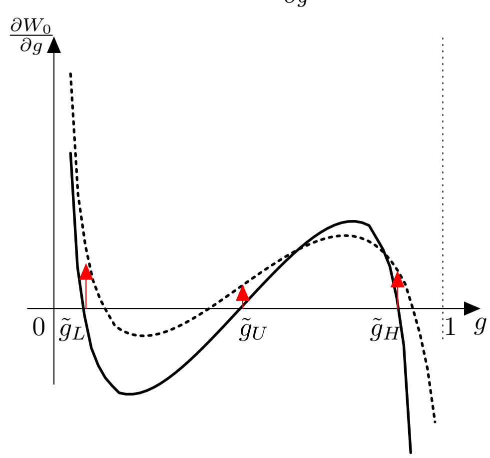
[21] To see this, denote $N(q_0) = 1-(1-q_0)^2(\eta+\overline{\phi}+q_0(\phi^2-\phi^1))$ the numerator of the first term in the RHS of (A.25). Hence, $N(q_0) > 1-(1-q)^2(\eta+(1+q_0)\max(\phi^1,\phi^2)$. Since $1-(1-q)^2(\eta+(1+q_0)\max(\phi^1,\phi^2)$ is a decreasing function of q_0, it is maximum in $q_0 = 0$. We deduce that $N(q_0) > 1-(\eta+\max(\phi^1,\phi^2) \geq 0$ when $\phi^i \in [0, 1/2]$ and $\eta \in [0, 1/4]$.

- *If $\beta > \tilde{\beta}$, there exist two threshold values of the initial fraction of type 1 individuals q_0, $\underline{q}$ and $\overline{q}$, with $0 < \underline{q} \leq \overline{q} < 1$ such that:*
  - *If $q_0 \geq \underline{q}$ or $q_0 \leq \overline{q}$, then there is a single stable solution $\tilde{g}$ that solves $\frac{\partial W_0(g)}{\partial g} = 0$.*
  - *If $q_0 \in (\underline{q}, \overline{q})$, then the equation $\frac{\partial W_0(g)}{\partial g} = 0$ admits two stable solutions $\tilde{g}_L$ and $\tilde{g}_H$, and one unstable solution $\tilde{g}_U$ with $\tilde{g}_L < \tilde{g}_U < \tilde{g}_H$ and $\tilde{g}_L, \tilde{g}_U, \tilde{g}_H \in (0, 1)$.*

In the case where $\beta \leq \tilde{\beta}$, then from Lemma 1, $\frac{\partial^2 W_0(g)}{\partial g^2} < 0$ for any $g \in [0, 1]$. Hence, the function $W_0(g)$ is concave, so the optimization problem admits a unique stable solution $\tilde{g}_a$. This solution belongs to $(0, 1)$, as $\frac{\partial W_0(g)}{\partial g} \to -\infty$ when $g \to 0$, and $\frac{\partial W_0(g)}{\partial g} \to +\infty$ when $g \to 1$. This concludes the proof of the first point of Lemma 4.

In the case where $\beta > \tilde{\beta}$ the results of Lemma 4 can be illustrated with graphs. The effect of an increase in q_0 on the function $\frac{\partial W_0(.)}{\partial g}$ is represented in Figure A.5. Given our results in Lemma 2, the effect of an increase in q_0 on the function $\frac{\partial W_0(.)}{\partial g}$ can be represented as in Figure A.5. $\frac{\partial^2 W_0(g)}{\partial g \partial q_0} > 0$ when $\frac{\partial W_0(g)}{\partial g} = 0$, so $\frac{\partial W_0(.)}{\partial g}$ is shifted upwardly along the horizontal axis. We can deduce from Lemma 2 that $\tilde{g}_L$ and $\tilde{g}_H$ increase with q_0, and that $\tilde{g}_U$ necessarily decreases with q_0.

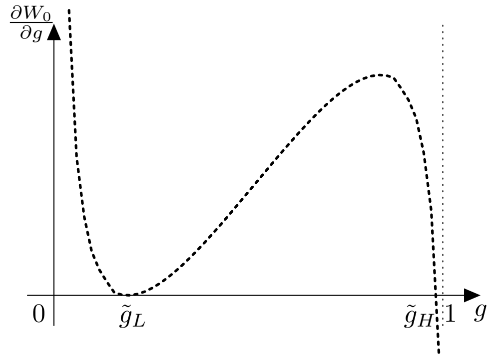
Figure A.5: Effect on $g \to \frac{\partial W_0(g)}{\partial g}$ of an increase in q_0.



Hence, when q_0 is sufficiently high, then the U-shaped part of the graph between $\tilde{g}_L$ and $\tilde{g}_U$ is shifted above the horizontal axis, as represented in figure A.6, and only one equilibrium remains: $\tilde{g}_H$. The value of q_0 such that the function $\frac{\partial W_0(.)}{\partial g}$ is exactly tangent to the horizontal axis in $\tilde{g}_L$ is denoted $\overline{q}$, and belongs to $(0, 1)$. Indeed, $\overline{q}$ is strictly in the segment $(0, 1)$, as when $q_0 = 0$, $\frac{\partial W_0(g)}{\partial g} < 0$ for any value of g, while when $q_0 = 1$, then $\frac{\partial W_0(g)}{\partial g} > 0$ for any value of g. Since $\frac{\partial W_0(.)}{\partial g}$ must switch sign when $q_0 = \overline{q}$, we deduce that $\overline{q} \in (0, 1)$.

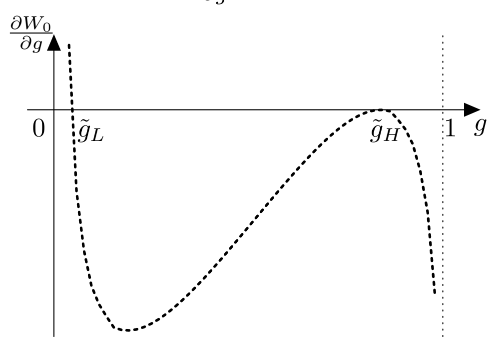
The reasoning is similar for $\underline{q}$. When q_0 decreases, then the inverted-U shape on the segment of the graph representing $\frac{\partial W_0(.)}{\partial g}$ on $[\tilde{g}_U, \tilde{g}_H]$ is shifted below the horizontal axis, as

Figure A.6: Effect on $g \rightarrow \frac{\partial W_0(g)}{\partial g}$ of an increase in q_0 when $q_0 = \overline{q}$.



represented in figure A.7, and only one equilibrium remains: $\tilde{g}_L$. The value of q_0 such that the function $\frac{\partial W_0(.)}{\partial g}$ is tangent to the horizontal axis in $\tilde{g}_H$ is denoted $\underline{q}$, and belongs to $(0, 1)$, given the same reasoning as the one developed above.

Figure A.7: Effect on $g \rightarrow \frac{\partial W_0(g)}{\partial g}$ of a decrease in q_0 when $q_0 = \underline{q}$.



Finally, when $q \in [\underline{q}, \overline{q}]$, then there are two stable solutions of the equation $\frac{\partial W_0}{\partial g} = 0$. These two solutions, $\tilde{g}_L$ and $\tilde{g}_H$, have already been represented in Figure A.3.[22]

To summarize, we have proven that depending on the initial cultural composition of the population, there can be either one or two stable Subgame Perfect Equilibria.

[22] In order to formally characterize $\underline{q}$ and $\overline{q}$, one can define the following system,

$$\begin{cases} \frac{\partial W_0}{\partial g} = 0 \\ \frac{\partial^2 W_0}{\partial g^2} = 0. \end{cases}$$

Given our analysis of the functions $\frac{\partial W_0(g)}{\partial g}$ and $\frac{\partial^2 W_0(g)}{\partial g^2}$, this system necessarily admits two solutions $(\tilde{g}_L, \underline{q})$, and $(\tilde{g}_H, \overline{q})$.

When $\beta \leq \tilde{\beta}$: the game admits a single stable Subgame Perfect Equilibria. The period-0 equilibrium allocation (g_0^{1*}, g_0^{2*}) is:

$$\begin{cases} g_0^{1*} = \tilde{g}_a, \text{ and} \\ g_0^{2*} = 1 - \tilde{g}_a \end{cases} \tag{A.26}$$

with $\tilde{g}_a$ is the unique solution of $\frac{\partial W_0(g)}{\partial g} = 0$, as established in Lemma 4.

In period 1, the elites choose:

$$\begin{cases} g_1^{1*} = q_1(\tilde{g}_a), \\ g_1^{2*} = 1 - q_1(\tilde{g}_a), \end{cases} \tag{A.27}$$

with

$$q_1(\tilde{g}_a) = \frac{q_0(1 - q_0(1 - q_0)(\eta + \phi^2)) + \eta q_0(1 - q_0)\{(1 - q_0)\tilde{g}_a - q_0(1 - \tilde{g}_a)\}}{1 - q_0(1 - q_0)(\eta + \overline{\phi})}, \tag{A.28}$$

The citizens socialize their offspring, and choose the equilibrium efforts

$$\begin{cases} \tau^{1*}(\tilde{g}_a) = (1 - q_0)\{\eta\tilde{g}_a + (\eta + \phi^1)q_1(\tilde{g}_a)\} \\ \tau^{2*}(\tilde{g}_a) = q_0\{\eta(1 - \tilde{g}_a) + (\eta + \phi^2)(1 - q_1(\tilde{g}_a))\}. \end{cases} \tag{A.29}$$

The SPE can then be written as: $\{(\tilde{g}_a, 1 - \tilde{g}_a); (q_1(\tilde{g}_a), 1 - q_1(\tilde{g}_a)); (\tau^{1*}(\tilde{g}_a), \tau^{2*}(\tilde{g}_a))\}$.

When $\beta > \tilde{\beta}$ and $q < \underline{q}$: The game admits a single stable Subgame Perfect Equilibria. The period-0 equilibrium allocation (g_0^{1*}, g_0^{2*}) is:

$$\begin{cases} g_0^{1*} = \tilde{g_L}, \text{ and} \\ g_0^{2*} = 1 - \tilde{g_L} \end{cases} \tag{A.30}$$

with $\tilde{g_L}$ the unique solution of $\frac{\partial W_0}{\partial g} = 0$.

In period 1, the elites choose:

$$\begin{cases} g_1^{1*} = q_1(\tilde{g_L}), \\ g_1^{2*} = 1 - q_1(\tilde{g_L}), \end{cases} \tag{A.31}$$

with

$$q_1(\tilde{g_L}) = \frac{q_0(1 - q_0(1 - q_0)(\eta + \phi^2)) + \eta q_0(1 - q_0)\{(1 - q_0)\tilde{g_L} - q_0(1 - \tilde{g_L})\}}{1 - q_0(1 - q_0)(\eta + \overline{\phi})}, \tag{A.32}$$

.

The citizens socialize their offspring, and choose the equilibrium efforts

$$\begin{cases} \tau_L^{1*} = (1-q_0)\{\eta\tilde{g_L} + (\eta+\phi^1)q_1(\tilde{g_L})\} \\ \tau_L^{2*} = q_0\{\eta(1-\tilde{g_L}) + (\eta+\phi^2)(1-q_1(\tilde{g_L}))\}. \end{cases} \tag{A.33}$$

The SPE can then be written as: $\{(\tilde{g_L}, 1-\tilde{g_L}); (q_1(\tilde{g_L}), 1-q_1(\tilde{g_L})); (\tau_L^{1*}, \tau_L^{2*})\}$.

When $\beta > \tilde{\beta}$ and $q > \underline{q}$: The game admits a single stable Subgame Perfect Equilibria.

The period-0 equilibrium allocation (g_0^{1*}, g_0^{2*}) is:

$$\begin{cases} g_0^{1*} = \tilde{g_H}, \text{ and} \\ g_0^{2*} = 1-\tilde{g_H} \end{cases} \tag{A.34}$$

with $\tilde{g}_H$ the unique solution of $\frac{\partial W_0}{\partial g} = 0$.

In period 1, the elites choose:

$$\begin{cases} g_1^{1*} = q_1(\tilde{g_H}), \\ g_1^{2*} = 1-q_1(\tilde{g_H}), \end{cases} \tag{A.35}$$

with

$$q_1(\tilde{g_H}) = \frac{q_0(1-q_0(1-q_0)(\eta+\phi^2)) + \eta q_0(1-q_0)\{(1-q_0)\tilde{g_H} - q_0(1-\tilde{g_H})\}}{1-q_0(1-q_0)(\eta+\overline{\phi})}, \tag{A.36}$$

.

The citizens socialize their offspring, and choose the equilibrium efforts

$$\begin{cases} \tau_H^{1*} = (1-q_0)\{\eta\tilde{g_H} + (\eta+\phi^1)q_1(\tilde{g_H})\} \\ \tau_H^{2*} = q_0\{\eta(1-\tilde{g_H}) + (\eta+\phi^2)(1-q_1(\tilde{g_H}))\}. \end{cases} \tag{A.37}$$

The SPE can then be written as: $\{(\tilde{g_H}, 1-\tilde{g_H}); (q_1(\tilde{g_H}), 1-q_1(\tilde{g_H})); (\tau_H^{1*}, \tau_H^{2*})\}$.

Finally, when $\beta > \tilde{\beta}$ and $q \in [\underline{q}, \overline{q}]$: the game admits two stable SPE (and one unstable SPE).

In the first stable SPE, the period-0 equilibrium allocation (g_0^{1*}, g_0^{2*}) is:

$$\begin{cases} g_0^{1*} = \tilde{g_H}, \text{ and} \\ g_0^{2*} = 1-\tilde{g_H} \end{cases} \tag{A.38}$$

with $\tilde{g}_H$ the first solution of $\frac{\partial W_0}{\partial g} = 0$ such that $\frac{\partial^2 W_0}{\partial g^2} < 0$.

In the second stable SPE, the period-0 equilibrium allocation (g_0^{1*}, g_0^{2*}) is:

$$\begin{cases} g_0^{1*} = \tilde{g}_L, \text{ and} \\ g_0^{2*} = 1 - \tilde{g}_L \end{cases} \tag{A.39}$$

with $\tilde{g}_H$ the second solution of $\frac{\partial W_0}{\partial g} = 0$ such that $\frac{\partial^2 W_0}{\partial g^2} < 0$.

In period 1, the elites choose:

$$\begin{cases} g_1^{1*} = q_1(\tilde{g}_K), \\ g_1^{2*} = 1 - q_1(\tilde{g}_K), \end{cases} \tag{A.40}$$

with

$$q_1(\tilde{g_K}) = \frac{q_0(1 - q_0(1 - q_0)(\eta + \phi^2)) + \eta q_0(1 - q_0)\{(1 - q_0)\tilde{g_K} - q_0(1 - \tilde{g_K})\}}{1 - q_0(1 - q_0)(\eta + \overline{\phi})}, \tag{A.41}$$

for $K \in \{L, H\}$. The citizens socialize their offspring, and choose the equilibrium efforts

$$\begin{cases} \tau_K^{1*} = (1 - q_0)\{\eta\tilde{g_K} + (\eta + \phi^1)q_1(\tilde{g_K})\} \\ \tau_K^{2*} = q_0\{\eta(1 - \tilde{g_K}) + (\eta + \phi^2)(1 - q_1(\tilde{g_K}))\}. \end{cases} \tag{A.42}$$

The two SPEs can then be written as: $\{(\tilde{g_H}, 1 - \tilde{g_H}); (q_1(\tilde{g_H}), 1 - q_1(\tilde{g_H})); (\tau_H^{1*}, \tau_H^{2*})\}$ and $\{(\tilde{g_L}, 1 - \tilde{g_L}); (q_1(\tilde{g_L}), 1 - q_1(\tilde{g_L})); (\tau_L^{1*}, \tau_L^{2*})\}$. This concludes the proof of Proposition 1.

A.3 Proof of Proposition 2

First, we establish the following intermediary result:

Lemma 5 *There exists two thresholds $\underline{q}^{DPE}$ and $\overline{q}^{DPE}$ in $[0, 1]$ such that if $q_0 \in (\underline{q}^{DPE}, \overline{q}^{DPE})$, then*

$$q_1(0, 1) = \min_{(g_0^1, g_0^2)} q_1(g_0^1, g_0^2) < \frac{\phi^2}{\phi^1 + \phi^2} < \max_{(g_0^1, g_0^2)} q_1(g_0^1, g_0^2) = q_1(1, 0), \tag{A.43}$$

with $g_0^1 + g_0^2 = 1$ and $q_1(g_0^1, g_0^2)$ the solution of (A.1).

First, notice that

$$\min_{(g_0^1, 1 - g_0^1)} q_1(g_0^1, 1 - g_0^1) = q_1(0, 1). \tag{A.44}$$

In words, the minimum value of $q_1(g_0^1, g_0^2)$ when $g_0^1 + g_0^2 = 1$ is such that only good two is provided in period 0. But since $q_0 \to q_1(0,1)$ is increasing in q_0, there exists a unique threshold $\overline{q}^{DPE}$ in $[0,1]$ such that if $q_0 \leq \overline{q}^{DPE}$, then $q_1(0,1) \leq \frac{\phi^2}{\phi^1+\phi^2}$.

Following the same reasoning, since

$$\max_{(g_0^1, 1-g_0^1)} q_1(g_0^1, 1-g_0^1) = q_1(1,0), \tag{A.45}$$

and that $q_0 \to q_1(1,0)$ is increasing in q_0, there exists a unique threshold $\underline{q}^{DPE}$ in $[0,1]$ such that if $q_0 \geq \underline{q}^{DPE}$, then $q_1(1,0) \geq \frac{\phi^2}{\phi^1+\phi^2}$.

Since $\max_{(g_0^1,1-g_0^1)} q_1(g_0^1, 1-g_0^1) = q_1(1,0) > \min_{(g_0^1,1-g_0^1)} q_1(g_0^1, 1-g_0^1) = q_1(0,1)$, then $\underline{q}^{DPE} < \overline{q}^{DPE}$ is necessarily true. This concludes the proof of Lemma 5.

Now, we solve the dynamically production efficient provision by backward induction. We assume in this proof that, when indifferent between producing goods 1 and 2, the default option is to produce good 2.

We distinguish three cases in this proof. Case A: $q_0 \in [\underline{q}^{DPE}, \overline{q}^{DPE}]$, case B: $q_0 < \underline{q}^{DPE}$, and Case C: $q_0 > \underline{q}^{DPE}$. In each case, we fully characterize the DPE of the model.

Case A: $q_0 \in [\underline{q}^{DPE}, \overline{q}^{DPE}]$. A dynamically efficient Production will necessarily be such that the constraint $g_t^1 + g_t^2 = 1$ is satisfied at equality in any period $t \in \{0,1\}$: all the available resources are used for the production along the efficient path. We find that in period 1,

$$\frac{\partial p_1(g_1^1, 1-g_1^1)}{\partial g_1^1} = q_1(g_0^1, g_0^2)\phi^1 - (1 - q_1(g_0^1, g_0^2))\phi^2, \tag{A.46}$$

so

$$\frac{\partial p_t(g_1^1, 1-g_1^1)}{\partial g_1^1} > 0 \text{ if and only if } q_1(g_0^1, 1-g_0^1) > \frac{\phi^2}{\phi^1+\phi^2}. \tag{A.47}$$

Hence,

$$\begin{cases} g_1^{1,DPE} = 1 \\ g_1^{2,DPE} = 0 \end{cases} \quad \text{if } q_1(g_0^1, 1-g_0^1) > \frac{\phi^2}{\phi^1+\phi^2}, \text{ and} \tag{A.48}$$

$$\begin{cases} g_1^{1,DPE} = 0 \\ g_1^{2,DPE} = 1 \end{cases} \quad \text{otherwise.} \tag{A.49}$$

From (A.2), since $\frac{\partial q_1}{\partial g_0^1} = \frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}$, we deduce that

$$\frac{\partial p_0(g_0^1, 1-g_0^1)}{\partial g_0^1} = q_0\phi^1 - (1-q_0)\phi^2 + \beta\frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}\{\phi^1 g_1^{1,DPE} - \phi^2(1-g_1^{1,DPE})\}, \tag{A.50}$$

Hence, there are two possible outcomes in period 1. In the first outcome, the equilibrium is such that $q_1(g_0^1, 1-g_0^1) > \frac{\phi^2}{\phi^1+\phi^2}$. In this case, the first-order condition in period 0 is:

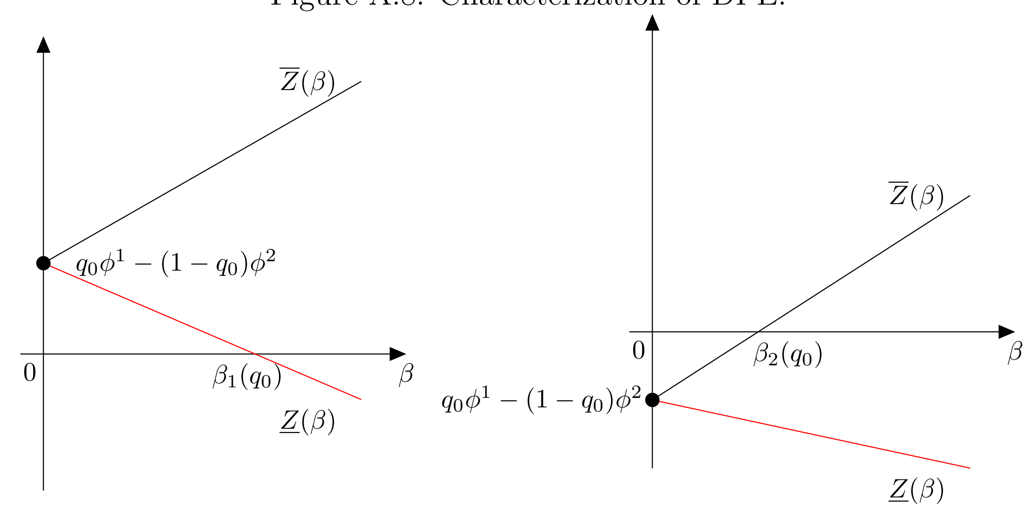
$$\frac{\partial p_0(g_0^1, 1-g_0^1)}{\partial g_0^1} = \overline{Z}(\beta) = q_0\phi^1 - (1-q_0)\phi^2 + \beta\frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}\phi^1. \tag{A.51}$$

In the second outcome, the equilibrium is such that $q_1(g_0^1, 1-g_0^1) \leq \frac{\phi^2}{\phi^1+\phi^2}$. In this case, the first-order condition in period 0 is:

$$\frac{\partial p_0(g_0^1, 1-g_0^1)}{\partial g_0^1} = \underline{Z}(\beta) = q_0\phi^1 - (1-q_0)\phi^2 - \beta\frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})}\phi^2. \tag{A.52}$$

In order to characterize the dynamic efficient production, we consider all the possible cases, depending on the parameter values.

Figure A.8: Characterization of DPE.



Case 1: $q_0\phi^1 - (1-q_0)\phi^2 > 0$ **and** $\beta < \beta_1(q_0)$**.** This case is depicted in the left panel of Figure A.8. There exists a certain threshold $\beta_1(q_0)$ such that if $\beta < \beta_1(q_0)$, then $\overline{Z}(\beta) > 0$ and $\underline{Z}(\beta) > 0$.

First, since $\underline{Z}(\beta) > 0$, then there is no equilibrium such that good 2 is provisioned in period 0. Since $\overline{Z}(\beta) > 0$, an equilibrium is necessarily such that

$$g_0^{1,DPE} = 1 \text{ and } g_0^{2,DPE} = 0. \tag{A.53}$$

As only good 1 is provided in period 0, the fraction of individuals of type 1 reaches $q_1(1,0)$ in period 1. From Lemma 5, $q_1(1,0) > \frac{\phi^2}{\phi^1+\phi^2}$ is satisfied, so

$$g_1^{1,DPE} = 1 \text{ and } g_1^{2,DPE} = 0. \tag{A.54}$$

Case 2: $q_0\phi^1 - (1-q_0)\phi^2 > 0$ and $\beta \geq \beta_1(q_0)$. From Figure A.8, we see that there can be two potential equilibrium outcomes. In the first outcome, since $\overline{Z}(\beta) \geq 0$,

$$g_0^{1,DPE} = 1,\ g_0^{2,DPE} = 0 \text{ and} \tag{A.55}$$
$$g_1^{1,DPE} = 1,\ g_1^{2,DPE} = 0 \tag{A.56}$$

from Lemma 5. If this outcome is realized, the production in period 0 will be

$$p_0(1,0) = q_0\phi^1 + \beta q(1,0)\phi^1. \tag{A.57}$$

In the second outcome, since $\underline{Z}(\beta) \leq 0$,

$$g_0^{1,DPE} = 0,\ g_0^{2,DPE} = 1 \text{ and} \tag{A.58}$$
$$g_1^{1,DPE} = 0,\ g_1^{2,DPE} = 1 \tag{A.59}$$

from Lemma 5. If this outcome is realized, the production in period 0 will be

$$p_0(0,1) = (1-q_0)\phi^2 + \beta(1-q(0,1))\phi^2. \tag{A.60}$$

Hence, the first outcome is realized if

$$p_0(1,0) > p_0(0,1), \tag{A.61}$$

or

$$q_0 > \frac{\phi^2}{\phi^1+\phi^2} + \frac{\beta}{\phi^1+\phi^2}\{(1-q(0,1))\phi^2 - q(1,0)\phi^1\}. \tag{A.62}$$

Let denote $G(q_0) = q_0 - \frac{\beta}{\phi^1+\phi^2}\{(1-q(0,1))\phi^2 - q(1,0)\phi^1\}$, so that the previous inequality rewrites

$$G(q_0) > \frac{\phi^2}{\phi^1+\phi^2}. \tag{A.63}$$

We find that

$$\frac{\partial G(q_0)}{\partial q_0} = 1 + \frac{\beta}{\phi^1+\phi^2}\{\frac{\partial q(0,1)}{\partial q_0}\phi^2 + \frac{\partial q(1,0)}{\partial q_0}\phi^1\}. \tag{A.64}$$

As $\frac{\partial q(0,1)}{\partial q_0} > 0$ and $\frac{\partial q(1,0)}{\partial q_0} > 0$, $\frac{\partial G(q_0)}{\partial q_0} > 0$. Hence, it is direct that there exists a unique threshold $\tilde{q}^{TEMP}$ such that

$$\begin{aligned} G(q_0) &> \frac{\phi^2}{\phi^1+\phi^2} \text{ if } q_0 > \tilde{q}^{TEMP}, \text{ and} \\ G(q_0) &\leq \frac{\phi^2}{\phi^1+\phi^2} \text{ otherwise.} \end{aligned} \tag{A.65}$$

Two important properties are worth stating before pursuing the proof. First, we find that $\tilde{q}^{TEMP}$ increases with β if and only if $(1-q(0,1))\phi^2 - q(1,0)\phi^1 > 0$. By symmetry, $(1-q(0,1))\phi^2 - q(1,0)\phi^1 > 0$ iff $\phi^2 > \phi^1$, we have demonstrated that $\tilde{q}^{TEMP}$ increases with β iff $\phi^2 > \phi^1$. Second, we can establish that $\tilde{q}^{TEMP} > \frac{\phi^2}{\phi^1+\phi^2}$ because $G(\frac{\phi^2}{\phi^1+\phi^2}) < \frac{\phi^2}{\phi^1+\phi^2}$.

Case 3: $q_0\phi^1 - (1-q_0)\phi^2 < 0$ **and** $\beta < \beta_2(q_0)$**.** As represented on the right panel of Figure A.8. There exists a certain threshold $\beta_2(q_0)$ such that if $\beta < \beta_2(q_0)$, $\overline{Z}(\beta) < 0$, and $\overline{Z}(\beta) \geq 0$ otherwise.

First, since $\overline{Z}(\beta) < 0$, then there is no equilibrium such that good 1 is provisioned in period 0. The equilibrium is necessarily such that

$$g_0^{1,DPE} = 0 \text{ and } g_0^{2,DPE} = 1. \tag{A.66}$$

As only good 2 is provided in period 0, the fraction of individuals of type 1 reaches $q_1(0,1)$ in period 1. From Lemma 5, $q_1(0,1) < \frac{\phi^2}{\phi^1+\phi^2}$ is satisfied, so

$$g_1^{1,DPE} = 0 \text{ and } g_1^{2,DPE} = 1. \tag{A.67}$$

Case 4: $q_0\phi^1 - (1-q_0)\phi^2 < 0$ **and** $\beta \geq \beta_2(q_0)$**.** From Figure A.8, we see that there can be two potential equilibrium outcomes. In the first outcome, since $\overline{Z}(\beta) \geq 0$,

$$g_0^{1,DPE} = 1,\ g_0^{2,DPE} = 0 \text{ and} \tag{A.68}$$

$$g_1^{1,DPE} = 1,\ g_1^{2,DPE} = 0 \tag{A.69}$$

from Lemma 5. If this outcome is realized, the production in period 0 will be

$$p_0(1,0) = q_0\phi^1 + \beta q(1,0)\phi^1. \tag{A.70}$$

In the second outcome, since $\underline{Z}(\beta) \leq 0$,

$$g_0^{1,DPE} = 0,\ g_0^{2,DPE} = 1 \text{ and} \tag{A.71}$$

$$g_1^{1,DPE} = 0,\ g_1^{2,DPE} = 1 \tag{A.72}$$

from Lemma 5. If this outcome is realized, the production in period 0 will be

$$p_0(0,1) = (1-q_0)\phi^2 + \beta(1-q(0,1))\phi^2. \tag{A.73}$$

Hence, the first outcome is realized if

$$p_0(1,0) > p_0(0,1), \tag{A.74}$$

or

$$q_0 > \frac{\phi^2}{\phi^1+\phi^2} + \frac{\beta}{\phi^1+\phi^2}\{(1-q(0,1))\phi^2 - q(1,0)\phi^1\}. \tag{A.75}$$

But since $\phi^2 > \phi^1$, $1-q(0,1) > q(1,0)$ by symmetry of the model. Hence, $(1-q(0,1))\phi^2 - q(1,0)\phi^1 > 0$. This implies that the inequalities

$$q_0 > \frac{\phi^2}{\phi^1+\phi^2} + \frac{\beta}{\phi^1+\phi^2}\{(1-q(0,1))\phi^2 - q(1,0)\phi^1\}. \tag{A.76}$$

and

$$\phi^1 q_0 + \phi^2(1-q_0) < 0 \text{ or equivalently } q_0 < \frac{\phi^2}{\phi^1+\phi^2} \tag{A.77}$$

cannot be simultaneously satisfied. We deduce that

$$p_0(1,0) \leq p_0(0,1) \tag{A.78}$$

necessarily holds. In case 4, the only equilibrium is then such that

$$g_0^{1,DPE} = 0,\ g_0^{2,DPE} = 1 \text{ and} \tag{A.79}$$

$$g_1^{1,DPE} = 0,\ g_1^{2,DPE} = 1 \tag{A.80}$$

Case B: $q_0 < \underline{q}^{DPE}$. In this case, independently from what is provided by the elites in period 0, $q(1,0) < \frac{\phi^2}{\phi^1+\phi^2}$, so the elites always provide good 2 in period 1. In period 0, the elites will provide good 1 if and only if $\beta < \beta_1(q_0)$, and good 2 otherwise.

Case C: $q_0 > \overline{q}^{DPE}$**.** In this case, independently from what is provided by the elites in period 0, $q(0,1) > \frac{\phi^2}{\phi^1+\phi^2}$, so the elites always provide good 1 in period 1. In period 0, the elites will provide good 1 if and only if $\beta > \beta_1(q_0)$, and good 2 otherwise.

This concludes the full characterization of the DPE.

In order to focus on a case where the DPE is continuous relative to the parameter values, we assume that β is sufficiently large, in that $\underline{Z}(q_0) < 0$ and $\overline{Z}(q_0) > 0$ for any $q_0 \in [0,1]$. Equivalently, $\beta \geq \overline{\beta}^{DPE} = \max_{q_0 \in [0,1]}(\beta_1(q_0), \beta_2(q_0))$, with

$$\begin{cases} \beta_1(q_0) = \max_{q_0 \in [0,1]} \left[(q_0\phi^1 - (1-q_0)\phi^2) \frac{1-q_0(1-q_0)(\eta+\overline{\phi})}{\eta q_0(1-q_0)\phi^2} \right] \\ \beta_2(q_0) = \max_{q_0 \in [0,1]} \left[(-q_0\phi^1 + (1-q_0)\phi^2) \frac{1-q_0(1-q_0)(\eta+\overline{\phi})}{\eta q_0(1-q_0)\phi^1} \right] . \end{cases} \tag{A.81}$$

When $\beta \geq \overline{\beta}^{DPE}$ and $q_0 < \underline{q}^{DPE}$ (Case B), the equilibrium is necessarily such that

$$g_0^{1,DPE} = 0,\ g_0^{2,DPE} = 1 \text{ and} \tag{A.82}$$

$$g_1^{1,DPE} = 0,\ g_1^{2,DPE} = 1. \tag{A.83}$$

When $\beta \geq \overline{\beta}^{DPE}$ and $q_0 > \overline{q}^{DPE}$ (Case C), the equilibrium is necessarily such that

$$g_0^{1,DPE} = 1,\ g_0^{2,DPE} = 0 \text{ and} \tag{A.84}$$

$$g_1^{1,DPE} = 1,,\ g_1^{2,DPE} = 0. \tag{A.85}$$

When $\beta \geq \overline{\beta}^{DPE}$ and $q_0 \in [\underline{q}^{DPE}, \overline{q}^{DPE}]$ (Case A), the equilibrium is necessarily such that

$$g_0^{1,DPE} = 1,\ g_0^{2,DPE} = 0 \text{ and} \tag{A.86}$$

$$g_1^{1,DPE} = 1,\ g_1^{2,DPE} = 0 \tag{A.87}$$

if $q_0 \geq \tilde{q}^{TEMP}$, and

$$g_0^{1,DPE} = 1,\ g_0^{2,DPE} = 0 \text{ and} \tag{A.88}$$

$$g_1^{1,DPE} = 1,\ g_1^{2,DPE} = 0 \tag{A.89}$$

otherwise. Hence, denoting $\tilde{q}^{DPE}$ the threshold such that

$$\tilde{q}^{DPE} = \begin{cases} \tilde{q}^{TEMP} \text{ if } \tilde{q}^{TEMP} \in [\underline{q}^{DPE}, \overline{q}^{DPE}] \\ \underline{q}^{DPE} \text{ if } \tilde{q}^{TEMP} < \underline{q}^{DPE} \\ \overline{q}^{DPE} \text{ otherwise,} \end{cases} \tag{A.90}$$

we have established that for any $q_0 \in [0,1]$,

$$g_0^{1,DPE} = 1,\ g_0^{2,DPE} = 0 \text{ and} \tag{A.91}$$
$$g_1^{1,DPE} = 1,\ g_1^{2,DPE} = 0 \tag{A.92}$$

if $q_0 \geq \tilde{q}^{DPE}$, and

$$g_0^{1,DPE} = 0,\ g_0^{2,DPE} = 1 \text{ and} \tag{A.93}$$
$$g_1^{1,DPE} = 0,\ g_1^{2,DPE} = 1 \tag{A.94}$$

otherwise. This concludes the proof of Proposition 2.

A.4 Proof of Proposition 3

Consider a SPE $\{(g_0^{1*}, g_0^{2*}); (g_1^{1*}, g_1^{2*}); (\tau^{1*}, \tau^{2*})\}$. Given the cultural dynamics (A.2),

$$q_1(g_0^{1*}, 1 - g_0^{1*}) > q_0 \text{ if and only if } g_0^{1*} > g_0. \tag{A.95}$$

with

$$g_0 = \begin{cases} \frac{q_0 + (\phi^2 - \phi^1)(1-q_0)}{\eta} \text{ if } \frac{q_0 + (\phi^2 - \phi^1)(1-q_0)}{\eta} < 1, \text{ and} \\ 1 \text{ otherwise.} \end{cases} . \tag{A.96}$$

We define an "excess provision" function $Z(\beta)$ as

$$Z(\beta) = g_0^{1*} - g_0. \tag{A.97}$$

From Definition 2, a revival occurs when:

$$\begin{cases} Z(\beta) > 0 \\ q_0 < \tilde{q}^{DPE}. \end{cases} \tag{A.98}$$

We will denote $g_0^1 = g$ to simplify the notations. We compute $\frac{\partial Z(\beta)}{\partial \beta}$:

$$\frac{\partial Z(\beta)}{\partial \beta} = \frac{\partial g_0^{1*}}{\partial \beta} = \frac{\partial^2 W_0(g_0^{1*}, 1 - g_0^{1*})/\partial g \partial \beta}{-\partial^2 W_0(g_0^{1*}, 1 - g_0^{1*})/\partial g^2}. \tag{A.99}$$

Since $-\partial^2 W_0(g_0^{1*}, 1-g_0^{1*})/\partial g^2 > 0$ in the SPE, $\frac{\partial Z(\beta)}{\partial \beta}$ and $\partial^2 W_0(g_0^{1*}, 1-g_0^{1*})/\partial g \partial \beta$ have the same sign, with

$$\frac{\partial^2 W_0(g_0^{1*}, 1-g_0^{1*})}{\partial g \partial \beta} = \frac{\eta q_0(1-q_0)}{1-q_0(1-q_0)(\eta+\overline{\phi})} \log\left(\frac{\phi^1 q_1(g_0^{1*}, 1-g_0^{1*})}{\phi^2(1-q_1(g_0^{1*}, 1-g_0^{1*}))}\right). \tag{A.100}$$

Hence,

$$\frac{\partial Z(\beta)}{\partial \beta} > 0 \tag{A.101}$$

iff

$$\phi^1 q_1(g_0^{1*}, 1-g_0^{1*}) > \phi^2(1-q_1(g_0^{1*}, 1-g_0^{1*})), \tag{A.102}$$

or equivalently iff

$$q_1(g_0^{1*}, 1-g_0^{1*}) > \frac{\phi^2}{\phi^1+\phi^2}. \tag{A.103}$$

We deduce the following intermediary result:

Lemma 6 $\frac{\partial g_0^{1*}}{\partial \beta} > 0$ *in at least one SPE if* $q_0 > k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$*, with*

$$k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0) = \begin{cases} p(q_0) \text{ if } p(q_0) \in [0,1] \\ 1 \text{ if } p(q_0) > 1 \\ 0 \text{ otherwise,} \end{cases} \tag{A.104}$$

with

$$p(q_0) = q_0 + \frac{1}{\eta q_0(1-q_0)}\left\{\frac{\phi^2}{\phi^1+\phi^2}(1-q_0(1-q_0)(\eta+\overline{\phi}) - q_0(1-q_0(1-q_0)(\eta+\overline{\phi})\right\}. \tag{A.105}$$

Proof. First, we define $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$ as the value of g_0^{1*} such that $q_1(g_0^{1*}, 1-g_0^{1*}) > \frac{\phi^2}{\phi^1+\phi^2}$ if and only if $g_0^{1*} > k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$.

The determination of $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$ is represented in Figure A.9 in the case where $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0) \in [0,1]$. Since $q_1(g_0^{1*}, 1-g_0^{1*})$ is linearly increasing in g_0^{1*}, there exists a unique threshold $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$ such that $q_1(g_0^{1*}, 1-g_0^{1*}) > \frac{\phi^2}{\phi^1+\phi^2}$ if and only if $g_0^{1*} > k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$.

Using the expression of $q_1(g_0^{1*}, 1-g_0^{1*})$ in (A.2), we deduce that $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$ can be expressed as in (A.104).

The main idea of the proof of the previous Lemma is represented in Figure A.10 in the case where $q_0 \notin [\underline{q}, \overline{q}]$. In this case, given Proposition 1, there is a unique SPE for any value of β. As represented in Figure A.10, if $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0) < q_0$, then $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0) < g_0^{1*}$ in $\beta = 0$,

Figure A.9: Determination of $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$.

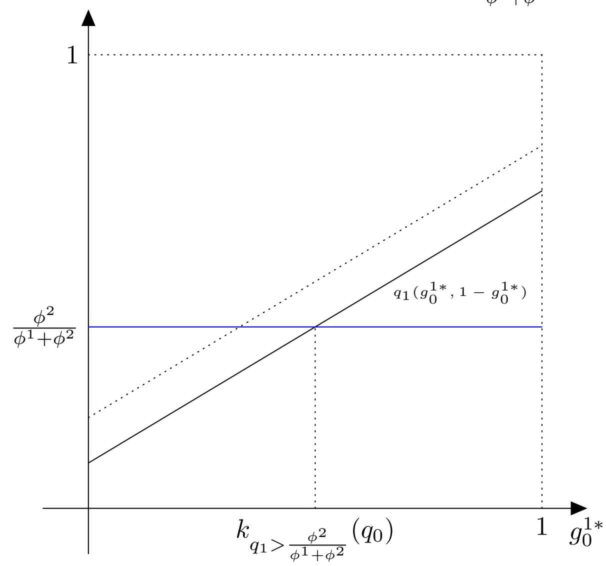
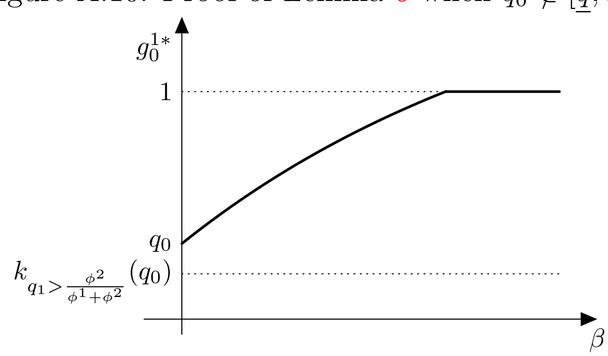


Figure A.10: Proof of Lemma 6 when $q_0 \notin [\underline{q}, \overline{q}]$



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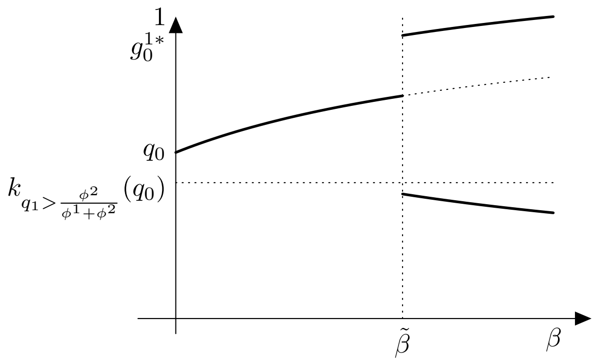
as $g_0^{1*} = q_0$ when $\beta = 0$. Hence, $\frac{\partial g_0^{1*}}{\partial \beta} > 0$ initially, and then $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0) < g_0^{1*}$ remains satisfied for $\beta > 0$ by monotonicity and continuity.

By contrast, if $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0) > q_0$, then $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0) > g_0^{1*}$ in $\beta = 0$, as $g_0^{1*} = q_0$ when $\beta = 0$. Hence, $\frac{\partial g_0^{1*}}{\partial \beta} < 0$ initially, and then $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0) > g_0^{1*}$ remains satisfied for $\beta > 0$ by monotonicity and continuity.

When $q_0 \notin [\underline{q}, \overline{q}]$, the proof of the Lemma is illustrated in Figure A.11. When $\beta < \tilde{\beta}$, there is a unique SPE (Proposition 1). By contrast, there is a bifurcation at $\beta = \tilde{\beta}$. The stable equilibrium when $\beta < \tilde{\beta}$ becomes unstable, and two stable equilibria emerge on each side of the unstable equilibrium, as represented in Figure A.11. One equilibrium is necessarily such that $\frac{\partial g_0^{1*}}{\partial \beta} > 0$. The other can be such that $\frac{\partial g_0^{1*}}{\partial \beta} < 0$, because it can be such that $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0) > g_0^{1*}$, as represented.

Figure A.11: Proof of Lemma 6 when $q_0 \in [\underline{q}, \overline{q}]$



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We have demonstrated that $\frac{\partial g_0^{1*}}{\partial \beta} > 0$ for at least one SPE if $q_0 > k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$, with $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$ given by (A.104). ■

The second step of the proof consists in proving the following result:

Lemma 7 *$q_0 > k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$ if and only if $q_0 > \tilde{q}_0$, with $\tilde{q}_0 \in (0, 1)$.*

Proof. This result is based on the fact that the function $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(.)$ is decreasing in q_0. Indeed, $q_1(g_0^{1*}, 1 - g_0^{1*})$ is increasing in q_0. Hence, when q_0 increases, the linear black curve representing $q_1(g_0^{1*}, 1 - g_0^{1*})$ in Figure A.9 is shifted upward. Hence, it is direct that the threshold $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$ decreases.

Given that $\lim_{q_0 \to 0} p(q_0) = +\infty$ and $\lim_{q_0 \to 1} p(q_0) = -\infty$, $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(0) = 1$ and $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(1) = 0$. Furthermore, as $k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$ is a decreasing function of q_0, we can represent it as in Figure A.12.

From Figure A.12, it is direct that there exists a threshold $\tilde{q}_0 \in (0, 1)$ such that $q_0 > k_{q_1 > \frac{\phi^2}{\phi^1+\phi^2}}(q_0)$ if and only if $q_0 > \tilde{q}_0$. ■

At $\beta = 0$,

$$Z(0) = q_0 - \frac{q_0 + (\phi^2 - \phi^1)(1 - q_0)}{\eta}, \tag{A.106}$$

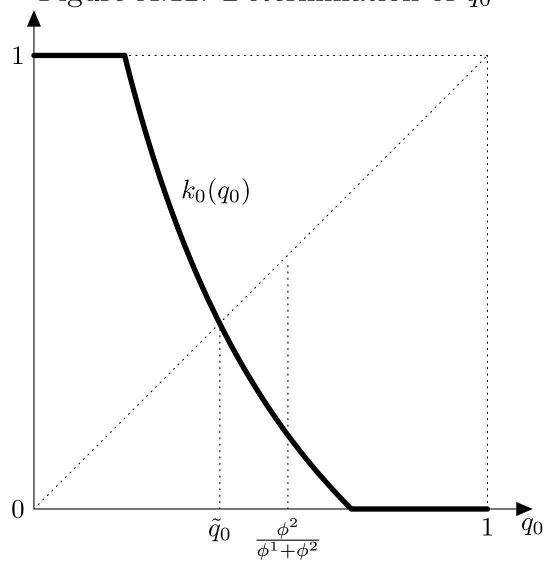
so

$$Z(0) < 0 \tag{A.107}$$

when $\phi^2 > \phi^1$ and $\eta \in [0, 1/4]$. Since $Z(.)$ is increasing with β when $q_0 > \tilde{q}_0$,

$$\lim_{\beta \to \infty} Z(\beta) = 1 - g_0 \geq 0. \tag{A.108}$$

Figure A.12: Determination of $\tilde{q}_0$



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Hence, there exists a unique threshold value $\tilde{\beta}_1 > 0$ such that if $\beta > \tilde{\beta}_1$, then $Z(\beta) > 0$. Importantly, from the definition of a cultural revival, it must be that $q_0 < \tilde{q}^{DPE}$. Hence, we have demonstrated that when $\tilde{q}^{DPE} > q_0 > \tilde{q}_0$, the conditions for a revival are fulfilled.

Last but not least, a revival occurs for a positive measure of parameters if $\tilde{q}^{DPE} > \tilde{q}_0$. This inequality is satisfied when β is sufficiently high. Indeed, $\tilde{q}^{DPE}$ is non-decreasing in β (Proposition 2) and $\tilde{q}_0$ is independent from β. Hence, there exists some threshold $\tilde{\beta}_2$ such that $\tilde{q}^{DPE} > \tilde{q}_0$ holds iif $\beta > \tilde{\beta}_2$.

We have demonstrated that there exists a threshold $\tilde{q}_0 \in (0,1)$, and a threshold $\overline{\beta} = \max(\tilde{\beta}_1, \tilde{\beta}_2) > 0$ such that if $\tilde{q}^{DPE} > q_0 > \tilde{q}_0$ and $\beta > \overline{\beta}$, then $Z(\beta) > 0$. Given that $\phi^2 > \phi^1$, there is a cultural revival favoring type 1 in the SPE. We have proven that $\tilde{q}^{DPE} > q_0 > \tilde{q}_0$ and $\beta > \tilde{\beta}$ are sufficient conditions for cultural revivals favoring type 1.