Implications of Static Modeling for Out-of-Sample Prediction and Welfare Estimation when the Underlying Decision Problem is Dynamic: An Application to Recreation Demand

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1. Introduction

Two of the more practical applications of recreation demand modeling are forecasting 1) the behavioral responses and 2) the welfare impacts of site quality changes. This is particularly true for resource managers who must allocate their limited budgets across multiple management activities, including investments in site quality improvements. In a recent paper that focuses on the role of preference heterogeneity in out-of-sample prediction, Provencher and Bishop (2003) demonstrate for recreational angling that static random utility models (RUMs) tend to overstate the impact of interseasonal changes in site quality on trip frequency. This implies the actual number of trips taken during a season is less elastic than the predicted number of trips and therefore any welfare estimates based on those predictions may be exaggerated.

Although several explanations for this result are possible, perhaps the most reasonable is that angler behavior is constrained on a seasonal basis in a way not considered by static RUMs. One approach that addresses this possibility is the Kuhn-Tucker (KT) model which imposes a seasonal budget constraint on each agent, as in Phaneuf, Kling and Herriges (2000). However, although appealing for its utility-theoretic basis, KT models do not account for the temporal allocation of trips throughout a season and therefore overlook useful information that might be gleaned from observing the impact of intraseasonal variability on recreation behavior.

An integrated utility-theoretic model linking a seasonal recreation budget with a dynamic (forward-looking) RUM therefore would seem to be appropriate, but empirical estimation of a structural dynamic model is both complicated and time consuming. As evidence of this, one need look no further than the existing literature on recreation demand which contains a preponderance of static RUMs (see Herriges and Kling, 1999, for an overview) despite the fact that the recreation decision clearly involves the evolution of predictable state variables and the expenditure of limited resources through time, thus making the decision appropriate for modeling in a dynamic framework.

With this in mind, the purpose of this paper is to examine the consequences of using a much simpler static model of trip-taking behavior when a more complicated dynamic one is appropriate. In particular, we examine the implications for out-of-sample prediction and welfare estimation using Lake Michigan salmon angling data for the 1996 and 1997 fishing seasons. Our results suggest that although a static model can predict out-of-sample behavior at least as well as a dynamic model, the welfare estimates can be drastically different.

2. Theoretical Considerations

We model the trip decision as a simple binary process in which the angler decides on each day of the season whether to fish for salmon, or instead spend the day doing other things. This is a reasonable representation of the decision faced by our Lake Michigan salmon anglers: on any given day the variability in catch rates along the Lake Michigan shore is difficult to detect and, because fishing is most often done far from shore, aesthetic differences among launch sites are few. Thus the vast majority of anglers in the study took the bulk of his (all anglers are male) trips from one or two favorite launch sites. Moreover, the vast majority of fishing trips taken by anglers in the sample were salmon trips on Lake Michigan. For example, of the 1504 total trips taken by sample anglers in 1996, 92 (6.1%) were non-Lake Michigan (usually inland lake) fishing trips; 56 (3.7%) were non-salmonid trips on Lake Michigan; and the remaining 1356 (90.2%) were salmonid trips on Lake Michigan. Of these salmonid trips, 1258 (83.6% of all trips, 92.8% of Lake Michigan salmonid trips) were from boat launches along the Milwaukee-Racine waters of Lake Michigan.

We present the basic model first. Letting $\mathbf{x}_{nt}$ denote a vector of state variables affecting the trip utility of angler (n) on day (t), the net utility from a fishing trip on day (t) can be expressed as:

$$u_{nt} \equiv \beta \mathbf{x}_{nt} + \varepsilon_{nt}^{*} \tag{1}$$

where β is a conformable vector of preference parameters. The vector of state variables $\mathbf{x}_{nt}$ is observable in the sense that both the angler and the analyst observe the value of these variables at the start of day (t). The random state variable ε_{nt}^{*} is assumed to be iid standard logistic (arising from the difference between two iid standard Gumbel-distributed random variables $\boldsymbol{\varepsilon}_{nt} \equiv \left\{\varepsilon_{nt}^{1}, \varepsilon_{nt}^{0}\right\}$, the first representing the unobserved utility from taking a trip, the second representing the unobserved utility from not taking a trip) and is observed contemporaneously by the angler but never observed by the analyst.

In the analysis, the vector $\mathbf{x}_{nt}$ includes an intercept and the following variables: the average (expected) money cost of a trip, $cost_n$, taken as the sum of driving costs, ramp fees, and boat operation costs and food costs, less the donations made by other anglers on the trip to defray trip costs; a variable denoting whether the angler is employed full-time during the fishing season, job_n; a variable denoting the average site-wide catch on day (t), $catch_t$; the weather variables $temp_t$, denoting the high temperature on day (t), and $wind_t$, denoting average wind speed on day (t); the time cost variable $weekday_t$, a dummy variable taking a value of one if the day is Monday through Friday; the time cost interaction term $job_n \cdot workday_{nt}$ taking a value of one on days when a fully employed angler is scheduled to work; a dummy variable $derby_t$ taking a value of one if day (t) falls within the run of a popular annual 9-day fishing derby based in

Racine, called Salmon-O-Rama; a dummy variable $prev\_day_{nt}$ taking a value of one if the angler took a trip on the previous day; and $elapsed_{nt}$, recording the number of days elapsed since the last salmon trip taken by angler (n).

In a static model, this net utility function denotes the difference between the utility on day (t) with a trip and without a trip. The income allocated for consumption on day (t) nets out under the assumption that income not spent fishing is spent in the consumption of other goods. In the absence of this assumption, the model must accommodate income dynamics. However, the model nonetheless includes two variables with implications for dynamic decision-making: $prev\_day_{nt}$ and $elapsed_{nt}$. Arguably a forward-looking angler recognizes that a trip on day (t) affects utility in the future via these variables. Yet these variables do not provide a compelling case for dynamic analysis. For example, to argue that the presence of $elapsed_{nt}$ in the utility function compels modeling the decision process as dynamic is to argue, given a positive marginal effect of elapsed on utility, that an angler on day (t) may choose to postpone a trip because he understands that postponement increases the utility of future trips; this is akin to postponing opening a gift in the knowledge that the joy of consumption will be greater for the wait. Though we do not doubt the existence of such effects, given the great difficulty of structural estimation of dynamic decision processes it strikes us as a relatively weak basis for dynamic modeling. By contrast, the dynamics introduced by a seasonal budget constraint pertain not to marginal changes in trip utility, but to the very opportunity to take a trip in the future. It seems eminently defensible to argue that an angler constrained to, say, eight trips during the season, performs forward calculations especially when only two or three trips remain.

2.1 The Seasonal Budget Constraint

An important question arises, though, regarding specification of this seasonal budget constraint. To be consistent with classical theory, the analyst should model the complete allocation of scarce resources across all activities in all time periods, including salmon fishing during the 1996 and 1997 seasons, order to ensure the optimization condition that the marginal utility of each resource be equated across activities. But this approach clearly raises problems of empirical tractability and routinely is ignored by practitioners. An alternative approach that has gained currency in the behavioral economics and marketing literatures (Thaler, 1985 and 1990; Heath and Soll, 1996; Read, Lowenstein and Rabin, 1999; Moon and Casey, 1999) is that of "mental accounting." Thaler (1999) observes,

> "A primary reason for studying mental accounting is to enhance our understanding of the psychology of choice. In general, understanding mental accounting processes helps us understand choice because mental accounting rules are not neutral. That is, accounting decisions such as to which category to assign a purchase, whether to combine an outcome with others in that category, and how often to balance the "books", can affect the perceived attractiveness of choices. They do so because mental accounting violates the economic notion of fungibility. Money in one mental account is not a perfect substitute for money in another account. Because of violations of fungability, mental accounting matters" (p.185).

Mental accounting provides an explanation for behavior inconsistent with the life cycle hypothesis. Thaler (1990) observes that the life cycle hypothesis, in which current consumption is the outcome of an optimal allocation of consumption over time, does not fare well in real-world tests. For instance, the young tend to consume too little and the old tend to consume too much. In the context of recreation models, the life cycle hypothesis essentially argues that the decision to take a recreation trip is a dynamic problem in which a trip at time (t) subtracts from lifetime expected wealth, and the consumer determines whether this is a worthwhile tradeoff.

Few people would argue that this is a useful or realistic conception of the trip decision. Yet what is the alternative?

The alternative is a model of mental accounting in which recreation trips fall within one of a consumer's several mental accounts, with the account then allocated among a designated set of goods and services. This is the approach taken implicitly or explicitly by all published utility-consistent recreation studies that attempt to explain the demand for recreation trips over a specified horizon. Mental accounts are defined by their time frames, their sizes, and the set of goods that draw on them.

In the KT model, the lifetime dynamic budget is trimmed to an annual budget equal to annual income, which is then allocated across all consumption for the year. Provided relevant aspects of the trip decision are unchanging over the course of a year (for instance, there are no intraseasonal changes in site quality), this model seems quite reasonable and indeed it is to be favored for its simplicity and elegance.

But it does make a particular assumption about the psychology of choice, one economists make willingly but psychologists caution against, namely mental accounts are quite broad in the set of goods that draws on them. Suppose, for instance, that hunting trips and food purchases are not perfectly fungible; that due to either household rules or norms, or perhaps due to the hunter's psychological bracketing, the hunter does not calculate, "If I take another hunting trip today, I'll have to cut back on gourmet foods". Far more likely is a narrow calculation suggestive of narrow bracketing, such as, "If I take a trip today, I won't be able to go next week", or a calculation consistent with narrow bracketing over goods, but broad bracketing over time, such as, "If I take a cruise this year, I won't be able to go next year".

The issue of mental accounting is especially apparent in trip occasion models attempting to examine the intraseasonal allocation of recreation trips. Such models have the potential to address economic issues which are of great concern to resource managers, but which have received relatively little attention from resource economists, such as season length, intraseasonal variation in site quality, and bag limits. In these models, the only way to avoid a model with a dynamic budget constraint is to assume that the time frame of mental accounts is the time frame of the choice occasion. In the repeated nested logit model of Morey, Rowe and Watson (1993), the relevant mental account allocates income to the consumption of all goods, but the duration of the account is the duration of a choice occasion (one week), and the size of the account is annual income divided by the number of choice occasions. Buchanan *et al*. (1998) assume that the relevant account is a daily budget defined by monthly disposable income divided by thirty. The account applies to all goods except those essential purchases (such as mortgage payments) already netted from monthly income. Provencher and Bishop (1997) and Provencher, Baerenklau and Bishop (2001) define the relevant budget as a daily budget that is conditional on a set of state variables and is random from the perspective of the analyst, though due to the linear form of the utility function, defining the exact value of the daily budget is not necessary for estimation of the choice model.

Misspecifying the budget constraint for recreation trips generates bad trip forecasts and bad welfare estimates impacting management decisions. Suppose, for instance, that managers of a fishery are interested in the economics of increasing catch rates at a number of sites, as would occur from habitat restoration and/or fish stocking programs. What would be the effect on angler welfare and trip behavior? In the presence of seasonal recreation budgets effectively constraining the number of trips taken in the season, the trip occasion models used in Morey, Rowe and Watson (1993) and Provencher, Baerenklau, and Bishop (2001) would overstate angler trip response to the improvement, because these models do not impose the (true) constraint on the number of trips for the season.

This tendency for static trip occasion models to overstate the trip response of anglers to changes in catch rates was observed by Provencher and Bishop (2003) in a comparison of the forecasting performance of various static models of angler heterogeneity. The models compared included a number of finite mixture (latent class) and random parameters logit models. The application was to salmon fishing on Lake Michigan, and forecasts were compared for 1996 and 1997. Catch rates in 1997 averaged 31% greater in 1996 than in 1997. Generally the models estimated on the 1996 data overforecasted trips in 1997, and the models on the 1997 data underforecasted trips in 1996. Although several explanations for this result are possible, one of the most reasonable – arguably the most reasonable – is that angler behavior during the season is constrained in a way not considered by the static models. A dynamic budget constraint, in which trips are allocated from a seasonal trip budget, is a strong and reasonable candidate.

In the next sections we present two estimable models of trip taking behavior. The first model uses a common random utility specification for each choice occasion, but treats anglers as forward-looking decision-makers allocating a fixed seasonal trip budget. This budget is presumed to arise from an optimization problem we present later, analogous to that of KT model, solved by each angler at the start of the season. Specifically, we assume each angler allocates income between fishing and other consumption with the understanding that future decisions about when to fish will be made optimally. The model is utility-theoretic and yet addresses the trip occasion decision without the untenable assumption that all income allocated to the day in question must be consumed. The second model is a reduced-form static version of the first (similar to that examined by Provencher and Bishop) in which anglers are treated as myopic, deciding on each day of the season whether to take a trip. As with other such models found in the literature, it is understood that if a trip is not taken then the budget allocated to the day is spent on other consumption that day.

2.2 Statement of the Structural Dynamic Model

In the static model given by (1), the optimal decision on day (t) is to take a trip if $u_{nt} > 0$. Letting y_{nt} take a value of one if angler (n) takes a trip on day (t) and zero otherwise, and assuming iid standard Gumbel-distributed error terms, the probability of observing angler (n) taking a trip on day (t) is given by the usual logit expression:

$$\text{prob}\left(y_{nt} = 1\right) = \frac{\exp\left(\beta \mathbf{x}_{nt}\right)}{1 + \exp\left(\beta \mathbf{x}_{nt}\right)}. \tag{2}$$

The likelihood of angler (n)'s trip sequence for the season follows as:

$$L_n^{static} \equiv \prod_t \text{prob}\left(y_{nt}\right). \tag{3}$$

In the dynamic model, the decision problem is complicated by the state equations

$$\mathbf{x}_{n,t+1} \equiv \mathbf{f}\left(\mathbf{x}_{nt}, y_{nt}\right) \tag{4}$$

and

$$s_{n,t+1} \equiv s_{nt} - y_{nt}, \tag{5}$$

where s_{nt} is the stock of trips remaining for the season. The state equation in (4) is strictly relevant for only two variables in our model:

$$prev\_day_{n,t+1} \equiv \begin{cases} 1 & \text{if } y_{nt} = 1 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$elapsed_{n,t+1} \equiv \begin{cases} 1 & \text{if } y_{nt} = 1 \\ elapsed_{nt} + 1 & \text{otherwise} \end{cases} \tag{7}$$

For the sake of simplicity of notation, we use the general form presented in (4). Regarding (5), in this binary decision problem there is a one-to-one correspondence between the number of trips remaining in the season and the fishing budget available to angler (n) for fishing on day (t); so we express the budget constraint in terms of trips. Also, there is no justification for the presence of $cost_n$ in the utility function in (1) because, in contrast to the static model, it is not true that the opportunity cost of a trip on day (t) is $cost_n$ less consumption of other goods on day (t); rather, the opportunity cost is one fewer trip available to take in the future. Nonetheless, we leave $cost_n$ in the utility function to test the null hypothesis that its coefficient is equal to zero (and later we show how welfare estimation remains possible with this model).

In the dynamic model the angler's decision problem is solved recursively. In the last period (T) angler (n)'s indirect utility may be written as:

$$\nu_{nT}\left(\mathbf{x}_{nT}, s_{nT}, \boldsymbol{\varepsilon}_{nT}\right) \equiv \begin{cases} \max\left[\beta\mathbf{x}_{nT} + \varepsilon_{nT}^{1}, \varepsilon_{nT}^{0}\right] & \text{if } s_{nT} \geq 1 \\ \varepsilon_{nT}^{0} & \text{otherwise} \end{cases}, \tag{8}$$

where all right-hand side terms are as defined previously. This specification recognizes that the angler cannot fish if he has no income in his fishing account, and that any income left in the account at the end of the season is used in other activities. The properties of Gumbel-distributed random variables give the result (Ben-Akiva and Lerman):

$$V_{nT}\left(\mathbf{x}_{nT}, s_{nT}\right) \equiv E_{\boldsymbol{\varepsilon}}\left[\nu_{nT}\left(\mathbf{x}_{nT}, s_{nT}, \boldsymbol{\varepsilon}_{nT}\right)\right] = \begin{cases} \ln\left(\exp\left(\beta\mathbf{x}_{nT}\right) + 1\right) + \gamma & \text{if } s_{nT} \geq 1 \\ \gamma & \text{otherwise} \end{cases}, \tag{9}$$

where γ is Euler's constant and represents the expected value of each Gumbel-distributed random variable. Furthermore, the probability of observing a trip is given by:

$$\text{prob}\left(y_{nT}=1\right)=\begin{cases}\dfrac{\exp\left(\beta\mathbf{x}_{nT}\right)}{1+\exp\left(\beta\mathbf{x}_{nT}\right)} & \text{if } s_{nT}\geq 1\\ 0 & \text{otherwise}\end{cases}. \tag{10}$$

Stepping backwards, in period (T-1) each angler's indirect utility may be written as:

$$\begin{aligned}&\nu_{n,T-1}\left(\mathbf{x}_{n,T-1},s_{n,T-1},\boldsymbol{\varepsilon}_{n,T-1}\right)\\ &\equiv\begin{cases}\max\begin{bmatrix}\beta\mathbf{x}_{n,T-1}+\varepsilon^{1}_{n,T-1}+\rho V_{nT}\left(f\left(\mathbf{x}_{n,T-1},1\right),s_{n,T-1}-1\right),\\ \rho V_{nT}\left(f\left(\mathbf{x}_{n,T-1},0\right),s_{n,T-1}\right)+\varepsilon^{0}_{n,T-1}\end{bmatrix} & \text{if } s_{n,T-1}\geq 1\\ \rho V_{nT}\left(f\left(\mathbf{x}_{n,T-1},0\right),s_{n,T-1}\right)+\varepsilon^{0}_{n,T-1} & \text{otherwise}\end{cases},\end{aligned} \tag{11}$$

where ρ is the discount factor. Similarly to (9), taking the expectation of $\nu_{n,T-1}$ with respect to the random portion of utility at time (T-1) yields:

$$\begin{aligned}&V_{n,T-1}\left(\mathbf{x}_{n,T-1},s_{n,T-1}\right)\equiv E_{\boldsymbol{\varepsilon}}\left[\nu_{n,T-1}\left(\mathbf{x}_{nT},s_{nT},\boldsymbol{\varepsilon}_{nT}\right)\right]\\ &=\begin{cases}\ln\left\{\exp\left[\beta\mathbf{x}_{n,T-1}+\rho V_{nT}\left(f\left(\mathbf{x}_{n,T-1},1\right),s_{n,T-1}-1\right)\right]+\exp\left[\rho V_{nT}\left(f\left(\mathbf{x}_{n,T-1},0\right),s_{n,T-1}\right)\right]\right\}+\gamma & \text{if } s_{n,T-1}\geq 1\\ \rho V_{nT}\left(f\left(\mathbf{x}_{n,T-1},0\right),s_{n,T-1}\right)+\gamma & \text{otherwise}\end{cases}.\end{aligned} \tag{12}$$

And the probability of observing a trip (conditional on $s_{n,T-1}$) is given by:

$$\begin{aligned}&\text{prob}\left(y_{n,T-1}=1\right)\\ &=\begin{cases}\dfrac{\exp\left(\beta\mathbf{x}_{n,T-1}+\rho V_{nT}\left(f\left(\mathbf{x}_{n,T-1},1\right),s_{n,T-1}-1\right)-\rho V_{nT}\left(f\left(\mathbf{x}_{n,T-1},0\right),s_{n,T-1}\right)\right)}{1+\exp\left(\beta\mathbf{x}_{n,T-1}+\rho V_{nT}\left(f\left(\mathbf{x}_{n,T-1},1\right),s_{n,T-1}-1\right)-\rho V_{nT}\left(f\left(\mathbf{x}_{n,T-1},0\right),s_{n,T-1}\right)\right)} & \text{if } s_{n,T-1}\geq 1\\ 0 & \text{otherwise}\end{cases}.\end{aligned} \tag{13}$$

Continuing with the recursion gives the indirect utility on any day (t):

$$\begin{aligned}&\nu_{nt}\left(\mathbf{x}_{nt},s_{nt},\boldsymbol{\varepsilon}_{nt}\right)\\ &\equiv\begin{cases}\max\begin{bmatrix}\beta\mathbf{x}_{nt}+\varepsilon^{1}_{nt}+\rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt},1\right),s_{nt}-1\right),\\ \rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt},0\right),s_{nt}\right)+\varepsilon^{0}_{nt}\end{bmatrix} & \text{if } s_{nt}\geq 1\\ \rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt},0\right),s_{nt}\right)+\varepsilon^{0}_{nt} & \text{otherwise}\end{cases},\end{aligned} \tag{14}$$

with the probability of observing a trip on that day given by:

$$\text{prob}\left(y_{nt}=1\right) = \begin{cases} \dfrac{\exp\left(\beta\mathbf{x}_{nt}+\rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt},1\right),s_{nt}-1\right)-\rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt},0\right),s_{nt}\right)\right)}{1+\exp\left(\beta\mathbf{x}_{nt}+\rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt},1\right),s_{nt}-1\right)-\rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt},0\right),s_{nt}\right)\right)} & \text{if } s_{nt}\geq 1 \\ 0 & \text{otherwise} \end{cases} \cdot \qquad (15)$$

Conditional on s_{n0}, the likelihood of angler (n)'s trip behavior is then given by:

$$L_n^{dynamic}\left(\cdot\,|\,s_{n0}\right)\equiv\prod_t \text{prob}\left(y_{nt}\right), \qquad (16)$$

where $\text{prob}\left(y_{nt}\right)$ is calculated as shown in (10) and (15).

As mentioned previously, we assume each angler selects s_{n0} as the solution to an optimization problem solved at the start of the season, with the understanding that future decisions about when to fish will be made optimally. In this sense, our approach is analogous to the KT model. Specifically, each angler chooses s_{n0} to maximize the difference between the expected seasonal utility of fishing and the opportunity cost of taking fishing trips:

$$s_{n0}=\max_{0\leq s\leq\bar{s}}\left\{V_{n0}\left(\mathbf{x}_{n0},s\right)-\mu\cdot\left(c_n+\theta\right)\cdot s+\eta_{ns}\right\}, \qquad (17)$$

where μ is the marginal utility of income; θ is a trip cost premium (positive or negative) to be estimated; η_{ns} are random components interpreted as increments to the seasonal value of fishing, observed by each angler at the time of the decision, but never observed by the analyst, and $\bar{s}$ is an upper bound. This approach is consistent with other utility-theoretic models of recreation demand which assume decisions regarding the total number of trips to take during a season are made at the start of the season. It reflects a type of mental accounting in which annual income is allocated across different categories of consumption, but here this budget allocation is made while explicitly acknowledging the dynamics of trip allocation throughout the season. The model readily generalizes to the case where the choice occasion involves multiple sites, heretofore the province of KT models.

Treating η_{ns} as iid standard Gumbel-distributed random variables, the probability that an angler chooses any quantity of trips is given by the familiar multinomial logit expression:

$$\text{prob}\left(s = s_{n0}\right) = \frac{\exp\left(V_{n0}\left(\mathbf{x}_{n0}, s_{n0}\right) - \mu \cdot \left(c_n + \theta\right) \cdot s_{n0}\right)}{\sum_{s=0}^{\bar{s}} \exp\left(V_{n0}\left(\mathbf{x}_{n0}, s\right) - \mu \cdot \left(c_n + \theta\right) \cdot s\right)}. \tag{18}$$

The likelihood of observing angler (n)'s behavior is then given by:

$$L_n^{\text{dynamic}}\left(\cdot, s_{n0}\right) \equiv \text{prob}\left(s = s_{n0}\right) \cdot \prod_t \text{prob}\left(y_{nt}\right) \tag{19}$$

and the sample log likelihood is the sum across all anglers of the logs of (19).

2.3 Statement of the Reduced-Form Static Model

With the structural dynamic model fully specified, we now address the task of determining a simpler reduced-form static model that we believe has the potential to closely mimic the results of the dynamic model but with far less modeling effort. To begin, we first revisit equation (15) for the case of $s_{nt} \geq 1$ (because there is no seasonal budget to exhaust in the static model):

$$\text{prob}\left(y_{nt} = 1\right) = \frac{\exp\left(\beta \mathbf{x}_{nt} + \rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt}, 1\right), s_{nt} - 1\right) - \rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt}, 0\right), s_{nt}\right)\right)}{1 + \exp\left(\beta \mathbf{x}_{nt} + \rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt}, 1\right), s_{nt} - 1\right) - \rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt}, 0\right), s_{nt}\right)\right)}. \tag{20}$$

Defining:

$$A_{nt} \equiv \rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt}, 1\right), s_{nt} - 1\right) - \rho V_{n,t+1}\left(f\left(\mathbf{x}_{nt}, 0\right), s_{nt}\right), \tag{21}$$

we can rewrite (20) as follows:

$$\text{prob}\left(y_{nt} = 1\right) = \frac{\exp\left(\beta \mathbf{x}_{nt} + A_{nt}\right)}{1 + \exp\left(\beta \mathbf{x}_{nt} + A_{nt}\right)}. \tag{22}$$

Notice that equation (22) takes the form of a standard logit model with $u_{nt} \equiv \beta \mathbf{x}_{nt} + A_{nt} + \varepsilon^*_{nt}$, where A_{nt} can be interpreted as an intercept term that varies across individuals and through time. We therefore propose to substitute a reduced form for A_{nt}, denoted by A^*_{nt}, which we will specify later. If A^*_{nt} can be made arbitrarily close to A_{nt}, then when $s_{nt} \geq 1$ the much simpler static model in (22) will be empirically indistinguishable from the more complicated dynamic model in (10), (15) and (17) and will yield identical predictions regarding trip-taking behavior. Only on days when the seasonal budget has been exhausted will the dynamic model yield superior predictions.

This is a promising result for the predictive ability of the reduced-form model, but unfortunately a similar result cannot be derived for its welfare estimates. To see why, first consider the differences in model structures. In the static model, the marginal utility of income (MUI) is given by the β coefficient on the variable $cost_n$ in equation (1) and the opportunity cost of a trip is $cost_n$ less consumption of other goods on the day of the trip. But in the dynamic model, we have argued the β coefficient on $cost_n$ should equal zero. Instead, the MUI is given by μ in equation (17) and the opportunity cost of a trip is one fewer trip available to take in the future. These differences alone suggest that the welfare estimates of the static model will differ from those of the dynamic model, but the problems with welfare calculations using the static model appear to be deeper than this.

To examine this issue further, recall that the first step in welfare estimation is to derive the money metric version of the per-period indirect utility function (ν_{nt}). Assuming as we do here that the per-period direct utility function (u_{nt}) is linear in income, this requires first dividing u_{nt} by the MUI. In the structural dynamic model, we then use the money metric version of ν_{nt}

to calculate recursively V_{n0} as shown in the preceding equations. This quantity is, by definition, the expected present value of the fishing season – a theoretically consistent welfare measure.

In the reduced-form static model, our per-period direct utility function includes an extra term: A_{nt}. While it is tempting to simply treat A_{nt} as a variable intercept term and follow the same procedure for calculating welfare (i.e., divide u_{nt} by the MUI and calculate the discounted expected seasonal value), this is not a theoretically consistent approach when the underlying decision problem is dynamic. To understand why, note that A_{nt} is the reduced form of an expression involving *future* utility; in other words, it represents a function of utility derived from decisions made *after* period (t). Therefore in a static model this term should not be included in the calculation of utility derived from the decision made at time (t). Erroneously including this term will produce biased (and theoretically inconsistent) welfare calculations.

A possible alternative approach would be to omit A_{nt} from the per-period welfare calculation and proceed as before; but this, too, is problematic because the static estimation method necessarily sweeps the mean value for A_{nt} into the constant term of $\mathbf{x}_{nt}$ and expresses the results for A_{nt} as deviations from this (unknown) mean. In other words, we have an identification problem: the static model cannot simultaneously recover unbiased estimates of A_{nt} (as it is defined by the dynamic model) and of the constant term in $\mathbf{x}_{nt}$. It is the additional structure imposed on the problem by the dynamic model that permits simultaneous identification; but unfortunately this identification cannot be achieved without deriving and estimating the more complicated dynamic model. Of course, when the underlying decision problem is static then $\rho = A_{nt} = 0$ and welfare estimation can proceed as usual.

These observations raise an important issue regarding the interpretation of variable intercept terms in static RUMs. Frequently, analysis of panel data employs a static model with some sort of variable intercept specification such as "fixed-effects" or "random-effects" in order to address unobserved heterogeneity in the sample population (Hsiao, 1986). When the underlying decision problem is static, these coefficients are appropriately interpreted as preference parameters that contribute to per-period utility and should be included in welfare calculations. But when the underlying decision problem is dynamic, our preceding discussion shows that this interpretation is theoretically inconsistent – behavioral prediction may be quite good, but welfare estimates will be biased; unfortunately, the literature has not previously recognized this.

To demonstrate these claims empirically, we estimate both the structural dynamic model and the reduced form static model presented above using Lake Michigan salmon angling data for the 1996 and 1997 fishing seasons. We then examine the trip predictions and welfare estimates derived from each model. The exact functional specification we choose for A_{nt} in the static model is motivated by our results for the dynamic model which we present in the following section.

3. Estimation and Results

Our dataset is composed of 97 anglers surveyed over the two-year period from 1996 to 1997. Anglers were queried about their fishing activity by telephone approximately every two weeks from May through September. Although the season for salmon fishing on Lake Michigan typically begins around April 1 and continues until December 31, anglers rarely fish before May 1 or past October 1. For the 1996 season angler activity was charted from May 1 to September 15, and for the 1997 season from May 15 to October 1. These dates give the effective seasons used in the empirical analysis. A mail survey was conducted at the end of each season to collect additional demographic information from the study participants.

We first use the data to estimate the dynamic model described in Section 2 for each season. The likelihood function for the dynamic model (equation 19) is maximized using the GQOPT routine in the Fortran programming language. The coefficient estimates and standard errors are shown in Table 1.

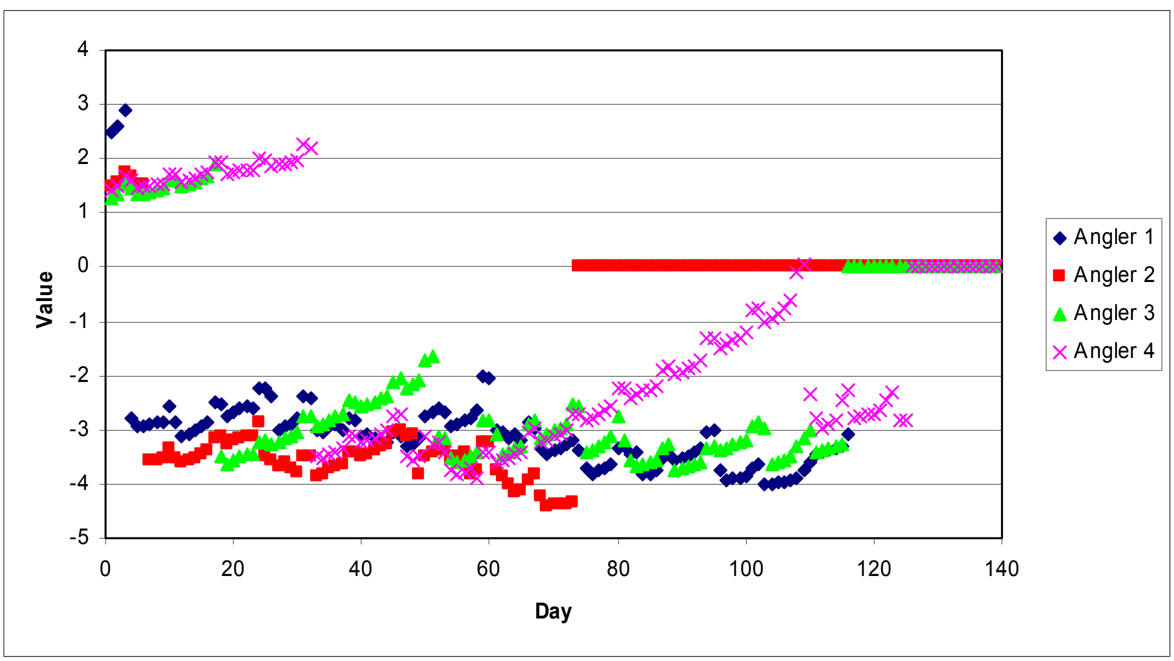
Table 1: Coefficient Estimates and Standard Errors for the Dynamic Model, 1996 and 1997 Seasons

Variable	1996 Season		1997 Season	
	Coefficient Estimate	Standard Error	Coefficient Estimate	Standard Error
constant	0.2491	0.5896	-0.5654	0.3687
$cost_n$	-0.0388	0.0078	-0.0494	0.0070
job_n	-0.1693	0.0763	-0.1964	0.0738
$catch_t$	0.4397	0.0916	0.1030	0.0546
$temp_t$	0.0189	0.0034	0.0267	0.0032
$wind_t$	-0.0545	0.0103	-0.1463	0.0130
$weekday_t$	-1.4606	0.0618	-1.1852	0.0592
$job_n \cdot workday_{nt}$	0.0961	0.0577	-0.0308	0.0642
$derby_t$	0.7458	0.0793	0.5619	0.0819
$prev\_day_{nt}$	1.1633	0.0791	1.4913	0.0768
$elapsed_{nt}$	-0.0411	0.0042	-0.0294	0.0030
Other Coefficients	**Estimate**	**Standard Error**	**Estimate**	**Standard Error**
Marginal utility of income (μ)	0.0491	0.0073	0.0620	0.0062
Trip cost premium (θ)	64.0975	1.0060	35.2143	1.1042

With these estimates in hand, we now turn to the task of specifying an appropriate reduced-form static model. To do this, we first use equation (21) to derive estimates of A_{nt} for each angler in the sample. Because estimation of the dynamic model requires that both quantities on the right-hand side of equation (21) be calculated for each angler in each time period, estimates of A_{nt} are easily obtained.

Figure 1 shows estimated values of A_{nt} for four sample anglers during the 1996 season. For each angler, A_{nt} is positive and increasing at the beginning of the season; then it becomes negative after each angler takes his first trip and typically remains negative until the seasonal budget has been exhausted at which time it becomes zero. The effect of the variable $elapsed_{nt}$ on trip taking behavior can be seen in the pattern exhibited by A_{nt} before the budget is exhausted. To see this, note that A_{nt} represents the difference in the expected value of the remainder of the season if a trip is taken at time (t) and the expected value if a trip is not taken. Because we have chosen a relatively large starting value for $elapsed_{nt}$ (motivated by the fact that very few anglers have fished since the end of the previous season) and because the coefficient on $elapsed_{nt}$ is negative, A_{nt} initially is positive – the expected value of the remainder of the

Figure 1: Values of A_{nt} for Five Sample Anglers in 1996



season is larger if a trip is taken because $elapsed_{nt}$ will be reset from a large positive number to zero. After this initial trip is taken, however, the more intuitive case emerges: A_{nt} is negative because the expected value of the remainder of the season tends to be smaller when a trip is taken due to the budget constraint; and the effect of resetting $elapsed_{nt}$ to zero is not as great when its current value is smaller. But the trend for Angler 4 shows that if a trip is not taken for a very long time, A_{nt} eventually will become positive again as $elapsed_{nt}$ becomes large.

Figure 1 suggests that for anglers who fish frequently and throughout the entire season (such as Anglers 1 and 3), A_{nt} could be approximated reasonably well by an individual-specific constant (i.e., no temporal component). But for anglers who fish infrequently and/or for only a short part of the season (Anglers 2 and 4), this specification for A_{nt} would tend to under-predict trips early in the season and over-predict later in the season, even though the net effect on the total number of trips predicted for the season may be negligible. Because this type of individual-specific constant is commonly used by practitioners working with panel data (Hsiao, 1986) and because it provides a useful baseline from which to compare other more complicated specifications, we adopt it here and redefine A_{nt} as A_n for clarity. Furthermore, because our model includes lagged endogenous variables ($prev\_day_{nt}$ and $elapsed_{nt}$) as regressors, we employ a "random effects" specification for A_n. In other words, we assume each A_n is drawn independently from a common distribution with mean and variance to be estimated. If our model did not include these lagged endogenous variables, we also could examine a "fixed effects" specification using Chamberlain's (1980) conditional likelihood approach; but inclusion of these two variables unfortunately renders this method invalid (Grether and Maddala, 1982; Card and

Sullivan, 1988) and we are not aware of any other feasible estimation approach for a dataset as large as ours.

Again assuming iid standard Gumbel-distributed error terms, the probability of observing a trip by angler (n) on day (t) is now given by:

$$\operatorname{prob}\left(y_{nt}=1\right)=\frac{\exp\left(\beta\mathbf{x}_{nt}+A_n\right)}{1+\exp\left(\beta\mathbf{x}_{nt}+A_n\right)}. \qquad (23)$$

And the likelihood of observing angler (n)'s trip sequence for the season is:

$$L_n^{static}\equiv\int_{A_n}\prod_t \operatorname{prob}\left(y_{nt}\mid A_n\right)\cdot dF\left(A_n\right), \qquad (24)$$

where $F\left(A_n\right)$ is the distribution of the individual-specific effects and is assumed to be $N\left(\alpha,\sigma^2\right)$. The likelihood function in (24) is maximized using the CO optimization routine in the Gauss programming language and employs Gaussian Quadrature to evaluate the integral. The coefficient estimates and standard errors are shown in Table 2.

Generally the estimates are similar to those for the dynamic model, but there are some notable differences. Anglers in the static model appear to be more affected by the catch rate, workdays and the fishing derby; and less affected by past trip-taking behavior. It is also notable that the estimate for the mean of the distribution of the random effects (α) is around -3, which appears to be the value around which most estimates of A_{nt} are clustered in Figure 1. Our main focus here, though, is to compare the predictive ability and welfare estimates of this static model with those of the dynamic model. The following section presents these comparisons.

Table 2: Coefficient Estimates and Standard Errors for the Static Model, 1996 and 1997 Seasons

Variable	1996 Season		1997 Season	
	Coefficient Estimate	Standard Error	Coefficient Estimate	Standard Error
constant (α)	-3.2006	0.3574	-2.8803	0.4299
$cost_n$	-0.0223	0.0084	-0.0189	0.0116
job_n	-0.0263	0.1586	0.1692	0.2654
$catch_t$	0.5652	0.1156	0.3753	0.0726
$temp_t$	0.0241	0.0042	0.0248	0.0043
$wind_t$	-0.0654	0.0128	-0.1363	0.0142
$weekday_t$	-1.2920	0.1014	-1.0151	0.0930
$job_n \cdot workday_{nt}$	-0.4520	0.1257	-0.7203	0.1305
$derby_t$	1.0191	0.0975	0.9084	0.1066
$prev\_day_{nt}$	0.8931	0.0852	0.8231	0.0834
$elapsed_{nt}$	-0.0052	0.0010	-0.0068	0.0010
Other Coefficients	**Estimate**	**Standard Error**	**Estimate**	**Standard Error**
Variance for random effects (σ^2)	0.2898	0.0499	0.5384	0.0878

4. Prediction and Welfare Estimation

A total of 1216 trips were observed during the 1996 season and 1261 during the 1997 season. Table 3 summarizes the trip prediction results on a seasonal basis and Figures 2 through 5 present the results on a daily basis. Figures 2 and 3 show the cumulative in-sample prediction error for the 1996 and 1997 seasons; Figure 4 shows the cumulative error when the 1997 coefficients are used with the 1996 data to predict 1996 behavior; and Figure 5 shows the cumulative error when the 1996 coefficients are used to predict 1997 behavior. Downward movements in these cumulative errors represent under-prediction and upward movements represent over-prediction. Perfect prediction throughout the season would be represented by a horizontal line at zero.

Table 3: Number of Trips Observed and Predicted, Entire Season

	Observed	Static Prediction	Dynamic Prediction
1996 In-Sample	1216	1204	1161
1997 In-Sample	1261	1257	1253
1996 Out-of-Sample	1216	1117	1083
1997 Out-of-Sample	1261	1386	1488

The in-sample results for 1996 are most consistent with our intuition regarding the static model: Figure 2 shows this model tends to under-predict trips early in the season and over-predict later in the season, but by the end of the season it is only 12 trips below the actual number taken (1% error). Its largest absolute prediction error occurs on the 95th day of the season (-101 trips, 11% error) and its largest relative error occurs on the 9th day of the season (12 trips, 90% error) when the total number of trips taken is still relatively small. Its average daily prediction error for the season is 16%. The in-sample results for 1997 static model also exhibit a general tendency for under-prediction early in the season followed by over-prediction later, but overall the predictions are better – the average daily error is only 10%, and the error for the entire season is only -4 trips (less than 0.5%).

The out-of-sample results for the static model in 1996 (Figure 4) are not as accurate as the in-sample results, but they, too, show the same U-shaped pattern. At the end of the season, the prediction error is larger than before (-99 trips, 8% error) but still reasonable. The out-of-sample results for the static model in 1997 are the most inaccurate and show persistent over-prediction. At the end of the season, the prediction error is 125 trips (10% of the total). Both of these results are consistent with those reported by Provencher and Bishop (2003) and suggest behavior may be constrained in a way the static model cannot address.

The 1996 in-sample and out-of-sample prediction results for the dynamic model (Figures 2 and 4) show that, although this model underperforms when compared to the static model on a

Figure 2: Cumulative In-Sample Prediction Error, Dynamic and Static Models, 1996 Season

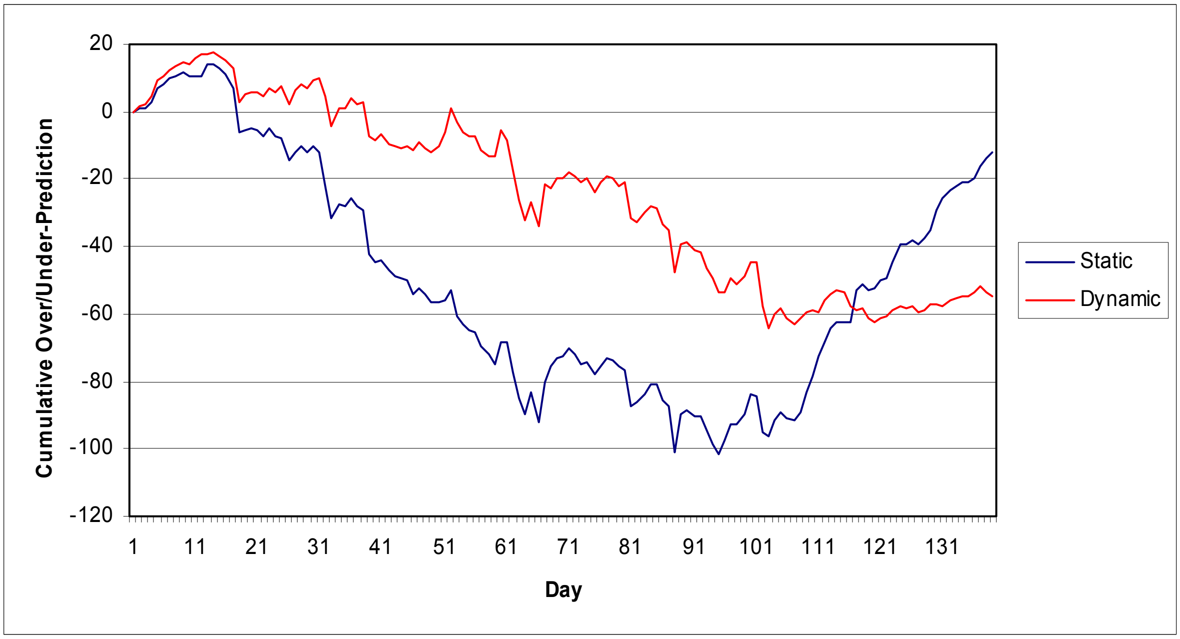


Figure 3: Cumulative In-Sample Prediction Error, Dynamic and Static Models, 1997 Season

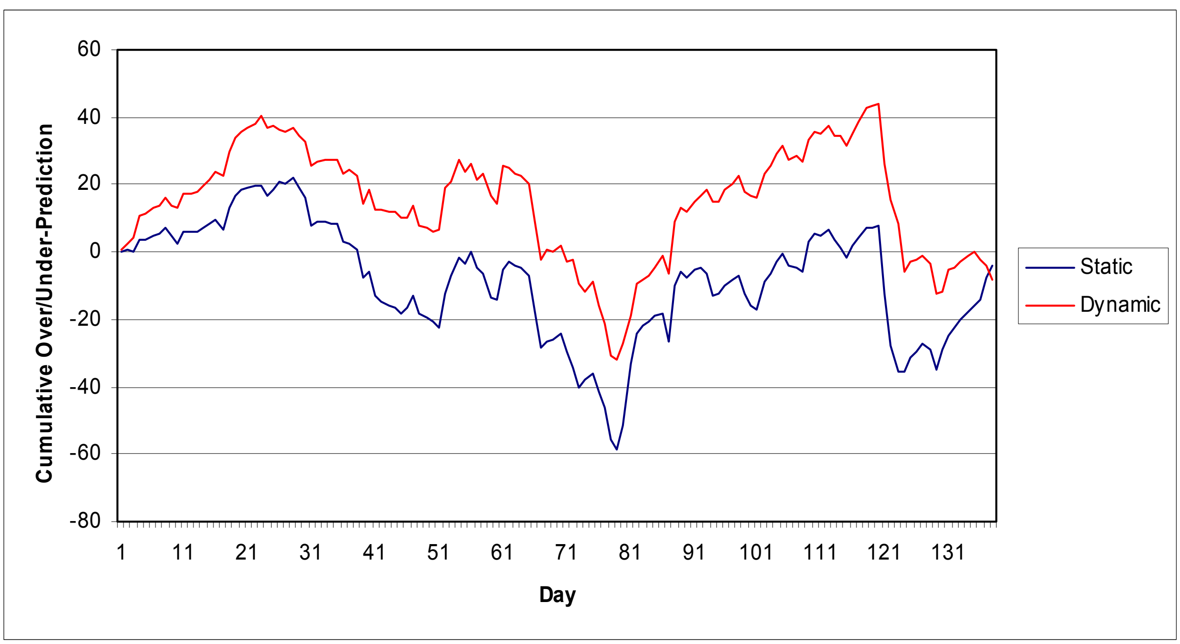


Figure 4: Cumulative Out-of-Sample Prediction Error,
Dynamic and Static Models, 1996 Season

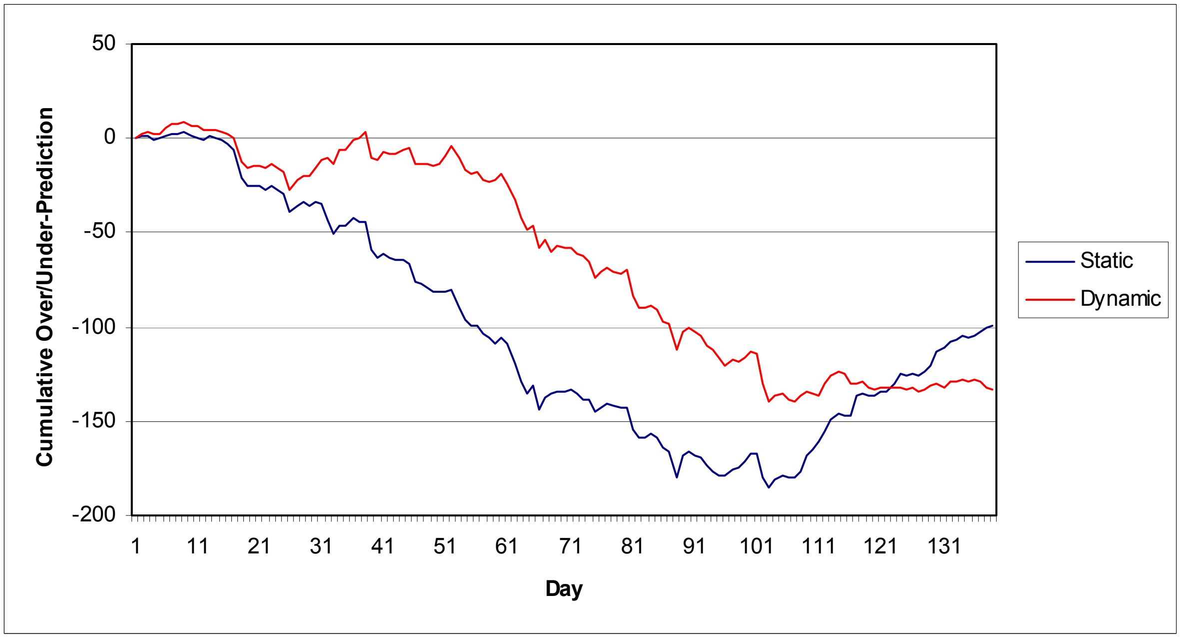
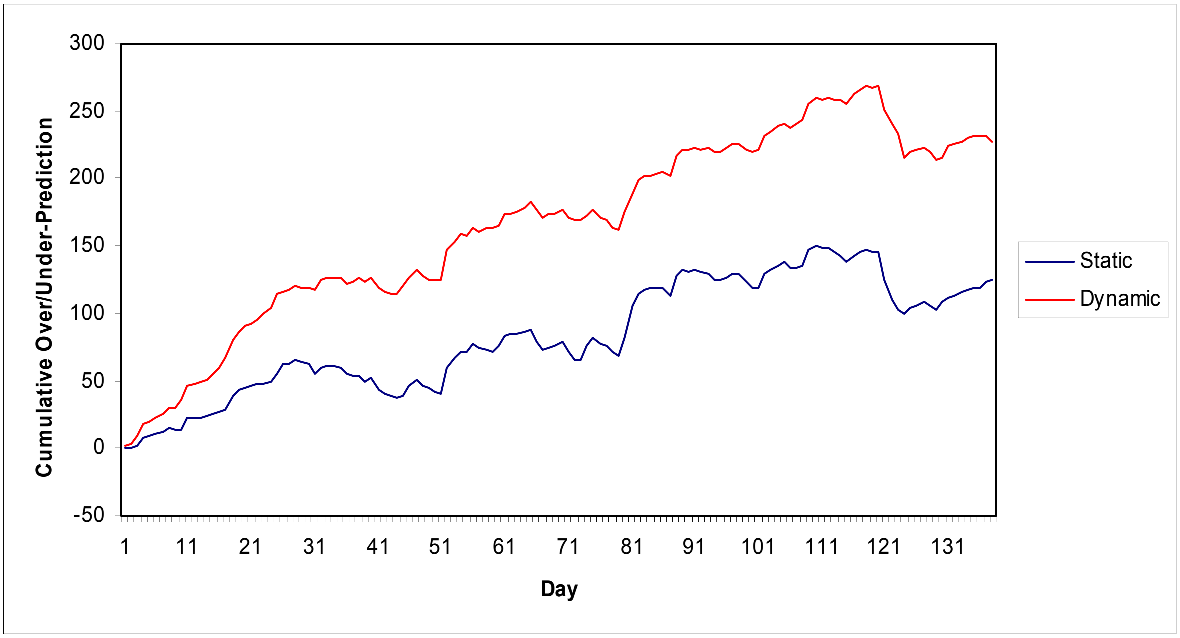


Figure 5: Cumulative Out-of-Sample Prediction Error,
Dynamic and Static Models, 1997 Season



seasonal basis (see Table 3), it produces better predictions during most of the season. The static model "recovers" late in the season and therefore has better seasonal predictions, but the dynamic model tends to be more accurate during the season. The 1997 in-sample results (Figure 3) are more mixed in this regard, and the 1997 out-of-sample results (Figure 5) do not follow this pattern, but it is clearly inappropriate to characterize the predictive performance of these models on a seasonal basis only.

Table 4 presents each model's aggregate seasonal welfare estimate for our sample. As we have argued previously, the welfare estimates for the static model are theoretically inconsistent under the assumption that the true behavioral model is dynamic, but we report these results to demonstrate the potential error associated with using this reduced-form approach. As the table shows, the static model significantly underestimates the value of the fishing season in each year: by 49% in 1996 and by 19% in 1997. This is not surprising given that each angler's A_{nt} tends to be negative for most of the season, but unfortunately a purely static modeling exercise cannot correct for this bias due to the inherent identification problem. To examine these discrepancies in more detail, Figures 6 and 7 present the cross-model differences in seasonal welfare estimates for each angler, sorted from lowest to highest static estimate. These figures show that the static model persistently underestimates individual welfare except at very high levels when it tends to overestimate.

Table 4: Seasonal Welfare Estimates, Entire Sample

	Static Estimate	Dynamic Estimate
1996 Season	$62,850	$124,380
1997 Season	$80,660	$99,120

Figure 6: Differences in Seasonal Welfare Estimates, 1996 Season, Sorted from Lowest to Highest Static Estimate

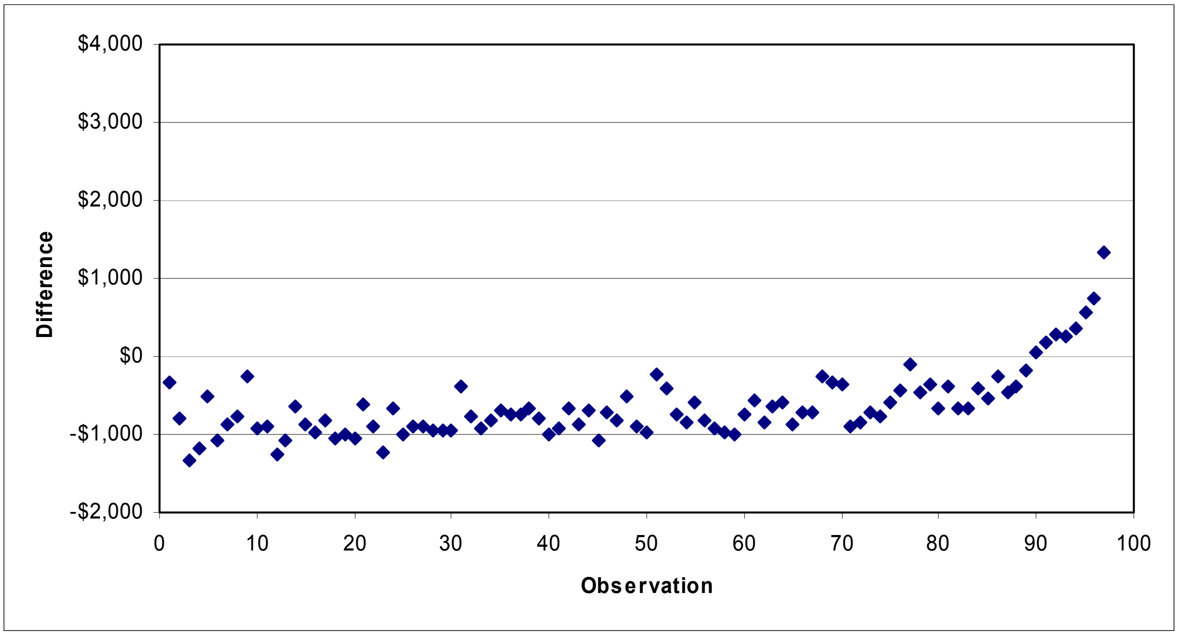
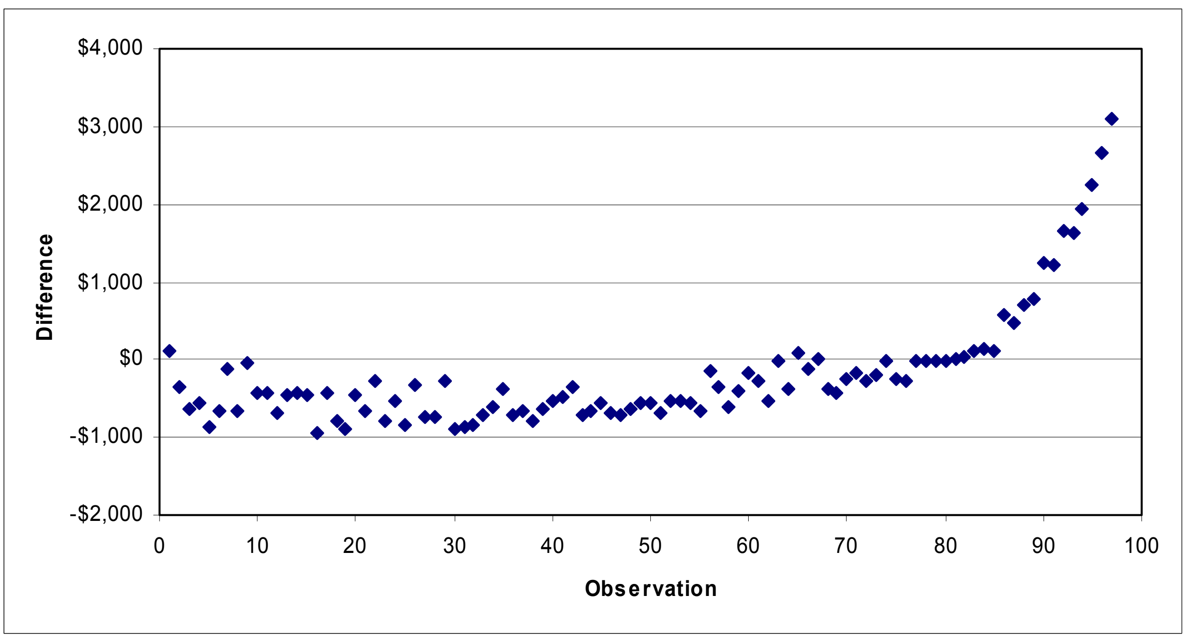


Figure 7: Differences in Seasonal Welfare Estimates, 1997 Season, Sorted from Lowest to Highest Static Estimate



5. Conclusions

The dynamic nature of behavior, whether in recreation or otherwise, tends to be overlooked by most empirical studies in order to simplify the behavioral model and reduce the computational complexity of the estimation. We have demonstrated here that when the underlying structural model is truly dynamic, a reduced-form static model may fit the data fairly well but it cannot provide unbiased welfare estimates due to problems of identification. And furthermore, our empirical application suggests this bias can be significant. Additional research focusing on this identification problem would be useful to the extent it might produce a reduced-form estimation approach that could provide reliable welfare estimates.

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