

Noise Traders and the Exchange Rate Disconnect Puzzle

JOB MARKET PAPER

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March 2004

Abstract

This paper proposes a framework to explain why the nominal and real exchange rates are highly volatile and seem to be disconnected from the macroeconomic fundamentals. Two types of foreign exchange traders, rational traders and noise traders with erroneous stochastic beliefs, are introduced into the dynamic general equilibrium framework of the new open economy macroeconomic literature. The presence of noise traders creates deviations from the uncovered interest parity. As a result, exchange rates can diverge significantly from the fundamental values. Combined with local currency pricing and consumption smoothing behavior in an infinite horizon model, the presence of noise traders can help to explain the “exchange rate disconnect puzzle”. Then we show that the excess exchange rate volatility caused by the presence of noise traders can be reduced by the ‘Tobin tax’ type of exchange rate policies.

JEL classification numbers: F41, F31, G15

Keywords: Noise traders, exchange rate disconnect, macroeconomic fundamentals, volatility.

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1 Introduction

A central puzzle in international macroeconomics over the last 20 years is that real exchange rates are volatile and persistent. Furthermore, as Flood and Rose (1995) have elegantly documented, the exchange rate seems to “have a life of its own”, being disconnected from other macroeconomic variables. For example, Mussa (1986), Baxter and Stockman (1989) and Flood and Rose (1995) have all found that both nominal and real exchange rates are highly volatile, especially when compared to the macroeconomic fundamentals, such as relative price level, consumption and outputs. Exchange rate volatility also varies substantially over time. Obstfeld and Rogoff (2000) state this kind of “exceedingly weak relationship between the exchange rate and virtually *any* macroeconomic aggregates” as the “exchange rate disconnect puzzle”.

This irregularity casts doubts on the traditional monetary macroeconomic model of exchange rates, which assumes that purchasing power parity (PPP) holds. With PPP, the “expenditure-switching” effect of exchange rate changes will lead to substitution between domestically-produced goods and internationally-produced goods. It implies that the exchange rate volatility will be transferred to macroeconomic fundamentals. Nevertheless, empirical evidence ¹ indicates that nominal exchange rate changes are not fully passed through to goods prices. Motivated by this evidence, Betts and Devereux (1996, 2000) introduced local currency pricing into the baseline *Redux* model developed by Obstfeld and Rogoff (1995). They assume that firms can charge different prices for the same goods in home and foreign markets and that the prices are sticky in each country in terms of the *local* currency. This allows the real exchange rate to fluctuate, and delinks the home and foreign price levels.

Although the new open economy macroeconomic models with sticky prices, imperfect competition and local currency pricing can generate volatile exchange rate movements, they typically predict a strong counterfactual relationship between the real exchange rate and relative consumption ². A monetary shock simultaneously raises domestic consumption (by more than it raises foreign consumption) and creates a (temporary) depreciation of home currency. Consequently, these models almost generically predict a strong positive correlation between depreciation and relative consumption, which is not observed empirically. ³

¹See Engel (1993, 1999) and Parsley and Wei (2001) for details.

²See, for example, Chari, Kehoe and McGrattan (2002).

³Benigno and Thoenissen (2003) report the correlation between bilateral exchange rate and bilateral relative consumption for seven countries (Canada, France, West Germany, Italy, Japan, U.K. and U.S.) for the periods starting from 1970 until 2002. The cross-correlation varies between -0.45 and 0.42 .

One explanation for this discrepancy might lie in the fact that the nominal exchange rate is also an asset price, and therefore will be inevitably affected by imperfections in the financial markets. These imperfections may include herd behavior, momentum investing and noise traders. Working together with sticky prices, these are all important reasons to explain why the real exchange rate persistently deviates from the level predicted by the fundamentals-based models. A large body of evidence has documented deviations from rational expectations in the foreign exchange markets⁴. Evans and Lyons (2002) show that most of the short-run exchange rate volatility is related to order flow, which also reflects the heterogeneity in investors' expectations. Although financial economists care about high frequency data, while international macroeconomists focus more on low frequency data, it is still surprising how little the microstructure of real world foreign exchange markets has been considered in the *macroeconomic theory* of exchange rates.

This raises another question: if exchange rate volatility is caused by erroneous beliefs and could be reduced without incurring costs due to other macroeconomic volatilities, then floating exchange rates may be too volatile and costly from a welfare point of view. However, it is impossible to make any policy recommendations in the absence of a *welfare-based* model which can explain exchange rate volatility and its relationship with macroeconomic fundamentals.

Therefore, our paper intends to propose a new approach to study exchange rates, that combines the macroeconomic model of exchange rates and the microstructure approach of foreign exchange markets. This approach is implemented within a specific model, where noise traders are introduced into the new open economy macroeconomic framework. The combination is helpful for understanding the behavior of exchange rates and their relationship with macroeconomic fundamentals better. It also gives more rigorous microeconomic foundations to the "noise trader" approach and enriches the new open economy macroeconomic framework with a more realistic setting of the microstructure of foreign exchange market. In addition, it provides a well-defined framework for policy evaluations, especially for those policies that are designed to control non-fundamental volatilities.

We adapt the overlapping-generation noise trader model of De Long *et al.* (1990). Two types of foreign exchange traders are introduced into the general equilibrium framework. One type is the "rational/informed trader", which has rational expectations about future investment returns, while the other type cannot forecast the future returns correctly and is called the "noise trader".

The results from the model show that when the number of noise traders increases, so does the

⁴See Frankel and Froot (1987,1990), Chinn and Frankel(1994), Taylor and Allen (1992) and Cheung and Wong (1998) for details.

exchange rate volatility. Nevertheless, the volatilities of macroeconomic fundamentals (except for the net foreign assets) are completely independent of the noise component on the foreign exchange market. Moreover, since in this model nominal and real exchange rate fluctuations can be generated by the erroneous belief of noise traders, our model does not predict a strong comovement of exchange rates and fundamentals. Therefore, the “exchange rate disconnect puzzle” may be explained by the approach described in this paper.

The basic intuition behind our results is as follows. The heterogeneity in beliefs among foreign exchange traders creates the basis for trading volume and deviations from the uncovered interest parity. Arbitrage does not eliminate the effect of noise here because noise itself creates risk: short-horizon investors must bear the risk that they may be required to liquidate their positions at a time when asset prices are pushed even further away (by noise traders) from the fundamental values than when investment was made. Therefore, exchange rates can diverge significantly from the fundamental values. The greater the number of noise traders, the more volatile will be the exchange rates.

Nevertheless, why is the exchange rate volatility not transferred to macroeconomic fundamentals? Normally, there are two channels through which the exchange rate affects the macroeconomic variables: the expenditure-switching effect and the wealth effect (through firms’ profits). Under the assumption of local currency pricing, the expenditure-switching effect is eliminated as the relative price of home and foreign goods does not change. Although the wealth effect still exists, it turns out to be quite small quantitatively. This is because the wealth effect of exchange rate change is spread out over current and future periods through intertemporal consumption smoothing, and so tends to be very small.

Many economists have suggested that increasing the trading cost on the foreign exchange market might reduce the exchange rate volatility. To understand the effect of this kind of exchange rate policies, the size of the noise component is endogenized by introducing a heterogenous entry cost for noise traders. Only noise traders having entry costs that are sufficiently low will choose to enter the foreign exchange market. We find that given the number of noise traders, increasing the entry cost will reduce the exchange rate volatility. We also analyze the ‘Tobin tax’ type of exchange rate policy suggested by Tobin (1978) and Eichengreen, Tobin and Wyplosz (1995) in an extension of the baseline model. We find that a Tobin tax will decrease the exchange rate volatility, however, the impact of a Tobin tax on exchange rate volatility depends crucially on the structure of the foreign exchange market and the interaction of the Tobin tax with other trading costs.

The microstructure of the exchange rate market in this paper follows the noise trader literature,

especially the work of Jeanne and Rose (2002), which also focuses on the relationship between exchange rate volatility and noise traders. However, the macroeconomic part of their model is a simple monetary model of exchange rates with PPP. Neither nominal rigidities nor pricing to market is considered. Moreover, intertemporal optimizing agents and profit maximizing firms are not considered in their model. This implies that most channels through which the exchange rate affects macroeconomic fundamentals are overlooked. Another feature of their model is that it is a partial equilibrium model without explicit welfare specifications for households, so rigorous policy evaluation is impossible.

This paper is also closely related to the new open economy macroeconomic literature. The paper that is closest, in spirit, to our analysis of exchange rate disconnect puzzle is Devereux and Engel (2002). They stated that the key ingredients to explain the exchange rate disconnect puzzle include: local currency pricing to eliminate the expenditure-switching effect, a special structure of international pricing and product distribution to minimize the wealth effect, incomplete international financial markets, and stochastic deviations from the uncovered interest parity. The analysis in this paper differs in the following aspects. First, more microeconomic foundations of noise traders are explored. Both noise traders and rational traders in our model are risk averse and utility maximizing agents, therefore, policy analysis is possible in our model. Second, we show that, the wealth effect of exchange rate changes may be quite small, *quantitatively*, in an infinite horizon model. Therefore, the exchange rate disconnect puzzle can be explained even without a specific assumption of production and distribution structure to remove wealth effects.

This paper is organized as follows. In Section 2, we construct a model that embeds noise traders into a new open economy macroeconomic framework. Both the exogenous entry and endogenous entry specifications are explored. In Section 3 features of the solution to the model are discussed. Section 4 gives the results of the model. Section 5 extends the baseline model to analyze the implications of Tobin tax. The paper concludes with a brief summary and suggestions for subsequent research.

2 The model

The world economy consists of two countries, denominated by home and foreign. Each country specializes in the production of a composite traded good. Variables in the foreign country are denoted by an asterisk. In addition, a subscript h denotes a variable originating from the home country; a subscript f denotes a variable used in the foreign country.

This model is analogous to most new open economy macroeconomic models except for the foreign exchange market. Each country is populated by a large number of atomistic households, a continuum of firms that set prices in advance, and a government (a combined fiscal and monetary authority). However, we assume that home and foreign households can only trade nominal bonds denominated in their domestic currency. Although home households cannot access the international bond market, the foreign exchange traders can carry out the international bond trading to maximize their utility. Thus, besides the infinitely lived household, a second type of representative agent is introduced into the model, namely, the foreign exchange trader, who lives in an overlapping-generation demographic structure. Hereafter, a superscript H denotes households and a superscript T stands for traders. In the foreign country, only one type of representative agent is present; the foreign household.⁵

Since this kind of model has been well covered in previous papers, here we will just briefly sketch out its main elements and emphasize those parts that make our model different from others; the foreign exchange market.

2.1 Households, Firms and Government

Households The lifetime expected utility of the home representative household is:

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^H)^{1-\rho}}{1-\rho} + \frac{1}{1-\epsilon} \left(\frac{M_t}{P_t} \right)^{1-\epsilon} - \frac{\eta}{1+\psi} L_t^{1+\psi} \right] \right\} \quad (2.1)$$

Subject to

$$P_t C_t^H + B_{t+1} + M_t = W_t L_t + \Pi_t + M_{t-1} + T_t + B_t(1+r_t) \quad (2.2)$$

where C_t^H is the time t composite consumption of home households, composed by a continuum of home goods and foreign goods; both are of measure 1. Let C_t^T denote the composite consumption of traders, then $C_t^T + C_t^H = C_t$, where C_t is the composite consumption of the home country and is given by:

$$C_t = \left[\omega^{\frac{1}{\gamma}} C_{h,t}^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}} C_{f,t}^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} \quad (2.3)$$

⁵Here we assume that there are only foreign exchange traders in the home country. The model could be easily extended to the case where there are foreign exchange traders in both countries. This might affect the welfare implication of traders, but the main results in this paper will not change. This is because we solve the model by log-linearizing around a non-stochastic steady state with zero net foreign assets. See Appendix D.1 for details.

where $C_{h,t} = \left(\int_0^1 C_{h,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$, $C_{f,t} = \left(\int_0^1 C_{f,t}(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$, and the weight $\omega \in (0,1)$ determines the home representative agent's bias for the domestic composite good. Note that θ is the elasticity of substitution between individual home(or foreign) goods and γ is the elasticity of substitution across home and foreign composite goods.

P_t is a consumption based price index for period t , which is defined by:

$$P_t = \left[\omega P_{h,t}^{1-\gamma} + (1-\omega) P_{f,t}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \quad (2.4)$$

where $P_{h,t} = \left(\int_0^1 P_{h,t}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$ and $P_{f,t} = \left(\int_0^1 P_{f,t}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}$.

In each period every household is endowed with one unit of time, which is divided between leisure and work. His income is derived from the labor income $W_t L_t$, profits from domestic goods producers (which is assumed to be owned by domestic households) Π_t , interest received on domestic bonds $B_t(1+r_t)$ and lump-sum government transfer T_t .

Solving the household's problem, the optimal money demand schedule can be derived as:

$$\left(\frac{M_t}{P_t} \right)^\epsilon = \frac{(C_t^H)^\rho}{1 - \frac{1}{1+r_{t+1}}} \quad (2.5)$$

The optimal labor supply decision is characterized by:

$$\eta L_t^\psi = \frac{W_t}{P_t (C_t^H)^\rho} \quad (2.6)$$

Finally, the household's intertemporal consumption stream is chosen such that

$$\beta E_t \frac{(C_t^H)^\rho}{(C_{t+1}^H)^\rho} \frac{P_t}{P_{t+1}} = \frac{1}{1+r_{t+1}} \quad (2.7)$$

The optimality conditions of the foreign households are entirely analogous, except that foreign household's consumption is denoted by C_t^* , as there is only one type of representative agent in the foreign country.

Firms We assume firms have linear technologies, for each home good i :

$$y_t(i) = L_t(i) \quad (2.8)$$

It is also assumed that, due to high costs of arbitrage for consumers, each individual monopolist can price discriminate across countries. Furthermore, as in Betts and Devereux (1996) and Chari,

Kehoe and McGrattan (2002), we assume local currency pricing: firms set prices (separately) in the currencies of buyers. Finally, prices are assumed to be set one period in advance and cannot be revised until the following period. That is, the home monopolist sets $P_{h,t}(i)$ and $P_{h,t}^*(i)$ optimally at the end of period $t - 1$, and these prices cannot be changed during time t .

Appendix A gives the details of the optimal pricing policies of firms. The firms will just set the price so that it equals to a mark-up over the expected marginal cost and a risk premium term arising from the covariance of the firm's profits with the marginal utility of consumption. The following equations give the prices of the representative home and foreign goods sold in home and foreign markets, respectively.

$$P_{h,t} = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D_t W_t C_t]}{E_{t-1} [D_t C_t]} \quad (2.9)$$

$$P_{h,t}^* = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D_t W_t C_t^*]}{E_{t-1} [D_t S_t C_t^*]} \quad (2.10)$$

$$P_{f,t} = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D_t^* W_t^* C_t]}{E_{t-1} \left[\frac{D_t^*}{S_t} C_t \right]} \quad (2.11)$$

$$P_{f,t}^* = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D_t^* W_t^* C_t^*]}{E_{t-1} [D_t^* C_t^*]} \quad (2.12)$$

where D_t and D_t^* denote the pricing kernels households used to value date t profits. Because all home firms are assumed to be owned by the domestic households, it follows that in equilibrium D_t is the intertemporal marginal rate of substitution in consumption between time $t - 1$ and t :

$$D_t = \beta \frac{(C_t^H)^{-\rho} P_{t-1}}{(C_{t-1}^H)^{-\rho} P_t} \quad (2.13)$$

D_t^* is defined analogously. S_t is the nominal exchange rate at time t .

Government The home government issues the local currency, has no expenditures, and runs a balanced budget every period. The nominal transfer is then given by:

$$T_t = M_t - M_{t-1} \quad (2.14)$$

The stochastic process that describes the evolution of the domestic money supply is:

$$M_t^s = \mu_t M_{t-1}^s \quad (2.15)$$

$$\log(\mu_t) = \varepsilon_{\mu,t} \quad (2.16)$$

where $\varepsilon_{\mu,t} \sim N(0, \sigma_{\varepsilon_\mu}^2)$ is a normally distributed random variable. The stochastic process of money supply in the foreign country is entirely analogous. Also, the home monetary shock and the foreign monetary shock are assumed to be independently distributed, that is, $Cov(\varepsilon_\mu, \varepsilon_\mu^*) = 0$.

2.2 Foreign Exchange Market

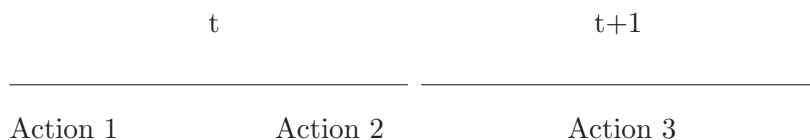
2.2.1 Foreign Exchange Traders

Following closely the work of De Long *et al.* (1990) and Jeanne and Rose (2002), the foreign exchange traders are modelled as overlapping generations of investors who decide how many one-period foreign nominal bonds to buy in the first period of their lives. Traders have the same taste, but differ in their abilities to trade in the foreign bond market. Some of them are able to form accurate expectations on risk and returns, while others have noisy expectation about future returns. The former are referred as the “rational trader” and the latter as the “noise traders”. Hereafter, the informed trader is denoted by a superscript I and the noise trader is denoted by a superscript N .

Two specifications of the model are developed. In the first specification, the number of incumbent noise traders is exogenously determined. In the second one, the traders have to pay a fixed entry cost to trade on the foreign exchange market. The introduction of an entry cost helps to endogenize the noise component of the market. This makes the policy analysis possible as policy makers can affect the composition of traders through the entry cost. In this paper, we focus on the effect of the exchange rate policy on noise traders, so it is assumed that only noise traders need to pay a positive entry cost, which can be affected by policy makers.

In the foreign exchange market, at each period, a generation of foreign exchange traders is born. The continuum of the traders is indexed by $i \in [0, 1]$. Assuming that in each generation of traders, N_I of them are rational traders, and $1 - N_I$ are noise traders. The timing of the model is illustrated in Figure 1.

Figure 1: Timing of Model



Action 1: Time t foreign exchange trader i is born; Time t shocks and nominal interest rates are revealed; The time t born trader i decides if he should enter the foreign bond market.

Action 2: He decides the number of foreign currency bonds $B_{h,t+1}^*(i)$ to purchase based on his expectation about future exchange rate S_{t+1} . To finance his purchase, he borrows $B_{h,t+1}^*(i)S_t$ from the home bond market.

Action 3: Time $t + 1$ exchange rate S_{t+1} is revealed, so the return of his investment in terms of home currency is realized, which equals $S_{t+1}B_{h,t+1}^*(i)(1 + r_{t+1}^*)$. He pays back the principle and interest of his borrowing ($B_{h,t+1}^*(i)S_t(1 + r_{t+1})$), gets the excess return, consumes, and dies.

Let φ_t^i denote the dummy variable characterizing the market-entry condition of period t born foreign exchange trader i . If $\varphi_t^i = 0$, trader i will not enter the foreign bond market and if $\varphi_t^i = 1$, he will enter. At the beginning of period t , trader i will enter the market as long as the expected utility of entering the market is higher than that of not entering:

$$E_t^i(U_t^i \mid \varphi_t^i = 1) \geq E_t^i(U_t^i \mid \varphi_t^i = 0) \quad (2.17)$$

A foreign exchange trader who has entered the foreign bond market maximizes a mean-variance utility function:

$$\max_{B_{h,t+1}^*(i)} E_t^i(C_{t+1}^T(i)) - \frac{a}{2} \text{Var}_t^i(C_{t+1}^T(i)) \quad (2.18)$$

Subject to

$$P_{t+1}C_{t+1}^T = [B_{h,t+1}^*(i)(1 + r_{t+1}^*)S_{t+1} - B_{h,t+1}^*(i)S_t(1 + r_{t+1})] - P_{t+1}c_i \quad (2.19)$$

where $B_{h,t+1}^*(i)$ denotes the amount of one-period foreign currency bonds held by trader i from time t to time $t + 1$, a is the absolute risk aversion coefficient, the cost c_i reflects the costs associated with entering the foreign bond market for trader i .

The entry costs may include tax, information costs for investment in the foreign bond market, and other costs when investing abroad.⁶ To formalize this heterogeneity, here we follow the specification used by Jeanne and Rose (2002). Rational traders are assumed to have a larger stock of knowledge about the economy and thus, do not need to invest in the acquisition of information. Their entry costs are therefore zero. For noise traders, they do not have a natural ability to acquire and process the information about the economy and therefore have to pay an entry cost that is greater than zero.

Although the preferences of the noise traders are the same, the noise traders are assumed to be distinguished from each other by their entry costs. Without loss of generality, the noise traders are

⁶These costs may be modelled in many ways. In this paper, the entry costs are assumed to be resource-consuming in the sense that it consumes the composite consumption good.

indexed by increasing entry costs:

$$c_i = \bar{c} \left\{ \frac{i}{1 - N_I} \right\}^\alpha \quad \text{for } i \in [0, 1 - N_I] \quad (2.20)$$

where $\alpha > 0$ is the curvature parameter and \bar{c} is the parameter characterizing the scale or level of the entry cost of the noise traders. Thus, the noise trader at the left end of the continuum (i near 0) tends to have a lower entry cost and the noise trader towards the right end of the continuum (i near $1 - N_I$) has a higher entry cost.

2.2.2 Optimal demand for foreign bond

Once the traders have decided to enter the market, the optimal demand for foreign bonds of each type of traders can be derived. Substituting Equation 2.19 into Equation 5.1, gives:

$$\max_{B_{h,t+1}^*(i)} E_t^i \left[\frac{B_{h,t+1}^*(i)}{P_{t+1}} S_t (1 + r_{t+1}) \rho_{t+1} - c_i \right] - \frac{a}{2} \text{Var}_t^i \left[\frac{B_{h,t+1}^*(i)}{P_{t+1}} S_t (1 + r_{t+1}) \rho_{t+1} - c_i \right] \quad (2.21)$$

where

$$\rho_{t+1} = \left[\frac{S_{t+1}(1 + r_{t+1}^*)}{S_t(1 + r_{t+1})} - 1 \right] \quad (2.22)$$

is the excess return.

We now discuss the information structure of traders. Specifically, we make the following assumptions about the subjective distribution over ρ_{t+1} . The rational traders can predict ρ_{t+1} correctly⁷; while the noise traders cannot predict the future excess return correctly. That is, for informed traders:

$$E_t^I[\rho_{t+1}] = E_t[\rho_{t+1}] \quad (2.23)$$

$$\text{Var}_t^I[\rho_{t+1}] = \text{Var}_t[\rho_{t+1}] \quad (2.24)$$

For noise traders, following the work of De Long *et al.* (1990), we assume:

$$E_t^N[\rho_{t+1}] = E_t[\rho_{t+1}] + v_t \quad (2.25)$$

$$\text{Var}_t^N[\rho_{t+1}] = \text{Var}_t[\rho_{t+1}] \quad (2.26)$$

$$\text{Var}(v_t) = \lambda \text{Var}(s_t) \quad \text{where } \lambda \in (0, +\infty) \quad (2.27)$$

⁷In other words, for rational traders, the subjective distributions of ρ_{t+1} are the same as the objective distributions.

where v_t is assumed to be *i.i.d* and normally distributed with zero mean. λ can be considered as a parameter characterizing the relative magnitude of noise traders' erroneous beliefs to exchange rate volatility.

From Equations 2.25 and 2.23, compared to the rational trader's expectation, the noise traders' expectation of ρ_{t+1} based on time t information is biased from the true conditional expectation by a random error. Nevertheless, noise traders can correctly forecast the conditional variance of the exchange rate. From Equation 2.27, another assumption is that the unconditional variance of v_t is proportional to the unconditional variance of the exchange rate itself. This assumption helps to tie down the scale of the volatility of noise traders' erroneous beliefs.⁸

From Equation 2.21, the optimal bond holding of trader i can be solved, which is given by:

$$B_{h,t+1}^*(i) = \frac{E_t^i[\rho_{t+1}]}{a \frac{S_t}{P_{t+1}} (1 + r_{t+1}) \text{Var}_t[\rho_{t+1}]} \quad (2.28)$$

Therefore, for each informed trader:

$$B_{h,t+1}^{I,*} = \frac{E_t[\rho_{t+1}]}{a \frac{S_t}{P_{t+1}} (1 + r_{t+1}) \text{Var}_t[\rho_{t+1}]} \quad (2.29)$$

For every noise trader:

$$B_{h,t+1}^{N,*} = \frac{E_t^N[\rho_{t+1}]}{a \frac{S_t}{P_{t+1}} (1 + r_{t+1}) \text{Var}_t[\rho_{t+1}]} \quad (2.30)$$

Obviously, the lower the expected excess return, the higher the excess return volatility and the risk coefficient, the less bond traders (both rational traders and noise traders) will hold. Thus, the traders account for risk when taking positions on assets. At the margin, the return from enlarging one's position in an asset that is mispriced (the expected excess return) is offset by the additional price risk (the volatility of excess return) that must be borne.

Having derived the optimal demand for foreign bonds of each type of traders, we can analyze the equilibrium condition of the foreign bond market.

⁸The logic behind this assumption is that the bias in noise traders' expectation must be related to the volatility of the exchange rate itself, otherwise noise traders might expect the future exchange rate to be volatile even under a fixed exchange rate regime.

2.2.3 Equilibrium condition of the foreign exchange market

Analysis with no entry costs We first analyze a simple case where $\bar{c} = 0$. Thus, all the noise traders will enter the market and the noise component of the market is exogenously determined by the number of existing noise traders on the market.

In each period t , $1 - N_I$ noise traders are present on the foreign bond market. Then, the aggregate demand for foreign bonds by foreign exchange traders of the home country can be denoted as:

$$\begin{aligned} B_{h,t+1}^* &= N_I B_{h,t+1}^{I,*} + (1 - N_I) B_{h,t+1}^{N,*} \\ &= \frac{E_t \left[\frac{S_{t+1}(1+r_{t+1}^*)}{S_t(1+r_{t+1})} - 1 \right] + (1 - N_I)v_t}{a \frac{S_t}{P_{t+1}} (1+r_{t+1}) \text{Var}_t(\rho_{t+1})} \end{aligned} \quad (2.31)$$

$$\Rightarrow E_t \left[\frac{S_{t+1}(1+r_{t+1}^*)}{S_t(1+r_{t+1})} - 1 \right] + \underbrace{(1 - N_I)v_t}_{\text{First Part}} - \underbrace{a \frac{S_t}{P_{t+1}} (1+r_{t+1}) \text{Var}_t(\rho_{t+1}) B_{h,t+1}^*}_{\text{Second Part}} = 0 \quad (2.32)$$

Endogenous entry of noise traders We now endogenize the composition of traders who enter the market in each period by introducing positive entry costs for noise traders.

The entry decision for informed traders is trivial. They bear no entry cost and always enter the foreign bonds market in equilibrium. A noise trader, however, enters if and only if Equation 2.17 is satisfied. As shown in Appendix B, for trader i , this condition takes the form:

$$c_i \leq GB_t^N \quad (2.33)$$

where GB_t^N can be considered as the gross benefit of entry for noise traders and is given by:

$$GB_t^N = \frac{[E_t^N(\rho_{t+1})]^2}{2a \text{Var}_t(\rho_{t+1})} \quad (2.34)$$

The gross benefit of entry can be interpreted intuitively. For noise traders, it increases with the expected excess return and decreases with the conditional time $t + 1$ exchange rate volatility. Note that in our general equilibrium setting, both terms are functions of the number of incumbent noise traders.

Let $c_t^* = GB_t^N$ be the cut-off value of entry cost. From Equation 2.33, for noise trader i

$$\text{if } c_i \leq c_t^*, \quad \varphi_t^i = 1 \quad (2.35)$$

$$\text{if } c_i \geq c_t^*, \quad \varphi_t^i = 0 \quad (2.36)$$

The number of incumbent noise traders n_t is then given by:

$$n_t = \left(\frac{c_t^*}{\bar{c}} \right)^{\frac{1}{\alpha}} (1 - N_I) = \left(\frac{[E_t^i(\rho_{t+1})]^2}{2a \text{Var}_t(\rho_{t+1}) \bar{c}} \right)^{\frac{1}{\alpha}} (1 - N_I) \quad (2.37)$$

Apparently, the number of active noise traders on the market increases with the square of the expected excess return and the number of existing noise traders, and decreases with the entry cost, the risk aversion coefficient a , and the excess return volatility. The economic intuition behind Equation 2.37 is as follows. The presence of more active noise traders creates a higher expected excess return and incentives for other noise traders to enter the market, however, the extra volatility brought about by their entry will reduce the gross benefit of entry for noise traders. In equilibrium, the two effects balance and no more noise traders will enter.

Substituting Equation 2.37 into $B_{h,t+1}^* = N_I B_{h,t+1}^{I,*} + n_t B_{h,t+1}^{N,*}$, we derive the equilibrium condition of the foreign bond market when the entry decision of traders is endogenized:

$$E_t \left[\frac{S_{t+1}(1 + r_{t+1}^*)}{S_t(1 + r_{t+1})} - 1 \right] + \underbrace{\frac{\left(\frac{c_t^*}{\bar{c}} \right)^{\frac{1}{\alpha}} (1 - N_I) v_t}{N_I + \left(\frac{c_t^*}{\bar{c}} \right)^{\frac{1}{\alpha}} (1 - N_I)}}_{\text{First Part}} - a \underbrace{\frac{S_t(1 + r_{t+1})}{P_{t+1} \left[N_I + \left(\frac{c_t^*}{\bar{c}} \right)^{\frac{1}{\alpha}} (1 - N_I) \right]}}_{\text{Second Part}} \text{Var}_t(\rho_{t+1}) B_{h,t+1}^* = 0 \quad (2.38)$$

Equations 2.32 and 2.38 represent the interest parity conditions in this economy. Note that the uncovered interest parity does not hold in this model. The last two terms in Equations 2.32 and Equation 2.38 show the deviation from the uncovered interest parity when noise traders are present in the market. This deviation consists of two parts: the expectation error of the noise traders, and the risk premium term, since the foreign exchange traders are risk averse.

In our model, as in De Long *et al.* (1990), the noise traders can “create their own space”: the uncertainty of the noise traders’ expectations over the future exchange rate increases the risk borne by informed traders engaged in arbitrage against noise traders. The aversion to this risk will severely limit arbitrage, especially in an overlapping-generation framework. Short-horizon investors must bear the risk that they may be required to liquidate their positions at a time when asset prices are pushed even further away (by noise traders) from the fundamental values than when the investment was made. Therefore, as shown in Section 3, exchange rate can diverge significantly from the fundamental values.

2.3 Equilibrium Condition

Equilibrium for this economy is a collection of 26 sequences $(P_t, P_t^*, P_{h,t}, P_{h,t}^*, P_{f,t}, P_{h,t}^*, C_t, C_t^T, C_t^H, C_t^*, C_{h,t}, C_{h,t}^*, C_{f,t}, C_{h,t}^*, S_t, r_t, r_t^*, D_t, D_t^*, W_t, W_t^*, B_t, B_t^*, B_{h,t}^*, L_t, L_t^*)$ satisfying 26 equilibrium conditions. They include the six household optimality conditions (Equations 2.5, 2.6, 2.7 and their foreign counterparts), the definition of the price indexes (Equation 2.4 and its foreign analogy), the definition of the pricing kernel (Equation 2.13 and its foreign analogy), the interest parity conditions (Equation 2.32 or Equation 2.38), the four individual demand equations, the four pricing conditions, and the four market clearing conditions for the bonds and goods markets:

$$B_{t+1} = S_t B_{h,t+1}^* \quad \forall t \quad (2.39)$$

$$B_t^* + B_{h,t}^* = 0 \quad (2.40)$$

$$L_t = C_{h,t} + C_{h,t}^* \quad (2.41)$$

$$L_t^* = C_{f,t} + C_{f,t}^* \quad (2.42)$$

Finally, the budget constraint of the foreign exchange traders. For the exogenous entry specification;

$$P_t C_t^T = B_{h,t}^*(1 + r_t^*)S_t - B_{h,t}^* S_{t-1}(1 + r_t) \quad (2.43)$$

while for the endogenous entry specification:

$$P_t C_t^T = [B_{h,t}^*(1 + r_t^*)S_t - B_{h,t}^* S_{t-1}(1 + r_t)] - P_t \sum_{i=0}^{n_t} c_i \quad (2.44)$$

And the home country aggregate consumption equation:

$$C_t = C_t^H + C_t^T \quad \text{Exogenous Entry} \quad (2.45)$$

$$C_t = C_t^H + C_t^T + \sum_{i=0}^{n_t} c_i \quad \text{Endogenous Entry} \quad (2.46)$$

Then the above two equations, the budget constraints of the home households 2.2, Equation 2.39 and its one-period lag can be combined to get the national budget constraint of the home country:

$$P_t C_t = W_t L_t + \Pi_t + S_t B_{h,t}^*(1 + r_t^*) - S_t B_{h,t+1}^* \quad (2.47)$$

where

$$\Pi_t = \omega \left[(P_{h,t} - W_t) \left(\frac{P_{h,t}}{P_t} \right)^{-\gamma} C_t + (P_{h,t}^* S_t - W_t) \left(\frac{P_{h,t}^*}{P_t^*} \right)^{-\gamma} C_t^* \right] \quad (2.48)$$

3 Model Solution

The model can be solved by log-linearization around a non-stochastic, symmetric steady state (as described in Appendix C), where net foreign assets are zero, all prices are equal, and the exchange rate is unity. Given the log-linearized system, the deviations of the exchange rate and the macroeconomic variables from their $t - 1$ expectations are solved in terms of exogenous money supply shocks and the expectation error shocks. Appendix D describes the complete solution of the model. Here, the important and intuitive features of the solution are outlined.

Hereafter, $\hat{x}_t = \log(X_t) - \log(\bar{X})$, where \bar{X} is the non-stochastic steady state value of variable X_t .

3.1 Log-linearization

Appendix D.1 gives the complete log-linearized system. Here just the economic intuition of the log-linearized price indexes and the parity conditions (Equations 2.32 and 2.38) are discussed.

Price Indexes From the log-linearization of the pricing equation of firms, we may find that the firms will set prices equal to the anticipated marginal costs:

$$p_{\hat{h},t} = E_{t-1}[\hat{w}_t], \quad p_{\hat{h},t}^* = E_{t-1}[\hat{w}_t] - E_{t-1}[\hat{s}_t] \quad (3.1)$$

$$p_{\hat{f},t} = E_{t-1}[\hat{w}_t^*] + E_{t-1}[\hat{s}_t], \quad p_{\hat{f},t}^* = E_{t-1}[\hat{w}_t^*] \quad (3.2)$$

Together, these equations give the price index for the home and foreign country:

$$\hat{p}_t = \frac{1}{2}(E_{t-1}[\hat{w}_t] + E_{t-1}[\hat{w}_t^*] + E_{t-1}[\hat{s}_t]) \quad (3.3)$$

$$\hat{p}_t^* = \frac{1}{2}(E_{t-1}[\hat{w}_t] - E_{t-1}[\hat{s}_t] + E_{t-1}[\hat{w}_t^*]) \quad (3.4)$$

Equations 3.3 and 3.4 establish that in an expected sense, PPP holds. This is not surprising, as the prices can fully adjust to all shocks after one period.

Parity Equations Since the non-linearities of the interest parity equations are important for understanding the dynamics of the economy, especially for the exchange rate, the variance term and

expectation error term will be kept through second-order approximation when the parity equations are log-linearized. The detailed derivation is given in Appendix D.1.

Linearizing the interest parity condition for the exogenous entry specification (2.32) gives

$$\hat{s}_t = E_t(s_{t+1}) - \beta(dr_{t+1} - dr_{t+1}^*) + (1 - N_I)v_t - a \frac{(1 + \bar{r})\bar{S}}{\bar{P}} Var_t[s_{t+1}] dB_{h,t+1}^* \quad (3.5)$$

where $dX_t = X_t - \bar{X}$.

Linearizing the interest parity condition for the endogenous entry specification (2.38) gives:

$$\hat{s}_t = E_t(s_{t+1}) - \beta(dr_{t+1} - dr_{t+1}^*) + \frac{1}{N_I} n_t v_t - a \frac{(1 + \bar{r})\bar{S}}{\bar{P}N_I} Var_t[s_{t+1}] dB_{h,t+1}^* \quad (3.6)$$

where n_t , the number of incumbent noise traders is given by⁹:

$$n_t = \frac{\{E_t(s_{t+1}) - \hat{s}_t - \beta(dr_{t+1}^* - dr_{t+1}) + v_t\}^2 (1 - N_I)}{2a Var_t(s_{t+1}) \bar{c}} \quad (3.7)$$

Similar to Equations 2.32 and 2.38, Equations 3.5 and 3.6 show that the biased expectation of noise traders causes a stochastic deviation from uncovered interest rate parity. This deviation is composed of two parts: the noise traders' expectation errors and a risk premium term. The former, as discussed intensively by Devereux and Engel (2002), is different from the traditional risk premium term that arises from the risk aversion of *households*. It captures the fluctuations in the exchange rate due to the variation of noise traders' misperceptions. As one would expect, the greater the number of noise traders, the greater is the impact of the noise traders' expectation error on the exchange rate. For example, when a positive expectation error shock occurs, the noise trader will have a higher demand for foreign bonds and foreign currency, which leads to a domestic currency depreciation. Therefore, this term tends to increase exchange rate volatility.

In contrast to Devereux and Engel (2002), there is also a risk premium term in the parity condition because of the assumption that traders are risk averse. Intuitively, when exchange rate volatility increases, traders would not hold the foreign bonds unless compensated for bearing the extra risk. Consequently, the price of the foreign currency (risky asset) should fall. Thus, the risk aversion of traders (both informed traders and noise traders) will tend to reduce exchange rate volatility.

⁹Hereafter, the curvature parameter of entry costs α is set to be equal to 1. The model can be easily extended to the case where $\alpha > 1$ or $0 < \alpha < 1$, and the main results will not change.

Solve for $T - 1$ Expectations Taking a linear approximation of the budget constraint of home household¹⁰, using the pricing indexes (Equations 3.3 and 3.4), the relationship between \hat{c}_t and \hat{c}_t^H , and the fact that (as will hold in equilibrium) in an expected sense, any initial change in net foreign assets is persistent, gives:

$$E_{t-1}(\hat{c}_t^H - \hat{c}_t^*) = E_{t-1}\left(\frac{dC_t^T}{C}\right) + (1 - \gamma)E_{t-1}(\hat{w}_t - \hat{w}_t^* - \hat{s}_t) + \frac{2}{\bar{P}\bar{C}}\left(\frac{1}{\beta} - 1\right)dB_t \quad (3.8)$$

Use $\bar{r} = \frac{1}{\beta} - 1$ and the fact that $\frac{dC_t^T}{C} = 0$ ¹¹, we get:

$$E_{t-1}(\hat{c}_t^H - \hat{c}_t^*) = (1 - \gamma)E_{t-1}(\hat{w}_t - \hat{w}_t^* - \hat{s}_t) + \frac{2\bar{r}}{\bar{P}\bar{C}}dB_t \quad (3.9)$$

Equation 3.9 shows that the relative home consumption increases in changes of the initial net foreign assets position and decreases in the expected terms of trade, as long as the elasticity of substitution between home and foreign composite goods γ is greater than 1.

From the linear approximation of the goods market clearing conditions (Equations 2.41 and 2.42), and the labor supply equation 2.6 and its foreign equivalent, using price indexes (Equations 3.3 and 3.4), and taking expectations at $t - 1$, gives:

$$E_{t-1}(\hat{w}_t - \hat{w}_t^* - \hat{s}_t) = \frac{\rho}{1 + \psi\gamma}E_{t-1}(\hat{c}_t^H - \hat{c}_t^*) \quad (3.10)$$

Equations 3.10 and 3.9 give a relationship between the initial net foreign assets and the expected relative consumption:

$$E_{t-1}(\hat{c}_t^H - \hat{c}_t^*) = \frac{2\bar{r}}{\bar{P}\bar{C}}\frac{1}{\sigma}dB_t \quad (3.11)$$

where $\sigma = 1 - \frac{(1-\gamma)\rho}{1+\psi\gamma}$. An increase in the home country's net foreign assets leads to an expected increase in the home relative consumption.

Finally, hereafter, we assume that the elasticity of the money demand $\epsilon = 1$.¹² In equilibrium, given the random walk assumption of money supply process and the log money utility function, a

¹⁰After the home money market equilibrium condition $M_t = M_{t-1} + T_t$ and the profit condition 2.48 are imposed.

¹¹This is because the traders' income is derived from the product of the net foreign assets ($B_{h,t+1}^*$) and the excess return (ρ_{t+1}). At the steady state, both the net foreign assets and the excess return are equal to zero. If we log-linearize the budget constraint of the traders (Equation 2.19) around the steady state, dC_T will be a product of two small deviations from zero, and tends to be very small. See Appendix D.1 for details.

¹²An estimate of the consumption elasticity of the demand for money (equal to $\frac{1}{\epsilon}$ in the model) is very close to unity, as reported by Mankiw and Summers(1986).

very convenient property is that the nominal interest rate will be constant. This is because, if the log of the money stock follows a random walk, so does the log of the term $P_t(C_t^H)^\rho$. Using this fact and pricing indexes (Equations 3.3 and 3.4), from the linear approximation of the money demand function and its foreign equivalent we could get:

$$E_{t-1}(\hat{m}_t - \hat{m}_t^*) = \rho E_{t-1}(\hat{c}_t^H - \hat{c}_t^*) + E_{t-1}(\hat{s}_t) \quad (3.12)$$

Therefore, in an expected sense, the exchange rate is consistent with the standard monetary model.

3.2 Model 1: Exogenous Entry

Response of Exchange rates to Money shocks Hereafter, let $x_{t+j} \tilde{=} x_{t+j} - E_{t-1}(x_{t+j})$, $j \geq 0$ denote the deviation of a variable from its date $t-1$ expectation. Then, the log-linearized home household's budget constraint minus its $t-1$ expectation gives:

$$\tilde{c}_t^H - \tilde{c}_t^* + \frac{2}{\bar{P}\bar{C}} dB_{t+1} = \tilde{s}_t \quad (3.13)$$

or

$$dB_{t+1} = \frac{\bar{P}\bar{C}}{2} [\tilde{s}_t - (\tilde{c}_t^H - \tilde{c}_t^*)] \quad (3.14)$$

The right-hand side of Equation 3.13 represents the relative wealth effect of an unanticipated shock to the exchange rate through firms' profits. This relative wealth increase will then be spread between an increase in relative home consumption and net foreign assets accumulation.

Using Equation 3.11 (updated to period $t+1$) and Equation 3.14, we may establish that :

$$(\tilde{c}_t^H - \tilde{c}_t^*) + \frac{\sigma}{\bar{r}} E_t(\tilde{c}_{t+1}^H - \tilde{c}_{t+1}^*) = \tilde{s}_t \quad (3.15)$$

This equation gives a relationship between current relative consumption, expected period $t+1$ relative consumption, and the unanticipated shock to the exchange rate. It represents the constraints on these three variables implied by the intertemporal current account.

Then, substituting the log-linearized intertemporal optimality equations into the interest parity condition, we may obtain the consumption-based interest parity condition:

$$\begin{aligned} \rho E_t(c_{t+1} - \hat{c}_t) + E_t(p_{t+1} - \hat{p}_t) &= \rho E_t(\hat{c}_{t+1}^* - \hat{c}_t^*) + E_t(\hat{p}_{t+1}^* - \hat{p}_t^*) \\ + E_t(s_{t+1}) - \hat{s}_t + (1 - N_I)v_t - a \frac{(1 + \bar{r})\bar{S}}{\bar{P}} \text{Var}_t[s_{t+1}] dB_{h,t+1}^* & \end{aligned} \quad (3.16)$$

where the left-hand side is the domestic nominal interest rate and the first two terms on the right-hand side represent the foreign nominal interest rate. Using the price indexes (Equations 3.3 and 3.4) and subtracting Equation 3.16 from its $t - 1$ expectation, we may get the relationship between current relative consumption and anticipated future relative consumption. Appendix D.2 gives the detailed derivation.

$$E_t(c_{t+1}^{\tilde{H}} - c_{t+1}^*) = (c_t^{\tilde{H}} - c_t^*) - \frac{1}{\rho}[\tilde{s}_t - (1 - N_I)v_t + a\frac{(1 + \bar{r})\bar{S}}{\bar{P}}Var_t[s_{t+1}]dB_{h,t+1}^*] \quad (3.17)$$

In this equation, expected consumption growth in the home country decreases in response to an unanticipated exchange rate depreciation, since this generates an unanticipated real depreciation, and therefore reduces the home country's real interest rate. From Equation 3.16, a positive shock to foreign exchange traders' expectations of the future exchange rate will increase the home real interest rate and lead to an increase in expected consumption growth of the home country. The last term in Equations 3.16 and 3.17, which denotes a risk premium term due to the risk-aversion of the foreign exchange traders, tends to reduce the real interest rate. Therefore, it has a negative effect on the expected home relative consumption growth.

Finally, the relation between relative money supply and relative consumption can be derived from the money demand equations¹³ :

$$\tilde{m}_t - \tilde{m}_t^* = \rho(c_t^{\tilde{H}} - c_t^*) \quad (3.18)$$

Putting Equations 3.18, 3.15, 3.17 and 3.14 together, we can get a system of equilibrium conditions that characterizes $\{\tilde{s}_t, \tilde{c}_t - c_t^*, dB_{t+1}\}$. We may solve for the deviation of the exchange rate from its $t - 1$ expectation (\tilde{s}_t) in terms of the exogenous money shock and expectation error of the noise traders. The details of derivation is given in Appendix D.2.

$$\tilde{s}_t = (\tilde{m}_t - \tilde{m}_t^*)\frac{1 + \frac{\sigma}{\bar{r}} + \frac{\phi}{\rho}Var_t(s_{t+1})}{\rho + \frac{\sigma}{\bar{r}} + \phi Var_t(s_{t+1})} + \frac{\frac{\sigma}{\bar{r}}}{\rho + \frac{\sigma}{\bar{r}} + \phi Var_t(s_{t+1})}(1 - N_I)v_t \quad (3.19)$$

where

$$\phi = \frac{a(1 + \bar{r})\bar{S}\bar{C}}{2} \frac{\sigma}{\bar{r}} \quad (3.20)$$

From Equation 3.19, the conditional mean: $E_t(s_{t+1})$ and the variance of the future exchange rate deviation: $Var_t(s_{t+1})$ may be solved. Since \tilde{s}_t is linear in \tilde{m}_t , \tilde{m}_t^* and v_t and the monetary shocks and expectation error shocks are normally distributed with zero mean and constant variance,

¹³Notice that $\tilde{m}_t = \varepsilon_{\mu,t}$, and $\tilde{m}_t^* = \varepsilon_{\mu,t}^*$. We use \tilde{m}_t and \tilde{m}_t^* for notational convenience.

$E_t(s_{t+1}) = E(s_{t+1}) = 0$, $Var_t(s_{t+1}) = Var(s_{t+1}) = \text{constant} \equiv V_s$, and V_s is determined by the following implicit function:

$$V_s = \frac{\left(\frac{1+\frac{\sigma}{\bar{r}}+\frac{\phi}{\rho}V_s}{\rho+\frac{\sigma}{\bar{r}}+\phi V_s}\right)^2}{1 - \left(\frac{\frac{\sigma}{\bar{r}}}{\rho+\frac{\sigma}{\bar{r}}+\phi V_s}\right)^2 (1 - N_I)^2 \lambda} [Var(\tilde{m}_t) + Var(\tilde{m}_t^*)] \quad (3.21)$$

If the noise traders are risk neutral, then $a = 0$, which implies $\phi = \frac{\phi}{\rho} = 0$. Then when $N_I = 0$, Equation 3.21 becomes:

$$V_s = \frac{(1 + \frac{\sigma}{\bar{r}})^2 (Var(\tilde{m}_t) + Var(\tilde{m}_t^*))}{(\rho + \frac{\sigma}{\bar{r}})^2 [1 - (\frac{\sigma}{\rho\bar{r} + \sigma})^2 \lambda]} \quad (3.22)$$

which is exactly the same as Equation 3.20 from Devereux and Engel(2002).¹⁴

Therefore, the coefficient ϕ is associated with the risk-aversion of traders. The higher the risk aversion coefficient, the lower will be the exchange rate volatility. For both types of traders, their aversion to risk prevents exchange rate volatility from increasing too much. To see this, it can be shown that as long as $\rho > 1$, the numerator $\left(\frac{1+\frac{\sigma}{\bar{r}}+\frac{\phi}{\rho}V_s}{\rho+\frac{\sigma}{\bar{r}}+\phi V_s}\right)^2$ on the right-hand side of Equation 3.21 is decreasing in V_s and the denominator $1 - \left(\frac{\frac{\sigma}{\bar{r}}}{\rho+\frac{\sigma}{\bar{r}}+\phi V_s}\right)^2 (1 - N_I)^2 \lambda$ is increasing in V_s .

Can the exchange rate display ‘excess volatility’ in this model? When $\rho = 1$, the coefficient of $(\tilde{m}_t - \tilde{m}_t^*)$ is exactly 1 in Equation 3.19. Therefore, with no noise traders, the exchange rate volatility will be equal to that of the fundamentals. If noise traders are present on the market, the exchange rate volatility may be much higher than the fundamental volatility, even when $\rho = 1$.

Volatilities of macroeconomic fundamentals Now we will find out the response of the macroeconomic fundamentals such as consumption, labor and wage to the exogenous monetary shocks and expectation error shocks. From the log-linearized goods market clearing condition, labor supply condition and the money demand condition, we can derive the functions that characterize the response of macroeconomic fundamentals to these shocks.

$$\tilde{w}_t = \frac{\psi}{2\rho}(\tilde{m}_t + \tilde{m}_t^*) + \tilde{m}_t \quad (3.23)$$

$$\tilde{l}_t = \frac{1}{2\rho}(\tilde{m}_t + \tilde{m}_t^*) \quad (3.24)$$

¹⁴Notice that no distributors are present in our model.

$$\tilde{c}_t = \frac{1}{\rho} \tilde{m}_t \quad \tilde{c}_t^* = \frac{1}{\rho} \tilde{m}_t^* \quad (3.25)$$

From the above equations:

$$\text{Var}(\tilde{c}_t) = \text{Var}(\tilde{c}_t^*) = \frac{1}{\rho^2} \text{Var}(\tilde{m}_t) \quad (3.26)$$

$$\text{Var}(\tilde{w}_t) = \left(\frac{\psi}{2\rho} + 1\right)^2 \text{Var}(\tilde{m}_t) + \left(\frac{\psi}{2\rho}\right)^2 \text{Var}(\tilde{m}_t^*) \quad (3.27)$$

$$\text{Var}(\tilde{l}_t) = \left(\frac{1}{2\rho}\right)^2 (\text{Var}(\tilde{m}_t) + \text{Var}(\tilde{m}_t^*)) \quad (3.28)$$

In other words, the volatilities of the macroeconomic fundamentals are only decided by the volatility of the relative monetary shock and the values of the parameters, but not by the volatility of the expectation error and the number of incumbent noise traders in the market.

Finally, from Equations 3.14 and 3.18, the net foreign assets are given by:

$$d\tilde{B}_{t+1} = \frac{\bar{P}\bar{C}}{2} \left[\tilde{s}_t - \frac{1}{\rho} (\tilde{m}_t - \tilde{m}_t^*) \right] \quad (3.29)$$

Therefore, the volatility of the net foreign assets will be affected by the number of incumbent noise traders.

3.3 Model 2: Endogenous Entry

The endogenous entry case is similar to the exogenous entry case except for the interest parity equation.

Substituting the log-linearized intertemporal optimality conditions into the endogenous interest parity condition (Equation 3.6), we may get the consumption-based interest parity condition:

$$\begin{aligned} \rho E_t(c_{t+1} - \hat{c}_t) + E_t(p_{t+1} - \hat{p}_t) &= \rho E_t(c_{t+1}^* - \hat{c}_t^*) + E_t(p_{t+1}^* - \hat{p}_t^*) \\ &+ E_t(s_{t+1}) - \hat{s}_t + \frac{1}{N_I} n_t v_t - a \frac{(1+\bar{r})\bar{S}}{\bar{P}N_I} \text{Var}_t[s_{t+1}] dB_{h,t+1}^* \end{aligned} \quad (3.30)$$

Using the price indexes, we may find a condition analogous to Equation 3.17:

$$E_t(c_{t+1}^{\tilde{H}} - c_{t+1}^*) = (c_t^{\tilde{H}} - c_t^*) - \frac{1}{\rho} [\tilde{s}_t - \frac{1}{N_I} n_t v_t + a \frac{(1+\bar{r})\bar{S}}{\bar{P}N_I} \text{Var}_t[s_{t+1}] dB_{h,t+1}^*] \quad (3.31)$$

where ¹⁵

$$n_t = \frac{\{E_t(s_{t+1}) - \tilde{s}_t + v_t\}^2 (1 - N_I)}{2a \text{Var}_t(s_{t+1}) \bar{c}} \quad (3.32)$$

¹⁵Here we use the fact that nominal interest rate are constant and $\tilde{s}_t = \hat{s}_t - E_{t-1}(\hat{s}_t)$.

Equations 3.18, 3.15, 3.14 and Equations 3.31, 3.32 give the solution of the endogenous entry model, and the derivation is entirely analogous to Equation 3.19 :

$$\tilde{s}_t = (\tilde{m}_t - \tilde{m}_t^*) \frac{1 + \frac{\sigma}{\bar{r}} + \frac{\phi'}{\rho} Var_t(s_{t+1})}{\rho + \frac{\sigma}{\bar{r}} + \phi' Var_t(s_{t+1})} + \frac{\frac{\sigma}{\bar{r}}}{\rho + \frac{\sigma}{\bar{r}} + \phi' Var_t(s_{t+1})} \frac{\{E_t(s_{t+1}) - \tilde{s}_t + v_t\}^2 (1 - N_I)}{2a Var_t(s_{t+1})} \frac{(1 - N_I)}{N_I \bar{c}} v_t \quad (3.33)$$

where

$$\phi' = \frac{a(1 + \bar{r})\bar{S}\bar{C}}{2N_I} \frac{\sigma}{\bar{r}} \quad (3.34)$$

As shown in Appendix F, the process of \tilde{s}_t can be simulated, then we may establish that $Var_t(s_{t+1}) = Var(s_{t+1}) = \text{constant} \equiv V_s$ and $E_t(s_{t+1}) = E(s_{t+1}) = \text{constant} \equiv E_s$. By the numerical undetermined coefficient method described in Appendix F, we can solve for \tilde{s}_t , V_s and E_s .

Analogous to the exogenous entry case, when $\rho = 1$ and no noise traders are present ($N_I = 1$), $\tilde{s}_t = (\tilde{m}_t - \tilde{m}_t^*)$, exchange rate volatility will be identical to that of the fundamental. When there are noise traders in the foreign exchange market, the exchange rate may diverge significantly from the fundamental values.

The expression for net foreign assets, consumption, labor, and wage are exactly the same as in the exogenous case.

4 Results

Equations 3.19 and 3.33 are too complicated to be solved analytically, so the numerical undetermined coefficient method (as described in Appendix F) are used to solve for \tilde{s}_t , V_s and E_s .

Table 1 gives the parameter values that are used in the numerical simulation. We choose $\beta = 0.94$, which produces a steady state real interest rate of six percent, about the average long-run real return on stocks. The parameters η and ψ are set so that the elasticity of labor supply is 1 and the time devoted to work is one quarter of the total time in the steady state. The business cycle literature has a wide range of estimates for the curvature parameter ρ . Chari, Kehoe, and McGratten(2002) set $\rho = 5$ to generate a high volatility of the real exchange rate. In our model, a high exchange rate volatility can be obtained without high risk aversion of households, so it is set to equal 2. For the final goods technology parameters, the elasticity of substitution between domestic produced goods θ is set to 11 following Betts and Devereux(2000). This gives a wage-price mark-up of about 1.1, which

is consistent with the finding of Basu and Fernald(1994). The elasticity of substitution between home goods and foreign goods γ is set to be 1.5, following Chari, Kehoe, and McGratten(2002) and Backus, Kehoe, and Kydland(1994). Note that, other parameters, such as the money supply process, number of informed traders on the market, and entry costs, are not fully calibrated as the purpose of this paper is just qualitative analysis.

TABLE 1
Parameter Values

<i>Exogenous Case</i>	
Preferences	$\beta = 0.94, \rho = 2, \epsilon = 1, \eta = 58.2, \psi = 1$
Final goods technology	$\theta = 11, \gamma = 1.5, \omega = 0.5$
Money Growth Process	$corr(\varepsilon_\mu, \varepsilon_\mu^*) = 0, \sigma_{\varepsilon_\mu}^2 = \sigma_{\varepsilon_\mu^*}^2 = 0.01$
Foreign exchange traders	$\bar{c} = 0, N_I \in [0, 1], \lambda = 1, 1.5$
Steady State Values	$\mu_{ss} = \mu_{ss}^* = 1, M_{ss} = M_{ss}^* = 2$
<i>Endogenous Case^a</i>	
Foreign exchange traders	$\bar{c} > 0, N_I \in [0, 1], \lambda = 1.5$

^aOther parameters in the endogenous case are the same as in the exogenous case.

4.1 Exogenous Case

First the exogenous entry case is solved. Tables 2 and 3 illustrate the results of the simulations, for different values of λ . The first ten rows show the changes in the volatilities of the exchange rate and the net foreign assets when the number of noise traders increases from 0 to 1. The last three rows report the volatilities of the macroeconomic fundamental variables, given the calibrated parameter values.

From Tables 2 and 3, three important findings are: First, the exchange rate volatility increases when the number of noise traders increases, while the volatilities of macroeconomic fundamentals remain constant. Moreover, the exchange rate volatility is much higher than that of the macroeconomic fundamentals. Second, from the functional form of \tilde{s}_t , listed in the first column of Tables 2 and 3, the impact of fundamental monetary shocks on the exchange rate (coefficient of \tilde{m}_t or \tilde{m}_t^*) decreases in the number of noise traders. Meanwhile, the effect of expectation error on exchange rate (coefficient of v_t) increases when more noise traders are present on the market. Third, the exchange rate volatility is higher when the magnification coefficient λ increases.

Therefore, the critical implication is that ‘disconnection’ does exist between the exchange rate and the macroeconomic fundamentals in this model. Thus, the presence of noise traders in the foreign exchange market, combined with local currency pricing, generates a degree of exchange rate volatility that may be much higher than that of the underlying fundamental shocks. In other words, the “exchange rate disconnect puzzle” may be explained by the approach suggested in this paper.

To understand intuitively why the disconnect puzzle can be solved in such a model, we may examine the case without noise traders. Obviously, the presence of local currency pricing tends to remove the expenditure-switching or substitution effects of exchange rate movements. Nevertheless, with just local currency pricing, the dynamic model will not generate a highly volatile exchange rate and the disconnection, as an exchange rate shock also affects the home real interest rates through the interest rate parity condition.

Rewriting Equation 3.17 by omitting the expectation error and the risk premium term, gives:

$$\rho E_t(c_{t+1}^{\tilde{H}} - c_{t+1}^*) = \rho(c_t^{\tilde{H}} - c_t^*) - \tilde{s}_t \quad (4.1)$$

Together with Equation 3.15,

$$(c_t^{\tilde{H}} - c_t^*) + \frac{\sigma}{\bar{r}} E_t(c_{t+1}^{\tilde{H}} - c_{t+1}^*) = \tilde{s}_t$$

Equation 4.1 illustrates why exchange rate volatility is limited without noise traders. When a depreciation of home currency occurs, the domestic currency value of foreign sales will increase, giving rise to an increase in home wealth. Equation 3.15 indicates that this positive wealth effect increases both current and future relative consumption in the home country.

Meanwhile, as an arbitrage condition, the interest parity condition (Equation 4.1) implies that a depreciation of home currency today will reduce the relative real interest rate in the home country and change the path of consumption, so that the current home consumption will increase, relative to the expected future consumption (holding foreign consumption constant). If the change in exchange rate is large and a disconnection between consumption and exchange rate is needed (i.e., the change in current consumption has to be small), the only possible way suggested by Equation 4.1 is for the expected future consumption to drop a lot. Nevertheless, Equation 3.15 implies that the future relative consumption of the home country should increase when a depreciation of home currency occurs. Therefore, with no noise traders, the only way to explain the difference between the exchange rate volatility and the fundamental volatility is by introducing a high value of ρ , which is exactly the mechanism emphasized by Chari, Kehoe, and McGratten(2002).

When the noise traders are introduced into the interest parity condition, we can see from Equation 3.17 that now a large increase of exchange rate and a small change in current consumption do not necessarily imply a large drop of expected future consumption, because the presence of the expectation errors and the risk premium term of noise traders also drive wedges between the home and foreign real interest rates. This could be called the “level effects” created by the noise traders.

The presence of noise traders also creates a “volatility effect”, which is due to the assumption that the volatility of v_t itself is proportional to the exchange rate volatility. This assumption “magnifies” the response of the exchange rate to the expectation error of noise traders. When the nominal exchange rate volatility increases, so does the expectation error volatility, which further increases the exchange rate volatility until the system reaches an equilibrium where $Var(v_t) = \lambda Var(s_t)$.¹⁶ Thus, λ is the parameter characterizing this magnification effect. The higher λ , the higher is the exchange rate volatility.

Therefore, volatile exchange rates can be obtained in this model. Still, why is the high volatility not transferred to other macroeconomic variables (except for the net foreign assets)?

Normally, there are two channels through which the exchange rate affects other macroeconomic variables: expenditure-switching effects and wealth effects. We assume local currency pricing stickiness in this model, that is, the firm can set prices separately in the currencies of buyers and the prices are sticky. Since the prices of the import goods are sticky in terms of the local currency, the relative price of home-produced goods to foreign-produced goods will remain unchanged when the exchange rate changes. Therefore, the expenditure-switching channel is completely shut down in the present model.
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With regard to the wealth effect, from Equation 3.15, the increase in wealth that comes from an unexpected depreciation will be spread between an increase in relative home consumption and the net foreign assets accumulation. From Equation 3.18, however, the increase in relative home consumption is limited by the relative money shocks due to the real balance effect. Therefore, the net foreign assets

¹⁶Please refer to Appendix E for the method used in the simulation to make $Var(v_t) = \lambda Var(s_t)$.

¹⁷In another paper, the assumption of local currency pricing is relaxed and the baseline model is analyzed under producer currency pricing or a mixture of local currency pricing and producer currency pricing (i.e., the exchange rate pass-through is between 0 and 1). As expected, with positive exchange rate pass-through, the volatilities of macroeconomic fundamentals (consumption, labor and wage) depend on the exchange rate volatility. The higher the exchange rate pass-through, the higher is the correlation between exchange rate volatility and the fundamental volatilities.

will absorb most of the wealth increase. This is actually shown in Tables 2 and 3, when the volatility of the nominal exchange rate increases, so does the volatility of the net foreign assets. Nevertheless, the magnitude of the volatility of the net foreign asset and the expected future relative consumption are small quantitatively, especially when compared to that of the exchange rate. That implies the wealth effect is also small quantitatively. From Equation 3.15,

$$E_t(c_{t+1}^{\tilde{H}} - c_{t+1}^*) = \frac{\bar{r}}{\sigma}[\tilde{s}_t - (c_t^{\tilde{H}} - c_t^*)] \quad (4.2)$$

It can be seen that the volatility of the change in expected future consumption is quantitatively small because $\frac{\bar{r}}{\sigma}$ is small given reasonable parameter values.¹⁸ The economic intuition is that the consumption-smoothing behavior of infinitely lived households limits the wealth effect in this model. When a shock occurs that leads to an exchange rate depreciation occurs, the households increase their holdings of net foreign assets. This increase will be spread over many future periods because the households want to smooth their future consumption. The increase in the expected consumption of the next period is then quite small. Therefore, the more risk averse the households (the higher ρ), the bigger will be $\sigma = 1 - \frac{(1-\gamma)\rho}{1+\psi\gamma}$ (suppose that $\gamma > 1$), and the smaller will be the wealth effect.

Moreover, in the monetary model of exchange rate without noise traders, the monetary shocks lead to movements in both macroeconomic fundamentals and exchange rates, as shown by the following equations:¹⁹

$$\tilde{s}_t = \frac{1 + \frac{\sigma}{\bar{r}}}{\rho + \frac{\sigma}{\bar{r}}}(\tilde{m}_t - \tilde{m}_t^*) \quad (4.3)$$

$$c_t^{\tilde{H}} - c_t^* = \frac{1}{\rho}(\tilde{m}_t - \tilde{m}_t^*) \quad (4.4)$$

Therefore, it generically predicts a strong comovement and a high and positive correlation between the exchange rate and relative consumption.²⁰ From empirical evidence, however, there is no clear path in the observed cross-correlation. Chari, Kehoe and McGrattan (2002) find that this correlation is negative for U.S. and Europe while it ranges between small and positive to somewhat negative for other country pairs.

Nevertheless, in our model, since exchange rate movements can be generated by the expectation error shocks, our model does not predict a strong comovement of the exchange rate and relative consumption. The functional form of \tilde{s}_t (listed in the first column of Tables 2 and 3) shows that

¹⁸For current calibration, $\frac{\bar{r}}{\sigma} = \frac{0.06}{1.4} = 0.0429$. Recall that $\sigma = 1 - \frac{(1-\gamma)\rho}{1+\psi\gamma}$, so as long as the elasticity of substitution between home and foreign goods γ is greater than 1, σ is greater than 1. Thus, $\frac{\bar{r}}{\sigma} < 0.06$.

¹⁹With no traders on the foreign exchange market, Equation 3.19 could be rewritten as Equation 4.3.

²⁰For example, in Chari, Kehoe and McGrattan (2002) the correlation is equal to 1.

the exchange rate can move even when the realization of the fundamentals shocks are equal to zero. Further more, as shown by the last column of Tables 3, the cross-correlation between the exchange rate and relative consumption decreases when more noise traders are present on the foreign exchange market. Intuitively, this is because the introduction of noise traders generate deviations from the uncovered interest parity condition and thus breaks the link between the real exchange rate and relative consumption. Thus, we may get a small and positive correlation in our model.

4.2 Endogenous Entry

The exogenous entry specification gives important implications for the model, however, a natural question to ask is what can the monetary authorities do to get rid of the excess volatility in the nominal exchange rate? In this section, we consider ways to endogenize the entry of noise traders, which will help to evaluate the implications of policies that target the non-fundamental risk.

Table 4 illustrates the simulation result of the endogenous entry specification: First, the exchange rate disconnection still holds in this specification. Second, given the number of noise traders on the market: $1 - N_I$, increasing the entry cost \bar{c} (within a reasonable domain of \bar{c}) will reduce the exchange rate volatility.

The first finding is not surprising. In Equation 3.31, as in Equation 3.17, the presence of noise traders generates a wedge between home and foreign real interest rates. This wedge, by analogy, is also composed of two parts, the expectation error of incumbent noise traders and the risk premium term. The only difference is that now the number of incumbent noise traders is endogenously decided, and as is the expectation error part. Nevertheless, this does not alter any of the results from the theoretical analysis in Section 4.1. This wedge creates the “level effects” and the “volatility effects”, which, in turn, imply a degree of exchange rate volatility that is much higher than the fundamental volatility. Meanwhile, the expenditure-switching effect is eliminated because of the LCP pricing behavior. The wealth effect is quantitatively small because of the households’ consumption smoothing behavior in an infinite horizon model. Therefore, the exchange rate volatility will not be transferred to the macroeconomic fundamentals except for the net foreign assets.

The second finding is very interesting and has important policy implications. Although the model is complicated and can only be solved numerically, this result is quite intuitive. The higher the entry cost, the fewer noise traders will enter the market and therefore fewer noise components will be present. Thus, it shows that the exchange rate policies that aim at eliminating the non-fundamental

risk can be justified theoretically. It also suggests possible approaches that could be taken by the monetary authorities to reduce the excess exchange rate volatility, to discourage the entrance of noise traders by increasing the entry cost or to “educate” the market to reduce the number of noise traders on the foreign exchange market. Furthermore, monetary authorities could reduce this kind of excess exchange rate volatility by commitments to low exchange rate volatility. In this way, the volatility of expectation error of noise traders will be reduced, as would the exchange rate volatility. A self-contained equilibrium with low exchange rate volatility would then be established.

Finally, the noise traders in our paper are risk-averse. Therefore, they have two counter-acting roles. On one hand, their presence creates risk, but on the other hand, their presence lowers the risk because they are risk-averse. To better understand their roles, we change the value of the coefficient of absolute risk aversion a to study the relationship between traders’ risk aversion and the exchange rate volatility. Table 5 gives the results of that experiment for both specifications. As expected, the higher the value of a (the more risk averse are the noise traders), the lower is the exchange rate volatility. Obviously, the risk aversion of the noise traders helps to keep exchange rate volatility from increasing without a limit.

5 An Extension - Tobin Tax

Tobin (1978) and Eichengreen, Tobin and Wyplosz (1995) suggested that an international transaction tax on purchases and sales of foreign exchange would be one way to “throw sand in the wheels of super-efficient financial vehicles”. They argue that a transaction tax might diminish excess volatility. Even a small transaction tax would deter the fast round trip into a foreign money market.²¹

A Tobin tax is different from the entry cost we analyzed in the benchmark model. First, it is a common cost for both rational and noise traders. Second, it is not a fixed cost, but increases with the amount of foreign currency bond traded. In this extension, the benchmark model is extended to include a transaction tax for analyzing the implication of the Tobin tax on exchange rate volatility in our model.

²¹A small transaction tax would be a negligible consideration in long-term portfolios or direct investments in other economies. Relative to ordinary commercial and transportation costs, it would be too small to have much effect on commodity trade.

When a transaction tax is imposed, the trader i 's problem can be written as:²²

$$\max_{B_{h,t+1}^*(i)} E_t^i(C_{t+1}^T(i)) - \frac{a}{2} \text{Var}_t^i(C_{t+1}^T(i)) \quad (5.1)$$

Subject to²³

$$P_{t+1}C_{t+1}^T = [B_{h,t+1}^*(i)(1+r_{t+1}^*)S_{t+1} - B_{h,t+1}^*(i)S_t(1+r_{t+1})] - P_{t+1}c_i - P_{t+1}\tau \frac{B_{h,t+1}^*(i)^2}{2} \quad (5.2)$$

where τ is the rate of the transaction tax on foreign bond trading. Solving the traders' problem,

$$B_{h,t+1}^{I,*} = \frac{E_t[\rho_{t+1}]}{\frac{P_{t+1}\tau}{S_t(1+r_{t+1})} + a\frac{S_t}{P_{t+1}}(1+r_{t+1})\text{Var}_t[\rho_{t+1}]} \quad (5.3)$$

$$B_{h,t+1}^{N,*} = \frac{E_t^N[\rho_{t+1}]}{\frac{P_{t+1}\tau}{S_t(1+r_{t+1})} + a\frac{S_t}{P_{t+1}}(1+r_{t+1})\text{Var}_t[\rho_{t+1}]} \quad (5.4)$$

From Equations 5.2 and 5.4, it can be seen that the introduction of a Tobin tax reduces the bond trading of both types of traders. This is quite intuitive, as foreign exchange traders will tend to trade less foreign currency bonds when there is a tax on transactions.

When there are only transaction costs, as shown in Appendix G, traders will always choose to enter the market. This is because the transaction cost is convex in the bonds traded, the traders can always choose to hold a small amount of foreign bonds and get a positive expected utility, regardless of how large is τ . Therefore, similar to the benchmark model, two cases are analyzed in this extension. In the exogenous entry case, we focus on the transaction cost only. In the endogenous entry case, we assume that noise traders have to pay two costs to trade in the foreign exchange market: the transaction cost and a fixed information cost as in previous sections. Nevertheless, the informed traders only need to pay a Tobin tax. The analysis of the second case will help in better understanding the role of the Tobin tax in the economy.

Using Equations 5.3 and 5.4, we could get the interest parity condition when there exists a transaction tax in the foreign exchange market. For the exogenous entry case:

$$\hat{s}_t = E_t(s_{t+1}) - \beta(dr_{t+1} - dr_{t+1}) + (1 - N_I)v_t - \frac{\bar{P}\tau dB_{h,t+1}^*}{\bar{S}(1+\bar{r})} - a\frac{(1+\bar{r})\bar{S}}{\bar{P}}\text{Var}_t[s_{t+1}]dB_{h,t+1}^* \quad (5.5)$$

²²The transaction cost could not be modelled as linear in $B_{h,t+1}^*(i)$, because this would imply that trader i will gain when selling foreign bonds. Thus, we assume a convex transaction cost.

²³We assume that the Tobin tax is a real tax and is resource-consuming in the sense that it consumes the composite consumption good.

When the noise traders must pay both the transaction cost and the fixed information cost, the gross benefit of entry for noise traders can be derived:

$$GB \equiv \frac{[E_t^N(\rho_{t+1})]^2}{2a\text{Var}_t(\rho_{t+1}) + 2\tau[\frac{P_{t+1}}{S_t(1+r_{t+1})}]^2} \quad (5.6)$$

From Equation 5.6, it can be seen that the Tobin tax reduces the gross benefit of entry for noise traders. Therefore, increasing the transaction cost will deter the noise traders from entering the market. Given that, we could get the interest parity condition for the endogenous entry case:

$$\hat{s}_t = E_t(s_{t+1}) - \beta(dr_{t+1} - dr_{t+1}^*) + \frac{1}{N_I}n_tv_t - \frac{\bar{P}\tau dB_{h,t+1}^*}{N_I\bar{S}(1+\bar{r})} - a\frac{(1+\bar{r})\bar{S}}{\bar{P}N_I}\text{Var}_t[s_{t+1}]dB_{h,t+1}^* \quad (5.7)$$

where n_t , the number of incumbent noise traders is given by:

$$n_t = dn_t \approx \frac{\{E_t(s_{t+1}) - \hat{s}_t - \beta(dr_{t+1}^* - dr_{t+1}) + v_t\}^2 (1 - N_I)}{2a\text{Var}_t(s_{t+1}) + \frac{2\bar{P}^2}{\bar{S}^2(1+\bar{r})^2}\tau} \frac{1}{\bar{c}} \quad (5.8)$$

Analogous to Equations 3.5 and 3.6, Equations 5.5 and 5.7 give the deviation from the uncovered interest parity. This deviation is composed of three parts. Besides the expectation error term and the risk premium term, there is an extra term that comes from the transaction tax. Even in the absence of noise traders, this term still exists. As emphasized by Eichengreen, Tobin and Wyplosz (1995), this term creates room in the interest parity condition and expands the autonomy of monetary policies.

To find out if the introduction of the Tobin tax will reduce excess exchange rate volatility, we solve the extended model by the approach described in Section 3.²⁴ Then, the solution of the exogenous entry model is given by:

$$\tilde{s}_t = (\tilde{m}_t - \tilde{m}_t^*) \frac{1 + \frac{\sigma}{\bar{r}} + \frac{\phi}{\rho}\text{Var}_t(s_{t+1}) + \frac{\xi\tau}{\rho}}{\rho + \frac{\sigma}{\bar{r}} + \phi\text{Var}_t(s_{t+1}) + \xi\tau} + \frac{\frac{\sigma}{\bar{r}}}{\rho + \frac{\sigma}{\bar{r}} + \phi\text{Var}_t(s_{t+1}) + \xi\tau} (1 - N_I)v_t \quad (5.9)$$

²⁴The only equation that has been changed besides the interest parity condition is the home country aggregate consumption equation, which now becomes:

$$\begin{aligned} C_t &= C_t^H + C_t^T + \tau B_{h,t+1}^* && \text{Exogenous Entry} \\ C_t &= C_t^H + C_t^T + \tau B_{h,t+1}^* + \sum_{i=0}^{n_t} c_i && \text{Endogenous Entry} \end{aligned}$$

Once we log-linearize the above equations around the steady state, the log-linearized equations remain unchanged.

where

$$\phi = \frac{a(1+\bar{r})\bar{S}\bar{C}}{2} \frac{\sigma}{\bar{r}} \quad \xi = \frac{\bar{P}^2\bar{C}}{2\bar{S}(1+\bar{r})} \frac{\sigma}{\bar{r}} \quad (5.10)$$

From Equation 5.9 we can solve for the exchange rate volatility V_s :

$$V_s = \frac{\left(\frac{1+\frac{\sigma}{\bar{r}}+\frac{\phi}{\rho}V_s+\frac{\xi}{\rho}\tau}{\rho+\frac{\sigma}{\bar{r}}+\phi V_s+\xi\tau}\right)^2}{1-\left(\frac{\frac{\sigma}{\bar{r}}}{\rho+\frac{\sigma}{\bar{r}}+\phi V_s+\xi\tau}\right)^2(1-N_I)^2\lambda} [Var(\tilde{m}_t) + Var(\tilde{m}_t^*)] \quad (5.11)$$

It can easily be shown that if the other variables are kept constant, the numerator on the right-hand side of Equation 5.11 decreases in τ , the rate of transaction tax, while the denominator increases in τ . Since Equation 5.11 is an implicit function of V_s , we solve it numerically to get the relationship between V_s and τ , which is given in Table 6. It can be seen that the higher the rate of the transaction tax, the lower is the exchange rate volatility.

Intuitively, this is because the introduction of the transaction tax reduces the bond trading. In our model, when an exchange rate change occurs, the real balance effect prevents the current consumption from increasing/decreasing more than the changes in the relative real money supply, so the bond holding of households will absorb most of the wealth effect caused by the exchange rate change. If the bond trading is deterred by the transaction tax, in equilibrium, the exchange rate volatility must decrease.

For the endogenous entry model, the solution may be derived analogously using Equation 5.7:

$$\tilde{s}_t = (\tilde{m}_t - \tilde{m}_t^*) \frac{1 + \frac{\sigma}{\bar{r}} + \frac{\phi'}{\rho} Var_t(s_{t+1}) + \frac{\xi'}{\rho} \tau}{\rho + \frac{\sigma}{\bar{r}} + \phi' Var_t(s_{t+1}) + \xi' \tau} + \frac{\frac{\sigma}{\bar{r}}}{\rho + \frac{\sigma}{\bar{r}} + \phi' Var_t(s_{t+1}) + \xi' \tau} \frac{1}{N_I} n_t v_t \quad (5.12)$$

where²⁵

$$n_t = \frac{\{E_t(s_{t+1}) - \tilde{s}_t + v_t\}^2}{2a Var_t(s_{t+1}) + \frac{2\bar{P}^2}{\bar{S}^2(1+\bar{r})^2} \tau} \frac{(1-N_I)}{\bar{c}} \quad (5.13)$$

$$\phi' = \frac{a(1+\bar{r})\bar{S}\bar{C}}{2N_I} \frac{\sigma}{\bar{r}} \quad \xi' = \frac{\bar{P}^2\bar{C}}{2N_I\bar{S}(1+\bar{r})} \frac{\sigma}{\bar{r}} \quad (5.14)$$

Solving Equation 5.12 numerically by the approach described in Appendix F, we find that exchange rate volatility also decreases in the transaction tax, as in the exogenous case. The results are given in Table 6.

²⁵The derivation of Equation 5.13 is analogous to that of Equation 3.32.

In the endogenous entry case, the transaction cost reduces the exchange rate volatility through two channels. First, as in the exogenous case, it reduces the bond trading of both types of traders, which in turn decreases the exchange rate volatility. Second, as shown by Equation 5.6, the Tobin tax reduces the gross benefit of entry for noise traders, which consequently reduces the noise component of the foreign exchange market. Therefore, the mechanism through which the transaction cost affects the exchange rate volatility is different when the noise component on the market is endogenously determined. The effect of the Tobin tax will thus be different as well. This can be seen from Table 6, for the same level of increase in τ , the decrease in the exchange rate volatility in the endogenous entry case is larger than that in the exogenous entry case. This finding has important policy implications. It shows that the impact of a Tobin tax on exchange rate volatility depends crucially on the structure of the foreign exchange market and the interaction of the Tobin tax with other trading costs.

6 Conclusions and Subsequent Research

In this paper we present a model of exchange rate determination which combines the new open economy macroeconomics approach and the noise trader approach for exchange rate behavior. This model emphasizes the interaction of the macroeconomic fundamentals of exchange rate and the microstructure channel through which exchange rates are determined. The latter is often ignored by conventional macroeconomic research on exchange rate and the literature on policy evaluation. Therefore, our work has important implications for understanding exchange rate behavior and exchange rate policies.

Two important and promising findings from this model are: 1. Models that take both the macroeconomic and microeconomic factors of exchange rate determination into consideration *can* explain the “exchange rate disconnect puzzle”. 2. The exchange rate volatility caused by *irrational market behavior or non-fundamental shocks* could be controlled by exchange rate policies. We analyze two kinds of policies. One focuses on the entry cost of noise traders, while the other is a ‘Tobin tax’ type of exchange rate policy. We find that both policies can reduce the exchange rate volatility, however, the effect of a Tobin tax on exchange rate volatility depends crucially on the structure of the foreign exchange market and the interaction of the Tobin tax with other trading costs.

Thus, subsequent research should focus on the policy implication of this model. What kind of exchange rate regime is better when non-fundamental shocks to the exchange rates are present – flexible or fixed? If the real exchange rate volatility is primarily affected by non-fundamental factors and most exchange rate volatility is useless, then would the fixed exchange rate regime or a single

currency area be better than the flexible exchange regime? The presentation of explicit utility and the profit maximization problem in our model could provide an answer to these questions based on rigorous analysis.

This model could also be used to evaluate the welfare implications of exchange rate policies such as the Tobin tax or other policies that discourage the entry of noise traders. These policies are discussed widely, but due to the lack of a *welfare-based* model which can explain exchange rate volatility and its relationship with macroeconomic fundamentals, they have not been evaluated on a welfare basis. The new open economy macroeconomic framework of our model provides a rigorous context for the welfare analysis of these policies.

Although this model could help to explain the exchange rate disconnect puzzle, it is still not a fully developed model that could be used to explain all empirical features of exchange rate. To explain the persistence of the real exchange rate, more persistent price setting or ‘sticky’ information of traders would be needed. For example, the information structure of the noise traders could be changed so that the expectation error is more persistent.

Another interesting direction for future research would be the empirical implications suggested by this model. We have developed testable hypothesis about the nature of exchange rate volatility and exchange rate disconnect. The higher the degree of local currency pricing, the greater is the disconnection between exchange rate and macroeconomic fundamentals. Also, our model implies the following prediction about the foreign exchange market: The deviations from uncovered interest parity will be greater and the trading volume of foreign exchange will be higher, when more noise traders are present on the market.

TABLE 2

Exogenous Case ($\lambda = 1, a = 2$)

No. of Noise Trader	\tilde{s}_t	$Var(\tilde{s}_t)$	Increase of $Var(\tilde{s}_t)$	$Var(dB_{t+1})$
0	$\tilde{s}_t = 0.9562\tilde{m}_t - 0.9562\tilde{m}_t^*$	0.0183	0.00%	2.4861E-04
0.1	$\tilde{s}_t = 0.9562\tilde{m}_t - 0.9562\tilde{m}_t^* + 0.0912v_t$	0.0184	0.84%	2.4861E-04
0.2	$\tilde{s}_t = 0.9561\tilde{m}_t - 0.9561\tilde{m}_t^* + 0.1824v_t$	0.0189	3.43%	2.7598E-04
0.3	$\tilde{s}_t = 0.9560\tilde{m}_t - 0.9560\tilde{m}_t^* + 0.2736v_t$	0.0198	8.06%	3.2484E-04
0.4	$\tilde{s}_t = 0.9559\tilde{m}_t - 0.9559\tilde{m}_t^* + 0.3647v_t$	0.0211	15.27%	4.0097E-04
0.5	$\tilde{s}_t = 0.9557\tilde{m}_t - 0.9557\tilde{m}_t^* + 0.4557v_t$	0.0231	26.06%	5.1497E-04
0.6	$\tilde{s}_t = 0.9553\tilde{m}_t - 0.9553\tilde{m}_t^* + 0.5464v_t$	0.0260	42.31%	6.8646E-04
0.7	$\tilde{s}_t = 0.9548\tilde{m}_t - 0.9548\tilde{m}_t^* + 0.6367v_t$	0.0307	67.71%	9.5470E-04
0.8	$\tilde{s}_t = 0.9540\tilde{m}_t - 0.9540\tilde{m}_t^* + 0.7263v_t$	0.0385	110.68%	1.4083E-03
0.9	$\tilde{s}_t = 0.9523\tilde{m}_t - 0.9523\tilde{m}_t^* + 0.8141v_t$	0.0538	194.14%	2.2895E-03
1.0	$\tilde{s}_t = 0.9482\tilde{m}_t - 0.9482\tilde{m}_t^* + 0.8964v_t$	0.0916	400.69%	4.4702E-03
Consumption	$Var(\tilde{c}_t) = Var(\tilde{c}_t^*) = 0.0025$			
Home wage	$Var(\tilde{w}_t) = 0.0163$			
Home Labor	$Var(\tilde{l}_t) = 0.0013$			

TABLE 3
Exogenous Case ($\lambda = 1.5, a = 2$)

No. of Noise Traders	\tilde{s}_t	$Var(\tilde{s}_t)$	Increase of $Var(\tilde{s}_t)$	$Var(dB_{t+1})$	$Corr(\tilde{s}_t, \tilde{c}_t - \tilde{c}_t^*)$
0	$\tilde{s}_t = 0.9562\tilde{m}_t - 0.9562\tilde{m}_t^*$	0.0183	0.00%	2.3979E-04	1.0000
0.1	$\tilde{s}_t = 0.9562\tilde{m}_t - 0.9562\tilde{m}_t^* + 0.0912v_t$	0.0185	1.26%	2.5308E-04	0.9937
0.2	$\tilde{s}_t = 0.9561\tilde{m}_t - 0.9561\tilde{m}_t^* + 0.1824v_t$	0.0192	5.23%	2.9502E-04	0.9747
0.3	$\tilde{s}_t = 0.9559\tilde{m}_t - 0.9559\tilde{m}_t^* + 0.2736v_t$	0.0206	12.58%	3.7265E-04	0.9422
0.4	$\tilde{s}_t = 0.9557\tilde{m}_t - 0.9557\tilde{m}_t^* + 0.3645v_t$	0.0228	24.77%	5.0127E-04	0.8948
0.5	$\tilde{s}_t = 0.9553\tilde{m}_t - 0.9553\tilde{m}_t^* + 0.4553v_t$	0.0265	44.85%	7.1327E-04	0.8301
0.6	$\tilde{s}_t = 0.9546\tilde{m}_t - 0.9546\tilde{m}_t^* + 0.5455v_t$	0.0329	80.00%	1.0845E-03	0.7441
0.7	$\tilde{s}_t = 0.9532\tilde{m}_t - 0.9532\tilde{m}_t^* + 0.6344v_t$	0.0459	150.75%	1.8314E-03	0.6295
0.8	$\tilde{s}_t = 0.9494\tilde{m}_t - 0.9494\tilde{m}_t^* + 0.7191v_t$	0.0803	339.33%	3.8224E-03	0.4737
0.9	$\tilde{s}_t = 0.9351\tilde{m}_t - 0.9351\tilde{m}_t^* + 0.7831v_t$	0.2183	1094.09%	1.1790E-02	0.2830
1.0	$\tilde{s}_t = 0.9024\tilde{m}_t - 0.9024\tilde{m}_t^* + 0.8047v_t$	0.5695	3014.60%	3.2060E-02	0.1691

Consumption

$$Var(\tilde{c}_t) = Var(\tilde{c}_t^*) = 0.0025$$

Home wage

$$Var(\tilde{w}_t) = 0.0163$$

Home Labor

$$Var(\tilde{l}_t) = 0.0013$$

TABLE 4
Endogenous Case($\lambda = 1.5, a = 2$)^a

	$\bar{c} = 0.1$	$\bar{c} = 0.15$	$\bar{c} = 0.25$	$\bar{c} = \infty$	$N_I = 0.2$	$\bar{c} = 0.1$	$\bar{c} = 0.15$	$\bar{c} = 0.25$	$\bar{c} = \infty$
$N_I = 0.1$									
$Var(\tilde{s}_t)$	0.7381	0.3697	0.1197	0.0168	$Var(\tilde{s}_t)$	0.2505	0.0728	0.0299	0.0171
$Var(dB_{t+1})$	4.23E-02	2.12E-02	6.80E-03	2.11E-04	$Var(dB_{t+1})$	1.43E-02	4.10E-03	1.60E-03	2.199E-04
$Mean(n)$	0.1741	0.1216	0.0931	0	$Mean(n)$	0.1743	0.1663	0.1749	0
$N_I = 0.4$									
$Var(\tilde{s}_t)$	0.0313	0.0239	0.0199	0.0173	$N_I = 0.5$	$\bar{c} = 0.1$	$\bar{c} = 0.15$	$\bar{c} = 0.25$	$\bar{c} = \infty$
$Var(dB_{t+1})$	1.60E-03	1.10E-03	8.00E-04	2.246E-04	$Var(\tilde{s}_t)$	0.021	0.0201	0.0193	0.0173
$Mean(n)$	0.2938	0.2834	0.2629	0	$Var(dB_{t+1})$	8.00E-04	7.00E-04	6.00E-04	2.256E-04
					$Mean(n)$	0.3067	0.2888	0.2604	0
$N_I = 0.8$									
$Var(\tilde{s}_t)$	0.0173	0.0171	0.0169	0.0173	$N_I = 0.99$	$\bar{c} = 0.1$	$\bar{c} = 0.15$	$\bar{c} = 0.25$	$\bar{c} = \infty$
$Var(dB_{t+1})$	3.00E-04	3.00E-04	2.00E-04	2.00E-04	$Var(\tilde{s}_t)$	0.0173	0.0173	0.0173	0.0173
$Mean(n)$	0.1494	0.14	0.1259	0	$Var(dB_{t+1})$	2.00E-04	2.00E-04	2.00E-04	2.00E-04
Consumption					$Mean(n)$	0.0076	0.0072	0.0065	0
Home wage					$Var(\tilde{c}_t) = Var(\tilde{c}_t^*) = 0.0025$				
Home Labor					$Var(\tilde{w}_t) = 0.0163$				
					$Var(\tilde{l}_t) = 0.0013$				

^a Here we only consider a reasonable range of \bar{c} : $\bar{c} \in (0, 0.25]$, as the steady state consumption in this model is 0.25.

TABLE 5

The Risk Aversion of the Traders

Coefficient of Absolute Risk Aversion: a	$Var(\hat{s}_t)$ (Exogenous Case, $\lambda = 1.5$)			$Var(\hat{s}_t)$ (Endogenous Case, $\lambda = 1.5, \bar{c} = 0.1$)				
	$N_I = 0.1$	$N_I = 0.2$	$N_I = 0.3$	$N_I = 0.4$	$N_I = 0.1$	$N_I = 0.2$	$N_I = 0.3$	$N_I = 0.4$
$a = 2$	0.2183	0.0803	0.0459	0.0329	0.7381	0.2505	0.0576	0.0313
$a = 3$	0.1753	0.0756	0.0449	0.0326	0.2565	0.0649	0.0306	0.0236
$a = 5$	0.1338	0.0687	0.0433	0.0320	0.0697	0.0275	0.0211	0.0196
$a = 10$	0.0935	0.0580	0.0400	0.0307	0.0232	0.0174	0.0172	0.0176
$a = 1000$	0.0116	0.0112	0.0109	0.0107	0.0049	0.005	0.0051	0.0052

TABLE 6
The impact of Tobin Tax

Tobin Tax	$Var(\tilde{s}_t)$ (Exogenous Case, $\lambda = 1.5, a = 2$)									
	$N_I = 0$	$N_I = 0.1$	$N_I = 0.2$	$N_I = 0.3$	$N_I = 0.4$	$N_I = 0.5$	$N_I = 0.6$	$N_I = 0.7$	$N_I = 0.8$	$N_I = 1$
$\tau = 0$	0.5695	0.2183	0.0803	0.0459	0.0329	0.0265	0.0228	0.0206	0.0192	0.0183
$\tau = 0.5\%$	0.5623	0.2139	0.0793	0.0455	0.0328	0.0264	0.0227	0.0205	0.0192	0.0183
$\tau = 1\%$	0.555	0.2096	0.0783	0.0452	0.0326	0.0263	0.0227	0.0205	0.0192	0.0182
$\tau = 1.5\%$	0.5478	0.2054	0.0772	0.0448	0.0324	0.0262	0.0226	0.0204	0.0191	0.0182
$\tau = 2.5\%$	0.5334	0.1971	0.0753	0.0442	0.0321	0.026	0.0225	0.0203	0.019	0.0181
Tobin Tax	$Var(\tilde{s}_t)$ (Endogenous Case, $\lambda = 1.5, a = 2, \bar{c} = 0.1$)									
	$N_I = 0.001$	$N_I = 0.1$	$N_I = 0.2$	$N_I = 0.3$	$N_I = 0.4$	$N_I = 0.5$	$N_I = 0.6$	$N_I = 0.7$	$N_I = 0.8$	$N_I = 0.99$
$\tau = 0$	1.2723	0.7381	0.2505	0.0576	0.0313	0.021	0.0178	0.0173	0.0173	0.0173
$\tau = 0.5\%$	1.2543	0.7152	0.2131	0.0404	0.0250	0.0201	0.0178	0.0173	0.0171	0.0173
$\tau = 1.0\%$	1.2375	0.6934	0.1747	0.0276	0.0212	0.0194	0.0178	0.0173	0.0169	0.0173
$\tau = 1.5\%$	1.2203	0.6709	0.1230	0.0222	0.0198	0.0191	0.0177	0.0171	0.0168	0.0172
$\tau = 2.5\%$	1.1856	0.6243	0.0239	0.0185	0.0186	0.0181	0.0172	0.0166	0.0165	0.0171

APPENDIX

A Optimal Pricing Schedule of Firms

All goods are imperfect substitutes in consumption, so each individual firm has some market power determined by the parameter θ . Taking prices for all individual goods as given, the optimal demand function of the consumer for each individual good can be derived, which implies that in each period the consumer allocates a given level of total consumption among the differentiated goods.

$$C_{h,t}(i) = \omega \left(\frac{P_{h,t}}{P_{h,t}(i)} \right)^\theta \left(\frac{P_t}{P_{h,t}} \right)^\gamma C_t \quad (\text{A.15})$$

and

$$C_{f,t}(j) = (1 - \omega) \left(\frac{P_{f,t}}{P_{f,t}(j)} \right)^\theta \left(\frac{P_t}{P_{f,t}} \right)^\gamma C_t. \quad (\text{A.16})$$

The price setting problem of monopolist i is to maximize expected profit conditional on $t - 1$ information, by choosing $P_{h,t}(i)$ and $P_{h,t}^*(i)$. That is, firm i solves

$$\max_{P_{h,t}(i), P_{h,t}^*(i)} E_{t-1} \{ D_t [P_{h,t}(i) C_{h,t}(i) + S_t P_{h,t}^*(i) C_{h,t}^*(i) - W_t L_t] \} \quad (\text{A.17})$$

subject to

$$L_t(i) = C_{h,t}(i) + C_{h,t}^*(i) \quad (\text{A.18})$$

and the downward-sloping demand functions for $C_{h,t}(i)$ and $C_{h,t}^*(i)$, as in Equation A.15 and the foreign analogue. Note that $P_{h,t}(i)$ and $P_{h,t}^*(i)$ are denoted in the home and foreign currency, respectively.

The optimal price setting schedules of firm i are:

$$P_{h,t}(i) = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D_t W_t \Lambda_{h,t}]}{E_{t-1} [D_t \Lambda_{h,t}]} \quad (\text{A.19})$$

$$P_{h,t}^*(i) = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D_t W_t \Lambda_{h,t}^*]}{E_{t-1} [D_t S_t \Lambda_{h,t}^*]} \quad (\text{A.20})$$

where

$$\Lambda_{h,t} = (P_{h,t})^\theta \left(\frac{P_{h,t}}{P_t} \right)^{-\gamma} C_t \quad \Lambda_{h,t}^* = (P_{h,t}^*)^\theta \left(\frac{P_{h,t}^*}{P_t^*} \right)^{-\gamma} C_t^* \quad (\text{A.21})$$

Analogously, we could derive the pricing setting equations for the intermediate goods producer j in the foreign country:

$$P_{f,t}(j) = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D_t^* W_t^* \Lambda_{f,t}]}{E_{t-1} \left[\frac{D_t^*}{S_t} \Lambda_{f,t} \right]} \quad (\text{A.22})$$

$$P_{f,t}^*(j) = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D_t^* W_t^* \Lambda_{f,t}^*]}{E_{t-1} [D_t^* \Lambda_{f,t}^*]} \quad (\text{A.23})$$

where

$$\Lambda_{f,t} = (P_{f,t})^\theta \left(\frac{P_{f,t}}{P_t} \right)^{-\gamma} C_t \quad \Lambda_{f,t}^* = (P_{f,t}^*)^\theta \left(\frac{P_{f,t}^*}{P_t^*} \right)^{-\gamma} C_t^* \quad (\text{A.24})$$

Using the fact that all prices are preset at time $t - 1$ and applying symmetry, we can derive Equations 2.9 -2.12.

B Entry Condition of Noise Traders

This appendix derives the entry condition (Equation 2.35) for noise traders to enter the foreign bond market. The noise trader i will enter the foreign bond market if and only if:

$$E_t^i(U_t^i | \varphi_t^i = 1) \geq E_t^i(U_t^i | \varphi_t^i = 0) \quad (\text{B.1})$$

If trader i does not enter the market, his expected utility is given by:

$$E_t^i(U_t^i | \varphi_t^i = 0) = 0 \quad (\text{B.2})$$

While if he enters, it is given by:

$$E_t^i(U_t^i | \varphi_t^i = 1) = E_t^i \left\{ \max_{B_{h,t+1}^*(i)} \left\{ E_t^i \left[\frac{B_{h,t+1}^*(i) S_t (1 + r_{t+1})}{P_{t+1}} \rho_{t+1} - c_i \right] - \frac{a}{2} \text{Var}_t^i \left[\frac{B_{h,t+1}^*(i) S_t (1 + r_{t+1})}{P_{t+1}} \rho_{t+1} \right] \right\} \right\} \quad (\text{B.3})$$

Substituting the optimal demand for foreign bonds of noise traders, $B_{h,t+1}^*(i) = \frac{E_t^N[\rho_{t+1}]}{a \frac{S_t}{P_{t+1}} (1 + r_{t+1}) \text{Var}_t[\rho_{t+1}]}$, into the above equation, we may establish that B.1 is equivalent to:

$$\varphi_t^i = 1 \iff E_t^i \left[\frac{E_t^N[\rho_{t+1}] \rho_{t+1}}{a \text{Var}_t[\rho_{t+1}]} - c_i \right] - \frac{a}{2} \text{Var}_t^i \left[\frac{E_t^N[\rho_{t+1}] \rho_{t+1}}{a \text{Var}_t[\rho_{t+1}]} \right] \geq 0 \quad (\text{B.4})$$

By the property of noise traders' subjective expectation of ρ_{t+1} , Equation B.4 can be rewritten as:

$$\varphi_t^i = 1 \iff \left[\frac{(E_t^N[\rho_{t+1}])^2}{a \text{Var}_t[\rho_{t+1}]} - c_i \right] - \frac{(E_t^N[\rho_{t+1}])^2}{2a(\text{Var}_t[\rho_{t+1}])^2} \text{Var}_t[\rho_{t+1}] \geq 0 \quad (\text{B.5})$$

or:

$$\varphi_t^i = 1 \iff c_i \leq \frac{[E_t^N(\rho_{t+1})]^2}{2a \text{Var}_t(\rho_{t+1})} \quad (\text{B.6})$$

C A Symmetric Steady State

In a non-stochastic steady state, all shocks are equal to zero. Hereafter, steady state values are marked by overbars.

As the consumption is constant at the steady state, the steady state world interest rate r is tied down by the intertemporal optimality equation (Equation 2.7):

$$\bar{r} = \bar{r}^* = \frac{1 - \beta}{\beta} \quad (\text{C.1})$$

From the pricing equation, at the steady state, all the prices are equal and steady state exchange rate $\bar{S} = 1$. Then the steady state excess return $\bar{\rho} = \frac{\bar{S}(1+\bar{r}^*)}{\bar{S}(1+\bar{r})} - 1 = 0$. From Equation 2.29, we will have ²⁶

$$B_h^*(i) = 0 \quad \forall i \in [0, 1] \quad (\text{C.2})$$

The economic intuition behind C.2 is that traders are not going to hold foreign bonds because they know that the excess return will be zero, and thus no trade takes place. The only way that no trade will occur in equilibrium is for the uncovered interest parity to hold.

Therefore, from the bond market clearing condition, net foreign assets are zero.

$$\bar{B} = \bar{B}^* = 0 \quad (\text{C.3})$$

The steady state values of other variables are straight forward. Since $\bar{B} = 0$, a closed-form solution exists for the steady state, in which the countries have identical outputs, consumption and real money holdings:

$$\bar{L} = \bar{L}^* = \bar{C} = \bar{C}^* = \left(\frac{\theta\eta}{\theta - 1} \right)^{-\frac{1}{\rho+1}} \quad (\text{C.4})$$

$$\frac{\bar{M}}{\bar{P}} = \frac{\bar{M}^*}{\bar{P}^*} = \left(\frac{\bar{C}}{1 - \beta} \right)^{\frac{1}{\epsilon}} \quad (\text{C.5})$$

²⁶Note that since $v_t = 0$, only rational traders are present on the market.

D Model Solution

The full model can be described by the 26 equilibrium conditions listed in Section 2.3. To solve the model, we take a log-linearization around the initial non-stochastic steady state described in Appendix C. Given the log-linearized system, the deviations of the exchange rate and the macroeconomic variables from their $t - 1$ expectations are solved in terms of exogenous money supply shocks and the expectation error shocks.

D.1 Log-linearized System

Money demand function

$$\hat{m}_t - \hat{p}_t = \frac{\rho}{\epsilon} \hat{c}_t^H - \frac{\beta}{\bar{r}\epsilon} dr_{t+1} \quad \hat{m}_t^* - \hat{p}_t^* = \frac{\rho}{\epsilon} \hat{c}_t^* - \frac{\beta}{\bar{r}\epsilon} dr_{t+1}^* \quad (\text{D.1})$$

Labor supply function

$$\hat{w}_t = \psi \hat{l}_t + \rho \hat{c}_t^H + \hat{p}_t \quad \hat{w}_t^* = \psi \hat{l}_t^* + \rho \hat{c}_t^* + \hat{p}_t^* \quad (\text{D.2})$$

Euler equations

$$-\beta dr_{t+1} = \rho \hat{c}_t^H - \rho E_t c_{t+1}^H + \hat{p}_t - E_t p_{t+1} \quad -\beta dr_{t+1}^* = \rho \hat{c}_t^* - \rho E_t c_{t+1}^* + \hat{p}_t^* - E_t p_{t+1}^* \quad (\text{D.3})$$

Home household budget constraint ($\omega = \frac{1}{2}$)

$$\hat{p}_t + \hat{c}_t^H = \frac{1}{2}(p_{h,t} + c_{h,t}) + \frac{1}{2}(p_{h,t}^* + c_{h,t}^* + \hat{s}_t) + \frac{1}{\beta PC} dB_t - \frac{1}{PC} dB_{t+1} \quad (\text{D.4})$$

Pricing Equations

$$p_{h,t} = E_{t-1}[\hat{w}_t] \quad p_{h,t}^* = E_{t-1}[\hat{w}_t] - E_{t-1}[\hat{s}_t] \quad p_{f,t} = E_{t-1}[\hat{w}_t^*] + E_{t-1}[\hat{s}_t] \quad p_{f,t}^* = E_{t-1}[\hat{w}_t^*] \quad (\text{D.5})$$

Price Indexes ($\omega = \frac{1}{2}$)

$$\hat{p}_t = \frac{1}{2} p_{h,t} + \frac{1}{2} p_{f,t} \quad \hat{p}_t^* = \frac{1}{2} p_{h,t}^* + \frac{1}{2} p_{f,t}^* \quad (\text{D.6})$$

Individual Goods Demand

$$c_{h,t} = -\gamma(p_{h,t} - \hat{p}_t) + \hat{c}_t \quad c_{h,t}^* = -\gamma(p_{h,t}^* - \hat{p}_t^*) + \hat{c}_t^* \quad (\text{D.7})$$

$$c_{f,t} = -\gamma(p_{f,t} - \hat{p}_t) + \hat{c}_t \quad c_{f,t}^* = -\gamma(p_{f,t}^* - \hat{p}_t^*) + \hat{c}_t^* \quad (\text{D.8})$$

Market Clearing Conditions

$$dB_{t+1} = \bar{S}dB_{h,t+1}^* \quad (\text{D.9})$$

$$\hat{l}_t = \frac{1}{2}c_{h,t}^{\hat{}} + \frac{1}{2}c_{h,t}^{\hat{*}} \quad \hat{l}_t^* = \frac{1}{2}c_{f,t}^{\hat{}} + \frac{1}{2}c_{f,t}^{\hat{*}} \quad (\text{D.10})$$

Home Country Aggregate Consumption (for both exogenous entry and endogenous entry cases)

$$\hat{c}_t = c_t^{\hat{H}} + \frac{dC_t^T}{C} \quad (\text{D.11})$$

Budget Constraints of Traders

Exogenous Entry Case:

The budget constraint of foreign exchange traders is given by:

$$P_{t+1}C_{t+1}^T = [(1 + r_{t+1}^*)S_{t+1} - (1 + r_{t+1})S_t] = B_{h,t+1}^*S_t(1 + r_{t+1})\rho_{t+1} \quad (\text{D.12})$$

Linearizing the above equation around $\bar{B}_h^* = 0$ and $\bar{\rho} = 0$ gives $dC_{t+1}^T = 0$.

Endogenous Case:

The budget constraint of foreign exchange traders is given by

$$P_{t+1}C_{t+1}^T = B_{h,t+1}^*[(1 + r_{t+1}^*)S_{t+1} - (1 + r_{t+1})S_t] - P_{t+1}\sum_{i=0}^{n_t} c_i = B_{h,t+1}^*S_t(1 + r_{t+1})\rho_{t+1} - P_{t+1}\sum_{i=0}^{n_t} c_i \quad (\text{D.13})$$

Linearizing the above equation around $\bar{B}_h^* = 0$, $\bar{\rho} = 0$ and $\bar{n} = 0$ gives $dC_{t+1}^T = 0$.

Interest Parity Conditions

Before linearizing the interest parity equation, an approximation ²⁷ is used to rewrite the excess return in log-terms:

$$\rho_{t+1} = \frac{S_{t+1}(1 + r_{t+1}^*)}{S_t(1 + r_{t+1})} - 1 \approx \ln \left[\frac{S_{t+1}(1 + r_{t+1}^*)}{S_t(1 + r_{t+1})} \right] = s_{t+1} + \ln(1 + r_{t+1}^*) - s_t - \ln(1 + r_{t+1}) \quad (\text{D.14})$$

Exogenous Case: Using D.14, Equation 2.32 can be rewritten as:

$$s_t = E_t(s_{t+1}) + \ln(1 + r_{t+1}^*) - \ln(1 + r_{t+1}) + (1 - N_I)v_t - a\frac{S_t}{P_{t+1}}(1 + r_{t+1})\text{Var}_t(s_{t+1})B_{h,t+1}^* \quad (\text{D.15})$$

²⁷If ξ is small enough, then $\ln(1 + \xi) \approx \xi$. This approximation of the excess return is widely used in the finance literature.

Linearizing the above equation around the steady state, but using the second-order approximation to approximate the variance term (as the second order terms are important for understanding the dynamics of the model), we have

$$\hat{s}_t = E_t(s_{t+1}) - \beta(dr_{t+1} - dr_{t+1}^*) + (1 - N_I)v_t - a \frac{(1 + \bar{r})\bar{S}}{\bar{P}} Var_t[s_{t+1}] dB_{h,t+1}^* \quad (D.16)$$

Endogenous Entry Case:

Using D.14, Equation 2.38 can be rewritten as:

$$s_t = E_t(s_{t+1}) + \ln(1 + r_{t+1}^*) - \ln(1 + r_{t+1}) + \frac{n_t}{N_I + n_t} v_t - a \frac{B_{h,t+1}^* S_t (1 + r_{t+1})}{P_{t+1} (N_I + n_t)} Var_t[s_{t+1}] \quad (D.17)$$

Linearizing above equation around the steady state, but using the second-order approximation to approximate $\frac{n_t}{N_I + n_t} v_t$ and the variance term, we have

$$\hat{s}_t = E_t(s_{t+1}) - \beta(dr_{t+1} - dr_{t+1}^*) + \frac{1}{N_I} n_t v_t - a \frac{(1 + \bar{r})\bar{S}}{\bar{P} N_I} Var_t[s_{t+1}] dB_{h,t+1}^* \quad (D.18)$$

where²⁸

$$\begin{aligned} n_t &= \frac{\{E_t[s_{t+1}] - s_t + \ln(1 + r_{t+1}^*) - \ln(1 + r_{t+1}) + v_t\}^2 \frac{1 - N_I}{\bar{c}}}{2a Var_t[s_{t+1}]} \\ &\approx \frac{\{E_t(s_{t+1}) - \hat{s}_t - \beta(dr_{t+1}^* - dr_{t+1}) + v_t\}^2 \frac{(1 - N_I)}{\bar{c}}}{2a Var_t(s_{t+1})} \end{aligned} \quad (D.20)$$

Money supply processes

$$m_{t+1} = \hat{m}_t + \varepsilon_{\mu,t} \quad m_{t+1}^* = \hat{m}_t^* + \varepsilon_{\mu,t}^* \quad (D.21)$$

where $\varepsilon_{\mu,t}^* \sim N(0, \sigma_{\varepsilon_{\mu}^*}^2)$ and $\varepsilon_{\mu,t} \sim N(0, \sigma_{\varepsilon_{\mu}}^2)$.

D.2 Derivation of Equations

Derivation of Equations 3.17 and 3.31 From the log-linearized intertemporal optimality conditions D.3, we have:

$$\rho(c_t^{\hat{H}} - \hat{c}_t^*) - \rho E_t(c_{t+1}^{\hat{H}} - \hat{c}_{t+1}^*) + E_{t-1}(\hat{s}_t) - E_t(s_{t+1}) = -\beta(dr_{t+1} - dr_{t+1}^*) \quad (D.22)$$

²⁸Note that at the steady state $\bar{S} = 1$. Here we also use the fact that approximating $\ln(1 + r_{t+1}^*) - \ln(1 + r_{t+1})$ around the steady state gives:

$$\ln(1 + r_{t+1}^*) - \ln(1 + r_{t+1}) = -\beta(dr_{t+1}^* - dr_{t+1}) \quad (D.19)$$

Equation D.22 minus its $t-1$ expectation gives:

$$\rho(\tilde{c}_t^H - \tilde{c}_t^*) - \rho E_t(c_{t+1}^H - c_{t+1}^*) - E_t(s_{t+1}) = -\beta(dr_{t+1} - dr_{t+1}^*) \quad (\text{D.23})$$

Taking $t-1$ expectation of Equation D.16 and using the fact that at $E_{t-1}(v_t) = 0$,²⁹ give:

$$E_{t-1}(\hat{s}_t) = E_{t-1}(s_{t+1}) - \beta E_{t-1}(dr_{t+1} - dr_{t+1}^*) - a \frac{(1+\bar{r})\bar{S}}{\bar{P}} \text{Var}[s_{t+1}] E_{t-1}[dB_{h,t+1}^*] \quad (\text{D.24})$$

Equation D.16 minus Equation D.24 gives:

$$-\beta(dr_{t+1} - dr_{t+1}^*) = \tilde{s}_t - E_t(s_{t+1}) - (1 - N_I)v_t + a \frac{(1+\bar{r})\bar{S}}{\bar{P}} \text{Var}[s_{t+1}] dB_{h,t+1}^* \quad (\text{D.25})$$

Equation D.23 and D.25 give Equation 3.17.

The derivation of Equation 3.31 is analogous. The only difference is that when deriving the analogy of D.24, we conjecture that $E_{t-1}(n_t v_t) = \text{constant}$ ³⁰. Since this term only affects the level of exchange rate and we are interested in the exchange rate volatility, it could be assumed to equal 0.

Derivation of Equations 3.19 and 3.33 To derive Equation 3.19, first substituting Equation 3.17 into Equation 3.15

$$(\tilde{c}_t^H - \tilde{c}_t^*) = \frac{[(1 + \frac{\sigma}{\rho\bar{r}})\tilde{s}_t - \frac{\sigma}{\rho\bar{r}}(1 - N_I)v_t + \frac{a(1+\bar{r})\bar{S}}{\bar{P}} \frac{\sigma}{\rho\bar{r}} \text{Var}_t[s_{t+1}] dB_{h,t+1}^*]}{1 + \frac{\sigma}{\bar{r}}} \quad (\text{D.26})$$

Substituting D.26 into 3.18,

$$\tilde{m}_t - \tilde{m}_t^* = \frac{\rho}{1 + \frac{\sigma}{\bar{r}}} [(1 + \frac{\sigma}{\rho\bar{r}})\tilde{s}_t - \frac{\sigma}{\rho\bar{r}}(1 - N_I)v_t + \frac{a(1+\bar{r})\bar{S}}{\bar{P}} \frac{\sigma}{\rho\bar{r}} \text{Var}_t[s_{t+1}] dB_{h,t+1}^*] \quad (\text{D.27})$$

Note that from Equations 3.14 and 3.18, $d\tilde{B}_{t+1} = \frac{\bar{P}\bar{C}}{2}[\tilde{s}_t - \frac{1}{\rho}(\tilde{m}_t - \tilde{m}_t^*)]$. Using the fact that $d\tilde{B}_{t+1} = \bar{S}dB_{h,t+1}^* = dB_{h,t+1}^*$, we could get:

$$\begin{aligned} (\tilde{m}_t - \tilde{m}_t^*) [1 + \frac{a(1+\bar{r})\bar{S}\bar{C}}{2} \frac{\sigma}{\rho(\sigma + \bar{r})} \text{Var}_t(s_{t+1})] &= [\frac{\rho\bar{r} + \sigma}{\sigma + \bar{r}} + \frac{a(1+\bar{r})\bar{S}\bar{C}}{2} \frac{\sigma}{\sigma + \bar{r}} \text{Var}_t(s_{t+1})] \tilde{s}_t \\ &\quad - \frac{\sigma}{\sigma + \bar{r}} (1 - N_I)v_t \end{aligned} \quad (\text{D.28})$$

Rewriting Equation D.28 gives Equation 3.19. The derivation of Equation 3.33 is entirely analogous.

²⁹At this stage, we conjecture that $\text{Var}_t[s_{t+1}] = \text{Var}_t[s_{t+1}] = \text{constant} = V_s$. This conjecture is verified in Section 3.2.

³⁰From 3.32 and the functional form of \tilde{s}_t (Equation F.1), our conjecture can be easily verified.

E The simulation of $Var(v_t) = \lambda Var(s_t)$

First, for a given distribution of fundamentals, $L_0(\varepsilon_{\mu,t}, \varepsilon_{\mu,t}^*)$, the variance of the exchange rate when the expectation errors of the noise traders are zero can be calculated. It can be denoted as $\sigma_{s_0}^2$.

Then, we assume that the stochastic expectation error v_t is given by:

$$v_t = \sqrt{\lambda \sigma_{s_0}^2} \varepsilon_t \quad (\text{E.1})$$

where ε_t is a random variable which satisfies the following three conditions:³¹

$$Cov(\varepsilon_t, \varepsilon_{\mu,t}) = Cov(\varepsilon_t, \varepsilon_{\mu,t}^*) = 0 \quad \sigma_\varepsilon^2 = 1 \quad (\text{E.2})$$

Equation E.1 implies $\sigma_v^2 = \lambda \sigma_{s_0}^2$. Given E.1, and the distribution of fundamentals $L_0(\varepsilon_{\mu,t}, \varepsilon_{\mu,t}^*)$, the variance of exchange rate: σ_s^2 can be computed. Let it be denoted as $\sigma_{s_1}^2$.

Compare $\sigma_{s_1}^2$ and $\sigma_{s_0}^2$, if

$$|\sigma_{s_0}^2 - \sigma_{s_1}^2| \leq \epsilon, \quad \epsilon \rightarrow 0 \quad (\text{E.3})$$

The procedure stops at this point, otherwise, we will redefine the stochastic process of v_t as:

$$v_t = \sqrt{\lambda \sigma_{s_1}^2} \varepsilon_t \quad (\text{E.4})$$

Notice that now $\sigma_v^2 = \lambda \sigma_{s_1}^2$. Using E.4 and $L_0(\varepsilon_{\mu,t}, \varepsilon_{\mu,t}^*)$, unconditional exchange rate volatility could be computed again and would be called $\sigma_{s_2}^2$. If $|\sigma_{s_1}^2 - \sigma_{s_2}^2| \leq \epsilon$, and $\epsilon \rightarrow 0$, the procedure stops here, otherwise, the procedure described above will be repeated to get $\sigma_{s_3}^2, \dots, \sigma_{s_n}^2, \sigma_{s_{n+1}}^2$ until $\sigma_{s_{n+1}}^2 - \sigma_{s_n}^2 \leq \epsilon$.

F Numerical Undetermined Coefficient Method

This section gives details for the undetermined coefficient method used to solve for the functional form of \tilde{s}_t in Equation 3.33.

First, guess a functional form for \tilde{s}_t :

$$\tilde{s}_t = \alpha_0 + \alpha_1 \tilde{m}_t + \alpha_2 \tilde{m}_t^* + \alpha_3 v_t + \alpha_4 \tilde{m}_t^2 + \alpha_5 \tilde{m}_t^{*2} + \alpha_6 v_t^2 + \alpha_7 \tilde{m}_t v_t + \alpha_8 \tilde{m}_t^* v_t + \alpha_9 \tilde{m}_t \tilde{m}_t^* \quad (\text{F.1})$$

³¹ $Cov(\varepsilon_{\mu,t}, v_t) = Cov(\varepsilon_{\mu,t}^*, v_t)$ must be equal to zero, as v_t is some noise and should not have any fundamental content.

Given that, we could get $E_t(s_{t+1})$ and $Var_t(s_{t+1})$ easily. Using the facts that $\tilde{m}_t = \varepsilon_{\mu,t}$, $\tilde{m}_t^* = \varepsilon_{\mu,t}^*$ and $Cov(\varepsilon_{\mu,t}, \varepsilon_{\mu,t}^*) = 0$, and that v_t is noise and should not have any fundamental content, gives:

$$Cov(\tilde{m}_t, \tilde{m}_t^*) = Cov(\tilde{m}_t, v_t) = Cov(\tilde{m}_t^*, v_t) = 0 \quad (\text{F.2})$$

Therefore,

$$E_t(s_{t+1}) = \alpha_4 \sigma_{\varepsilon_\mu}^2 + \alpha_5 \sigma_{\varepsilon_\mu^*}^2 + \alpha_6 \sigma_v^2 \quad (\text{F.3})$$

To get the conditional variance of the exchange rate, the properties of the normally distributed variables³² and the fact that the three random variables are independent are used.

$$\begin{aligned} Var_t(s_{t+1}) &= \alpha_1^2 Var(\tilde{m}_t) + \alpha_2^2 Var(\tilde{m}_t^*) + \alpha_3^2 Var(v_t) + \alpha_4^2 Var(\tilde{m}_t^2) + \alpha_5^2 Var(\tilde{m}_t^{*2}) \\ &\quad + \alpha_6^2 Var(v_t^2) + \alpha_7^2 Var(\tilde{m}_t v_t) + \alpha_8^2 Var(\tilde{m}_t^* v_t) + \alpha_9 (\tilde{m}_t \tilde{m}_t^*) + \text{Covariance terms} \\ &= \alpha_1^2 \sigma_{\varepsilon_\mu}^2 + \alpha_2^2 \sigma_{\varepsilon_\mu^*}^2 + \alpha_3^2 v \sigma_v^2 + 2\alpha_4^2 \sigma_{\varepsilon_\mu}^4 + 2\alpha_5^2 \sigma_{\varepsilon_\mu^*}^4 \\ &\quad + 2\alpha_6^2 \sigma_v^4 + \alpha_7^2 \sigma_{\varepsilon_\mu}^2 \sigma_v^2 + \alpha_8^2 \sigma_{\varepsilon_\mu^*}^2 \sigma_v^2 + \alpha_9 \sigma_{\varepsilon_\mu}^2 \sigma_{\varepsilon_\mu^*}^2 \end{aligned} \quad (\text{F.5})$$

That is, $Var_t(s_{t+1}) = Var(s_{t+1}) = \text{constant} \equiv V_s$ and $E_t(s_{t+1}) = E(s_{t+1}) = \text{constant} \equiv E_s$. Using the parameterized E_s and V_s from Equations F.3 and F.5, we might solve for \tilde{s}_t from Equation 3.33 given any exogenous shocks \tilde{m}_t , \tilde{m}_t^* and v_t . To test if our initial guess is a good guess, we can do simulations and regress the \tilde{s}_t we get from above process on \tilde{m}_t , \tilde{m}_t^* and v_t . If the coefficients (α 's) are close enough to the initial guess, the process is stopped. Otherwise, the above procedure will be repeated. This method is actually an undetermined coefficient method, and is also known as the “parameterized estimation approach” in numerical methods.

G Entry Condition of Traders with Tobin Tax

When the traders only need to pay transaction cost to trade in the foreign exchange market, trader i will enter the market if and only if:

$$E_t^i(U_t^i | \varphi_t^i = 1) \geq E_t^i(U_t^i | \varphi_t^i = 0) = 0 \quad (\text{G.1})$$

³²Notice that, if x_t is normally distributed with variance σ_x^2 , then

$$E[(x_t)^{2k}] = (2k-1)(\sigma_x^2)^k \quad E[(x_t)^{2k+1}] = 0 \quad \text{where } k = 1, 2, \dots, n \quad (\text{F.4})$$

Or

$$E_t^i \left\{ \max_{B_{h,t+1}^*(i)} \left\{ E_t^i \left[\frac{B_{h,t+1}^*(i)S_t(1+r_{t+1})}{P_{t+1}} \rho_{t+1} - \tau \frac{B_{h,t+1}^*(i)^2}{2} \right] - \frac{a}{2} \text{Var}_t^i \left[\frac{B_{h,t+1}^*(i)S_t(1+r_{t+1})}{P_{t+1}} \rho_{t+1} \right] \right\} \right\} \geq 0 \quad (\text{G.2})$$

Substituting $B_{h,t+1}^{i,*} = \frac{E_t^i[\rho_{t+1}]}{\frac{P_{t+1}\tau}{S_t(1+r_{t+1})} + a\frac{S_t}{P_{t+1}}(1+r_{t+1})\text{Var}_t[\rho_{t+1}]}$ into the above equation, it can be shown that Equation G.2 is equivalent to:

$$\zeta \left\{ 2 \frac{S_t(1+r_t)}{P_{t+1}} \left[\frac{P_{t+1}\tau}{S_t(1+r_{t+1})} + a \frac{S_t}{P_{t+1}} (1+r_{t+1}) \text{Var}_t(\rho_{t+1}) \right] - \tau - a \text{Var}_t(\rho_{t+1}) \left[\frac{S_t(1+r_t)}{P_{t+1}} \right]^2 \right\} \geq 0 \quad (\text{G.3})$$

where

$$\zeta = \frac{[E_t^i(\rho_{t+1})]^2}{2 \left[\frac{P_{t+1}\tau}{S_t(1+r_{t+1})} + a \frac{S_t}{P_{t+1}} (1+r_{t+1}) \text{Var}_t(\rho_{t+1}) \right]^2} \geq 0 \quad (\text{G.4})$$

It can be shown that the terms in the big bracket of Equation G.3 are equal to:

$$\tau + \left[\frac{S_t(1+r_t)}{P_{t+1}} \right]^2 \text{Var}_t(\rho_{t+1}) \geq 0 \quad (\text{G.5})$$

Therefore, regardless of how large is the rate of Tobin tax (τ), the traders will always enter the foreign bond market. This is because the transaction cost is convex in the bond traded, the trader can always choose to hold a small amount of foreign bonds and get a positive expected utility.

When the noise traders has to pay two costs to trade on the foreign exchange market, for noise trader i , he will enter the market if and only if:

$$E_t^i \left\{ \max_{B_{h,t+1}^*(i)} \left\{ E_t^i \left[\frac{B_{h,t+1}^*(i)S_t(1+r_{t+1})}{P_{t+1}} \rho_{t+1} - \tau \frac{B_{h,t+1}^*(i)^2}{2} - c_i \right] - \frac{a}{2} \text{Var}_t^i \left[\frac{B_{h,t+1}^*(i)S_t(1+r_{t+1})}{P_{t+1}} \rho_{t+1} \right] \right\} \right\} \geq 0 \quad (\text{G.6})$$

Following the steps in Appendix B, we could get the following entry condition for noise trader i :

$$\varphi_t^i = 1 \iff c_i \leq \frac{[E_t^N(\rho_{t+1})]^2}{2a\text{Var}_t(\rho_{t+1}) + 2\tau \left[\frac{P_{t+1}}{S_t(1+r_{t+1})} \right]^2} \equiv GB \quad (\text{G.7})$$

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