

# Heterogeneity, Frictional Assignment and Home-ownership\*

Allen Head<sup>†</sup>

Huw Lloyd-Ellis<sup>†</sup>

Derek Stacey<sup>‡</sup>

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## Abstract

A model of frictional assignment is developed to study the composition of housing units and households across city-level ownership and rental markets. Heterogeneous houses are built by a competitive development industry and either rented competitively or sold through directed search to households which differ in wealth and sort over housing types. Even in the absence of financial restrictions and constraints on house characteristics, higher income households are more likely to own and lower quality housing is more likely to be rented. When calibrated to match average features of housing markets *within* U.S. cities, the model is qualitatively consistent with U.S. data on the relationships between observed differences in median income, inequality, median household age, and construction/land costs *across* cities and both home-ownership and the average cost of owning vs. renting. Policies designed to improve housing affordability raise both housing quality and ownership for lower income households while lowering housing quality (but not ownership) for high income ones.

*Journal of Economic Literature Classification:* E30, R31, R10

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<sup>†</sup>Queen's University, Department of Economics, Kingston, Ontario, Canada, K7L 3N6.  
Email: heada@econ.queensu.ca, lloydell@econ.queensu.ca

<sup>‡</sup>Ryerson University, Department of Economics, Toronto, Ontario, Canada, M5B 2K3.  
Email: dstacey@economics.ryerson.ca

# 1 Introduction

Search and matching models have been used extensively to understand housing market microstructure (*e.g.* Wheaton, 1990; Krainer, 2001; Albrecht, Anderson, Smith and Vroman, 2007; Halket and Pignatti Morano di Custoza, 2015; see Lu and Strange (2015) for a survey) and to study various aspects of the dynamics of housing markets (*e.g.* Diaz and Jerez, 2013; Head, Lloyd-Ellis and Sun, 2014; Ngai and Sheedy, 2017; Hedlund, 2015; Garriga and Hedlund, 2017; Anenberg and Bayer, 2018). Relatively little, however, has been done regarding the implications of search frictions for the composition of the housing stock across ownership and rental markets and the relative cost of owning versus renting. Rather, search-theoretic models typically focus either on one of these markets in isolation or, where both are incorporated, assume that they are largely disconnected on the supply side.<sup>1</sup> This contrasts with the standard user-cost framework in which the incentives for heterogeneous housing units to be sold or rented are a primary concern.<sup>2</sup>

In this paper, we develop a dynamic equilibrium model of *frictional assignment* in which the quality distribution of the housing stock across owner-occupied and rental markets is determined by the endogenous decisions of households, landlords and developers. The main assumptions of our theory are motivated by two broad observations: First, the likelihood of home-ownership is strongly increasing in household income and wealth after controlling for other household characteristics (including age and family composition), neighbourhood characteristics and cyclical factors.<sup>3</sup> Second, the likelihood that a given housing unit is owner-occupied rather than rented rises with the value of the unit.<sup>4</sup> Halket, Nesheim and Oswald (2015), for example, summarize their findings as follows: “Despite their relatively high gross yield in the rental sector, properties with high value physical characteristics are less likely to be bought up by landlords and supplied to renters.” Notwithstanding these overall tendencies, home-ownership is significant even for the lowest income quintile and some low value housing units are owned while many high value ones are rented.

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<sup>1</sup>Head, Lloyd-Ellis and Sun (2014) develop a search model with a rent vs. sell decision but where housing is homogeneous.

<sup>2</sup>A common argument is that nature of the housing stock in these markets are so different, we can treat them as completely segmented. It seems reasonable, however, to understand the reasons why these housing stocks are so different.

<sup>3</sup>See, for example, Rosen (1979), Goodman (1988), Kan (2000) and Carter (2011).

<sup>4</sup>Glaeser and Gyourko (2007) document that the composition of rental housing is systematically different from that which is owner-occupied. For example, owned units often consist of single-family detached dwellings, while rental units are more commonly part of multi-unit buildings. The average owner-occupied unit is roughly double the size of the typical rental unit.

Our model consists of a city comprised of housing units differentiated by quality (taken to represent size, proximity to amenities, etc.) and inhabited by a growing population of households with stochastic lifetimes who are differentiated permanently by income/wealth. New houses may be of any type and are built by a development industry comprised of a large number of firms with free entry. Construction/land costs increase with quality and, once built, a house's quality is permanently fixed. All households require housing, but may choose whether to rent or own. We assume that asset markets are complete and focus on a balanced growth path where new housing is built every period.<sup>5</sup>

The surplus associated with ownership, as opposed to rental, of a given housing unit rises with its quality. In our baseline specification, this occurs because maintenance costs incurred by landlords increase more rapidly with quality than those for owner-occupiers. This assumption is intended to reflect the idea that the costs of moral hazard associated with renting increase with house quality (*e.g.* Sweeney, 1974; Henderson and Ioannides, 1983). It may also reflect economies of scale in maintaining buildings with multiple low-quality apartments in comparison with detached houses. The model allows for other potential sources of this rising surplus, including preference for ownership that increases with quality or the implications of mortgage interest deductibility. The exact source, however, matters very little for our main results (see Section 4.4).

In equilibrium, ownership patterns solve a frictional assignment problem in the sense of Shi (2001, 2005). Vacant houses of any quality may be either rented in Walrasian markets or offered for sale through directed search.<sup>6</sup> Unmatched households either rent or purchase an affordable home of their preferred quality. Since searching for a vacant house is *costless*, whether or not households own depends on their willingness/ability to pay and the incentives facing *supply-side* participants. Specifically, the cost of holding a house vacant-for-sale is the foregone rent net of maintenance costs that could have been earned in the rental market. A price premium is therefore needed to induce sellers to offer homes for sale, and the premium is greater in tighter sub-markets with shorter expected time to buy. While owning is more desirable than renting for any given house quality,<sup>7</sup> proportionately more poor households end up renting because they *cannot afford* housing in market segments where housing for

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<sup>5</sup>The assumption of complete markets here functions mainly to avoid complications of *ex post* heterogeneity arising from idiosyncratic shocks at the household level. Households, particularly lower income ones, remain constrained with regard to their ability to afford housing. See below.

<sup>6</sup>Search is motivated by the idea that while houses of a given objective quality are in some sense alike, they have idiosyncratic differences which appeal only to certain households.

<sup>7</sup>This may be either because maintenance costs are lower or there is an ownership premium.

sale is more plentiful.

In the United States, average rates of home-ownership and the relative average costs of owning and renting vary dramatically across cities. For a sample of 366 metropolitan statistical areas (MSA's) in the 2010 American Community Survey (ACS), for example, home-ownership rates vary from 51% to 81% and ratios of average prices to average rents from 8.7 to 54.2.<sup>8</sup> To study these phenomena, we calibrate the model's balanced growth path to match several median features of U.S. MSA housing markets, including average time to sell, average ownership duration, the ratio of average prices to average rents and the distribution of ownership across income quintiles.<sup>9</sup> We use the calibrated model to characterize the steady state effects of variation in several key fundamentals. In particular, the model predicts that the ownership rate increases with median income and age and decreases with inequality and construction costs. The ratios of average prices to average rents increase with median income, inequality and construction costs.<sup>10</sup>

Using the 2010 ACS, we characterize the empirical relationships between both ownership rates and ratios of average prices to average rents and median income, inequality, age and land costs using cross-city regressions.<sup>11</sup> Controlling for other factors affecting the desirability of living in a given city, we find that, qualitatively, the patterns observed in cross-city data are remarkably consistent with the predictions of our model. Moreover, these patterns appear robust to alternative specifications and samples.

To study further the theory's quantitative predictions, we use the calibrated model to generate predicted cross-city variation in outcomes resulting from observed and inferred variation in MSA-level characteristics. We find that, while the distribution of income and age play a key role, differences in construction/land costs and average amenities across cities are the most important factor in accounting for observed cross-city variation in both ownership and ratios of average prices to average rents.

Given the observed variation in fundamentals, the model generates substantial variation in the *affordability* of housing across cities. Having less affordable housing reduces housing quality for all households and, importantly, makes ownership less

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<sup>8</sup>For this calculation we use the ratio of the mean price-asked for each MSA to mean annual rent (see Appendix B). Note that this is not the relative price and rent of a specific unit. Indeed, much of the large variation in the data, as in our model, is due to composition effects.

<sup>9</sup>The assumed income distribution is log-normal, with inequality measured by the Gini coefficient.

<sup>10</sup>The relationship between median age and the price-rent ratio is ambiguous.

<sup>11</sup>For our sample of 366 MSA's, median incomes range from \$31,264 to \$86,286 and income Gini coefficients from .388 to .537 (see also Glaeser, Resseger, and Tobin, 2009).

attainable for relatively low income ones. We consider policies aimed at improving housing affordability by subsidizing the provision of relatively low quality/size units to both the rental and owner-occupied markets. Such policies improve the well-being of lower income households relative to that of higher income ones, mainly by increasing housing quality. These policies nevertheless also increase home-ownership in spite of not targeting it directly. High income households (who effectively bear the cost of the policy) continue to own at roughly the same rate, but live in lower quality houses. They can compensate for this to a large extent by increasing their non-housing consumption.

In order to focus on the role of search frictions, our model abstracts from numerous other factors that are potentially important in understanding the relative incentives to own and rent. These include supply constraints, differential tax treatment, risk factors, life cycle issues and transactions costs. Many of their effects on incentives could be replicated by our assumptions regarding maintenance costs and/or preferences. Of course, in order to consider explicitly the effects of changes in these factors, the model must, and could be, extended to incorporate them.

Households in our model face binding financial limitations imposed by their intertemporal budget constraints. Much of the macroeconomic housing literature focuses on the role of additional financial frictions in the form of exogenous borrowing constraints and minimum downpayment requirements.<sup>12</sup> The critical assumption in these models, however, is that low quality housing cannot be owned.<sup>13</sup> Without such a restriction, such constraints of these types normally will affect house size or quality rather than ownership *per se*. In our theory, ownership patterns are driven by the optimal decisions of buyers and sellers faced with a choice between liquid rental markets and frictional markets for owner-occupied houses. Rental housing is more likely to be of low quality because the price incentive to supply such housing to the owner-occupied market is weak. Nevertheless, we do study the implications of an extended version that incorporates a simple financial constraint.

The remainder of the paper is organized as follows. Section 2 describes the theoretical environment which will be used to study housing tenure in equilibrium and for comparisons across cities, both qualitative and quantitative. Section 3 defines a stationary balanced growth path in a directed search equilibrium. The calibration is detailed in Section 4 along with the implied characteristics of housing markets within

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<sup>12</sup>See, for example, Gervais (2002), Iacovello and Pavan (2013), Sommer, Sullivan and Verbrugge (2013), Floetotto, Kirker and Stroebel (2016) and Sommer and Sullivan (2018).

<sup>13</sup>Moreover, quantitative analyses often impose a substantial minimum downpayment of 20%, which appears counterfactual.

the city. Section 5 describes variation across cities with regard to housing tenure and the ratio of average prices to average rents in both the data and the model. The affordability of housing and the effects of policies designed to improve it are considered in Section 6 and additional implications are discussed in Section 7. Section 8 concludes and outlines future work.

## 2 A Model of Construction and Housing Tenure

### 2.1 The environment

Consider a dynamic economy in discrete time, consisting of a single city populated by a growing number of households with stochastic life-times. Each period new households enter the city either through migration from elsewhere or by its members attaining an age at which they live independently. The rate of entry/household formation is constant, and denoted by  $\nu$ . Households die with probability  $\delta$  each period.<sup>14</sup> The population of the city,  $L_t$ , thus evolves:

$$L_{t+1} = (1 + \nu - \delta)L_t. \quad (1)$$

Households differ *ex ante* with regard only to their lifetime income. For simplicity, we think of this coming in the form of a constant income,  $y$ .<sup>15</sup> Household incomes are distributed according to cumulative distribution function,  $F$ , with positive and continuous support.

Households consume both goods and housing services. In particular, each period they must live in a single *house*, which they may either rent or own. Houses differ with regard to their characteristics, and we represent these by a single index of *quality*,  $q \in \mathbb{R}_+$ . Households maximize expected utility over their stochastic lifetimes. Preferences are represented by

$$U = \sum_{t=0}^{\infty} \beta^t [u(c_t) + h(z_t, q_t)], \quad (2)$$

where  $\beta$  is the household's discount factor adjusted for the probability of survival.

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<sup>14</sup>Rather than dying, households could leave the city for elsewhere randomly. This would make little difference for our results, although it would require a re-interpretation of certain parameters.

<sup>15</sup>Given our assumption below of complete markets, households could face idiosyncratic shocks to their income flow with no changes to our analysis or results.

That is,  $\beta/(1 - \delta)$  reflects the pure rate of time preference. The discount factor satisfies  $\beta = (1 - \delta)/(1 + \rho)$  with  $\rho$  the exogenous world interest rate.<sup>16</sup>

In (2),  $h(z_t, q_t)$  represents the current period utility flow from living in a house of quality  $q_t$ . Here  $z_t \in \{0, 1\}$  is an indicator of housing tenure;  $z_t = 1$  if the household owns the house in which it lives in period  $t$  and  $z_t = 0$  if it rents. This formulation allows for the potential existence of an *ownership premium*: an additional utility benefit to a household from owning the house in which it lives. We assume that  $h(z, q)$  is increasing and strictly concave in  $q$  and that  $h(z, 0) = 0$ , for  $z \in \{0, 1\}$ . Also, for all  $q$ ,  $h(1, q) \geq h(0, q)$ .<sup>17</sup>

Each period, with probability  $\pi$ , a household receives an idiosyncratic preference shock which results in them no longer liking their current house. Specifically, they no longer receive housing services from living in *that particular house*. The household can, however, obtain housing services by moving to a *different* house of their preferred quality. This *mobility shock* is intended to capture a household's evolving taste for the idiosyncratic features of a house, and generates turnover in housing markets.<sup>18</sup>

Houses of different qualities are built using a construction technology through which the cost of land and construction required to build a house of quality  $q$  is  $T(q)$ , where  $T'(q) > 0$ . Construction is undertaken by an industry comprised of a large number of identical, risk neutral *developers* under conditions of free entry. Once produced, the quality of a given house is fixed, permanently. Construction of a house takes one period, and development firms are owned by households, remitting their profits (if any) lump-sum. Because of free entry, firms build houses of each type as long as the discounted future value of a house exceeds the current cost of construction.

An owner of a vacant house, whether a developer or household, may either rent it to a prospective tenant or offer it for sale. Rental markets are perfectly competitive, and  $x_t(q)$  represents the current rent for a house of quality  $q$ .<sup>19</sup> In contrast, house

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<sup>16</sup>This assumption is necessary for there to be a stationary balanced growth path. One justification is that  $\rho$  is set in a stationary “rest of the world”, and taken as given in the city.

<sup>17</sup>In our baseline calibration below, we set this premium to zero, so that  $h(1, q) = h(0, q)$ . See Section 4.4 for discussion of an alternative formulation with a non-zero ownership premium.

<sup>18</sup>In general, mobility risk may depend on house quality and/or age. For example, a case in which  $\pi'(q) < 0$  is consistent with the finding of Piazzesi, Schneider and Stroebel (2013) that in the San Francisco Bay area, less expensive market segments tend to be less “stable” (*i.e.* moving shocks occur more frequently). For simplicity, however, here we hold  $\pi$  constant across house types.

<sup>19</sup>While there may be search frictions in the rental market as well, we assume that they are such that matching *always* occurs well within the period (which is a quarter in our calibration). While we do not model this process explicitly, we assume it implies that the rental market is Walrasian to a first approximation.

sales take place through a directed search process. Vacant houses of a given quality (a market *segment*) are offered for sale in sub-markets characterized by a posted price and a pair of matching probabilities, one each for both buyers and sellers. We assume a CRS matching function and refer to the ratio of buyers to sellers as market tightness, denoted  $\theta_t$ . The matching rates for buyers and sellers ( $\lambda$  and  $\gamma$ , respectively) are functions of tightness and satisfy:<sup>20</sup>

**Assumption 1.** *The matching probabilities have the following properties:*

- (i)  $\lambda(\theta) \in [0, 1]$  and  $\gamma(\theta) \in [0, 1]$  for all  $\theta \in [0, \infty]$ ;
- (ii)  $\lambda'(\theta) < 0$  and  $\gamma'(\theta) > 0$  for all  $\theta \in (0, \infty)$ ; and
- (iii)  $\lim_{\theta \rightarrow 0} \gamma(\theta) = 0$  and  $\lim_{\theta \rightarrow \infty} \gamma(\theta) = \bar{\gamma} \leq 1$ .

There is no restriction on how many houses a household can own. Each household must, however, live in (and thereby receive housing services from) one, and only one, house at a time.

Occupancy of houses results in depreciation which we assume to be completely offset by maintenance, the cost of which depends on quality and whether or not the house is occupied by its owner. Specifically,  $Z_R(q)$  represents the per period cost of maintaining a house of quality  $q$  when rented and  $Z_N(q)$  denotes that cost when owner-occupied. We assume that  $Z_R(q) \geq Z_N(q)$  and that  $Z'_R(q) \geq Z'_N(q)$ . Houses depreciate when rented or owner-occupied, but not while vacant.

**Assumption 2.** *The joint properties of the functions  $u$ ,  $h$ ,  $T$ ,  $\lambda$ ,  $\gamma$ ,  $Z_R$  and  $Z_N$  are such that the optimal choices of households and developers in equilibrium generate a bijective mapping between household income,  $y$ , and house quality,  $q$ , in the owner-occupied market.*

While we do not characterize these properties in general, in Section 4 we specify functional forms and use them to compute an equilibrium with this feature. Eeckhout and Kircher (2010) discuss the general properties required for positive assortative matching in a related but static environment.

At each point in time, the total stock of housing in the city is given by

$$M_t = \int_0^\infty M_t(q) dq \quad (3)$$

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<sup>20</sup>The likelihood of a match could, in principle, depend on the quality of the house. This could reflect, for instance, higher quality houses being more diverse and thus specifically appealing to a smaller fraction of buyers who visit them. Here we assume that the matching probabilities depend only on market tightness.



Each of these houses may be either owned by or rented to its occupant, or held vacant for sale. Let  $N_t$  denote the measure of owner-occupied houses (or, equivalently, the measure of homeowners),  $R_t$  denote the measure of houses for rent (or of renting households), and  $S_t$  the measure of houses vacant for sale. We then have

$$M_t = N_t + S_t + R_t \quad (4)$$

$$L_t = N_t + R_t. \quad (5)$$

Thus, in each period, the measure of houses in the city exceeds that of resident households by vacancies,  $S_t$ . Note that for each fixed house quality,  $q$ , equations analogous to (4) and (5) hold also.

Finally, there exist competitive markets in a complete set of one-period-ahead state-contingent claims paying off in units of the non-storable consumption good. These enable households to fully insure their idiosyncratic risks in the housing market (associated with  $\lambda$  and  $\gamma$ ) and of losing the housing services from their current house (associated with  $\pi$ ). Households are also required to purchase/issue contingent claims to settle their estate in the event of death (associated with  $\delta$ ). Households face no financial constraints beyond that implied by their life-time budgets.

### 3 Equilibrium

Throughout, we focus on balanced growth paths in which rents, house prices and values all remain constant. Along such a path, the distributions of both households and houses across rental and owner-occupied markets are time invariant.

#### 3.1 The supply-side decision problem

The most important choice in the economy is the rent vs. sell decision, made by owners of vacant houses, be they households or developers. Let  $V_t(q)$  denote the value of a vacant house of quality  $q$  at the beginning of period  $t$ . Such a house may be either rented or held vacant for sale in the current period. Its value is

$$V_t(q) = \max \left\{ \underbrace{x_t(q) - Z_R(q) + \frac{V_{t+1}(q)}{1 + \rho}}_{\text{rent}}, \underbrace{\frac{1}{1 + \rho} \max_p [\gamma(\theta_t(q, p))p + (1 - \gamma(\theta_t(q, p)))V_{t+1}(q)]}_{\text{sell}} \right\}. \quad (6)$$

The first term in brackets is the value of renting the house and the second is that of holding it vacant for sale. The maximization operator in the second term reflects the optimal choice of sub-market by the seller. The seller, taking as given the search behavior of buyers, anticipates how market tightness, and consequently the matching probability, responds to the price.

All sellers value vacant houses of a particular quality identically. Thus, their indifference across *active* sub-markets gives rise to an equilibrium relationship between price and tightness for houses in a given market segment:

$$\gamma(\theta_t(q, p)) = \frac{(1 + \rho)V_t(q) - V_{t+1}(q)}{p - V_{t+1}(q)}. \quad (7)$$

Moreover, as sellers may freely decide whether to rent or hold a house vacant-for-sale, we have

$$V_t(q) = x_t(q) - Z_R(q) + \frac{V_{t+1}(q)}{1 + \rho}. \quad (8)$$

Conditions (7) and (8) equate house values across rental and sales markets. Free entry then implies that house values and rents are determined by construction costs. On the balanced growth path, house values and rental costs can therefore be represented by time-invariant functions of house quality:

$$V(q) = (1 + \rho)T(q) \quad (9)$$

$$x(q) = Z_R(q) + \frac{\rho}{1 + \rho}V(q) = Z_R(q) + \rho T(q). \quad (10)$$

Moreover, tightness for active sub-markets is a time-invariant function of  $p$  and  $q$ :

$$\gamma(\theta(q, p)) = \frac{\rho V(q)}{p - V(q)}. \quad (11)$$

### 3.2 The household decision problem

As households are risk-averse, have separable utility and markets are complete, they carry out financial transactions to smooth consumption completely. Specifically, at the beginning of each period, through the purchase and sale of contingent claims, households insure themselves against risks associated with preference shocks (which determine whether the household remains happy with their house), matching outcomes and death, all of which are random. From this point on, where possible, time subscripts will be suppressed.

Consider first a homeowner. This household may purchase/issue  $w_S$  units of a security, each of which pays one unit of consumption good in the next period contingent on receiving a preference shock and becoming a renter, and  $w_N$  units of a security that pay contingent on remaining a homeowner.<sup>21</sup> The homeowner may also sell up to  $V(q)$  units of contingent claims which pay-off in the event that the household dies at the end of the period. The payment of these claims is financed by the sale of the household's then-vacant house.

Similarly, a household renting while searching to buy a house may purchase  $w_B$  units of insurance that pay contingent on buying a house, and  $w_R$  units of a security that pay contingent on having failed to buy and continuing to rent. Finally, we impose that a household searching to buy in sub-market  $p$  of segment  $q$  must purchase  $p - V(q)$  units of a claim that pays off contingent on the household committing to buy a house but then dying at the end of the period.

The prices of the contingent securities, in the order defined, are denoted  $\phi_S$ ,  $\phi_N$ ,  $\phi_D$ ,  $\phi_B(q, p)$ ,  $\phi_R(q, p)$ , and  $\phi_D(q, p)$ . The last three prices depend on the house quality and the price as they insure against outcomes in a particular sub-market of a particular housing market segment.

At the beginning of each period, renters, depending on their *total wealth*,<sup>22</sup> choose: (i) a type/quality of house to rent,  $q_R$ ; (ii) whether or not to search for a house to buy; and, if searching, (iii) a particular sub-market,  $p$ , (associated with a particular house type,  $q_S$ ) in which to search; and (iv) a consumption level,  $c$ , and savings in the form of a portfolio of claims,  $\{w_B, w_R\}$ . For simplicity, the decision not to search for a house to purchase will be represented by the choice of  $q_S = 0$  and  $p = 0$ . Accordingly,  $\theta(0, 0) = \infty$  and  $\lambda(\theta(0, 0)) = 0$ . A renter with wealth  $w$  then has value:

$$W^R(w) = \max_{c, w_R, w_B, q_R, q_S, p} \left\{ \begin{array}{l} u(c) + h(0, q_R) \\ + \beta [\lambda(\theta(q_S, p)) W^N(q_S, w_B - p) + (1 - \lambda(\theta(q_S, p))) W^R(w_R)] \end{array} \right\} \quad (12)$$

subject to:

$$c + \phi_B(q_S, p)w_B + \phi_R(q_S, p)w_R + \phi_D(q_S, p)(p - V(q_S)) + x(q_R) = w, \quad (13)$$

where  $W^N(q_S, w_B - p)$  is the value of entering next period as an owner of a house of

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<sup>21</sup>All households, on losing their access to housing services due to a shock, spend at least one period as a renter.

<sup>22</sup>Total wealth,  $w$ , is defined to include all available financial resources. In particular, the present discounted value of future labor income,  $y/(1 - \beta)$ , is contained in  $w$ . See Appendix A.1 for details.

quality  $q_S$  with wealth  $w_B - p$ . This value is given by:

$$W^N(q, w) = \max_{c, w_N, w_S, w_D} \left\{ \begin{array}{l} u(c) + h(1, q) \\ + \beta [\pi W^R(w_S + V(q)) + (1 - \pi) W^N(q, w_N)] \end{array} \right\} \quad (14)$$

subject to:

$$c + \phi_S w_S + \phi_N w_N + \phi_D w_D + Z_N(q) = w \quad (15)$$

$$w_D + V(q) \geq 0. \quad (16)$$

### 3.2.1 Consumption and portfolio choice

Appendix A.1 contains the solution to households' portfolio allocation problem and the derivation of the optimal consumption by income. Let  $q_R$ ,  $q_S$ , and  $p$  denote the optimal rental and search choices for a household with permanent income  $y$ .<sup>23</sup> This household's (constant) per period consumption is given by

$$\begin{aligned} c = y - & \frac{[1 - \beta(1 - \pi)] x(q_R) + \beta \lambda(\theta(q_S, p(y))) x(q_S)}{1 - \beta(1 - \pi - \lambda(\theta(q_S, p)))} \\ & - \frac{\beta \lambda(\theta(q_S, p)) [1 - \beta(1 - \pi)] (p - V(q_S))}{(1 - \delta) [1 - \beta(1 - \pi - \lambda(\theta(q_S, p)))]}. \end{aligned} \quad (17)$$

This is the highest attainable constant consumption sequence satisfying the present value budget constraint given the cost of insuring against both preference shocks and the matching risks associated with housing-related transactions.

### 3.2.2 Renting

The rent decision is determined by the *intratemporal* Euler equation associated with the choice of  $q_R$  in (12):

$$u'(c) x'(q_R) = \frac{\partial h(0, q_R)}{\partial q}. \quad (18)$$

Here  $x'(q_R)$  is the derivative of the time-invariant rental cost function from equation (10). First order condition (18) combined with (17) pins down the quality of housing rented by a household with permanent income  $y$ .

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<sup>23</sup>We suppress the dependence of  $q_R$ ,  $q_S$ ,  $p$  and  $c$  on  $y$  for brevity.

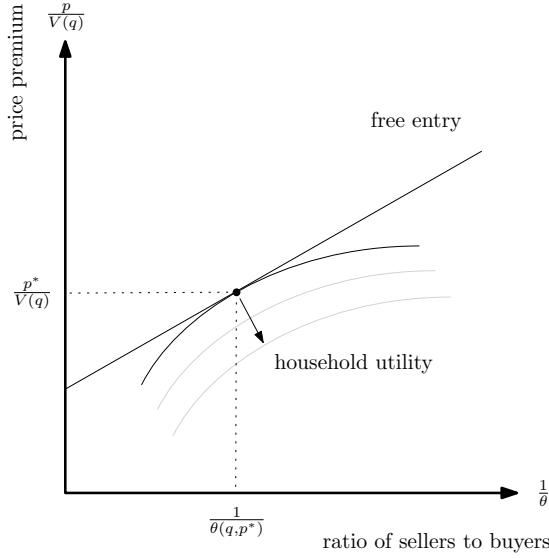


Figure 1: Equilibrium submarket.

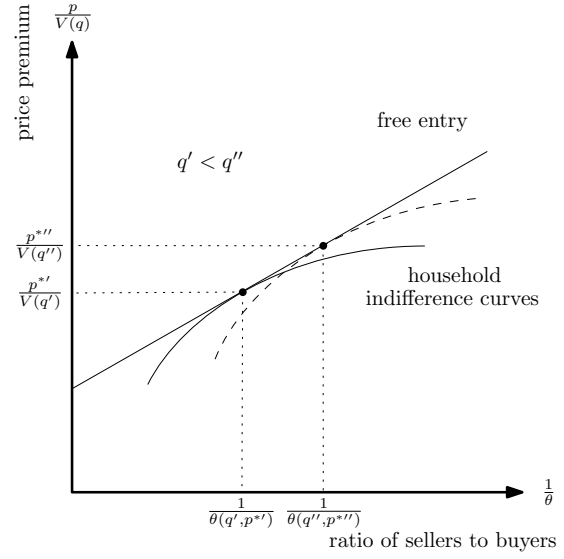


Figure 2: Equilibrium segments.

### 3.2.3 Home-ownership

Maximizing with respect to  $q_S$  and  $p$  in (12) reflects optimal search decisions with regard to house type and sub-market, where tightness  $\theta(q, p)$  is determined by (11). A pair of *intertemporal* Euler equations (one for segment choice,  $q_S$ , the other for choice of sub-market,  $p$ ) characterize the solution to the household's search problem, derived in Appendix A.2. In steady state, the optimal choice of segment for a given sub-market is characterized by

$$\frac{\partial h(1, q_S)}{\partial q} = u'(c) \left\{ \frac{[1 - \beta(1 - \pi)]p - (\beta\pi + \delta)V(q_S)}{(1 - \delta)V(q_S)} V'(q_S) + Z'_N(q_S) \right\}. \quad (19)$$

The optimal sub-market within a given segment reflects the household's understanding that higher priced sub-markets will attract more units for sale relative to buyers (higher  $1/\theta$ ). The free entry condition (11) for sellers is illustrated in Figure 1. For a given unit, this condition governs how the ratio of price to its present discounted value in the rental market must vary with the ratio of units for sale to searchers. For a given quality of house, households prefer lower prices and a higher ratio of vacant houses for sale to buyers, as illustrated by the indifference curves. Given that all households in the sub-market are identical, they will therefore optimally select the same unique sub-market  $(p, \theta)$ .

The implications of this directed search equilibrium are summarized by the price

premium that the household pays to acquire owner-occupied housing.

$$p - V(q_S) = \mathcal{L}(\theta(q_S, p)) \left\{ x(q_R) - x(q_S) + \frac{h(1, q_S) - h(0, q_R)}{u'(c)} \right\} \quad (20)$$

where

$$\mathcal{L}(\theta) = \frac{(1 - \delta)(1 - \eta(\theta))}{1 - \beta(1 - \pi - \eta(\theta)\lambda(\theta))}, \quad (21)$$

and  $\eta(\theta) = \theta\gamma'(\theta)/\gamma(\theta)$ . The term in curly brackets on the right-hand side of (20) is the overall flow surplus associated with the change in the household's living arrangements from renting a unit of quality  $q_R$  to owning a unit of quality  $q_S$ . The term  $\mathcal{L}(\theta)$  is the present value factor for this stream of ownership surplus and reflects the frictions in sub-market  $p$  of segment  $q_S$ .

As we move to higher segments, the trade-off in household preferences between price and sub-market tightness can change as a result of how the surplus from ownership varies by segment.<sup>24</sup> As a result, both the equilibrium price premium and likelihood of finding a house to own,  $\lambda(\theta)$ , can increase with  $q$ .<sup>25</sup> Even if owning is more desirable, lower income households may not be willing and able to afford houses in market segments where many are offered for sale and are therefore more likely to rent. This outcome is illustrated by the shift in the indifference curves in Figure 2.

Household optimization is summarized by the decision rules,  $c(y)$ ,  $q_R(y)$ ,  $q_S(y)$  and  $p(y)$ . These satisfy (17), (18), (19) and (20), with market tightness,  $\theta(y) \equiv \theta(q_S(y), p(y))$ , determined by (11).

### 3.3 An Equilibrium with Balanced Growth

The measure of renters with income level  $y$  evolves according to:

$$R_{t+1}(y) = (1 - \lambda(\theta(y)))(1 - \delta)R_t(y) + \pi(1 - \delta)N_t(y) + \nu f(y)L_t. \quad (22)$$

Here the first term is the measure of unsuccessful, surviving searchers who remain as renters, the second term that of mismatched surviving owners who enter the renter pool and the last is that of new entrants into the housing market.

Similarly, the measure of owners with income level  $y$  evolves according to:

$$N_{t+1}(y) = (1 - \pi)(1 - \delta)N_t(y) + \lambda(\theta(y))(1 - \delta)R_t(y). \quad (23)$$

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<sup>24</sup>This can occur either because of the maintenance cost or ownership premium.

<sup>25</sup>This implication is consistent with the findings of Piazzesi, Schneider and Strobel (2019).

This consists of surviving owners who remain well-matched and surviving renters who successfully match and buy a home. Dividing all quantities by the population,  $L_t$ , and using lower case letters to represent *per capita* values, the relative measures along a balanced growth path can be expressed as

$$r(y) = \frac{\nu + \pi(1 - \delta)}{\nu + (1 - \delta)[\pi + \lambda(\theta(y))]} f(y) \quad (24)$$

$$n(y) = \frac{(1 - \delta)\lambda(\theta(y))}{\nu + (1 - \delta)[\pi + \lambda(\theta(y))]} f(y). \quad (25)$$

Given (24) and (25), the (normalized) stocks of owner-occupied and rental housing of type  $q$  are  $n(q_S^{-1}(q))$  and  $r(q_R^{-1}(q))$ , where  $q_S^{-1}(q)$  and  $q_R^{-1}(q)$  denote the income levels for households that buy and rent in segment  $q$ , respectively. Similarly, the (normalized) stock of vacant houses for sale in segment  $q$  is  $r(q_S^{-1}(q))/\theta(q, p(q_S^{-1}(q)))$ . The per capita total stock of housing of each type therefore satisfies:

$$m(q) = n(q_S^{-1}(q)) + r(q_R^{-1}(q)) + \frac{r(q_S^{-1}(q))}{\theta(q, p(q_S^{-1}(q)))}. \quad (26)$$

We then have the following:

**Definition 1.** *A Directed Search Equilibrium Balanced Growth Path is a list of time-invariant functions of income  $y \in \mathbb{R}_+$  and house quality,  $q \in \mathbb{R}_+$ :*

- i. *household values,  $W^R(w)$  and  $W^N(q, w)$ , and decision rules:  $w_R(y)$ ,  $w_B(y)$ ,  $w_N(y)$ ,  $w_S(y)$ ,  $w_D(y)$ ,  $c(y)$ ,  $q_R(y)$ ,  $q_S(y)$ , and  $p(y)$ ;*
- ii. *house values,  $V(q)$ , and rents,  $x(q)$ ;*
- iii. *a function for market tightness,  $\theta(q, p)$ ;*
- iv. *shares of households renting and living in owner-occupied housing  $r(y)$  and  $n(y)$ ;*
- v. *per capita stocks of housing,  $m(q)$ ;*

such that

1.  *$W^R$  and  $W^S$  satisfy the household Bellman equations, (12) and (14), with the associated policies  $q_R(y)$ ,  $q_S(y)$ ,  $p(y)$ ,  $c(y)$ ,  $w_R(y)$ ,  $w_N(y)$ ,  $w_B(y)$ ,  $w_S(y)$  and  $w_D(y)$  satisfying (17)–(20) and (A.16)–(A.20);*
2. *free entry into new housing construction and rental markets: that is,  $V(q)$  and  $x(q)$  satisfy (9) and (10);*
3. *optimal price posting strategies by sellers of houses in the owner-occupied market: that is,  $\theta(q, p)$  satisfies (11);*
4. *the per capita measures of households,  $r(y)$  and  $n(y)$ , satisfy (24) and (25);*
5. *the stock of each type of housing grows at the rate of population growth: that is,  $m(q)$  satisfies (26).*

## 4 A Calibrated Economy

We now parameterize the model to match several characteristics of the median MSA in a sample of U.S. cities. We then assess the extent to which the predictions of the model are consistent with observations of the within-city distributions of houses and households across rental and owner-occupied markets. In Section 5, we compare the cross-city predictions of the model with the data.

### 4.1 Baseline Calibration

#### 4.1.1 Functional Forms

We use the following form for preferences:

$$u(c) = (1 - \alpha) \ln c \quad \text{and} \quad h(q) = \alpha \ln q. \quad (27)$$

This form is consistent with the observation of Davis and Ortalo-Magné (2011) that the share of income allocated to rent is roughly constant across renting households. Note that in (27), we set the ownership premium to zero so that there is no utility benefit to owning one's home, *per se*. In this case, households own their own residences only because of cost advantages to doing so, if any. Along those lines, we assume that the maintenance costs incurred by owner-occupiers are proportional to  $q$  and that those incurred by landlords are greater and rise more than proportionately with  $q$ :

$$Z_N(q) = \zeta_0 q \quad \text{and} \quad Z_R(q) = Z_N(q) + \zeta_1 q + \zeta_2 (e^q - 1). \quad (28)$$

The exact functional forms assumed allow us to fit ownership rates by income quintile (see below). In Section 4.4, we discuss the equivalence of an alternative calibration where maintenance costs are equal across owned and rented units, but an ownership utility premium varies with  $q$ .

Construction costs are assumed to be linear in house quality:

$$T(q) = \tau q. \quad (29)$$

Note that since  $q$  is itself an index, the assumption of linearity in (29) is not very restrictive. For example, any homogeneous function of  $q$  will generate identical results with an appropriate adjustment of the preference specification.



Our matching probabilities correspond to the so-called “telephone line” function derived from a congestion and coordination problem (see Cox and Miller, 1965; and Stevens, 2007):

$$\lambda(\theta) = \frac{\chi}{1 + \chi\theta} \quad \text{and} \quad \gamma(\theta) = \frac{\chi\theta}{1 + \chi\theta} \quad (30)$$

Here  $\chi \in (0, 1)$  is a matching efficiency parameter reflecting buyers’ search intensity. Note that these matching probabilities satisfy Assumption 1.<sup>26</sup>

We model income as a quarterly flow distributed log-normally:

$$y \sim \log\mathcal{N}(\mu, \sigma^2). \quad (31)$$

This implies that incomes are bounded below by zero and that their distribution can be summarized by its first two moments. It also implies a one-to-one relationship between the standard deviation and the implied Gini coefficient.<sup>27</sup> The log-normal distribution is a common and convenient approximation for real world income distributions, which are typically left-skewed with a long upper tail.

#### 4.1.2 Parameterization

Table 1 contains calibrated parameter values, together with the economy statistics with which each is most closely associated. The mean of log quarterly income,  $\mu$ , is chosen so that median annual income is normalized to unity and the standard deviation,  $\sigma$ , is set so that the Gini coefficient corresponds to the average across MSA’s. Given a value of  $\delta$  chosen to deliver an annual death rate of 5%, the entry rate,  $\nu$ , is chosen so that steady state population growth is 1%. Given these parameters, the moving probability,  $\pi$ , is chosen to match an average ownership duration of 10 years and the matching parameter,  $\chi$ , is set to generate an average time to sell of 1.25 quarters. The preference parameter,  $\alpha$ , is set so that the model generates a mean rent to median income ratio equal to the corresponding average across MSA’s.

The parameters of the maintenance cost functions,  $\{\zeta_0, \zeta_1, \zeta_2\}$ , were chosen to match the average city-level ratio of average prices to average rents, and to minimize the sum of the squared differences between the equilibrium ownership rates by income quintile and those reported in Table 1 which are computed using census

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<sup>26</sup>Of course, other commonly-applied matching technologies also satisfy these assumptions (*e.g.* the “urn-ball” matching function). We do not, however, believe that our calibrated balanced growth path is very sensitive to these alternatives.

<sup>27</sup>Below we use observed Gini coefficients computed for MSA’s by the U.S. Census Bureau.

summary tables for all U.S. households in 2010.<sup>28</sup> Table 2 displays the extent to which the calibrated economy successfully matches these targets. Figure 3 depicts the maintenance cost by market segment for both rentals and owner-occupied houses.

Table 1: Calibrated Parameters

statistic	targeted value	calibrated parameter	calibrated value
median annual income (normalization)	1	$\mu$	-1.3863
Gini coefficient for income	0.445	$\sigma$	0.8348
population growth rate (%)	1.0	$\nu$	0.0152
probability of death/exit (%)	5.0	$\delta$	0.0127
average ownership duration (years)	10	$\pi$	0.0250
average time to sell (quarters)	1.25	$\chi$	0.7839
average price-rent ratio	24	$\zeta_0$	0.0109
annual interest rate (%)	4.0	$\beta$	0.9776
ratio of mean rent to median income	0.186	$\alpha$	0.2234
normalization	1	$\tau$	1.0000
average ownership rate, Q1 (%)	44		
average ownership rate, Q2 (%)	56		
average ownership rate, Q3 (%)	67	$\zeta_1$	0.0004
average ownership rate, Q4 (%)	77	$\zeta_2$	0.0001
average ownership rate, Q5 (%)	87		

## 4.2 The City in Equilibrium

Figure 4 illustrates the housing decisions of households by income. Clearly, quality is strictly increasing in income for both rental ( $q_R$ ) and owner-occupied ( $q_S$ ) housing. At low incomes, households search to own houses of slightly lower quality than those they rent. At income levels above the median, however, households of a given income search for houses to buy of higher quality than they rent. This reflects the fact that higher quality houses are relatively expensive to rent, given their high relative maintenance costs.

<sup>28</sup>See <https://www.census.gov/data/tables/time-series/demo/income-poverty/cps-hinc/hinc-05.2010.html>). For the U.S. economy as a whole, ownership rates for the bottom quintiles are likely higher than for households residing in MSA's. A disproportionate fraction of lower-income households live outside MSA's and own at a higher rate than those in MSA's.

Table 2: Calibration Results

statistic	target value	model value
average ownership rate, Q1	0.44	0.4401
average ownership rate, Q2	0.56	0.5606
average ownership rate, Q3	0.67	0.6669
average ownership rate, Q4	0.77	0.7736
average ownership rate, Q5	0.87	0.8688

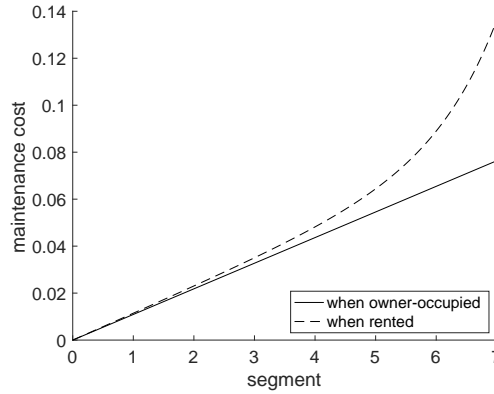


Figure 3: Maintenance costs by housing segment.

Figure 5 plots home-ownership by income. The supply of owner-occupied housing to the very poorest households is low: these households choose to search in sub-markets with low prices and hence low matching rates. Beyond a point, however, home-ownership rises rapidly and then flattens out at high incomes. Intuitively, an increasing ownership rate manifests because the relatively high cost of maintaining higher quality rental houses translates into higher equilibrium rents. High income households seek high quality homes and thus search aggressively in the owner-occupied market by targeting high price sub-markets with better buying probabilities.

Figure 6 plots the relative cost of owning vs. renting by household income. For a household with a given permanent income, the figure compares the price of the house for which they search to buy to their annual rental cost while searching. The relationship in the figure reflects the differences in households' choice of house quality

when renting versus owning (see Figure 4). Due to the scarcity of high quality rentals that results from their high rental costs, higher income households search to buy houses of much higher quality than they rent while searching. At levels of income below the median, the effect is small, even negligible, reflecting the relatively small gap in maintenance costs between rented and owner-occupied homes of a given quality.

The rapid rise in relative maintenance costs as quality increases results in the relationship depicted in Figure 7, which plots the price-rent ratio by market segment. These maintenance costs (which depend on an interaction between occupant household's tenure and the physical characteristics of the house) may be seen as an example of "unobservable costs of renting" in the language of Halket, Nesheim and Oswald (2015) who, as noted above, observe a price-rent ratio declining in house quality. Figure 7 also plots the ratio of the price to rent net of maintenance cost by market segment, which is rising slightly with quality.

The price-rent ratio for a house of a given quality is low, yet the calibrated model delivers an aggregate price-rent ratio of 24. This reflects households' heterogeneous search strategies in the owner-occupied market. Figure 8 displays the histogram of housing market transactions by income for both sales and rentals. High income households are responsible for a large share of transactions in the owner-occupied market, whereas the opposite is true for the rental market. Both of these contribute to a high average price to average rent ratio.<sup>29</sup> The distributions of transactions by income play a crucial role in the calculation of aggregate statistics and are important determinants of the cross-city implications derived below.

### 4.3 Comparisons across equilibrium balanced-growth paths

We now consider the effects of changes in median income, income inequality (*i.e.* the Gini coefficient), median age (or population growth) and land/construction costs on both home-ownership and the relative cost of owner-occupied vs. rental housing (measured by the ratio of average prices to average rents) along the balanced growth path. As above, these relationships depend on the relative costs of owning and renting, and reflect the equilibrium implications of both rent vs. buy decisions on the demand side and rent vs. sell decisions on the supply side.

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<sup>29</sup>The ratio of two weighted averages can be a lot higher than the average of the underlying ratios if the weights are inversely related.

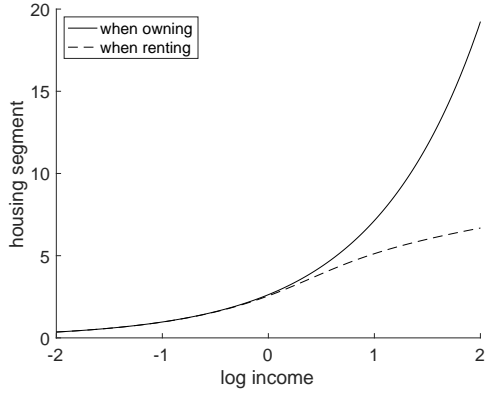


Figure 4: Market segment vs. income by housing tenure.

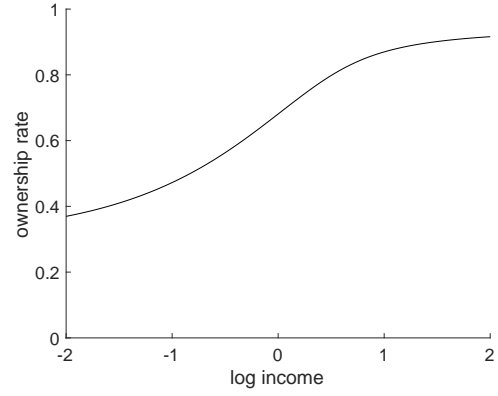


Figure 5: Ownership rate vs. household income.

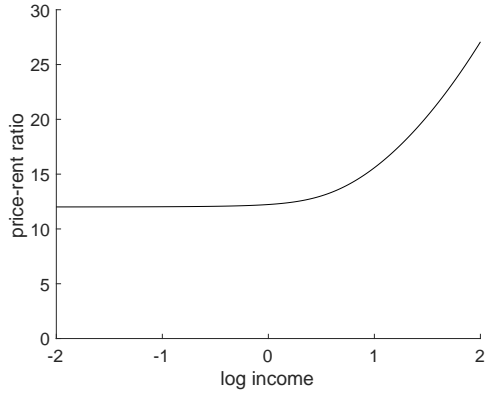


Figure 6: Price-rent ratio by household income.

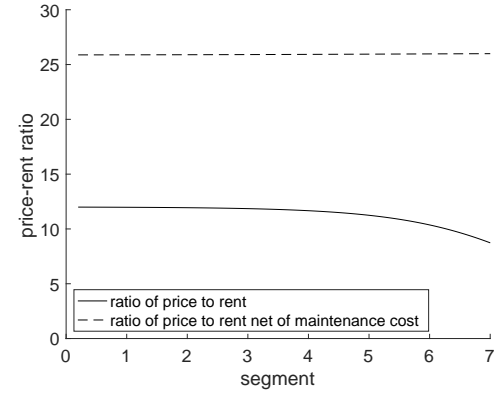


Figure 7: Price-rent ratio by housing segment.

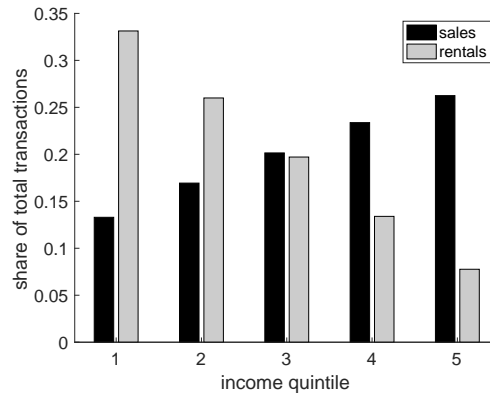


Figure 8: Histograms of house purchases and rentals by income.

### 4.3.1 Median income

As city median income increases holding inequality constant, the quality of housing desired by households rises. Moreover, since the cost of maintaining rental properties increases with quality, so also does the fraction of developers and moving households who sell rather than rent their vacant houses. In equilibrium, the search frictions in the housing market allow the increasing costs of renting to support a selling probability that *falls* (and a buying probability that *rises*) with quality. Consequently, (see Figure 9a), the rate of home-ownership increases with median income.

In general, the implication of rising city median income for the ratio of average prices to average rents is ambiguous. Since the quality of housing rises and more of it is owned, the average purchase price must increase. Average rents, however, also increase as households demand higher quality housing. Which increase is larger depends on the distribution of the home-ownership rate by income. On the balanced growth path, this depends on the relationship between matching probabilities and income, which in turn is dictated by the relative costs of renting, owning and selling. Under our calibration, the ratio of average prices to average rents rises with median income. If relative maintenance costs were independent of quality, neither home-ownership nor the ratio of average prices to average rents would vary with median income.

### 4.3.2 Inequality

As inequality increases holding median income constant, the quality of housing desired by relatively high income households increases while that desired by relatively low income households declines. Since the relative costs of renting rise sharply for increases in quality at the upper end, but fall only minimally as quality declines at the lower end, the fraction of high quality houses supplied to the owner-occupied market rises while that of low quality houses falls.

The impact on the aggregate home-ownership rate is in general ambiguous and again depends on the relationship between ownership rates and household income. In our calibration, the relative costs of owning and renting imply a concave relationship over most of the income distribution (see Figure 5). Consequently (see Figure 9b), home-ownership falls with inequality as measured by the income Gini coefficient.

At the same time, increased income inequality *raises* the ratio of average prices

to average rents through a composition effect. Having more low-income households results in the construction of more low quality houses. While this lowers *both* prices and rents, the effect on the former is mitigated by the fact that low income households buy houses at a low rate. Similarly, having more high-income households results in more high quality houses being built and drives *up* both prices and rents. In this case, however, the effect on the *latter* is minor as high-income households rarely rent. Overall, as shown in Figure 10b the increase in the purchase prices of high quality homes and the reduction in rents together result in an increase in the ratio of average prices to average rents.

### 4.3.3 Median age

To the extent that death rates do not vary much across cities, variation in median age largely reflects variation in entry rates and steady state population growth. Figures 9c and 10c depict the relationships between median age and home-ownership and the ratio of average prices to average rents, respectively, resulting from variation in the entry rate,  $\nu$ . Ownership increases monotonically with median age. An older city has a lower rate of entry, and as such a smaller fraction of the population renting while searching for an initial house. This accounts directly for the effect of age on ownership. The impact of median age on the price-rent ratio is small and, while for this calibration it is negative, for others it can be positive.

### 4.3.4 Construction costs and city-wide amenities

The parameter  $\tau$  represents the cost of building per unit of housing quality. As such, it reflects city-wide amenities (*e.g.* climate) and costs (*e.g.* regulatory hurdles) as well as the choices of developers (*e.g.* land, size, construction materials, etc.). Variations in  $\tau$  capture all the costs of providing housing that are *independent* of whether the occupying household owns or rents.

An increase in  $\tau$  translates into a proportional increase in the value of vacant housing required to induce competitive developers to supply new housing of any quality. The resulting rise in both rents and purchase prices causes households at every given income level to choose lower quality housing which, in turn, reduces the relative costs of renting. Consequently, as shown in Figure 9d, the aggregate ownership rate declines as  $\tau$  increases.

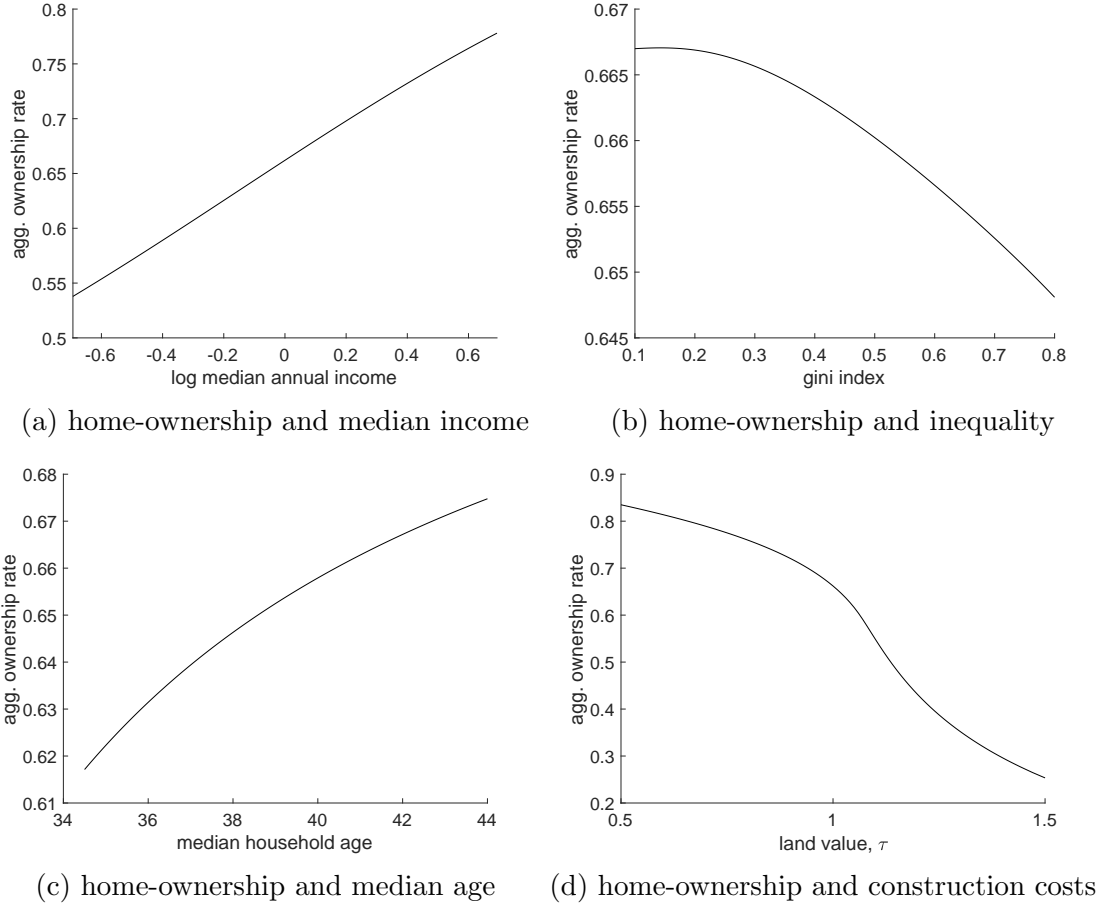


Figure 9: Cross-city relationships between home-ownership and fundamentals

In general, the impact of an increase in  $\tau$  on the ratio of average prices to average rents is more ambiguous. While the purchase prices and rents paid for houses of a given quality rise, this is largely offset at the household level by the reduced quality of such houses. A more important factor determining the relative impact on average prices and average rents is the implied distribution of changes in the ownership rate across income levels. This, in turn, depends on the shift in the steady state mappings between income and matching probabilities.

As may be seen in Figure 10d, our calibration implies the ratio of average prices to average rents increases with  $\tau$ . This results from the combination of two effects. First, overall the quality distribution shifts to the left as houses become less affordable. This results in a general reduction of relative rental costs, and hence an increase in the price-rent ratios, segment by segment (see Figure 7). The second, and more significant



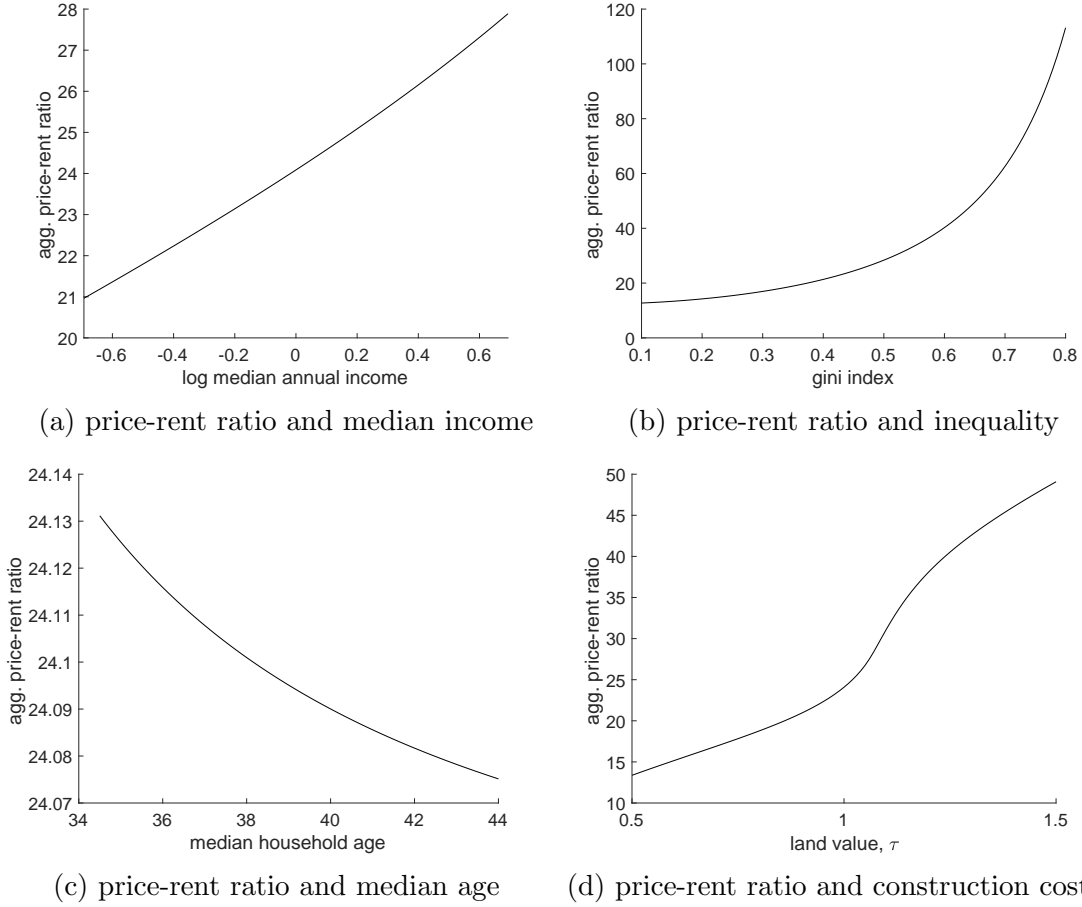


Figure 10: Cross-city relationships between the price-rent ratio and fundamentals

effect, comes from the fact that any reduction in home-ownership is overwhelmingly concentrated amongst low-income households. High-income ones continue to own, paying relatively high prices simply because they live in high quality houses.

Note, once again, that if the relative cost of renting (through maintenance) were independent of quality, neither home-ownership nor the ratio of average prices to average rents would vary with the supply side factors represented by  $\tau$ .

## 4.4 An Alternative Calibration

In our baseline calibration, the net surplus associated with home-ownership increases with quality due to the specification of maintenance costs. Here we describe briefly an alternative calibration of the baseline model with an explicit utility benefit to home ownership that rises with quality.<sup>30</sup> Specifically, we assume that the utility from housing services is given by:

$$h(z, q) = \alpha(\ln q + zg(q)), \quad (32)$$

where the ownership premium is

$$g(q) = \psi_0 + \frac{\psi_1 q}{1 + e^{\psi_2 - \psi_3 q}} \quad q > 0. \quad (33)$$

Maintenance costs are now assumed to be equivalent across owned and rented units of a given quality:

$$Z_N(q) = Z_R(q) = \zeta q. \quad (34)$$

When we calibrate this version of the model to match the same targets as above, we find that the implications are qualitatively unchanged and quantitatively very similar.<sup>31</sup> The only substantive difference is that the price-rent ratio for equivalent housing units now rises with quality, in contrast to the negative relationship depicted in Figure 7.<sup>32</sup> Empirical studies of the relationship between price-rent ratios and housing quality find mixed results. A number of studies, using various approaches and in different locations, have suggested an increasing price-rent ratio by quality.<sup>33</sup> Using data from London, Halket, Nesheim and Oswald (2015), however, attribute much of this to the effects of selection on unobservable quality. Once they correct for this bias, they find that the price-rent ratio falls with value.

The equivalence between these alternative calibrations for our main results and the additional implications reported below, demonstrate a more general observation. There are many other possible differences in the benefits and costs of owning versus

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<sup>30</sup>That owners derive greater utility than renters from a given dwelling is a common assumption (Rosen, 1985; Poterba, 1992; Kiyotaki et al., 2011; Iacovello and Pavan, 2013; Floetotto, Kirker, and Stroebel, 2016). One explanation is that owners can customize the unit to suit their own preferences.

<sup>31</sup>These results are available upon request.

<sup>32</sup>Note that a positive relationship also arises in the baseline calibration when we replace the rent with the rent net of maintenance costs (see Figure 7).

<sup>33</sup>See, for examples, Verbrugge (2008), Heston and Nakamura (2009), Garner and Verbrugge (2009), Verbrugge and Poole (2010), Epple, et al. (2013), Landvoigt et al. (2015), Bracke, (2015) and Hill and Syed (2016).

renting. These include transactions costs of buying that are unrelated to liquidity, differential income tax treatment (e.g. mortgage interest deductibility), differences in property tax implications, etc. If we were to incorporate all of these into the model and then re-calibrate preference and/or maintenance cost parameters to target the distribution of ownership by income as above, it would not change the main implications.

## 5 Cross-City Variation in the Data and the Theory

We now document observed variation across U.S. cities with regard to home-ownership and the relative costs of owning and renting (price-rent ratios) and consider the extent to which it is associated with variation in median incomes, inequality, age and land values, controlling for several other factors. We then compare the corresponding variation generated by the model to these characteristics of the data.

### 5.1 Variation Among a Sample of U.S. Cities

Our base sample consists of the 366 primary MSA's from the 2010 American Community Survey (ACS).<sup>34</sup> For all of these MSA's ownership rates, average price-rent ratios, median incomes, Gini coefficients, median age and population density can be computed from the ACS. Since it affects average lot size, population density should be inversely related to average city-wide housing quality.

We take the view that land values are the main source of variation in overall construction costs across cities. Land values are taken from Albouy *et al.* (2017) who compute them at the MSA level using land transactions data, adjusted to account for geographic selection in location and limited sample sizes. Their calculations are based on the 1999 OMB definitions of MSA's. There is not an exact match between MSA's in the two samples for several reasons. For example, there were several new primary MSA's in the 2010 census resulting from population growth, some MSA's were subdivided and others experienced name changes. After matching the MSA's as closely as possible and, where they were subdivided, applying the same land values to each, we were left with 332 MSA's.<sup>35</sup>

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<sup>34</sup>An MSA is an urban agglomeration containing at least 50,000 households. These 366 MSA's contain over 83% of U.S. households.

<sup>35</sup>See Appendix B for details.

To isolate the role of costs per unit of housing quality we must control for average quality (density and amenities). The amenity controls that we use are “natural” amenities taken from Albouy (2016). This data is also based on the 1999 OMB definitions of MSA’s. Moreover, in this case some of the observations are for consolidated MSA’s, which combine congruent primary MSA’s. In order to maximize our sample size, we use the amenity controls computed for the consolidated MSA’s to approximate those for each constituent primary MSA.

Table 3: U.S. Metropolitan Statistical Areas (2010)  
(Median income not adjusted for local non-housing cost of living)

	Ownership Rate		Price-Rent ratio			
			(price-asked)		(estimated-value)	
<b>Log Median Income</b>	0.022	0.076	23.560	14.497	22.141	13.088
Robust s.e.	(0.021)	(0.021***)	(2.458***)	(2.283***)	(1.972***)	(1.653***)
Clustered s.e.	(0.024)	(0.030**)	(2.410***)	(2.833***)	(2.057***)	(2.114***)
<b>Gini index</b>	-0.511	-0.449	89.809	90.286	53.287	65.427
Robust s.e.	(0.116***)	(0.130***)	(13.445***)	(13.677***)	(9.682***)	(8.240***)
Clustered s.e.	(0.155***)	(0.138***)	(18.925***)	(13.395***)	(16.261***)	(8.523***)
<b>Age</b>	0.007	0.007	0.205	0.032	0.239	0.122
Robust s.e.	(0.001***)	(0.001***)	(0.084**)	(0.077)	(.057***)	(0.039***)
Clustered s.e.	(0.001***)	(0.001***)	(0.119*)	(0.083)	(.096**)	(0.044***)
<b>Log Density</b>	-0.028	-0.024	-1.631	-3.585	0.226	-1.447
Robust s.e.	(0.005***)	(0.005***)	(0.679**)	(0.680***)	(0.492)	(0.407***)
Clustered s.e.	(0.009***)	(0.006***)	(0.991)	(0.787***)	(0.947)	(0.336***)
<b>Log Land Value</b>		-0.010		2.680		2.321
Robust s.e.		(0.004**)		(0.606***)		(0.392***)
Clustered s.e.		(0.005**)		(0.736***)		(0.651***)
<b>Amenity Controls</b>	No	Yes	No	Yes	No	Yes
$\overline{R}^2$	0.52	0.63	0.26	0.56	0.42	0.79
# obs	366	332	366	332	366	332

Notes: (1) Sources and definitions of variables may be found in Appendix B.

(2) Standard errors are provided in parenthesis:

heteroskedasticity-robust and clustered at the state level.

(3) \*, \*\* and \*\*\* indicate statistical significance at the 10, 5, and 1% levels, respectively.

In the first column of Table 3 the dependent variable is the ownership rate by MSA, computed as the ratio of owner-occupied units to total (owner and renter) occupied units. As predicted by our model, the ownership rate is positively associated with median income and age and negatively associated with inequality. These three variables along with population density account for over 50% of the variation in the ownership rate across cities. In the second column we add land values (\$ per acre)

Table 4: U.S. Metropolitan Statistical Areas (2010)  
(Median income adjusted for local non-housing cost of living)

	Ownership Rate		Price-Rent ratio			
			(price-asked)		(estimated-value)	
<b>Log Median Income</b>	0.089	0.093	4.613	5.874	7.163	5.945
Robust s.e.	(0.030***)	(0.025***)	(3.582)	(3.020*)	(3.208**)	(2.105***)
Clustered s.e.	(0.031***)	(0.030***)	(2.924)	(3.120*)	(2.527***)	(2.720**)
<b>Gini index</b>	-0.380	-0.448	27.740	56.147	3.119	46.824
Robust s.e.	(0.134***)	(0.160***)	(19.005)	(18.544***)	(14.495)	(11.085***)
Clustered s.e.	(0.157**)	(0.148***)	(25.661)	(17.899***)	(25.131)	(14.960***)
<b>Age</b>	0.006	0.008	0.215	-0.066	0.324	0.097
Robust s.e.	(0.001***)	(0.001***)	(0.118*)	(0.091)	(0.086***)	(.061)
Clustered s.e.	(0.001***)	(0.001***)	(0.133)	(0.101)	(0.118***)	(.062)
<b>Log Density</b>	-0.038	-0.025	1.605	-2.766	2.681	-1.175
Robust s.e.	(0.005***)	(0.005***)	(0.875*)	(0.735***)	(0.754***)	(0.500**)
Clustered s.e.	(0.008***)	(0.006***)	(1.200)	(0.808***)	(1.193**)	(0.468**)
<b>Log Land Value</b>		-0.012		3.851		3.979
Robust s.e.		(0.005***)		(0.685***)		(0.494***)
Clustered s.e.		(0.005***)		(0.769***)		(0.874***)
<b>Amenity controls</b>	No	Yes	No	Yes	No	Yes
$\bar{R}^2$	0.52	0.65	0.07	0.49	0.24	0.77
# obs	219	199	219	199	219	199

Notes: See Table 3

and the amenity controls.<sup>36</sup> As predicted by the model, ownership is negatively and significantly associated with land values, controlling for amenities and density.

For the remaining columns of Table 3 the dependent variable is the ratio of average prices to average rents by MSA. In all cases, *rent* refers to the mean gross rent of renter-occupied housing units. In the second two columns *price* refers to the mean price asked for vacant units for sale whereas in the last two columns, it refers to the mean estimated value of all owner-occupied housing units. Again, as predicted by the model, the estimates in the second two columns imply that average price-rent ratios are positively and significantly associated with median income, inequality and land values.<sup>37,38</sup> The effect of median age is small and less robust.

<sup>36</sup>Full estimation results are provided in Appendix C.

<sup>37</sup>We also considered controlling for city-size as measured by the total number of households in each MSA. The results are robust to the inclusion of this variable and, while statistically significant in some cases, it does not add any explanatory power to the regressions.

<sup>38</sup>The results in Table 3 are also robust to controlling for average property tax rates across cities using data from the Lincoln Institute of Land Policy. Table 13 in Appendix C documents the results

Table 4 presents the same regressions but where median incomes are adjusted using a local cost of living index (COLI) for non-housing expenditures. This index is not available for every MSA, so this reduces the base sample size to 219 MSA's. When combined with the land value and amenity data, the sample size is further reduced to 199 MSA's. Nonetheless, the correlations with median income, the Gini index, age and land values remain much the same as in Table 3 and the impacts of controlling for amenities are very similar.

Overall, these results conform very well qualitatively with the cross-city predictions of our model. In Appendix C, we show that this conformity extends to smaller urban areas (micropolitan areas and urban clusters). While these estimates cannot be interpreted as causal, they do suggest that our stylized model replicates qualitatively the patterns observed in the data.

## 5.2 Cross-city Variation in the Theory

Here we attempt to quantify the relative importance of variation in MSA-level characteristics in driving cross-city variation in ownership rates and price-rent ratios that is implied by our theory. While we can set median income, the Gini coefficient and median age in the model to match those observed for the corresponding city in the 2010 ACS,<sup>39</sup> there is no single variable in the data that corresponds to  $\tau$ : construction costs per unit of quality.

Variation in the model parameter  $\tau$  ultimately depends, in the data, on variation in multiple variables related to construction/land costs and city-wide amenities. We therefore essentially impose our theory on the data in order to infer the distribution of  $\tau$ 's across cities. Specifically, we compute city-specific  $\tau$ 's to minimize the weighted distance between actual and predicted values of home-ownership and the ratio of average prices to average rents, where the weights are inversely proportional to their variance.<sup>40</sup> Figures 11 and 12 depict the results. The ratio of the explained sum of squares to the total sum of squares implied by this procedure is 0.57.

To assess the validity of this procedure, Table 5 reports estimates of a least-squares of estimating the same regressions as in Table 3, but with the inclusion of our estimated tax rate variable.

<sup>39</sup>The sample here is of the 219 U.S. MSA's for which COLI's are available. Average age is controlled by population growth, and is subject to the requirement that growth be neither less than 0% nor greater than 5% annually.

<sup>40</sup>For details on the procedure used, see Appendix B.

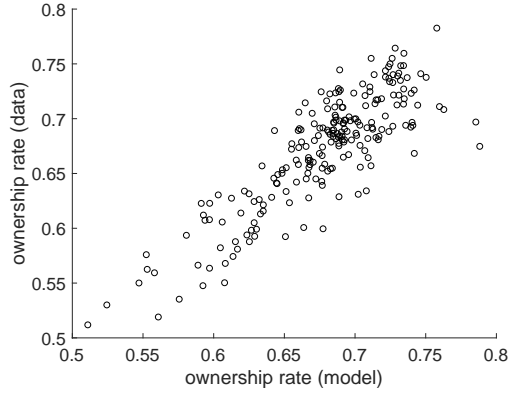


Figure 11: Ownership in the data and the model with variation in construction/land costs.

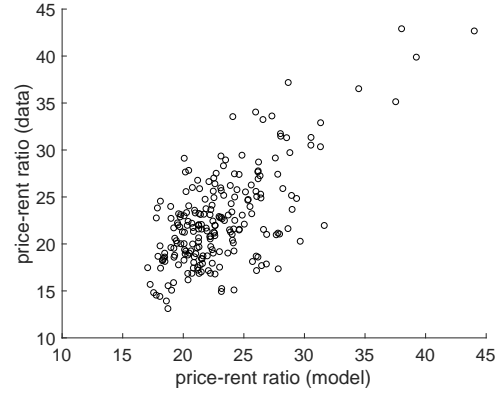


Figure 12: Price-rent ratio in the data and the model with variation in construction/land costs.

regression across MSA's of our inferred  $\tau$ 's on average land values (from Albouy *et al.*, 2017), population density and various natural amenity measures (from Albouy, 2016). As seen in the first three columns of Table 5 these explanatory variables are statistically significant and account for almost half of the variation in  $\tau$ . The last column replaces the direct amenity measures with a city-level amenity index constructed by Albouy (2016). As may be seen, this more parsimonious representation accounts for almost as much of the overall variation as before. While there are obviously numerous unobserved factors which determine the variation in these inferred  $\tau$ 's, it seems clear that it does indeed reflect variation in actual land costs and amenities.

Table 6 documents the changes implied by the model in the ownership and the price-rent ratio as a result of a one standard deviation change in each of the MSA-level characteristics. Overall, these results demonstrate that while variation in income, inequality and population growth play a role in determining variation in the ownership rates and ratios of average prices to average rents rate across cities, variation in land costs and average amenities is the most important factor, quantitatively, for understanding these relationships.

Table 5: Relationship between Inferred  $\tau$ 's and Land Costs and Amenities

	Log of Inferred $\tau$					
<b>Cdd65</b>	0.048 (0.014***)	0.049 (0.012***)	0.039 (0.012***)	—	—	
<b>Hdd65</b>	0.031 (0.005***)	0.026 (0.005***)	0.012 (0.006**)	—	—	
<b>Sun_a</b>	0.332 (0.082***)	0.169 (0.084**)	0.096 (0.081)	—	—	
<b>Slope_pct</b>	0.007 (0.002***)	0.008 (0.002***)	0.005 (0.002***)	—	—	
<b>Inv_water</b>	0.545 (0.122***)	0.216 (0.135)	-0.004 (0.145)	—	—	
<b>Latitude_w</b>	0.010 (0.002***)	0.006 (0.002**)	0.002 (0.002)	—	—	
<b>Log Density</b>	—	0.048 (0.008***)	0.023 (0.011**)	—	—	
<b>Log Land value</b>	—	—	0.059 (0.011***)	0.031 (0.013**)		
<b>Log Amenity</b>	—	—	—	0.470 (0.100***)		
<b>Constant</b>	-0.467 (0.122***)	-0.570 (0.103***)	-0.576 (0.105***)	-0.198 (0.072***)		
$R^2$	0.29	0.37	0.45	0.43		
# obs	199	199	199	201		

Notes: Heteroskedasticity-robust standard errors in parenthesis.

Table 6: Relative Importance of MSA-level Characteristics

	One Standard Deviation (SD) Increase			
	Median Income	Gini Index	Median Age	Construction Cost
Ownership Rate (in SD's)	0.4528	-0.0134	0.5219	-0.9152
Price-Rent Ratio (in SD's)	0.1239	0.3071	-0.0049	0.5093

Notes: This table displays the effects on the ownership rate and the price-rent ratio of a one SD increase (from .5 SD's below to .5 SD's above the mean) for each of the four factors indicated in the column headers. These effects are expressed in SD's of the dependent variable of interest.

## 6 The Affordability of Housing

In the model, all households would prefer to own since the overall costs of owning a house of any given quality level are lower than renting.<sup>41</sup> However, lower income households can only afford housing in market segments where, in equilibrium, there are relatively few housing units for sale. Consequently, they are more likely to rent. In this way, overall affordability of housing is inextricably linked to home-ownership. The distribution of income and the cost of supplying housing interact to determine the type of housing that households at various positions in the income distribution can afford, both to rent and to buy. Exogenous wealth inequality generates inequality of well-being, in part attributable to differential access to housing. As such, redistribution can be beneficial depending on one's definition of social welfare. To study these issues, we

<sup>41</sup>In the alternative formulation described in Section 4.4 there is a utility premium derived from ownership.



consider the implications of policies designed to increase the *affordability* of housing, using a standard definition taken from the National Association of Realtors (NAR).

## 6.1 The Housing Affordability Index (HAI)

The Housing Affordability index (HAI) is defined as:

$$\text{HAI} = \frac{y_m}{y_q} \times 100; \quad (35)$$

where  $y_m$  denotes median (annual) household income,<sup>42</sup> and  $y_q$  denotes *qualifying* income. The latter is taken to represent the annual income flow required to afford, “reasonably”, the mortgage payment on the median-priced home.<sup>43</sup> Consequently, the HAI measures, more specifically, the affordability of *home-ownership*.

Our notion of “reasonableness”, with regard to affording a mortgage payment, is based on the following assumptions:

- (i) The household makes a 20% downpayment. Of course, in our model, households need not do this, nor would they choose to given the concavity of utility and the existence of complete financial markets.
- (ii) The mortgage interest rate is 4% annually; the monthly rate is thus  $i_m = .04/12$ .
- (iii) The amortization period is 30 years or 360 months.
- (iv) The household spends no more than 25% of its income on its mortgage payment.

Let  $p_m$  denote the price of a house of median quality (equivalently in the model, the median price of a house). We then have:

$$y_q = 4 \times \underbrace{\frac{(1 - 0.2)p_m i_m}{1 - \left(\frac{1}{1+i_m}\right)^{360}}}_{\text{monthly payment}} \times 12. \quad (36)$$

A lower HAI indicates that housing is *less* affordable.

---

<sup>42</sup>As we assume a log-normal distribution of income,  $y$ , with mean  $\mu$ , and interpret it as a quarterly flow,  $y_m = 4e^\mu$ .

<sup>43</sup>The method used by the NAR in constructing HAI’s for various housing markets is described online at: <https://www.nar.realtor/topics/housing-affordability-index/methodology>.

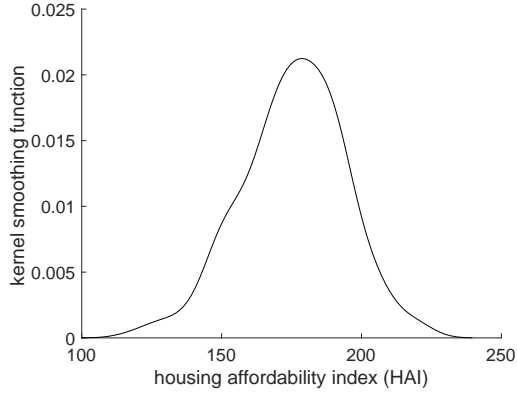


Figure 13: Housing affordability across cities (kernel density estimate).

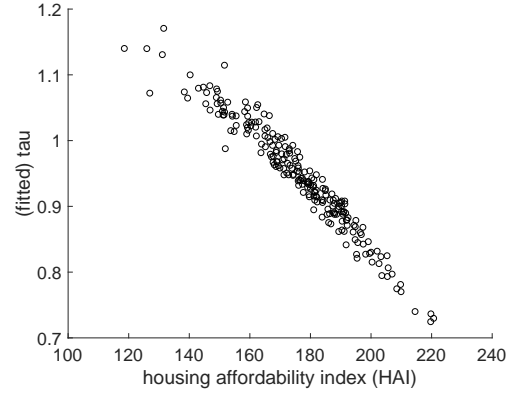


Figure 14: Fitted measure of construction costs and housing affordability.

In our baseline calibration the  $\text{HAI} = 166$ . That is, a household with median income has a quarterly income flow 1.66 times that of the qualifying income. Using the values of  $\tau$  fitted to match the variation in ownership rates and price-rent ratios across U.S. MSA's (see Figures 11 and 12 in Section 5), the HAI ranges from  $\underline{\text{HAI}} = 119$  to  $\overline{\text{HAI}} = 221$ . Figure 13 depicts a kernel density estimate for the distribution of HAI's across the 219 cities in our sample. Figure 14 plots the HAI for each of these cities against the fitted value of  $\tau$ . Clearly, housing affordability in the model is closely related negatively to the cost of producing a unit of housing quality.

In the model, city-level housing affordability affects *home-ownership* significantly for lower income households only. Figure 15 plots home-ownership rates for households in the 20th, 50th and 80th income percentiles against the HAI for each city in the sample. In the figure it is clear that affordability is effectively uncorrelated with home-ownership for households in the 80th percentile, and shows only a slight positive correlation for those in the 50th. For households in the bottom quintile, however, the correlation of affordability with ownership is striking.

## 6.2 A Policy to Improve Housing Affordability

We now consider the effects of a simple policy designed to increase the affordability of housing, with emphasis on houses targeted by low income households. We have in mind a policy through which government subsidies reduce the cost of producing housing in the market segments targeted by these households. Specifically, consider

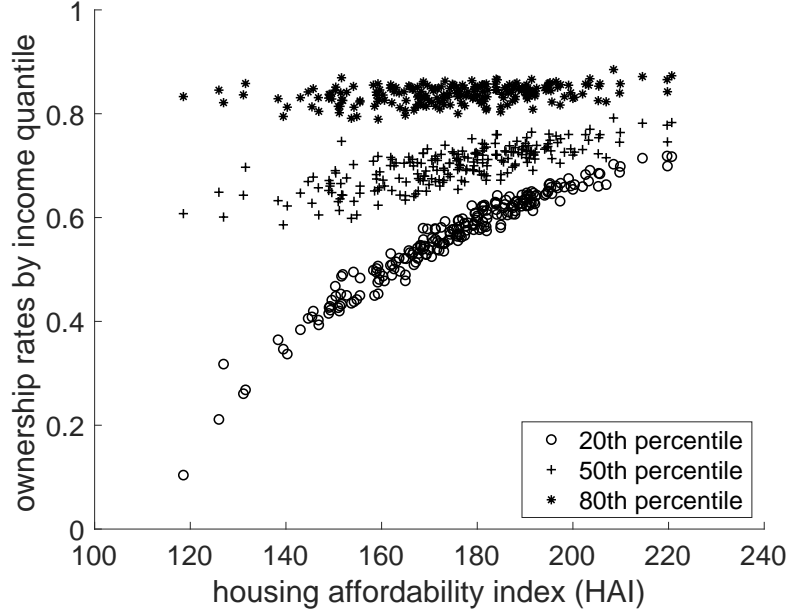


Figure 15: Ownership across cities by income quantile.

a tax/subsidy scheme that changes the cost of construction from  $T(q) = \tau q$  to

$$\hat{T}(q) = [\hat{\tau}_0 + \hat{\tau}_1(q - \hat{q})] \tau q \quad (37)$$

where  $\hat{\tau}_0 \leq 1$ ,  $\hat{\tau}_1 > 0$ , and  $\hat{q} > 0$ . Under the policy, developers building houses of quality  $q < \hat{q} + (1 - \hat{\tau}_0)/\hat{\tau}_1$  receive subsidies, while those building houses of quality  $q > \hat{q} + (1 - \hat{\tau}_0)/\hat{\tau}_1$  are taxed. This policy is chosen not to mimic any specific policy observed, but rather to illustrate the implications of policies of a general form. The construction cost changes its effects are passed through to home buyers through competition in the directed search equilibrium.

We think of (37) as representative of a number of different possible policies designed to make housing more affordable in the sense of raising the HAI. For example, alternatively we could specify a subsidy to construction overall, coupled with an appropriately progressive property tax on both homeowners and rental properties.<sup>44</sup> That said, it is not representative of all policies that change the relative costs of supplying and/or occupying houses of different qualities. For example, the policy treats rental and owner-occupied houses symmetrically, and as such neither encourages nor discourages home-ownership directly. In any case, the particular allocation

<sup>44</sup>A proportional income tax will make no difference; such a tax is effectively already present in the baseline model.

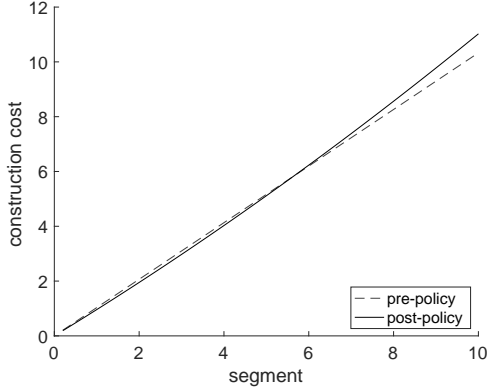


Figure 16: Construction costs.

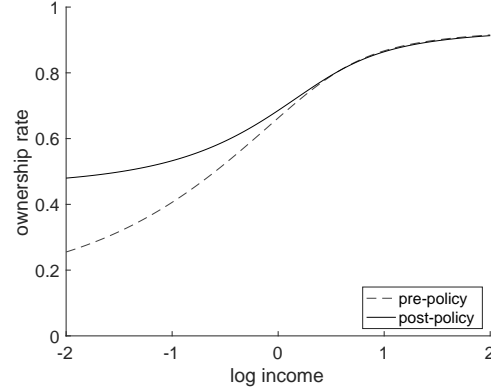


Figure 17: Ownership rate vs. income.

of consumption, housing quality, ownership rates, etc. that arises in the presence of the policy clearly could be implemented via a number of different policies with observationally equivalent results.

Now, consider a relatively “unaffordable” city with a construction cost parameter  $\tau = 1.0329$ . This corresponds to an HAI one standard deviation below the average for the 219 simulated MSA’s in Section 5.2. Specifically, the city has an HAI = 157 whereas the average HAI = 176. We construct a policy of the form (37) that

- (i) raises the HAI for the city to the *average* affordability, and
- (ii) balances the budget at the city level. That is, the total taxes collected in equilibrium equal the total subsidies paid out.

The following policy satisfies these requirements in the stationary equilibrium:

$$\{\hat{\tau}_0, \hat{\tau}_1, \hat{q}\} = \{0.9336, 0.0154, 1.3252\}. \quad (38)$$

Construction costs with and without the affordability policy are displayed in Figure 16. As may be seen, under this particular policy, the effective costs of relatively high quality housing must be increased substantially in order to achieve somewhat lower costs of relatively low quality housing. This reflects first the fact that given the distribution of households across housing quality levels (which depends on the distribution of income) the majority of households are effectively subsidized while relatively few are taxed. Second, it reflects, as we will see, substitution of relatively high income households into lower quality housing units.

The affordability policy affects the levels and distributions of home-ownership, housing quality, consumption, and welfare. It generally redistributes from upper to lower income households. It has, however, effects throughout the distribution.

### 6.2.1 Home-ownership

While the net effect on construction costs is small (as the budget is balanced) overall home-ownership rises significantly, from 63.39% in the baseline to 67.99% under the policy. This occurs in spite of the fact that the policy does not favor home-ownership *per se*. The increase in home-ownership is concentrated overwhelmingly in the bottom half of the income distribution, as can be seen in Figure 17.

The increase in home-ownership is driven by the relative cost of renting vs. owning. In equilibrium, the cost of renting depends on the both the construction cost *and* the cost of maintenance. Construction costs, of course, are changed directly by the policy. The cost advantage to owning, then, lies in avoiding increased relative costs of renting, which stem from relative maintenance costs that increase with quality.

For low quality housing, the relative cost of renting is small and importantly, does not change much with  $q$  (see Figure 3). A small reduction in the construction cost lowers the cost of housing services (regardless of tenure), induces households to demand higher quality houses, and thus increases the relative cost of renting. This, together with the fact that a significant share of the population lives in relatively low quality rental housing, leads to a large increase in home-ownership for this segment of the population.

In contrast, houses occupied by above median income households are sold (rather than rented) at high rates even in the baseline. The policy *increases* both rents and prices for these houses. Market tightness rises, sending prospective buyers into lower quality sub-markets. While the policy reduces the *quality* of their housing, it has only very minor effects on their rates of ownership.

### 6.2.2 Housing quality and costs

While the affordability policy changes home-ownership significantly only for households in the bottom 50% of the income distribution, it affects housing quality and both prices and rents throughout the distribution. Table 7 documents the average quality of housing consumed by a “representative” household at the midpoint of each quintile of the income distribution both before and after the policy change. As can be seen, the average quality of housing for the representative household rises for the bottom three quintiles and falls for the top two quintiles. The decline in housing quality is large for households in the top quintile.

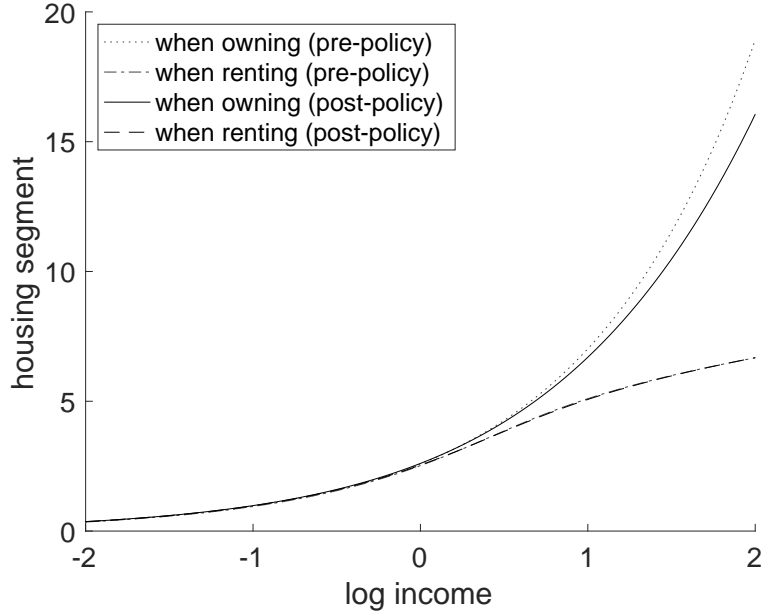


Figure 18: Market segment vs. income by housing tenure.

The effects of the policy are not surprising given that it affects directly the construction cost and hence the rent and return to selling differentially for low and high quality houses. As noted above, the rents on high quality houses (inhabited by high income households) rise, sending households into lower quality segments as both renters (due to the direct increase in rent) and as prospective owners (due to the increase in tightness associated with diminished returns to selling in this quality range). Similar forces are at work (in reverse) for lower quality houses which see their construction costs reduced; rents fall, inducing lower income households to rent better houses and to search in higher quality segments where tightness has declined.

Overall, the city-wide average quality of rented housing rises while that of owner-occupied housing declines. The former effect reflects mainly the increase in housing quality for low income households who rent at a high rate. The latter reflects the combined effects of the increase in ownership (of low quality housing) by low income households and the decline in quality for high income ones.

The affordability policy lowers the average sale price from 4.67 to 4.29, by reducing the cost of building low quality homes. The policy, however, *raises* average rent slightly (from 0.18 to 0.19) by raising the average quality of homes rented, most commonly by low-income households. The policy reduces the average price/rent ratio

from 25.69 to 22.49 and, as intended, raises the HAI from 157 to 175.

Table 7: Housing Quality under the Policy to Improve Housing Affordability

Household Income	$q$ When Renting		$q$ When Owning		Average $q$	
	<i>Pre-</i>	<i>Post-</i>	<i>Pre-</i>	<i>Post-</i>	<i>Pre-</i>	<i>Post-</i>
$F^{-1}(0.1)$	0.8809	0.9072	0.8836	0.9109	0.8820	0.9092
$F^{-1}(0.3)$	1.6510	1.6825	1.6661	1.6991	1.6592	1.6925
$F^{-1}(0.5)$	2.5179	2.5372	2.5817	2.6006	2.5602	2.5807
$F^{-1}(0.7)$	3.6755	3.6617	3.9985	3.9544	3.9263	3.8899
$F^{-1}(0.9)$	5.2327	5.2060	7.4931	7.1066	7.2060	6.8598
Agg. Average $q$	2.0459	2.1876	4.4099	3.9918	3.5501	3.4191

### 6.2.3 Consumption and welfare

The affordability policy affects the consumption of both goods and housing services for all households. The bottom row of Table 8 measures the impact on welfare for the median household in each quintile using the percentage change in goods consumption that would have the effect on household utility equivalent to that of the policy. Overall, the policy increases welfare for the median household in each of the bottom four quintiles while reducing welfare for that of the top quintile. The percentage welfare gain is largest for the bottom quintile and falls as income rises.

The first five rows of Table 8 report the welfare effects of the affordability policy emanating from changes in goods consumption (row one), consumption of housing services (overall, row two; while renting and owning respectively, rows three and four) and home-ownership (row five).

For households in the bottom two quintiles, welfare gains come mainly from increases in housing quality. This is true even for the bottom quintile which experiences a large increase in home-ownership. Given the quality of the houses these households own, home-ownership *per se* does not contribute much to welfare.

For households in the top quintile, the effects of reduced housing quality are paramount. The overall welfare losses to these households are mitigated significantly, however, by a relatively large increase in goods consumption. The results for the

second and third quintiles diverge from those of the first in that they gain from the affordability policy overall; and from those for the bottom two quintiles in that their gains are driven by increased goods consumption. These households experience small changes in housing quality and negligible effects on home-ownership.

Table 8: Percentage Welfare Effects (Expressed in Consumption Equivalents)

Post-Policy Outcome(s)	Household Income				
	$F^{-1}(0.1)$	$F^{-1}(0.3)$	$F^{-1}(0.5)$	$F^{-1}(0.7)$	$F^{-1}(0.9)$
Consumption	0.1513	0.2812	0.4253	0.6230	1.0564
Housing Services	0.8597	0.5547	0.2141	-0.2625	-1.3013
Quality While Renting	0.5622	0.2818	0.0864	-0.0288	-0.0228
Quality While Owning	0.2957	0.2721	0.1276	-0.2338	-1.2787
Home Ownership	0.0115	0.0165	0.0178	0.0080	-0.0352
All Post-Policy Outcomes	1.0279	0.8554	0.6580	0.3661	-0.2890

## 7 Extensions and other implications

### 7.1 A Version with Financial Constraints

Households in our model face binding financial limitations imposed by their intertemporal budget constraints. This is why poorer households cannot afford to search for housing to own in segments where houses for sale are relatively abundant and why cheap housing is more likely to be rented. As we have noted in the introduction, models that emphasize the role of additional tighter borrowing constraints rest on the assumption that no one is allowed to sell cheap housing units. Although, households in our model have access to complete asset markets, we can explore the implications of financial frictions by imposing a simple type of payment-to-income (PTI) constraint for home buyers:

$$Z_N(q_S) + \rho T(q_S) \leq 0.3y \quad (39)$$

While it is not, strictly-speaking a borrowing constraint, the PTI constraint does have a similar effect by restricting searchers to market segments in which the imputed flow cost of home-ownership is no more than 30 percent of their income.



We introduce this constraint into a version of our model with a utility premium for ownership that rises with quality (see Section 4.4) and calibrate it to match the same targets as before.<sup>45</sup> When we do so, our main qualitative results remain largely unchanged.<sup>46</sup> In this equilibrium, the PTI constraint is binding for relatively high income households. Consequently, and more realistically, the share of income spent on consumption does not rise dramatically with income as it does in the unconstrained version of the model.

## 7.2 The price-rent ratio of the median quality house

The main focus of this paper is on the role of search frictions in driving cross-city variation in housing composition across owned and rental markets. We have used the ratio of average prices to average rents across cities because a large part of the variation in this measure is driven by housing composition. However, our model also has implications for cross-city variation in the price-rent ratio of the average quality house. This concept is often viewed by housing economists as a potential gauge of the role of financial or speculative factors in driving relative activity in the owned housing market (see, for example, Gilbukh, Haughwout and Tracy, 2017). Empirically, however, it is very challenging to estimate the price to rent ratio of equivalent housing units, precisely because the composition of owned and rented housing is so different. A substantial literature has used various approaches to adjust for quality and the implications are somewhat mixed. For example, Zillow Research provides MSA-level estimates of the price-rent ratio of the median quality house based on a hedonic approach.

As seen in Figure 19 in Appendix C, there is a significant positive empirical correlation between the price-rent ratio of the median quality house provided by Zillow and the ratio of average prices to average rents estimated from the ACS data.<sup>47</sup> When we incorporate cross-city variation in the fundamentals, all versions of our model also predict a positive correlation between the two price-rent ratio concepts. This is true even for the baseline calibration that exhibits a declining price-rent ratio by quality (see Figure 20) because a large part of the cross-city variation comes from supply-side variation driven by housing production costs per unit of quality,  $\tau$ . Both price-rent

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<sup>45</sup>Using the baseline version is not very interesting because the share of income spent on housing is roughly constant. Consequently, the constraint binds either for all households or for none.

<sup>46</sup>Details are available upon request.

<sup>47</sup>There are fewer observations in Figure 19 than in Figure 20) because there are missing Zillow values for some of the MSA's.

ratio concepts are strongly increasing with  $\tau$ .<sup>48</sup>

## 8 Conclusion

We have developed a theory of rent vs. own and rent vs. sell decisions based on search and frictional assignment. Owning a house of a given quality is more desirable than renting, either due to the relatively high cost of maintaining a rental home, as in our baseline calibration, or to a preference for ownership, or both. Proportionately more housing units are put up for sale, rather than rented out, in high quality market segments where buyers are willing and able to pay a price premium to compensate sellers for the cost of holding a house vacant-for-sale. Proportionately more poor households end up renting because they cannot afford housing in sub-markets where housing for sale is relatively more plentiful.

The model can be calibrated to replicate relevant features of the median U.S. city's housing market: specifically, the average time to sell, ownership duration, price-rent ratio and distribution of ownership by income quintile. In the calibrated model, within the city average cost of renting vs. owning rises with house quality. Across cities, we find that variation in median income, inequality, average age (population growth) and land/amenity values generates variation in both rates of ownership and ratios of average prices to average rents that is qualitatively consistent with those observed across U.S. MSA's.

We use the calibrated model to study the effects of policies designed to increase the affordability of housing. Specifically, we consider the implications of a policy which taxes construction of high quality housing and uses the proceeds to subsidize that of low quality housing in a way which lowers the cost of the median quality house. The main impacts of such a policy are (1) to raise the quality of housing for low income households and lower it for high income ones and (2) to raise the rate at which housing is sold rather than rented to low income households. The particular policy modeled increases the welfare of households in the bottom four quintiles at the expense of those in the top quintile.

In future work, we intend to relax some of the theory's limiting assumptions: in particular, that household incomes are certain and permanent, that moves both

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<sup>48</sup>When we run cross-city regressions like those in Table 3 but using the Zillow price-rent ratio index as a dependant variable, the qualitative results are similar to those using the ratio of average prices to average rents.

within the city and to/from outside are exogenous, and that there is no aggregate uncertainty. The transitional dynamics of the model are complex but potentially very interesting. At the same time, we plan to consider empirical applications of the current environment which take account of specific amenities exhibited and constraints existing in particular cities.

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## A Additional Model Details

### A.1 The consumption-saving decision

Owners issue claims to achieve zero wealth in the event of death:  $w_D = -V(q_S)$ . These claims work essentially as a reverse mortgage. The intertemporal Euler equations are

$$\phi_R(q_S, p) = \beta(1 - \lambda(\theta(q_S, p))) \frac{u'(c_R^{+1})}{u'(c_R)} \quad (\text{A.1})$$

$$\phi_B(q_S, p) = \beta\lambda(\theta(q_S, p)) \frac{u'(c_B^{+1})}{u'(c_R)} \quad (\text{A.2})$$

$$\phi_N = \beta(1 - \pi) \frac{u'(c_N^{+1})}{u'(c_N)} \quad (\text{A.3})$$

$$\phi_S = \beta\pi \frac{u'(c_S^{+1})}{u'(c_N)} \quad (\text{A.4})$$

The no-arbitrage conditions are:

$$\phi_N = \beta(1 - \pi) \quad (\text{A.5})$$

$$\phi_S = \beta\pi \quad (\text{A.6})$$

$$\phi_D = \frac{\beta\delta}{1 - \delta} \quad (\text{A.7})$$

$$\phi_R(q, p) = \beta(1 - \lambda(\theta(p; q))) \quad (\text{A.8})$$

$$\phi_B(q, p) = \beta\lambda(\theta(p; q)) \quad (\text{A.9})$$

$$\phi_D(q, p) = \frac{\beta\delta\lambda(\theta(p; q))}{1 - \lambda} \quad (\text{A.10})$$

The Euler equations then imply a constant consumption stream. To achieve this, wealth positions,  $\{w_R, w_B, w_N, w_S\}$ , must satisfy  $w_B = w_N + p$ ,  $w_S = w_R - V(q_S)$  and the following budget constraints:

$$c + \beta\lambda(\theta) [w_N + p] + \beta(1 - \lambda(\theta))w_R + \frac{\beta\delta\lambda(\theta)}{1 - \delta} [p - V_S] + x_R = w_R \quad (\text{A.11})$$

$$c + \beta\pi [w_R - V_S] + \beta(1 - \pi)w_N - \frac{\beta\delta}{1 - \delta} V_S + \zeta q = w_N \quad (\text{A.12})$$

For convenience, the notation here has been simplified by replacing, for example,  $\theta(q_S, p)$  with  $\theta$ ,  $V(q_S)$  with  $V_S$ , and  $x(q_R)$  with  $x_R$ . Combining both budget constraints



to eliminate  $c$  yields

$$w_R - w_N = V_S + \frac{\beta\lambda(\theta)(p - V_S) + (1 - \delta)(x_R - x_S)}{(1 - \delta)[1 - \beta(1 - \pi - \lambda(\theta))]} \quad (\text{A.13})$$

This relationship and a budget constraint determine the level of consumption:

$$c = (1 - \beta)w_R - x_R - \frac{\beta\lambda(\theta)}{1 - \delta} \left( \frac{[1 - \beta(1 - \pi)](p - V_S) - (1 - \delta)(x_R - x_S)}{1 - \beta(1 - \pi - \lambda(\theta))} \right) \quad (\text{A.14})$$

A household with permanent income  $y$  initially enters the city as a renter with wealth equal to the present discounted value of lifetime income:  $w_R = y/(1 - \beta)$ . The constant consumption level for this household is therefore

$$c = y - \frac{[1 - \beta(1 - \pi)]x_R + \beta\lambda(\theta)x_S}{1 - \beta(1 - \pi - \lambda(\theta))} - \frac{\beta\lambda(\theta)[1 - \beta(1 - \pi)](p - V_S)}{(1 - \delta)[1 - \beta(1 - \pi - \lambda(\theta))]} \quad (\text{A.15})$$

Note that (A.15) is (17) in the text. The financial wealth of a household changes with every transition to a different ownership status and in the event of death according to

$$w_R = \frac{y}{1 - \beta} \quad (\text{A.16})$$

$$w_B = \frac{y}{1 - \beta} + p - V_S - \frac{\beta\lambda(\theta)(p - V_S) + (1 - \delta)(x_R - x_S)}{(1 - \delta)[1 - \beta(1 - \pi - \lambda(\theta))]} \quad (\text{A.17})$$

$$w_N = \frac{y}{1 - \beta} - V_S - \frac{\beta\lambda(\theta)(p - V_S) + (1 - \delta)(x_R - x_S)}{(1 - \delta)[1 - \beta(1 - \pi - \lambda(\theta))]} \quad (\text{A.18})$$

$$w_S = \frac{y}{1 - \beta} - V_S \quad (\text{A.19})$$

$$w_D = -V_S \quad (\text{A.20})$$

$$(\text{A.21})$$

## A.2 The search decision

Households direct their search by choosing a particular sub-market (price) and market segment (quality). The search decision appears in the Bellman equation for a household that is currently renting:

$$W^R(w) = \max_{q_S, p} \left\{ \begin{array}{l} u(c) + h(0, q_R) \\ + \beta\lambda(\theta(q_S, p))W^N(q_S, w_B - p) \\ + \beta[1 - \lambda(\theta(q_S, p))]W^R(w_R) \end{array} \right\} \quad (\text{A.22})$$

subject to

$$c = w - x(q_R) - \beta\lambda(\theta(q_S, p))w_B - \beta(1 - \lambda(\theta(q_S, p)))w_R - \frac{\beta\delta\lambda(\theta(q_S, p))}{1 - \delta}(p - V(q_S))$$

and

$$\beta\gamma(\theta(q_S, p)) [p - V(q_S)] = (1 - \delta - \beta)V(q_S). \quad (\text{A.23})$$

where the latter constraint imposes that the searching household correctly anticipates that market tightness,  $\theta$ , is determined by the free entry of sellers according to (11). By substituting this constraint into the household's objective function, the optimal choice of segment and sub-market solve the following:

$$W^R(w) = \max_{q_S, \theta} \left\{ \begin{array}{l} u(c) + h(0, q_R) + \beta [1 - \lambda(\theta)] W^R(w_R) \\ + \beta\lambda(\theta)W^N \left( q_S, w_B - V(q_S) - \frac{1-\delta-\beta}{\beta\gamma(\theta)}V(q_S) \right) \end{array} \right\} \quad (\text{A.24})$$

subject to

$$c = w - x(q_R) - \beta\lambda(\theta)w_B - \beta(1 - \lambda(\theta))w_R - \frac{\delta(1 - \delta - \beta)}{\theta(1 - \delta)}V(q_S).$$

The first order condition with respect to  $q_S$  is

$$u'(c) \frac{\delta[1 - \delta - \beta]V'(q_S)}{1 - \delta} + \beta\gamma(\theta) \left[ \frac{\partial W^N}{\partial q} - \frac{\partial W^N}{\partial w} \left( 1 + \frac{1 - \delta - \beta}{\beta\gamma(\theta)} \right) V'(q_S) \right] = 0 \quad (\text{A.25})$$

and the first order condition with respect to  $\theta$  is

$$\begin{aligned} & \beta \left[ \frac{1 - \alpha}{c} (w_R - w_B) + W^N - W^R \right] \frac{\theta\lambda'(\theta)}{\lambda(\theta)} \\ & + \frac{[1 - \delta - \beta]V(q_S)}{\gamma(\theta)} \frac{\theta\gamma'(\theta)}{\gamma(\theta)} \frac{\partial W^N}{\partial w} + u'(c) \frac{\delta[1 - \delta - \beta]V(q_S)}{(1 - \delta)\gamma(\theta)} = 0. \end{aligned} \quad (\text{A.26})$$

The value associated with home-ownership satisfies

$$W^N(q_S, w) = u(c) + h(1, q_S) + \beta\pi W^R(w_s + V(q_S)) + \beta(1 - \pi)W^N(q_S, w_N) \quad (\text{A.27})$$

subject to

$$c = w + \beta\pi w_s - \beta(1 - \pi)w_N + \frac{\beta\delta}{1 - \delta}V(q_S) - Z_N(q_S).$$

The Benveniste-Scheinkman conditions are therefore

$$\frac{\partial W^N(q_S, w)}{\partial w} = u'(c) \quad (\text{A.28})$$

$$\frac{\partial W^N(q_S, w)}{\partial q} = \frac{1}{1 - \beta(1 - \pi)} \left\{ \frac{\partial h(1, q_S)}{\partial q} + u'(c) \left[ \left( \beta\pi + \frac{\beta\delta}{1 - \delta} \right) V'(q_S) - Z'_N(q_S) \right] \right\} \quad (\text{A.29})$$

Substituting (A.23), (A.28) and (A.29) into first order conditions (A.25) and (A.26) yields

$$\begin{aligned} & \left( \frac{1 - \alpha}{c} \right) \left[ \frac{p}{(1 - \delta)V(q_S)} - \frac{\delta}{1 - \delta} \right] V'(q_S) \\ &= \frac{1}{1 - \beta(1 - \pi)} \left\{ \frac{\partial h(1, q_S)}{\partial q} + u'(c) \left[ \left( \beta\pi + \frac{\beta\delta}{1 - \delta} \right) V'(q_S) - Z'_N(q_S) \right] \right\} \end{aligned} \quad (\text{A.30})$$

and

$$\left[ u'(c)(w_R - w_B) + W^N - W^R \right] (1 - \eta(\theta)) = u'(c) \left[ \eta(\theta) + \frac{\delta}{1 - \delta} \right] (p - V(q_S)) \quad (\text{A.31})$$

where  $\eta(\theta) = \theta\gamma'(\theta)/\gamma(\theta) = \theta\lambda'(\theta)/\lambda(\theta) + 1$ . Condition (A.30) simplifies to

$$u'(c) \left[ \frac{[1 - \beta(1 - \pi)]p - (\beta\pi + \delta)V(q_S)}{(1 - \delta)V(q_S)} V'(q_S) + Z'_N(q_S) \right] = \frac{\partial h(1, q_S)}{\partial q}.$$

With substitutions from (A.16), (A.17), (A.24), (A.27), condition (A.31) simplifies to

$$p - V(q_S) = \frac{(1 - \delta)(1 - \eta(\theta))}{1 - \beta(1 - \pi - \eta(\theta)\lambda(\theta))} \left[ x(q_R) - x(q_S) + \frac{h(1, q_S) - h(0, q_R)}{u'(c)} \right].$$

These correspond to conditions (19) and (20) in the text.

## B Data Definitions, Sources and Calculations

### B.1 Definitions and sources

All data on housing and households is from the American Community Survey (2010, 5-year estimates) accessed via the Census Bureau at <http://factfinder.census.gov>.

**House price:** To compute mean house prices by urban area, we use the data on housing value obtained from Housing Question 16 in the ACS. The question was asked at housing units that were owned, being bought, vacant for sale, or sold and not occupied at the time of the survey. The estimated value is the respondent’s estimate of how much the property (house and lot, mobile home and lot, or condominium unit) would sell for if it were for sale. We also used the average price-asked on vacant housing for sale only and on housing sold but unoccupied. This was calculated by dividing the aggregate price-asked by the number for sale and sold but unoccupied.

**Rent:** To compute mean rent for each urban area, we use the data on gross rent obtained from answers to Housing Questions 11a-d and 15a in the ACS. Gross rent is the contract rent plus the estimated average monthly cost of utilities if these are paid by the renter.

**Housing tenure:** To compute the number of owning and renting households we use data obtained from Housing Question 14 in the ACS. The question was asked at occupied housing units. Occupied housing units are classified as either owner occupied or renter occupied:

Owner occupied – A housing unit is owner occupied if the owner or co-owner lives in the unit even if it is mortgaged or not fully paid for.

Renter occupied – All occupied housing units which are not owner occupied, whether they are rented or occupied without payment of rent, are classified as renter occupied.

Households – the number of households was computed as the sum of owner and renter occupied units.

Vacant housing units – A housing unit is vacant if no one is living in it at the time of interview. Units occupied at the time of interview entirely by persons who are staying two months or less and who have a more permanent residence elsewhere are considered to be temporarily occupied, and are classified as “vacant.” New units not

yet occupied are classified as vacant housing units if construction has reached a point where all exterior windows and doors are installed and final usable floors are in place.

**Median household income:** The data on income during the last 12 months were derived from answers to Questions 47 and 48, which were asked of the population 15 years old and over. Household income includes the income of the householder and all other individuals 15 years old and over in the household. “Total income” is the sum of the amounts reported separately for wage or salary income; net self-employment income; interest, dividends, or net rental or royalty income or income from estates and trusts; Social Security or Railroad Retirement income; Supplemental Security Income (SSI); public assistance or welfare payments; retirement, survivor, or disability pensions; and all other income

**Gini coefficient:** The Gini index of income inequality for each urban area comes from the ACS and measures the dispersion of the household income distribution.

**Population-weighted density** for metropolitan and micropolitan areas is from [http://www.census.gov/population/metro/data/pop\\_pro.html](http://www.census.gov/population/metro/data/pop_pro.html). It is calculated as a weighted-average of population densities for each census tract where the weights are the share of the total area population in each census tract. It is intended to reflect the population density experienced by the average person in the urban area.

**Median age** refers to the median age of the population taken from the 2010 ACS. (It is not the median age of the head of the household, which does not appear to be easily obtainable.)

**Land values** are taken from column 5 of Albouy *et al.* (2017) appendix Table A2. The MSA’s for which they provide these estimates are based on the 1999 OMB definitions. The MSA’s used in the current paper are from the 2010 census and are slightly different and greater in number. This is in part because some MSA’s have been subdivided and also because there are some new MSA’s.<sup>49</sup> Where MSA’s have been subdivided, we use the same average land values for both. Where we could find no match, we dropped the MSA from the data set.

**COLI:** The local cost of living index is the ACCRA Cost of Living Index provided by Council for Community and Economic Research (<https://www.c2er.org/>). It measures relative price levels for consumer goods and services in participating areas for

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<sup>49</sup>The MSA’s in 2010 are based on the concept of Core Based Statistical Areas.

a mid-management standard of living. Weights are based on the Bureau of Labor Statistics’ 2004 Consumer Expenditure Survey. We use these weights to extract a COLI for non-housing expenditures only. Data are for selected urban areas within the larger MSA. In the few cases where there are multiple areas within a given MSA, we used a simple average.

**Amenity controls:** “Natural” amenities taken from Albouy (2016):

Heating and cooling degree days (Annual) – Degree day data are used to estimate amounts of energy required to maintain comfortable indoor temperature levels. Daily values are computed from each day’s mean temperature,  $(\max + \min)/2$ . Daily heating degree days (**hdd65**) are equal to  $\max\{0; 65 - \text{mean temp}\}$  and daily cooling degree days (**cdd65**) are  $\max\{0; \text{mean temp} - 65\}$ . Annual degree days are the sum of daily degree days over the year. The data here refer to averages from 1970 to 1999.

Sunshine – Average percentage of possible sunshine (**sun\_a**). The total time that sunshine reaches the surface of the earth is expressed as the percentage of the maximum amount possible from sunrise to sunset with clear sky conditions.

Average slope (**slope\_pct**) – The slope of the land in the metropolitan area (percent), using an average maximum slope technique based on a 30 arcsec x 30 arcsec grid.

Coastal proximity (**inv\_water**) – Equal to the logarithm of the inverse distance in miles to the nearest coastline from PUMA centroid.

Latitude (**latitude\_w**) – Measured in degrees from the equator.

## B.2 Fitting construction costs ( $\tau$ ) to cross-city data

We compute MSA-level construction cost parameter values that best match the observed ownership rates and price-rent ratios. We allow median income, the income Gini and median age to match the characteristics of each MSA. For a given construction cost parameter value  $\tau$ , the resulting simulated economies yield predicted ownership rates and aggregate price-rent ratios. For each MSA, we choose the value for  $\tau$  that best matches the observed MSA-level statistics in the following sense: we minimize a weighted average of the squared differences between the simulated statistics and their empirical counterparts. For the weights, we use the inverse of the variances of the MSA ownership rates and price-rent ratios.

## C Additional Empirical Results

Tables 9 and 10 contain full parameter estimates for Tables 3 and 4, respectively.

Table 9: Full results for Table 3

	Ownership Rate		Price-Rent Ratio			
			(price-asked)		(estimated-value)	
<b>Log Median Income</b>	0.076	(0.030**)	14.497	(2.833***)	13.088	(2.114***)
<b>Gini index</b>	-0.449	(0.138***)	90.286	(13.395***)	65.427	(8.523***)
<b>Age</b>	0.007	(0.001***)	0.032	(0.083)	0.122	(0.044***)
<b>Log Density</b>	-0.024	(0.006***)	-3.585	(0.787***)	-1.447	(0.336***)
<b>Log Land Value</b>	-0.010	(0.005**)	2.680	(0.736***)	2.321	(0.651***)
<b>cdd65</b>	-0.024	(0.006***)	2.710	(0.790***)	2.998	(0.455***)
<b>hdd65</b>	-0.013	(0.003***)	0.554	(0.319*)	0.903	(0.236***)
<b>sun_a</b>	-0.067	(0.035*)	18.869	(6.491***)	19.239	(5.357***)
<b>slope_pct</b>	-0.001	(0.002)	0.692	(0.184***)	0.717	(0.104***)
<b>inv_water</b>	-0.064	(0.056)	41.339	(8.294***)	28.713	(4.453***)
<b>latitude_w</b>	-0.003	(0.001*)	0.082	(0.182)	0.263	(0.114**)
<b>Constant</b>	-0.076	(0.302)	-174.27	(33.08***)	-168.5	(25.35***)
$\overline{R}^2$	0.63		0.56		0.79	
# obs	332		332		332	

Notes: Standard errors (in parentheses) are clustered at the state level.

In Tables 11 and 12 we consider samples which also include smaller urban agglomerations. Table 11 includes *Micropolitan* areas which are urban agglomerations containing fewer than 50,000 households but more than 10,000. This increases the number of urban areas to 942 but we have no data for land values or amenities. Table 12 includes all urban areas with more than 1000 households, which increases the number of observations to 3360. Controlling for population density, the results documented in Table 11 are qualitatively similar to those of Table 3: (1) The price-rent ratio is positively correlated with median income, the Gini index and age and (2) the ownership rate is positively correlated with median income and age but negatively correlated with the Gini index. In Table 12, we have no data on density for smaller urban clusters and so cannot control for it. Nevertheless, the same qualitative correlations for median incomes, inequality and median age are obtained.

A potentially important factor that varies considerably across U.S. cities are property taxes. Property taxes are incurred by both owner-occupiers and landlords and therefore enter into the effective cost of supplying housing of a given quality,  $\tau$ . Unfortunately, we have not been able to obtain details data on average property taxes

Table 10: Full results for Table 4

	Ownership Rate		Price-Rent Ratio			
			(price-asked)		(estimated-value)	
<b>Log Median Income</b>	0.093	(0.030***)	5.874	(3.120*)	5.945	(2.720**)
<b>Gini index</b>	-0.448	(0.148***)	56.147	(17.899***)	46.824	(14.960***)
<b>Age</b>	0.008	(0.001***)	-0.066	(0.101)	0.097	(.062)
<b>Log Density</b>	-0.025	(0.006***)	-2.766	(0.808***)	-1.175	(0.468**)
<b>Log Land Value</b>	-0.012	(0.005***)	3.851	(0.769***)	3.979	(0.874***)
<b>cdd65</b>	-0.024	(0.005***)	3.216	(1.064***)	3.419	(0.537***)
<b>hdd65</b>	-0.013	(0.004***)	0.433	(0.449)	0.732	(0.374*)
<b>sun_a</b>	-0.028	(0.036)	14.454	(8.165*)	14.872	(6.111**)
<b>slope_pct</b>	-0.002	(0.001**)	0.745	(0.260***)	0.639	(0.111***)
<b>inv_water</b>	-0.095	(0.053*)	50.33	(10.33***)	29.017	(5.816***)
<b>latitude_w</b>	-0.003	(0.002)	-0.059	(0.165)	0.175	(0.150)
<b>Constant</b>	0.299	(0.002)	-40.138	(22.633*)	-60.04	(22.58**)
$\overline{R}^2$	0.65		0.49		0.77	
# obs	199		199		199	

Notes: Standard errors (in parentheses) are clustered at the state level.

at the MSA level. However, the Lincoln Institute of Land Policy does provide data on the property taxes paid per person within 150 or so “Fiscally Standardized Cities” (FSC).<sup>50</sup> While these do not correspond to MSAs, it is possible to match them with the main cities within many MSA’s. We then estimate an index of the average property tax as a percentage of land value in 2010 using available data as follows:

*Estimated MSA average property tax rate index =*

$$\frac{\text{Property Tax rate per person for FSC} \times \text{population density of MSA}}{\text{Estimated Land value per acre in MSA containing FSC}}.$$

Table 13 documents the results of estimating the same regressions as in Table 3, but now including the estimated tax rate variable. If we do not adjust incomes for the non-housing cost of living, the resulting sample contains 118 MSA’s. As may be seen, the qualitative nature of the results regarding the other factors are unchanged from Table 3. As is consistent with our model, ownership rates exhibit a statistically significant negative partial correlation with tax rates whereas price-rent ratios exhibit a positive one.

<sup>50</sup>Lincoln Institute of Land Policy. Fiscally Standardized Cities database. <http://www.lincolninst.edu/subcenters/fiscally-standardized-cities/>.



Table 11: All U.S. Metropolitan and Micropolitan Areas

Dependent Variable	Ownership Rate	Price-Rent Ratio	
		(price-asked)	(estimated-value)
<b>Log Median Income</b>	0.011	14.315	21.396
Robust s.e.	(.012)	(2.369***)	(1.395***)
Clustered s.e.	(.018)	(2.579***)	(2.346***)
<b>Gini index</b>	-0.599	59.731	45.261
Robust s.e.	(.070***)	(13.009***)	(6.848***)
Clustered s.e.	(.113***)	(18.471***)	(15.951**)
<b>Median age</b>	0.006	0.120	0.101
Robust s.e.	(.000***)	(0.096)	(.038***)
Clustered s.e.	(.001***)	(0.126)	(.051**)
<b>Log Density</b>	-0.017	-1.078	-0.238
Robust s.e.	(.002***)	(0.442**)	(.262)
Clustered s.e.	(.003***)	(0.537**)	(.407)
Adjusted $R^2$	0.47	0.05	0.43
# obs	942	937	942

Table 12: U.S. Urban Areas with more than 1000 Households

(Median income not adjusted for local non-housing cost of living)

	<b>Ownership Rate</b>		<b>Price-Rent ratio</b>	
<b>Log Median Income</b>	0.103	0.119	13.328	13.222
Robust s.e.	(.006***)	(.008***)	(.522***)	(.615***)
Clustered s.e.	(.012***)	(.016***)	(1.013***)	(1.280***)
<b>Gini index</b>	-0.530	-0.438	37.639	37.023
Robust s.e.	(.038***)	(.040***)	(2.851***)	(3.328***)
Clustered s.e.	(.038***)	(.044***)	(4.185***)	(5.944***)
<b>Median age</b>	0.007	0.006	0.044	0.046
Robust s.e.	(.000***)	(.000***)	(.018**)	(.018**)
Clustered s.e.	(.000***)	(.000***)	(.035)	(.037)
<b>Log Households</b>	—	-0.007	—	0.050
Robust s.e.		(.001***)		(.0.091)
Clustered s.e.		(.002***)		(.183)
Adjusted $R^2$	0.42	0.43	0.29	0.29
# obs	3360	3360	3360	3360

Table 13: U.S. Metropolitan Statistical Areas (2010)

(Median income not adjusted for local non-housing cost of living)

	<b>Ownership Rate</b>	<b>Price-Rent ratio</b>	
		(price-asked)	(estimated-value)
<b>Log Median Income</b>	0.111 (0.038***)	12.549 (3.034***)	9.929 (1.964***)
<b>Gini index</b>	-0.413 (0.282)	102.333 (20.030***)	62.695 (12.259***)
<b>Age</b>	0.004 (0.002**)	-0.231 (0.140)	0.062 (0.083***)
<b>Log Density</b>	-0.034 (0.010***)	-4.442 (1.010***)	-2.784 (0.713***)
<b>Log Land Value</b>	-0.016 (0.009*)	4.043 (0.865***)	4.161 (0.613***)
<b>Property Tax Rate</b>	-0.008 (0.004**)	0.990 (0.398**)	0.825 (0.321***)
<b>Amenity Controls</b>	Yes	Yes	Yes
$R^2$	0.60	0.73	0.87
# obs	118	118	118

Notes: (1) Sources and definitions of variables may be found in the appendix.

(2) Heteroskedasticity-robust standard errors are provided in parenthesis.

(3) \*, \*\* and \*\*\* indicate statistical significance at the 10, 5, and 1% levels, respectively.

## C.1 The price-rent ratio of equivalent units

Figures 19 and 20 plot two different approaches to calculating price-rent ratios using empirical and simulated data. The price-rent ratio of equivalent units is plotted relative to the ratio of average prices to average rents. A positive correlation between the two price-rent ratio concepts is evident both empirically (Figure 19), and in our model simulations with variation in construction/land costs (Figure 20).

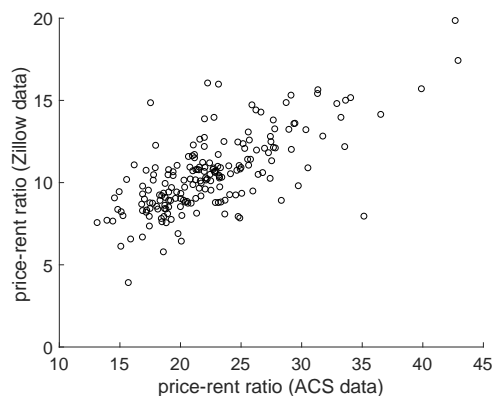


Figure 19: The two price-rent ratio concepts in the data: the price-rent ratio of the median quality house provided by Zillow and the ratio of average prices to average rents estimated from ACS data.

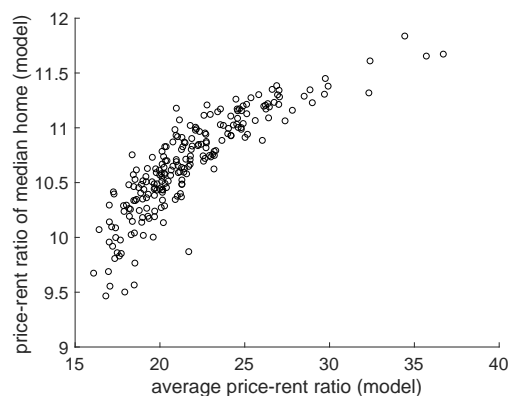


Figure 20: The two price-rent ratio concepts in the model: the price-rent ratio of the median quality house and the ratio of average prices to average rents.