Pareto Principle and Intergenerational Equity: Immediate Impatience, Universal Indifference and Impossibility*

Yongsheng Xu
Georgia State University

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1 Introduction

In an important contribution to the problem of aggregating infinite utility streams, Svensson (1980) shows the existence of a *social welfare relation*—a reflexive, transitive and complete binary relation over all possible infinite utility streams—that accommodates the axioms of *Pareto* and *intergenerational equity*. This possibility result is in sharp contrast with the seminal contribution by Diamond (1965) in the same context of aggregating infinite utility streams, who established the non-existence of a *social welfare function*—a function which aggregates an infinite utility stream into a real number—that satisfies the axioms of *Pareto*, *intergenerational equity* and *continuity* (in the sup metric). The axiom of continuity in Diamond's result is shown to be redundant by Basu and Mitra (2003) recently: they show that, in aggregating infinite utility streams, there exists no social welfare function satisfying the axioms of Pareto and intergenerational equity.

The possibility result by Svensson suggests the compatibility of the Pareto principle and intergenerational equity for a social welfare relation, while the impossibility results by Diamond, and Basu and Mitra suggest that the compatibility of the Pareto principle and intergenerational equity breaks down when a social welfare relation is replaced by a social welfare function. Though the possibility of accommodating both the Pareto principle and intergenerational equity for a social welfare relation in aggregating infinite utility streams can be obtained, it is not clear what structure such a social welfare relation may have and to what extent the possibility may be obtained. This is because, in proving his possibility result, Svensson uses a non-constructive method by making use of Szpilrajn's lemma on extending a reflexive and transitive binary relation to a reflexive, transitive and complete binary relation.

The purpose of this paper is therefore two-fold. First, we examine the scope of obtaining the possibility result for a social welfare relation to be both Paretian and intergenerationally equitable. We show that, under a very mild restriction on social welfare relations, it is not possible to accommodate both the Pareto principle and intergenerational equity. Therefore, the scope of social welfare relations that are both Paretian and intergenerationally equitable is rather limited. Secondly, we examine, under a set of common restrictions on a social welfare relation, the respective implications of the axiom of Pareto and of the axiom of intergenerational equity. We show that, the axiom of Pareto implies *immediate impatience*—an impatience for any period t over its very next period, and the axiom of intergenerational equity implies *universal indifference*—every infinite utility stream is indifferent to any other infinite utility stream. Therefore, to some extent, our results clarify the structure of a social welfare relation satisfying the Pareto principle and intergenerational equity.

The organization of the remaining of the paper is as follows. Section 2 presents the basic notation and definitions. Section 3 introduces our basic axioms and presents our impossibility results. Section 4 examines the respective implications of the axioms of Pareto and intergenerational equity. A brief conclusion is contained in Section 5.

2 Notation

$\mathbb{R}$ is to denote the set of all real numbers, and $\mathbb{N}$ is to denote the set of all positive integers. Let X be a non-empty subset of $\mathbb{R}$ containing at least two elements. The set of all infinite utility streams is to be denoted by X^∞. The elements in X^∞ are the infinite utility streams and for $x = (x_1, x_2, \ldots, x_m, \ldots) \in X^\infty$, x_m is the utility of generation $m \in \mathbb{N}$. For all $x = (x_1, x_2, \ldots, x_m, \ldots)$ and $y = (y_1, y_2, \ldots, y_m, \ldots) \in X^\infty$, $x > y$ if and only if $[x_m \geq y_m$ for all $m \in \mathbb{N}$ and $x \neq y]$.

For any $t \in \mathbb{N}$, and for any $x_1, \ldots, x_t \in X$, let ${}_1x_t = (x_1, \ldots, x_t)$. For any $t \in \mathbb{N}$, any ${}_1x_t$, and any $y, z \in X^\infty$, an infinite stream $({}_1x_t, y)$ will mean $(x_1, x_2, \ldots, x_t, y_1, y_2, \ldots, y_m, \ldots)$. For any $t \in \mathbb{N}$ and any $x_1, \ldots, x_t \in X$, let $({}_1x_{trep})$ denote the infinite utility stream where ${}_1x_t$ repeats infinite times. For any $m \in \mathbb{N} \cup \{0\}$, any $n \in \mathbb{N}$ and any $({}_1x_{nrep}) \in X^\infty$, let $({}_1a_m,\ {}_1x_{nrep})$ denote $(a_1, \ldots, a_m,\ {}_1x_{nrep})$ if $m \geq 1$, where $a_1, \ldots, a_m \in X$, and $({}_1x_{nrep})$ if $m = 0$. For all $i, j \in \mathbb{N}$ and all $x \in X^\infty$, $x(ij)$ is the infinite utility stream obtained from x by switching utilities of generations i and j while keeping utilities of all other generations unchanged.

Let $\succeq$ be a reflexive, transitive and complete binary relation over X^∞. The symmetric and asymmetric of $\succeq$ will be denoted by $\sim$ and $\succ$, respectively. $\succeq$ is referred to as a *social welfare relation*.

3 Basic axioms and impossibility results

3.1 Basic axioms

Consider the following axioms to be imposed on a social welfare relation $\succeq$.

Pareto (P) For all $x, y \in X^\infty$, if $x > y$ then $x \succ y$.

Weak Dominance (WD) For all $x, y \in X^\infty$, if $[x_1 > y_1$ and $x_i = y_i$ for all $i > 1]$, then $x \succ y$.

Intergenerational Equity (IE) For all $x \in X^\infty$ and all $i, j \in N$, $x \sim x(ij)$.

The axiom of Pareto is the standard Pareto principle used in the literature on evaluating infinite utility streams. Weak dominance requires that if for two infinite utility streams, x and y, the first generation in x enjoys a higher utility than the first generation in y, and the respective utility levels for all other generations under x and y are the same, then the infinite utility stream x is ranked higher than the infinite utility stream y. It is clear that weak dominance is weaker than Pareto. It turns out that weak dominance is sufficient for our results in this paper. It is also interesting to note that the axiom of weak dominance is weaker than the axiom of partial Pareto, which requires that if $x, y \in X^\infty$ are such that either [for some $i \in \mathbb{N}, x_i > y_i$ and $x_k = y_k$ for all $k \in \mathbb{N} \setminus \{i\}$] or [for all $i \in \mathbb{N}, x_i > y_i$], then $x \succ y$, used in Basu and Mitra (2005) in establishing their possibility result. The axiom of intergenerational equity is the standard intergenerational equity condition used in the literature. It is also known as finite anonymity.

To analyze the scope and the structure of social welfare relations satisfying Pareto and intergeneral equity, we consider the following structural properties to be imposed on a social welfare relation.

Minimal Support (I) (MS(I)) For all $x, y \in X^\infty$, if $x = ({}_1a_m,\ {}_1x_{nrep})$ and $y = ({}_1a_m,\ {}_1y_{nrep})$ for some ${}_1a_m$, ${}_1x_n$, ${}_1y_n$, where $m \geq 0$ and $n \geq 1$, then $x \succ y$ implies that there exist $t \in \mathbb{N}$, $z \in X^\infty$ such that $({}_1a_m,\ {}_1x_n, \ldots,\ {}_1x_n, z) \succ ({}_1a_m,\ {}_1y_n, \ldots,\ {}_1y_n, z)$, where ${}_1x_n$ and ${}_1y_n$, respectively, repeat

t times in their respective infinite utility streams $({}_1a_m,\ {}_1x_n, \ldots,\ {}_1x_n, z)$ and $({}_1a_m,\ {}_1y_n, \ldots,\ {}_1y_n, z)$, .

Minimal Support (II) (MS(II)) For all $x = (x_1, \ldots, x_m, \ldots)$, $y = (y_1, \ldots, y_m, \ldots) \in X^\infty$, if $x \succ y$, then there exist $t \in \mathbb{N}$ and $z \in X^\infty$ such that $(x_1, \ldots, x_t, z) \succ (y_1, \ldots, y_t, z)$ and $(x_1, \ldots, x_t, x_{t+1}, z) \succ (y_1, \ldots, y_t, y_{t+1}, z)$.

The axioms of minimal support (I) and (II) can be regarded as a principle of minimal requirement for ranking one infinite utility stream strictly higher than another infinite utility stream. Each of them essentially requires that, whenever an infinite utility stream x is ranked strictly higher than another infinite utility stream y, then there exist a generation t and an infinite utility stream z such that by replacing the "tails" of both x and y with the common z after the generation t will preserve the corresponding strict ranking.

To further our understanding of the axioms of minimal support (I) and (II), we consider, for example, MS(I). Suppose that MS(I) is not true. Then, for some two infinite utility streams x and y such that $x = (x_1, \ldots, x_m, \ldots)$, $y = (y_1, \ldots, y_m, \ldots)$ and $x \succ y$, we must have that, for all $z \in X^\infty$ and for all $t \in \mathbb{N}$, $({}_1a_m,\ {}_1y_n, \ldots,\ {}_1y_n, z) \succeq ({}_1a_m,\ {}_1x_n, \ldots,\ {}_1x_n, z)$, where ${}_1y_n$ and ${}_1x_n$ repeat, respectively, t times in their respective infinite utility stream. This seems rather unacceptable. As a matter of fact, Fleurbaey and Michel (2003) propose a condition called *Limit Ranking*, which requires that, for all $x, y \in X^\infty$, if there exists $z \in X^\infty$ such that $({}_1x_t, z) \succeq ({}_1y_t, z)$ for all $t \in \mathbb{N}$, then $x \succeq y$. Clearly, Limit Ranking demands much more than either (MS(I)) or (MS(II)) suggesting that our (MS(I)) and (MS(II)) are very weak conditions.

3.2 Impossibility of a social welfare relation being Paretian and intergenerationally equitable

We now turn to the first main results of this paper, each of which shows the incompatibility of axioms weak Pareto, intergenerational equity, and either minimal support(I) or minimal support (II).

Theorem 1. There is no $\succeq$ satisfying the axioms of (WD), (IE) and MS(I).
Proof. Suppose to the contrary that there exists $\succeq$ satisfying the axioms of (WP), (IE), and (MS(I)). Let $a, b \in X$ with $a > b$. Consider $x = (a, b, a, b, a, b, \ldots)$ and $y = (b, a, b, a, b, a, \ldots)$, where a, b, and b, a repeat infinitely times in their respective infinite utility streams. Since $\succeq$ is

complete, there are three cases to be considered: (i) $(a, b, a, b, a, b, \dots) \succ (b, a, b, a, b, a, \dots)$; (ii) $(b, a, b, a, b, a, \dots) \succ (a, b, a, b, a, b, \dots)$; and (iii) $(a, b, a, b, a, b, \dots) \sim (b, a, b, a, b, a, \dots)$.

Case (i): $(a, b, a, b, a, b, \dots) \succ (b, a, b, a, b, a, \dots)$. If $(a, b, a, b, a, b, \dots) \succ (b, a, b, a, b, a, \dots)$, then (MS(I)) implies that there exist $z \in X^\infty$ and $t \in \mathbb{N}$ such that, $(a, b, \dots, a, b, z) \succ (b, a, \dots, b, a, z)$, where a, b repeat t times in $(a, b, \dots, a, b, z)$ and b, a appear t times in $(b, a, \dots, b, a, z)$. On the other hand, by the repeated use of (IE) and the transitivity of $\succeq$, we must have $(a, b, \dots, a, b, z) \sim (b, a, \dots, b, a, z)$, a contradiction.

Case (ii): $(b, a, b, a, b, a, \dots) \succ (a, b, a, b, a, b, \dots)$. In this case, from $(b, a, b, a, b, a, \dots) \succ (a, b, a, b, a, b, \dots)$, by (MS(I)), we derive another contradiction with (IE) and the transitivity of $\succeq$.

Case (iii): $(a, b, a, b, a, b, \dots) \sim (b, a, b, a, b, a, \dots)$. By (WD), $(a, b, a, b, a, b, a, \dots) \succ (b, b, a, b, a, b, a, \dots)$. Noting that $(a, b, a, b, a, b, \dots) \sim (b, a, b, a, b, a, \dots)$ and that $(a, b, a, b, a, b, a, \dots) = (a, b, a, b, a, b, \dots)$, the transitivity of $\succeq$ implies that $(b, a, b, a, b, a, \dots) = (b, a, b, a, b, a, \dots) \succ (b, b, a, b, a, a, \dots)$. Let $v = (b, a, b, a, b, a, \dots)$ and $w = (b, b, a, b, a, a, \dots)$. By (MS(I)), there exists $u \in X^\infty$ and $t \in \mathbb{N}$ such that $(b, a, b, a, b, \dots, a, b, u) \succ (b, b, a, b, a, \dots, b, a, u)$ where a, b repeat t times in $(b, a, b, a, b, \dots, a, b, u)$ and b, a appear t times in $(b, b, a, b, a, \dots, b, a, u)$. By the repeated use of (IE) and the transitivity of $\succeq$, $(b, a, b, \dots, a, b, u) \sim (b, b, a, dots, b, a, u)$, a contradiction.

The above three cases exhaust all possibilities. Therefore, there is no $\succeq$ satisfying (WD), (IE) and (MS(I)). ■

Theorem 2. There is no $\succeq$ satisfying the axioms of (WD), (IE) and MS(II).
Proof. Suppose to the contrary that there exists $\succeq$ satisfying (WD), (IE) and (MS(II)). Let $a, b \in X$ with $a > b$. Consider $x = (a, b, a, b, a, b, \dots)$ and $y = (b, a, b, a, b, a, \dots)$, where a, b and b, a repeat infinitely times in their respective infinite utility streams. If $x = (a, b, a, b, a, b, \dots) \succ (b, a, b, a, b, a, \dots) = y$, then (MS(II)) implies that there exist $t \in \mathbb{N}$ and $z, z' \in X^\infty$ such that, $(x_1, \dots, x_t, z) \succ (y_1, \dots, y_t, z)$ and $(x_1, \dots, x_t, x_{t+1}, z') \succ (y_1, \dots, y_t, y_{t+1}, z')$. Clearly, either t or $t + 1$ is even. Without loss of generality, let t be even. Then, we have $(a, b, a, b, a, b, \dots a, b, z) \succ (b, a, b, a, b, a, \dots b, a, z)$, where a, b and b, a appear $t/2$ times in their respective infinite utility streams. Since t is finite, by the repeated use of (IE) and the transitivity of $\succeq$, we must have $(a, b, a, b, a, b, \dots a, b, z) \sim (b, a, b, a, b, a, \dots b, a, z)$, a contradiction. Similarly, if $(b, a, b, a, b, a, \dots) \succ (a, b, a, b, a, b, \dots)$, by (MS(II)), we derive another contradiction with (IE) and the transitivity of $\succeq$. Therefore, $(a, b, a, b, a, b, \dots) \sim$

$(b,a,b,a,b,a,\ldots)$. By (WD), $(a,b,a,b,a,b,a,\ldots) \succ (b,b,a,b,a,b,a,\ldots)$. Noting that $(a,b,a,b,a,b,\ldots) \sim (b,a,b,a,b,a,\ldots)$ and that $(a,b,a,b,a,b,a,\ldots) = (a,b,a,b,a,b,\ldots)$, the transitivity of $\succeq$ implies that $(b,a,b,a,b,a,\ldots) = (b,a,b,a,b,a,\ldots) \succ (b,b,a,b,a,a,\ldots)$. Let $v = (b,a,b,a,b,a,\ldots)$ and $w = (b,b,a,b,a,a,\ldots)$. By (MS(II)), there exist $s \in \mathbb{N}$ and $u, u' \in X^\infty$ such that $(v_1,\ldots,v_s,u) \succ (w_1,\ldots,w_s,u)$ and $(v_1,\ldots,v_s,v_{s+1},u') \succ (w_1,\ldots,w_s,w_{s+1},u')$. By the reflexivity of $\succeq$, $s > 1$. Note that either s or $s+1$ is odd. Without loss of generality, let s be odd. Then, we have $(b,a,b,a,b,a,b,\ldots a,b,u) \succ (b,b,a,b,a,b,a,\ldots b,a,u)$, where $a,b,a,b,a,b,\ldots a,b$ and $b,a,b,a,b,a,\ldots b,a$, respectively, are such that a,b and b,a, respectively, appear $(s-1)/2 \geq 1$ times. Since s is finite, by the repeated use of (IE) and the transitivity of $\succeq$, $(b,a,b,a,b,a,b,\ldots a,b,u) \sim (b,b,a,b,a,b,a,\ldots b,a,u)$, a contradiction. Therefore, there is no $\succeq$ satisfying (WD), (IE) and (MS(II)). ■

Remark 1. Since the axiom of Pareto implies the axiom of weak dominance, it is clear that there is no social welfare relation that simultaneously satisfies (P), (IE), and either (MS(I)) or (MS(II)).

Remark 2. We note that, in establishing our impossibility results, it is sufficient that X contains just different two elements.

Remark 3. We note that a social welfare relation satisfying (MS(I)) or (MS(II)) may or may not be representable by a social welfare function. For example, the social welfare relation $\succeq^2_{lex}$ defined below satisfies (MS(I)) and (MS(II)), but is not representable by a social welfare function:

for all $x, y \in X^\infty$, let $x \succeq^2_{lex} y$ iff $[x_1 > y_1$ or $(x_1 = y_1$ and $x_2 \geq y_2)]$.

$\succeq^2_{lex}$ satisfies (WD), but violates (IE).

Remark 4. On the other hand, as shown by Basu and Mitra (2005), there exists a social welfare function that satisfies the axiom of partial Pareto (see Section 2), (IE) when X is the set of all integers. Note that the axiom of partial Pareto is a stronger condition than (WD). Therefore, in view of our impossibility results and the possibility result obtained in Basu and Mitra (2005), either (MS(I)) or (MS(II)) (or both) are not necessary for the representability. In view of Remark 3, (MS(I)) or (MS(II)) are independent properties from the representability of a social welfare relation by a social welfare function.

4 Immediate impatience and universal indifference

As shown in the last section, in the presence of either (MS(I)) or (MS(II)), any Paretian social welfare relation is not intergenerationally equitable. (MS(I)) and (MS(II)) are very reasonable properties for a social welfare relation to satisfy. Inspired by our impossibility results, in this section, we examine the respective implications of (WD) and (IE) under some common axioms. First, we consider three more axioms introduced below.

Independence (I) (IND(I)) For all $a \in X$, and all $x, y \in X^\infty$, $[x \succeq y \Rightarrow (a, x) \succeq (a, y)]$.

Independence (II) (IND(II) For all $t \in \mathbb{N}$, all $x, y \in X^\infty$ and all $u_1, \ldots, u_t, v_1, \ldots, v_t \in X$, $[(u_1, \ldots, u_t, x) \succeq (v_1, \ldots, v_t, x) \Leftrightarrow (u_1, \ldots, u_t, y) \succeq (v_1, \ldots, v_t, y)]$.

Minimal Support (III) (MS(III)) For all $x, y \in X^\infty$, if $x \succ y$, then there exist $t \in \mathbb{N}$ and $z \in X^\infty$ such that $({}_1x_t, z) \succ ({}_1y_t, z)$.

(IND(I)) and (IND(II)) correspond, respectively, to Diamond's axioms of (NC1) and (NC2). Similar axioms are used and discussed in Koopmans (1960) as well. As pointed out by Diamond, these two axioms reflect 'a certain type of noncomplementarity of the preferences over time or that the "preference" over part of the time horizon are independent of the utility levels achieved in other times' (Diamond (1965, pp. 175)).

(MS(III)) has the same intuition as either (MS(I)) or (MS(II)) and once again requires that, in a very weak sense, the ranking of two infinite utility streams is independent of distant tails of the two utility streams.

4.1 Pareto principle and immediate impatience

We now explore implications of the Pareto principle in the presence of IND(I), IND(II) and either (MS(I)) and (MS(II)). The consequences of the implications are two immediate impatience results which are summarized in the following theorems.

Theorem 3. Suppose $\succeq$ satisfies (WD), (MS(I)), IND(I) and IND(II). Then, for all $i \in \mathbb{N}$, all $x \in X^\infty$,

$$x_i > x_{i+1} \Rightarrow x \succ x(i\, i+1).$$

Proof. Suppose $\succeq$ satisfies (WD), (MS(I)), IND(I) and IND(II). Let $a, b \in X$ with $a > b$. Consider $x = (a, b, a, b, a, b, \ldots)$ and $y = (b, a, b, a, b, a, \ldots)$. If $(b, a, b, a, b, a, \ldots) \succeq (a, b, a, b, a, b, \ldots)$, then, by IND(I), $(a, b, a, b, a, b, a, \ldots) \succeq (a, a, b, a, b, a, b, \ldots)$. The transitivity of $\succeq$ implies that $(b, a, b, a, b, a, \ldots) \succeq (a, a, b, a, b, a, b, \ldots)$, a contradiction to (WD), which implies that $(a, a, b, a, b, a, b, \ldots) \succ (b, a, b, a, b, a, b, \ldots)$. Therefore, from the completeness of $\succeq$, $(a, b, a, b, a, b, \ldots) \succ (b, a, b, a, b, a, \ldots)$. By (MS(I)), there exists $z \in X^\infty$ and $t \in \mathbb{N}$ such that $(a, b, \ldots, a, b, z) \succ (b, a, \ldots, b, a, z)$, where a, b repeat t times in $(a, b, \ldots, a, b, z)$ and b, a repeat t times in $(b, a, \ldots, b, a, z)$. We now show that $(a, b, z) \succ (b, a, z)$. Suppose not, then, by the completeness of $\succeq$, we have $(b, a, z) \succeq (a, b, z)$. By using IND(I) twice, from $(b, a, z) \succeq (a, b, z)$, we obtain $(b, a, b, a, z) \succeq (b, a, a, b, z)$. By IND(II), from $(b, a, z) \succeq (a, b, z)$ and considering $(a, b, z) \in X^\infty$, we obtain $(b, a, a, b, z) \succeq (a, b, a, b, z)$. The transitivity of $\succeq$ implies that $(b, a, b, a, z) \succeq (a, b, a, b, z)$. Repeating the above procedures, if necessary, we obtain $(b, a, b, a, \ldots b, a, z) \succeq (a, b, a, b, \ldots a, b, z)$ where b, a and a, b repeat t times in their respective infinite utility streams, a contradiction with the previously established fact that $(a, b, a, b, \ldots a, b, z) \succ (b, a, b, a, \ldots b, a, z)$. Therefore, $(a, b, z) \succ (b, a, z)$. From IND(II), it follows that, for all $x' \in X^\infty$, $(a, b, x') \succ (b, a, x')$. Thus, we have shown that,

$$\text{for all } a, b \in X \text{ and all } x' \in X^\infty, \text{ if } a > b \text{ then } (a, b, x') \succ (b, a, x').$$

Consider any $x' \in X^\infty$. If $x'_i > x'_{i+1}$, from the above analysis, we have $(x'_i, x'_{i+1}, x'_{i+2} \ldots) \succ (x'_{i+1}, x'_i, x'_{i+2} \ldots)$. By the repeated application of IND(I), we then obtain $x' \succ x'(i\, i+1)$. ■

Theorem 4. Suppose $\succeq$ satisfies (WD), (MS(II)), IND(I) and IND(II). Then, for all $i \in \mathbb{N}$, all $x \in X^\infty$,

$$x_i > x_{i+1} \Rightarrow x \succ x(i\, i+1).$$

Proof. The proof is similar to that of Theorem 3 and we omit it. ■

It should be noted that Theorems 3 and 4 are not vacuous. For example, the lexicographic relation $\succeq_{lex}$ defined below:

> for all $x, y \in X^\infty$, $x \succ_{lex} y$ if $x_1 > y_1$, or there exists $m \in N$ such that $[x_m > y_m$ and $x_j = y_j$ for all $j < m]$, and $x \sim_{lex} y$ if $x = y$,

satisfies (WD), (MS(I)), (MS(II)), IND(I) and IND(II), and thus exhibits immediate impatience. Another social welfare relation, to be called the additive relation and to be denoted by $\succeq_{additive}$, defined below also satisfies (WD), (MS(I)), (MS(II)), IND(I) and IND(II):

For each and every $i \in N$, there exists $f_i : X \to \mathbb{R}$ such that,
(i) for all $a \in X$ and all $j, m \in N$, $j < m \Rightarrow f_j(a) > f_m(a)$,
(ii) for all $a, b \in X$ and all $j \in N$, $a \leq b \Leftrightarrow f_j(a) \leq f_j(b)$,
and
(iii) for all $x, y \in X$, $x \succeq_{additive} y \Leftrightarrow \sum_{i=1}^{\infty} f_i(x_i) \geq \sum_{i=1}^{\infty} f_i(y_i)$.

We also note that Diamond (1965) shows that if a social welfare function satisfies the (corresponding) axioms of Pareto, independence (I), independence (II), and the axiom of continuity (in the sup metric), then the social welfare function exhibits *eventual impatience*—an impatience for the first period over the tth period for all t sufficiently far in the future. Our results of Theorems 3 and 4 thus strengthen Diamond's result in two respects. First, we show that there is an immediate impatience. Second, we obtain our results without insisting on a social welfare function satisfying the axiom of continuity.

4.2 Intergenerational equity and universal indifference

In this subsection, we examine the implication of the axiom of intergenerational equity under (MS(I)), (MS(II)), (MS(III), (IND(I)) and (IND(II)). The implications are summarized in the following theorems.

Theorem 5. Suppose that $\succeq$ satisfies (IE), (MS(I)), (MS(III)), IND(I) and (IND(II)). Then, for all $x, y \in X^{\infty}$, $x \sim y$.
Proof. Suppose that $\succeq$ satisfies (IE), (MS(I)), (MS(III)), IND(I) and (IND(II)). First, we show that

$$\text{for all } a, b \in X,\ u = (a, b, a, b, a, b, \ldots) \sim v = (b, a, b, a, b, a, \ldots). \quad (1)$$

Let $a, b \in X$. If $(a, b, a, b, a, b, \ldots) \succ (b, a, b, a, b, a, \ldots)$, then (MS(I)) implies that there exists $z \in X^{\infty}$ and $t \in \mathbb{N}$ such that, $(a, b, \ldots, a, b, z) \succ (b, a, \ldots, b, a, z)$, where a, b repeat t times in $(a, b, \ldots, a, b, z)$, and b, a repeat t times in $(b, a, \ldots, b, a, z)$. By the repeated used of (IE) and the transitivity of $\succeq$, we have $(a, b, \dots, a, b, z) \sim (b, a, \dots, b, a, z)$, a contradiction. Similarly, if $(b, a, b, a, b, a, \dots) \succ (a, b, a, b, a, b, \dots)$, by (MS(I)), we derive another contradiction with (IE) and the transitivity of $\succeq$. Therefore, (1) is established.

We next show that

$$\text{for every } x \in X^\infty, (a, x) \sim (b, x). \tag{2}$$

To show (2), we first note that, by IND(I), from (1), we obtain

$$(b, a, b, a, b, a, b, \dots) \sim (b, b, a, b, a, b, a, \dots), \text{ and } (a, b, a, b, a, b, a, \dots) \sim (a, a, b, a, b, a, b, \dots). \tag{3}$$

From (1) and the first part of (3), by the transitivity of $\succeq$, we obtain

$$(a, b, a, b, a, b, a, \dots) \sim (b, b, a, b, a, b, a, \dots). \tag{4}$$

From (1) and the second part of (3), the transitivity of $\succeq$ implies that

$$(b, a, b, a, b, a, b, \dots) \sim (a, a, b, a, b, a, b, \dots). \tag{5}$$

If $(a, x) \succ (b, x)$, by (IND(II)), $(a, b, a, b, a, b, a, \dots) \succ (b, b, a, b, a, b, a, \dots)$, which contradicts (4). If $(b, x) \succ (a, x)$, by (IND(II)), $(b, a, b, a, b, a, b, \dots) \succ (a, a, b, a, b, a, b, \dots)$, which contradicts (5). Since $\succeq$ is complete, (2) then follows easily.

Consider any $(x_1 x_2, x')$ and $(y_1 y_2, x') \in X^\infty$. From (2), $(x_2, x') \sim (y_2, x')$. By (IND(I)), $(x_1, x_2, x') \sim (x_1, y_2, x')$. Similarly, $(x_1, x') \sim (y_1, x')$. By (IND(II)), $(x_1, y_2, x') \sim (y_1, y_2, x')$. Therefore, $(x_1, x_2, x') \sim (y_1, y_2, x')$ follows from the transitivity of $\succeq$. Similarly, it can be shown that

$$\text{for all } t \in \mathbb{N}, \text{ all } x' \in X^\infty, \text{ all } x_1, \dots, x_t, y_1, \dots, y_t \in X, ({}_1x_t, x') \sim ({}_1y_t, x'). \tag{6}$$

Consider $x, y \in X^\infty$. If $x \succ y$, then, by (MS(III)), there exist $t \in \mathbb{N}$ and $z \in X^\infty$ such that $(x_1, \dots, x_t, z) \succ (y_1, \dots, y_t, z)$, which contradicts (6). By (MS(III)), $y \succ x$ leads to another contradiction to (6). Therefore, for all $x, y \in X^\infty, x \sim y$. ■

Theorem 6. Suppose that $\succeq$ satisfies (IE), (MS(II)), (MS(III)), IND(I) and (IND(II)). Then, for all $x, y \in X^\infty$, $x \sim y$.

Proof. The proof is similar to that of Theorem 5 and we omit it. ■

5 Conclusion

The possibility of combining both the Pareto principle and intergenerational equity in a social welfare relation established by Svensson (1980) sounded very promising. Upon a further examination, however, the scope of constructing a social welfare relation that is both Paretian and intergenerationally equitable is very limited. The impossibility results of Theorems 1 and 2 show that, under very mild restrictions on a social welfare relation, it is not possible to accommodate both Pareto and intergenerational equity in a social welfare relation.

Both (MS(I)) and (MS(II)) are structural properties and are very reasonable. In the literature on evaluating infinite utility streams, apart from the possibility results established by Svensson (1980), there are several other possibility results obtained by several authors. For example, in the approach that uses social welfare relations to evaluate infinite utility streams Fleurbaey and Michel (2003) discuss extensions of the Ramsey principle, and Bossert, Sprumont and Suzumura (2005) and Asheim and Tungodden (2004) discuss extensions based on transfer-sensitive quasi orderings and of leximin. In the social welfare function approach to the problem of evaluating infinite utility streams, Basu and Mitra (2003a) consider an infinite-horizon version of utilitarianism and discuss extensions of the overtaking criterion by von Weizsäcker (1965). It is fair to say that, in all those possibility results, the methods used for proving are not constructive: either the axiom of choice is invoked or Szpilrajn's (1930) lemma on extending a quasi ordering to a complete ordering is used. Given that all the possibility results available up to now satisfy both (WD) and (IE), clearly, they all fail to satisfy (MS(I)) and (MS(II)). As a future research agenda, it is then interesting to investigate the precise reason why they all fail (MS(I)) and (MS(II)).

Given the simplicity of (MS(I)) and (MS(II)), both (MS(I)) and (MS(II)) can be served as a handy tool to check if a *constructed* social welfare function or social welfare relation indeed satisfies both (WD) and (IE).

Finally, our results of Theorems 3, 4, 5 and 6 suggest that, the axiom of intergenerational equity, together with the axioms of minimal support and independence (I) and (II), puts severe restrictions on possible social welfare relations: There is just one way of ranking all infinite utility streams, which is that they all must be indifferent. On the other hand, though the axiom of weak Pareto implies immediate impatience in the presence of the axioms of minimal support, and independence (I) and (II), it offers more possibilities.

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