

# The Natural Rate of Structure Depreciation: Decoupling Capital Gains and Maintenance Improvement from Property Value Variation

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## Abstract

This paper, motivated by the observations that the land of a property does not depreciate, along with taking into account the capital gains from housing boom and bust cycles, provides a novel approach to separate housing structure depreciation and capital gains on land into two distinct components that make up overall changes in property value. This analysis is further developed by distinguishing between the natural rate of structure depreciation (the rate that would occur in the absence of maintenance) and the effects of maintenance on structure value over time. We utilize this approach to estimate the natural depreciation rate of housing structure value and homeowner's maintenance technology of single-family detached housing in the New York metropolitan area using data from the American Housing Survey. This method estimates the natural rate of depreciation of structures to be roughly 9 percent per year; this shows a significant, and often overlooked economic cost is associated with real estate ownership due to value loss from structure depreciation and maintenance expenses.

keywords: housing, real estate, structure depreciation, land, home maintenance, returns to scale

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# 1 Introduction

Scholars and investors in the United States have long debated whether investing in real estate assets is a wise decision compared to investment in the financial market. The expected return on a real estate investment is commonly projected by relying on macroeconomic conditions and government policies (e.g., property tax rates). However, the value loss from structure depreciation and the hidden cost of maintenance improvements are often neglected. These two issues are especially important in the US, where houses are traditionally built with relatively less durable materials like wood and lumber, as compared to brick or concrete. The cost of maintenance builds up over time to have a significant impact on homeowners, particularly in the US. Despite its importance, the natural depreciation rate of housing structures (the rate that would occur in the absence of maintenance) has been overlooked in previous literature. Due to this, we aim to present a method to estimate the exogenous natural rate of structure depreciation by disentangling the various elements of changes in housing property values and explaining their relationships. Not only does this research expand real estate investment and housing depreciation literature, but our model for properly measuring the natural rate of depreciation also contributes to applications such as housing capital valuation and implementation of optimal property tax rates. Moreover, our research has led to an estimated maintenance function that will be useful for developing policies that promote affordable housing.

Our approach is motivated by the following facts. First, not every component of housing capital depreciates. Only the structure of the property depreciates, but not the land associated with it. A direct estimation of structure depreciation (instead of the property value depreciation) is very necessary since the ratio between land value and the property value varies with different houses, different neighborhoods, and different regions. Second, homeowners routinely spend money on housing maintenance. This slows down the observed rate of de-

preciation from its natural rate. Third, land is non-reproducible. The inelastic supply of non-reproducible land leads to particularly volatile changes in capital values of housing property due to variations in economic activity and job opportunities associated with the locations of the property that stem from the business cycle.

Consider a house whose market value is observed at two points in time. The measured property value change captures change in the quality of the structure, change in the desirability of the house's location, and macroeconomic conditions. It is, therefore, challenging to achieve an accurate estimation of the depreciation of housing structures. The housing depreciation estimation is complicated by several facets. The first facet is that even after adjusting for inflation, housing itself (with controlling its quality of the structure) experiences capital gains in the housing boom and bust cycle. Without decoupling capital gains from total change in value of housing property, the housing structure depreciation rate will be understated.

The second facet is the fact that not all the components in a given housing property depreciate. The land of a housing property does not depreciate, while the value of the housing structure do. Since there is no separable market for the housing structure and its lot due to the fact that structure and land are sold as a bundle in real estate transactions, we can not directly observe the change of housing structure values from transaction data. Moreover, the land value of lots differ across different metropolitan areas. The different weights of non-depreciating parts of the properties due to regional market differences could bias the estimation of depreciation rate if we use only the housing property value for depreciation. For a particular housing unit, the higher the lot value of the property is, the lower the depreciation rate will be estimated if we do not subtract out the non-depreciable land value of that property.

The third facet is the effect of maintenance expenditures on the depreciation of housing structure value. Sweeney (1974) and Arnott et al. (1983) provide a theoretical analysis of hous-

ing maintenance in the context of housing filtering theory. Housing maintenance expenditures made by the rational landlord or homeowner per year are not a constant number; it is determined by the state of the housing quality. The maintenance expenses will not cover all the depreciation loss unless the housing quality is in a steady state as illustrated in Arnott et al. (1983) and Arnott and Zhao (2019). The estimated depreciation rate without considering the maintenance effect is, in actuality, the net depreciation rate under the effects of maintenance. Much of the existing literature on depreciation rate estimation does not take the homeowner's maintenance expenses into account leading to a blind spot in depreciation analysis this paper purports to correct.

The fourth facet regards age-related depreciation. Literature documents that housing depreciation (net depreciation when considering the maintenance effect) is nonlinear as well as age dependent. Shilling et al. (1991) document that housing capital depreciates fast in its early years. Examining this further, we argue that one important factor that contributes to this nonlinear depreciation rate is the effect of a homeowner's various levels of maintenance expenses at different ages of the housing. Aged houses require higher maintenance expenditures, while homeowners do not usually spend much money on maintenance for newly built housing units. As a result, the net depreciation (after taking into account maintenance expenditures) for housing units in their early years is higher than when they are older. Therefore, the fact that a housing structure depreciates quickly in its early years can be explained by the homeowner spending less on maintenance in the early years of ownership. This observation of housing maintenance and the resulting nonlinear depreciation are also consistent with the simulation results generated by Arnott and Zhao (2019) using our estimated natural rate of structure depreciation and the estimated maintenance technology. Therefore, this aged-related depreciation is partially mitigated in our approach since our method admits a nonlinear (via considering maintenance) depreciation rate. The other factor, in the relatively long

run, that contributes to age-related nonlinear depreciation is the obsolescence effect (property potentially loses value due to its outdated housing style).

The last facet we consider is the variation in the natural rate of structure depreciation itself. This arises from the discrepancies associated with the various types of building and construction materials. The structural durability and soundness of various construction materials are different. This fact leads to the natural rate of structure depreciation having some potential variability due to the varying physical durability of different construction materials. With controlling types of housing units to be single-family detached houses, we assume a homogeneous natural depreciation rate for this type of home in the US.<sup>1</sup> The other potential source of variability arises from regional differences in natural conditions such as weather, geographic activities (e.g., earthquake, hurricane). However, by focusing on a particular region, this issue can be accounted for as well.

In the current literature, there are mainly three approaches with respect to housing depreciation. The first approach is a modified repeat sales model approach. For example, Harding et al. (2007) estimate the depreciation rates of housing capital (structure and land) based on the housing asset values provided in the data of the American Housing Survey. One innovation of their approach is that they integrate the housing depreciation and maintenance effect into the repeat sales model; they also estimate the depreciation rate of average housing capital in the US with and without the effect of maintenance. The second approach is based on the hedonic pricing model. Smith (2004) identifies that it is important to remove land values from depreciation estimation and that housing locations play an important role in the depreciation rates of housing capital. Yoshida (2016) proposes a method based on the hedonic approach to estimate the structure value depreciation based on his estimated structure and land ratio. The

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<sup>1</sup>It may still be possible that for very high-end luxury and customized houses, the natural rate of depreciation may be slightly different than conventional houses.

third approach is the national account approach, used by Davis and Heathcote (2005). Each approach carries with it its shortcomings as well as benefits.

Previous research variously realizes the need to consider individual elements discussed previously: take the maintenance effect into consideration (Shilling et al., 1991; Harding et al., 2007), separate structure value (Yoshida, 2016), recognize that land will not depreciate, and emphasize the regional heterogeneity of land (Smith, 2004; Yoshida, 2016). However, none accounts for every factor presented in this paper. Among them, it is the hedonic pricing approach that has been employed as a foundation for researchers. However, the works based on the hedonic pricing model did not seem to mention how they can integrate the capital gains on land into their models. By contrast, the modified repeat sales approach incorporates the maintenance effect and housing capital gains (Harding et al., 2007), but it does not separate land value from property value and does not account for regional differences in land. Davis and Heathcote (2005), in their national account approach, make contributions in separating the treatment of housing structures and land, but they did not take into account the effect of homeowner maintenance at different ages of a housing unit.

This paper introduces a novel method to overcome these issues and advances housing depreciation research by providing an approach to estimate the housing maintenance technology and to directly measure the housing structure depreciation while taking account of the housing capital gains, the maintenance effect, the non-depreciability of land, and the regional heterogeneity in the land. In summary, our research contributes to the literature in the following respects. First, we recognize the land value of the lot in a property does not depreciate and housing maintenance slows down the depreciation. By decomposing the property value into land value and structure value, then focusing on the depreciation of housing structure value rather than the housing property value, we mitigate the bias (which before arose

from heterogeneity in land price) in estimating the structure depreciation rate. In this way, we can separate the exogenous depreciation of housing structures from the slowing effect of housing maintenance on decreases in structure value. Second, as far as we know, the American Housing Survey (AHS) is the only database that provides maintenance expenditure data. However, AHS does not provide useful information about the land value of the lot, which creates difficulty with directly ascertaining the structure value of the house.<sup>2</sup> The effect of capital gains from the housing boom and bust cycles creates further difficulties in estimating the depreciation rate. We impute land values, taking into account the plattage effect (larger parcels have a lower value per unit area of land) and the lot size information of each property provided by AHS. We then identify the individual contributions from structure depreciation and capital gains derived from changes in land values. Third, we utilize the characteristics of the New York housing market and the plattage effect to account for the heterogeneity of land value. Plattage effect refers to the empirical findings that, with controlling general accessibility, the price of a land parcel rises less than proportionally with its parcel size. Specifically, an important condition to properly apply the plattage effect to impute land value is to control the general urban accessibility. Among the New York metropolitan area, when comparing the ‘urban center’ (Manhattan) with its peripheral regions, the urban accessibility is quite different. We remove the Manhattan housing market (the highest land value in the nation) from our New York metropolitan area dataset, justified by the fact that there are almost no single-family houses on Manhattan used for residential occupancy.<sup>3</sup> Fourth, the estimated exogenous housing de-

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<sup>2</sup>Technically, AHS has a data entry for the land value of a lot. However, possibly due to the difficulty for a homeowner to know the lot value of his property, it is extremely rare to see house lot value reported in the AHS survey. For comparison, in the 2011 AHS, only less than 1 percent of total observations come with the land value while 50 percent of total observations report property value.

<sup>3</sup>The New York metropolitan area gives us the largest amount of data from the AHS national survey; this unique characteristic of the New York housing market (there is almost no single family house in Manhattan, the ‘urban center’ of the New York metropolitan area) is the main reason we choose the New York metropolitan area as our example to measure depreciation. Technically, in AHS surveys, it does contain a geographic variable (METRO3) to distinguish the central city and its suburban areas. However, in the AHS codebook, it clearly warns the users to treat this variable with caution for many reasons, such as METRO3 having been altered or assigned with different values to mask the locations of some housing units. The sample unit’s geographic category in-

preciation rate and maintenance technology can be applied to the numerical simulations of housing policy using the housing filtering models. In the next section, we describe our model and method. In section 3, we describe our data and data filtering process. In section 4, we present our findings and results. In section 5, we present empirical discussion, robustness checks, and an extended model (with its estimation results). In particular, in the extended model, by introducing time dummies and assuming the maintenance technology keeps the same from the year 2005 to the year 2013, we show the robustness of our model as that estimation of the natural depreciation rates and maintenance technology (with combining the survey year 2005 and year 2007) are quite consistent with the estimations in section 4. We conclude this paper in Section 6.

## 2 Model

The model contains two stages. In the inner stage, the net depreciation of housing structure value is decomposed into two opposing effects: the natural depreciation of the housing structure itself and the appreciation due to housing maintenance. In the outer stage, the value of depreciable and reproducible housing structure is derived from the difference between the value of housing property and the value of the non-depreciable/non-reproducible land associated with the property, with the land value being modeled using the plattage effect. Section 2.1 lists a notational glossary for the model parameters. Section 2.2 describes the derivation procedures of our model.

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formation varied widely depending on the time when it was added to the survey due to various definitions of century cities /urban areas were adopted at different times.



## 2.1 Notational Glossary

Variable	Definition
$HV_{it}$	real property value of house $i$ <sup>1</sup> at time $t$
$SV_{it}$	real structure value of house $i$ at $t$
$sv_{it}$	real structure value per unit floor area of house $i$ at $t$
$LV_{it}$	real land value of house $i$ at time $t$
$a$	age of house $i$
$f_i$	floor area of house $i$
$m_{it}$	real maintenance expenditure per unit floor area for house $i$ at $t$
$p_t$	real normalized price of land
$LS_i$	lot size of house $i$
$NLS_i$	normalized lot size of house $i$
$nls_i$	ratio of normalized lot size to floor area of house $i$
$\alpha$	shift-scaling parameter in maintenance function
$\beta$	elasticity of rate of structure depreciation with respect to maintenance
$\delta$	natural rate of structure depreciation
$\gamma$	elasticity of land value with respect to land area
$c_{t,t+1}$	rate of real capital gains on land from $t$ to $t+1$
$c_t$	average rate of real capital gains on land from $t$ to $t+2$

<sup>1</sup> Single-family house, detached from any other building

TABLE 1: Notational Glossary

## 2.2 Structure Depreciation and Maintenance

Housing structure, like capital, depreciates every year. In the meantime, homeowners pay for maintenance expenses (including routine maintenance services and home improvement projects), which cause the housing structure value to ‘appreciate.’ The model in the inner stage, allows us to identify and separate the changes in housing structure value from these two different effects: natural depreciation of a housing structure and maintenance improvement. Since housing floor space does not depreciate, the depreciation rate of housing structure value is equal to that of housing structure value per unit floor area.

After adjusting for inflation<sup>4</sup>, let  $sv_{it}$  be house  $i$ ’s real structure value per unit floor area at

<sup>4</sup>As being discussed later, under the assumption we make, inflation in quality-controlled replacement cost is equal to inflation at the general level.

$t$ ,  $sv_{i,t+1}$  be house  $i$ 's real structure value per unit floor area at  $t + 1$ ,  $\delta$  be the annual natural depreciation rate on the structure,  $m_{it}$  be the real maintenance expenditure per unit floor area of house  $i$  at  $t$ , and  $a$  be the age of house  $i$ . Then, the difference in structure value per unit floor area between two adjacent years is given by:

$$sv_{i,t+1} - sv_{i,t} = g(sv_{it}, m_{it}, \delta, a), \quad (1)$$

where  $g(sv_{it}, m_{it}, \delta, a)$  is a general function form which reflects the effects from the natural depreciation of the housing itself, homeowner's maintenance expenditure, and obsolescence change associated with the age of the house. The age of house  $i$ ,  $a$ , enters into Eq.(1) through the obsolescence.<sup>5</sup>

Here, for convenience, we assume the above general function form is independent of the age of the house. This simplification is valid with the assumption that there is no value loss due to obsolescence over the short-run period we study. While the above discussion is for the age effect on the particular house  $i$  from the perspective of our model setup, in the empirical analysis, although it is not related to Eq.(1), another effect associated with age can arise from considering multiple houses with different ages at time  $t$ . Different houses may be built with different types of construction materials depending on their construction years. Different construction material will lead to variations in the natural rate of depreciation across observations. Our object of study is single-family detached houses in the US; in particular, in the New York metro region. Developers have a long tradition of using wood as the predominant construction material for single-family houses in the US where there is an abundance of wood and timbers. Therefore, we assume a homogeneous natural rate of structure depreciation across

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<sup>5</sup>Obsolescence refers to the value loss associated with outdated housing styles (old-fashioned architectural design, outmoded amenities, etc.) which is likely caused by changes in consumers' tastes or sudden changes derived from external factors, such as sudden breakthroughs in housing construction technology or changes in housing regulations (e.g., code revisions of earthquake-resistant design of buildings and housing structures).

the years of construction in our sample. That being said, we can ignore the age of the house provided the above conditions are satisfied and write Eq.(1) as:

$$sv_{i,t+1} - sv_{i,t} = g(sv_{it}, m_{it}, \delta). \quad (2)$$

Eq.(2) is similar to the housing quality depreciation function in Arnott et.al (1983) in which housing quality is cardinized as housing (re)construction cost per unit floor area. In this paper, we can also represent the housing structure value per unit of floor area as (re)construction cost per unit floor area. In the real world, with housing being constructed with a multitude of qualities, this induces (re)construction cost (per unit floor area) to be equal to the structure value (per unit floor area) of the house in this wide range. The above arguments, implicitly carry with them the assumption that the quality-controlled and inflation-adjusted replacement costs are stable in the short run. Therefore, housing structure value can be represented as the housing (re)construction cost (in other words, replacement cost), which is also consistent with Davis and Heathcote (2007) and Case (2007).

Strong (additive) separability between housing structure depreciation and housing maintenance associated with the Eq.(2) is assumed,

$$g(sv_{it}, m_{it}, \delta) = g_1(sv_{it}, \delta) + g_2(m_{it}), \quad (3)$$

where  $g_1(sv_{it}, \delta)$  is the function representing the natural depreciation of housing structure and  $g_2(m_{it})$  is the maintenance function.

The basis of the above separation is by recognizing that natural depreciation of housing structure and homeowner's maintenance improvements are two separate effects on changes

in housing structure value and, more importantly, by assuming the cross partials associated are zero. That is, the cross effect of the maintenance expense per unit floor area  $m_{it}$  over the marginal effect of the structure value per unit floor area  $sv_{it}$  on the depreciation function  $g_1$  is 0,  $\frac{\partial}{\partial m_{it}} \left( \frac{\partial g_1}{\partial sv_{it}} \right) = 0$ ; the cross effect of structure value per unit floor area  $sv_{it}$  over the marginal effect of the maintenance expense per unit floor area  $m_{it}$  on the maintenance function  $g_2$  is 0,  $\frac{\partial}{\partial sv_{it}} \left( \frac{\partial g_2}{\partial m_{it}} \right) = 0$ ; and the cross effect of natural depreciation rate itself  $\delta$  over the marginal effect of the maintenance expense per unit floor area  $m_{it}$  on the maintenance function  $g_2$  is 0,  $\frac{\partial}{\partial \delta} \left( \frac{\partial g_2}{\partial m_{it}} \right) = 0$ . The above arguments can be intuitively illustrated as maintenance expense has no effect on the exogenous process of natural depreciation of housing structure, marginal productivity of maintenance function is independent of the structure value per unit floor area, and natural depreciation itself has no influence on the marginal productivity of the maintenance function. Then Eq.(2) can be written as:

$$sv_{i,t+1} - sv_{i,t} = g_1(sv_{it}, \delta) + g_2(m_{it}). \quad (4)$$

In the above equation,  $g_1(sv_{it}, \delta)$  characterizes the housing structure depreciation in the absence of maintenance. The natural rate of housing structure depreciation is assumed to, just as the depreciation of capital stock, follow exponential decay (i.e., depreciation under a constant geometric rate).<sup>6</sup> It follows that, without maintenance, housing structure value decreases by a fixed percent each year rather than by a fixed and equal dollar amount each year. Geometric depreciation of housing structure we assume is in line with Arnott and Braid (1991) and Davis and Heathcote (2005). With the assumption of exponential decay, we have:

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<sup>6</sup>Although most of physical depreciation follow exponential decay process - it decreases at a rate proportional to its present amount- with a constant geometric depreciation rate, some external factors (e.g., unusual usage of housing, weather and climate change and etc.) can cause fluctuation in the 'constant' geometric depreciation rate we assumed. In order to avoid complication, we assume the processes of external factors that possibly influence the natural rate of depreciation itself is stochastic so that on average, we have a constant geometric depreciation rate.

$$g_1(sv_{it}, \delta) = -\delta sv_{it}. \quad (5)$$

Following Arnott et al. (1983), the housing maintenance function is assumed to be diminishing returns to maintenance expenditure per unit floor area.<sup>7</sup> We also assume, in the short run, there is no technological change in the maintenance function; the function of maintenance technology keeps the same for the time duration we study. Based on the above analysis, a standard function form of diminishing returns to scale, which also carries with it a well-defined, unit-less elasticity,  $\beta$ , is assumed for housing maintenance technology,

$$g_2(m_{it}) = \alpha m_{it}^\beta, \quad (6)$$

where  $g_2(m_{it})$  is the maintenance function and it measures ‘appreciation’ due to maintenance in the unit of structure value per unit area.  $m_{it}$  is maintenance expenditure per unit area at year  $t$ ;  $\alpha$  is the coefficient of maintenance function, whose value is governed by the relative cost of maintenance;  $\beta$  is the elasticity of maintenance expenditure per unit floor area on structure value improvement per unit floor area. Based on the above analysis, we should expect to see that  $\beta \in (0, 1)$  and  $\alpha > 0$  from data. The validity of our assumptions is supported by the estimation results in sections 4 and 5, which both confirm this conjecture.

It is also worth noting that the above simplifications, the exclusion of the obsolescence change associated with house  $i$ 's age,  $a$ , in Eq.(2) and the assumption of constant geometric depreciation rate  $\delta$  in Eq.(5), do not shy away from the empirical regularity of the age-related non-linear depreciation of housing structure. The observed depreciation rate in the literature

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<sup>7</sup>Diminishing returns to maintenance is a crucial assumption, which guarantees that there is unique and interior optimal level of maintenance expenditure per unit floor area for a house at a particular age from the time of construction.

is the net depreciation rate with the effect of housing maintenance. The net depreciation (after taking into account maintenance expenditures) for housing units in their early years is higher than when they are older. Therefore, the observed endogenous depreciation rate under the effect of maintenance is still nonlinear in our model.

By plugging Eq.(5) and Eq.(6) into Eq.(4), the difference in structure value per unit floor area between two adjacent years is given by

$$sv_{i,t+1} - sv_{it} = -\delta sv_{it} + \alpha m_{it}^{\beta}. \quad (7)$$

Then,

$$sv_{i,t+1} = (1 - \delta)sv_{it} + \alpha m_{it}^{\beta}. \quad (8)$$

The above equation concludes the inner stage of our model, which elucidates the change in housing structure value resulting from both the natural depreciation and the appreciation from maintenance.

### 2.3 Land Value, Capital Gains and Plattage Effect

The outer stage of our model is presented in this subsection. Since the housing unit is sold as a bundle in real estate transactions, the housing structure value is not directly observable, only housing property value is observable. However, we can treat housing structure value as a residual - the difference between property value and the value of the plot of land which the house is built on. The structure value of house  $i$  at time  $t$  is

$$SV_{it} = HV_{it} - LV_{it}. \quad (9)$$

where  $HV_{it}$  denotes house  $i$ 's property value at time  $t$ ,  $SV_{it}$  denotes house  $i$ 's structure value at time  $t$ , and  $LV_{it}$  is house  $i$ 's land value at time  $t$ .

The housing structure is durable (with depreciation) and reproducible, whereas the land of the housing unit is non-depreciable and non-reproducible. As discussed earlier, structure value can be represented by housing structure replacement cost. With the assumption that real replacement costs are stable in the short-run, we argue the durable structure is not contributing to the real capital gains.<sup>8</sup> In the short run, all the value change due to capital gains falls on land for the following reasons. First, land is scarce and the supply of land is relatively inelastic. Second, land is non-reproducible and each parcel of land and its location is unique. Third, land embodies the value of the property's geographic location. Various geographic locations provide differing access to a variety of quality of natural and social amenities, such as employment opportunities, shopping malls, schools, medical facilities, etc. In cyclical fluctuations, the social amenities, including the employment opportunity in that location will change, which then causes a change of the land value in that geographic location. Liu et al. (2016) provide empirical evidence that land value and unemployment move in opposite directions over the business cycle. Using the above reasoning, the land value of house  $i$  between time  $t$  and  $t + 1$  is:

$$LV_{i,t+1} = (1 + c_{t,t+1})LV_{it}, \quad (10)$$

where  $c_t$  is the rate of real capital gains of the land from time  $t$  to time  $t + 1$  for house  $i$ .

The land value of a house can be imputed by utilizing the plattage effect, which was first studied by Colwell and Sirmans (1978). The plattage effect refers to the fact that while controlling for general urban accessibility, the value of a land parcel increases less than proportionally

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<sup>8</sup>That is to say, we assume, in the short run, the inflation of quality-controlled replacement costs is the same as the inflation in general.

with the size of the lot. That is, the price per square foot of a lot decreases as lot size increases. A normalized lot size is then introduced so that the price per unit of normalized land is the same for different sizes of the lot. Let  $\gamma$  denote the elasticity of land value with respect to lot size and  $p_t$  denote the price per normalized land at time  $t$ . We can represent the normalized lot size of house  $i$ ,  $NLS_i$  as,

$$NLS_i = (LS_i)^\gamma. \quad (11)$$

Many works in the literature have estimated the elasticity of land value with respect to the lot size under the plattage effect using different methods. Isakson (1997) estimates the value of  $\gamma$  to be 0.6343, 0.6495 and 0.7007 using his three empirical models. Zhang and Arnott (2015) estimate  $\gamma = 0.639$  for the value of a developable vacant parcel with respect to lot size. These results are very close to each other which suggests that the elasticity of land value with respect to lot size is very stable across time and space. We take the  $\gamma = 0.639$  for land size normalization for the reason that this estimated value is particularly for the developable land.<sup>9</sup>

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<sup>9</sup>Technically, there is a subtle distinction here. The land we study is the developed land – the land on which there is a structure built above. However, since the value of developed land is not directly observable, the datasets employed by the literature to estimate the plattage effect are usually obtained from information on vacant land sales. The value of vacant land is also referred to as the raw value of the site (the land on which there is no structure). An important reason we choose to use the estimation results from Zhang and Arnott (2015) is that they classified land as either undevelopable or developable and reported the estimation results based on this classification. Developable land is quite close to the developed land we study with the exception that developed land has been leveled and connected to utilities (such as water, sewage, gas and etc) whereas developable land has not been serviced and connected to such infrastructure. We assume in our model that the value of the developed land is equal to the value of developable land since the land value we refer to in our model is the location value; this assumption is justified when several factors are considered. First, the value derived from the land grading and utility infrastructure connections are associated with wages and materials; it is not associated with the location of the land parcel, therefore it is not subject to capital gains from the location of the parcel. Second, the land grading and utility infrastructure connections are in essence subsumed within the structure value since both of them depreciate and require maintenance. (e.g., backfilled soil may cause house settling, and connected water and gas pumps may leak.) Thirdly, because we treat the structure value as the (re)construction cost, and the site preparation cost (which includes the cost of land grading) is included in the construction cost according to RSMMeans Estimating Handbook.



The land value of house  $i$  at time  $t$ ,  $LV_{it}$ , is

$$LV_{it} = p_t NLS_i. \quad (12)$$

We should also note that urban accessibility is quite different in the center of a metropolitan area compared to its periphery. For example, in the New York metro area, people living in Manhattan enjoy a short walking distance to a high density of quality restaurants, and other social amenities. As a result, the price of the normalized land is very different in Manhattan compared with that in its surrounding areas. By focusing on single-family detached houses in our data in the New York metro area, this issue will be greatly mitigated by using the fact that there are almost no single-family detached houses on Manhattan, limiting this phenomenon's impact on our analysis.<sup>10</sup>

Structure value of house  $i$ ,  $SV_{it}$ , equals structure value of house  $i$  per unit floor area,  $sv_{it}$ , multiplied by the total floor area of house  $i$ ,  $f_i$ . Then:

$$SV_{it} = sv_{it} \times f_i. \quad (13)$$

Substituting Eq.(9), Eq.(10), Eq.(12) and Eq.(13) into Eq.(8) yields

$$\frac{HV_{i,t+1}}{f_i} = (1 - \delta) \frac{HV_{it}}{f_i} + ((1 + c_{i,t+1}) - (1 - \delta)) p_{it} \frac{NLS_i}{f_i} + \alpha (m_{it})^\beta, \quad (14)$$

where the notation is as follows:

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<sup>10</sup>Technically, there are a few exceptions. For example, there are some historical detached houses in Manhattan, e.g., Alexander Hamilton's House, however, they are no longer belong to residential housing and as a result, they are not in the American housing survey.

- $HV_{i,t+1}$  = real property value of house  $i$  at time  $t + 1$ ;  
 $HV_{it}$  = real property value of house  $i$  at time  $t$ ;  
 $LV_{it}$  = real land value of house  $i$  at time  $t$ ;  
 $p_t$  = real normalized land price per unit area at time  $t$ ;  
 $NLS_i$  = normalized lot size of house  $i$ ;  
 $m_{it}$  = real maintenance cost per unit floor area of house  $i$  at time  $t$ ;  
 $f_i$  = floor size of house  $i$ ;  
 $\delta$  = housing structure (natural) depreciation rate;  
 $c_{t,t+1}$  = rate of real capital gains on land from  $t$  to  $t + 1$ ;  
 $\gamma$  = elasticity of land value with respect to lot size;  
 $\alpha$  = shift-scaling parameter of maintenance function;  
 $\beta$  = elasticity of improvement of housing structure value per floor area with respect to  $m_{it}$ .

## 2.4 Empirical Model

AHS - national survey is conducted every other year, iterate Eq.(14) one period forward while assuming  $m_{it} = m_{i,t+1}$ :

$$\frac{HV_{i,t+2}}{f_i} = (1 - \delta)^2 \frac{HV_{it}}{f_i} + ((1 + c_t)^2 - (1 - \delta)^2) p_t \frac{NLS_i}{f_i} + (2 - \delta) \alpha (m_{it})^\beta. \quad (15)$$

where  $c_t$  denotes the average rate of real land capital gains between period  $t$  to  $t + 2$ ; that is,  $(1 + c_t)^2 = (1 + c_{t,t+1})(1 + c_{t+1,t+2})$ . When the land experiences a capital gain over these two years, then  $c_t$  is a positive. When the land experiences negative economic shocks such as resulted from the financial crisis, then the rate of capital gains  $c_t$  is negative.

The model is then formulated into a nonlinear regression for estimation:

$$\frac{HV_{i,t+2}}{f_i} = a \frac{HV_{it}}{f_i} + b \frac{NLS_i}{f_i} + k(m_{it})^\beta + u_{i,t+2}. \quad (16)$$

where  $u_{i,t+2}$  is the error term.

The relationship between the model parameters and the estimated parameters are:

$$\begin{aligned} a &= (1 - \delta)^2 \rightarrow \delta = 1 - \sqrt{a}; \\ b &= ((1 + c_t)^2 - (1 - \delta)^2)p_t; \\ k &= (2 - \delta)\alpha \rightarrow \alpha = \frac{k}{2 - \delta}; \\ \beta &= \beta. \end{aligned} \quad (17)$$

Based on our model and assumptions, we should expect to see the following results from our data. The structure depreciation rate,  $\delta$ , should be in the range of  $0 < \delta < 1$ , thus,  $0 < a < 1$ . Since housing maintenance will slow down the structure depreciation, shift-scaling parameter,  $\alpha$ , should be positive, thus,  $k > 0$ . Elasticity of maintenance expenditure, should be in the range of  $0 < \beta < 1$ . The sign of  $b$  is an interesting indicator; recall from Eq.(15),  $b = ((1 + c_t)^2 - (1 - \delta)^2)p_t$  with  $p_t > 0$  and  $0 < \delta < 1$ . In the situation of a housing boom period, the rate of land capital gain,  $c_t$ , is positive and in the range of  $0 < c_t < 1$ , thus,  $b$  must be a positive number; in the situation of a housing bust period, land capital gain is negative, and  $c_t$  is in the range of  $-1 < c_t < 0$ . Therefore, the sign of  $b$  depends on the magnitude of  $c_t$  and  $\delta$ . In particular, when in the situation of  $-c_t = \delta$ ,  $b$  is 0.

## 3 Data

### 3.1 Data Description

The data set we use to test our model and to perform the empirical analysis is the American Housing Survey (AHS). In particular, AHS 2013 national survey and AHS 2011 national survey (in the parameter estimations of the extended model in Section 5, we also use AHS 2007 national survey and AHS 2005 national survey together with AHS 2011 and AHS 2013). We focus on the New York metropolitan area (New York-Northern New Jersey-Long Island). AHS national survey is a biennial survey and it also follows every single house by its unique house IDs. AHS contains homeowners' self-reported property value, maintenance expenditures, lot size, and floor area. More importantly, to the best of our knowledge, AHS is the only database that contains the household maintenance data, which includes both project-specific maintenance (for home improvement) and routine maintenance data.<sup>11</sup> All dollar value variables are converted to the year 2011 dollars according to the annual average Consumer Price Index published by the Federal Reserve Bank of Minneapolis.

### 3.2 Data Cleaning and Filtering

Since AHS survey follows every single house by its unique house IDs, the 2011 and 2013 survey year data are merged according to this unique ID. At the same time, we select the New

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<sup>11</sup>The routine maintenance data (AHS variable CSTMNT) contains the regular maintenance expenditure homeowners spent last year or in a typical year. The home improvement data (AHS variable RAC) contains the aggregate expenditure in all home improvement projects since the last survey. (Technically, the home improvement data is measured either since the last year's survey time or since the move-in date for homes that changed owners. However, we restrict move-in time data to 2011 or earlier. ) In our sample, we use maintenance data from the 2013 survey; therefore within our sample, home improvement data covers all projects since the 2011 year survey. Therefore, real maintenance expenditure per unit floor area in Eq.(15),  $m_{it} = (\text{real routine maintenance expense} + 1/2 \times \text{real home improvement expenditure})/\text{floor area}$ .

York metro housing units and omit observations which contain missing values for housing value data, lot size, floor size or maintenance data. House prices in the AHS may have been top-coded. As indicated in AHS, high values are usually replaced with maximum values to ensure confidentiality. We then omit the maximum values of house prices of New York Metro in the year 2011 and year 2013. Moreover, there are some data quality issues at the bottom level of the housing price distributions in the AHS. In our sample, roughly 1% of the housing value data is below ten thousands dollars. We then drop the housing units whose values are below ten thousands dollar from our analysis. Then the following necessary data filtering procedures are employed and the final sample includes 377 observations. This restricted and filtered sample will be used in our later analysis. Table 2 sums up the statistics for our final sample.

*Select only the single-family owner-occupied detached houses*

For apartment complexes or condos, which typically have shared community spaces, AHS does not report the individual allocated shared space; thus, we cannot impute the land value for the apartment complex. Renters usually do not spend their own money on the maintenance, so we cannot use their data to estimate the maintenance effect. This procedure also eliminates the sharp effect of the Manhattan housing market in the sample. There are almost no single-family detached houses in Manhattan.

*Filter out mobile homes*

Mobile homes are, in many aspects of differences compared to regular housing units. In particular, in the American Housing Survey, house values reported by homeowners of mobile homes are different from that of all other regular housing units. The value for mobile homes is the structure value, which does not include the land beneath.

*Filter out the data if the floor size or lot size was different across years*

In some rare cases, the maintenance costs are used to expand housing space; In order to avoid considering space extension, we filtered out the data if the floor size or lot size was different in the data in the 2011 and 2013 survey years.

*Filter out houses whose owners have recently changed*

The 2013 AHS survey reports homeowner's maintenance expenses for two years, 2012 and 2013. Homeowner's maintenance includes two types of expenses, the cost of routine maintenance and the cost of home improvement. In some cases, a new owner moved into the house during that two-year period, which could cause a significant discrepancy between the homeowners reported expenses and the actual maintenance cost of that house in the last two years (e.g., the home improvement data is measured either since the last survey year or since the move-in date for homes that changed owners). Because of this potential discrepancy, our final dataset excludes houses where the owners had moved into the house in 2012 or 2013.

### 3.3 Data Summary

	Std Dev	Mean	25th	50th	75th
<i>House Value</i>					
House Value (2013)	171,863	382,074	289,570	366,789	463,313
House Value (2011)	172,285	406,442	300,000	380,000	490,000
<i>House Info.</i>					
Floor Area (Sq.ft)	832	1,983	1,480	1,900	2,476
Lot Size (Sq.ft)	16,570	13,304	5,500	8,000	14,000
<i>Maintenance</i>					
Home Improvement (2012 and 2013)	17,240	5,425	0	43	4,537
Routine Maintenance (2013)	1,413	1,221	290	772	1,930

TABLE 2: Summary Statistics for the final sample (all monetary units are in 2011 dollars)

## 4 Results

Our model is estimated by nonlinear least square estimation.<sup>12</sup> Table 3 shows the estimated primary coefficients from the model given in Eq.(16) and the calculated model coefficients based on the primary coefficients.

	Pri. Coeff.	Std. Err.	<i>P</i> -values	Model Coeff.	
<i>a</i>	0.839	0.013	0.000	$\delta$	0.084
<i>b</i>	-24.376	21.817	0.265	$\alpha$	16.16
<i>k</i>	30.961	5.836	0.000	$\beta$	0.233
$\beta$	0.233	0.086	0.007		
# of Obs.	377				
R-square	0.968				

TABLE 3: Estimation Results

<sup>12</sup>Nonlinear least square regression based on the procedures described by Davidson and MacKinnon (2004). The validity of hypothesis testing associated with nonlinear least square regression is also provided in Davidson and MacKinnon (2004).

As shown in the above table, the signs of model parameters generated from the data confirmed with our expectations and reasoning discussed in the end of section 2.

## 5 Empirical Discussion

### 5.1 Price Level Irrelevance

Recall our empirical model for nonlinear regression is

$$\frac{HV_{i,t+2}}{f_i} = \underbrace{(1-\delta)^2}_a \frac{HV_{it}}{f_i} + \underbrace{((1+c_t)^2-(1-\delta)^2)p_t}_b \frac{NLS_i}{f_i} + \underbrace{(2-\delta)\alpha}_k (m_{it})^\beta + u_{i,t+2}. \quad (18)$$

Our previous regression results are based on the 2011 price level. Intuitively and analytically, in our nonlinear model, if we convert the nominal terms (house values and maintenance expenses) with a different base year, we should get the same value for the primary parameter  $a$ , the depreciation rate  $\delta$ , and elasticity  $\beta$  since they are not involved with the price level. The primary parameter  $b$  will change accordingly since it contains normalized land price  $p_t$  and annual rate of capital gains  $c_t$ . Shift-scaling parameter  $\alpha$  shall also be expected to change because of the change of base year for real maintenance expense  $m_{it}$ . The regression results using the year 2018 as the base year shown in table 4 confirmed this conjecture.



	Pri. Coeff.	Std. Err.	P-values		
$a$	0.839	0.013	0.000		
$b$	-27.216	24.359	0.265		
$k$	33.690	6.515	0.000		
$\beta$	0.233	0.086	0.007		
# of Obs.	377				
R-square	0.968				
					Model Coeff.
				$\delta$	0.084
				$\alpha$	17.58
				$\beta$	0.233

TABLE 4: Estimation Results

## 5.2 Estimation of Plattage Effect from Data

We can also estimate the plattage effect from our model. Substituting Eq.(11) into Eq.(16) yields

$$\frac{HV_{i,t+2}}{f_i} = \underbrace{(1-\delta)^2}_{a} \frac{HV_{it}}{f_i} + \underbrace{((1+c_t)^2 - (1-\delta)^2)p_t}_{b} \frac{(LS_i)^\gamma}{f_i} + \underbrace{(2-\delta)\alpha}_{k} (m_{it})^\beta + u_{i,t+2}. \quad (19)$$

It is worth noting that if let data determine the elasticity of land value with respect to lot size,  $\gamma$ , instead of setting  $\gamma = 0.639$  from literature, a set of similar results including  $\gamma$  generated although the estimation of  $\gamma$  is not of significance.

	Pri. Coeff.	Std. Err.	P-values		
$a$	0.838	0.019	0.000		
$b$	-18.022	130.703	0.890		
$k$	30.965	5.848	0.000		
$\gamma$	0.668	0.685	0.330		
$\beta$	0.233	0.086	0.007		
# of Obs.	377				
R-square	0.968				
					Model Coeff.
				$\delta$	0.085
				$\alpha$	16.17
				$\beta$	0.233

TABLE 5: Estimation Results

### 5.3 Drop Regressor

$P$ -value on  $b$  indicates that the rate of land capital gains,  $c_t$ , of the housing units in the New York metropolitan area from 2011 and 2013 survey years are negative and its absolute value is equal or very close to the housing structure depreciation rate ( $\delta$ ). If it is true, then the model can be written as,

$$\frac{HV_{i,t+2}}{f_i} = \underbrace{(1-\delta)^2}_a \frac{HV_{it}}{f_i} + \underbrace{(2-\delta)\alpha}_k (m_{it})^\beta + u_{i,t+2}. \quad (20)$$

If the above conjecture is correct, the regression results should be close to the previous results. Table 6 presents the regression results and it indeed shows the conjecture is valid. The intuition for Eq.(20) is that only when the natural rate of structure depreciation is equal to the rate of land capital losses, then the natural rate of structure depreciation equals the natural rate of property depreciation (the depreciation rate that would occur for a particular property in the absence of maintenance).

	Pri. Coeff.	Std. Err.	$P$ -values		Model Coeff.
$a$	0.829	0.010	0.000	$\delta$	0.090
$k$	27.604	4.987	0.000	$\alpha$	14.45
$\beta$	0.256	0.092	0.006	$\beta$	0.256
# of Obs.	377				
R-square	0.968				

TABLE 6: Estimation Results

### 5.4 An Extended Model

In this section, an extended version of this paper's model is provided. The post-financial crisis years illustrate a slow recovery from the economic shock of the 2008 financial crisis. By assuming technology related to housing construction and maintenance remains the same

across survey years pre and post the financial crisis, time dummies can be introduced to incorporate various years into our model. This extension can also be utilized to examine the robustness of the estimations based on our approach. By incorporating multiple neighboring survey years into our model, it can also increase the sample size and decrease the sampling noise. In the following example, we add the 2005 and 2007 survey years into our original data set. This yields a total of four survey years (2005, 2007, 2011, and 2013) representing years both pre and post financial crisis. We expect that housing maintenance technology in the period from 2005 to 2007, in the aspect of comparing with that from 2011 to 2013 keeps almost unchanged. It is clear from our regression model specified by Eq. (16) that the same maintenance technology in our observations is required. It is worth mentioning that the identical housing construction technology is also required. In our model, housing capital gains are reflected by the capital gains on land. This assumption, no capital gains on housing structure across years, is primarily based on the reasoning that non-reproducible land is the most sensitive as well as the predominant component contributing to real estate capital gains, however the accuracy of this assumption is predicated on the condition that construction technology (with its quality-controlled real construction cost) stays the same across the years we study. In this way, by inserting two time dummies into Eq.(16), we have:

$$\frac{HV_{i,t+2}}{f_i} = a \frac{HV_{it}}{f_i} + D_1 b_1 \frac{NLS_i}{f_i} + D_2 b_2 \frac{NLS_i}{f_i} + k(m_{it})^\beta + u_{i,t+2}. \quad (21)$$

where  $b_1 = ((1 + c_{2005})^2 - (1 - \delta)^2)p_{2005}$  and  $b_2 = ((1 + c_{2011})^2 - (1 - \delta)^2)p_{2011}$ ;  $D_1$  indicates the observation is from the year 2005 while  $D_2$  indicates the observation is from the year 2011. Price level is set to the year of 2011.

We employ the same data cleaning and filtering procedures as described in section 3.2 on the 2005 and 2007 survey years and then append the dataset to our prepared dataset from the

2011 and 2013 survey years. Table 7 reports the regression results.

	Pri. Coef.	Std. Err.	<i>P</i> -values		
<i>a</i>	0.827	0.013	0.000		
<i>b</i> <sub>1</sub>	60.292	22.444	0.007	Model Coef.	
<i>b</i> <sub>2</sub>	-13.356	21.149	0.528	$\delta$	0.090
<i>k</i>	30.944	5.442	0.000	$\alpha$	16.20
$\beta$	0.268	0.074	0.000	$\beta$	0.268
# of Obs.	599				
R-square	0.953				

TABLE 7: Estimation Results

The regression results support our previous estimates in Section 4. The estimation in Section 4 is consistent with the that of the natural rate of housing structure depreciation, elasticity of maintenance expenditure, and the shift-scaling parameter of the maintenance function derived from the regression analysis of this extended model. More importantly, from the historical observations around the financial crisis, from the year 2005 to 2007 (which are the years pre-financial crisis), land has (positive) capital gains; then, the primary coefficient  $b_1$  in Eq.(21) should be positive according to our model, which is indeed supported by the estimation results of  $b_1$  in table 7. The significance test on  $b_2$  in Eq.(21) is consistent with that in Section 4, which once again indicates that land capital gains from 2011 to 2013 (which are the years post-financial crisis) is negative and the rate of capital loss on land is roughly equal to the rate of structure depreciation.

In the extended model we propose, there could be issues of reverse causality. Some houses in the sample of the 2011 and 2013 survey years come from that of the 2005 and 2007 survey years. The value of such a house in 2007 may have had effects on the house value in 2011 and the maintenance expense in 2011. We hereby exclude the 'repeat observations' in the 2005 and 2007 survey years to avoid this possible reverse causality issue; no house in the 2005 and 2007

survey year appears in the 2011 and 2013 survey years in our sample. We report the estimation results in table 8. The regression results are similar and reflects the robustness of our model.

	Pri. Coef.	Std. Err.	<i>P</i> -values		
<i>a</i>	0.830	0.015	0.000		
<i>b</i> <sub>1</sub>	52.002	24.404	0.034	Model Coef.	
<i>b</i> <sub>2</sub>	-17.549	24.480	0.474	$\delta$	0.089
<i>k</i>	33.973	6.274	0.000	$\alpha$	17.78
$\beta$	0.237	0.079	0.003	$\beta$	0.237
# of Obs.	479				
R-square	0.955				

TABLE 8: Estimation Results

## 6 Conclusion

In this paper, we study the natural depreciation rate of housing structure value, which has been overlooked in the previous literature. We elucidate a method to estimate this exogenous structure depreciation rate by separating the effect from housing capital gains and housing maintenance improvement from changes in housing property value. Using data from the biennial American Housing Survey for the survey years surrounding the 2008 financial crisis (2005, 2007, 2011 and 2013), the natural rate of structure depreciation is estimated to be roughly 9 percent per year in the New York metropolitan area. It is important to mention that the natural rate of structure depreciation that we study is the exogenous depreciation rate that would occur on the structure value in the absence of maintenance; it is different from the depreciation rate of property value since housing property value also subsumes non-depreciable land. As anticipated, this exogenous depreciation rate of the housing structure itself that we estimate is higher than the rate of property depreciation or the net depreciation rate of the housing structure under the effect of maintenance improvement, which are often estimated

in the previous literature studying housing depreciation. This rate of depreciation might vary across regions depending on the construction techniques and building materials employed for various types of housing, reflecting differences in weather, climate, and/or geologic conditions. A more accurate estimation of the rate of structure depreciation which takes into account the principal factors influencing housing value allows for more effective implementation of housing policy and lets real estate investors make more informed investment decisions.

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