# Adjustment of Inputs and Measurement of Technical Efficiency: A Dynamic Panel Data Analysis

**Aditi Bhattacharyya**
Department of Economics
University of California, Riverside

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## Abstract

The purpose of this paper is to construct a dynamic stochastic production frontier incorporating the sluggish adjustment of inputs, to measure the speed of adjustment of output, and to compare the technical efficiency estimates from this dynamic model to those from a static model. Assuming that the adjustment speed of all inputs is similar for every production unit, a linear partial adjustment scheme for output characterizes the dynamic frontier. We provide the estimation method for technical efficiency of production units and the speed of adjustment of output for cases when they are time-invariant and when they vary over time. We apply our methods to a panel dataset spanning nine years of data on private manufacturing establishments in Egypt. The dynamic frontiers with time-invariant and time-varying technical efficiency are estimated using the system GMM (generalized method of moments) estimator and the GLS (generalized least squares) estimator with instrumental variables, respectively. The results show that 1) the speed of adjustment of output is significantly lower than unity, 2) the static model overestimates technical efficiency on average, 3) the magnitude of this overestimation varies from 1.5 percentage points to 28.8 percentage points, depending upon the model under consideration, and 4) the dynamic model captures more variation in the time pattern of technical efficiency of a production unit. Further, the ranking of production units based on their technical efficiency measures changes when the lagged adjustment process of inputs is taken into account.

**Key words:** Technical efficiency, stochastic production frontier, dynamic panel data models, adjustment of inputs

**JEL Classification:** C23, D24, L60

## 1. Introduction

Production frontier estimation and the measurement of technical efficiency of production systems have been important areas of research for more than half a century. Following the pioneering work of Farrell (1957), this field has further grown with important contributions by many others, including Aigner, Lovell, and Schmidt (1977); Meeusen and Broeck (1977); and Kumbhakar (1988, 1991). These studies have posited two main causes for the deviation of actual output from the maximum possible output (potential output), given a certain set of inputs. A part of this deviation is attributed to the symmetric random shocks to a production system that are not under the control of a producer (for e.g., uncertainty about the weather, and input market conditions). The other reason for the failure to produce the potential output, given a set of inputs, is the presence of technical inefficiency caused by factors such as managerial error, and ignorance. Accordingly, a firm is said to be technically inefficient if it produces below the production frontier, and the corresponding technical inefficiency is measured by the deviation of the actual output from the frontier, after accounting for the random shocks to the system. Based on this concept, the literature has expanded to include both time-invariant and time-varying technical efficiency measures (see Cornwell, Schimdt and Sickles (1990); Kumbhakar (1990); Battese and Coelli (1992); Lee and Schmidt (1993); Ahn, Lee, and Schimdt (1994); and Kumbhakar, Heshmati, and Hjalmarsson (1997)), as well as cross sectional and panel data models of stochastic frontier estimation. Detailed discussions on the measurement of productive efficiency can be found in Lovell (1996), Kumbhakar and Lovell (2000), and Coelli et al. (2005).

Most of the existing studies on stochastic frontiers and technical efficiency focus on the static analysis of a producer's behavior, and therefore, fail to capture the dynamic nature of a firm's optimization process. In other words, these studies assume that when a unit of input is introduced into the production system, it immediately contributes to production at its maximum possible level. Accordingly, any shortfall from the potential output results from random shocks and the technical inefficiency of the production unit. However, it is reasonable to assume that following its introduction into a production system, an input requires some time for adjustment within the system. For example, consider a firm using labor and capital as inputs to produce a single output. If new capital is introduced, then the existing labor force has to be reassigned to the new capital stock, and during this process, output cannot be produced at the maximum level. Similarly, a newly hired labor unit or an employee will take time to get familiar with the production process. Given such a process of adjustment of inputs within the production system, it may not be possible for a firm to catch up with the production frontier instantaneously following the introduction of a new input, even in the absence of any other source of inefficiency. A vast literature on the source, structure, size and specification of adjustment costs (Lucas (1967a, 1967b); Treadway (1971); and Hamermesh and Pfann (1996)) has established the importance of such adjustment in the theory of production.

Evidently, behind the productivity change of a firm, there is a dynamic process at work in terms of input adjustment. The static production frontier model ignores the effect of input adjustment on output, and hence misspecifies the process of output generation. Consequently, technical efficiency measures from such a misspecified model are likely to be biased. Assuming that the speed of adjustment of inputs is similar for all inputs and production units, the actual change in the output of a firm is likely to be a fraction of the change in output that is needed to catch up with the potential output, at any given time period. Let us refer to the change in output that is needed in any period to catch up with the potential output, as the 'desired change' in output. The difference between the actual and the desired change in output is likely to depend on the speed of adjustment of inputs. If the speed of adjustment is lower than unity, then the change in actual output will be lower than the desired change. In such a scenario, a static model is likely to overestimate the technical efficiency of production units on average, by assuming that the speed of adjustment is unity. Also, the ranking of firms based on their technical efficiency estimates will be biased if the ranking is obtained from a similarly misspecified static model. Therefore, in the presence of sluggish adjustment of inputs, a static model cannot identify the true process of output generation or the technical efficiency of a production system. A dynamic model is more suitable for this purpose.

Little work has so far been done to incorporate the dynamic adjustment process of inputs while measuring technical efficiency of a production unit. Among studies that have considered dynamic models for technical efficiency using panel data, Ahn, Good, and Sickles (2000) allow for the lagged adjustment of inputs to explain the autoregressive nature of the technical efficiency component that varies with time. They also measure the speed of sluggish adoption of technological innovations and the associated efficiency loss. However, sluggish adjustment of inputs not only affects the adoption of technological innovations, but can also affect the whole production process by restricting output from reaching its maximum possible level. Further, the specification of Ahn, Good, and Sickles (2000) does not capture the possibility of variation in the speed of adjustment of inputs over time. Another study by Kumbhakar, Hesmati, and Hjalmarsson (2002) formulates the input requirement frontier for a firm in a dynamic set-up, but it does not shed any light on specifying a stochastic frontier for measuring technical efficiency and speed of adjustment of inputs, when inputs need time for adjustment. Ayed-Mouelhi and Goaied (2003) follow Ahn, Good, and Sickles (2000) to measure technical efficiency in a dynamic framework. However, they restrict the speed of adjustment to be constant over time while measuring only time-invariant technical efficiency. Finally, a recent paper by Asche, Kumbhakar, and Tveteras (2006) analyzes the dynamic profit maximization process of a firm by considering time series data, but once again, fails to fully exploit the panel structure of the data in its efficiency measurement.

The objective of this paper is to examine the effect of the input adjustment process on a firm's technical efficiency estimates. Specifically, this paper presents a dynamic, stochastic production frontier incorporating the lagged adjustment of inputs, and also provides methods to estimate technical efficiency – both when the speed of adjustment is constant, and when the speed of adjustment changes over time. Accordingly, both time-invariant and time-varying technical efficiency are measured, using a panel dataset on private manufacturing establishments in Egypt from the Industrial Production Statistics of the Central Agency for Public Mobilization and Statistics (CAPMAS). Results from this main analysis are further compared with those from the static versions of the models that assume instantaneous adjustment of all inputs.

The remainder of this paper is organized as follows. The main theoretical and econometric models are presented in sections 2 and 3 respectively. Section 4 elaborates on the estimation methods. Results from the empirical analysis are described in section 5, and finally, section 6 presents concluding remarks.

## 2. The Theoretical Model

Let $y_{it}^{*}$ be the maximum possible production level of firm $i$ that uses a vector of inputs $X_{it}$ at time $t$. However, after introduction of inputs, it is logical to have a time lag before they produce at their maximum possible level. Therefore, because of the underlying adjustment process of inputs, it is likely that a newer input will contribute less to the output than the older ones. Accordingly, the actual output of firm $i$ at time $t$, given by $y_{it}$, is determined by the speed of adjustment of inputs and the history of input usage. Thus, the actual output is a function of the current and past input levels, and the speed of adjustment of inputs.

Let $\lambda$ $(0 \le \lambda \le 1)$ be the speed of adjustment of inputs. We assume that $\lambda$ is constant over time and identical for all inputs and for every production unit. Then the actual output of firm $i$ at time $t$ is given by-

$$y_{it} = f(\lambda, X_{it}, X_{it-1}, X_{it-2}, ...) \tag{2.1}$$

The change in actual output between any two periods is a combined result of contribution of new inputs, a part of which is adjusted during the period, and contribution of the adjusted old inputs. Therefore, during the adjustment process of inputs, the current output $y_{it}$ is higher than $y_{i(t-1)}$ but lower than $y_{it}^{*}$, and change in the actual output is a fraction of the desired change. In other words, the dynamic production process of output generation can be represented by the partial adjustment scheme -

$$y_{it} - y_{i(t-1)} = \lambda(y_{it}^{*} - y_{i(t-1)}) \tag{2.2}$$

To further analyze the production model, let us consider a Cobb-Douglas function for the production of potential output[1]-

$$\ln y_{it}^{*} = \beta_0 + \sum_{m=1}^{M} \beta_m \ln x_{mit} + \sum_{t=2}^{T} \delta_t D_t \tag{2.3}$$

where, $i$ = 1,…,$N$, denotes the total number of firms, $t$ = 1,…,$T$, represents the total number of time periods considered, $m$ = *1*,…,$M$, represents the inputs used in production, $\beta_m$ is the marginal effect of the $m$th input on the potential output, and $\beta_0$ is the frontier intercept. We introduce the time dummy variables $D_t$ in the production model to

[1] The analysis is valid for more general production functions.

incorporate the pure technological change as proposed by Baltagi and Griffin (1988). Thus, no specific structure is imposed on the behavior of the technological change. $\delta_t$ captures the effect of technological changes on the potential output. The partial adjustment scheme for output generation is then given by-

$$\ln y_{it} = (1-\lambda)\ln y_{i(t-1)} + \lambda(\beta_0 + \sum_{m=1}^{M}\beta_m \ln x_{mit} + \sum_{t=1}^{T}\delta_t D_t) \tag{2.4}$$

Using (2.4) for different time periods, the partial adjustment scheme of output as given in (2.2) can further be illustrated as follows-

$$\ln y_{it} = \lambda \ln y_{it}^{*} + (1-\lambda)(\lambda \ln y_{i(t-1)}^{*} + (1-\lambda)\ln y_{i(t-2)})$$

$$\text{or, } \ln y_{it} = \lambda\beta_0 + \lambda(1-\lambda)\beta_0 + \lambda(1-\lambda)^2\beta_0 + \lambda(1-\lambda)^3\beta_0 + \ldots + \lambda\sum_{m=1}^{M}\beta_m \ln x_{mit} +$$
$$\lambda(1-\lambda)\sum_{m=1}^{M}\beta_m \ln x_{mi(t-1)} + \lambda(1-\lambda)^2\sum_{m=1}^{M}\beta_m \ln x_{mi(t-2)} + \lambda(1-\lambda)^3\sum_{m=1}^{M}\beta_m \ln x_{mi(t-3)} + \ldots\ldots$$
$$+\lambda\delta_2 D_2 + \lambda(1-\lambda)\delta_2 D_2 + \lambda(1-\lambda)^2\delta_2 D_2 + \lambda(1-\lambda)^3\delta_2 D_2 + \ldots..$$
$$+\lambda\delta_3 D_3 + \lambda(1-\lambda)\delta_3 D_3 + \lambda(1-\lambda)^2\delta_3 D_3 + \lambda(1-\lambda)^3\delta_3 D_3 + \ldots\ldots \tag{2.5}$$

Therefore, the partial adjustment scheme for actual output at time *t* demonstrates that the current output depends on the current and past inputs, and on the speed of adjustment of inputs. A fraction $\lambda$ of an input $x_{mi(t-k)}$, introduced by firm *i* in the period $t - k$, $(0 < k < t)$, contributes to the output in that period. In period $t - k + 1$, $\lambda$ fraction of the remaining (1-$\lambda$) $x_{mi(t-k)}$ adds to the output, and again $\lambda$ fraction of the unadjusted $(1-\lambda)^2 x_{mi(t-k)}$ contributes to output in $t - k + 2$. Following this process, $\lambda$ fraction of $(1-\lambda)^k x_{mi(t-k)}$ contributes to output at time *t*. Therefore, the marginal effects of current inputs on the current output is higher than that for the inputs from previous periods. With a speed of adjustment that is less than unity, these marginal effects are declining in a geometric progression as we consider older inputs. In other words, the most recent past of input usage receives the greatest weight in determining the current output, and influence of past inputs will fade out uniformly with the passage of time. Therefore, the distant past receives arbitrarily small weight.

Further, Following Kennan (1979), we can draw implications of rational expectation equilibrium from the partial adjustment scheme of output as specified in our framework. Under the rational expectation hypothesis, it can be shown that the solution to a problem that minimizes a quadratic loss function, results in a partial adjustment model for the current output. The total loss in output is generated by the loss due to deviation of current output from the potential output, and the loss due to the lagged adjustment of inputs.

In our subsequent analysis, we relax the assumption of constant $\lambda$ and allow the speed of adjustment of output to vary over time. However, every production unit is assumed to follow a similar time pattern of the speed of adjustment for every input. Thus the dynamic output generation process with a speed of adjustment of output that changes over time, is described by –

$$y_{it} - y_{i(t-1)} = \lambda_t (y_{it}^* - y_{i(t-1)}), \ 0 \le \lambda_t \le 1 \qquad (2.6)$$

## 3. Econometric Model

The potential output is a hypothetical characterization of the maximum possible output and is not observed in reality. The actual output is always above or below the potential output because of random shocks to the production system. Moreover, a production unit is likely to suffer from technical inefficiency that further lowers the actual output. The stochastic versions of (2.2) and (2.6), which are more realistic, consider a composite error term that accounts for the random shocks to a production unit, and the technical inefficiency of that unit. We obtain the stochastic versions of the dynamic output generation process (2.2) and (2.6) by considering a composite error term ($\varepsilon_{it}$) consisting of symmetric random shocks $v_{it}$ to firm *i* at time *t*, and the producer specific technical inefficiencies that can either be time-invariant or can vary with time. Let the time-invariant technical inefficiency of firm *i* be represented by $u_i$, and $u_{it}$ is the time-varying inefficiency component.

### 3.1. Time-invariant technical inefficiency model

The dynamic stochastic production frontier that incorporates the sluggish adjustments of inputs and time-invariant technical inefficiency, is given by-

$$\ln y_{it} = (1-\lambda)\ln y_{i(t-1)} + \lambda(\beta_0 + \sum_{m=1}^{M}\beta_m \ln x_{mit} + \sum_{t=2}^{T}\delta_t D_t) + \varepsilon_{it} \quad (3.1)$$

where, $\varepsilon_{it} = v_{it} - u_i$, and $u_i \geq 0$ capture the producer specific, time-invariant, non-negative inefficiency effects with $E(u_i) = \mu$, and variance $\sigma_u^2$. $v_{it}$ is the random shock to the production system with zero mean and variance $\sigma_v^2$. We further assume that $\lambda\beta_0 - \mu = \beta_0^*$, $u_i^* = u_i - \mu$ such that $u_i^* \sim iid(0,\sigma_u^2)$. The time dummies, $D_t$, have value equals unity for year $t$ and zero otherwise. The standard structure of the error component as discussed in Blundell and Bond (1998) is also maintained as follows-

1. $u_i^*$ is uncorrelated with $v_{it}$, i.e. $E(v_{it}u_i^*) = 0$ for all $i$ =1,…,$N$, and $t$ =1,…,$T$.
2. $v_{it}$ is serially uncorrelated, i.e. $E(v_{it}v_{is}) = 0$ for all $i$ =1,…,$N$, and $t \neq s$.
3. $E(y_{i1}v_{it}) = 0$ for $i$ =1,…,$N$, and $t$ =2,…,$T$.
4. Input $x_{mit}$ is strictly exogenous[2] for $i$ =1,…,$N$, $t$ =1,…,$T$, and $m$ =1,…,$M$.

In the dynamic model (3.1), the parameter $\lambda$, which is invariant over time, producer, and inputs, reflects the fraction of the desired change in output that is realized in any period. Following Schmidt and Sickles (1984), the most efficient production unit in the sample is assumed to be 100% efficient, and technical efficiency of other units are measured relative to the most efficient one as -

$$TE_i = \exp(-(\max_i \hat{u}_i^* - \hat{u}_i^*)) \quad (3.2)$$

where $$\hat{u}_i^* = \frac{1}{T-1}\sum_{t=2}^{T}\left(\ln y_{it} - (1-\hat{\lambda})\ln y_{i(t-1)} - \hat{\beta}_0^* - \hat{\lambda}\sum_{m=1}^{M}\hat{\beta}_m \ln x_{mit} - \hat{\lambda}\sum_{t=2}^{T}\hat{\delta}_t D_t\right) \quad (3.3)$$

The conventional static specification of the stochastic production frontier assumes instantaneous adjustment of inputs while catching up with the potential output and hence $\lambda = 1$ for the static version of (3.1). Formally, the static frontier is -

$$\ln y_{it} = \beta_0 + \sum_{m=1}^{M}\beta_m \ln x_{mit} + \sum_{t=2}^{T}\delta_t D_t - \eta_i + v_{it} \quad (3.4)$$

Here, $\eta_i$ represents the non-negative technical inefficiency effects. Therefore, the technical efficiency is measured from (3.4) as

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[2] Strict exogeneity of regressors can be verified with Hausman's specification test (1978).

$$T\tilde{E}_i = \exp(-(\max_i \hat{\eta}_i^* - \hat{\eta}_i^*)) \tag{3.5}$$

where $\eta_i^* = \eta_i - E(\eta_i)$, $\beta_0^* = \beta_0 - E(\eta_i)$, $\eta_i^* \sim iid(0, \sigma_\eta^2)$, and $\upsilon_{it} \sim iid(0, \sigma_\upsilon^2)$. If the producer specific effects are correlated with the inputs, then (3.4) is estimated as a fixed effects model and the producer specific effects are estimated as

$$\hat{\eta}_i^* = \frac{1}{T}\sum_t \left( \ln y_{it} - \hat{\beta}_0^* - \sum_{m=1}^{M} \hat{\beta}_m \ln x_{mit} - \sum_{t=2}^{T} \hat{\delta}_t D_t \right) \tag{3.6}$$

However, if the producer specific effects are uncorrelated with the inputs, then (3.4) is estimated as a random effects model[3] and the estimates of producer specific effects are given by-

$$\hat{\eta}_i^* = \frac{\sigma_\eta^2}{T\sigma_\eta^2 + \sigma_\upsilon^2} \sum_t \left( \ln y_{it} - \hat{\beta}_0^* - \sum_{m=1}^{M} \hat{\beta}_m \ln x_{mit} - \sum_{t=2}^{T} \hat{\delta}_t D_t \right) \tag{3.7}$$

The static model as represented in (3.4) omits the lagged adjustment phenomenon of inputs and is likely to provide biased estimates of technical efficiencies of the production units, if the true process of output generation is dynamic. On the other hand, the dynamic model as given in (3.1) captures the effect of the underlying adjustment process of inputs on outputs and provides measure for true technical efficiency of the production units.

## 3.2. Time-varying technical inefficiency model

The model given in section 3.1 is restrictive in the sense that the technical inefficiencies of producers and the speed of adjustment of output are assumed to be constant over time. However, for large $T$ and sufficiently competitive structure, the inefficiency effects are likely to change over time (Kumbhakar and Lovell, 2000). Moreover, as the inputs get more time to adjust within the production system, their speed of adjustment may change and consequently, the speed of adjustment of output may also vary with time. Therefore, we consider a dynamic stochastic production frontier in which the speed of adjustment of output and the technical efficiency vary over time, and the frontier is given by -

$$\ln y_{it} = (1-\lambda_t)\ln y_{i(t-1)} + \lambda_t \ln y_{it}^* + e_{it} \tag{3.8}$$

[3] A detailed discussion on the model specification and related prediction procedures can be found in Baltagi (1995).

or, $\ln y_{it} = (1-\lambda_t)\ln y_{i(t-1)} + \lambda_t(\beta_0 + \partial t + \sum_{m=1}^{M}\beta_m \ln x_{mit}) + e_{it}$ (3.9)

where $\partial$ captures the effect of technological changes on output[4]. $\lambda_t$ ($0 \le \lambda_t \le 1$) is the fraction of desired change in output that is realized in period *t*. For any particular time period, this speed of adjustment is assumed to be the same for all production units. The composed error term can be decomposed into the technical inefficiency term, $\theta_t f_i$, that varies with time and the symmetric random shock, $\tau_{it}$, i.e., $e_{it} = -\theta_t f_i + \tau_{it}$, where $\tau_{it} \sim iid(0, \sigma_\tau^2)$. $\theta_t$ captures the time varying influence of the producer specific inefficiency $f_i$ on the current output. In this formulation, the temporal pattern of technical inefficiency is the same for all production units. However, as discussed by Lee and Schmidt (1993), this structure is less restricted than the structures proposed by Cornwell, Schmidt, and Sickles (1990) and Kumbhakar (1990). The estimation method for (3.9) is discussed in the next section. However, once the parameters $\theta_t$, and the firm specific effect $f_i$ are estimated, the technical efficiency is measured as -

$TE_{it} = \exp(-(\max_j \hat{\theta}_t \hat{f}_j - \hat{\theta}_t \hat{f}_i))$ (3.10)

If the speed of adjustment of all inputs is assumed to be unity, as in the static specification of equation (3.9), we have the following static stochastic frontier model -

$\ln y_{it} = \beta_0 + \partial t + \sum_{m=1}^{M}\beta_m \ln x_{mit} + \pi_{it}$ (3.11)

where $\pi_{it} = -\rho_t \kappa_i + w_{it}$, and the symmetric statistical noise $w_{it} \sim iid(0, \sigma_w^2)$. Since the regressors are strictly exogenous for our sample that is considered for the empirical analysis, (3.11) represents a random effects model and we estimate it using the methods suggested by Lee and Schmidt (1993). We assume $\kappa_i^* = \kappa_i - z$, $E(\kappa_i) = z$, $\kappa_i^* \sim iid(0, \sigma_\kappa^2)$, and $\kappa_i^*$ is uncorrelated with $w_{it}$ and the regressors. The estimation procedure for (3.11) is discussed in detail in the next section. The technical efficiency from (3.11) is calculated as -

[4] Unlike the time-invariant technical efficiency model (3.1), here we use only time trend to capture the effect of technological changes. Since all the parameters in (3.9) vary with time, this simplification considerably reduces the number of parameter estimates.

$$T\tilde{E}_{it} = \exp(-(\max_j \hat{\rho}_t \hat{\kappa}_j^* - \hat{\rho}_t \hat{\kappa}_i^*)) \qquad (3.12)$$

The technical efficiency estimates using the static model (3.11) is expected to be biased and different than those obtained from the dynamic model (3.9), because of similar reasons as mentioned before.

# 4. Estimation Methods

## 4.1. Time-invariant technical inefficiency model

The dynamic model of production as given in (3.1) includes one period lagged dependent variable as a regressor along with other strictly exogenous variables. Both $y_{it}$ and $y_{it-1}$ are functions of $u_i$, leading to a correlation between one of the regressors and the error term. Thus the OLS estimator and the fixed effects estimator are biased and inconsistent even if $v_{it}$ are not serially correlated. Arellano and Bond (1991) suggested a generalized method of moments (GMM) estimator for the dynamic panel data model that gives consistent estimator of the coefficients. The GMM estimator uses dependent variable with two period lags or more lags as valid instruments to derive a more efficient estimator than the Anderson-Hsiao (1982) instrumental variable estimator. Ahn and Schimdt (1995) derived additional non-linear moment restrictions and the estimation method is further generalized and extended by Arellano and Bover (1995) and Blundell and Bond (1998).

We use the system GMM estimator proposed by Blundell and Bond (1998) that uses a set of moment conditions relating to the first differenced regression equation and another set of moment conditions for the regression equation in levels. Thus, first differences of the two or more period lagged depended variable as well as the first differences of the strictly exogenous variables are valid instruments for the equation in levels. Similarly, two or more period lagged dependent variable and the strictly exogenous variables are relevant instruments for the equation in first differences. We use $\ln y_{it-2}, \ln x_{mit}$, $m = 1,...M$, as instruments for the lagged dependent variable in the equation in first difference and $(\ln y_{it-1} - \ln y_{it-2})$, $(\ln x_{mit-1} - \ln x_{mit-2})$, $m = 1,..,M$, are the instruments used for the equation in levels. Validity of the orthogonality conditions is tested by Sargan test, and exogeneity of the instruments are verified with the Hansen test.

The static model with time-invariant technical efficiency as given in equation (3.4) is estimated as a random effects model[5] and accordingly the technical efficiency is estimated using (3.7).

### 4.2. Time-varying technical efficiency model

To estimate the dynamic panel data model with time-varying technical efficiency, as given in equation (3.9), we adapt the method described by Holtz-Eakin, Newey, and Rosen (1988). For identification purposes, we assume that

$$E[\ln y_{is}\tau_{it}] = E[\ln x_{mis}\tau_{it}] = E[f_i\tau_{it}] = 0, \ (s < t), \ (m = 1, ..., M) \tag{4.1}$$

The error term in (3.9) does not have a mean of zero. Therefore, we transform equation (3.9) to eliminate the individual effects in the following way. Let $r_t = \frac{\theta_t}{\theta_{t-1}}$. We consider (3.9) for period (t-1), multiply it by $r_t$, and take the difference of the derived equation from (3.9) for period t. This gives us the following quasi-transformed equation-

$$\ln y_{it} = \lambda_t\beta_0 - r_t\lambda_{t-1}\beta_0 + r_t\lambda_{t-1}\partial + (1 + r_t - \lambda_t)\ln y_{it-1} - r_t(1-\lambda_{t-1})\ln y_{it-2} + \lambda_t\sum_{m=1}^{M}\beta_m \ln x_{mit}$$

$$+\lambda_t\partial t - r_t\lambda_{t-1}\partial t - r_t\lambda_{t-1}\sum_{m=1}^{M}\beta_m \ln x_{mit-1} + e_{it} - r_t e_{it-1} \tag{4.2}$$

The regressors in (4.2) involve one period lagged dependent variable that is correlated with the error term. However, the orthogonality conditions in (4.1) imply that the error term in (4.2) satisfies the following conditions -

$$E[\ln y_{is}\varepsilon_{it}] = E[\ln x_{mis}\varepsilon_{it}] = 0 \text{ for } s < t-1, \ m = 1,..,M \tag{4.3}$$

where, $\varepsilon_{it} = e_{it} - r_t e_{it-1}$ . Therefore, the vector of instrumental variables that is available to identify the parameters is $Z_{it} = [1, \ln y_{it-2}, \ln y_{it-3}, ..., \ln y_{i1}, \ln x_{mit-2}, \ln x_{mit-3}, ..., \ln x_{mi1}]$.

Equation (4.2) is estimated by the method of GLS (generalized least squares) with $\ln y_{it-3}$ as the instrumental variable[6]. Since the number of sectors in our sample is not large, we do not use all the available instruments, in order to avoid the problem of too

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[5] Hausman's specification test for our sample (1978) suggests not to reject the null hypothesis that the random effects estimator is consistent.

[6] For further details on the estimation method, see Holtz-Ekin, Newey, and Rosen (1988).

many instruments[7]. Note, that the parameters of (4.2) vary with time and these parameters are only identified for $t \geq 3$. Given the choice of our instrumental variable, (4.2) is estimated for $t \geq 4$.

Holtz-Eakin, Newey, and Rosen (1988) do not discuss about estimation of the individual specific effects that vary with time. However, the main objective of our analysis is to estimate the time-varying technical efficiency of a production unit, which is a part of the composite error term. For this purpose, we estimate (4.2) and get estimates for (2*M* + 4) parameters, where each of these parameters is a nonlinear function of (*M* + 5) distinct parameters given by $r_t$, $\lambda_t$, $\lambda_{t-1}$, $\beta_0$, $\partial$, and $\beta_1,...,\beta_M$. Thus, to identify *M*+5 parameters, we have an over identified system of (2*M* + 4) equations, for $M \geq 1$. We denote the vector of (*M* + 5) parameters by $\varphi_t$ and the system of equations by $g(\varphi_t)$, and solve the following optimization problem[8]

$$\underset{\varphi_t}{Min}\, g(\varphi_t)' g(\varphi_t)$$

where, $0 \leq \lambda_t \leq 1$, $\beta_m \geq 0$ for *m* = 1,…,*M* , and (2*M* + 4) estimates from (4.2) are given by $a_t$, $b_t$, $c_t$, $d_{1t},...,d_{Mt}$, $f_t$, and $h_{1t},...,h_{Mt}$. Then,

$$g(\varphi_t) = \begin{bmatrix} (\lambda_t - r_t\lambda_{t-1})\beta_0 + r_t\lambda_{t-1}\partial - a_t \\ 1 + r_t - \lambda_t - b_t \\ -r_t(1-\lambda_{t-1}) - c_t \\ \lambda_t\beta_1 - d_{1t} \\ \vdots \\ \lambda_t\beta_M - d_{Mt} \\ \lambda_t - r_t\lambda_{t-1} - f_t \\ r_t\lambda_{t-1}\beta_1 - h_{1t} \\ \vdots \\ r_t\lambda_{t-1}\beta_M - h_{Mt} \end{bmatrix}$$

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[7] The estimates are robust to the choice of instruments.

[8] $\underset{\varphi_t}{Min}\, g(\varphi_t)'V(\varphi_t)g(\varphi_t)$, where V(.) represents variance, makes no considerable changes in the results.

Thus, we get unique estimates for the parameters in the original model as given in (3.9) and also for $r_t = \frac{\theta_t}{\theta_{t-1}}$. Further, to identify $\theta_t$, we normalize $\theta_T = 1$[9] and accordingly identify $\theta_t$ for the periods for which (4.2) is estimated. Finally, we estimate the sector specific effect $f_i$ by the ordinary least squares method for each sector using the following equation

$$\hat{\phi}_{it} = \hat{\theta}_t f_i + \tau_{it} \quad (4.4)$$

where, $$\hat{\phi}_{it} = \ln y_{it} - (1-\hat{\lambda}_t)\ln y_{i(t-1)} - \hat{\lambda}_t(\hat{\beta}_0 + \hat{\partial} t + \sum_{m=1}^{M} \hat{\beta}_m \ln x_{mit}) \quad (4.5)$$

The time-varying technical efficiency is estimated following equation (3.10)

To compare the technical efficiency estimates from (3.9) with those from the static version of the model that assumes the speed of adjustment is constant and equals unity, we estimate equation (3.11), following the method suggested by Lee and Schmidt (1993). According to the results of Hausman's specification test (1978), we estimate (3.11) as a random effects model. In a general notation the model can be summarized as

$$\ln y_{it} = X'_{it}\beta + \pi_{it} \quad (4.6)$$

where, $\pi_{it} = -\rho_t \kappa_i + w_{it}$, $X_{it}$ is the vector of regressors including a constant term, years, and M inputs in logarithmic term. $\beta$ is the vector of marginal effects of the regressors, $\kappa_i^* = \kappa_i - z$, $E(\kappa_i) = z$. The explicit solution for the OLS estimator that is consistent is given as -

$$\hat{\beta} = [\sum_i (X_i - \bar{X})'(X_i - \bar{X})]^{-1} \sum_i (X_i - \bar{X})'(\ln y_i - \ln \bar{y}) \quad (4.7)$$

where, $\ln y_i = (\ln y_{i1,\dots,} \ln y_{iT})'$, $X_i = (X_{i1,\dots,} X_{iT})'$, $\bar{X} = \sum_i X_i / N$, and $\ln \bar{y} = \sum_i \ln y_i / N$.

The vector of the mean residuals is defined as $\bar{\pi} = \ln \bar{y} - \bar{X}\hat{\beta}$, and $\xi' = (1, \rho_2, \dots, \rho_T)$[10].

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[9] Lee and Schmidt (1993) suggest the normalization $\theta_1 = 1$ for the static model with similar time-varying technical efficiency structure. However, our model being a dynamic one, the parameters cannot be estimated for the initial period and we choose the normalization with respect to the last period.

[10] $\theta_T = 1$ for normalization. The normalization $\theta_1 = 1$ does not change the results.

Let $\bar{\pi}_1$ be the first element of $\bar{\pi}$. Then $\hat{\xi} = \frac{\bar{\pi}}{\bar{\pi}_1}$, and $\hat{z} = \bar{\pi}_1$. The firm specific effects are estimated as $\hat{k}_i^* = \hat{\xi}'(\ln y_i - X_i\hat{\beta} - \hat{\xi}\hat{z}) / \hat{\xi}'\hat{\xi}$ and finally, the time-varying technical efficiency is estimated from (3.12).

# 5. Empirical Analysis

## 5.1. Data

To illustrate the theoretical model empirically, we use the panel data for nine years (1987/88 – 1995/96) on the private sector manufacturing establishments in Egypt, obtained from the Industrial Production Statistics of the Central Agency for Public Mobilization and Statistics (CAPMAS). The data is in three-digit ISIC (International Standard Industrial Classification) level and for 28 sectors with the total number of observation being 252. The broader categories of output include food, tobacco, wood, paper, chemicals, non-metallic products, metallic product, engineering products, and other manufacturing products. This data set is directly taken from a study by Getachew and Sickles (2007) and details about the data can be found in their paper. They use the superlative index number approach to aggregate the data to the three-digit level, such that the establishments in each sector can be viewed as homogeneous in terms of production technology. To get a single aggregate measure of output from heterogeneous and multi-product firms, they consider total revenue from these firms for goods sold, industrial services provided to others, and so on. Finally, they obtain the quantity indices for output and inputs by deflating the total value of output and inputs by the relevant price indices.

Capital, labor, energy, and material are the inputs for the manufacturing sectors' output. As found by Getachew and Sickles (2007) the quantity indices for output and inputs grew over the period under consideration. The summary statistics of the indices are presented in table 1 in the appendix. Getachew and Sickles (2007) use this data set to analyze relative price efficiency of the Egyptian manufacturing sectors, but they do not measure technical efficiency of these sectors, especially in a dynamic framework.

## 5.2. Results

We consider a Cobb-Douglas production function for the potential output of the manufacturing sectors[11]. Therefore, the dynamic model corresponding to equation (3.1) is

$$\ln y_{it} = (1-\lambda)\ln y_{i(t-1)} + \lambda(\beta_0 + \sum_{m=1}^{4}\beta_m \ln x_{mit} + \sum_{t=2}^{9}\delta_t D_t) + \varepsilon_{it} \quad (5.1)$$

where, $\varepsilon_{it} = v_{it} - u_i$, $u_i \geq 0$. The inputs are capital, labor, energy and material with $m$ = 1 for capital, $m$ = 2 for labor, $m$ = 3 for energy, and $m$ = 4 for material. The estimation results for this model are derived using the Blundell and Bond (1998) system GMM[12] estimator and are given in table 2. The standard errors of the estimates are robust to heteroskedasticity and arbitrary patterns of autocorrelation within sectors, and we also incorporate the small-sample corrections to the covariance matrix estimate. From the estimation results we find that the one period lagged output has a significant[13] positive effect on the current output, where output is measured in logarithm. Using the estimated value of $1-\hat{\lambda} = 0.17$, we calculate the fraction of desired change in output that is realized as, $\hat{\lambda} = 0.83$. Therefore, the actual change in output of a sector in any period is 83% of the change in output that is needed to catch up with the potential output in that period. Further the p-value for $(1-\hat{\lambda})$ suggests that $\hat{\lambda}$ is significantly different from unity at 5% level of significance. Therefore, assuming similar speed of adjustment for inputs across sectors, this result supports the partial adjustment of output that is generated by the sluggish adjustment of inputs in the production system.

Consistency of the system GMM estimator relies upon the fact that the idiosyncratic errors are not serially correlated. Following the test procedure proposed by Arellano and Bond (1991), we expect a negative first order serial correlation in the equation in first differences. However, to check for the absence of first order serial correlation in the equation in levels, we need to test for the absence of second order serial correlation in the equation in differences. Results for the Arellano-Bond test, as reported in table 2, establish that the idiosyncratic errors are not serially correlated.

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[11] We compared a Cobb-Douglas and a more general translog production function for the data. Based on the information criterion (AIC and BIC) from these two models, we select the Cobb-Douglas production function.

[12] We use Stata command xtabond2 developed by David Roodman (2006).

[13] Estimated coefficient of the lagged level of output = $\hat{\lambda}$ =0.17, significant at the 5% level.

The validity of the GMM estimates also depends on the assumption of exogeneity of the instruments. If the system is over identified, a Wald test can be used to test for the joint validity of the moment conditions and the test statistic follows a $\chi^2$ distribution with degrees of freedom equal to the degrees of over identification. The test statistics for Sargan and Hansen tests, as reported in table 2, support the validity of the instruments used in the estimation. However, too many instruments can over fit the endogenous variables and weaken the power of the Hansen's test. Following the suggestions of Roodman (2006), we restrict the number of instruments to be less than the number of sectors considered in our analysis.

The static stochastic frontier corresponding to equation (3.4) that assumes instantaneous adjustment of all inputs, i.e., $\lambda = 1$, is given as follows-

$$\ln y_{it} = \beta_0 + \sum_{m=1}^{4} \beta_m \ln x_{mit} + \sum_{t=2}^{9} \delta_t D_t - \eta_i + \upsilon_{it} \tag{5.2}$$

We estimate (5.2) as a random effects model, following the Housman's specification test (1978) results. The estimation results are also presented in table 2. Finally the average of the time-invariant technical efficiency estimates from the dynamic (5.1) and the static (5.2) models are shown in table 3 and graph 1. We find that the technical efficiency estimates from the dynamic model is 68.5% for a sector on average, whereas, the estimates from the static model are 70.02% on average. Thus, the technical efficiency estimates from these two models are not similar, and the absolute difference between the estimates is 5.8 percentage points on average, with a maximum of 13.8 percentage points.

Further, we find that the static model overestimates the technical efficiency of sectors by 1.5 percentage points on average, and this overestimation can be as high as 9.9 percentage points. Thus, in the presence of sluggish adjustment of inputs, the conventional static model is likely to overestimate the technical efficiency of production units and our results show that it overestimates the technical efficiency of a production unit by 28.9% on average. However, due to model misspecification, the technical efficiency estimates from static model can be higher or lower than the estimates from the dynamic model, for a particular sector. Closer looks at the results presented in table 2 reveal that the elasticity of inputs as estimated from the dynamic and static specifications are also quite different. The elasticity of an input as estimated from the dynamic model, captures the sluggish adjustment of the input and hence is more reasonable. While analyzing the time-invariant technical efficiency, we do not allow for the speed of adjustment to vary over time or sectors. In our subsequent analysis we relax this assumption and consider a speed of adjustment of output that changes with time.

The significant effect of the one period lagged output on the current output level clearly suggests that the static model is misspecified. Consequently, we find that the ranking of sectors according to the dynamic and static model specifications are also not the same. The ranking of sectors based on their technical efficiency estimates are given in table 4. The Spearman's correlation coefficient for these ranks from the dynamic and static model is found to be 0.64.

As discussed before, the technical efficiency as well as the speed of adjustment of output may vary over time. More specifically, it is likely that the rate of adjustment of an input improves over time by the process of learning and doing, and as a result, the speed of adjustment of output increases as well. Consequently, the technical efficiency of a production unit is likely to increase with time. To measure the time-varying technical efficiency and speed of adjustment of output, we consider a Cobb-Douglas production function with constant elasticity of inputs for the potential output. Since we do not expect the elasticities to vary when the inputs are producing at their maximum possible level, the assumption of constant elasticities of inputs for the potential output, is reasonable. because. This assumption also assures a considerable reduction in the number of parameter estimates from a small sample. The effect of technological change on the potential output is captured by a time trend. The dynamic specification as given in equation (3.9) is estimated as -

$$\ln y_{it} = (1-\lambda_t)\ln y_{i(t-1)} + \lambda_t(\beta_0 + \partial t + \sum_{m=1}^{4}\beta_m \ln x_{mit}) + e_{it} \qquad (5.3)$$

where $e_{it} = -\theta_t f_i + \tau_{it}$.

Following the method described in section 4, we estimate the speed of adjustment of output for time periods $t =$ 4,…,9, and the time varying technical efficiency for each sector $i$ = 1,…,28. We use the two-stage least squares results that are consistent and find that the coefficient of the lagged dependent variable is positive and significant for every period, implying that the true data generating process is dynamic. The coefficient estimates[14] of the lagged dependent variable are given in table 5 along with their *t*-ratios that use heteroskedasticity corrected standard errors[15]. Finally, we recover the parameter estimates from the original model for each period by minimizing an over identified system of equation.

The estimation results show that the speed of adjustment of output ranges from 0.43 to 0.55 for our sample over the period under consideration (given in graph 2), with an average of 0.49. Thus, on average, the actual change in output in any period is 49% of the change in output that is needed to catch up with the potential output and this speed of adjustment of output changes over time. More specifically, the speed of adjustment has a positive time trend, showing that it is increasing over time.

We find that the average time-varying technical efficiency as measured from (5.3) is 67.3 %, and is given in table 6. We also estimate the time-varying technical efficiency from the static model –

$$\ln y_{it} = \beta_0 + \partial t + \sum_{m=1}^{4} \beta_m \ln x_{mit} + \pi_{it} \tag{5.4}$$

where $\pi_{it} = -\rho_t \kappa_i + w_{it}$. The average technical efficiency as estimated from (5.4) is 93.98% and is much higher than the average efficiency estimates from the dynamic model. The absolute difference between the efficiency estimates from the static and the dynamic model is found to be 28.76 percentage points on average, and can be as high as 89.59 percentage points for a sector in a period. Since the static model seems to overestimate the technical efficiency, we present the magnitude of this overestimation in table 6 as well. We find that the static model overestimates the technical efficiency of production units by 26.68 percentage points on average, i.e., the static model overestimates technical efficiency of a sector in a period by 39.6%, on average.

Instead of presenting the technical efficiency for all observations we present the average for each sector in table 7. Graph 3 further illustrates the contents of this table. Graph 3 makes it evident that by ignoring the adjustment process of inputs, the static model overestimates the technical efficiency on average.

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[14] The coefficient of the lagged dependent variable is given by $(1 + r_t - \lambda_t)$ for each *t*.

[15] The total number of parameter estimates from the quasi-transformed model is 72 and we present only the relevant ones.

We also find that the ranking of sectors from the dynamic and the static model are markedly different, except for the most efficient sector. The ranking of sectors are given in table 8. Further investigation on the ranks reveals that the Spearman's correlation coefficient is 0.0148 for them, and we cannot reject the hypothesis that the ranks from the static and dynamic model are independent.

Finally, we look into the pattern of variation of technical efficiency over time. The basic assumption behind the efficiency models (5.3) and (5.4) is that the pattern (though not the magnitude) of time variation of technical efficiency is similar for all production units. We choose three sectors from our sample to illustrate this idea. In graph 4, 5, and 6, we present the technical efficiency estimates from the static and the dynamic model for sector 6, sector 12, and sector 18, respectively[16]. We find that though the pattern of time variation is similar for different sectors, the dynamic model identifies more variation in this pattern, for each sector. Thus, by ignoring the lagged adjustment of inputs, the static model not only overestimates technical efficiency, but it also fails to capture the temporal variation in the efficiency measures.

## 6. Conclusion

This paper outlined a theory for a dynamic stochastic production frontier that described the process of output generation in the presence of lagged adjustment of inputs. It also discussed estimation methods for both time-invariant and time-varying technical efficiency measures of production units, based on a dynamic model. It further illustrated the methods of estimation using data from the private manufacturing sectors in Egypt, and found that the speed of adjustment of output was significantly lower than unity. This, in turn, suggests that the conventional static model that assumes instantaneous adjustment of inputs is missspecified, and provides biased estimates of technical efficiency. Comparing the technical efficiency estimates from the dynamic model with those from a static model, we found that the static model overestimates technical efficiency of different sectors by 1.5 percentage points to 28.76 percentage points, on average, and provides an inappropriate ranking of sectors based on these biased estimates.

[16] Sector 6 is the most efficient according to both models; sector 12 acquires rank 4 from the dynamic model and rank 20 from the static model, sector 18 has rank 7 from the dynamic model and rank 15 from the static model.

Estimation of technical efficiency and ranking of production units according to their efficiency levels are important aspects of productivity analysis. Producers often take critical decisions about their production plans that are, in part based on such technical efficiency measures. This paper has provided a more realistic and rigorous approach for capturing the dynamics of a production system, and measuring technical efficiency. In particular, this paper offers a novel approach towards estimating the speed of adjustment of output and technical efficiency that vary over time, and thereby, captures the effect of lagged adjustment of inputs on output.

The theoretical and econometric models, as discussed in this paper, are based on the simplifying assumption that the speed of adjustment of inputs is similar for all inputs and every production unit. However, different production units are likely to have different speeds of adjustment. Similarly, the adjustment processes of different inputs are also likely to be different. While this paper does not discuss methods to estimate technical efficiency under less restrictive assumptions, these should be interesting areas of exploration for future research in this field

# Appendix

### Table 1: Variable Descriptions and Summary Statistics

| Variable | Description | Observation | Mean | Standard deviation | Minimum | Maximum |
|---|---|---|---|---|---|---|
| **Yearid** | id number for 9 years of data for each sector | 252 | 5 | 2.59 | 1 | 9 |
| **Sectorid** | id numbers for the 28 three digit manufacturing sectors | 252 | 14.5 | 8.09 | 1 | 28 |
| **Output** | Output quantity index | 252 | 2888.90 | 3333.39 | 67 | 19236 |
| **Capital** | Capital quantity index | 252 | 288.84 | 475.29 | 1 | 3437 |
| **Labor** | Labor quantity index | 252 | 273.34 | 344.06 | 10.50 | 1689.2 |
| **Energy** | Energy quantity index | 252 | 61.97 | 116.56 | 0.20 | 860.1 |
| **Material** | Material quantity index | 252 | 1823.44 | 2168.83 | 44.8 | 11853.8 |

Source: Getachew and Sickles (2007).

**Table 2: Estimation Results from Dynamic and Static Specifications (time-invariant technical efficiency model)**

| Coefficients | Dynamic Specification<br>Blundell-Bond System GMM Estimator | Static Specification<br>Random Effects Estimator |
|---|---|---|
| lag_ln(output) | 0.17**<br>[0.075] | - |
| ln(capital) | 0.021<br>[0.023] | 0.014<br>[0.014] |
| ln(labor) | 0.221***<br>[0.05] | 0.123***<br>[0.038] |
| ln(energy) | 0.002<br>[0.03] | 0.044**<br>[0.020] |
| ln(material) | 0.67***<br>[0.06] | 0.833***<br>[0.035] |
| Yeardummy2 | - | -.027<br>[0.051] |
| Yeardummy3 | - | -0.080<br>[0.052] |
| Yeardummy4 | -0.06**<br>[0.03] | -0.098<br>[0.052] |
| Yeardummy5 | -0.04<br>[0.03] | 0.041<br>[0.053] |
| Yeardummy6 | 0.003<br>[0.06] | 0.035<br>[0.054] |
| yeardummy 7 | -0.018<br>[0.05] | 0.002<br>[0.053] |
| yeardummy 8 | 0.021<br>[0.06] | 0.027<br>[0.054] |
| yeardummy 9 | -0.015<br>[0.07] | 0.035<br>[0.055] |
| Constant | 0.301<br>[0.29] | 0.803***<br>[0.148] |
| AR(1) | -2.89*** | - |
| AR(2) | 0.11 | - |
| Sargan | 6.98 | - |
| Hansen | 5.57 | - |
| Observations | 196 | 252 |
| Number of sectors | 28 | 28 |
| Number of instruments | 23 | - |
| R-squared | - | 0.973 |
| F (10, 27) | 162.06*** | - |

Notes: Robust standard errors are reported in parentheses; * significant at 10%; ** significant at 5%; *** significant at 1%; AR(1) and AR(2) represent the Arellano-Bond (1991) test statistics for the first order and second order serial correlation in the first differenced errors respectively; Sargan and Hansen are the test statistic for the over identification; the instruments used are $\ln(output)_{it-2}$, $\ln(capital)_{it}$, $\ln(labor)_{it}$, $\ln(energy)_{it}$, and $\ln(material)_{it}$ for the eqation in first differences, and are $\ln(output)_{it-1} - \ln(output)_{it-2}$, $\ln(capital)_{it-2} - \ln(capital)_{it-2}$, $\ln(labor)_{it-1} - \ln(labor)_{it-2}$, $\ln(energy)_{it-1} - \ln(energy)_{it-2}$, and $\ln(material)_{it-1} - \ln(material)_{it-2}$ for the equation in levels.

**Table 3: Difference in the Time-invariant Technical Efficiency Estimates from Static and Dynamic Specifications**

| Variables | Mean | Maximum |
|---|---|---|
| Technical Efficiency_Dynamic | 68.5 | 100 |
| Technical Efficiency_Static | 70.02 | 100 |
| Difference in Efficiency Estimates | 5.8 | 13.8 |
| Overestimation by Static Model | 1.5 | 9.9 |

Notes: The technical efficiency estimates from the dynamic and the static models are presented in percentage terms. These estimates show the efficiency level of a production unit relative to the most efficient unit in the sample.

The difference in efficiency estimates is calculated by taking the absolute difference in the technical efficiency estimates from the dynamic and the static model. The difference in efficiency estimates and the overestimation by static model are presented in terms of percentage points.

**Table 4: Time-invariant Technical Efficiency Estimates and Ranking of Firms under Dynamic and Static Specifications**

| Sectorid | Technical Efficiency from Dynamic Specification (%) | Technical Efficiency from Static Specification (%) | Rank_Dynamic Specification | Rank_Static Specification |
|---|---|---|---|---|
| 1 | 54.93 | 64.35 | 26 | 21 |
| 2 | 72.34 | 70.94 | 9 | 11 |
| 3 | 63.90 | 66.88 | 19 | 17 |
| 4 | 75.00 | 74.28 | 4 | 6 |
| 5 | 52.70 | 61.84 | 28 | 25 |
| 6 | 60.80 | 65.69 | 23 | 20 |
| 7 | 63.43 | 59.56 | 21 | 28 |
| 8 | 63.82 | 65.87 | 20 | 19 |
| 9 | 72.56 | 66.34 | 8 | 18 |
| 10 | 69.31 | 73.58 | 13 | 8 |
| 11 | 93.10 | 100.00 | 2 | 1 |
| 12 | 58.63 | 68.50 | 24 | 14 |
| 13 | 68.76 | 67.90 | 14 | 16 |
| 14 | 66.86 | 71.96 | 16 | 9 |
| 15 | 71.49 | 61.42 | 10 | 26 |
| 16 | 70.70 | 62.66 | 12 | 24 |
| 17 | 57.32 | 64.30 | 25 | 22 |
| 18 | 73.75 | 71.78 | 7 | 10 |
| 19 | 74.22 | 77.72 | 6 | 4 |
| 20 | 71.00 | 78.14 | 11 | 3 |
| 21 | 100.00 | 86.20 | 1 | 2 |
| 22 | 67.77 | 60.19 | 15 | 27 |
| 23 | 61.26 | 70.23 | 22 | 12 |
| 24 | 54.12 | 63.29 | 27 | 23 |
| 25 | 64.43 | 68.10 | 18 | 15 |
| 26 | 76.12 | 76.20 | 3 | 5 |
| 27 | 65.46 | 73.80 | 17 | 7 |
| 28 | 74.35 | 68.81 | 5 | 13 |

Note: Technical efficiency of a sector is measured relative to the most efficient sector.

**Table 5: Coefficient of the Lagged Dependent Variable in the Time-varying Technical Efficiency Model**

| Year | Coefficient of lagged dependent variable | t-ratio |
|---|---|---|
| 4 | 0.069 | 5.61 |
| 5 | 0.073 | 8.43 |
| 6 | 0.084 | 7.83 |
| 7 | 0.095 | 10.77 |
| 8 | 0.087 | 12.26 |
| 9 | 0.082 | 20.41 |

Notes: The results are from the two-stage least squares analysis. The standard errors are corrected for heteroskedasticity.

**Table 6: Difference in the Time-varying Technical Efficiency Estimates from Dynamic and Static Specifications**

| Variables | Mean | Maximum |
|---|---|---|
| Technical Efficiency_Dynamic | 67.3 | 100 |
| Technical Efficiency_Static | 93.98 | 100 |
| Difference in Efficiency Estimates | 28.76 | 89.59 |
| Overestimation by the Static Model | 26.68 | 89.59 |

Notes: The technical efficiency estimates from the dynamic and the static models are presented in percentage terms. These estimates show the efficiency level of a production unit relative to the most efficient unit in the sample.

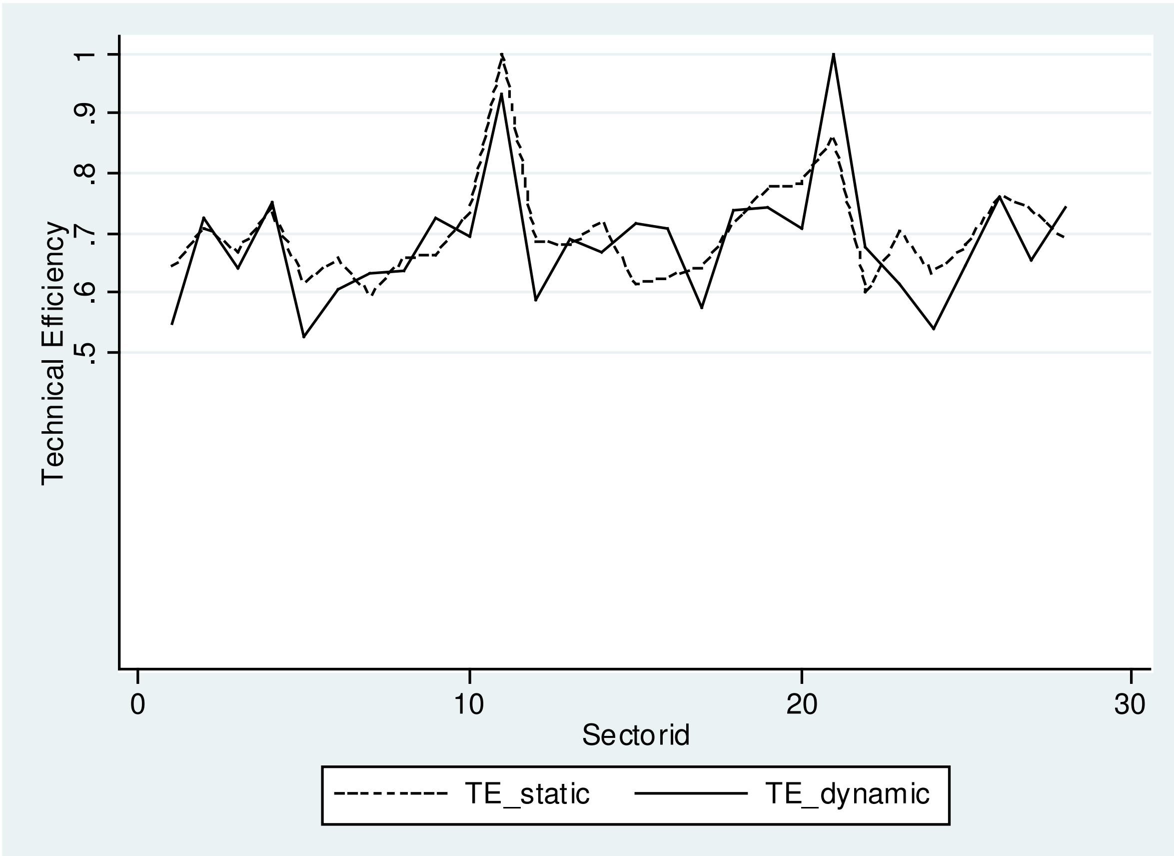
The difference in efficiency estimates is calculated by taking the absolute difference in the technical efficiency estimates from the dynamic and the static model. The difference in efficiency estimates and the overestimation by static model are presented in terms of percentage points.

**Table 7: Average Time-varying Technical Efficiency Estimates and Ranking of Firms under Dynamic and Static Specifications**

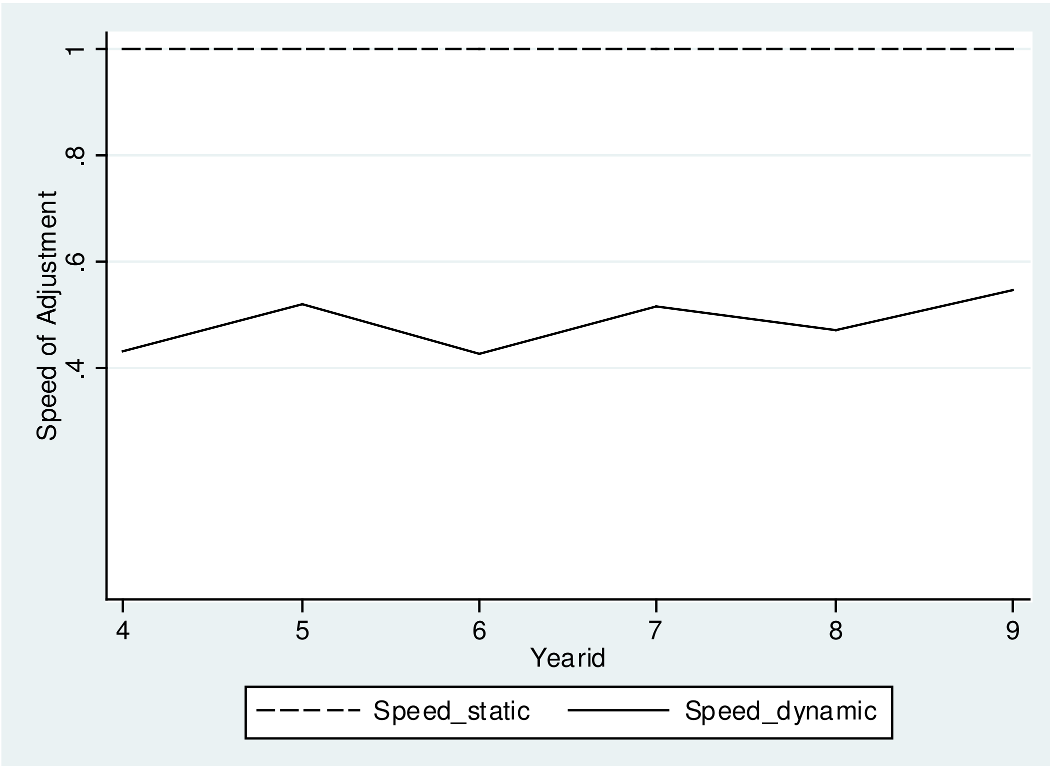
| Sectorid | Technical Efficiency from Dynamic Specification (%) | Technical Efficiency from Static Specification (%) | Rank_Dynamic Specification | Rank_Static Specification |
|---|---|---|---|---|
| 1 | 80.11 | 91.22 | 5 | 21 |
| 2 | 46.83 | 94.67 | 28 | 14 |
| 3 | 63.44 | 92.58 | 16 | 18 |
| 4 | 49.63 | 93.56 | 27 | 17 |
| 5 | 72.78 | 87.22 | 10 | 26 |
| 6 | 92.18 | 99.93 | 1 | 1 |
| 7 | 56.00 | 86.95 | 23 | 27 |
| 8 | 66.65 | 96.10 | 13 | 12 |
| 9 | 88.13 | 98.47 | 3 | 6 |
| 10 | 58.28 | 99.05 | 22 | 4 |
| 11 | 67.12 | 96.58 | 12 | 11 |
| 12 | 85.98 | 92.02 | 4 | 20 |
| 13 | 65.11 | 98.02 | 14 | 7 |
| 14 | 61.07 | 99.57 | 17 | 2 |
| 15 | 63.85 | 96.80 | 15 | 9 |
| 16 | 60.45 | 99.50 | 19 | 3 |
| 17 | 74.70 | 89.62 | 8 | 22 |
| 18 | 78.10 | 94.30 | 7 | 15 |
| 19 | 91.70 | 87.94 | 2 | 25 |
| 20 | 79.80 | 94.85 | 6 | 13 |
| 21 | 72.73 | 86.02 | 11 | 28 |
| 22 | 59.64 | 88.38 | 20 | 24 |
| 23 | 74.22 | 97.44 | 9 | 8 |
| 24 | 59.09 | 92.43 | 21 | 19 |
| 25 | 60.69 | 98.83 | 18 | 5 |
| 26 | 52.79 | 93.94 | 24 | 16 |
| 27 | 50.70 | 89.25 | 26 | 23 |
| 28 | 52.63 | 96.77 | 25 | 10 |

Note: Technical efficiency of a sector is measured relative to the most efficient sector.

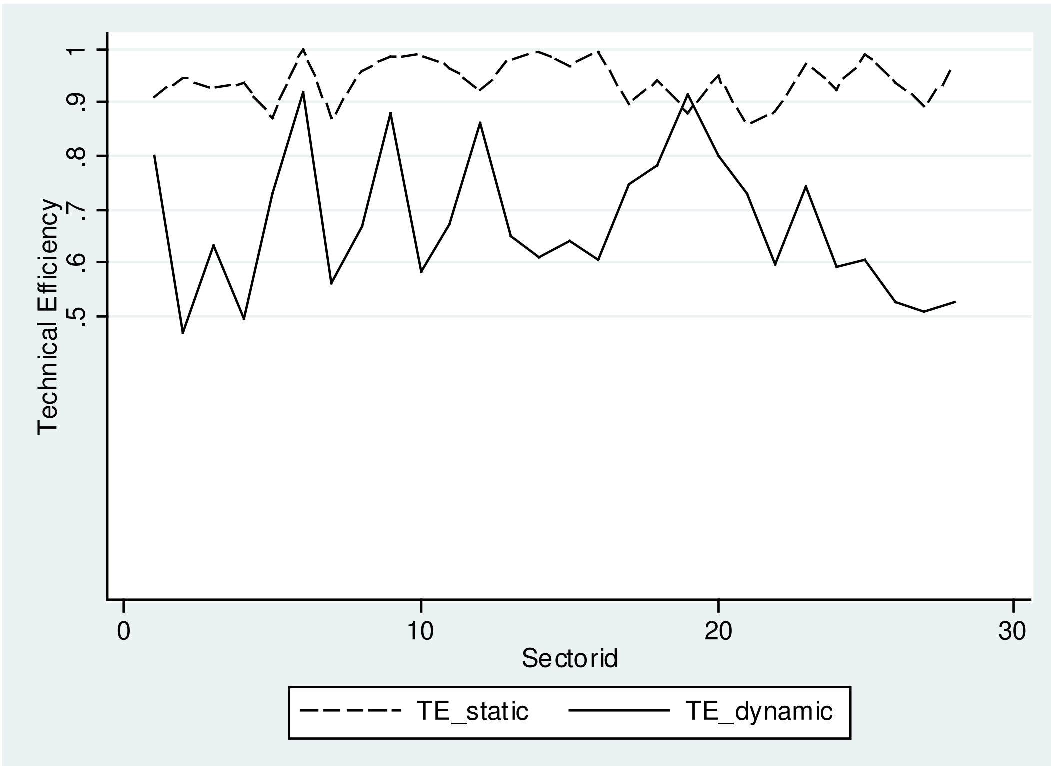
**Graph 1: Time-invariant Technical Efficiency Estimates from Dynamic and Static Specifications**



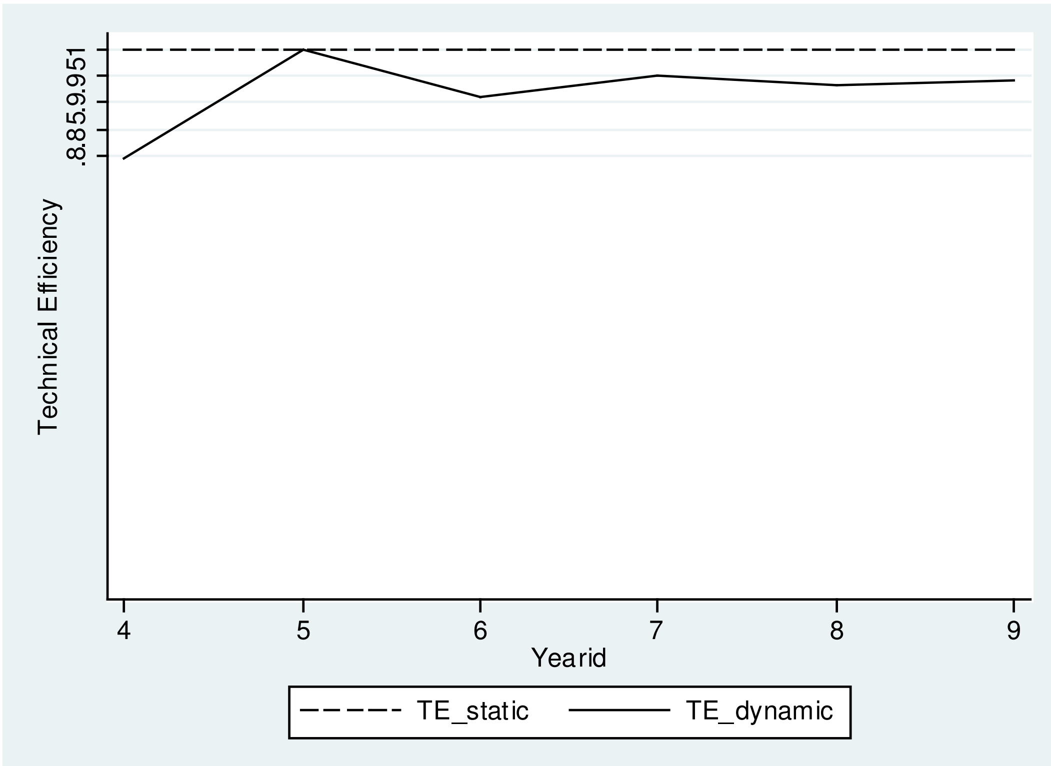
**Graph 2: Speed of Adjustment from Dynamic and Static Specifications**



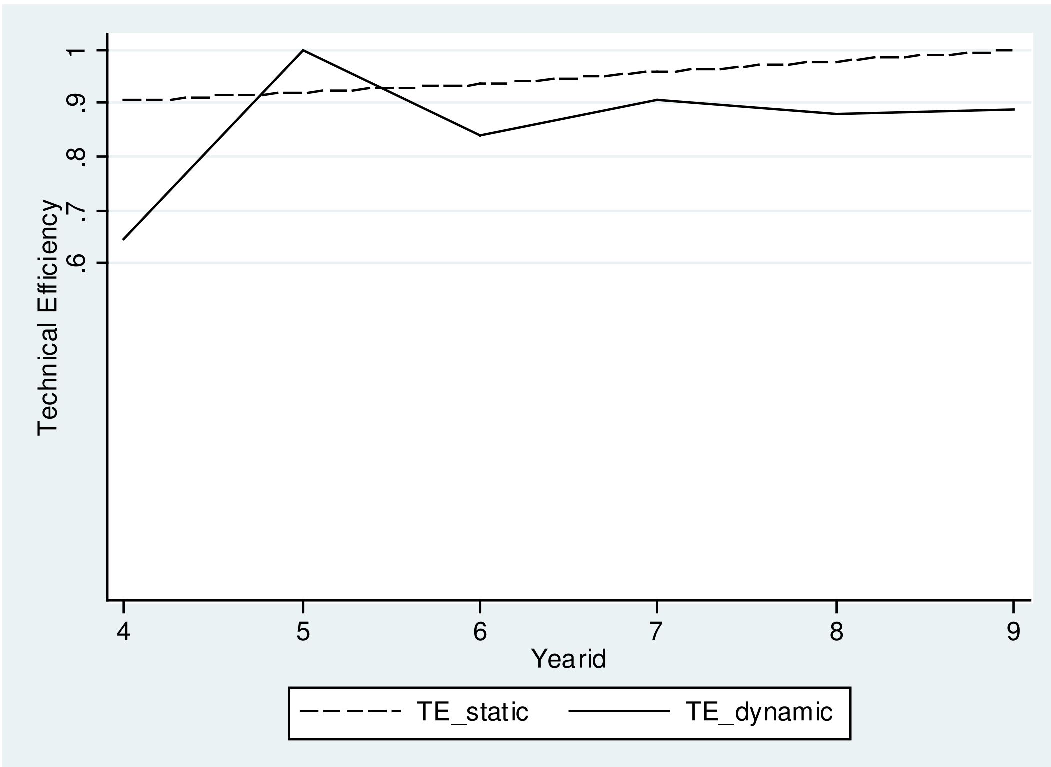
**Graph 3: Average Time-varying Technical Efficiency for Sectors from Dynamic and Static Specifications**



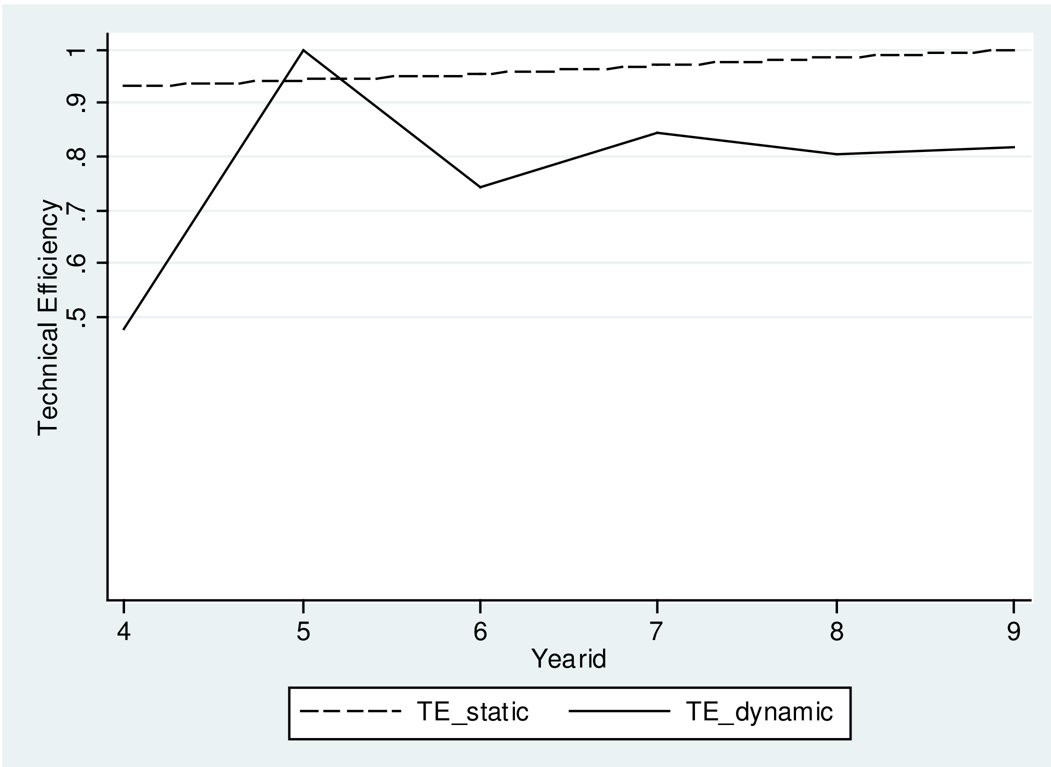
**Graph 4: Time-varying Technical Efficiency for Sector 6**



**Graph 5: Time-varying Technical Efficiency for Sector 12**



**Graph 6: Time-varying Technical Efficiency for Sector 18**



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