Optimal Second Best Taxation of Addictive Goods[*]

Luca Bossi[†]   Pere Gomis-Porqueras[‡]   David L. Kelly[§]

Abstract

In this paper we derive conditions under which optimal tax rates for addictive goods exceed tax rates for non-addictive consumption goods in a rational addiction framework where exogenous government spending cannot be financed with lump sum taxes. Standard static models that consider a revenue raising motive predict taxing addictive goods at a rate in excess of that observed in the data. In contrast, our dynamic results imply tax rates on addictive goods which are smaller than tax rates implied by the static framework. This is the case because high current tax rates on addictive goods tend to reduce future tax revenues, by making households less addicted in the future. Finally, we consider features of addictive goods such as complementarity to leisure that, while unrelated to addiction itself, are nonetheless common among some addictive goods. In general, such effects are weaker in our dynamic setting since if taxing addictive goods has strong positive revenue effects today, then taxing goods has a strong offsetting effect on future tax revenues.

[†]All the authors are affiliated with the Department of Economics, University of Miami. P.O. Box 248126, Coral Gables, FL 33124-6550, USA. E-mail: l.bossi@miami.edu.
[‡]E-mail: gomis@miami.edu.
[§]E-mail: dkelly@miami.edu.

1 Introduction

A popular and increasingly common way for local, state, and federal governments to raise revenue is through taxation of addictive goods, including cigarettes, alcohol, and gambling. What is the optimal excise tax for addictive goods, when the government must raise revenue to finance a stream of exogenous government expenditures? The goal of this paper is to determine the optimal tax on addictive goods and characterize and analyze the conditions under which taxation of addictive goods might differ from taxes on labor and non-addictive consumption goods (hereafter *ordinary goods*).

We extend classic results of optimal commodity taxation (e.g. Atkinson and Stiglitz (1972)) to the case of addictive goods for common cases such as homothetic and separable utility. For example, if utility is homothetic in ordinary and addictive consumption, then the classical result of uniform commodity taxation holds only for special cases. Intuitively, homotheticity implies that ordinary consumption goods taxation yields marginal tax revenue equal to the marginal tax revenue of addictive consumption in the current period. However, some of the marginal tax revenue of addictive consumption is realized in the next period, when elasticities may be different. Suppose for example that addictive consumption is becoming more income inelastic over time. By homotheticity, taxation of ordinary consumption results in a distortion equal to the distortion caused by addictive consumption at current elasticities. However, if addictive consumption is taxed at a higher rate than ordinary consumption today, then households will be less addicted in the future. This is attractive because, since the income elasticity is falling, the government will still be able to tax addictive consumption without too much distortion in the next period. The increase in income elasticity caused by a lower addiction level is at least partially offset by the falling income elasticity.

In other words, optimal addictive goods taxation deviates from ordinary consumption taxation so as to smooth intertemporal distortions caused by taxation for revenue raising. In this sense, our results are related to those on capital taxation (Chamley 1986, Chari and Kehoe 1998). Nonetheless, our results differ because addictive consumption acts like both a finished good (current addictive consumption) and as an intermediate good (current

addictive consumption affects future addictive consumption). For example, optimal steady state tax rates on addictive goods equal tax rates on ordinary consumption in some cases where optimal capital tax rates are zero, because addictive consumption acts like a finished good.

As noted by Becker and Murphy (1988), addictive goods are characterized by *tolerance* (also known as harmful addiction): past consumption lowers current utility. We show that tolerance makes taxing addictive goods less attractive from a revenue raising perspective. Suppose, for example, that consumption in excess of that required to sustain the addiction (hereafter *effective consumption*) is complementary with leisure. Standard public finance theory suggests that the tax rate on addictive goods should be relatively high, since reduced consumption of addictive goods will increase labor supply, thus raising labor income tax revenues. However, if the good is addictive, then reduced current consumption of addictive goods raises future effective consumption. But then future labor supply falls, and future labor income tax revenues fall, offsetting some of the revenue gains in the current period. This type of dynamic effect is not captured by standard static models that compute optimal tax rates on addictive goods. Thus ignoring the dynamic nature of addiction when designing optimal fiscal policy may result in lower welfare.

The literature typically models effective consumption in one of two ways: the subtractive specification (e.g. Campbell and Cochrane 1999) and the multiplicative specification (e.g. Abel 1990). As pointed out by Bossi and Gomis-Porqueras (2008), the two models differ in terms of their homogeneity properties.[1] In this paper, we show that the optimal tax policy depends crucially on the degree of homogeneity of the addiction function. In particular, we show that the income elasticity of the addictive good is decreasing in the degree of homogeneity, given separable or homothetic utility with constant relative risk aversion. Thus, taxation of addictive goods is more attractive if the addiction model is homogeneous of degree

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[1] A good is habit forming if the marginal utility of the good is increasing in past consumption. We use the standard definition of addiction, which is when current consumption is increasing in past consumption, holding fixed the marginal utility of wealth and prices. Habit formation is often used in the macro literature, whereas addiction was introduced by Becker and Murphy (1988). It is straightforward to show that the subtractive model of habit formation implies the good is addictive, and the multiplicative model of habit formation implies the good is addictive with an additional restriction.

one, as in the subtractive case, than if the addiction model is homogeneous of degree less than one, as in the multiplicative case, since it is optimal to tax necessities at a higher rate. Further, strong tolerance in the multiplicative model decreases the degree of homogeneity, making addictive goods more income elastic, which therefore *lowers* the optimal tax rate on addictive goods.

In the next section we describe the three main motives for taxing addictive goods found in the literature. We then develop a dynamic, rational addiction model in order to determine the conditions under which optimal tax rates for addictive goods exceed tax rates for non-addictive consumption goods.

2   Taxing Addictive Goods

Three classical motivations exist in the literature for taxing addictive goods differently than ordinary goods. The first is to lower the external costs often associated with consumption of addictive goods. The second is because some consumers fail to take into account some private costs and thus over-consume. The third motivation is to raise revenue.

2.1   Addictive Goods and Externalities

The standard economic rationale for taxation of addictive goods is that their consumption is often associated with external costs, such as second-hand smoke, drunk driving, and crime. However, it is well known (Kenkel 1996, Pogue and Sgontz 1989) that taxing an addictive good (e.g. alcohol) whose consumption is imperfectly correlated with an externality is a second-best solution. Taxing the actual behavior causing the externality (e.g. make the punishment for drunk driving more severe) is more efficient. Indeed, Parry, Laxminarayan and West (2006) show that welfare gains from increasing drunk driving penalties exceed those from raising taxes on alcohol, even when implementation costs and dead-weight losses associated with incarceration are included.

The literature tends to find that most addictive goods are taxed at a rate less than the rate which is second best in the sense that the government cannot discriminate between

consumers who generate external costs and responsible consumers.[2] This literature differs from our paper in that the focus is on Pigouvian concerns, rather than revenue raising.

2.2 Addictive Goods and Non-market Internal Costs

Another source of non-market costs occurs if addiction is modeled as non-fully rational excess consumption. Suppose consumers fail to take into account the self-adverse health effects caused by consumption of addictive goods, either because they are unaware that addictive goods consumption has adverse health effects (e.g. Kenkel 1996) or because some consumers are exogenously assumed to be unable to take into account the health gains from reducing addictive goods consumption (e.g. Pogue and Sgontz 1989). When some consumers are exogenously assumed not to consider some private costs, they over-consume. The resulting "internality" causes the optimal second best (again, in the sense that the government cannot distinguish between naive and rational consumers) tax rate to rise considerably.[3]

A related, subsequent literature makes excess consumption endogenous and rational by defining "sin goods" as goods for which preferences are time inconsistent (Gruber and Koszegi 2001, Gruber and Koszegi 2004, O'Donoghue and Rabin 2003, O'Donoghue and Rabin 2006). In this approach, consumers optimally choose to consume more now and less in the future. However, next period consumers also optimally choose to consume more now and less in the future. Hence consumers are rational, but over-consume in the sense that consumer welfare increases with a tax that reduces consumption to a level which consumers would choose if they could pre-commit to consume less in the future.[4]

Internalities are a dynamic feature of addiction. Results in this literature find that

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[2]For example Gruber and Koszegi (2001) estimate external costs of smoking at $0.94 to $1.75 per pack, versus an average excise tax of about $0.65. Kenkel (1996) finds that a tax rate on alcohol of about 42% is optimal for the drunk driving externality, while the actual average tax rate ranges from over 50% in 1954 to 20% in the 1980s. Moreover, Grinols and Mustard (2006) estimate external costs of casino gambling are 47% of revenues, thus the optimal tax would be higher than 47% if demand for casino gambling is inelastic, or less than 47% if a significant fraction of casino gamblers do not impose external costs. Anderson (2005) reports that casinos pay 16% of gross revenues in taxes.

[3] Kenkel (1996) finds the optimal tax rate on alcohol rises to about 106% while Pogue and Sgontz (1989) find the optimal tax rate on alcohol rises to 306%.

[4] O'Donoghue and Rabin (2006) compute numerical examples where the optimal tax on unhealthy foods ranges from 1-72%. Gruber and Koszegi (2001) show that the optimal tax on cigarettes rises to at least $1 per pack when the time inconsistency problem is included.

optimal tax rates are generally greater than tax rates observed in the data. In this paper, we focus on another dynamic aspect of addiction: tolerance. In particular, we study the dynamic revenue raising properties of addictive goods taxation in a rational addiction framework.

2.3   Addictive Goods and Fiscal Concerns

A final motivation for taxing of addictive goods is revenue raising. Taxation of many addictive goods, such as lotteries, have an obvious revenue raising component. Taxes on many other addictive goods have at least a stated goal of raising revenue. For example, Parry et al. (2006) note that the last two increases in federal alcohol taxes were part of deficit reduction packages.[5]

A few papers consider the revenue raising motivation by treating addictive goods in a static way as simply goods with external costs and which are possibly complementary with leisure. If so, one can apply the ideas from the "double dividend" literature (e.g. Bovenberg and Goulder 1996). Taxing a good with external costs raises revenues which can be used to reduce taxes on labor income (the "revenue recycling effect"). If taxing addictive goods results in lower dead-weight losses than taxing labor (say if demand for addictive goods was very inelastic), then the revenue recycling effect is positive and it is optimal to tax addictive goods at a relatively high rate. Moreover, a good with external costs may also be taxed above its Pigouvian rate for revenue raising if it is complementary with leisure, since the tax therefore increases labor supply and labor income tax revenues (the "tax interaction effect").[6] This literature models addiction in a static way as simply a good with external costs; the dynamic nature of addiction is ignored. It remains unclear how dynamic addictive properties such as tolerance affect optimal revenue raising.

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[5]For lotteries, external costs are presumably small, but the nationwide average lottery tax ranges from 40% in 1989 (Clotfelter and Cook 1990) to 31% in 2003 (Hansen 2004), accounting for 2% of state tax revenues. States spent about $272 million on lottery advertising in 1989, which is at least a strong indication that states are motivated by revenue concerns, rather than the external costs of lotteries and other forms of gambling.

[6]Sgontz (1993) finds the revenue recycling effect to be positive, and Parry et al. (2006) finds both the revenue recycling effect and the tax interaction effect to be positive: alcohol is complementary to leisure and also reduces labor productivity. Therefore, they find it is optimal to tax alcohol above it's Pigouvian rate as part of the optimal revenue raising package.

This paper tries to fill this gap in the literature by considering a dynamic model of rational addiction while explicitly considering a revenue raising motive. Throughout the rest of the paper we model addiction using the rational addiction framework of Becker and Murphy (1988) and others. In this approach, consumption of the addictive good is specifically related to past consumption. Although not conclusive, some evidence for rational addiction exists in that current consumption of cigarettes,[7] alcohol,[8], and caffeine (Olekalns and Bardsley 1996) respond to announced future price changes, as predicted by the rational addiction model. Gruber and Koszegi (2001), however, show that evidence of rational addiction does not preclude time inconsistent preferences.[9] The main alternative, modeling addiction as either rational or irrational excess consumption, has intuitive appeal but also some practical difficulties. First, it is difficult to determine the degree of excess consumption, especially since it must be heterogeneous across the population. The optimal tax is sensitive to both the degree of excess consumption and the fraction of the population that suffers from excess consumption. Furthermore, computational difficulties of time inconsistent preferences require separability in addictive and ordinary goods, no savings, and often quadratic utility functions. All of these assumptions affect the optimal tax rates, especially if the government has a revenue raising requirement.

The Becker and Murphy framework has no internality motivation for taxation of addictive goods, but a fiscal motivation can still exist. Thus we examine the revenue-raising motivation, using the long standing tradition of the Ramsey approach (see for example Chari and Kehoe 1998). Unlike static models, in our dynamic framework changes in tax rates on addictive goods affects future revenues, by changing future elasticities.

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[7]See for example Gruber and Koszegi (2001), Becker, Grossman and Murphy (1994), Chaloupka (1991) and Sung, Hu and Keeler (1994).

[8]See for example, Grossman, Chaloupka and Sirtalan (1998) Baltagi and Griffin (2002), Bentzen, Eriksson and Smith (1999), Baltagi and Geishecker (2006), and Waters and Sloan (1995).

[9]Although laboratory evidence of time-inconsistent preferences are strong, little formal econometric evidence exists for or against time inconsistent preferences in actual markets.

3 Model

We consider an infinite horizon closed economy in discrete time. The economy is populated by a continuum of identical households of measure one who maximize the discounted sum of instantaneous utilities. A large number of identical firms produce both addictive and ordinary goods using a constant return to scale technology. Finally, there is a government that needs to finance a constant stream of government expenditures through fiscal policy.

3.1 Firms

A large number of identical firms at time t rent capital k_t and labor h_t from households to produce a composite good using a technology $F(k_t, h_t)$. We assume throughout the paper that:

Assumption 1 $F(.,.)$ *is constant returns to scale and increasing, concave, and satisfies Inada conditions in each input.*

Let w_t denote the wage rate and r_t the rental rate of capital, then the objective of the firm is to maximize profits, which equal:

$$\max_{k_t, h_t} \left\{ F(k_t, h_t) - r_t k_t - w_t h_t \right\}. \tag{1}$$

Let subscripts on functions denote corresponding partial derivatives. The equilibrium rental rate and wage rate are:

$$r_t = F_k(k_t, h_t), \tag{2}$$

$$w_t = F_h(k_t, h_t). \tag{3}$$

For simplicity we assume that the composite good can be used for either addictive or non-addictive goods consumption or investment.[10]

3.2 Households

A representative household derives utility from consumption of an ordinary (non-addictive) good, c_t, the fraction of time allocated to leisure, $1 - h_t \equiv l_t \in [0,1]$, and consumption of an addictive good, d_t. Let $s_t = s(d_t, d_{t-1})$ denote effective consumption, i.e. consumption in excess of that required to sustain the addiction. The per period utility depends on consumption of ordinary goods, effective consumption, and leisure through the utility function $u(c_t, s_t, l_t)$.[11] We assume throughout the paper that:

Assumption 2 $u(.,.,.)$ *is strictly increasing, concave, and satisfies the Inada conditions in each argument.*

Lifetime utility is:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, s_t, l_t);$$
(4)

where β is the discount factor with rate of time preference $\rho = \frac{1-\beta}{\beta}$.

For effective consumption, we assume throughout the paper that:

Assumption 3 $s(.,.)$ *is homogeneous of degree α in $[d_t, d_{t-1}]$ (HD-α) and satisfies $s_1 > 0$, $s_2 < 0$, $s_{11} \leq 0$.*

The first inequality states that households get positive utility from consumption of the addictive good. The second inequality states that the addictive good has the *tolerance* property, meaning past consumption lowers current utility, which is also known a harmful addiction. The third inequality is a sufficient condition which ensures that the household

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[10]Note that it is possible (but cumbersome) to extend the analysis to allow the production technology to differ by consumption goods.

[11]This specification is clearly equivalent to Becker et al. (1994), who assume a utility function of the form $u(c_t, d_t, d_{t-1})$, except they assume no preferences for leisure. Our assumption below that s is homogeneous is the main restriction we impose on their utility specification.

return is globally concave in the choice set $[c_t, l_t, d_t]$ if the return function is concave when $s_t = d_t$ (i.e. the standard problem with no addiction is concave), which we also assume throughout. The role of homogeneity will be discussed below.

3.2.1 Habits versus Addiction

The term habit formation is often used in the macro literature, whereas addiction was introduced by Becker and Murphy (1988). These two notions are equivalent under certain conditions, which are spelled out below.

Gruber and Koszegi (2004) and others define habit formation as past consumption increasing the taste for current consumption.[12] Therefore, a good is *habit forming* if and only if:

$$\frac{\partial^2 u}{\partial d_t \partial d_{t-1}} > 0. \tag{5}$$

From the assumptions on s, a good is habit forming if and only if:

$$\sigma_s\left(c_t, s_t, l_t\right) \equiv \frac{-u_{ss}\left(c_t, s_t, l_t\right) s_t}{u_s\left(c_t, s_t, l_t\right)} > \frac{s_t s_{12}\left(d_t, d_{t-1}\right)}{s_1\left(d_t, d_{t-1}\right) s_2\left(d_t, d_{t-1}\right)}. \tag{6}$$

Becker and Murphy (1988) and others define *addiction* as when past consumption increases current consumption, holding fixed prices and the marginal utility of ordinary consumption. Let $c_t = y_t - p_t d_t$, where y_t represents income in period t and p_t is the price of d in period t, then d is addictive if and only if:

$$\frac{\partial d_t}{\partial d_{t-1}} = \frac{\frac{\partial^2 U}{\partial d_t \partial d_{t-1}}}{-\frac{\partial^2 U}{\partial d_t^2}} > 0, \tag{7}$$

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[12]Becker and Murphy (1988) define reinforcement as when past consumption increases the taste for current consumption.

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holding fixed the marginal utility of consumption. Using the concavity assumptions, equation (7) simplifies to:

$$\frac{\partial^2 U}{\partial d_t \partial d_{t-1}} = \frac{\partial^2 u}{\partial d_t \partial d_{t-1}} > 0. \tag{8}$$

Thus d is addictive if and only if d is habit forming given our one-lag specification of effective consumption, and our concavity assumptions.[13]

The two most commonly used specifications of effective consumption, s, in the literature are the *subtractive model* (see for example Campbell and Cochrane 1999), where effective consumption is:

$$s_t = d_t - \gamma d_{t-1}, \tag{9}$$

and the *multiplicative model* (see for example Abel 1990), which specifies effective consumption as:

$$s_t = \frac{d_t}{d_{t-1}^\gamma}. \tag{10}$$

In both models $\gamma \geq 0$ denotes the strength of tolerance. If $\gamma = 0$, then past consumption has no weight at all, in which case the model reduces to the standard time separable model, and utility is fully determined by absolute consumption levels and not by the changes in consumption.

Both specifications satisfy our assumptions on s, but two key differences exist. In the subtractive model, effective consumption is HD-1. In the multiplicative model, effective consumption is HD-$(1 - \gamma)$, and the degree of homogeneity depends on the degree of tolerance. Moreover, equation (6) implies that if s is subtractive, then d is addictive for all $\gamma > 0$. Furthermore, if s is multiplicative, then d is addictive if and only if $\sigma_s(c_t, s_t, l_t) > 1$ for all $[c_t, s_t, l_t]$.

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[13]In general, if s has more than one lag, addiction is more restrictive than habit formation. Thus, for example, habit formation and addiction are not equivalent in Becker and Murphy (1988), but are equivalent in Becker et al. (1994).

3.2.2 Household Resources and Optimal Decisions

The household budget constraint sets after tax wage and rental income and government bond redemptions (equal to $R_t^b b_t$, where b_t are bonds issued in $t-1$ and redeemed in t) equal to after tax expenditures on government bond issues and consumption of addictive, ordinary, and investment goods given by $i_t = k_{t+1} - (1-\delta) k_t$, where δ is the depreciation rate. Since consumption of ordinary, addictive, and investment goods all have the same production technology, they have the same pre-tax price, which is normalized to one. Let τ_c and τ_d be the tax rates on consumption of ordinary and addictive goods, respectively and let τ_h be the tax rate on labor income. The household budget constraint is then:

$$R_t^b b_t + r_t k_t + (1 - \tau_{h,t}) w_t h_t = (1 + \tau_{c,t}) c_t + (1 + \tau_{d,t}) d_t + i_t + b_{t+1}. \tag{11}$$

Let λ_t denote the Lagrange multiplier on the budget constraint. The resulting household first order conditions are:

$$(1 + \tau_{c,t}) \lambda_t = \beta^t u_c (c_t, s_t, l_t), \tag{12}$$

$$(1 - \tau_{h,t}) w_t \lambda_t = \beta^t u_l (c_t, s_t, l_t), \tag{13}$$

$$(1 + \tau_{d,t}) \lambda_t = \beta^t u_s (c_t, s_t, l_t) s_1 (d_t, d_{t-1}) + \beta^{t+1} u_s (c_{t+1}, s_{t+1}, l_{t+1}) s_2 (d_{t+1}, d_t), \tag{14}$$

$$\lambda_t R_t = \lambda_{t-1} , \ t \geq 1, \tag{15}$$

$$\lambda_t R_t^b = \lambda_{t-1} , \ t \geq 1, \tag{16}$$

$$R_t = r_t + 1 - \delta. \tag{17}$$

Equations (12)-(16), the budget constraint (11), initial conditions k_0 and d_{-1}, and the appropriate transversality conditions characterize the optimal household decisions k_t, b_t, h_t, c_t, d_t, λ_t. In equation (14), the household increases effective consumption by increasing d_t (first term on the right hand side), but also increases tolerance and therefore reduces future effective consumption (second term on the right hand side). From equations (12) and (14) we have a dynamic Ramsey rule:

$$\frac{1 + \tau_{d,t}}{1 + \tau_{c,t}} = \frac{u_s\left(c_t, s_t, l_t\right) s_1\left(d_t, d_{t-1}\right) + \beta u_s\left(c_{t+1}, s_{t+1}, l_{t+1}\right) s_2\left(d_{t+1}, d_t\right)}{u_c\left(c_t, s_t, l_t\right)} \equiv \frac{MU_{d,t}}{MU_{c,t}}; \qquad (18)$$

where $MU_{i,t}$ represents the marginal utility of good i at time t.

Any difference in tax rates drives a wedge between the marginal utilities of the consumption of ordinary and addictive goods. Thus the optimal tax rate of addictive goods exceeds the tax rate of ordinary consumption goods ($\tau_{d,t} > \tau_{c,t}$) if and only if $MU_{d,t} > MU_{c,t}$. The goal of this paper is to find conditions under which the marginal utility of addictive goods differs from that of ordinary goods.

3.3 Government

The government finances an exogenous sequence of expenditures, g_t, with bond issues and consumption and labor income tax revenues. The government budget constraint is:

$$g_t = \tau_{h,t} w_t h_t + \tau_{c,t} c_t + \tau_{d,t} d_t + b_{t+1} - R_t^b b_t. \qquad (19)$$

As will be clear below, three wedges exist in our model: one between the marginal rate of substitution between addictive and ordinary consumption, a second between the marginal utilities of leisure and working (the after tax wage times the marginal utility of consumption), and a third between the intertemporal marginal rate of substitution and the rate of interest. Thus we need only three tax instruments for a complete tax system. We therefore set interest taxes equal to zero, noting that the government can affect all three margins by setting a time-varying consumption tax (to alter the intertemporal marginal rate

of substitution), a wage tax, and an addictive goods tax.[14] The government optimally uses bonds to smooth tax burdens over time. In the absence of bonds, the government may favor the tax with better smoothing properties.[15] Changes in current addictive goods tax rates affects both current and future tax revenue. The existence of government bonds enables us to conveniently summarize the effect of a change in current addictive tax rates on all periods as the effect on the infinite horizon version of the government's budget constraint.

Let $\pi = \left[(\tau_{c,t})_{t=0}^{\infty}, (\tau_{d,t})_{t=0}^{\infty}, (\tau_{h,t})_{t=0}^{\infty}, (g_t)_{t=0}^{\infty} \right]$ denote an infinite sequence of government policies. As is standard in the literature (e.g. Gruber and Koszegi 2001) we assume the existence of a commitment technology, so that the government commits to all future policies at time zero.[16]

4 Equilibrium and Ramsey Problem

Equations (2), (3), (11), (12) - (15), and (19) form a system of nine non-linear equations that characterize the competitive equilibrium. Hence:

Definition 1 *Given initial values k_0 and d_{-1}, a competitive equilibrium is a set of allocations $\{c_t, d_t, h_t, \lambda_t, k_t\}$, prices $\{w_t, r_t, R_t^b\}$ and a sequence of policies π that satisfy the household budget constraint (11), firm profit maximization (1), the government budget constraint (19), and household maximization of (4) for all t.*

In order to determine optimal taxation we use the primal approach (see for example Chari and Kehoe 1998). The primal approach uses household and firm first order conditions eliminate prices and policies from the equations that define the competitive equilibrium. The planner then chooses allocations which maximize welfare subject to the remaining equations

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[14]We also do not allow a tax on effective consumption, since informational asymmetries rule out taxes on effective consumption in practice.

[15]However, addictive taxes are common at the state and local level, which frequently have constitutional borrowing restrictions. We leave this interesting case to future research.

[16]In principle the government could promise low future taxes on addictive goods, and then find it optimal to renege on the promise once households become addicted. For example, Bossi and Petkov (2007) show that time inconsistency may occur in the regulation of monopolies which sell addictive goods and examine time consistent policies.

from the competitive equilibrium. These equations are the resource constraint:

$$F(k_t, h_t) = c_t + d_t + k_{t+1} - (1 - \delta) k_t + g_t, \tag{20}$$

and the *implementability constraint* (IMC):

$$\frac{u_c(c_0, s_0, l_0) R_0 (k_0 + b_0)}{1 + \tau_{c,0}} = \sum_{t=0}^{\infty} \beta^t \{ u_c(c_t, s_t, l_t) c_t + [u_s(c_t, s_t, l_t) s_1(d_t, d_{t-1}) + \tag{21}$$

$$\beta u_s(c_{t+1}, s_{t+1}, l_{t+1}) s_2(d_{t+1}, d_t)] d_t - u_l(c_t, s_t, l_t) h_t \}.$$

The IMC uses the household first order conditions to substitute out for all prices and policies in the budget constraint and then recursively eliminates λ_t. Thus, the IMC is the infinite horizon version of the household budget constraint where all prices and policies have been written in terms of their corresponding marginal utilities. It is immediate from Walras Law and the resource constraint that the IMC can also be thought of as the infinite horizon version of the government budget constraint. The Ramsey approach is therefore very convenient in that the planner can, through the IMC, determine the effect of a change in d_t on government revenues over the infinite horizon.

The first proposition gives the relationship between the competitive equilibrium and the IMC and resource constraint.

Proposition 2 *Let assumptions (1)-(3) hold. Given k_0, d_{-1}, $\tau_{h,0}$, and $\tau_{c,0}$, the allocations of a competitive equilibrium satisfy (20) and (22). In addition, given k_0, d_{-1}, $\tau_{h,0}$, and $\tau_{c,0}$, and allocations which satisfy (20) and (22), prices and polices exist which, together with the allocations, are a competitive equilibrium.*

All proofs are in the appendix.

The Ramsey Problem (RAM) determines the optimal tax package that maximizes welfare subject to the IMC and resource constraint:

$$
\begin{aligned}
\text{RAM} \quad = \quad & \max_{c_t, d_t, h_t, k_t} \left\{ \sum_{t=0}^{\infty} \beta^t \left(u\left(c_t, s_t, 1 - h_t\right) + \mu \left[u_c\left(c_t, s_t, 1 - h_t\right) c_t + \right. \right. \right. \\
& \left(u_s\left(c_t, s_t, 1 - h_t\right) s_1\left(d_t, d_{t-1}\right) + \beta u_s\left(c_{t+1}, s_{t+1}, l_{t+1}\right) s_2\left(d_{t+1}, d_t\right)\right) d_t - \\
& \left. u_l\left(c_t, s_t, 1 - h_t\right) h_t\right] \right) - \mu \frac{u_c\left(c_0, s_0, l_0\right) R_0\left(k_0 + b_0\right)}{1 + \tau_{c,0}} + \\
& \left. \phi_t \left[F\left(k_t, h_t\right) - c_t - d_t - k_{t+1} + (1 - \delta) k_t - g_t\right] \right\} .
\end{aligned}
\tag{22}
$$

The term multiplied by β^t in problem (22) is the social welfare in period t consisting of private welfare $u\left(c_t, s_t, 1 - h_t\right)$ plus public welfare which is discounted tax revenue (expressed as marginal utilities) multiplied by the marginal value of public funds μ.

The first order conditions that characterize optimal taxation are:

$$
\frac{\phi_t}{\beta^t} = MU_{c,t} + \mu \frac{\partial IMC}{\partial c_t},
\tag{23}
$$

$$
\frac{\phi_t}{\beta^t} = MU_{d,t} + \mu \frac{\partial IMC}{\partial d_t},
\tag{24}
$$

$$
\frac{\phi_t F_h\left(k_t, h_t\right)}{\beta^t} = u_l\left(c_t, s_t, 1 - h_t\right) - \mu \frac{\partial IMC}{\partial h_t},
\tag{25}
$$

$$
\phi_t \left(F_k\left(k_t, h_t\right) + 1 - \delta\right) = \phi_{t-1}.
\tag{26}
$$

First order conditions (23)-(25) equate the marginal social welfare of c, d, and l with the marginal resource cost ϕ. Equation (26) equates the return on capital with the intertemporal marginal rate of substitution.

From equations (23) and (24):

$$
MU_{c,t} - MU_{d,t} = \mu \left(\frac{\partial IMC}{\partial d_t} - \frac{\partial IMC}{\partial c_t} \right).
\tag{27}
$$

Hence using equation (18), we find that addictive goods are taxed at a higher rate than ordinary goods if and only if the derivative of the IMC with respect to d_t is smaller than the derivative with respect to c_t:

$$\tau_{d,t} > \tau_{c,t} \text{ iff } \frac{\partial IMC}{\partial d_t} < \frac{\partial IMC}{\partial c_t}. \tag{28}$$

Since the marginal rate of transformation between c and d is one, the marginal rate of substitution is the tax wedge. The Ramsey problem computes the optimal wedges between the marginal utilities as:

$$\text{Wedge } = \frac{MU_{d,t}}{MU_{c,t}} = 1 + \frac{\mu}{MU_{c,t}} \left(\frac{\partial IMC}{\partial c_t} - \frac{\partial IMC}{\partial d_t} \right). \tag{29}$$

From equation (29), if $MU_{d,t} > MU_{c,t}$, reallocating a marginal resource from ordinary to addictive consumption raises private welfare by $MU_{d,t}$-$MU_{c,t}$. Thus, tax revenue must fall by $\frac{\partial IMC}{\partial c_t} - \frac{\partial IMC}{\partial d_t}$, resulting in a loss of public welfare of μ times the loss of tax revenue. Hence addictive goods are taxed at a higher rate than ordinary goods if and only if moving a resource unit from addictive to ordinary consumption raises revenue, that is, if the marginal tax revenue of ordinary goods exceeds that of addictive goods.[17]

In turn, the marginal tax revenue of ordinary consumption depends on how a small change in ordinary consumption affects ordinary consumption tax revenue, addictive tax revenue, and labor income tax revenue:

$$\frac{\partial IMC}{\partial c_t} = u_{c,t} + u_{cc,t}c_t + \alpha u_{cs,t}s_t - u_{cl,t}h_t. \tag{30}$$

An increase in c_t directly increases ordinary consumption tax revenue (first term), but decreases the marginal utility of consumption and thus requires the planner to lower the ordinary consumption tax rate in order to maintain equilibrium, which lowers tax revenues (second term). The third term contains two offsetting effects. Suppose for example that

---

[17]Note that μ can also be interpreted as the marginal welfare cost of the distorted margins. Thus, addictive goods are taxed at a higher rate if and only if the welfare distortion induced by addictive taxation is less than that of ordinary taxation.

$u_{cs}{>}0$. Then an increase in c_t raises $MU_{d,t}$, so the planner must raise the tax on d_t to maintain equilibrium, which increases addictive tax revenues. In addition, an increase in c_t lowers $MU_{d,t-1}$: consuming d_{t-1} is less attractive because it causes effective consumption to fall in t (tolerance), which lowers utility since $u_{cs}{>}0$. Thus, the planner must also lower $\tau_{d,t-1}$, reducing revenues. Thus an increase in c_t has offsetting addictive tax revenue effects, but both work through the u_{cs} term. Given the homogeneity assumption, these two effects can be combined into a single effect, as if a smaller tax on s_t, rather than d_t, existed. Finally, the fourth term implies an increase in c_t increases preferences for leisure, and thus causes the planner to decrease the labor income tax rate to maintain equilibrium, if and only if $u_{cl}{>}0$.

The marginal tax revenue of addictive goods depends on how a change in addictive consumption affects ordinary consumption tax revenue, addictive consumption tax revenue, and labor income tax revenue:

$$
\begin{aligned}
\frac{\partial IMC}{\partial d_t} = & \ \alpha \left(MU_{d,t} + u_{ss,t}s_{1,t}s_t + \beta u_{ss,t+1}s_{2,t+1}s_{t+1} \right) + \\
& \ u_{cs,t}s_{1,t}c_t + \beta u_{cs,t+1}s_{2,t+1}c_{t+1} - u_{sl,t}s_{1,t}h_t - \beta u_{sl,t+1}s_{2,t+1}h_{t+1}
\end{aligned}
\tag{31}
$$

A small increase in d_t directly increases addictive goods tax revenue, but reduces the marginal utility of effective consumption, which requires the planner to decrease $\tau_{d,t}$. An increase in d_t increases the marginal utility of ordinary consumption and thus increases $\tau_{c,t}$ if and only if $u_{cs} > 0$. An increase in d_t increases the marginal utility of leisure and thus decreases $\tau_{h,t}$ if and only if $u_{sl}{>}0$. An increase in d_t also increases tolerance in period $t+1$, thus reducing s_{t+1}. Thus an increase in d_t affects all three types of tax revenue in period $t+1$ as well but *in the opposite direction.*

In summary then, simple static results and intuition might indicate that taxing addictive goods is a good revenue raiser because addictive goods tend to be income inelastic and complementary to leisure. However, the dynamic results are likely to be more moderate. For example, if leisure and effective consumption are highly complementary, then a decrease in d_t raises labor income tax revenues in period t, but increases s_{t+1}, reducing labor income tax revenues in period $t+1$. In addition, the stronger the tolerance, the stronger is the

dynamic effect. To obtain further results requires more specific preference assumptions. These assumptions shed further light on the optimal tax rates on addictive goods in a dynamic setting.

5   An Analytical Example: The Quadratic Case

In this section, we consider a linear-quadratic utility function. The linear-quadratic utility assumption allows for an analytic solution, which in turn allows us to derive how optimal taxation changes the dynamics of addictive consumption and to derive an exact relationship between tolerance and addictive taxation. On the other hand, the linear-quadratic utility function has only trivial income effects and no labor supply effects.[18]

Suppose the subtractive specification for effective consumption and that the utility and production function are:

$$u\left(c_t, s_t, l_t\right) = c_t + \nu s_t - \frac{s_t^2}{2} + e l_t - \frac{l_t^2}{2}, \quad e < 1, \nu > \frac{1}{1 - \beta\gamma}, \tag{32}$$

$$F\left(k_t, h_t\right) = k_t^\theta h_t^{1-\theta}. \tag{33}$$

Inspection of equations (23) and (24), given the utility function (32), reveals that the marginal utility of d_t is constant in the optimal second best allocation. In particular:

$$\left(1 + \mu\right) = \left(1 + 2\mu\right) MU_{d,t} - \mu\nu\left(1 - \beta\gamma\right). \tag{34}$$

Hence:

$$MU_{d,t} = \frac{1 + \mu\left(1 + \nu\left(1 - \beta\gamma\right)\right)}{1 + 2\mu}. \tag{35}$$

Since the marginal utility of c_t is unity, the marginal utility of d_t equals the tax ratio

---

[18]In the next section we will consider utility functions with non-trivial labor supply and income effects. Nonetheless, the linear-quadratic utility is a common specification in the literature on intertemporally non-separable preferences (Becker et al. 1994, Gruber and Koszegi 2001).

given by equation (18). Hence the tax ratio is constant over time. Furthermore, inspection of equations (12), (15), (23), and (26) indicates that τ_c is constant over time. Therefore, τ_d and τ_h are also constant over time. Thus the implicit interest tax rate is zero for all t.

Given the assumptions on ν (necessary for consumption of d_t to be positive at the steady state), equation (35) implies $MU_{d,t} > 1$ and thus $\tau_{d,t} > \tau_{c,t}$ for all t. Hence we have shown:

Proposition 3 *Let $u(.,.,.)$ and $F(.,.)$ be given by equations (32) and (33) and let effective consumption be given by the subtractive model. Then $\tau_{d,t} > \tau_{c,t}$ for all t and the ratio of tax rates $\frac{1+\tau_{d,t}}{1+\tau_{c,t}}$ is constant over time.*

In the static version of the model without addiction, $d_t = s_t$ has an income elasticity equal to zero whereas the income elasticity of c_t is positive. Since no cross price or labor supply effects exist and since c_t is everywhere more income elastic than d_t, it is optimal to tax d_t at a higher rate regardless of k_t or d_{t-1}, because d_t is a necessity.

It is also clear from equation (35) that the second best optimal d_t is the solution to a linear second order difference equation and that the second best optimal s_t is the solution to a linear first order difference equation. However, before computing the solution to d_t, we must verify that a solution exists for μ. As the next proposition shows, a unique, positive solution exists if government spending is not so large as to exhaust the maximum feasible revenue in the economy, and not so small that given initial tax rates are sufficient to pay for all current and future government expenditures.

Proposition 4 *Let $u(.,.,.)$ and $F(.,.)$ be given by equations (32) and (33) and let effective consumption be given by the subtractive model. Let g_t be a stationary sequence with limiting value $\bar{g}$. Then there exists an interval $[\zeta_l, \zeta_h]$, with $0 < \zeta_l < \zeta_h < \infty$ such that if $G \equiv \sum_{t=0}^{\infty} \beta^t g_t \in [\zeta_l, \zeta_h]$, then a unique positive solution for μ exists.*

Given a unique solution for μ, d_t is the solution to the second order difference characterized by equation (35).

Proposition 5 *Let the conditions for Proposition 4 hold. Then the explicit solution for d_t*

is:

$$d_t = \frac{\nu(1 - \beta\gamma) - 1}{(1 - \gamma)(1 - \beta\gamma)}\left(\frac{1 + \mu}{1 + 2\mu}\right)\left(1 - \gamma^{t+1}\right) + d_{-1}\gamma^{t+1}. \tag{36}$$

The solution for d_t, given by equation (36), allows us to derive some interesting properties of the second best solution, both over time and as compared to the first best solution ($\mu = 0$). First, optimal consumption of d_t increases over time, assuming d_{-1} is less than the steady state. The planner decreases d_t relative to the first best solution and the growth rate of d_t through the tax. However, d_t in the second best optimum is at at least one third of the first best level in the steady state, and greater fraction of the first best solution along the transition.

We can also explore how the strength of tolerance affects second best addictive consumption. Since the solution for μ is unique, we can use the implicit function theorem to derive comparative statics using equation (35). Our intuition is that strong tolerance should moderate the optimal tax ratio, as gains in current tax revenue from taxation of addictive goods are offset by losses in future tax revenues. If d_{-1} is sufficiently large, it is indeed true that the optimal tax ratio is inversely related to the degree of tolerance. In particular, we have:

$$d_{-1} \geq \frac{\beta}{(1 - \beta\gamma)(1 - \beta)} \quad \Rightarrow \quad \frac{\partial \frac{1+\tau_d}{1+\tau_c}}{\partial \gamma} < 0. \tag{37}$$

Condition (37) is a sufficient condition calculated assuming $\mu = \infty$. In practice, for μ small, the optimal tax ratio is decreasing in the degree of tolerance under much less restrictive conditions.

Table 1 gives parameter values for a simple numerical example. Table 1 indicates that the optimal tax ratio is decreasing in the degree of tolerance, even though condition 37 is violated, since the parameter G, set to 30% of GDP for all t, generates at most a value of only $\mu = 4.74$. The planner relies increasingly on labor taxes and less on addictive taxes as the degree of tolerance increases. For $\gamma = 0.55$, taxation is nearly uniform. Figure 1 shows the time path of the first and second best levels of d for various values of γ. Increasing the level of tolerance severely reduces addictive consumption since the future costs of current

consumption are higher. As expected, the difference between first and second best addictive consumption is widest at the steady state.

6   Results for Specific Preferences

As in the literature on optimal commodity taxation, characteristics of the utility function play an important role in determining any deviations from uniform taxation. Two well-known cases are homothetic and separable utility functions. In the next sections we explore these two cases as well as the possible interaction between consumption of addictive goods and leisure.

6.1   Homothetic Utility

In this section we assume utility takes the form:

$$u\left(c_t, s_t, l_t\right) = q\left(v\left(c_t, s_t\right), l_t\right), \tag{38}$$

where $v(.)$ is homothetic and $q(.)$ is an increasing function.

To determine whether or not addictive goods should be taxed at a higher rate than ordinary goods, we combine equations (28), (30), and (31), assuming (38). Let us define the following elasticities:

$$\sigma_{cs,t} \equiv \frac{u_{cs}\left(c_t, s_t, l_t\right) c_t}{u_s\left(c_t, s_t, l_t\right)}, \ \ \sigma_{sc,t} \equiv \frac{u_{cs}\left(c_t, s_t, l_t\right) s_t}{u_c\left(c_t, s_t, l_t\right)}, \ \ \sigma_{hs,t} \equiv \frac{u_{sl}\left(c_t, s_t, l_t\right) h_t}{u_s\left(c_t, s_t, l_t\right)}, \tag{39}$$

and let $\sigma_{hc,t}$ be defined analogously. Then we have the following result.

Proposition 6 *Let assumptions (1)-(3) hold. In addition, let $u(.)$ be of the form given in equation (38). Then $\tau_{d,t} > \tau_{c,t}$ if and only if:*

$$\left(1 - \alpha\right)\left(1 - \sigma_{s,t} - \sigma_{sc,t}\right) > -\frac{\beta u_{s,t+1} s_{2,t+1}}{MU_{d,t}} \left(\begin{array}{c} \alpha\left(\sigma_{s,t+1} - \sigma_{s,t}\right) - \left(\sigma_{cs,t+1} - \sigma_{cs,t}\right) \\ + \left(\sigma_{hs,t+1} - \sigma_{hs,t}\right) \end{array} \right) \tag{40}$$

Homotheticity in c and s does not generally imply uniform taxation of c and d for two reasons: tolerance and because d is taxed but homotheticity is in c and s. Even if no tolerance exists ($s_2 = 0$), homotheticity in c and s does not imply homotheticity of c and d unless s is HD-1 in d only. To see this, suppose without loss of generality that s is HD-η in d only and rewrite equation (40) as:

$$(1 - \eta) + (\eta - \alpha)(1 - \sigma_{s,t} - \sigma_{sc,t}) > -s_{2,t+1}J, \tag{41}$$

where J is the right hand side equation (40), excluding the term $-s_{2,t+1}$. Now let $s_2 \to 0$, so no tolerance exists. The right hand of equation (41) approaches zero, and under our maintained assumptions $s_2 \to 0$ implies $\alpha \to \eta$. The remaining term thus represents the difference between homotheticity of c and s versus c and d. Clearly this term is zero if $\eta = 1$, since $\eta = 1$ if and only if homotheticity of c and s is equivalent to homotheticity of c and d. Thus $s_2 = 0$ and homotheticity in c and d implies uniform taxation. If $\eta \neq 1$, then homotheticity in c and s implies d is a necessity (luxury) if $\sigma_s + \sigma_{sc} < (>) 1$ implying d (c) should be taxed at a higher rate. This is the classical result that it is optimal to tax necessities at a higher rate than luxuries.[19]

To see the effect of tolerance, assume now that $\eta = 1$ or that homotheticity of c and s is equivalent to homotheticity of c and d. Then all terms in equation (40) arise because of tolerance. Consider the labor supply terms only, and suppose both c and d are becoming more complementary with leisure over time ($\sigma'_{hs} < \sigma_{hs}$). Considering only the labor supply terms of equation (40) and rewriting results in:

$$u_{s,t}s_{1,t}\sigma_{hs,t} + \beta u_{s,t+1}s_{2,t+1}\sigma_{hs,t+1} > \sigma_{hs,t}\left(u_{s,t}s_{1,t} + \beta u_{s,t+1}s_{2,t+1}\right). \tag{42}$$

If taxes were equal, then the right hand side of equation (42), or the increase in labor income tax revenues resulting from taxing c, would be identical to the increase in labor income tax

[19]Let M denote non-labor income and $M + h = p_c c + p_d d$ be the budget constraint for a static version of the model. Then equation (41) when $s_2 = 0$ is positive if and only if d has a lower elasticity with respect to non-labor income than c in the static version of the model. Note that the simple partial equilibrium intuition that goods that are more price inelastic should be taxed at a greater rate does not hold in general equilibrium, unless utility is separable and no income effects exist (see for example Chari and Kehoe 1998).

23

revenues resulting from taxing d. But all of the change in labor income tax revenue occurs in the current period for c, whereas some of the change in labor income tax revenues from taxing d come in the next period for d, when the complementarity of addictive consumption and leisure is higher (the left hand side of equation 42). Taxing d at a higher rate means the loss of revenues resulting from households being less addicted in the future will be offset to some extent by the increase in future labor income tax revenues because leisure and addictive consumption are more complementary. Thus taxing d more than c today better smooths distortions over time.

A similar intuition holds for the other elasticities. Tolerance by itself only implies taxation at rates which exceed ordinary goods if the distortions caused by revenue raising are falling over time, so that addictive taxation smooths distortions, because taxing addictive goods now makes taxing addictive goods more distortionary later. As shown in the following corollaries, however, some common specifications for v and s induce constant elasticities which imply uniform taxation.

Corollary 7 *Let the conditions of Proposition 6 hold, and let $q(.) = z(l) + \frac{\left(c^{\delta}s^{1-\delta}\right)^{1-\sigma}-1}{1-\sigma}$, and $z(.)$ be concave, then $\tau_d = \tau_c$ for all t.*

Although we have assumed here that $v(.)$ is constant relative risk aversion (CRR), this corollary is considerably more realistic than the existing literature which assumes a static utility function and/or separable quadratic utility for tractability. Hence, ignoring labor supply effects and making utility CRR in c and s, we find that it is optimal not to tax addictive goods at a higher rate than ordinary goods. However, if labor supply effects are present then the results could shift in favor of taxing addictive goods at a higher rate.

Let $\bar{x}$ denote the steady state value of any variable x, then we have the following result.

Corollary 8 *Let the conditions of Proposition 6 hold. Then $\bar{\tau}_d > \bar{\tau}_c$ if and only if:*

$$(1-\alpha)\left(1-\bar{\sigma}_s-\bar{\sigma}_{sc}\right) > 0. \tag{43}$$

Equation (43) holds if and only if the steady state income elasticity of d is less than the steady state income elasticity of c. For HD-1 addiction functions, including the subtractive

model, Corollary 8 indicates that the steady state tax rates are uniform. An increase in $\bar{d}$ increases $\bar{s}$ by the same percentage, preserving homotheticity. For the multiplicative case, the degree of homogeneity is decreasing in the strength of tolerance. Since $\sigma_s > 1$ is required for addiction in the multiplicative case, strong tolerance tends to reduce the tax rate on the addictive good, unless addictive and ordinary consumption are sufficiently strong substitutes (that is, unless $-\sigma_{sc} > \sigma_s - 1$), because stronger tolerance implies strong offsetting future revenue effects from current addictive taxation. When the degree of homogeneity is less than one, as is the case for the multiplicative model, then equation (43) reduces to:

$$\bar{\sigma}_s - 1 < -\bar{\sigma}_{sc} \tag{44}$$

Since with multiplicative habits addiction and reinforcement occur if and only if $\bar{\sigma}_s > 1$, it is optimal to tax addictive goods at a higher rate only if addictive and ordinary consumption goods are sufficiently strong substitutes. Finally, Corollary 7 and Proposition 6 indicate that the choice of addiction function is not innocuous when designing optimal tax policies. In particular, given condition (44), the subtractive model implies a higher tax rate on the addictive good than the multiplicative model, since $\alpha = 1$ in the subtractive model.

6.2   Additively Separable Utility

n this section we consider the case in which utility is additively separable. Following a similar procedure as with Proposition 6, we have:

Proposition 9 *Let assumptions (1)-(3) hold. In addition, let $u(.)$ be additively separable in c, s, and l. Then $\tau_{d,t} > \tau_{c,t}$ if and only if:*

$$\alpha\sigma_{s,t} + 1 - \alpha - \sigma_{c,t} > -\frac{\beta\alpha u_{s,t+1}s_{2,t+1}}{MU_{d,t}}\left(\sigma_{s,t+1} - \sigma_{s,t}\right). \tag{45}$$

In a static model, separable utility removes all labor supply and cross price revenue effects, so we have two effects: c and d have potentially different income elasticities in a static sense and the effect of tolerance.

To see the static income elasticity terms, suppose without loss of generality that s is HD-η in d only and rewrite equation (45) as:

$$(\eta\sigma_{s,t} + 1 - \eta) - \sigma_{c,t} + (\alpha - \eta)(\sigma_{s,t} - 1) > -s_{2,t+1}J, \tag{46}$$

where J is the right hand side equation (40), excluding the term $-s_{2,t+1}$. Now let $s_2 \to 0$, so no tolerance exists. The right hand of equation (46) approaches zero, and under our maintained assumptions $s_2 \to 0$ implies $\alpha \to \eta$. The remaining term is:

$$\eta\sigma_{s,t} + 1 - \eta > \sigma_{c,t}, \tag{47}$$

$$\frac{\eta\sigma_{s,t} + 1 - \eta}{\sigma_{c,t}} > 1. \tag{48}$$

The left hand side equals the income elasticity of c divided by the income elasticity of d in the static version of the model. Condition (48) says to tax the more income inelastic (the necessity) good at a higher rate. If $\eta = 1$ then $s = d$ and the income elasticity ratio is $\sigma_{s,t}/\sigma_{c,t}$. If $\eta < 1$, then η affects the concavity of u in d and thus the income elasticity.

To see the effect of tolerance, rewrite equation (45) as:

$$(\alpha\sigma_{s,t} + 1 - \alpha)\, u_{s,t}s_{1,t} + (\alpha\sigma_{s,t+1} + 1 - \alpha)\, \beta u_{s,t+1}s_{2,t+1} >$$
$$\sigma_{c,t}\, (u_{s,t}s_{1,t} + \beta u_{s,t+1}s_{2,t+1})\,. \tag{49}$$

Suppose further that d is becoming more income inelastic relative to c over time ($\sigma_{s,t+1} > \sigma_{s,t}$). If tax rates were equal, then the right hand side of equation (49), or the increase in tax revenues resulting from taxing c, would be identical to the increase in tax revenues resulting from taxing d. But all of the change in tax revenues comes in the current period for c, whereas some of the change in tax revenues from taxing d come in the next period for d, when d is more income inelastic (the left hand side of equation 49). Taxing d at a higher rate today means d will become more income elastic in the future, offsetting to some

degree the decrease in income elasticity resulting from $\sigma_{s,t+1} > \sigma_{s,t}$. Thus, as in Proposition 6, the existence of tolerance implies it is optimal to use the tax on d to smooth intertemporal distortions.

As shown in the following corollary, however, the common CRR specification has constant elasticities which implies tolerance affects optimal taxation only through it's effect on the income elasticity.

Corollary 10 *Let the conditions of Proposition 9 hold, and let* $u(.) = v^1(c) + v^2(s) + v^3(l)$, *with* v^1 *and* $v^2(.)$ *CRR. Then* $\tau_{d,t} > \tau_{c,t}$ *for all t if and only if:*

$$\alpha\sigma_s + 1 - \alpha > \sigma_c. \tag{50}$$

Condition (50) holds if and only if d has a lower income elasticity than c. In turn, tolerance only affects the ratio of income elasticities if it affects the homogeneity of s. For s functions such that α is independent of tolerance, such as the subtractive model, addiction has no effect on the optimal tax rates. With constant elasticities, no distortion smoothing motivation exists. However, for s functions such that α depends on tolerance, such as the multiplicative model, stronger tolerance reduces the income elasticity and thus moderates the optimal tax rate on addictive goods, since $\sigma_s > 1$. In particular, if d is a necessity in period t, then taxing d at a high rate raises revenue with less distortions in period t, but will nonetheless reduce future addiction and thus d_{t+1} and revenue in $t + 1$. Since the income elasticity is constant, the revenue effects tend to cancel. That is, the stronger the revenue effect in period t, in general the stronger the offsetting revenue effect in period $t + 1$. Thus strong tolerance makes taxing addictive consumption less attractive. Finally, the subtractive model implies a higher tax rate on the addictive good than the multiplicative model, since $\alpha = 1$ in the subtractive model.

In general, the optimal tax on addictive goods depends on the degree of tolerance and the relative income elasticities. The relative tax rate on the addictive good rises as the income elasticity of the addictive good falls, irrespective of the degree of addiction. Tolerance allows the planner to smooth intertemporal distortions by increasing the tax rate on addictive

consumption if the income elasticity is falling (if σ_s is rising). Taxation of addictive goods reduces future tolerance and thus makes addictive consumption more income elastic, offsetting the fall in income elasticity. However, since σ_s is constant for CRR utility functions and at the steady state, no distortion smoothing motivation exists. However, strong tolerance still tends to moderate the optimal tax on addictive goods, since taxing d reduces future tax revenues.

7   Labor Supply Effects

The homothetic and separable cases examined above assume that changes in c and s have identical effects on labor supply. However, some addictive goods are highly complementary with leisure. For example, alcohol is complementary with leisure and reduces productivity (Parry et al. 2006).[20] In this section we consider the possibility that ordinary and addictive goods differ in their static labor supply effects. Addiction, as defined here, is related to reinforcement and tolerance, neither of which depends on complementarity with leisure. Thus we should expect to see extra terms which reflect how addictive goods consumption affects labor income tax revenues in addition to the relative income elasticity and degree of tolerance.

To see this in a concise way, let us consider the following class of utility functions:

$$u\left(s_t, d_t, l_t\right) = q\left(c_t\right) + v\left(s_t, l_t\right). \tag{51}$$

Given the specification (51), addictive consumption is potentially more complementary with leisure than ordinary consumption. For this specification, we find:

Proposition 11 *Let assumptions (1)-(3) hold. In addition, let $u(.)$ be of the form given by (51). Then $\tau_{d,t} > \tau_{c,t}$ if and only if:*

$$1 - \alpha + \alpha\sigma_{s,t} + \sigma_{hs,t} > \sigma_{c,t} - \frac{\beta u_{s,t+1} s_{2,t+1}}{MU_{d,t}} \left(\sigma_{hs,t+1} - \sigma_{hs,t} + \alpha\left(\sigma_{s,t+1} - \sigma_{s,t}\right)\right). \tag{52}$$

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[20]On the other hand, goods like lotteries do not seem to exhibit strong labor supply effects.

The left hand side of equation (52) indicates that, ignoring the dynamic effect caused by tolerance, strong complementarity between addictive consumption and leisure tends to increase the relative tax on addictive goods.

However, tolerance may change this effect. To see how, note that equation (52) can be rewritten as:

$$\frac{u_{s,t}s_{1,t}\left(1 - \alpha + \alpha\sigma_{s,t} + \sigma_{hs,t}\right) + \beta u_{s,t+1}s_{2,t+1}\left(1 - \alpha + \alpha\sigma_{s,t+1} + \sigma_{hs,t+1}\right)}{MU_{d,t}} > \sigma_{c,t}. \tag{53}$$

Since utility is separable in c, the right hand side of equation (53) incorporates only two effects: an increase in c directly raises revenue but requires the planner to lower the tax rate on c to maintain equilibrium. Conversely, the tax on d raises labor supply if $\sigma_{hs} > 0$, which raises labor income tax revenues. However, taxing d is not as attractive as taxing s, since taxing d creates an offsetting effect on next period's labor income tax revenue, since $s_2 < 0$. Further, the stronger the effect on today's labor income tax revenues, the stronger the offsetting effect on future labor income tax revenues. Ignoring the dynamic effect on future labor income tax revenues results in a tax rate on addictive goods that exceeds the optimal rate.

Equation (52) simplifies in the steady state or for CRR preferences.

Corollary 12 *Let the conditions of Proposition 11 hold, and let* $v(.) = \frac{\left(s^\delta l^{1-\delta}\right)^{1-\sigma} - 1}{1-\sigma}$, *with* $\sigma \geq 1$. *Then* $\tau_{d,t} < \tau_{c,t}$ *for all* t.

Corollary 12 is an example with complementarity between leisure and consumption of addictive goods, which nonetheless has a lower tax rate on d_t than c_t.

Corollary 13 *Let the conditions of Proposition 11 hold. Then* $\bar{\tau}_d > \bar{\tau}_c$ *if and only if:*

$$\alpha\left(\bar{\sigma}_s - 1\right) + \bar{\sigma}_{hs} > \bar{\sigma}_c - 1. \tag{54}$$

In this case only the revenue effects matter.

8 Conclusions

This paper is the first attempt in the literature to characterize and analyze the conditions under which taxation of addictive goods might differ from taxes on labor and ordinary consumption goods in a dynamic, rational addiction setting. In particular, we derive conditions under which tax rates for addictive goods exceeds tax rates for non-addictive goods in an environment where exogenous government spending cannot be financed with lump sum taxes.

We find that if taxing addictive goods has strong positive revenue effects today, then a strong offsetting effect on future tax revenues also exists. This dynamic property of addictive goods might explain why static models, which do not model addiction, but incorporate revenue raising and externalities motives, tend to derive tax rates in excess of that observed in the data.

For the homothetic and separable cases that we consider, the dynamic nature of tolerance justifies tax rates in excess of ordinary consumption goods for the purpose of smoothing intertemporal distortions. For example, if the income elasticity of addictive consumption is falling over time, then a higher tax rate on the addictive good today reduces future addiction and thus tends to offset some of the decrease in the income elasticity. It is also possible that the offsetting revenue effects of addictive taxation makes addictive goods more income elastic, thus decreasing the optimal tax rate on addictive goods.

Finally, we also consider features of addictive goods such as complementarity to leisure that, while unrelated to addiction itself, are nonetheless common among some addictive goods. In general, such effects are weaker in our dynamic setting.

Consideration of other intertemporal goods, such as durable goods or storable goods, while outside the scope of this paper, is a natural extension. Some properties of the s function would likely change with these intertemporal goods, but our basic methodology would still apply.

Our results come with several caveats. First, one common feature of addictive goods, the presence of externalities, has not been considered in this paper. Still, the "double dividend" literature (e.g. Bovenberg and Goulder 1996) indicates external effects alone is not a reason to tax such goods above the first best level which corrects the externality. Thus, the optimal

tax may rise in a model that includes externalities, but our qualitative conclusions would likely be similar, if interpreted as relative to the first best tax rate. We leave this case for future research.

Second, we consider only the optimal tax package, not the optimal addictive goods tax taking as given other taxes. Certain features of the tax code including balanced budget rules and positive capital tax rates, if taken as given, may change our results. Third, we have no heterogeneity in addictive consumption or wealth. However, if we assume that the poor are more likely to consume addictive goods, then our results would likely strengthen, because consumers of addictive goods would have a higher marginal utility of income. Finally, following the Ramsey approach, we allow for distortionary linear taxes only. Our results might change if we allowed nonlinear tax rates within a Mirrles framework. However, the capital and income taxes studied by Mirrles are nonlinear in the data, whereas addictive taxes are typically linear. We leave these issues to further research as well.

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9 Appendix: Proofs

9.1 Proof of Proposition 2

To see that a competitive equilibrium satisfies the IMC and resource constraint, we substitute the factor prices (2) and (3) into the budget constraint (11). Using constant returns to scale, we then have:

$$R_t^b b_t + F(k_t, h_t) - \tau_{h,t} F_h(k_t, h_t) h_t = (1 + \tau_{c,t}) c_t +$$
$$(1 + \tau_{d,t}) d_t + i_t + b_{t+1}. \tag{55}$$

Combining the above equation with the government budget constraint (19) gives the resource constraint (20).

To derive the IMC from the budget constraint, we substitute the household first order conditions (12)-(14) into the budget constraint (11), eliminating the tax rates, so that:

$$\lambda_t R_t k_t + \lambda_t R_t^b b_t - \lambda_t k_{t+1} - \lambda_t b_{t+1} = \beta^t \left(u_c(c_t, s_t, l_t) c_t + \right.$$
$$\left. MU_{d,t} d_t + u_l(c_t, s_t, l_t) h_t \right). \tag{56}$$

Next using the first order conditions (15) and (16), we have:

$$\lambda_t R_t (k_t + b_t) - \lambda_{t+1} R_{t+1} (k_{t+1} + b_{t+1}) =$$
$$\beta^t \left(u_c(c_t, s_t, l_t) c_t + MU_{d,t} d_t + u_l(c_t, s_t, l_t) h_t \right). \tag{57}$$

The above equation characterizes a sequence of budget constraints that can be used to recursively eliminate $\lambda_t R_t (k_t + b_t)$, yielding:

$$\lambda_0 R_0 (k_0 + b_0) - \lim_{t \to \infty} \lambda_{t+1} R_{t+1} (k_{t+1} + b_{t+1}) =$$
$$\sum_{t=0}^{\infty} \beta^t \left(u_c(c_t, s_t, l_t) c_t + MU_{d,t} d_t - u_l(c_t, s_t, l_t) h_t \right). \tag{58}$$

The transversality conditions imply the second term on the left hand side equals zero. Again using the household first order conditions at period zero gives:

$$\frac{u_{c,0}\left(k_0 + b_0\right)}{1 + \tau_{c,0}}\left(F_{k,0} + 1 - \delta\right) = \sum_{t=0}^{\infty} \beta^t \left(u_c\left(c_t, s_t, l_t\right) c_t + MU_{d,t} d_t - u_l\left(c_t, s_t, l_t\right) h_t\right), \quad (59)$$

which is the IMC.

We next show that, given allocations which satisfy the IMC and resource constraint, prices and policies exist which, along with the allocations, are a competitive equilibrium. Let $\{c_t, k_t, h_t, d_t\}$ be a sequence which satisfies the IMC and resource constraint. Then r_t and w_t are defined using equations (2) and (3). Since $\tau_{c,0}$ is given, we can define λ_0 using equation (12). Then λ_t can be defined recursively using equation (15). Then R_t^b is defined using equation (16). Next, we define the government policies:

$$\left(1 + \tau_{c,t}\right) = \frac{\beta^t u_c\left(c_t, s_t, l_t\right)}{\lambda_t}, \quad (60)$$

$$\left(1 - \tau_{h,t}\right) = \frac{\beta^t u_l\left(c_t, s_t, l_t\right)}{\lambda_t F_h\left(k_t, h_t\right)}, \quad (61)$$

$$\left(1 + \tau_{d,t}\right) = \frac{\beta^t MU_{d,t}}{\lambda_t}, \quad (62)$$

$$g_t = \tau_{h,t} w_t h_t + \tau_{c,t} c_t + \tau_{d,t} d_t \quad (63)$$

Given the above prices and policies, all equations which define a competitive equilibrium are satisfied except the household and government budget constraints. We use b_t to satisfy the household budget constraint:

$$\begin{aligned} b_t &= \frac{1}{R_t^b}\left(-r_t k_t - \left(1 - \tau_{h,t}\right) w_t h_t + \left(1 + \tau_{c,t}\right) c_t + \right. \\ &\quad \left. \left(1 + \tau_{d,t}\right) d_t + i_t + b_{t+1}\right). \end{aligned} \quad (64)$$

We can multiply the above equation by λ_t and recursively eliminate b_{t+1} from the above equation. After eliminating prices and policies using the household first order conditions (12)-(14), b_t is a function of the allocations:

$$b_t = \left(\prod_{i=0}^{t-1} \left(F_k \left(K_i, H_i \right) + 1 - \delta \right) \right) \frac{1 + \tau_{c,0}}{\tau_{c,0}} \sum_{i=t}^{\infty} \beta^i \left(u_{c,i} c_i + MU_{d,i} d_i - u_{l,i} h_i \right) - k_t \qquad (65)$$

The above equation is the debt allocation which implies the household budget constraint is satisfied.

Since the budget constraint is satisfied, we simply substitute the resource constraint into the budget constraint to see that the government budget constraint is satisfied. Finally, by substituting the prices and policies into the IMC and reversing the derivation of the IMC, we see that the transversality conditions are satisfied.

9.2   Proof of Propositions 3-5

Proposition 3 was proved in the text. For Proposition 4, we derive the solution for μ as follows. First, for the quadratic case, the first order conditions for the Ramsey problem (23)-(26) are now:

$$\frac{\phi_t}{\beta^t} = 1 + \mu, \qquad (66)$$

$$\frac{\phi_t}{\beta^t} = \nu \left(1 - \beta\gamma \right) \left(1 + \mu \right) - \left(1 + 2\mu \right) \left(s_t - \beta\gamma s_{t+1} \right), \qquad (67)$$

$$\frac{\phi_t \left(1 - \theta \right) \left(\frac{k_t}{h_t} \right)^{\theta}}{\beta^t} = \left(e - 1 \right) \left(1 + \mu \right) + \left(1 + 2\mu \right) h_t, \qquad (68)$$

$$\phi_t \left(\theta \left(\frac{k_t}{h_t} \right)^{\theta - 1} + 1 - \delta \right) = \phi_{t-1}. \qquad (69)$$

Using equation (66) to eliminate ϕ_t gives:

$$1 + \mu = (\nu (1 + \mu) - \mu)(s_t - \beta\gamma s_{t+1}),\tag{70}$$

$$(1 + \mu)(1 - \theta)\left(\frac{k_t}{h_t}\right)^\theta = (e - 1)(1 + \mu) + (1 + 2\mu)h_t,\tag{71}$$

$$\beta\left(\theta\left(\frac{k_t}{h_t}\right)^{\theta-1} + 1 - \delta\right) = 1.\tag{72}$$

For the subtractive model equations (66) and (70) imply:

$$1 + \mu = \nu(1 - \beta\gamma)(1 + \mu) - (1 + 2\mu)(d_t - \gamma d_{t-1} - \beta\gamma(d_{t+1} - \gamma d_t)),\tag{73}$$

$$-\beta\gamma d_{t+1} + (1 + \beta\gamma^2)d_t - \gamma d_{t-1} = \frac{1 + \mu}{1 + 2\mu}(\nu(1 - \beta\gamma) - 1),\tag{74}$$

which has general solution given by (36). Proposition (5) thus holds if a non-zero and finite solution for μ exists, which we now show.

Equation (72) implies the capital to labor ratio, denoted by A, is a constant equal to:

$$A \equiv \left(\frac{\theta}{\rho + \delta}\right)^{\frac{1}{1-\theta}}.\tag{75}$$

Thus, equation (71) implies

$$h_t = \frac{1 + \mu}{1 + 2\mu}\hat{h}\ ,\ \hat{h} \equiv 1 - e + (1 - \theta)A^\theta,\tag{76}$$

is constant. Thus, $k_t = Ah_t$ is constant and equation (36) implies s_t and thus $MU_{d,t}$ is constant as well. Since elasticity of substitution of consumption over time is infinite, the planner absorbs all changes in g_t by varying c_t. Combining these results with resource

constraint (20) yields a solution for c_t:

$$c_t = \frac{1+\mu}{1+2\mu} \left(\hat{h} \left(A^\theta - \delta A \right) - \hat{d} \left(1 - \gamma^{t+1} \right) \right) - \gamma^{t+1} d_{-1} - g_t, \tag{77}$$

$$\hat{d} \equiv \frac{\nu \left(1 - \beta\gamma \right) - 1}{\left(1 - \gamma \right) \left(1 - \beta\gamma \right)}. \tag{78}$$

Now since h_0 enters into the left hand side of the IMC (22) and k_0 is given, the solutions for h_0, k_0 and therefore c_0 generally differ from the solutions for $t \geq 1$. Therefore, we let $x \equiv \frac{1+\mu}{1+2\mu}$ and insert the solutions for c_t, d_t, and h_t into the IMC (22) for $t \geq 1$, so that:

$$
\begin{aligned}
\frac{R_0 \left(k_0 + b_0 \right)}{1 + \tau_{c,0}} \quad = \quad & \sum_{t=1}^{\infty} \beta^t \left[x \left(\hat{h} \left(A^\theta - \delta A \right) - \hat{d} \left(1 - \gamma^{t+1} \right) \right) - \gamma^{t+1} d_{-1} + \right. \\
& \left. \left(1 - \beta\gamma \right) \left(\nu - \left(1 - \gamma \right) \hat{d}x \right) \left(\hat{d}x \left(1 - \gamma_{t+1} \right) + \gamma^{t+1} d_{-1} \right) - \left(e - 1 + \hat{h}x \right) \hat{h}x \right] + \\
& c_0 + MU_{d,0} d_0 - u_{l,0} h_0 + g_0 - G.
\end{aligned} \tag{79}
$$

Next, recall from Proposition 2 that $\tau_{c,0}$ and $\tau_{h,0}$ are given. It follows from equations (12) and (13) that the planner cannot choose h_0 in this example, and instead takes the solution for h_0 from the competitive model as given. Further, the terms inside the summation depend on time only through γ^{t+1} and β^t, and the equation is quadratic in x. Therefore, after evaluating the summation we can write the equation as:

$$-\zeta_2 x^2 + \zeta_1 x + \zeta_0 = 0, \tag{80}$$

$$\zeta_2 \equiv \hat{h}^2 + \frac{\left(1 - \gamma \right)^2}{\beta} \hat{d}^2, \tag{81}$$

$$\zeta_1 \equiv \zeta_2 - \left(\frac{1-\beta}{\beta} \right) \left(1 - \gamma \right) \gamma \hat{d} d_{-1}, \tag{82}$$

39

$$\zeta_0 \equiv \zeta_2 - \zeta_1 + \zeta_3, \tag{83}$$

$$\zeta_3 \equiv \frac{1-\beta}{\beta} \left(\frac{\tau_{c,0} \left(1-\delta\right) k_0 - R_0 b_0}{1+\tau_{c,0}} + \frac{1+\tau_{c,0}-\theta}{1+\tau_{c,0}} k_0^\theta h_0^{1-\theta} - u_{l,0} h_0 - G \right). \tag{84}$$

A solution such that $\mu > 0$ is a solution in the range $\frac{1}{2} < x < 1$. Note that equation (80) can be written as:

$$\left(\zeta_2 x + \zeta_2 - \zeta_1\right)\left(1-x\right) = -\zeta_3. \tag{85}$$

Hence it is immediate that $\zeta_3 < 0$ is necessary for $x < 1$. From equation (84), $\zeta_3 < 0$ if and only if:

$$G > \zeta_l \equiv \frac{\tau_{c,0}\left(1-\delta\right) k_0 - R_0 b_0}{1+\tau_{c,0}} + \frac{1+\tau_{c,0}-\theta}{1+\tau_{c,0}} k_0^\theta h_0^{1-\theta} - u_{l,0} h_0. \tag{86}$$

This is the lower bound for G.

Condition (86) implies that $\zeta_3 < 0$ which implies both roots are less than one. It remains to show that the roots are real and that one root is greater than one half. The roots are real if and only if:

$$\zeta_1^2 + 4\zeta_2 \left(\zeta_2 - \zeta_1 + \zeta_3\right) > 0, \tag{87}$$

$$\left(2\zeta_2 - \zeta_1\right)^2 > -4\zeta_2 \zeta_3, \tag{88}$$

$$-\zeta_3 < \frac{\left(2\zeta_2 - \zeta_1\right)^2}{4\zeta_2}, \tag{89}$$

$$G < \zeta_l + \frac{\beta}{1-\beta} \left(\frac{\left(2\zeta_2 - \zeta_1\right)^2}{4\zeta_2} \right). \tag{90}$$

40

Finally, the roots are greater than one half if and only if:

$$\frac{\zeta_1}{2\zeta_2} \pm \frac{\sqrt{(2\zeta_2 - \zeta_1)^2 + 4\zeta_2\zeta_3}}{2\zeta_2} > \frac{1}{2}, \tag{91}$$

$$\pm \sqrt{(2\zeta_2 - \zeta_1)^2 + 4\zeta_2\zeta_3} > \zeta_2 - \zeta_1. \tag{92}$$

The right hand side is positive, so the smaller root cannot be bigger than one half. The larger root is bigger than one half if and only if:

$$(2\zeta_2 - \zeta_1)^2 + 4\zeta_2\zeta_3 > (\zeta_2 - \zeta_1)^2, \tag{93}$$

$$-\zeta_3 < \frac{3\zeta_2 - 2\zeta_1}{4}, \tag{94}$$

$$G < \zeta_l + \frac{\beta}{1-\beta}\left(\frac{3\zeta_2 - 2\zeta_1}{4}\right). \tag{95}$$

The final bound for a unique solution is thus the intersection of conditions (86), (90), and (95):

$$\zeta_l < G < \zeta_l + \frac{\beta}{1-\beta}\frac{3\zeta_2 - 2\zeta_1}{4}. \tag{96}$$

Defining ζ_h as the right hand side of equation (96) completes the proof.

For Proposition 5, we solve the second order difference equation (35), using equation (18):

$$(\nu - s_t) - \beta\gamma(\nu - s_{t+1}) = \frac{1 + \mu(1 + \nu(1 - \beta\gamma))}{1 + 2\mu}, \tag{97}$$

$$\nu\left(1-\beta\gamma\right)-d_t+\gamma d_{t-1}+\beta\gamma\left(d_{t+1}-\gamma d_t\right)=\frac{1+\mu\left(1+\nu\left(1-\beta\gamma\right)\right)}{1+2\mu}, \tag{98}$$

$$d_{t+1}-\left(\frac{1+\beta\gamma^2}{\beta\gamma}\right)d_t+\frac{1}{\beta}d_{t-1}=-\frac{1+\mu}{1+2\mu}\left(\frac{\nu\left(1-\beta\gamma\right)-1}{\beta\gamma}\right). \tag{99}$$

It is straightforward to show the general solution of the above difference equation is:

$$d_t=D_p+A_0\gamma^t+A_1\left(\beta\gamma\right)^{-t}, \tag{100}$$

$$D_p\equiv\frac{1+\mu}{1+2\mu}\left(\frac{\nu\left(1-\beta\gamma\right)-1}{\left(1-\beta\gamma\right)\left(1-\gamma\right)}\right). \tag{101}$$

Following convention, we rule out the explosive, bubble solutions which requires $A_1=0$. Letting $t=-1$ implies $A_0=\gamma\left(d_{-1}+D_p\right)$. Substituting for A_0 and simplifying gives the desired solution.

9.3   Proof of Proposition 6

First, we rewrite equations (30) and (31), using the σ definitions, so that:

$$\frac{\partial IMC}{\partial c_t}=u_{c,t}\left(1-\sigma_c+\alpha\sigma_{sc,t}-\sigma_{hc,t}\right), \tag{102}$$

$$\begin{aligned}\frac{\partial IMC}{\partial d_t}=&\ MU_{d,t}\left(\alpha-\alpha\sigma_{s,t}+\sigma_{cs,t}-\sigma_{hs,t}+\right.\\&\left.\frac{\beta u_{s,t+1}s_{2,t+1}}{MU_{d,t}}\left(\alpha\sigma_{s,t}-\alpha\sigma_{s,t+1}-\sigma_{cs,t}+\sigma_{cs,t+1}+\sigma_{hs,t}-\sigma_{hs,t+1}\right)\right).\end{aligned} \tag{103}$$

Now since v is homothetic, we know that:

$$\frac{v_c\left(\psi c,\psi s\right)}{v_s\left(\psi c,\psi s\right)}=\frac{v_c\left(\psi c,\psi s\right)}{v_s\left(\psi c,\psi s\right)}, \tag{104}$$

which implies:

$$\frac{v_{cc}\left(c,s\right)c}{v_c\left(c,s\right)}+\frac{v_{cs}\left(c,s\right)s}{v_c\left(c,s\right)}=\frac{v_{ss}\left(c,s\right)s}{v_s\left(c,s\right)}+\frac{v_{cs}\left(c,s\right)c}{v_s\left(c,s\right)}, \tag{105}$$

which, using the definition of $u(.)$ in equation (38), implies:

$$\sigma_{sc}-\sigma_c=\sigma_{cs}-\sigma_s. \tag{106}$$

It is also immediate from the definition of $u(.)$ that $\sigma_{hc}=\sigma_{hs}$. These facts and equations (23), (24), (102), and (103) together imply:

$$\frac{MU_{d,t}}{u_{c,t}}=\frac{1+\mu\left(1-\sigma_{s,t}+\sigma_{cs,t}-\left(1-\alpha\right)\sigma_{sc,t}-\sigma_{hs,t}\right)}{1+\mu\left(\alpha-\alpha\sigma_{s,t}+\sigma_{cs,t}-\sigma_{hs,t}+\frac{\beta u_{s,t+1}s_{2,t+1}}{MU_{d,t}}j_t\right)}, \tag{107}$$

where $j_t\equiv\alpha\sigma_{s,t}-\alpha\sigma_{s,t+1}-\sigma_{cs,t}+\sigma_{cs,t+1}+\sigma_{hs,t}-\sigma_{hs,t+1}$. Hence, $\tau_{d,t}>\tau_{c,t}$ if and only if the right hand side is greater than one, or:

$$1\quad-\left(1-\alpha\right)\sigma_{sc,t}-\sigma_{s,t}>$$
$$\alpha-\alpha\sigma_{s,t}+\frac{\beta u_{s,t+1}s_{2,t+1}}{MU_{d,t}}\left(\sigma_{s,t}-\sigma_{s,t+1}-\sigma_{cs,t}+\sigma_{cs,t+1}+\sigma_{hs,t}-\sigma_{hs,t+1}\right), \tag{108}$$

which simplifies to the desired result.

9.4   Proof of Corollaries 7-8

For the CRR case, note that $\sigma_{sl}=0$, $\sigma_s=1-\delta\left(1-\sigma\right)$, and $\sigma_{sc}=\left(1-\delta\right)\left(1-\sigma\right)$, which implies the left hand side of condition (40) is zero. The right hand side of (40) is also zero since σ_{cs} and σ_s are constant.

For the steady state case, $\sigma_{i,t}=\sigma_{i,t+1}$ for all $i\in\{s,sc,cs,hs\}$, so the result follows immediately from condition (40).

9.5 Proof of Proposition 9

If utility is separable, equations (102) and (103) become:

$$\frac{\partial IMC}{\partial c_t} = u_{c,t}\left(1 - \sigma_c\right), \tag{109}$$

$$\frac{\partial IMC}{\partial d_t} = MU_{d,t}\left(\alpha - \alpha\sigma_{s,t} + \frac{\beta u_{s,t+1}s_{2,t+1}}{MU_{d,t}}\left(\alpha\sigma_{s,t} - \alpha\sigma_{s,t+1}\right)\right). \tag{110}$$

Equations (23), (24), (109), and (110) together imply:

$$\frac{MU_{d,t}}{u_{c,t}} = \frac{1 + \mu\left(1 - \sigma_{c,t}\right)}{1 + \mu\left(\alpha\left(1 - \sigma_{s,t}\right) + \frac{\beta\alpha u_{s,t+1}s_{2,t+1}}{MU_{d,t}}\left(\sigma_{s,t} - \sigma_{s,t+1}\right)\right)}. \tag{111}$$

Hence, d is taxed at a higher rate if and only if the right hand side is greater than one, or:

$$1 - \sigma_{c,t} > \alpha\left(1 - \sigma_{s,t}\right) + \frac{\beta\alpha u_{s,t+1}s_{2,t+1}}{MU_{d,t}}\left(\sigma_{s,t} - \sigma_{s,t+1}\right), \tag{112}$$

which simplifies to the desired result.

9.6 Proof of Corollary 10

For CRR preferences, $\sigma_{i,t} = \sigma_{i,t+1}$ for all $i \in \{s, c\}$, so the results follow immediately from condition (45).

9.7 Proof of Proposition 11

If utility is given by equation (51), equations (102) and (103) become:

$$\frac{\partial IMC}{\partial c_t} = u_{c,t}\left(1 - \sigma_{c,t}\right), \tag{113}$$

$$\frac{\partial IMC}{\partial d_t} = MU_{d,t}\left(\alpha - \alpha\sigma_{s,t} - \sigma_{hs,t}+ \right.$$
$$\left. \frac{\beta u_{s,t+1}s_{2,t+1}}{MU_{d,t}}\left(\alpha\left(\sigma_{s,t} - \sigma_{s,t+1}\right) + \sigma_{hs,t} - \sigma_{hs,t+1}\right)\right). \tag{114}$$

Equations (23), (24), (113), and (114) together imply:

$$\frac{MU_{d,t}}{u_{c,t}} = \frac{1 + \mu\left(1 - \sigma_{c,t}\right)}{1 + \mu\left(\alpha\left(1 - \sigma_{s,t}\right) - \sigma_{hs,t} + \frac{\beta u_{s,t+1}s_{2,t+1}}{MU_{d,t}}\left(\alpha\left(\sigma_{s,t} - \sigma_{s,t+1}\right) + \sigma_{hs,t} - \sigma_{hs,t+1}\right)\right)}. \tag{115}$$

Hence, d is taxed at a higher rate if and only if the right hand side is greater than one, or:

$$1 - \sigma_{c,t} > \alpha\left(1 - \sigma_{s,t}\right) - \sigma_{hs,t} + \frac{\beta u_{s,t+1}s_{2,t+1}}{MU_{d,t}}\left(\alpha\left(\sigma_{s,t} - \sigma_{s,t+1}\right)\sigma_{hs,t} - \sigma_{hs,t+1}\right), \tag{116}$$

which simplifies to the desired result.

9.8 Proof of Corollaries 12 and 13

For the CRR case, note that $\sigma_s = 1 - \delta\left(1 - \sigma\right)$, and $\sigma_{sl} = \left(1 - \delta\right)\left(1 - \sigma\right)\frac{h}{1-h}$. Substituting in these conditions into equation (52) gives:

$$\delta\left(1 - \alpha\right)\left(1 - \sigma\right) - \sigma_{c,t} > -\frac{\left(1 - \delta\right)\left(1 - \sigma\right)}{MU_{d,t}}\left(\frac{u_{s,t}s_{1,t}}{l_t} + \frac{\beta u_{s,t+1}s_{2,t+1}}{l_{t+1}}\right). \tag{117}$$

For $\sigma \geq 1$, the left hand side is negative. Further the right hand side is positive if:

$$\frac{u_{s,t}s_{1,t}}{l_t} + \frac{\beta u_{s,t+1}s_{2,t+1}}{l_{t+1}} > 0, \tag{118}$$

$$u_{s,t}s_{1,t}l_{t+1} > -\beta u_{s,t+1}s_{2,t+1}l_t. \tag{119}$$

If $l_{t+1} \geq l_t$, then the above inequality is satisfied since marginal utility is positive. But stability analysis indicates l_t is an increasing sequence along the transition path. Therefore, the right hand side is indeed positive, and so the inequality (52) cannot be satisfied, and

hence $\tau_{c,t} \geq \tau_{d,t}$.

For the steady state case, $\sigma_{i,t} = \sigma_{i,t+1}$ for all $i \in \{s, hs\}$, so the result follows immediately from condition (52).

10   Appendix: Tables and Figures

Parameter	Value	variable	Results for Constant Variables		
			$\gamma = 0.45$	$\gamma = 0.5$	$\gamma = 0.55$
β	0.9	k_t	1.66	1.58	1.54
ν	2	h_t	0.57	0.55	0.53
θ	0.4	y_t	0.88	0.83	0.81
g_t	0.3	i_t	0.17	0.16	0.15
δ	0.1	μ	2.23	3.48	4.74
e	0.95	x	0.59	0.56	0.55
d_{-1}	0.02	τ_c	0.07	0.07	0.07
$\tau_{c,0}$	0.07	τ_d	0.15	0.12	0.075
b_0	0	τ_h	0.39	0.42	0.44

Table 1: Parameter values and results for variables which are constant over time. The parameters h_0 and k_0 are set equal to h_t using equation (76) and $k_t = Ah_t$, respectively. The parameter g_t is set equal to 30% of GDP for all t.
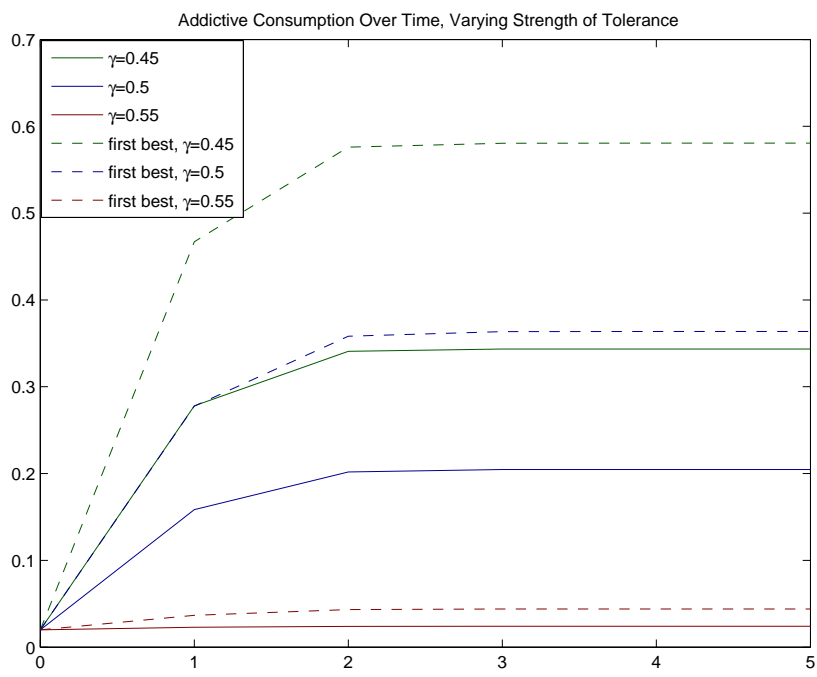
Figure 1: Dynamics of first and second best addictive consumption for various values of γ.