Membership in Citizen Groups*

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Abstract

We analyze the coordination problem of agents deciding to join a group that uses membership revenues to provide a discrete public good and excludable benefits. The public good and the benefits are jointly produced, so that benefits are valued only if the group succeeds in providing the public good. With asymmetric information about the cost of provision, the membership game admits a unique equilibrium and we characterize the optimal membership fee. In a static context, we show that heterogeneity in agents' valuations for the excludable benefits is always detrimental to the group. We also consider a dynamic version of the model where heterogeneity arises endogenously: returning members receive additional seniority benefits at the expense of new members. The dynamic game has a unique equilibrium in the space of monotone strategies, and we characterize the ex-ante optimal dynamic contract. In this context, we show that offering seniority benefits is beneficial for the group despite the heterogeneity in valuations created, and we prove that the optimal level of seniority benefits increases when asymmetries in information among agents become small.

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1 Introduction

The National Association Study shows that the mean and median membership size of US voluntary associations are 27,575 and 750, respectively. These numbers suggest that, although not many, very large associations exist. Examples are environmentalist groups like the National Wildlife Federation (NWF), or the World Wildlife Fund (WWF), professional and business groups like the American Farm Bureau Federation, citizens' groups like the American Association of Retired People, and trade unions.[1] The main activity of these associations is lobbying for public policy, and their financial resources mostly derive from due-paying members.[2] Since the benefits of lobbying (environmental legislation, farm subsidies, tax reliefs, minimum wage laws) are clearly non-excludable to non-members, it follows that all these groups were able to overcome a potentially severe free-rider problem.

In "The Logic of Collective Action," Olson [25] suggests that the existence of large voluntary associations can be explained if the group is able to provide selective incentives: goods and services excludable to non-members. These benefits can provide utility directly - e.g., publications, information services, insurance policies, legal advice, advocacy - or they can acquire value through social interaction, as in the case of reputation or peer pressure.[3] Interestingly, the value that prospective members attach to these excludable incentives is often correlated with the success of the association in providing a collective good. For example, as in the case of discounts, the value of

[1] The WWF and the NWF have more than a million members each. The American Association of Retired People (AARP) is the largest nonprofit association in the US with 23 million members. The American Farm Bureau Federation (AFBF) has 6 million members. The largest union in the AFL-CIO is the American Federation of State, County and Municipal Employees (AFSCME), with more than a million members. The data for the National Association Study are reported in Knoke [20].

[2] See, e.g., Knoke [21], and Walker [28].

[3] Indeed, a majority of voluntary associations that have been successful in providing a collective good offer selective incentives for their members. See, e.g., Walker [28].

selective benefits may be directly related to the size of the association: a larger group is able to negotiate better terms with various vendors. At the same time, membership size is a critical factor in determining a group's success in its lobbying efforts for public policy. Moreover, the success in accomplishing some environmental protection projects (a public good) may enhance the quality of organized hiking and animal watching activities by members (a selective incentive).[4] As a result, strategic complementarities in joining decisions may arise, i.e., the more people join the association, the higher the value of being a member.[5] However, models with strategic complementarities are often associated with multiple "extreme" equilibria (i.e., either nobody joins or everybody joins) which are not particularly interesting, not responsive to fundamentals, and not suited to analyze questions of optimal design of a membership contract.[6]

To solve the multiplicity problem, in this paper we present a natural application of the global game approach pioneered by Carlsson and Van Damme [4] and Morris and Shin [23] to a membership game with strategic complementarities. In particular, we study the decision of agents to join a group that uses membership revenues to provide a discrete public good and excludable benefits, in the presence of asymmetric information about the cost of providing the public good. Following Cornes and Sandler [5], [6], [7], we assume that the public good and the selective incentives are jointly produced, so that excludable benefits acquire value only if the group is successful in securing enough revenue to cover the provision cost of the public good. This approach captures a fundamental characteristic of the incentives packages we observe in reality, and uncovers the coordination problem agents face, since their payoff of joining displays strategic complementarities.

The membership game admits a unique equilibrium. Moreover, despite the pres-

[4] See King and Walker [19].

[5] This observation is well known, see , e.g., the trade union membership model of Booth [3].

[6] One possibility is to focus on asymmetric (or mixed-strategy) equilibria that yield an "interior" membership. However, as we show in Section 2, these equilibria typically deliver counterintuitive comparative statics properties.

ence of positive externalities in membership and asymmetric information, finding the optimal membership fee reduces to solving a simple monopoly pricing problem. The model is indeed rather tractable and delivers very intuitive comparative statics, especially in comparison with the properties of the mixed-strategy equilibrium of the model without asymmetric information. In contrast to the latter, for example, in our model collective action is more successful when the net benefit from joining increases. We then interpret the comparative statics in light of some empirical regularities singled out in the political-science literature on membership in large voluntary groups. In particular, we first consider the observation that membership tends to be larger when groups face threats to rights they already enjoy. We then show how our model can be straightforwardly applied to analyze the effects of challenges to the not-for-profit status of associations, a common practice of antagonistic politicians.[7]

Our first contribution is to show that in a static context an increase in heterogeneity among prospective members is always detrimental for the group. To show this, we first characterize the unique equilibrium of the membership game when there are two categories of agents: those with a high valuation for selective incentives, and those with a low valuation. We then consider a mean-preserving spread of valuations, and show that such an increase in heterogeneity decreases the equilibrium size, the optimal membership fee, and ultimately the probability of success of the group. This result obtains because low-valuation agents respond in larger numbers to the perturbation than high-valuation agents, thus reducing the group's total revenue. Indeed, even before the perturbation, low-valuation agents face a greater strategic uncertainty. They must rely on a larger proportion of agents joining and they must believe the group more likely to succeed than high-valuation agents do, to be willing to pay the same cost of membership. Therefore, because benefits are valued only in case of success, low-valuation members are more affected by the mean preserving spread than high-valuation members, coeteris paribus. The negative externality imposed by low-valuation agents lowers

[7] See Walker [28], [29], and Hansen [15].

the incentive to join for all other potential members. Our results are consistent with the empirical evidence in Alesina and La Ferrara [1], where they show that, after controlling for individual characteristics, participation in social activities is significantly lower in more heterogeneous localities.[8]

Our second contribution is to show that in a dynamic context some form of heterogeneity may in fact be beneficial for the group. Dynamic considerations are relevant for many membership decisions. Indeed, if attracting new members is very important for many organizations, retaining existing members is regarded with the same if not larger concern.[9] In this respect, a common practice is the preferential assignment of resources to returning members in the form of seniority benefits.[10] This practice is particularly interesting because it is a choice of the organization's management that endogenously creates heterogeneity among potential members. It appears surprising and potentially counterproductive in light of our previous result and of the received wisdom on the disadvantages of heterogeneity.

To investigate the effects of seniority benefits, we analyze a simple two-period version of the model. The first-period game is our initial membership game with homogenous agents. In the second-period game, heterogeneity among agents arises endogenously: returning members receive additional "seniority" benefits at the expense of new members. This implies that the extra-benefit received by senior members decreases in

[8] For a survey of the empirical literature on the effect of community heterogeneity on social capital, see, e.g., Costa and Kahn [8].

[9] Quoting Rothenberg [27]: "Organizational maintenance is a fact of life all group leaders confront. For the majority of interest group entrepreneurs, who depend on constituent dues as a prime funding source, maintenance dictates the need to keep members contributing [...]. Even seemingly small drops in numbers [...] are viewed with great alarm; and the loss of long-time contributors is perceived as a threat to the entity's survival."

[10] A typical seniority benefit is the practice of reserving office positions to returning members (see, e.g., Moe [22]). Interestingly enough, in the case of citizens' groups like Common Cause, where about a third of the members report that they have politicial aspirations (see, e.g., Rothenberg [27]), the value of seniority benefits is clearly related to the success of the group in its lobbying effort.

first-period membership and, as a result, payoffs are not monotonic in membership. In this context, we prove existence and uniqueness of the equilibrium in the space of monotone strategies. In other words, we show that a unique equilibrium exists if more favorable information implies that each agent is more likely to join. More importantly, when the group maximizes a weighted sum of the probabilities of success in the first and second period, we characterize the ex-ante optimal membership contract, we show that offering seniority is always optimal, and we prove that the optimal level of seniority benefits increases when asymmetries in information among agents become small.

The sharp difference in the effects of heterogeneity between the static and the dynamic model obtains because in the dynamic model the role played by seniority benefits is twofold. On the one hand, seniority benefits affect the value of membership in the first period. On the other hand, they introduce heterogeneity between second-period prospective members. Offering seniority benefits is always optimal for the group because, when the level of seniority benefits is zero, the negative marginal effect on second-period membership is only of second-order magnitude as compared to the positive marginal direct effect on first-period membership. In fact, when no seniority benefits are offered, agents are homogeneous in the second period. All potential members face the same strategic uncertainty, since there are no high or low-valuation agents. Therefore, the overall marginal effect of increasing heterogeneity on second-period membership is zero. On the contrary, the first-period marginal effect of introducing seniority benefits remains always strictly positive.

1.1 Related Literature

Three strands of literature are related to our work. The first deals with impure public goods, the second with dynamic global games, and the third more directly with heterogeneity in membership.

Cornes and Sandler [5], [6], [7] analyze an impure public good model in which the purchase of any quantity of an intermediate good makes available, through a joint production function, fixed proportions of public good and private characteristic. With sufficiently strong complementarities between the private characteristic and the public good, individual demand for the intermediate good may be increasing in the quantity demanded by others, thus alleviating the free rider problem. However, they do not address directly the issue of coordination among agents.

As for dynamic applications of global games, relevant papers include Dasgupta [9], Heidhues and Melissas [16], Giannitsaru and Toxvaerd [12], and Goldstein and Pauzner [13]. Heidhues and Melissas [16] focus on cohort effects, while Dasgupta [9] focuses on social learning. In both papers, contrary to our paper, the decision to contribute is once and for all, therefore there is no heterogeneity among all agents that can take an action in any one period. Giannitsaru and Toxvaerd [12] prove uniqueness of an equilibrium in a general class of dynamic global games under the assumption of strict supermodularity of the payoff of taking an action at time $t+1$ with respect to the number of agents that took the action in t. Since in our model this assumption is violated, their results do not apply.[11] The closest work is Goldstein and Pauzner [13]. Indeed, their proof of uniqueness of an equilibrium with heterogeneous agents applies in our model as well. Their goal is to explain contagion of financial crises across countries. Therefore, they investigate a "first-order" perturbation where, following an earlier crisis in one country, one set of agents becomes poorer and more risk-averse, and hence more likely to run in a second country. On the contrary, in our analysis of heterogeneity we investigate a "second-order" perturbation, where the rewards to the risky action increases for one set of agents, and decreases for the others. Moreover, our perturbation changes the utility of joining directly, not its argument. One may effectively consider agents to be risk-neutral in our analysis of heterogeneity.[12]

Finally, regarding the effect of heterogeneity on membership decisions, the closest paper is Alesina and La Ferrara [1]. In a static model, they show that homogeneity within a community leads to higher participation in social activities. In their model

[11] A violation of supermodularity appears in Goldstein and Pauzner [14] as well.

[12] A formal definition of the perturbation we analyze appears in Section 2.

membership is costless, group size has no effect on individual utility, and individuals have an exogenous preference for homogeneity within a social group.

The remainder of the paper is organized as follows. Section 2 presents the basic structure of the static model and contains our results on the effect of heterogeneity on the equilibrium group size. Section 3 contains a dynamic version of the model in which heterogeneity emerges endogenously. Technical proofs can be found in the appendix.

2 The Model

Consider a continuum of agents of size 1. They decide independently and simultaneously whether or not to join a group. Let $k > 0$ be the cost of membership and $e \in [0,1]$ be the proportion of agents joining the group.[13] The group's total revenues ke are used as an input in a binary production function $f(ke, \theta)$. If total revenues are above a threshold θ, the production function jointly generates an amount G of a pure public good, and an amount x of a non-rival club good that agents enjoy only if they are members. Henceforth, we say that the group is successful when $ke \geq \theta$. Otherwise, $G = x = 0$. Formally,

$$f(ke, \theta) = \begin{cases} (x, G) & \text{if } ke \geq \theta \\ (0,0) & \text{otherwise.} \end{cases}$$

Let $u_i(x, G)$ denote agent i's value for the club good and the public good. We assume that $u_i(x, G)$ is increasing in both arguments, and we adopt the normalization $u_i(0,0) = 0$. Finally, we assume that money enters linearly in agents' utility functions. Payoffs can then be represented in the following table:

	join	not join
$ke \geq \theta$	$u_i(x, G) - k$	$u_i(0, G)$
$ke < \theta$	$-k$	0

[13] In assuming a continuum of agents we follow the literature, and ignore the technical issues discussed in Judd [17] and Feldman and Gilles [10].

What determines agent i's decision to join is the expected net utility from joining:

$$b_i \Pr(ke \geq \theta) - k,$$

where b_i denote the difference between the utility of joining the group and the utility of not joining conditional on the public good being provided. Namely

$$b_i = u_i(x, G) - u_i(0, G).$$

Following Olson [25], we assume that the success in providing a public good is a by-product of the operation of selective incentives: it is not the reason for joining but it is a consequence of members joining. Although one criticism to this argument is that a competing firm, not burdened by the cost of producing the public good, can offer just the private benefit at a lower price, we believe that establishing a brand name through success in providing the public good gives the association some monopoly power over the private good.[14] Moreover, notice that for our purposes, excludability of selective benefits does not need to be absolute, just partial.[15] Our specification of $f(ke, \theta)$ above is a convenient way to formalize the idea that the value of selective benefits that are offered by citizens' associations is often tied to the success of the association in providing the public good, through the standard notion of joint-production.[16]

Consider first the case where all agents are homogeneous, that is $b_i = b$, and assume $b > k$ to rule out the uninteresting case where joining the group is a dominated strategy.

[14] An interesting example is the case of groups that offer free advertising space on the group's magazine to its own members as a selective incentive, see Moe [22]. Moreover, a survey in Walker [28] shows that virtually every group in a sample of 206 citizen associations offers some kind of publication which is considered one of the most important benefit by members. Other instances in which the provision of a collective good gives a cost advantage in the production of the selective benefits include discounts on postal rates, preferred tax treatments or free interns.

[15] Indeed, some benefits appear at first easily obtainable by non-members: it is of course possible to borrow a newsletter or the detailed program of a conference from a friend. However, the lender may enjoy a timing and convenience advantage over the borrower, may earn reciprocation of the favor, or may have the chance of handing out a sly comment to her leeching friend.

[16] Another relevant specification for the success of a groups is simply group size. For example, lobbying activity may be carried out not just by using the groups' revenue to hire professional lobbyist,

The value of the threshold θ is not observable, it is drawn from a uniform distribution on $\left[\underline{\theta}, \overline{\theta}\right]$, and each agent i receives a signal θ_i of the realization of θ.[17] In particular, we assume that $\theta_i = \theta + \varepsilon_i$, where ε_i is a noise drawn from a uniform distribution on $[-\varepsilon, \varepsilon]$ independent across agents, and independent of θ.[18] We also assume that ε is "small" with respect to the support of θ, namely $\underline{\theta} < -2\varepsilon$, and $\overline{\theta} > b + 2\varepsilon$. The

but through the coordinated grassroot efforts of members. Our model can easily encompass such situation by fixing k equal to 1. In this case, success solely depends on membership, and b can be reinterpreted as a normalized benefit to cost ratio of becoming a member. Our results on heterogeneity are qualitatively unaffected.

[17] Without asymmetric information, our game shares with other games of strategic complementarities the feature of multiplicity of (non-interesting) equilibria. For example, if θ is perfectly observable and smaller than k, the simultaneous-move game has two pure strategy equilibria: one in which nobody joins the group, and one in which everybody joins. If instead θ is not observable, it is drawn from a uniform distribution on $\left[\underline{\theta}, \overline{\theta}\right]$, and no private information is received by the agents, under some conditions (i.e., $0 < \left(\overline{\theta} - \underline{\theta}\right)/b + \underline{\theta}/k < 1$) the equilibrium outcomes of the simultaneous-move game are $e = \left\{0, \left(\overline{\theta} - \underline{\theta}\right)/b + \underline{\theta}/k, 1\right\}$. The unattractive feature of the extreme equilibria is not multiplicity per-se, but non-responsiveness to fundamentals. As Goldstein and Pauzner [14] point out, such equilibria are not suited to analyze questions of optimal design. As for the interior equilibrium outcome $e = \left(\overline{\theta} - \underline{\theta}\right)/b + \underline{\theta}/k$, its comparative statics properties are very unattractive, as we will later explicitly analyze. Moreover, (in)stability is an issue.

[18] The assumption that θ is drawn from a uniform distribution allows us to derive closed form solutions but is not essential for our results as long as the prior probability distribution satisfies the conditions in Morris and Shin [23]. Closed form solutions can be obtained also if θ is normally distributed as it is shown in an earlier version of this paper [2].

unconditional distributions of θ and θ_i are depicted in Figure 1.

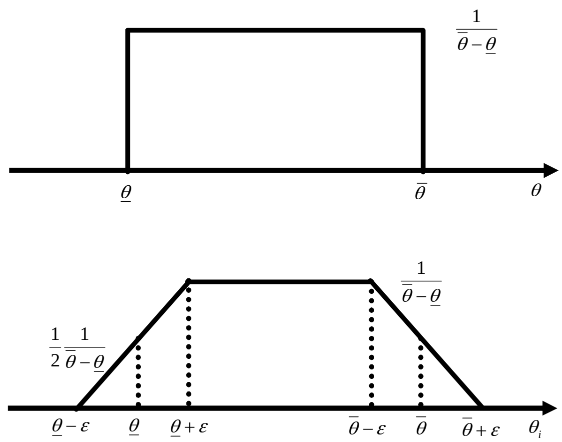

Figure 1

The expected net utility from joining, conditional on having received signal θ_i is

$$b\,\mathrm{Pr}\left(ke \geq \theta|\theta_i\right) - k, \tag{1}$$

where now e represents individual i's belief about the proportion of agents joining the group, conditional on θ_i. This game admits a unique equilibrium in which players follow a cutoff strategy around θ^b, i.e., they join the group if $\theta_i < \theta^b$ and stay out otherwise. The uniqueness result derives from iterated deletion of strictly dominated strategies, it follows [23], [24] and [13], and therefore we omit a proof. Note that for the first round of deletion we need regions of the signal space where, for sufficiently unfavorable (favorable) signals, staying out (joining) is a strictly dominant strategy. Indeed, when $e = 1$, i.e. under the most optimistic belief about the group, (1) is strictly negative for any $\theta_i \geq \overline{\theta} - \varepsilon > b + \varepsilon > k$. Likewise, under the most pessimistic belief about the group, i.e., for $e = 0$, (1) is strictly positive for any $\theta_i \leq \underline{\theta} + \varepsilon < -\varepsilon < 0$. In order to characterize the equilibrium cutoff θ^b, we first define the critical state θ^* as the highest value of the threshold cost θ for which the group is successful, or

$$k\,\mathrm{Pr}\left(\theta_i \leq \theta^b|\theta = \theta^*\right) = \theta^*, \tag{2}$$

and equation (2) further implies that θ^* is the total revenue for the group conditional on state θ^* (k times membership conditional on θ^*). Using equation (1) and the definition of θ^*, since in equilibrium type θ^b must be indifferent between joining and staying out, the equilibrium values of θ^* and θ^b must satisfy (2) and

$$b \Pr\left(\theta \leq \theta^* | \theta_i = \theta^b\right) = k, \tag{3}$$

where $\Pr\left(\theta \leq \theta^* | \theta_i = \theta^b\right)$ is the probability of success perceived by the indifferent type.

The dominance regions described above imply that θ^b must lie inside the interval $\left(\underline{\theta} + \varepsilon, \overline{\theta} - \varepsilon\right)$ and therefore, conditional on $\theta_i = \theta^b$, the distribution of θ is uniform on $\left[\theta^b - \varepsilon, \theta^b + \varepsilon\right]$. In turn, $\theta^* \in \left[\theta^b - \varepsilon, \theta^b + \varepsilon\right]$, for (3) to admit a solution. Therefore,

$$\Pr\left(\theta_i \leq \theta^b | \theta = \theta^*\right) = \frac{\theta^b - \theta^*}{2\varepsilon} + \frac{1}{2}, \tag{4}$$

and

$$\Pr\left(\theta \leq \theta^* | \theta_i = \theta^b\right) = \frac{1}{2} - \frac{\theta^b - \theta^*}{2\varepsilon}. \tag{5}$$

Adding (4) and (5) yields

$$\Pr\left(\theta_i \leq \theta^b | \theta = \theta^*\right) = 1 - \Pr\left(\theta \leq \theta^* | \theta_i = \theta^b\right), \tag{6}$$

and in equilibrium, using (3), we obtain

$$\Pr\left(\theta_i \leq \theta^b | \theta = \theta^*\right) = 1 - \frac{k}{b}. \tag{7}$$

Substituting (7) in (2) yields

$$\theta^* = k\left(1 - \frac{k}{b}\right), \tag{8}$$

and further substitution in (3) results in

$$\theta^b = \theta^* + \varepsilon\left(1 - \frac{2k}{b}\right). \tag{9}$$

In equilibrium, the value of $\theta^b - \theta^*$ is determined by the last term of equation (9), it may be positive or negative, and it captures the fact that joiners pay the membership fee k for sure, and receive the benefit b only with some probability. The relationship between $\theta^b - \theta^*$ and k is rather intuitive. When joining is relatively inexpensive ($k \to 0$), an agent needs a relatively small probability of success and expected benefit of joining to be indifferent between actions. In fact, (9) implies that when k is small, θ^b is larger than θ^* and, since the posterior probability distribution of θ conditional on $\theta_i = \theta^b$ is centered around θ^b, the probability of success for θ^b is smaller than $1/2$, from (5). The opposite occurs when joining is relatively expensive ($k \to b$). In light of this, we can interpret the difference between θ^* and θ^b as a measure of the strategic uncertainty agents are willing to bear in equilibrium.

Since our global game framework is a clear departure from the standard provision of a public good literature, it is interesting to analyze the simple comparative statics properties of our model, and compare our predictions with those resulting from an interior equilibrium of the game without asymmetric information. For example, the existing political science literature on group membership emphasizes the fact that collective action tends to be more successful if individuals face a threat to their status-quo level enjoyment of a public good.[19] Various theories have been advanced to explain such "loss-averse" behavior, and a full analysis from basic principles is outside the scope of this paper.[20] However, a reasonable reduced-form conjecture to account for this phenomenon in our framework is to assume that the net benefit b is perceived by agents as being larger when the group is trying to avoid a loss rather than obtain a

[19]See, e.g., Walker [29]: "When persons face a threat [...] to rights they already enjoy, they are more likely to engage in collective action to protect these gains despite the problems posed by the public goods dilemma." See also Hansen [15].

[20]See, e.g., Kahneman and Tversky [18] among many others.

gain in the level of public good provided.[21] We then see straightforwardly from (9) that membership is larger when the group is trying to avoid a loss. On the contrary, at the interior equilibrium of the game without asymmetric information, we have the opposite result because membership is always, counterintuitively, decreasing in the net benefit b.[22]

So far we have assumed that the membership fee k is exogenous. Before proceeding further in our comparative statics, we analyze the problem of finding the optimal membership fee. Typically, in standard global games, the threshold for success depends only on the measure of agents taking the risky action. In our model, θ^* depends on the total amount of financial resources raised by the group. When k goes to zero, θ^* converges to zero because per-capita payments are zero. When k goes to b, θ^* converges to zero because agents find it very risky to join, and equilibrium membership conditional on $\theta = \theta^*$ in (7) approaches zero. The maximum θ^* obtains for a level of k that balances out the positive effect on per-capita payment and the negative effect on membership. A graphical illustration, similar to the textbook analysis of a one-price monopoly with zero marginal cost, is presented in Figure 2. The interpretation is that equation (7) describes a linear demand curve D, where the fee k is the price, and expected membership conditional on θ^* represents the quantity.

[21] More specifically, by denoting the status-quo provision of public good as G_{SQ}, the net utility from joining the association is then

$$b = u\left(x, G + G_{SQ}\right) - u\left(0, G + G_{SQ}\right),$$

and, if the cross derivative u_{12} is negative, we have that b is larger when avoiding a loss, that is $G = 0$, rather then when obtaining a gain, that is for $G > 0$. The assumption of $u_{12} < 0$ implies that the utility of the selective benefit is decreasing in the status quo level of public good provision. It can be justified if we consider selective benefits like representation before government.

[22] Recall that the interior equilibrium outcome of the game without asymmetric information is $e = \left(\overline{\theta} - \underline{\theta}\right)/b + \underline{\theta}/k$. Moreover, as it will become clear in the next paragraph, a similar argument holds if insted of assuming that the membership fee k is fixed we consider the case in which k is optimally chosen by the group.

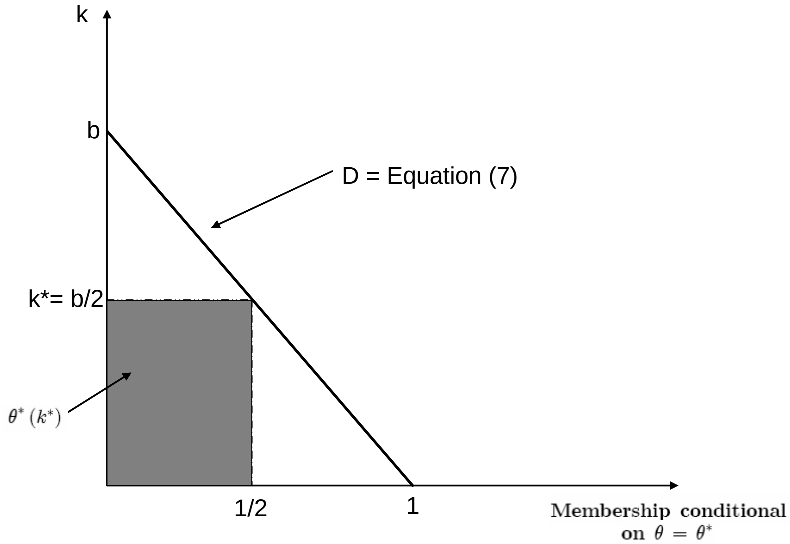

Figure 2

Equation (2) implies that $\theta^*(k)$ is the area of the shaded rectangle. Conditional total revenue, which equals $\theta^*(k)$, is maximized at the midpoint of the demand curve, i.e., for $k^* = b/2$, where conditional membership is equal to 1/2. In equilibrium, the ex-ante expected probability that the group is successful is

$$W \equiv \frac{\theta^*(k) - \underline{\theta}}{\overline{\theta} - \underline{\theta}},$$

and $k^* = b/2$ maximizes the probability of success.

Consider now the ex-ante expected probability of success evaluated at the optimal k^*, which is

$$W(k^*) \equiv \frac{\frac{b}{4} - \underline{\theta}}{\overline{\theta} - \underline{\theta}} = \frac{1}{2}\frac{\frac{b}{4} - \underline{\theta}}{\frac{\overline{\theta}+\underline{\theta}}{2} - \underline{\theta}}.$$

Note that $W(k^*)$ is increasing in b, and it is decreasing in the mean of the cost θ holding its variance constant (i.e., when the support of θ shifts to the right). It is also worth noting that such intuitive comparative statics cannot be generated by an interior equilibrium of the game without asymmetric information. By letting $S(k)$ denote the ex-ante expected size of the group, we have

$$S(k) \equiv \frac{\theta^b(k) - \underline{\theta}}{\overline{\theta} - \underline{\theta}}, \tag{10}$$

and it is easy to show that $k^{**} = k^* - \varepsilon$ is the optimal interior membership fee that maximizes $S(k)$.

To see why k^{**} must be smaller than k^*, note that, using (9), $S(k)$ can be expressed as a linear increasing function of equilibrium conditional total revenue $\theta^*(k)$, and equilibrium conditional membership $1-k/b$. Maximizing $S(k)$ can then be interpreted as having a monopoly that maximizes a weighted average of revenue and membership. Therefore, the optimal k will be lower than the one that maximizes revenue alone. Finally, note that $S(k^{**})$, that is the ex-ante expected size of the group evaluated at the optimal k^{**}, displays analogous comparative statics properties to those for $W(k^*)$ analyzed above.

As an illustration of the comparative statics properties of our model when k adjusts optimally, we consider another empirical regularity pointed out in Walker [28] regarding the attempts to frustrate antagonist associations by politicians through different means like challenges to their not-for-profit status, or by raising postal rates. We examine explicitly the case of a raise in postal rates. For simplicity, we focus only on the increase in the cost of the pre-paid response envelope, and we model it as an additional expense for the group equal to $t \in (0, k)$ per member. The condition that characterize the equilibrium become

$$\theta^* = (k-t)(1-k/b) \text{ and } \theta^b = \theta^* + \varepsilon(1-2k/b),$$

so that the fee that maximizes the probability of success and the fee that maximizes membership increases respectively to

$$k^*(t) = (b+t)/2 \text{ and } k^{**}(t) = k^*(t) - \varepsilon.$$

At these optimal fees, we have that

$$W(k^*) \equiv \frac{\frac{1}{b}\left(\frac{b-t}{2}\right)^2 - \underline{\theta}}{\overline{\theta} - \underline{\theta}} \text{ and } S(k^{**}) \equiv \frac{\frac{1}{b}\left(\left(\frac{b-t}{2}\right)^2 + \varepsilon(\varepsilon - t)\right) - \underline{\theta}}{\overline{\theta} - \underline{\theta}}.$$

Hence, since $t < k < b$, an increase in postal rate decreases the expected probability of providing the public good and the expected size of the group. In the case of an interior equilibrium of the game without asymmetric information we obtain, once again, the opposite results.

2.1 Heterogeneous Agents

Consider now the case in which the population is heterogeneous with respect to the benefit they derive from joining. We assume that the population is divided into two classes: for a fraction $p \in (0,1)$ of the population, the difference between the value of joining the group and the value of not joining conditional on the public good being provided is equal to n, while for the remaining $(1-p)$ it is equal to s. Moreover, assume that $s \geq b \geq n > k$ and, to save notation, let $\alpha \equiv ps + (1-p)n$. Our simple form of heterogeneity describes a situation where an exogenous proportion of agents receives more value from the selective benefit given the same level of public good.[23] Our objective is to explore the effect of increasing heterogeneity in the population on the equilibrium probability of providing the public good, the equilibrium size of the group, and the optimal fee.

Similarly to the homogeneous benefit case, we can show that a unique equilibrium exists. Players with benefit n (s) follow a cutoff strategy around θ^n (θ^s), i.e., they join the group if $\theta_i < \theta^n$ ($\theta_i < \theta^s$) and stay out otherwise. Proposition 1 characterizes the equilibrium cutoffs.

Proposition 1 *An equilibrium of the membership game exists and it is unique. In*

[23] A sufficient condition on the utility functions is that the partial derivative of u_i with respect to x for some agents is larger than the one for the rest of the population.

equilibrium

$$\theta^* = k\left(1 - k\frac{\alpha}{ns}\right), \tag{11}$$
$$\theta^n = \theta^* - 2\varepsilon\left(\frac{k}{n} - \frac{1}{2}\right),$$
$$\theta^s = \theta^* - 2\varepsilon\left(\frac{k}{s} - \frac{1}{2}\right).$$

Proof of Proposition 1. Existence and uniqueness follow by Proposition 1 in Goldstein [13]. The characterization is similar to the homogenous case. In particular, the critical state θ^* is again determined as conditional revenue, or k times average conditional membership:

$$\theta^* = k\left(p\Pr\left(\theta_i \leq \theta^n | \theta = \theta^*\right) + (1-p)\Pr\left(\theta_i \leq \theta^s | \theta = \theta^*\right)\right). \tag{12}$$

Moreover, in such a cutoff equilibrium, the indifferent type in class s, θ^s, will satisfy

$$s\Pr\left(\theta \leq \theta^* | \theta_i = \theta^s\right) = k, \tag{13}$$

and the indifferent type θ^n will satisfy

$$n\Pr\left(\theta \leq \theta^* | \theta_i = \theta^n\right) = k. \tag{14}$$

The existence of strict dominance regions can be established following exactly the same steps as for the homogenous population case. It is useful not only to establish uniqueness of an equilibrium, but it simplifies the characterization as well. Indeed, since the expected payoff from joining is strictly negative for any $\theta_i \geq \overline{\theta} - \varepsilon$, while it is strictly positive for any $\theta_i \leq \underline{\theta} + \varepsilon$, it follows that θ^s and θ^n must belong to the interval $\left(\underline{\theta} + \varepsilon, \overline{\theta} - \varepsilon\right)$ and, therefore, conditional on $\theta_i = \theta^s$ $(\theta_i = \theta^n)$, θ is uniformly distributed in $[\theta^s - \varepsilon, \theta^s + \varepsilon]$ $\left([\theta^n - \varepsilon, \theta^n + \varepsilon]\right)$. This implies that $\theta^* \in [\theta^s - \varepsilon, \theta^s + \varepsilon]$, otherwise (13) does not admit a solution, and that $\theta^* \in [\theta^n - \varepsilon, \theta^n + \varepsilon]$, otherwise (14) does not admit a solution. Therefore, a condition similar to (6) holds for each class of agents:

$$\Pr\left(\theta_i \leq \theta^v | \theta = \theta^*\right) = 1 - \Pr\left(\theta \leq \theta^* | \theta_i = \theta^v\right), \text{ for } v \in \{s, n\}, \tag{15}$$

and equilibrium average membership conditional on θ^* will be

$$p \Pr(\theta_i \leq \theta^n | \theta = \theta^*) + (1-p) \Pr(\theta_i \leq \theta^s | \theta = \theta^*) = 1 - k\alpha/(ns), \tag{16}$$

where we used (15), and the indifference conditions (13) and (14). The expression for θ^*, θ^n and θ^s in (11) then follow from (12) and recursive substitutions in (13) and (14). ■

The analysis of the optimal k is analogous to the homogenous population case: the level of k that maximizes the probability of success of the group is $k^*_{het} = ns/2\alpha$, while the level of k that maximizes the size of the group is $k^{**}_{het} = k^*_{het} - \varepsilon$.[24] The comparative statics property of these optimal fees are intuitive, and similar to those for the homogenous population case.

We now investigate the equilibrium effects of increasing heterogeneity among agents when k is set at the value that maximizes the probability of success of the group.[25] In particular, we increase the net payoff of the $1-p$ agents in class s by Δ and we decrease it for the remaining p agents by $\Delta(1-p)/p$. This spread holds constant the population mean net payoff. We obtain the following result:

Proposition 2 *Increased heterogeneity in the form of a mean preserving spread in net payoffs decreases the equilibrium probability of success, the ex-ante size of the group, and the optimal fee charged.*

We leave the complete proof to the appendix and outline the argument here. Figure 3 provides an illustration of the intuition behind this result using the same monopoly analogy as before. In Figure 3 we depict the demand curve in the homogenous case, D, derived from equation (7), and the demand curve in the heterogenous case, D_{het}, derived from equation (16). To provide meaningful comparisons between the results of

[24] With the provision that parameter values are such that the resulting optimal k is indeed smaller than n, so that the group caters to both kind of agents. A sufficient condition is $n > s/2$, as the Proof of Proposition 2 establishes.

[25] The results are similar if we instead consider the fee that maximizes the size of the group.

homogenous and heterogenous cases, we are assuming $s = b + \Delta$ and $n = b - \Delta(1-p)/p$, to maintain the population mean net payoff constant at b.

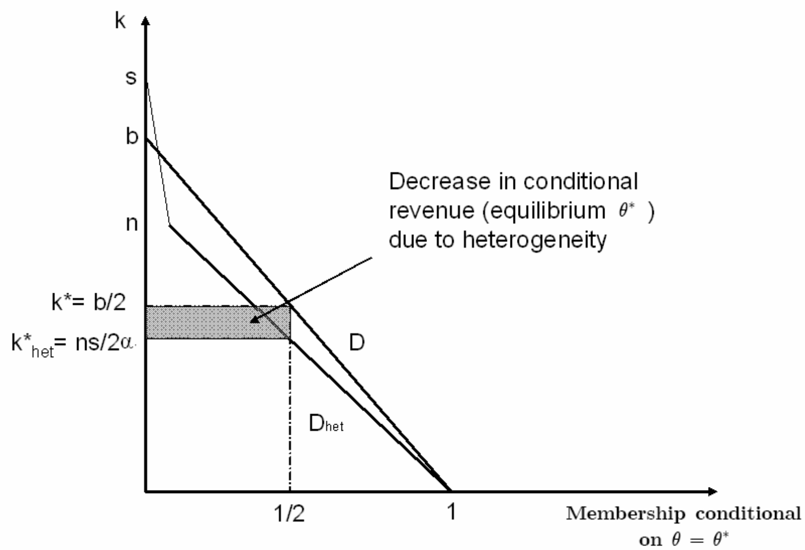

Figure 3

The important observation is that D_{het} turns out to be smaller than D for the relevant range $k < n$, and, as we will show momentarily, the larger Δ, the smaller D_{het}. Since monopoly revenue is maximized at the fee that makes conditional membership equal to 1/2, the optimal fee k^*_{het} is lower than k^*. Moreover, the conditional revenue and the level of θ^* (see Figure 2) are smaller on D_{het}, and so is the ex-ante expected membership. To complete the proof of this proposition, we show that D_{het} is indeed smaller than D. We proceed by demonstrating that, for $s > n > k$, membership conditional on θ^*, that is demand in Figure 3, is smaller after the mean preserving spread. The difference between D_{het} and D then simply results from aggregation of mean preserving spreads, for any k fixed smaller than n. To further help our graphical exposition, consider $p = 1/2$, so that s increases to $s + \Delta$, and n decreases to $n - \Delta$. Intuitively, two opposing externalities come into play. Class s (n) agents' net payoff in case of success increases (decreases), so they should join more (less) often, and by strategic complementarities, all other agents' should enter more (less) often. Therefore, the overall result depends on the relative strength of such externalities. Since we are interested in membership conditional on θ^* in (12), what drives the result are the changes in $\theta^n - \theta^*$ and $\theta^s - \theta^*$. The key observation is that, compared to class-s agents, low-valuation agents face a larger strategic uncertainty: to be willing to pay the same fee, they must believe the group more likely to succeed. Figure 4 illustrates this observation using equations (13) and (14). These two equations imply that the area of the regions ABCD in the top and bottom halves of Figure 4 must be equal to each other (and to $2\varepsilon k$). Hence, since $s > n$, we must have $\theta^* - (\theta^n - \varepsilon) > \theta^* - (\theta^s - \varepsilon)$. Therefore, since benefits have value only in case of success, the change in interim expected payoff for the (formerly) indifferent type θ^n is larger than for θ^s, i.e., the area of the region EFBC depicted in the top half of Figure 4 is strictly larger than the area of the region EFBC depicted in the bottom half. It then follows that class-n agents react more to the mean preserving spread than class-s agents, that is $\theta^n - \theta^*$ changes more than $\theta^s - \theta^*$ in order to restore (13) and (14), so that D_{het} decreases.

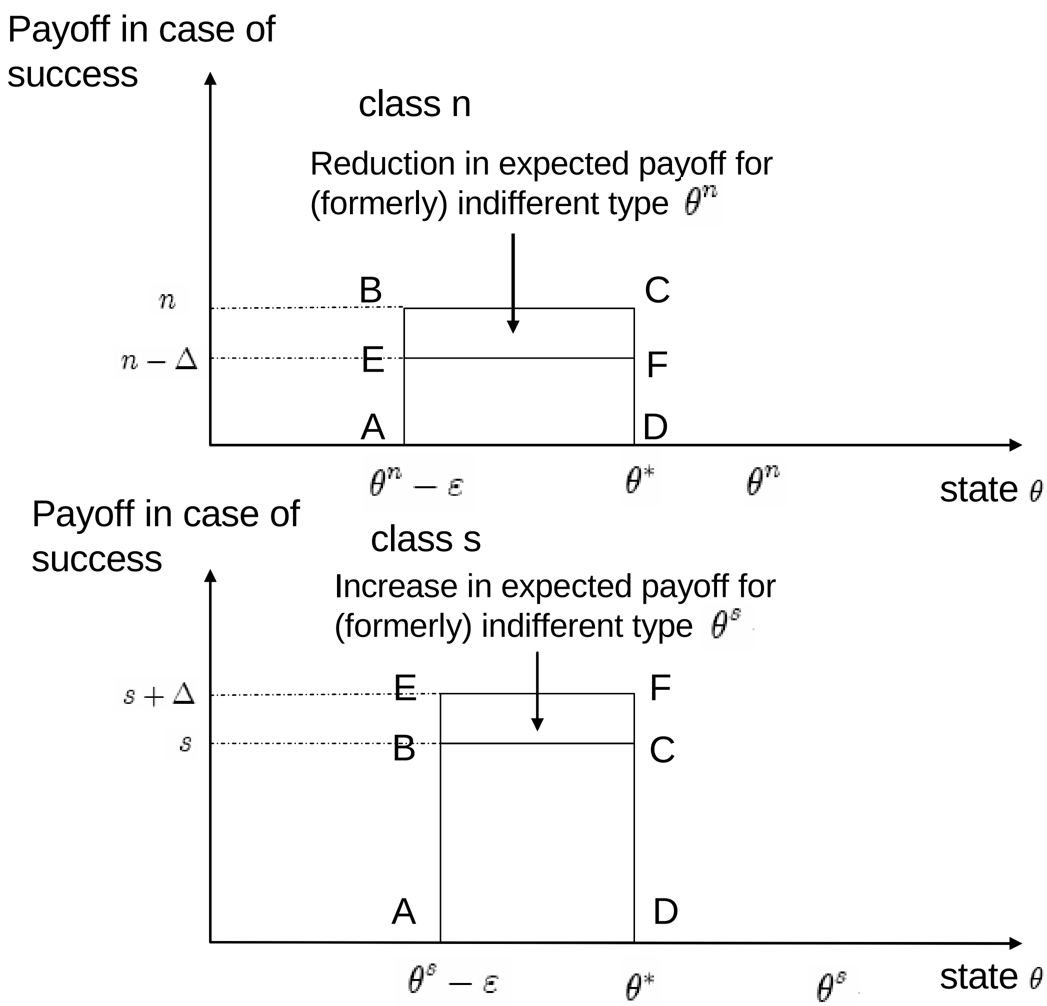

Figure 4

We conclude this section by noticing that the result of Proposition 3 is in line with the existing literature showing that exogenous heterogeneity typically hampers participation to social activities.[26] In the next section we will show that this conclusion can be reversed when heterogeneity arises endogenously in a dynamic setting.

[26] See, among others, Alesina and La Ferrara [1], and Costa and Kahn [8].

3 The Dynamic Model

We consider a two-period dynamic extension of our model to explore the effect of seniority benefits on membership and retention decisions. In each period $t = \{1, 2\}$ a threshold level θ_t is drawn, agents observe a noisy signal of the true threshold and decide whether to pay a membership fee k_t to join the group or not. The first period is similar to the homogeneous case in Section 2: all members receive b when the group is successful. The second period is similar to the heterogenous case in Section 2: in case of success returning members receive s while new members receive n, with $s \geq b \geq n$. For simplicity, we model seniority benefits as an endogenous mean-preserving spread, that is we assume

$$s\,(p; \Delta) = b + p\Delta$$
$$n\,(p; \Delta) = b - (1 - p)\,\Delta,$$

where $(1 - p)$ is the endogenous fraction of agents who joins in the first period. The difference $s\,(p; \Delta) - n\,(p; \Delta) = \Delta \geq 0$ represents the total utility value of seniority benefits, and we assume that it is distributed among new and senior members so that the average utility of selective incentives is unchanged:

$$(1 - p)\, s\,(p; \Delta) + pn\,(p; \Delta) = b.$$

This assumption facilitates the comparison with our results on the exogenous heterogeneity case.[27]

We assume that k_1 and Δ are chosen optimally by the group at the beginning of the game, in order to maximize a weighted sum of the probabilities of providing the public good in the first and second period. Further, we assume that, at the beginning of the second period, the membership fee k_2 is chosen to maximize the probability of

[27] Note that the extra-benefit senior members receive with respect to the first period, that is $s\,(p; \Delta) - b = p\Delta$, is decreasing in first-period membership. This is one way to capture the notion of preferential assignment of limited resources to returning members.

success for any realized $(1-p)$. While all our results are qualitatively unaffected under reasonable alternative extensive forms, a critical assumption regards the credibility of committing to the Δ chosen at the beginning of the game. It is immediate from our previous results that with no commitment power, that is when the association can revise Δ in the second period at no cost, the only credible Δ is zero. Clearly, some degree of commitment seems both plausible and realistic. Here, for simplicity, we assume perfect commitment. With such assumption, one rationale for the use of seniority benefits is to effectively bundle admission for the two periods. Indeed, it is possible to choose Δ so large that, at the same time, agents do not join in the first period on the merits of its fundamentals but just not to be excluded in the second period, and the association does not try to obtain new members in the second period but only caters to returning members. To avoid such a radical departure from our earlier framework, along with perfect commitment we confine our analysis to cases where:

A1) $b > k_1 > \Delta$, so that first-period fundamentals are the deciding factor in first-period membership decisions, and

A2) $\Delta \leq b/2$, so that the associations optimally caters to both new and returning members in the second period.

Let $\theta_{1i} = \theta_1 + \varepsilon_{i1}$, and $\theta_{2i} = \theta_2 + \varepsilon_{i2}$ be the signals in the first and second period, respectively, where θ_1 and θ_2 are uniformly distributed on $\left[\underline{\theta}, \overline{\theta}\right]$, while ε_{i1} and ε_{i2} are uniformly distributed on $[-\varepsilon, \varepsilon]$. All these random variables are assumed to be mutually independent.[28] Finally, we assume that at the beginning of the second period agents observe the proportion of those who joined in the first period $(1-p)$.

Note that seniority benefits directly increase the value of retaining membership status conditional on reaching the production threshold. However, given the result

[28] If the distributions of the first and second-period states are not independent, the fraction of agents who joins in the first period may convey information about the realization of the second period state. This information contagion has been investigated in a number of different papers; here, for clarity and simplicity, we consider the new state independently drawn. For a model considering the information contagion effect see, e.g., Dasgupta [9].

of Proposition 2, heterogeneity among agents reduces the probability of reaching this threshold in the second period. Since these opposite effects spill over to the first period, payoffs may be non-monotonic in the signal, hence we cannot apply existing results to show existence and uniqueness of an equilibrium.[29] In the next proposition we fix Δ and k_1 and we show that an equilibrium exists and is unique in the space of monotone strategies, that is when more favorable information implies that each agent is more likely to join in equilibrium.

Proposition 3 *Under assumptions A1 and A2, in any subgame following a choice of* $k_1 > 0$ *and* Δ*, if agents use monotone strategies a unique equilibrium of the dynamic membership game exists. In equilibrium players follow a switching strategy around* $\theta_1^b (k_1; \Delta)$ *in the first period, the group sets* k_2 *optimally to* $k^*_{2het} (p; \Delta) < n (p; \Delta)$ *and, in the second period, players follow a switching strategy around* $\theta_2^s (p; \Delta)$ *if they joined in the first period, and around* $\theta_2^n (p; \Delta)$ *otherwise.*

The proof is in the appendix. The logic of the proof is simple and proceeds by backwards induction. Proposition 1 ensures existence and uniqueness of an equilibrium in the second period, for any p. The group then chooses the optimal k_2 in order to serve both groups of potential members, because $\Delta \leq b/2$. Moving back to the first period, payoffs to joining have then two components. One is related to first-period fundamentals, and is identical to the one in the static homogenous case. The second is related to the expected difference in equilibrium payoffs between entering as a senior or a new members in the second period. The non-monotonicity in p of this second component precludes convergence to a unique equilibrium of the standard iterated deletion of strictly dominated strategies process. However, if we restrict attention to

[29] A paper that tackles a similar problem is Giannitsaru and Toxvaerd [12], where uniqueness of an equilibrium is proven in a general class of dynamic global games. However, our problem does not satisfy one assumption for their results, namely strict supermodularity of the payoff to joining in the second period with respect to the number of agents that joined in the first. In our case, we have an inverse relation for senior members.

monotone strategies, the marginal effect of p on the first component of payoffs is always the dominant one, so existence and uniqueness of equilibrium (in cutoff strategies) is preserved.

At this point, a natural question to ask is whether offering seniority benefits is ever an optimal strategy and, if this is the case, what determines the optimal level of Δ. We let w_1 and w_2 denote the weights that the group attaches to the probability of providing the public good in the first and second period, respectively. Therefore, the objective function of the group is

$$W(k_1;\Delta) \equiv w_1 \Pr\left(\theta_1 < \theta_1^*(k_1;\Delta)\right) + w_2 E_{\theta_1}\left(\Pr\left(\theta_2 < \theta_2^*(p,\Delta)\right)\right), \tag{17}$$

where $\theta_1^*(k_1;\Delta)$ is the threshold below which the group is successful in the first period and $\theta_2^*(p,\Delta)$ is the threshold below which the group is successful in the second period.[30] The number of potential new members in the second period, p, is itself a function of the cutoff value $\theta_1^b(k_1;\Delta)$ and of the actual first-period state θ_1. Our first result is to establish that not offering seniority benefits is never optimal for the group. Indeed, denoting the problem facing the association as

$$\begin{array}{ll} \max_{k_1,\Delta} & W(k_1;\Delta) \\ & \\ s.t. & \begin{array}{c} \Delta \leq b/2 \\ \Delta < k_1 < b \end{array} \end{array} \tag{M}$$

we have the following:

Proposition 4 *There exists a unique solution (k_1^*,Δ^*) to problem (M). Moreover, $\Delta^* > 0$ and $k_1^* > b/2$.*

Existence and uniqueness follow from continuity and convexity arguments. The proof that $\Delta^* > 0$ in the appendix relies on the following intuition. The role of

[30] Clearly, the value of θ_2^* is a function of k_2 as well. We suppress this argument because the optimal k_2 is itself a function of p and Δ.

seniority benefits is twofold: they directly increase the value of membership in the first period, and they introduce heterogeneity between prospective members. When $\Delta = 0$, the marginal effect of seniority benefits on second-period equilibrium values is zero. Indeed, without seniority benefits, there is no agent that receives a smaller utility of membership in the second period, i.e., all agents are facing the same strategic uncertainty. Therefore, at $\Delta = 0$, the endogenous mean-preserving spread generated by offering some seniority benefits produces marginal effects on senior and returning members that exactly counterbalance. On the contrary, the marginal effect of Δ on first-period equilibrium values remains positive at $\Delta = 0$. Seniority benefits add an extra-term to the payoff of joining in the first period in (1): the expected value of re-entering as a senior member and receiving s, versus joining as a new member and receiving n in the second-period. The difference $s - n = \Delta$ is zero at $\Delta = 0$, but its derivative remains strictly positive.[31] Therefore, at $\Delta = 0$ the marginal positive effect of Δ on first-period equilibrium values dominates the marginal negative effect on the second-period ones. As for the optimal fee k_1^*, quite intuitively we have that the association charges more than in the static case of Section 2, that is more than $b/2$, since membership is more valuable because of seniority benefits. It is worth noting that the optimal Δ^* is non-negative even when membership fees are exogenously fixed. Hence, the result that offering seniority benefits is optimal obtains as well for the interpretation of our model where success is determined only by the size of the association.

Proposition 4 leaves open the possibility of a corner solution at $\Delta^* = b/2$. The following quite intuitive lemma establishes that when the weight on the second period is sufficiently large Δ^* is interior.

Lemma 1 *For any $w_1 > 0$ and any $\varepsilon > 0$, there exists $\overline{w} > w_1$ such that, for*

[31] Clearly, the difference $(s - n)$ is received only with some probability. In the proof we show that the extra-term to be added to equation (1) is the expected value of $\Delta \Pr(\theta_2 < \theta_2^*(p; \Delta)(1 - \varepsilon/\alpha))$, where θ_2^* is the critical second-period state (see equation (21) in the appendix). The important observation here is that this probability is always strictly positive because of the existence of lower dominance regions, so that the marginal effect of Δ is always positive.

$w_2 > \overline{w}$, we have $\Delta^* < b/2$.

When Lemma 1 holds, it is straightforward to establish that Δ^* is increasing in b. Moreover, the group reacts to a smaller asymmetry in information among agents by increasing the level of seniority benefits, as the next proposition shows.

Proposition 5 *If $w_2 > \overline{w}$, the optimal level of seniority benefits Δ^* is decreasing in ε.*

The intuition relies again on the twofold role of seniority benefits. Consider first the negative effects in the second period. When ε decreases, it is more likely that agents receive similar signals, therefore it is more likely that they choose the same action. Indeed, in our model, for all realizations of θ_1 not in an ε-neighborhood of the cutoff θ_1^b, all agents either join the group or stay out.[32] Therefore, since Δ is chosen at the beginning of the first period, the group's ex-ante expectation about the degree of heterogeneity induced by any Δ in the second period decreases with ε. Therefore, the smaller ε, the smaller the negative marginal effect of seniority benefits on the second period. On the contrary, the positive effect of seniority benefits in the first period increases when ε becomes smaller. When ε decreases, it is more likely that agents choose the correct action, that is entering only when the group is successful. This increases the expected payoff of both senior and junior agents in the second-period, but more so for senior members, because they receive an extra $(s - n) = \Delta$, which is unaffected by ε.[33]

[32] Indeed, it is a well-known property of global games that when the noise in agents' information tends to zero, almost all agents take the same action.

[33] This is the reason why the extra-term seniority benefits add to the expected first-period payoff of joining in equation (1) is decreasing in ε, (cf. footnote [30]). As a technical remark, it is worth noting that first-period membership $1 - p$ is constant at $1/2$ in the proof of Proposition 5. This is because in our cutoff-strategy equilibrium, the only types swayed by a marginal change in ε are those near the cutoff $\theta_1^b(k_1, \theta)$, and it is enough for the association to consider how this cutoff changes with ε to determine changes in first-period membership. Since signals are conditionally independent, the cutoff type $\theta_1^b(k_1, \theta)$ believes other agents' signals to be higher or lower than $\theta_1^b(k_1, \theta)$ with equal probability for any $\varepsilon > 0$. As a result, by definition of the cutoff type, the relevant belief about first-period membership for the group to consider is $1/2$ for any ε.

In conclusion, and contrary to the one-shot game in Section 2, the precision of agents' signals does affect the optimal dynamic membership contract, even when the group is only maximizing the probability of success. In a model where the precision of information is an endogenous variable, our result in Proposition 5 provides an incentive for groups to publicize their efforts. This and other extensions of our work are subjects of current research.

Appendix

Proof of Proposition 2.

We first consider the effect of increased heterogeneity in the form of a mean preserving spread on the ex-ante expected probability that the group is successful in providing the public good. If $p > (s-2n)/2(s-n)$, the optimal k, i.e., the level of the fee that maximizes the probability of success, is interior, that is smaller than n, and it is equal to $k^*_{het} = ns/2\alpha$. Since $p > 0$, an immediate sufficient condition for the group to cater to both classes of agents is $2n > s$. When $k = k^*$, taking the total differential of $(\theta^* - \underline{\theta}) / (\overline{\theta} - \underline{\theta})$, and substituting $dn = -ds(1-p)/p$ and $ds = 1$, yields, using Proposition 1,

$$d\left(\frac{\theta^* - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right) = \frac{d\theta^*}{\overline{\theta} - \underline{\theta}} = \frac{dk^*}{2\left(\overline{\theta} - \underline{\theta}\right)} = d\left(\frac{ns}{4\alpha}\right) = -\frac{(1-p)\left(s^2 - n^2\right)}{4\alpha^2} < 0.$$

As for the effect of increased heterogeneity on the expected size of the group, when $k = k^*$, using Proposition 1, we have

$$p\theta^n + (1-p)\,\theta^s = \theta^*,$$

and the result follows as above. The analysis of exogenous heterogeneity when the group caters only to class-s agents is not interesting. However, the endogenous heterogeneity case is analyzed in Proposition 3. Finally, all qualitative predictions are trivially maintained for a group that maximizes expected membership, that is a group that sets k to $k^{**}_{het} = k^*_{het} - \varepsilon$.

Proof of Proposition 3.

We start with the group's optimal choice of k_2 in the second period. Given any proportion $1 - p$ of agents joining in the first period, the association may decide not to seek new members by setting $k_2 > n(p;\Delta)$. In this case, the analysis is similar to the homogenous case in Section 2: the resulting probability of success is $\Pr\left(\theta_2 < (1-p)\,s(p;\Delta)/4\right)$. Alternatively, the association may decide to cater to both new and returning members by setting $k_2 \leq n(p;\Delta)$. In this case we know from

Proposition 1 that, in equilibrium, senior (new) members enter if their signal is below $\theta_2^s(p;\Delta)$ $(\theta_2^n(p;\Delta))$, and the critical state below which the group is successful is $\theta_2^*(p;\Delta)$. The group will then set k_2 to its optimal level

$$k_{2het}^* = \frac{n(p;\Delta)\, s(p;\Delta)}{2\alpha(p;\Delta)} = \frac{1}{2}\frac{(b-(1-p)\Delta)(b+p\Delta)}{p(b+p\Delta)+(1-p)(b-(1-p)\Delta)}.$$

Note that k_{2het}^*, $\theta_2^*(p;\Delta)$, $\theta_2^n(p;\Delta)$ and $\theta_2^s(p;\Delta)$ are all functions of the proportion of agents joining in the first period and of the utility value of seniority benefits (Δ) through $s(p;\Delta)$ and $n(p;\Delta)$ (and therefore $\alpha(p;\Delta) = ps(p;\Delta) + (1-p)\,n(p;\Delta)$). Keeping this in mind, we henceforth suppress the argument $(p;\Delta)$ to save notation. Using Proposition 1, we then have

$$\begin{cases} \theta_2^* = k_{2het}^*\left(1 - k_{2het}^*\left(\frac{p}{n} + \frac{1-p}{s}\right)\right) = \frac{ns}{4\alpha} \\ \theta_2^n = \theta_2^* - 2\varepsilon\left(\frac{k_{2het}^*}{n} - \frac{1}{2}\right) = \theta_2^* - \varepsilon\frac{s}{\alpha} + \varepsilon \\ \theta_2^s = \theta_2^* - 2\varepsilon\left(\frac{k_{2het}^*}{s} - \frac{1}{2}\right) = \theta_2^* - \varepsilon\frac{n}{\alpha} + \varepsilon. \end{cases} \tag{18}$$

Simple algebra shows that the condition $\Delta \leq b/2$ implies that, for any realized p, we have $k_{2het}^* \leq n$ and $ns/4\alpha \geq (1-p)\,s/4$. Therefore the optimal choice of the association is indeed to serve both classes of potential members, and set $k_2 = k_{2het}^*$. To bridge first and second periods, we now define $Q(p,\Delta)$ as the expected difference in equilibrium payoffs between senior and new members in the second period, before θ_{i2} is realized, that is

$$\int_{\underline{\theta}-\varepsilon}^{\theta_2^s} \left(s \Pr(\theta_2 < \theta_2^* | \theta_{2i}) - k_{2het}^*\right) dF(\theta_{2i}) - \int_{\underline{\theta}-\varepsilon}^{\theta_2^n} \left(n \Pr(\theta_2 < \theta_2^* | \theta_{2i}) - k_{2het}^*\right) dF(\theta_{2i}). \tag{19}$$

Using (18), and

$$\Pr(\theta_2 < \theta_2^* | \theta_{2i}) = \begin{cases} 0 & \text{if } \theta_2^* < \theta_{2i} - \varepsilon \\ \frac{\theta_2^* - (\theta_{2i}-\varepsilon)}{2\varepsilon} & \text{if } \theta_{2i} - \varepsilon < \theta_2^* < \theta_{2i} + \varepsilon \\ 1 & \text{if } \theta_2^* > \theta_{2i} + \varepsilon, \end{cases} \tag{20}$$

$Q(p,\Delta)$ simplifies as

$$Q(p,\Delta) = \frac{\Delta}{\overline{\theta} - \underline{\theta}}\left(\theta_2^*\left(1 - \frac{\varepsilon}{\alpha}\right) - \underline{\theta}\right). \tag{21}$$

Indeed, $Q(p,\Delta)$ in (19) can be written, using (20), as

$$\Delta\left(F\left(\theta_{2i}=\theta_2^*-\varepsilon\right)+\int_{\theta_2^*-\varepsilon}^{\theta_2^n}\frac{\theta_2^*-(\theta_{2i}-\varepsilon)}{2\varepsilon}dF\left(\theta_{2i}\right)\right)+$$
$$+s\int_{\theta_2^n}^{\theta_2^s}\left(\frac{\theta_2^*-(\theta_{2i}-\varepsilon)}{2\varepsilon}-\frac{k_{2het}^*}{s}\right)dF\left(\theta_{2i}\right),$$

so that, since $\int_{x_1}^{x_2}\left(\frac{\theta_2^*-(\theta_{2i}-\varepsilon)}{2\varepsilon}\right)d\theta_i^2=\frac{1}{2\varepsilon}\left(\theta_2^*+\varepsilon-\frac{x_2+x_1}{2}\right)(x_2-x_1)$, and calculating the terms (x_2+x_1) and (x_2-x_1) according to (18), we have

$$\left(\overline{\theta}-\underline{\theta}\right)Q(p,\Delta)=\Delta\left(\left(\theta_2^*-\varepsilon-\underline{\theta}\right)+\varepsilon\left(1-\left(\frac{1}{2}\frac{s}{\alpha}\right)^2\right)\right)+s\frac{\varepsilon}{4}\frac{s^2-n^2}{\alpha^2}-k_{2het}^*\varepsilon\left(\frac{s-n}{\alpha}\right).$$

Using $s-n=\Delta$, $k_{2het}^*=ns/2\alpha$ and $\theta_2^*=k_{2het}^*/2$, we obtain the expression in (21). Using (18), one may verify that $Q(p,\Delta)$ in (21) is non-monotonic in p so, moving back to the first period, the standard iterated deletion of strictly dominated strategies does not yield a unique equilibrium. However, it is possible to show that dominance regions still exist. To see this, let $\pi(\theta_{1i},e)$ denote the net benefit from joining in the first period for agent i conditional on receiving signal θ_{1i}, for any strategy followed by all other agents that induces a proportion e of agents joining in the first period, that is

$$\pi(\theta_{1i},e)=E\left(b\Pr\left(k_1e>\theta_1\right)-k_1+Q\left(1-e,\Delta\right)|\theta_{1i}\right).$$

Using (21), we have $Q(p,\Delta)\in[0,\Delta]$, and since $\Delta<k_1<b$, the existence of dominance regions for θ_{1i} follows as in Section 2. Indeed, if $\theta_{1i}<\underline{\theta}+\varepsilon$,

$$\pi(\theta_{1i},e)>b\Pr\left(k_1\cdot 0>\theta_1|\theta_{1i}\right)-k_1+0>b-k_1>0,$$

and if $\theta_{1i}>\overline{\theta}-\varepsilon$,

$$\pi(\theta_{1i},e)<b\Pr\left(k_1\cdot 1>\theta_1|\theta_{1i}\right)-k_1+\Delta<-k_1+\Delta<0.$$

The existence of dominance regions is very useful in establishing existence and uniqueness of an equilibrium in monotone (cutoff) strategies. Suppose that all agents follow a cutoff strategy around θ_1^b. Let θ_1^* be the value of θ_1 below which the group is successful in providing benefits in the first period, which is determined by

$$k_1 \Pr\left(\theta_{1i} \leq \theta_1^b | \theta_1 = \theta_1^*\right) = \theta_1^*. \tag{22}$$

The net benefit from joining in the first period for agent i conditional on receiving signal θ_{1i} is

$$\pi\left(\theta_{1i}, \theta_1^b\right) = b \Pr\left(\theta_1 < \theta_1^* | \theta_{1i}\right) - k_1 + Q\left(\int_{\theta_1^b}^{\overline{\theta}+\varepsilon} f\left(\theta_{1i'} | \theta_{1i}\right) d\theta_{1i'}, \Delta\right), \tag{23}$$

where the first argument of Q is the proportion of agents that did not join in the first period from the point of view of an agent with private signal θ_{1i}, which is non-stochastic because of the continuum of agents assumption. The existence of dominance regions implies that $\pi\left(\theta_1^b, \theta_1^b\right) > 0$ for $\theta_1^b < \underline{\theta} + \varepsilon$, and that $\pi\left(\theta_1^b, \theta_1^b\right) < 0$ for $\theta_1^b > \overline{\theta} - \varepsilon$. Since $\pi\left(\theta_1^b, \theta_1^b\right)$ is continuous in θ_1^b, a solution to $\pi\left(\theta_1^b, \theta_1^b\right) = 0$ exists, with $\theta_1^b \in \left(\underline{\theta} + \varepsilon, \overline{\theta} - \varepsilon\right)$. Uniqueness of a solution to $\pi\left(\theta_1^b, \theta_1^b\right) = 0$ follows because $d\pi\left(\theta_1^b, \theta_1^b\right)/d\theta_1^b < 0$. To see this, note that given $\theta_1^b \in \left(\underline{\theta} + \varepsilon, \overline{\theta} - \varepsilon\right)$ the distribution of θ_1 conditional on $\theta_{1i} = \theta_1^b$ is uniform in $\left[\theta_1^b - \varepsilon, \theta_1^b + \varepsilon\right]$, and the distribution of $\theta_{1i'}$ conditional on $\theta_{1i} = \theta_1^b$ is a symmetric triangular distribution centered on θ_1^b, with support $\left[\theta_1^b - 2\varepsilon, \theta_1^b + 2\varepsilon\right]$. Hence, $\int_{\theta_1^b}^{\overline{\theta}+\varepsilon} f\left(\theta_{1i'} | \theta_{1i} = \theta_1^b\right) d\theta_{1i'} = 1/2$ for any $\theta_1^b \in \left(\underline{\theta} + \varepsilon, \overline{\theta} - \varepsilon\right)$, so that the expected proportion of agents that does not join in the first period from the point of view of type θ_1^b is constant and equal to $1/2$. Therefore,

$$\pi\left(\theta_1^b, \theta_1^b\right) = b \Pr\left(\theta_1 < \theta_1^* | \theta_1^b\right) - k_1 + Q\left(\frac{1}{2}, \Delta\right),$$

so that

$$\frac{d\pi\left(\theta_1^b, \theta_1^b\right)}{d\theta_1^b} = \begin{cases} \frac{b}{2\varepsilon}\left(\frac{d\theta_1^*}{d\theta_1^b} - 1\right) & \text{if } \left|\theta_1^* - \theta_1^b\right| < \varepsilon \\ 0 & \text{otherwise.} \end{cases}$$

However, if $\left|\theta_1^* - \theta_1^b\right| \geq \varepsilon$, we have that either

$$\pi\left(\theta_1^b, \theta_1^b\right) = (b - k_1) + Q\left(\frac{1}{2}, \Delta\right) > b - k_1 > 0,$$

or

$$\pi\left(\theta_1^b,\theta_1^b\right)=(0-k_1)+Q\left(\frac{1}{2},\Delta\right)<-k_1+\Delta<0,$$

contradicting the fact that $\pi\left(\theta_1^b,\theta_1^b\right)=0$. Therefore, it must be $\left|\theta_1^*-\theta_1^b\right|<\varepsilon$, implying

$$\frac{d\pi\left(\theta_1^b,\theta_1^b\right)}{d\theta_1^b}=\frac{b}{2\varepsilon}\left(\frac{d\theta_1^*}{d\theta_1^b}-1\right)=\frac{b}{2\varepsilon}\left(\frac{k_1}{k_1+2\varepsilon}-1\right)<0,$$

where $d\theta_1^*/d\theta_1^b$ is calculated using (22).

The proof is completed by showing that a cutoff strategy around θ_1^b is a best response to cutoff strategies around θ_1^b, i.e., by showing that

$$\left.\frac{d\pi\left(\theta_{1i},\theta_1^b\right)}{d\theta_{1i}}\right|_{\theta_{1i}=\theta_{1c}}<0, \tag{24}$$

where θ_{1c} is any value of the signal θ_{1i} for which $\pi\left(\theta_{1c},\theta_1^b\right)=0$. Note first that, by the definition of θ_{1c}, it must be the case that $|\theta_1^*-\theta_{1c}|<\varepsilon$, and that $\theta_{1c}\in\left(\underline{\theta}+\varepsilon,\overline{\theta}-\varepsilon\right)$. Therefore,

$$\pi\left(\theta_{1i}=\theta_{1c},\theta_1^b\right)=\left(b\frac{\theta_1^*-\theta_{1c}+\varepsilon}{2\varepsilon}-k_1\right)+Q\left(\int_{\theta_1^b}^{\overline{\theta}+\varepsilon}f\left(\theta_{1i'}|\theta_{1i}=\theta_{1c}\right)d\theta_{1i'},\Delta\right),$$

so that

$$\left.\frac{d\pi\left(\theta_{1i},\theta_1^b\right)}{d\theta_{1i}}\right|_{\theta_{1i}=\theta_{1c}}=-\frac{b}{2\varepsilon}+\frac{dQ\left(p,\Delta\right)}{dp}\left.\frac{d\left(\int_{\theta_1^b}^{\overline{\theta}+\varepsilon}f\left(\theta_{1i'}|\theta_{1i}\right)d\theta_{1i'}\right)}{d\theta_{1i}}\right|_{\theta_{1i}=\theta_{1c}}.$$

Since

$$\frac{d\left(\int_{\theta_1^b}^{\overline{\theta}+\varepsilon}f\left(\theta_{1i'}|\theta_{1i}\right)d\theta_{1i'}\right)}{d\theta_{1i}}\leq\frac{1}{2\varepsilon},$$

and

$$\frac{\partial Q\left(p,\Delta\right)}{\partial p}<b,$$

as we will show momentarily, we have

$$\left.\frac{d\pi\left(\theta_{1i},\theta_1^b\right)}{d\theta_{1i}}\right|_{\theta_{1i}=\theta_{1c}}<-\frac{b}{2\varepsilon}+\frac{b}{2\varepsilon}<0.$$

The proof that the derivative of $Q\left(p,\Delta\right)$ with respect to p is always smaller than b follows because, from (21), we have

$$\frac{\partial Q\left(p,\Delta\right)}{\partial p}=\frac{\Delta}{\overline{\theta}-\underline{\theta}}\left(\frac{\partial\theta_2^*}{\partial p}\left(1-\frac{\varepsilon}{\alpha}\right)+\varepsilon\frac{\theta_2^*}{\alpha^2}\frac{\partial\alpha}{\partial p}\right), \tag{25}$$

so that, using $\frac{\partial\alpha}{\partial p}=2\Delta$ and $\frac{\partial\theta_2^*}{\partial p}=\Delta\frac{\alpha(n+s)-2ns}{4\alpha^2}$, we obtain

$$\frac{\partial Q\left(p,\Delta\right)}{\partial p}=\frac{\Delta^2}{4\alpha\left(\overline{\theta}-\underline{\theta}\right)}\left(\Delta\frac{ps-\left(1-p\right)n}{\alpha}+\varepsilon\frac{2bs-\left(b+\Delta\right)}{\alpha^2}\right),$$

yielding

$$\begin{aligned}
&\tfrac{\partial Q(p,\Delta)}{\partial p}\\
&<\tfrac{\Delta^2}{4\alpha\left(\overline{\theta}-\underline{\theta}\right)}\left(\Delta+\varepsilon\tfrac{2bs}{\alpha^2}\right) && \text{using } \alpha=ps+\left(1-p\right)n\\
&<\tfrac{\Delta^2}{4\alpha\left(\overline{\theta}-\underline{\theta}\right)}\left(\Delta+2\varepsilon\tfrac{s}{\alpha}\right) && \text{since } b<s, \text{ and } s/\alpha>1\\
&<\tfrac{\Delta^2}{4\alpha\left(\overline{\theta}-\underline{\theta}\right)}\left(\Delta+2\varepsilon\tfrac{b}{b-\Delta}\right) && \text{using } s/\alpha \text{ decreasing in } p\\
&<\tfrac{\Delta^2}{4(b-\Delta)\left(\overline{\theta}-\underline{\theta}\right)}\left(\Delta+2\varepsilon\tfrac{b}{b-\Delta}\right) && \text{since } \alpha \text{ is increasing in } p\\
&<\tfrac{\Delta^2}{4(b-\Delta)(b+4\varepsilon)}\left(\Delta+2\varepsilon\tfrac{b}{b-\Delta}\right) && \text{using } \left(\overline{\theta}-\underline{\theta}\right)>b+4\varepsilon,
\end{aligned}$$

so that, using $\Delta<b/2$, we have $b/(b-\Delta)<2$ and eventually

$$\frac{\partial Q\left(p,\Delta\right)}{\partial p}<\frac{\Delta^2}{4\left(b-\Delta\right)}\leq\frac{b}{8}<b.$$

A similar argument can be used to rule out asymmetric equilibria in cutoff strategies, since the above bound continues to hold.

Proof of Proposition 4.

We proceed ignoring the strict inequality constraint $\Delta<k_1<b$, and then verify it is satisfied. Usual continuity arguments ensure existence of a solution to this relaxed maximization problem. After eliminating constants, maximizing (17) is equivalent to maximizing

$$\theta_1^*+\hat{w}_2\int_{\underline{\theta}}^{\overline{\theta}}\theta_2^*\left(p,\Delta\right)d\theta_1, \tag{26}$$

subject to the equilibrium constraints for the first-period cutoff strategy, namely

$$b \Pr\left(\theta_1 \leq \theta_1^* | \theta_{1i} = \theta_1^b\right) + Q\left(\frac{1}{2}, \Delta\right) = k_1 \tag{27}$$
$$\theta_1^* = k_1 \Pr\left(\theta_{1i} \leq \theta_1^b | \theta_1 = \theta_1^*\right),$$

and where, from (18),

$$\theta_2^*(p, \Delta) = \frac{1}{4} \frac{n(p, \Delta) s(p, \Delta)}{\alpha(p, \Delta)}. \tag{28}$$

The function p in the maximand and in (28) describes the agents that do not join in the first period. As a function of the realized state θ_1 and equilibrium cutoff θ_1^b, p is

$$p = \begin{cases} 0 & \text{for} \quad \theta_1 \in \left(\underline{\theta}, \theta_1^b - \varepsilon\right) \\ 1 - \frac{\theta_1^b - \theta_1 + \varepsilon}{2\varepsilon} & \text{for} \quad \theta_1 \in \left(\theta_1^b - \varepsilon, \theta_1^b + \varepsilon\right) \\ 1 & \text{for} \quad \theta_1 \in \left(\theta_1^b + \varepsilon, \overline{\theta}\right). \end{cases} \tag{29}$$

Remember that $Q(p, \Delta)$ is the expected value of seniority benefits. In (27) it is calculated at $p = 1/2$ because the indifferent agent θ_1^b always believes that the measure of agents joining is $1/2$. Using (21),

$$Q\left(\frac{1}{2}, \Delta\right) = \frac{\Delta}{\overline{\theta} - \underline{\theta}} \left(\frac{(b - \varepsilon)(4b^2 - \Delta^2)}{16b^2} - \underline{\theta}\right).$$

Finally, $\hat{w}_2 > 0$ in (26) is just a rescaling of w_2 by w_1, and $\left(\overline{\theta} - \underline{\theta}\right)$. The restrictions in (27) uniquely define θ_1^* and θ_1^b as functions of k_1 and Δ. The derivative of the objective function for k_1 then is

$$\frac{\partial \theta_1^*(\Delta, k_1)}{\partial k_1} + \hat{w}_2 \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial \theta_2^*(p, \Delta)}{\partial p} \frac{\partial p}{\partial \theta_1^b} \frac{\partial \theta_1^b}{\partial k_1} d\theta_1.$$

Using (29) to change the variable of integration from θ_1 to p

$$\frac{\partial \theta_1^*(\Delta, k_1)}{\partial k_1} + \hat{w}_2 (2\varepsilon) \int_0^1 \frac{\partial \theta_2^*(p, \Delta)}{\partial p} \left(\frac{\partial p}{\partial \theta_1^b} \frac{\partial \theta_1^b}{\partial k_1}\right) dp,$$

and, noting that $\frac{\partial p}{\partial \theta_1^b}$ and $\frac{\partial \theta_1^b}{\partial k_1}$ do not depend on p, we have

$$\frac{\partial \theta_1^*(\Delta, k_1)}{\partial k_1} + \hat{w}_2 (2\varepsilon) \left(\frac{\partial p}{\partial \theta_1^b} \frac{\partial \theta_1^b}{\partial k_1}\right) \int_0^1 \frac{\partial \theta_2^*(p, \Delta)}{\partial p} dp.$$

Since $\theta_2^*(1,\Delta) = \theta_2^*(0,\Delta) = b/4$ from (28), the optimal k_1 then solves

$$\frac{\partial \theta_1^*(\Delta, k_1)}{\partial k_1} = 0,$$

or, using (27),

$$k_1^* = \frac{b + Q\left(\frac{1}{2}, \Delta\right)}{2}. \tag{30}$$

Note how this level of k_1 will be strictly larger than $b/2$ as soon as $\Delta^* > 0$. Moreover, k_1^* is strictly smaller than b, since $Q\left(\frac{1}{2}, \Delta\right) \leq \Delta/4$, because $\overline{\theta} > b$, and $\Delta \leq b/2$. Therefore, the constraint $\Delta < k_1 < b$ is always satisfied, since, as we show below, $\Delta^* > 0$. With a similar procedure, the first derivative of (26) with respect to Δ yields

$$\frac{\partial \theta_1^*(\Delta, k_1)}{\partial \Delta} + \hat{w}_2 \int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{\partial \theta_2^*(p,\Delta)}{\partial p} \frac{\partial p}{\partial \theta_1^b} \frac{\partial \theta_1^b}{\partial k_1} + \frac{\partial \theta_2^*(p,\Delta)}{\partial \Delta} \right) d\theta_1 =$$
$$= \frac{\partial \theta_1^*(\Delta, k_1)}{\partial \Delta} + \hat{w}_2 2\varepsilon \int_0^1 \frac{\partial \theta_2^*(p,\Delta)}{\partial \Delta} dp,$$

and using (27) and (28), we obtain

$$\frac{k_1}{b} \frac{\partial Q\left(\frac{1}{2}, \Delta\right)}{\partial \Delta} + \hat{w}_2 2\varepsilon \int_0^1 \left(-\frac{\Delta}{4} p(1-p) \frac{b + (ps + (1-p)n)}{(ps + (1-p)n)^2} \right) dp =$$
$$= \frac{k_1}{b\left(\overline{\theta} - \underline{\theta}\right)} \left((b - \varepsilon) \frac{4b^2 - 3\Delta^2}{16b^2} - \underline{\theta} \right) - \hat{w}_2 \varepsilon \frac{(b^2 + \Delta^2)\log\left(\frac{b+\Delta}{b-\Delta}\right) - 2\Delta b}{(4\Delta)^2}.$$

Substituting the optimal level of k_1 in (30), we have that the first derivative of the objective function (26) with respect to Δ is

$$\Phi^{all}(\Delta;\varepsilon) \equiv \Phi^{one}(\Delta;\varepsilon) \cdot \Phi^{two}(\Delta;\varepsilon) - \hat{w}_2 \varepsilon \cdot \Phi^{three}(\Delta;\varepsilon), \tag{31}$$

where

$$\Phi^{one}(\Delta;\varepsilon) \equiv \frac{1}{2b\left(\overline{\theta} - \underline{\theta}\right)} \left(b + \frac{\Delta}{\left(\overline{\theta} - \underline{\theta}\right)} \left(\frac{(b-\varepsilon)(4b^2 - \Delta^2)}{16b^2} - \underline{\theta} \right) \right) \geq 0, \tag{32}$$
$$\Phi^{two}(\Delta;\varepsilon) \equiv (b - \varepsilon) \frac{4b^2 - 3\Delta^2}{16b^2} - \underline{\theta} \geq 0,$$
$$\Phi^{three}(\Delta;\varepsilon) \equiv \frac{(b^2 + \Delta^2)\log\left(\frac{b+\Delta}{b-\Delta}\right) - 2\Delta b}{(4\Delta)^2} \geq 0.$$

Note how

$$\lim_{\Delta\to 0}\Phi^{one}(\Delta;\varepsilon)\cdot\Phi^{two}(\Delta;\varepsilon)=\frac{1}{2\left(\overline{\theta}-\underline{\theta}\right)}\left(\frac{b-\varepsilon}{4}-\underline{\theta}\right)>0,$$

since $\underline{\theta}+\varepsilon<0$. As for $\Phi^{three}(\Delta;\varepsilon)$, we have, using de l'Hopital's rule

$$\begin{aligned}\lim_{\Delta\to 0}\Phi^{three}(\Delta;\varepsilon)&=\lim_{\Delta\to 0}\frac{2\Delta\left(\log\left(\frac{b+\Delta}{b-\Delta}\right)+\frac{2b\Delta}{b^2-\Delta^2}\right)}{32\Delta}=\\&=\frac{1}{16}\lim_{\Delta\to 0}\left(\log\left(\frac{b+\Delta}{b-\Delta}\right)+\frac{2b\Delta}{b^2-\Delta^2}\right)=0.\end{aligned}$$

Therefore, for Δ close to zero, the objective function (26) is strictly increasing in Δ, so $\Delta^*>0$. Finally, note that to show uniqueness of Δ^*, it is enough to show that at any $\hat{\Delta}$ such that $\Phi^{all}\left(\hat{\Delta};\varepsilon\right)=0$, we have

$$\left.\frac{\partial\Phi^{all}(\Delta;\varepsilon)}{\partial\Delta}\right|_{\Delta=\hat{\Delta}}<0.$$

Indeed, note that

$$\frac{\partial\Phi^{one}(\Delta;\varepsilon)}{\partial\Delta}=\frac{1}{\Delta}\left(\Phi^{one}(\Delta;\varepsilon)-\frac{1}{2\left(\overline{\theta}-\underline{\theta}\right)}\right)-\frac{1}{2b\left(\overline{\theta}-\underline{\theta}\right)}\frac{2\Delta^2(b-\varepsilon)}{\left(\overline{\theta}-\underline{\theta}\right)16b^2}<\frac{\Phi^{one}(\Delta;\varepsilon)}{\Delta},$$

$$\frac{\partial\Phi^{two}(\Delta;\varepsilon)}{\partial\Delta}=-\frac{(b-\varepsilon)\,6\Delta}{16b^2}<0,$$

and

$$\frac{\partial\Phi^{three}(\Delta;\varepsilon)}{\partial\Delta}=\frac{1}{\Delta}\frac{2b^2}{b^2-\Delta^2}\frac{2\Delta b-(b^2-\Delta^2)\log\left(\frac{b+\Delta}{b-\Delta}\right)}{(4\Delta)^2}.$$

Since

$$\frac{\partial\Phi^{three}(\Delta;\varepsilon)/\partial\Delta}{\Phi^{three}(\Delta;\varepsilon)/\Delta}=\frac{2}{1-x^2}\frac{2x-(1-x^2)\left(\log(1+x)-\log(1-x)\right)}{2x-(1+x^2)\left(\log(1+x)-\log(1-x)\right)},$$

which can be shown to be larger than one for $x=\Delta/b\in[0,1/2]$, we have

$$\frac{\partial\Phi^{three}(\Delta;\varepsilon)}{\partial\Delta}>\frac{\Phi^{three}(\Delta;\varepsilon)}{\Delta}.$$

Therefore,

$$\begin{aligned}\frac{\partial\Phi^{all}(\Delta;\varepsilon)}{\partial\Delta}&=\frac{\partial\Phi^{one}(\Delta;\varepsilon)}{\partial\Delta}\Phi^{two}(\Delta;\varepsilon)+\Phi^{one}(\Delta;\varepsilon)\frac{\partial\Phi^{two}(\Delta;\varepsilon)}{\partial\Delta}-\hat{w}_2\varepsilon\frac{\partial\Phi^{three}(\Delta;\varepsilon)}{\partial\Delta}<\\&<\frac{\Phi^{one}(\Delta;\varepsilon)}{\Delta}\Phi^{two}(\Delta;\varepsilon)-\frac{\partial\Phi^{three}(\Delta;\varepsilon)}{\partial\Delta}\hat{w}_2\varepsilon,\end{aligned}$$

which, when evaluated at a $\hat{\Delta}$ that makes $\Phi^{all}\left(\hat{\Delta};\varepsilon\right)=0$, yields

$$\left.\frac{\partial\Phi^{all}\left(\Delta;\varepsilon\right)}{\partial\Delta}\right|_{\Delta=\hat{\Delta}}<\frac{\hat{w}_2\varepsilon}{\hat{\Delta}}\left(\Phi^{three}\left(\hat{\Delta};\varepsilon\right)-\frac{\partial\Phi^{three}\left(\Delta;\varepsilon\right)}{\partial\Delta}\right)<$$
$$<\frac{\hat{w}_2\varepsilon}{\hat{\Delta}}\left(\Phi^{three}\left(\hat{\Delta};\varepsilon\right)-\Phi^{three}\left(\hat{\Delta};\varepsilon\right)\right)=0,$$

and therefore the optimal Δ^* is unique.

Proof of Lemma 1.

The claim follows from

$$\Phi^{three}\left(\Delta=b/2;\varepsilon\right)=\frac{1}{4}\left(\frac{5}{4}\log 3-1\right)>0,$$

using (32), so that, for $\hat{w}_2$ large enough, the derivative of the group's objective function with respect to Δ, that is $\Phi^{all}\left(\Delta;\varepsilon\right)$ in (31), is negative at $\Delta=b/2$.

Proof of Proposition 5.

The result follows by applying the implicit function theorem to (31), and noting that $\frac{\partial\Phi^{all}(\Delta;\varepsilon)}{\partial\varepsilon}<0$ since $\frac{\partial\Phi^{one}(\Delta;\varepsilon)}{\partial\varepsilon}$ and $\frac{\partial\Phi^{two}(\Delta;\varepsilon)}{\partial\varepsilon}$ are negative, while $\frac{\partial\Phi^{three}(\Delta;\varepsilon)}{\partial\varepsilon}=0$ and $\Phi^{three}\left(\Delta^*;\varepsilon\right)>0$.

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