

# PAPER OR PLASTIC? MONEY AND CREDIT AS MEANS OF PAYMENT\*

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## Abstract

This paper studies the choice of payment instruments in a simple model where both money and credit can be used as means of payment. We endogenize the acceptability of credit by allowing retailers to invest in a costly record-keeping technology. Our framework captures the two-sided market interaction between consumers and retailers, leading to strategic complementarities that can generate multiple steady-state equilibria. In addition, limited commitment makes debt contracts self-enforcing and yields an endogenous upper bound on credit use. Our model can explain why the demand for credit declines as inflation falls, and how hold-up problems in technological adoption can prevent retailers from accepting credit as consumers continue to coordinate on cash usage. We show that whenever money and credit coexist, equilibrium is generically inefficient and optimal policy entails an inflation rate strictly above the Friedman rule. We also discuss the extent to which our model can reconcile some key patterns in the use of cash and credit in retail transactions.

*Keywords:* coexistence of money and credit, inflation, costly record-keeping, credit constraints  
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# 1 Introduction

Consumers now have more payment instruments than ever to choose from, ranging from cash, credit cards, prepaid cards, smart cards, mobile account payments, and electronic money (“e-money”).<sup>1</sup> According to Gerdes, Walton, Liu, and Parke (2005), innovations in the retail landscape have generated a payments transformation, as card payments now dominate more traditional paper-based ones.<sup>2</sup> In particular, technological improvements in electronic record-keeping have made credit cards as ubiquitous as cash as means of payment in many OECD countries. A recent study by the Federal Reserve finds that the number of payments made by general-purpose credit cards rose from 15.2 billion to 19.0 billion between 2003 and 2006 in the United States, for a growth rate of 7.6% a year. During this same period, the number of ATM cash withdrawals dropped slightly from 5.9 billion to 5.8 billion (Gerdes (2008)). This suggests that while consumers are indeed adopting new payment instruments, they are not completely abandoning older ones (Schuh (2012)).

As consumers change the way they pay and businesses change the way they accept payments, it is increasingly important to understand how consumer demand affects merchant behavior and vice versa. In fact, the payment system is a classic example of a two-sided market where both consumers and firms must make choices that affect one other. This dynamic often generates complementarities and network externalities, which is a key characteristic of the retail payment market (Rysman (2009), BIS (2012)).<sup>3</sup> Moreover, the recent trends in retail payments raise many interesting and challenging questions for central banks and policymakers. In particular, how does the availability of alternative means of payment, such as credit cards, affect the role of money? And if both money and credit can be used, how does policy and inflation affect the money-credit margin?

We investigate the possible substitution away from cash to electronic payments such as credit cards using a simple model where money and credit can coexist as means of payment. Our objective is to determine the impact that retail payment innovations can have on future cash usage. Understanding how people substitute between cash and credit is a key policy concern for central banks when setting an inflation target, as well as legislators when developing new regulation on credit card fees. As it is the sole issuer of bank notes, central banks also need to understand substitution patterns to predict consumers’ demand for cash.

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<sup>1</sup>Evidence from Foster, Meijer, Schuh, and Zabek (2009) reveals that in 2009, the average U.S. consumer holds 5.0 of the nine most common payment instruments and used 3.8 of them during a typical month.

<sup>2</sup>48.8% of transactions recorded in the Federal Reserve Bank of Boston’s 2010 Survey of Consumer Payment Choice were conducted with payment cards, while 40.8% of transactions used paper instruments, such as cash or check.

<sup>3</sup>Network externalities exist when the value of a good or service to a potential user increases with the number of other users using the same product. Credit cards are a good example of network good, where its adoption and use can be below the socially optimal level because consumers or firms do not internalize the benefit of their own use on others’ use. For evidence and a discussion of the empirical issues, see Gowrisankaran and Stavins (2004).

To capture the two-sided nature of actual payment systems, our model focuses on the market interaction between consumers (buyers, or borrowers) and retailers (sellers, or lenders). A vital distinction between monetary and credit trades is that the former is *quid pro quo* and settled on the spot while the latter involves delayed settlement.<sup>4</sup> For credit to have a role, we introduce a costly record-keeping technology that allows transactions to be recorded.<sup>5</sup> A retailer that invests in this technology will thus be able to accept an IOU from a consumer. In this way, credit allow retailers to sell to illiquid consumers or to those paying with future income. Due to limited commitment and enforcement however, lenders cannot force borrowers to repay their debts. In order to motivate voluntary debt repayment, we assume that default by the borrower triggers a global punishment that banishes agents from all future credit transactions. In that case, a defaulter can only trade with money. Consequently, debt contracts must be self-enforcing and the possibility of strategic default generates an endogenous upper-bound on credit use.<sup>6</sup>

When enforcement is imperfect, inflation has two effects: a higher inflation rate both lowers the rate of return on money and makes default more costly. This relaxes the credit constraint and induces agents to shift from money to credit to finance their consumption. Consequently, consumers decrease their borrowing as inflation falls. When lump-sum taxes can be enforced and the monetary authority implements the Friedman rule, deflation completely crowds out credit and there is a flight to liquidity where all borrowing and lending ceases to exist. In that case, efficient monetary policy drives out credit.<sup>7</sup> However when both money and credit are used, the Friedman rule is not feasible to sustain voluntary debt repayment. When borrowers are not patient enough, inflation has a hump-shaped effect on welfare and optimal policy entails a strictly positive inflation rate. While equilibrium is typically inefficient when money and credit coexist, the first-best allocation can still be achieved in a pure credit economy, provided that agents are patient enough. However equilibrium is not socially efficient since sellers must incur the real cost of technological adoption.

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<sup>4</sup>This distinction separates debit cards, which are “pay now” cards, from credit cards, or “pay later” cards. For debit cards, funds are typically debited from the cardholder’s account within a day or two of purchase, while credit cards allow consumers to access credit lines at their bank which are repaid at a future date. In this paper, we interpret credit trades as occurring with credit cards and monetary trades as those occurring with cash or debit.

<sup>5</sup>As is well established by now, the same frictions that render money essential make credit arrangements impossible. These frictions include imperfect record-keeping over individual trading histories, lack of commitment, and lack of enforcement. See e.g. Kocherlakota and Wallace (1998).

<sup>6</sup>This is in the spirit of Kehoe and Levine (1993) and Alvarez and Jermann (2000) where the threat of banishment from future credit transactions motivate voluntary debt repayment.

<sup>7</sup>In our model, the presence of multiple steady-state equilibria where either money, credit, or both are used makes the choice of optimal policy difficult to analyze in full generality. If the monetary authority must choose an inflation rate before it knows which equilibrium will obtain, policy will affect the equilibrium selection process. Instead of specifying equilibrium selection rules, we suppose the existence of a particular equilibrium and then analyze the optimal policy in that equilibrium. In models where the fraction of credit trades is fixed, limited commitment and imperfect enforcement can also lead to a positive optimal inflation rate; see e.g. Berentsen, Camera, and Waller (2007), Antinolfi, Azariadis, and Bullard (2009), and Gomis-Porqueras and Sanches (2011).

The channel through which monetary policy affects macroeconomic outcomes is through buyers' choice of portfolio holdings, sellers' decision to invest in the record-keeping technology, and the endogenously determined credit constraint. If sellers must invest ex-ante in a costly technology to record credit transactions, there are strategic complementarities between the seller's decision to invest and the buyer's ability to repay. When more sellers accept credit, the gain for buyers from using and redeeming credit increases, which relaxes the credit constraint. At the same time, an increase in the buyer's ability to repay raises the incentive to invest in the record-keeping technology and hence the fraction of credit trades. This complementarity leads to feedback effects that can generate multiple steady-state equilibria, including outcomes where both money and credit are used.<sup>8</sup>

Moreover, this channel mimics the mechanism behind two-sided markets in actual payment systems as described by McAndrews and Zhu (2008): merchants are more willing to accept credit cards that have many cardholders, and cardholders want cards that are accepted at many establishments. Just as in our model, the payment network benefits the merchant and the consumer jointly, leading to the same kind of complementarities and network externalities highlighted in the industrial organization literature. At the same time, consumers may still coordinate on using cash due to a hold-up problem in technological adoption. Since retailers do not receive the full surplus associated with technological adoption, they fail to internalize the total benefit of accepting credit. The choice of payment instruments will therefore depend on fundamentals, as well as history and social conventions.

This potential for coordination failures also raises new concerns for policymakers. In contrast with conventional wisdom, our theory suggests that economies with similar technologies, institutions, and policies can still end up with very different payment systems, some being better in terms of social welfare than others. If for example society prefers a payment system with only credit, the government may want to introduce special policies such as information campaigns, advertisements, or even financial literacy programs that help coordinate agents on using credit. These measures that enhance communication are especially relevant for many emerging economies where the lack of financial infrastructure and intermediation makes new forms of mobile credit payments particularly appealing.<sup>9</sup> Hence our model not only provides policymakers a useful framework for understanding how consumers substitute between money and credit, but also makes clear the channels through which their policies affect prices, trade, and social welfare.

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<sup>8</sup>This multiplicity is consistent with empirical evidence from Humphreys, Pulley, and Vesala (1996) that finds inertia in the adoption of new payment instruments.

<sup>9</sup>For example, the November 17, 2012 *Economist* article "War of the Virtual Wallets" predicts that "the biggest prize of all lies in emerging markets where a lack of financial infrastructure is hastening the rise of phone-based payments systems."

The remainder of the paper proceeds as follows. Section 1.1 reviews the related literature. Section 2 describes the basic environment with limited enforcement. Section 3 then determines equilibrium where an exogenous fraction of sellers accept credit. Section 4 determines the endogenous debt limit and characterizes properties of monetary and non-monetary equilibrium with credit. Section 5 endogenizes the fraction of credit trades and discusses multiplicity. Section 6 turns to normative considerations and discusses welfare and optimal monetary policy, and Section 7 relates our model with the empirical evidence on consumer payments. Finally, Section 8 concludes.

## 1.1 Related Literature

Within modern monetary theory, there is a strong tradition of studying the coexistence of money and credit. Shi (1996) provides the first model with bargaining, money, and credit and shows that money can coexist with credit that yields a higher rate of return. In Kocherlakota and Wallace (1998), an equilibrium with money and credit can be sustained if individual histories are made public with a lag.<sup>10</sup> In another approach, Berentsen, Camera, and Waller (2007) models credit as bank loans in an environment with limited enforcement. However money remains the only means of payment since goods transactions remain private information for banks.<sup>11</sup> Sanches and Williamson (2010) get money and credit to coexist in a divisible money model with imperfect memory, limited commitment, and theft, while Bethune, Rocheteau, and Rupert (2013) develop a model with credit and liquid assets to examine the relationship between unsecured debt and unemployment. Gu and Wright (2012) use a related framework to study dynamics in a pure credit economy. However in all these approaches, only an exogenous subset of agents can use credit while the choice of using credit is endogenous in this paper.<sup>12</sup>

Our model of costly record-keeping is based on the model of money and costly credit in Nosal and Rocheteau (2010), though a key novelty is that we derive an endogenous debt limit under limited commitment instead of assuming that loan repayments can be perfectly enforced. Dong (2011) also introduces costly record-keeping, but focuses on the buyer's choice of payments used in bilateral meetings.<sup>13</sup> Our paper is the first to model the two-sided nature of accepting payments in an environment where money and credit can coexist and enforcement is limited.

Closely related to our paper is Gomis-Porqueras and Sanches (2011), who also discuss the role of

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<sup>10</sup>A related approach assumes limited participation to allow for monetary and credit transactions; see e.g. Calvanti and Wallace (1999), Williamson (1999), Williamson (2004), and Mills (2007), among many others.

<sup>11</sup>See also Telyukova and Wright (2008) for a model with divisible money and IOUs issued in a competitive market.

<sup>12</sup>In a similar vein, Schreft (1992) and Dotsey and Ireland (1996) introduce costs paid to financial intermediaries to endogenize the composition of trades that use money or credit, and Prescott (1987) and Freeman and Kydland (2000) feature a fixed record-keeping cost for transactions made with demand deposits.

<sup>13</sup>Arango and Taylor (2008) and Turban (2008) find that record-keeping or other technological costs associated with accepting credit are incurred by the seller.

money and credit in a model with anonymity, limited commitment, and imperfect record-keeping. A key difference in the set-up is that they adopt a different pricing mechanism by assuming a buyer-take-all bargaining solution. While this assumption may seem innocuous, our assumption of proportional bargaining allows us to take the analysis further in two important ways.<sup>14</sup> By giving the seller some bargaining power, proportional bargaining allows us to endogenize the fraction of sellers that can accept credit by allowing them to invest in a costly record-keeping technology. This also allows us to discuss hold-up problems on the seller's side which will lead to complementarities with the buyer's borrowing limit. This generates interesting multiplicities and network effects that the previous study cannot discuss.

This paper is also related to a growing strand in the industrial organization literature that examines the costs and benefits of credit cards to network participants. In particular, recent work by Wright (2003) and Rochet and Tirole (2011) models the bilateral transactions between consumers and retailers to study the effects of regulatory policies and market structure in the credit card industry.<sup>15</sup> However, this literature abstracts from a critical distinction between monetary and credit transactions by ignoring the actual borrowing component of credit transactions.

By contrast, our paper is explicit about the intertemporal nature of credit transactions by allowing consumers to issue an IOU to the seller, or paying on the spot with cash. In turn, our framework can be used to determine the conditions under which consumers prefer one type of payment instrument over the other, and how this can be affected by policy. As new forms of payment develop and become increasingly prevalent, these issues are central concerns that both central banks and policymakers need to understand.

## 2 Environment

The economy consists of a continuum  $[0, 2]$  of infinitely lived agents, evenly divided between *buyers* (or consumers) and *sellers* (or retailers). Time is discrete and continues forever. Each period is divided into two sub-periods where economic activity will differ. In the first sub-period, agents meet pairwise and at random in a decentralized market, called the *DM*. Sellers can produce output,  $q \in \mathbb{R}_+$ , but do not want to consume, while buyers want to consume but cannot produce. Agents' identities as buyers or sellers are permanent, exogenous, and determined at the beginning of the

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<sup>14</sup>More generally, proportional bargaining guarantees that trade is pairwise Pareto efficient and has several desirable features that cannot be guaranteed with Nash bargaining, as discussed in Aruoba, Rocheteau, and Waller (2007). First, it guarantees the concavity of agent's value functions. Second, the proportional solution is monotonic and hence does not suffer from a shortcoming of Nash bargaining that an agent can end up with a lower individual surplus even if the size of the total surplus increases.

<sup>15</sup>Chakravorti (2003) provides a theoretical survey of the industrial organization approach to credit card networks, and Rysman (2009) gives an overview of the economics of two-sided markets.

DM. In the second sub-period, trade occurs in a frictionless centralized market, called the *CM*, where all agents can consume a numéraire good,  $x \in \mathbb{R}_+$ , by supplying labor,  $y$ , one-for-one using a linear technology.

Instantaneous utility functions of buyers, ( $U^b$ ), and sellers, ( $U^s$ ), are assumed to be separable between sub-periods and linear in the CM:

$$\begin{aligned} U^b(q, x, y) &= u(q) + x - y, \\ U^s(q, x, y) &= -c(q) + x - y. \end{aligned}$$

Functional forms for utility and cost functions in the DM,  $u(q)$  and  $c(q)$  respectively, are assumed to be  $C^2$  with  $u' > 0$ ,  $u'' < 0$ ,  $c' > 0$ ,  $c'' > 0$ ,  $u(0) = c(0) = c'(0) = 0$ , and  $u'(0) = \infty$ . Also, let  $q^* \equiv \{q : u'(q^*) = c'(q^*)\}$ . All agents discount the future between periods, but not sub-periods, with a discount factor  $\beta \in (0, 1)$ .

The only asset in this economy is fiat money, which is perfectly divisible and storable. Money  $m \in \mathbb{R}_+$  is valued at  $\phi$ , the price of money in terms of numéraire. Its aggregate stock in the economy,  $M$ , can grow or shrink each period at a constant gross rate  $\gamma \equiv \frac{M_{t+1}}{M_t}$ . Changes in the money supply are facilitated through lump-sum transfer or taxes in the CM to buyers. In the latter case, we assume that the government has sufficient enforcement so that agents will repay the lump-sum tax.<sup>16</sup>

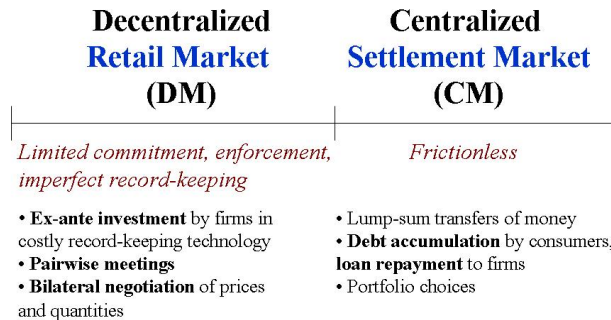
To purchase goods in the DM, both monetary and credit transactions are feasible due to the availability of a record-keeping technology that can record agent's transactions. However this technology is only available to a fraction  $\Lambda \in [0, 1]$  of sellers, while the remaining  $1 - \Lambda$  sellers can only accept money. For example, the cost function is such that investment in this technology is infinitely costly for a fraction  $1 - \Lambda$  of firms while costless for the remaining  $\Lambda$  firms. In Section 5, we endogenize  $\Lambda$  by considering an alternative cost function where sellers have heterogenous costs of investing.

We assume that contracts written in the DM can be repaid in the subsequent CM. Buyers can issue  $b \in \mathbb{R}_+$  units of one-period IOUs that we normalize to be worth one unit of the numéraire good. While the record-keeping technology can identify agents and record their transactions, enforcement may be imperfect. This leads to the possibility of strategic default by the borrower. In order to

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<sup>16</sup>While the government can never observe agents' real balances, it has the authority to impose arbitrarily harsh penalties on agents who do not pay taxes when  $\gamma < 1$ . Alternatively, Andolfatto (2007) considers an environment where the government's enforcement power is limited and the payment of lump-sum taxes is voluntary. This induces a lower bound on the deflation rate in which case the Friedman rule fails to be incentive feasible even though it is desirable. Alternatively, if the monetary authority does not want to implement deflation, it can still achieve the first-best allocation by paying interest on currency financed by increases in the money supply, as discussed in Gomis Porqueras and Peralta-Alva (2010) and Nosal and Rocheteau (2011).

Figure 1: **Timing of Representative Period**



support trade in a credit economy, potential borrowers must be punished if they do not deliver on their promise to repay. We assume that punishment for default entails permanent exclusion from the credit system.<sup>17</sup> In that case, a borrower who defaults can only use money for all future transactions.

The timing of events in a typical period is summarized in Figure 1. At the beginning of the DM, a measure  $\sigma$  of buyers and sellers are randomly matched, where the buyer has  $m \in \mathbb{R}^+$  units of money, or equivalently,  $z = \phi m$  units of real balances. Terms of trade are determined using a proportional bargaining rule. In the CM, buyers produce the numéraire good, redeem their loan, and acquire money, while sellers can purchase the numéraire with money and can get their loan repaid. We focus on stationary equilibria where real balances are constant over time.

### 3 Equilibrium

The model can be solved in four steps. First, we characterize properties of agents' value functions in the CM. Using these properties, we then determine the terms of trade in the DM. Third, we determine the buyer's choice of asset holdings, and in Section 4 we characterize equilibrium with the endogenous debt limit. Then in Section 5, we determine  $\Lambda$  endogenously by allowing sellers to

<sup>17</sup>Starting with Kehoe and Levine (1993) and Alvarez and Jermann (2000), off-equilibrium path punishments are also considered by Aiyagari and Williamson (2000), Antinolfi, Azariadis, and Bullard (2007), Berentsen, Camera, and Waller (2007), Camera and Li (2008), Sanches and Williamson (2010), Gomis-Porqueras and Sanches (2011), Venkateswaran and Wright (2012), and Bethune, Rocheteau, and Rupert (2013), among many others.



invest in the costly record-keeping technology.

### 3.1 Centralized Market

In the beginning of the CM, agents consume the numéraire  $x$ , supply labor  $y$ , and readjust their portfolios. Let  $W^b(z, -b)$  denote the value function of a buyer who holds  $z = \phi m$  units of real balances and has issued  $b$  units of IOUs in the previous DM. The buyer's maximization problem at the beginning of the CM,  $W^b(z, -b)$ , is

$$W^b(z, -b) = \max_{x, y, z' \geq 0} \left\{ x - y + \beta V^b(z') \right\} \quad (1)$$

$$\text{s.t. } x + b + \phi m' = y + z + T \quad (2)$$

$$z' = \phi' m' \quad (3)$$

where  $V^b$  is the buyer's continuation value in the next DM and  $T \equiv (\gamma - 1)\phi M$  is the lump-sum transfer from the government (in units of numéraire). According to (2), the buyer finances his net consumption of numéraire ( $x - y$ ), the repayment of his IOUs ( $b$ ), and his following period real balances ( $\gamma z'$ ) with his current real balances ( $z$ ) and the lump-sum transfer ( $T$ ). Substituting  $m' = z'/\phi'$  from (3) into (2), and then substituting  $x - y$  from (2) into (1) yields

$$W^b(z, -b) = z - b + T + \max_{z' \geq 0} \left\{ -\gamma z' + \beta V^b(z') \right\}. \quad (4)$$

The buyer's lifetime utility in the CM is the sum of his real balances net of any IOUs to be repaid, the lump-sum transfer from the government, and his continuation value at the beginning of the next DM net of the investment in real balances. The gross rate of return of money is  $\frac{\phi_{t+1}}{\phi_t} = \frac{M_t}{M_{t+1}} = \gamma^{-1}$ . Hence in order to hold  $z'$  units of real balances in the following period, the buyer must acquire  $\gamma z'$  units of real balances in the current period.

Notice that  $W^b(z, -b)$  is linear in the buyer's current portfolio:  $W^b(z, -b) = z - b + W^b(0, 0)$ . In addition, the choice of real balances next period is independent of current real balances. Identically, the value function  $W^s(z, b)$  of a seller who holds  $z$  units of real balances and  $b$  units of IOUs can be written:

$$W^s(z, b) = z + b + \beta V^s(0)$$

where  $V^s(0)$  is the value function of a seller at the beginning of the following DM since they have no incentive to accumulate real balances in the DM.

### 3.2 Terms of Trade

We now turn to the terms of trade in the DM. Agents meet bilaterally, and bargain over the units of money or IOUs to be exchanged for goods. We adopt Kalai (1977)'s proportional bargaining solution where the buyer receives a constant share  $\theta \in (0, 1)$  of the match surplus, while the seller gets the remaining share,  $(1 - \theta) > 0$ .<sup>18</sup>

We will show that the terms of trade depend only on buyers' portfolios and what sellers accept. First consider a match where the seller accepts credit. In that case, the buyer holding  $z$  units of real balances proposes a contract  $(q, b, d)$  that maximizes his expected surplus such that the seller gets a constant share  $1 - \theta$  of the total surplus. To apply the pricing mechanism, notice that the surplus of a buyer who gets  $q$  for payment  $d + b$  to the seller is  $u(q) + W^b(z - d - b) - W^b(z) = u(q) - b - d$ , by the linearity of  $W^b$ . Similarly, the surplus of a seller is  $-c(q) + d + b$ . The bargaining problem then becomes

$$(q, d, b) = \arg \max_{q, d, b} \{u(q) - d - b\} \quad (5)$$

$$\text{s.t. } -c(q) + d + b = \frac{1 - \theta}{\theta} [u(q) - d - b] \quad (6)$$

$$d \leq z \quad (7)$$

$$b \leq \bar{b}. \quad (8)$$

According to (5) – (8), the buyer's offer maximizes his trade surplus such that (i) the seller's payoff cannot be less than a constant share  $\frac{1-\theta}{\theta}$  of the buyer's payoff, (ii) the buyer cannot transfer more money than he has, and (iii) the buyer cannot borrow more than he can repay. Condition (7) is a feasibility constraint on the amount the buyer can transfer to the seller, while condition (8) is the buyer's incentive constraint that motivates voluntary debt repayment. The threshold  $\bar{b}$  is an equilibrium object and represents the endogenous borrowing limit faced by the buyer, which is taken as given in the bargaining problem but is determined endogenously in the next section.

Combining the feasibility constraint (7) and the buyer's incentive constraint (8) then results in the payment constraint

$$d + b \leq z + \bar{b} \quad (9)$$

which says the total payment to the seller,  $d + b$ , cannot exceed what the buyer holds, which is  $z + \bar{b}$  when the seller accepts credit. The solution to the bargaining problem will depend on whether the payment constraint, (9), binds. If (9) does not bind, then the buyer will have sufficient wealth to

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<sup>18</sup>There are also strategic foundations for the proportional bargaining solution. In Dutta (2012), Kalai (1977)'s solution emerges as a unique equilibrium outcome in a limiting case of a Nash demand game.

purchase the first-best level of output,  $q^*$ . In that case, payment to the seller will be exactly

$$d + b = (1 - \theta)u(q^*) + \theta c(q^*).$$

If (9) binds, then the buyer simply hands over what he has,

$$z + \bar{b} = (1 - \theta)u(q^c) + \theta c(q^c) \tag{10}$$

and gets in return  $q^c \equiv q(z + \bar{b})$ . Hence a buyer who does not have enough payment capacity will just pay with their cash on hand and borrow up to their limit in order to purchase the maximum quantity of output,  $q^c < q^*$ .

If the seller does not have access to record-keeping, credit cannot be used. In that case,  $b = \bar{b} = 0$  and the bargaining problem can be described by (5) – (7). If  $z \geq z^* \equiv (1 - \theta)u(q^*) + \theta c(q^*)$ , then the buyer has enough payment capacity to obtain  $q^*$ . Otherwise, the buyer just hands over his real balances,

$$z = (1 - \theta)u(q) + \theta c(q) \tag{11}$$

where  $q \equiv q(z) < q^*$ .

### 3.3 Decentralized Market

We next characterize agents' value functions the DM. After simplification, the expected discounted utility of a buyer holding  $z$  units of real balances at the beginning of the period is:

$$V^b(z) = \sigma(1 - \Lambda)\theta[u(q) - c(q)] + \sigma\Lambda\theta[u(q^c) - c(q^c)] + z + W^b(0, 0), \tag{12}$$

where we have used the bargaining solution and the fact that the buyer will never accumulate more balances than he would spend in the DM. According to (12), a buyer in the DM is randomly matched with a seller who does not have access to record-keeping with probability  $\sigma(1 - \Lambda)$ , receives  $\theta$  of the match surplus,  $u(q) - c(q)$ , and can only pay with money. With probability  $\sigma\Lambda$ , a buyer matches with a seller with access to record-keeping, in which case he gets  $\theta$  of  $u(q^c) - c(q^c)$  and can pay with both money and credit. The last two terms result from the linearity of  $W^b$  and is the value of proceeding to the CM with one's portfolio intact.

### 3.4 Optimal Portfolio Choice

Next, we determine the buyer's choice of real balances. Given the linearity of  $W^b$ , the buyer's bargaining problem (5) – (7), and substituting  $V^b(z)$  from (12) into (4), the buyer's choice of real

balances must satisfy:

$$\max_{z \geq 0} \{-iz + \sigma(1 - \Lambda)\theta[u(q) - c(q)] + \sigma\Lambda\theta[u(q^c) - c(q^c)]\} \quad (13)$$

where  $i = \frac{\gamma - \beta}{\beta}$  is the cost of holding real balances. As a result, the buyer chooses his real balances  $z$  in order to maximize his expected surplus in the *DM*, net of the cost of holding real balances,  $i$ .

Since the objective function (13) is continuous and maximizes over a compact set, a solution exists. We further assume  $u(q^*) - z(q^*) > 0$  in order to guarantee the existence of a monetary equilibrium. In the Appendix, we show that (13) is concave. The first-order condition for problem (13) when  $z \geq 0$  is

$$-i + \sigma(1 - \Lambda) \frac{\theta[u'(q) - c'(q)]}{\theta c'(q) + (1 - \theta)u'(q)} + \sigma\Lambda \frac{\theta[u'(q^c) - c'(q^c)]}{\theta c'(q^c) + (1 - \theta)u'(q^c)} \leq 0, \quad (14)$$

and with equality if  $z > 0$ .

We first make a few remarks on equilibrium when enforcement is perfect. With full enforcement, buyers are never constrained by  $b \leq \bar{b}$  and can borrow as much as they want to finance consumption in the first-best,  $q^*$ . When  $z > 0$ , the second term on the right-hand-side of (14) equals to zero since at  $q^*$ ,  $u'(q^*) = c'(q^*)$ . In that case, the right-hand-side is increasing with  $\Lambda$ , meaning that an increase in the fraction of credit trades  $\Lambda$  decreases  $q(z)$  and hence real balances  $z$ .

## 4 Limited Enforcement and Credit Limits

When the government's ability to force repayment is limited, borrowers have an incentive to renege on their debts. In order to support trade in a credit economy, we assume that punishment for default entails permanent exclusion from the credit system. In that case, debt-contracts must be self-enforcing, and a borrower that defaults can no longer use credit and can only use money for all future transactions.

The borrowing limit,  $\bar{b}$ , is determined in order to satisfy the buyer's incentive constraint to voluntarily repay his debt in the CM:

$$W^b(z, -b) \geq \widetilde{W}^b(z),$$

where  $W^b(z, -b)$  is the value function of a buyer who repays his debt at the beginning of the CM, and  $\widetilde{W}^b(z)$  is the value function of a buyer who defaults. By the linearity of  $W^b$ , the value function

of a buyer who repays his debt in the CM is

$$W^b(z, -b) = z - b + W^b(0, 0).$$

On the other hand, the value function of a buyer who defaults,  $\widetilde{W}^b(z)$  must satisfy

$$\begin{aligned}\widetilde{W}^b(z) &= z + T + \max_{z' \geq 0} \left\{ -\gamma z' + \beta \widetilde{V}^b(z') \right\} \\ &= z + \widetilde{W}^b(0)\end{aligned}$$

where  $\widetilde{z} \geq 0$  is the choice of real balances for a buyer without access to credit such that

$$-i + \sigma \frac{\theta[u'(\widetilde{q}) - c'(\widetilde{q})]}{\theta c'(\widetilde{q}) + (1 - \theta)u'(\widetilde{q})} \leq 0, \quad (15)$$

and with equality if  $\widetilde{z} > 0$ . In addition,  $\widetilde{q}$  solves  $\widetilde{z} = (1 - \theta)u(\widetilde{q}) + \theta c(\widetilde{q})$  if  $\widetilde{z} < (1 - \theta)c(q^*) + \theta u(q^*)$  and  $\widetilde{z} = (1 - \theta)c(q^*) + \theta u(q^*)$  if  $\widetilde{z} \geq (1 - \theta)c(q^*) + \theta u(q^*)$ . By the linearity of  $W^b(z, -b)$  and  $\widetilde{W}^b(z)$ , a buyer will repay his debt if

$$b \leq \bar{b} \equiv W^b(0, 0) - \widetilde{W}^b(0)$$

where  $\bar{b}$  is the endogenous debt limit. In other words, the amount borrowed can be no larger than the cost of defaulting, which is the difference between the lifetime utility of a buyer with access to credit and the lifetime utility of a buyer permanently excluded from using credit.

**Lemma 1.** *The equilibrium debt limit,  $\bar{b}$ , is a solution to*

$$r\bar{b} = \max_{z \geq 0} \left\{ -iz + \sigma\theta \left[ (1 - \Lambda)S(z) + \Lambda S(z + \bar{b}) \right] \right\} - \max_{\widetilde{z} \geq 0} \left\{ -i\widetilde{z} + \sigma\theta S(\widetilde{z}) \right\} \equiv \Omega(\bar{b}) \quad (16)$$

where  $r = \frac{1-\beta}{\beta}$ , and  $S(\cdot) \equiv u[q(\cdot)] - c[q(\cdot)]$ .

The left-side of (16),  $r\bar{b}$ , represents the return from borrowing a loan of size  $\bar{b}$ . The right-side,  $\Omega(\bar{b})$ , is the flow cost of defaulting, which equals the surplus from not having access to credit. To characterize equilibrium under limited enforcement, we start by establishing some key properties of  $\Omega(\bar{b})$ .

**Lemma 2.** *The function  $\Omega(\bar{b}) \equiv \max_{z \geq 0} \left\{ -iz + \sigma\theta \left[ (1 - \Lambda)S(z) + \Lambda S(z + \bar{b}) \right] \right\} - \max_{\widetilde{z} \geq 0} \left\{ -i\widetilde{z} + \sigma\theta S(\widetilde{z}) \right\}$  has the following key properties:*

1.  $\Omega(0) = 0$ ,
2.  $\Omega'(0) = i\Lambda \geq 0$ ,

3.  $\Omega'(\bar{b}) \begin{cases} > 0 & \text{when } \bar{b} < (1 - \theta)u(q^*) + \theta c(q^*) \\ = 0 & \text{when } \bar{b} \geq (1 - \theta)u(q^*) + \theta c(q^*) \end{cases}$
4.  $\Omega(\bar{b})$  is a concave function  $\forall \bar{b} \in (0, (1 - \theta)u(q^*) + \theta c(q^*))$ ,
5.  $\Omega(\bar{b})$  is continuous for all  $z > 0$  and becomes discontinuous at  $z = 0$ .

To describe how the debt limit affects the value of money and output in the DM, we first define two critical values for the debt limit. The value  $\bar{b}_0$  is the threshold for the debt limit, above which money is no longer valued and solves

$$r\bar{b}_0 = \sigma\theta\Lambda S(\bar{b}_0).$$

The value  $\bar{b}_1$  is the threshold for the debt limit, above which the buyer can borrow enough to finance consumption of the first-best,  $q^*$ , and is given by:

$$\bar{b}_1 = (1 - \theta)u(q^*) + \theta c(q^*).$$

For all  $\bar{b} > \bar{b}_1$ ,  $r\bar{b} = \sigma\theta\Lambda S(q^*)$ . Consequently when  $z = 0$ ,  $\bar{b}_0 \leq \bar{b}_1$ .

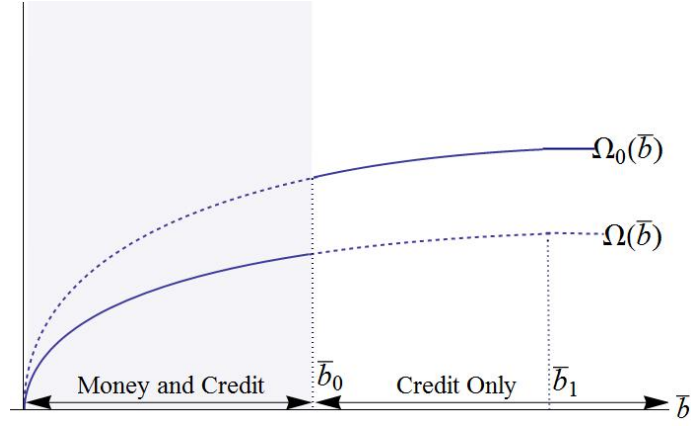
**Lemma 3.** *Equilibrium with limited enforcement will be such that*

1. If  $\bar{b} \in [0, \bar{b}_0)$ , then  $z > 0$  and  $q(z + \bar{b}) < q^*$ ,
2. If  $\bar{b} \in [\bar{b}_0, \bar{b}_1)$ , then  $z = 0$  and  $q(\bar{b}) < q^*$ ,
3. If  $\bar{b} \in [\bar{b}_1, \infty)$ , then  $z = 0$  and  $q(\bar{b}) = q^*$ .

At  $\bar{b} = 0$ , credit is not used and the buyer can only use money. Since  $\Omega(0) = 0$ , an equilibrium without credit always exists. If  $\bar{b} \in (0, \bar{b}_0)$ , the buyer can use both money and credit, but cannot borrow enough to obtain the first-best,  $q^*$ . When  $\bar{b} \in [\bar{b}_0, \bar{b}_1)$ , money is no longer valued and the buyer still cannot borrow enough to finance  $q^*$ . In this range, only credit is used. Finally when  $\bar{b} \in [\bar{b}_1, \infty)$ , money is not valued and the borrowing constraint no longer binds, in which case the buyer can borrow enough to finance consumption of the first-best,  $q^*$ .

The function  $\Omega(\bar{b})$  represents the flow cost of not having access to credit and is increasing in the size of the loan,  $\bar{b}$ . For  $z > 0$ , the right side of (16) is continuous for all  $\bar{b} \in [0, \bar{b}_0)$  and becomes discontinuous at  $\bar{b}_0$  when money is no longer valued. When  $z = 0$  and  $\bar{b} \in [\bar{b}_0, \bar{b}_1)$ , the right side of (16) is given by  $\Omega_0(\bar{b})$  and becomes linear at  $\bar{b}_1$  when the buyer can borrow enough to obtain the first-best,  $q^*$ . Furthermore, from Lemma 2, the slope of  $\Omega(\bar{b})$  at  $z > 0$  is  $\sigma\theta\Lambda S'(z)$ , which is smaller

Figure 2: Monetary and Credit Trades:  $\Omega(\bar{b})$  and  $\Omega_0(\bar{b})$



than the slope of  $\Omega_0(\bar{b})$  at  $z = 0$ , which is  $\sigma\theta\Lambda S'(0)$ . Consequently,  $\Omega(\bar{b})$  when  $z > 0$  is less than  $\Omega_0(\bar{b})$  when  $z = 0$ . This is shown in Figure 2 where  $\Omega(\bar{b})$  lies strictly below  $\Omega_0(\bar{b})$ . Money can only be valued in the shaded region where  $\bar{b} < \bar{b}_0$ , while equilibria is non-monetary for all  $\bar{b} > \bar{b}_0$ .

**Definition 1.** Given  $\Lambda$ , a steady-state equilibrium with limited enforcement is a list  $(q, q^c, z, \tilde{z}, \bar{b})$  that satisfy (10), (11), (14), (15), and (16).

We now turn to characterizing three types of equilibria that can arise in the model: (i) a pure credit equilibrium, (ii) a pure monetary equilibrium, and (iii) a money and credit equilibrium.

#### 4.1 Pure Credit Equilibrium

Figure 3: Pure Credit Equilibrium

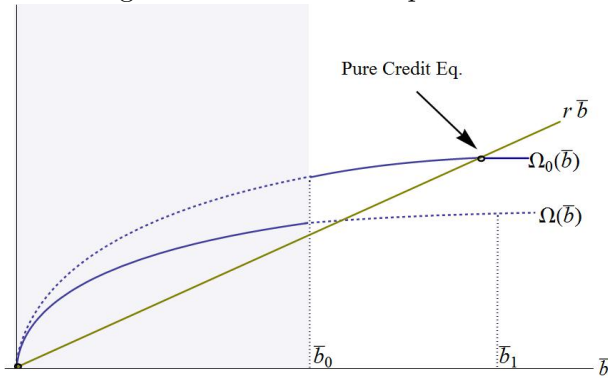
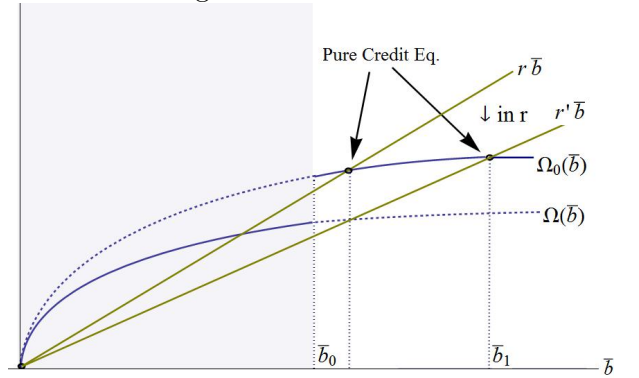


Figure 4: Decrease in  $r$



A non-monetary equilibrium with credit exists when  $z = \tilde{z} = 0$  and  $\bar{b} \in [\bar{b}_0, \infty)$ . When money is not valued ( $z = \tilde{z} = 0$ ), the debt limit  $\bar{b}$  must satisfy

$$r\bar{b} = \sigma\theta\Lambda S(\bar{b}) \equiv \Omega_0(\bar{b}). \quad (17)$$

A necessary condition for there to be credit is that the slope of  $r\bar{b}$  is less than the slope of  $\Omega_0(\bar{b})$  at  $\bar{b} = 0$ . Differentiating  $\Omega_0(\bar{b})$ , with respect to  $\bar{b}$  at  $\bar{b} = 0$  yields

$$\begin{aligned} \left. \frac{\partial \Omega_0(\bar{b})}{\partial \bar{b}} \right|_{\bar{b}=0} &= \sigma\theta\Lambda S'(0) \\ &= \sigma\Lambda \frac{\theta}{1-\theta}. \end{aligned}$$

An equilibrium with only credit will exist if

$$r < \sigma\Lambda \frac{\theta}{1-\theta}. \quad (18)$$

When the fraction of sellers accepting credit is exogenous, there exists a threshold for the fraction of credit trades, below which  $\bar{b} = 0$ . From (18), credit is feasible if

$$\Lambda > \frac{r(1-\theta)}{\sigma\theta} \equiv \bar{\Lambda}.$$

Figure 3 shows the determination of the debt limit. Notice that an equilibrium without credit always exists since  $\bar{b} = 0$  is always a solution to (16). This captures the idea that an equilibrium without credit is self-fulfilling and can arise under the expectation that borrowers will not repay their debts in the future.

In addition, there exists a critical value for the rate of time preference,  $\bar{r}$ , below which the debt limit stops binding and borrowers can borrow enough to purchase the first-best,  $q^*$ . The borrowing constraint will not bind if  $\bar{b} \geq (1-\theta)u(q^*) + \theta c(q^*)$ , in which case  $\bar{r}$  must satisfy  $\bar{r}[(1-\theta)u(q^*) + \theta c(q^*)] = \sigma\theta\Lambda[u(q^*) - c(q^*)]$ . Hence borrowers will be unconstrained if

$$r \leq \frac{\sigma\theta\Lambda[u(q^*) - c(q^*)]}{(1-\theta)u(q^*) + \theta c(q^*)} \equiv \bar{r}.$$

Hence the first-best is more likely to be attained if agents are more patient, trading frictions are small, buyers have enough market power in the DM, or the fraction of the economy with access to record-keeping is large. Since  $\bar{r} < \frac{\sigma\Lambda\theta}{1-\theta}$  is always satisfied whenever a pure credit equilibrium exists, the borrowing constraint binds if  $\bar{r} < r < \sigma\Lambda \frac{\theta}{1-\theta}$  and does not bind if  $r < \bar{r} < \sigma\Lambda \frac{\theta}{1-\theta}$ .



Figure 5: Pure Monetary Equilibrium

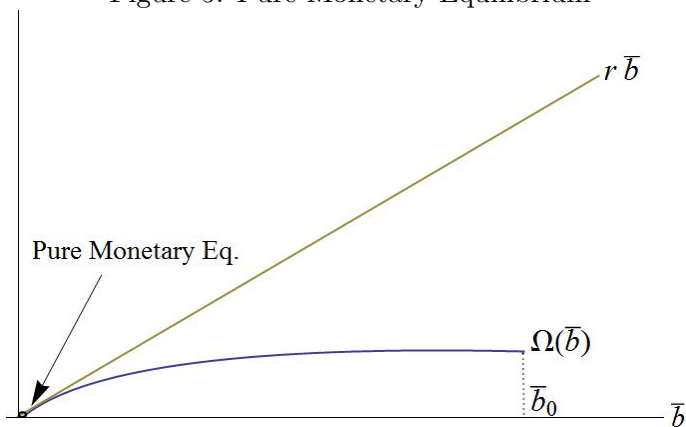


Figure 4 depicts the pure credit equilibrium and shows the effects of a decrease in  $r$ , or as agents become more patient. When  $r$  decreases to  $r' = \bar{r}$ , the debt limit increases to  $\bar{b}_1$  and quantity traded increases from  $q < q^*$  to  $q = q^*$ . Intuitively, the borrowing limit relaxes as agents become more patient since buyers can credibly promise to repay more. If on the other hand  $r$  increases above  $\sigma\Lambda\frac{\theta}{1-\theta}$ , the borrowing limit is driven to zero as borrowers are not patient enough to sustain credit use. As a result, a pure credit equilibrium will cease to exist.

More generally,  $\Omega_0(\bar{b})$  shifts up as the measure of sellers with access to record-keeping,  $\Lambda$ , increases, trading frictions,  $\sigma^{-1}$ , decrease, or the buyer's bargaining power,  $\theta$ , increases, each of which relaxes the debt limit and thereby increasing  $\bar{b}$ . Moreover, notice that in a pure credit equilibrium, inflation has no effect on the debt limit or equilibrium allocations.

## 4.2 Pure Monetary Equilibrium

In a pure monetary equilibrium, money is valued ( $z > 0$ ) while credit is not used ( $\bar{b} = 0$ ). At  $\bar{b} = 0$ ,  $\Omega(0) = 0$  by Lemma 2. Further, since  $S(z)$  is concave and  $S'(0) = \frac{1}{(1-\theta)}$  by the Envelope Theorem, money is valued if and only if

$$\begin{aligned} i &= \sigma\theta S'(z), \\ i &< \sigma\theta S'(0), \\ i &< \frac{\sigma\theta}{1-\theta} \equiv \bar{i}. \end{aligned}$$

The critical value,  $\bar{i}$  is the upper-bound for the cost of holding money, above which money is no longer valued.

In addition, a pure monetary equilibrium will exist uniquely so long as credit is not feasible, or  $i\Lambda < r$ , so that the slope of  $\Omega(\bar{b})$  at  $\bar{b} = 0$  is less than the slope of  $r\bar{b}$ . Consequently, there exists a critical value  $\underline{i} \equiv \frac{r}{\Lambda}$ , below which credit is not incentive-feasible. Figure 5 plots  $\Omega(\bar{b})$  as a function of  $\bar{b}$  when  $i < \underline{i}$  and  $i < \bar{i}$ . In that case,  $\Omega(\bar{b})$  and  $r\bar{b}$  intersect once at  $\bar{b} = 0$ , and the unique equilibrium is one where only money is used.

The next proposition characterizes how a key policy variable, the money growth rate  $\gamma$ , affects the existence of monetary equilibrium.

**Proposition 1.** *Define  $\underline{\gamma} \equiv \beta(1 + \underline{i})$  and  $\bar{\gamma} \equiv \beta(1 + \bar{i})$ , where  $\underline{i} \equiv \frac{r}{\Lambda}$  and  $\bar{i} \equiv \frac{\sigma\theta}{1-\theta}$ . If  $\underline{\gamma} < \bar{\gamma}$ , then  $r < \sigma\Lambda\frac{\theta}{1-\theta}$  and the following steady-state equilibria are possible:*

1. *If  $\gamma = \beta$ , a pure monetary equilibrium with  $q = q^*$ ,  $z = \tilde{z} = (1 - \theta)u(q^*) + \theta c(q^*)$ , and  $\bar{b} = 0$  exists uniquely.*
2. *If  $\gamma \in (\beta, \underline{\gamma})$ , a pure monetary equilibrium with  $q < q^*$ ,  $z = \tilde{z} \in (0, (1 - \theta)u(q^*) + \theta c(q^*))$ , and  $\bar{b} = 0$  exists uniquely.*
3. *If  $\gamma \in (\underline{\gamma}, \bar{\gamma})$ , a pure monetary equilibrium coexists with a pure credit equilibrium. If  $\bar{b} = 0$ , then  $z = \tilde{z} \in (0, (1 - \theta)u(q^*) + \theta c(q^*))$ , and  $q < q^*$ . If  $z = \tilde{z} = 0$ , then  $\bar{b} > 0$ . If in addition,  $r \in (0, \bar{r}]$ , equilibrium is unconstrained with  $q^c = q^*$  and  $\bar{b} = (1 - \theta)u(q^*) + \theta c(q^*)$ . If  $r \in (\bar{r}, \Lambda\bar{i})$ , equilibrium is constrained with  $q^c < q^*$  and  $\bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$ .*
4. *If  $\gamma \geq \bar{\gamma}$ , a pure monetary equilibrium ceases to exist, and a pure credit equilibrium will exist if  $\Lambda > \bar{\Lambda}$ . If  $r \in (0, \bar{r}]$ , then  $q^c = q^*$  and  $\bar{b} = (1 - \theta)u(q^*) + \theta c(q^*)$ . If  $r \in (\bar{r}, \Lambda\bar{i})$ , then  $q^c < q^*$  and  $\bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$ .*

The first part of Proposition 1 is very intuitive and simply says that when  $\gamma = \beta$ , the rate of return on money is high enough so that there is no need to use credit. This is because when  $\gamma$ , or equivalently  $i$ , decreases, the expected surplus from defaulting increases which raises the incentive to renege on debt repayment. This in turn tightens the credit constraint and leads the debt limit  $\bar{b}$  to fall. When  $\gamma \rightarrow \beta$  or  $i \rightarrow 0$ , money becomes costless to hold and the incentive to renege is too high to support voluntary debt repayment. Efficient monetary policy drives out credit, and money alone is enough to finance the first-best.

In addition, the Friedman rule is sufficient but not necessary to permit the uniqueness of a pure monetary equilibrium. Proposition 1 also shows that so long as  $\gamma < \underline{\gamma}$ , credit can never be sustained since the incentive to renege is too high. To take the most extreme case, suppose that

$\Lambda = 1$  so that all sellers accept credit. In that case, a pure monetary equilibrium will exist if  $i < r$ , or equivalently,  $\gamma < \underline{\gamma} = 1$ . Even though all sellers accept credit, buyers choose to only hold real balances since the incentives to renege on debt repayment is too high.

It is possible for a pure monetary equilibrium to coexist with a pure credit equilibrium when  $\gamma \in (\underline{\gamma}, \bar{\gamma})$ . In this region, the cost of money is high enough for debt repayment to be feasible and low enough so that money can still be valued.

When  $\gamma > \bar{\gamma}$ , money is too costly to hold and only credit is feasible. The first-best allocation can be achieved provided that agents are patient enough, or if  $r \in (0, \bar{r}]$ . This can be implemented with any inflation rate such that  $\gamma > \bar{\gamma} \equiv \beta(1 + \bar{i})$ . In that case, equilibrium is unconstrained and a pure credit economy ensures that agents trade the first-best level of output.

### 4.3 Money and Credit Equilibrium

In an equilibrium with both money and credit,  $z > 0$ ,  $\tilde{z} > 0$ , and  $\bar{b} \in (0, \bar{b}_0)$ . When  $\bar{b} > 0$ ,  $z$  has an interior solution so long as

$$\begin{aligned} i &= \sigma\theta(1 - \Lambda)S'(z) + \sigma\theta\Lambda S'(z + \bar{b}), \\ i &< \sigma\theta(1 - \Lambda)S'(0) + \sigma\theta\Lambda S'(\bar{b}), \\ i &< \frac{\sigma\theta(1 - \Lambda)}{1 - \theta} + \sigma\theta\Lambda S'(\bar{b}), \\ i &< (1 - \Lambda)\bar{i} + \sigma\theta\Lambda S'(\bar{b}) \equiv \tilde{i}. \end{aligned}$$

The critical value  $\tilde{i}$  is the upper-bound on the nominal interest rate, below which both money and credit can be sustained. In addition, there is also a lower-bound on  $i$ , above which debt repayment is incentive-feasible. This requires that the slope of  $\Omega(\bar{b})$  from (16) is higher than the slope of  $r\bar{b}$  at  $\bar{b} = 0$ , or

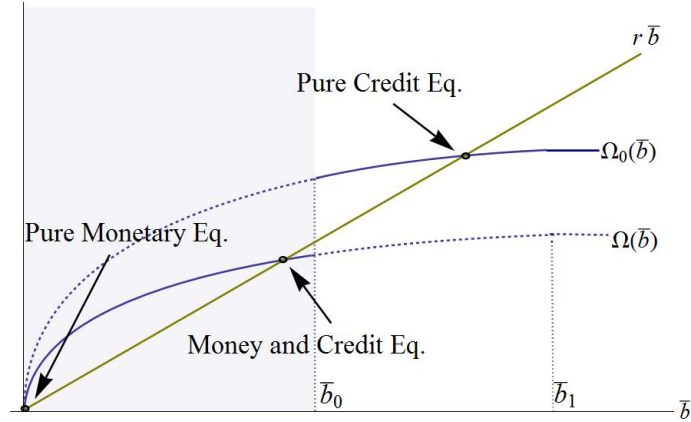
$$i\Lambda > r. \tag{19}$$

Consequently, there exists an equilibrium where both money and credit are used if  $i \in (\underline{i}, \tilde{i})$  where  $\underline{i} \equiv \frac{r}{\Lambda}$  and  $\tilde{i} \equiv (1 - \Lambda)\bar{i} + \sigma\theta\Lambda S'(\bar{b})$ .

An equilibrium with both money and credit is depicted in Figure 6. Notice that a pure monetary equilibrium will always exist whenever there is an equilibrium with both money and credit since the condition for a pure monetary equilibrium  $i < \bar{i}$  is always satisfied if  $i \in (\underline{i}, \tilde{i})$ .<sup>19</sup> Since only a fraction of sellers accept credit, money maintains a social role since it allows buyers to insure

<sup>19</sup>This can also be seen in Figure 6 and follows directly from Lemma 1: since  $\Omega(0) = 0$ , the function  $\Omega(\bar{b})$  will always intersect  $r\bar{b}$  at the origin  $\bar{b} = 0$  (i.e. a pure monetary equilibrium) whenever there is also an interior solution (i.e. a money and credit equilibrium if  $i\Lambda > r$ ).

Figure 6: Pure Monetary, Money and Credit, and Pure Credit Equilibria



against the possibility of not being able to use credit in some transactions.

The following table summarizes some comparative statics for effects on the debt limit.

$\frac{\partial \bar{b}}{\partial \Lambda}$	$\frac{\partial \bar{b}}{\partial \sigma}$	$\frac{\partial \bar{b}}{\partial i}$	$\frac{\partial \bar{b}}{\partial r}$	$\frac{\partial \bar{b}}{\partial \theta}$
+	+	+	-	+

An increase in  $\Lambda$  increases the right-hand-side of (16), which shifts  $\Omega(\bar{b})$  up and induces an increase in  $\bar{b}$ . When more sellers accept credit, the gain for buyers from using and redeeming credit increases, which relaxes the payment constraint  $b \leq \bar{b}$ . The increase in  $\Lambda$  can be high enough so that credit starts to drive money out of circulation. This can cause the economy to shift to a pure credit equilibrium where money is no longer valued.

An increase in inflation (analogously,  $i$ ) generates the same qualitative effect: credit is more profitable than money in the sense that the value of money decreases over time. In this way, inflation has two effects in this model: first, is the usual effect on reducing the purchasing power of money, which reduces trade and hence welfare; second, is the effect on reducing agents' incentive to default. Intuitively, an increase in inflation relaxes the credit constraint by increasing the cost of default, since defaulters need to bring enough money to finance their consumption.

In sum, the debt limit depends on the fraction of credit trades, the extent of trading frictions, the rate of return on money, agents' patience, and the buyer's bargaining power. The larger the fraction of sellers that accept credit, the lower the rate of return on money, or the more patient agents become, the less likely the credit constraint will be binding. In these cases, the buyer can credibly promise to repay more, which induces cooperation in credit arrangements thereby relaxing the debt limit.

The next proposition summarizes the conditions under which an equilibrium with money and credit exists and establishes a particularly interesting case where a money and credit equilibrium ceases to exist.

**Proposition 2.** *When  $i \in (\underline{i}, \tilde{i})$  and  $\Lambda \in (0, 1)$ , a money and credit equilibrium coexists with a pure monetary equilibrium and a pure credit equilibrium. If  $\Lambda = 1$ , there can either be a pure credit equilibrium where  $\bar{b} > 0$  and  $z = \tilde{z} = 0$  or a pure monetary equilibrium where  $\bar{b} = 0$  and  $z > 0$ , but there cannot be an equilibrium where both money and credit are used.*

PROOF. If  $\Lambda = 1$  and money is valued, (14) and (15) imply that  $i = \sigma\theta S'[q(z + \bar{b})] = \sigma\theta S'[q(\tilde{z})]$ , or  $q(z + \bar{b}) = q(\tilde{z})$ . Then since  $z + \bar{b} = \tilde{z} = (1 - \theta)u(q) + \theta c(q)$  from the bargaining solution, the left side of the debt limit (16) becomes  $-i[z - \tilde{z}] = -i[\tilde{z} - \bar{b} - \tilde{z}] = i\bar{b}$ . Consequently, (16) implies that  $r\bar{b} = i\bar{b}$ , or  $\bar{b} = 0$ .  $\square$

Proposition 2 highlights an important dichotomy between monetary and credit trades when  $\Lambda = 1$ : there can be trades with credit only or trades with money only, but never trades with both money and credit. At  $\Lambda = 1$ , (14) and (15) implies that if money is valued, the debt limit must be zero:  $z = \tilde{z} > 0$  implies  $\bar{b} = 0$ . Since buyers obtain the same surplus whether or not they default, there cannot exist a positive debt limit that supports voluntary debt repayment. Consequently, the debt limit is driven to zero and there cannot be a monetary equilibrium where credit is also used. This special case also points to the difficulty of getting both money and credit to be used when all trades are identical and record-keeping is costless: either only credit is used as money becomes inessential, or only money is used since the incentive to renege on debt repayment is too high.

#### 4.4 Multiple Equilibria

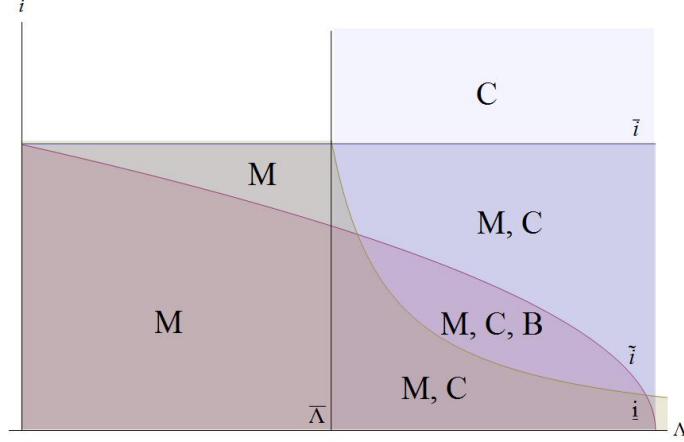
A particularly striking feature of the model is that there can be a multiplicity of equilibria even without any changes in fundamentals. The next proposition establishes the possible cases for multiple equilibria, which the remainder of this subsection discusses.

**Proposition 3.** *When  $i > \bar{i}$ , equilibrium will be non-monetary and there will either be (i) autarky where neither money nor credit is used if  $\Lambda < \bar{\Lambda}$  or (ii) a pure credit equilibrium if  $\Lambda > \bar{\Lambda}$ . When  $i < \bar{i}$ , a pure monetary equilibrium either (iii) exists uniquely if  $i < \underline{i}$ , (iv) coexists with a pure credit equilibrium if  $i > \underline{i}$ , or (v) coexists with both a pure credit equilibrium and a money and credit equilibrium if  $\underline{i} < i < \tilde{i}$ .*

Proposition 3 is illustrated in Figure 7, which plots existence conditions for different types of equilibria in  $(\Lambda, i)$ -space.<sup>20</sup> We have shown in the previous sub-sections that a necessary (but

<sup>20</sup>The types of equilibria in Figure 7 are pure credit (C) equilibrium where only credit is used, a pure monetary (M) equilibrium where only money is used, and a mixed equilibrium (B) where both money and credit are used.

Figure 7: Multiple Equilibria in  $(\Lambda, i)$ -Space



not sufficient) condition for credit is  $\Lambda > \bar{\Lambda}$ . A pure monetary equilibrium will exist if and only if  $i < \bar{i} \equiv \frac{\sigma\theta}{1-\theta}$ . For both money and credit to be used, it must be that  $\underline{i} < i < \tilde{i} \equiv (1-\Lambda)\bar{i} + \sigma\theta\Lambda S'(\bar{b})$ . Consequently, there will be a unique equilibrium with credit only when  $i > \bar{i}$  and  $\Lambda > \bar{\Lambda}$ ; a unique equilibrium with money only when  $i < \bar{i}$  and  $\Lambda < \bar{\Lambda}$ ; or multiple equilibria when  $\underline{i} < i < \bar{i}$ .

Figure 7 also shows how payment systems depend not just on fundamentals but also on histories and social conventions. Suppose that inflation is initially low and the economy is in an equilibrium where a pure monetary equilibrium coexists with a pure credit equilibrium (region  $M, C$ ). As inflation increases above  $\bar{i}$ , the pure monetary equilibrium disappears and only credit is used (region  $C$ ). But when inflation goes back down to its initial level, it is possible that agents may still coordinate on the pure credit equilibrium. The economy therefore displays hysteresis and inertia: when there are many possible types of equilibria, social conventions and histories can dictate the equilibrium that prevails.

When agents get less patient ( $r$  increases), both the threshold for credit to be used,  $\bar{\Lambda}$ , and the condition for both money and credit to be used,  $\frac{r}{\Lambda}$ , increases. In Figure 7, the vertical line  $\bar{\Lambda}$  shifts to the right while the curve  $\frac{r}{\Lambda}$  shifts up. An increase in  $r$  therefore decreases the possibility of any equilibrium with credit. Intuitively, less patient buyers find it more difficult to credibly promise to repay their debts, which decreases their borrowing limit  $\bar{b}$ .

## 5 Costly Record-Keeping

We now consider the choice of accepting credit by making  $\Lambda \in [0, 1]$  endogenous. In order to accept credit, sellers must invest *ex-ante* in a costly record-keeping technology that records

and authenticates an *IOU* proposed by the buyer.<sup>21</sup> The per-period cost of this investment is  $\kappa > 0$ , which is drawn from a cumulative distribution  $F(\kappa) : \mathbb{R}_+ \rightarrow [0, 1]$ . Sellers are heterogeneous according to their record-keeping cost and are indexed by  $\kappa$ .<sup>22</sup> Hence for some sellers this cost will be close to zero, so that they will always accept credit, while for others this cost will be very large and they will never accept credit. The distribution of costs across sellers is known by all agents and is assumed to be continuous.

At the beginning of each period before trades occur, sellers choose whether or not to invest in the costly record-keeping technology. When making this decision, sellers take as given buyer's choice of real balances,  $z$ , and the debt limit,  $\bar{b}$ . The seller's problem is given by

$$\max\{-\kappa + \sigma(1 - \theta)S(z + \bar{b}), \sigma(1 - \theta)S(z)\}. \quad (20)$$

According to (20), if the seller decides to invest, he incurs the disutility cost  $\kappa > 0$  that allows him to extend a loan to the buyer. In that case, the seller extracts a constant fraction  $(1 - \theta)$  of the total surplus,  $S(z + \bar{b}) \equiv u[q(z + \bar{b})] - c[q(z + \bar{b})]$ . If the seller does not invest, then he can only accept money, and gets  $(1 - \theta)$  of  $S(z) \equiv u[q(z)] - c[q(z)]$ . Since total surplus is increasing in the buyer's total wealth  $z + \bar{b}$ ,  $S(z + \bar{b}) > S(z)$ . Further,  $S(z + \bar{b})$  and hence the first term in the seller's maximization problem (20) increases with  $\bar{b}$ , and both terms of (20) increase with  $z$ .

There exists a threshold for the record-keeping cost,  $\bar{\kappa}$  below which the seller invests in the record-keeping technology and above which they do not invest. From (20), this threshold is given by

$$\bar{\kappa} \equiv \sigma(1 - \theta)[S(z + \bar{b}) - S(z)], \quad (21)$$

and gives the seller's expected benefit of accepting credit. Since  $S(z + \bar{b})$  increases with  $\bar{b}$ , the seller's expected benefit  $\bar{\kappa}$  increases with  $\bar{b}$ . Given  $\kappa$ , let  $\lambda(\kappa) \in [0, 1]$  denote an individual seller's decision to invest. This decision problem is given by

$$\lambda(\kappa) = \begin{cases} 1 & \text{if } \kappa < \bar{\kappa} \\ [0, 1] & \text{if } \kappa = \bar{\kappa} \\ 0 & \text{if } \kappa > \bar{\kappa} \end{cases} \quad (22)$$

Condition (22) simply says that all sellers with  $\kappa < \bar{\kappa}$  will invest in the costly record-keeping

<sup>21</sup>This cost can also reflect issues of fraud and information problems that currently permeate the credit industry. In fact, the credit card industry is facing serious challenges in the form of credit card fraud, identity theft, and the need to secure confidential information. Besides being a costly drain on banks and retailers that accept credit, these problems may erode consumer confidence in the credit card industry.

<sup>22</sup>Arango and Taylor (2008) find that merchants perceive cash as the least costly form of payment while credit cards stand out as the most costly due to relatively high processing fees.

technology, since the benefit exceeds the cost; sellers with  $\kappa > \bar{\kappa}$  do not invest; and any seller with  $\kappa = \bar{\kappa}$  will invest with an arbitrary probability since they are indifferent.

Consequently, since  $F(\kappa)$  is continuous, the aggregate measure of sellers that invest is

$$\Lambda \equiv \int_0^\infty \lambda(\kappa) dF(\kappa) = F(\bar{\kappa}). \quad (23)$$

That is, the measure of sellers that invest is given by the measure of sellers with  $\kappa \leq \bar{\kappa}$ .

**Definition 2.** *A stationary monetary equilibrium with limited enforcement and endogenous  $\Lambda$  is a list  $(q, q^c, z, \bar{z}, \bar{b}, \Lambda)$  that satisfy (10), (11), (14), (15), (16), and (23).*

To determine equilibrium when  $\Lambda$  is endogenous, we first determine buyers' choice of real balances and how much they want to borrow, given sellers' investment decisions. Next we determine sellers' investment decisions, given buyers' choice of real balances and decision to repay their debts. These decisions are then depicted as reaction functions for buyers and sellers, respectively.

## 5.1 Buyers' Reaction Function

Given sellers' investment decisions  $\Lambda$ , buyers must decide how much money to hold and how much to borrow. Indeed, the buyer's choice of real balances,  $z$ , and the amount borrowed,  $\bar{b}$ , are each functions of the measure of sellers accepting credit,  $\Lambda$ . In what follows, we characterize equilibria where  $i < \bar{i}$ , otherwise money has no value.

When  $\Lambda \in [0, \frac{r}{i})$ , buyers only use money, and we show in Section 4.2 that a pure monetary equilibrium with  $z > 0$  and  $\bar{b} = 0$  will exist uniquely. When  $\Lambda \in (\frac{r}{i}, 1]$ , there can either be a money and credit equilibrium with  $z > 0$  and  $\bar{b} \in (0, \bar{b}_0)$ , or a pure credit equilibrium with  $z = 0$  and  $\bar{b} \in (\bar{b}_0, \bar{b}_1)$ . However when  $\Lambda = 1$ , it follows from Proposition 3 that the only equilibrium with a positive debt limit is a pure credit equilibrium with  $z = 0$  and  $\bar{b} = b_1$ .

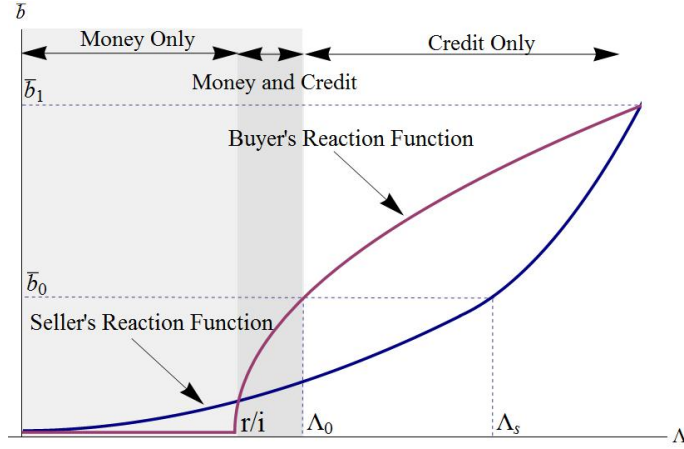
The following lemma establishes some key properties of the buyer's reaction function.

**Lemma 4.** *When  $\Lambda \in (0, \frac{r}{i}]$ ,  $\bar{b} = 0$ . When  $\Lambda \in (\frac{r}{i}, 1]$ , the debt limit  $\bar{b}$  is a strictly increasing and concave function of  $\Lambda$ .*

At  $\bar{b}_0$ , the corresponding fraction of sellers that accept credit is defined as  $\Lambda_0 \equiv \frac{r\bar{b}_0}{\sigma\theta S(\bar{b}_0)}$ . Clearly when  $\Lambda \in [0, \frac{r}{i})$ , credit is not used:  $\bar{b} = 0$ . When  $\Lambda \in (\frac{r}{i}, \Lambda_0)$ , the debt limit becomes positive with  $\bar{b} \in (0, \bar{b}_0)$ . Finally when  $\Lambda \in (\Lambda_0, 1)$ , money is no longer valued and the debt limit is  $\bar{b} \in (\bar{b}_0, \bar{b}_1)$ . The buyer's reaction function is depicted in Figure 8, which shows the buyer's choice of  $\bar{b}$  for a given  $\Lambda$ .



Figure 8: Buyers' and Sellers' Reaction Functions: Multiple Equilibria



## 5.2 Sellers' Reaction Function

Given buyers' decision to hold money and whether to repay their debts, sellers must decide whether or not to invest in the costly technology to record credit arrangements. Indeed, sellers' investment decisions,  $\Lambda$ , depend on the buyer's choice of real balances  $z$ , and the amount borrowed,  $\bar{b}$ .

When  $\bar{b} = 0$ , the debt limit is zero and buyers only use money for DM trades. Clearly from (21), the seller's expected benefit of accepting credit at  $\bar{b} = 0$  is zero:  $\bar{\kappa} = 0$ . Consequently, no sellers will invest:  $\Lambda = 0$ . When  $\bar{b} = \bar{b}_1$ , money is no longer valued and buyers only use credit. In that case, the seller's expected benefit of accepting credit is at its maximum:  $\bar{\kappa}_{\max} \equiv \sigma(1 - \theta)S^*$  where  $S^* \equiv S(\bar{b}_1)$ . We assume that all sellers have a record-keeping cost smaller than this maximum value,  $\bar{\kappa}_{\max}$ . Consequently, all sellers will invest at  $\bar{b} = \bar{b}_1$ , in which case  $\Lambda = 1$ .<sup>23</sup>

We now determine the equilibrium measure of sellers who invest when  $\bar{b} \in (0, \bar{b}_1)$ . In what follows, we assume that sellers' record-keeping costs are drawn from a uniform distribution of  $\kappa$  over  $[0, \bar{\kappa}_{\max}]$ .<sup>24</sup> The following lemma establishes how the aggregate measure of sellers who invest depends on the debt limit.

**Lemma 5.** *The fraction of sellers who invest,  $\Lambda$ , is a strictly increasing function of the credit limit,  $\bar{b}$ .*

<sup>23</sup>Without assuming an upper bound on the record-keeping cost, it is also possible to have a pure credit equilibrium with  $\Lambda < 1$ . In that case, the seller's reaction function in Figure 8 can intersect with the buyer's at  $\bar{b} \in (\bar{b}_0, \bar{b}_1)$  so that total surplus in the pure credit equilibrium will be less than  $S^*$ . This distinction does not matter for our qualitative results but will matter for welfare, as we discuss in Section 7.

<sup>24</sup>Our results go through more generally, with any continuous distribution  $F(\kappa) : \mathbb{R}_+ \rightarrow [0, 1]$ . Here, the assumption of the uniform distribution permits us to determine the value of  $\Lambda$  at  $\bar{b}_0$ .

Together, Lemma 4 and Lemma 5 allow us to characterize equilibrium as a function of the measure of sellers who invest,  $\Lambda$ , and the debt limit,  $\bar{b}$ .

**Proposition 4.** *When  $\gamma \in [\beta, \bar{\gamma})$  and  $\Lambda$  is endogenous, there are multiple equilibria. There exists (i) a pure monetary equilibrium with  $\Lambda = 0$ ,  $z > 0$ , and  $\bar{b} = 0$ , (ii) a pure credit equilibrium with  $\Lambda = 1$ ,  $z = 0$ , and  $\bar{b} = 0$ ; and (iii) a money and credit equilibrium with  $\Lambda < \Lambda_0 \in (0, 1)$ ,  $z > 0$ , and  $\bar{b} > 0$ .*

Proposition 4 is illustrated in Figure 8. The buyers' and sellers' reaction functions intersect three times, corresponding to three different types of equilibria: a pure monetary equilibrium where only money is used ( $\Lambda = 0$ ), a money and credit equilibrium where a fraction  $\Lambda < \Lambda_0$  of sellers accept both money and credit while the remaining  $(1 - \Lambda)$  sellers only accept money, and a pure credit equilibrium where money is not valued and all sellers accept credit ( $\Lambda = 1$ ).

Multiplicity arises through general equilibrium effects in the trading environment that produce strategic complementarities between buyers' and sellers' decisions. When more sellers invest in the costly record-keeping technology, the gain for buyers from using credit also increase. This lowers the incentive to default and relaxes the debt limit. As a result, buyers use more credit and hold less money. Intuitively, if it is more likely that sellers accept credit, then money is needed in a smaller fraction of matches. So long as it is costly to hold money, buyers will therefor carry fewer real balances. As a result, sellers have even more incentive to invest to accept credit, which in turn raises the debt limit and hence the buyer's real balances. These feedback effects produce network externalities that have frequently been described to characterize retail payment systems.<sup>25</sup>

## 6 Welfare

We now turn to examining some of the model's normative implications and begin by comparing the different types of equilibria in terms of social welfare. Society's welfare is measured as the steady-state sum of buyers' and sellers' utilities in the DM:  $\mathcal{W} \equiv (1 - \beta)V^b(z) + (1 - \beta)V^s(0)$ . This is given by

$$\mathcal{W} \equiv \sigma[\Lambda S(z + \bar{b}) + (1 - \Lambda)S(z)] - k \quad (24)$$

where  $k \equiv \int_0^{\bar{\kappa}} \kappa dF(\kappa)$  is defined as the aggregate record-keeping cost averaged across all sellers. There can be a pure monetary equilibrium with  $\bar{b} = 0$ ,  $z > 0$ , and  $\Lambda = 0$ ; a pure credit equilibrium with  $\bar{b} > 0$ ,  $z = 0$ , and  $\Lambda = 1$ ; and finally a money and credit equilibrium with  $\bar{b} \in (0, \bar{b}_0)$  and  $z > 0$

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<sup>25</sup>Strategic complementarities between the seller's decision to invest and the buyer's choice of real balances would still exist even under perfect enforcement where borrowers can always borrow enough to finance purchase of the first-best. See Nosal and Rocheteau (2010) for an analysis assuming loan repayments are always perfectly enforced.

where a fraction  $\Lambda \in (0, 1)$  of sellers accept both money and credit while the remaining  $(1 - \Lambda)$  sellers only accept money. Table 1 summarizes social welfare across these types of equilibria.

Table 1: **Welfare Across Equilibria**

EQUILIBRIUM	WELFARE
Pure Monetary	$\mathcal{W}_m = \sigma S(z)$
Pure Credit	$\mathcal{W}_c = \sigma S(z + \bar{b}) - k$
Money and Credit	$\mathcal{W}_{mc} = \sigma[\Lambda S(z + \bar{b}) + (1 - \Lambda)S(z)] - k$

For the pure credit equilibrium to dominate the pure monetary equilibrium, the aggregate record-keeping cost must be low enough; that is,  $\mathcal{W}_c > \mathcal{W}_m$  if  $k \in (0, \sigma S(z + \bar{b}) - S(z))$ . Even with a low-record keeping cost however, it is still possible for the welfare-dominated monetary equilibrium to prevail due to a rent-sharing externality: since sellers must incur the full cost of adopting the record-keeping technology but only obtains a fraction  $(1 - \theta)$  of the total surplus, they fail to internalize the full benefit of accepting credit. Consequently, there can be coordination failures and excess inertia in the decision to accept credit, in which case the economy can end up in the Pareto-inferior monetary equilibrium.

## 6.1 Optimal Monetary Policy

We now consider the issue of optimal monetary policy for the three types of equilibria examined above. To fix ideas, we start by assuming  $\Lambda$  is exogenous. The presence of multiple equilibria for the same fundamentals makes the choice of optimal policy difficult to analyze in full generality since we must deal with the issue of equilibrium selection. In regions with multiplicity, we will assume agents coordinate on a particular equilibrium and then analyze the optimal policy of that equilibrium.

When  $i < \underline{i}$  and  $\underline{i} < \bar{i}$ , agents only use money and the model reduces to the pure-currency economy of Rocheteau and Wright (2005). In that case, social welfare is decreasing in the inflation rate:  $\frac{d\mathcal{W}_m}{di} = \sigma(1 - \Lambda)S'(z)\frac{dz}{di} < 0$  since from (14),  $\frac{dz}{di} = [\sigma\theta(1 - \Lambda)S''(z)]^{-1} < 0$ . Since inflation is a tax on money holdings, an increase in inflation will reduce the purchasing power of money, and hence output and welfare. If lump-sum taxes can be enforced, the optimal policy in a pure monetary equilibrium corresponds to the Friedman rule.

Now suppose that  $\Lambda > \bar{\Lambda}$  and  $i \geq \bar{i}$  so that the economy is in a pure credit equilibrium. Since money is not valued, inflation has no effect on welfare, irrespective of whether or not equilibrium is efficient. When  $r \in (0, \bar{r}]$ , borrowers are patient enough to finance consumption of the first-best, in which case welfare is at its maximum for all inflation rates. Otherwise, equilibrium is inefficient

and welfare is strictly dominated by welfare in a pure monetary equilibrium at the Friedman rule.

Finally, suppose that  $\tilde{i} < i < \bar{i}$  and agents coordinate on an equilibrium where both money and credit are used. In that case, the overall effect of inflation on welfare will be ambiguous and depend on two effects: a real balance effect and the debt limit effect. In a money and credit equilibrium, the effect of inflation on welfare is given by

$$\frac{dW_{mc}}{di} = \sigma \left[ \Lambda S'(z + \bar{b}) \underbrace{\frac{d(z + \bar{b})}{di}}_{(+)} + (1 - \Lambda) S'(z) \underbrace{\frac{dz}{di}}_{(-)} \right].$$

In the  $(1 - \Lambda)$  of transactions involving money only, inflating is simply a tax on buyers' real balances, which decreases welfare. However in the  $\Lambda$  of transactions with both money and credit, an increase in inflation can be welfare improving by relaxing agents' borrowing constraints. Intuitively, higher inflation makes default more costly which reduces the incentive to default. The overall effect of inflation on welfare will be positive if  $\Lambda$  is sufficiently large, or if  $\frac{\Lambda}{1 - \Lambda} > \frac{u'(q) - c'(q)}{u'(q^c) - c'(q^c)}$  since it can be shown that  $\left| \frac{d(z + \bar{b})}{di} \right| > \left| \frac{dz}{di} \right|$ . Inflation can therefore have redistributive effects across agents by lowering the consumption in the  $(1 - \Lambda)$  transactions with money only while raising consumption for the fraction  $\Lambda$  of credit users.<sup>26</sup>

Our findings are summarized in Figures 9 and 10, which depict social welfare as a function of the money growth rate,  $\gamma$ . The welfare function is continuous and concave within a particular equilibrium, but can be discontinuous at the transition from one equilibrium to another. Figure 9 is an example where a pure monetary equilibrium exists uniquely for  $\gamma < \bar{\gamma}$  and then ceases to exist at  $\bar{\gamma}$  when money is no longer valued. In the region where only money is used, welfare is maximized at the Friedman rule,  $\gamma = \beta$ , in which case agents trade  $q^*$ . When  $\gamma > \bar{\gamma}$  there will exist a pure credit equilibrium. Moreover when  $\Lambda$  is endogenous, welfare in a pure credit equilibrium under any inflation rate will always be dominated by a pure monetary equilibrium under the Friedman rule since sellers must incur the real cost of technological adoption.

We now compare welfare in a pure monetary equilibrium versus an equilibrium with both money and credit. Whenever an equilibrium with both money and credit exist, there also exists a pure monetary equilibrium and a pure credit equilibrium, as shown in Figure 7. While there is a multiplicity of equilibria, we assume in Figure 10 that agents coordinate on the pure monetary equilibrium when  $\gamma \in [\beta, \underline{\gamma})$  and coordinate on the money and credit equilibrium when  $\gamma > \bar{\gamma}$ . As before, the Friedman rule maximizes welfare in the pure monetary equilibrium and implements the

<sup>26</sup>The redistributive effects of inflation are also discussed in Chiu, Dong, and Shao (2012) in a competitive environment. However, competitive pricing implies that deviations from the Friedman rule is still sub-optimal and inflation is always welfare reducing, whereas inflation can be welfare improving in our model.

Figure 9: Welfare in a Pure Monetary vs. Pure Credit Equilibrium

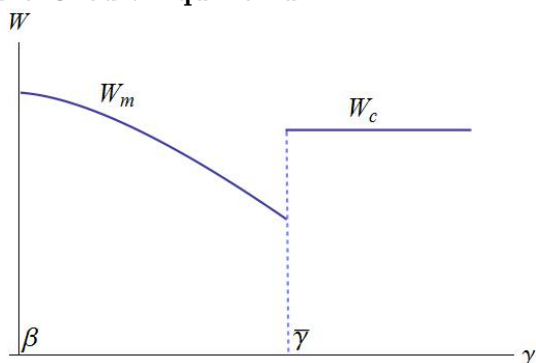
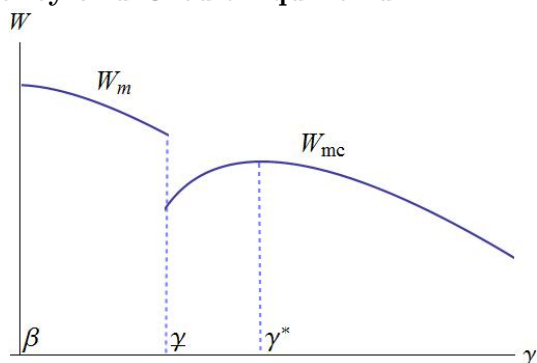


Figure 10: Welfare in a Pure Monetary vs. Money and Credit Equilibrium



first-best allocation. When both money and credit are used, there exists an interior money growth rate that is strictly above the Friedman rule that maximizes welfare. However notice that this equilibrium is strictly dominated in terms of welfare by a pure monetary equilibrium under the Friedman rule.

When  $\Lambda$  is endogenous, the positive effect of inflation on welfare is amplified and generates feedback effects. Since an increase in inflation raises the debt limit, sellers now have an even greater incentive to accept credit, which further relaxes borrowing constraints. When the debt limit relaxes to the point where it no longer binds, all sellers accept credit and money is no longer valued. In that case, agents can still trade the first-best level of output even when monetary authorities do not implement the Friedman rule. Notice however this equilibrium is not socially efficient since sellers must still incur the real cost of technological adoption.

## 7 Discussion

In this section, we discuss relevant policy issues and the extent to which our model is able to reconcile some of the empirical evidence on the use of money and credit in retail transactions. Understanding the substitution between cash and other payment instruments such as credit cards is a vital policy concern as this mechanism is crucial for policymakers when developing regulation on surcharges and interchange fees to implement the socially optimal payment system. As it is the sole issuer of bank notes, central banks also need to understand substitution patterns to predict cash demand.

## 7.1 Entrenchment of Cash

While consumers are adopting new instruments such as credit cards and electronic payments, they are not necessarily discarding older ones such as cash (Schuh (2012)). Even with falling costs in electronic record-keeping, our model predicts that agents may still coordinate on using cash due to a hold-up externality in technological adoption. Since retailers do not receive the full surplus associated with technological adoption, they fail to internalize the total benefit of adopting credit. Consequently, there may be inertia in the adoption of new forms of payment.

This can explain why some merchants have been slow to adopt new technologies for accepting credit. For example, Gerdes (2008) reports that the vast majority of card payments made within the United States are still being made using magnetic stripe technology even though advanced chip-based technology on “smart cards” are available. Adoption of new technologies remains limited because merchants have not extensively adopted terminals that can read them. As we discuss more below, policymakers must therefore remember to take into account this sluggish response when designing policies or regulations geared towards merchant behavior.

## 7.2 Consumer Borrowing and Inflation

The recent recession from 2008 and 2009 in the United States provides a good recent example of our theory in practice. During this period, the economy went from an average inflation rate of 3.85% to deflation at -0.34% per year. Similarly, short-term interest rates available to consumers through the rate on one-month certificates of deposit, fell from about 5% before the recession to nearly zero. These changes in economic conditions also lead to changes in payment behavior. For the United States, Foster, Meijer, Chuh, and Zabek (2011) find that between 2008 and 2009, cash payments increased by 26.9%, cash holdings increased by 25.5%, while credit card payments decreased by 21.9%.<sup>27</sup>

This shift in consumer behavior during the 2008–2009 recession can be explained by our theory. According to Proposition 1, consumer borrowing declines as inflation falls since this raises the incentive to default. Deflation completely crowds out credit and as a result, consumers shift towards using cash. As the evidence suggests, since both the use of cash and credit is indexed to inflation, agents treat the two as substitutes as economic conditions change. Similarly, Kahn, Senhadji, and Smith (2006) find that for countries with low inflation, decreases in inflation increases cash use and decreases credit use.

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<sup>27</sup>Besides deflation, the authors also discuss other factors that may explain this shift in consumer payments such as changes in government regulations toward credit and debit cards, and changes in consumers’ assessment about the security of electronic payments. Our model has a role for these factors as well, as we discuss below.

### 7.3 Hysteresis and Coordination Failures

Under certain conditions, our theory predicts that two economies with similar technologies, constraints, and policies can still end up with very different payment systems. Indeed, Proposition 2 and Figure 7 show that multiple equilibria can arise even without any changes in fundamentals. To take one example, suppose there are two countries, A and B. In each country, inflation is initially very low and all agents coordinate on an equilibrium where only money is used. Now suppose that both countries experience a temporary period of high inflation. If inflation increases above some threshold, the pure monetary equilibrium ceases to exist and agents turn to using credit to avoid the inflation tax. Now suppose that in both countries, inflation goes back down to its initial low level. It is possible that agents in country A still coordinate on using credit while agents in country B go back to using only money. Since both outcomes are consistent with economic fundamentals, which equilibrium an economy ends up in will depend in large part on the beliefs and expectations of market participants.

The reason why coordination failures can arise is due to the two-sided nature of the payment system and the beliefs of market participants. As is evident from agents' upward-sloping reaction functions in Figure 8, what the seller accepts affects what the buyer holds and vice versa. Coordination failures such as the kind described above can therefore arise since the maximum the buyer can borrow and the measure of sellers that adopt the record-keeping technology are complements.<sup>28</sup>

Consequently, economies with similar fundamentals can still end up with drastically different payment systems, some being better from society's perspective than others. This therefore provides rationale for policy interventions that help individuals coordinate on the better outcome. For instance, the welfare analysis in Section 6 implies that a pure credit equilibrium dominates a pure monetary one in terms of social welfare if inflation is high enough or the record-keeping cost is low enough. In that case, policymakers may want to introduce mechanisms that help coordinate agents on using credit, such as information campaigns, advertisements, or even financial literacy programs.

### 7.4 Payment Regulations and Merchant Fees

Besides monetary policy intervention through changes in the nominal interest rate or inflation rate, government policies that affect the provision of credit cards through fees and regulations also affects consumer demand for different payment instruments. For instance, new restrictions were recently imposed on banks and payment card networks in the United States that lead to declines in credit

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<sup>28</sup>Coordination failure frequently arise in economies with frictions, and show up in many different contexts such as goods markets and labor markets, as in Diamond (1982), Blanchard and Summer (1987), and discussed in Rocheteau and Tasci (2007).

card acceptance by merchants as well as decreases in credit usage by consumers. According to Foster, Meijer, Chuh, and Zabek (2011), businesses pay merchant discount fees on credit cards to their banks that amount to as much as 2% of total sales. Regulations that increased these fees decreased the number of merchants that choose to accept credit, and have lead some merchants to attempt at steering consumers toward lower cost payment instruments such as cash.<sup>29</sup>

Our theory suggests that since policy affects both retailers and consumers jointly, regulations of this kind may have multiplier effects. For instance, suppose that policymakers pass a new law that decreases credit card fees for merchants. In the model, a fall in  $\kappa$  gives merchants stronger incentives to invest in the technology that allows for credit. At the same time, when more sellers accept credit, the gain for buyers from using and redeeming credit also increases. This relaxes the borrowing constraint and permits buyers to use more credit relative to cash for transactions, which in turn amplifies the initial policy change. Through this feedback effect, policy changes such as the wave of new regulation imposed on payment card networks in 2008 and 2009 may end up inducing sizable shifts in consumer behavior.<sup>30</sup>

## 7.5 Payment Systems and Market Structure

Our framework can be used to determine how changes in market structure in retail markets impact the adoption of payment instruments by both firms and consumers. The parameter  $(1 - \theta)$  can be interpreted as the merchant’s pricing power, or the degree of competitiveness merchants face, which is directly related to the price mark-up: greater competition in retail markets implies  $\theta \rightarrow 1$  and a mark-up close to 1. When the acceptability of credit is endogenous, the expected benefit of investing in costly record-keeping is a decreasing function of  $\theta$ .<sup>31</sup> When  $\theta \rightarrow 1$ , sellers have no pricing power and hence no long invest. More generally, increased competitiveness in the retailer market (an increase in  $\theta$ ) reduces the equilibrium measure of sellers that invest and hence the fraction of credit transactions.

This could explain why a country like the U.S. with a relatively more competitive goods markets than the Euro Area, currently lags behind in credit-card technology. For example, the December 10, 2012 edition of *Time* magazine notes that: “In France and in much of the rest of the world... banks developed chip-and-PIN, which allows merchants to authenticate transactions locally. It became standard everywhere but in the U.S. Now we’re sub-standard.” Consequently, our theory

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<sup>29</sup>Schuh and Stavins (2012) find that these costs significantly affect debit card use. Similarly, the recent increase in the cost of credit cards issued by some banks may reduce consumers’ reliance on card payments for transactions.

<sup>30</sup>For instance, the Credit Cardholder’s Bill of Rights and Regulation AA were introduced in 2008 to protect consumers from unexpected rate increases on pre-existing credit card balances and limit the fees that reduce the availability of credit. See Foster, Meijer, Chuh, and Zabek (2011) for more discussion on these legislative reforms.

<sup>31</sup>To verify, derive  $\bar{\kappa} = \sigma(1 - \theta)[S(z + \bar{b}) - S(z)]$  to obtain  $\frac{d\bar{\kappa}}{d\theta} = -\sigma[S(z + \bar{b}) - S(z)] < 0$ .



predicts that economies like Europe with less concentrated markets are more likely to adopt new electronic payments than countries where the retail market is more competitive, such as the United States.<sup>32</sup>

## 7.6 Payment Innovations

In many countries, there are now new mobile payments in the form of text or SMS messages that authorizes a non-bank third party, such as a cellular phone carrier, to make a payment for the consumer. Since this new type of mobile payment may involve short-term credit extended by a cellular carrier or another third party, it serves the same role as credit cards that allow consumers to “buy now and pay later” (Foster, Meijer, Chuh, and Zabek (2011)).

Perhaps the largest potential gain from these payment innovations lies in emerging markets, where the lack of financial infrastructure makes mobile payments especially appealing (The Economist, 2012). For example, phone-based payments such as M-Pesa currency serve Kenya and several other markets. Large credit card companies such as Visa and Mastercard have set up joint ventures with many payment services in emerging countries such as Fundamo, which specialises in payment services for the un-banked and under-banked, and Telefonica, which aims to boost mobile payments across Latin America. The results from our paper suggests that in order for these innovations to be successful, there should be more information campaigns that promote their use in order to successfully coordinate agents on making the substitution away from cash.

## 8 Conclusion

As many economies now feature new forms of payment such as credit cards, smart cards, and electronic money, it is increasingly important for policymakers to understand how consumers substitute between cash and competing media of exchange. For that purpose, we investigate the choice of payment instruments in a simple model featuring costly record-keeping and limited commitment. Inflation triggers agents to substitute from money to credit for two reasons: a higher inflation rate both lowers the rate of return on money and makes default more costly, which relaxes agents’ borrowing limits.

Our model also highlights a strategic complementarity between consumers’ credit limit and retailers’ decision to invest. Multiple equilibria and coordination failures can therefore arise due to the two-sided market nature of payment systems. This poses new challenges for policymakers and

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<sup>32</sup>Bayoumi, Laxton, and Pesenti (2004) estimate an average price mark-up of 1.30 for the Euro Area and 1.16 for the U.S., which reflects a relatively more competitive goods market in the United States. Stavins (2003) presents a similar market structure argument regarding the market for electronic check products.

we discuss some mechanisms such as information campaigns and financial literacy programs that can help coordinate society on the socially preferred outcome. Our theory also suggests that future research should include more empirical studies on consumer preferences and merchant adoption to help policymakers enact the most beneficial regulations, laws, and educational programs that protect and support both consumer's and firm's payment choices.

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## Appendix

### Concavity of Buyer's Objective Function

The buyer's objective function is

$$\Psi(z) = -iz + \sigma\theta\Lambda[u(q^c) - c(q^c)] + \sigma\theta(1 - \Lambda)[u(q) - c(q)],$$

where  $q^c$  and  $q$  are given by  $z + \bar{b} = (1 - \theta)u(q^c) + \theta c(q^c)$  and  $z = (1 - \theta)u(q) + \theta c(q)$ , respectively, and  $b$  is given by (16). The partial derivative of the buyer's objective function with respect to the choice of real balances are

$$\Psi'(z) = -i + \sigma\theta\Lambda \left[ \frac{u'(q^c) - c'(q^c)}{\theta c'(q^c) + (1 - \theta)u'(q^c)} \right] + \sigma\theta(1 - \Lambda) \left[ \frac{u'(q) - c'(q)}{\theta c'(q) + (1 - \theta)u'(q)} \right],$$

if  $z + \bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$  and  $\Psi'(z) = 0$  if  $z + \bar{b} \geq (1 - \theta)u(q^*) + \theta c(q^*)$ . Consider the case where  $z + \bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$  and hence  $q^c < q^*$  and  $q < q^*$ . The second partial derivative is

$$\Psi''(z) = \sigma\theta[\Lambda\Delta^c + (1 - \Lambda)\Delta] < 0,$$

where  $\Delta^c = \frac{u''(q^c)c'(q^c) - u'(q^c)c''(q^c)}{[\theta c'(q^c) + (1 - \theta)u'(q^c)]^3}$  and  $\Delta = \frac{u''(q)c'(q) - u'(q)c''(q)}{[\theta c'(q) + (1 - \theta)u'(q)]^3}$ . Hence for all  $(z, \bar{b})$  such that  $\bar{b}$  solve (16) and  $z + \bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$ , the objective function  $\Psi(z)$  is strictly concave.

### Proof of Lemma 1

The borrowing limit,  $\bar{b}$ , is determined in order to satisfy the buyer's incentive constraint to voluntarily repay his debt in the CM. The buyer will repay his debt if

$$W^b(z, -b) \geq \widetilde{W}^b(z),$$

where  $W^b(z, -b)$  is the value function of a buyer who chooses to repay his debt at the beginning of the CM, and  $\widetilde{W}^b(z)$  is the value function of a buyer who chooses to default. By the linearity of  $W^b$ , the value function of a buyer who repays his debt in the CM is

$$W^b(z, -b) = z - b + W^b(0, 0).$$

On the other hand, the value function of a buyer who defaults,  $\widetilde{W}^b(z)$  must satisfy

$$\begin{aligned}\widetilde{W}^b(z) &= z + T + \max_{z' \geq 0} \left\{ -\gamma z' + \beta \widetilde{V}^b(z') \right\} \\ &= z + \widetilde{W}^b(0)\end{aligned}$$

since by defaulting, he is excluded from the use of credit for all future trades and can only use  $\widetilde{z}$  real balances for trades in the next DM. As a result, a buyer will repay his debt if

$$\begin{aligned}W^b(z, -b) - \widetilde{W}^b(z) &\geq 0, \\ W^b(0, 0) - \widetilde{W}^b(0) &\geq b, \\ \bar{b} &\geq b,\end{aligned}$$

where  $\bar{b} \equiv W^b(0, 0) - \widetilde{W}^b(0)$  is the endogenous debt limit determined such that buyers will always repay their debt. Therefore, the repayment constraint takes the form of an upper bound on credit use. To obtain the expression for  $\bar{b}$ , we now determine  $W^b(0, 0) - \widetilde{W}^b(0)$ . The value functions of a buyer who does not default in the CM can be rewritten as

$$\begin{aligned}W^b(0, 0) &= T + \max_{z \geq 0} \left\{ -\gamma z + \beta V^b(z) \right\} \\ &= T + \max_{z \geq 0} \left\{ -\gamma z + \beta \left[ \sigma(1 - \Lambda)\theta S(z) + \sigma\Lambda\theta S(z + \bar{b}) + z + W^b(0, 0) \right] \right\},\end{aligned}$$

where  $S(z) \equiv u[q(z)] - c[q(z)]$  is the total trade surplus when sellers only accept money, and  $S(z + \bar{b}) \equiv u[q(z + \bar{b})] - c[q(z + \bar{b})]$  is the total trade surplus when both money and credit are accepted. Rearranging and dividing both sides of the equality by  $\beta$  results in

$$\frac{(1 - \beta)}{\beta} W^b(0, 0) = \frac{T}{\beta} + \max_{z \geq 0} \left\{ -\frac{(\gamma - \beta)}{\beta} z + \sigma\theta [(1 - \Lambda)S(z) + \Lambda S(z + \bar{b})] \right\}.$$

Similarly, the value functions of a buyer who defaults in the CM can be rewritten as

$$\begin{aligned}\widetilde{W}^b(0) &= T + \max_{\widetilde{z} \geq 0} \left\{ -\gamma \widetilde{z} + \beta \widetilde{V}^b(\widetilde{z}) \right\} \\ &= T + \max_{\widetilde{z} \geq 0} \left\{ -\gamma \widetilde{z} + \beta \left[ \sigma\theta S(\widetilde{z}) + \widetilde{z} + \widetilde{W}^b(0) \right] \right\},\end{aligned}$$

where  $\widetilde{z} > 0$  solves

$$i = \sigma \frac{\theta[u'(q(\widetilde{z})) - c'(q(\widetilde{z}))]}{\theta c'(q(\widetilde{z})) + (1 - \theta)u'(q(\widetilde{z}))}, \quad (25)$$

since a buyer who defaults can only use money irrespective of what the seller accepts. Rearranging and dividing both sides of the equality by  $\beta$  results in

$$\frac{(1-\beta)}{\beta} \widetilde{W}^b(0) = \frac{T}{\beta} + \max_{\widetilde{z} \geq 0} \left\{ -\frac{(\gamma-\beta)}{\beta} \widetilde{z} + \sigma \theta S(\widetilde{z}) \right\}.$$

Therefore, the debt limit,  $\bar{b}$ , must satisfy

$$\frac{(1-\beta)}{\beta} \bar{b} = \frac{(1-\beta)}{\beta} [W^b(0,0) - \widetilde{W}^b(0)].$$

Substituting in the expressions for  $W^b(0,0)$  and  $\widetilde{W}^b(0)$  then leads to the following expression for  $\bar{b}$ :

$$r\bar{b} = \max_{z \geq 0} \{-iz + \sigma \theta [(1-\Lambda)S(z) + \Lambda S(z+\bar{b})]\} - \max_{\widetilde{z} \geq 0} \{-i\widetilde{z} + \sigma \theta S(\widetilde{z})\} \equiv \Omega(\bar{b}) \quad (26)$$

where  $r = \frac{1-\beta}{\beta}$ .

### Proof of Lemma 2

First, we show that  $\Omega(0) = 0$ . If  $\bar{b} = 0$ , it must be that  $z = \widetilde{z}$  since

$$-iz + \sigma \theta S(z) = -i\widetilde{z} + \sigma \theta S(\widetilde{z}).$$

As a result,  $\Omega(0) = 0$ . Next, to verify that  $\Omega(\bar{b})$  is increasing in  $\bar{b}$ , differentiate  $\Omega(\bar{b})$  with respect to  $\bar{b}$  to obtain

$$\begin{aligned} \frac{\partial \Omega(\bar{b})}{\partial \bar{b}} &= \sigma \theta \Lambda S'(z + \bar{b}) \\ &= \sigma \theta \Lambda \frac{u'(q^c) - c'(q^c)}{(1-\theta)u'(q^c) + \theta c'(q^c)} > 0, \end{aligned}$$

where  $q^c \equiv q(z + \bar{b})$  is output traded when the seller accepts both money and credit and  $z > 0$  satisfies

$$i = \sigma \theta [(1-\Lambda)S'(z) + \Lambda S'(z + \bar{b})].$$

The slope of  $\Omega(\bar{b})$  with respect to  $\bar{b}$  is strictly positive for all  $\bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$  and becomes zero when  $\bar{b} > (1 - \theta)u(q^*) + \theta c(q^*)$ . Further, the slope of  $\Omega(\bar{b})$  at  $\bar{b} = 0$  is

$$\begin{aligned} \left. \frac{\partial \Omega(\bar{b})}{\partial \bar{b}} \right|_{\bar{b}=0} &= \sigma \theta \Lambda S'(z) \\ &= i \Lambda \geq 0. \end{aligned}$$

A necessary for a monetary equilibrium with a positive debt limit is that the slope of  $r\bar{b}$  from (16) is less than the slope of  $\Omega(\bar{b})$  at  $\bar{b} = 0$ . Consequently a monetary equilibrium with  $\bar{b} > 0$  will exist so long as  $r < \Omega'(\bar{b}) = i\Lambda$ .

Next,  $\Omega(\bar{b})$  is concave since total surplus  $S(z + \bar{b})$  as a function of the buyer's liquid wealth,  $z + \bar{b}$ , is concave and strictly concave if  $z + \bar{b} < \theta c(q^*) + (1 - \theta)u(q^*)$ . Consequently,

$$\begin{aligned} \frac{\partial^2 \Omega(\bar{b})}{\partial \bar{b}^2} &= \frac{\partial [\sigma \theta \Lambda S'(z + \bar{b})]}{\partial \bar{b}} \\ &= \sigma \theta \Lambda S''(z + \bar{b}) \leq 0, \end{aligned}$$

since  $S''(z + \bar{b}) \leq 0$ . Finally,  $\Omega(\bar{b})$  is continuous for all  $z > 0$ , and for  $i > 0$ , the solution to (14) and (15) must lie in the interval  $[0, z^*]$ . Then from the Theorem of the Maximum, there exists a solution to (14) and (15), and consequently (16).  $\square$

#### Proof of Lemma 4

First we differentiate (16) to obtain

$$r d\bar{b} = \sigma \theta [-S(z) d\Lambda + S(z + \bar{b}) d\Lambda + \Lambda S'(z + \bar{b}) d\bar{b}],$$

$$\frac{d\bar{b}}{d\Lambda} = \frac{\sigma \theta [S(z + \bar{b}) - S(z)]}{r - \sigma \theta \Lambda S'(z + \bar{b})} > 0,$$

since  $S(z + \bar{b}) > S(z)$  and  $r > \sigma \theta \Lambda S'(\bar{b}_0)$ . Next, we verify that  $\bar{b}$  is a concave function of  $\Lambda$ :

$$\frac{d^2 \bar{b}}{d\Lambda^2} = \frac{-(\sigma \theta)^2 [S(z + \bar{b}) - S(z)] S'(z + \bar{b})}{[r - \sigma \theta \Lambda S'(z + \bar{b})]^2} < 0$$

since  $S'(z + \bar{b}) > 0$ . Therefore,  $\left. \frac{d^2 \bar{b}}{d\Lambda^2} \right|_{\bar{b} < \bar{b}_0} > \left. \frac{d^2 \bar{b}}{d\Lambda^2} \right|_{\bar{b}_0}$ .  $\square$

### Proof of Lemma 5

We first establish sellers' investment decisions at  $\bar{b} = \bar{b}_0$  where money is not valued. For a given  $\bar{b} = \bar{b}_0$ , the fraction of sellers who invest in the record-keeping technology depends on the expected benefit of accepting credit when money is not used in trade by buyers:

$$\bar{\kappa}_0 \equiv \sigma(1 - \theta)S(\bar{b}_0).$$

Clearly, a seller will invest so long as the expected benefit of doing so outweighs the cost:  $\bar{\kappa}_0 > \kappa$ . Assuming a uniform distribution of  $\kappa$  over  $[0, \bar{\kappa}_{\max}]$ , the aggregate measure of sellers who invest at  $\bar{b}$  is given by  $\Lambda_s = \frac{S(\bar{b}_0)}{S^*}$ , where  $S^* \equiv S(\bar{b}_1)$  is the maximum surplus obtained when  $q = q^*$ .

For all  $\bar{b} \in (\bar{b}_0, \bar{b}_1)$ , the expected benefit of investing is

$$\bar{\kappa} = \sigma(1 - \theta)S(\bar{b})$$

where  $\bar{\kappa} > \bar{\kappa}_0$ . Then since  $\Lambda$  is increasing in  $\bar{\kappa}$ , it follows that  $\Lambda > \Lambda_s$ . Moreover, the expected benefit of investing is a strictly increasing function of the debt limit,  $\bar{b}$ :  $\frac{d\bar{\kappa}}{d\bar{b}} \Big|_{\bar{b} > \bar{b}_0} > 0$  and  $\frac{d^2\bar{\kappa}}{d\bar{b}^2} \Big|_{\bar{b} > \bar{b}_0} < 0$ . To verify, differentiate  $\bar{\kappa}$  to obtain

$$\frac{d\bar{\kappa}}{d\bar{b}} \Big|_{\bar{b} > \bar{b}_0} = \sigma(1 - \theta)S'(\bar{b}) > 0$$

since  $S'(\bar{b}) > 0$ . Then differentiating again, it is easy to see that

$$\frac{d^2\bar{\kappa}}{d\bar{b}^2} \Big|_{\bar{b} > \bar{b}_0} = \sigma(1 - \theta)S''(\bar{b}) \leq 0$$

since  $S'(\bar{b}) \leq 0$ . Next, for all  $\bar{b} \in (0, \bar{b}_0)$ , the expected benefit of investing is

$$\bar{\kappa} = \sigma(1 - \theta)[S(z + \bar{b}) - S(z)].$$

Consequently,  $\bar{\kappa} < \bar{\kappa}_0$  and  $\Lambda < \Lambda_s$ . In this region, the expected benefit of investing is also an increasing function of the debt limit,  $\bar{b}$ . To verify, we differentiate to obtain

$$\frac{d\bar{\kappa}}{d\bar{b}} \Big|_{\bar{b} < \bar{b}_0} = \sigma(1 - \theta) \left\{ S'(z + \bar{b}) + [S'(z + \bar{b}) - S'(z)] \frac{dz}{d\bar{b}} \right\}.$$

Then using the first order condition (14),

$$i = \sigma\theta [(1 - \Lambda)S'(z) + \Lambda S'(z + \bar{b})],$$

we obtain

$$\frac{dz}{d\bar{b}} = -\frac{1}{1 + \frac{(1-\Lambda)S''(z)}{\Lambda S''(z+\bar{b})}} < 0$$

since  $S''(z) \leq 0$  and  $S''(z + \bar{b}) \leq 0$ . Consequently,  $\frac{d\bar{\kappa}}{d\bar{b}}|_{\bar{b} < \bar{b}_0} > 0$ .<sup>33</sup>  $\square$

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<sup>33</sup>We do not determine  $\frac{d^2\bar{\kappa}}{d\bar{b}^2}$  when  $\bar{b} < \bar{b}_0$  since this will not affect our results and will depend on third derivative  $S'''(z + \bar{b})$ , which we do not know the sign of.