Labor Market Dynamics: A Model of Search and Human Capital Accumulation *

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Abstract

Informed by new measurements of labor market dynamics, I develop and estimate an equilibrium search model of worker mobility. I first describe new facts about the dynamics of wages across unemployment spells in Denmark. These dynamics imply that the job-ladder search model cannot by itself explain all the observed movements of workers between firms. I construct a new model of worker mobility which combines search and human capital accumulation. Workers in the model accumulate skills via learning-by-doing which has decreasing returns for a given job. Workers must either be promoted or find a job at a new firm in order to continue learning new skills. I show that by including this incentive to change jobs, career development not only helps explain wage dispersion, but also contributes to a more complete understanding of labor market dynamics. I structurally estimate the job-ladder search model using matched employer-employee data from Denmark. The estimates show that the job-ladder search model explains less than 10% of the worker mobility seen in the data. In addition, worker heterogeneity explains about 65% of the wage variance for college graduates and about 45% for all other workers.

1 Introduction

How can we explain the great inequality in earnings observed in the US and all over the world? In the US, the 90th percentile in income earns almost 16 times what the tenth percentile in income earns. Even in Denmark, where inequality is relatively low, the 90/10 earnings ratio is a little more than 8. Much work has been done to explain the differences in wages using worker's characteristics, but typical human capital regressions using observable characteristics explain only between 20% and 40% of the variance in wages[1] (Mortensen, 2005). The main

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[1] While some studies have been able to explain up to 70% of the wage variance by focusing on certain populations and using detailed characteristics, it is not clear if these results can be extended to the full population.

explanations for wage dispersion in the literature can be put into the categories of worker attributes, firm attributes and frictions in the marketplace (Mortensen, 2005; Rubinstein and Weiss, 2006). There can be differences in worker's preferences or productivity (human capital), and firms offer different non-pecuniary benefits (hedonics or compensating differentials). At the same time there could be frictions in the monitoring of employees (efficiency wages) and in the matching of workers and firms (search). The contributions that have gotten the most attention are human capital and search. They are important for understanding wage dispersion and growth (Rubinstein and Weiss, 2006), and they also lead to clear policy implications. The goal of this paper is then to understand the sources of wage inequality and labor market dynamics from the point of view of the search theoretic research program.

The workhorse search model for understanding wage dispersion and labor market dynamics is the job-ladder model[2]. In this model, unemployed workers search for jobs by drawing random job offers from a distribution of firms. Workers continue to search while they are employed and accept job offers with higher wages. So identical workers will earn different wages depending on their search history. The job-ladder model is also used to explain worker mobility and wage growth as workers move up the "ladder" to higher paying firms.

I present new facts on the labor market dynamics of unemployment that imply that the job-ladder model can only explain a small fraction of the observed job-to-job movements. Given that workers are quite mobile, it is hard to explain in the context of the job-ladder model why almost half of all workers draw a better paying job after an involuntary unemployment spell. For if all the job-to-job changes are motivated by finding a higher paying firm, only a small fraction should find better paying jobs in their first draw out of unemployment[3]. Understanding the motivations for job-to-job movements is important as it explains at least a third of a worker's early career wage growth (Topel and Ward, 1992) and is important from a macro-economic perspective[4]. The fact that so many workers earn higher wages after unemployment is the main motivation for extending the job-ladder model by adding a second motivation for worker mobility.

In the basic job-ladder search model, unemployment shocks treat worker and firm specific components of wages differently. If the separation into unemployment is involuntary and permanent (i.e. no reemployment), then firm-specific search capital is lost. Workers accumulate firm-specific capital by searching on the job and finding higher-paying firms. Firm-specific capi-

[2]The job-ladder search model refers to the basic random search model with on-the-job search. Lentz and Mortensen (2010) provide a nice survey of this literature.

[3]It is not important if workers actually accept their first employment offer, the argument only requires that the average employment spell is long relative to the average unemployment spell.

[4]The job-ladder model provides a natural way for the economy to reallocate workers from low productivity jobs or firms to more productive ones (Mortensen, 2005).

tal increases over time as a worker receives more job offers and moves from lower to higher paying employers. In the basic job-ladder search model, workers completely lose their firm-specific capital when they enter unemployment and do not return to the previous firm. The same is not true when a worker moves with no intervening unemployment spell. This is because employed workers have the default option to stay at their current firm. But once a worker becomes unemployed, their default option is independent of their previous work history[5]. On the other hand, the worker-specific component is persistent[6] across *short* unemployment spells.

The Danish matched employee-employer dataset is exceptionally well-suited for looking at the labor market dynamics of unemployment. Involuntary separations can be identified by checking to see if a worker is receiving unemployment benefits, because of the eligibility requirements in Denmark. It is easy to see if a worker returns to the same employer since there is a high quality record of the establishment for each job. The wages are measured with little error as they are determined from tax records. The dataset contains the full population of Denmark, so it is possible to learn a lot from splitting the data along different dimensions. Finally, the data is also very rich as it includes most government records of workers and firms.

I begin by investigating the fraction of Danish workers who earn higher wages after involuntary and permanent unemployment spells. There is a large literature estimating the mean change in log-wages[7], but the literature has not featured the large variance in the difference between pre- and post-unemployment spell log-wages. I find about 47% of workers earn higher wages after an unemployment spell. I simulate the basic job-ladder model to calculate the offer rate for employed workers implied by the 47%, and it is essentially zero. This is not possible given that about 15% of Danish workers change employers in a year. Nagypál (2005) also finds that the job-ladder model has a problem in explaining worker mobility in US data. She finds that in order to explain the observed job-to-job flows using the job-ladder model, the offer arrival rate for employed workers would have to be higher than that for unemployed workers. She points out that there is no evidence for such high offer rates for employed workers in survey data. This indicates that there are likely other important motives for job-to-job transitions.

This naturally leads to the question: what fraction of job-to-job movements can be explained

[5]One concern is that workers may save and have higher reservation wages as a result. In this way a worker's future choices of firms would be influenced by their pre-unemployment work history. For this to be a significant effect, it would have the implication that workers who have longer unemployment spells will on average find jobs that pay higher wages. I will show that there is no evidence for this in the data.

[6]One might worry about the effect of human capital depreciation. To the extent that the depreciation depends on the time spent unemployed, the effect is minimized by restricting the analysis to short unemployment spells. In either case, significant human capital depreciation will only strengthen the main argument of the paper.

[7]Kim and Polachek (1994); Albrecht, Edin, Sundström, and Vroman (1999); Corcoran and Duncan (1979); Corcoran, Duncan, and Ponza (1983); Light and Ureta (1995)

by the job-ladder model? If the fraction is small, using the raw job-to-job flow when estimating job-ladder models can lead to biases in the estimate of wage growth due to search. There are recent analyses that study a search model with human capital accumulation[8], where the wage growth is decomposed into a search component and a human capital accumulation component. When estimating a structural search model, the rate of job-to-job transitions is a key input in the wage growth due to search. If only a fraction of job-to-job movements are due to this motive, the growth due to search might be overestimated.

While there is a large literature studying the dynamics and persistence of log-wages over the lifecycle[9], none of these papers focus on the persistence of log-wages across unemployment spells. Restricting the dataset to short unemployment spells, I measure the correlation between pre- and post-unemployment spell log-wages to be 0.39, 0.44, and 0.62 for workers with primary, secondary, and a four-year college education, respectively[10]. I find that these correlations also increase on average about 0.1 over the first 25 years of labor market experience, which indicates that the persistence in wages increases over the lifecycle.

The fact that wages are so persistent across unemployment spells is puzzling given recent results in the search literature. Postel-Vinay and Robin (2002) estimate an equilibrium search model using French data, and find different implications. In their model, firms bargain over workers, and this process can lead firms to pay different wages to identical workers depending on their search history. They find that for all but the highest ability occupations, worker heterogeneity contributes very little to wage dispersion[11]. Worker heterogeneity explains between 20%-40% of the variance for managers, engineers, and supervisors, but very little of the variance in wages of the remaining occupations (0%-10%). I show that search models with piece-rate wage contracts, such as used in Postel-Vinay and Robin (2002), imply that the correlation between pre- and post-unemployment spell log-wages is a direct measure of the fraction of wage dispersion explained by the worker effect.

These empirical patterns, which cannot be explained by the job-ladder model alone, motivate extending the job-ladder model. The principal purpose of this paper is to develop an equilibrium search model which includes a career development motive for job-to-job mobility. Specifically, workers accumulate "skills" via learning-by-doing while employed, but this process

[8] Bunzel, Christensen, Kiefer, and Korsholm (1999); Rubinstein and Weiss (2006); Barlevy (2008); Burdett, Carrillo-Tudela, and Coles (2009); Yamaguchi (2010); Bagger, Fontaine, Postel-Vinay, and Robin (2011)

[9] Lillard and Weiss (1979); Hause (1980); Hall and Mishkin (1982); MaCurdy (1982); Abowd and Card (1989); Moffitt and Gottschalk (2002); Topel and Ward (1992); Baker (1997); Meghir and Pistaferri (2004); Guiso, Pistaferri, and Schivardi (2005); Gladden and Taber (2006); Guvenen (2007); Gottschalk and Moffitt (2009)

[10] These are significantly different with $p < 0.01$.

[11] Postel-Vinay and Robin (2002) explain that one possible reason the worker effect is so small is that there is some unmodelled French institution, like collective bargaining or minimum wages, that are driving their results.

has decreasing returns in a given job. At some point workers stop accumulating skills and become "qualified" for a job at the next level in the job hierarchy. At that point, the worker must find a more productive job within their firm or move to a different firm. One can imagine a cashier at a restaurant who has learned how to use the register and interact with custumers while observing how their supervisor did their job. At some point, the cashier will have nothing more to learn at their current job and will need either a promotion or to find a job at another firm to continue their career. Jovanovic and Nyarko (1997) provide micro-foundations for a career development motive for worker mobility as well as evidence from datasets on occupation ladders that workers are indeed changing jobs in order to learn new skills. Skills are observable from the resumes of workers, and so there are different labor markets for each skill level. It is natural in this model that the labor market dynamics of unemployment change with the experience of workers, which is something that is not explicitly accounted for in most search models. This feature is a compelling reason to adopt the career development motive for worker mobility, since many other motives would not reproduce this relationship.

Workers in this model are also heterogeneous in their ability at labor market entry. While ability does not change over the career of the worker, the returns to their ability can depend on their current level in the job hierarchy. Workers decide whether or not to search for a different job, where searching comes with a fixed cost. This will lead many workers not to search while they are employed and unqualified, as the expected payoff of searching for a higher paying firm is not worth the search cost. Once they become qualified, on the other hand, they will choose to search because finding a job at the next level has the option value of continuing their career development. This choice will help explain the job-to-job flows seen in the data, while at the same time explain why there are not as many job-to-job transitions due to searching for a higher paying firm. Qualified workers who are unemployed also have the option to search at the next level. This will explain a portion of the workers who earn higher wages after an unemployment spell. Firms are also heterogeneous in their productivity and pay a convex cost to recruit in the separate skill markets, but cannot observe a worker's ability until the interview. Since they know that qualified workers will be searching for jobs at the next level, qualified workers will be much less valuable to the firms at the previous level. This creates an overqualified effect where the offer rate for qualified workers at the previous level is many times smaller than for unqualified workers. One can imagine a cashier who has worked many years at one firm applying for a cashier position at other firms. The other firms will prefer to hire someone with less experience, since they are less likely to quickly leave for a job as a supervisor.

Finally, I structurally estimate the job-ladder model with indirect inference. This is an preliminary step to estimating the full model with both motives for worker mobility. The model is estimated using employment and unemployment durations, the wage distribution and the dynamics of wages across unemployment spells. I do not include information from job-to-job flows since the point of the exercise is to compare the predictions of the model to what is observed in the data. The estimated model predicts very low worker mobility compared to what is observed in the data, where the inference is coming from the fraction of unemployed workers earning higher wages. The main result is that only 7.8%, 9.9%, and 6.5% of the observed job-to-job flows are explained by the basic job-ladder model for primary, secondary, and college graduates, respectively. The estimated model can be used to compute the contribution of worker heterogeneity and search frictions to wage dispersion, where the inference is coming from the correlation of pre- and post-unemployment spell wages. Worker heterogeneity explains about 43%, 48%, and 67% of the wage variance for primary, secondary, and college graduates, respectively. I am in the process of extending the analysis to estimate the model with career development, but it is already clear from this preliminary estimation that the job-ladder model, by itself, cannot explain both worker mobility and the labor market dynamics of unemployment observed in the data.

The paper is divided into three parts. First, I discuss the labor market dynamics of unemployment in Section 2. Having established the necessity of a new model, I develop the model in Section 3. I discuss the data, identification and estimation in Section 4. A final section presents the conclusion.

2 Labor Market Dynamics of Unemployment in Denmark

In this section, I present two moments of the labor market dynamics (LMD) of unemployment estimated in Danish data. First, I will discuss how unemployment is defined in Denmark and describe the Danish data. Then for each moment, I discuss the measurements and their implications for existing search models.

2.1 Unemployment in Denmark

Unemployed workers in the Danish data are defined as those workers receiving unemployment benefits. One advantage of working with the Danish data is that workers can only receive unemployment benefits if they were involuntary fired or laid off from their job. As employers need to give on average two months advanced notice to a worker and the government for a layoff, it would be very difficult for a worker to voluntarily quit and be registered as unemployed. For that to happen the employer would have to collude with the worker two months ahead of the quit date to fool the government.

One concern of investigating the LMD of unemployment is the effect of unemployment benefits. If the benefit depends on the previous job's wage, then the unemployment benefit could produce differences in search behavior. Workers in Denmark receive generous unemployment benefits. If they are fired, they are paid about 83% of the weekly wage of their previous job up to a maximum weekly wage of 3760 DKK (∼US$700) for up to 4 years. Although this benefit might seem high, it has to be compared to the minimum wage, which is about 100 DKK/hour (∼US$18/hour). A simple calculation shows that the benefit for a full-time minimum wage worker is very close to the ceiling. Indeed, the benefit ceiling is binding for nearly all full-time workers, or 94% of all unemployed workers. So although workers receive large benefits, they are generally independent of their previous wage.

2.1.1 Dataset on unemployment

A dataset on unemployment has been constructed starting from the dataset of full-time private sector workers described in section 4.1. The unemployment dataset consists of workers that are employed in a given November, become registered as unemployed and then are registered as employed in a different establishment at the November following the return to employment. I require that the worker does not return to the same establishment in order to avoid workers returning to the same firm where their new wages will likely be related to their pre-unemployment wages.

There are certain empirical features of unemployment that I am not modeling and so I select the data so as to avoid them in the analysis. First, as explained in Akerlof and Main (1980), there are certain occupations and industries (e.g. construction) where "spells of unemployment are inherent in the organization of their industry." People working in these industries experience multiple employment and unemployment spells in a given year and account for a large number of the total unemployment spells. As I am not trying to model these dynamics, I make the further requirement that workers spend at least a year in their pre-unemployment job and spend enough time in their post-unemployment spell job to register a wage in the subsequent November. Secondly, loss of skills and other forms of human capital depreciation becomes an issue for longer unemployment spells. So when studying the correlation between pre- and post-unemployment spell wages, I focus on workers with short unemployment spells (< 26 weeks).

The unemployment dataset consists of 151,1433 unemployment spells of private-sector and full-time workers, of which 54,273 had unemployment spells less than 26 weeks. Table 1 describes the statistical properties of the data.

2.2 Difference in pre- and post-unemployment wages

I study the distribution of the difference in pre- and post-unemployment spell wages. The distribution has a large variance (Table 2), where the standard deviation of log-wages is 0.289 for college graduates. While this fact can be found in the summary statistics in other papers on unemployment, it has not been featured, to my knowledge. As a result, a surprisingly large fraction of workers experience an increase in wages after an unemployment spell compared to what they were earning before. As can be seen in Table 2, unemployed workers find a job that pays more about 47% of the time. In order to see if there is any effect from unemployment benefits, I construct the distribution of the percentage change in wages relative to the pre-unemployment job wage in Figure 2. There is no noticeable threshold at which unemployed workers refuse to take lower paying jobs. Given that the wages are very well measured in this dataset[12], this dispersion represents real dynamics in the Danish economy. Noise would have to completely dominate the wage measurement in order to produce this distribution. I plan to investigate this using U.S. data in the near future, but there is evidence that a large proportion of workers in the U.S. also earn larger wages in Hu and Taber (2011). They find that the mean difference in log-wages to be between -0.047 and -0.135, but the standard deviation of these differences is very large, between 0.426 and 0.492. This indicates that a large fraction of workers in the U.S. also earn more after an unemployment spell.

The fact that post-unemployment wages are larger than pre-unemployment spell wages about half the time is difficult for job-ladder search models to explain. Search models with random matching where wages are determined through wage-posting (Burdett and Mortensen, 1998) or bargaining (Pissarides, 2000) do generate a significant fraction of workers who earn higher wages after an unemployment spell, but cannot reconcile the rate of job-to-job transitions and the fact that half of the unemployed find higher paying jobs.

In the job ladder model workers search and draw job offers randomly from the wage offer

[12] Earnings are measured from the data sent by firms directly to the tax authorities and are considered very reliable. The hours worked is collected from pension data again sent directly from the firm. I require that the hours-worked data is of sufficient quality (as defined by Statistics Denmark) such that the wage is very well measured. The hours measurement has also been compared between two different sources and has been found to have little measurement error.

distribution. Once they find their first job, they continue to search while employed. As they receive offers they will move to any job that offers a higher wage. This is what generates worker mobility in the job-ladder model. Over continuous periods of employment, they accumulate job offers and find the higher paying jobs in the job-offer distribution. The puzzle then is that once they become unemployed they take the first job offer they receive. If workers receive many offers during the previous period of employment, it is very unlikely that they will receive a better offer on their first draw out of unemployment. The fraction of workers earning higher wages only depends on the ratio of the job offer rate to the job destruction rate[13]. If the job-offer rate is high, then workers will receive more offers while they are employed and it is less likely that their first job-offer out of unemployment will be better. If the separation rate is higher, their employment spells will be shorter and so they will receive fewer offers before becoming unemployed. So more unemployed workers will earn higher wages as the separation rate increases. This relationship can be inverted so that the fraction of workers earning higher wages after unemployment implies a certain ratio of job-offer rate to job-destruction rate. This relationship is calculated based on simulations and is shown in figure 3. Since earning a higher wage only depends on receiving an higher ranking offer, this relationship does not depend on the actual distribution of offers. The figure confirms the intuition that as the fraction of workers earning higher wages gets close to 50%, the implied job-offer rate goes to zero. What is interesting is that the implied job-offer ratio for 42% is 3.3 times larger than the implied rate 47%. If a modification of the model could explain just 5% of the workers who earn higher wages after unemployment spells it would have a large effect on the implied job-offer rate for the job-ladder model. An average on-the-job offer rate that is about one-fifth of the separation rate is clearly rejected by the data (see section 4.2).

The unemployed worker in sequential-auction models (Postel-Vinay and Robin, 2002) faces a very different environment. Firms are only willing to pay a worker their outside value. Firms will eventually pay a worker higher wages when they receive a second offer and at that point the more productive firm will pay the outside value of the highest paid job at the less productive firm. An unemployed worker will only be paid a wage such that the value of the job is equivalent to their unemployment value. Since working for a productive firm has a positive option value, the post-unemployment wage will actually be lower than their unemployment income. The more productive the firm, the higher the option value, the lower the post-unemployment wage. So the sequential-auction model does predict that a very small fraction of workers will receive a marginally higher wage. The workers that receive a higher wage out of unemployment are those

[13]The job destruction rate is the rate at which workers become unemployed.

that worked for a very productive firm and had no other offers[14], became unemployed and then got a job at a less productive firm. Of course, a sequential-auction model that includes rent-sharing (Cahuc, Postel-Vinay, and Robin, 2006) could also generate a positive fraction of workers earning higher wages after an unemployment spell, but these models would imply an even lower job-offer rate since workers in these models will often see their wages increase when they are contacted by less productive firms.

The fact that half of the unemployed workers find jobs with higher wages motivates extending the job-ladder model to include new dynamics that can help reconcile both the high fraction of workers earning higher wages after unemployment spells and the rate of job to job transitions. I present such a model in section 3.2.

2.3 The persistence of wages across unemployment spells

The persistence of wages across short unemployment spells is measured by estimating the correlation of the pre- and post-unemployment spell wages. The estimates of the correlations for the Danish data are shown in table 3 for unemployment spells less than 26 weeks. In general, I find a strong correlation between pre- and post-unemployment spell wages. As expected, these correlations are increasing in education. More education gives access to higher-skilled jobs. The productivity of higher-skilled jobs is likely to be more sensitive to each individuals ability.

I also find that the correlation is increasing in experience for all educational levels as seen in figure 1. This indicates that the worker-specific contribution to wage dispersion is growing with labor market experience. This raises the question, what is driving the persistent heterogeneous wage growth that is present in the data. The model in section 3.2 is partly motivated by this fact. Workers must search for jobs in new firms for new human capital accumulation opportunities. The heterogeneity in wage growth will be partly driven by the fact that some workers will quickly find these jobs while others will take longer.

A significant persistence of wages across unemployment spells would be interpreted as an important contribution from worker heterogeneity in a search model with firm and worker heterogeneity. As discussed in the introduction, if a worker in a random search model suffers an involuntary and permanent separation from their employer they lose their firm-specific capital. In this case, the worker's firm-specific capital in his pre- and post-unemployment jobs would be uncorrelated, as the worker would randomly draw from the firm offer distribution to exit out of unemployment. On the other hand, the worker-specific component would be persistent across

[14] or they only received offers from firms with very low productivity

short unemployment spells if the worker-specific component does not depreciate very quickly. I study the correlation between pre- and post-unemployment spell wages since it is a moment that is sensitive to the fraction of wage dispersion explained by worker attributes or general human capital. In a model where all of wage dispersion is explained by worker attributes, this correlation will be one. Likewise, in a model where wage dispersion is completely explained by firm-specific capital, the correlation should be zero if there is no re-employment and workers do not differ in their search options. This moment will play an important role in the identification of the equilibrium search model described in section 3.1.

Appendices A, A.1, and A.2 show how this wage correlation can be directly interpreted in the context of the AKM model(Abowd, Kramarz, and Margolis, 1999) and search models with piece-rate wages.

3 Model

I begin by describing the canonical job-ladder model with worker and firm heterogeneity as the different features of the model are easier to describe in this setting. I then extend this model to add human capital accumulation and can then discuss what this extension contributes to wage dispersion and dynamics beyond the canonical model.

3.1 Canonical model with on-the-job search and worker-firm heterogeneity

The model adopts the Stole-Zwiebel bargaining environment for wages where the contracts are non-binding (Stole and Zwiebel, 1996; Wolinsky, 2000). I follow Helpman, Itskhoki, and Redding (2009); Mortensen (2009, 2010) in using this in a search and matching model. In this application, firms have constant returns to scale production, convex costs for posting vacancies, and there are heterogeneous workers. This environment is a special case of Mortensen (2009, 2010) (CRS), but has the addition of heterogeneous workers.

3.1.1 Wage bargaining with non-binding contracts

Non-binding contracts are motivated by the observation that the usual form of labor contracts are employment at will. Stole and Zwiebel argue that because of liquidity constraints firms cannot ask for payments in the event of a breakdown, and they cannot force the employees to perform the work because of involuntary servitude restrictions. The sequence of events in the no commitment model goes as follows. An employee and firm first search and recruit to begin an employment relationship. Given that they have formed an employment relationship the employee and firm then negotiate over production and the wage paid. The only consequence of a breakdown in negotiations is that the employee does not produce, so the firm receives zero profits and the worker receives their value of leisure. This is different to binding contracts where the outside values of the firm is posting a vacancy, and the employee's outside value is unemployment and search.

The wage bargaining outcome is taken to be the generalized Nash bargaining outcome with complete information. Unlike in Stole and Zwiebel, the firms in our model have constant returns to scale for each additional worker. A firm-worker match produces $f(p, \kappa) = a + p + \kappa$, where a, p, and κ are the productivities of the job, firm, and worker, respectively.

So the wage paid at each moment becomes

$$w(p, \kappa) = \beta(a + p + \kappa) + (1 - \beta)b, \tag{1}$$

where β represents the employee's bargaining power and b is the workers unemployment income. And the firm's profit from the match with a worker with ability κ is

$$\pi(p, \kappa) = (1 - \beta)(a + p + \kappa - b). \tag{2}$$

There is also a one-to-one relationship between the productivity of a firm and the wage it pays a worker with ability κ. Given a wage w and ability κ, p can be solved for in equation 1 to define $p(w, \kappa) = \frac{1}{\beta}(w - \beta(a + \kappa) - (1 - \beta)b)$. Let $F^p(p)$ be the cumulative distribution function of offers by firms with productivity of p or less. For a worker with a given ability κ, this can be directly related to the distribution of wage offers $F^w(w, \kappa)$

$$F^w(w, \kappa) = F^p(p(w, \kappa)) = F^p\left(\frac{1}{\beta}(w - \beta(a + \kappa) - (1 - \beta)b)\right). \tag{3}$$

In order to avoid the feature of exogenous unemployment, it is assumed that the least productive firm and least productive worker produce more than the worker's home production (b), or in other words,

$$a + \underline{p} + \underline{\kappa} \geq b. \tag{4}$$

Finally, a is normalized such that $E[\kappa] = 0$.

3.1.2 Worker's problem

A worker receives income b and searches for a job while unemployed. They receive offers at rate $\lambda = \theta q(\theta)$, where $q(\theta)$ is the matching function of workers to vacancies and $\theta = \nu/z$ is the market tightness (ratio of searching workers to vacancies). An employed worker earns $w(\kappa, p)$ while employed, sees the match dissolved at the exogenous rate δ, and meanwhile searches for a job with a higher wage (i.e. a more productive employer), receiving offers at the same rate, λ. Workers die at rate ρ, and the same amount of workers are born into unemployment. Finally, there is a fixed cost of search (χ) and workers can choose whether or not to search ($\phi \in \{0, 1\}$).

A worker's value of unemployment (U) and employment (W) then solves the following Bellman equations

$$\begin{aligned} (r+\rho)W(\kappa,p) &= w(\kappa,p) + \delta(U(\kappa) - W(\kappa,p)) \\ &\quad + \max_{\phi\in\{0,1\}} \left\{ \phi\lambda \int_{\underline{p}}^{\overline{p}} \max\langle W(\kappa,x) - W(\kappa,p), 0\rangle dF^p(x) - \chi\phi \right\} \\ (r+\rho)U(\kappa) &= b + \max_{\phi\in\{0,1\}} \left\{ \phi\lambda \int_{\underline{p}}^{\overline{p}} \max\langle W(\kappa,x) - U(\kappa), 0\rangle dF^p(x) - \chi\phi \right\}. \end{aligned}$$

These equations imply that $W(\kappa, p)$ is strictly increasing in p for both workers that search ($W_p(\kappa, p) = W_p(p) = \frac{\beta}{r+\rho+\delta+\lambda[1-F^p(p)]}$) and those that choose not to search ($W_p(\kappa, p) = W_p(p) = \frac{\beta}{r+\rho+\delta}$). So the worker's Bellman equations can be rewritten as

$$\begin{aligned} (r+\rho)W(\kappa,p) &= w(\kappa,p) + \delta(U(\kappa) - W(\kappa,p)) \\ &\quad + \max_{\phi\in\{0,1\}} \left\{ \phi\lambda \int_{p}^{\overline{p}} W(\kappa,x) - W(\kappa,p) dF^p(x) - \chi\phi \right\} \\ (r+\rho)U(\kappa) &= b + \max_{\phi\in\{0,1\}} \left\{ \phi\lambda \int_{R_p}^{\overline{p}} W(\kappa,x) - U(\kappa) dF^p(x) - \chi\phi \right\}. \end{aligned}$$

Where R_p is the reservation productivity that can be related to the reservation wage using equation 1 ($w(\kappa, R_p) = \beta(a + k + R_p) + (1 - \beta)b$). These equations and the definition of the reservation productivity $U(\kappa) = W(\kappa, R_p(\kappa))$ imply that the reservation wage is $R_w(\kappa) = w(k, R_p(\kappa)) = b$, and so the reservation productivity is $R_p(\kappa) = b - a - \kappa \le \underline{p}$. So in this economy all workers and firms form productive matches.

Finally, I can solve for the worker's asset equations in terms of the offer distribution and market tightness by integrating by parts and using the fact that $F^p(p) = 0 \; \forall \; p \leq \underline{p}$

$$\begin{aligned}
(r+\rho)U(\kappa) &= b + \max_{\phi \in \{0,1\}} \left\{ \phi\lambda \int_{R_p}^{\overline{p}} \frac{\beta[1-F^p(x)]}{r+\delta+\phi(x)\lambda[1-F^p(x)]} dx - \chi\phi \right\} \\
(r+\rho+\delta)W(\kappa,p) &= w(\kappa,p) + \delta U(\kappa) \\
&\quad + \max_{\phi \in \{0,1\}} \left\{ \phi\lambda \int_{p}^{\overline{p}} \frac{\beta[1-F^p(x)]}{r+\delta+\phi(x)\lambda[1-F^p(x)]} dx - \chi\phi \right\}.
\end{aligned}$$

Since workers are observed to search while unemployed and employed, I assume χ is not so large that workers never search. In this case, $\phi = 1$ for unemployed workers. Of course, employed workers will choose to search in order to maximize their expected future income. If their current employer is near the top of the productivity distribution, they will not gain much from searching. Let the reservation productivity for search (S_p) be defined by

$$\lambda \int_{S_p}^{\overline{p}} \frac{\beta[1-F^p(x)]}{r+\delta} dx = \chi, \tag{5}$$

where the left-hand side is the expected gain from search and the right-hand side is obviously the cost. A worker who is employed at an employer with $p \geq S_p$ will stop searching and those at lower productivity firms will continue to search. The decision of the worker is then

$$\phi(p) = \begin{cases} 1 & \text{for} \quad p < S_p \\ 0 & \text{for} \quad p \geq S_p \end{cases}$$

3.1.3 Firm's problem

The firms must pay a cost in order to post a vacancy (ν). The cost of vacancies is a convex function of the number of open vacancies. Firms must decide the size of their labor force (i.e. how many vacancies to post) before they know the ability of the workers they come into contact with. So they maximize their profit based on the expected ability (κ) of the workers. Firms meet workers for a given vacancy at rate $\eta = q(\theta)$.

The hiring rate per vacancy ($h(p,\kappa)$), and the separation rate per worker ($s(p)$) is

$$s(p) = \delta + \phi(p)\lambda(1 - F^p(p))$$
$$h(\kappa,p) = \eta(u(\kappa) + (1-u(\kappa))G^p(\min\{p, S_p\}|\kappa))$$

where u is the unemployment rate and $G^p(p|\kappa)$ is the fraction of employed workers with ability

κ at firms with productivity p or less. The law of motion of the firm's work force is then

$$\dot{n}(\kappa, p) = \upsilon h(\kappa, p) - s(p)n(\kappa, p)$$

The value of a worker with ability κ to a firm is

$$rJ(\kappa, p) = (1-\beta)[a+p+\kappa-b] - (\delta + \phi(p)\lambda[1-F^p(p)])\, J(\kappa, p)$$

This can be rewritten using the definition of the separation rate.

$$J(\kappa, p) = \frac{(1-\beta)[a+p+\kappa-b]}{r+s(p)}$$

The firm will maximize the total profits of its workforce. The Bellman equation describing the firm's value is

$$rV(n, p) = \max_{\upsilon \geq 0} E_\kappa[n(\kappa, p)J(\kappa, p) - c(\upsilon) + J(\kappa, p)(\upsilon h(\kappa, p) - s(p)n(\kappa, p)] \tag{6}$$

where $c(\upsilon)$ is the cost of posting υ vacancies, which is assumed to follow the regularity conditions: $c(0) = c'(0) = 0$ and $\lim_{\upsilon\to\infty} c'(\upsilon) = \infty$, in addition to being convex ($c''(\upsilon) > 0$).

The solution to the firm problem is to equate the cost of opening vacancies to the benefit of hiring additional workers. The first order necessary condition (FONC) of equation 6 is

$$c'(\upsilon) = E_\kappa[h(\kappa, p)J(\kappa, p)]$$

which determines the number of vacancies a firm with productivity p will open given market tightness and the offer distribution. The number of vacancies opened implies that, in steady state, the firm size will converge to

$$n(\kappa, p) = \frac{h(\kappa, p)\upsilon(p)}{s(p)}.$$

3.1.4 Steady State Equilibria

The flow into and out of unemployment will be equal:

$$\delta(1-u) = \lambda u.$$

This implies that unemployment does not depend on ability ($u(\kappa) = u$) in steady state.

The flow of workers into and out of firms with productivity less that p will be equal:

$$\lambda F^p(p)u = \delta G^p(p|\kappa)(1-u) + \lambda(1-F^p(p))G^p(\min\{p, S_p\}|\kappa)(1-u)$$

The second term on the right-hand side is the outflow of workers at firms with productivity less than p who are searching ($G^p(\min\{p, S_p\}|\kappa)(1-u)$) who also find jobs at firms with productivity greater than p ($1-F^p(p)$). This can be used to derive $G^p(p|\kappa)$ in terms of $F^p(p)$:

$$G^p(p|\kappa) = G^p(p) = \begin{cases} \frac{\delta F^p(p)}{\delta+\lambda(1-F^p(p))} & \text{for} \quad p \leq S_p \\ F^p(p) - \frac{\lambda}{\delta}(1-F^p(p))G^p(S_p) & \text{for} \quad p > S_p \end{cases}. \tag{7}$$

Here you can see that the difference between the offer distribution ($F^p(p)$) and the working distribution ($G^p(p)$) is completely determined by the ratio of the offer rate to the separation rate (λ/δ). If the ratio is zero, then $G^p(p) = F^p(p)$ and the productivity distribution of job offers and jobs that are held is the same. In this case, half the workers will earn higher wages after an unemployment spell. Since in steady state, $G^p(p)$ does not depend on κ it follows that the hiring rate is also independent of κ in steady state ($h(\kappa, p) = h(p)$). This simplifies finding the vacancies opened by a firm:

$$c'(\upsilon) = \frac{(1-\beta)[a+p-b]}{r+s(p)}$$

using the fact that a has been normalized such that $E[\kappa] = 0$.

Let I(p) be the cumulative distribution function of firms with productivity p, which is exogenous to the model. Then the distribution of job offers or vacancies is

$$F^p(p) = \frac{1}{N_\upsilon}\int_{\underline{p}}^{p} \upsilon(x)dI(x)$$

where $N_\upsilon = \int_{\underline{p}}^{\overline{p}} \upsilon(x)dI(x)$ is the total number of vacancies.

Finally, the last condition to close the model (i.e. determine market tightness θ) is to make sure the total number of workers in the economy is equal to the sum of the workers at each firm:

$$(1-u)N = \frac{\lambda}{\delta+\lambda}N = \int_{\underline{p}}^{\overline{p}} n(x)dI(x).$$

where N is the number of workers in the economy.

3.1.5 Equilibrium Wage Dispersion

With $F^p(p)$ and $G^p(p)$ in hand, the wage distributions can be calculated. The wage offer distribution conditional on ability was already found in equation 3. The analogous wage earned distribution is

$$G^w(w,\kappa) = G^p(p(w,\kappa)) = G^p\left(\frac{1}{\beta}(w - \beta(a+\kappa) - (1-\beta)b)\right).$$

Let $H(\kappa)$ be the cumulative distribution for ability. Then the cumulative distribution for wages is

$$\begin{aligned} F^w(w) &= \int_{\underline{\kappa}}^{\overline{\kappa}} F^w(w,\kappa)dH(\kappa) \\ G^w(w) &= \int_{\underline{\kappa}}^{\overline{\kappa}} G^w(w,\kappa)dH(\kappa) \end{aligned}$$

I can also write down the variance decomposition of the wages, knowing that in steady state κ and p are not correlated. The variance of the wage from equation 1 is then just:

$$Var(w) = Var(\beta(a+p+\kappa) + (1-\beta)b) = \beta^2\left[Var(p) + Var(\kappa)\right]$$

where the variance of firm productivities is calculated using the employment weighted distribution $G^p(p)$.

3.2 Model with firm-worker heterogeneity and human capital accumulation

In the final model, workers are heterogeneous in two dimensions. The first is the ability that they have when they enter the labor force which does not change over time. This endowment can be thought of as the accumulation of parental investments and education. The second dimension is the human capital accumulated in the workplace. This dimension will be called "skills". This dimension can be thought of as the general skills the workers accumulate while in their job.

By adding a second dimension, the canonical model is extended to include human capital accumulation via learning-by-doing. This is achieved through a multi-level hierarchy of jobs. Employees begin their lives unemployed in the first level with their endowed ability and they immediately begin searching for a job. Once they find their first job, they must work there until they have accumulated enough skills to be qualified for a job at the next level. The learning-by-doing process has diminishing returns, so that after a certain amount of time they have learned all they can at that job and their skill growth stagnates. Once they are qualified, they begin searching for a job at the next level both within their current firm and in other firms. If they find a job in their current firm, they are promoted. If they find a job in another firm, then they quit their current job and take the next-level job at the new firm. Jobs become more and more productive as the employee takes jobs at higher and higher levels. Firms can evaluate the skills of candidate employees from their resumes and determine if an employee has the requisite amount of skills. If a candidate is under-qualified they will have a very low productivity for that job, and the firm will not hire them until they have the the required skills. For this reason workers do not bother applying for jobs for which they are under-qualified. Since lower-level jobs have lower productivity, and hence pay lower wages, they also do not apply for jobs at lower levels than they have worked. This essentially sets up distinct markets from the point of view of the employer, where they search for employees separately in each level. There is a final level at which point they stay in the job with the highest productivity. This multi-level model is appealing, since it provides an additional incentive for on-the-job search: that is to find a higher level position and continue accumulating skills. A diagram of the worker's problem is shown in figure 4.

The model from the previous section represents the building blocks of this model, where each level of skills is represented by a separate market i. Workers search while unemployed and on the job, and become unemployed at a rate δ_i. Firms participate by opening vacancies in all markets, and workers search for jobs in the market for which they are qualified. There is a fixed cost of search (χ) and workers can choose whether or not to search ($\phi \in \{0, 1\}$).

3.2.1 Worker's problem

There are three unique states. The first state ("unqualified"), represented by Bellman equations W_1 and U_1, is where the worker is accumulating skills and becomes qualified for the next level at rate γ. In the second state ("qualified"), the worker is searching for a higher level job at another firm and is waiting to be promoted at rate ψ and is represented by Bellman equations $\hat{W}_1$ and $\hat{U}_1$. "Qualified" and "unqualified" refer to their readiness to advance to the next level. The job-specific parameters in these two states are identical since they represent the same job. These two states are repeated for each level in the job hierarchy, except for the final level. The final level ("terminal") is represented by Bellman equations W_2 and U_2. It is assumed that the productivity in each level is significantly higher than the previous one and so the value of working in a given level is higher than working in a lower level ($W_T(\kappa,p) \geq \hat{W}_{T-1}(\kappa,p) \geq W_{T-1}(\kappa,p) ... \geq \hat{W}_1(\kappa,p) \geq W_1(\kappa,p)\ \forall p,\kappa$). So workers will always want to seek the higher level job. It is again assumed that any firm-worker pair produces at least b at any level in which a worker is qualified. The Bellman equations for the terminal level are the same as the canonical job-ladder model. The Bellman equations for the three unique states are

$$
\begin{aligned}
(r+\rho)W_2(\kappa,p) &= w_2(\kappa,p) + \delta_2(U_2(\kappa) - W_2(\kappa,p)) \\
&\quad + \max_{\phi\in\{0,1\}} \left\{ \phi\lambda_2 \int_{\underline{p}}^{\overline{p}} \max\langle W_2(\kappa,x) - W_2(\kappa,p), 0\rangle dF_2^p(x) - \chi\phi \right\} \\
(r+\rho)U_2(\kappa) &= b + \max_{\phi\in\{0,1\}} \left\{ \phi\lambda_2 \int_{\underline{p}}^{\overline{p}} \max\langle W_2(\kappa,x) - U_2(\kappa), 0\rangle dF_2^p(x) - \chi\phi \right\}
\end{aligned}
$$

$$
\begin{aligned}
(r+\rho)\hat{W}_1(\kappa,p) &= \hat{w}_1(\kappa,p) + \delta_1(\hat{U}_1(\kappa) - \hat{W}_1(\kappa,p)) \underbrace{+ \psi_{\mathbf{1}}(\mathbf{W_2}(\kappa,\mathbf{p}) - \hat{\mathbf{W}}_{\mathbf{1}}(\kappa,\mathbf{p}))} \\
&\quad + \max_{\phi\in\{0,1\}} \Bigg\{ \phi\hat{\lambda}_1 \int_{\underline{p}}^{\overline{p}} \max\langle \hat{W}_1(\kappa,x) - \hat{W}_1(\kappa,p), 0\rangle d\hat{F}_1^p(x) - \chi\phi \\
&\quad \underbrace{+\phi\varepsilon\lambda_{\mathbf{2}} \int_{\underline{\mathbf{p}}}^{\overline{\mathbf{p}}} \max\langle \mathbf{W_2}(\kappa,\mathbf{x}) - \hat{\mathbf{W}}_{\mathbf{1}}(\kappa,\mathbf{p}), \mathbf{0}\rangle \mathbf{dF_2^p}(\mathbf{x})} \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
(r+\rho)\hat{U}_1(\kappa) &= b + \max_{\phi\in\{0,1\}} \Bigg\{ \phi\hat{\lambda}_1 \int_{\underline{p}}^{\overline{p}} \max\langle \hat{W}_1(\kappa,x) - \hat{U}_1(\kappa), 0\rangle d\hat{F}_1^p(x) - \chi\phi \\
&\quad \underbrace{+\phi\varepsilon\lambda_{\mathbf{2}} \int_{\underline{\mathbf{p}}}^{\overline{\mathbf{p}}} \max\langle \mathbf{W_2}(\kappa,\mathbf{x}) - \hat{\mathbf{U}}_{\mathbf{1}}(\kappa), \mathbf{0}\rangle \mathbf{dF_2^p}(\mathbf{x})} \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
(r+\rho)W_1(\kappa,p) &= w_1(\kappa,p) + \delta_1(U_1(\kappa) - W_1(\kappa,p)) \\
&\quad + \max_{\phi\in\{0,1\}} \left\{ \phi\lambda_1 \int_{\underline{p}}^{\overline{p}} \max\langle W_1(\kappa,x) - W_1(\kappa,p), 0\rangle dF_1^p(x) - \chi\phi \right\} \\
&\quad \underbrace{+\gamma_{\mathbf{1}}(\hat{\mathbf{W}}_{\mathbf{1}}(\kappa,\mathbf{p}) - \mathbf{W_1}(\kappa,\mathbf{p}))} \\
(r+\rho)U_1(\kappa) &= b + \max_{\phi\in\{0,1\}} \left\{ \phi\lambda_1 \int_{\underline{p}}^{\overline{p}} \max\langle W_1(\kappa,x) - U_1(\kappa), 0\rangle dF_1^p(x) - \chi\phi \right\}.
\end{aligned}
$$

Although this appears complicated, there are only four new terms compared to the canonical job-ladder model. The new terms have underbraces and are set in bold font to more easily see them. In the qualified state ($\hat{W}$ and $\hat{U}$), one can see the two paths through which a worker attains the next level, namely promotion (ψ_1) and on-the-job search ($\varepsilon\lambda_2$). The worker has the option of searching for a job in the next level while unemployed as well. ε ($0 \leq \varepsilon < 1$) is an additional exogenous friction in searching due to the fact that the worker is looking for a job he has never held and firms may be reluctant to hire them compared to someone who has worked at that position before. Also notice that this model nests into a model with exogenous human capital accumulation by setting $\varepsilon = 0$. The only difference between the unqualified and terminal model is that workers attain the qualified level at rate γ_1.

The Bellman equations imply that the value of working is strictly increasing in p for all states. The first derivative for workers that are searching is

$$W_{2p}(\kappa, p) = W_{2p}(p) = \frac{\beta}{r + \rho + \delta_2 + \lambda_2[1 - F_2^p(p)]}$$

$$\hat{W}_{1p}(\kappa, p) = \frac{\beta + \psi_1 W_{2p}(p)}{r + \rho + \delta_1 + \psi_1 + \hat{\lambda}_1[1 - \hat{F}_1^p(p)] + \varepsilon\lambda_2[1 - F_2^p(\breve{p})]} \quad (8)$$

$$W_{1p}(\kappa, p) = \frac{\beta + \gamma_1 \hat{W}_{1p}(\kappa, p)}{r + \rho + \delta_1 + \gamma_1 + \lambda_1[1 - F_1^p(p)]}. \quad (9)$$

where $\breve{p}$ represents the lowest productivity that a qualified worker is willing to accept for a job in the next level, given that he has a job with productivity p (i.e. $W_2(\kappa, \breve{p}) = \hat{W}_1(\kappa, p)$). It could depend on the worker's ability. The fact that the value functions are strictly increasing in p along with the fact that $W_2(\kappa, p) \geq \hat{W}_1(\kappa, p)\ \forall \kappa, p$ imply that $\breve{p} \leq p$. So workers are willing to accept a job at a less productive firm if it allows them to attain the next level.

These equations and the definition of the reservation productivity $U(\kappa) = W(\kappa, R_p)$ imply that the reservation wage is determined from the following equations

$$R_{2w} \equiv w_2(\kappa, R_{2p}) = b$$

$$\hat{R}_{1w} \equiv \hat{w}_1(\kappa, \hat{R}_{1p}(\kappa)) = b - \psi_1[W_2(\kappa, \hat{R}_{1p}(\kappa)) - \hat{W}_1(\kappa, \hat{R}_{1p}(\kappa))] \quad (10)$$

$$R_{1w} \equiv w_1(\kappa, R_{1p}(\kappa)) = b - \gamma_1[\hat{W}_1(\kappa, R_{1p}(\kappa)) - W_1(\kappa, R_{1p}(\kappa))] \quad (11)$$

where $R_{1p}(\kappa) = w_1(\kappa, R_{1p}(\kappa)) - a_1 - \kappa$, from the wage bargaining outcome. Since $W_2(\kappa, p) \geq \hat{W}_1(\kappa, p) \geq W_1(\kappa, p)\ \forall p$, equations 10 and 11 imply that both $\hat{R}_{1w}, R_{1w} \leq b$, which in turn implies that both $\hat{R}_{1p}, R_{1p} \leq \underline{p}$. The workers lower their reservation wage to below their unemployment income since working gives them the option value of skill accumulation or a chance for a promotion.

The asset equations can be written in terms of the wage offer distribution functions and market tightness by applying integration by parts as was done with the canonical model. The choice of the reservation productivity for searching (S_p) for the unqualified state and the terminal level are the similar to the canonical model (see equation 5). The worker in these states considers the tradeoff of a more productive firm with the search cost. On the other hand search has a much larger payoff for qualified workers. A qualified worker gains not only from finding a more productive firm, but also gains from finding a job at the next level. Of course a job at the next level leads to a permanent increase in earnings. For this reason, qualified workers are much more likely to search than unqualified workers.

3.2.2 Distribution of workers in jobs and unemployment

One of the goals of solving the model is to characterize the distribution of workers with ability κ, working for firms with productivity p and in jobs of level i. Or, in other words, the goal is to characterize the joint distribution, $g(i, \kappa, p)$. It turns out it is easier to write down the laws of motion for the conditional distributions. In this case, conditional on κ and i. The distribution of workers with ability κ and working in level i that are working for a firm with productivity of at least p is

$$G_i^p(p|\kappa) \equiv \int_{\underline{p}}^{p} g(x|\kappa, i)dx.$$

Likewise, the fraction of workers with ability κ working in level i is

$$e_i(\kappa) \equiv g(i|\kappa). \tag{12}$$

The total number of workers working in level i is then

$$e_i \equiv g(i) = \int_{\underline{\kappa}}^{\overline{\kappa}} e_i(x)dH(x)$$

where $H(\kappa)$ is the exogenous cumulative distribution of ability. Finally, Bayes Rule and the law of total probability can be used to find the distribution of ability of workers in a certain level ($h_i(\kappa) \equiv g(k|i)$), the distribution of ability of workers working for a firm of productivity p ($h_i(\kappa|p) \equiv g(\kappa|p, i)$), and the distribution of workers in jobs with productivity p ($G_i^p(p) \equiv \int_{\underline{p}}^{p} g(x|i)dx$).

3.2.3 Firm's problem

The firm's problem is very similar to the canonical model, except it hires workers at all levels. So the firms must decide how many vacancies to open at each level. Firms observe skills and ability once they meet an employee. At the same time the employee is able to observe the firm's productivity. All firms promote qualified workers at rate ψ. It may seem strange that firms have to wait to promote workers while they are paying to open vacancies at the same level. In this case I'm abstracting from the fact that a lot of skills are occupation specific (Kambourov and Manovskii, 2009) and firms have capacity constraints. A firm may open a vacancy for a supervisor in their human resources department but at the same time have a qualified engineer waiting for promotion in another department.

The hiring rate per vacancy ($h(p)$), and the separation rate per worker ($s(p)$) for each level and given ability are

$$\begin{aligned}
s_2(\kappa,p) = s_2(p) &= \rho + \delta_2 + \phi_2(p)\lambda_2[1 - F_2^p(p)] \\
h_2(\kappa,p) &= \eta_2[u_2(\kappa) + e_2(\kappa)G_2^p(\min\{p, S_{2p}\}|\kappa) + \varepsilon(\hat{u}_1(\kappa) + \hat{e}_1(\kappa)\hat{G}_1^p(\breve{p}^{-1}(\kappa,p)|\kappa))] \\
\hat{s}_1(\kappa,p) &= \rho + \delta_1 + \hat{\phi}_1(p)\hat{\lambda}_1[1 - \hat{F}_1^p(p)] + \psi_1 + \hat{\phi}_1(p)\varepsilon\lambda_2[1 - F_2^p(\breve{p}(\kappa,p))] \\
\hat{h}_1(\kappa,p) &= \hat{\eta}_1(\hat{u}_1(\kappa) + \hat{e}_1(\kappa)\hat{G}_1^p(\min\{p, \hat{S}_{2p}\}|\kappa)) \\
s_1(\kappa,p) = s_1(p) &= \rho + \delta_1 + \gamma_1 + \phi_1(p)\lambda_1[1 - F_1^p(p)] \\
h_1(\kappa,p) &= \eta_1(u_1(\kappa) + e_1(\kappa)G_1^p(\min\{p, S_{1p}\}|\kappa))
\end{aligned}$$

where $u_i(\kappa)$ is the fraction of workers with ability κ unemployed at level i, $e_i(\kappa)$ is the fraction of workers with ability κ employed at level i and $G_i^p(p|\kappa)$ is the fraction of level i employed workers with ability κ working at at firms with productivity p or less. The law of motion of the firm's work force for a given κ is then:

$$\begin{aligned}
\dot{n}_2(\kappa) &= \upsilon_2 h_2(\kappa,p) - s_2(\kappa,p)n_2(\kappa) + \psi_1\hat{n}_1(\kappa) \\
\dot{\hat{n}}_1(\kappa) &= \hat{\upsilon}_1\hat{h}_1(\kappa,p) - \hat{s}_1(\kappa,p)\hat{n}_1(\kappa) + \gamma_1 n_1(\kappa) \\
\dot{n}_1(\kappa) &= \upsilon_1 h_1(\kappa,p) - s_1(\kappa,p)n_1(\kappa)
\end{aligned}$$

The value of a worker with ability κ to a firm is

$$\begin{aligned}
rJ_2(\kappa,p) &= (1-\beta)[a_2 + p + \kappa - b] - (\rho + \delta_2 + \phi_2(p)\lambda_2[1 - F_2^p(p)])\, J_2(\kappa,p) \\
r\hat{J}_1(\kappa,p) &= (1-\beta)[a_1 + p + \kappa - b] + \psi_1(J_2(\kappa,p) - \hat{J}_1(\kappa,p)) \\
&\quad - \Big(\rho + \delta_1 + \hat{\phi}_1(p)\hat{\lambda}_1[1 - \hat{F}_1^p(p)] + \hat{\phi}_1(p)\varepsilon\lambda_2[1 - F_2^p(\breve{p}(p|\kappa))]\Big)\, \hat{J}_1(\kappa,p) \\
rJ_1(\kappa,p) &= (1-\beta)[a_1 + p + \kappa - b] + \gamma_1(\hat{J}_1(\kappa,p) - J_1(\kappa,p)) \\
&\quad - (\rho + \delta_1 + \phi_1(p)\lambda_1[1 - F_1^p(p)])\, J_1(\kappa,p)
\end{aligned}$$

These can be rewritten using the definitions of the separation rate.

$$
\begin{aligned}
J_2(\kappa, p) &= \frac{(1-\beta)[a_2 + p + \kappa - b]}{r + s_2(p)} \\
\hat{J}_1(\kappa, p) &= \frac{(1-\beta)[a_1 + p + \kappa - b] + \psi_1 J_2(\kappa, p)}{r + \hat{s}_1(\kappa, p)} \\
J_1(\kappa, p) &= \frac{(1-\beta)[a_1 + p + \kappa - b] + \gamma_1 \hat{J}_1(\kappa, p)}{r + s_1(p)},
\end{aligned}
$$

Notice that by starting with the terminal level and working backwards, the rest of the value functions can be expressed in terms of just the parameters, the offer distributions and market tightness.

The firm will maximize the profits for its entire workforce level by level. Let $c(\upsilon)$ be the cost of posting υ vacancies, which is assumed to follow the regularity conditions: $c(0) = c'(0) = 0$ and $\lim_{\upsilon \to \infty} c'(\upsilon) = \infty$, in addition to being convex ($c''(\upsilon) > 0$). The Bellman equation describing the firm's value is

$$
\begin{aligned}
rV(\mathbf{n}, p) = \max_{\{\upsilon_i\} \geq 0} E_\kappa[\quad & n_2(\kappa, p) J_2(\kappa, p) + \hat{n}_1(\kappa, p) \hat{J}_1(\kappa, p) + n_1(\kappa, p) J_1(\kappa, p) \\
& - c(\upsilon_2) - c(\hat{\upsilon}_1) - c(\upsilon_1) \\
& + J_2(\kappa, p)(\upsilon_2 h_2(\kappa, p) - s_2(p) n_2(\kappa, p) + \psi_1 \hat{n}_1(\kappa, p)) \\
& + \hat{J}_1(\kappa, p)(\hat{\upsilon}_1 \hat{h}_1(\kappa, p) - \hat{s}_1(\kappa, p) \hat{n}_1(\kappa, p) + \gamma_1 n_1(\kappa, p)) \\
& + J_1(\kappa, p)(\upsilon_1 h_1(\kappa, p) - s_1(p) n_1(\kappa, p))]
\end{aligned} \tag{13}
$$

The solution to the firm problem is to equate the cost of opening vacancies to the benefit of hiring additional workers. The first order necessary conditions (FONC) of equation 13 are

$$
\begin{aligned}
c'(\upsilon_2(p)) &= E_\kappa[h_2(\kappa, p) J_2(\kappa, p)] \\
c'(\hat{\upsilon}_1(p)) &= E_\kappa[\hat{h}_1(\kappa, p) \hat{J}_1(\kappa, p)] \\
c'(\upsilon_1(p)) &= E_\kappa[h_1(\kappa, p) J_1(\kappa, p)]
\end{aligned}
$$

which determines the number of vacancies a firm with productivity p will open at each level given market tightness and the offer distribution. The number of vacancies opened implies that the firm will maintain the following labor force sizes

$$\begin{aligned} n_2(\kappa,p) &= \frac{h_2(\kappa,p)\upsilon_2(p)+\psi_1\hat{n}_1(\kappa,p)}{s_2(p)} \\ \hat{n}_1(\kappa,p) &= \frac{\hat{h}_1(\kappa,p)\hat{\upsilon}_1(p)+\gamma_1 n_1(\kappa,p)}{\hat{s}_1(\kappa,p)} \\ n_1(\kappa,p) &= \frac{h_1(\kappa,p)\upsilon_1(p)}{s_1(p)}. \end{aligned}$$

3.2.4 Steady State Equilibria

The flow into and out of each employment and unemployment state expressed as a fraction of the total population of workers is

$$\begin{aligned} u_1(\kappa): &\quad \rho+\delta_1 e_1(\kappa)=(\rho+\lambda_1)u_1(\kappa) \\ e_1(\kappa): &\quad \lambda_1 u_1(\kappa)=(\rho+\delta_1+\gamma_1)e_1(\kappa) \\ \hat{u}_1(\kappa): &\quad \delta_1\hat{e}_1(\kappa)=(\rho+\hat{\lambda}_1+\varepsilon\lambda_2)\hat{u}_1(\kappa) \\ \hat{e}_1(\kappa): &\quad \hat{\lambda}_1\hat{u}_1(\kappa)+\gamma_1 e_1(\kappa)=(\rho+\delta_1+\psi_1+\varepsilon\lambda_2\Phi_1(x))\hat{e}_1(\kappa) \\ u_2(\kappa): &\quad \delta_2 e_2(\kappa)=(\rho+\lambda_2)u_2(\kappa) \\ e_2(\kappa): &\quad \lambda_2 u_2(\kappa)+(\psi_1+\varepsilon\lambda_2\Phi_1)\hat{e}_1(\kappa)+\varepsilon\lambda_2\hat{u}_1(\kappa)=(\rho+\delta_2)e_2(\kappa) \end{aligned}$$

where $\Phi_1(x)\equiv\int_{\underline{p}}^{\overline{p}}[1-F_2^p(x)]d\hat{G}_1^p(x|\kappa)$.

Equating the flow of workers into and out of firms with productivity less that p allows me to express $G_i^p(p)$ as a function of $F_i^p(p)$, λ_i and parameters as in the canonical model. **RECOMMENT THIS OUT!!**

$$\begin{aligned} \{s_1(p)G_1^p(p|\kappa)+1(p>S_p)\lambda_1[1-F_1^p(p)]G_1^p(S_p|\kappa)\}\,e_1(\kappa) &= \lambda_1 F_1^p(p)u_1(\kappa) \\ \hat{s}_1(p)\hat{G}_1^p(p|\kappa)\hat{e}_1(\kappa) &= \hat{\lambda}_1\hat{F}_1^p(p)\hat{u}_1(\kappa)+\gamma_1 G_1^p(p|\kappa)e_1(\kappa) \\ \{s_2(p)G_2^p(p|\kappa)+(1-\phi(p))\lambda_2[1-F_2^p(p)]G_2^p(S_p|\kappa)\}\,e_2(\kappa) &= \lambda_2 F_2^p(p)u_2(\kappa)+\varepsilon\lambda_2 F_2^p(p)\hat{u}_1(\kappa)+\psi_1\hat{G}_1^p(p|\kappa)\hat{e}_1(\kappa) \\ &\quad +\varepsilon\lambda_2\int_{\underline{p}}^{\breve{p}^{-1}(p|\kappa)}[F_2^p(p)-F_2^p(\breve{p}(x|\kappa))]d\hat{G}_1^p(x|\kappa)\hat{e}_1(\kappa) \end{aligned}$$

This can be used to derive $G_i^p(p|\kappa)$ in terms of $F_i^p(p)$:

$$
\begin{aligned}
G_1^p(p|\kappa) &= \frac{(\rho+\delta_1+\gamma_1)F_1^p(p) - 1(p > S_p)\lambda_1[1-F_1^p(p)]G_1^p(S_p|\kappa)}{s_1(p)} \\
\hat{G}_1^p(p|\kappa) &= \frac{\hat{\lambda}_1 \hat{F}_1^p(p)\frac{\delta_1}{\rho+\hat{\lambda}_1+\varepsilon\lambda_2} + \gamma_1 G_1^p(p|\kappa)\frac{e_1(\kappa)}{\hat{e}_1(\kappa)}}{\hat{s}_1(\kappa, p)} \\
G_2^p(p|\kappa) &= \frac{\frac{\lambda_2\delta_2}{\rho+\lambda_2}F_2^p(p) + \left[\frac{\varepsilon\lambda_2\delta_1}{\rho+\hat{\lambda}_1+\varepsilon\lambda_2}F_2^p(p) + \psi_1\hat{G}_1^p(p|\kappa) + \varepsilon\lambda_2\int_{\underline{p}}^{\breve{p}^{-1}(p|\kappa)}\hat{G}_1^p(x|\kappa)dF_2^p(x)\right]\frac{\hat{e}_1(\kappa)}{e_2(\kappa)}}{s_2(p)} \\
&\quad - \frac{1(p > S_p)\lambda_2[1-F_2^p(p)]G_2^p(S_p|\kappa)}{s_2(p)}
\end{aligned}
$$

Let I(p) be the cumulative distribution function of firms with productivity p, which is exogenous to the model. Then the distribution of job offers or vacancies is

$$F_i^p(p) = \frac{1}{N_{\upsilon_i}}\int_{\underline{p}}^{p}\upsilon_i(x)dI(x) \tag{14}$$

where $N_{\upsilon_i} = \int_{\underline{p}}^{\overline{p}}\upsilon_i(x)dI(x)$ is the total number of vacancies. This defines a system of three differential equations from which the offer distributions can be derived for a given set of market tightness values.

Finally, the last condition to close the model (i.e. determine market tightness θ) is to make sure the total number of workers in the economy at each level is equal to the sum of the workers at each firm:

$$e_i N = \int_{\underline{p}}^{\overline{p}} n_i(x)dI(x). \tag{15}$$

where N is the number of workers in the economy. This defines a system of three non-linear equations and three market tightness values.

3.2.5 Equilibrium Wage Dispersion

With $F_i^p(p)$ and $G_i^p(p)$ in hand, the wage distributions for each level can be calculated. The wage offer distribution conditional on ability was already found in equation 3. The analogous wage earned distribution is

$$G_i^w(w,\kappa) = G_i^p(p_i(w,\kappa)) = G_i^p\left(\frac{1}{\beta}(w - \beta(a_i+\kappa) - (1-\beta)b)\right). \tag{16}$$

Let $H(\kappa)$ be the cumulative distribution for ability. Then the cumulative distribution for wages at each level is

$$F_i^w(w) = \int_{\underline{\kappa}}^{\overline{\kappa}} F_i^w(w,\kappa)dH(\kappa)$$
$$G_i^w(w) = \int_{\underline{\kappa}}^{\overline{\kappa}} G_i^w(w,\kappa)dH(\kappa)$$

Finally, the distribution of wages for the full economy is

$$F^w(w) = \frac{N}{\sum_i e_i}\sum_i e_i F_i^w(w)$$
$$G^w(w) = \frac{N}{\sum_i e_i}\sum_i e_i G_i^w(w)$$

4 Estimation

This section presents preliminary estimates of the model. In this first step, I have estimated the parameters of the canonical job-ladder model. One advantage of the model is that the solution involves solving initial value problems. Although, initial value problems still need to be calculated numerically, the solutions are found very quickly and always converge. The other common alternative is value function iteration, where solutions are often slow to converge.

4.1 Data

The data used in the estimation are based on the Danish Integrated Database for Labor Market Research (IDA)[15] and weekly spell data. Both the IDA and spell data are register-based datasets that contain the full population of Denmark. The data are confidential, but my access is not exclusive. IDA is a matched employer-employee longitudinal database containing socio-economic information on the entire Danish population, the population's attachment to the labor market, and at which firms the worker is employed. The persons and firms are monitored from 1980 and onward, but I will use 19 years of the IDA and spell data (1985-2003) in this study. The weekly spell data set is a longitudinal data set that contains information on labor market transitions for each person in the Danish population. All individuals are at first assigned to one of sixteen mutually exclusive labor market states in each week over the years 1985-2004 using a number of different register data sets. These states are then narrowed down to three states: employed, unemployed, and not participating. Descriptive statistics of the number of workers, firms and

[15] For detailed documentation on the IDA dataset, see:
http://www.dst.dk/HomeUK/Guide/documentation/Varedeklarationer/emnegruppe/emne.aspx?sysrid=1013

cross sections are shown in table 4.

The full sample (table 4) is used to calculate moments on wages and the rate that an employed worker becomes unemployed ($\Pr(E \rightarrow U)$). The wage data has been cleaned by trimming highest and lowest 1% of wages and by requiring that the hours data is of sufficient quality[16] as defined by Statistics Denmark. The mean wage is measured using the cross section from 2000. The data from 1985-2003 is used for all other wage measures, where year fixed effects have been removed by education level. The constant hazard rate at which workers lose their jobs is measured by beginning with the sample of workers who are employed in each November cross section. I then measure the fraction which become unemployed before the next November cross section. The hazard rate for unemployment is easily calculated from this fraction assuming a homogeneous Poisson process[17].

The moments on unemployment are measured in the unemployment dataset (see section 2.1.1). The calculation of the correlation and difference between pre and post unemployment spell wages has already been described. The measurement of the unemployed job-finding rate is similar to the separation rate. Assuming a homogeneous Poisson process, the hazard rate for finding a job is calculated from the fraction of the unemployed who find a job within a year of becoming unemployed. Since Poisson processes are memoryless, the calculation is not biased by the fact that workers are given two months notice before they are laid off.

4.2 Estimation Strategy

Given the complexity of the model, I use indirect inference (Gourieroux, Monfort, and Renault, 1993) to do the estimation. Indirect inference is essentially a moment matching estimation technique. Simple models (usually called auxiliary models) are chosen that capture the important features of the full model. The data is then estimated with these simple models even though they might be mis-specified. This estimation yields estimated auxiliary parameters ($\hat{\beta}$). The full model is then used to generate simulated data, and these same auxiliary models are estimated with the simulated data ($\hat{\beta}^s$). The indirect estimator is then defined by

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}}(\hat{\beta}^s - \hat{\beta})'\Sigma(\hat{\beta}^s - \hat{\beta})$$

where Σ is a symmetric positive definite matrix. The model will be estimated in the just-identified case, where the number of moments and parameters are the same. In this case, Σ can

[16] If a worker is employed at a firm for a short period of time, the hours measurement might not be reliable.

[17] $\delta = -ln(1 - f_U)$, where f_U are the fraction of employed workers that become unemployed in the following year and δ is the annual separation rate.

be set to the identity matrix without any loss of efficiency. Since it is unlikely that workers of different schooling levels search for the same jobs, I estimate the model separately for primary, secondary, and four-year college graduates.

Finally, the productivity distributions of the worker and firms, the vacancy cost function and the matching function are defined parametrically. The Gamma distribution is used to model the productivity distributions of workers and firms. Power functions are used to model the vacancy cost ($c(\nu) = c\nu^2$) and the matching function ($q(\theta) = \theta^{0.5}$). Finally, the discount rate (r) does not have an effect on the wage setting and only a very weak effect on the labor market dynamics. I follow Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005) by setting it to $r = 0.05$. The procedure for numerically solving the model and simulating the moments is described in the appendix.

4.2.1 Auxiliary models and identification of parameters

Although it is difficult to prove identification with a structural model, having a full-rank Jacobian of the moments and the parameters is a necessary condition. Table 5 shows the normalized Jacobian evaluated at the estimated parameters for secondary school graduates. This matrix is full rank and the matrix elements are helpful in confirming the intuition behind the identification of the model. The Jacobians calculated at the estimates for college graduates and primary school graduates are similar and are also full rank.

Each moment is sensitive to multiple parameters, but I will highlight the most intuitive and important links. Each element in Table 5 is normalized ($\frac{\partial y}{\partial x}\frac{x}{y}$) so that the different elements can be interpreted as elasticities and directly compared. The mean wage was chosen to give information about the job productivity (a). The standard deviation of wages and the 90th percentile of wages are informative about the worker and firm productivity parameters. The 90th percentile was chosen as a moment in order to reproduce the positive tail in wages. What allows the firm and worker heterogeneity to be separately identified are the wage dynamics across unemployment spells. Whereas both the firm and worker productivity parameters have a positive relationship with the wage moments, they have different relationships with the correlation between pre- and post-unemployment wages and the 90th percentile of the difference in pre- and post-unemployment spell wages. This set of four moments (σ_w, 90%-w, $corr(w_{pre}, w_{post})$, 90%-Δ_w) provides identification of the worker and firm productivity distribution parameters.

The search cost determines the fraction of workers who search on the job for higher-paying firms. As the search cost increases, fewer workers perform on-the-job search and move up the job-ladder. If fewer workers are moving up the job-ladder, more of them will find higher paying jobs after unemployment as explained in section 2.2. So there is a positive relationship between the search cost and the fraction of unemployed workers earning higher wages. Finally, the vacancy cost determines how many vacancies firms open and hence, how easy it is to find a job (market tightness). This is the reason for the negative relationship between the vacancy cost and the unemployed job-finding rate.

4.3 Empirical estimates of the job-ladder model

I present preliminary estimates of the job-ladder model for three different education groups. Table 6 shows the moments from the data and those calculated from the model estimates. The job-separation rate (Pr(U $\rightarrow$ E)) is an exogenous parameter of the model (δ) and so I get a perfect fit. On the other hand, the job-finding rate is endogenous in the model. The two rates predict the average employment and unemployment spell durations. The average predicted unemployment spell duration is 1.71, 1.47, and 1.31 years for workers with primary, secondary, and college educations, respectively. The average predicted employment spell, with no intervening unemployment, is 8.77, 15.2, and 32.3 years for primary, secondary, and college educated workers. So in general, not only do more educated workers earn more, but the unemployment risk they face is smaller as are the unemployment spell durations. The model matches the moments from wages fairly well, although it seems to not do as well reproducing the tails fo both the wage distribution and the Δ_w distribution. I plan to investigate alternative functional forms for the productivity distribution of firms and workers to see if this can be improved.

The model parameter estimates are shown in Table 7. The difference in job-finding rates between education groups is directly interpreted by the model to reflect differences in the cost of vacancies for each group, where the cost is almost three times higher for primary-educated workers as for college-educated workers. This may be due to the fact that I made a poor choice in the parametric function for the vacancy costs. In future estimations, I will include the distribution of firm sizes, which will be informative of the curvature of the vacancy cost function. I express the estimate of the search cost in terms of the fraction of workers searching while employed. The model predicts that only 2.8% to 4.8% of workers are searching because of the search cost, depending on education level. I also report the mean and standard deviation of the firm and worker productivities. The standard deviation of the firm productivity does not differ much between education groups even though there is no restriction in the model or estimation procedure that requires this. On the other hand, worker heterogeneity significantly increases with education level. This explains why wages are more persistent across unemployment spells for college educated workers.

The main result of the estimation is the rate of job to job movements predicted by the model. Table 8 shows the probability that a worker changes employers in a given year. In the data, this is calculated by measuring the fraction of employed workers that change jobs between consecutive November cross-sections with no intervening unemployment spell[18]. In the data, about 15% of the workers change jobs each year. On the other hand, the model predicts that less than 1.5% of workers change jobs in a given year. The other result is the decomposition of wages due to worker and firm heterogeneity in the model. The estimates show that worker heterogeneity explains 43.3%, 47.8% and 66.6% of the variance in wages for primary, secondary and college educated workers, with the remaining variance explained by firm heterogeneity.

In summary, the preliminary estimates of the model do a good job of matching the moments being used for estimation. The job-ladder model on the other hand explains only a fraction of the observed worker mobility. This is because very few workers are searching on the job for higher paying firms. The next step is to estimate the model with a career motive for worker mobility and see if it can explain the observed worker flows between firms.

5 Conclusions

The main contribution of this paper is a model of search and human capital accumulation that can provide a better understanding of worker mobility. The paper presents new measurements of the labor market dynamics of unemployment using Danish data. The measurements reveal that almost half of workers earn higher wages after an unemployment spell and that the persistence of wages across unemployment spells increase with experience.

These two facts motivate extending the workhorse job-ladder model. The new model adds a career development motive for job-to-job movements. Workers must be promoted or find a job at a new firm if they want to continue accumulating skills. The addition of search costs explain why only a small fraction of workers search for jobs at a higher paying firms. The benefit from working at a higher paying firm is lost if a worker becomes unemployed. On the other hand, the benefit from career development is a permanent increase to a worker's expected future income. So while most workers will choose to not search for higher paying firms, they will choose to search when presented with an opportunity to further develop their career. The model also

[18] The probabilities have been corrected for workers who become unemployed, but find a job before they are registered as unemployed in the data. Workers continue to be paid for about two months after they are laid off by their previous employer. They cannot receive unemployment benefits until this period ends.

contributes to the fraction of unemployed workers earning higher wages. These are unemployed workers who are qualified to work at the next level in the career hierarchy.

As a first step towards estimating the full model, I structurally estimate the job-ladder search model with search costs using the Danish matched employee-employer dataset. The moments from the labor market dynamics of unemployment play an important role in the inference. The estimates show that very few employed workers search for new jobs. This implies that the job-ladder model explains less than 10% of the observed worker mobility. I also find that in the context of the model, worker heterogeneity explains 67% of the variance in wages for college graduates and about 45% for primary and secondary graduates.

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A Labor Market Dynamics of Unemployment and the Search Literature

The purpose of sections A, A.1, and A.2 is to show how studying unemployment dynamics contributes to two specific questions in the search literature. Namely, does the AKM model (Abowd, Kramarz, and Margolis, 1999) suffer a strong bias from endogenous labor mobility and what does unemployment dynamics say about the worker-specific contribution to wage dispersion in search models with piece-rate wages.

I start by considering a linear model of log-wages (w_{it}), worker-specific capital (θ_i), firm-specific capital ($\psi_{J(i,t)}$) and a residual (ε_{it}) motivated by the AKM model (Abowd, Kramarz, and Margolis, 1999).

$$w_{it} = \theta_{it} + \psi_{J(i,t)} + \varepsilon_{it} \tag{17}$$

where $t = 0$ if it is the pre-unemployment spell wage and $t = 1$ if it is the post-unemployment spell wage. The identifying assumptions are that worker specific capital is persistent across short unemployment spells ($\theta_{i0} = \theta_{i1} = \theta_i$), and that the residual has mean zero and is independent across unemployment spells ($E[\varepsilon_{it}|i, t, J(i, t)] = 0 \ \forall i, t$ and $Cov(\varepsilon_{i0}, \varepsilon_{i1}) = 0 \ \forall i$). Finally, I make the stronger assumption that there is no sorting between worker and firm components coming out of unemployment ($Cov(\theta_i, \psi_{J(i,1)}) = 0$). In this case, random search models predict that the pre- and post-unemployment firm effect will be independent ($Cov(\psi_{J(i,0)}, \psi_{J(i,1)}) = 0$), as an unemployed worker has no reason to accept or reject a job offer based on his previous wage, if it is not based on his characteristics. This is supported in theories of firm-specific human capital, most search theories with no sorting, and a class of search theories that have sorting in the overall labor market as a result of workers choosing different search intensities based on their characteristics (Lentz, 2010).

The covariance between pre- and post-unemployment spell wages under these assumptions is

$$Cov(w_{i0}, w_{i1}) = Var(\theta_i) + Cov(\psi_{J(i,0)}, \theta_i) \tag{18}$$

where the second term is zero if there is no sorting between workers and firms or if firm-specific human capital accumulation is independent of a worker's human capital. The second term describes the covariance between human capital and firm-specific capital in the general labor market. The correlation is then

$$Corr(w_{i0}, w_{i1}) = \frac{1}{\sigma(w_{i0})\sigma(w_{i1})}\big(Var(\theta_i) + Cov(\psi_{J(i,0)}, \theta_i)\big) \tag{19}$$

Finally, I make the assumption that the variance in wages of the job offer distribution is roughly equal to the variance of the wages in the labor market ($\sigma(w_{i,1}) \approx \sigma_w \equiv \sigma(w_{i,0})$), which is empirically supported in the Danish data by the findings in Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005). The correlation is then

$$Corr(w_{i0}, w_{i1}) = \frac{Var(\theta_i)}{Var(w_i)} + \frac{Cov(\psi_{J(i,0)}, \theta_i)}{Var(w_i)} \tag{20}$$

A.1 Comparisons to estimates from fixed effects regressions (AKM)

I will now compare the results of the wage correlation with a fixed effects model. Equation 17 is a version of the AKM model (Abowd, Kramarz, and Margolis, 1999). Although the AKM model does not make any assumption about the sorting of workers and firms out of unemployment, the results from the wage correlations and the AKM estimation can be directly compared if the sorting out of unemployment is believed to be small. This is informative, because the AKM model and the wage correlations are sensitive to the firm-specific capital in different ways. Any persistent within-firm heterogeneity will be assigned to the worker effect in the AKM model, even if it is due to labor market frictions. The correlation between pre- and post-unemployment spell wages does not suffer from this bias since it all heterogeneity due to labor market frictions is reset when entering unemployment. The comparison of the fraction of the wage variance due to worker effects between the two methods is informative of the amount of within-firm heterogeneity due to labor market frictions. If labor market frictions make a large contribution to the within-firm heterogeneity, then I would expect the AKM model estimate to be far larger than the wage correlation estimate. If not, then I would expect them to be similar.

Sørensen and Vejlin (2011) estimate the AKM model using Danish data from 1980-2006. Since they have multiple observations for each worker they estimate a time-invariant worker effect (θ_{AKM}) and a time-varying worker effect, i.e. experience ($X\beta$). In the wage correlation (see section A) there is only one observation per individual, so the worker effect is the sum of both of these effects ($\theta = \theta_{AKM} + X\beta$). The decomposition of the wage dispersion is reported for both $\frac{Cov(w,\theta_{AKM})}{Var(w)}$ and $\frac{Cov(w,X\beta)}{Var(w)}$, the sum of which is

$$\begin{aligned}
\frac{Cov(w,\theta_{AKM})}{Var(w)} + \frac{Cov(w,X\beta)}{Var(w)} &= \frac{Cov(w,\theta_{AKM}+X\beta)}{Var(w)} \\
&= \frac{Var(\theta_{AKM}+X\beta)}{Var(w)} + \frac{Cov(\psi,\theta_{AKM}+X\beta)}{Var(w)} \\
&= \frac{Var(\theta)}{Var(w)} + \frac{Cov(\psi,\theta)}{Var(w)} \\
&= Corr(w_0,w_1)
\end{aligned}$$

which is exactly what is being measured by the wage correlation (see equation 20).

Table 9 compares the results from the wage variance decomposition in Sørensen and Vejlin (2011) to the correlation of pre- and post-unemployment spell wages for different educational groups. One would expect that if the AKM estimates are biased, then the fraction of the wage variance explained by the worker effect would be much larger than the measurement from the wage correlations. Since the wage correlation is comparing wages from before and after unemployment spells, there is no bias from within-firm wage heterogeneity. In order to make the comparison, I match the education groups in Sørensen and Vejlin (2011) where "low", "medium" and "high" correspond to less than 12 years of schooling, between 12 and 14 years of schooling and more than 14 years of schooling. In all cases the AKM estimate is larger than the wage correlations as expected, but not by a large amount. The results for the high and low educated workers are nearly the same, where the AKM model estimate for the worker effect explains only 1% and 5% more than the estimate from the correlations. The difference between the two estimates for medium educated workers is 9%. These results indicate that if there is no sorting out of unemployment, the intra-firm heterogeneity due to labor-market frictions does not play a large role in biasing the worker effect in the AKM estimation.

A.2 Comparisons to estimates of search models with piece-rate wage contracts

There is now a large literature on the structural estimation of search models with with worker and firm heterogeneity[19]. One way to add worker heterogeneity to a equilibrium search model is to assume firms post piece rate contracts or a wage per unit of effective labor[20]. This simplifies the problem since worker heterogeneity does not affect the dynamics of the search model. Firms post wages per unit of effective labor ($\hat{w}_j$), and these wages are offered to workers regardless of their productivity (ℓ_i). The observed wage for worker i working at firm j is then

$$w_{ij} = \ell_i \hat{w}_j, \tag{21}$$

where ℓ is the productivity of a worker. You will notice that the log-wage is then

$$\ln w_{ij} = \ln \ell_i + \ln \hat{w}_j \tag{22}$$

In this context, log-wages are linear in worker productivity and the wage offered by the firm. So I can interpret the human capital as the worker fixed effect from section A ($\theta_i = \ln \ell_i$), the wage per unit of effective labor as the firm fixed effect ($\psi = \ln \hat{w}_j$) and the residual term is not needed ($\varepsilon = 0$). Since there is no sorting in these models ($Cov(\ln \ell_i, \ln \hat{w}_j) = 0$), the correlation of the pre- and post-unemployment spell wages is exactly the fraction of wage variance due to human capital.

$$\begin{aligned} Corr(w_{i0}, w_{i1}) &= \frac{Var(\theta_i)}{Var(w)} \\ &= \frac{Var(\ln \ell_i)}{Var(\ln w_{ij})} \end{aligned}$$

[19]see Lentz and Mortensen (2010) for a recent survey

[20]Some examples are Postel-Vinay and Robin (2002); Barlevy (2003); Bagger, Fontaine, Postel-Vinay, and Robin (2011)

Table 1: Description of unemployment dataset

Variable	N	Mean	Std. Dev.	Min	Max
All U Spells					
Spell Length (weeks)	151143	74.07091	78.19746	3	408
pre-U log-wage (w_{pre})	151143	-.0345924	.2608598	-1.366396	1.374735
$w_{post} - w_{pre}$	151143	-.0340036	.2857224	-2.225117	1.73589
Experience (years)	151143	8.367406	6.058807	0	32
Short U Spells					
Spell Length (weeks)	54273	11.43349	6.097654	3	25
pre-U log-wage (w_{pre})	54273	-.0205783	.2625781	-1.306286	1.345193
$w_{post} - w_{pre}$	54273	-.0190641	.2669616	-2.01069	1.73589
Experience (years)	54273	8.07047	5.890936	0	30.122

Notes: From Danish matched employer-employee data (1985-2003). Unemployment spells that follow an employment spell of at least one year and that end in an employment spell for a different employer that lasts at least until the following November cross section. Short unemployment spells are defined as those that last less than 26 weeks. Log-wages have had year fixed effects removed by education level.

Table 2: Difference between pre- and post-unemployment log-wages

Educ	Mean	Std Dev	Frac > 0
Denmark - Primary	-0.010	0.269	0.489
Denmark - Secondary	-0.023	0.266	0.475
Denmark - Bachelors	-0.029	0.264	0.461
US White Male - White Collar	-0.065	0.427	0.440[a]
US White Male - Blue Collar	-0.088	0.456	0.432[a]

Notes: US estimates for workers who lost their job due to plant closings from Hu and Taber (2011). a) The fraction of workers earning higher wages in US data is calculated assuming a normal distribution.

Table 3: Correlation between pre- and post-unemployment spell log-wages by education

Education	N	corr(w_{pre},w_{post})
Primary	18307	0.394
		(0.007)
Secondary	28762	0.436
		(0.005)
Bachelors	7204	0.621
		(0.009)

Notes: Standard Errors are in parenthesis.

Table 4: Description of Danish matched employee-employer dataset (1985-2003)

	Primary Grad.	Secondary Grad.	College Grad.	Total
N Workers	517,012	800,080	393,783	1,710,875
Nov. cross-sections	6,409,670	10,332,836	4,525,936	21,268,442
Employed in Nov.	3,964,954	8,015,601	3,693,125	15,673,125
N Unemp within a year	423,435	507,613	113,014	1,044,062
N Firms				396,972

Notes: Number of private sector workers and firms in the Danish data. Cross sections of the full danish populations are constructed the last week of every November. Self-employed workers are not included in the employed.

Table 5: Identification: Jacobian of moments and parameters (High School Graduates)

Param.	$E(w)$	σ_w	90% w	$corr(w_{pre}, w_{post})$	90% Δ_w	$\Delta_w > 0$	Pr(U $\rightarrow$ E)
Job product. (χ)	0.11	-0.60	2.57	0.15	-1.12	-0.02	0.24
Search Cost (c)	-0.01	-0.02	-0.28	0.09	0.01	0.08	-0.01
Vacancy Cost (c)	-0.00	-0.00	-0.07	0.04	0.00	0.02	-0.24
Worker Prod. k_κ	0.03	0.24	1.27	0.84	-0.18	0.00	0.00
Worker Prod. θ_κ	0.03	0.07	1.00	0.48	-0.21	0.00	0.00
Firm Prod. k_p	0.03	0.16	1.09	-0.55	0.79	-0.01	0.05
Firm Prod. θ_p	0.03	0.04	0.86	-0.34	0.42	-0.01	0.09

Notes: Normalized Jacobian ($\frac{\partial y}{\partial x}\frac{x}{y}$) for the job-ladder model, calculated at the estimate for high school graduates. This Jacobian is full-rank. Jacobians for other points are similar and are also full rank. 90% refers to the 90th percentile, Δ_w is the difference between post and pre unemployment spell log-wages, $corr(w_{pre}, w_{post})$ is the correlation between pre and post unemployment spell log-wages, Pr(U $\rightarrow$ E) is the rate at which the unemployed find jobs.

Table 6: Moments used for estimation by education level

Moment	Primary Grad IDA data	Primary Grad Model	Secondary Grad IDA data	Secondary Grad Model	College Grad IDA data	College Grad Model
Pr(E $\rightarrow$ U)	0.114	0.114	0.066	0.066	0.031	0.031
Pr(U $\rightarrow$ E)	0.593	0.586	0.691	0.678	0.761	0.765
$E(w)$	5.075	5.057	5.161	5.083	5.355	5.363
σ_w	0.271	0.261	0.279	0.273	0.302	0.286
90th percentile w	0.1	0.116	0.1	0.064	0.1	0.077
$\Delta_w > 0$	0.489	0.478	0.475	0.456	0.461	0.456
90th percentile Δ_w	0.1	0.121	0.1	0.127	0.1	0.109
$corr(w_{pre}, w_{post})$	0.393	0.412	0.437	0.446	0.621	0.634

Notes: IDA data is estimated from the Danish matched employee-employer dataset. Moments from the estimatd model are based on the simulation of 10 million workers. Pr(E $\rightarrow$ U) is exogenous to the model (δ) and so I obtain an exact fit. 90% refers to the 90th percentile, Δ_w is the difference between post and pre unemployment spell log-wages, $corr(w_{pre}, w_{post})$ is the correlation between pre and post unemployment spell log-wages, Pr(U $\rightarrow$ E) is the rate at which the unemployed find jobs. Both Pr(E $\rightarrow$ U) and Pr(U $\rightarrow$ E) are measured in annual rates. Wages are denominated in kroner.

Table 7: Estimates of model parameters by education level

Parameter	Estimates Primary Grad.	Estimates Secondary Grad	Estimates College Grad
Job Productivity	225.0	227.0	227.1
Separation Rate	0.114	0.066	0.031
Vacancy Cost	450.8	260.7	166.8
Search Cost (fraction searching)*	0.037	0.048	0.0275
Worker Prod. $\kappa \sim \Gamma(k_\kappa, \theta_\kappa)$			
$E(\kappa)^*$	42.71	59.16	148.98
σ_κ^*	62.08	67.97	107.17
Firm Prod. $p \sim \Gamma(k_p, \theta_p)$			
$E(p)^*$	57.37	70.06	89.42
σ_p^*	71.02	70.98	75.92

Notes: Parameter estimates of job-ladder model using the Danish matched employee-employer dataset. The Discount rate ($r = 0.05$) was not estimated. *The search cost, firm and worker productivity estimates are expressed in terms of interpretable parameters (Fraction of employed workers searching for jobs; the mean and standard deviation of the distributions).

Table 8: Worker mobility

Education	P(J→J) (Data)	P(J→J) (Model)	Model/Data
Primary Graduate	0.133	0.0104	0.0783
Secondary Graduate	0.153	0.0152	0.0993
College Graduate	0.152	0.0098	0.0645

Notes: P(J→J) is the probability that a worker changes employers in the next year. The data is calculated from the Danish matched employer-employee dataset where the contribution from unemployed workers that find new jobs before they are registered as unemployed have been removed (contributes 0.0086, 0.006, and .0031 to the probability for primary, secondary and college graduates, respectively). The model represents the predictions of the job-ladder model estimated with the Danish employer-employee dataset.

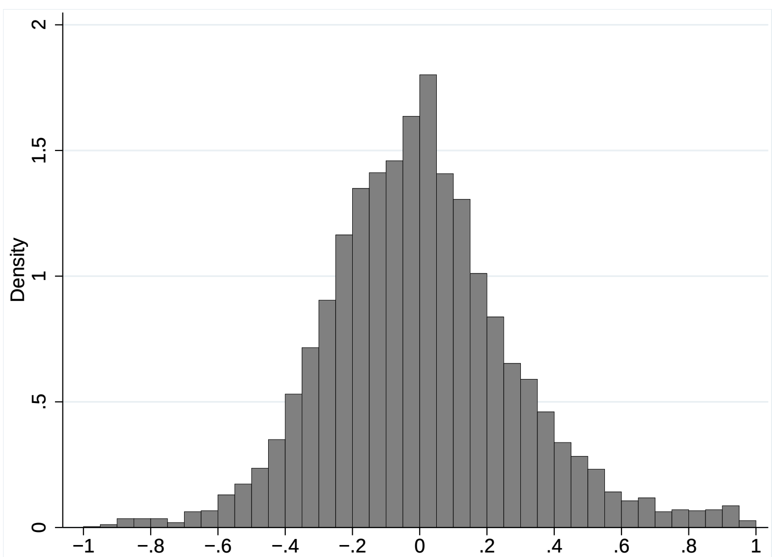
Table 9: Comparing different estimates of the fraction of the variance in log-wages ($\frac{cov(w,\theta)}{var(w)}$) explained by worker effects in Danish data

Education[a]	AKM[b]	$corr(w_{pre}, w_{post})^c$
Low	0.451	0.394
		(0.007)
Medium	0.554	0.436
		(0.005)
High	0.699	0.621
		(0.009)

Notes: (a) High, medium and low correspond to more than 14, between 12 and 14, and less than 12 years of schooling, respectively (b) Based on fixed effects estimates in Sørensen and Vejlin (2011) using Danish data from 1980-2006. (c) Calculation based on Danish data from 1998 and 1999.

Figure 1: Correlation between pre- and post-unemployment spell log-wages by experience

Figure 2: The percent difference between post- and pre-unemployment wages ($\frac{w_{post}-w_{pre}}{w_{pre}}$)



Note: Figure restricted to show the range (-1.0,1.0). There is a long positive tail.

Figure 3: Implied job-offer rate from fraction of workers earning higher wages after unemployment

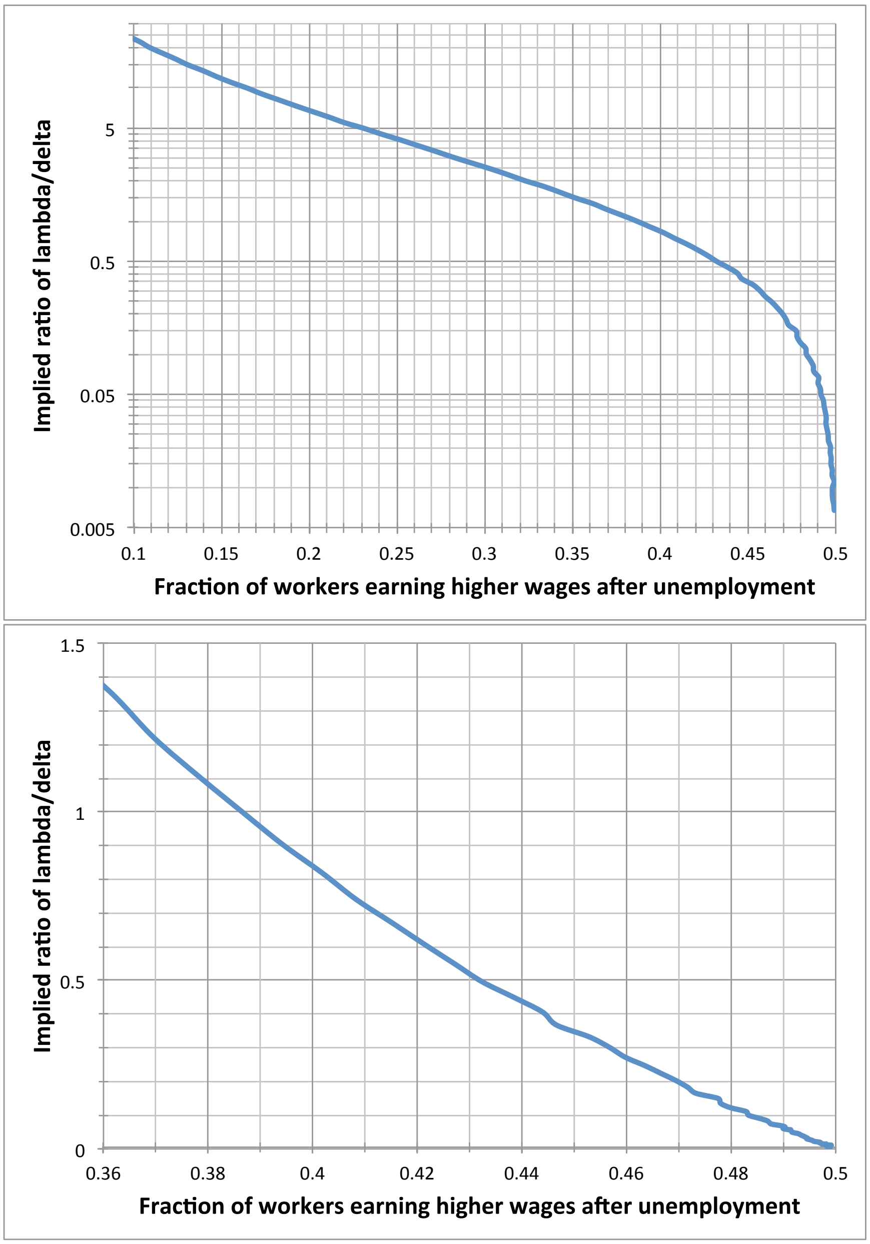


Figure 4: Worker's Problem