

Equilibrium Underemployment*

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Abstract

This paper develops and calibrates a model of human capital investment in a frictional labor market with two-sided heterogeneity. The model generates underemployment in equilibrium: highly-educated workers are employed in jobs that do not require human capital to be productive. The decentralized equilibrium is never constrained efficient and exhibits an inefficient amount of human capital investment and underemployment. The model is calibrated to the U.S. economy to compare the decentralized and constrained-efficient allocations and to perform counterfactual policy experiments. The baseline calibration implies that the U.S. economy exhibits under-investment in human capital and an inefficiently high underemployment rate. Fully subsidizing education increases both human capital investment and the underemployment rate. The benefit of increasing investment in human capital outweighs the inefficiencies associated with a higher underemployment rate, leading to a net increase in welfare.

JEL Classification: E24, J24, J64, I22

Keywords: underemployment, human capital, education subsidies, student loans

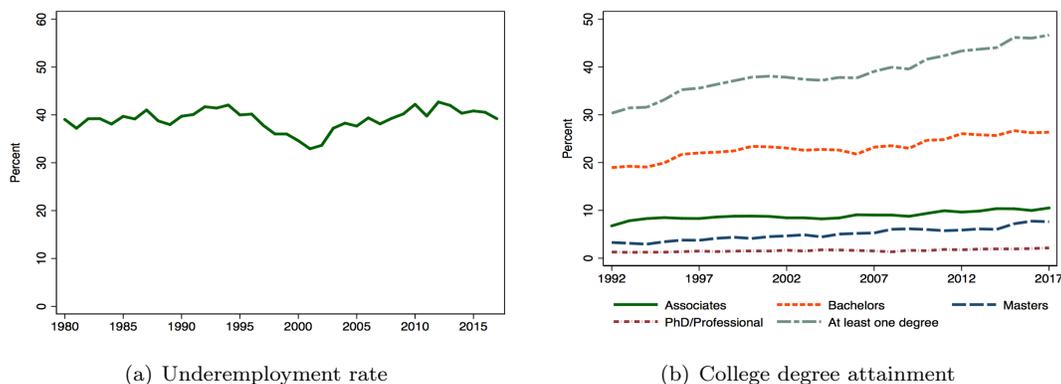
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1 Introduction

College graduates in the U.S. are frequently underemployed, i.e. working in occupations that do not require a college degree.¹ As seen in Figure 1, nearly 40% of recent graduates are underemployed.² Moreover, nearly 60% of underemployment durations last at least 1 year (Barnichon and Zylberberg, 2019) and 48% of those who begin their career underemployed remain so 10 years later (BGT and SI, 2018). Meanwhile, as seen in panel (b) of Figure 1, attainment of college degrees continues to expand and there has been an enduring priority to further increase their attainment via policy levers such as offering free tuition and relaxing student loan borrowing limits.³

Figure 1: Underemployment and college degree attainment



Notes: Data come from the Current Population Survey and O*NET survey. Panel (a) shows the fraction of recent graduates who are employed in non-college occupations. Each line in Panel (b) represents the percentage of 25-30 year olds whose highest degree completed is the corresponding degree. Section 2 provides details on definitions and calculations.

Underemployment is typically viewed as an inefficient outcome. The prominent example that exemplifies this sentiment is the story of ordering from a barista who has an advanced degree.⁴ Building on that logic, one may oppose the idea of subsidizing education as many graduates will ultimately spend portions of their career underemployed. This rationale, however, neglects that underemployment is an outcome that arises from (i) education choices based on factors such as the college earnings premium and the composition of job complexity and (ii) firms choosing their job’s complexity based on the supply of college educated workers. With this perspective, the positive impact of education policies on underemployment is not trivial, as the ultimate effect on the *underemployment rate* depends on the policy’s

¹This has been extensively discussed in the media. See, for example, recent articles in *The Washington Post*: “First jobs matter: Avoiding the underemployment trap” by Michelle Weise and “College students say they want a degree for a job. Are they getting what they want?” by Jeffrey J. Selingo.

²Abel et al. (2014); Barnichon and Zylberberg (2019); BGT and SI (2018) also find that nearly 40% of recent graduates are underemployed.

³Section 6 presents more details on trends in Federal student loans and grants.

⁴See, for example “Welcome to the Well-Educated-Barista Economy” by William A. Galston in the *Wall Street Journal*.

impact on educational attainment, the returns from education, and job creation decisions of firms. Additionally, the normative implications of such policies likely depends on whether the economy exhibits under-investment, over-investment, or the socially efficient level of human capital investment.

In this paper, I develop a model of equilibrium underemployment and study the model's implications for aggregate underemployment, job creation, the supply of human capital, and efficiency of equilibrium allocations. I then calibrate the model to quantify the effects of increasing education subsidies and student loan borrowing limits on underemployment, human capital investment, and welfare. My theory builds on previous literature that has emphasized two channels in studying the effects of education policy on aggregate outcomes. The first, emphasized by [Heckman et al. \(1998\)](#), [Lee \(2005\)](#), [Krueger and Ludwig \(2016\)](#), [Abbott et al. \(2018\)](#), and others, study competitive environments with an aggregate production technology that exhibits diminishing returns to labor. These frameworks highlight the effect increasing the supply of highly-educated workers on the returns to labor and human capital investment. The second, emphasized in [Shephard and Sidibé \(2019\)](#), considers an environment with search and matching frictions where firms choose their job's skill requirements based on the supply of education. In this setting, an increase in the supply of highly-educated workers causes shifts in the composition of job complexity to be directed to more skill intensive occupations. What has not been studied, to date, is a theory that accounts for both channels.

The model features a frictional labor market with two-sided heterogeneity. There are two types of jobs (simple and complex) and two education groups among workers (less- and highly-educated) which determines their capacity to be productive in complex jobs ([Albrecht and Vroman, 2002](#)). The labor market is unsegmented and, due to random matching, highly-educated workers meet simple jobs according to a Poisson process. If this meeting turns into a match, the worker forms a *cross-skill match* ([Albrecht and Vroman, 2002](#)), becomes underemployed, and searches on the job to eventually meet a complex job ([Dolado et al., 2009](#)). The decision to become underemployed is endogenous and is a function of two quantities. The first is the relative productivity of simple to complex jobs, which is made endogenous through a final goods technology as in [Acemoglu \(2001\)](#). The second is the worker's opportunity cost of giving up their job search that is determined by how much faster they expect to meet a complex job searching from unemployment than employment.

There are overlapping generations of workers who face a constant risk of death ([Blanchard, 1985](#)). Newborn workers are ex-ante heterogenous along two dimensions and make an extensive-margin human capital investment decision before entering the labor market. Workers differ in their innate ability, which affects their productivity and returns to human capital. They are also endowed with a technology to produce the final good that can be

used to finance human capital. The notion of differences in familiar wealth and transfers is introduced by assuming workers differ in their cost to produce the final good.

The set of equilibria contains pure- and mixed-strategy equilibria in the formation of cross-skill matches. Additionally, the model exhibits multiplicity of steady-state equilibria in some regions of the parameter space that is driven by two coordination problems. The first is a two-sided entry problem: workers choose how much human capital to accumulate whereas firms choose their vacancy's skill level. There is a complementarity between the supply of highly-educated workers and complex jobs: firms create more complex jobs if more workers invest in human capital, whereas more workers will invest in human capital if firms create more complex jobs. The second coordination problem is in the formation of cross-skill matches. Cross-skill matches are more likely to be formed if there are more simple jobs, as this reduces the worker's opportunity cost of giving up their job search, while firms will create more simple jobs if highly-educated workers are more likely match with them.⁵

The model can be used to study a wide variety of comparative statics on the *underemployment rate*, i.e. the fraction of employed, highly-educated workers who are employed in simple jobs. To build intuition, I study a simplified version of the model where output from the two sectors are perfect substitutes and there are no differences across workers in their innate ability. Increasing the productivity of complex jobs increases the expected profits of posting a complex vacancy and benefit of investing in human capital. As more workers invest in human capital, the vacancy filling rate of firms with complex vacancies increases which further incentivizes the creation of complex jobs. As more complex jobs are created, the underemployment rate decreases as highly-educated workers are less likely to match with simple jobs. I then relax the simplifying assumptions and numerically compute comparative statics. By a similar intuition, increasing the productivity of complex jobs causes more complex jobs to be created and increases the benefits of investing in human capital. However, with imperfect substitutability between sectors, the relative price of output produced in simple jobs to complex jobs increases as the supply of highly-educated workers increases. This increases the probability that highly-educated workers form cross-skill matches and ultimately increases the underemployment rate.

The decentralized equilibrium is never constrained efficient.⁶ There are several inefficiencies in human capital investment and the formation of cross-skill matches that give rise to this. The first is a hold-up problem where workers incur the full cost of human capital and earn a share of the returns (Acemoglu, 1996; Moen, 1998). The second is that workers do not internalize that the magnitude of thick market and congestion externalities they generate as a job seeker differs across education groups. For example, if the planner does not form

⁵This coordination problem is discussed in Albrecht and Vroman (2002) and also generates multiplicity of steady-state equilibria in their environment.

⁶Blázquez and Jansen (2008) find the same result in a similar environment but one where the supply of highly-educated workers is exogenous.

cross-skill matches and creates a majority of jobs that are simple, highly-educated workers create more congestion in the labor market relative to less-educated workers. Finally, the planner is typically choosier in the formation of cross-skill matches, as the planner considers the total expected forgone surplus of a match between a highly-educated worker and complex job when deciding to form a cross-skill match or not. Workers in the decentralized equilibrium, however, only consider their private share of the forgone surplus.

With these inefficiencies in hand, the focus of the paper narrows to compare the constrained efficient and decentralized allocations and to study the effects of education policies, changes to education subsidies and student loan borrowing limits, on underemployment and welfare. I begin by identifying three channels through which these policies affect the underemployment rate. The first is a supply channel where increasing the supply of highly-educated workers shifts the composition of unemployed workers towards highly-educated workers, causing firms to create more complex jobs. The second, the composition channel, occurs when there are shifts in the average innate ability within the pools of less- and highly-educated workers. To illustrate, suppose that only high-ability workers invest in human capital. Decreasing the price of human capital will increase the supply of complex jobs through the supply channel to a point where low-ability workers begin to invest in human capital. This decreases the average ability among the pool of highly-educated workers and reduces the creation of complex jobs. I show, however, that the supply channel always outweighs the composition channel. The final channel, the relative price channel, is active if the final goods technology is not linear: policies that increase the supply of highly-educated workers decrease the price of output produced in complex jobs, leading to a decline in the creation of complex jobs and higher underemployment. I show existence of cases where the relative price channel outweighs the supply channel and therefore where underemployment increases following an increase in the supply of highly-educated workers.

Having identified these mechanisms, I calibrate the model to the U.S. economy over the period 1992-2017 and compute the constrained efficient and decentralized allocations. The baseline calibration implies that workers under-invest in human capital and form cross-skill matches at an inefficiently high rate, leading to an inefficiently high underemployment rate. As workers under-invest in human capital, I use the model to study the effects of education policies which aim to increase the supply of highly-educated workers.

In the first experiment, I study the effects of fully subsidizing human capital through lump-sum taxes. I find that this policy increases the supply of highly-educated workers by 4.828%, which in turn increases the price of output produced in simple jobs relative to complex jobs. Through this change in the relative prices, the probability of forming a cross-skill match increases from 83.5% to 100% and thereby increases the underemployment rate from 26.5% to 30.0%. While the policy increases underemployment, it also increases welfare

by 1.171%.⁷ This result illustrates that while education subsidies can further increase the underemployment rate, which was already at an inefficiently high level, subsidies can improve welfare if the initial level of human capital investment is inefficiently low.

In a second experiment, I increase the maximum amount that workers can produce to be equal to the price of human capital so that there is no borrowing limit. This policy reduces the underemployment rate by 8.67%, reduces human capital investment by 2.78%, and reduces welfare by 0.749%. The reason for these effects is that the workers who are initially constrained are those with a relatively high innate ability. When they become unconstrained and invest in human capital, they crowd out the returns from human capital investment for all other workers, particularly those with a low innate ability, leading to a net decrease in human capital investment, the underemployment rate, and welfare. The quantitative effects of relaxing borrowing limits are relatively small due to the fact that less than 1% of workers are constrained by the initial borrowing limit.⁸

The rest of this paper is organized as follows. The remaining part of the introduction reviews the related literature. Section 2 provides details on measuring underemployment and presents an overview of underemployment in the data since 1980. Section 3 presents the environment. Section 4 derives and defines the set of equilibria. Section 5 derives the constrained efficient allocation. Section 6 studies, empirically and analytically, channels through which education policies affect underemployment. Section 7 presents the calibration and quantitative analysis. Finally, Section 8 concludes. Details on data sources and construction are in Appendix A while proofs and derivations are delegated to Appendix B.

1.1 Related literature

This paper contributes to several literatures. The first is search models with heterogeneity among workers and firms. [Albrecht and Vroman \(2002\)](#) developed a model of heterogeneous jobs and workers where highly-educated workers can end up working in low-skill jobs and characterize two equilibrium regimes: cross-skill matching and ex-post segmentation.⁹ [Dolado et al. \(2009\)](#) extend [Albrecht and Vroman \(2002\)](#)'s model by allowing underemployed workers to search on the job. [Barnichon and Zylberberg \(2019\)](#) develop a model with segmented labor markets and non-random matching in which high-skill workers are preferred to lower-skill competitors. Underemployment is generated in this model as high-skill workers can escape competition from their highly-skilled peers and can more easily find jobs for which they are over-qualified.¹⁰ This paper builds on these studies by endogenizing

⁷Welfare is measured as the economy's net output, i.e. production of the final good and home production from unemployed workers net of vacancy creation costs and costs incurred investing in human capital.

⁸This is consistent with previous work that has tested for, and found little evidence on the importance of borrowing constraints. See [Lochner and Monge-Naranjo \(2012\)](#) for a survey.

⁹See [Gautier \(2002\)](#) for a similar framework as [Albrecht and Vroman \(2002\)](#).

¹⁰Examples related models with segmented labor markets, directed search with heterogeneous workers and firms include [Shi \(2001, 2002\)](#), and [Shimer \(2005a\)](#). The models of [Shi \(2001, 2002\)](#) do not generate mismatch between highly-skilled workers

the supply of highly-educated workers through a human capital decision and the relative productivity of jobs through a final goods technology as [Acemoglu \(2001\)](#). This enriches a theory of underemployment in several ways. The first is that I am able to characterize when workers (i) invest in human capital and (ii) form cross-skill matches. The second is my model highlights additional inefficiencies that can lead to an inefficiently high or low amount of underemployment that is tied to workers' human capital decision. Finally, my theory illustrates the importance of allowing the productivity of jobs to respond to changes in the supply of highly-educated workers.

This paper is also related to models of human capital investment in frictional labor markets. [Acemoglu \(1996\)](#) and [Moen \(1998\)](#) study the hold-up problem that arises in these environments. [Moen \(1999\)](#) studied an environment where, due to a particular form for the matching technology, investing in human capital increased worker's job-finding rates. The models of [Charlot and Decreuse \(2005\)](#), [Flinn and Mullins \(2015\)](#), and [Macera and Tsujiyama \(2017\)](#) also have workers who are heterogenous in ability and invest in human capital before entering the labor market. Relative to these studies, this paper is the first to characterize the efficient allocation of human capital investment and composition of jobs in an unsegmented labor market where the productivity of jobs is endogenous.

The normative analysis in this paper is related to the literature which studies efficiency in frictional labor markets. The most relevant paper is [Blázquez and Jansen \(2008\)](#) who show that the decentralized equilibrium in the same environment as [Albrecht and Vroman \(2002\)](#) can never be efficient.¹¹ [Acemoglu \(2001\)](#) shows that in a model with heterogenous jobs and homogenous workers that there can be a composition in the bias of jobs towards low-wage jobs. [Charlot and Decreuse \(2005\)](#) show that due to the composition effects associated with low-ability workers investing in human capital that there can be over-education relative to what a social planner would choose.¹² This paper advances this literature by emphasizing a wedge between the social and private returns to human capital that results from workers not internalizing the net search externalities they generate by investing in human capital.

Finally, this paper contributes to the growing literature which studies the effects education policies on aggregate labor market outcomes. [Ji \(2018\)](#) studies student loan repayment plans and emphasizes how the structure of student loan repayment plans impacts the decision to accept low-wage jobs. [Shephard and Sidibé \(2019\)](#) study the effect of education subsidies and compulsory schooling on wage inequality and mismatch within a framework where the distribution of job complexity responds endogenously to the supply of education. This paper

and unskilled jobs in equilibrium, while [Shimer \(2005a\)](#) does.

¹¹They show that at the [Hosios \(1990\)](#) condition, that the total number of vacancies is efficient, but that there is a bias in the composition of jobs.

¹²More generally, the normative analysis is related to recent work by [Mangin and Julien \(2018\)](#) who study efficiency in economies where the productivity of matches depend on market tightness. This situation is relevant in my model as the output produced in jobs is a function of market tightness through the linkage generated by the final goods technology.

advances this literature by developing a framework that emphasizes both the job creation channel studied in [Shephard and Sidibé \(2019\)](#) and a relative price channel. Additionally, this paper connects the effects of education policies on welfare to inefficiencies identified in comparing the centralized and decentralized allocations.¹³

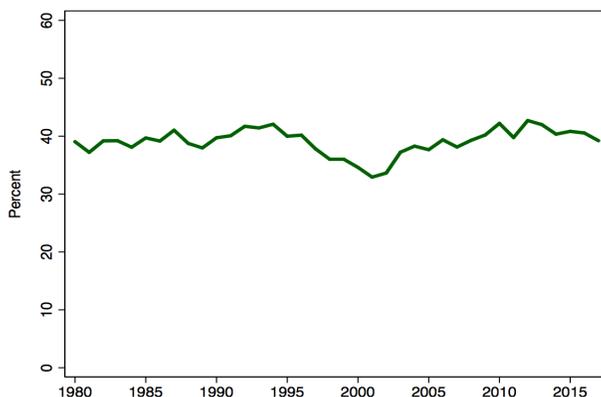
2 Underemployment in the data

This section presents the empirical definition of the aggregate underemployment rate. Section [2.2](#) illustrates differences in the underemployment rate across education and demographic groups while Section [2.3](#) summarizes evidence on the duration of underemployment.

2.1 Measuring underemployment

I define a recent graduate (ages 22-27) with at least a Bachelors degree to be underemployed if they work in an occupation that requires less than a Bachelors degree.¹⁴ An occupation is defined to require at least a Bachelors degree if at least 50% of respondents in the O*NET survey respond that a Bachelors degree or above is required to perform that occupation. Figure [2](#) presents the aggregate underemployment rate.

Figure 2: Underemployment among recent college graduates



Notes: Data come from the March Annual Social and Economic Supplement (ASEC) to the Current Population Survey (CPS), the U.S. Department of Labor’s Occupational Information Network (O*NET), and the Bureau of Labor Statistics (BLS). A college graduate is defined to be underemployed if they work in an occupation where less than 50% of respondents in the O*NET survey respond that a college degree is necessary to perform that occupation. The graph shows the fraction of recent graduates (ages 22-27) with a bachelors degree and above who are underemployed, where educational attainment comes from the ASEC. All calculations use the ASEC person weight.

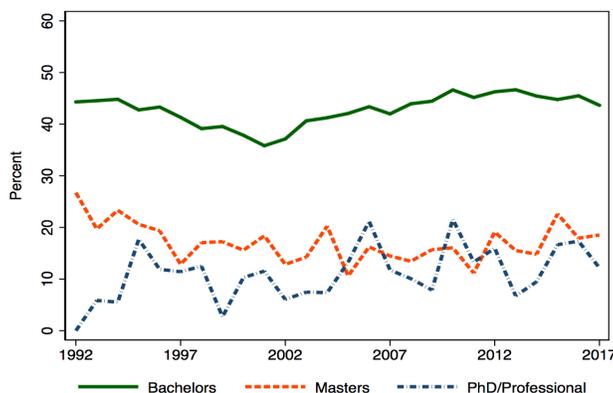
¹³[Ionescu \(2009\)](#), [Ionescu and Simpson \(2016\)](#) and [Abbott et al. \(2018\)](#) develop heterogenous agent life-cycle models to study the effects of various student loan policies on, among other outcomes, college enrollment, borrowing, and defaults on student loans. While these studies address many interesting questions, they do not analyze underemployment.

¹⁴This definition follows from [Abel et al. \(2014\)](#).

2.2 Heterogeneity in underemployment

As with the unemployment, the aggregate underemployment rate in Figure 2 masks differences across education and demographic groups. Starting with highest degree obtained, Figure 3 illustrates that the underemployment rate among those whose highest degree is a Bachelors is typically between 40-50%. As one may expect, the underemployment rate is substantially lower among those with a Masters, PhD, or Professional degree.

Figure 3: Underemployment by college degree



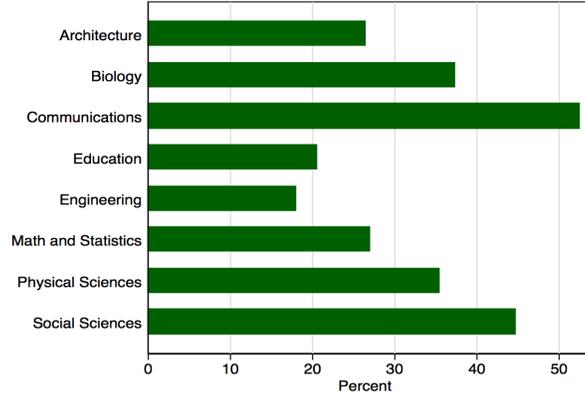
Notes: Data come from the March Annual Social and Economic Supplement (ASEC) to the Current Population Survey (CPS), the U.S. Department of Labor’s Occupational Information Network (O*NET), and the Bureau of Labor Statistics (BLS). All calculations use the ASEC person weight.

While Figure 3 shows that the underemployment rate among those with a Bachelors degree is much larger than those with advanced degrees, there can also be substantial heterogeneity among those with a Bachelors degree. Figure 4 illustrates this by showing the underemployment rate for several undergraduate majors. For these majors, the underemployment rate varies from 17.9% (Engineering) to 52.46% (Communications).¹⁵

With an overview of underemployment across several measures of education attainment, I proceed to present the underemployment rate across several demographic variables. Starting with panel (a) of Figure 5, one can see that the underemployment rate is decreasing in age. This is consistent with evidence that it takes time for young workers to find suitable matches early in their career and they may need to change employers several times to do so (Topel and Ward, 1992). Panel (b) presents the underemployment rate for different racial groups and shows that whites and asians are typically less likely to be underemployed. Finally, panel (c) shows that there has been relatively little differences in underemployment rates across females and males.

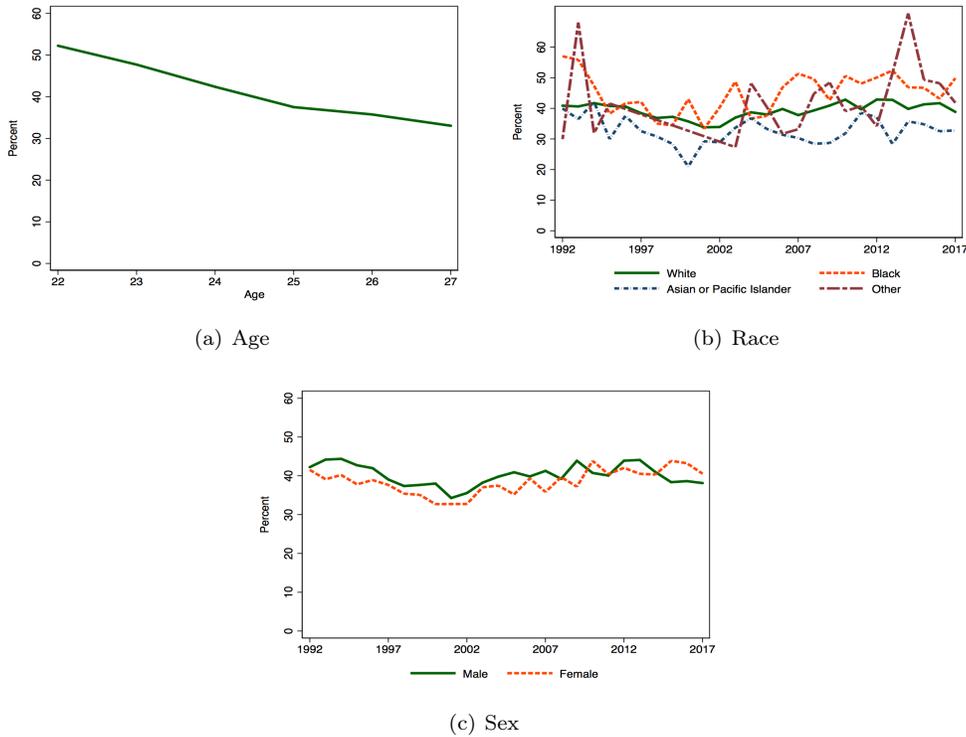
¹⁵The full list of underemployment rates by major for those available in the ACS is available by request.

Figure 4: Underemployment by undergraduate major



Notes: Data come from the American Community Survey (ACS). The graph shows the fraction of recent graduates (ages 22-27) with a bachelors degree and above who are underemployed between 2009-2016. Calculations use the ACS person weight.

Figure 5: Underemployment across demographic groups



Notes: Notes: Data come from the March Annual Social and Economic Supplement (ASEC) to the Current Population Survey (CPS), the U.S. Department of Labor’s Occupational Information Network (O*NET), and the Bureau of Labor Statistics (BLS). All calculations use the ASEC person weight.

2.3 The duration of underemployment

The degree to which underemployment is viewed as an inefficient outcome may rest on how transitory underemployment is as many college graduates may take a temporary job that they are overqualified for as they transition from college to a career in their field. A recent report by Burning Glass Technologies and the Strada Institute, [BGT and SI \(2018\)](#), sheds light on this. They find that (i) nearly 43% of college graduates begin their career underemployed, (ii) out of those initially underemployed, 67% are underemployed five years later, (iii) ten years after entering the labor market, 72% of the group that is underemployed after five years remain underemployed, and (iv) overall, 48% of those initially underemployed are still underemployed ten years after entering the labor market. Additionally, [Barnichon and Zylberberg \(2019\)](#) find that nearly 60% of workers who become underemployed are underemployed one year later.

3 Environment

Time is continuous and indexed by $t \in \mathbb{R}_+$. There are two types of agents: workers and intermediate-good firms. There are three goods: two intermediate goods and a final good. All agents are risk neutral, discount the future at rate $\rho > 0$, and only value consumption of the final good. The final good is taken as the numeraire. The lifetime discounted utility of a worker born at time t is given by

$$\mathbb{E} \int_t^{t+T} e^{-\rho(\tau-t)} c_\tau d\tau, \quad (1)$$

where c_τ is consumption of the numeraire and T is the worker's lifespan that is exponentially distributed with mean $1/\sigma$, i.e. workers die at Poisson rate σ .¹⁶ I define $r \equiv \rho + \sigma$ as the effective discount rate.

Over an infinitesimal time interval dt , a measure σdt of workers are born. Each flow of newborn workers at time t are comprised of different combinations of three characteristics: innate ability, human capital, and a technology to produce the numeraire. Workers draw their innate ability $a \in \{a_L, a_H\}$, where $a_H > a_L$ and a_L is drawn with probability π .¹⁷ There are two values of human capital denoted $h \in \{0, 1\}$.¹⁸ When workers are born, they are endowed with $h = 0$ and must make an irreversible decision of how much to invest in human capital. Following the human capital decision, workers enter the labor market as unemployed where they receive a flow utility ba while unemployed.

¹⁶This gives the feature of “perpetual youth” as in [Blanchard \(1985\)](#).

¹⁷Innate ability is innate characteristics, parental investments, and any other factors which affect the returns to human capital.

¹⁸Human capital represents a worker's stock of skills which determine their capacity to be productive in the labor market.

There are two costs to human capital: a pecuniary cost in terms of the numeraire, p_h , and psychic cost, ς . When workers are young, i.e. have not entered the labor market, they can produce ℓ units of the numeraire at cost

$$\varphi(\ell) = \begin{cases} \ell & \text{if } \ell \leq \underline{\ell}, \\ c(\ell) & \text{if } \underline{\ell} < \ell < \bar{\ell}, \end{cases} \quad (2)$$

where $c(\underline{\ell}) = \underline{\ell}$, $c'(\underline{\ell}) = 1$, $c'(\ell) > 0$, $c''(\ell) > 0$, and $c'(\bar{\ell}) = \infty$. The production cost is linear for $\ell \in [0, \underline{\ell}]$, where $\underline{\ell}$ is drawn from the cumulative distribution $F(\underline{\ell})$ over $[0, \infty)$ when the worker is born and represents familiar transfers/wealth that reduce their need to finance human capital through borrowing. The second component of $\varphi(\ell)$ that is strictly convex and approaches ∞ as $\ell \rightarrow \bar{\ell}$ is interpreted as borrowing costs incurred once the worker exceeds a production of $\underline{\ell}$. The value of $\bar{\ell}$ is common to all workers and is interpreted as a policy parameter that represents either relaxing ($\bar{\ell}$ increasing) or tightening ($\bar{\ell}$ decreasing) of borrowing limits. After workers enter the labor market, they can produce the numeraire at unit cost.

Intermediate-good firms are infinitely lived, comprised of one job that can be one of two types indexed by $\chi \in \{s, c\}$: simple (s) and complex (c), and incur a flow cost γ while searching for a worker.¹⁹ A worker and firm produce flow output $y_\chi(a, h)$ of intermediate goods as shown in Table 1:

Table 1: Production of intermediate goods

Human capital	Simple	Complex
0	$x_s a$	0
1	$x_s a$	$x_c a$

with $x_c > x_s$. Intermediate goods are sold to competitive firms who produce the final good according to

$$Y = [\mu(Y_s)^\epsilon + (1 - \mu)(Y_c)^\epsilon]^{\frac{1}{\epsilon}}, \quad (3)$$

where Y_χ is the aggregate output from type χ jobs, $\mu \in [0, 1]$ measures the relative importance of Y_s , and $1/[1 - \epsilon]$ is the elasticity of substitution between Y_s and Y_c .

The labor market is unsegmented. The flow of contacts between workers and vacancies is given by the aggregate meeting technology

$$\mathcal{M} = \mathcal{M}\left(\int_{i \in \mathbb{W}} e(i) di, v\right), \quad (4)$$

where \mathbb{W} is the set of all workers workers alive at a point in time, $e(i)$ is worker i 's search effort, and v is the measure of vacancies. Unemployed workers are endowed with 1 unit

¹⁹Hereafter, I refer to intermediate-good firms as "firms".

of search intensity whereas employed workers are endowed with $\lambda \in [0, 1]$ units of search intensity.²⁰ The meeting technology is continuous, strictly increasing and concave in each argument, and exhibits constant returns to scale. Defining $\Omega \equiv \int_{i \in \mathbb{W}} e(i) di$ as the aggregate search effort and $\theta \equiv v/\Omega$ as labor market tightness, firms meet workers at rate $q(\theta) = \mathcal{M}/v = \mathcal{M}(\theta^{-1}, 1)$. Unemployed workers meet firms at rate $f(\theta) = \mathcal{M}/\Omega = \mathcal{M}(1, \theta)$ and employed workers meet firms at rate $\lambda f(\theta)$. Upon meeting, the worker's ability and human capital are observable to the firm. Filled jobs are destroyed at rate δ .

4 Equilibrium

The description of the equilibrium is presented as follows. I start by defining the flow Bellman equations in Section 4.1. Section 4.2 then presents the optimal employment contracts that are bargained over in a meeting between a worker and firm. The two subsection sections, 4.3 and 4.4, describe the entry of firms and human capital investment among workers. Section 4.5 then characterizes the formation of cross-skill matches. The final equilibrium conditions that determine the distribution of workers across their states are presented in Section 4.6. Section 4.7 then defines a steady-state equilibrium and characterizes the set of equilibria. Finally, Section 4.8 presents comparative statics.

4.1 Bellman equations

The lifetime discounted utility of a worker when they are born, $W(a, \underline{\ell})$, solves:

$$W(a, \underline{\ell}) = \max_{c, \ell, h \in \{0, 1\}} \{c - \varphi(\ell; \underline{\ell}) + h[U(a, 1) - \varsigma] + (1 - h)U(a, 0)\}, \quad (5)$$

$$\text{s.t. } c + p_h h = \ell. \quad (6)$$

Workers choose their consumption, c , production of the numeraire, ℓ , and human capital, h , to maximize their lifetime discounted utility. The budget constraint shows that workers allocate their production of the numeraire between consumption and the pecuniary cost of human capital.

Let $\zeta = v_s/[v_s + v_c]$ denote the share of vacancies that are simple. The flow Bellman

²⁰If there is an infinitely small fixed cost to searching for a job, only unemployed and underemployed workers (those with $h = 1$ and employed at $\chi = s$ jobs) will search for jobs.

equations for workers in the labor market are given by

$$rU(a, h) = ba + f(\theta) \left\{ \zeta (\mathbb{I}_{h=0} + \mathbb{I}_{h=1} \max_{\kappa \in [0,1]} \kappa) [E_s(a, h) - U(a, h) - \phi_s^u(a, h)] \right. \\ \left. + (1 - \zeta) \mathbb{I}_{h=1} [E_c(a, h) - U(a, h) - \phi_c^u(a, h)] \right\}, \quad (7)$$

$$rE_\chi(a, h) = w_\chi(a, h) + \lambda f(\theta) (1 - \zeta) \mathbb{I}_{h=1, \chi=s} [E_c(a, h) - E_s(a, h) - \phi_c^e(a, h)] \\ + \delta [U(a, h) - E_\chi(a, h)], \quad (8)$$

where $\phi_\chi^{lf}(a, h)$ is a hiring fee paid by a worker who is hired from labor force status lf (unemployed or employed).²¹ Equation (7) shows that unemployed workers earn a flow utility ba and meet firms at rate $f(\theta)$. With probability ζ , they meet a simple vacancy. If they are highly-educated, they form a cross-skill match with probability κ .²² With probability $1 - \zeta$ they meet a complex vacancy and become employed if they are highly-educated. From (8), workers earn a wage $w_\chi(a, h)$, lose their job at rate δ , and meet a complex job at rate $\lambda f(\theta)(1 - \zeta)$.

Denoting $\psi = u/\Omega$ as the share of job seekers who are unemployed and η as the fraction of unemployed workers who are less-educated, the flow Bellman equations for firms are

$$rV_\chi = -\gamma + q(\theta) \left\{ \psi \left\{ \eta \mathbb{I}_{\chi=s} \mathbb{E}_{a|h=0} [J_\chi(a, 0) - V_\chi + \phi_\chi^u(a, 0)] \right. \right. \\ \left. \left. + (1 - \eta) \left(\mathbb{I}_{\chi=s} \max_{\kappa \in [0,1]} \kappa + \mathbb{I}_{\chi=c} \right) \mathbb{E}_{a|h=1} [J_\chi(a, 1) - V_\chi + \phi_\chi^u(a, 1)] \right\} \right. \\ \left. + (1 - \psi) \mathbb{I}_{\chi=c} \mathbb{E}_{a|h=1} [J_\chi(a, 1) - V_\chi + \phi_\chi^e(a, 1)] \right\}, \quad (9)$$

$$rJ_\chi(a, h) = p_\chi y_\chi(a, h) - w_\chi(a, h) + [\lambda f(\theta)(1 - \zeta) \mathbb{I}_{h=1, \chi=s} + \delta] [V_\chi - J_\chi(a, h)], \quad (10)$$

where p_χ is the price of output produced in type χ jobs. According to (9), firms pay a flow cost γ until they meet a worker at rate $q(\theta)$. With probability ψ , they meet an unemployed worker. Conditional on meeting an unemployed worker, they meet a less-educated worker with probability η , where $\mathbb{E}_{a|h}$ is the expected value with respect to innate ability within education group h . Firms initially meet an employed worker with probability $1 - \psi$ and form the match if they have a complex vacancy. Equation (10) shows that firms earn flow profits of the output net of the wage until either the job is destroyed or a highly-educated worker quits.

²¹The determination of the hiring fee is presented in Section 4.2.

²²Equation (7) assumes that the probability of forming a cross-skill match is independent of the worker's innate ability. Lemma 3 proves this. One can interpret κ as being chosen by the worker subject to a participation constraint for the firm, where the firm forms a match if and only if it generates a positive surplus.

4.2 Optimal employment contracts

In this section I show that, with no loss in generality, an employment contract can be reduced to a pair (w, ϕ) that specifies a wage paid to the worker by the firm and a one-time hiring fee paid by the worker to the firm.²³ To determine the employment optimal contract, let $S_\chi^u(a, h) = E_\chi(a, h) - U(a, h) + J_\chi(a, h) - V_\chi$ be the total surplus of a match between a firm and worker hired from unemployment. It follows that $S_\chi^u(a, h)$ solves

$$rS_s^u(a, h) = p_s y_s(a, h) - rU(a, h) - \delta S_s^u(a, h) + \lambda f(\theta)(1 - \zeta) \mathbb{I}_{h=1} [E_c(a, h) - E_s(a, h) - \phi_c^e(a, h) - (J_s(a, h) - V_s)], \quad (11)$$

Equation (11) has the following interpretation: a match at a simple job generates a flow surplus $p_s y_s(a, h) - rU(a, h)$ and the match is destroyed at rate δ . A highly-educated worker quits at rate $\lambda f(\theta)(1 - \zeta)$, gains the surplus $E_c(a, h) - E_s(a, h) - \phi_c^e(a, h)$, and the firm incurs the capital loss $J_s(a, h) - V_s$.

If the worker and firm could jointly decide when the worker quits, they would choose the opportunities for which (11) is maximized, which occurs if

$$E_c(a, h) - E_s(a, h) - \phi_c^e(a, h) \geq J_s(a, h) - V_s. \quad (12)$$

However, I assume that the decision to separate is non-contractable. It follows that when the worker makes the quit decision on their own, they will quit if the private net benefit from doing so is positive, i.e. if

$$E_c(a, h) - E_s(a, h) - \phi_c^e(a, h) \geq 0. \quad (13)$$

Comparing (12) and (13) shows that if $J_s(a, h) - V_s > 0$, the worker's private decision rule and the choice that maximizes the match surplus differs. That is, the match surplus is not maximized because workers do not internalize the negative externality that they impose on the incumbent firm when they quit. Only when $J_s(a, h) = V_s$ will the worker's decision to quit be aligned with the choice that maximizes the match surplus.

The worker and firm can reach a pairwise agreement over an employment contract that achieves efficient separations. The contract satisfies the following generalized Nash solution

²³Stevens (2004) shows that, in a model with on-the-job search, the bargaining set over a contract with only a flat wage may not achieve a pairwise Pareto-efficient outcome. This is because a worker does not account for the turnover costs paid by the firm following a quit. Also, the feasible set payoffs when bargaining over a flat wage may not be convex (Shimer, 2006). An employment contract that specifies a one-time hiring fee paid by the worker to the firm and a flat wage is Pareto-efficient as the hiring fee compensates a firm when they hire a worker who searches on the job and eventually quits. While this may seem empirically unrealistic, it is a simplified version of contracts where wages increase with tenure as in Pissarides (1994) and Burdett and Coles (2003).

where $\beta \in [0, 1]$ is the worker's bargaining power and $w_\chi(a, h)$ is the wage:

$$w_s(a, h), \phi_s^u(a, h) \in \arg \max [E_s(a, h) - U(a, h) - \phi_s^u(a, h)]^\beta [J_s(a, h) - V_s + \phi_s^u(a, h)]^{1-\beta}. \quad (14)$$

Lemma 1. *The employment contract as the solution to (14) is*

$$w_s(a, h) = p_s y_s(a, h), \quad (15)$$

$$\phi_s^u(a, h) = (1 - \beta)[E_s(a, h) - U(a, h)]. \quad (16)$$

Proof. See Appendix B.1. ■

The worker earns a wage that is equated with their marginal product so that they earn the entire flow surplus and fully internalize their decision to quit on the match surplus. The hiring fee is then used to split the total match surplus according to the agent's bargaining power.²⁴

4.3 Entry of firms

Firms post vacancies until the expected profits of doing so are equal to zero, i.e. $V_\chi = 0$ for $\chi \in \{s, c\}$. This gives the free-entry condition for type χ jobs:

$$\frac{\gamma}{q(\theta)} = (1 - \beta)\mathbb{E}[\psi S_\chi^u(a, h) + (1 - \psi)S_\chi^e(a, h)]. \quad (17)$$

The left side of (17) is the expected costs to meet a worker whereas the right side is the expected surplus from meeting a worker. The expected value is taken with respect to the heterogeneity within the pool of unemployed workers (less- and highly-educated) and differences in innate ability within education groups.²⁵

4.4 Human capital investment

A worker will invest in human capital if the benefits outweigh the opportunity costs, i.e. if

$$U(a, 1) - U(a, 0) \geq \varsigma + \varphi(p_h; \underline{\ell}). \quad (18)$$

Lemma 2 characterizes the worker's optimal choice of human capital investment.

²⁴There are a variety of matches for which the worker does not search on the job. It may seem unnecessary to specify the two-part contracts in these matches. However, in these matches, the employment contracts are payoff-equivalent to the Nash bargaining solution over a contract which only specifies a flat wage. For consistency, I allow the generalized Nash solution described above for all meetings between workers and firms. The solution to the optimal employment contracts in these other types of meetings is delegated to Appendix B.2.

²⁵A closed-form derivation of (17) is delegated to Appendix B.3.

Lemma 2. Define $\Gamma(a, \underline{\ell}) \equiv U(a, 1) - U(a, 0) - (\varsigma + \varphi(p_h; \underline{\ell}))$ as the net gain of investing in human capital and $\underline{\ell}^*(a)$ as the solution to $\Gamma(a, \underline{\ell}^*(a)) = 0$.

1. If $U(a, 1) - U(a, 0) > 0$, then $\partial\Gamma(a, \underline{\ell})/\partial a > 0$.
2. $\partial\underline{\ell}^*(a)/\partial a \leq 0$.
3. $\partial\Gamma(a, \underline{\ell})/\partial\underline{\ell} > 0$ if $\underline{\ell} < p_h$.

Proof. See Appendix B.4. ■

Equation (18) shows that workers choose $h = 1$ if the capital gain of doing outweighs the sum of the psychic and production costs. The reservation property in innate ability follows from the complementarity between the worker's ability and productivity of job, x_χ .²⁶ This reservation property leads to the next result which states that higher ability workers are willing to incur higher costs to invest in human capital. The third result shows that the capital gain is increasing in the worker's endowment if $\underline{\ell} < p_h$ as this reduces the costs the costs incurred in the strictly convex region of $\varphi(\ell)$.

The aggregate supply of highly-educated workers, H , is given by

$$H = \pi h(a_L) + (1 - \pi)h(a_H), \quad (19)$$

where $h(a)$ is the fraction within an ability group who invest in human capital and is given by

$$h(a) = \begin{cases} 0 & \text{if } \Gamma(a, p_h) < 0, \\ 1 - F(\underline{\ell}^*(a)) & \text{if } \Gamma(a, p_h) \geq 0. \end{cases} \quad (20)$$

Equation (20) illustrates that any worker, of ability a , who draws an endowment below the critical value, $\underline{\ell}^*(a)$, will not invest in human capital.

4.5 Cross-skill matches

I have assumed that the probability of forming a cross-skill match is independent of the worker's innate ability. Lemma 3 proves this and characterizes the optimal choice of κ .

Lemma 3. *The formation of cross-skill matches is independent of a . Moreover, $\kappa \in [0, 1]$ if*

$$\frac{p_s x_s - b}{p_c x_c - b} = \frac{\beta f(\theta)(1 - \zeta)(1 - \lambda)}{r + \delta + \beta f(\theta)(1 - \zeta)}. \quad (21)$$

Proof. See Appendix B.5. ■

²⁶See Table 1.

When deciding to form a cross-skill match, a worker and firm compare the relative productivities of simple and complex jobs to the opportunity cost of the worker giving up their job search. The relative productivities of the job, the right side of (3), is independent of the worker's ability as both their productivity and flow value of unemployment are scaled by their innate ability. This implies that what is important for determining the relative productivities of the jobs is the differences between the unique components of the output in a match, $p_\chi x_\chi$, and the flow value of unemployment, b . Both the frequency at which the worker meets vacancies, $f(\theta)$, and composition of vacancies determine the worker's opportunity cost.

4.6 Distribution of workers

The remaining equilibrium conditions are steady-state flow conditions that determine the distribution of workers across states:

$$\delta[1 - H - \eta u] = f(\theta)\zeta\eta u, \quad (22)$$

$$\delta[H - (1 - \eta)u] = f(\theta)(\zeta\kappa + 1 - \zeta)(1 - \eta)u, \quad (23)$$

$$f(\theta)(1 - \eta)u = [\delta + \lambda f(\theta)(1 - \zeta)][H - (1 - \eta)u]. \quad (24)$$

Equation (22) states the flow of less-educated workers from employment to unemployment is equal to the flow from unemployment to employment, where ηu is the measure of less-educated unemployed workers. Equation (23) is the same condition for highly-educated workers. Equation (24) states that the flow of workers into underemployment is equal to the separations and quits among underemployed workers.

With the steady-state conditions above, one can define the steady-state underemployment rate, i.e. the fraction of employed highly-educated workers in simple jobs. I denote the underemployment rate by \underline{u} and it is given by

$$\underline{u} = \frac{(\delta + \sigma)\zeta\kappa}{(\delta + \sigma + \lambda f(\theta)(1 - \zeta))(\zeta\kappa + 1 - \zeta)}, \quad (25)$$

which is decreasing in θ , as an increase in market tightness increases the flow out of underemployment through job-to-job transitions. The underemployment rate is increase in both ζ and κ , as both increase the flow into employment at simple jobs.

The aggregate unemployment rate, u , is given by

$$u = \frac{(\delta + \sigma)(1 - H)}{\delta + \sigma + f(\theta)\zeta} + \frac{(\delta + \sigma)H}{\delta + \sigma + f(\theta)(\zeta\kappa + 1 - \zeta)}, \quad (26)$$

where the first term on the right side of (26) is the measure of less-educated workers who

are unemployed and the second term is the measure of unemployed, highly-educated workers. Comparing equations (25) and (26) shows that both the underemployment and unemployment rates are decreasing in market tightness, θ , while an increase in $\zeta\kappa$ increases the underemployment rate while decreasing the unemployment rate.

4.7 Definition and characterization of equilibria

Definition 1. A steady-state equilibrium is a list of value functions $\{W(\cdot), U(\cdot), E_\chi(\cdot), V_\chi, J_\chi(\cdot)\}$ and prices p_χ for $\chi \in \{s, c\}$, aggregate supply of highly-educated workers H , the probability to form a cross-skill match κ , a vector $\{\theta, \zeta, \eta, \psi\}$, and distribution of workers across their states such that: The value functions satisfy (5)-(10), intermediate good prices are equated with marginal product,²⁷ the supply of highly-educated workers is given by (19), the probability to form a cross-skill match satisfies (21), and the vector $\{\theta, \zeta, \eta, \psi\}$ and distribution of workers satisfies the free-entry condition, (17) for $\chi \in \{s, c\}$, and steady-state conditions (22)-(24).

Proposition 1. *The following results describe the existence of steady-state equilibria.*

- (i) *Assume $b < \min\{\mu x_s, (1 - \mu)x_c\}$. An active steady-state equilibrium with $\theta > 0$ exists.*
- (ii) *If $\epsilon < 1$, then $\zeta \in (0, 1)$ and $H > 0$.*
- (iii) *If $\epsilon = 1$ and $p_h + \varsigma > \underline{\iota}$, where $\underline{\iota}$ is defined in Appendix B.6, then $\zeta = 1$ and $H = 0$.*

Proof. See Appendix B.6. ■

The first result in Proposition 1 shows that if the flow utility while unemployed is sufficiently small, then a positive measure of firms will create vacancies. The set of equilibria contains various combinations of human capital investment, job creation, and matching patterns within the labor market. Proposition 1 shows that if the final goods technology is not linear, then both types of jobs are created and a positive amount of workers invest in human capital. There can be equilibria where no workers invest in human capital, $H = 0$, and only simple jobs are created, $\zeta = 1$. A necessary condition for this to occur is that the final goods technology is linear. Workers may still find it optimal to invest in human capital if the cost to acquire human capital is relatively small. Proposition 1 shows that if the final goods technology is linear and the cost of human capital is sufficiently large, then no workers will invest in human capital and only simple jobs will be created.

Within equilibria with $H > 0$, there can be cross-skill matching equilibria with $\kappa \in (0, 1]$ and ex-post segmentation equilibria with $\kappa = 0$. Proposition 2 establishes that if employed workers are endowed with enough search intensity, then cross-skill matches will always be

²⁷It is straightforward to show that $p_s = \mu(Y_s)^{\epsilon-1}Y^{1-\epsilon}$ and $p_c = (1 - \mu)(Y_c)^{\epsilon-1}Y^{1-\epsilon}$.

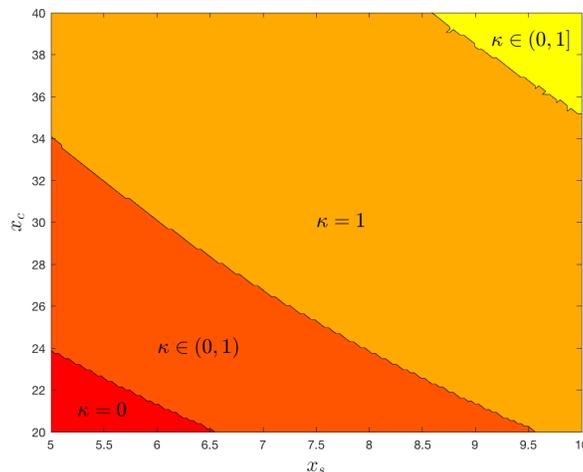
formed and the underemployment rate will be positive. This is because having a higher search intensity while in a cross-skill match increases the rate at which underemployed workers can meet complex jobs relative to unemployed workers, thus reducing the opportunity cost of forming a cross-skill match.²⁸ If however, the search intensity of underemployed works is low enough, and the productivity of complex jobs is large, then there is a large opportunity cost of forming a cross-skill matches, resulting in an ex-post segmentation equilibrium with no underemployment.

Proposition 2. *If $\lambda \geq \underline{\lambda}$, then $\underline{u} > 0$. If $\lambda < \underline{\lambda}$ and $x_c \geq \underline{x}_c$, then $\underline{u} = 0$.*

Proof. See Appendix B.7. ■

While an equilibrium with $\theta > 0$ typically exists, it is not always unique. This is illustrated in Figure 6 which shows the equilibrium regime in the (x_s, x_c) parameter space.²⁹ Starting on the left side of the figure, the equilibrium is a unique mixed-strategy equilibrium in the formation of cross-skill matches, i.e. $\kappa \in (0, 1)$. As x_s increases, the economy switches to a region with a unique pure-strategy equilibria in the formation of cross-skill matches, $\kappa = 1$. As x_s continues to increase, the economy enters a region of the parameter space that exhibits both a pure- and mixed-strategy equilibria in the formation of cross-skill matches, $\kappa \in (0, 1]$.

Figure 6: Topology of equilibria



Multiplicity arises from two coordination problems. The first is the complementarity between the firm’s entry decision and the worker’s human capital decision. If firms create more complex vacancies, then the value of investing in human capital is larger. Additionally, the expected profits of posting a complex vacancy are increasing in the supply of highly-educated workers. The second coordination problem is in the formation of cross-skill matches.

²⁸See Dolado et al. (2009) for a complete analysis of how search intensity in cross-skill matches affects the formation of cross-skill matches.

²⁹The parameter values used to construct Figure 6 are the same as in the calibration presented in Section 7, except I set $p_h = 0$ in the construction of Figure 6.

If highly-educated workers match with any job, then the composition of vacancies will shift towards simple jobs. If firms create more simple jobs, then cross-skill matches will be formed with a higher probability.

4.8 Comparative statics

The effects of a change in the model's parameters can (i) move the economy from a pure-strategy equilibrium, $\kappa \in \{0, 1\}$, to another pure-strategy equilibrium, (ii) cause the economy to shift from a pure-strategy equilibrium to a mixed-strategy equilibrium, or (iii) switch the economy from a mixed- to a pure-strategy equilibrium. To illustrate the model's key mechanisms, I study comparative statics within a pure-strategy cross-skill matching equilibrium. After studying a few cases analytically, I present numerical examples that allow for changes to the equilibrium regime.

The outcome of interest is the underemployment rate, \underline{u} . I first study comparative statics within a simplified version of the model. Specifically, I assume that the supply of highly educated workers is fixed at $H \in (0, 1)$, shut down search on the job, $\lambda = 0$, consider a final goods technology that is linear, $\epsilon = 1$, eliminate heterogeneity in the workers' innate ability, $a_L = a_H = 1$, and assume $\beta \approx 0$. I also assume parameter values are such that $\zeta \in (0, 1)$. From (25), in the case of a $\kappa = 1$ and $\lambda = 0$, the underemployment rate is simply given by ζ . Proposition 3 summarizes comparative statics on market tightness and the underemployment rate.

Proposition 3. *Assume that $H \in (0, 1)$ and is exogenous, $\lambda = 0$, $\epsilon = 1$, $a_L = a_H = 1$, $\beta \approx 0$, and the remaining parameters are such that $\kappa = 1$. Comparative statics are summarized in the table below.*

	μ	x_s	x_c	γ	H
θ	+	+	0	-	0
\underline{u}	+	+	-	+	-

Proof. See Appendix B.8. ■

An increase in the relative importance of simple jobs, μ , or the productivity of simple jobs, x_s , increases the expected profits of posting a simple job, causing the composition of vacancies to shift towards simple jobs and for the underemployment rate to increase. This also increases the outside option of highly-educated workers in meetings with complex vacancies, thereby reducing the expected profits of posting a complex vacancy. However, the increased supply of simple jobs outweighs the decrease in complex jobs to result in a larger value of market tightness. Increasing the productivity of complex jobs has the opposite effect: the expected profits of posting a complex (simple) vacancy increase (decrease), as highly-educated workers

have a larger outside option when bargaining with simple jobs. This causes the composition of vacancies to shift towards complex jobs and for the underemployment rate to decrease. The increase in complex vacancies and decrease in simple vacancies cancel each other out to leave market tightness unchanged.³⁰ If the vacancy flow cost increases, it becomes more costly for firms to fill a vacancy, reducing market tightness. The composition of vacancies shifts towards simple jobs and the underemployment rate increases because as firms with complex vacancies expect to incur the vacancy costs over a longer duration. An increase in the supply of highly-educated workers shifts the composition of unemployed workers towards highly-educated workers which increases the vacancy filling rate of complex vacancies. In this simplified case, market tightness is independent of the composition of unemployed workers but due to the increased vacancy filling rate, the composition of vacancies shifts towards complex jobs and the underemployment rate decreases.

In the next set of comparative statics, I allow for H to be endogenous and consider comparative statics with respect to the same parameters in Proposition 3 in addition to the effects of changes to the cost of human capital. Proposition 4 summarizes the results.

Proposition 4. *Assume that $\lambda = 0$, $\epsilon = 1$, $a_L = a_H = 1$, $\beta \approx 0$, and the remaining parameters are such that $\kappa = 1$. Comparative statics are summarized in the table below.*

	μ	x_s	x_c	γ	p_h	ζ
θ	+	+	0	-	0	0
\underline{u}	+/-	+/-	-	+	+	+
H	+/-	+/-	+	-	-	-

Proof. See Appendix B.9. ■

With the supply of human capital endogenous, an increase in either μ or x_s causes market tightness to increase and has ambiguous effects on the the underemployment rate and supply of highly-educated workers. This is because, as discussed in Proposition 3, an increase in μ or x_s causes ζ to increase. However, an increase in market tightness (simple jobs) increases (decreases) the benefits of investing in human capital. If the increased supply of simple jobs outweighs the effect of a higher market tightness on the benefits of human capital, then the supply of highly-educated workers will decrease. Alternatively, if the market tightness effect dominates, then the supply of highly-educated workers will increase which causes the underemployment rate, ζ , to decrease.

³⁰In order for both jobs to be created, the effective productivity of the two jobs has to be equalized. As $x_c > x_s$, the effective productivities are equalized when accounting for the fact that it is more difficult to fill complex vacancies. Changes to x_c or H affect the effective productivity of complex jobs. However, since the effective productivity of complex jobs must be equal to that of simple jobs, a change in x_c or H is accounted for by a shift in the composition of jobs. Market tightness is independent of changes to x_c or H because there is no change to the effective productivities of jobs after accounting for a shift in the composition of vacancies to equalize the productivities of the jobs.

An increase in x_c increases both the benefits of investing in human capital and posting a complex vacancy, causing the underemployment rate to decrease and for the supply of highly-educated workers to increase. If the vacancy flow cost increases, market tightness will decrease and cause more simple jobs to be created and the underemployment rate to increase, as in Proposition 3, which decreases the benefits of human capital. Finally, as market tightness is still independent of the composition of jobs seekers, changes to p_h or ς have no effect on θ . An increase to either p_h or ς reduces the net benefit of human capital. As H decreases, the composition of unemployed workers shifts towards less-educated workers and increases the vacancy filling probability of firms with a simple job, causing the underemployment rate to increase.

With the mechanisms in hand from these simplified cases, I proceed to demonstrate a few numerical examples that relax the simplifying assumptions made in the previous examples.³¹ As mentioned above, I also allow for all types of equilibria, $\kappa \in [0, 1]$. To understand the effects of changes to parameters on the underemployment rate, it is helpful to present the effects on market tightness, θ , and the prices of intermediate goods, p_χ .

Figure 7 shows the effects of changes to the productivity in complex jobs, x_c . As x_c increases, more firms post vacancies and more workers invest in human capital. As the supply of highly-educated workers increases, the composition of vacancies shifts towards complex jobs and the ratio p_s/p_c increases. Through the changes to the intermediate-good prices, an increase in x_c causes the probability of forming a cross-skill match and the underemployment rate to increase. This result differs from the previous analytical results, where an increase in x_c caused a reduction in the underemployment rate, and is driven by the endogenous response of the relative prices, p_χ . This channel was shut down in the analytical cases by assuming a linear final goods technology.

Consider the effects of changes to the relative importance of simple jobs, μ . Figure 8 shows increasing μ can cause a decrease in market tightness. This differs from previous results because market tightness is no longer independent of the composition of unemployed workers when workers are heterogenous in their innate ability and the final goods technology is not linear. Increasing μ decreases the benefit of investing in human capital, causing H to decrease, and for the composition of vacancies to shift towards simple jobs. The effects on the probability of forming a cross-skill match, κ are non-monotonic as well. This is because, as μ increases, the increased price of output produced in complex jobs outweighs the effects of an increase in ζ on the worker's opportunity cost of giving up their job search, causing κ to decrease. Eventually, as μ increases, the increase in ζ outweighs the effects of changes to p_χ and causes κ to increase. For most of the parameter space, an increase in ζ and decrease in θ cause the underemployment rate to increase.

³¹Numerical examples not presented in this section are available upon request. The parameter values used to construct these examples are the same as those in Table 4 with the exception of $x_s = 5$, $x_c = 20$, and $p_h = 0$ in the numerical examples.

Figure 7: Comparative statics with respect to x_c

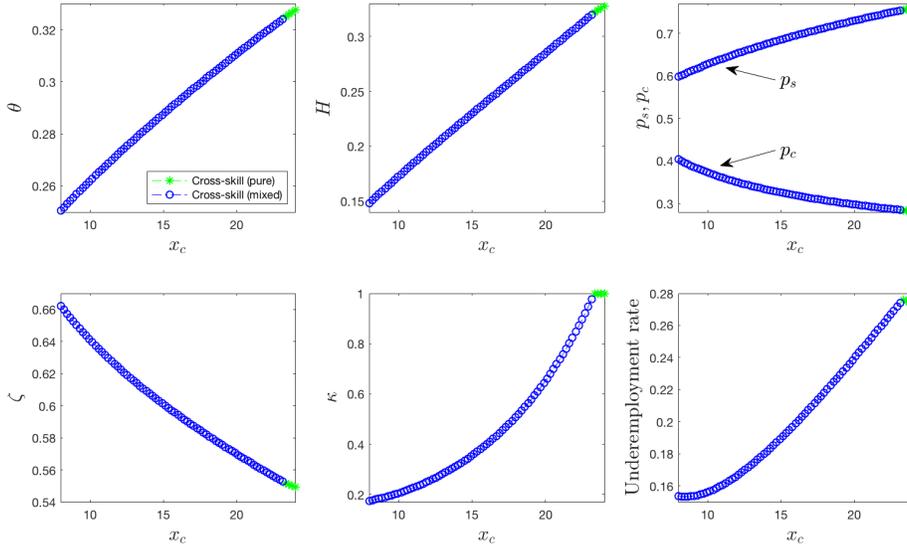
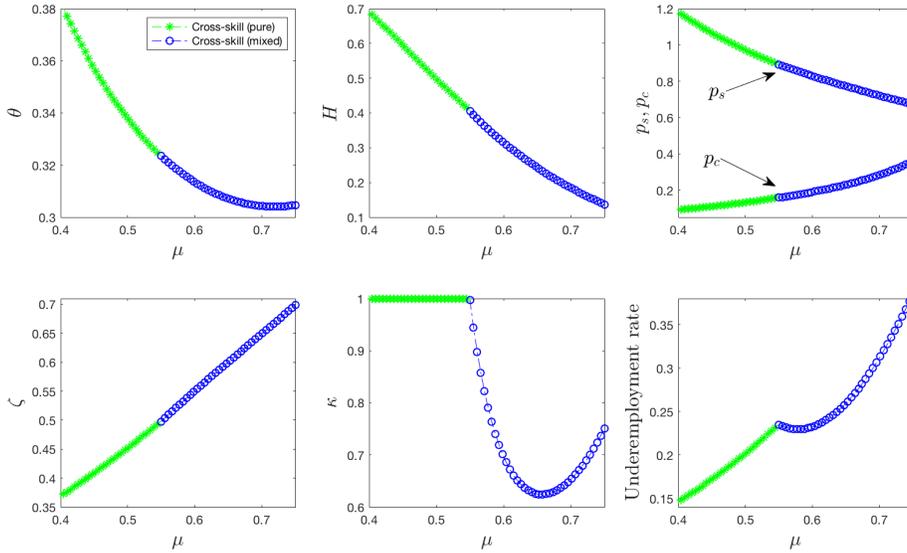
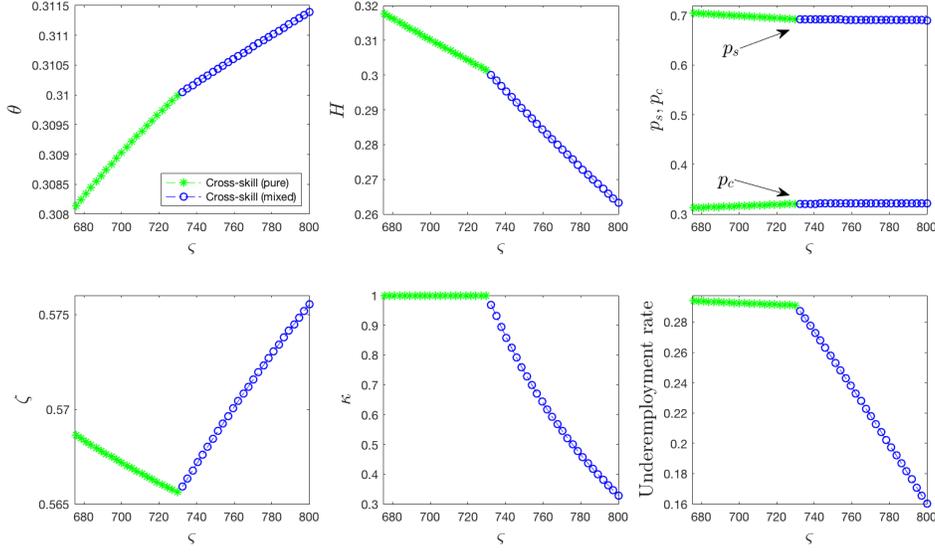


Figure 8: Comparative statics with respect to μ



The last example that I present is the effects of changes to the psychic cost of education, ζ . As seen in Figure 9, increasing ζ causes the supply of highly-educated workers to decrease. As H decreases, the composition of vacancies shifts towards simple jobs. The increase in θ outweighs the effect of an increase in ζ on the opportunity cost of forming a cross-skill match and eventually causes κ to decrease. The bottom right panel shows that the decline in κ outweighs the increase in ζ , ultimately causing the underemployment rate to decrease.

Figure 9: Comparative statics with respect to ς



5 Planner's problem

Consider a social planner whose objective is to maximize society's net output subject to the search frictions that agents face in a decentralized equilibrium. The planner chooses the amount of simple and complex vacancies to open, v_s and v_c , whether a worker endowed with the pair $(a_i, \underline{\ell})$ should invest in human capital, $h(a_i, \underline{\ell}) \in [0, 1]$, and the fraction of meetings between highly-educated workers of ability a_i and simple jobs that should become matches, $\kappa_i \in [0, 1]$. The planner's objective function is given by

$$\max_{\{v_\chi, h(a_i, \underline{\ell}), \kappa_i\}} \int_0^\infty e^{-\rho t} \left[Y + \sum_i \sum_h u_i^h b a_i - \gamma(v_s + v_c) - \sigma \sum_i \pi_i \int_0^\infty h(a_i, \underline{\ell}) [\varsigma + \varphi(p_h; \underline{\ell})] dF(\underline{\ell}) \right] dt, \quad (27)$$

for $\chi \in \{s, c\}$, $i \in \{L, H\}$, $h \in \{0, 1\}$, and where u_i^h is the measure of unemployed workers with human capital h and ability a_i . From (27), the planner maximizes production of the final good, Y , and home production from unemployment net of vacancy costs and costs incurred to produce highly-educated workers. The planner maximizes (27) subject to the laws of motion of workers across the states of employment and unemployment. Proposition 5 compares the decentralized and efficient steady-states under the simplifying assumptions that there is no search on the job and the final goods technology is linear.

Proposition 5. *Suppose that $\lambda = 0$ and $\epsilon = 1$. A decentralized steady-state equilibrium never coincides with the efficient steady-state.*

Proof. See Appendix B.10. ■

There are several inefficiencies in each of the agents’s key decisions (human capital investment, vacancy creation, and formation of cross-skill matches) which lead to the result shown in Proposition 5. The first is a hold-up problem in the worker’s human capital investment decision.³² Workers only obtain a share β of the total returns associated with investing in human capital due to ex-post surplus sharing with firms. Thus, there is a share, $1 - \beta$, of the total gains from accumulating human capital that workers do not internalize when they make their investment decision. This can be seen by comparing equation (B.12), a private agent’s benefit of human capital, to equation (B.49), the social benefit of investing in human capital, as the private agent’s benefits to human capital are scaled by their share of the match surplus, β .

A second inefficiency relates to the thick market and congestion externalities generated by job seekers in frictional labor markets. Job seekers produce congestion externalities as an additional job seeker reduces the job-finding rate of all other job seekers while the thick market externality arises as job seekers increase firms’ vacancy filling rate. In a model of homogenous workers, these externalities cancel each other out when the Hosios (1990) condition holds. As shown in Blázquez and Jansen (2008), this is not true in an unsegmented labor market with heterogenous workers as the search externalities generated by a job seeker differ across education groups. When cross-skill matches are formed, highly-educated workers improve the vacancy filling rates of both simple and complex vacancies, whereas less-educated workers only improve the vacancy filling rate of simple vacancies. This can be seen in the social benefits of human capital investment, (B.49), by a term that I define as the *net thick market externality*, Θ , where

$$\Theta \equiv \frac{f(\theta)(1 - \nu)\Psi[1 + \zeta(\kappa_i - 2)]}{(r + \delta + f(\theta)(1 - \zeta + \zeta\kappa_i))(r + \delta + f(\theta)\zeta)}, \quad (28)$$

where ν is the elasticity of the meeting technology with respect to job seekers and Ψ is the average value of a match.³³ It is straitforward to see that $\Theta > 0$ if $\kappa_i = 1$ or $\zeta < 1/2$, i.e. if it is relatively easy for the planner to form matches with highly-educated workers.

There are additional differences between the decentralized equilibrium and efficient steady-state in the conditions that govern the formation of cross-skill matches. The first difference is similar to a hold-up problem: workers in the decentralized equilibrium weigh the benefits of accepting a simple job offer against the opportunity cost of giving up their job search. The opportunity cost is given by the right hand side of equation (21) and is scaled by the worker’s bargaining power, β . The opportunity cost to the planner, however, is not scaled by

³²See Acemoglu (1996) and Moen (1998) for earlier discussions of hold-up problems in human capital investment.

³³See Appendix B.10 for a formal definition of Ψ .

β as the planner considers the total expected surplus that is forgone by forming a cross-skill match.

A second difference between the decentralized and centralized solutions in the formation of cross-skill matches is that the planner accounts for the fact that forming a cross-skill match reduces the congestion faced by other unemployed workers. This is seen by the term $(1 - \nu)f(\theta)\Psi$ in (B.42). It is due to this that the rate at which the planner forms cross-skill matches is a function of the worker’s ability, whereas the formation of cross-skill matches was independent of the worker’s ability in the decentralized equilibrium. From (B.42), the benefit from reducing congestion by forming cross-skill matches is larger for low-ability workers. The intuition for this is simple: the planner forms cross-skill matches among low-ability workers at a higher rate because this reduces congestion and allows for highly-educated, high-ability workers.

6 Education Policy

6.1 Background and empirical evidence

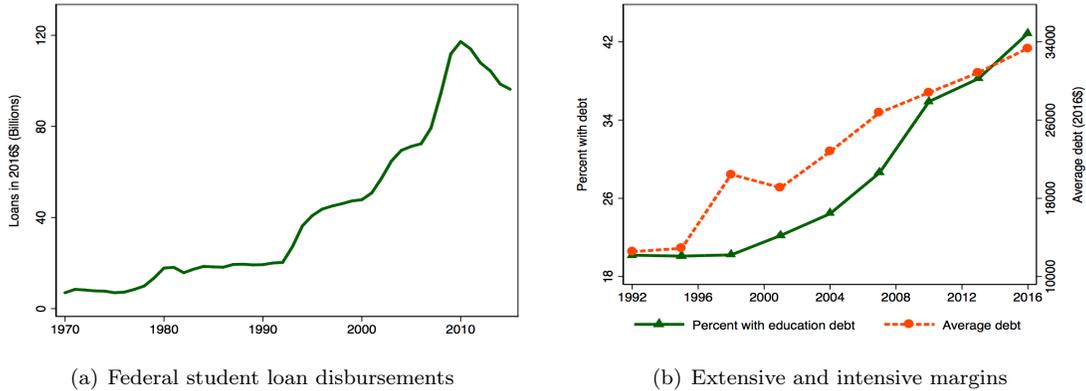
One of the most striking developments in the attainment college degrees is the use of student loans. In fact, borrowing to finance college has increased to the point where student debt is the second largest type of consumer debt behind only mortgage debt (FRBNY, 2018). To illustrate, panel (a) in Figure 10 shows that federal student loan disbursements have been increasing since the 1970s and that the pace of disbursements increased in the early 1990s and continued until 2010. Panel (b) shows how the extensive and intensive margins of borrowing have evolved since 1992. The solid line (left axis) shows that the percentage of U.S. households with education debt increased from 20% to 43% while the dashed line (right axis) shows that, among those with a positive amount of education debt, the average amount borrowed increased by nearly \$20,000.

I focus on Federal student loans because they made up 87% of all student loan disbursements between 1995 and 2015 (College Board, 2017). The Federal student loan program offers Stafford, Perkins, and PLUS/GradPLUS loans. I focus on Stafford Loans as they made up an average of 87% of Federal loan disbursements between the 1992-93 and 2015-16 academic years (College Board, 2017).³⁴ There are two types of Stafford loans: subsidized and unsubsidized. Interest accrues on unsubsidized loans while enrolled in school, whereas it does not on subsidized loans. Both undergraduates and graduate students can obtain unsubsidized loans without demonstrating financial need, while eligibility for subsidized loans is restricted to undergraduates who demonstrate financial need.³⁵

³⁴For more details on the other types of loans available through the Federal student loan program, see Lochner and Monge-Naranjo (2016).

³⁵Factors determining eligibility for subsidized loans include dependency status, family income, and cost of the institution

Figure 10: Trends in student loans



Notes: The data in panel (a) come from [College Board \(2017\)](#) and shows the total amount of Federal student loans disbursed in an academic year. Data in panel (b) comes from the Survey of Consumer Finances (SCF). The solid line (left axis) shows the percentage of respondents who report having a positive amount of education debt. The dashed line (right axis) shows, among those with a positive amount of education debt, the average amount of education debt. Calculations only include households where the head of the household is between 20-40 years old. All calculations use SCF weights where, per recommendation of the Federal Reserve Board, the weights are divided by 5 before performing calculations.

Stafford loans have both annual and cumulative borrowing limits that are determined by the student’s dependency status and year in school. The borrowing limits are set by congress and are fixed in-between policy changes. Table 2 shows the cumulative limit based on a student’s dependency status and their loan type and also illustrates that the only change to the cumulative limits in 2008 increased the borrowing limit for dependent (independent) students by 34.7% (25%).³⁶

Table 2: Stafford loan cumulative borrowing limits

	Dependent			Independent		
	Subsidized	Unsubsidized	Combined	Subsidized	Unsubsidized	Combined
1993-2008	23,000	23,000	23,000	23,000	46,000	46,000
2008-09 and after	23,000	31,000	31,000	23,000	57,500	57,500

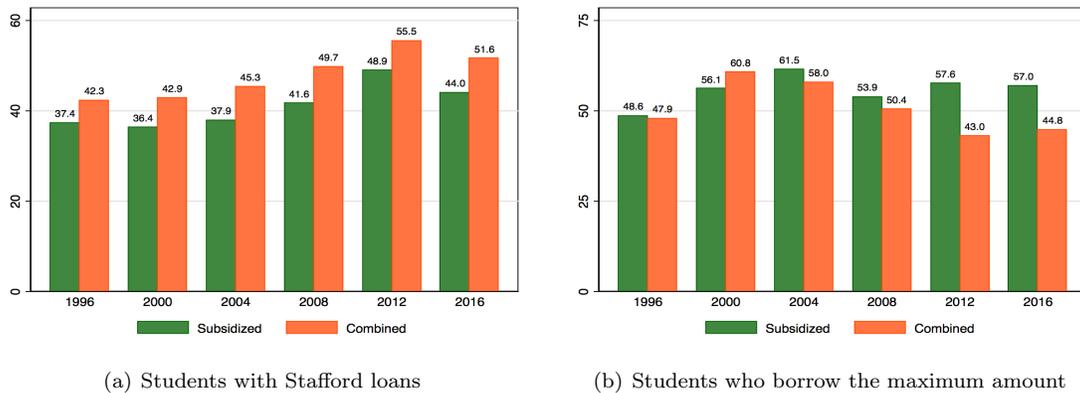
Notes: Each column shows the maximum amount that a student can borrow based on their year dependency status and loan type. Students whose parents do not qualify for PLUS loans are eligible to borrow up to the limit for independent students. Prior to 1993, independent students and some dependent students could borrow from the Supplemental Student Loan for Students (SLS) program.

attended. Students under age 24 are considered to be dependent.

³⁶The borrowing limits in Table 2 are often referred to as “program limits” as opposed to “individual limits”. An individual limit specifies that a student may not borrow more than their student budget (total price of attendance) or financial need (student budget net expected family contribution). A student is therefore constrained by the individual limit if it is less than the program limit. See Table 2 of [Wei and Skomsvold \(2011\)](#) for borrowing limits by year in school.

To provide some context for the relevance of these limits, panel (a) in Figure 11 shows that nearly 50% of undergraduates use Stafford loans.³⁷ Panel (b) shows that, among those who use Stafford loans, approximately 50% borrow the maximum that they are eligible for.³⁸ These borrowing limits are also relevant because students who hit the maximum are more likely to have to take out private student loans which often have higher interest rates and less flexible repayment options (Wei and Skomsvold, 2011).

Figure 11: The use of Stafford loans



Notes: Data come from the National Postsecondary Student Aid Survey (NPSAS). Panel (a) shows the fraction of undergraduates who borrow a positive amount of Stafford loans. Panel (b) shows the percentage of undergraduates who borrowed the “usual maximum” amount of Stafford loans. Sample include students who were enrolled full-time, full-year. Percentages are calculated using the NPSAS weights.

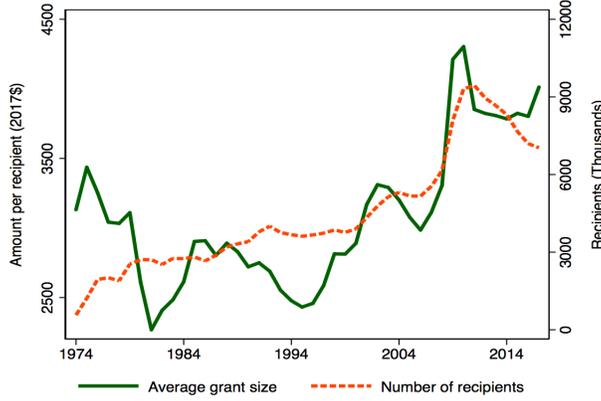
Another education policy that has been extensively discussed are subsidies/grants. While most of the discussion and debate in recent years has centered around whether college should be fully subsidized, Federal pell grants have increasingly been used a policy tool since the mid 1990s. The solid line (left axis) in Figure 12 shows that, between 1994 and 2017, the average grant amount per recipient increased from nearly \$2500 to \$4000, a 60% increase. The dashed line (right axis) shows that the number of grant recipients steadily has steadily increased.

To more formally test for whether higher education policy is useful for predicting the underemployment rate, I perform a VAR analysis and subsequently perform tests for Granger causality. Consider the five variable VAR:

³⁷See Figure 14 in Appendix A for the fraction of students who borrow Stafford loans and borrow the maximum by dependency status.

³⁸As for before 1996, Berkner (2000) found that 17.8% of full-time, full-year undergraduates borrowed the maximum combined amount of Stafford loans in the 1989-90 academic year.

Figure 12: Trends in Federal pell grants



Notes: Data come from [College Board \(2017\)](#). The left axis (solid line) shows the average grant amount per recipient in 2017\$. The right axis (dashed line) shows the amount of grant recipients per academic year (measured in thousands).

$$\begin{bmatrix} \Delta BA_t \\ \Delta \text{Underemp}_t \\ \Delta \text{Disburse}_t \\ \Delta \text{Recipients}_t \\ \Delta \text{Grant}_t \end{bmatrix} = \beta_0 + \beta_1 \begin{bmatrix} \Delta BA_{t-1} \\ \Delta \text{Underemp}_{t-1} \\ \Delta \text{Disburse}_{t-1} \\ \Delta \text{Recipients}_{t-1} \\ \Delta \text{Grant}_{t-1} \end{bmatrix} + \dots + \beta_k \begin{bmatrix} \Delta BA_{t-k} \\ \Delta \text{Underemp}_{t-k} \\ \Delta \text{Disburse}_{t-k} \\ \Delta \text{Recipients}_{t-k} \\ \Delta \text{Grant}_{t-k} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \end{bmatrix}, \quad (29)$$

where BA_t is the fraction of 25-30 year olds in year t who have at least a Bachelors degree, Underemp_t is the fraction of underemployed college graduates in year t who work in occupations with an average annual salary below \$25,000, Disburse_t is the total amount of Federal student loans disbursed in the academic year $t - t + 1$, Recipients_t is the number of recipients of a Federal Pell Grant in the academic year $t - t + 1$, and Grant_t is the average per-capita Federal Pell grant award in the academic year $t - t + 1$, β_0 is a matrix of intercept terms, and β_k is a matrix of coefficients for $t \in \{1, \dots, k\}$. I estimate (29) with $k = 3$ (per the Akaike Information Criterion) and using data between 1974 and 2015.³⁹

Using the estimates of (29), I test the null hypothesis that Federal loan disbursements, number of grant recipients, and per-capita grant awarded do not Granger-cause the underemployment rate. I find that the amount of Federal loan disbursements and per-capita grant amount Granger cause the underemployment rate at the 1% significance level and fail to reject the null hypothesis that the number of grant recipients Granger causes the underemployment rate.⁴⁰ These results indicate that changes in the amount of Federal loans

³⁹These results are available upon request.

⁴⁰See Table 9 in Appendix A.3 for test statistics generated by the Wald tests of joint significance.

disbursed and changes to average grant sizes are useful for predicting future changes to the underemployment rate.

6.2 Analytical channels

With an overview of developments in higher-education policy in recent decades and evidence of a connection between education policy and underemployment, I return to the model to isolate the channels through which changes to education policy affect the equilibrium underemployment rate. These channels can be illustrated through studying comparative statics with respect to the pecuniary cost of human capital, p_h . I proceed by outlining the intuition behind these channels and summarize the formal results in Proposition 6.

Suppose that $\epsilon = 1$ and p_h decreases. This causes the net benefit of investing in human capital to increase and for more workers to invest in human capital. As the supply of highly-educated workers increases, the composition of unemployed workers shifts from less- to highly-educated workers. From equation (9), as η decreases, the expected profits of posting a complex vacancy increase. When more complex vacancies are created, highly-educated workers have a higher opportunity cost of forming a cross-skill match. Thus, the *supply channel* induces more complex jobs to be created and for highly-educated workers to become less-likely to form a cross-skill match.

Recall, from Lemma 2, that the net benefit of investing in human capital is increasing in the worker's innate ability. Consider an equilibrium in which only high ability workers invest in human capital. As p_h declines and the composition of vacancies shifts towards complex jobs through the supply channel, the net benefit of investing in human capital will increase. Eventually, low-ability workers will find it beneficial to invest in human capital. When low-ability workers invest, the average innate ability within highly-educated workers decreases which, from (9), decreases the expected profits of posting a complex vacancy. It follows that the *composition channel* induces less complex jobs to be created, the opposite effect of the supply channel.

Despite their competing effects on the expected profits of posting a complex vacancy, it can be shown that the supply channel outweighs the composition channel. The intuition for this is the fact that when low-ability workers invest in human capital, they enter a group of highly-educated workers which already contains high-ability workers, which diminishes the impact of low-ability workers on the average ability within highly-educated workers.

Now suppose that $a_L = a_H = 1$, which shuts down the composition channel, and there is curvature in the final goods technology, i.e. $\epsilon < 1$. As p_h decreases and more workers to invest in human capital, the price of output produced in complex jobs decreases, as there are diminishing returns to production of the final good, which reduces the expected profits of posting a complex vacancy. Thus, the *relative price channel* causes the composition of jobs

to shift towards simple vacancies, the opposite effect of the supply channel. Proposition 6 shows that as there is a stronger complementarity between output from simple and complex jobs, that the effect of the relative price channel can outweigh the effect of the supply channel on the expected profits of a complex job.

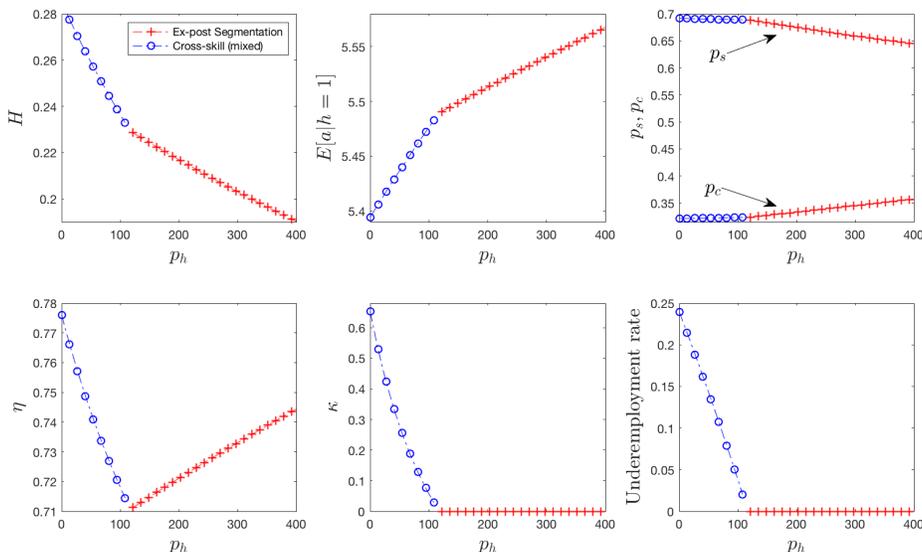
Proposition 6. *The following cases summarize the results mentioned above:*

1. Suppose that $\epsilon = 1$. The effect of an increase in the supply of highly-educated workers on the expected profits of posting a complex vacancy outweighs the effects of changes to the average innate ability within highly-educated workers.
2. Suppose that $a_L = a_H = 1$ and $\epsilon < 1$. As $\epsilon \rightarrow -\infty$, the effect of an increase in the supply of highly-educated workers on the expected profits of posting a complex vacancy through the relative price channel outweighs the effect of the supply channel.

Proof. See Appendix B.11. ■

Figure 13 illustrates comparative statics with respect to p_h and the aforementioned channels. The top row shows that as p_h increases and less workers invest in human capital that the average innate ability within highly-educated workers increases and the relative prices adjust. The bottom row shows that as less workers invest in human capital, the composition of unemployed workers shifts towards less-educated workers. The effects of the relative price and composition channels outweigh the effect of the supply channel workers become less likely to form cross-skill matches and the underemployment rate decreases.

Figure 13: Comparative statics with respect to p_h



7 Calibration and Policy Experiments

This section presents the calibrated version of the model and performs counterfactual policy experiments. In Section 7.1, I introduce a few modifications to the model that are unique to the quantitative version of the model. Section 7.2 details the calibration strategy, Section 7.3 compares the decentralized and constrained efficient allocations, and Section 7.4 performs education policy experiments.

7.1 Quantitative version of the model

I introduce two differences in the quantitative model relative to the baseline environment in Section 3. The first is that a worker’s innate ability is drawn from a continuous distribution $G(a)$. The second modification is that I assume the following structure for the production costs, $\varphi(\ell)$:

$$\varphi(\ell) = \begin{cases} 0 & \text{if } \ell \leq \underline{\ell}, \\ \alpha\ell & \text{if } \underline{\ell} < \ell < \bar{\ell}, \end{cases} \quad (30)$$

with $\alpha > 1$. The interpretation of (30) is that a worker’s endowment, $\underline{\ell}$, is a familiar transfer that only can be used for educational expenses. The linear portion of $\varphi(\ell)$ is now interpreted as the borrowing costs incurred to finance human capital if their endowment is less than the pecuniary cost of human capital, i.e. if $\underline{\ell} < p_h$.

7.2 Calibration strategy

The model is calibrated to the U.S. economy between 1992-2017. A unit of time is interpreted as one month. I assume that the aggregate meeting technology is Cobb-Douglas: $\mathcal{M}(\Omega, v) = A(\Omega)^\nu v^{1-\nu}$. The elasticity of the meeting technology is set to $\nu = 0.5$, as this is within an empirically supported range (Petrongolo and Pissarides, 2001) and I subsequently assume $\beta = 0.5$. The elasticity of substitution between simple and complex jobs, $1/_{1-\epsilon}$, is set equal to 1.41 following Katz and Murphy (1992), which implies $\epsilon = 0.29$.⁴¹ The death and birth rate, σ , is calculated as the average mortality rate among 15-34 year olds in 2007 from the Centers for Disease Control (CDC) National Vital Statistics System and gives $\sigma = 0.000924$. The separation rate is set equal to the monthly separation rate among 22-27 year olds in the Current Population Survey (CPS). Following Shimer (2012)’s method for constructing transition rates, I find $s = 0.021$.

The parameter which determines borrowing costs, α in (30), is calculated by equating the production costs $\alpha\ell$ to the total cost incurred by a borrower who borrowed the amount

⁴¹This value for the elasticity of substitution is also used in Krueger and Ludwig (2016). Borjas (2003) finds a similar estimate for the elasticity of substitution.

$p_h - \underline{\ell}$ and made monthly repayments over 10 years at an annual interest rate of 5%.⁴² This gives the following form for $\varphi(\underline{\ell})$:

$$\varphi(\underline{\ell}) = \frac{120 \max\{\hat{p}_h - \underline{\ell}, 0\} \left[\frac{.05}{12} \left(1 + \frac{.05}{12} \right)^{120} \right]}{\left[1 + \frac{.05}{12} \right]^{120} - 1}, \quad (31)$$

where \hat{p}_h is the net price of human capital.

The strategy for choosing the distribution of innate ability follows Braun (2019) who matches the distribution of ASVAB scores in the NLSY and estimates that $a-1$ is distributed log-normal with a mean of 4.62 and a standard deviation of 0.62.

The rest of the model's parameters are chosen to target empirical moments. The first five targets from the data are the following: (i) The average value of market tightness from the Job Openings and Labor Turnover Survey (JOLTS) between December 2000 and December 2017 of 0.3857 (ii) an underemployment rate of 24.6%,⁴³ and three estimates of the college earnings premium.⁴⁴ I estimate these premia by estimating variations of the following regression:

$$y_{ist} = \alpha + \beta_1 \text{college}_i + \beta X_i + \lambda_s + \delta_t + \varepsilon_{ist}, \quad (32)$$

where the subscript ist refers to individual i in state s and year t , y is an outcome of interest (log earnings), college is an indicator for whether the individual has at least a bachelors degree, X is a vector of individual characteristics (e.g., demographics and industry), λ_s is a year fixed effect, δ_t is a year fixed effect, and ε_{ist} is an error term that captures shocks and omitted variables. I estimate variations of (32) by ordinary least squares.

Table 3 reports the estimates of the college earnings premium. Column (1) includes all individuals in the sample and shows that on average a college degree is associated with an increase in earnings of 43.8%. Column (2) restricts the sample by excluding workers with at least a bachelors degree who work in college occupations. It shows that within non-college occupations that a college degree is associated with an earnings premium of 19.4%. Column (3) excludes those with at least a bachelors degree who work in non-college occupations and shows that a college degree is associated with a 60.7% increase in earnings in occupations that typically require a college degree. Five parameters, $(x_s, x_c, \mu, \gamma, b)$, are chosen to match

⁴²Ten-year repayment plans are typical for Federal student loans and a 5% interest rate is within the range of interest rates seen over the last decade. See <https://studentaid.ed.gov/sa/types/loans/interest-rates> for an overview of Federal student loan interest rates.

⁴³The target I use for the underemployment rate is the average of the average fraction of workers who work in occupations where less than 50% of respondents say that a college degree is required to perform that occupation (39.6%) and the fraction who work in occupations where less than 5% of respondents say that a college degree is required to perform that occupation (9.6%).

⁴⁴I focus on annual earnings rather than hourly wages because it is more transparent to interpret a worker's expected earnings in a job due to the two-part employment contract rather than the flow wage earned by a worker. Appendix A.4 contains estimates of the same estimation strategy with hourly wages and shows that the gaps between the estimated premia are relatively unchanged when considering hourly wages.

Table 3: Regression estimates: college earnings premia

	(1)	(2)	(3)
	Overall	All non-college occupations	Appropriately matched
College	0.438*** (0.039)	0.194*** (0.036)	0.607*** (0.058)
N	213,778	182,125	193,917
R^2	0.118	0.085	0.134

Notes: All regressions include state fixed effects, year fixed effects, control for demographics (age, sex, race, marital status), whether the individual works in a city, and the individual’s industry of employment. The sample covers 1992-2017 and is composed of individuals between the ages of 22-27 who are not currently enrolled in school. Column (1) includes all individuals in the constructed sample. Column (2) excludes workers with at least a bachelors degree who work in a college occupation. Column (3) excludes workers with a college degree who work in non-college occupations. Standard errors are clustered at the occupation level and are in parentheses. Levels of statistical significance are denoted by *** $p < 0.01$.

these five targets. I find $x_s = 7.97$, $x_c = 22.06$, $\mu = 0.619$, $\gamma = 86.12$, and $b = -2.73$.

The value of the matching efficiency, A , is chosen to match the monthly job-finding rate of 0.504 among college educated workers ages 22-27 in the CPS. Combining with the target of $\theta = 0.3857$, I find $A = 0.943$. The search intensity of employed workers, λ , is chosen to match the ratio of the monthly job-to-job transition rate among mismatched college educated workers (0.0379) to the monthly job-finding rate of college educated workers (0.504) in the CPS. This gives $\lambda = 0.125$. The rate of time preference is chosen to target an annual effective discount factor of 0.953 (Shimer, 2005b). Combining with σ gives $\rho = 0.003076$.

The pecuniary cost is chosen to match the estimated rate of return of college of 15% in Abel and Deitz (2014), which gives $p_h = 799.80$. The debt limit, $\bar{\ell}$, is chosen to match the ratio of the cumulative Stafford loan borrowing limits to the average four-year sticker price of public universities in the U.S. of 43%. This gives $\bar{\ell} = 343.91$. The psychic cost is chosen to match the fraction of 25-30 year olds with at least a bachelors degree between 1992-2017 in the CPS of 30.5%. This corresponds to targeting $H = 0.305$ and gives $\varsigma = 755.60$.

The distribution $F(\ell)$ is a Generalized Pareto distribution with location parameter 0. The shape and scale parameters are chosen to match the mean and median of expected family contributions (EFC) for education purposes relative to the average sticker price of public universities.⁴⁵ This gives a shape parameter of 0.2136 and a scale parameter of 4278.4. Table 4 summarizes the parameter values and Table 5 shows that the model is able to closely

⁴⁵Data on EFC comes from the National Postsecondary Student Aid Survey (NPSAS) for the survey years 1996, 2000, 2004, 2008, 2012, and 2016. The estimates of the mean and median only includes dependent students and includes those students who had an EFC of 0. The mean (median) in 2017\$ was \$14,684 (\$8,627). Combined with data on average sticker prices of four-year public universities from the College Board implies that the ratio of the mean (median) ω to p_h is 0.9423 (0.5536).

match the empirical targets.

Table 4: Parameter values

Parameter	Definition	Value
A. Externally calibrated		
σ	Death & birth rate	0.000924
δ	Separation rate	0.021
ν	Elasticity of meeting function w.r.t. job seekers	0.50
ϵ	Elasticity of substitution in final goods technology	0.29
β	Worker's bargaining power	0.50
B. Internally calibrated		
x_s	Productivity of simple jobs	7.97
x_c	Productivity of complex jobs	22.06
μ	Relative importance of simple jobs	0.619
γ	Vacancy flow cost	86.12
b	Unemployed flow utility	-2.73
A	Efficiency of meeting function	0.943
λ	Search intensity of mismatched workers	0.125
ρ	Discount rate	0.003076
p_h	Price of education	799.80
$\bar{\ell}$	Borrowing limit	343.91
ς	Psychic cost of education	755.60

Table 5: Targeted moments

	Data	Model
Market tightness	0.385	0.388
Underemployment rate	0.246	0.265
Earnings premia: non-college	0.194	0.192
Earnings premia: college only	0.607	0.585
Earnings premia: overall	0.438	0.414
Job-finding rate: highly-educated	0.504	0.504
JtJ rate among underemp.	0.037	0.030
Fraction with a BA	0.305	0.308
College RoR	1.150	1.150
Debt limit/sticker price	0.430	0.430
Mean EFC/sticker price	0.942	0.940
Median EFC/sticker price	0.553	0.552

7.3 Centralized vs. decentralized allocations

In this section, I compare the constrained efficient and decentralized allocations under the calibrated parameters presented in Table 4. For the constrained efficient allocation, I compute the planner's choice of market tightness, θ , the composition of vacancies, ζ , human capital choice $h(a_i, \ell)$, and cross-skill matching rules κ_i to maximize steady-state net output.⁴⁶ Table 6 summarizes the results.

⁴⁶Shimer and Smith (2001) show that the efficient allocation with ex-ante heterogeneous agents may be a limit cycle if both the production function is supermodular and the planner has the option to break up existing matches. I abstract from allowing

Table 6: Comparison of allocations

	Centralized	Decentralized
Market tightness	0.320	0.388
Composition of vacancies	0.570	0.580
Supply of highly-educated workers	0.325	0.308
$Pr(\text{form cross-skill match})$	0.000	0.835
Underemployment rate	0.000	0.265
Aggregate unemployment rate	0.098	0.072
Net output	20.56	18.13

As seen in Table 6, the constrained efficient allocation exhibits a lower amount of job creation and a composition of vacancies that consists of less simple jobs. Relative to the constrained efficient allocation, the decentralized equilibrium exhibits under-investment in human capital. Moreover, the underemployment rate in the constrained efficient allocation is 0%, due to the fact that the social planner chooses not to form cross-skill matches. Finally, the aggregate unemployment rate is higher and net output is 13.4% larger under the constrained efficient allocation relative to decentralized outcome.

The discrepancies between the constrained efficient and decentralized outcomes can be tied to the inefficiencies discussed in Section 5. We see that workers under-invest in human capital, which results from the hold-up problem in human capital investment. Moreover, workers in the decentralized equilibrium are not choosy enough in the formation of cross-skill matches. This is due to the fact that their opportunity cost of forming a cross-skill match is scaled by their share of the match surplus, β , whereas the social planner considers the total forgone surplus of forming a cross-skill match. As workers in the decentralized outcome form cross-skill matches at an inefficiently high rate, the composition of jobs creates too many simple jobs and leads to an inefficiently high amount of job creation.

In the subsequent policy experiments, I study the effect of policies that have a goal of increasing investment in human capital to address the under-investment seen in Table 6. In particular, I study the effect of subsidizing education through lump-sum taxes and relaxing borrowing constraints by increasing student loan borrowing limits.

7.4 Policy experiments

The first set of policy experiments that I consider are varying the size of education subsidies. I assume that the subsidies are financed by lump-sum taxes on workers.⁴⁷ The tax revenue is equally distributed among the workers who invest in human capital and the subsidy is not correlated with a worker's endowments. The first experiment is to implement subsidies that

the planner to break up existing matches. Moreover, the steady-state output may not be the optimal solution but I show that the steady-state output produced under the planner's choice is larger than that of the decentralized equilibrium.

⁴⁷The amount of the tax is independent of a worker's employment status.

finance 27% of the pecuniary cost of human capital, which is the average amount observed in the U.S. between 1992-2016 (College Board, 2017). The second experiment is one where the tax is increased to the point where education is fully subsidized. Table 7 contains the results of both experiments.

Table 7: Policy experiment: education subsidies

	Benchmark	1992-2017 increase		Fully subsidized education	
		Level	% change	Level	% change
Underemployment					
P (form cross-skill match)	0.835	1.000	19.76	1.000	19.76
Share of simple vacancies	0.580	0.584	0.585	0.593	2.220
Underemployment rate	0.265	0.291	9.811	0.300	13.20
Unemployment rates					
Less-educated	0.081	0.081	-0.122	0.080	-0.859
Highly-educated	0.053	0.049	-8.905	0.049	-7.977
Aggregate	0.072	0.071	-2.606	0.070	-3.017
Earnings premia					
Non-college	0.192	0.190	-1.400	0.190	-1.244
College only	0.585	0.574	-1.878	0.578	-1.297
Overall	0.414	0.368	-11.08	0.366	-11.56
Education sector					
Supply of human capital	0.308	0.321	4.082	0.323	4.828
Net price/sticker price	1.000	0.730	-27.00	0.000	-100.0
Welfare					
	18.13	18.26	0.693	18.34	1.171

As seen in columns (2) and (3), when 27% of p_h is subsidized, the probability to form a cross skill match increases from 85.3% to 100%. Combined with little change to the composition of vacancies, this increases the underemployment rate by 9.81%. Rows (3) through (6) show that the subsidy decreases the unemployment rate for highly-educated workers as they are much less likely to become employed at simple jobs. The bottom half of the table shows that the subsidy increases the supply of human capital by 4.08%. As the supply of highly-educated workers increases, the relative prices p_χ adjust and decreases the overall college earnings premium by 11.08%. The last row shows that this policy increases welfare by 0.693% due to its effect on human capital accumulation and reduction in unemployment.

Columns (4) and (5) of Table 7 show the effects of fully subsidizing human capital. This policy also causes κ to increase from 0.835 to 1 and for the composition of vacancies to slightly shift towards simple jobs. These changes cause the underemployment rate to increase by 13.20% and for the unemployment rate among highly-educated workers to decrease by

7.977% relative to the baseline allocation. The supply of highly-educated workers increases by 4.828% and due to the changes in the ratio p_s/p_c , the earnings premium in non-college jobs decreases by 11.56%. Overall, fully subsidizing human capital increases welfare by 1.17% and, taken together, Table 7 illustrates that while subsidizing education can cause the underemployment rate to increase, these policies also increase welfare.

The second set of policies studies changes to the borrowing limit $\bar{\ell}$. The first change that I consider is eliminating the option to borrow to finance human capital by setting $\bar{\ell} = 0$. Secondly, I consider the opposite extreme of fully relaxing borrowing limits and setting $\bar{\ell} = p_h$. Table 8 contains the results.

Table 8: Policy experiment: borrowing limits

	Benchmark	No borrowing		No borrowing limit	
		Level	% change	Level	% change
Underemployment					
P (form cross-skill match)	0.835	0.244	-70.77	0.684	-18.08
Share of simple vacancies	0.580	0.592	1.910	0.582	0.189
Underemployment rate	0.265	0.131	-50.566	0.242	-8.679
Unemployment rates					
Less-educated	0.081	0.080	-1.71	0.081	-0.491
Highly-educated	0.053	0.085	57.69	0.059	9.461
Aggregate	0.072	0.081	11.11	0.074	1.508
Earnings premia					
Non-college	0.192	0.117	-39.31	0.190	-1.452
College only	0.585	0.582	-0.631	0.586	0.051
Overall	0.414	0.526	26.99	0.446	7.677
Education sector					
Supply of human capital	0.308	0.250	-18.98	0.300	-2.786
Net price/sticker price	1.000	1.000	0.000	1.000	0.000
Welfare					
	18.13	16.51	-8.965	18.00	-0.749

Columns (2) and (3), which correspond to $\bar{\ell} = 0$, show that tightening borrowing constraints causes the underemployment rate to decrease from 26.5% to 13.1%. This is driven by the decrease in the supply of highly-educated workers, H , due to the tightened borrowing constraints, as this policy causes H to decrease by 18.98%. This policy causes a decline in welfare of 8.96%. Columns (4) and (5) illustrate that the quantitative effects of relaxing borrowing constraints to the point where $\bar{\ell} = p_h$ are relatively small. This is because, in the baseline calibration, less than 1% of workers are constrained by the original borrowing limit. The workers who are constrained have a relatively high innate ability (see Lemma

2). When these workers begin to invest in human capital, it reduces the benefits of human capital investment for other workers, particularly those with a relatively low innate ability. In net, the supply of highly-educated workers decreases, which leads to a decline in the underemployment rate of 8.67% and a 0.749% decrease in welfare.

8 Conclusion

This paper has developed a theory of equilibrium underemployment. The model generates a rich set of equilibria, including the multiplicity of equilibria with different combinations of matching and human capital investment patterns. The introduction of a human capital investment decision among workers allows for a positive and normative analysis of the effects of increasing student loan borrowing limits and education subsidies. The analytical results show that the effects educational policies on the labor market are driven by a supply, composition, and relative price channels. A normative analysis shows that the decentralized equilibrium is never efficient and can exhibit an inefficiently high or low amount of underemployment. A calibrated version of the model shows that there is under-investment in human capital and an inefficiently high amount of underemployment in the U.S. Increasing education subsidies can increase welfare by inducing more workers to investment in human capital, despite the fact that this policy also increases the underemployment rate.

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Appendices

A Empirical appendix

A.1 Data sources and construction

Data on underemployment and college degree attainment comes from four sources: (i) the ASEC, (ii) the U.S. Department of Labor’s Occupational Information Network (O*NET), (iii) The Bureau of Labor Statistics (BLS), and (iv) The American Community Survey (ACS).⁴⁸ Occupations are defined to require a college degree if at least 50% of respondents in the August 2017 O*NET survey (release 222) respond that a college degree is necessary to perform that occupation. The sample of recent college graduates comes from the ASEC and only includes workers who are ages 22-27 (inclusive) and have at least a Bachelors degree, where educational attainment is based on the EDUC variable. For the years 1992-2017, Bachelors recipients are those with $EDUC \geq 111$. Prior to 1992, Bachelors recipients are those with at least four years of college ($EDUC \geq 110$). A recent graduate is defined to be underemployed if the respondent’s primary occupation is one in which less than 50% of respondents in the O*NET survey say that a college degree is required to perform that job. Occupations are matched between the ASEC and O*NET survey using 2010 Census Bureau occupation codes. In Figure 2 panel (b), high-wage (low-wage) jobs are those where the average annual salary of that occupation is more than \$35,000 (less than \$25,000), where data on average salary by occupation is based on 2012 data published by the BLS.⁴⁹

The sample in panel (b) of 1 comes from the ASEC and is restricted to ages 25 to 30 (inclusive). Educational attainment is derived from the variable EDUC. Associates are those where the highest degree attained is an Associates degree occupational/vocational program or Associates degree academic program ($EDUC = 91$ or $EDUC = 92$). Bachelors corresponds to $EDUC = 111$ and Masters degree is $EDUC = 123$. Ph.D. & Professional is those with a Professional School Degree or Doctorate Degree ($EDUC = 124$ or $EDUC = 125$). Percentages are calculated using the ASEC person weight (ASECWT).

Data on Federal student loan disbursements in Figure 10 panel (a) comes from [College Board \(2017\)](#). Data for Figure 10 panel (b) comes from the Survey of Consumer Finances. The average amount of debt is calculated by summing the amount owed on education loans among those who report having a positive amount of education debt. The sample contains households where the head of the household is between 20-40 years old (inclusive), where age is calculated by subtracting birth year from year of the survey. Calculations use SCF weights where the weights are divided by 5.⁵⁰

⁴⁸The ASEC and ACS data was download from the IPUMS at <https://ipums.org>.

⁴⁹This data is downloaded from <https://www.bls.gov/oes/tables.htm>.

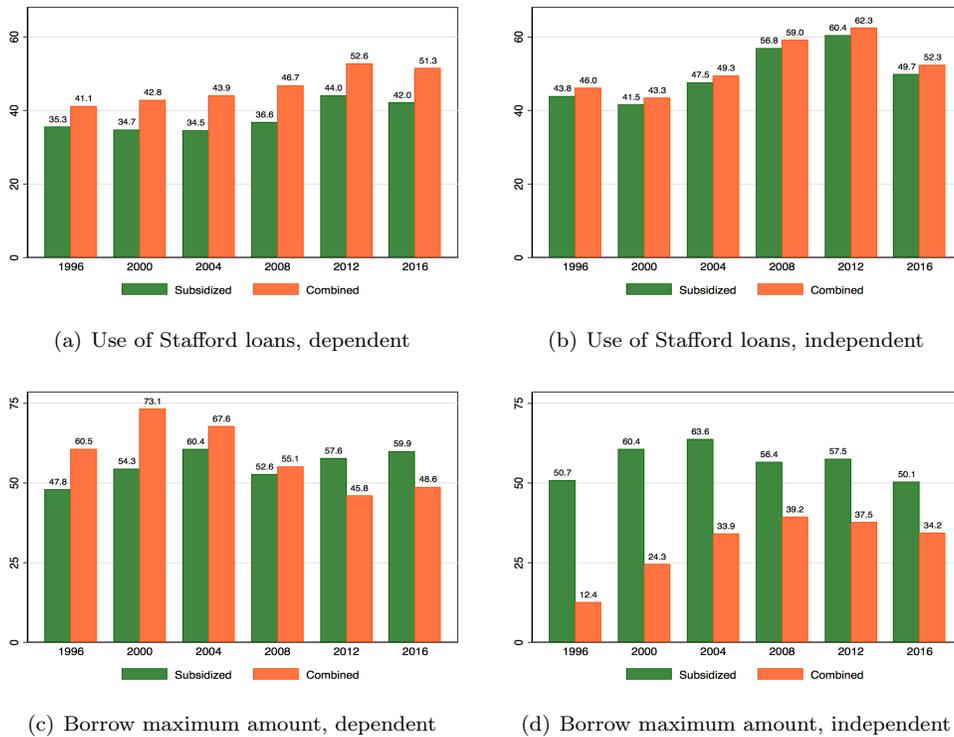
⁵⁰This follows the recommendation made by the Federal Reserve Board.

The regressions in Table 3 use data on annual earnings from ASEC data. College graduates are those with at least a Bachelors degree and follows the same definition as described above. The definition of underemployment follows the same definition described above. The sample is restricted to ages 22-27 (inclusive) and those who report having positive earnings in the previous year ($INCWAGE > 0$). Earnings observations in the top and bottom 1% of the earnings distribution in each year are dropped. Real earnings are in 2016\$ and are deflated using the CPI-U, annual average.

A.2 The use of Stafford Loans by Dependency Status

Panels (a) and (b) in Figure 14 illustrates the percentage of full-time, full-year undergraduate students who borrow a positive amount using Stafford loans by dependency status. Panels (c) and (d) show the percentage, among full-time, full-year students who borrow using Stafford loans, who borrow the maximum amount that they are eligible for by dependency status.

Figure 14: The use of Stafford loans by dependency status



Notes: Data come from the National Postsecondary Student Aid Survey (NPSAS). These calculations only include students who were enrolled full-time, full-year in that particular academic year. Panels (c) and (d) show the percentage of undergraduates, by dependency status, who borrowed the “usual maximum” amount of Stafford loans. Percentages are calculated using the NPSAS weights.

A.3 Granger Causality Tests

Table 9 presents results from testing for Granger causality. Specifically, the second column presents the χ^2 statistic that is generated by performing a Wald test with the null hypothesis that the coefficients of the variable in the first column are jointly zero in the equation for the underemployment rate that is estimated in (29).

Table 9: Granger Causality Wald tests

Excluded variable	χ^2	P-value
Δ Federal	11.498	0.009
Δ Recipients	11.74	0.008
Δ Grants Size	4.9837	0.173

A.4 College wage premiums

Table 10 presents estimates of the college hourly wage premium and is also constructed using ASEC data. The sample is the same as the annual earnings regressions. Hourly wages are calculated by dividing total income (INCWAGE) by total hours worked in the last year ($WKSWORK1 \times UHRWORKLY$). Wage observations in the top and bottom 1% of the wage distribution in each year are dropped. Real wages are in 2016\$ and are deflated using the CPI-U, annual average.

Table 10: Regression estimates (wages)

	(1)	(2)	(3)
	Overall	All non-college occupations	Appropriately matched
College	0.342*** (0.022)	0.179*** (0.018)	0.454*** (0.030)
N	213,778	182,125	193,917
R^2	0.136	0.076	0.156

Notes: All regressions include state fixed effects, year fixed effects, control for demographics (age, sex, race, marital status), whether the individual works in a city, and the individual's industry of employment. The sample covers 1992-2017 and is composed of individuals between the ages of 22-27 who are not currently enrolled in school. Column (1) includes all individuals in the constructed sample. Column (2) excludes workers with at least a bachelors degree who work in a college occupation. Column (3) excludes workers with a college degree who work in non-college occupations. Standard errors are clustered at the occupation level and are in parentheses. Levels of statistical significance are denoted by *** $p < 0.01$.

B Proofs and derivations

B.1 Proof of Lemma 1

The contract $w_s(a, h), \phi_s^u(a, h)$ must be pairwise Pareto efficient. After the hiring fee is transferred from the worker to the firm the wage is chosen to maximize the surplus of the match. The surplus is maximized if and only if $J_s(a, h) = V_s$. This is true if and only if $w_s(a, h) = p_s y_s(a, h)$. The hiring fee in (16) is derived by taking the first order condition of (14) with respect to $\phi_s^u(a, h)$. ■

B.2 Additional employment contracts

There are two other types of meetings to consider.

1. Highly-educated workers and complex jobs (via employment).
2. Highly-educated workers and complex jobs (via unemployment).

The flow surpluses of each match type are given by:

$$rS_c^e(a, h) = p_c y_c(a, h) - rE_s(a, h) - sS_c^e(a, h), \quad (\text{B.1})$$

$$rS_c^u(a, h) = p_c y_c(a, h) - rU(a, h) - sS_c^u(a, h). \quad (\text{B.2})$$

Equations (B.1)-(B.2) have a similar interpretation as equation (11), except that there are no voluntary separations in the three types of meetings described above. One feature that is unique to equation (B.1) is that the worker's outside option in a meeting between a complex job and an employed, highly-educated worker is the value of employment at the simple job. The contracts solve:

$$w_c(a, h), \phi_c^e(a, h) \in \arg \max [E_c(a, h) - E_s(a, h) - \phi_c^e(a, h)]^\beta [J_c(a, h) - V_c + \phi_c^e(a, h)]^{1-\beta}, \quad (\text{B.3})$$

$$w_c(a, h), \phi_c^u(a, h) \in \arg \max [E_c(a, h) - U(a, h) - \phi_c^u(a, h)]^\beta [J_c(a, h) - V_c + \phi_c^u(a, h)]^{1-\beta}. \quad (\text{B.4})$$

The solution to the wage contracts follows the same logic as in the Proof to Lemma 1. The wage is chosen to maximize the joint surplus after the hiring fee is transferred from the worker to the firm. From equations (B.1)-(B.2), the joint surplus will be maximized independent of the wage (because there are no voluntary separations in these meetings). It follows that, without loss of generality, the wage can be set equal to the marginal product and the hiring

fee splits the match surplus according to the respective bargaining weights:

$$w_c(a, h) = p_c y_c(a, h); \phi_c^u(a, h) = (1 - \beta)[E_c(a, h) - U(a, h)];$$

$$\phi_c^e(a, h) = (1 - \beta)[E_c(a, h) - E_s(a, h)]. \quad (\text{B.5})$$

It is straitforward to show that the solutions in (B.5) are payoff equivalent to Nash bargaining over an employment contract which only specifies a flat wage as the wage does not affect the joint surplus in a match without voluntary separations.

B.3 Derivation of closed-form free-entry conditions

Combining equations (7)-(8) with the optimal contracts in Section 4.2 gives the values of unemployment for less- and highly-educated workers:

$$rU(a, 0) = \frac{a[b(r + \delta) + \beta f(\theta)\zeta p_s x_s]}{\rho_0}, \quad (\text{B.6})$$

$$rU(a, 1) = \frac{a[b(r + \delta)\rho_2 + \beta f(\theta)\{\zeta \kappa p_s x_s(r + \delta) + (1 - \zeta)p_c x_c \rho_3\}]}{\rho_1 \rho_2}, \quad (\text{B.7})$$

where $\rho_0 \equiv r + \delta + \beta f(\theta)\zeta$, $\rho_1 \equiv r + \delta + \beta f(\theta)(\zeta \kappa + 1 - \zeta)$, $\rho_2 \equiv r + \delta + \beta \lambda f(\theta)(1 - \zeta)$, and $\rho_3 = r + \delta + \beta \lambda f(\theta)(\zeta \kappa + 1 - \zeta)$ are discount factors.

Next, combining equations (9)-(10), the optimal employment contracts, and equations (B.6)-(B.7), the free-entry conditions are given by

$$\frac{\gamma}{q(\theta)(1 - \beta)} = \psi \left\{ \eta \frac{p_s x_s - b}{\rho_0} \mathbb{E}[a|h = 0] \right.$$

$$\left. + (1 - \eta) \kappa \frac{p_s x_s(r + \delta) - \beta f(\theta)(1 - \zeta)[(1 - \lambda)p_c x_c - p_s x_s] - b \rho_2}{\rho_1 \rho_2} \mathbb{E}[a|h = 1] \right\}, \quad (\text{B.8})$$

$$\frac{\gamma}{q(\theta)(1 - \beta)} = \mathbb{E}[a|h = 1] \left\{ \psi(1 - \eta) \frac{p_c x_c \rho_4 - \beta f(\theta)\zeta[\kappa p_s x_s - (\kappa - \lambda)p_c x_c] - b \rho_2}{\rho_1 \rho_2} \right.$$

$$\left. + (1 - \psi) \frac{p_c x_c - p_s x_s}{\rho_2} \right\}, \quad (\text{B.9})$$

where $\rho_4 \equiv r + \delta + \beta \lambda f(\theta)$.

B.4 Proof of Lemma 2

Substituting the worker's budget constraint, (6), into (5) reduces their maximization problem to consumption and human capital investment:

$$W(a, \underline{\ell}) = \max_{c, h} \{c - \varphi(c + p_h h; \underline{\ell}) + h[U(a, 1) - \varsigma] + (1 - h)U(a, 0)\}. \quad (\text{B.10})$$

The first order condition with respect to consumption given by

$$1 \leq \varphi'(c + p_h h; \underline{\ell}). \quad (\text{B.11})$$

From (B.11), a worker will only produce the numeraire good for consumption if they are at the linear portion of $\varphi(\ell)$. Even then, a worker is indifferent between producing the numeraire for consumption, as the marginal benefit is equal to the marginal cost. I consider an equilibrium where workers choose $c = 0$. It follows that a worker will choose $h = 1$ if (18) is satisfied.

From equations (B.6)-(B.7), the net gain of investing in human capital is given by

$$\Gamma(a, \underline{\ell}) = -[\varphi(p_h; \underline{\ell}) + \varsigma] + \frac{a}{r} \left\{ \beta f(\theta) \left[\frac{\zeta \kappa p_s x_s (r + \delta) + (1 - \zeta) p_c x_c \rho_3}{\rho_1 \rho_2} - \frac{\zeta p_s x_s}{\rho_0} \right] + b(r + \delta) \frac{\rho_0 - \rho_1}{\rho_0 \rho_1} \right\}. \quad (\text{B.12})$$

From equation (B.12), the capital gain of investing in education can only be positive when

$$\beta f(\theta) \left[\frac{\zeta \kappa p_s x_s (r + \delta) + (1 - \zeta) p_c x_c \rho_3}{\rho_1 \rho_2} - \frac{\zeta p_s x_s}{\rho_0} \right] + b(r + \delta) \frac{\rho_0 - \rho_1}{\rho_0 \rho_1} > 0. \quad (\text{B.13})$$

It follows that (B.13) is a necessary condition to have a positive measure of workers invest in human capital. It also follows that $\partial \Gamma(a, \underline{\ell}) / \partial a > 0$ if (B.13) is satisfied.

From equation (B.12), $\underline{\ell}^*(a)$ satisfies:

$$\varphi(p_h, \underline{\ell}^*(a)) + \varsigma = \frac{a}{r} \left\{ \beta f(\theta) \left[\frac{\zeta \kappa p_s x_s (r + \delta) + (1 - \zeta) p_c x_c \rho_3}{\rho_1 \rho_2} - \frac{\zeta p_s x_s}{\rho_0} \right] + b(r + \delta) \frac{\rho_0 - \rho_1}{\rho_0 \rho_1} \right\}. \quad (\text{B.14})$$

Assume that (B.13) is satisfied which implies that the right hand side of (B.14) is increasing in a . Thus, if a increases, $\varphi(p_h, \underline{\ell}^*(a))$ must increase so that (B.14) is satisfied. As $\varphi(p_h; \underline{\ell})$ is weakly decreasing in $\underline{\ell}$, $\partial \underline{\ell}^*(a) / \partial a \leq 0$. ■

It is also clear from (B.12) that $\partial \Gamma(a, \underline{\ell}) / \partial \underline{\ell} > 0$ if $\underline{\ell} < p_h$ as $\partial \varphi(\ell) / \partial \underline{\ell} < 0$ if $\underline{\ell} < p_h$.

B.5 Proof of Lemma 3

Cross-skill matches are formed if they generate a positive surplus. From equation (11), the surplus in a cross skill match is positive if

$$p_s x_s a + \beta \lambda f(\theta)(1 - \zeta) \frac{p_c x_c a - p_s x_s a}{\rho_2} > rU(a, 1). \quad (\text{B.15})$$

Combining with (B.7) and simplifying shows that the surplus for forming a cross-skill match is positive if

$$\frac{p_s x_s - b}{p_c x_c - b} > \frac{\beta f(\theta)(1 - \zeta)(1 - \lambda)}{r + \delta + \beta f(\theta)(1 - \zeta)}. \quad (\text{B.16})$$

Thus, $\kappa = 1$ if (B.16) is satisfied, $\kappa = 0$ if (B.16) holds with opposite inequality, and $\kappa \in [0, 1]$ if (B.16) holds at equality. It is straightforward to see from (B.16) that the decision to form a cross-skill match is independent of the worker's ability and financial wealth. ■

B.6 Proof of Proposition 1

Part (i): The proof of existence of an active steady-state equilibrium with $\theta > 0$ is done by contradiction.

Case 1: $\epsilon < 1$. Suppose that $\theta = 0$. If $\theta \rightarrow 0$, then the left hand side of the free-entry conditions, (B.8)-(B.9), approach 0 as $q(\theta) \rightarrow \infty$ as $\theta \rightarrow 0$. However, if $\theta \rightarrow 0$, the right-hand side of at least one of the free-entry conditions is strictly positive for all $H \in [0, 1]$ as $p_\chi \rightarrow \infty$ as $\theta \rightarrow 0$. Thus, firms have an incentive to post at least one type of a vacancy giving $\theta > 0$.

Case 2: $\epsilon = 1$. As $\theta \rightarrow 0$, the right-hand side of at least one of the free-entry conditions is strictly positive for all $H \in [0, 1]$ if $b < \min\{\mu x_s, (1 - \mu)x_c\}$. Thus, firms have an incentive to post at least one type of a vacancy giving $\theta > 0$.

Part (ii): I now proceed to prove that if $\epsilon < 1$, then $\zeta \in (0, 1)$ and $H > 0$. Suppose that firms only post simple vacancies ($\zeta = 1$). It follows that the value of a simple vacancy is given by

$$V_s = -\gamma + q(\theta)(1 - \beta)\mathbb{E}_{a,h} \left[\frac{p_s y_s(a, h) - rU(a, h)}{r + \delta} \right] = 0, \quad (\text{B.17})$$

where

$$rU(a, h) = ba + \beta f(\theta) \left[\frac{p_s x_s a - rU(a, h)}{r + \delta} \right], \quad (\text{B.18})$$

defines the worker's reservation wage which is independent of the worker's human capital if $\zeta = 1$. Define θ^* as the unique value of market tightness that jointly solves equations (B.17) and (B.18).

Now suppose that a small measure of firms deviate to post a complex vacancy so that the fraction of complex vacancies is given by $1 - \zeta = \varepsilon$ where ε is positive and small. Additionally, let $\Gamma^*(a, \underline{\ell})$ denote the capital gain of investing in human capital for $\theta = \theta^*$ and $1 - \zeta = \varepsilon$. If $\Gamma^*(a_H, p_h) \geq 0$, then the minimum supply of highly-educated workers will be given by $(1 - \pi)[1 - F(p_h)]$.⁵¹ Firms have an incentive to deviate and post a complex vacancy if the expected profits of doing so are positive, i.e. if

$$\rho V_c = -\gamma + q(\theta)(1 - \beta)(1 - \pi)[1 - F(p_h)]\mathbb{E}_{a,h}\left[\frac{p_c x_c - rU(a, h)}{r + \delta}\right] > 0. \quad (\text{B.19})$$

Combining equation (B.18) with equation (B.19), it follows that (B.19) is satisfied if

$$(1 - \pi)[1 - F(p_h)]\mathbb{E}_{a,h}\left[\frac{p_c x_c - rU(a, h)}{r + \delta}\right] > \mathbb{E}_{a,h}\left[\frac{p_s y_s(a, h) - rU(a, h)}{r + \delta}\right]. \quad (\text{B.20})$$

Solving for $rU(a, h)$ from equation (B.18), substituting into (B.20) and simplifying shows that (B.20) is satisfied if

$$\frac{(1 - \pi)[1 - F(p_h)][(r + \delta)(p_c x_c - b) + \beta f(\theta^*)(p_c x_c - p_s x_s)]\mathbb{E}[a|h = 1]}{(r + \delta)(p_s x_s - b)\mathbb{E}[a]} > 1. \quad (\text{B.21})$$

The prices of intermediate goods are given by

$$p_s = \mu \left[\mu + (1 - \mu) \left(\frac{Y_c}{Y_s} \right)^\epsilon \right]^{\frac{1-\epsilon}{\epsilon}}, \quad (\text{B.22})$$

$$p_c = (1 - \mu) \left[\mu \left(\frac{Y_s}{Y_c} \right)^\epsilon + (1 - \mu) \right]^{\frac{1-\epsilon}{\epsilon}}, \quad (\text{B.23})$$

where, from the steady-state conditions (22)-(24), the aggregate output from type χ matches is given by

$$Y_s = x_s \left[\frac{(1 - H)f(\theta)\zeta\mathbb{E}[a|h = 0]}{\delta + f(\theta)\zeta} + \frac{Hsf(\theta)\zeta\kappa\mathbb{E}[a|h = 1]}{(s + f(\theta)[\zeta\kappa + 1 - \zeta])[\delta + \lambda f(\theta)(1 - \zeta)]} \right], \quad (\text{B.24})$$

$$Y_c = x_c \frac{Hf(\theta)(1 - \zeta)\mathbb{E}[a|h = 1]}{\delta + f(\theta)(\zeta\kappa + 1 - \zeta)} \left[\frac{\delta + \lambda f(\theta)(\zeta\kappa + 1 - \zeta)}{\delta + \lambda f(\theta)(1 - \zeta)} \right]. \quad (\text{B.25})$$

Suppose that $\epsilon < 1$. From equation (B.23),

$$\lim_{Y_c/Y_s \rightarrow 0} p_c = \infty. \quad (\text{B.26})$$

⁵¹See equation (20)

Combining (B.26) with the fact that $Y_c \rightarrow 0$ as $\zeta \rightarrow 1$,⁵² it follows that

$$\lim_{\zeta \rightarrow 1} p_c = \infty. \quad (\text{B.27})$$

It is straitforward to see that (B.21) is always satisfied if $p_c \rightarrow \infty$ as $\zeta \rightarrow 1$.

Part (iii): From the Proof of Lemma 2, the first workers to invest in human capital are those with (i) the high ability and (ii) lowest cost to finance the pecuniary cost of human capital. It follows that the minimum supply of highly-educated workers is given by $H^* = (1 - \pi)(1 - F(\bar{\ell}))$. Suppose that the supply of highly-educated workers is given by H^* . Define θ^* , ζ^* , and κ^* as the corresponding market tightness, composition of vacancies, and cross-skill match formation rule that solve the entry conditions, (B.8)-(B.9), and (21) when the supply of highly-educated workers is given by H^* . Workers with the high innate ability and low cost to produce the numeraire will not invest in human capital if

$$p_h + \varsigma > \frac{a_H}{r} \left\{ \beta f(\theta^*) \left[\frac{\zeta^* \kappa^* \mu x_s (r + \delta) + (1 - \zeta^*)(1 - \mu)x_c \rho_3}{\rho_1 \rho_2} - \frac{\zeta^* \mu x_s}{\rho_0} \right] + b(r + \delta) \frac{\rho_0 - \rho_1}{\rho_0 \rho_1} \right\}. \quad (\text{B.28})$$

It follows that if $\iota \equiv p_h + \varsigma$ is greater than $\underline{\iota}$, which is defined as the right side of (B.28), then no workers will invest in human capital. If $H = 0$, then the right hand side of (B.9) is equal to zero and no complex jobs will be created leaving $\zeta = 1$. ■

B.7 Proof of Proposition 2

Assume that parameter values are such that $H > 0$ and $\zeta \in (0, 1)$. Recall that cross-skill matches are formed, i.e. $\kappa \in (0, 1]$ if

$$\frac{p_s x_s - b}{p_c x_c - b} > \frac{\beta f(\theta)(1 - \zeta)(1 - \lambda)}{r + \delta + \beta f(\theta)(1 - \zeta)}. \quad (\text{B.29})$$

It is clear that as $\lambda \rightarrow 1$, the right hand side of (B.29) approaches 0. It follows that as $\lambda \rightarrow 1$, then $\kappa \in (0, 1]$. From (25), $\underline{u} > 0$ if $\zeta \kappa > 0$.

Cross-skill matches are not formed, i.e. $\kappa = 0$, if

$$\frac{p_s x_s - b}{p_c x_c - b} < \frac{\beta f(\theta)(1 - \zeta)(1 - \lambda)}{r + \delta + \beta f(\theta)(1 - \zeta)}. \quad (\text{B.30})$$

Suppose that $\lambda < \underline{\lambda}$. As x_c increases, the left-hand side of (B.30) decreases, making it more likely that (B.30) is satisfied. In the extreme case of $x_c \rightarrow \infty$, it is clear that (B.30) will be satisfied and cross-skill matches will not be formed and $\underline{u} = 0$. ■

⁵²This follows from equation (B.25).

B.8 Proof of Proposition 3

With a fixed supply of highly-educated workers and $\kappa = 1$, the two-equilibrium conditions that simultaneously determine θ and ζ are given by the following entry conditions.

$$\frac{\gamma}{q(\theta)(1-\beta)} = \eta \frac{\mu x_s - b}{\rho_0} + (1-\eta) \frac{(\mu x_s - b)(r + \delta) - \beta f(\theta)(1-\zeta)[(1-\mu)x_c - \mu x_s]}{\rho_1 \rho_2}, \quad (\text{B.31})$$

$$\frac{\gamma}{q(\theta)(1-\beta)} = (1-\eta) \frac{[(1-\mu)x_c - b](r + \delta) - \beta f(\theta)\zeta[\mu x_s - (1-\mu)x_c]}{\rho_1 \rho_2}. \quad (\text{B.32})$$

It is strait-forward to make use of $V_s = V_c$ to show

$$(r + \delta)(\mu x_s - b) = (1-\eta) \left[[(1-\mu)x_c - b](r + \delta) + \beta f(\theta)\zeta[(1-\mu)x_c - \mu x_s] \right], \quad (\text{B.33})$$

where

$$1 - \eta = \frac{H[\delta + f(\theta)\zeta]}{\delta + Hf(\theta)\zeta + (1-H)f(\theta)}. \quad (\text{B.34})$$

Substituting (B.33) into (B.32) and simplifying gives

$$\frac{\gamma}{q(\theta)(1-\beta)} = \frac{(r + \delta)(\mu x_s - b)}{r + \delta + \beta f(\theta)}, \quad (\text{B.35})$$

which defines a unique value of θ that is independent of ζ . Given θ that is determined by (B.35), (B.33) determines the equilibrium value of ζ . It is strait-forward to show that the right hand side of (B.33) is decreasing in θ (recall that I have assumed $\beta \approx 0$). It is also straitforward to see that the right hand side of (B.33) is increasing in ζ , as $1 - \eta$ is decreasing in ζ .

If μ or x_s increases, then, from (B.33), θ will increase. As μ or x_s and θ increase, the left side of (B.33) increases while the right side decreases. It follows that ζ must increase to satisfy (B.33).

From (B.35), θ is independent of x_c and H . From (B.33), if x_c or H increases, then the right side will increase. It follows that ζ must decrease to satisfy (B.33).

Finally, if γ increases, then θ will decrease to satisfy (B.35). If θ decreases, the left side of (B.33) decreases which implies that ζ must increase so that (B.33) is satisfied. ■

B.9 Proof of Proposition 4

The net benefits of human capital, under the simplifying assumptions, is given by

$$\Gamma(\underline{\ell}) = -[\varphi(p_h; \underline{\ell}) + \varsigma] + \frac{\beta f(\theta)}{r} \left\{ \frac{\zeta \mu x_s + (1 - \zeta)(1 - \mu)x_c}{r + \delta + \beta f(\theta)} - \frac{\zeta \mu x_s}{r + \delta + \beta f(\theta)\zeta} - \frac{b(r + \delta)(1 - \zeta)}{(r + \delta + \beta f(\theta))(r + \delta + \beta f(\theta)\zeta)} \right\}, \quad (\text{B.36})$$

with $\partial\Gamma(\underline{\ell})/\partial\theta > 0$ and $\partial\Gamma(\underline{\ell})/\partial\zeta < 0$. The remaining equilibrium conditions that determine ζ and θ are given by equations (B.33)-(B.35).

Suppose that μ or x_s increases. From (B.33), it is clear that θ will increase. From the Proof of Proposition 3, this causes ζ to increase. An increase in θ and ζ have opposite effects on the net benefits of human capital, (B.36). If the effect of θ outweighs the effect of ζ on $\Gamma(\underline{\ell})$, then the supply of highly-educated workers will increase, which puts downward pressure on ζ . On the other hand, if the effect of ζ increasing dominates the effect of θ , then H will decrease which further causes ζ to increase. Thus, the end result on both H and ζ is ambiguous.

If γ increases, then θ will decrease (from (B.35)). From the Proof of Proposition 3, this increase ζ . A decrease in θ and increase in ζ both reduce the benefits from investing in human capital, causing H to decrease. A decrease in H subsequently causes ζ to decline (see the Proof of Proposition 3). Thus, an increase in γ will cause both θ and H to decrease and for ζ to increase.

Market tightness is independent of the remaining parameters, x_c , p_h , and ς , as neither enter equation (B.35). From the Proof of Proposition 3, if x_c increases, then ζ decreases. As ζ decreases, the benefits of investing in human capital increase and thus H increases, which further causes ζ to decrease. It follows that if x_c increases, then θ is unaffected, ζ decreases, and H increases. Now suppose that p_h or ς increase. From (B.36), this decreases the benefits of investing in human capital. As H decreases, ζ will increase (see the Proof of Proposition 3), which further decreases the benefits of investing in human capital. It follows that ζ increases while H decreases following an increase in p_h or ς . ■

B.10 Proof of Proposition 5

I proceed to derive the efficient steady-state allocations by writing down the current-value Hamiltonian, where $\Upsilon_{\chi,i}^h$ is the multiplier on the $\dot{e}_{\chi,i}^h$ laws of motion and Λ_i^h is the multiplier

on the \dot{u}_i^h laws of motion:

$$\begin{aligned}
\mathcal{H} = & Y(e_{s,i}^h, e_{c,i}^1) + \sum_i \sum_h u_i^h b a_i - \gamma v - \sigma \sum_i \pi_i \int_0^\infty h(a_i, \underline{\ell}) [\zeta + \varphi(p_h; \underline{\ell})] dF(\underline{\ell}) \\
& + \sum_i \left\{ \sum_h \Upsilon_{s,i}^h [\mathcal{M}(\cdot, \cdot) \frac{v_s u_i^h}{v u} [\kappa_i h + (1-h)] - (\delta + \sigma) e_{s,i}^h] + \Upsilon_{c,i}^1 [\mathcal{M}(\cdot, \cdot) \frac{v_c u_i^1}{v u} - (\delta + \sigma) e_{c,i}^1] \right. \\
& + \Lambda_i^0 \left[\sigma \pi_i \int_0^\infty [1 - h(a_i, \underline{\ell})] dF(\underline{\ell}) + \delta e_{s,i}^0 - \mathcal{M}(\cdot, \cdot) \frac{v_s u_i^0}{v u} - \sigma u_i^0 \right] \\
& \left. + \Lambda_i^1 \left[\sigma \pi_i \int_0^\infty h(a_i, \underline{\ell}) dF(\underline{\ell}) + \delta [e_{s,i}^1 + e_{c,i}^1] - \mathcal{M}(\cdot, \cdot) \left(\frac{v_s}{v} \kappa_i + \frac{v_c}{v} \right) \frac{u_i^1}{u} - \sigma u_i^1 \right] \right\}.
\end{aligned} \tag{B.37}$$

I begin by characterizing the total surplus generated in each type of match which are derived from the standard co-state equations. They are given by:

$$\Upsilon_{s,i}^0 - \Lambda_i^0 = \frac{a_i [\mu x_s - b] + (1 - \nu) f(\theta) \Psi}{r + \delta + f(\theta) \zeta}, \tag{B.38}$$

$$\Upsilon_{s,i}^1 - \Lambda_i^1 = \frac{a_i [(\mu x_s - b)(r + \delta) - f(\theta)(1 - \zeta)((1 - \mu)x_c - \mu x_s)] + (r + \delta) f(\theta)(1 - \nu) \Psi}{(r + \delta)(r + \delta + f(\theta)(1 - \zeta + \zeta \kappa_i))}, \tag{B.39}$$

$$\Upsilon_{c,i}^1 - \Lambda_i^1 = \frac{a_i [((1 - \mu)x_c - b)(r + \delta) - f(\theta) \zeta \kappa_i (\mu x_s - (1 - \mu)x_c)] + (r + \delta) f(\theta)(1 - \nu) \Psi}{(r + \delta)(r + \delta + f(\theta)(1 - \zeta + \zeta \kappa_i))}, \tag{B.40}$$

where $\nu \equiv \frac{\theta q'(\theta)}{q(\theta)}$ is the elasticity of the matching function with respect to job seekers and Ψ is the average value of a match:

$$\Psi \equiv \eta \zeta \sum_i \hat{\pi}_i^0 [\Upsilon_{s,i}^0 - \Lambda_i^0] + (1 - \eta) \left\{ \zeta \sum_i \hat{\pi}_i^1 \kappa_i [\Upsilon_{s,i}^1 - \Lambda_i^1] + (1 - \zeta) \sum_i \hat{\pi}_i^1 [\Upsilon_{c,i}^1 - \Lambda_i^1] \right\}, \tag{B.41}$$

and $\hat{\pi}_i^h$ is the probability of matching a worker with ability i conditional on meeting a worker with human capital h . Equation (B.38) is the surplus generated in a match between a less-educated worker and simple job, (B.39) shows the surplus in cross-skill matches between highly-educated workers and simple jobs, and finally equation (B.40) is the total surplus in a match between a highly-educated worker in a complex job.

The planner's choice of forming cross-skill matches is governed by the sign of $\partial \mathcal{H} / \partial \kappa_i$,

which gives $\kappa_i \in [0, 1]$ if

$$\frac{\mu x_s - b}{(1 - \mu)x_c - b} + \frac{(1 - \nu)f(\theta)\Psi}{a_i((1 - \mu)x_c - b)(r + \delta + f(\theta)(1 - \zeta))} = \frac{f(\theta)(1 - \zeta)}{r + \delta + f(\theta)(1 - \zeta)}. \quad (\text{B.42})$$

The planner's optimal choice for vacancies satisfies $\partial \mathcal{H} / \partial v_\chi = 0$ for $\chi \in \{s, c\}$:

$$\frac{\gamma}{q(\theta)} = \eta \sum_i \hat{\pi}_i^0 [\Upsilon_{s,i}^0 - \Lambda_i^0] + (1 - \eta) \sum_i \hat{\pi}_i^1 \kappa_i [\Upsilon_{s,i}^1 - \Lambda_i^1] - \nu \Psi, \quad (\text{B.43})$$

$$\frac{\gamma}{q(\theta)} = (1 - \eta) \sum_i \hat{\pi}_i^1 [\Upsilon_{c,i}^1 - \Lambda_i^1] - \nu \Psi. \quad (\text{B.44})$$

The planner opens vacancies until the expected cost to fill the vacancy is equal to the expected surplus generated when the vacancy is filled. Equations (B.43) and (B.44) can be combined to show that

$$\eta \sum_i \hat{\pi}_i^0 [\Upsilon_{s,i}^0 - \Lambda_i^0] + (1 - \eta) \sum_i \hat{\pi}_i^1 \kappa_i [\Upsilon_{s,i}^1 - \Lambda_i^1] = (1 - \eta) \sum_i \hat{\pi}_i^1 [\Upsilon_{c,i}^1 - \Lambda_i^1] = \Psi, \quad (\text{B.45})$$

which states the the expected surplus generated by each type of vacancy is equal to the average surplus generated by a match. Combining the surplus terms, (B.38)-(B.40), and (B.45) with (B.43)-(B.44) gives

$$\begin{aligned} \frac{\gamma}{q(\theta)(1 - \nu)} &= \eta \sum_i \hat{\pi}_i^0 \frac{a_i[\mu x_s - b] + (1 - \nu)f(\theta)\Psi}{r + \delta + f(\theta)\zeta} + \\ (1 - \eta) \sum_i \hat{\pi}_i^1 \kappa_i &\frac{a_i[(\mu x_s - b)(r + \delta) - f(\theta)(1 - \zeta)((1 - \mu)x_c - \mu x_s)] + (r + \delta)f(\theta)(1 - \nu)\Psi}{(r + \delta)(r + \delta + f(\theta)(1 - \zeta + \zeta\kappa_i))}, \end{aligned} \quad (\text{B.46})$$

$$\frac{\gamma}{q(\theta)(1 - \nu)} = (1 - \eta) \sum_i \hat{\pi}_{1,i} \frac{a_i[((1 - \mu)x_c - b)(r + \delta) - f(\theta)\zeta\kappa_i(\mu x_s - (1 - \mu)x_c)] + (r + \delta)f(\theta)(1 - \nu)\Psi}{(r + \delta)(r + \delta + f(\theta)(1 - \zeta + \zeta\kappa_i))}. \quad (\text{B.47})$$

Equations (B.46)-(B.47) are analogous to the free-entry conditions in the decentralized equilibrium.

The final component of the solution to the planner's problem is the choice of human capital investment, which is governed by the signed of $\partial \mathcal{H} / \partial h(a_i, \omega)$:

$$h(a_i, \omega) = \begin{cases} 0 & \text{if } \Lambda_i^1 - \Lambda_i^0 < \varsigma + \varphi(\ell), \\ \in [0, 1] & \text{if } \Lambda_i^1 - \Lambda_i^0 = \varsigma + \varphi(\ell), \\ 1 & \text{if } \Lambda_i^1 - \Lambda_i^0 > \varsigma + \varphi(\ell), \end{cases} \quad (\text{B.48})$$

where

$$\Lambda_i^1 - \Lambda_i^0 = \frac{a_i f(\theta)}{r} \left\{ \frac{\zeta \kappa_i \mu x_s + (1 - \zeta)(1 - \mu)x_c}{r + \delta + f(\theta)(1 - \zeta + \zeta \kappa_i)} - \frac{\zeta \mu x_s}{r + \delta + f(\theta)\zeta} + \frac{b(r + \delta)[\zeta(2 - \kappa_i) - 1]}{(r + \delta + f(\theta)(1 - \zeta + \zeta \kappa_i))(r + \delta + f(\theta)\zeta)} + \frac{f(\theta)(1 - \nu)\Psi[1 + \zeta(\kappa_i - 2)]}{(r + \delta + f(\theta)(1 - \zeta + \zeta \kappa_i))(r + \delta + f(\theta)\zeta)} \right\}. \quad (\text{B.49})$$

Suppose that parameter values are such that all cross-skill matches are formed in both the decentralized equilibrium and efficient steady-state (i.e., $\kappa_i = 1$ for $i \in \{L, H\}$). A comparison of (B.8)-(B.9) and (B.46)-(B.47) show that these pairs of equations may only coincide with each other when $(\nu, \beta) = (1, 1)$. But in the case where $\beta = 1$, there is no interior solution as firms have no incentive to post vacancies. ■

B.11 Proof of Proposition 6

To help illustrate the results, I make an additional simplifying assumption of $\lambda = 0$. Suppose that $\epsilon = 1$. I begin by showing that the supply channel outweighs the composition channel. This can be illustrating by showing

$$\frac{(1 - \eta)\mathbb{E}[a|h = 1]}{\partial H} > 0, \quad (\text{B.50})$$

where

$$\eta(\theta, \zeta) = \frac{(1 - H)(\delta + f(\theta))}{\delta + Hf(\theta)\zeta + (1 - H)f(\theta)}, \quad (\text{B.51})$$

and

$$\mathbb{E}[a|h = 1] = \frac{\pi h(a_L)a_L + (1 - \pi)h(a_H)a_H}{\pi h(a_L) + (1 - \pi)h(a_H)}. \quad (\text{B.52})$$

It is straitforward to show that (B.50) is true if and only if

$$\frac{(1 - \pi)h(a_L)(a_H - a_L)}{\pi h(a_L)a_L + (1 - \pi)h(a_H)a_H} < \frac{\delta + f(\theta)}{\delta + Hf(\theta)\zeta + (1 - H)f(\theta)}. \quad (\text{B.53})$$

The right hand side of equation (B.53) is always greater than 1. The left hand side is greater than 1 if

$$a_H \left[\frac{h(a_L) - h(a_H)}{h(a_L)} \right] > \frac{a_L}{1 - \pi}. \quad (\text{B.54})$$

From Lemma 2, we know that $h(a_H) \geq h(a_L)$. Therefore, (B.54) is never satisfied, the left hand side of (B.53) is never greater than 1, and (B.53) always holds.

Now suppose that $a_L = a_H = 1$ and $\epsilon < 1$. Consider the term $(1 - \eta(\theta, \zeta))p_c$, where the prices of intermediate goods are given by equations (B.22) and (B.23). The sign of

$\partial(1 - \eta(\theta, \zeta))p_c/\partial H$ determines the net effect of the supply and relative price channels on the expected profits of posting a complex vacancy. Differentiating with respect to H , one can show that

$$\frac{\partial(1 - \eta(\theta, \zeta))p_c}{\partial H} < 0 \tag{B.55}$$

if and only if

$$(1 - \eta(\theta, \zeta)) \left(\mu^{(Y_s/Y_c)} + 1 - \mu \right) < \frac{x_s \zeta (1 - \epsilon) \mu}{x_c (1 - \zeta)} (Y_s/Y_c)^{\epsilon-1}, \tag{B.56}$$

which is true as $\epsilon \rightarrow -\infty$. ■

C Segmented markets

It may be unrealistic to assume a completely unsegmented labor market. For example, if the differences between simple and complex jobs are large enough, it may be more reasonable to assume that the markets for these types of jobs are completely segmented into their own submarkets. To study the implications of this type of market structure, this appendix analyzes a version of the model where there are separate submarkets for simple and complex vacancies. Workers direct their search into a submarket to where, within each market, there is still random search and ex-post bargaining over employment contracts. That is, firms cannot commit to a posted wage or a single type of worker to hire. To simplify the exposition while maintaining the same insights, I consider the case without on-the-job search.

Suppose that there are two markets indexed by job type $\chi \in \{s, c\}$. I assume that the flow cost to post vacancies differs across markets and is given by γ_χ with $\gamma_s < \gamma_c$. Meetings within each market are generated by the same matching technology as in the baseline model.

Let ϱ denote the probability that the worker permanently enters the market for simple jobs. It follows that market tightness in the market for skilled jobs is given by $\theta_s \equiv v_s/[u_0 + \varrho u_1]$ and market tightness in the market for complex jobs is given by $\theta_c \equiv v_c/[(1 - \varrho)u_1]$. The share of less-educated workers among all unemployed workers searching in the market for simple jobs is given by $\eta \equiv u_0/[u_0 + \varrho u_1]$. All other assumptions are the same as in Section 3.

The main changes in this version of the model's equilibrium begin with the value of being an unemployed worker, as seen below:

$$rU(a, 0) = ba + \beta f(\theta_s)[E_s(a, 0) - U(a, 0)], \quad (\text{C.1})$$

$$rU(a, 1) = ba + \max_{\varrho \in [0, 1]} \beta (\varrho f(\theta_s)[E_s(a, 1) - U(a, 1)] + (1 - \varrho) f(\theta_c)[E_c(a, 1) - U(a, 1)]), \quad (\text{C.2})$$

where $f(\theta_\chi)$ is the job finding rate in market for type χ jobs.⁵³ According to (C.1), less-educated workers only search in the market for simple jobs. This is because less-educated workers produce zero output in complex jobs and thus would never search in that market with a positive probability.

Equation (C.2) shows that highly-educated workers choose the probability of permanently searching in the market for simple jobs, ϱ , to maximize their lifetime discounted utility. I assume that highly-educated workers always form matches with simple jobs as they would never enter the market for simple jobs (i.e. $\varrho = 0$) if cross-skill matches generated a negative surplus.

⁵³Equations (C.1) and (C.2) already account for the solution to the optimal employment contracts and show that the worker receives a share β of the total match surplus.

Free entry of firms into each sub-market implies:

$$\frac{\gamma_s}{q(\theta_s)(1-\beta)} = \eta \sum_i \hat{\pi}_{i,0} [E_s(a_i, 0) - U(a_i, 0)] + (1-\eta) \sum_i \hat{\pi}_{i,1} [E_s(a_i, 1) - U(a_i, 1)], \quad (\text{C.3})$$

$$\frac{\gamma_c}{q(\theta_c)(1-\beta)} = \sum_i \hat{\pi}_{i,1} [E_c(a_i, 1) - U(a_i, 1)]. \quad (\text{C.4})$$

Conditions (C.3) and (C.4) simply state that market tightness in each submarket adjusts so that the expected costs to fill a vacancy in each market are equal to the surplus of filling a vacancy.

Taking the first order condition of (C.2) with respect to ϱ gives

$$\varrho = \begin{cases} 0 & \text{if } f(\theta_s) [E_s(a, 1) - U(a, 1)] < f(\theta_c) [E_c(a, 1) - U(a, 1)], \\ \in [0, 1] & \text{if } f(\theta_s) [E_s(a, 1) - U(a, 1)] = f(\theta_c) [E_c(a, 1) - U(a, 1)], \\ 1 & \text{if } f(\theta_s) [E_s(a, 1) - U(a, 1)] > f(\theta_c) [E_c(a, 1) - U(a, 1)]. \end{cases} \quad (\text{C.5})$$

From (C.5), highly-educated workers only search in the market for complex jobs ($\varrho = 0$) if the interaction between the job-finding rate and surplus in that type of match is larger than the same term in the market for simple jobs. This type of equilibrium is referred to as complete separation: highly-educated workers only search for complex jobs.⁵⁴ An equilibrium of partial mixing, $\varrho \in (0, 1)$, occurs if the highly-educated worker is indifferent between the two markets. After some simple algebra, one can obtain the following surplus terms:

$$E_s(a, 1) - U(a, 1) = \frac{a\{p_s x_s (r + \delta + \beta(1-\varrho)f(\theta_c)) - \beta(1-\varrho)f(\theta_c)p_c x_c - (r + \delta)b\}}{(r + \delta)(r + \delta + \beta[\varrho f(\theta_s) + (1-\varrho)f(\theta_c)])}, \quad (\text{C.6})$$

$$E_c(a, 1) - U(a, 1) = \frac{a\{p_c x_c (r + \delta + \beta\varrho f(\theta_s)) - \beta\varrho f(\theta_s)p_s x_s - (r + \delta)b\}}{(r + \delta)(r + \delta + \beta[\varrho f(\theta_s) + (1-\varrho)f(\theta_c)])}. \quad (\text{C.7})$$

From equations (C.6) and (C.7), it is straightforward to show that the surplus generated in a complex match is greater than the surplus in a simple match if $p_c x_c > p_s x_s$, which I assume is always true. Combining this result with equation (C.5) reveals that a necessary condition for highly-educated workers to be indifferent between searching in the simple or complex markets and choose $\varrho \in (0, 1)$ if $f(\theta_s) > f(\theta_c)$. The intuition for this is simple: highly-educated workers must be compensated with a higher job-finding rate in the market with simple jobs in order to search there to offset the lower surplus that they earn from a match. This is the standard tradeoff found in models of directed search whereby workers can either search in markets with a low (high) wage and a high (low) job-finding rate.

Suppose that workers are indifferent between searching for simple and complex jobs. It

⁵⁴I do not consider the case of $\varrho = 1$ as workers would never invest in human capital to only search for simple jobs.

follows that the workers are then indifferent between the pure strategies of $\varrho = 0$ and $\varrho = 1$ and that $U(a, 0) = U(a, 1)$, as less- and highly-educated workers are equally productive in simple jobs. It is easy to show that under these assumptions that the free-entry conditions are given by

$$\frac{r + \delta + \beta f(\theta_s)}{q(\theta_s)} = \frac{(1 - \beta)(p_s x_s - b) \sum_i \pi_i a_i}{\gamma_s}, \quad (\text{C.8})$$

$$\frac{r + \delta + \beta f(\theta_c)}{q(\theta_c)} = \frac{(1 - \beta)(p_c x_c - b) \sum_i \hat{\pi}_{1,i} a_i}{\gamma_c}. \quad (\text{C.9})$$

The left hand side of equations (C.8) and (C.9) are increasing in θ_s and θ_c , respectively. It follows that $\theta_s > \theta_c$ if and only if

$$\frac{\gamma_c}{\gamma_s} > \frac{(p_c x_c - b) \sum_i \hat{\pi}_{1,i} a_i}{(p_s x_s - b) \sum_i \pi_i a_i}. \quad (\text{C.10})$$

Equation (C.10) shows that if the relative flow cost to fill a complex vacancy is larger than the relative productivity of a complex vacancy, then $\theta_s > \theta_c$ and it is possible to have a partial mixing equilibrium.⁵⁵

Consider the case where (C.10) holds and $\theta_s > \theta_c$. From equations (C.6) and (C.7), workers are indifferent between searching for simple and complex jobs if and only if

$$f(\theta_c)\{p_c x_c(r + \delta + \beta f(\theta_s)) - (r + \delta)b\} = f(\theta_s)\{p_s x_s(r + \delta + \beta f(\theta_c)) - (r + \delta)b\}. \quad (\text{C.11})$$

Equation (C.11) defines an increasing relationship between θ_c and θ_s that will ensure a highly-educated worker's indifference between searching for simple and complex jobs. Thus, in an environment with segmented markets, it is only in a knife-edge case where (C.11) holds that highly-educated workers are indifferent between searching for simple and complex jobs. This is in addition to the necessary condition that (C.10) holds and $\theta_s > \theta_c$.

Moreover, recall the worker's human capital decision upon birth, (B.10). It is straitforward to see that if $U(a, 0) = U(a, 1)$, then there is no incentive to invest in human capital. Taken together, with a segmented labor market, an equilibrium with partial mixing and underemployment is not robust as it is only sustained in a knife-edge case and workers would not invest in human capital if they were indifferent between searching in both submarkets.⁵⁶

⁵⁵Blázquez and Jansen (2008) consider the case where the flow vacancy costs are equal across jobs and show that there can never be a partial mixing equilibrium.

⁵⁶This equilibrium also requires that the job-finding rate among less-educated workers is higher than the job-finding rate of highly-educated workers, which is counterfactual to U.S. data.