Currency Substitution, Price, Exchange Rate, and Welfare*

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September 30, 2020

Abstract

The distributed ledger technology, which can eliminate the third party in a transaction, has been developing rapidly in recent years, especially in the private cryptocurrency sector, with strong implications for monetary policy and payment system. This paper examines the potential welfare effect of currency substitution between fiat currency and private cryptocurrency when both can be used as a medium of exchange. A dynamic general equilibrium model is developed, which captures novel features of a currently operating private cryptocurrency payment processor and uses the relevant data of bitcoin. The findings indicate that a private cryptocurrency with high rate of return and low stable exchange rate not only can compete with legal fiat currency but also has the potential of crowding it out. This significantly impacts the effectiveness of monetary policy. Changes in price have a small positive effect on consumer's welfare while the effect of the exchange rate is significant and mixed. The results also suggest that more R&D is necessary to improve the currently operating blockchain network and online cryptocurrency exchange market to increase users' welfare.

Keywords: Monetary policy, Welfare, Currency substitution, Cryptocurrency, Payment system, Technology and innovations

JEL Classification: E4, E5, G2, O3

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*I sincerely appreciate my dissertation advisor Marcelle Chauvet, for her insightful advice and help. I also thank Huseyin Murat Ozbilgin and seminar participants at UC Riverside for clarification, helpful discussions, comments, and suggestions. All errors are my own.

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1 Introduction

Bitcoin (BTC) is a decentralized privately issued cryptocurrency in which the transaction takes place without intermediaries, using Distributed Ledger Technology (DLT).[1] This technology has been developing rapidly in recent years, especially in the private cryptocurrency sector, with strong implications for monetary policy and the payment system.

The goal of this paper is to examine the potential welfare effect of currency substitution when both privately issued cryptocurrency and domestic legal fiat currency co-exist in the economy. In particular, it investigates the welfare and policy implications associated with the transaction cost of using private cryptocurrency as a medium of exchange, and with the cost of replenishing monetary assets from nonmonetary assets.

A dynamic general equilibrium model is proposed to study the impact of cryptocurrency in the economy and in the monetary system. The model contains three agents: a representative household, a representative firm, and a government that plays the role of a central bank. Privately issued cryptocurrency (Bitcoin) and domestic legal fiat currency (U.S. dollar) can both be used to purchase goods and services, while only fiat currency has a unit of account function. When the cryptocurrency is used to make purchases, a private payment operator processes the transaction and charges an exogenous, time-varying transaction or network fee, which is independent of the transaction amount. The transaction fee for using fiat currency is assumed to be zero. A Cash-in-Advance (CIA) model along the lines of Freeman and Kydland (2000) and Özbilgin (2012) is constructed by introducing an asset market, which is used to replenish cryptocurrency and fiat currency balances from nonmonetary assets. The introduction of an asset market for cryptocurrency is a novel deviation from the classic CIA model, in which consumption expenditure is assumed to be financed by gross returns on saved monetary assets from the previous period. Firms operate in a competitive market and produce according to a regular Cobb-Douglas production function. The government conducts monetary policy by injecting lump-sum fiat currency into the economy. The paper conducts a welfare analysis of the cost of changes in price and nominal exchange rate, and other core variables of the model, based on several assumptions related to private cryptocurrency. First, private cryptocurrency is universally available to both consumers and merchants and its supply is exogenous. Second, the rate of return on cryptocurrency is measured by using the median value of the gross appreciation rate of the cryptocurrency. Third, the steady-state or long-run nominal exchange rate of cryptocurrency is assumed to be one since actual bitcoin values are very high and volatile, which discourages consumers from using it to purchase goods. Fourth, the rate of return on cryptocurrency is assumed to be independent of the nominal exchange rate of cryptocurrency. Bitcoin data are used for the quantitative analysis. Also, it is assumed that there is a private cryptocurrency payment processor such as BitPay in the economy to process cryptocurrency transactions.

This paper has several remarkable contributions. First, to the best of my knowledge, this is the first paper that incorporates the currently existing and operating private cryptocurrency payment processor features into a dynamic general equilibrium macroeconomic model. Second, this is the first paper to examine the welfare implication of currency substitution between fiat and private cryptocurrency by using an extended version of CIA model. Third, similar to Freeman and Kydland (2000) and Özbilgin (2012), the choice of payment instrument when purchasing goods and services is endogenously determined by the consumer by comparing the

[1] Bitcoin is the first-ever issued cryptocurrency and has been issued since 2009. Currently bitcoin has the highest market capitalization value among all private cryptocurrencies. The detailed working principle of bitcoin is described in Nakamoto (2019) while Böhme et al. (2015) present a thorough review of the bitcoin and related issues. Now there are many private cryptocurrencies with different designs. The coinmarketcap.com reports 5161 listed cryptocurrencies (as of March 04, 2020) and this number can be different on alternative tracking or exchange markets.

expected opportunity cost of using fiat currency and cryptocurrency. Fourth, inspired by the mining fees of the bitcoin blockchain network, an exogenous and time-varying transaction cost for each payment made by using cryptocurrency is included in the model. Fifth, different from the classic CIA model, an asset market is introduced to replenish money balances.[2] Therefore, money balances saved for the next period do not have to be the same as the money amount spent on purchasing goods and services in the current period. Sixth, this paper also examines the potential welfare impact of the transaction cost of using cryptocurrency as a payment instrument, and the replenishing cost of both currencies from nonmonetary assets. This is particularly important, given their striking policy and innovations implications for the cryptocurrency exchange platforms, cryptocurrency payment processors, and for the blockchain-powered Central Bank issued Digital Currency (CBDC). The proposed model is carefully calibrated to the U.S. economy. The model uses the exchange rate to denote the value of a CBDC or cryptocurrency in legal fiat currencies. Any changes in the price or exchange rate can impact the consumption and leisure choice of the consumer through wealth or substitution effect. With the availability of another currency that has the same or similar functions, the effect on the consumer's welfare is likely to be amplified. Until now, researchers mostly focused on the substitution or competition between the bank deposit and cryptocurrency or cash and cryptocurrency, which includes both the CBDC and privately issued cryptocurrency. However, the question of the effect of currency substitution on welfare when a cryptocurrency is used to purchase goods like legal fiat currency has not been thoroughly examined yet.[3]

The findings of this paper are as follows. First, changes in price have a positive impact on consumer welfare and cryptocurrency balance, but a significant negative impact on the fiat currency balance and traveling times to the asset market to replenish money balances from nonmonetary assets. The welfare cost decreases slightly as price increases. However, the nominal exchange rate between bitcoin and the U.S. dollar has a mixed effect on consumer welfare, cryptocurrency balance, and fiat currency balance. For example, for a 20% increase in the nominal exchange rate between cryptocurrency and fiat currency, the welfare cost rises by 6%, and fiat currency balance increases more than threefold, while private cryptocurrency balance decreases by around 100% at first. As the economy adjusts to the shock, fiat currency balance decreases some but continues to be higher than before the shock. Further, the welfare gains or losses are not as large as the corresponding increase in the price and nominal exchange rate given the relatively high appealing gross return on the cryptocurrency and very low transaction cost, which is the mean value of the time-varying transaction cost of the cryptocurrency. Second, the findings indicate that a private cryptocurrency with relatively high rate of return and low stable exchange rate can compete with legal fiat currency and has the potential to crowd it out. Third, the availability of the substitutable currency is likely to mitigate welfare losses. Both the substitution and wealth effect play an important role during the process. Fourth, in general, the replenishing cost of both currencies has a relatively big impact on the welfare of the consumers than the transaction cost of using cryptocurrency as a payment instrument. Thus, making it cheaper and convenient to buy or sell cryptocurrencies can enhance the usage of cryptocurrency as a payment instrument by decreasing the welfare losses of users. Additionally, having a stable and low transaction cost is also important to increase users' welfare.

The results have several important implications. First, countries experiencing higher infla-

[2]The proposed model follows the currency substitution model of Özbilgin (2012) and the endogenous fluctuations in monetary aggregates framework as in Freeman and Kydland (2000). Bank deposit and fiat currency are used as means of payment in Freeman and Kydland (2000) while domestic currency, domestic bank deposits, and foreign currency are used as payment instruments in Özbilgin (2012). Similarly, in this paper both domestic fiat currency and privately issued cryptocurrency are used as payment instruments.

[3]As discussed in more detailed in Section (3), several papers study the case of bitcoin or assumed central bank cryptocurrency having the function of a medium of exchange, but with different approaches, models, or in different environments.

tion or high prices should be cautious about privately issued cryptocurrencies that can compete with the legal fiat currency both as a medium of exchange and a store of value. Second, the effect of the cryptocurrency exchange rate on the consumer welfare is negative and significant while the effect of changes in price is small and positive. Therefore, potential CBDC and private cryptocurrency issuers are recommended to closely monitor the fluctuations in the cryptocurrency value vis-à-vis changes in prices. Third, private cryptocurrency issuers can reduce consumer welfare losses by investing more in R&D to increase network capabilities and decrease the cost of exchanges between different types of currencies.

The paper is organized as follows. Section (2) gives a background on the developments of cryptocurrency and the associated technology. Section (3) discusses related literature. Section (4) describes the proposed general equilibrium macroeconomic model and the calibration of the model to U.S. data. Section (5) discusses the quantitative analysis of the welfare cost of price changes and nominal exchange rate related to currency substitution. It also reports sensitivity analysis and technology implications. Section (6) concludes and makes final remarks.

2 Background

The Distributed Ledger Technology is a database shared by independent computers (i.e., nodes) in different sites and geographical locations, by individual or institutions that are used to record and synchronize transactions in their electronic ledgers (see, e.g., World Bank 2018). That is, instead of keeping data centralized as in a traditional ledger, the data are decentralized in multiple locations by multiple parties. The use of DLT is becoming increasingly widespread and is starting to bring pervasive changes in an array of sectors.[4] The blockchain technology is the most well known type of DLT and it has gained wide application in many sectors. Decentralization is the key feature of blockchain technology, and cryptocurrency, especially Bitcoin, is the most important and well-known application of blockchain technology.[5]

Bitcoin is a decentralized privately issued cryptocurrency in which transaction is carried out without intermediaries. Since the 2008 financial crisis, the confidence in the traditional banking system has declined (see, e.g., Mccarthy 2016, Bowman 2018). The popularity of decentralized DLT and the increasing demand for seamless, real-time, independent domestic or international payment system are the main driving forces behind the new central bank trend of considering or implementing projects related to the blockchain-powered CBDC (see, e.g., Boar et al. 2020). As Yermack (2015) and Baur et al. (2018) point out, bitcoin and other private cryptocurrencies are mostly treated as a speculative asset by holders rather than as currency. The main reasons are the extremely high volatility of bitcoin, lack of trust in the bitcoin system, and lack of wide merchant acceptance.[6] Figure (1) and Table (1) depict the volatility of bitcoin price compared to the USD/EUR exchange rate. The advantages of a blockchain-powered CBDC include preventing tax evasion and fighting crimes, decreasing the unbanked section of the population, and reducing the cost of maintaining the payment system.[7] The users' welfare and the stability of the financial system are always of great importance to central bank regulators. It is highly unlikely that fiat currency will be replaced with CBDC in

[4]For example, land registration and blockchain government in the public service sector, risk management, insurance, and cryptocurrency application in the financial sector, sharing economies, global authentication, and ownership in the data management sector (see, e.g., Labazova et al. 2019, Chen et al. 2018, Zheng et al. 2018).

[5]Major financial institutions and technological corporations have already realized the financial importance of blockchain technology. Some of them are conducting research while some others have even adopted it already. For example, JPM coin of JPMorgan Chase and Libra of Facebook. For more details of these two specific cryptocurrencies, check JPMorgan (2019) and Libra Associations (2020).

[6]See, e.g., Dumitrescu (2017), Schuh and Shy (2016), Henry et al. (2019), Jonker (2018), etc.

[7]For the cost of maintaining the U.S. fiat payment system, see Board of Governors of the Federal Reserve System (2019b) and Chakravorti and Mazzotta (2013).

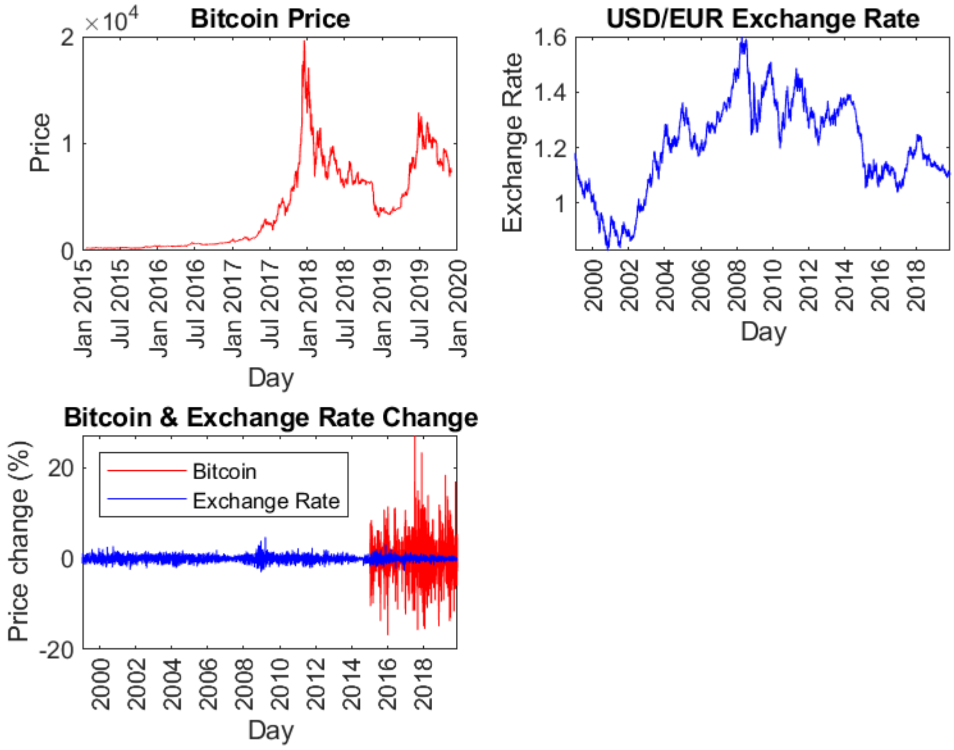

Figure 1: Volatility comparison of bitcoin price and USD/EUR exchange rate. Note: The daily Coinbase bitcoin price data is from Federal Reserve Economic Data (2019) and the period is 01/19/2015-12/03/2019. The daily USD/EUR exchange rate is also obtained from Federal Reserve Economic Data (2019) and the period is 01/04/1999-11/29/2019. Some not available (NA) values are dropped. Price changes are net increase in the percentage of both bitcoin and exchange rate values from the previous day. Coinbase is a major online cryptocurrency exchange market.

Table 1: Statistics of bitcoin and USD/EUR exchange rate values

	BTC price	BTC price change (%)	Exchange rate	Exchange rate change (%)
Mean	3930.2	0.2693	1.2031	0.00051678
SD	3978.3	3.8502	0.1662	0.6070
AC	0.9970	-0.0167	0.9990	0.0077

Note: This table shows descriptive statistics of Figure (1). SD stands for standard deviation, AC stands for the lag 1 autocorrelation. Here USD/EUR exchange rate is chosen to compare volatility with bitcoin since the U.S. and the European Union are the world's two largest economic entities and their currencies are relatively more stable than most other currencies. Further, according to International Monetary Fund (2020), the U.S. dollar and Euro are the two major reserve currencies for foreign exchange.

the very short run. In addition, fiat currency may still be used and valued by some sections of the population for quite some time. Therefore, the coexistence between domestic fiat currency and CBDC is expected.

Significant progress has been made by private payment processors to increase the wider adoption of private cryptocurrency as a payment instrument. One of the leading private cryptocurrency payment processors in the U.S. is the BitPay, which works as an intermediary and the exchange rate shock absorber between the two sides of a transaction.[8] All the goods and services are priced with domestic legal currency and merchants are guaranteed to receive the exact amount in domestic currency for sold goods and services. Even though BitPay is not an

[8]See BitPay (2019) for more details.

ideal CBDC payment processor, it helps private cryptocurrency to capture one more function of money - a medium of exchange. Therefore, private cryptocurrency like bitcoin can have the role of a medium of exchange and a store of value, which is one of the major motivations of this paper.

The second motivation is the necessity of modernizing the currently operating and widely used Real-Time Gross Settlement (RTGS) systems across the world.[9] Even though the transaction process is regarded as being in real-time, it does not complete in a second, and merchants have to bear most or all of the interchange fees according to the different local regulations.[10] When it comes to international transactions, the time needed for a complete transaction is much longer and the fees are higher.

The third motivation is the changing landscape of the payment instrument preference among consumers. Consumer-friendly mobile payments, card payments, and online payments have been gaining popularity and market shares thanks to the rapid technological advancement and the convenience they bring to users. Cash usages have already been low and still declining for many countries, particularly Sweden and Norway, while mobile payments like Wechat or Alipay have been crowding out cash in China.[11] However, there is a small decrease in cash usage in the U.S. from 2015 to 2018, while the cash payment value is relatively stable during this period.[12] Given the innovations in the blockchain technology accompanied with the increasing demand for modernizing payment systems, it is not prudent to ignore the possibility of employing a privately issued cryptocurrency with some basic money functions or a CBDC to meet the needs of the public.

3 Literature Review

Since the DLT and CBDC are still at the early age of development, there is a limited amount of research on currency substitution and welfare regarding cryptocurrency. Barrdear and Kumhof (2016) examine the effect of issuing an interest-bearing and universally accessible CBDC, which competes with bank deposits, on the macroeconomy by using a rich Dynamic Stochastic General Equilibrium (DSGE) model. Keister and Sanches (2019) also focus on the competition between CBDC and bank deposit, both of which are used as a medium of exchange, while studying the optimal design of cryptocurrency and conclude that CBDC can improve welfare, which is measured by the utility. Andolfatto (2018) examines the impact of CBDC on monopolistic private banks when bank deposit competes with cryptocurrency by using an overlapping generations model. Davoodalhosseini (2018) examines the optimal monetary policy under different combination of cash and interest-bearing CBDC with a discrete two-subperiod model and find that both cash and CBDC availability could reduce the overall welfare compared with when cash or CBDC is available exclusively. Both Keister and Sanches (2019) and Davoodalhosseini (2018)'s models stress the micro-foundation of money. Hong et al. (2018) study currency substitution between fiat currency and privately issued cryptocurrency with a search and match approach and investigate the crowding out effect. Hendry and Zhu (2019) study the interaction between the central bank and the private e-money issuer when the legal fiat currency competes with privately issued e-money. Kim and Kwon (2019) examine the CBDC's implication on the stability of the financial system with a simple overlapping gener-

[9] For example, the FedNew service in the U.S., Request to Pay in the U.K. See Board of Governors of the Federal Reserve System (2019a) and Faster Payments (2020) for details.

[10] To have a better understanding of different types of inter-bank real-time retail payment system, see Lai (2018). To understand the payment system and issues regarding the U.S. domestic payment system, refer to Cooper et al. (2019).

[11] See, e.g., Riksbank (2018), Khiaonarong and Humphrey (2019), Abkowitz (2018), Shan (2019).

[12] See, e.g., Kumar and O'Brien (2019), Kumar, Maktabi, et al. (2018).

ations model and the interesting point in this paper is the direct competition between bank deposit and CBDC, which is directly accessible by consumers at their central bank account. What is more, there are also other papers like Fernández-Villaverde and Sanches (2019) and Gandal and Halaburda (2014) that examine the competition between privately issued cryptocurrencies.

Kang and Lee (2019) examine the welfare implication of inflation when bitcoin and fiat currency are both used as a medium of exchange by using a search model, in which miners and the bitcoin transaction cost is modeled. Asimakopoulos et al. (2019) examine the unintended response of currency substitution between government fiat currency and private cryptocurrency to the technology, monetary, and preference shocks with a DSGE model, in which both currency balances are in the consumer's utility function and cryptocurrency producing firms, intermediate and final good producers are included. Schilling and Uhlig (2019b) analyze some basics of bitcoin pricing with the availability of U.S. dollars, both of them can be used for transactions, in a simple model that central banks target a stochastic U.S. dollar inflation through money injection while the bitcoin supply is deterministic in time. Benigno et al. (2019) study the currency competition in a two-country model when two national currencies and a global cryptocurrency are available. Schilling and Uhlig (2019a) is a theoretical paper that examines the relationship between currency substitution, asymmetric transaction costs, and the exchange fees. Schilling and Uhlig (2019a) is the most relevant paper in terms of using cryptocurrency as a means of payment and considering the endogenous determination of payment instrument choice between fiat currency (Dollar) and cryptocurrency (Bitcoin). However, it is also different. First, my paper does not consider the possibility of exchanging currencies in either direction between the fiat currency and cryptocurrency in a period and the choice of payment instrument is based on comparing the expected opportunity cost of using respective currencies. Besides, the usage of the fiat currency does not incur transaction costs. Second, the private cryptocurrency transaction cost in my paper is time-varying and irrelevant to the amount of purchase. Third, there is no specific exchange fee in my paper and the fluctuation in the cryptocurrency value is absorbed by the private cryptocurrency payment processor.

Without considering the availability of the cryptocurrency, currency substitution such as dollarization is old literature and there are plenty of papers that investigate different effects of the dual currency or asset competition. For money and credit as payment, please check Gillman (1993), Lucas Jr and Stokey (1983), and Lucas Jr and Stokey (1985). For domestic and foreign currency, please check Felices and Tuesta (2013), Minford (1995), Martin (2006), and Özbilgin (2012). For fiat currency and bank deposit competition, please check Henriksen and Kydland (2010), Freeman and Kydland (2000), and Özbilgin (2012).

4 Model

4.1 Households

The representative of a large number of infinitely lived identical households is endowed with one unit of time in each period and a stock of capital in the initial period, which is the period 0. The representative household values both consumption goods and leisure. In each period $t \geq 0$, a continuum of goods, of which types are indexed by $j \in [0, 1]$, are consumed. The representative household is forward-looking and maximizes expected discounted lifetime utility. In each period t, the representative household consumes $c_t(j)$ from each type of goods j and enjoys leisure l_t. Therefore, the household maximizes:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u\Big[min\Big\{\frac{c_t(j)}{(1-\omega)j^{-\omega}}\Big\}, l_t\Big], \quad \omega \in R_- \tag{1}$$

The representative household's period utility function $u(c_t, l_t)$ is in Leontief form and the Leontief parameter ω captures the curvature of the consumption amount $c_t(j)$ of each type of good. The utility function $u(c_t, l_t)$ is assumed to be increasing in both c_t and l_t, quasi-concave, twice continuously differentiable and satisfy Inada conditions (Freeman and Kydland 2000).

The representative household consumes $c_t(j)$ amount of each type of consumption good $j \in [0, 1]$ according to the optimization condition of Leontief ordering of the consumption goods:

$$\frac{c_t(j)}{(1-\omega)j^{-\omega}} = c_t \tag{2}$$

Replacing the first item of the utility function in eq (1) with (2), then the representative household's optimization problem becomes:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \tag{3}$$

What is more, private cryptocurrency CC_t, fiat currency M_t, and capital K_t are the available assets for the household. Both private cryptocurrency and fiat currency can be used as a means of payment for daily transactions. Fiat currency satisfies all three functions, a unit of account, a store of value, and a medium of exchange, of money while it is assumed that private cryptocurrency satisfies only two functions except for the unit of account. Therefore, the value of the goods is measured with domestic fiat currency (U.S. dollar). Whenever a household wants to pay with private cryptocurrency, the specific amount of cryptocurrency is needed to be converted into domestic fiat currency instantaneously, which is the point where the private cryptocurrency payment processor is needed. In this paper, I will not discuss the incentives for the household to use cryptocurrency as a payment instrument rather than just holding it as a speculative asset. Currently, most of the cryptocurrency users hold it because they expect to benefit from the value fluctuations. As far as I know, there is no any reliable survey or data that shows exactly what percentage of the cryptocurrency holders use it frequently to buy goods and services and the reasoning behind it.

Even though using fiat currency to purchase goods and services incur private and social transaction costs, it is simply assumed that the fiat currency transaction cost is equal to zero in this paper. But when it comes to the private cryptocurrency, from purchasing it on the online cryptocurrency exchange market and using it for purchasing consumption goods and services, it incurs several fees. If a consumer does not own any cryptocurrency, he can mine it or buy it using fiat currency (Online payment method such as PayPal and card payments), and it incurs transaction costs. Once the consumer possesses cryptocurrency and BitPay Prepaid Debit Card (After application, which costs \$9.99), then he can use a different variety of BitPay supported cryptocurrencies to make a purchase. It should be highlighted that BitPay is a payment processor, not a blockchain network and it still operates on the specific cryptocurrency's network. As BitPay (2019) describes, when a consumer decides to buy an item, the BitPay instantaneously locks the cryptocurrency exchange rate at the spot price and keeps at that rate for fifteen minutes. Therefore, the merchant will receive the exact amount in terms of fiat currency (The merchant can also choose the composition of cryptocurrency and fiat currency acceptance) while the consumer receives the goods and enjoys using this new technology. What is more, similar to the regular bitcoin transaction, the consumer needs to add "tips", which is mining fees for the miner's transaction verification effort on the blockchain network, during the purchasing process and twice. One is for the consumer to BitPay period and the second, which is called "Network fee" by BitPay, for the period from BitPay to the merchant. It is also worth noting that those tips and network fees, which are in terms of cryptocurrency, are not calculated as the percentage of the transaction amount. Instead, they are determined by the network environment and file size (Blockchain 2020(b)). Thus, it is a one-time transaction cost regardless of the transaction amount. For more details of BitPay, please check BitPay (2019).

Therefore, whenever cryptocurrency is used to purchase goods and services, it incurs the transaction cost τ_{cc} that fiat currency transaction is free from. For simplicity reasons, it is assumed that τ_{cc} is incurred only once during the whole transaction process.

At the beginning of each period $t \geq 0$, the representative household decides how much cryptocurrency and fiat currency to hold and the distribution ratio of monetary assets among the fiat currency and cryptocurrency will be kept constant until the beginning of the next period.[13] As it is mentioned earlier, fiat currency and cryptocurrency are both used as a payment instrument and a store of value. It is possible that the amount of money spent on purchasing goods and services may be more or less than the amount saved for the next period. Therefore, an asset market, which can be understood as a certain type of exchange market similar to private cryptocurrency online exchanges, is introduced to incorporate this issue. The representative household can replenish both fiat currency balance and cryptocurrency balance using nonmonetary assets in the asset market.[14] The n_t denotes the number of visits to the asset market and ϕ denotes the time cost of traveling to the asset market. The streams of income from the previous period are used to purchase consumption goods. After coming back from visiting the asset market each time in period t, it is assumed that the household makes symmetric purchases: buying the same combination of goods with different currencies each time. As Freeman and Kydland (2000) states "ϕ represents not the cost of going to the ATM, but the cost of replenishing all deposit and cash balances from nonmonetary assets" when studying the usage of bank deposit and cash as a means of payment. Similarly, ϕ measures the cost of replenishing both cryptocurrency and fiat currency balances from nonmonetary assets. If a dollar is replenished n_t times, then consumption goods worth n_t dollars can be purchased in a single period. Each trip costs ϕ units of time, and thus ϕn_t units of the time is spent on replenishing the money balances. What is more, for a cryptocurrency exchange market in the real world, ϕ can be understood as a "convenience" parameter that measures the degree of convenience when conducting cryptocurrency exchanges by using other monetary assets. Therefore, it is the cost that private cryptocurrency issuers should decrease to attract more users.

Furthermore, the optimal consumption level c_t^* can be gained by integrating $c_t(j)$ in eq (2) from 0 to 1. Before conducting any purchase, the household needs to decide which currency to use for a given optimal consumption level of c_t^*. Therefore, the household needs to compare the expected opportunity cost of using a private cryptocurrency to the opportunity cost of using fiat currency. Let $\theta_{t+1} = \frac{\Theta_{t+1}}{P_{t+1}}$ to denote the real gross appreciation rate of the cryptocurrency between period t and t+1 and $\bar{r}_{t+1}^k$ to represent the gross real rate of return on nonintermediated assets, which is capital acquired at time t in this model, net of depreciation rate.[15] After considering the time-varying transaction cost τ_{cct} for buying each item $c_t(j)$ and n_t times in

[13]For example, if the agent decides to hold 30 dollars fiat currency and 70 dollars value of the cryptocurrency at the beginning of the period t, then the ratio of the 30:70 will be kept the same during this period t.

[14]Please note that exchange between the domestic fiat currency and the cryptocurrency within a period is not considered here since the household can change their mind of holding any type of currency during any time point in a period t because of the value fluctuation of cryptocurrency. Also, the frequency of the data used for the calibration is quarterly.

[15]θ_t and $\Theta_t = \frac{S_t}{S_{t-1}}$ are the real and nominal gross appreciation rate of a cryptocurrency respectively. I treat Θ as a variable independent from the nominal exchange rate S as Özbilgin (2012) did. Otherwise, the gross appreciation rate of private cryptocurrency has to be one at the steady-state, which makes threshold level j^* as expressed in eq (8) infinite and cryptocurrency is not used during the transaction at all. What needs to be stressed is that I focus on the case when both the fiat currency and cryptocurrency are used as a transaction instrument, which means $j_t^* \in [0, 1]$.

each period, the expected opportunity cost of making the purchase with the cryptocurrency is:

$$E_t\Big[\frac{\Theta_{t+1}}{\pi_{t+1}} - \frac{S_t\tau_{cct}}{P_t}\frac{\bar{r}^k_{t+1}n_t}{c_t(j)}\Big] \tag{4}$$

where S_t is the nominal exchange rate of cryptocurrency and P_t is the price of consumption goods. It is assumed that the purchasing power parity holds in each period, which is consistent with the assumption made in Özbilgin (2012). The nominal exchange rate S_t is defined as:

$$S_t = \frac{Price\ of\ Domestic\ Fiat\ Currency_t}{Price\ of\ Private\ Cryptocurrency_t} \tag{5}$$

For example, for bitcoin, if $S_t = 10$, then it means one unit of bitcoin is worth 10 U.S. dollars in America. What is more, purchasing consumption goods with fiat currency is free from direct transaction cost and the only value change comes from inflation. Thus, the expected opportunity cost of using fiat currency is:

$$E_t\frac{P_t}{P_{t+1}} = E_t\frac{1}{\pi_{t+1}} \tag{6}$$

where the inflation rate is expressed as $\pi_t = \frac{P_t}{P_{t-1}}$. From the expression (2), it is easy to observe that $c_t(j)$ is an increasing function of the payment instrument choice threshold j_t. Therefore, as the size of purchased consumption goods j_t increases, the per capita transaction cost of using cryptocurrency to purchase goods goes down and the opportunity cost of using cryptocurrency increases. Thus, the expression (4) is an increasing function of j_t while expression (6) is irrelevant to the purchase size. Thus, it is obvious that there is a threshold level j_t^* such that the household will use cryptocurrency for purchases when $j_t^* < j_t \leq 1$ and use fiat currency if $0 \leq j_t < j_t^*$. The representative household is indifferent between using cryptocurrency and fiat currency to conduct transactions only if the expected opportunity cost of using cryptocurrency in a transaction is the same as the expected opportunity cost of using fiat currency. This condition can be expressed as:

$$E_t\Big[\frac{\Theta_{t+1}}{\pi_{t+1}} - \frac{S_t\tau_{cct}}{P_t}\frac{\bar{r}^k_{t+1}n_t}{c_t(j)}\Big] = E_t\frac{1}{\pi_{t+1}} \tag{7}$$

which then implies the optimal threshold level j_t^* is:

$$j_t^* = (\frac{c_t}{n_t})^{\frac{1}{\omega}}\left[E_t\frac{\frac{1}{\pi_{t+1}}(1-\omega)(\Theta_{t+1}-1)}{\frac{S_t}{P_t}\tau_{cct}\bar{r}^k_{t+1}}\right]^{\frac{1}{\omega}} \tag{8}$$

Threshold choice level j_t^* is positively related to n_t, τ_{cct}, $\bar{r}^k_t$, and nominal exchange rate S_t while negatively related to c_t, P_t, and Θ_{t+1}. Any increase in τ_{cct} or S_t while keeping other variables constant makes the domestic currency value of the cryptocurrency transaction cost expenditure expensive. It implies an increase in the value of j_t^* to maintain equality in eq (7). Therefore, consumers will move to buy more goods with fiat currency. However, any increase in the price level P_t makes the real transaction cost of using cryptocurrency less expensive and can change the expected inflation rate of π_{t+1} at the same time because of the possible update in expected price level P_{t+1}. The increases or decrease in the expected opportunity cost of using a cryptocurrency or fiat currency for a payment, which then leads to the corresponding change in j_t^*, demand for cryptocurrency and fiat currency, depends on the magnitude of changes in both P_t and expected π_{t+1}. What is more, any increase in the gross nominal appreciation rate Θ_{t+1} of a cryptocurrency will increase the expected gross real return net of transaction cost on the cryptocurrency and induce the consumer to use or demand more cryptocurrency, which implies j_t^* is lowered to maintain the equality in eq (7).

The time-varying transaction cost τ_{cct} is an exogenous variable and here it is simply assumed that the deviation of τ_{cct} from its steady-state level τ_{cc} follows a simple autoregressive process of order one (AR(1)), which is:

$$\tilde{\tau_{cct}} = \rho_{cc}\tilde{\tau_{cct-1}} + \varepsilon_t^{cc}, \quad \varepsilon_t^{cc} \sim \mathcal{N}(0, \sigma_{cc}^2) \\ , \quad \tilde{\tau_{cct}} = (\tau_{cct} - \tau_{cc}) \tag{9}$$

where $\rho_{cc} \in [0, 1]$ and measures the persistence of the transaction cost while ε_t^{cc} is an innovation shock drawn from a normal distribution with mean zero and variance σ_{cc}^2.

Both fiat currency and cryptocurrency are used to purchase consumption goods and services. As the assumption made earlier, the household conducts symmetric purchases and the money balances are replenished n_t times in each period. Therefore, the Cash-in-Advance constraints can be written as below for both the fiat currency and cryptocurrency:

$$\int_0^{j_t^*} c_t(j)dj \leq n_t \frac{M_t}{P_t} \tag{10}$$

$$\int_{j_t^*}^1 c_t(j)dj \leq n_t \frac{CC_t S_t}{P_t} \tag{11}$$

The above constraints are binding. Plugging in $c_t(j)$ from eq (2), eq (10) and (11) can be simplified to:

$$c_t j_t^{*(1-\omega)} = n_t \frac{M_t}{P_t} \tag{12}$$

$$c_t(1 - j_t^{*(1-\omega)}) = n_t \frac{CC_t S_t}{P_t} \tag{13}$$

The budget constraint at period t is:

$$c_t + k_t - (1-\delta)k_{t-1} + \frac{M_t}{P_t} + CC_t \frac{S_t}{P_t} + \frac{S_t \tau_{cct}(1 - j_t^*)}{P_t} = \\ r_t^k k_{t-1} + w_t h_t + CC_{t-1}\frac{S_t}{P_t} + tr_t + \frac{M_{t-1}}{P_t} \tag{14}$$

where k_t is the real capital lent to the producer and h_t is the working time supplied to the production sector at time t. The r_t^k and w_t are the real rate of return on capital and the real wage paid to a unit of labor employed at time t respectively. The tr_t is the real lump-sum fiat money transferred by the government to the household in each period t. Since the cryptocurrency is privately issued and the supply is exogenous in this model, the government can only supply and control fiat currency. The quantity of the private cryptocurrency is freely determined by the market or issuers or a cryptocurrency protocol. It is assumed that the demand for the cryptocurrency is always satisfied. The left side of eq (14) is the total expenditure, which includes consumption, investment, and savings of different currencies, of the representative households at time t with including the extra transaction expenditure involved by using the privately issued cryptocurrency. However, the right side of eq (14) is the total income received at time t. The income includes wage income, rental income, and return on money savings from the previous period.

The representative household is endowed with one unit of total time in each period and is distributed among leisure, working hours, and time spent on going to the asset market to replenish money balances. Therefore, the time constraint is:

$$l_t + h_t + \phi n_t = 1 \tag{15}$$

4.2 Firms

There are a large number of firms operating in the production sector at time t. Therefore, any firm in this sector is operating in a competitive market. A representative good producer employs capital k_t and hires labor h_t at rates of r_t^k and w_t at time t. It is assumed that production technology is given by a constant-returns-to-scale Cobb-Douglas production function, which is:

$$y_t = z_t k_t^{\alpha} h_t^{1-\alpha}, \quad \alpha \in (0,1) \tag{16}$$

where z_t is the exogenous productivity shocks and α measures the share of capital stock in the production. It is assumed that z_t follows an AR(1) process as in Özbilgin (2012) and Walsh (2010).

$$\ln z_t = \rho_z \ln z_{t-1} + \varepsilon_t^z, \quad \varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2) \tag{17}$$

where $\rho_z \in [0,1]$ measures the persistence of the shocks while ϵ_t^z is the innovation with mean zero and variance σ_z^2. In each period t, the firm owner optimizes his profit Π_t, which is:

$$\Pi_t = y_t - r_t^k k_t - w_t h_t \tag{18}$$

Since the firm is operating in a competitive market, the profit Π_t is zero.

4.3 Government

The government in this model plays the role of the central bank and is responsible for the monetary policy as in Özbilgin (2012), Henriksen and Kydland (2010), and Freeman and Kydland (2000). However, there is a representative financial institution in all of those three models and this paper does not incorporate a bank in the model. Thus, stock of nominal fiat money rather than monetary base, which includes required reserves stored at a central bank, follows a certain growth path as in Walsh (2010). Nominal fiat money balance M_t growth at the gross rate of g_{mt} and the path can be expressed as:

$$M_t = g_{mt} M_{t-1} \tag{19}$$

The lump-sum transfer or injection TR_t of fiat currency is the net change in the fiat money balance between the period t and $t-1$. Therefore, it can be expressed as:

$$TR_t = (g_{mt} - 1) M_{t-1} \tag{20}$$

Adjusted by the price P_t, the real transfer amount is:

$$tr_t = (1 - g_{mt}) \frac{M_{t-1}}{P_t} \tag{21}$$

It is assumed that the deviation of nominal fiat money balance growth rate from its steady-state level g_m, which is $\tilde{g_{mt}} = (g_{mt} - g_m)$, follows a simple AR(1) process as in Özbilgin (2012):

$$\tilde{g_{mt}} = \rho_m \tilde{g_{mt-1}} + \varepsilon_t^m, \quad \varepsilon_t^m \sim \mathcal{N}(0, \sigma_m^2) \tag{22}$$

where ρ_m and ε_t^m represents the persistence of the monetary policy and shocks (innovation) to the monetary policy. What is more, ε_t^m follows a normal distribution with mean zero and variance σ_m^2.

4.4 Equilibrium

In this model, there are three agents: a representative household, a representative firm, and the government. At any period t, the competitive equilibrium is a sequences of quantities $Q = \left\{ c_t, k_t, M_t, CC_t, h_t, n_t, j_t^* \right\}_{t=0}^{\infty}$, a sequences of prices $V = \left\{ P_t, r_t^k, w_t, \Theta_t, S_t \right\}_{t=0}^{\infty}$, and the initial given values of k_0, CC_0, and M_0 such that for any given price V and exogenous shock process z_t, $\tilde{\tau_{cct}}$, and $\tilde{g_{mt}}$:

- $\left\{ c_t^d, k_t^s, M_t^d, CC_t^d, h_t^s, n_t, j_t^* \right\}_{t=0}^{\infty}$ solves the representative household's maximization problem.[16]
- $\left\{ h_t^d, k_t^d, y_t^s \right\}_{t=0}^{\infty}$ solves firm's profit maximization problem.
- Transversality conditions hold.
- Markets are clear. This market includes the goods market, capital market, and money market.
  - Goods market: $y_t^s = c_t^d + k_t^d - (1-\delta)k_{t-1}^d$;
  - Capital market: $k_t^s = k_t^d$;
  - Labor market: $h_t^s = h_t^d$;
  - Fiat currency market: $M_t^s = M_t^d$;
  - Private cryptocurrency market: $CC_t^s = CC_t^d + \tau_{cct}(1 - j_t^*)$;[17]

4.5 Steady State and Calibration

For the calibration of the relevant parameters, quarterly U.S. data of period 2010Q4-2019Q3 is used and all the macroeconomic data about the U.S. economy and some of the cryptocurrency data are from Federal Reserve Economic Data (2019) while some other private cryptocurrency data is gained from Blockchain (2020[a]).[18] To the specific details of data set, please check the Appendix (7.2).[19]

The utility function is assumed to be in the following form:

$$u(c_t, l_t) = \frac{1}{1-\nu} \left[c_t^{\gamma} l_t^{1-\gamma} \right]^{1-\nu}, \quad \gamma \in (0,1), \quad \nu > 0 \tag{23}$$

where γ and ν are the share parameter and risk aversion parameter of the utility function respectively. The Leontief parameter ω captures the curvature of the consumption amount of each type of good, which is a function of the size of the good. Consumption amount $c_t(j)$ curves for multiple ω are shown in Figure (2).

When $\omega = -1$, consumption amount is linear in purchase size $j_t \in [0,1]$. Thus, the household is indifferent with spending on different sizes of purchase. When $|\omega| > 1$, the

[16] Please note that lowercase s stands for supply while lowercase d denotes for the demand. These notations s and d are only used here to differentiate demand and supply sides. The lowercase s has nothing to do with the uppercase S that denotes the nominal exchange rate of a cryptocurrency.

[17] The supply of the private cryptocurrency is assumed to be exogenous. Theoretically, private firms can supply as much cryptocurrency as demanded by consumers. In reality, there is a limit and demand affects exchange rate S.

[18] I sincerely appreciate Blockchain.com for making their data available for research.

[19] Regarding the calibration and simulation techniques, I refer to both Sims (2010) and Walsh (2010).

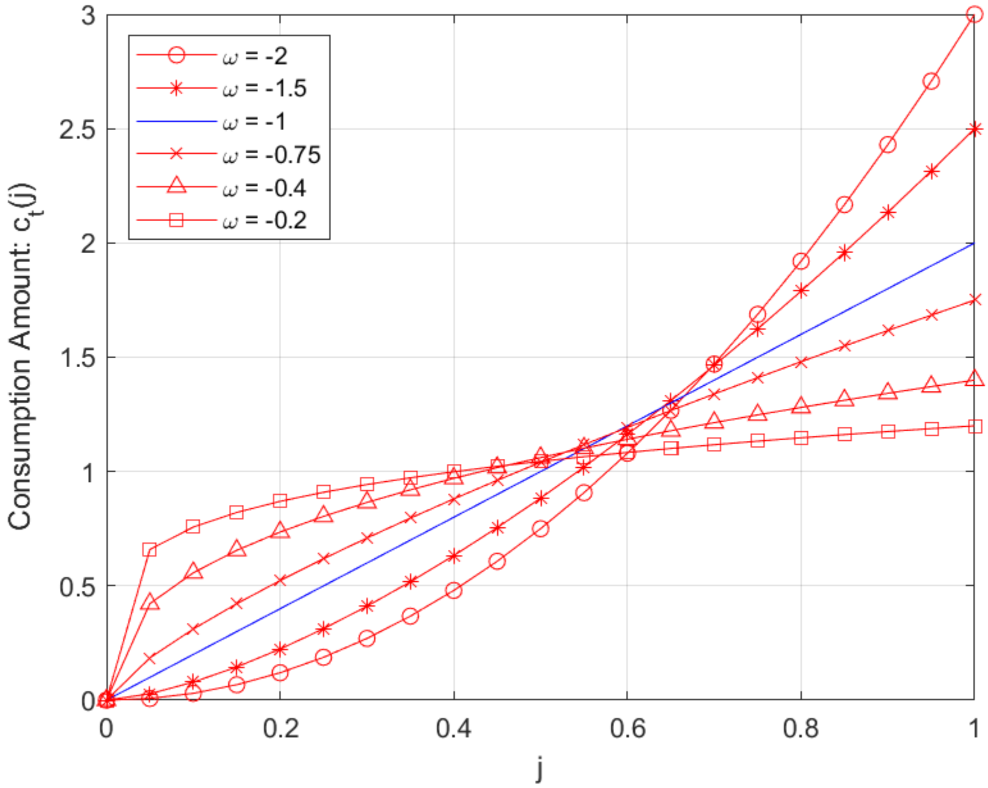

Figure 2: Consumption amount, size j, and ω

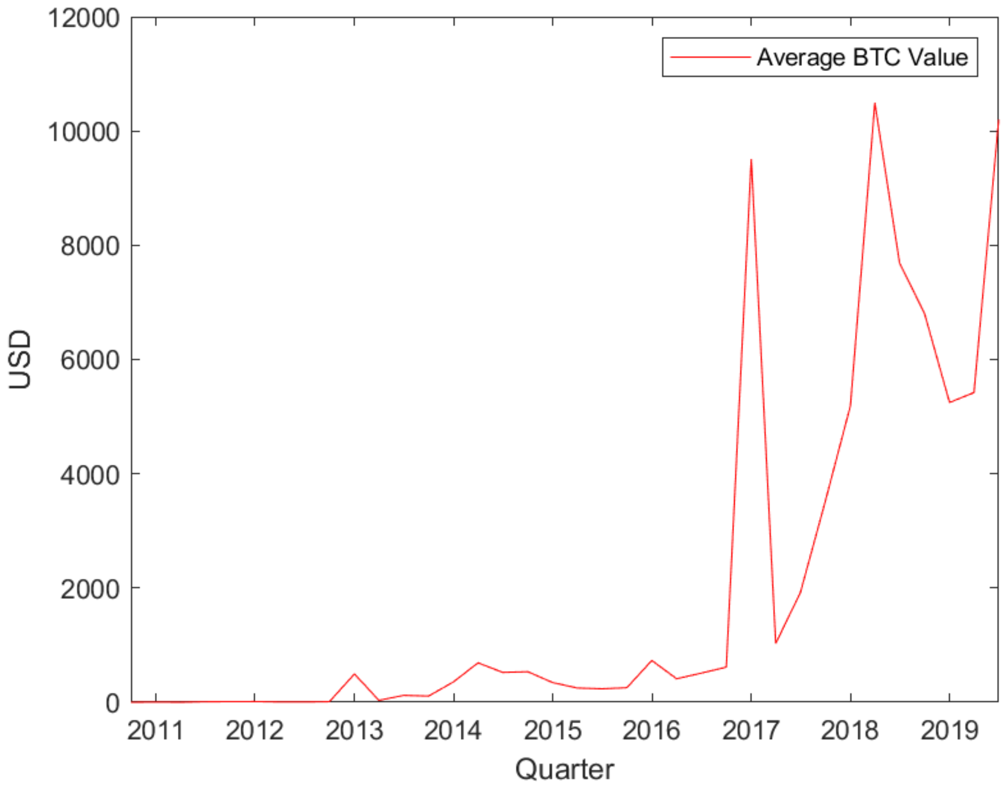

Figure 3: Quarterly average BTC market value in U.S. Dollars (USD). Note: The data is daily data from Blockchain (2020[a]) and the frequency is adjusted from daily into quarterly.

consumption amount curve is convex and convexity increases as $|\omega|$ goes up. It implies that consumers are likely to buy more of the bigger size goods when $|\omega| > 1$. When $|\omega| < 1$, the consumption amount curve $c_t(j)$ is concave and concavity increase as $|\omega|$ goes down. Hence, consumers are likely to buy more of the smaller size goods. Freeman and Kydland (2000) simply study the case of $\omega = -1$ and Henriksen and Kydland (2010) choose to set $\omega = -1.5$ after analyzing the cross-correlation between the price and output under three different policy regimes, different ω, and find that the price gets more counter-cyclical as $|\omega|$ increases. Özbilgin (2012) follows Henriksen and Kydland (2010). In this paper, considering the mathematical feasibility of solving the model, I follow Freeman and Kydland (2000) and simply set $\omega = -1$.

As Freeman and Kydland (2000), I set the capital depreciation rate $\delta = 0.025$, which is consistent with the long-run investment to output ratio of 0.25 and capital to output ratio of 10, risk aversion parameter $\nu = 2$, and average time that the representative household allocates to work $h = 0.3$. The steady-state net real rate of return r_t^k on capital is set to be 0.04 as in Henriksen and Kydland (2010) and it is consistent with the value of β.

The capital stock share parameter α is calibrated such that the labor share of national income is 0.5938. Thus, α is approximated to be 0.40. Since $h = 0.3$, the share parameter γ of the utility function is restricted to 0.3537. The value of the representative household's discount factor β is calibrated to be 0.9852. The rate of return on capital $\bar{r}^k$ can be expressed as $r^k + 1 - \delta$ and it is 1.0150. The steady-state value of price level P is set to equal to the mean value of the core personal consumption expenditure price index and is 1.0431. Since the inflation is $\pi_t = \frac{P_t}{P_{t-1}}$, steady-state value of the inflation π is one.[20]

Whether it is a stablecoin (a type of private cryptocurrency that has relatively stable value and lower volatility or has designated specific target) or a volatile cryptocurrency such as Bitcoin, the net gain or loss is determined by the cryptocurrency value difference between the two different time points. Therefore, there is no any specific interest rate like bank deposit rate in Freeman and Kydland (2000), Henriksen and Kydland (2010), and Özbilgin (2012) designated for the private cryptocurrency.[21] However, the appreciation rate can be considered as the interest rate of a cryptocurrency. The challenge to calibrate the value of the Θ also comes from the fact that the domestic currency value of a cryptocurrency transaction cost τ_{cc} is also related to the exchange rate S_t of a cryptocurrency. Therefore, as I mentioned earlier, I will treat Θ_t as an independent variable and the nominal exchange rate S_t of a private cryptocurrency as another independent variable. This assumption enables this model to capture the net gains or losses from the cryptocurrency value fluctuations at the steady-state while also considering the domestic currency value of the transaction cost. Since bitcoin is the most widely known cryptocurrency and ranked first in market value, I will employ most of the features of bitcoin and bitcoin blockchain network except the bitcoin exchange rate as the features of an appealing private cryptocurrency that I study here.[22] I simply assume the steady-state nominal exchange rate S between the private cryptocurrency and the domestic legal fiat currency is one.

Figure (5) shows the quarterly average bitcoin value of the cost of the per-transaction conducted on BTC blockchain network and Figure (3) displays the quarterly average value of bitcoin. The steady-state value of the cost per-transaction τ_{cc} is set to equal to the mean bitcoin

[20]Please note that since the values at the steady-state are very small, to get the possible highest accuracy, I will not approximate values during the coding process.

[21]In Özbilgin (2012), he treats the domestic currency depreciation rate as return on foreign currency and the transaction cost of using aggregated foreign currency and bank deposit is a parameter pinned down by some ratio. Therefore, the transaction cost is irrelevant to the foreign currency exchange rate.

[22]As shown in Figure (1), the daily value of BTC is ranging from near zero to as high as \$20000. If the mean value of that exchange rate is used as a steady-state value of the private cryptocurrency, it will make the domestic fiat currency value of the transaction cost of using BTC as a payment instrument so high even the BTC value of transaction cost is nearly negligible. This will greatly discourage consumers from using BTC for payment purposes.

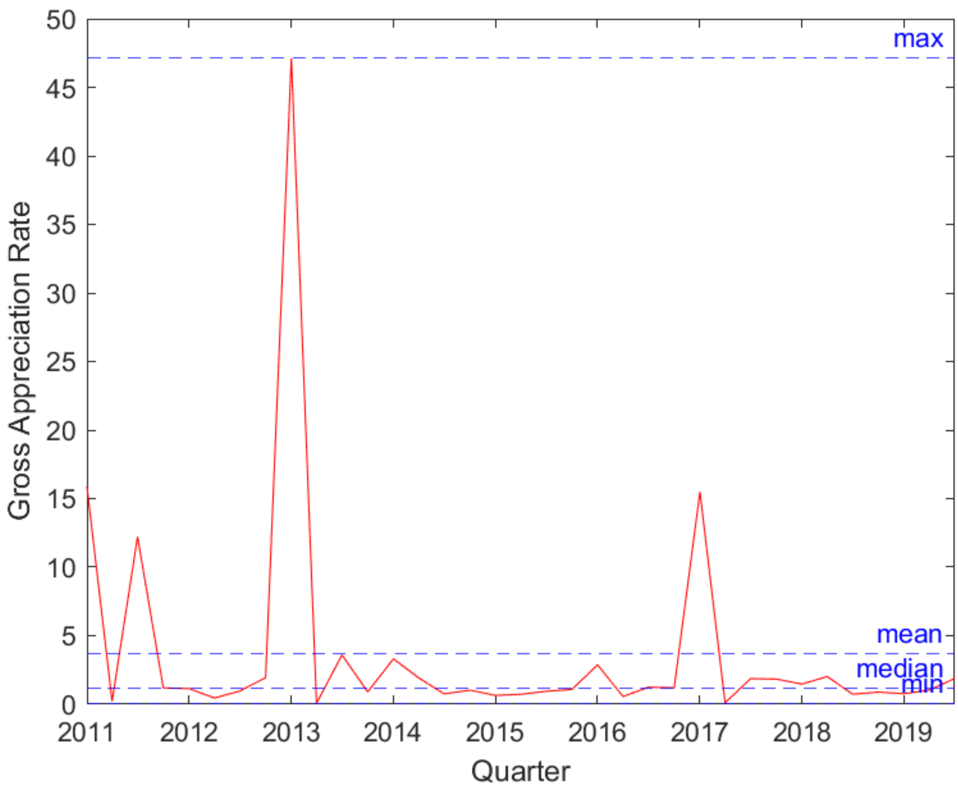

Figure 4: Quarterly BTC gross appreciation rate (Not in percentage). Note: The maximum, mean, median, and minimum values of the gross appreciation rate are 47.1247, 3.7166, 1.1350, and 0.0674 respectively.

	Minimum	Medium	Mean	Maximum
ϕ	-0.00015	0.0014	0.00005	0.000003
n	177.1590	3.7129	1503.2	433340
j^*	-2.5918	0.3752	7.5499	128.1863
M	0.0413	0.0413	0.0413	0.0413
CC	-0.0352	0.2521	-0.0406	-0.0413

Table 2: Comparative outcomes for the different BTC gross appreciation rate at the steady-state. The minimum, medium, mean, and maximum values are from Figure (4).

cost per-transaction and it is 0.0293. By comparing bitcoin values in Figure (1) and (3), it is easy to observe that the peak of the quarterly average BTC market value is nearly the half of the daily value peak shown in Figure (1).

Figure (4) shows the quarterly gross appreciation rate of the bitcoin, and the highest value reaches as much as 47.1247. What is more, the appreciation rate of cryptocurrency can change the incentive of using bitcoin for purchasing goods and this paper only focus on the case that both the fiat currency and private cryptocurrency are used as a medium of exchange, which suggests $j^* \in [0, 1]$. Therefore, the selection of a proper appreciation rate of bitcoin is necessary. Table (2) shows the steady-state values of the important variables corresponding to the minimum, median, mean, and maximum values of the bitcoin gross appreciation rate. As the appreciation rate goes up, the threshold level j^* goes up. Only the median value of the bitcoin gross appreciation rate offers a reasonable j^* that both the fiat currency and private cryptocurrency are used for trading goods and services. Therefore, the steady-state gross nominal rate of return Θ on the private cryptocurrency is set to equal to the median value of

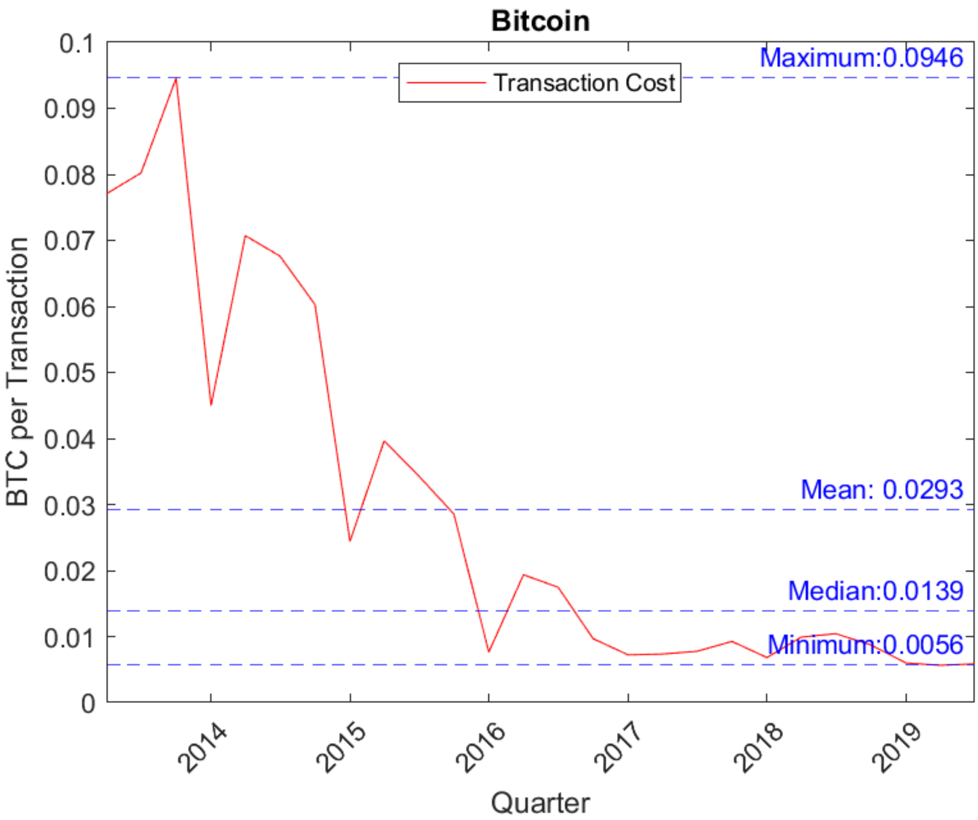

Figure 5: Per-transaction cost. Note: The transaction cost data is available at Blockchain (2020[a]). It is calculated by dividing the daily dollar value of the cost per-transaction by the market value of bitcoin. Several outliers are dropped and the figure starts from 2013Q2. The exchange rate used here is the average across major exchange markets since the prices across different exchange markets differ slightly. The BTC market value data from Federal Reserve Economic Data (2019) is just from one single exchange market and only used once in Figure (1).

the quarterly gross appreciation rate of the bitcoin exchange rate.[23] As a result, Θ is 1.1350.

The time cost ϕ of visiting the asset market to replenish money balances is not a common term used in the classic CIA models. I will pin down the value of ϕ as Freeman and Kydland (2000) and Özbilgin (2012) did by setting the steady-state domestic currency to consumption ratio equal to the sample average. At the steady-state, ϕ can be expressed as:

$$\phi = \frac{\frac{\tau_{cc}^4(1-\beta)}{c^2} + (\frac{M}{Pc})^2 \frac{S}{P}\tau_{cc}^2 \left[\frac{2\frac{1}{\pi}(\Theta-1)}{\frac{S}{P}\bar{r}^k}\right]^3}{(\frac{M}{Pc})^2 wc \left[\frac{2\frac{1}{\pi}(\Theta-1)}{\frac{S}{P}\bar{r}^k}\right]^4} \tag{24}$$

As Judson (2017) estimates, around 60% of the U.S. dollar is circulating outside of the United States and this can be even higher for high-denomination bills such as $100 and this ratio has been steadily increasing since the 1960s. I will simply set the domestic currency ratio equal to one-third as in Freeman and Kydland (2000). Therefore, the steady-state real domestic currency to consumption ratio $\frac{M}{Pc}$ is 0.0379. Then the value of ϕ is pinned down to 0.0492, which implies that the representative household spends 5.1902 minutes each day for portfolio management. The time cost of visiting an asset market, which will be used to replenish bank deposit and fiat currency balances, in both Freeman and Kydland (2000) and Henriksen and Kydland (2010) is 0.00076, which can be explained as the representative household spends around one hour quarterly for managing their assets. The much higher value of ϕ in this model can be explained by a relatively high transaction cost of using private cryptocurrency and a higher rate of return on the cryptocurrency than the bank deposit. It suggests that if private cryptocurrency issuers want more customers to hold or use their currency while spending less time on managing their asset portfolio, then they are advised to keep the transaction cost τ_{cc} low and the price of their cryptocurrency less volatile and more stable.

The persistence parameters $\rho_m, \rho_z, \rho_{cc}$ and innovation variances $\sigma_m^2, \sigma_z^2, \sigma_{cc}^2$ of the shock processes are estimated by using the U.S. data and linear detrending method. Calibrated values of persistence parameters are $\rho_m = -0.2543$, $\rho_z = 0.80196$, and $\rho_{cc} = 0.4436$. Values of standard deviations are $\sigma_m = 0.0042$, $\sigma_z = 0.0035$, and $\sigma_{cc} = 0.0124$. The calibrated parameters are summarized in Table (3).

5 Quantitative Analysis

Table (4) shows the steady-state values of some important variables. The fraction of the time spent on managing monetary assets, which is $n\phi$, is 1.3931. This value corresponds to the average daily and quarterly portfolio management time of 5.1902 and 472.3125 minutes respectively.

5.1 Welfare Analysis

This goal of this paper is to examine the welfare effect of currency substitution in an environment where both the fiat currency and private cryptocurrency are used to conduct transactions. I focus on the change in price and nominal exchange rate and their ultimate effect on consumer welfare through different channels. In this paper, the cryptocurrency has some appealing features such as unit nominal exchange rate and a stable high rate of return.

[23] Several important and well-known statistical values of gross appreciation rate are tested in Table (2). It is clear that there is a range of gross appreciation rates that give a reasonable threshold level j^*. Therefore, the selection of the median gross appreciation rate is random.

Parameter	Description	Value
δ	Capital depreciation rate	0.025
γ	Utility function consumption share parameter	0.3537
ω	Leontief utility parameter	-1
ν	Coefficient of relative risk aversion	2
α	Share of capital stock	0.4
β	Subjective discount factor	0.9852
τ_{cc}	Cost per-transaction cryptocurrency	0.0293
ϕ	Asset market trip cost	0.0014
P	Core PCE price level	1.0431
Θ	Gross rate of return on cryptocurrency (Bitcoin)	1.1350
ρ_z	Persistence of productivity shocks	0.8012
σ_z	Standard deviation of the productivity shock	0.0035
ρ_{cc}	Persistence of cryptocurrency transaction cost shock	0.4436
σ_{cc}	Standard deviation of the cryptocurrency transaction cost shock	0.0124
ρ_m	Persistence of the growth rate of money balance shock	-0.2543
σ_m	Standard deviation of the growth rate of money balance shock	0.0042

Table 3: Baseline calibration

Variable	c	j^*	n	M	CC
Value	1.0444	0.3752	3.7129	0.0413	0.2521

Table 4: Steady-state values of some critical variables.

By following Özbilgin (2012), the welfare cost function $\Gamma(\tilde{P})$ is defined as below:

$$u[(1+\Gamma(\tilde{P}))c(\tilde{P}), l(\tilde{P})] = u[c(P), l(P)] \tag{25}$$

where $c(P)$, $l(P)$, and P are the baseline steady-state values while $\tilde{P}$ is the new varying price level, which can be also understood as the inflation, that deviates from the steady-state price level P. The optimal level of labor and consumption are functions of the price level P. Therefore, any change in the price level will affect the consumer's leisure-consumption decision and thus it will affect the welfare of the representative household. The welfare cost definition implies that the representative household needs consumption compensation under different price levels $\tilde{P}$ so that he still enjoys the same level of utility gained at the steady-state price level P. Using the definition expression (25) and the utility function form (23), the welfare cost function can be expressed as:

$$\Gamma(\tilde{P}) = \frac{c(P)l(P)^{\frac{1-\gamma}{\gamma}}}{c(\tilde{P})l(\tilde{P})^{\frac{1-\gamma}{\gamma}}} - 1 \tag{26}$$

If the $\Gamma(\tilde{P})$ is negative, it means the change in price level is welfare-enhancing and there is a welfare gain. However, if the $\Gamma(\tilde{P})$ is positive, then it suggests that the net change in the price level reduces the utility of the representative household and the household needs some additional consumption to maintain the original utility level. Therefore, it is a welfare loss.

Figure (6) shows how the welfare cost as a percentage, which implies $100 * \Gamma(\tilde{P})$, and in the form of the consumption good will be impacted by the increase in the price level. The welfare cost of the price is not as strong as I expected. The highest welfare cost corresponding to a 40% net increase in price level is just -0.9549%. Therefore, an increase in the price

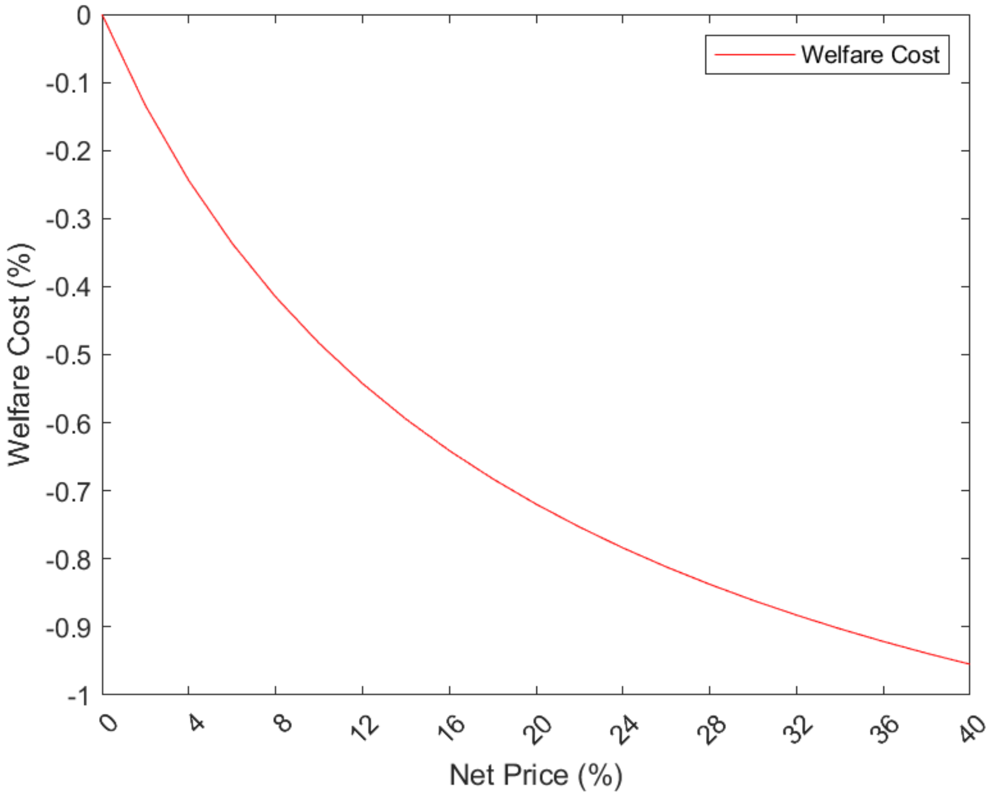

Figure 6: Welfare cost of the price.

enhances the welfare of the consumer.[24] The relatively high welfare cost, which is compared to the approximately 0.25% welfare loss of inflation in Henriksen and Kydland (2010) if the net inflation goes up from 0% to 20% when bank deposit is a substitute to the fiat currency, is likely caused by very stable and low exchange rate S, relatively high rate of return Θ, and higher transaction cost of using the private cryptocurrency. If we compare the Figure (6) here and Figure (20) in Appendix (7.3), it is easy to observe that the convexity (to the origin) of the welfare cost curve increases as the price level goes up further. Therefore, the marginal welfare cost decreases as the price level goes up. The relative flatness of the welfare cost curve at the high price level can be explained by the relatively low real transaction cost τ_{cc}, the fixed opportunity cost of using fiat currency, and a possible "already adjusted" mind to the price shocks because of the decaying effect of price on real return net of transaction cost of using private cryptocurrency. Besides, the availability of the currency substitution against the inflation tax on the fiat currency and the relatively big marginal increase in the real return net of transaction cost of using private cryptocurrency can account for the steepness of the welfare cost curve at the low price level.

The substitution effect drives consumers away from using or holding fiat currency while the wealth effect caused by increasing cryptocurrency balance with a high rate of return also induces consumers to demand more cryptocurrency. When the net price level goes up by 40 percent, the consumer chooses to consume more consumption goods as shown in Figure (17).[25]

[24] The setting of this model is related to this low welfare cost. Chiu and Koeppl (2017) point out that bitcoin is nearly 500 times more costly than using fiat currency in a low inflation economy when both of the currencies are used as a medium of exchange. Chiu and Koeppl (2017) find that 0.08% welfare cost under improved optimal bitcoin design is equivalent to a fiat currency system with moderate inflation. Kang and Lee (2019) also point out that only when the inflation is sufficiently high, then bitcoin can compete with fiat currency as a medium of exchange.

[25] This is caused by the availability of another safe asset since the consumer can move to buy more of them

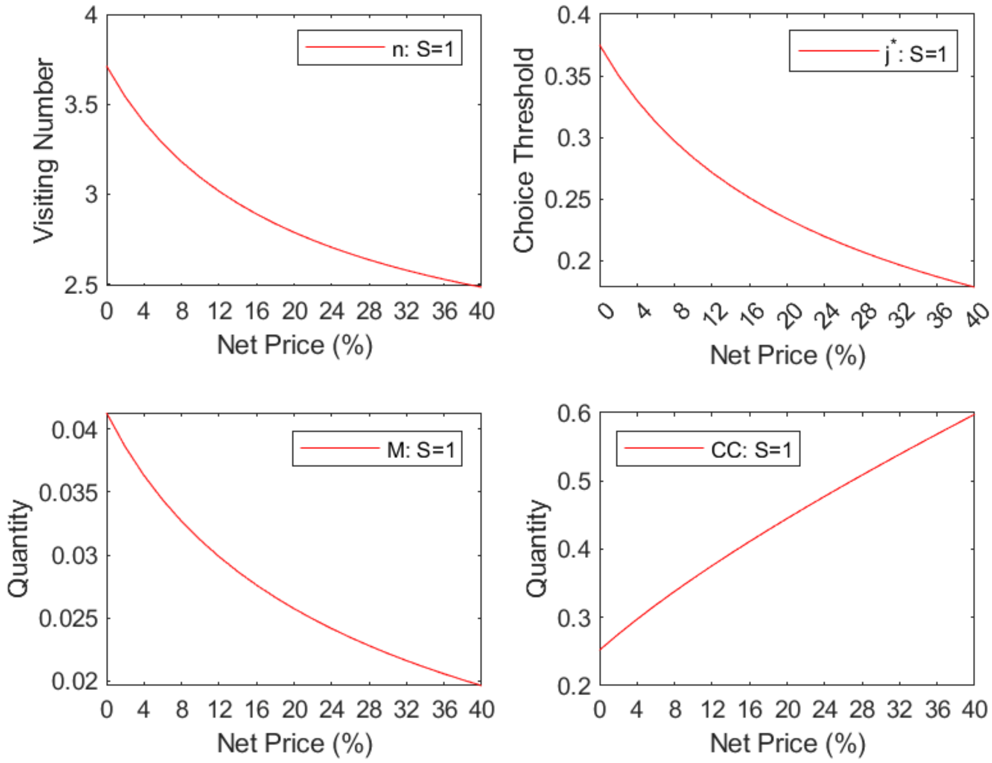

Figure 7: Changes in critical variables when price level changes.

Once the amount of consumption goods is decided, then the representative household decides the times of travel to the asset market, which is given by:

$$n = \left[\frac{c}{\frac{\phi w}{1-\beta} - \frac{1}{1-\beta} \frac{S^2}{P^2} \frac{\tau_{cc}^2}{c} \frac{\pi \bar{r}^k}{2(\Theta-1)}} \right]^{\frac{1}{2}} \tag{27}$$

The travel times n decreases as P goes up because the increase in the price dominates the increase in the consumption amount and forces the agent to travel less to the asset market. After the determination of the consumption amount c and the travel times n to the asset market, the threshold level j^* of the purchase instrument choice will also respond to all changes in P, c, and n. At the steady-state, j^* is given by:

$$j^* = \frac{n}{c} \frac{\pi S \tau_{cc} \bar{r}^k}{2P(\Theta - 1)} \tag{28}$$

Since both P and c increase, n decreases while keeping other variables in eq (28) constant, the threshold level j^* goes down further. The decrease in j^* implies that the fraction of the goods purchased by using private cryptocurrency CC increases while fewer goods are bought by using fiat currency M. The visual explanation of this analysis is shown in Figure (7) here and (25) in Appendix (7.3). What is more, the fiat currency balance decreases by 52.48%,

while keeping less of the fiat currency. As in Figure (7), the private cryptocurrency balance has increased by 137.12% while fiat currency balance has decreased by 52.48% as the price level increases by 40%. Since the opportunity cost of making purchases with the private cryptocurrency goes up because of the lower real transaction cost as the price level goes up, the consumer can buy more consumption goods by using private cryptocurrency while still getting the same gross real return net of transaction cost from it. Both wealth and substitution effects play an important role in the welfare cost of the price. For the optimal consumption, please check the Figure (17) and (19) in Appendix (7.3) private cryptocurrency balance increases by 137.12%, threshold level j^* decreases by 52.48%, the number of visits to the asset market decreases by 33.13%, and the consumption amount increases by 0.50% for a net 40% increase in the price level.

These findings are also consistent with economic intuition. Since an increase in the steady-state price level decreases the real transaction cost of using private cryptocurrency as a payment instrument and with the availability of another currency with a relatively high rate of return and low extra transaction cost, the rational agent will move to hold more of his wealth in the form of the appealing private cryptocurrency rather than the fiat currency. This finding has one important implication for countries considering issuing CBDC. Countries that suffer relatively high or volatile inflation or price should be very cautious when issuing CBDC while a private cryptocurrency (ones with similar features as in our model) is also available for the public. Even though the high price level enhances the welfare of the consumer when a private cryptocurrency like bitcoin is used as money, it weakens a central bank's role in implementing monetary policy.[26] Countries can ban private cryptocurrencies that have similar or the same functions as money whenever they begin to issue CBDC or else central banks will face the tough problem of implementing their monetary policy through legal currencies, especially in the case of high inflation. Central banks can also try to pay higher interest on CBDC than private cryptocurrency in the case of an increase in the price level, but this will likely not last long since central banks face the tough problem of securing the interest payment from themselves or other sources such as government tax revenues.

Countries like the U.S. that have stable inflation do not have to worry about the welfare cost of price as much as other countries that experience frequent inflation or price shocks since the welfare cost of the price is not significantly big for a 40% increase in price in this specific setting. As shown in Table (4), the steady-state private cryptocurrency balance is 0.2521, fiat currency balance is 0.0413, and the threshold level j^* is 0.3752. The representative household prefers storing the majority of his monetary assets in the form of private cryptocurrency rather than legal domestic currency. At the steady-state, the fraction of CC in the monetary asset, which is $M + SCC$, is 0.8592. If bitcoin has all the assumed features like the private cryptocurrency in this model, then bitcoin would dominate the U.S. dollar as both the main payment instrument and value storage of the monetary assets and the Federal Reserve would likely become obsolete. The details of the ratio of nominal cryptocurrency balance to the monetary assets corresponding to changes in the price or nominal exchange rate are shown in Figure (8).

As stated earlier, I simply assume the steady-state nominal exchange rate is one. But how do welfare and other critical variables respond to the increase in the nominal exchange rate?. Since it will affect the domestic currency value of the transaction cost when making purchases with private cryptocurrency. The domestic currency value of the saved cryptocurrency balance is also affected. On the one hand, an increase in the exchange rate induces the consumer to use fiat currency more often to conduct purchases, but on the other hand, the consumer is induced to demand more cryptocurrency. The welfare cost function $\Gamma(\tilde{S})$ can be similarly defined as in eq (25) for varying nominal exchange rates while the price stays at the steady-state level.

$$u[(1+\Gamma(\tilde{S}))c(\tilde{S}), l(\tilde{S})] = u[c(S), l(S)] \tag{29}$$

and the welfare cost function can be expressed as:

$$\Gamma(\tilde{S}) = \frac{c(S)l(S)^{\frac{1-\gamma}{\gamma}}}{c(\tilde{S})l(\tilde{S})^{\frac{1-\gamma}{\gamma}}} - 1 \tag{30}$$

[26] Weakening a central bank's role will likely be further strengthened in this cycle. Since once more consumers choose to use private cryptocurrency in the wake of inflation or high price, they will bid the value of a cryptocurrency to a new higher level, which means the gross nominal return (gross appreciation rate) on cryptocurrency and the fiat currency value of the transaction cost of cryptocurrency goes up and likely will further induce consumers to hold or demand more private cryptocurrency.

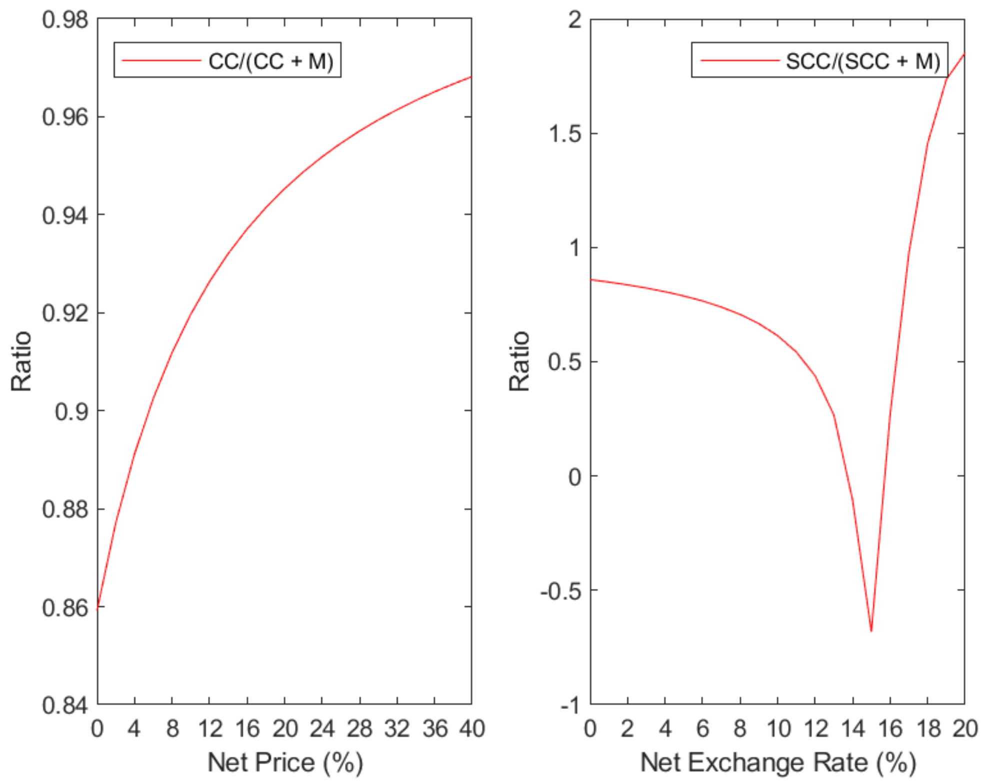

Figure 8: Ratio of CC. Note: Ratio of CC=$\frac{SCC}{SCC+M}$. Cryptocurrency values corresponding to the changes in exchange rate are real parts of complex numbers. The steady-state exchange rate is one, thus, S is ignored for the ratio of CC for net price change.

the welfare cost, which is $100 * \Gamma(\tilde{S})$, of the nominal exchange rate between private cryptocurrency and the domestic fiat currency is shown as in Figure (9). The meaning of the welfare cost is the same as before.[27] As the net exchange rate increases from 0% to 15%, the welfare cost increases from 0 to 6.3218%. However, the welfare cost decreases from 6.3218% to -0.2169% as the net exchange rate further increases to 20%. Here the welfare cost is positive until the net exchange rate increases by 19%, which implies that as the nominal exchange rate increases, the representative household needs some extra consumption goods to maintain the same level of utility when the nominal exchange rate is one. Thus, an increase in the nominal exchange rate S leads to an increase in the welfare cost of the representative household at first but then the trend reverses. It is obvious that the substitution effect dominates in the pre-15% net increase period and the wealth effect dominates the post-15% period. Figure (24) in Appendix (7.3) describes the welfare cost curve for a 0-100% range of net nominal exchange rate increase. The welfare cost increases at first but then it begins to decrease sharply. However, the decrease in the welfare cost will not last long and it stays nearly flat after the net exchange rate reaches around 40%. The minimum welfare cost, which corresponds to the 100% net exchange rate increase, is -2.8439%. Even with the relatively high rate of return Θ on the private cryptocurrency, the opportunity cost of using private cryptocurrency decreases significantly at the beginning until S goes up by 15%. Therefore, the change in S is big enough to "frighten" the representative household and discourage even stopping them from demanding private cryptocurrency at first as shown in Figure (10). What is more, the demand for the private cryptocurrency returns positive at some net exchange rate level between 20% and 24% and this low level of CC balance

[27]Please note that optimal consumption of varying nominal exchange rates turns up to be a complex number. As a result, the real part of the c, n, j^*, M, CC, and welfare cost is used for both graphing and analysis.

stays flat for the further increases as shown in Figure (26). However, both CC and M balances are very low and near to zero. It is probably because of the very high exchange rate of private cryptocurrency and zero net interest rate on the fiat currency. Therefore, holding a tiny unit of cryptocurrency is enough to store most of the household's monetary assets with preferable rate of returns.

The representative household determines the optimal consumption level once he observes the increase in the exchange rate. The optimal consumption level goes down for the first 15% net increase in the exchange rate as shown in Figure (21) in Appendix (7.3).[28] But the magnitude of the decrease in consumption level is nearly negligible. For the net 15% increase in the nominal exchange rate, the optimal consumption level goes down by 3.1915%. However, the optimal consumption level reverses the previous trend and goes up for the rest of the net 20% increase in the exchange rate. The magnitude of this increase is 3.2858%. The effect of a wider range of net exchange rate increase on the consumption is shown in Figure (23) in Appendix (7.3) and it is clear that after some increase, the optimal consumption level nearly stays constant. What is more, as shown in Figure (10), the increasing exchange rate and decreasing level of consumption in the denominator dominates the numerator in eq (27) and induces the consumer to travel more to the asset market, which implies n goes up at first. Once c, n choice is determined, then it is evident from eq (28) that all three variables put upward pressure on the threshold level j^*. As a result, the representative household decides to use more fiat currency M and less private cryptocurrency CC when purchasing consumption goods until the net increase in the nominal exchange rate is 15%. However, the trend for all five variables above is reversed for further increases in the nominal exchange rate. The most possible explanation for this sudden reversal of the trend is the wealth effect of the initial increase in the exchange rate and the household is inclined to purchase goods and services by spending big amounts of cryptocurrency with less frequency to avoid the extra total transaction expenditure and welfare loss for further increase in the exchange rate.

The availability of currency substitution plays an important role in mitigating the welfare cost of the exchange rate. Once the nominal exchange rate begins to go up, the representative household begins to feel the heat of rising transaction costs of purchasing goods with private cryptocurrency while the opportunity cost of using fiat currency is still the same.[29] Intuitively, as a rational agent, the representative household will move to hold a higher fraction of his monetary assets in the form of fiat currency rather than private cryptocurrency. As shown in Figure (10), the fiat currency balance increases by 322.37% at first but then decreases by 47.55% while the private cryptocurrency balance decreases by 104.40% initially but then increases by 24.28% as the nominal net exchange rate increases by 20%. What is more, the threshold level j^* also increases by 267.28% for the first 15% net increase in the exchange rate but then decreases by 49.74%. The times of travel to the asset market goes up by 210.81% first but then goes down by 49.24% further for a 20 percent net increase in the exchange rate.

The welfare cost of the nominal exchange rate also has important implications for countries that consider issuing CBDC. First, if a central bank offers CBDC in the future, will the banks or financial institutions that operate the payment system charge the transaction fees according to the amount of the transactions or the size of the CBDC file that the consumer sends?. If the banks charge on the amount it sends, any change in the nominal exchange rate is likely to

[28]Since the gross real appreciation rate of the cryptocurrency is fixed, the immediate effect of the increasing nominal exchange rate is on the real rate of return net of transaction cost on the private cryptocurrency and it will decrease because of the increasing transaction cost. Therefore, the consumer offsets the effect of high transaction costs on the opportunity cost of using private cryptocurrency by purchasing less using private cryptocurrency. Besides, an increase in the exchange rate also increases the value of saving in cryptocurrency. Thus, it induces the household to save more and consume less, which will create a wealth effect.

[29]Davoodalhosseini (2018) finds that the cost of using CBDC in the transaction is relevant to achieve the best welfare outcome when CBDC and cash are perfect substitute in conducting purchases.

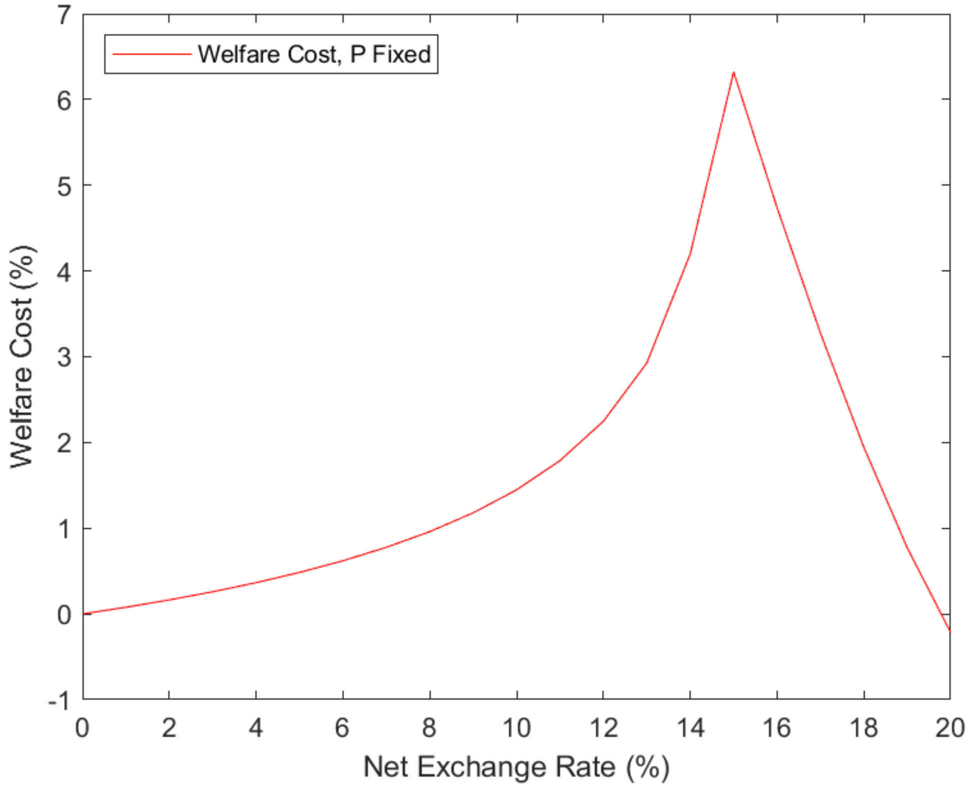

Figure 9: Welfare cost of the nominal exchange rate. Note: Real parts of complex numbers are plotted here.

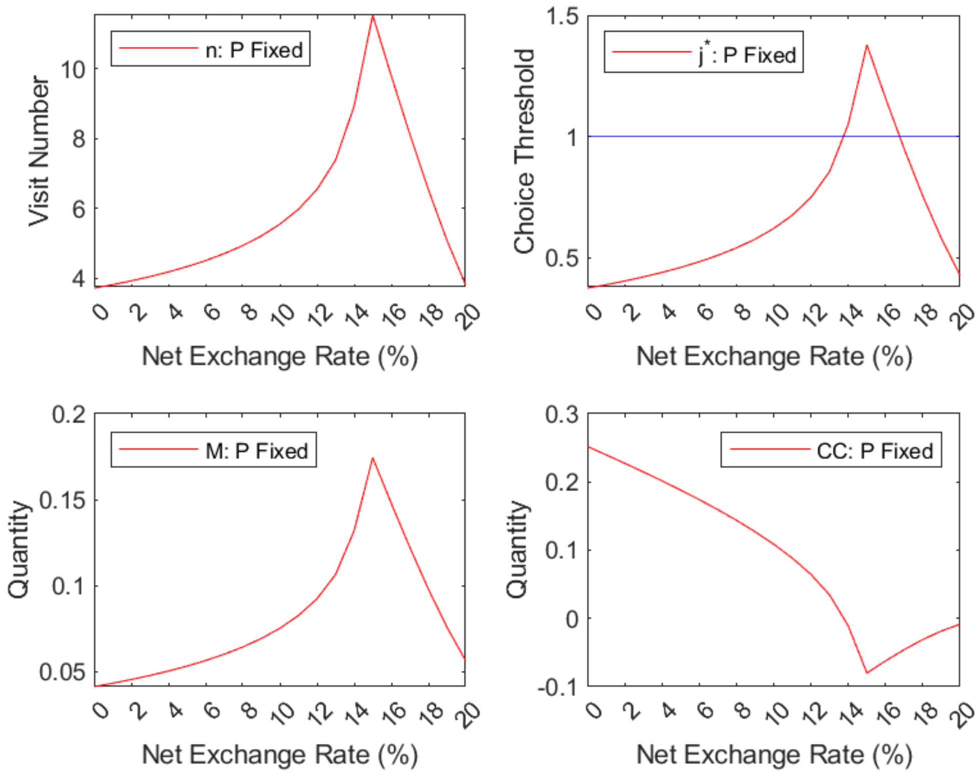

Figure 10: Changes in critical variables when nominal exchange rate changes. Note: Since the values of n, j^*, CC, and M are complex numbers, only the real parts are plotted.

influence the saving or holding amount of cryptocurrency more than the behavior of purchasing goods by using cryptocurrency. It is worth noting that more research is needed to understand the interaction between the nominal exchange rate, cryptocurrency balance, and fiat currency balance when both currencies are used as a payment instrument and transaction cost is charged based on the purchase amount. However, if banks decide to charge fees according to the CBDC file size and network environment, then any slight fluctuations in the value of CBDC (assume that the value of CBDC in domestic currency is freely determined by the market with a small fluctuation around a trend or central bank target rate and the transaction cost is in the cryptocurrency unit. To be consistent, it is simply assumed the CBDC has an independent interest rate, which can be net value deviation from the sample mean or the central bank target value, from its value.[30]) can change the demand for CBDC that be used for carrying out transactions. If the exchange rate increases by a large percentage, then it would make CBDC less preferable to the fiat currency or private cryptocurrency that offer the same or similar service. Second, with the availability of the private cryptocurrency with a relatively high rate of return that can be used for both storing values and making purchases like the cash, any slight increase in the nominal exchange rate of CBDC in an economy with stable inflation will likely to force CBDC holders or users to increase the ratio of the private cryptocurrency or fiat currency in their asset portfolio and the corresponding increase in fiat currency and private cryptocurrency combined is likely higher than the respective decrease in CBDC balance similar to the case in Figure (10). Third, in the case of the transaction fees charged based on the CBDC file size, the central bank can adjust the rate of return on the CBDC to offset the declining demand for CBDC by keeping the opportunity cost of using the CBDC relatively constant. Fourth, what is more, the welfare loss from the 15% increase in the exchange rate is significantly larger than welfare gains from the same increase in the price level. Therefore, Central Banks that manage CBDC should be more sensitive about the fluctuation of CBDC value than the inflation or price level.

Figure (11) displays the quantified analysis of the potential source, which includes the opportunity cost and transaction cost of holding different currencies, of welfare cost of both price and nominal exchange rate. The definition of those costs here is consistent with Özbilgin (2012) and Henriksen and Kydland (2010). The total transaction expenditure (or cost) of purchasing goods with private cryptocurrency is $\frac{S\tau_{cc}(1-j^*)}{P}$. The opportunity cost of replenishing money balances is $wn\phi$. The opportunity cost of holding domestic fiat currency is $(r^k+1-\delta-\frac{1}{\pi})\frac{M}{P}$. The opportunity cost of holding private cryptocurrency is $(r^k+1-\delta-\frac{\Theta}{\pi})\frac{SCC}{P}$.

In Figure (11), for the net increase in the price level from 0% to 40%, the opportunity cost of holding fiat currency, private cryptocurrency, the replenishment cost, and the summation of all costs all decrease by 66.06%, 69.37%, 33.13%, and 691.54% respectively while the transaction expenditure of using private cryptocurrency as a payment instrument increases by 4.27% until the net price increases by 10% and then it decreases by 9.91% for the further increase in the price. What is more, it is worth noting that the change in price level does not change the opportunity cost of holding unit private cryptocurrency or fiat currency. Thus, the significant change in the opportunity cost of holding fiat or cryptocurrency is mainly driven by the changes in the price level and the corresponding changes in the fiat or cryptocurrency balance. The change in the transaction expenditure is not as significant as other costs and it is probably the result of a combined effect of a decrease in j^* and increase in P. As we know from previous

[30] Not being able to express the return on the cryptocurrency as a function of the exchange rate is one shortcoming of this paper. Central banks can issue CBDC that value is freely determined by the market and net change in the value is the gain or loss to the holders or users. Because of the static nature of our analysis, if the gross return on CBDC is measured in terms of the exchange rate, then, at the steady-state, its gross return is one as I mentioned earlier. As a result, with the existing transaction cost of using CBDC on some types of blockchain network like bitcoin blockchain, the j^* goes to infinity, which means CBDC will never be used in purchasing consumption goods.

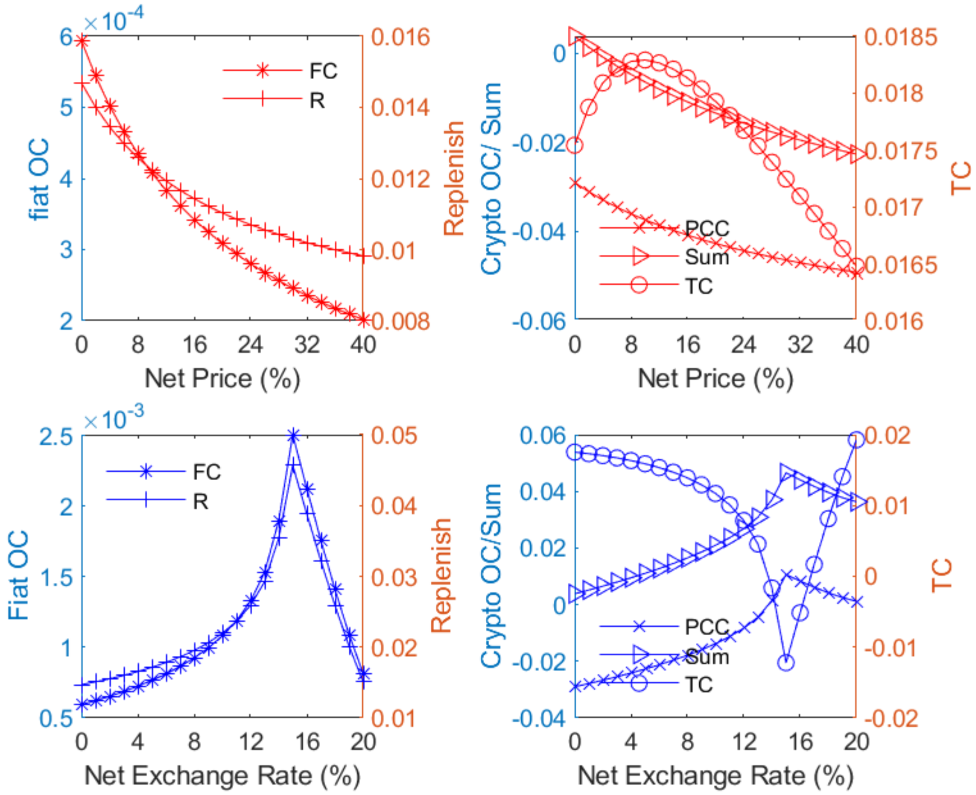

Figure 11: Source of the welfare cost of the price and nominal exchange rate. Note: FC stands for the fiat currency, R stands for the replenishment of money balances, PCC stands for the private cryptocurrency, TC stands for the total transaction expenditure, OC stands for the opportunity cost and Sum is the summation of all the costs. The net change in both price and exchange rate shows the net percentage increase from steady-state values. The opportunity costs are the values. The figure depicts the opportunity cost of replenishing money balances, transaction expenditure of using private cryptocurrency, opportunity cost of holding private cryptocurrency, and opportunity cost of holding fiat currency. Corresponding cost values to the varying price are real numbers while corresponding cost values to the varying exchange rate are real parts of complex numbers.

analysis, the threshold level decreases by 52.48% for a 40% increase in the price and the j^* is convex to the origin. Therefore, we can observe a concave transaction expenditure curve here. What is more, the change in the opportunity cost of holding cryptocurrency is the biggest among all the costs studied here (except the summation). It is probably because of the bigger change in the cryptocurrency balance. The decreasing opportunity cost of holding fiat currency is mainly driven by the combined effect of an increase in the price and decrease in the fiat money balance. It should be emphasized that the risk aversion character of the household can influence the money choice process when there is a shock to the price or nominal exchange rate and can impact the results here. It is apparent from the Figure (11) that currency substitution improves welfare for any increase in the price level through the wealth and substitution effect.[31]

What is more, for an increase in the exchange rate from 0% to 20%, the opportunity cost of holding the fiat currency, private cryptocurrency, replenishment cost, and the summation of all costs all increase by 322.37%, 136.52%, 210.81%, and 1117.6% respectively at first but then they reverse the rend and decrease by 67.62%, 89.1%, 67%, and 21.98% respectively. However, the transaction expenditure for using private cryptocurrency to purchase goods and services decreases by 169.59% at first but then increases by 257.95% for the further increase in the nominal exchange rate. Since Θ is assumed to be independent of the nominal exchange rate S, any change in the nominal exchange rate does not impact the opportunity cost of holding unit private cryptocurrency. Besides, the opportunity cost of holding fiat currency is solely driven by the increase in fiat currency balance. The fiat money balance responds to the increase in the nominal exchange rate dramatically as described earlier. Therefore, the change in the opportunity cost of holding fiat currency is initially significantly larger than other costs. However, the increasing exchange rate and decreasing cryptocurrency balance offset each other for some degree at first, but then they move in the same direction. Therefore, the opportunity cost of holding cryptocurrency is relatively smaller at first and bigger later. What is more, the transaction expenditure is affected by both the nominal exchange rate and threshold level j^*, which goes up first but goes down later and itself depends on n, c, and S. Therefore, the change in j^* offsets some motion of the increase in the nominal exchange rate at first but then enhances the effect of the increasing nominal exchange rate on the transaction expenditure later. In both cases of price and exchange rate, the change in the opportunity cost of replenishing money balance is simply and solely driven by the change in the times of the visit to the asset market. Both the wealth and substitution channels have mixed effects on welfare when there are relatively big changes in the nominal exchange rate.

5.2 Sensitivity

In this section, I will examine how the present results in the section (5.1) (referred to as benchmark case thereafter) would fare if critical parameters are re-calibrated to alternative values. In the benchmark case, the value of steady-state private cryptocurrency transaction cost τ_{cc} is assumed to equal to the sample average of the τ_{cct} over the period of 2013Q2-2019Q3. But it is easy to observe from the Figure (5) that there is a relatively large gap between the maximum and the average values of the τ_{cct}, not to mention already dropped extra large values. What is more, the value of the asset market trip cost ϕ is pinned down by using the sample average ratio of the real domestic currency to real consumption. As shown in Judson (2017), the share of U.S. currency of all denominations abroad over the period of 2000-2016 ranges from 40% to 60%, and the ratio ranges from 60% to approximately 80% for the \$100 bills. Freeman and Kydland (2000) also state that the ratio of the U.S. dollars held abroad ranges from two-thirds to three-quarters. Thus, alternative values for the domestic currency over the

[31] Please note that the opportunity cost of holding PCC is always negative, which implies the consumer is losing wealth by not holding PCC.

consumption ratio and τ_{cc} are used in this section to test the effectiveness of the benchmark case. For the τ_{cc}, the minimum, median, mean, and maximum values are chosen from the sample and the values are 0.0056, 0.0139, 0.0293, and 0.0946 respectively. The values of one-quarter, one-third, and one-half are used for the domestic currency to consumption ratio.

Table (5) displays the response of the key parameter and core variables to the alternative values of domestic currency to consumption ratio and the transaction cost of using the private cryptocurrency as a payment instrument. Please note that the values in the second row and third column in all panels of the Table (5) corresponds to the results of the benchmark case. Hence, any difference of the values from the coordinate $(2,3)$ implies deviation from the benchmark case. In panel A, any changes in $\frac{M}{Pc}$ does not have a significant impact on ϕ for the smaller values of τ_{cc}. However, the effect begins to magnify significantly as the transaction cost goes up further. The transaction cost has a significant positive effect on ϕ while domestic currency to consumption ratio has a varying degree, which depends on the value of the transaction cost, of negative impact on ϕ. The maximum value of ϕ is 0.088, which implies the household needs to spend more time on asset managing than the benchmark case. What is more, panel B shows the response of travel times to the asset market. The domestic currency to consumption ratio has a significant positive impact on the number of asset market visits n while the visit times goes down dramatically as the transaction cost τ_{cc} increases. The representative household visits the market with the highest frequency when half of the money is spent on the consumption goods and the transaction cost is the least, which is in line with the smallest value of ϕ. The minimum and maximum value of visit numbers are 0.26 and 134.79 respectively.

Panel C describes the response of the threshold level j^*. As the ratio of the domestic currency spent on the consumption goods increases, the threshold level begins to go up significantly. What is more, the transaction cost τ_{cc} has a negative impact on the j^*, which is also consistent with the theoretical and quantitative analysis in the benchmark case. The upward pressure placed by the tiny decrease in consumption and significant increase, of which absolute value is small, in the transaction cost is offset by the downward pressure put by the significant decrease in the travel times to the asset market. Therefore, from the eq (28), it is obvious that τ_{cc} has a negative impact on the threshold level j^*. The panel D and E display the response of the private cryptocurrency and fiat currency balances for alternative values of τ_{cc} and $\frac{M}{Pc}$. In both panels, an increase in the money to consumption ratio has opposite effects on the currencies: negative on private cryptocurrency balance and positive on fiat currency balance, which is consistent with the movement of j^*. What is more, the transaction cost has a significant positive effect on the cryptocurrency balance. The significant increase in the cryptocurrency balance is mainly driven by the upward pressure put by the dramatic decrease in the n and a significant increase in $1 - j^*$. It is probably that the wealth effect induces the consumer to hold more cryptocurrency. When it comes to the fiat currency balance, the effect of the transaction cost depends on the value of the domestic currency to consumption ratio. When domestic currency to consumption ratio is small like 0.25 and 0.33, transaction cost also has a positive effect on the fiat currency balance. However, when the ratio is big enough like 0.5, the transaction cost has a complicated effect on the fiat currency balance. The fiat currency balance first goes up, then goes down, and goes up slightly again later as the transaction cost goes up. Another interesting finding in panel E is that consumers are very sensitive to the initial "sudden" increase in the transaction cost for all values of the domestic currency to consumption ratio. Therefore, the fiat currency balance increases bigger when the transaction cost goes up from 0.0056 to 0.0139 than the further increases.[32]

Panel F shows the corresponding welfare costs. The 3D visualization, which is Figure

[32] Transaction cost of 0.0139 is approximately 2.5 times of the 0.0056 and 0.0946 is approximately 3.3 times of the 0.0293. But the corresponding change in fiat currency balance is much larger for the increase from 0.0056 to 0.0139.

Panel A: Asset market travel time cost: ϕ

$\frac{M}{Pc}$	Per-transaction cost: τ_{cc}			
	0.0056	0.0139	0.0293	0.0946
0.25	0.0000388	0.0002678	0.0017336	0.0883450
0.33	0.0000384	0.0002518	0.0014209	0.0543425
0.5	0.0000381	0.0002403	0.0011975	0.0300549

Panel B: Number of asset market visit: n

$\frac{M}{Pc}$	Per-transaction cost: τ_{cc}			
	0.0056	0.0139	0.0293	0.0946
0.25	66.1134269	12.0952707	2.7838824	0.2610872
0.33	84.9784282	16.0878488	3.7128563	0.3506887
0.5	134.790726	25.2654622	5.6255502	0.5304238

Panel C: Payment choice threshold: j^*

$\frac{M}{Pc}$	Per-transaction cost: τ_{cc}			
	0.0056	0.0139	0.0293	0.0946
0.25	1.2776944	0.5787289	0.2809733	0.0897886
0.33	1.6455360	0.7715483	0.3752167	0.1191584
0.5	2.6221437	1.2187332	0.5709052	0.1785673

Panel D: Private cryptocurrency balance: CC

$\frac{M}{Pc}$	Per-transaction cost: τ_{cc}			
	0.0056	0.0139	0.0293	0.0946
0.25	-0.0105027	0.0602488	0.3608768	3.9274091
0.33	-0.0220187	0.0275004	0.2520910	2.9410847
0.5	-0.0479399	-0.0208769	0.1299805	1.9273794

Panel E: Fiat currency balance: M

$\frac{M}{Pc}$	Per-transaction cost: τ_{cc}			
	0.0056	0.0139	0.0293	0.0946
0.25	0.0271076	0.0303410	0.0309317	0.0319201
0.33	0.0349118	0.0404499	0.0413068	0.0423611
0.5	0.0556316	0.0638945	0.0628497	0.0634812

Panel F: Welfare Cost (%)

$\frac{M}{Pc}$	Per-transaction cost: τ_{cc}			
	0.0056	0.0139	0.0293	0.0946
0.25	-1.4741437	-1.1109744	-0.2467725	10.4943005
0.33	-1.0983435	-0.6707314	0	7.9997439
0.5	-0.0806343	0.4384440	0.8081533	6.1150705

Table 5: Sensitivity analysis: core variables. Note: The transaction cost τ_{cc} is in the cryptocurrency unit. Since the difference between some values are very small, the values are rounded to 7 digits to observe the differences. The values of all the variables including the optimal consumption level except ϕ are complex numbers. Thus, real parts of complex numbers are used in this table.

(30), is shown in Appendix (7.3). As panel F describes, the domestic currency to consumption ratio has a positive impact on the welfare cost for small values of the transaction cost and the magnitude of the impact is relatively big. However, the ratio of domestic currency to consumption has a negative impact on the welfare costs when transaction cost is large enough like 0.0946. The increase in the domestic currency to consumption ratio reduces the welfare of the consumer for small transaction costs while enhances consumer's welfare for the large transaction costs. Therefore, central banks can reduce the welfare cost of consumers when the transaction cost of using a CBDC is relatively high by encouraging them to spend a higher fraction of their money in the domestic market. What is more, the transaction cost τ_{cc} has a positive impact on the welfare cost. The maximum welfare loss occurs when the transaction cost is at maximum and the domestic currency to consumption ratio is at minimum. Domestic currency to consumption ratio plays a critical role when $\tau_{cc} = 0.0293$. The slightest decrease in that ratio from the benchmark rate of 0.33 leads to welfare gains while an increase creates welfare losses. Figure (12) and (13) show the welfare cost of the price and nominal exchange rate for the different combination of the τ_{cc} and domestic currency to consumption ratios. The decreasing welfare cost of the price and the increasing first but then decreasing welfare cost of the nominal exchange rate is consistent with the trends in Figure (6) and (9) in section (5.1). From both figures, it is easy to observe that as the transaction cost increases, the domestic currency to consumption ratio generates a more visible and significant impact on the welfare cost of the representative household.

The effect of the domestic currency to consumption ratio on other core variables is generated solely through the travel time cost to the asset market. However, the transaction cost of the private cryptocurrency can impact the variables and welfare costs through both the travel time cost and the transaction cost involved while conducting transactions. As shown in Figure (11), the total transaction expenditure is one of the major sources of the welfare cost of both price and nominal exchange rate. The travel time cost to the asset market is not individually significantly sensitive to the change in $\frac{M}{Pc}$ ratio when the transaction cost is low. However, the sensitivity increases as the transaction cost goes up. What is more, the combined effect of the transaction cost on both the ϕ and the total transaction expenditure amplifies the impact and generates more significant changes in critical variables. That is the major reason why we observe the welfare cost values in Table (5).

In general, the change in the domestic currency to consumption ratio does generate an impact on parameter ϕ, variables n, j^*, CC, M, and the welfare cost. The significance of the impact on some variables depends on the transaction cost of using cryptocurrency. However, the change in transaction cost of using private cryptocurrency as a payment instrument can generate a more significant impact on the core variables and parameters of interest. The domestic currency to consumption ratio magnifies its impact on the welfare cost as the transaction cost increases. Therefore, our benchmark welfare costs and steady-state variables are very sensitive to the changes in the transaction cost of using the private cryptocurrency. However, it is less sensitive to the changes in the domestic currency to consumption ratio.

5.3 Technology Implications

As mentioned in section (5.2), the welfare results are sensitive to the transaction cost incurred while using private cryptocurrency as a payment instrument. The variables τ_{cct} and ϕ also have important implications for the cryptocurrency issuers and cryptocurrency payment system evolution. The value of ϕ measures the cost of converting nonmonetary assets like capital into monetary assets like private cryptocurrency and fiat currency in this model. The higher the cost of visiting the asset market, then the representative household will likely to visit less frequently or just quit visiting. For a cryptocurrency, ϕ can be understood as the fees that

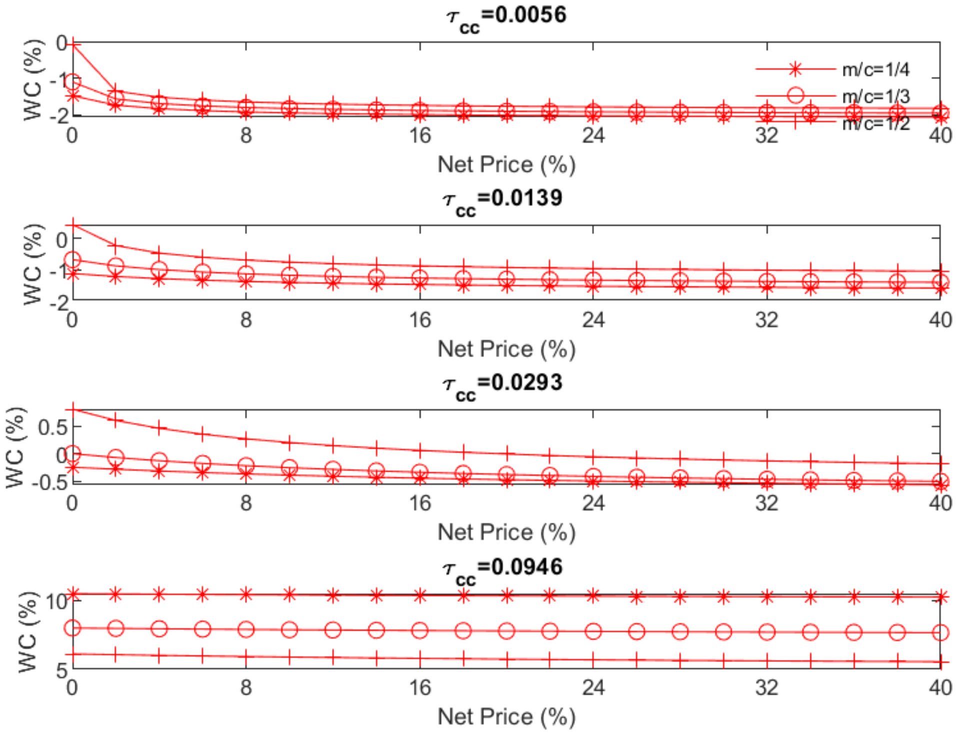

Figure 12: Sensitivity analysis: welfare cost of the price. Note: WC stands for welfare cost and m/c stands for domestic currency to consumption ratio. Welfare cost values are real parts of complex numbers.

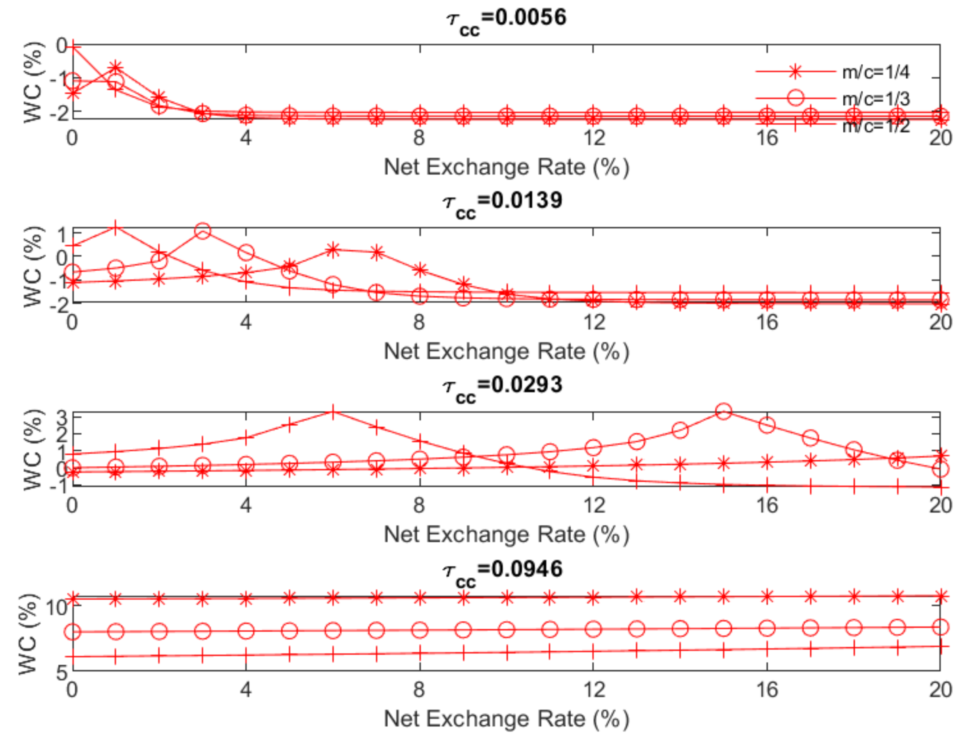

Figure 13: Sensitivity analysis: welfare cost of the exchange rate. Note: WC stands for welfare cost. Welfare cost values are real parts of complex numbers.

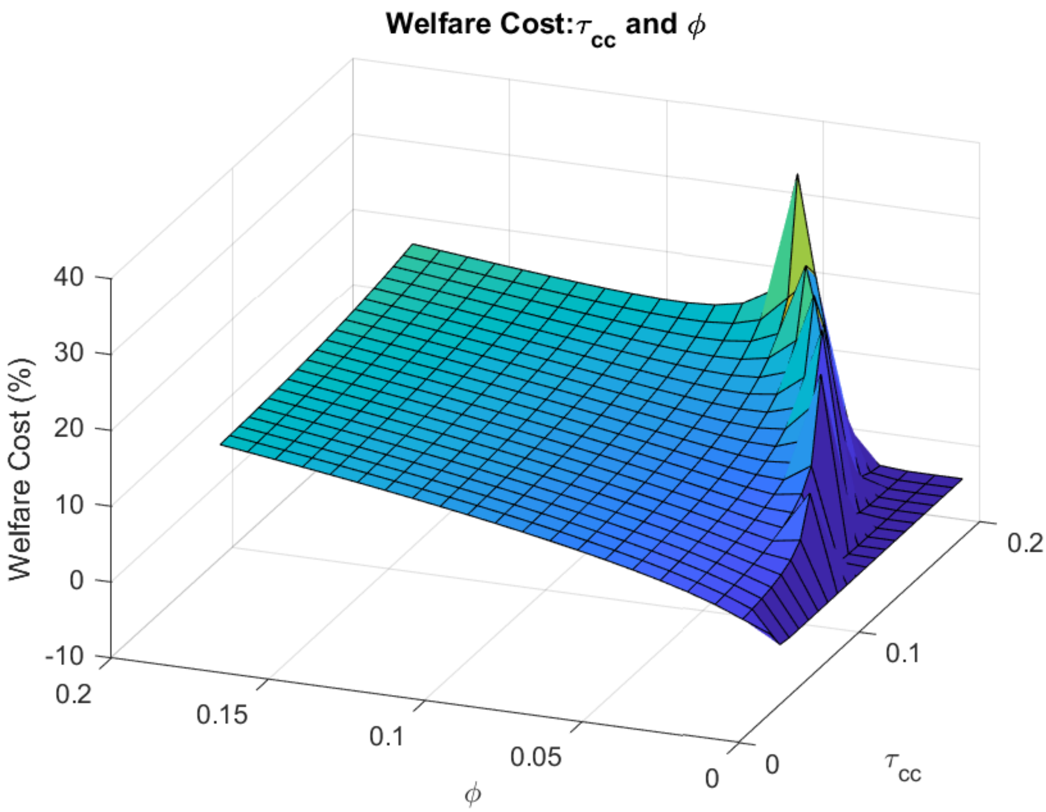

Figure 14: Welfare cost: ϕ and τ_{cc}. Note: Welfare cost values are real parts of complex numbers.

users have to pay when they buy or sell private cryptocurrency or converting between different supported cryptocurrencies on the online private cryptocurrency asset exchange market. For the fiat currency, it can be explained as the opportunity cost of the time spent on converting nonmonetary assets into fiat currency or other extra costs involved during the conversion process.[33] What is more, the transaction cost is the fee paid to miners or payment system operators that make and facilitate each transaction on the bitcoin or other cryptocurrency networks. If the blockchain network has robust hardware and high-quality miners or operators, it can cope with volatile transaction demands relatively smoothly and the fluctuations of the transaction cost become relatively flat. Therefore, I will examine the welfare implications of both the time cost of visiting the asset market and the transaction cost of using private cryptocurrency while keeping other parameters and variables the same as in the benchmark case.[34] For this analysis, the value of the transaction cost ranges from the sample average of 0.0293 to twice, which is 0.1892, the maximum value as shown in Figure (5) with an increment of 0.0080. The traveling time cost ϕ starts with the benchmark value of 0.0014 and the highest value is twice, which is 0.1767, of the maximum value of ϕ in panel A of Table (5). The increment of the ϕ is 0.0088.

Figure (14) shows how the welfare is affected by the travel time cost ϕ and the transaction cost τ_{cc} in a 3D graph. Except for the curve corresponding to the benchmark ϕ value of 0.0014, which will be discussed later, the welfare cost of the transaction cost shows an increasing trend in general for the other values of the ϕ. However, for the values of the ϕ between 0.0102 and 0.0452, the welfare cost of the transaction cost shows similar trends as the welfare cost of the

[33]Please note that it is possible that the representative household visits the asset market and converts nonmonetary assets into fiat currency first and then uses that fiat currency to buy private cryptocurrency. Or converting nonmonetary assets directly into both currencies can happen at the same time. Both cases do not have an impact on our analysis.

[34]Please note that value selection of ϕ and τ_{cc} is random. To keep the results comparable to the benchmark case, the values of ϕ and τ_{cc} are selected from Table (5).

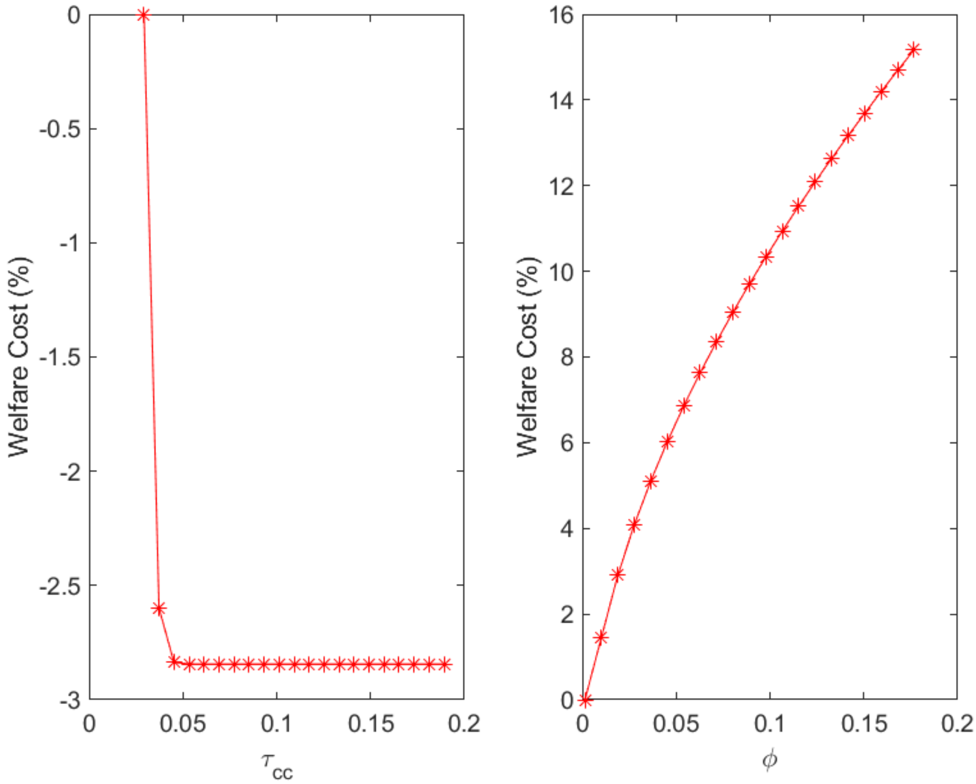

Figure 15: Welfare cost: ϕ and τ_{cc}. Note: The definition of the welfare cost is the same as in eq (25) and the only difference is that now ϕ or τ_{cc} are the changing variables rather than P. The welfare cost values of τ_{cc} are the real parts of complex numbers.

nominal exchange rate described in Figure (9). For example, when ϕ is 0.0452, the welfare cost of the transaction cost increases from 6.0169% to 25.1878% at first till $\tau_{cc} = 0.1732$ but then decreases dramatically to 14.1332% for the further increase in the transaction cost. For the initial small values of ϕ, there is even having welfare gains, which implies negative welfare cost, for the bigger values of the τ_{cc} at the end of the decreasing section of the welfare cost curve. What is more, the welfare cost of the transaction cost corresponding to ϕ larger than 0.0452 shows a steady increasing trend. Thus, the effect of the transaction cost on the welfare of the consumer depends on the cost of visiting the asset market, and mostly it reduces the welfare. Since an increase in the transaction cost reduces the real rate of return net of transaction cost on the private cryptocurrency, it induces the household to buy consumption goods using cryptocurrency less than before. In addition, it reduces the amount of the consumption goods that they can afford under the same income or wealth. What is more, the sudden decreasing trend of the welfare cost of the transaction cost for the small ϕ values and the bigger transaction costs is because of the relatively large increase in the fiat currency balance, threshold level j^*, and decrease in the cryptocurrency balance before the dramatic turn. Therefore, the consumer can afford to reduce the positive welfare cost with a small amount of accumulated wealth. However, after the welfare cost reaches the peak, the cryptocurrency balance stays relatively flat while j^* and M decrease dramatically, which implies the household is increasing the ratio of the private cryptocurrency that pays relatively higher interest in his monetary assets. Therefore, the household owns more resources to increase his consumption, which further decreases the welfare cost. Figures describing the corresponding changes in other core variables are given in Appendix (7.3).

However, the welfare cost curve of the travel time cost is concave for the small transaction costs and the impact is significant, which is shown in Figure (14) and (15). As the transaction

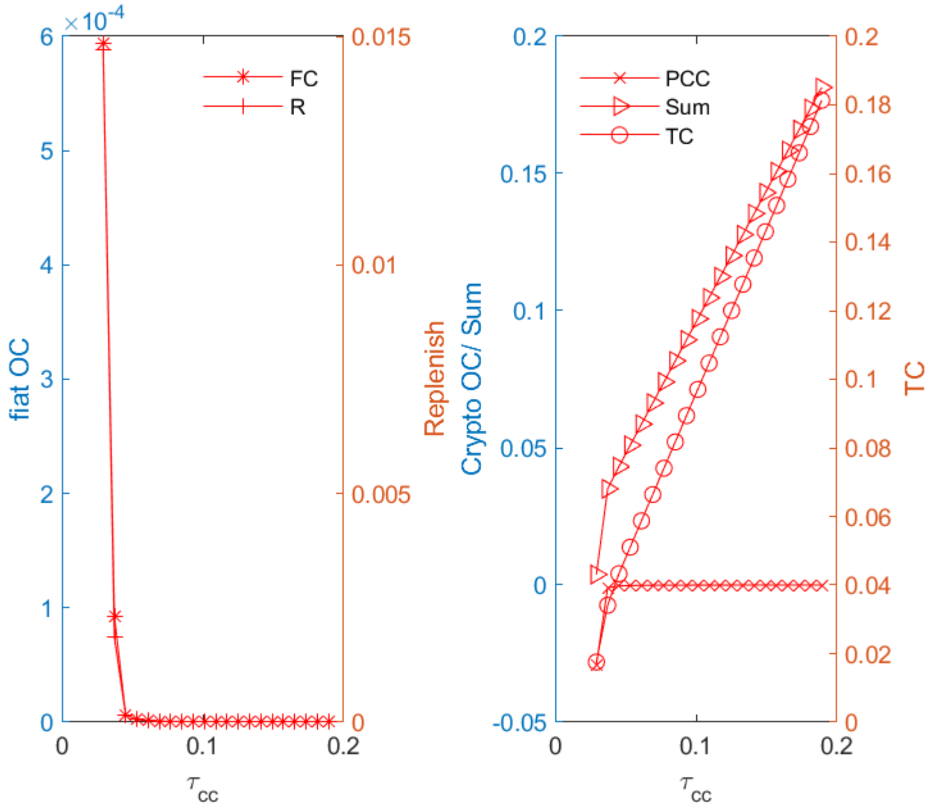

Figure 16: Source of the welfare cost when ϕ is at the benchmark level, Note: FC stands for the fiat currency, R stands for the replenishment of money balances, PCC stands for the private cryptocurrency, TC sands for the total transaction expenditure and OC stands for the opportunity cost and Sum stands for the summation of the TC, OC of PCC and FC, and R. The values are the real parts of complex numbers.

cost goes up further, the shape of the welfare cost of the travel time cost ϕ changes dramatically and becomes more obvious. The welfare cost of the travel time cost increases slightly at first and then it increases sharply to the possible maximum values. However, once it reaches the highest welfare cost, it decreases dramatically for a short range of ϕ. But, again it goes up steadily as the travel time cost goes up further. For example, when τ_{cc} is at its maximum of 0.1892, the welfare cost of the travel time cost increases slightly from -2.8435% to -2.5653% for the travel time cost range of 0.0014-0.0277. Then welfare cost goes up sharply to 34.4837% for another range of 0.0277-0.0452. This increasing trend does not last long and begins to decrease dramatically. The welfare cost drops down to 15.4784% for the travel time cost range of 0.0452-0.0891. However, the welfare cost goes up again slowly for the further increase in the travel time cost and the net increase is relatively small. What is more, the kink in the welfare cost curve of the travel time cost for the bigger τ_{cc} is mainly caused by the dramatic increase in the threshold level j^*, cryptocurrency and fiat currency balance, which have wealth effect, before the turning point. Since the household increases both the cryptocurrency and fiat currency balances even for the increasing threshold level j^* for the initial increase in the travel time cost, this slightly increases the wealth of the consumer and thus increases the consumption for a very short range of ϕ. Therefore, the welfare cost is reduced. However, this will not last long.

Figure (15) shows the welfare cost of the transaction cost and travel time cost while keeping other variables at the benchmark level. It is easy to understand the concavity of the welfare cost curve of the travel time cost. However, a dramatic decrease in the welfare cost of the transaction cost τ_{cc} when $\phi = 0.0014$ is just the opposite of what I expected since higher transaction costs are supposed to create welfare losses rather than welfare gains. To understand this puzzle, it is important to check the opportunity cost of holding different currencies. Figure (16) shows how the opportunity costs, total transaction expenditure of conducting transactions with cryptocurrency, and replenishment cost change when the transaction cost goes up. The opportunity cost of holding fiat currency and the replenishment cost both decreases by 99.9891% and 99.9847% respectively while the transaction expenditure, opportunity cost of holding cryptocurrency, and summation of all costs all increase by 933.7173%, 99.9708%, and 4642.1% respectively. The dramatic changes happen only for the initial transaction cost range of 0.0293-0.0453 for the replenishment cost and the opportunity cost of holding fiat currency and cryptocurrency. What is more, from the Figure (37) and (38) in Appendix (7.3), it is observed that cryptocurrency balance, fiat currency balance, threshold level j^*, and travel times to the asset market all decrease dramatically for the initial increase in the transaction cost while the consumption goes up dramatically. The dramatic response of the consumer to the initial small increase in the transaction cost by increasing consumption sharply is very likely caused by the consumer's fear of potential further increase in the transaction cost and shrinking of his expected wealth and consumption in the future. As a result, the household sharply increases consumption as a response to the initial small increase in the transaction cost and to take advantage of relatively lower transaction cost before it is too late. However, for the further increase in the transaction cost, the consumer's reaction is far more stable by slightly increasing the consumption level. This suggests that consumers are very sensitive to small deviations of transaction cost from the steady-state rate they get used to paying when the travel time cost ϕ is at the steady-state level. What is more, an initial increase in the transaction cost dominates the corresponding increase in the consumption level and thus the travel times initially goes down dramatically according to the eq (27).[35] The magnitude of changes in the c, n, and τ_{cc} determines the trend of other variables in a similar way.

The findings here have important implications for the future development direction of the private or a central bank issued cryptocurrency. First, the cost of traveling to the asset market has a significant effect on the representative household's consumption behavior in general and thus on the welfare cost. This effect becomes more obvious as the transaction cost goes up. Therefore, if private cryptocurrency firms want their products to be more consumer-friendly and to minimize the welfare loss of users, it is a wise decision to invest more in R&D to improve the network hardware and reduce the fees of buying, selling, or converting between different private cryptocurrencies. The same logic goes to central banks. Second, the transaction cost of using private cryptocurrency as a payment instrument has a varying impact on the welfare of the consumer and it depends on the travel time cost. For small values of the travel time cost, the consumer's response varies dramatically for the increase in the transaction cost while it is relatively stable for the bigger values of the travel time cost. Thus, if the private cryptocurrency network is robust enough to absorb most of the demand related shocks and travel time cost is relatively high, it will not seriously discourage consumers from using the network to make transactions. However, the transaction cost still has a dramatic effect on the welfare of consumers if the travel time cost is relatively small.[36] Third, If central banks issue CBDC, generally, they should be more sensitive about the potential change in the fees regarding the conversion between fiat currency and CBDC than the transaction cost of using the CBDC. Fourth, the results suggest that consumers react very dramatically to the slight deviation of the transaction cost from the level they got used to in the past. Therefore, CBDC issuers are advised not to react to the dramatic response of consumers to the initial tiny deviation of the

[35]Please note that τ_{cc}^2 in eq (27) plays an important role in determining the initial dramatic decreasing trend of n. Change in τ_{cc}^2 is totally dominated by the corresponding change in consumption for the initial increase in τ_{cc}.

[36]Catalini and Gans (2016) point out that the cost of verification, which is the transaction cost I use here, and the cost of networking is affected by the blockchain technology

transaction cost from the long-run stable level.

6 Conclusion

This paper is motivated by the fast development of the DLT applications like BitPay and its potential implications for monetary policy, financial system, and especially the currently outdated so-called “real-time” gross settlement system. Some novel and important features of the private cryptocurrency payment processor of BitPay are incorporated in the dynamic general equilibrium model. The main aim of this paper is to examine the welfare effect of the currency substitution between fiat currency and private cryptocurrency when both can be used as a medium of exchange. Compared to existing literature regarding cryptocurrency, this model has several novel features. For example, the choice of the payment instrument is endogenously determined by the consumer by comparing the expected opportunity cost of using fiat currency and cryptocurrency. The transaction cost of using cryptocurrency to make purchases is time-varying and irrelevant to the transaction amount. Also, consumers can travel multiple times to the asset market to replenish money balances to satisfy their demand for consumption goods and services.

The dynamic general equilibrium model in this paper is very simple and has three agents: a representative household, firm, and government. The government plays the role of a central bank and is responsible for monetary policy. The welfare cost of both the price and nominal exchange rate is examined at the steady-state. A net increase in the steady-state price level by 40% decreases the welfare cost by 0.9549%, which is little. The representative household decreases the fiat currency balance by 52.48% and increases the private cryptocurrency balance by 137.12%. The relatively large increase in the private cryptocurrency balance can be explained by the decreasing real transaction cost of using it as a medium of exchange and high net real return on private cryptocurrency while the same rate is zero for the fiat currency. The choice threshold of payment instruments goes down by 52.48%, which implies less fiat currency is used as a medium of exchange. However, for a net 20% increase in the nominal exchange rate, the welfare cost increases by 6.3218% for the first 15% net increase but then decreases by 6.5387% for the rest. An increase in the nominal exchange rate directly affects the domestic currency value of the cryptocurrency balance saved and the transaction cost of using a private cryptocurrency as a payment instrument, which implies that the gross real return net of transaction cost on the private cryptocurrency decreases. As the nominal exchange rate increases by 20%, the fiat currency balance increases by 322.37% first but then decreases by 47.55% while the private cryptocurrency balance decreases by 104.40% first but then increases by 24.28%. Bitcoin price is very volatile and if any CBDC is issued in the future, it is very unlikely that central banks pay such high net interest on it. Therefore, the welfare cost of price and nominal exchange rate can be amplified if the net appreciation rate of a cryptocurrency is small or similar to the bank deposit rates. For the future CBDC, it is appealing to have features like a specific exchange rate target with relatively stable fluctuations such that any value changes between buying and selling prices can be considered as the interest rate of the CBDC, which is also a useful additional monetary policy tool for central banks.

In this paper, the robustness of the results to some changes in specific parameters and variables is also examined. The steady-state welfare cost results are very sensitive to changes in the transaction cost of using private cryptocurrency as a payment instrument. What is more, the ratio of domestic currency to consumption, which is used to pin down the time cost of traveling to the asset market, can also generate a significant impact on the welfare cost. Except for the sensitivity, the impact of the time cost of traveling to the asset market and cryptocurrency transaction cost on welfare is also examined separately since both have significant implications for the payment system technological innovations. Both the traveling time cost and transaction cost have complicated effects on the welfare cost. In general, an increase in the travel time cost decreases the welfare of consumers and this effect is also influenced by the transaction cost. Therefore, for private cryptocurrency payment systems or online cryptocurrency exchange market operators, it is important to invest more in R&D to cut the cost of buying, selling, or converting between cryptocurrencies if they want to attract more customers and reduce the welfare loss of private cryptocurrency users. This also implies that to minimize the consumer's welfare losses, for any central bank that will issue CBDC, it is recommended to make it as convenient and cheap as possible to convert between domestic fiat currency and CBDC or other assets. What is more, keeping the transaction cost (in cryptocurrency units) relatively stable with small fluctuations is also a wise strategy for central banks if they seek to avoid a significant increase in the welfare loss of consumers.

Private cryptocurrency, cryptocurrency payment system, and blockchain technology are the newly developing areas and there are still uncertainties and legal barriers that exist for a central bank issued digital currency. It is necessary for us to scientifically understand its shortcomings and advantages by employing the important features and data of currently available private cryptocurrencies. There are several promising directions for future research. First, since an interest-bearing cryptocurrency is a potential competitor for the bank deposit, researchers can examine the welfare implications of the currency substitution when domestic fiat currency, private cryptocurrency, and bank deposits are all available for consumers. This case is very similar to Özbilgin (2012), in which domestic currency, bank deposit, and foreign currency are all available as a payment instrument for domestic users. Second, the importance of the data increases as artificial intelligence, blockchain technology, and other information technologies develop. Today, data is as important as oil in the 20th century. It is of great importance to understand whether the transaction fees will be charged as a percentage of the consumption amount or the same or similar setting as in this paper. Besides, examining the welfare implications from the perspective of both a private cryptocurrency payment system operating firm and a consumer is also a promising avenue for future research. Third, it is also worth examining the welfare-maximizing optimal rate of return on cryptocurrency and the policy rule of achieving such an optimal rate.

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7 Appendix

7.1 Appendix A1

7.1.1 Proof for the payment instrument choice condition

The equation (7) that determines threshold level j_t can be explained intuitively as in section (4.1), but it is also determined by the first-order optimality conditions of the system of equations implied by this paper. For a given optimal consumption level c_t^*, the consumer needs to solve the following Lagrangian optimization with rational expectation ($\omega = -1$ is already plugged in).

$$L = E_t\bigg\{\sum_{t=0}^{\infty}\bigg[\beta^t u\Big[r_t^k + w_t h_t + CC_{t-1}\frac{S_t}{P_t} + \frac{M_{t-1}}{P_t} + tr_t - k_t + (1-\delta)k_{t-1} \\ -\frac{M_t}{P_t} - CC_t\frac{S_t}{P_t} - \frac{S_t\tau_{cct}(1-j_t)}{P_t}, 1 - h_t - \phi n_t\Big] + \eta_{1t}\Big[n_t\frac{M_t}{P_t} - c_t j_t^2\Big] + \\ \eta_{2t}\Big[n_t CC_t\frac{S_t}{P_t} - c_t(1-j_t^2)\Big]\bigg]\bigg\} \tag{31}$$

The derivatives for the k_t, CC_t, M_t, j_t, n_t, and h_t (The first four derivatives is enough to get eq (7)) are as follows:

$$k_t: \quad E_t \bar{r^k_{t+1}} = E_t\frac{u_{ct}}{u_{ct+1}\beta} \tag{32}$$

$$CC_t: \quad \eta_{2t}E_t\left[\frac{n_t S_t}{\beta^{t+1}P_t u_{ct+1}} + \frac{S_{t+1}}{P_{t+1}}\right] = E_t\left[\frac{1}{\beta}\frac{S_t}{P_t}\frac{u_{ct}}{u_{ct+1}}\right] \tag{33}$$

$$M_t: \quad E_t\frac{1}{\beta}\frac{1}{P_t}\frac{u_{ct}}{u_{ct+1}} = E_t\left[\eta_{1t}\frac{n_t}{\beta^{t+1}P_t u_{ct+1}} + E_t\frac{1}{P_{t+1}}\right] \tag{34}$$

$$j_t: \quad \eta_{1t} = \eta_{2t} + \beta^t u_{ct}\frac{S_t\tau_{cct}}{2c_t j_t P_t} \tag{35}$$

After simplification, eq (33) will become:

$$E_t\left[\eta_{2t}\frac{n_t}{\beta^{t+1}u_{ct+1}} + \frac{S_{t+1}}{S_t}\frac{P_t}{P_{t+1}}\right] = E_t\frac{1}{\beta}\frac{u_{ct}}{u_{ct+1}} \tag{36}$$

Eq (34) becomes:

$$E_t\left[\frac{P_t}{P_{t+1}} + \eta_{1t}\frac{n_t}{\beta^{t+1}u_{ct+1}}\right] = E_t\frac{1}{\beta}\frac{u_{ct}}{u_{ct+1}} \tag{37}$$

Now subtracting eq (37) from eq (36), and then using eq (32), (35), definition of Θ_t and the inflation π_t, we can get:

$$E_t\left[\frac{\Theta_{t+1}}{\pi_{t+1}} - \frac{1}{\pi_{t+1}}\right] - E_t \bar{r^k_{t+1}}\frac{n_t S_t\tau_{cct}}{2c_t j_t P_t} = 0 \tag{38}$$

which is the same as eq (7):

$$E_t\Big[\frac{\Theta_{t+1}}{\pi_{t+1}} - \frac{S_t\tau_{cct}}{P_t}\frac{\bar{r}^k_{t+1}n_t}{c_t(j)}\Big] = E_t\frac{1}{\pi_{t+1}} \tag{39}$$

Please note that $\bar{r^k_t} = r_t^k + 1 - \delta$.

7.1.2 Model Solution

After plugging j_t^* expression into the budget constraint and CIA constraints, then the representative household solves the following Bellman equation (Please note that the same equation (39) can be extracted by using the Bellman approach):

$$
\begin{aligned}
V_t(k_{t-1}, M_{t-1}, CC_{t-1}) = \max_{c_t, k_t, M_t, CC_t, h_t, n_t} u(c_t, 1-h_t-\phi n_t) + \beta E_t V_{t+1}(k_t, M_t, CC_t) + \lambda_t \Bigg\{ r_t^k k_{t-1} + w_t h_t \\
+ CC_{t-1}\frac{S_t}{P_t} + tr_t + \frac{M_{t-1}}{P_t} - c_t - k_t + (1-\delta)k_{t-1} - \frac{M_t}{P_t} - CC_t\frac{S_t}{P_t} - \frac{S_t \tau_{cct}}{P_t} + \\
\frac{S_t}{P_t} c_t^{\frac{1}{\omega}} n_t^{\frac{-1}{\omega}} \tau_{cct}^{\frac{\omega-1}{\omega}} \left[E_t \frac{\frac{1}{\pi_{t+1}}(1-\omega)(\Theta_{t+1}-1)}{\frac{S_t}{P_t} r_{t+1}^{\bar{k}}} \right]^{\frac{1}{\omega}} \Bigg\} + \mu_{1t} \Bigg\{ n_t \frac{M_t}{P_t} - c_t^{\frac{1}{\omega}} n_t^{\frac{\omega-1}{\omega}} \left[E_t \frac{\frac{1}{\pi_{t+1}}(1-\omega)(\Theta_{t+1}-1)}{\frac{S_t}{P_t}\tau_{cct} r_{t+1}^{\bar{k}}} \right]^{\frac{1-\omega}{\omega}} \Bigg\} \\
+ \mu_{2t} \Bigg\{ n_t \frac{CC_t S_t}{P_t} - c_t + c_t^{\frac{1}{\omega}} n_t^{\frac{\omega-1}{\omega}} \left[E_t \frac{\frac{1}{\pi_{t+1}}(1-\omega)(\Theta_{t+1}-1)}{\frac{S_t}{P_t}\tau_{cct} r_{t+1}^{\bar{k}}} \right]^{\frac{1-\omega}{\omega}} \Bigg\}
\end{aligned} \tag{40}
$$

FOC:

$$
\begin{aligned}
c_t : u_{ct} + \lambda_t \Bigg\{ -1 + \frac{1}{\omega} c_t^{\frac{1}{\omega}-1} n_t^{\frac{-1}{\omega}} \tau_{cct}^{\frac{\omega-1}{\omega}} \frac{S_t}{P_t} \left[E_t \frac{\frac{1}{\pi_{t+1}}(1-\omega)(\Theta_{t+1}-1)}{\frac{S_t}{P_t} r_{t+1}^{\bar{k}}} \right]^{\frac{1}{\omega}} \Bigg\} + \\
\mu_{1t} \Bigg\{ -\frac{1}{\omega} c_t^{\frac{1}{\omega}-1} n_t^{1-\frac{1}{\omega}} \tau_{cct}^{\frac{\omega-1}{\omega}} \left[E_t \frac{\frac{1}{\pi_{t+1}}(1-\omega)(\Theta_{t+1}-1)}{\frac{S_t}{P_t} r_{t+1}^{\bar{k}}} \right]^{\frac{1-\omega}{\omega}} \Bigg\} + \\
\mu_{2t} \Bigg\{ -1 + \frac{1}{\omega} c_t^{\frac{1}{\omega}-1} n_t^{1-\frac{1}{\omega}} \tau_{cct}^{\frac{\omega-1}{\omega}} \left[E_t \frac{\frac{1}{\pi_{t+1}}(1-\omega)(\Theta_{t+1}-1)}{\frac{S_t}{P_t} r_{t+1}^{\bar{k}}} \right]^{\frac{1-\omega}{\omega}} \Bigg\} = 0
\end{aligned} \tag{41}
$$

$$
k_t : \beta E_t V_{t+1,k_t} = \lambda_t \tag{42}
$$

$$
M_t : \beta E_t V_{t+1,Mt} = \frac{1}{P_t}[\lambda_t - \mu_{1t} n_t] \tag{43}
$$

$$
CC_t : \beta E_t V_{t+1,cct} = \lambda_t \frac{S_t}{P_t} - \mu_{2t} n_t \frac{S_t}{P_t} \tag{44}
$$

$$
h_t : u_{lt} = \lambda_t w_t \tag{45}
$$

$$
\begin{aligned}
n_t : -\phi u_{lt} + \lambda_t \Bigg\{ -\frac{1}{\omega} c_t^{\frac{1}{\omega}} n_t^{-\frac{1}{\omega}-1} \tau_{cct}^{\frac{\omega-1}{\omega}} \frac{S_t}{P_t} \left[E_t \frac{\frac{1}{\pi_{t+1}}(1-\omega)(\Theta_{t+1}-1)}{\frac{S_t}{P_t} r_{t+1}^{\bar{k}}} \right]^{\frac{1}{\omega}} \Bigg\} + \\
\mu_{1t} \Bigg\{ \frac{M_t}{P_t} - \frac{\omega-1}{\omega} c_t^{\frac{1}{\omega}} n_t^{\frac{-1}{\omega}} \tau_{cct}^{\frac{\omega-1}{\omega}} \left[E_t \frac{\frac{1}{\pi_{t+1}}(1-\omega)(\Theta_{t+1}-1)}{\frac{S_t}{P_t} r_{t+1}^{\bar{k}}} \right]^{\frac{1-\omega}{\omega}} \Bigg\} + \\
\mu_{2t} \Bigg\{ \frac{CC_t S_t}{P_t} + \frac{\omega-1}{\omega} c_t^{\frac{1}{\omega}} n_t^{\frac{-1}{\omega}} \tau_{cct}^{\frac{\omega-1}{\omega}} \left[E_t \frac{\frac{1}{\pi_{t+1}}(1-\omega)(\Theta_{t+1}-1)}{\frac{S_t}{P_t} r_{t+1}^{\bar{k}}} \right]^{\frac{1-\omega}{\omega}} \Bigg\} = 0
\end{aligned} \tag{46}
$$

Envelope conditions:

$$
M_{t-1} : V_{t,M_{t-1}} = \lambda_t \frac{1}{P_t} \tag{47}
$$

$$k_{t-1} : V_{t,k_{t-1}} = \lambda_t[r_t^k + 1 - \delta] \tag{48}$$

$$CC_{t-1} : V_{t,cc_{t-1}} = \lambda_t \frac{S_t}{P_t} \tag{49}$$

Combining updated envelope conditions and first-order conditions:
Eq (42) + eq (48)

$$\lambda_t = \beta E_t \lambda_{t+1}(r_{t+1}^k + 1 - \delta) \tag{50}$$

Eq (43) + eq (47)

$$\beta E_t \lambda_{t+1} \frac{1}{P_{t+1}} = \frac{1}{P_t}(\lambda_t - \mu_{1t} n_t) \tag{51}$$

Eq (44) + eq (49)

$$\beta E_t \lambda_{t+1} \frac{S_{t+1}}{P_{t+1}} = \frac{S_t}{P_t}(\lambda_t - \mu_{2t} n_t) \tag{52}$$

At the steady-state, eq (51) and (52) become:

$$\mu_1 = \frac{1}{n}\lambda(1-\beta) \tag{53}$$

$$\mu_2 = \frac{1}{n}\lambda(1-\beta) \tag{54}$$

Thus, the following equations determine the corresponding values of the consumer's optimization problem: (41), (45), (46), (50), (51), (52).
The fiat money balance M_t growth rule can be simplified as:

$$\frac{M_t}{P_t} = \frac{g_{mt}}{\pi_t} \frac{M_{t-1}}{P_{t-1}} \tag{55}$$

Thus, at steady-state: $g_m = \pi = 1$.
At the steady-state, travel times can be expressed as:

$$n = \left[\frac{c}{\frac{\phi w}{1-\beta} - \frac{1}{1-\beta} \frac{S^2}{P^2} \frac{\tau_{cc}^2}{c} \frac{\pi \bar{r}^k}{2(\Theta-1)}} \right]^{\frac{1}{2}} \tag{56}$$

Simplifying eq (41) at the steady-state, we get:

$$\frac{u_c}{u_l} w - \frac{S}{P} \frac{\tau_{cc}^2}{c^2} \frac{n}{A} = 1 + \frac{1-\beta}{n} \tag{57}$$

After combing eq (56) and (57) and simplifying further, eq (57) becomes a cubic equation, which is

$$a_{11} c^3 + a_{12} c^2 + a_{13} c + a_{14} = 0 \tag{58}$$

where

$$a_{11} = (1-\beta)\phi w \left[\frac{w\gamma}{1-\gamma} \frac{1}{B} + 1 \right]^2 \tag{59}$$

$$a_{12} = -2(1-\beta)\phi w \frac{w\gamma}{1-\gamma} \left[\frac{w\gamma}{1-\gamma} \frac{1}{B} + 1 \right] - (1-\beta) \frac{S}{P} \frac{\tau_{cc}^2}{A} \left[\frac{w\gamma}{1-\gamma} \frac{1}{B} + 1 \right]^2 - \left[\frac{(1-\beta) w \phi}{1-\gamma} \right]^2 \tag{60}$$

$$a_{13} = (1-\beta)\phi w \left[\frac{w\gamma}{1-\gamma} \right]^2 + 2(1-\beta) \frac{S}{P} \frac{\tau_{cc}^2}{A} \frac{w\gamma}{1-\gamma} \left[\frac{w\gamma}{1-\gamma} \frac{1}{B} + 1 \right] \tag{61}$$

$$a_{14} = -(1-\beta) \frac{S}{P} \frac{\tau_{cc}^2}{A} \left[\frac{w\gamma}{1-\gamma} \right]^2 \tag{62}$$

where

$$A = \frac{2P(\Theta - 1)}{\pi S \bar{r}^k} \tag{63}$$

$$B = \left[\frac{\frac{1}{\beta} + \delta - 1}{\alpha}\right]^{\frac{\alpha}{\alpha-1}} - \delta\left[\frac{\frac{1}{\beta} + \delta - 1}{\alpha}\right]^{\frac{1}{\alpha-1}} \tag{64}$$

What is more,

$$M = \frac{c}{n}(j^*)^2 P \tag{65}$$

$$CC = \frac{c}{n}(1 - (j^*)^2)\frac{P}{S} \tag{66}$$

Besides,

$$\frac{u_c}{u_l} = \frac{\gamma}{1-\gamma}\frac{l}{c} \tag{67}$$

$$\frac{c}{h} = B \tag{68}$$

7.2 Appendix A2

The following table describes the data source.

Variable	Source
Real Personal Consumption Expenditure	Federal Reserve Economic Data (2019)
Gross Domestic Product	Federal Reserve Economic Data (2019)
Monetary Base; Currency in Circulation	Federal Reserve Economic Data (2019)
Core PCE Price Index	Federal Reserve Economic Data (2019)
Gross Fixed Capital Formation	Federal Reserve Economic Data (2019)
Total Non-farm employment	Federal Reserve Economic Data (2019)
Consumption of Fixed Capital	Federal Reserve Economic Data (2019)
Average Weekly Hours of All employees (Total Private)	Federal Reserve Economic Data (2019)
USD/EUR Exchange Rate	Federal Reserve Economic Data (2019)
Coinbase bitcoin Price	Federal Reserve Economic Data (2019)
Share of labor Compensation in GDP	Federal Reserve Economic Data (2019)
Bitcoin Network Cost Per-Transaction	Blockchain (2020[a])
Average bitcoin Market Value	Blockchain (2020[a])

7.3 Appendix A3

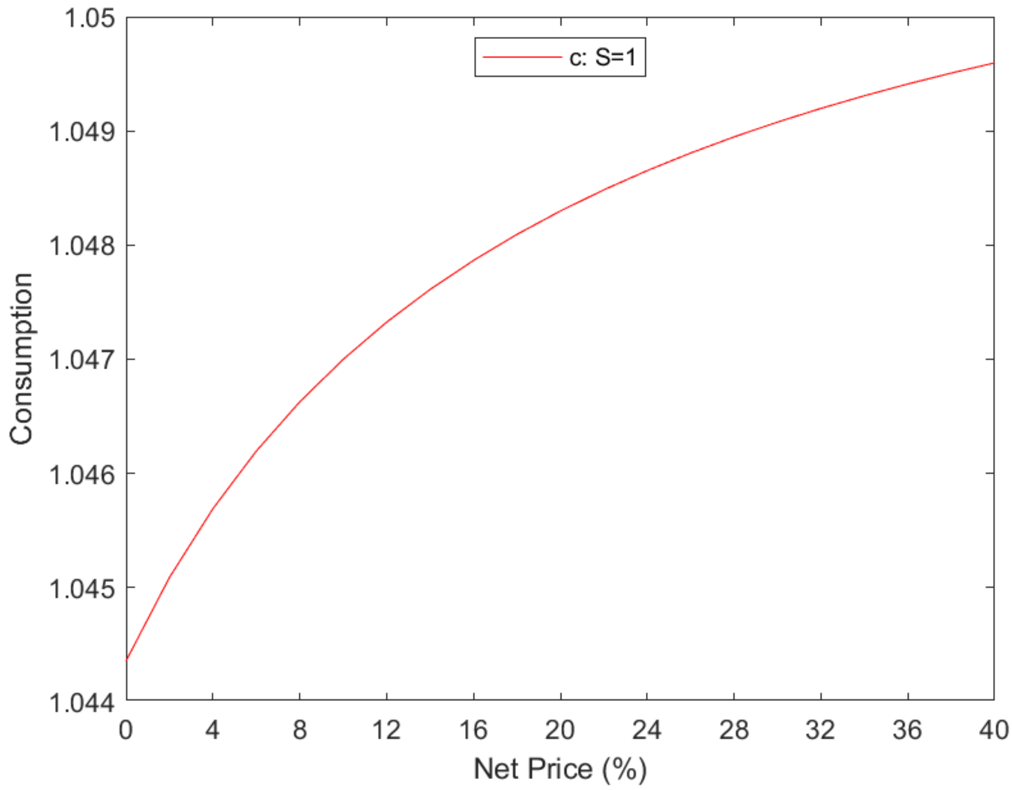

Figure 17: Optimal consumption response to changes in the price (%).

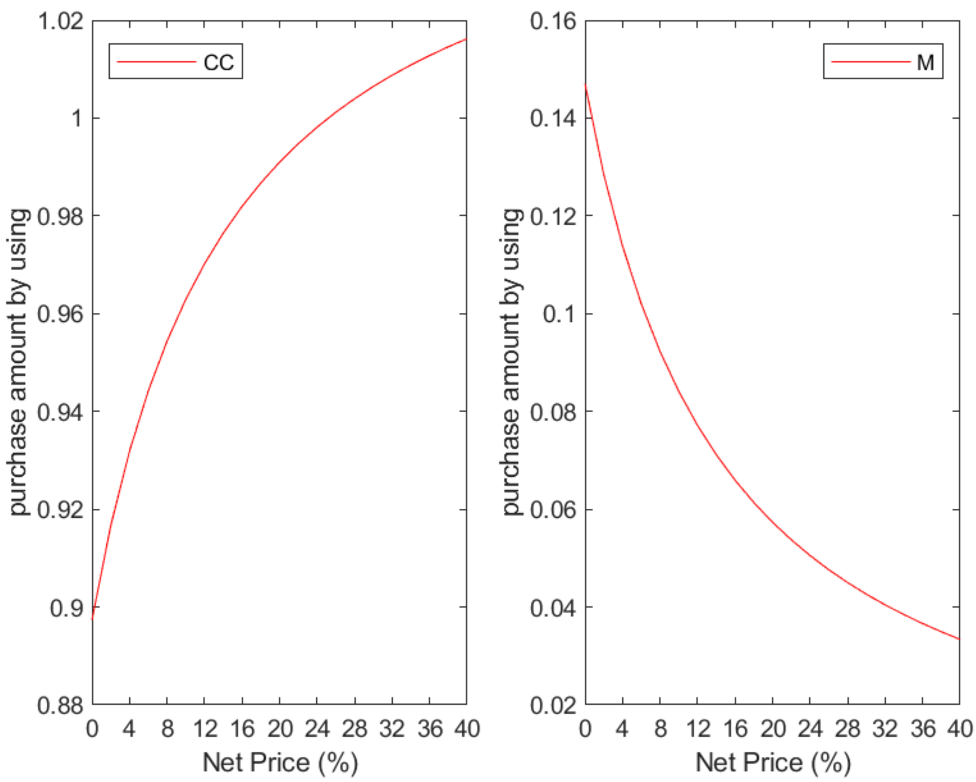

Figure 18: Consumption amount purchased by using different currencies: response to changes in the price. Note: The consumption amount by different currencies are defined as: $\frac{Mn}{P} = cj^*$, $\frac{CCSn}{P} = c(1 - j^*)$.

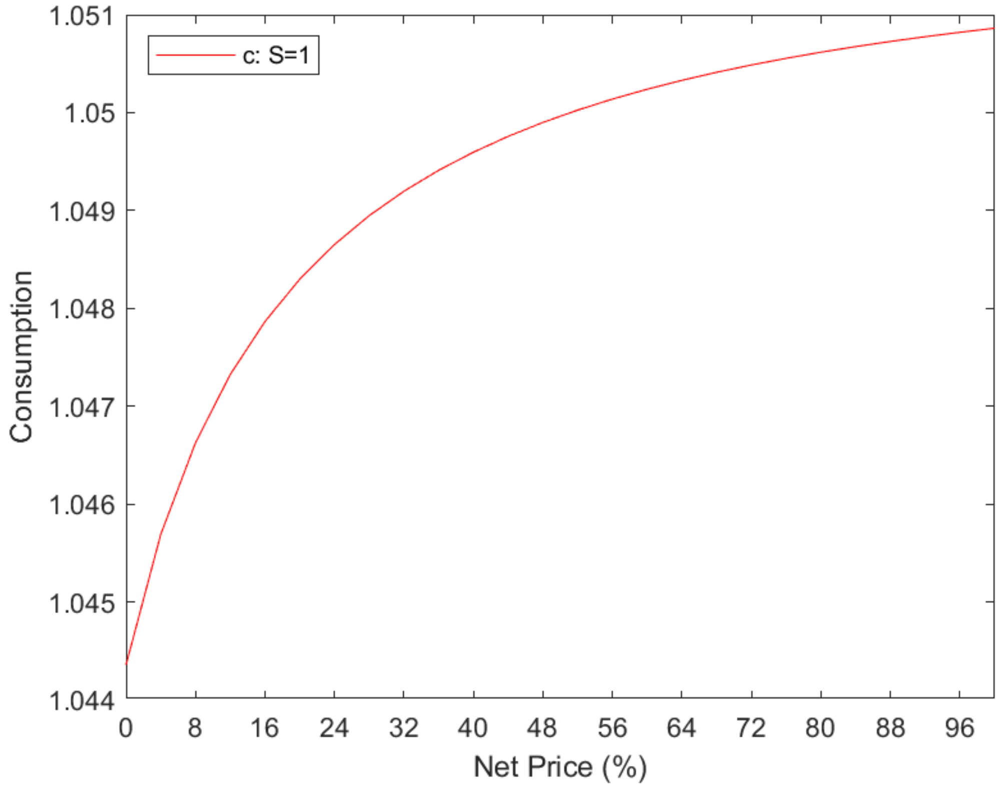

Figure 19: Optimal consumption response to changes in the price (0-100% range).

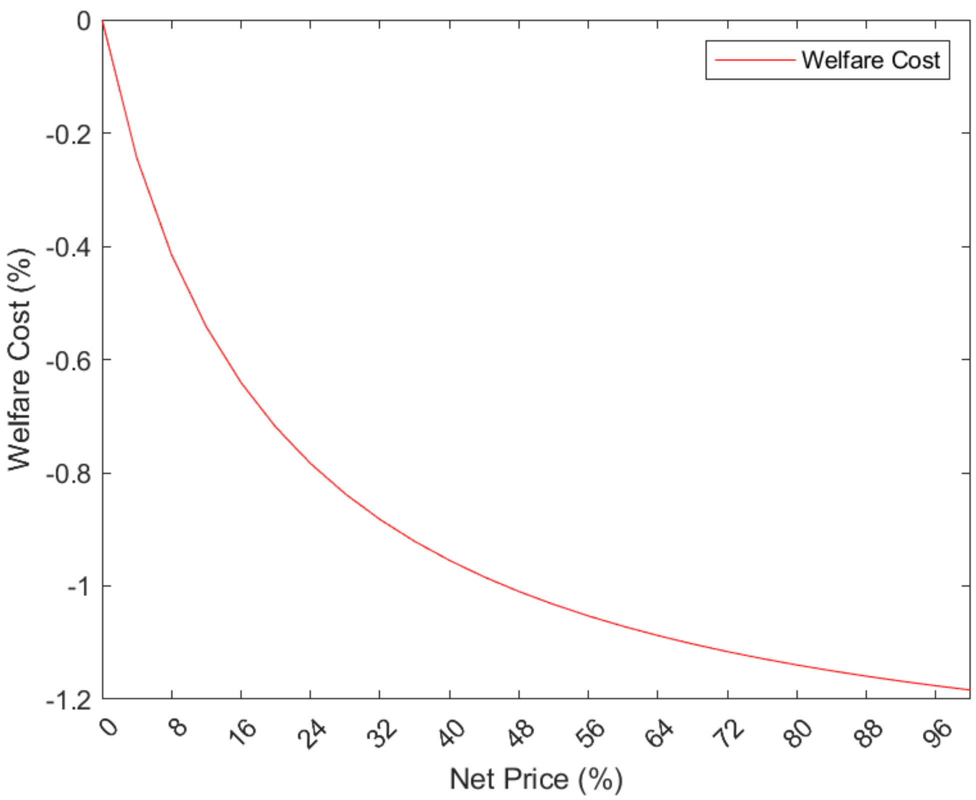

Figure 20: Welfare cost of the price.

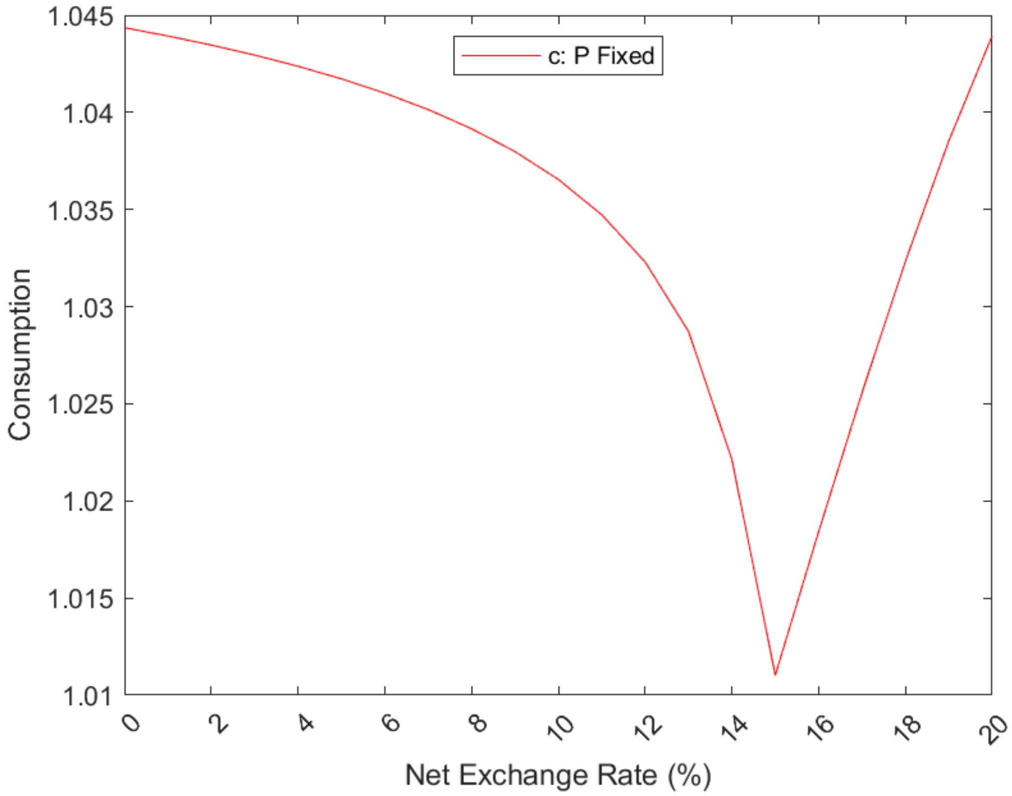

Figure 21: Optimal consumption response to change in the nominal exchange rate (%). Note: Consumption values are real parts of complex numbers.

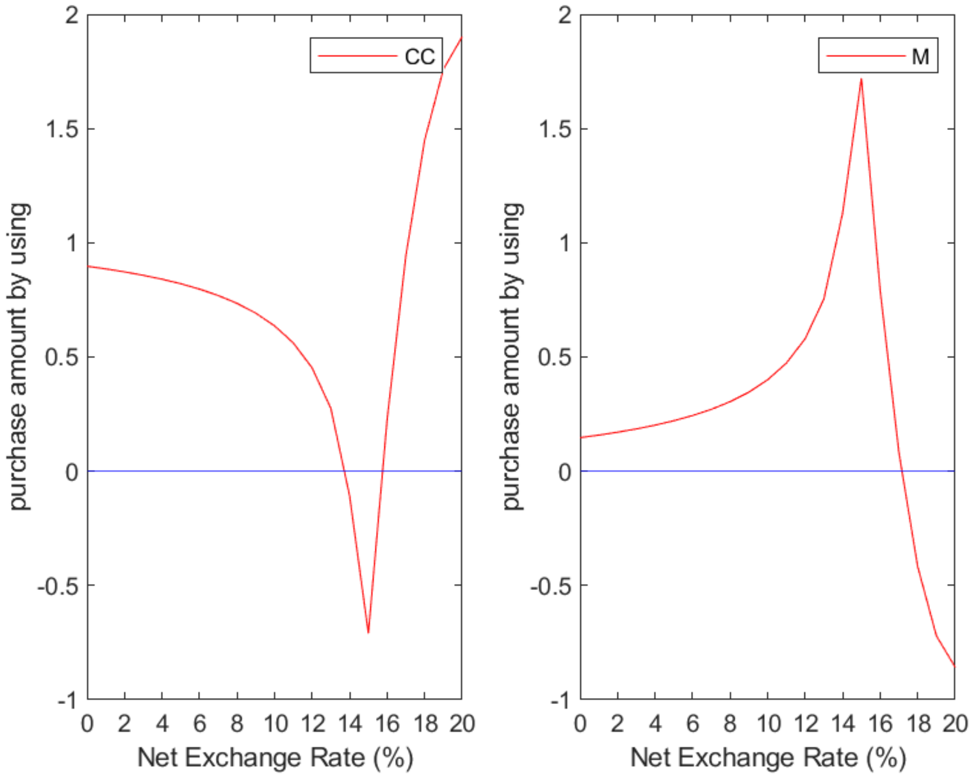

Figure 22: Consumption amount purchased by using different currencies: response to changes in the nominal exchange rate. Note: The consumption amount by different currencies are defined as: $\frac{Mn}{P} = cj^*$, $\frac{CCSn}{P} = c(1-j^*)$. Values of the purchase amount are real parts of complex numbers.

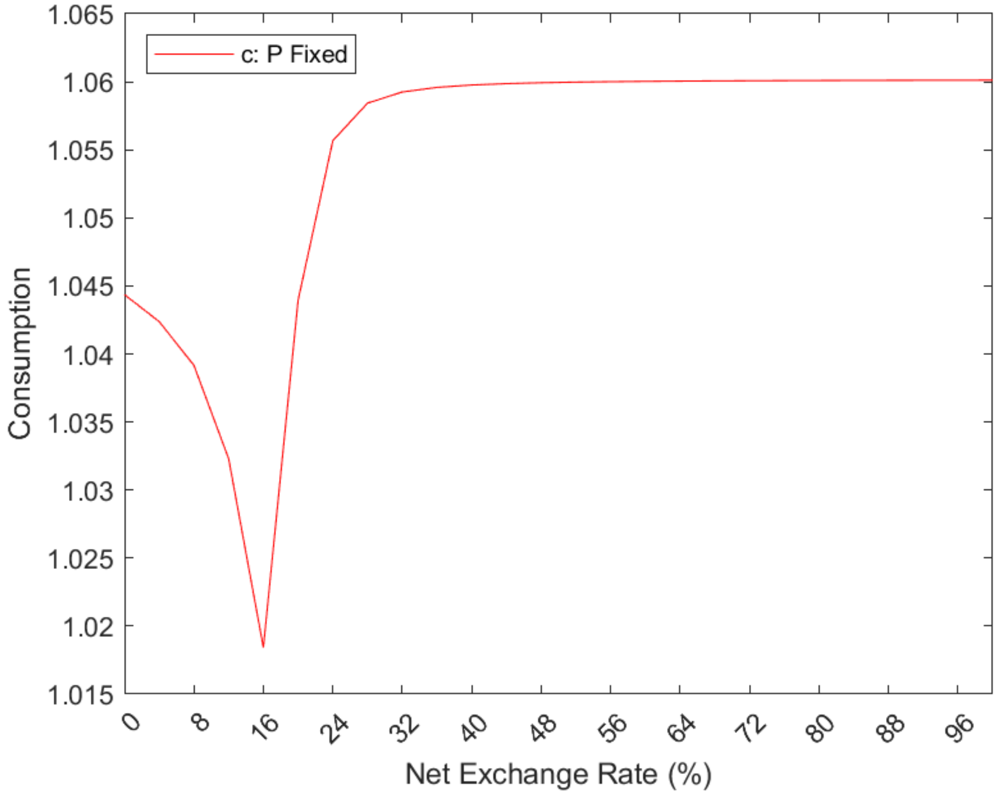

Figure 23: Optimal consumption response to changes in the nominal exchange rate (0-100% range). Note: Consumption values are real parts of complex numbers.

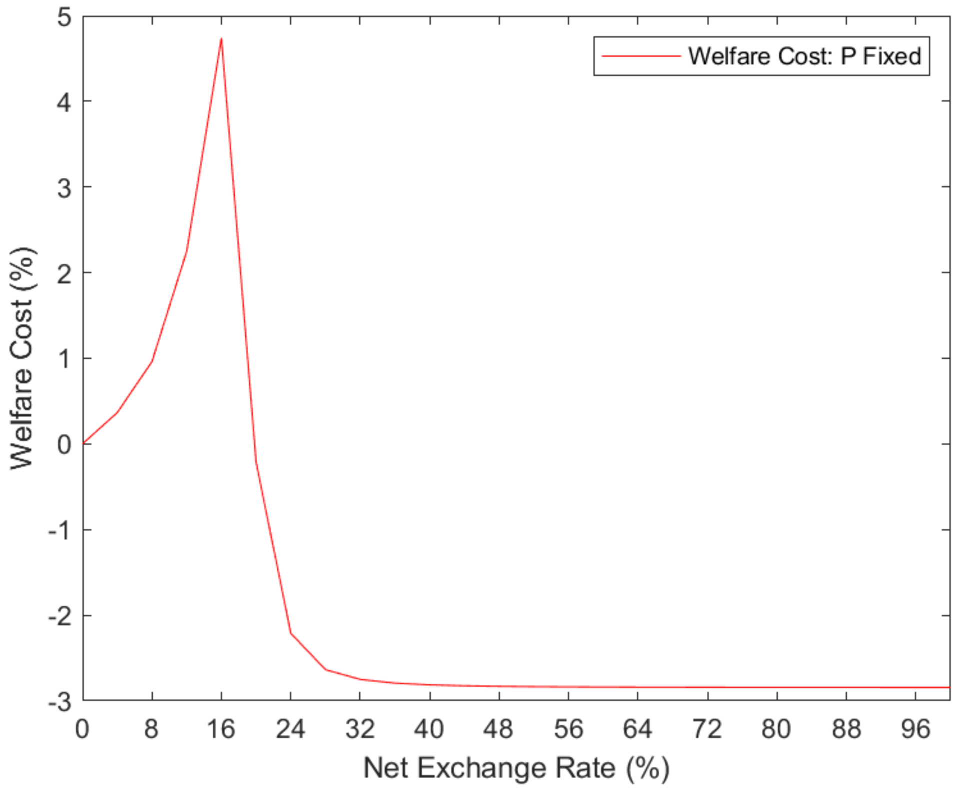

Figure 24: Welfare cost of the nominal exchange rate. Note: Welfare cost values are real parts of complex numbers.

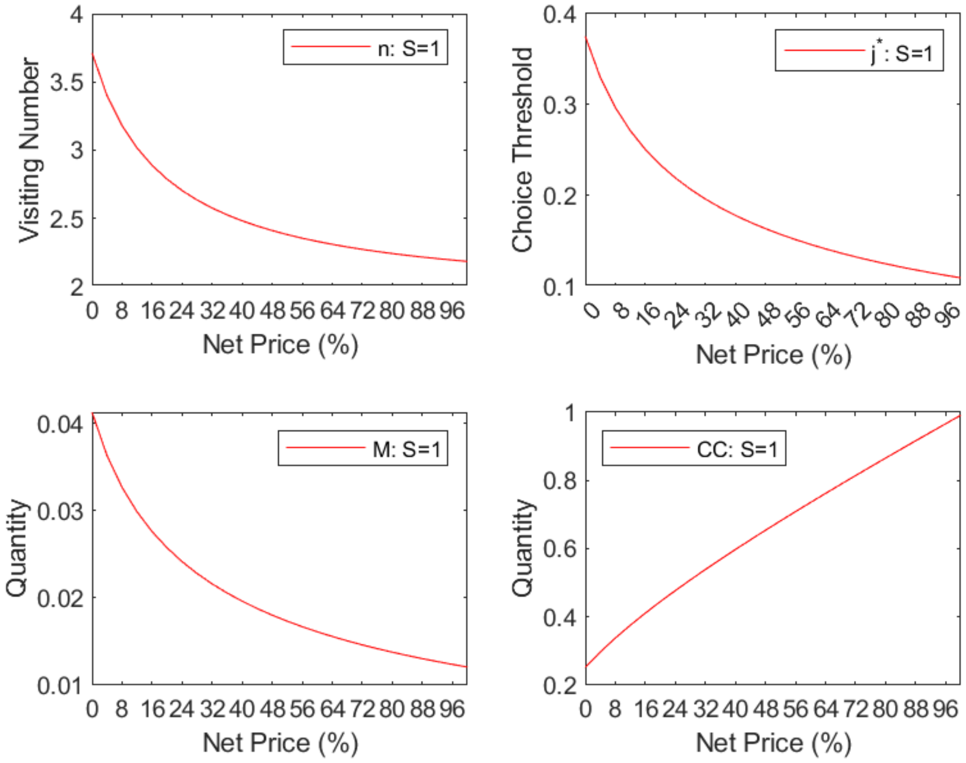

Figure 25: Changes in critical variables when the price changes.

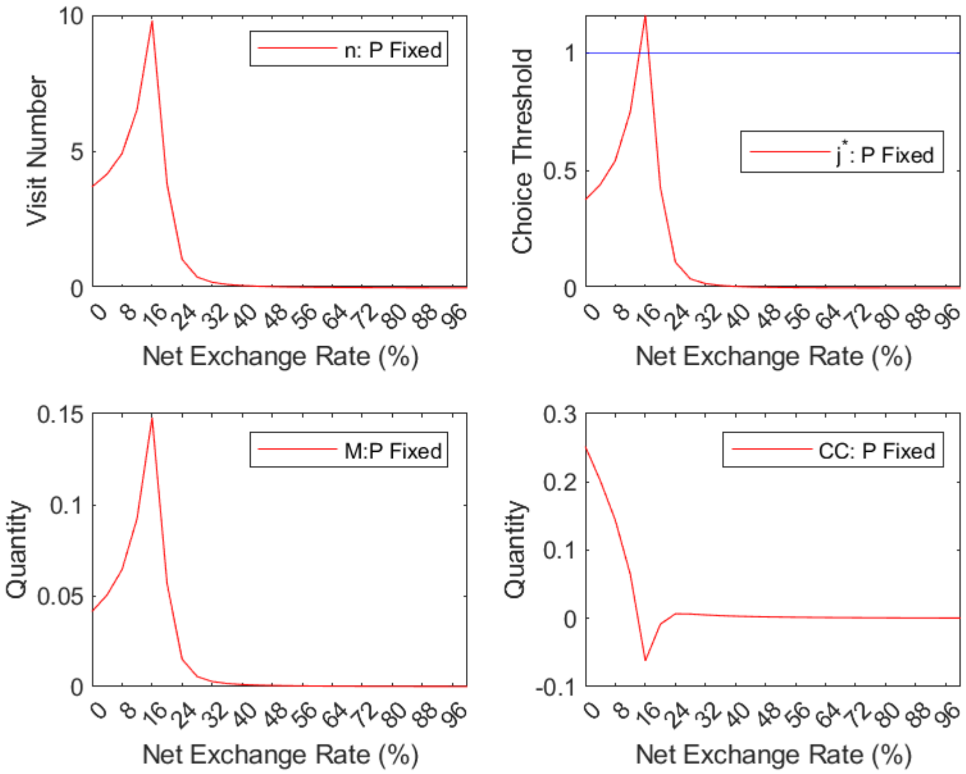

Figure 26: Changes in critical variables when nominal exchange rate changes. Note: Since the values of n, j^*, CC, and M are complex numbers, only the real parts are plotted.

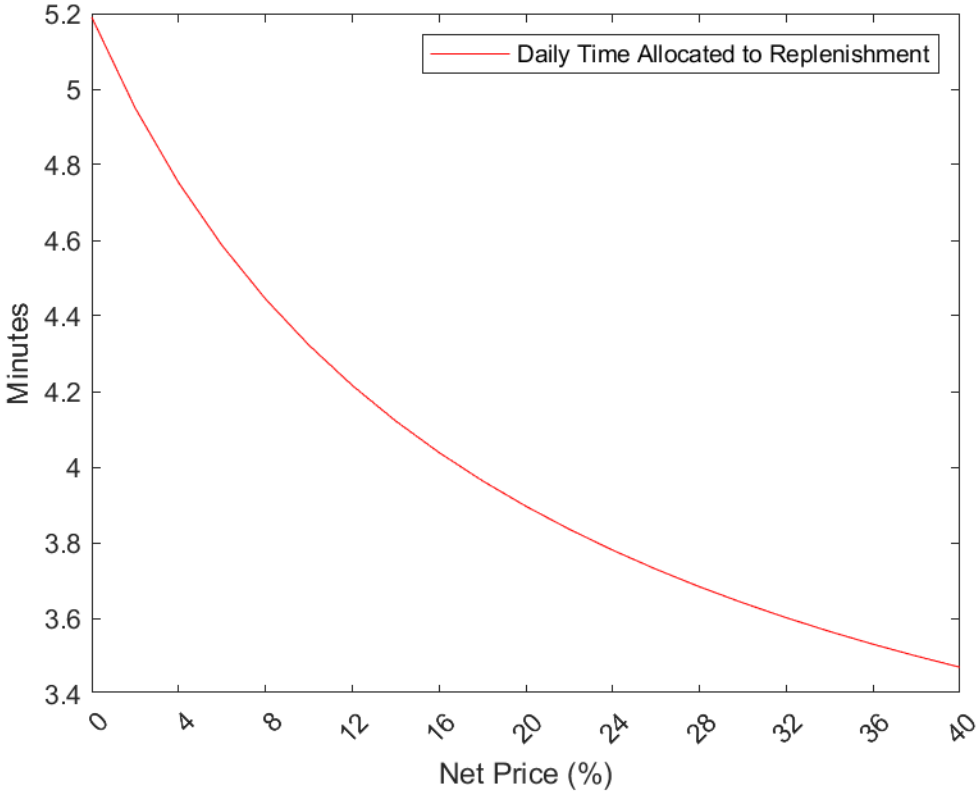

Figure 27: Average daily time spent on visiting the asset market (or managing asset portfolio) when the price changes. Note: Values of average daily time are real parts of complex numbers.

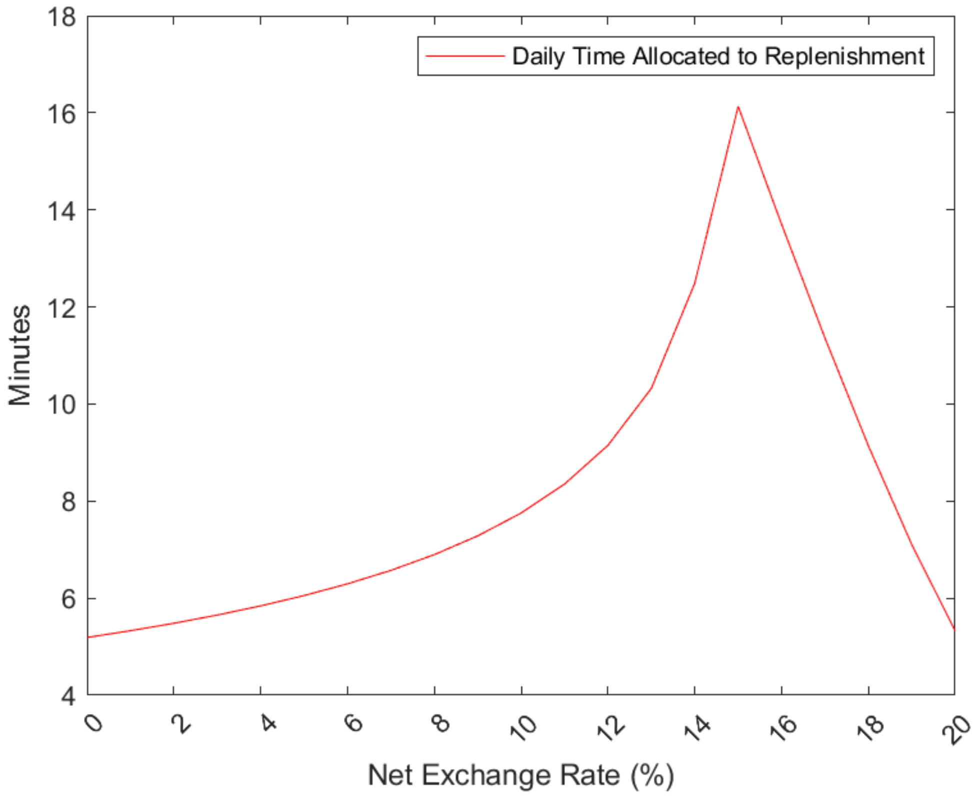

Figure 28: Average daily time spend on visiting the asset market (or managing asset portfolio) when the nominal exchange rate changes. Note: Values of average daily time are real parts of complex numbers.

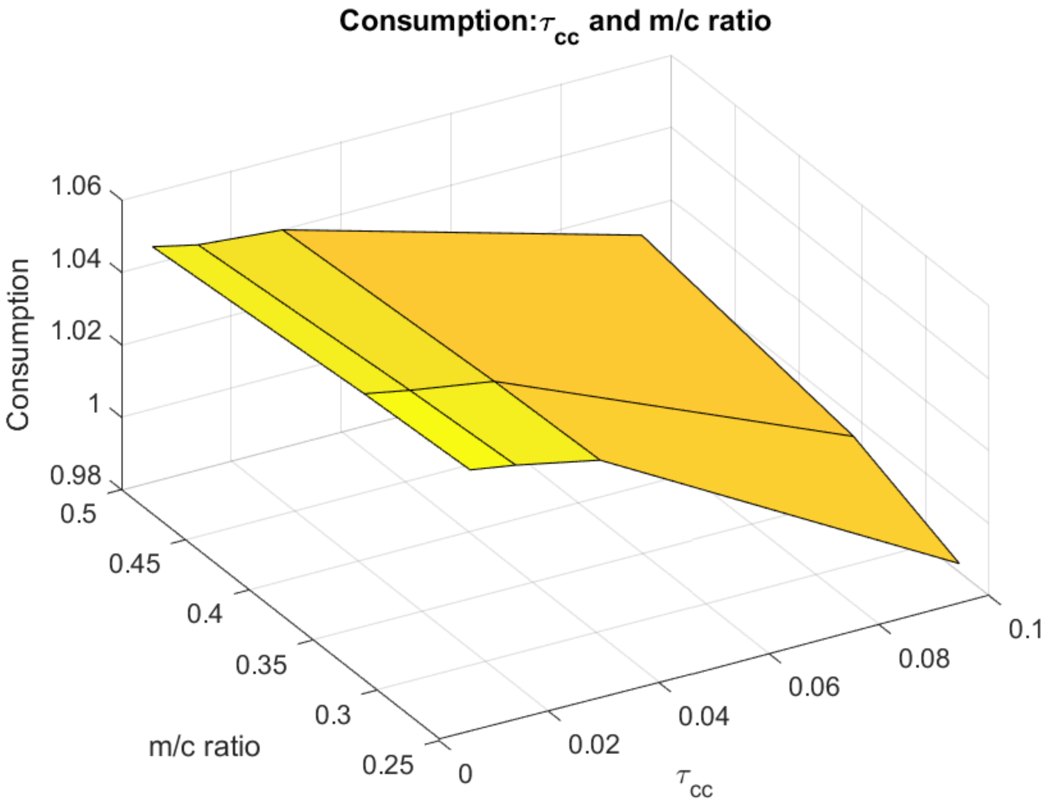

Figure 29: Sensitivity analysis - consumption: transaction cost of using cryptocurrency as payment instrument and domestic currency to consumption ratio. Note: Optimal consumption goes down slightly as the domestic currency to consumption ratio goes up. This trend is not obvious in this graph. Consumption values are real parts of complex numbers.

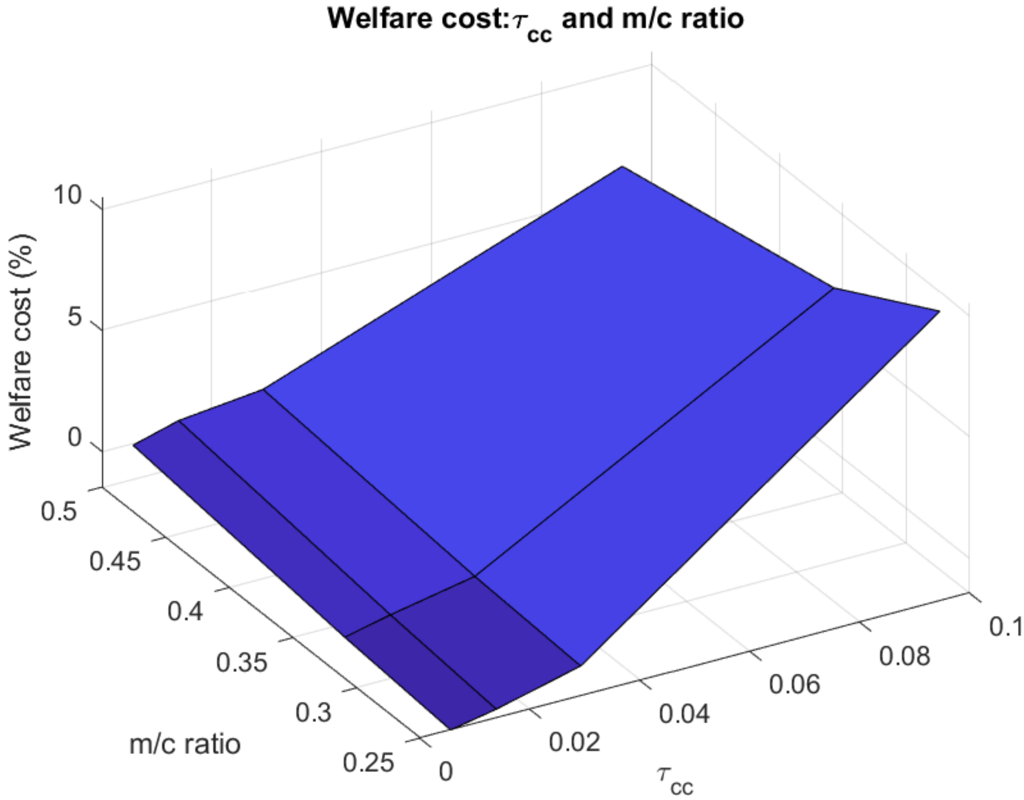

Figure 30: Sensitivity analysis - welfare cost: transaction cost of using cryptocurrency as payment instrument and domestic currency to consumption ratio. Note: Welfare cost values are real parts of complex numbers.

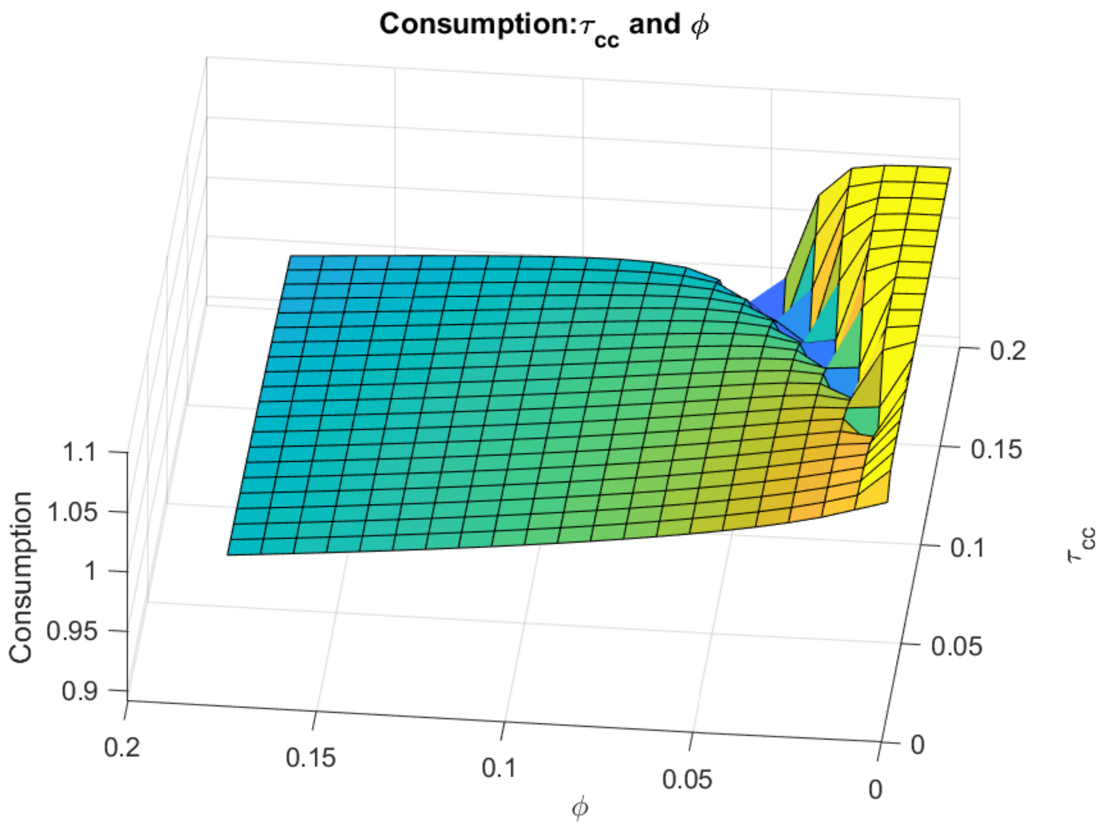

Figure 31: Technological implications - real consumption: transaction cost τ_{cc} and asset market travel time cost ϕ. Note: Consumption values are real parts of complex numbers.

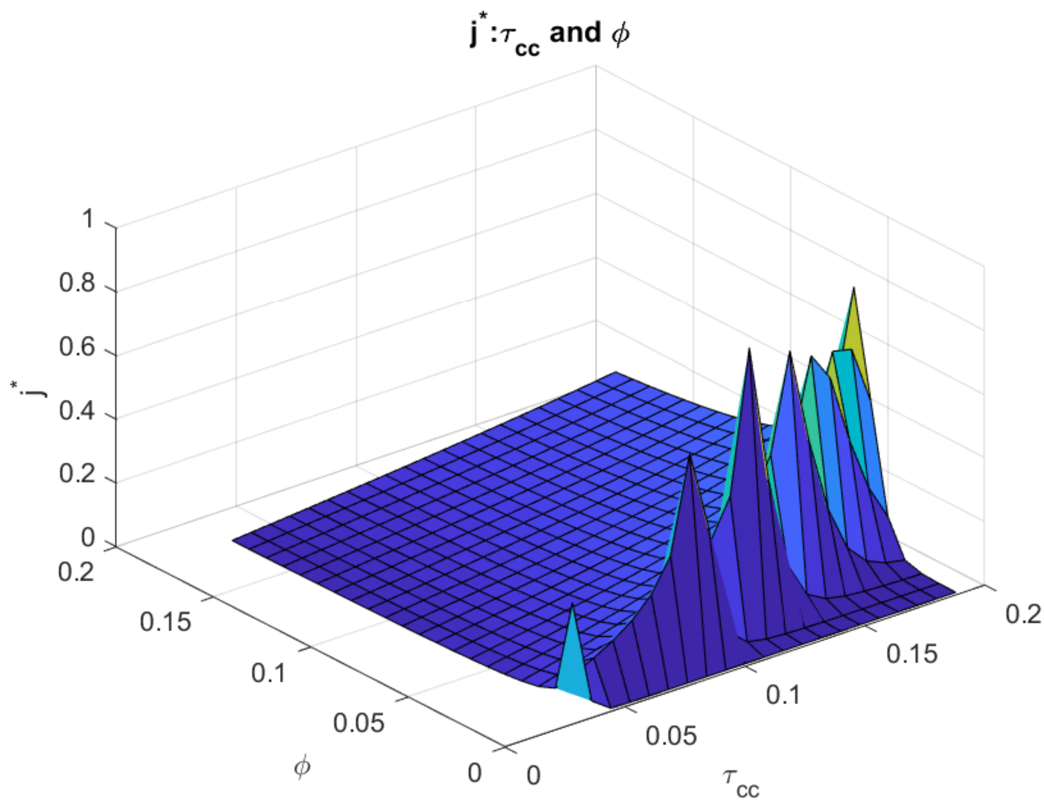

Figure 32: Technological implications - threshold j^*: transaction cost τ_{cc} and asset market travel time cost ϕ. Note: Threshold level values are real parts of complex numbers.

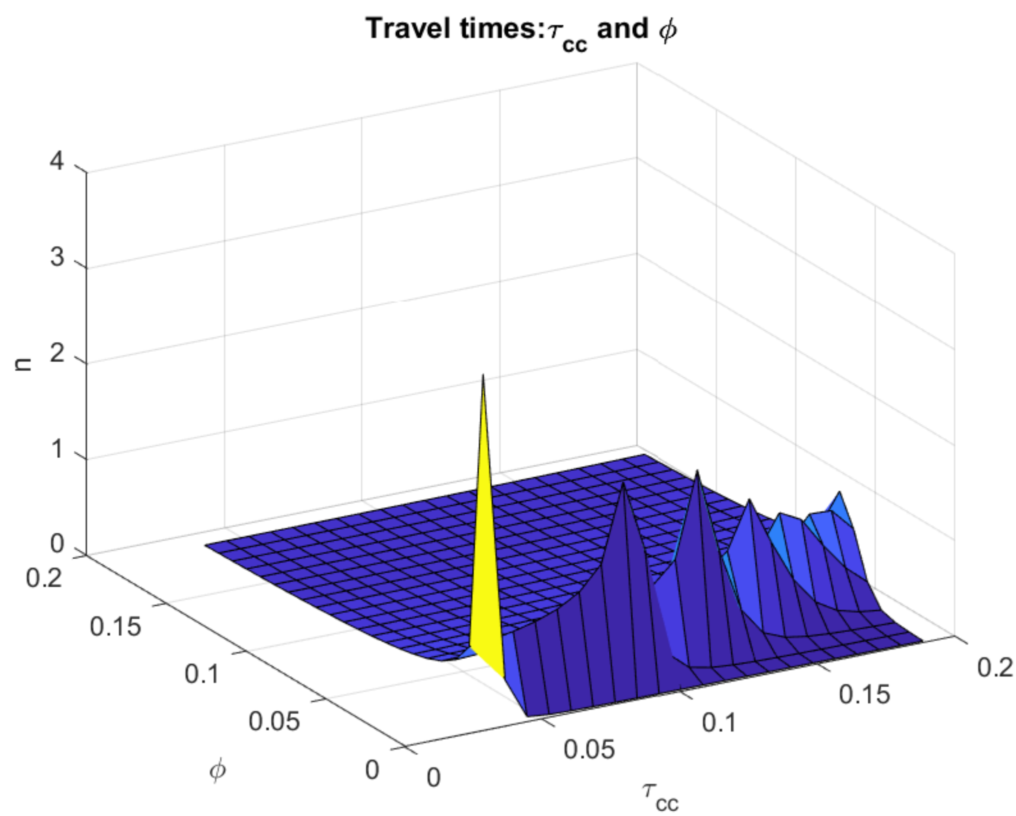

Figure 33: Technological implications - asset market travel times: transaction cost τ_{cc} and asset market travel time cost ϕ. Note: Values of n are real parts of complex numbers.

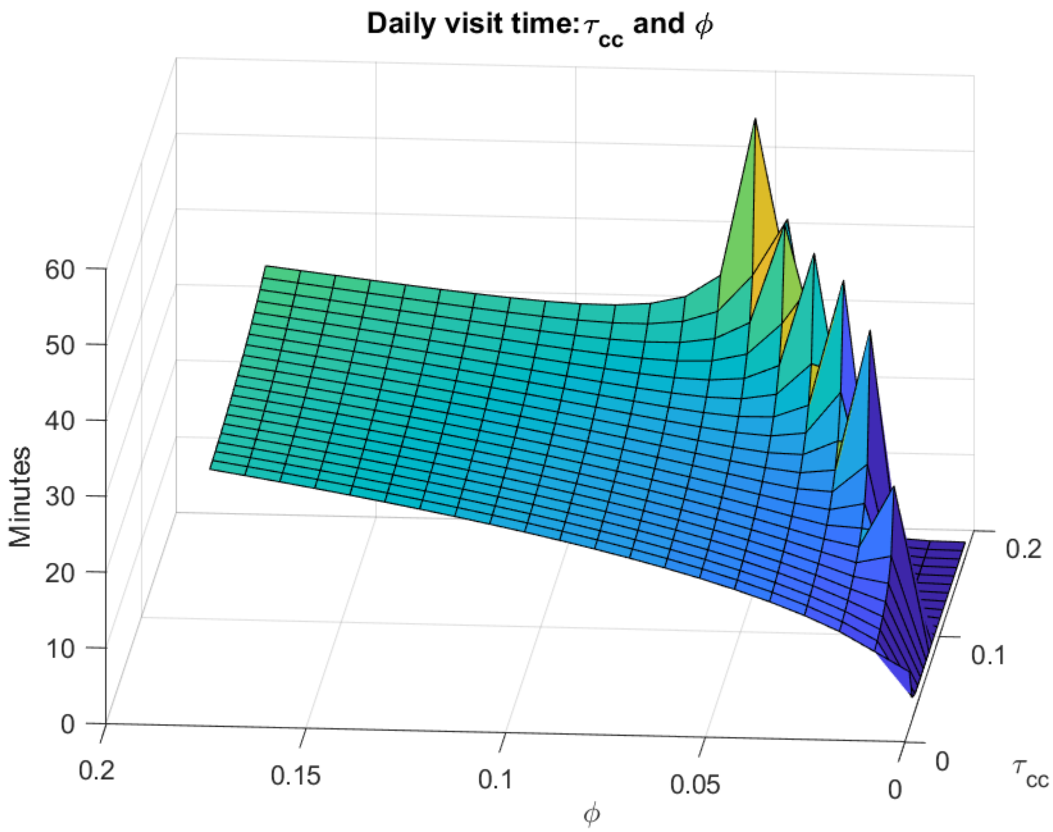

Figure 34: Technological implications - average daily time spent on asset managing: transaction cost τ_{cc} and asset market travel time cost ϕ. Note: Average daily time values are real parts of complex numbers.

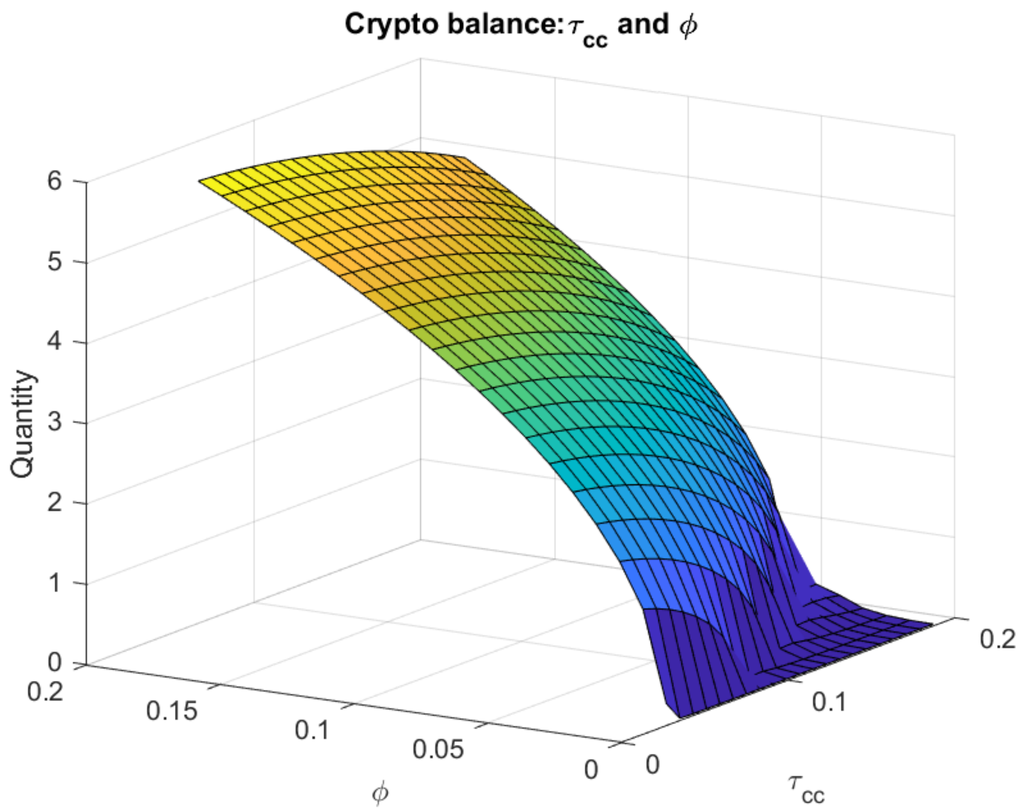

Figure 35: Technological implications - cryptocurrency balance: transaction cost τ_{cc} and asset market travel time cost ϕ. Note: Values of cryptocurrency balance are real parts of complex numbers.

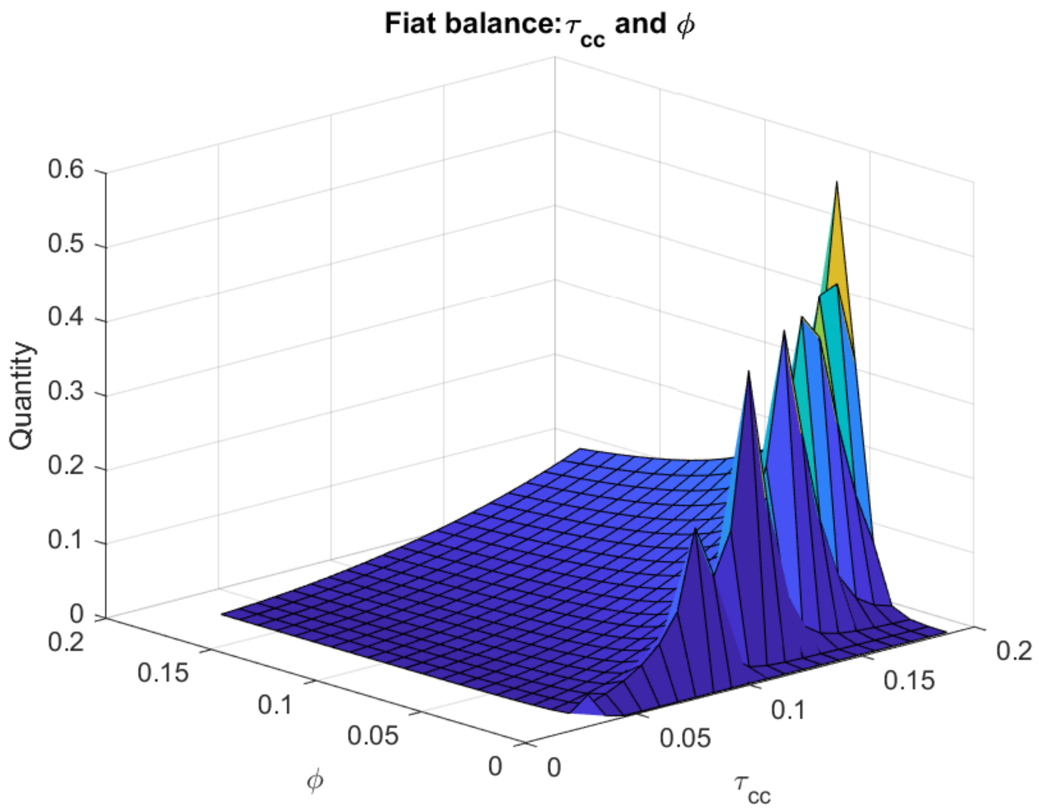

Figure 36: Technological implications - nominal fiat currency balance: transaction cost τ_{cc} and asset market travel time cost ϕ. Note: Values of nominal fiat currency balance are real parts of complex numbers.

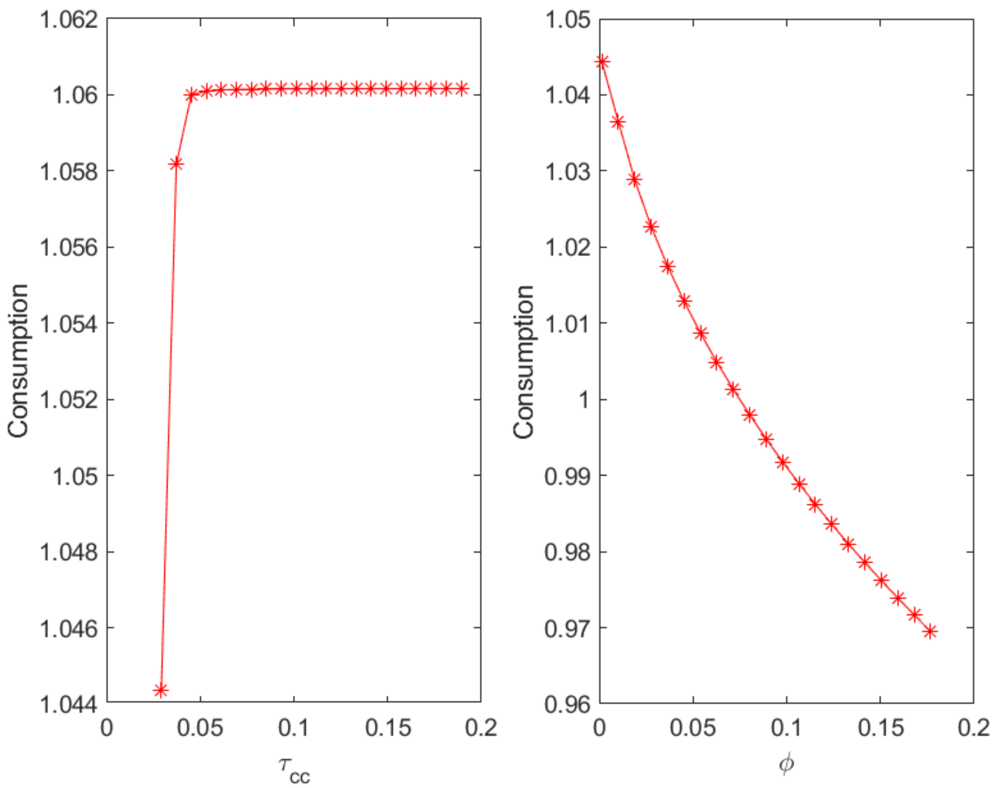

Figure 37: Technological implications - real consumption: τ_{cc} and ϕ at the steady-state (one of them is fixed in each case). Note: Values of consumption response to the τ_{cc} are real parts of complex numbers.

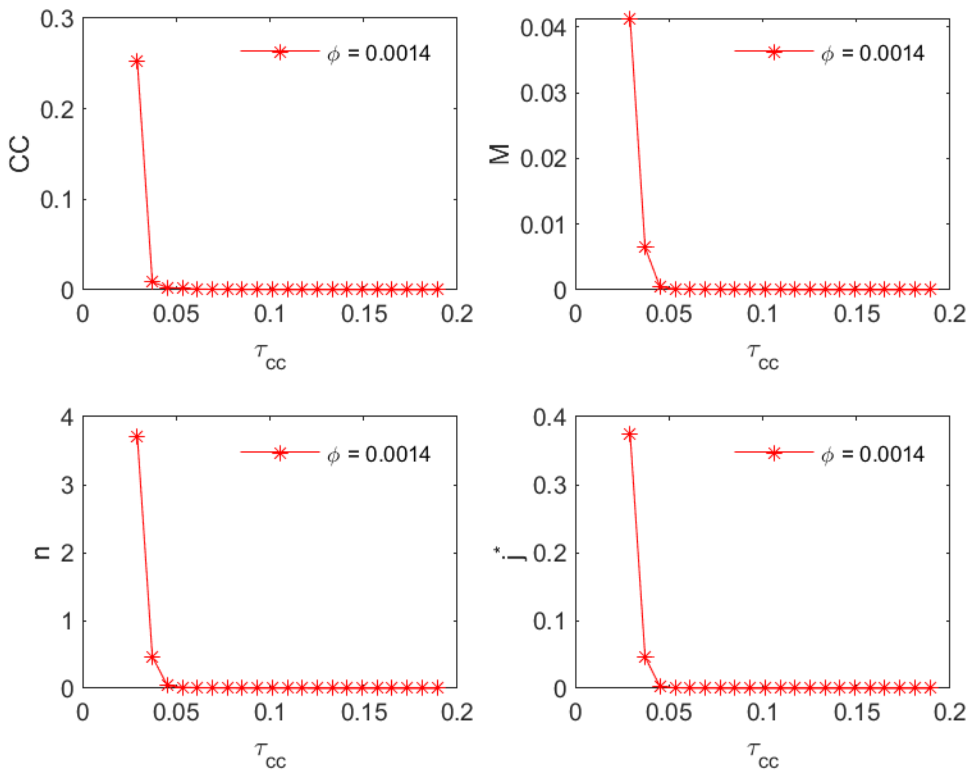

Figure 38: Response of critical variables to the transaction cost when $\phi = 0.0014$. Note: Values of critical variables are real parts of complex numbers.