

STICKY PRICES AND COSTLY CREDIT*

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Abstract

This paper has two related goals: (i) construct a model where money and credit coexist; (ii) pursue in this setting a theory of endogenous sticky prices that can be taken to the data. Search frictions generate price dispersion, and lead to monetary equilibria where profit-maximizing sellers set nominal prices they sometimes keep fixed when aggregate conditions change. Buyers can use cash or credit, where the former (latter) is subject to inflation (fixed costs), and hence is better for low (high) price transactions. This avoids technical problems in previous models. Calibrated versions match price-change data well. Yet policy implications differ markedly from exogenous sticky-price models.

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1 Introduction

This paper has two goals that are intimately related: (i) construct a model where money and credit coexist as alternative means of payment; (ii) pursue within this setting a theory of endogenous price stickiness and take it to the data. We build on the analysis of endogenous price dispersion in frictional goods markets by Burdett and Judd (1983), integrated with the general equilibrium monetary framework of Lagos and Wright (2005). As in Head et al. (2012), a nondegenerate price distribution means some firms may not adjust their nominal prices when the aggregate price level changes. It also means consumers sometimes pay a little and sometimes pay a lot, and they tend to use credit only in the latter case, because it entails a fixed cost, as in an earlier literature discussed below. Due to price dispersion, the endogenous choice of payment instrument is nontrivial, and at the same time decisions concerning payment instruments allow us to develop a tractable model of nominal price rigidity that avoids a real indeterminacy of equilibrium, with fewer ad hoc assumptions, as we now explain.

To begin, consider the model in Diamond (1971), where sellers (firms) of an indivisible good post prices, then buyers (households) sample one at a time until finding p below their chosen reservation price p^* . Clearly, for any seller the profit maximizing strategy is to post $p = p^*$. There is therefore a single price in the market, which turns out to give all gains from trade to sellers. Burdett and Judd (1983) make one change in the specification: buyers sometimes sample multiple prices simultaneously, and when they do they obviously choose the lowest. Now there cannot be a single p , or indeed any set of sellers with positive measure charging the same p , since if there were there is a profitable deviation to $p - \varepsilon$ for some $\varepsilon > 0$. One can compute explicitly the equilibrium distribution $F(p)$ on $[\underline{p}, \bar{p}]$ such that any $p \in [\underline{p}, \bar{p}]$ yields the same profit, since

sellers with lower p earn less per unit, but make it up on the volume, as they are more likely to sell to buyers that sample multiple p . This is not a “knife-edge” result: equilibrium is unique, and involves a distribution constructed so that profit is the same for all $p \in [\underline{p}, \bar{p}]$.

Burdett-Judd pricing is easily embed in the monetary economy of Lagos and Wright (2005). When firms sell goods for cash, it makes sense for them to post prices in dollars. If the money supply M increases, the aggregate price level increases and $F(p)$ shifts to the right, so the real distribution stays the same. Some sellers can keep p fixed, however, and hence some prices can be sticky in the face of increasing M , even though they are free to change whenever they like. While real prices fall for sellers not adjusting, their profits stay the same because they increase sales. This is true in Head et al. (2012), too, but they have a serious technical problem that we avoid: as discussed more below, the combination of indivisible goods and price posting in monetary economies entails a real indeterminacy (multiplicity) of equilibria. This led them to switch to divisible goods, but then, the problem is, it is not obvious what firms should post. Head et al. (2012) simply assume linear menus – i.e., a seller posts p , and buyers can choose any quantity q as long as they pay pq – but there is no reason to think that this maximizes profit.

Allowing buyers to choose between cash and credit turns out to eliminate indeterminacy with indivisible goods, and hence we can dispense with the ad hoc restriction to linear pricing. Intuitively, money and credit coexist because the former is subject to the inflation tax while the latter has a fixed cost. Given $F(p)$, random matching implies buyers might spend a little or a lot, and it is worth paying the fixed cost only in larger transactions. Holding more cash reduces the probability of needing to use credit, which delivers a well-behaved money demand function. This is consistent with a unique stationary equilibrium, where both cash and credit are used, for some parameters (if

the fixed cost is too high relative to inflation, naturally, money drives out credit, and vice versa). So, sticky prices arise endogenously with relatively “clean” assumptions.¹ But, in contrast to theories where stickiness is imposed exogenously, we get classical neutrality: an increase in M does not affect real activity. Whether or not money is neutral in actual economies is a different issue; the point is that sticky nominal prices do not imply nonneutrality.

As regards empirics, Head et al. (2012) calibrate their model to match the observed price-change distribution, and then show it matches several other facts reasonably well, including the average duration between price changes. Although the details differ in several ways, we perform a similar exercise, and are able to match the price-change distribution and other facts from the literature. In particular, we match the fraction of cash and credit purchases in the micro payment data, as well as the macro observations usually called the money demand observations. Thus, the ability of search-based models to account for the key observations is not compromised by changing assumptions about divisibility, to giving buyers a choice between cash and credit, and to bringing to bear more quantitative discipline. To put it differently, Head et al. (2012) assume retail transactions use only cash and do not confront money demand data, and it seems relevant to ask how the quantitative results change if one proceeds differently.

To summarize, search generates price dispersion, which motivates endogenous choices of payment instruments, and can generate nominal stickiness even when money is neutral. Head et al. (2012) discuss this in a pure-currency economy, but due to technical problems they use divisible goods and an ad hoc pricing mechanism. As regards methodological contributions, we avoid that

¹In case there is any doubt, restrictions on the divisibility of goods are assumptions about the physical environment, and hence are more “clean” than restrictions on pricing, which are assumptions about behavior. We do not take a stand on whether divisible or indivisible goods are more “realistic” because that should depend on the application.

issue because the cash-credit choice itself resolves the technical problems. In terms of substantive conclusions, we show the theory accounts well for salient features of pricing behavior and endogenous payment methods. In the rest of the paper, Section 2 reviews more literature. Section 3 presents the environment. Section 4 characterizes equilibrium. Section 5 calibrates the model and confronts the data. Section 6 considers extensions. Section 7 concludes.

2 Literature

Many papers with sticky prices follow Taylor (1980) or Calvo (1983) by only allowing sellers to adjust p at some points in time. Others follow Rotemberg (1982) or Mankiw (1985) by allowing sellers to change whenever they like at a cost. We allow them to adjust whenever they like for free. Other papers push imperfect-information or rational-inattention theories of price rigidity, including Mankiw and Reis (2002), Woodford (2002), Sims (2003) and Mackowiak and Wiederhold (2009). This is related to search theory, in that both concern incomplete, or costly-to-process, information. Although other features can be incorporated, we focus exclusively on search, because when that is combined with menu costs, as in Burdett and Menzies (2014), the analysis is much more complicated and delivers similar results (one can interpret our formulation as a version of theirs where menu costs are small). Moreover, Burdett and Menzies (2014) find the majority of price dispersion (about 70%) in the data is due to search, not menu costs. Hence, we abstract from menu costs and related devices because they complicate the analysis, do not change the basic message, and may be less important for our purposes.²

²Nonmonetary search models, where prices are sticky in unit of account due to menu costs, like Benabou (1988,1992) and Diamond (1993), can also be considered special cases of Burdett and Menzies (2014). Ours is almost a different special case, except we model money seriously. Craig and Rocheteau (2008) previously showed that putting genuine money in Benabou-Diamond matters – e.g., it is often said that model recommends inflation, but once money is added explicitly, the model actually predicts deflation is optimal.

The crucial ingredients in Burdett and Judd (1983) are pricing posting and random matching.³ The literature on these models is large, especially the labor-market applications discussed in Burdett and Mortensen (1998) and Mortensen and Pissarides (1999). Prior to Liu (2010), Wang (2011) and Head et al. (2012) putting Burdett-Judd into Lagos and Wright (2005), Head and Kumar (2005) and Head et al. (2010) put it into the related model of Shi (1997). Alternatives in terms of price dispersion include Albrecht and Axel (1984) and Diamond (1987), where instead of buyers differing in terms of the sets of prices from which they can choose, they differ with respect to intrinsic (e.g., preference) type. The monetary version of that model in Curtis and Wright (2004) generically delivers a two-point p distribution, independent of the number of types, and hence is less useful for our purposes. Also, that model, like Burdett et al. (2014), assumes diametrically from us that goods are divisible while money is not, following Shi (1995) and Trejos and Wright (1995). This is fine, although the assumption of indivisible money does hinder the discussion of many policy and empirical issues.

Caplin and Spulber (1987) and Eden (1994) have sticky-price models that are similar in spirit but also quite different. In addition using Burdett-Judd, we build on the relatively firm foundations for monetary economics in Lagos-Wright, which itself builds on Kiyotaki and Wright (1989,1993), Kochelakota (1998), Wallace (2001) and many others. See Wallace (2010), Williamson and Wright (2010), Nosal and Rocheteau (2011) or Lagos et al. (2014) for surveys of the approach, sometimes called New Monetarist economics, that endeavors

³For most of what we do, firms are homogeneous, in which case theory does not pin down who charges which p . With heterogeneous firms, Burdett-Judd implies lower cost firms prefer lower p . But for any subset of firms with the same marginal cost, it does not matter who in the set posts which p . This is perhaps especially descriptive of retail, where marginal cost is the wholesale price, and even if some retailers get better deals (e.g., quantity discounts) it is reasonable to think many face the same wholesale price. In any case, this is not a real indeterminacy: having different sellers post different p is merely a relabeling; by contrast, the indeterminacy of monetary equilibria discussed below affects payoffs.

to provide clear-cut descriptions of how agents trade and assumptions about specialization, limited commitment and imperfect information that can give money and related institutions a socially beneficial role. The premise is that it is desirable to analyze monetary phenomena in environments that are explicit about the frictions that money-like institutions are supposed to ameliorate. We follow the approach because the premise seems valid, even though similar points can be made in, say, CIA or MUF models if one were for some reason committed to reduced-form modeling.

Although one can find similar arguments elsewhere, we think it is fair to highlight Ball and Mankiw (1994), who champion the position that stickiness leads to nonneutrality: “We believe that sticky prices provide the most natural explanation of monetary nonneutrality since so many prices are, in fact, sticky.” They also state “As a matter of logic, nominal stickiness requires a cost of nominal adjustment.” Golosov and Lucas (2003) similarly state “menu costs are really there: The fact that many individual goods prices remain fixed for weeks or months in the face of continuously changing demand and supply conditions testifies conclusively to the existence of a fixed cost of repricing.” While we agree that prices are in fact sticky, the other statements are demonstrably incorrect: in reality, menu costs may or may not be important, and money may or may not be neutral, but a message we want to emphasize is that *as a matter of logic* sticky prices imply neither menu costs nor nonneutrality.

Empirical work on pricing is surveyed by Klenow and Malin (2010). Especially relevant for us is Campbell and Eden (2014). Using scanner data from grocery stores, they find an average duration between price changes of 10.3 weeks, and 17% (29%) of prices change in an average week (month). This suggests the probability of adjustment declines with the time since the last change – i.e., a downward sloping hazard function. Several papers study BLS data, going back to Bils and Klenow (2005). They conclude retail prices are

fairly flexible, with at least half lasting less than 4.3 months. Even excluding sales, about half last for less than 5.5 months. Their mean frequency of regular price changes is 26.1%. Klenow and Kryvtsov (2008) find durations in a range between 6.8 and 10.4 months, with an average of 8.6 months. Nakamura and Steinsson (2008) get longer durations, from 8 to 11 months, by excluding substitutions and sales. Also, these last two papers find a large fraction of negative and small price changes, and find some but relative weak evidence of a nonconstant hazard function.⁴

Midrigan (2011) accounts for some features of the data that give others trouble, including the fact that average price changes are big (suggesting high menu costs), but also many price changes are small (suggesting low menu costs). In his model, firms sell multiple goods, and paying a fixed cost to change one price lets them change the rest for free. This is fine, but it is also worth considering alternatives without such complications. We account for average duration, large average price changes plus many small changes, and prices changing more frequently at higher inflation. Simple Mankiw cost theories cannot account for large and small changes, while Calvo pricing cannot account for frequency depending on inflation. While extending those models potentially helps, even a very basic version of our model accounts for the facts. Also, our model generates price dispersion even at low or 0 inflation, consistent with the evidence (e.g., Campbell and Eden 2014); price dispersion arises in simple Calvo or Mankiw models only because of inflation.

⁴Earlier, Cecchetti (1986) studied magazine prices, finding the time between changes ranges from 1.8 months to 14 years. Carlton (1986) studied wholesale markets, also finding the average duration of prices varies widely across product groups, from 5.9 months for household appliances to 19.2 for chemicals. He also observed many small changes, with about 65% below 2%. More recently, Eichenbaum et al. (2011) study reference prices (the most-quoted prices in a quarter). Median duration is 3 weeks for regular prices but 6 months for reference prices. Using supermarket data, Eichenbaum et al. (2014) find the fraction of price changes below 5% in absolute value is 5.2% after correcting for measurement error. A similar exercise on CPI data set gets 3.6% for posted prices and 5% for regular prices, compared to 12.5% and 14% without removing problematic items. They argue the prevailing evidence on small price changes may be partly an artifact of measurement error.

The project is also intended as a contribution to monetary theory. Money demand here emerges as a cube-root function of interest rates, as compared to the square-root function famously associated with Baumol (1952) and Tobin (1956). Of course, Baumol-Tobin is a partial-equilibrium model, or more accurately a decision-theoretic model, that describes how to manage one’s money given that it (as opposed to credit, barter or something else) must be used for transactions. This does not mean those models are not still being applied to good effect – e.g., Alvarez and Lippi (2009) – but we admit to a preference for theories where the roles of money, credit and related institutions are not taken for granted. Reasons to study this carefully include the notion that credit conditions may be critical for economic performance (Gertler and Kiyotaki 2010), and that models with money plus credit are necessary for analyzing certain aspects of monetary policy (Wallace 2013).⁵

The multiplicity of equilibria in monetary economies with indivisible goods is discussed at length in a literature spawned by Green and Zhou (1998). See Jean et al. (2010) for citations and a discussion based on the following intuition: If all sellers post p then buyers’ best response is to bring $m = p$ dollars to market, at least as long as p is not too big. If all buyers bring m then sellers’ best response is post $p = m$, as long as m is not too small. Hence, any $p = m$ in some range is an equilibrium, and payoffs depend on the one that gets selected. While some substantive messages apply to all the equilibria, it is surely undesirable to admit such a plethora of outcomes. Jean et al. (2010) show how to refine this away in models where buyers sample at most one p per period, but it is not clear how to do it in Burdett-Judd, and even if it were,

⁵Additionally, applying Burdett-Judd pricing in the New Monetarist framework is a further step in understanding its generality and robustness. In other papers, Lagos and Wright (2005) use Nash bargaining; Aruoba et al. (2007) advocate Kalai bargaining; Lagos and Rocheteau (2005) use price posting with directed search; Rocheteau and Wright (2005) use competitive pricing; Galenianos and Kircher (2008) and Dutu et al (2012) use auctions; and Hu et al. (2009) use mechanism design. Some of these are driven by substantive issues, others by an interest in the robustness of results.

the refined outcome depends in a big way on whether buyers or sellers move first.

An option is to incorporate divisible goods as in Head et al. (2012), which works because then, even if buyers think all sellers post p , they need not bring $m = p$ when quantity is variable; but then it is not clear what sellers should post, and linear menus seem fairly arbitrary. While there may be other ways around this dilemma, our approach is based on the venerable notion that people use cash for small and credit for large purchases.⁶ Generally, the observation that credit is more common in bigger transactions motivates the assumption that it involves a fixed cost.⁷ This cost can be described as encompassing a variety of features, like the application process, time screening candidates, checking debt limits etc. To keep it simple, credit is assumed to be perfectly enforced once the cost is paid, although we think this could also be modeled as in Kehoe and Levine (1993) or Kiyotaki and Moore (1997). In any case, while there are undoubtedly papers we missed, this summarizes a lot of the related material.

3 Model

Each period in discrete time is divided into two subperiods: first, there is a decentralized market, called the BJ market for Burdett-Judd; second, there is a frictionless centralized market, called the AD market for Arrow-Debreu. There is a set of infinite-lived households with measure \bar{b} , and a set of firms with nor-

⁶Versions of this idea appear in Prescott (1987), Freeman and Huffman (1991), Lacker and Schreft (1996), Freeman and Kydland (2000), Dong (2010) and Lucas and Nicolini (2013); see Nosal and Rocheteau (2011, chapter 8) for an overview.

⁷For the record, this is also explained by He et al. (2005,2008), Sanches and Williamson (2010) or Hazra (2012), who assume cash is subject to theft while credit or demand deposits are not; then again, Kahn et al. (2005) and Kahn and Roberds (2008) assume the opposite in order to study information and identity theft. Berentsen et al. (2007), Ferraris and Watanabe (2008), Telyukova and Wright (2008), Ferraris (2010), Gomis-Porqueras and Sanches (2013), Li and Li (2013), Lotz and Zhang (2013), Araujo and Hu (2014), Chiu et al. (2014) and Gu et al. (2014) are other analyses of multiple payment methods in New Monetarist models.

malized measure 1. Households consume a divisible good in the AD market and an indivisible good in the BJ market. They produce only in the AD market, while BJ goods are produced at constant marginal cost c by firms that we think of as retail sellers. Agents are anonymous in the BJ market unless they take some action. Specifically, there is a technology to reveal/authenticate identities and record transactions at cost $\delta \in [0, u)$. By incurring δ , a household can get a BJ good on credit – i.e., in exchange for a perfectly-enforced promise to deliver d_t dollars in the next AD market. Otherwise, they pay with cash. Let M_t be the money supply per household at t , and assume $M_{t+1} = (1 + \pi) M_t$ with π constant. Changes in M occur in the AD market via lump-sum transfers (taxes) if $\pi > 0$ (if $\pi < 0$); if instead government uses new money to buy goods, nothing changes except labor supply.

For simplicity, the household's preferences within a period are separable across (x_t, y_t, ℓ_t) , as given by $U(x_t) + u\mathbf{1}(y_t) - v\ell_t$, where x_t is AD consumption, y_t is BJ consumption, ℓ_t is AD labor, U satisfies $U'(x) > 0 > U''(x)$, $\mathbf{1}(y_t)$ is an indicator function giving 1 if $y_t = 1$ and 0 otherwise, and $u, v > 0$ are parameters. Let $\beta = 1/(1 + r)$, $r > 0$, be the discount factor between today's AD market and tomorrow's BJ market. Assume $\pi > \beta - 1$, where in stationary equilibrium π is the inflation rate, and the nominal interest rate is given by the Fisher equation $1 + i = (1 + \pi)(1 + r)$; thus $\pi > \beta - 1$ iff $i > 0$, and the Friedman rule is the limiting case $i \rightarrow 0$. As usual, we can interpret $1 + i$ as the amount of money one would require in the next AD market to give up a dollar in AD today, even if nominal bonds are not actually traded in equilibrium. Let x_t be the AD numeraire at t , produced with a linear technology $x = \omega\ell$. By choosing units wlog we normalize $\omega = v$. Let ϕ_t be the AD price of money in terms of x_t , so $1/\phi_t$ is the nominal price level.

In AD, households maximize utility subject to a budget constraint. They also decide whether to enter the subsequent BJ market, at cost k . In the

baseline model, $k = 0$, so all households participate. Firms always enter the BJ market for free, and use the revenue to buy AD goods over which they have linear utility.⁸ Hence, firms maximize revenue, by posting prices, taking as given household behavior and the price distribution implied by other firms, described by CDF $F_t(p)$ with support \mathcal{F}_t . Every household that enters the BJ market randomly samples a subset of firms. Generally, a household contacts n firms – i.e., sees n independent prices drawn from $F_t(p)$ – with probability $\alpha_n(b_t) \geq 0$, generally depending on market tightness (the buyer-seller ratio) b_t . For simplicity, in the benchmark model households contact no more than two firms, so that $\alpha_0(b_t) + \alpha_1(b_t) + \alpha_2(b_t) = 1$.

3.1 Firms

In a monetary economy, it makes sense to have firms post prices in dollars. Expected real profit for a firm posting p at date t is

$$\Pi_t(p) = b_t \{ \alpha_1(b_t) + 2\alpha_2(b_t) [1 - F_t(p)] \} (p\phi_t - c). \quad (1)$$

Intuitively, real revenue per unit is $p\phi_t - c$, and the number of units is determined as follows: The measure of households that contact this firm and no other is $\alpha_1(b_t)$. In this case there is a sale for sure. The measure that contact this firm plus another is $2\alpha_2(b_t)$, as it can happen in two ways – the household contacts this one first and the other second, or vice versa. In this case, there is a sale iff this firm beats the other’s price, which happens with probability $1 - F_t(p)$. All of this is multiplied by market tightness b_t to convert buyer probabilities into seller probabilities.

Equilibrium in the price-posting game means every $p \in \mathcal{F}_t$ yields the same profit, and no $p \notin \mathcal{F}_t$ yields higher profit. As is standard (Burdett and Judd

⁸One could instead allow households to enter for free and have firms pay a cost; our approach follows earlier work (Liu et al. 2010), where we found it more interesting to have households make the participation decision.

1983; Head et al. 2012), there is a unique such outcome, and it has the property that $F_t(p)$ is continuous and \mathcal{F}_t is an interval.⁹ Taking as given for now the upper limit of the interval \bar{p}_t , profit from any $p \in \mathcal{F}_t$ must equal the profit from this price, which is

$$\Pi_t(\bar{p}_t) = b_t \alpha_1(b_t) (\bar{p}_t \phi_t - c). \quad (2)$$

Equating (1) and (2), for every $p \in \mathcal{F}_t$, we can solve for

$$F_t(p) = 1 - \frac{\alpha_1(b_t) \phi_t \bar{p}_t - \phi_t p}{2\alpha_2(b_t) (\phi_t p - c)}. \quad (3)$$

This is the closed-form solution for the BJ price distribution. Notice

$$F'_t(p) = \frac{\alpha_1(b_t) \phi_t}{2\alpha_2(b_t)} \frac{\phi_t \bar{p}_t - c}{(\phi_t p - c)^2} > 0 \quad (4)$$

$$F''_t(p) = -\frac{\alpha_1(b_t) \phi_t^2}{\alpha_2(b_t)} \frac{\phi_t \bar{p}_t - c}{(\phi_t p - c)^3} < 0. \quad (5)$$

Also, since the lower limit of the support satisfies $F(\underline{p}_t) = 0$ we have

$$\underline{p}_t = \frac{\alpha_1(b_t) \phi_t \bar{p}_t + 2\alpha_2(b_t) c}{\phi_t [\alpha_1(b_t) + 2\alpha_2(b_t)]}. \quad (6)$$

It remains to discuss the upper limit \bar{p}_t . In monetary equilibrium, there are three logical possibilities, depending on real balances of the representative buyer in the BJ market, $z_t = \phi_t m_t$, where m_t is nominal balances. First, if $z_t < u - \delta$ then the highest price is $\bar{p}_t = (u - \delta) / \phi_t > m_t$.¹⁰ In this case buyers use credit when they sample $p \in (m_t, \bar{p}_t]$ and cash otherwise. Second, if $u - \delta < z_t < u$ then $\bar{p}_t = m_t$. In this case, buyers always use cash. Finally, if $u < z_t$ then $\bar{p}_t = u / \phi_t$, but this never happens in equilibrium since households

⁹There cannot be a mass of firms posting the same p in equilibrium, because any one of them would have a profitable deviation to $p - \varepsilon$, since he would lose only ε per unit and make discretely more sales by undercutting others at the mass point. Similarly, if there were a gap between say p_1 and $p_2 > p_1$, a firm posting p_1 can deviate to $p_1 + \varepsilon$, and earn more per unit without losing sales.

¹⁰This follows because no buyer would pay more than this, and if $\bar{p}_t < (u - \delta) / \phi_t$ the highest price firm would have a profitable deviation to $(u - \delta) / \phi_t$.

never carry more money than they need. To translate prices from dollars to numeraire, let $q_t = \phi_t p_t$ and note the distribution of real BJ prices is

$$G_t(q) = F_t(\phi_t p) = 1 - \frac{\alpha_1(b_t)}{2\alpha_2(b_t)} \frac{\bar{q}_t - q}{q - c}. \quad (7)$$

In nonmonetary equilibrium, firms post a q and all transactions use credit. In this case, the bounds are $\bar{q}_t = u - \delta$ and

$$\underline{q}_t = \frac{\alpha_1(b_t)(u - \delta) + 2\alpha_2(b_t)c}{\alpha_1(b_t) + 2\alpha_2(b_t)}. \quad (8)$$

Below we focus on stationary equilibrium, where real variables are constant and nominal variables grow at rate π . There are three types of candidate equilibria: nonmonetary equilibrium, or NME, where $z = 0$ so only credit is used in BJ; mixed monetary equilibrium, or MME, where $0 < z < u - \delta$ so cash and credit are both used; and pure money equilibrium, or PME, where $u - \delta < z$ so only cash is used. The outcome generally depends on households, but in each case, firm behavior leads to one of the three possibilities described above: in NME, BJ prices are described in terms of numeraire, with the q distribution given by (7) where $\bar{q} = u - \delta$; in MME, prices can be described in dollars or numeraire, with the p and q distributions given by (3) and (7) where again $\bar{q} = \phi_t \bar{p}_t = u - \delta$; and in PME, prices can again be in dollars or numeraire, where $\bar{q} = \phi_t \bar{p}_t \in (u - \delta, u)$.

3.2 Households

In the interest of stationarity we frame the household problem in real terms. The state variable in the AD market is net worth, $A = \phi(m - d) - \delta \mathbf{1}(d) + \bar{A}$, where money m and debt d are brought in from the previous BJ market, $\mathbf{1}(d)$ is an indicator function equal to 1 when $d > 0$ and 0 otherwise, while \bar{A} is any other source of purchasing power, including net government transfers, or dividends if firms are owned by household. Since preferences are linear in ℓ ,

nothing depends on \bar{A} except labor supply, and wlog all debt is assumed to be settled in AD. The state variable in the next BJ market is real balances, $z = \phi m$. The AD and BJ value functions are $W(A)$ and $V(z)$.

The AD payoff for a household that decides to enter the next BJ market is

$$W^1(A) = \max_{x, \ell, z} \{U(x) - v\ell + \beta V(z)\} \text{ st } x = A - k + \omega\ell - (1 + \pi)z,$$

where k is the entry cost, and due to inflation the cost of having z next period, in terms of current numeraire, is $(1 + \pi)z$. Eliminating ℓ using the budget equation, with the normalization $\omega = v$ we get

$$W^1(A) = A - k + U(x^*) - x^* + \beta \max_z O_i(z), \quad (9)$$

where $U'(x^*) = 1$ and the objective function for choice of z is $O_i(z) \equiv V(z) - (1 + i)z$.¹¹ For a household that skips the next BJ market, $z = 0$ and

$$W^0(A) = A + U(x^*) - x^* + \beta W^0(0). \quad (10)$$

Then $W(A) = \max\{W^1(A), W^0(A)\} = W^1(A)$ as long as the BJ market is open (i.e., as long as some households enter).

What keeps the analysis tractable, as in Lagos and Wright (2005), is $W'(A) = 1$. This allows us to easily derive

$$V(z) = W(z) + [\alpha_1(b) + \alpha_2(b)] [u - \mathbb{E}_J q - \delta + \delta J(z)], \quad (11)$$

where $\mathbb{E}_J q = \int_{\underline{q}}^{\bar{q}} q dJ(q)$, and

$$J(q) = \frac{\alpha_1(b) G(q) + \alpha_2(b) \{1 - [1 - G(q)]^2\}}{\alpha_1(b) + \alpha_2(b)} \quad (12)$$

is the CDF of transaction prices.¹² When a buyer does not incur cost δ he is

¹¹This uses the Fisher equation, $(1 + i) = (1 + \pi)/\beta$, and nonnegativity constraints are assumed slack. Also, notice from (9) that it does not matter if k is in utils or numeraire. The same is true for the other costs, δ and c .

¹²To see this, first write

$$\begin{aligned} V(z) = & W(z) + \alpha_1(b) \int_{\underline{q}}^z (u - q) dG_1(q) + \alpha_1(b) \int_z^{\bar{q}} (u - q - \delta) dG_1(q) \\ & + \alpha_2(b) \int_{\underline{q}}^z (u - q) dG_2(q) + \alpha_2(b) \int_z^{\bar{q}} (u - q - \delta) dG_2(q), \end{aligned}$$

forced to use cash; when he does incur δ he can use cash or credit, at which point all parties are indifferent, so we assume wlog they use only credit. From (11), the benefit of holding z is that it reduces the probability of having to use credit, $1 - J(z)$.

Households also need to make a BJ entry decision. Algebra implies $\Phi = (1 + r)[W^1(A) - W^0(A)]$ is independent of A and satisfies

$$\Phi = [\alpha_1(b) + \alpha_2(b)] [u - \mathbb{E}_J q - \delta + \delta J(z)] - \kappa - (1 + i)z, \quad (13)$$

where $\kappa = k/\beta$. The first term is the expected benefit of participating in BJ; the others are the costs. In equilibrium,

$$b = \bar{b} \implies \Phi \geq 0; b = 0 \implies \Phi \leq 0; b \in (0, \bar{b}) \implies \Phi = 0. \quad (14)$$

3.3 Equilibrium

The following is standard:

Definition 1 *A stationary equilibrium is a list $\langle G(q), b, z \rangle$ solving (7), (14) and the money demand problem in (9).*

Definition 2 *A PME is an equilibrium with $z > u - \delta$ and $\bar{q} = z$. A MME is one with $u - \delta > z > 0$ and $\bar{q} = u - \delta$. A NME is one with $z = 0$ and $\bar{q} = u - \delta$.*

Given a monetary equilibrium, the nominal price distribution is $F_t(p) = G(\phi_t p)$. Other variables include AD consumption, $x = x^*$, and labor supply ℓ , which given by the budget equation and hence differs across agents depending on their experience in the previous BJ market.

where $G_n(q) = 1 - [1 - G(q)]^n$ is the CDF of the lowest of n independent draws from $G(q)$. The first term is the continuation value if the buyer meets no seller. The second is the probability of meeting a seller with $q \leq z$, so cash is used, times the expected surplus. The third is the probability of meeting a seller with $q > z$, so credit is used, which entails additional cost δ . The fourth or fifth term is the probability of meeting two sellers where the lower price is $q \leq z$ or $q > z$. The rest is algebra.

4 Characterization

In the baseline model $k = 0$, so $b = \bar{b}$ is fixed, and the arguments are omitted from the meeting probabilities α_1 and α_2 . It is then relatively easy to characterize equilibria. To begin consider these preliminary results:

Lemma 1 For $\underline{q} < z < \bar{q}$, $V(z)$ is smooth, with $V'(z) > 0$, $V''(z) < 0$. For $z < \underline{q}$ or $z > \bar{q}$, $V'(z) = 1$.

Proof: For $z \in (\underline{q}, \bar{q})$ we have

$$V'(z) = 1 + (\alpha_1 + \alpha_2) \delta J'(z) \quad (15)$$

$$V''(z) = (\alpha_1 + \alpha_2) \delta J''(z). \quad (16)$$

From (12) we have

$$J'(q) = \frac{\alpha_1 G'(q) + 2\alpha_2 [1 - G(q)] G'(q)}{\alpha_1 + \alpha_2} > 0 \quad (17)$$

$$J''(q) = \frac{\alpha_1 G'''(q) + 2\alpha_2 [1 - G(q)] G'''(q) - 2\alpha_2 G'(q)^2}{\alpha_1 + \alpha_2} < 0 \quad (18)$$

by virtue of (7). The rest is obvious. ■

We are most interested in MME, because it is the empirically relevant case, and because NME is similar to the original Burdett-Judd (1983) model, while PME is basically the case in Head et al. (2012), with its problematic indeterminacy. If MME were to exist, it would entail $z \in (\underline{q}, \bar{q})$, and since $V''(z) < 0$ on this interval, there is a unique $z_i = \arg \max_{z \in (\underline{q}, \bar{q})} O_i(z)$ where $O_i(z)$ is as in (9). It satisfies the FOC $O'_i(z_i) = 0$, or

$$(\alpha_1 + \alpha_2) \delta J'(z_i) = i, \quad (19)$$

by virtue of (15), with the obvious marginal interpretation. However, MME may or may not exist, and corner solutions are very relevant here.

To describe our method, let \hat{z}_i be the global maximizer of $O_i(z)$, and let $O_i^-(z)$ and $O_i^+(z)$ be the left and right derivatives. We first check $O_i^+(\underline{q})$. If

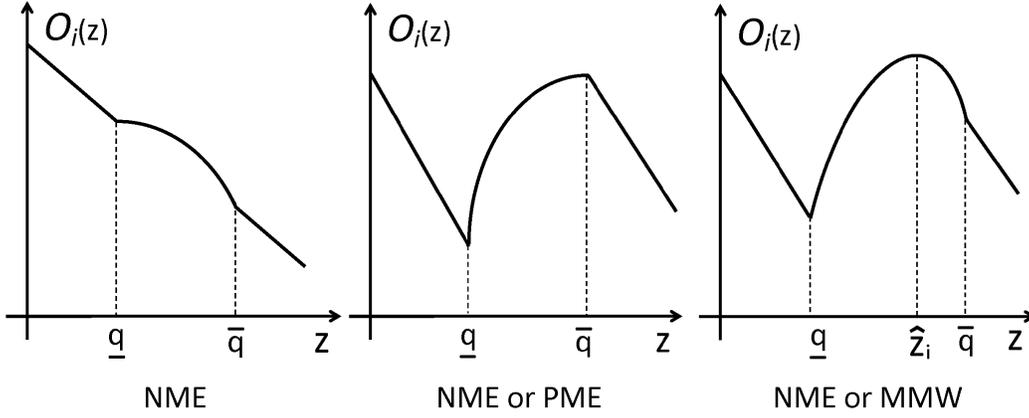


Figure 1: Different Types of Equilibria

$O_i^+(\underline{q}) \leq 0$ then $\hat{z}_i = 0$, as shown in the left panel of Figure 1. If $O_i^+(\underline{q}) > 0$, we then check $O_i^-(\bar{q})$. If $O_i^-(\bar{q}) \geq 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = \bar{q}$, as shown in the center panel. If $O_i^-(\bar{q}) < 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = z_i$, as shown in the right panel. While we are most interested in MME, all possibilities are discussed below.

Proposition 1 *There always exists a unique NME.*

Proof: As always, if people believe the value of fiat currency is $\phi = 0$, this is a self-fulfilling prophecy. The condition for equilibrium is then simply that $G(q)$ is consistent with profit maximization by firms. As is standard, the unique such $G(q)$ is the Burdett-Judd distribution in (7). ■

Proposition 2 *There exists a unique MME iff*

$$\delta < \bar{\delta} \equiv u - \frac{2\alpha_2^2 + 2\alpha_1\alpha_2}{2\alpha_2^2 + 2\alpha_1\alpha_2 - \alpha_1^2}c \quad (20)$$

and $i \in (\underline{i}, \bar{i})$, where $\underline{i} = \delta\alpha_1^2/2\alpha_2(u - \delta - c)$ and $\bar{i} \in (\underline{i}, \infty)$.

Proof: Based on the discussion surrounding Figure 1, we know that there is a unique candidate MME z_i , and in fact we can find the closed-form solution (see below). Necessary and sufficient conditions for MME are: (i) $O_i^-(\bar{q}) <$

0; (ii) $O_i^+(q) > 0$; and (iii) $O_i(z_i) > O_i(0)$. Condition (i) is equivalent to $(\alpha_1 + \alpha_2) \delta J^-(\bar{q}) < i$. Given $G(q)$ and z_i , a calculation implies this holds iff $i > \underline{i}$. Similarly, condition (ii) is equivalent to $(\alpha_1 + \alpha_2) \delta J^+(\underline{q}) > i$, and a calculation implies this holds iff $i < \tilde{i}$ where

$$\tilde{i} = \frac{\delta (\alpha_1 + 2\alpha_2)^3}{2\alpha_1\alpha_2 (u - \delta - c)} > \underline{i}. \quad (21)$$

Condition (iii) is equivalent to $(\alpha_1 + \alpha_2) \delta J(z_i) - iz_i > (\alpha_1 + \alpha_2) \delta J(0)$, and a calculation implies this holds iff $\Delta(i) > 0$ where

$$\Delta(i) = -ic + \frac{\delta (\alpha_1 + 2\alpha_2)^2}{4\alpha_2} - i^{\frac{2}{3}} \delta^{\frac{1}{3}} \alpha_1^{\frac{2}{3}} \alpha_2^{-\frac{1}{3}} (u - \delta - c)^{\frac{2}{3}} \left(2^{-\frac{1}{3}} + 2^{-\frac{4}{3}} \right).$$

Notice $\Delta(0) > 0 > \Delta(\tilde{i})$ and $\Delta'(i) < 0$. Hence $\exists! \bar{i}$ such that $\Delta(\bar{i}) = 0$, and $\Delta(i) > 0$ iff $i < \bar{i}$. It remains to verify that $\bar{i} > \underline{i}$, so that (i) and (iii) are not mutually exclusive. It can be checked that this is true iff $\delta < \bar{\delta}$. Therefore, there is a MME under the conditions given above. There is only one because in MME $\bar{q} = u - \delta$, and from this the unique distribution consistent with profit maximization is $G(q)$, and given, this there is a unique $\hat{z}_i = \arg \max_{z \in (q, \bar{q})} O_i(z)$ as discussed above. ■

The simplicity of the above result follows because MME implies $\bar{q} = u - \delta$, independent of monetary considerations. Hence, profit maximization pins down $G(q)$, and households have a unique best response \hat{z}_i to $G(q)$. The point is that we do not have to simultaneously worry about the firm's best response to \hat{z}_i , which greatly complicated the argument in Head et al. (2012). This is another reason to combine money and credit: not only does it generate interesting results, it elegantly simplifies existence and uniqueness arguments for the relevant type of equilibrium.

Finally, for completeness, consider PME, which is like Head et al. (2012) with indivisible goods, or Burdett and Judd (1983), with money but no credit. Now there is no hope of uniqueness: not only does there coexist a NME by

Proposition 1, more problematically there are a plethora of PME, as discussed above. Heuristically, this multiplicity (indeterminacy) emerges because, due to strong coordination effects, there is a continuum of fixed points where \bar{q} is a best response to z and vice-versa.

Proposition 3 *There exists a PME iff either: $\delta > \bar{\delta}$ and $i < \hat{i} = \delta(\alpha_1 + \alpha_2)/(u - \delta)$; or $\delta < \bar{\delta}$ and $i < \underline{i}$.*

Proof: Suppose we try to construct a PME, as in the middle panel of Figure 1. Conditional on money being valued, necessary and sufficient conditions are: (i) $O_i^-(\bar{q}) > 0$; (ii) $O_i^+(\underline{q}) > 0$; and (iii) $O_i(\bar{q}) > O_i(0)$. Condition (i) holds iff $i < \underline{i}$. Condition (ii) holds iff $i < \tilde{i}$, as defined above. Condition (iii) holds iff $i < \hat{i}$. For $\delta > \bar{\delta}$, it is easily checked that $\hat{i} < \underline{i}$, and $\underline{i} < \tilde{i}$ by (21), so the binding condition is $i < \hat{i}$. For $\delta < \bar{\delta}$, it is easily checked that $\hat{i} > \underline{i}$, and $\underline{i} < \tilde{i}$, so the binding condition is $i < \underline{i}$. ■

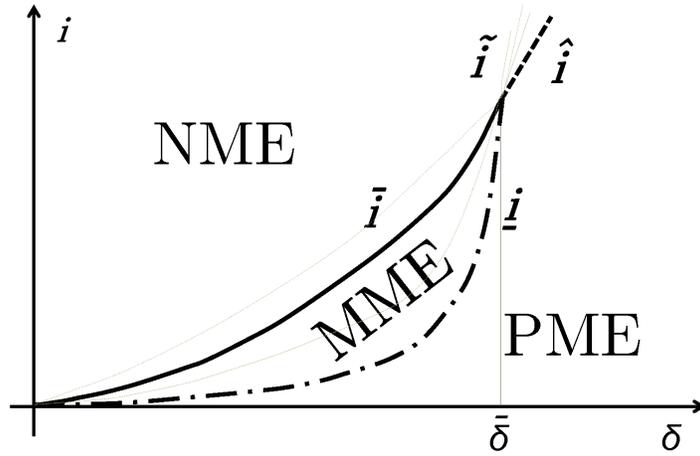


Figure 2: Different Equilibria in Parameter Space

As shown in Figure 2, these results partition parameter space into three regions. In one, characterized by low δ and high i , we have NME. In another, characterized by high δ and low i , we have (multiple) PME. In the final region, we have MME, which is the equilibrium on which we focus.

In MME (19), the FOC for \hat{z}_i , can be expanded into

$$i = \delta \{ \alpha_1 + 2\alpha_2 [1 - G(z)] \} G'(z) = \frac{\delta \alpha_1^2 (u - \delta - c)^2}{2\alpha_2 (z_i - c)^3},$$

or, after rearrangement,

$$z_i = c + \left[\frac{\alpha_1^2 \delta (u - \delta - c)^2}{2\alpha_2 i} \right]^{1/3}. \quad (22)$$

This expresses real balances in terms of the cube-root of $1/i$, which is worth comparing to Baumol (1952) and Tobin (1956), where the square-root of $1/i$ shows up. Heuristically, Baumol-Tobin agents sequentially incur expenditures requiring cash, and have a fixed cost of rebalancing z . The agents compare i , the opportunity cost of cash, with the benefit of reducing the number of financial transactions, usually interpreted as trips to the bank. Our buyers partake of at most one decentralized transaction a period, before rebalancing z for free, but expenditure size is random and credit is costly. Still, they also compare cost i with the benefit of reducing the number of financial transactions, interpreted as tapping credit. The economics is related.¹³

Now notice something interesting: the model *dichotomizes*, in the sense that the above discussion concerns the real side of the economy without reference to nominal variables, but after analyzing that we can introduce nominal considerations. In particular, in MME the nominal price distribution follows from (7),

$$F_t(p) = 1 - \frac{\alpha_1}{2\alpha_2} \frac{u - \delta - \phi_t p}{\phi_t p - c}.$$

Each period, $F_t(q)$ is uniquely determined, but price dynamics for individual firms are not. Figure 3 illustrates this when $\pi > 0$, where the density F'_{t+1} moves F'_t to the right. Again, $\Pi(p)$ is the same $\forall p \in \mathcal{F}_t = [\underline{p}_t, \bar{p}_t]$. Firms with $p < \underline{p}_{t+1}$ at t (region A Figure 3) must reprice at $t+1$, since while p maximized

¹³Our δ is like the cost of going to the bank to get more cash, except it involves genuine credit: households get a BJ good now and pay later, in the AD market. Hence, one can interpret it as the cost of going to the bank get a loan, rather than going to make a withdrawal.

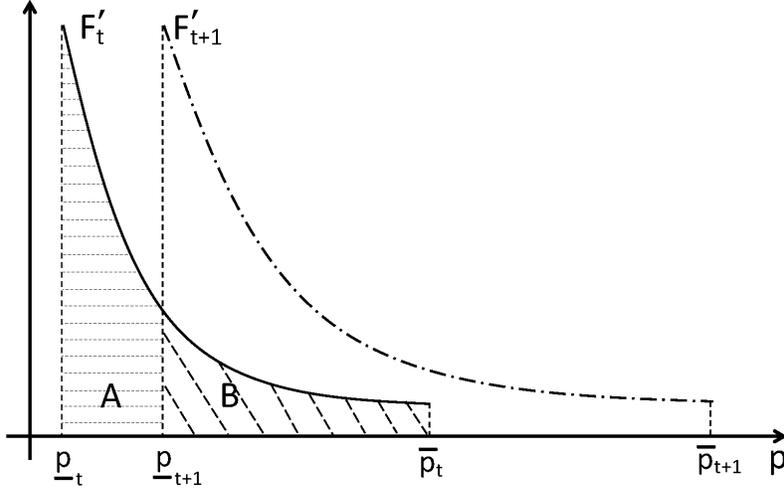


Figure 3: Change in Density of Nominal Prices with Inflation

Π at t it no longer maximizes Π at $t + 1$. But, as long as the supports \mathcal{F}_t and \mathcal{F}_{t+1} overlap, there are firms with $p > \underline{p}_{t+1}$ at t (region B in Figure 3) that can keep the same price at $t + 1$ without reducing profit – again, they earn less per unit, but make it up on the volume, by beating the competitive with higher probability.

Since equilibrium puts strong restrictions on the distribution, but weaker restrictions on individual prices, two conditions are imposed: first, we adopt a payoff-irrelevant tie-breaking rule; and second, we focus on symmetric equilibria. The first says firms that use repricing strategies of the following class: if $p_t \notin \mathcal{F}_{t+1}$ then $p_{t+1}(p_t) = \hat{p}$ where \hat{p} is a new price; but if $p_t \in \mathcal{F}_{t+1}$ then

$$p_{t+1}(p_t) = \begin{cases} p_t & \text{with prob } \sigma \\ \hat{p} & \text{with prob } 1 - \sigma \end{cases} \quad (23)$$

In other words, firms that are indifferent stick to their incumbent price with probability σ . In case it is not 100% obvious, this tie-breaking rule is very different from Calvo pricing, where firms desperately want to change prices but are not allowed: here they are indifferent. Given this, the symmetry restriction merely says that all repricing firms choose \hat{p} in the same way, which implies it must be a random draw from the same repricing distribution $H_{t+1}(\hat{p})$.

For a given σ , there is a unique repricing distribution $H_{t+1}(\hat{p})$ that generates $F_{t+1}(p)$. It is not hard to show (see the Appendix) it is given by:

$$H_{t+1}(p) = \begin{cases} \frac{F_t\left(\frac{p}{1+\pi}\right) - \sigma[F_t(p) - F_t(\underline{p}_{t+1})]}{1 - \sigma + \sigma F_t(\underline{p}_{t+1})} & \text{if } p \in [\underline{p}_{t+1}, \bar{p}_t) \\ \frac{F_t\left(\frac{p}{1+\pi}\right) - \sigma[1 - F_t(\underline{p}_{t+1})]}{1 - \sigma + \sigma F_t(\underline{p}_{t+1})} & \text{if } p \in [\bar{p}_t, \bar{p}_{t+1}] \end{cases} \quad (24)$$

As σ varies between 0 and 1, this class of repricing policies captures a range of possible price dynamics, from which we can compute statistics comparable to those in the empirical literature. In particular, we can compute average price duration, the distribution of changes $(p_{t+1} - p_t)/p_t$, and the hazard function, which gives the probability of a change conditioned on time since the last change. See Head et al. (2012) for derivations of formulae for these statistics.

5 Quantitative Analysis

Since we are interested in capturing realistic payment patterns, we concentrate on MME and ask how well can the model matches the evidence. While one can look at distribution of prices, the focus here is on the distribution of price changes, because that is what the literature emphasizes, as discussed above. Given the dichotomy in the theory, we proceed recursively: first analyze/calibrate the real part of the economy in terms of \hat{z}_i , $G(q)$, etc.; then study the distribution of nominal price changes. For the real part, in terms of preferences, with indivisible BJ goods we only need specify a utility function over AD goods, taken to be $\log(x)$. We calibrate u , which governs the trade off between the AD and BJ markets.¹⁴

Unlike Head et al. (2012), we calibrate the BJ matching technology α_1 and α_2 directly, instead of assuming a functional form. Given this matching

¹⁴Given utility $B \log(x) + u \mathbf{1}(y) - v \ell$, clearly $B = 1$ is a normalization. As usual, v can be set to match average hours $\bar{\ell}$, but the exercise below is independent of $\bar{\ell}$ and v , so these do not need to be specified.

technology, in MME $\bar{q} = u - \delta$ and

$$G(q) = 1 - \frac{\alpha_1}{2\alpha_2} \left(\frac{u - \delta - q}{q - c} \right).$$

The lower bound is $\underline{q} = (\alpha_1(u - \delta) + 2\alpha_2c) / (\alpha_1 + 2\alpha_2)$, which interestingly enough looks like the generalized Nash bargaining solution when the buyer has bargaining power $2\alpha_2 / (\alpha_1 + 2\alpha_2)$. The transaction-price distribution is

$$J(q) = 1 - \frac{\alpha_1^2}{4\alpha_2(\alpha_1 + \alpha_2)} \frac{(u - \delta - q)(u - \delta + q - 2c)}{(q - c)^2},$$

with the same \bar{q} and \bar{q} .¹⁵ Real balances become

$$\hat{z}_i = c + [\alpha_1^2 \delta (u - \delta - c)^2 / 2\alpha_2]^{1/3} i^{-1/3}.$$

Based on these results, we now construct some observable objects. The fraction of transactions that do not need credit simplifies to

$$J(\hat{z}_i) = \frac{(\alpha_1/2 + \alpha_2)^2 - [\alpha_1\alpha_2(u - \delta - c)i]^{2/3} (4\delta)^{-2/3}}{\alpha_2(\alpha_1 + \alpha_2)}. \quad (25)$$

The average markup using posted prices is¹⁶

$$\frac{\mathbb{E}_G q}{c} = \frac{\alpha_1(u - \delta - c) \log(1 + 2\alpha_2/\alpha_1) + 2\alpha_2c}{2\alpha_2c}. \quad (26)$$

As in Lucas (2000), Lagos and Wright (2005) and many other papers, a common notion of money demand (or inverse velocity) is $L(i) = M/PY = \hat{z}_i/Y$,

¹⁵For the record, the densities are

$$G'(q) = \frac{\alpha_1}{2\alpha_2} \frac{u - \delta - c}{(q - c)^2} \text{ and } J'(q) = \frac{\alpha_1^2 (u - \delta - c)^2}{2\alpha_2 (\alpha_1 + \alpha_2) (q - c)^3}.$$

We can also simplify the conditions needed to check for the existence of MME. Given

$$\bar{\delta} = u - \frac{2\alpha_1\alpha_2 + 2\alpha_2^2}{2\alpha_1\alpha_2 + 2\alpha_2^2 - \alpha_1^2} c \text{ and } \bar{i} = \frac{\delta\alpha_1^2}{2\alpha_2(u - \delta - c)}$$

where \bar{i} solves $\Delta(\bar{i}) = 0$, we need $\delta < \bar{\delta}$ and $i \in (\underline{i}, \bar{i})$ at the calibrated parameters.

¹⁶Since the markup data discussed below come from a retail survey, we think it is better to look at the markup defined using posted prices; one can of course also define it using transaction prices, which yields $\mathbb{E}_J q/c = (\alpha_1(u - \delta) + \alpha_2c) / (\alpha_1 + \alpha_2)c$.

where $Y = x + (\alpha_1 + \alpha_2) \mathbb{E}_J q$ is total output evaluated at transaction prices denominated in numeraire. Simplification yields

$$L(i) = \frac{c + [\alpha_1^2 \delta (u - \delta - c)^2 / 2\alpha_2]^{1/3} i^{-1/3}}{1 + \alpha_1 (u - \delta) + \alpha_2 c}. \quad (27)$$

This implies an elasticity of $\varepsilon(i) = -(1 - c/\hat{z}_i)/3$, or

$$\varepsilon(i) = \frac{-1}{3 + 3c [\alpha_1^2 \delta (u - \delta - c)^2 / 2\alpha_2]^{-1/3} i^{1/3}}. \quad (28)$$

In the calibration below we target $J(\hat{z}_i)$, $\mathbb{E}_G q/c$ and moments of money demand, $L(i)$ and $\varepsilon(i)$, at the mean interest rate.

5.1 Calibration

The model is calibrated with a focus on the period 1988-2004, because we are going to evaluate the theory by its ability to account for the empirical price-change distribution during that period. However, longer series can be used to calibrate some parameters. As in many related models, the definition of money is obviously important. We use the M1J definition in Lucas and Nicolini (2012), who adjust M1 by including money market deposit accounts, related to the way Cynamon et al. (2006) adjust for sweeps. Lucas and Nicolini provide annual series from 1915 to 2008, and quarterly series from 1984 to 2013, and make the case that M1J has fairly a stable relationship with nominal interest rates, which as a benchmark are measured using 3-month T-Bills. The average annualized nominal interest rate in their quarterly data is 4.80%, the mean of L is 0.2788, and the money demand elasticity at the mean is -0.1486 . For their annual data the mean nominal rate is 3.83%, the mean of L is 0.2574, and the elasticity at the mean is -0.1054 . We consider both sets of statistics, but focus mainly on the quarterly series since the period corresponds better to the data on price changes. We also consider truncating the sample to eliminate the recent financial crisis.

As regards markups, we use the U.S. Census Bureau Annual Retail Trade Report 1992-2008, cited in Faig and Jerez (2005) and used in many subsequent studies. At the low end, in Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations, gross margins over sales range between 1.17 and 1.21; at the high end, in Specialty Foods, Clothing, Footwear and Furniture, they range between 1.42 and 1.44. Our target is 1.3, right in the middle of these data, which implies a markup target of 1.39, as used in Aruoba et al. (2013), and elsewhere, although as they show the exact value does not matter too much – their results are similar to the those in Lagos and Wright (2005), where the target markup is 1.1, closer to what ones see in applications going back to Basu and Fernald (1997). Also note that those studies matched the markup by calibrating bargaining power using the generalized Nash solution, while markups here are due to the Burdett-Judd frictions (intuitively, as is well known in Burdett-Judd models, when $\alpha_2 \rightarrow 0$ we get monopoly pricing, and when $\alpha_1 \rightarrow 0$ we get competitive pricing).

For the percentage of transactions using money or credit, there are various sources of information. First, we follow much of the literature and interpret money purchases broadly to include cash, check and debit card purchases. One rationale for this practice is that checks and debit cards use demand deposits, which very much like cash are quite liquid and pay basically no interest, and it does not matter much for economic purposes whether your money is in your pocket or in your bank account, given vendors accept checks or debit cards. We think this is consistent with the use of the MIJ measure discussed above. Another rationale is that this notion of money has to be acquired before going to the market, while credit transactions allow one to acquire the purchasing power after the fact – which matters a lot when the probability of a purchase is not 1 and the size is random, as in our model.

In some older calibrations based on Fed surveys from the 1980s (see Co-

ley 1995, chapter 7), 84% of purchases use money monetary while 16% use credit. Much more information is now available. Klee (2008) finds in 2001 grocery-store scanner data that credit cards account for 11.6% of transactions. In a Boston Fed study discussed in Bennett et al. (2014) and Schuh and Stavins (2014), in 2012 credit cards account for 21.6% of purchases in survey data and 17.3% in diary data. Differences from Klee may be due to the fact that the Boston Fed numbers are comprehensive, not just for groceries, and are more recent, since there have been substantial changes in payment methods from 2001 to 2012 (see Schuh and Bringlevics 2014, Table 1). In comprehensive 2009 Bank of Canadian data, Arango and Welte (2012) report the fraction of credit card purchases is 19.3% by volume and 40.1% by value, since credit tends to be used in larger transactions.¹⁷ Further on this, Arango and Welte say “Cash prevails for transactions up to \$25, accounting for 76 per cent of all transactions below \$15, and for 49 per cent of those in the \$15 to \$25 dollar range ... whereas credit cards clearly dominate payments above \$50.” This is of course consistent with costly-credit theory.¹⁸ Based on all this, we target 20% credit transactions, between 19.3% from the Bank of Canadian and 21.6% from the Boston Fed survey, but we also check robustness on this.

This is what we need to calibrate the real side of the model. Thus, we solve

¹⁷In Boston Fed diary data, the number in terms of value is 16.4%, about the same as in terms of volume. We do not know why the Boston Fed volume versus value comparison is so different from the Bank of Canada; when asked, they did not know either. One can also look at the 2013 Federal Reserve Payment Study, where for the entire economy (households, business and government) the credit share by volume is in the same ballpark, 21-22%, but the average value of credit card payments is \$95, while it is on \$56 in the household diaries. We think the household numbers correspond better to our model.

¹⁸For more detail, in terms of volume we have the following: In Klee’s data, cash is used in 54% of transactions, debit cards 14.7% and checks 19.5%. In Boston Fed (diary) data, cash is 40.3%, debit is 24.5%, and checks or money orders is 6.8%. In the Canadian data, cash is 53.7%, debit is 24.7% and checks constitute a meager 0.8%. The Bank of Canadian and Boston Fed also report cash holdings. According to the latter, cash on person (in purse or wallet) plus cash on property (in the cookie jar or under the mattress, say) has a mean of \$340 and a median of \$70, higher than previous estimates mainly due to the inclusion of cash on property. We do not calibrate to these numbers, however, since it is unclear how to map them into the model.

for the parameters $(u, c, \delta, \alpha_1, \alpha_2)$ that match four observations: the fraction of money versus credit transactions as given in the model by (25); the markup as given by (26); and the level and elasticity of money demand given in (27) and (28) evaluated at the mean i . We solve these four conditions jointly, since some targets depend on more than one parameter (e.g., intuitively the markup naturally depends on the matching technology α_1, α_2 as well as cost c); however, u affects only the level of $L(i)$, and it can therefore be set to match the average level of money demand after $(c, \delta, \alpha_1, \alpha_2)$ are calibrated to match the other targets. Additionally, of course, we check the calibrated parameter to see if MME exists.

For the nominal side, the tie-breaking parameter σ is set to match average duration between price changes. This is different from Head et al. (2012), where they calibrate the comparable parameter jointly with the curvature of $u(y)$ to match the price-change distribution in Klenow and Kryvtsov (2008), then compare the predicted duration with the data to. We instead calibrate σ directly to duration, and evaluate model performance based on the match with the price-change distribution. While we think this is more stringent, although it does not affect the results very much: because the model fits quite well, it ends up not mattering dramatically if we fit duration and check the distribution or vice versa. Since we are using the Klenow and Kryvtsov (2008) empirical price-change distribution, we use their average duration as a benchmark. They report a range for duration between 6.8 and 10.4 months, depending on details, with an average of 8.6 months. So we target 8.6 months, although we consider alternatives, based on findings like Nakamura and Steinsson (2008), who report durations between 8 and 11 months.

5.2 Results

In the baseline calibration, we find a large preference coefficient for the DM consumption and marginal cost, $u = 17.45$ and $c = 10.02$, in order to match the monthly money demand. The cost of using credit is very small, $\delta = 0.092$, and this is implied by a relatively high share of monthly credit usage. The probability of sampling one price in a month is 13.49%, and the probability of sampling two is 14.55%, indicating a rather competitive BJ market. When a firm is indifferent between changing its price or not, it chooses to stay at the current price with probability $\rho = 90.17\%$.

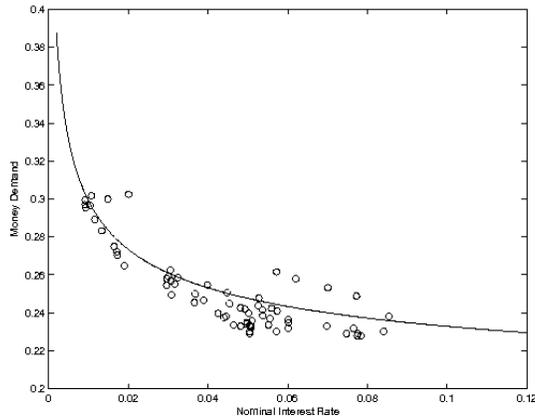


Figure 4: Fitted Money Demand

While these calibrated parameter values match the joint targets fairly well, there are still several other independent ways to check the ability of the model to match empirical data. Figure 4 shows that the model-predicted money demand function can match the actual money demand data pretty well, although calibration only targets on two moments. The model-predicted price change distribution is shown in Figure 5, comparing with the empirical evidence in Figure II of Klenow and Kryvtsov (2008). One can see that our model-generated distribution captures the overall shape of the empirical distribution as well as a few key features. First, Klenow and Kryvtsov (2008) find that the average

price change is large, about 11%. Our model predicts the average size of price changes is 11.12%. Second, there are many small price changes, and 44.3% of all price changes are less than 5%. Our model predicts the share to be 32.59%. Third, 37% of all price changes are negative, compared to 41.49% predicted by the model. While these facts are difficult to reconcile in a simple menu cost model, we are able to capture them pretty well in a monetary search model without idiosyncratic productivity shocks on the supply side. The shift of equilibrium price dispersion combined with tie-breaking rule for indifferent firms are able to generate large price adjustments on average and small price changes at the same time. Still, we are missing some small price changes and generate too many negative changes, but in principle the model does a decent job in matching the empirical evidence of price dynamics.

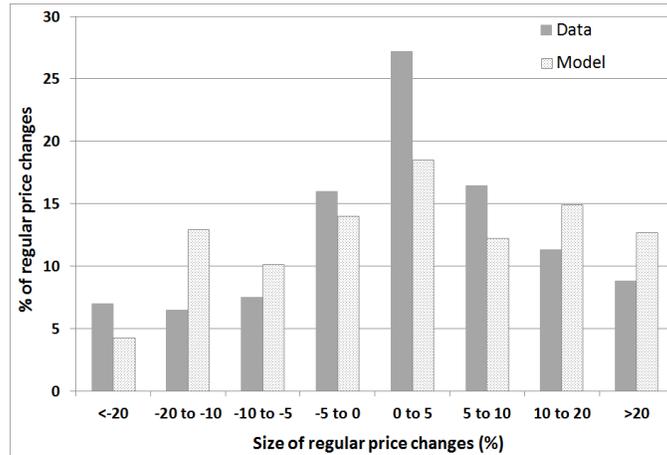


Figure 5: Distribution of Price Changes

Figure 6 shows the hazard function of price changes in 10 years, i.e. the relationship between the probability that a firm adjusts its price and the time since last price adjustment. For example, about 12% of price changes happen after a month. The price change hazard first decreases then increases in the long run. In Figure 7, our hazard function is compared to Figure V in Klenow and Kryvtsov (2008), and we also compare with Figure VIII-A in Nakamura

and Steinsson (2008) in Figure 8. While our model-predicted hazard function does not generate enough price changes in the first several months, it captures the overall trend found in empirical evidence. One can also observe in the data that, after the first several months, although the hazard function decreases with respect to time, except for a few blips, the downward-sloping trend is very gradual. In Figure 7, the model-predicted price change hazard captures this pattern by displaying a drop of about 3% over a period of 36 months. As pointed out by Nakamura and Steinsson (2008), baseline menu cost models fail to replicate these facts.

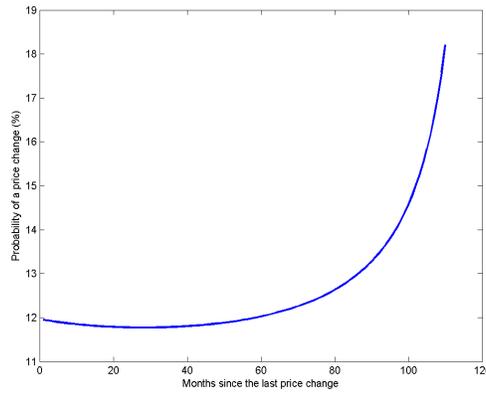


Figure 6: Price Change Hazard in 10 Years

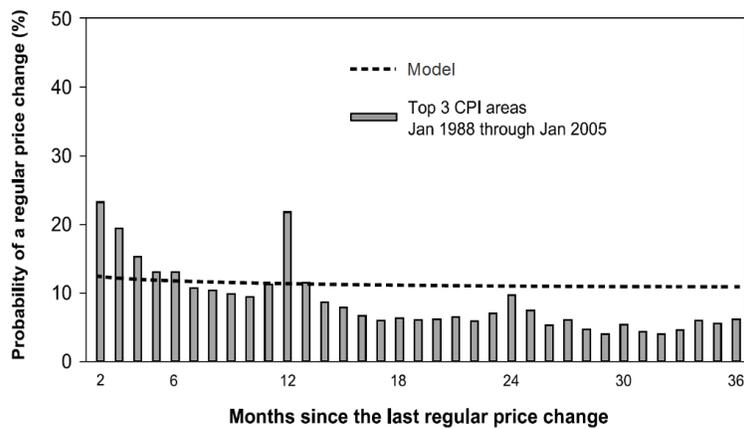


Figure 7: Price Change Hazard in 36 Months

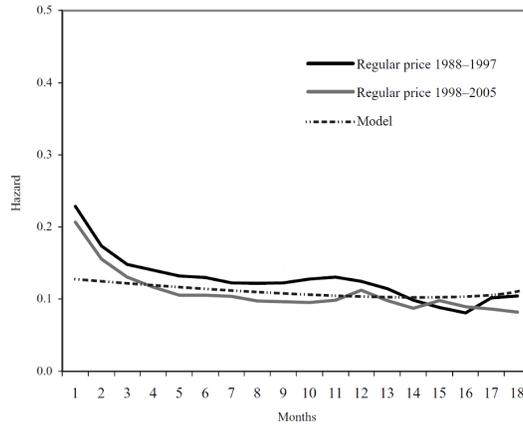


Figure 8: Price Change Hazard in 18 Months

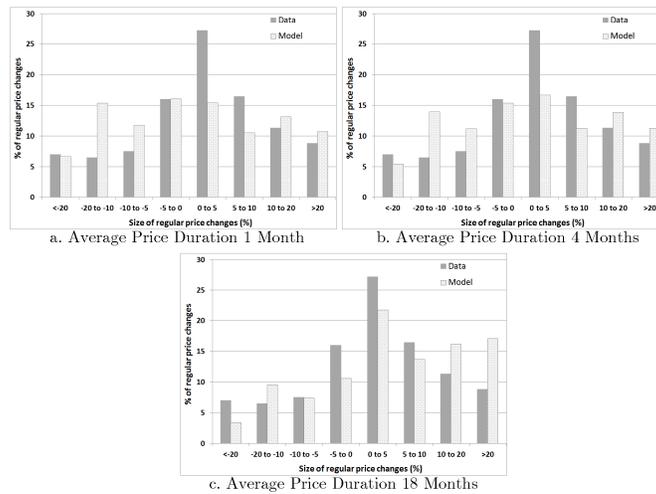


Figure 9: Distribution of Price Changes

Since there is a wide range for average price duration in the empirical literature, as a robustness check, we depart from the baseline calibration and target on different values. We recalibrate the Data model to match average price change duration of 1 month, 4 months, and 18 months. The predicted distributions of price changes are reported in Figure 9. First, we notice that the overall shape of the distribution does not change much. The fraction of small price changes, i.e. $< 5\%$, increases gradually with duration length, but the fraction of negative price changes decreases as the price duration target gets longer. Therefore, we can conclude that the model of price changes presented

in this paper can match the empirical evidence on price change dynamics very well.

6 Endogenous Participation

In this section, we extend the previous analysis by allowing positive entry cost, i.e. $k > 0$. For simplicity, rather than characterizing all possible equilibria in the entire parameter space, we mainly focus on the case in which both money and credit are used in a stationary equilibrium. If firms believe that households use both money and credit in equilibrium, the price distribution consistent with firm's optimization problem in the BJ market is

$$F_t(p) = 1 - \frac{\alpha_1(b)}{2\alpha_2(b)} \left(\frac{u - \delta - \phi_t p}{\phi_t p - c} \right) \quad (29)$$

with $\phi_t \bar{p}_t = u - \delta$ and $\phi_t \underline{p}_t = [\alpha_1(b)(u - \delta) + 2\alpha_2(b)c] / [\alpha_1(b) + 2\alpha_2(b)]$. The meeting probabilities α_1 and α_2 now depend on the measure of participating households in the BJ market.

Given the price distribution above, an equilibrium with both money and credit implies the household's optimal money balance falls in $(\underline{p}_t, \bar{p}_t)$. Based on the discussion in Section ??, a local optimal \hat{m}_t^* satisfies

$$\phi_{t+1} \hat{m}_t = \hat{z}_t = \left[\frac{\delta \alpha_1^2(b_{t+1})(u - \delta - c)^2}{2\alpha_2(b_{t+1})i} \right]^{\frac{1}{3}} + c. \quad (30)$$

Assume household's attempt to collect price quotes succeeds independently with probability $\lambda(b)$, which is twice differentiable in b , with $\lambda(0) = 1$, $\lambda(\bar{b}) = 0$, $\lambda'(b) < 0$, and $\lambda''(b) > 0$. Then, $\alpha_0(b_t) = [1 - \lambda(b_t)]^2$, $\alpha_1(b_t) = 2\lambda(b_t)[1 - \lambda(b_t)]$ and $\alpha_2(b_t) = \lambda(b_t)^2$. Then (30) becomes

$$\phi_{t+1} \hat{m}_t = \hat{z}_t = \left\{ \frac{2\delta [1 - \lambda(b_{t+1})]^2 (u - \delta - c)^2}{i} \right\}^{\frac{1}{3}} + c. \quad (31)$$

Call this the RB (real balance) curve. For $b = 0$, $\hat{z} = c$. For $b = \bar{b}$, $\hat{z} = [2\delta(u - \delta - c)^2]^{\frac{1}{3}} i^{-\frac{1}{3}} + c$. Moreover, $\partial \hat{z} / \partial b > 0$, and $\partial^2 \hat{z} / \partial b^2 < 0$.

The entry decision implies $\Phi(b, \hat{m}_t) = 0$ for $b \in (0, \bar{b})$. We can write this in real terms as

$$\frac{\alpha_1^2(b) \delta (u - \delta - \hat{z})(u - \delta + \hat{z} - 2c)}{4\alpha_2(b) (\hat{z} - c)^2} + \alpha_1(b) \delta + \alpha_2(b) (u - c) - i\hat{z} - \kappa = 0, \quad (32)$$

which can be further simplified to

$$\lambda(b)^2 (u - \delta - c) - \frac{\delta [1 - \lambda(b)]^2 (u - \delta - c)^2}{(\hat{z} - c)^2} + \delta - i\hat{z} - \kappa = 0. \quad (33)$$

We call this the FE (free entry) curve. It implies

$$\frac{\partial b}{\partial \hat{z}} = \frac{i - 2\delta [1 - \lambda(b)]^2 (u - \delta - c)^2 (\hat{z} - c)^{-3}}{\{2\lambda(b) (u - \delta - c) + 2\delta [1 - \lambda(b)] (u - \delta - c)^2 (\hat{z} - c)^{-2}\} \lambda'(b)}.$$

The denominator is negative while the numerator is negative (positive) if \hat{z} is less (greater) than the \hat{z} determined by (31). As shown in Figure ??, the FE curve is upward sloping to the left of the RB curve and downward sloping to the right. Moreover, (33) implies the RB curve intersects the z -axis at $\hat{z} = (u - c - \kappa) / i$ and the b -axis at $b > 0$. Therefore, there exists only one stationary monetary equilibrium in this economy.

In order for this intersection (b, \hat{z}) to be an equilibrium with both money and credit we still need to verify $\hat{z} \in (\phi_{t+1} \underline{p}_{t+1}, \phi_{t+1} \bar{p}_{t+1})$, i.e. at $\hat{z} = \phi_{t+1} \bar{p}_{t+1}$ the FE curve is below the RB curve, while at $\hat{z} = \phi_{t+1} \underline{p}_{t+1}$ the FE curve is above it. We also need to check that this yields higher utility than choosing $\hat{z} = 0$ or using credit alone, i.e. $\tilde{V}(\hat{m}, b) \geq \tilde{V}(\hat{m}_t = 0, b_{t+1}^*(0))$. For this we use computational methods. We solve the model numerically and calibrate the parameters to match the same targets as in Section ?? with one difference: since now households can endogenously choose to participate in the BJ market, we cannot directly calibrate λ . Instead, we follow Head et al. (2012) to parameterize the probability function as $\lambda(b) = \min\{(2b)^{-\frac{1}{2}}, 1\}$ and calibrate the entry cost κ , which determines the equilibrium b and hence the meeting probabilities. Thus we calibrate A and κ jointly to match money demand data.

The model-predicted distribution of price changes is shown in Figure 10, which is again close to the empirical evidence in Klenow and Kryvtsov (2008). The average size of a price changes is 10.9%, compared to 11% in the data. The model also predicts the fraction of price changes less than 5% is 46.68%, and the fraction of negative price changes is 38.9%, comparing to 44.3% and 37% in the data. Compared to the case without endogenous participation, there are more smaller and fewer extreme price changes. The baseline calibration without free entry is able to match data well, and there is not much gain by introducing buyer’s participation decision in terms of improving model’s fitness with data. Therefore, we share the view in Burdett and Menzio (2014) that search frictions are important in explaining price persistence and price change dynamics.

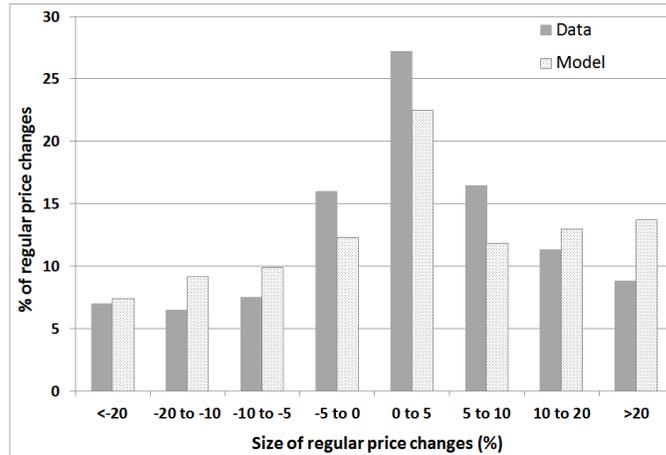


Figure 10: Distribution of Price Changes

7 Conclusion

We build a general equilibrium monetary model to study the behavior of nominal price. The model features indivisible goods, price posting, consumer search frictions, and costly credit. Since consumers make money holding decision before observing prices, there is an ex post demand for more means of payment,

if the consumer samples a price too high for his money balance. Therefore, money and credit can coexist in equilibrium, as long as neither of them is too costly to use.

We also calibrate the model to match the average duration of price changes as well as several other targets. Based on the calibrated parameters, the model-predicted price change distribution and other statistics match the empirical findings on micro-level data very well. The model can generate enough small price changes and negative price changes, and also a downward sloping hazard function. Inflation increases welfare cost by encouraging the use of costly credit, unless the welfare gain from extensive margin dominates.

The framework proposed in this paper certainly has a lot of potential to study more interesting and policy relevant questions. For example, we can introduce proportional interest charges on the outstanding credit balance, and the interest rate depends on central bank's monetary policy. Consider the case when central bank decides to raise interest rate. The cost of using credit increases and fewer households choose to use credit as means of payment. The response of households may then restrict firms from posting very high prices, which could change the price distribution and affect total output. The implication is that although price stickiness alone does not give rise to monetary non-neutrality, other type of frictions may do. We can then apply the extended model to understand and quantify the real effect of monetary policy through the interest rate channel or even unconventional type of liquidity provision.

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