

Precautionary Demand for Money in a Monetary Business Cycle Model*

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Abstract

We investigate the quantitative implications of precautionary demand for money for business cycle dynamics of velocity of money and other nominal aggregates. There is a standing challenge in monetary macroeconomics to account for such dynamics, as previous business cycle models that have tried to incorporate demand for money have failed to generate realistic predictions in this regard. Our stance is that part of this failure results from the fact that demand for money in those previous models is deterministic, since agents in them face only aggregate risk, whereas we believe idiosyncratic risk to be important as well. We conduct the exercise inside a stochastic cash-credit good model, where cash-good consumption is subject to idiosyncratic uninsurable preference shocks. We find that precautionary demand for money plays a substantial role in accounting for business cycle behavior of velocity of money and other nominal variables such as inflation and nominal interest rates. In the result analysis, we isolate the precautionary demand channel to demonstrate its quantitative role in the model. In addition, we compare our results to several existing monetary models that do not have idiosyncratic uncertainty, to show that we improve substantially on their performance.

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1 Introduction

In this paper, we study, theoretically and quantitatively, aggregate business cycle implications of precautionary demand for money. It is an outstanding challenge in the literature to account for business cycle behavior of nominal aggregates, their interaction with real aggregates, as well as to account for seemingly substantial real effects of monetary policy. Business cycle models that have tried to incorporate money through, for example, cash-in-advance constraints, have done so by assuming that agents face only aggregate risk, which has resulted in the demand for money being largely deterministic, in the sense that the cash-in-advance constraint almost always binds. These models had unrealistic implications for the dynamics of nominal variables, as well as for interaction between real and nominal variables, when compared to data (see, e.g., Cooley and Hansen, 1995).

Yet precautionary motive for holding liquidity seems to be strong in the data (Telyukova, 2008), implying that *idiosyncratic* risk may play a key role for money demand, and thus that its aggregate implications are important to investigate. The goal of this paper is to test this hypothesis. The set of questions we want to answer is: (a) What are the aggregate implications of precautionary demand for money? (b) Can it help account for business cycle dynamics of velocity of money, interest rates and inflation? (c) Can it help to account for real effects of monetary policy?

We start with a version of the standard cash-good-credit-good model that combines a productive real sector with a monetary retail sector. We incorporate into this model uninsurable idiosyncratic preference risk which creates a precautionary motive for holding liquidity. Telyukova (2008) demonstrates that this idiosyncratic uncertainty appears significant in the data, and in turn creates sizeable observed precautionary balance holdings, absent in most standard models. In a standard deterministic-demand setting, nonstochastic cash-credit good models, for example, calibrated to aggregate data cannot account for aggregate facts such as variability of velocity of money, correlation of velocity with output growth or money growth, correlation of inflation with nominal interest rates, and others, as Hodrick, Kocherlakota and Lucas (1991) have shown. The reason is that in such models, agents' money demand is almost entirely deterministic, because the only type of uncertainty the households face in these models is aggregate uncertainty, and the amount of aggregate uncertainty in the data is not large enough to generate significant precautionary motives for holding money in the model. Then, the cash-in-advance constraint almost always binds and money demand is made equivalent to cash-good consumption. This also implies that volatility of money demand is tightly linked to volatility of aggregate consumption. Aggregate consumption is not volatile enough in the data to generate enough volatility of money demand or other nominal aggregates.

We show that incorporating precautionary demand for money generated by unpredictable idiosyncratic variation, in combination with aggregate uncertainty, can

help account for monetary issues mentioned above, by breaking the link between money demand and aggregate consumption. Agents generally hold more money than they spend, and money demand is no longer linked to *average* aggregate consumption, but rather to consumption of agents whose preference shock realizations make them constrained (i.e. they spend all of their balances) in trade. We show that velocity of money can be significantly more volatile in this heterogeneous-agent setting, thanks to the *unconstrained* agents, who are absent in standard deterministic cash-in-advance models.

We study this link qualitatively and quantitatively in a stochastic version of a model that combines, in each period, two types of markets in a sequential manner, as in, for example, Lucas and Stokey (1987). The first market is a standard Walrasian market, which we will term, somewhat loosely, the “credit market”. The second market is also competitive, but characterized by anonymity and the absence of barter possibility, which makes a medium of exchange - money - essential in trade. We term this market the “cash market”. Agents’ utility function in the credit market features linear preferences on labor, which allows us to simplify the model by not having to keep track of the distribution of agents across wealth, but other features of the model will still render the model analytically intractable, which leads us to rely on computational methods to solve the model. The credit market is much like a standard real business cycle model, except that trade in this market can be conducted using either money or credit, and agents have to decide how much money to carry out of the market for future cash consumption. The production function in this market is subject to aggregate productivity shocks.

At the start of the cash market, agents are subject to preference shocks which determine how much they want to consume during the subperiod, but the realization of the shock is not known at the time that agents make their portfolio decisions. This generates precautionary motive for holding liquidity. One additional feature of our model that is not standard in search models of money is that we allow capital, which exists in the credit market, to be indirectly productive in the cash market, in the following way: we assume that the goods that are sold in the cash market are made out of goods produced, using capital and labor, in the credit markets. This introduces an explicit link between the real and monetary sectors of the economy.

One of the contributions of our work is the calibration of the model. To our knowledge, all the models of the types we mentioned above that have looked at aggregate behavior of nominal variables, have been calibrated to aggregate data. Instead, we use micro survey data on liquid consumption from the Consumption Expenditure Survey, like in Telyukova (2008), to calibrate idiosyncratic preference risk in our cash market. Using these data, we are able to constrain our calibration further than is commonly the case. In general, preference risk of the type that creates precautionary liquidity demand has not been measured in calibration of other models, and in the few contexts where precautionary liquidity demand has

appeared, it has been treated as a free parameter (e.g. Faig and Jerez, 2007). Our use of micro data allows us to be very disciplined in our approach, and thus makes our results more comparable to data.

Incorporating idiosyncratic preference uncertainty based on micro evidence appears to improve significantly the quantitative performance of the model along a number of dimensions. Once calibrated, we solve the model computationally to investigate the effects of real productivity shocks and monetary policy shocks. We find that precautionary demand for money alone makes a dramatic difference for the model in terms of helping it account for a variety of dynamic moments related to nominal aggregates in the data. We test these results by also computing a version of the model where we shut down the idiosyncratic risk, and find that without it, the model is incapable of reproducing any of the key moments in the data, much as previous literature has suggested.

Our results lead us to conclude that in many monetary contexts, especially those aimed at accounting for aggregate data facts, it is important not to omit idiosyncratic uncertainty that gives rise to precautionary demand for money. As one example, omitting this empirically relevant and quantitatively significant mechanism may cause the standard practice of calibrating monetary models to the aggregate money demand equation (as has been done in many cash-in-advance models and monetary search models) to produce incorrect results for parameters. If aggregate money demand includes sizeable precautionary balances, then a model that omits this mechanism may not align with data well.

This paper is related to several strands of literature. On the topic of precautionary demand for liquidity, the key mechanism in our model is close to Faig and Jerez (2007), Telyukova (2008) and Telyukova and Wright (2008). In Telyukova (2008), the idiosyncratic uncertainty about the need for liquid consumptions in the data is documented to be significant and shown to be quantitatively relevant for the household portfolio decisions with respect to the joint holding of liquidity and credit card debt, in a calibrated heterogeneous-agent model that is related in some respects to our model in this paper. In Telyukova and Wright (2008), preference shocks for liquidity are embedded in a Lagos-Wright type of model to study the impact of these shocks on the role of money and credit in a general equilibrium setting. Faig and Jerez (2007) look at the behavior of velocity and nominal interest rates over the long run. They find that the estimated time series of velocity over the last century, interpreted as the outcome of a sequence of steady states, each conditioned by the nominal interest rate, fits the empirical series well. We share with Faig and Jerez the interest in aggregate quantitative implications of precautionary demand for money. However, we concentrate on business cycle fluctuations and do so in an full-fledged business-cycle economy with capital and the other standard ingredients, such as technology and monetary shocks. Moreover, their estimation is based on aggregate level observations and does not constrain the parameterization of the

preference shocks, which naturally are an important determinant of the magnitude of precautionary liquidity demand, to fit the data. It is worth emphasizing, though, that the mechanisms in Faig and Jerez (2007) are closely related to our model, and their empirical insights are complementary to ours.¹

On the broad subject of accounting for aggregate behavior of nominal variables such as velocity of money, another paper that is related to ours in focus, although not in the mechanism, is Wang and Shi (2006). They investigate the business cycle properties of velocity of money in a monetary search model, but the main mechanism in their model that generates the fluctuations is connected to search intensity, rather than to precautionary liquidity demand. Another paper that focuses on aggregate properties of a monetary model is Chiu and Molico (2008) - theirs is also a monetary search model, like Wang and Shi (2006), without the variable search intensity mechanism. The do, however, model both aggregate and idiosyncratic risk (of a different kind). However, their focus is on studying distributional properties of inflation and monetary policy as well as on optimal monetary policy rules in a setting where shocks propagate slowly through the heterogeneous households. Naturally, they do not have a degenerate distribution of money holdings.

The paper is organized as follows. Section 2 describes the model and characterizes the equilibrium. Section 3 demonstrates analytically the impact of precautionary demand for money on the dynamic behavior of money, velocity and interest rates. Section 4 describes our calibration strategy, and section 5 details the solution algorithm. Section 6 presents our results and discusses the quantitative role of precautionary liquidity demand. Section 7 concludes.

2 Model

The economy is populated by a measure 1 of households, who live infinitely in discrete time. The households rent labor and capital to firms, consume goods bought from the firms, and save. There are two types of markets open sequentially during the period: a Walrasian market operates during the first subperiod, while a market in which money is essential operates during the second subperiod. In the Walrasian market, all parties involved in transactions are known and all trades can be enforced; intertemporal trade and asset trading are possible. The second market is competitive, in the sense that all agents are price takers, but everyone is anonymous. In addition, in our setup there is no scope for barter in this market – households are

¹Hagedorn (2008) also has looked at aggregate implications of precautionary demand for money. He demonstrates that strong liquidity effects that translate into significant aggregate effects can arise when precautionary demand for money is taken into account in an otherwise standard cash-credit good model. His setting is quite different from ours, in that he generates the liquidity effect using banks, which we abstract from; on the other hand, his model has no real sector and no aggregate uncertainty.

always buyers who have no goods to offer, by construction. As a result, no trade would occur in the absence of a medium of exchange, so that a medium of exchange – here, fiat money – is essential (Levine 1991, Temzelides and Yu 2003, Rocheteau and Wright 2005). Since in the first market households pay with either cash or credit for consumption, we will refer to this, slightly broadly, as the credit market, while the second subperiod - where payment takes place using money - will be termed the cash market.

There are also two types of firms in the economy. Production firms use capital and labor as inputs in production, and sell their output in the first subperiod credit market. This output is used for consumption and capital investment in the credit market. However, part of the output is also bought up in the credit market by retail firms,² who then transform the goods into retail goods to be sold in the cash market.

2.1 Households

Households maximize lifetime expected discounted utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t v_t(c_t, q_t, h_t, \vartheta_t), \quad (1)$$

where $0 < \beta < 1$. v_t , the utility achieved in each period, depends on consumption in the centralized market, c_t , and consumption in the decentralized market, q_t , time spent working h_t , and the consumption preference shock ϑ_t .³ In the first subperiod, utility follows the Hansen-Rogerson specification of indivisible labor with lotteries, and is given in reduced form by

$$U(c_t) - Ah_t. \quad (2)$$

Second-subperiod utility, upon consuming q_t of the cash good, is

$$\vartheta_t u(q_t). \quad (3)$$

The taste shock ϑ_t realizes when the credit market is already closed and money holdings can no longer be adjusted, as described below. This will lead to precautionary demand for money. The taste shock comes from a distribution with finite support.

²We chose the name “retailers” to indicate that the role of these firms is to buy up goods and make them available to households in the cash market. In the context of the model, we can think of credit goods being transformed one-for-one into goods that have to be sold locally in a market which is large enough to be competitive, and where trades are anonymous. Moreover, our retailers are not necessarily meant to correspond one-for-one to the retail sector in the data: some retailers in the data are better characterized as selling in the credit market, whereas some non-retail firms sell predominantly in the cash sector.

³Subscripting our variables by t is an abuse of notation, as we mean that each variable at time t is actually chosen conditional on the entire history the household has experienced up to that period, in principle. Moreover, the expectation is taken with respect to all possible histories.

In this section, the probability of a realization ϑ is implied by the expectation operator. Later we will assume the shock to come from a discrete set with the same support, and will also refer to it by its discrete probability $\mathbb{P}(\vartheta)$.

Households own capital k_t , hold money \hat{m} for future trade in the cash market, and own nominal bonds, sold to them by retail firms, as detailed below.⁴ We normalize the price of the credit good to one. We also normalize the household's money holdings by the aggregate money holdings. Given a measure 1 of households, m is then this normalized measure of money holdings, with \tilde{m} as the normalized counterpart of \hat{m} above. Let wage, capital rent (net of depreciation), value of one unit of normalized money, and the return on nominal bonds b_t be $w_t, r_t, \phi_t, i_{t-1}$. The budget constraint, expressed in real terms, is

$$\phi_t m_t + (1 + r_t)k_t + w_t h_t + \phi_t b_t (1 + i_{t-1}) = c_t + \phi_t \tilde{m}_t + k_{t+1} + \phi_t b_{t+1} \quad (4)$$

Moreover, the household chooses its money holdings \tilde{m} in the credit market before ϑ realizes. Given price ψ_t of the cash good, consumption q_ϑ in the cash market, conditional on the preference shock realization ϑ , has to satisfy

$$\psi_t q_{\vartheta,t} \leq \tilde{m}_t. \quad (5)$$

Finally, hours worked are constrained,

$$h \in [0, 1]. \quad (6)$$

At the beginning of the period the government makes a lump sum transfer $\varpi_t M_t$, where M_t is the aggregate money stock. The full household problem then is specified as

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(U(c_t) - A h_t + \vartheta_t u(q_{\vartheta,t}) \right) \right] \quad (7)$$

with respect to $c_t, h_t, k_{t+1}, \tilde{m}_t, b_{t+1}$, and the vector $\{q_{\vartheta,t}\}$, taking as given $w_t, r_t, i_t, \phi_t, \psi_t$, and subject to constraints (4)-(6) and the evolution of money holdings given by

$$m_{t+1} = \tilde{m}_t - \psi_t q_{\vartheta,t} + \varpi_t M_t. \quad (8)$$

The value (supremum) of the household's problem in the sequential formulation is finite under standard assumptions⁵. Bellman's Principle of Optimality holds, and we can equivalently formulate the problem recursively, which we do below.

⁴In principle, households can hold shares of firms as well. We will see that in our formulation all firms make zero profits, share holding is irrelevant. Alternatively, we can formulate the economy with firms selling shares instead of bonds; this leads to equivalent allocations of resources, but involves more notation.

⁵Each period, the amount of utility that can be achieved is bounded from above by $\sup_x U(x) + \sup_q u(q)$, where x, q are constrained by the boundedness of capital stock k .

2.2 Production Firms

The problem of the production firm is static and completely standard – to maximize its profits in each period. Given a constant returns to scale production function $y_t = e^{z_t} f(k_t, h_t)$, where z_t is the stochastic productivity level described in more detail below, the problem is: $\max_{k_t, h_t} \{e^{z_t} f(k_t, h_t) - w_t h_t - r_t k_t\}$, the solution of which is characterized by the usual first-order conditions.

2.3 Retail Firms

Retail firms are assumed exist for two periods: they buy the goods in the credit market, borrowing from the households to do so, sell these in the subsequent cash market, and settle their debt obligations in the following credit market, before disbanding. Free entry in the retail market yields the following condition, expressed in nominal terms at time t :

$$\Pi_{rt} = \max_{q_t} \frac{\psi_t q_t}{1 + i_t} - \frac{q_t}{\phi_t} = 0. \quad (9)$$

All cash receipts are going towards repayment in the following period, the value of which is discounted to the current period using the nominal interest rate. The repayment equals the nominal value in the current period for the q_t goods that were purchased in the credit market by the retailers. Since the cash market is competitive, retail firms will sell all their goods in equilibrium.⁶

2.4 Monetary Policy and Aggregate Shocks

The monetary authority follows an interest rate feedback rule

$$\frac{1 + i_{t+1}}{1 + i} = \left(\frac{1 + i_t}{1 + i} \right)^{\rho_i} \left(\frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\rho_\pi} \left(\frac{y_t}{\bar{y}} \right)^{\rho_y} \exp(\varepsilon_{t+1}^{mp}). \quad (10)$$

The term ε^{mp} denotes a stochastic monetary policy shock which realizes at the beginning of the period. Consistent with the movement in interest rates, the rate of money supply growth ϖ_t adjusts, thus determining the lump-sum injections $\varpi_t M_t$ paid out to the households, where ϖ_t refers to money supply growth from period t to $t + 1$. In addition, we will assume that the monetary policy shock ε^{mp} and the stochastic productivity shock z are correlated, so that the interest rate responds to changes in the productivity levels and vice versa.

2.5 Recursive Formulation of the Household Problem

We focus on Markov (payoff-relevant) decision strategies. The constraint set of the household depends only on the state variables discussed below, and we assume (and later prove) that all prices only depend on the aggregate state variables.

⁶We assume that goods are storable, so even at a zero expected real interest rate, this is without loss of generality.

From now on, we will conserve notation by omitting time subscripts, and using primes to denote $t + 1$. The aggregate state variables in this economy are the aggregate capital stock, K , the technology shock z , the previous interest rate in the economy i_{-1} , current interest rate i , and the term $(1 + \varpi_{-1})\phi_{-1}$, which denotes the previous period's post-injection real value of money, and which households need to know in order to determine the current rate of inflation in the economy. We will denote these as $S = (K, z, i_{-1}, i, (1 + \varpi_{-1})\phi_{-1})$. The individual state variables at the beginning of the centralized market are normalized money holdings m , capital holdings k , and bond holdings b . Recall that individual money holdings m are defined relative to total money stock M : if a household holds the average stock of money, then $m = 1$. This renders the money holdings stationary. We denote by ϕ the value of one unit of *normalized* money, which implies $\phi = M/P$, the real value of the total money stock.

Instead of writing this as a problem with separate value functions for centralized and decentralized subperiods, we can formulate the household's problem as a more transparent full-period problem. This means that in the first subperiod the household can make the choices for the second subperiod, contingent on its information at the start of the second subperiod, which is the realization of its preference shock ϑ_t . In sum, we have the following recursive maximization problem:

$$V(k, m, b, S) = \max_{c, h, \tilde{m}, k', b', \{q_\vartheta\}} \left\{ U(c) - Ah \right. \quad (11)$$

$$\left. + \mathbb{E}_\vartheta \left[\vartheta u(q_\vartheta) + \beta \mathbb{E}_{z', i'} V(k', m', \frac{b'}{1 + \varpi}, S') \right] \right\}$$

$$\text{s.t. } c + \phi \tilde{m} + k' + \phi b' = \phi m + \phi b(1 + i_{-1}) + (1 + r - \delta)k + wh \quad (12)$$

$$\psi q_\vartheta \leq \tilde{m} \quad (13)$$

$$\psi = \frac{1 + i}{\phi} \quad (14)$$

$$\pi = \frac{(1 + \varpi_{-1})\phi_{-1}}{\phi} \quad (15)$$

$$[z', i'] = \Xi_0 + \Xi_1[z, i, \pi, y] + [\varepsilon'_1, \varepsilon'_2] \quad (16)$$

$$1 + \varpi = \Omega(S) \quad (17)$$

$$m' = \frac{\tilde{m}}{1 + \varpi} + \frac{\varpi}{1 + \varpi} - \frac{\psi q_\vartheta}{1 + \varpi} \quad (18)$$

The term Ξ refers to a 2×4 matrix. Given today's aggregate states, in particular the nominal interest rate between today and tomorrow, i , the central bank will adjust the money growth rate to make i arise as an equilibrium price. As a result, we can write the money growth rate as a function of the current aggregate state in equation (17). Finally, equation (18) details the law of motion of the normalized money supply, incorporating expenditure in the cash market and the lump-sum transfer by the monetary authority.

Denote the household state variables as $s = (k, m, b)$. Denote the policy functions of the household's problem by $g(s, S)$, with $g_x(\cdot)$ as the policy function for the choice variable x .

2.6 Equilibrium

Definition 1. *A Symmetric Stationary Monetary Equilibrium is a set of pricing functions $\phi(S)$, $\psi(S)$, $w(S)$, $r(S)$; law of motions $K'(S)$ value function $V(s, S)$ and policy functions $g_c(s, S)$, $g_h(s, S)$, $g_k(s, S)$, $g_b(s, S)$, $g_m(s, S)$, $\{g_{q,\vartheta}(s, S)\}$, all ϑ , such that:*

1. *The value function solves the household optimization, in (11), with associated policy functions, given prices and laws of motion;*
2. *Production and retail firm optimize, given prices and laws of motion, as in sections 2.2 and 2.3.*
3. *Free entry of retailers: $\Pi_{rt} = 0$.*
4. *Consistent expectations: the aggregate law of motion follows from the sum of all individual decisions (index individual households by i) –*

$$K'(S) = \int_0^1 k^i(s, S) di$$

5. *Market clears in all markets (capital, labor, general goods, money, financial markets):*

$$\begin{aligned} \int_0^1 m^i(s, S) di &= 1 \\ \int_0^1 \phi \tilde{b}^i(s, S) di &= \int_0^1 \left[\sum_{\vartheta} \mathbb{P}(\vartheta) q_{\vartheta}(s, S) \right] di \\ \int_0^1 h^i(s, S) di &= H \\ (1 - \delta)K + e^z f(H, K) &= C + K' + \int_0^1 \left[\sum_{\vartheta} \mathbb{P}(\vartheta) q_{\vartheta}^i(s, S) \right] di \end{aligned} \quad (19)$$

2.7 Walrasian Market creates Homogeneity

For general utility functions, different realizations of the idiosyncratic preference shock would lead to a nontrivial distribution of wealth (with, for example, those who have recently experienced a sequence of high ϑ s, now being poorer on average).⁷ In turn, households with different wealth could make different portfolio decisions, and

⁷See Molico (2005) and Chiu and Molico (2008), who investigate the distributional impact of heterogeneity on e.g. the welfare effects of monetary policy when markets have search frictions.

hence the distribution across individual state variables would be relevant for the equilibrium prices.

However, the quasi-linear specification of the problem allows equilibria in which all heterogeneity created in the second subperiod washes out in the centralized market.⁸ This occurs if the boundary conditions of h are never hit, which we assume to be the relevant case below. Our quantitative strategy later is to solve the problem assuming that optimal choices of h are interior, and check (in our calibrated equilibrium) whether this is indeed the case.

After substituting the budget constraint for h into the household's value function, we can split the value function in two parts

$$\begin{aligned}
 V(s, S) = & \\
 & A \left(\frac{\phi m + (1 + r - \delta)k + \phi b(1 + i_{-1})}{w} \right) \\
 & + \max_{\dots} \left\{ U(c) - A \left(\frac{c + \phi \tilde{m} + k' + \phi b'}{w} \right) + \mathbb{E}_{\vartheta} [\vartheta u(q_{\vartheta}) + \beta \mathbb{E}_{z', i'} [V(s', S')]] \right\}.
 \end{aligned} \tag{20}$$

The following result is immediate, under the assumption of interiority on h .

Result 1. *The choice of c, \tilde{m}, k', b' only depends on the aggregate states S .*

Household wealth differs at the beginning of the Walrasian market, due to heterogeneous trading histories in the previous cash market, but households adjust their hours worked to be able to get the same amount of c, \tilde{m}, k', b' . The value function $V(\cdot)$ is differentiable in k, m, b (Benveniste-Scheinkman applies trivially). The envelope conditions are, then,

$$V_k(s, S) = \frac{A(1 + r(S) - \delta)}{w(S)} \tag{21}$$

$$V_m(s, S) = \frac{A\phi(S)}{w(S)} \tag{22}$$

$$V_b(s, S) = \frac{A\phi(S)(1 + i_{-1})}{w(S)} \tag{23}$$

Note that the envelope conditions are independent of the individual state variables. Hence, the expectation over ϑ does not matter for intertemporal choice variables, for example:

$$\mathbb{E}_{\vartheta} [\mathbb{E}_{z', i'} V_m(s, S)] = \mathbb{E}_{z', i'} V_m(s, S) = \mathbb{E}_{z', i'} \left[\frac{A\phi(S)}{w(S)} \right].$$

⁸This result has been used extensively in models that combine Walrasian markets with decentralized trade and idiosyncratic *matching* risk, such as Lagos and Wright (2005) and Rocheteau and Wright (2005). Here we use it to combine Walrasian markets with cash markets and idiosyncratic *preference* risk.

The problem is weakly concave in capital, labor and bond holdings, and the solution is interior by assumption that (6) is not binding. The Euler equations with respect to capital and bonds, and the first-order condition with respect to labor look as follows:

$$U'(c(S)) = \beta \mathbb{E}_{z',i'}[U'(c(S'))(1 + r(S') - \delta)] \quad (24)$$

$$U'(c(S)) = \frac{A}{w(S)} \quad (25)$$

$$\phi U'(c(S)) = \beta \mathbb{E}_{z',i'} \left[\frac{\phi'}{1 + \varpi} U'(c(S')) \right] (1 + i) \quad (26)$$

For future reference, we introduce the following notation, using marginal utilities defined in terms of the marginal productivity of labor (25):

$$\mathbb{E} \left[\frac{w(S)}{w(S')} \right] \equiv \widetilde{\mathbb{E}}, \quad \mathbb{E} \left[\frac{\phi'(S') w(S)}{1 + \varpi' w(S')} \right] \equiv \widetilde{\mathbb{E}\phi'}.$$

Note that $\frac{\phi}{\beta \mathbb{E}\phi'} = 1 + i$.

2.8 The Choice of Money Holdings and Cash Market Consumption

Above we have discussed the Euler equations that link consumption, capital investment and bond investment between periods. Taking as given that equations (24)-(26) are satisfied, money holdings solve the following problem (where without loss of generality, we can premultiply the maximization by the constant $\frac{w(S)}{A} = \frac{1}{U'(c)}$, which does not affect the maximization)

$$\max_{\tilde{m}, \{q_\vartheta\}} \left\{ -\phi \tilde{m} + \sum_{\vartheta} \mathbb{P}(\vartheta) \left(\frac{\vartheta u'(q_\vartheta)}{U'(c)} - \beta \widetilde{\mathbb{E}\phi'} \psi q_\vartheta \right) + \beta \widetilde{\mathbb{E}\phi'} \tilde{m} \right\} \quad (27)$$

subject to

$$\psi = \frac{1 + i}{\phi} \quad (28)$$

$$\psi q_\vartheta \leq \tilde{m} \quad \forall \vartheta \quad (29)$$

Substitute out (28), noting that $\beta \widetilde{\mathbb{E}\phi'} \psi = 1$. Denote μ_ϑ as the multipliers of (29), for each ϑ . First-order conditions then give

$$\mathbb{P}(\vartheta) \left(\frac{\vartheta u'(q_\vartheta)}{U'(c)} - 1 \right) - \psi \mu_\vartheta = 0 \quad (\text{FOC-}q_\vartheta)$$

and

$$-\phi + \sum_{\vartheta} \mu_\vartheta + \beta \widetilde{\mathbb{E}\phi'} = 0 \quad (\text{FOC-}\tilde{m})$$

while the complementary slackness (for each ϑ) gives

$$\left(\frac{\vartheta u'(q_\vartheta)}{U'(c)} - 1 \right) \left(\tilde{m} - q_\vartheta \frac{(1 + i)}{\phi} \right) = 0, \quad \tilde{m} - q_\vartheta \frac{(1 + i)}{\phi} \geq 0, \quad \frac{\vartheta u'(q_\vartheta)}{U'(c)} - 1 \geq 0$$

It is immediate that in this model cash balances are often not spent in full. Since a social planner would equate $U'(c)$ to $\vartheta u'(q_\vartheta)$, the following conclusions can be drawn:

Result 2. *If a shock ϑ results in a nonbinding constraint, then q_ϑ is the efficient quantity. Moreover, as long as the cash constraint does not bind, the quantity q_ϑ does not respond to the interest rate.*

Moreover, also observe that if $\hat{\vartheta}$ leads to a binding constraint, then for every $\vartheta > \hat{\vartheta}$, the cash constraint will bind. If $\hat{\vartheta}$ leads to a slack cash constraint, any $\vartheta < \hat{\vartheta}$ will lead to a nonbinding constraint. A binding cash constraint leads to underconsumption of the cash good relative to the social optimum.

2.9 Characterizing Equations

We summarize the above discussion in the system of first-order conditions and Euler equations that characterize the equilibrium of the problem:

$$\begin{aligned}
U'(c) &= \beta \mathbb{E}[U'(c')(1+r' - \delta)] & (30) \\
U'(c) &= \frac{A}{w} \\
\psi &= \frac{1+i}{\phi} \\
\tilde{\mu}_\vartheta &= \mathbb{P}(\vartheta_i) \left(\frac{\vartheta u'(q_\vartheta)}{U'(c)\psi} - \frac{\phi}{1+i} \right); \tilde{\mu}_\vartheta(\tilde{m} - \psi q_\vartheta) = 0 \forall \vartheta \\
\phi &= \sum_{\vartheta} \tilde{\mu}_\vartheta + \frac{\phi}{1+i} \\
\frac{\phi}{1+i} &= \frac{\beta \mathbb{E} \widetilde{\phi}'}{1+\varpi} \\
y + (1-\delta)k &= c + k' + \sum_{\vartheta} \mathbb{P}(\vartheta) q_\vartheta \\
[z', i'] &= \Xi_0 + \Xi_1[z, i, \pi, y] + [\varepsilon'_1, \varepsilon'_2]
\end{aligned}$$

3 Velocity and Precautionary Demand for Money with Idiosyncratic Uncertainty

In this section, we demonstrate that there are at least three ways in which idiosyncratic shocks to cash-good preferences can contribute to an improved quantitative performance of the model compared to other flexible-price monetary business-cycle models without the idiosyncratic shocks. First, the dynamic behavior of the value of money varies significantly with the probability that the marginal dollar is spent. Uncertainty about the need for cash leads to the following results, relative to an

environment with no idiosyncratic shocks: (a) we are no longer restricted to have an intertemporal elasticity of substitution higher than one to avoid counterfactual dynamic behavior of interest rates and prices, and (b) the elasticity of velocity that originates in the credit market (i.e. keeping the velocity in the cash market constant) with respect to nominal interest rates is raised for any σ , the rate of relative risk aversion. Second, part of the velocity fluctuation is now generated in the cash market, thus increasing the overall magnitude of velocity volatility, and velocity depends in an intuitive way on nominal interest rates. Third, the general equilibrium feedback of fluctuations in the real size of the cash market could be dampened relative to the no-shock model, because cash consumption only adjusts for the *binding* realization of the shock.

3.1 The Dynamic Behavior of the Value of Money

We start by studying the dynamic behavior of the value of the money stock; this relation will be an essential input for relating velocity to interest rates, for example. It is also, however, empirically relevant: one uncontroversial empirical regularity is the degree of persistence of interest rates, prices, and real balances (before and after detrending) over the business cycle. Nominal interest rates have an autocorrelation at quarterly frequency of 0.932; for real balances, it is 0.951.⁹ It seems a minimally necessary requirement that a monetary business cycle model can replicate the sign of these autocorrelations. This requirement turns out to imply, in the absence of precautionary money demand, stringent restrictions on the coefficient of relative risk aversion (RRA) σ . In fact, a standard range of parameters for the RRA coefficient (equivalently, coefficient of intertemporal elasticity of substitution - IES) used in real business cycle calibration by and large violates this restriction. With precautionary demand, it becomes possible to use parameters in this range.

For the sake of exposition, assume two shock realizations ϑ , where ϑ_h leads to a binding cash constraint, and ϑ_l to a nonbinding constraint. We write p for $\mathbb{P}(\vartheta_h)$; moreover, we simplify the utility function in the credit market to be fully linear, $u(c) = c$. (When we turn to general equilibrium effects, we generalize this). Reworking the characterizing equations (30) for the binding case, we find the following:

$$\phi = p \left(\vartheta_h u'(q_h) \frac{\phi}{1+i} - \frac{\phi}{1+i} \right) + \frac{\phi}{1+i}$$

Rewriting this, we get

$$\vartheta_h u'(q_h) = 1 + \frac{i}{p}; \tag{31}$$

without idiosyncratic uncertainty, the cash constraint always binds, and the right-hand side of the equation above equals $1 + i$.

⁹Detrended using the band pass filter, nominal interest rate from three-month treasury bonds, and real money balances from M2 and the GDP deflator (source: FRED2).

Now let us look at the behavior of real balances. From the system (30), we have that $q_h = \beta\widetilde{\mathbb{E}\phi}'$, with $(1+i) = \phi/(\beta\widetilde{\mathbb{E}\phi}')$. Then equation (31) turns into

$$\phi = p\vartheta u'(\beta\widetilde{\mathbb{E}\phi}')\beta\widetilde{\mathbb{E}\phi}' + (1-p)\beta\widetilde{\mathbb{E}\phi}'. \quad (32)$$

Now we can calculate what happens to the value of money today, ϕ , in response to a change in the discounted value of money tomorrow, $\widetilde{\mathbb{E}\phi}'$.

Lemma 1. *The elasticity of real balances today with respect to real balances tomorrow, $\varepsilon_{\phi, \beta\widetilde{\mathbb{E}\phi}'}$, evaluated at an equilibrium $\phi, \beta\widetilde{\mathbb{E}\phi}'$, equals*

$$\varepsilon_{\phi, \beta\widetilde{\mathbb{E}\phi}'} = \left(1 - \frac{1-p}{1+i}\right) (1-\sigma) + \frac{1-p}{1+i} \quad (33)$$

Proof. The derivative of ϕ with respect to $\beta\widetilde{\mathbb{E}\phi}'$, using (32), (26), (29) and $\tilde{m} = 1$, is

$$\frac{d\phi}{d(\beta\widetilde{\mathbb{E}\phi}')} = p\vartheta(u''(\beta\widetilde{\mathbb{E}\phi}')\beta\widetilde{\mathbb{E}\phi}' + u'(\beta\widetilde{\mathbb{E}\phi}')) + (1-p)$$

To get to the elasticity of ϕ with respect to $\beta\widetilde{\mathbb{E}\phi}'$, we divide both sides by $\phi/(\beta\widetilde{\mathbb{E}\phi}')$, and using that $\phi = p\vartheta_h u'(\beta\widetilde{\mathbb{E}\phi}')\beta\widetilde{\mathbb{E}\phi}' + (1-p)\beta\widetilde{\mathbb{E}\phi}'$, we find

$$\varepsilon_{\phi, \beta\widetilde{\mathbb{E}\phi}'} = p \frac{\vartheta(u''(\beta\widetilde{\mathbb{E}\phi}')\beta\widetilde{\mathbb{E}\phi}' + u'(\beta\widetilde{\mathbb{E}\phi}'))}{p\vartheta_h u'(\beta\widetilde{\mathbb{E}\phi}') + (1-p)} + (1-p) \frac{\beta\widetilde{\mathbb{E}\phi}'}{\phi}.$$

Rewriting this as a function of the interest rate $(\phi/\beta\widetilde{\mathbb{E}\phi}')$, this elasticity then becomes equation (33). \square

If this elasticity is negative, then a lower expected value of the money stock tomorrow translates into a higher value of the money stock today. (To go back one more period: in the period before today, it would then have resulted in a lower value of the money stock). Suppose we are in the steady state and have an *expected* one-time injection of money $1 + \varpi$ tomorrow. Then ϕ' will be unaffected in this stationary equilibrium, since the value of money next period adjusts immediately. If $\varepsilon_{\phi, \beta\widetilde{\mathbb{E}\phi}'} < 0$, this injection will *lower* the cash market prices today. Thus, if we inject $(1 + \varpi)$ in the economy next period, ϕ - the value of money today - will go up. This means that $P = 1/\phi$ - the nonnormalized credit market price - goes down while P' goes up, as ϕ' stayed constant, but nonnormalized money stock rose by $1 + \varpi$. The period before today, the prices will have gone up in response of a lower price today. If we look at nominal interest rates, interest will be higher than the steady state in the period before the injection (today), but lower than the steady state from yesterday to today. (The dynamic system is a so-called ‘sink’.) As a result, we get high volatility of prices and interest rates, and counterfactual responses, in a zigzag pattern, to an expected decline in the value of money in the future. This

in turn could mean that if $\varepsilon_{\phi, \widetilde{\mathbb{E}\phi}} < 0$, the model would produce the right size of velocity fluctuations, but the fundamental forces behind these fluctuations would be empirically invalid.¹⁰

When is this elasticity negative? If $p = 1$, this happens when the coefficient of relative risk aversion is $\sigma > 1$. To avoid the counterfactual behavior of prices and interest rates, we thus have to set $\sigma \leq 1$ in a model with no idiosyncratic risk. However, with the exception of $\sigma = 1$, this is smaller than is standard RRA coefficient on *consumption* in the real business cycle literature (although labor disutility here has $\sigma=0$, as in Hansen-Rogerson). A $\sigma < 1$ is equivalent to values of intertemporal elasticity of substitution larger than 1 ($IES = 1/\sigma$).

With idiosyncratic uncertainty we are able to put in higher σ (lower IES): the behavior sketched above only occurs in the setup with idiosyncratic shocks if

$$\sigma > \frac{1+i}{p+i},$$

which can be significantly higher than one, depending on p . The intuition for the less tight bound is the following: the marginal unit of money is only used with probability p , while with probability $(1-p)$ it is not used. The value of money today is an expected value of the value of money in use (with probability p) and the value of money when not used - that is, with probability $1-p$, the value of money tomorrow plays into today's value of money. So a drop in the value of money tomorrow will now, with a factor $1-p$, contribute to a drop in the value of money today. Above we showed that if $\sigma > 1$, then the marginal value *in use* will go up - the marginal utility increases faster than the drop in the value of one unit of money. However, this is now more than offset by the drop in the value of money when not used. This effect occurs in general in models where, with some probability, not all money is spent, also e.g. in search models.¹¹

3.2 Velocity and Idiosyncratic Uncertainty

The consumption velocity of money in the above example with two idiosyncratic shocks is given by

$$V_c = \frac{PC}{M} = \frac{c}{\phi} + (1-p)\frac{q_l(1+i)}{\phi} + p\frac{q_h(1+i)}{\phi},$$

while output velocity is

$$V_y = \frac{PY}{M} = \frac{(y - (1-p)q_l + pq_h)}{\phi} + (1-p)\frac{q_l(1+i)}{\phi} + p\frac{q_h(1+i)}{\phi}.$$

¹⁰In our calibration, we will have a nominal interest rate rule with persistence. In a setting with $p = 1$ (no idiosyncratic shocks) and $\sigma > 1$, persistence in the nominal interest rate could possibly be achieved by alternating expansions and contractions of the money supply. This provides the counterfactual observation.

¹¹In most calibrations in search models nevertheless $0 < \sigma < 1$ is chosen; this avoids issues with negative utility when doing e.g. Nash Bargaining.

Since $\frac{q_h(1+i)}{\phi} = \psi q_h = \tilde{m} = 1$, we have

$$V_c = \frac{PC}{M} = \frac{c}{\phi} + (1-p) \frac{q_l(1+i)}{\phi} + p$$

$$V_y = \frac{PY}{M} = \frac{(y - (1-p)q_l + pq_h)}{\phi} + (1-p) \frac{q_l(1+i)}{\phi} + p$$

Take consumption velocity. From the above equations, observe that, as is the case in standard cash-in-advance and cash-credit-good models, the constrained part of the cash market always contributes 1 to the level of consumption velocity, and nothing to velocity fluctuations. If $p = 1$, then the cash market does not contribute anything directly to velocity movements. That is, if we have no idiosyncratic uncertainty in our model, it essentially behaves like the standard cash-credit-good model. This means that essentially all velocity movement has to come from the credit market - that is, from c or ϕ .¹² In our model, we create velocity fluctuations in the cash market in addition to the credit market, and it is the idiosyncratic shocks that deliver this.¹³ The contribution towards velocity fluctuations from the cash market comes from the fluctuations in the cash-good price paid in case of the non-binding shock, which equals $\psi = \frac{1+i}{\phi}$, or equivalently, $\frac{1}{\beta \mathbb{E} \phi'}$. The elasticity of consumption velocity with respect to interest rates can then be derived as

$$\varepsilon_{V_c, 1+i} = s_c(\varepsilon_{c, 1+i} - \varepsilon_{\phi, 1+i}) + s_{\text{cash, nb}} \cdot \varepsilon_{\psi, 1+i}, \quad (34)$$

where s_c is the share of the credit market consumption in total consumption and $s_{\text{cash, nb}}$ is the share of cash consumption *expenditure* under non-binding preference shocks in total consumption. The last term captures the velocity fluctuations in the cash market, which we study first:

Lemma 2. *Elasticity of the cash-good price with respect to the interest rate is always positive and equals*

$$\varepsilon_{\psi, 1+i} = \varepsilon_{\frac{1+i}{\phi}, 1+i} = \frac{1+i}{\sigma(p+i)} > 0. \quad (35)$$

In addition, if $\varepsilon_{\phi, \beta \mathbb{E} \phi'} > 0$, then

$$\varepsilon_{\psi, 1+i} > 1.$$

Thus, the less risk averse the household is, the more velocity fluctuations originate in the cash market.

Proof. One can derive that $\varepsilon_{\frac{1+i}{\phi}, 1+i} = 1 - \varepsilon_{\phi, 1+i}$. Substituting in

$$\varepsilon_{\phi, 1+i} = \frac{1}{\varepsilon_{1+i, \phi}} = \frac{1}{1 - \varepsilon_{\beta \mathbb{E} \phi', \phi}} = \frac{\varepsilon_{\phi, \beta \mathbb{E} \phi'}}{\varepsilon_{\phi, \beta \mathbb{E} \phi'} - 1},$$

¹²In any standard search model with fixed match probabilities, this conclusion also holds.

¹³Models with variable search intensity also create velocity fluctuations in the cash market (Wang and Shi 2006).

we find

$$\varepsilon_{\psi,1+i} = \varepsilon_{\frac{1+i}{\phi},1+i} = \frac{-1}{\varepsilon_{\phi,\beta\widetilde{E}\phi'} - 1}$$

Putting the last equation together with (33) yields (35). \square

The above lemma shows that a higher nominal interest rate leads to a higher price in the cash market, while the quantity transacted in the case of any non-binding preference shock stays constant. The lower the degree of relative risk aversion, the more pronounced the fluctuations of the prices in the cash market become.

The remaining terms in (34) are also affected by the presence of idiosyncratic shocks. Fix a σ , and focus on $\varepsilon_{\phi,1+i}$, while keeping everything else constant. It is the case that idiosyncratic uncertainty raises $\varepsilon_{\phi,1+i}$, the responsiveness of prices to changes in the interest rate, because $\varepsilon_{\phi,\beta\phi'}$ is increasing in p , and $\varepsilon_{\phi,1+i}$ is increasingly negative in $\varepsilon_{\phi,\beta\phi'}$. It does not necessarily raise the *absolute value* of the elasticity, though, but for the relevant parameterizations, it can. A higher σ diminishes the responsiveness of prices to nominal interest changes.

The analysis of output velocity is similar to our analysis above. To conclude our analytical discussion, we now turn to the response of c to changes in prices and interest rates.

3.3 General Equilibrium Effects

In the above, we assumed $U(c)$ to be linear, and hence $U'(c)$ was constant. In this section, we generalize this, to take into account indirect effects. For example, a smaller cash market means that more consumption in the credit market is possible, when hours are kept constant. Alternatively, hours could be lowered. It turns out that we can derive analytically elasticities that take into account general equilibrium effects in a relatively straightforward way in our model. The only assumption that we need for analytical tractability is that capital is constant - while this shuts down one equilibrium effect, it does not alter the other effects that we focus on. Once we let capital fluctuate, we will turn to the numerical solution of the model. In this section we once again calculate the elasticity of velocity with respect to nominal interest rates, when this increase is caused by a one-time fully anticipated money injection. It is equivalent to consider a change in $\beta U'(c)\phi'$, which is what we'll use below. First, we derive how h varies with c in the equilibrium. From equation (25), the elasticity is

$$\varepsilon_{c,h} = -\frac{\alpha}{\sigma}, \quad \varepsilon_{U'(c),h} = \alpha \tag{36}$$

Now, from the household budget constraint, we use (36) to derive

$$\varepsilon_{c,q_h} = -\frac{s_{q_h}}{s_c + \frac{1-\alpha}{\alpha}\sigma}, \quad \varepsilon_{U'(c),q_h} = \frac{s_{q_h}\sigma}{s_c + \frac{1-\alpha}{\alpha}\sigma} > 0, \tag{37}$$

where s_{q_h} is the share of total output going to q_h consumption, $s_{q_h} = (pq_h)/Y$; likewise $s_c = c/Y$. Equation (37) captures the general equilibrium effect from changes in q_h : a shift away from cash consumption will lead to an increase in credit market consumption. This effect is proportional to the share of cash consumption under the binding shock in total consumption. In case of idiosyncratic uncertainty, s_{q_h} is smaller (by a factor smaller than p) than total cash market consumption, and hence the elasticity in equation (37) is smaller.

The free entry condition now allows us to link tomorrow's value of money $\mathbb{E}U'(c')\phi'$ to today's movements in q_h . Free entry gives $U'(c)q = \beta\mathbb{E}U'(c')\phi'$, which means that

$$\varepsilon_{q_h, U'(c)\phi'} = \frac{1}{\varepsilon_{U'(c), q_h} + 1} \quad (38)$$

Finally, let us relate ϕ to $U'(c')\phi'$, from $U'(c)\phi = p\vartheta_h u'(q_h)q_h + (1-p)\beta U'(c')\phi'$. We find

$$\begin{aligned} \frac{d \ln \phi}{d \ln U'(c')\phi'} &= \frac{p\vartheta_h u'(q_h)q_h}{p\vartheta_h u'(q_h)q_h + (1-p)\beta U'(c')\phi'} \frac{d \ln(u'(q_h)q_h)}{d \ln q_h} \frac{d \ln q_h}{d \ln U'(c')\phi'} \\ &\quad + \frac{(1-p)\beta U'(c')\phi'}{p\vartheta_h u'(q_h)q_h + (1-p)\beta U'(c')\phi'} - \frac{d \ln U'(c)}{d \ln q_h} \frac{d \ln q_h}{d \ln U'(c')\phi'}, \end{aligned} \quad (39)$$

which can be rewritten as

$$\varepsilon_{\phi, \beta U'(c')\phi'} = \left(\frac{p+i}{1+i}(1-\sigma) - \varepsilon_{U'(c), q_h} \right) \frac{1}{\varepsilon_{U'(c), q_h} + 1} + \frac{1-p}{1+i}. \quad (40)$$

Likewise, we can calculate

$$\varepsilon_{U'(c)\phi, \beta U'(c')\phi'} = \frac{d \ln U'(c)\phi}{d \ln \beta U'(c')\phi'} = \left(\frac{p+i}{1+i}(1-\sigma) \right) \frac{1}{\varepsilon_{U'(c), q_h} + 1} + \frac{1-p}{1+i}. \quad (41)$$

Note that the only difference between (41) and (33) is the presence of the term $(\varepsilon_{U'(c), q_h} + 1)^{-1}$, which captures the general equilibrium feedback of movements of q_h on c , taking into account the optimal labor supply decision. Again in equation (41), if $p = 1$, we would need $\sigma \leq 1$ to get a positive sign for the autocorrelations of prices, real money stock and interest rates, and again, this constraint is relaxed if $p < 1$.

Now let us calculate the elasticity of consumption velocity with respect to the nominal interest rate. We start by calculating the elasticity of velocity with respect

to $\beta U'(c')\phi'$.

$$\begin{aligned}
\varepsilon_{V_c, \beta U'(c')\phi'} &= \frac{d \ln V_c}{d \ln \beta U'(c')\phi'} = -\frac{d \ln V}{d \ln 1 + \varpi} = -\varepsilon_{V_c, 1+\varpi} \\
&= s_c \left(\frac{d \ln c}{d \ln \beta U'(c')\phi'} - \frac{d \ln \phi}{d \ln \beta U'(c')\phi'} \right) + s_{cash, nb} \frac{d \ln \psi}{d \ln \beta U'(c')\phi'} \\
&= s_c \left(\left(\left(-\frac{1}{\sigma} + 1 \right) \varepsilon_{U'(c), q_h} - \frac{p+i}{1+i} (1-\sigma) \right) \frac{1}{\varepsilon_{U'(c), q_h} + 1} - \frac{1-p}{1+i} \right) \\
&\quad - s_{cash, nb} \frac{1}{\varepsilon_{u'(c), q_h} + 1} \tag{42}
\end{aligned}$$

From (42) it follows that, for $p = 1$, this elasticity is negative if $\sigma < 1$; for $p < 1$, a larger σ will also lead to a negative elasticity. The equilibrium elasticity of $\beta U'(c')\phi'$ with respect to $1+i$ can be calculated as $(\varepsilon_{U'(c)\phi, \beta U'(c')\phi'} - 1)^{-1}$, so

$$\begin{aligned}
\varepsilon_{V_c, 1+i} &= \left(\frac{1+i}{p+i} \cdot \frac{\varepsilon_{U'(c), q_h} + 1}{\varepsilon_{U'(c), q_h} + \sigma} \right) \times \tag{43} \\
&\quad \left(s_c \left\{ \left[\left(\frac{1}{\sigma} - 1 \right) \varepsilon_{U'(c), q_h} + \frac{p+i}{1+i} (1-\sigma) \right] \frac{1}{\varepsilon_{U'(c), q_h} + 1} + \frac{1-p}{1+i} \right\} \right. \\
&\quad \left. + s_{cash, nb} \frac{1}{\varepsilon_{u'(c), q_h} + 1} \right),
\end{aligned}$$

which simplifies to

$$\varepsilon_{V_c, 1+i} = s_c \left(\frac{1}{\sigma} \frac{1+i}{p+i} - 1 \right) + s_{cash, nb} \left(\frac{1+i}{p+i} \right) \left(\frac{1}{\varepsilon_{U'(c), q_h} + \sigma} \right) \tag{44}$$

In (44), we can recognize the different channels: (i) general equilibrium channel, through $\varepsilon_{U'(c), q_h}$, (ii) cash market channel through the right-most term, $s_{cash, nb}$, and (iii) credit market effects through the rightmost $(p+i)/(1+i)$ term.

We show these components graphically, as a function of σ , in figure 1. We see that idiosyncratic shocks raise the elasticity of velocity with respect to interest rates dramatically, as signified by the vertical difference between the second (grey dashed) and third (black dashed) lines in the graph, and allow for a positive elasticity for a much larger range of σ . We also observe, in the difference between the top dotted line and the top solid line, that the general equilibrium effect is small, but works to raise the elasticity of velocity with respect the nominal interest rate. Keeping the size of the cash market the same, and lowering p , the share of cash consumption bought with a shock ϑ_h drops. Hence, $\varepsilon_{u'(c), q_h}$ drops and, in turn, this will lead to a drop in the denominator in the rightmost fraction in (44). As a result, the sensitivity of the velocity to interest rates through the cash market channel is raised.

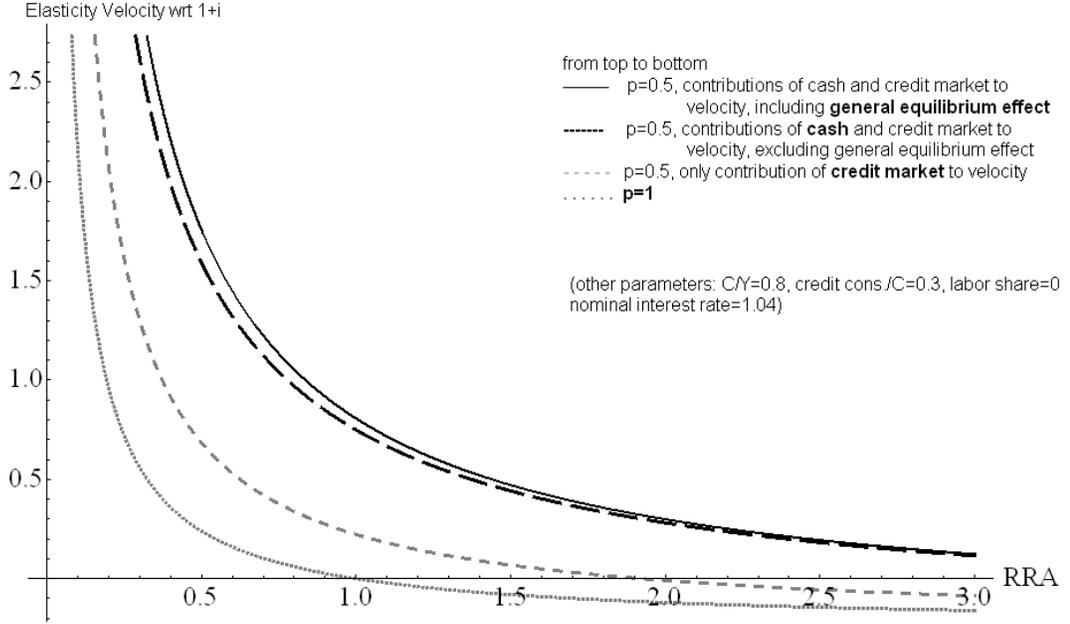


Figure 1: Contributions of Idiosyncratic Uncertainty.

4 Calibration

The model period is a quarter. The functional forms that we choose are as follows:

$$\begin{aligned}
 U(c) &= \frac{c^{1-\sigma}}{1-\sigma} \\
 u(q) &= \frac{x_1 q^{1-\sigma}}{1-\sigma} \\
 f(k, h) &= k^\theta h^{1-\theta}
 \end{aligned}$$

In total, we need to calibrate the following parameters, given our functional form choices: on the real side, we have β , σ , A , θ , δ . On the nominal side, the parameters to calibrate are x_1 , and most importantly, the process for the idiosyncratic shock ϑ . Finally, Ξ , which is a 2×4 matrix, the standard deviations σ_{ε_1} and σ_{ε_2} , as well as the covariance of the two shocks, $\sigma_{\varepsilon_1, \varepsilon_2}$ have to be calibrated to parameterize the real and monetary aggregate shock processes.

We parameterize the model as follows. $\beta = 0.9901$ matches the annual capital-output ratio of 3. $\sigma = 2$ is chosen within the standard range of 1.5-3 in the literature; for now, we hold it fixed; later, we will analyze sensitivity of our results to this choice. $A = 34$ is chosen to match aggregate labor supply of 0.3, given the other parameter choices. $\theta = 0.36$ is the capital share of output as measured in the data. $\delta = 0.02$ gives quarterly depreciation rate of 2%, consistent with existing estimates in the data.

The constant $x_1 = 6$ gives us the size of the retail market, given other parameters, at 72% of total consumption, consistent with the aggregate fact, as in Telyukova(2008), that roughly 75% of the total value of consumer transactions in 2001 took place using liquid payments methods - cash, checks, and debit cards. This number, for payments taking place by check and cash, was quoted at 82% in 1986 in Wang and Shi (2006). For now, we remain close to the 2001 target, and it is also the number that Wang and Shi use in their calibration. We will analyze the sensitivity of results to this target.

To calibrate the process for the preference shock, we use the same methodology as Telyukova (2008), which estimates a similar preference shock process by matching time series properties of survey data on liquid household expenditures. Liquid consumption is measured in the Consumer Expenditure Survey (CEX), by dividing all goods reported in that survey into credit goods and cash goods. For the moment, we do this estimation only on the CEX data for 2000-2002. We thus bias the target against our model: notice that before the 1990's, credit cards were not a ubiquitous payment method (and their prevalence increased in the 1990's), so that many more goods could be considered cash goods than would be today, and would thus likely contribute to a higher volatility estimate. The extension of the CEX sample is currently in progress. In all cases, we separate out consumption of durables from the list, since their consumption appears lumpy in the data on the one hand, thus overestimating volatility of consumption, while they are unlikely to be unpredictable to the household on the other hand. While the survey collects data on monthly consumption, the interviews are conducted quarterly, and we use quarterly data for our purposes.

Using the measure of expenditure on cash goods (liquid consumption), we take the following steps to separate out the idiosyncratic uncertainty component. First, we estimate a regression of log-liquid consumption, controlling for a household fixed effect and quarter and year dummies, to separate out seasonality of consumption. We also do not include in this measure any consumption that is reported to be a gift, rather than for the household's own consumption.

On the residual of this regression, we estimate an AR(1), thus measuring the persistence and volatility of the process. Since in this paper, we model the preference shocks as i.i.d., we use only the standard deviation of the residual of the AR(1) as a measure of the idiosyncratic volatility of consumption, which is measured at 0.1807 in our sample. From here, we assume that liquid consumption in the data (as in the model) responds to preference shocks, and we calibrate the preference shock process such that the standard deviation of simulated household log-liquid consumption in the model matches the standard deviation of the residual of the AR(1). This is done by minimizing the squared distance between the respective model and data moment, as in the simulated method of moments. The resulting estimate of the standard deviation of the preference shock, under the i.i.d. assumption, is 0.6145.

Table 1: Calibration

β	σ	A	θ	δ	x_1	x_2	κ
0.990	2	34	0.36	0.02	6	35	2

Table 2: Preference Shock Process

ϑ_1	ϑ_2	ϑ_3	ϑ_4	ϑ_5	$\mathbb{P}(\vartheta_1)$	$\mathbb{P}(\vartheta_2)$	$\mathbb{P}(\vartheta_3)$	$\mathbb{P}(\vartheta_4)$	$\mathbb{P}(\vartheta_5)$
0.29	0.54	1	1.85	3.42	0.07	0.24	0.38	0.24	0.07

The shocks, based on the assumption of normality of the process, are discretized by the method of Tauchen (1986). The above discussion is summarized in tables 1 and 2.

Finally, we calibrate technology and monetary policy shocks. We model these as a joint stochastic process, and parameterize it by estimating a VAR of the following form:

$$\begin{aligned}
 z_t &= \xi_{zz} z_{t-1} + \xi_{zi} \ln \left(\frac{1 + i_{t-1}}{1 + \bar{i}} \right) + \xi_{z\pi} \ln \left(\frac{1 + \pi_{t-1}}{1 + \bar{\pi}} \right) + \varepsilon_1 \\
 \ln \left(\frac{1 + i_t}{1 + \bar{i}} \right) &= \xi_{ii} \ln \left(\frac{1 + i_{t-1}}{1 + \bar{i}} \right) + \xi_{i\pi} \ln \left(\frac{1 + \pi_{t-1}}{1 + \bar{\pi}} \right) + \xi_{iy} \ln \left(\frac{y}{\bar{y}} \right) + \varepsilon_2
 \end{aligned}$$

where the Solow residual is measured in the standard way, and we take out the linear trend from both the Solow residual and the output series. The variables with bars over them capture long-term averages of the respective variables in our sample period, as is standard in estimating central banks' targets in policy rules. The sample of data on which we estimate this process is from 1984 until 2007, to capture the period when the Federal Reserve is perceived to have begun using (implicit) inflation targeting. Notice that our productivity process and the interest rate rule both depend on endogenous variables. We use the Federal Funds rate as the measure of choice of interest rates in the data. The resulting VAR coefficients are in table 3.

Table 3: VAR Estimates of the Aggregate Shock Process, 1984-2007

ξ_{zz}	ξ_{zi}	$\xi_{z\pi}$	ξ_{ii}	$\xi_{i\pi}$	ξ_{iy}	σ_{ε_1}	σ_{ε_2}	$\sigma_{\varepsilon_1, \varepsilon_2}$
0.937	-0.144	0.060	0.780	0.120	0.008	0.004	0.001	-0.0000002

5 Computation

We employ the Parameterized Expectations Approach (PEA) to solve the model. The main idea of the method is to approximate the expectations terms in our Euler equation system - two in total - by polynomial functions of the state variables. The coefficients of the polynomials form the basis of an iterative approach pioneered by Den Haan and described in our context below. In order for the algorithm to converge, we need a good first guess of these coefficients, which we will derive by using a version of homotopy - that is, by first solving a second-order approximation of the model and deriving the first set of polynomial coefficients from the solution.

Let χ_t denote the state variables of the problem (known at time t that help predict the expectations terms), ζ_t denote all the endogenous and exogenous variables that appear inside the expectation terms, and u_t denote the shocks of the problem. We have

$$\begin{aligned}\chi_t &= \{k_t, z_t, i_{t-1}, i_t, \phi_{t-1}(1 + \varpi_{t-1})\} \\ \zeta_t &= \{k_t, h_t, q_t, \phi_t, z_t\} \\ u_t &= z_t, i_t.\end{aligned}$$

Note that c_t does not appear in ζ_t because it can be determined from the other variables and the budget constraint. The variables we are solving for are $\{c_t, k_{t+1}, h_t, q_t, \phi_t, \psi_t\}$.

The approximating functions for the expectation terms are always just functions of x_t . We choose the following forms:

$$\begin{aligned}\mathbb{E} \left[(c')^{-\sigma} (1 + e^{z'} \theta (k')^{\theta-1} (h')^{1-\theta} - \delta) \right] &= \psi^1(\chi; \gamma^1) \\ \mathbb{E} \left[\frac{1}{w'} \right] &= \frac{\tilde{\mathbb{E}}}{w} = \psi^2(\chi; \gamma^2)\end{aligned}\tag{45}$$

where, for example,

$$\psi^j(\chi; \gamma^j) = \gamma_1^j \exp(\gamma_2^j \log k + \gamma_3^j \log z + \gamma_4^j \log i_{-1} + \gamma_5^j \log i + \gamma_6^j \log[\phi_{-1}(1 + \varpi_{-1})])$$

The accuracy of approximation can be increased by raising the degree of approximating polynomials above. We substitute the expressions in (45) into the system of Euler equations, to obtain the system of equations that we use in solving the model. The full iterative algorithm is described in the appendix. We find that the convergence properties of our model are good: convergence is monotone and very robust. In order to compute moments from the model, we re-run the model solution 100 times for each parameter combination, and the simulations within each run are for 10,000 periods.

6 Results

6.1 The Role of Idiosyncratic Shocks

We choose M2 as the basis for analysis of the data, to follow Wang and Shi (2006) and Hodrick, Kocherlakota and Lucas (1991). Part of the reason of using this series is that it exhibits much more stationarity over time than does M1. Table 4 summarizes the results concerning the dynamic properties of some key nominal variables. The first column of each table presents results in the data, and the last - in our benchmark model. The middle column presents the results for the version of our model with the idiosyncratic preference shocks shut off, so that the model more closely replicates a standard cash-credit good model with only aggregate risk.

It is important to emphasize that we are not targeting any of these moments in choosing the parameters of the model. As is clear from the table, introducing precautionary motive for holding money into our model makes an enormous difference for the performance of the model: without it, the model is not able to capture any of the moments in the data, while introducing precautionary demand makes the model align quite successfully on nearly all of the dimensions listed. We over-predict correlation of consumption velocity with gross nominal interest rates, and, on a related note, underpredict volatility of nominal interest rates, but we do better on these moments with the preference shocks than we would without. We also do not target expected value of velocity, so we do not match it, but neither do we expect to, since we only model household money demand, while in the data firms and other entities also demand it. Aside from these moments, our model is in the ballpark of the data numbers.

The main contribution of the preference shocks in the model - that is, of introducing the precautionary motive for holding money - is that it adds dynamics to velocity of money that would not be there otherwise. This is what leads to the vast difference in the results of the two models in the table above, especially in moments like volatility of velocity and correlation of velocity with interest rates. This is further clarified by the impulse response functions generated by the model, shown in figure 3. The top two panels show the response of consumption and total money spent to the orthogonalized productivity shock in the event of each of the possible shock realizations. The bottom two show the same variables' responses to the monetary policy shock. These impulse responses reinforce our analysis of the implications of the idiosyncratic shocks in the model section, as demonstrated by the marginal rates of substitution between the cash and credit good and the subsequent discussion.

We have two types of agents in the model - those who face binding preference shocks (ϑ_5), with corresponding consumption levels q_5 and total money spending $\psi q_5 = d_5$ in the decentralized market), and those who face non-binding shocks (in

Table 4: Dynamic Properties of the Model

Moment	Data	No-Shock Model	Full Model
$\mathbb{E}(V_y)$	1.897	1.812	1.122
$\mathbb{E}(V_c)$	1.120	1.380	0.855
$\sigma(V_y)$	0.017	0.007	0.014
$\sigma(V_c)$	0.014	0.0002	0.011
$\sigma(y)$	0.009	0.008	0.008
$\sigma(1+i)$	0.0026	0.001	0.002
$corr(V_y, y)$	0.638	0.994	0.503
$corr(V_y, g_c)$	0.127	0.511	0.053
$corr(V_c, g_c)$	-0.027	0.195	-0.177
$corr(V_y, g_y)$	0.059	0.286	0.095
$corr(V_c, g_y)$	-0.094	0.104	-0.058
$corr(V_y, 1+i)$	0.714	-0.138	0.758
$corr(V_c, 1+i)$	0.690	-0.894	0.897
$\varepsilon_{V_y, 1+i}$	5.072	-0.817	5.227
$\varepsilon_{V_c, 1+i}$	4.158	-0.123	5.014
$corr(1+\pi, 1+i)$	0.529	0.881	0.371

All data logged and BP filtered. Data period: 1984-2007. Velocity moments calculated based on M2. Interest rate is the Fed Funds rate. Inflation measured based on GDP deflator. g_y refers to output growth. “No-Shock” model is the version of the model with idiosyncratic preference shocks shut down. All model moments are computed from 100 repetitions of model solution, with simulations of 10,000 periods.

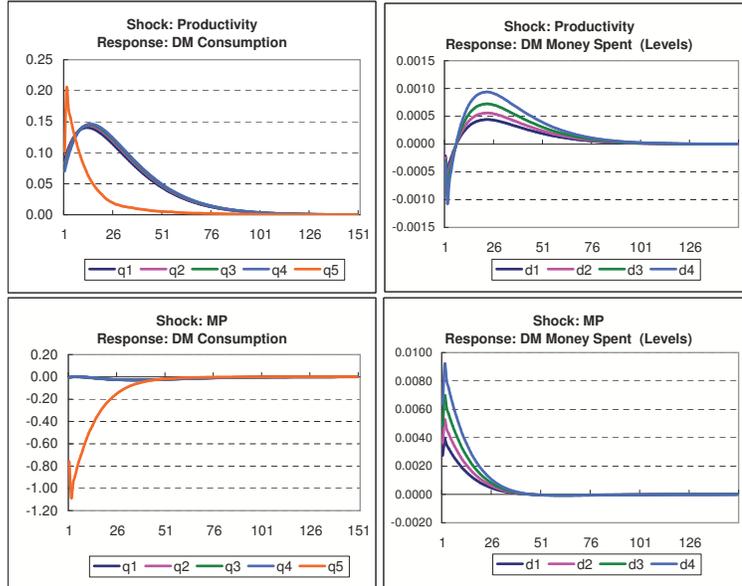


Figure 2: The idiosyncratic risk mechanism - the response of cash-good consumption and the total expenditure in the cash market ($d_\vartheta = \psi q_\vartheta$) to productivity and policy shocks.

the model, shocks $\vartheta_1 - \vartheta_4$). Those whose shocks are binding always spend all of their money. Thus, in response to a decrease in the value of money (through a positive nominal interest rate shock, for example, bottom two panels), when prices of consumption goods increase, these agents are constrained in terms of how much money they spend - they cannot respond to increased prices by spending more money, so their consumption responds, just as we showed before, while their money spending does not. Hence, the fall in the impulse response for q_5 , with the flat line (not shown) for d_5 . In contrast, those who are not constrained (all remaining shocks) always attain their optimal consumption levels (they have no nominal wedge affecting their marginal rate of substitution between centralized and decentralized market goods). Thus, even with a price increase that results from a monetary policy shock, they can maintain their consumption levels, but respond by spending more of their money. Hence, there is no response on impact in $q_1 - q_4$, but a pronounced increase on impact in $d_1 - d_4$. This response in d contributes to a response of velocity of money to a change in interest rates that would not be there if there were no precautionary demand motive in the model: in that case, everyone's cash-market consumption would go down, as q_5 does, with no adjustment in money spent or velocity, upon the impact of the shock.

The dynamics of the cash-market variables in the quarters after the shock hits are due to the interaction of productivity and interest rates. Even though the shocks themselves are orthogonalized, the laws of motion for both the productivity shock

and the interest rate rule have endogenous components. For example, the nominal interest rate enters the productivity process with a negative coefficient. So once interest rates increase due to a monetary policy shock, the level of productivity drops, which in turn has propagation effects similar to what a negative productivity shock would do. Namely, once the productivity level drops in the quarters following the monetary policy shock, it has negative effects on output, investment and consumption in both credit and cash markets. This is where the subsequent drop in $q_1 - q_4$ comes from in figure 1. As the interest rates decrease and the productivity level starts to recover, prices in the cash market decrease, allowing both constrained and unconstrained individuals to increase their consumption, while those who are unconstrained also decrease their spending.

Responses to the productivity shock, upon impact, are of similarly diverse nature for those who are constrained and those who are not. When the productivity shock hits, everyone wants to consume more, as prices of cash goods have fallen. This is reflected in the fall on impact of $d_1 - d_4$ and the increase on impact of $q_1 - q_5$. Again, notice that this creates a dynamic response of velocity of money that would not occur if all the individuals were equally constrained: in that case, only their consumption would respond. In the quarters following initial impact, the increase in output that has resulted from the high productivity shock feeds into the interest rate rule, causing prices to rise, which in turn means that constrained households decrease their cash consumption q_5 much more rapidly than the unconstrained households, while the unconstrained can continue to increase their consumption optimally along with their spending. The hump-shaped response in both the credit-good and cash-good consumption is standard in the business cycle literature, in response to productivity shocks.

6.2 Some Other Aggregate Facts

We now assess the performance of our model according to an additional set of facts, listed by Cooley and Hansen (1995) as the most significant monetary features of business cycles. The first two columns list the performance of our model against the data, while the second show the comparable moments from the Cooley and Hansen sample, and their own performance along this dimension. Many of these facts are listed in table 5. The facts are that monetary aggregates (M1 and M2) and prices are countercyclical, while velocity is procyclical; and that correlation of output and inflation with the growth of money supply is negative. We match these facts in the model and get the magnitudes of the correlations about right.

Notice that in the Cooley-Hansen sample, which is from the 1950's until 1991, monetary aggregates appear procyclical. In our analysis of the data, we find that the procyclicality disappears after the policy changes in 1983, as shown in figures 4 and 5.

Table 5: The Cooley-Hansen Facts

Moment	Data	Model	CH data	CH
$corr(M, y)$	-0.17	-0.09	0.33	n/a
$corr(V, y)$	0.64	0.50	0.37	0.948
$corr(p, y)$	-0.13	-0.19	-0.57	-0.22
$corr(g_m, y)$	-0.11	-0.09	-0.12	-0.01
$corr(g_m, \pi)$	-0.06	-0.10	-0.29	0.92

All data logged and BP filtered. All model moments are computed from 100 repetitions of model solution, with simulations of 10,000 periods.

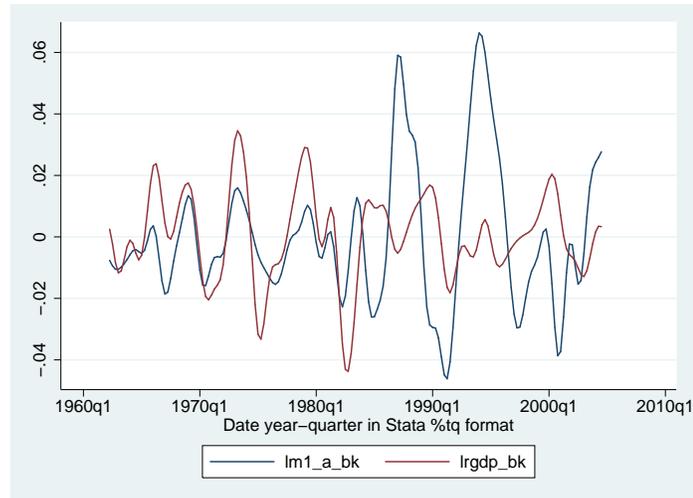


Figure 3: M1 in the data.

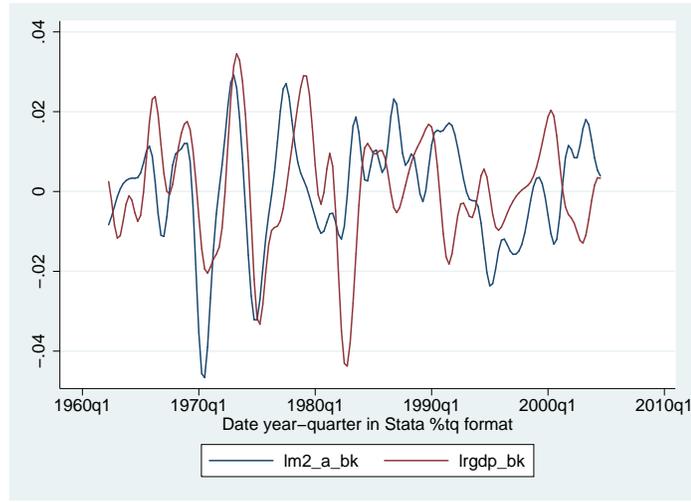


Figure 4: M2 in the data.

To analyze our performance further with respect to facts highlighted by Cooley and Hansen, we present cross-correlations of several endogenous variables with output in graphical form, as in figure 4. Where possible, we also graph the cross-correlations presented by Cooley and Hansen from their model. With respect to these correlations, we do as well or better as they do. A notable improvement in our model relative to theirs concerns the dynamic pattern of output velocity: we match the data for M2 velocity a lot more closely than they did.

To finish, we want to highlight some aspects of the data that we are not so successful in capturing. These appear in table 6 and in figure 6. As is clear from table 6, our model, as the previous models in this class, misses the liquidity effect - the negative correlation of nominal interest rates with money growth. In addition, our prices and inflation are too flexible. All of this produces the series of moments replicated in this table. Observe, however, that the Cooley-Hansen cash-in-advance model similarly misses these moments. In general, it is not surprising that our prices are completely flexible - we do not build in any frictions to change the speed and magnitude of price adjustment - and that we miss the liquidity effect, partly as a result of this. Sluggish adjustment of prices, as for example in Alvarez, Atkeson and Edmond (2008), is difficult to incorporate without a mechanism like market segmentation. We do not target this mechanism and hence did not expect to get the liquidity effect right.

Finally, looking at figure 6, we present some further cross-correlation results on endogenous variables that we get less well. While we get the dynamic pattern of money supply half-right (although our cross-correlation bottoms out far later than the data suggest), and we improve on Cooley and Hansen's cross-correlation of

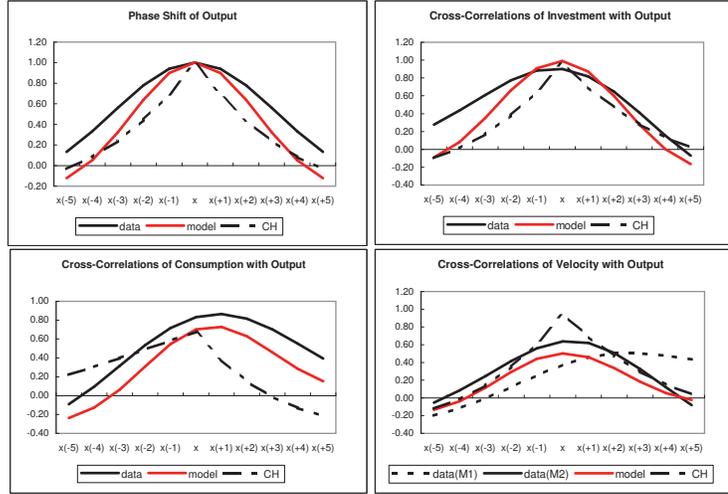


Figure 5: Cross-Correlations of Endogenous Variables with Output

Table 6: Liquidity Effect

Moment	Data	Model	CH data	CH
$corr(g_m, i)$	-0.7(M1)/0.07(M2)	0.82	-0.27	0.72
$corr(y, i)$	0.54	-0.03	0.40	-0.01
$corr(y, \pi)$	0.37	-0.07	0.34	-0.14
$corr(g_m, p)$	0.03	0.63	-0.16	0.43

All data logged and BP filtered. All model moments are computed from 100 repetitions of model solution, with simulations of 10,000 periods.

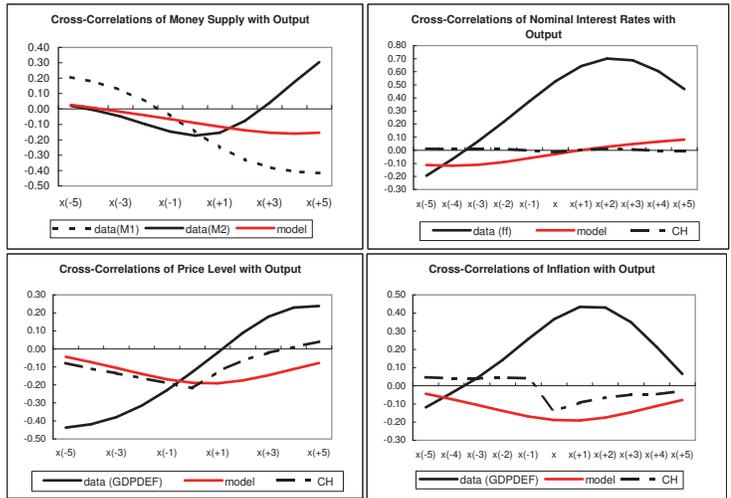


Figure 6: Cross-Correlations of Endogenous Variables with Output

nominal interest rates, we get neither these two, nor the dynamic patterns of prices and inflation very close to the data. (One thing to note here is that our model's price pattern replicates that in the earlier model sample much more closely; but for the post-1983 sample, the dynamic pattern in the data changes). Notice that our performance on the bottom two panels is fairly close to Cooley and Hansen's. Again, we do not expect to get the patterns of prices to replicate the data with a price adjustment mechanism that is as flexible as ours.

7 Conclusion

In this paper, we study the aggregate implications of precautionary demand for money, an empirically significant phenomenon. We highlight the importance of modeling idiosyncratic risk in expenses as a cause of precautionary motive for holding liquidity. By incorporating this idiosyncratic risk in the form of preference shocks into a standard cash-credit good model with aggregate risk, and by carefully calibrating the idiosyncratic shocks to data, we find that the model matches many dynamic moments of nominal variables well, and greatly improves on the performance of existing monetary models that do not incorporate idiosyncratic shocks to preferences. Thus, we see much promise in the explicit treatment of precautionary demand for money resulting from idiosyncratic uncertainty for accounting for business cycle behavior of the data and other aggregate monetary facts.

This opens up several promising venues of future research. In ongoing work, we incorporate the same type of idiosyncratic shocks into a monetary *search* business cycle model, and study its aggregate implications, especially pertaining to the

dynamic behavior of inventory investment and retail markups. We also are investigating better ways of measuring idiosyncratic risk to expenses, an area that may be as contentious as measurement of idiosyncratic income shocks has been in the macroeconomic literature.

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