Risky Investments with Limited Commitment*

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April 15, 2013

Abstract

Over the last three decades there has been a dramatic increase in the size of the financial sector and in the compensation of financial executives. This increase has been associated with greater risk-taking with the use of more complex financial instruments. Parallel to this trend, the organizational structure of the financial sector has changed with the traditional partnership replaced by public companies that compete for managerial talent and have weaker forms of commitment between investors and managers. In this paper we show that the increased competition and the weaker commitment associated to the organizational change has facilitated more entry into the sector and increased the managerial incentives to undertake risky investment. In the general equilibrium, this change results in a larger financial sector, greater risk-taking and higher income inequality.

1 Introduction

The past several decades have been characterized by dramatic changes in the size and structure of financial firms in the United States and elsewhere. What was once an industry dominated by partnerships has evolved into a much more concentrated sector dominated by large public firms. In this paper we argue that this evolution has altered the structure of contractual arrangements between investors and managers in ways that weakened commitment and increased the managers' incentives to undertake risky investments. At the aggregate level, the change resulted in a larger financial sector and greater income inequality.

The increase in the size and importance of the financial sector in the United States has been documented by Phillipon (2008) and Phillipon and Resheff (2009). This is also shown in Figure 1 which plots the shares of the financial industry in value added and employment since the late 1940's. As can be seen from the figure, the contribution of the finance industry to GDP doubled in size between 1970 and 2006. The share of employment has also increased but by less than the contribution to value added. This

*We would like to thank Facundo Piguillem and other seminar participants at the Banca d'Italia and European University Institute for their comments on previous versions of this work.

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is especially noticeable starting in the mid 1980s when the share of employment stopped growing while the share of value added continued to expand. Acordingly, we observe a significant increase in productivity compared to the remaining sectors of the economy.

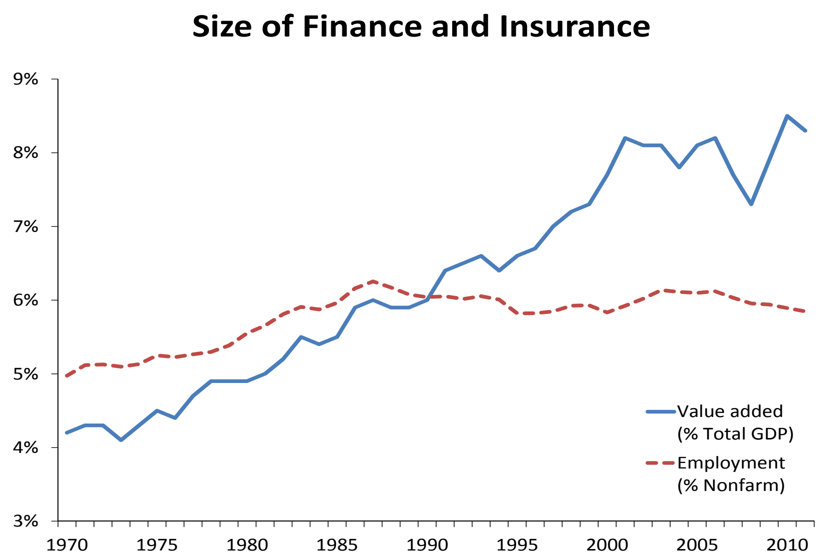

Figure 1: Share of Value Added and Employment

The increase in size was also associated with a sharp increase in compensation. Clementi and Cooley (2009) show that between 1980 and 2007 the average compensation levels in the financial sector increased from parity with other sectors of the economy to 181%. At the same time compensation of managers became more unequal in the financial sector. Figure 2 shows the evolution of the income share of the top 5% of managerial positions in the sector compared to other occupations.

This period of increased size and importance of the financial sector followed significant changes in the organizational form of financial firms and had two important effects. The first effect was to facilitate entry into the financial sector increasing the demand (competition) for managers. The second effect was to alter the structure of contractual arrangements between investors and managers in ways that weakened commitments. As we will see in the theoretical section of the paper, the combination of these two effects increased the managers' incentives to undertake risky investments and generated greater income inequality.

Historically, it was common for investment firms to be organized as partnerships. Many argued that this was a preferred form of organization because in a partnership, managers and investors were the same people and it was the partners own assets that were at risk when more risky investments were taken. A partnership can be interpreted as an organizational structure where the separation between ownership and investment control is minimized. From this perspective the partnership can be viewed as the organi-

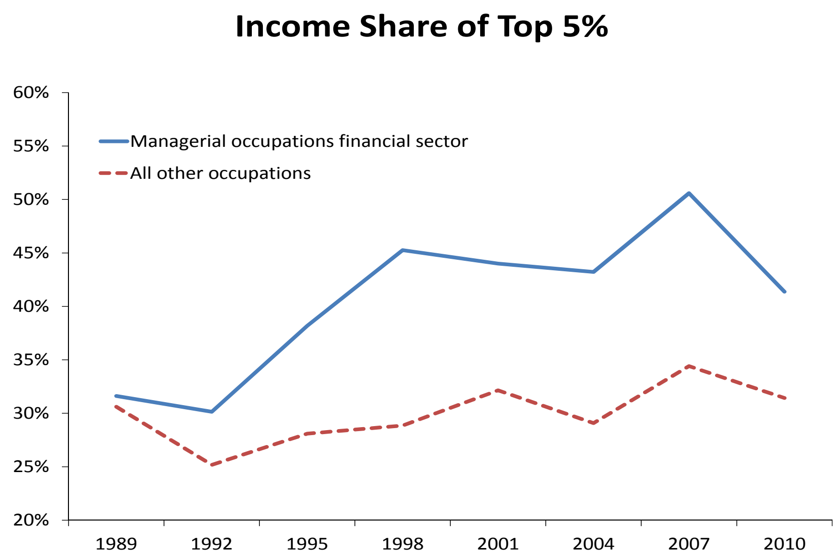

Figure 2: Share of the top 5%

zational form that minimizes the agency issues studied in the contract theory literature.[1] Public companies, on the other hand, are organizational structures with significant separation between ownership (shareholders) and investment control (managers). It is well understood that this organizational form is characterized by significant agency issues.

Until 1970 the New York Stock Exchange prohibited member firms from being public companies. When the organizational restriction on financial companies was relaxed, there was a movement to go public and partnerships began to disappear. Merrill Lynch went public in 1971, followed by Bear Stearns in 1985, Morgan Stanley in 1985, Lehman Brothers in 1994 and Goldman Sachs in 1999. Other venerable investment banks were taken public and either absorbed by commercial banks or converted to bank holding companies. The same evolution occurred in Britain where the closed ownership Merchant Banks virtually disappeared.

The fact that member firms were allowed to become public companies does not tell us why they chose to do so. In several cases firms were simply acquired by public companies but in others it was an important strategic decision. Charles Ellis (2008) in his history of Goldman Sachs—the last major firm to go public—indicates that the major motive for financial partnerships to become public was that an IPO could provide greater capital for their proprietary trading.

The partnership form and its customs had some important implications for managerial mobility. The capital in a partnership and the ownership shares are typically relatively illiquid so it was difficult for partners to liquidate their ownership positions and move to other firms. Also important is the process of becoming a partner. In the typical firm,

[1] This was largely consistent with the view that emerges from the incomplete contracting literature of Grossman and Hart (1986) and Hart and Moore (1990) that more efficient organizational forms are those where the agents who control the allocation of the surplus from investments own more of the assets.

new professionals are hired as associates and, after a trial period, they are either chosen to be partners or released. In this environment separation is viewed as a signal of inferior performance, thus affecting the external option of a financial professional. Becoming a partner, on the other hand, represented a firm commitment to continued employment on the part of the other partners. Thus, the change in organizational form was quite significant for the nature of contracts and competition in the financial sector.[2]

As the structure of financial firms changed, so did the evaluation of them. The market does not seem to value highly the large complex financial institutions. Figure 3 shows the evolution of the ratio of average market value of equity to book value of equity for publicly listed financial and nonfinancial firms since 1970 and shows that, starting in the early 1980's, the ratio for financial firms has been flat while for nonfinancial firms it has continued to grow. The fact that the market values the financial sector relatively less may be a reflection of compensation practices in firms where managers retain so much of the surplus.[3]

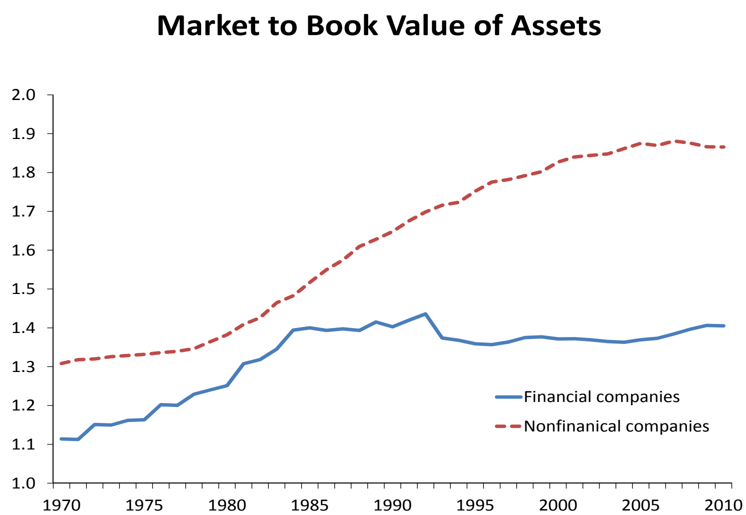

Figure 3: Average Market Value of Equity/Book Values of Equity

To understand the implications of the change in the organizational structure, we study a model where investors compete for and hire managers to run investment projects, with

[2]Roy Smith, a former partner at Goldman Sachs described the evolution of the relationship between compensation and firm structure as follows: "In time there was an erosion of the simple principles of the partnership days. Compensation for top managers followed the trend into excess set by other public companies. Competition for talent made recruitment and retention more difficult and thus tilted negotiating power further in favor of stars. You had to pay everyone well because you never knew what next year would bring, and because there was always someone trying to poach your best trained people, whom you didn't want to lose even if they were not superstars. Consequently, bonuses in general became more automatic and less tied to superior performance. Compensation became the industry's largest expense, accounting for about 50% of net revenues." *Wall Street Journal* February 7, 2009

[3]Since the financial crisis, compensation in the securities industry has increased by 8.7% annually. Currently nearly half of all revenues are earmarked for compensation and it has been higher in the past.

each investor-manager pair representing a financial firm. A key feature of the model is that production depends on the human capital of the manager which can be enhanced, within the firm, with costly investment. Human capital accumulation can be understood as acquiring new skills by engaging in risky financial innovations (e.g. implementing new financial instruments which may or may not have positive returns). Since part of the accumulated human capital can be transferred outside the firm by the manager, there is a conflict of interest between the investor and the manager. In this environment, the investment desired by the investor may be smaller than the investment desired by the manager because the cost is incurred by the firm while the benefits are shared. This implies that, if the investor cannot control the investment policy either directly or indirectly through a credible compensation scheme, the manager has an incentive to deviate from the optimal policy simply because she does not internalize the full cost of the investment. The goal of the paper is to characterize the investment and compensation policies that result from the (constrained) optimal contract and show how these policies change when the competition for managers increases.

The basic framework that is often used to study executive compensation is adapted from the principle-agent model of dynamic moral hazard by Spear and Srivastava (1987).[4] An assumption typically made in this class of models is that the outside option of the agent is exogenous. As argued above, however, an important consequence of the demise of the partnership form is that financial managers are no longer constrained by the limited liquidity of the portion of their wealth that is tied to their firm and it is easier for them to seek outside employment. Since the value of seeking outside employment depends on the market conditions for managers, it becomes important to derive these conditions endogenously in general equilibrium. A second feature of principal-agent models is the assumption that investors fully commit to the optimal contract. However, as argued above, the clearer separation between investors and managers that followed the transformation of financial partnerships to public companies, could have also reduced the commitment of investors. Therefore, in this paper we relax both assumptions: we endogeneize the outside option of managers which will be determined in general equilibrium and we allow for the limited commitment of investors.[5]

To make the outside value of managers endogenous and to study the implications for the whole economy, we embed the micro structure in a general equilibrium model with two sectors—financial and nonfinancial—and two types of workers—skilled and unskilled. Skilled workers can work in both sectors but they have the capacity to innovate only in the financial sector. Unskilled workers have no role in the financial sector. With this general

[4]Among the models in this class see, for example, Wang (1997), Quadrini (2004), Clementi and Hopenhayn (2006), Fishman and DeMarzo (2007). Albuquerque and Hopenhayn (2004) can also be considered within this class of models although the frictions are based on limited enforcement rather than information asymmetry.

[5]Although in a different set-up, Cooley, Marimon and Quadrini (2004) endogeneized the outside value of entrepreneurs but kept the assumption that investors commit to the long-term contract. Marimon and Quadrini (2011) relaxed both assumptions and, using a model without uncertainty, showed that differences in 'barriers to competition' can result in income differences across countries. In the current paper, instead, uncertainty is central to the analysis. The focus is on risk-taking and how this is affected by changes in the organization of financial firms.

framework we study the consequences of the organizational changes which, as discussed above, had two effects: it facilitated entry into the financial sector increasing the demand (competition) for managers and it altered the structure of contractual arrangements between investors and managers in ways that weakened commitments. These two effects are formalized in the model by lowering the cost of entry in the financial sector and by shifting to a regime where investors no longer commit to the contract (double-sided limited commitment). We then show that these structural changes can generate: (*i*) greater risk-taking; (*ii*) larger share (and higher relative productivity) of the financial sector; (*iii*) lower stock market valuation of financial institutions; (*iv*) greater income inequality within and between sectors. Thus, the model can capture the key empirical facts outlined in this paper. Facts that cannot be easily reconciled with more standard dynamic contractual models.

The organization of the paper is as follows. In Section 2 we describe the environment and characterize the optimal contract under different assumptions about commitment. Section 3 embeds the micro structure in a general equilibrium model. Section 4.1 provides a numerical characterization of the equilibrium and relates its properties to the empirical facts that motivate the paper. Section 5 concludes.

2 The model

We start with the description of the financial sector and the contracting relationships that are at the core of the model. After the characterization of the financial sector, we will embed it in a general equilibrium framework in Section 3.

The financial sector is characterized by firms regulated by a contract between an investor, the owner of the firm, and a manager. We should think of managers as skilled workers who have the ability to run the firm and implement innovative projects.

The expected lifetime utility of managers is

$$Q_0 = E_t \sum_{t=0}^{\infty} \beta^t \Big[u(C_t) - e(\lambda_t) \Big],$$

where C_t is consumption and $e(\lambda_t)$ is the dis-utility from effort required for the implementation of investment projects as described below. The period utility satisfies $u' > 0$, $u'' < 0$ and $e' > 0$, $e'' > 0$, $e(0) = 0$ and $e(1) = \infty$. Investors are the residual claimants of the income of the firm, Y_t, with expected lifetime utility

$$V_0 = E_t \sum_{t=0}^{\infty} \beta^t (Y_t - C_t).$$

The output produced by the firm in period t is equal to the human capital of the manager, h_t, which can be enhanced with the development of new implementable projects or ideas i_t according to the technology

$$i_{t+1} = h_t \lambda_t \varepsilon_{t+1},$$

where $\varepsilon_{t+1} \in \{0, \bar{\varepsilon}\}$ is an i.i.d. stochastic variable that takes the value $\bar{\varepsilon} > 0$ with probability p and zero with probability $1 - p$.

We think of λ_t as the investment to generate new implementable projects or ideas i_{t+1}. If the manager continues to work in a financial firm and the new project is implemented, then her human capital is enhanced to $h_{t+1} = h_t + i_{t+1}$. However, if the new project is not implemented in a financial firm—for instance, if the manager leaves the financial sector—the human capital of the manager remains h_t. So in order for the human capital of the manager to increase it is not enough that a new project is developed. It must also be implemented. In other words, if new ideas or projects are implemented after their development stage, they become *embedded* human capital. Otherwise they fully depreciate. The importance of this assumption will become clear later.[6].

There are two types of cost associated with investment λ_t. The first is the manager's dis-utility $e(\lambda_t)$. The second is a pecuniary cost $\kappa h_t \lambda_t$. A key difference between these two costs is that the first is incurred by the manager while the second is incurred by the firm. This creates a wedge between who pays the cost of the investment and who enjoys the benefit: If the manager chooses to quit, the pecuniary cost is paid by the firm but the benefit could go to the manager in the form of increased human capital (provided that the manager finds occupation in another financial firm).

To use a compact notation, we define $y(\lambda_t) = 1 - \kappa\lambda_t$ the income per unit of human capital net of the investment cost, and $g(\lambda_t, \varepsilon_{t+1}) = 1 + \lambda_t \varepsilon_{t+1}$ the gross growth rate of human capital of the manager, provided she remains employed. Then, the net income generated by the firm and the evolution of human capital can be written as

$$Y_t = y(\lambda_t) h_t, \tag{1}$$
$$h_{t+1} = g(\lambda_t, \varepsilon_{t+1}) h_t. \tag{2}$$

Managers have the option to quit and search for an offer from a new firm/employer. If they choose to quit, they will receive an offer with probability $\rho \in [0, 1]$. The probability ρ captures the degree of *competition* for managers, that is, the ease with which they can start a new contract after quitting the firm. Higher values of ρ denote more competitive economies. Since we are assuming that an implementable project of size i_{t+1} fully depreciates if not implemented in a firm, the human capital of a manager who chooses to quit at the beginning of period $t + 1$ will be $h_t + i_{t+1}$ only if she receives an offer in a financial firm. Otherwise, her human capital remains h_t.

Denote by $\underline{Q}_{t+1}(h_t)$ the outside value of a manager at the beginning of period $t + 1$ without an external offer and by $\overline{Q}_{t+1}(h_{t+1})$ the outside value with an offer. Then, the expected outside value at $t + 1$ of a manager with previous human capital h_t is equal to

$$D(h_t, h_{t+1}, \rho) = (1 - \rho) \cdot \underline{Q}_{t+1}(h_t) + \rho \cdot \overline{Q}_{t+1}(h_{t+1}),$$

where $h_{t+1} = h_t(1 + \lambda_t \varepsilon_{t+1})$

[6]We only need to assume that newly developed projects depreciate faster than embedded human capital. In the general model presented in Section 3, we assume that embedded human capital is a productive factor outside the financial sector.

For the moment we take ρ, $\underline{Q}_{t+1}(h_t)$ and $\overline{Q}_{t+1}(h_{t+1})$ as given. At this stage we only assume that $\underline{Q}_{t+1}(h_t)$ and $\overline{Q}_{t+1}(h_{t+1})$ are strictly increasing and differentiable, which implies $D_{2,3} > 0$. However, when we extend the model to a general equilibrium in Section 3, the probability of an external offer ρ and the outside values $\underline{Q}_{t+1}(h_t)$ and $\overline{Q}_{t+1}(h_t)$ will be derived endogenously. This is an important innovation of our model and will be key for some of the results.

In addition to having the ability to quit, the manager controls the investment λ_t and she could choose a different investment from the one that maximizes the surplus of the partnership. Therefore, there are two sources of limited enforcement for managers: the ability to quit and discretion about the choice of λ_t.

Definition 1 *A **contract** between an investor and a manager with initial human capital h_0 consists of sequences of payments to the manager $\{C(h^t)\}_{t=0}^{\infty}$ and investments $\{\lambda(h^t)\}_{t=0}^{\infty}$, conditional on the observed history of human capital $h^t = (h_0, \ldots, h_t)$.*

Implicit, in this definition, is the assumption that the payments made to the manager in period t cannot be conditioned on the 'actual' investment λ_t chosen by the manager in period t. This can be rationalized by assuming that the actual λ_t chosen by the manager and the pecuniary cost $\kappa h_t \lambda_t$ incurred by the firm become public information only at the end of period, after making the payment C_t to the manager.[7]

2.1 Optimal contract with one-sided limited commitment

We first characterize the optimal contract when the investor commits to the contract but the manager does not (with one-sided limited commitment). The manager could quit the firm at any point in time and could choose any investment λ_t. The optimal contract can be characterized by solving a planner's problem that maximizes the weighted sum of utilities for the investor and the manager but subject to a set of constraints. These constraints guarantee that the allocation chosen by the planner is enforceable in the sense that both parties choose to participate and the manager has no incentive to take actions other than the ones prescribed by the contract. We first characterize the key constraints and then we specify the optimization problem.

The allocation chosen by the planner must be such that the value of the contract for the manager is not smaller than the value of quitting. This gives rise to the ***enforcement constraint***

$$E_{t+1} \sum_{n=0}^{\infty} \beta^n \Big[u(C_{t+1+n}) - e(\lambda_{t+1+n}) \Big] \geq D(h_t, h_{t+1}, \rho), \qquad t \geq 0. \tag{3}$$

A second constraint takes into account that the manager has full control of the investment λ_t. The manager could deviate from the λ_t recommended by the planner since,

[7] Alternatively, we could assume that λ_t becomes public information in period $t+1$ after the realization of ε_{t+1}. The pecuniary cost $\kappa h_t \lambda_t$ is also incurred at the beginning of period $t+1$. This alternative assumption requires only the redefinition of $y(\lambda_t) = 1 - \beta\kappa\lambda_t$, where the investment cost is discounted by β because incurred in period $t+1$. The whole analysis will go through without significant changes.

through the choice of λ_t, she can affect the outside value. Thus, the allocation must satisfy an incentive-compatibility constraint insuring that the manager does not deviate from the investment policy recommended by the planner.

Denote by $\hat{\lambda}_t$ the investment chosen by the manager when she deviates from the recommended λ_t. This maximizes the outside value net of the cost of effort, that is,

$$\hat{\lambda}_t = \arg\max_{\lambda\in[0,1]} \left\{ -e(\lambda) + \beta ED\Big(h_t, g(\lambda,\varepsilon_{t+1})h_t,\rho\Big)\right\}. \tag{4}$$

Since the outside value of the manager is differentiable, the optimal deviation solves the first-order condition

$$e'(\hat{\lambda}_t) \geq \beta p\bar{\varepsilon}h_t D_2\Big(h_t,(1+\bar{\varepsilon}\hat{\lambda}_t)h_t,\rho\Big), \tag{5}$$

with is satisfied with equality if $\hat{\lambda}_t > 0$.

Given the optimal deviation $\hat{\lambda}_t$, the ***incentive-compatibility constraint*** at t is

$$-e(\lambda_t) + \beta E_t \sum_{n=0}^{\infty} \beta^n \Big(u(C_{t+n+1}) - e(\lambda_{t+n+1})\Big) \geq -e(\hat{\lambda}_t) + \beta ED\Big(h_t, g(\hat{\lambda}_t, h_t\varepsilon_{t+1}),\rho\Big). \tag{6}$$

Notice that C_t does not appear in (6) since we are assuming that current consumption cannot be contingent on current investment $\hat{\lambda}_t$.

We now have all the ingredients to write down the optimization problem solved by the planner in the regime with one-sided limited commitment. Let $\tilde{\mu}_0$ be the planner's weight assigned to the manager and 1 the weight assigned to the investor. We can then write the planner's problem as

$$\max_{\{C_t,\lambda_t\}_{t=0}^{\infty}} \quad E_0\left\{\sum_{t=0}^{\infty}\beta^t\Big(y(\lambda_t)h_t - C_t\Big) + \tilde{\mu}_0\sum_{t=0}^{\infty}\beta^t u(C_t)\right\} \tag{7}$$

$$\textbf{s.t.} \quad (2),\ (3),\ (6).$$

The problem is also subject to initial participation constraints for both the investor and the manager which, for simplicity, we have omitted. They only restrict the admissible values of the weight $\tilde{\mu}_0$.

Following Marcet and Marimon (2011), the problem can be written recursively as

$$\begin{aligned} W(h,\tilde{\mu}) \quad = \quad \min_{\tilde{\chi},\tilde{\gamma}(\varepsilon')}\max_{C,\lambda} \Bigg\{ & y(\lambda)h - C + \tilde{\mu}\Big(u(C)-e(\lambda)\Big) - \tilde{\chi}\Big(e(\lambda)-e(\hat{\lambda})\Big) + \\ & \beta E\Big[W(h',\tilde{\mu}') - \tilde{\chi}D\Big(h,g(\hat{\lambda},\varepsilon')h,\rho\Big) - \tilde{\gamma}(\varepsilon')D(h,h',\rho)\Big]\Bigg\} \end{aligned} \tag{8}$$

$$\textbf{s.t.} \quad h' = g(\lambda,\varepsilon')h, \quad \tilde{\mu}' = \tilde{\mu} + \tilde{\chi} + \tilde{\gamma}(\varepsilon'),$$

where $\tilde{\gamma}(\varepsilon')$ is the Lagrange multiplier for the enforcement constraint (3) and $\tilde{\chi}$ is the Lagrange multiplier for the incentive-compatibility constraint (6).

The function $W(h, \tilde{\mu})$ is related to the value of the contract for the investor, $V(h, \tilde{\mu})$, and to the value for the manager, $Q(h, \tilde{\mu})$, by the equation $W(h, \tilde{\mu}) = V(h, \tilde{\mu}) + \tilde{\mu}Q(h, \tilde{\mu})$. An environment with full commitment is just a special case with $\tilde{\gamma}(\varepsilon') = \tilde{\chi} = 0$. Another special case is when λ_t is controlled by the investor, in which case $\tilde{\chi} = 0$.

Differentiating problem (8) by C we obtain the optimality condition

$$C_t = u'^{-1}\left(\frac{1}{\tilde{\mu}_t}\right), \tag{9}$$

which characterizes the ***consumption policy*** as a function with the state variable $\tilde{\mu}_t$. The state variable evolves according to the law of motion $\tilde{\mu}_{t+1} = \tilde{\mu}_t + \tilde{\chi}_t + \tilde{\gamma}_t(\varepsilon_{t+1})$.

It is useful to consider the *normalized* manager's weight, or manager's share of the surplus, $\mu_t = \tilde{\mu}_t/h_t$. Since, income is proportional to h_t (recall (1)), this inverse relation between the manager's share and her human capital plays a key role in our analysis. In particular, $Eh_{t+1} = (1 + p\lambda_t\bar{\varepsilon})\, h_t$. Therefore, as long as the contract prescribes $\lambda_t > 0$, then $Eh_{t+1} > h_t$, that is, human capital increases on average over time. If contracts were perfectly enforceable, $\tilde{\chi}_t$ and $\tilde{\gamma}_t(\varepsilon_{t+1})$ would be both equal to zero and $\tilde{\mu}_{t+1} = \tilde{\mu}_t$. This implies that the normalized share of the surplus μ_t converges to zero. However, with one-sided limited commitment, the enforcement and incentive-compatibility constraints set a lower bound on μ_t. In particular, $\tilde{\mu}_{t+1}$ increases when either the enforcement constraint binds ($\tilde{\gamma}_t(\varepsilon_{t+1}) > 0$) or the incentive-compatibility constraint binds ($\tilde{\chi}_t > 0$). Thus, with one-sided limited commitment the manager has a minimum share guaranteed. The consequence of this is that the contract does not provide full insurance since the manager's consumption increases stochastically.

The ***investment policy*** is characterized by the first-order condition with respect to λ. Using $g(\lambda_t, \varepsilon_{t+1}) = 1 + \lambda_t\varepsilon_{t+1}$, the optimality condition can be rewritten as

$$\kappa + \left(\frac{\tilde{\mu}_t + \tilde{\chi}_t}{h_t}\right) e'(\lambda_t) \quad \geq \quad \beta p\bar{\varepsilon}\Big[W_1\Big((1+\bar{\varepsilon}\lambda_t)h_t, \tilde{\mu}_{t+1}\Big) - \gamma_t(\bar{\varepsilon})D_2\Big(h_t, (1+\bar{\varepsilon}\lambda_t)h_t, \rho\Big)\Big], \tag{10}$$

which is satisfied with equality if $\lambda_t > 0$.

The left-hand side of (10) is the marginal cost of investment per unit of human capital. This is increasing in λ_t, μ_t and $\chi_t = \tilde{\chi}_t/h_t$. The right-hand-side is the expected benefit from investing. With full commitment, λ_t is increasing because $\chi_t = 0$, $\tilde{\mu}_t = \tilde{\mu}_0$ and μ_t converges to zero. With limited commitment, however, the limited enforcement and/or the incentive compatibility constraints could be binding, raising the marginal cost. Looking now at the marginal benefit, we can see that the increase in W_1 induced by the increase in human capital is counterbalanced by the increase in D_2. Thus, the effect of binding constraints is a lower λ_t (lower risk), since the marginal cost incurred by the manager is higher (when $\tilde{\gamma}_{t-1}(\varepsilon_t) + \tilde{\chi}_t > 0$) and the marginal benefit is curtailed by the fact that the outside value is higher (when $\gamma_t(\bar{\varepsilon}) > 0$). More importantly, since $D_{2,3} > 0$, an increase in the degree of competition as captured by the parameter ρ, increases the effect of $\gamma_t(\bar{\varepsilon}) > 0$. More formally,

Proposition 1 *Suppose that the optimal investment is interior, that is $\lambda_t^* \in (0,1)$. More competition for managers (higher ρ) affects investment by increasing the weight given to the manager when the limited enforcement and incentive compatibility constraints are binding. It affects investment directly only when the enforcement constraint is binding, in which case it lowers λ_t^*.*

2.2 Optimal contract with double-sided limited commitment

The law of motion $\tilde{\mu}_{t+1} = \tilde{\mu}_t + \tilde{\chi}_t + \tilde{\gamma}_t(\varepsilon_{t+1})$ captures the investor's commitment to fulfill promises made to the manager. With double-sided limited commitment the investor does not commit to fulfill the promises made to the manager and renegotiates whenever the value of the contract for the manager exceeds her outside value. This implies that the value of $\tilde{\mu}_t$ chosen in the previous period becomes irrelevant for the new $\tilde{\mu}_{t+1}$ chosen in the current period. Under these conditions, the manager has the incentive to choose the investment that maximizes the outside value as defined in (4), that is, $\lambda_t = \hat{\lambda}_t$. Thus, the incentive-compatibility constraint and the multiplier $\tilde{\chi}_t$ become irrelevant, and the optimal contract solves

$$\begin{aligned} W(h,\tilde{\mu}) \quad = \quad & \min_{\tilde{\gamma}(\varepsilon')} \max_{C} \Bigg\{ y(\hat{\lambda})h - C + \tilde{\mu}\Big(u(C) - e(\hat{\lambda})\Big) + \\ & \qquad \beta E\Big[W\Big(g(\hat{\lambda},\varepsilon')h,\tilde{\mu}'\Big) - \tilde{\gamma}(\varepsilon')D\Big(h, g(\hat{\lambda},\varepsilon')h,\rho\Big)\Big]\Bigg\} \qquad (11) \\ & \textbf{s.t.} \quad \tilde{\mu}' = \tilde{\gamma}(\varepsilon'). \end{aligned}$$

The optimal contract with double-sided limited commitment simply prescribes a consumption plan, according to (9) with $\tilde{\mu}' = \tilde{\gamma}(\varepsilon')$, and an investment plan given by $\hat{\lambda}$, which is the solution to (5). Since $D_{2,3} > 0$, an increase in competitiveness captured by the parameter ρ, increases the right-hand of (5), that is, the the marginal benefit of investing for the manager. This is formally stated in the next proposition.

Proposition 2 *Consider the environment with double-sided limited commitment and suppose that $\hat{\lambda} \in (0,1)$. Then a higher ρ is associated with higher investment $\hat{\lambda}$.*

Propositions 1 and 2 show that the effect of increasing competition on risk-taking depends crucially on whether both agents can commit to the contract. We should expect increasing competition to result in increased risk-taking only when there is limited commitment from both investors and managers.

2.3 The normalized contract

Since human capital grows on average over time, so does the values of the contract for the manager and the investor. It will then be convenient to normalize the growing variables so that we can study the properties of the contract using stationary variables. The normalization will be especially convenient in Section 3 when we embed the financial sector in a general equilibrium set-up. To facilitate this, we first specialize the utility of managers to take the log-form and make special assumptions about the outside values of managers.

Assumption 1 *The utility function and the outside values of managers take the form*

$$\begin{aligned} u(C) - e(\lambda) &= \ln(C) + \alpha \ln(1-\lambda), \\ \underline{Q}_{t+1}(h_t) &= \underline{q} + \mathcal{B} \ln(h_t), \\ \overline{Q}_{t+1}(h_{t+1}) &= \overline{q} + \mathcal{B} \ln(h_{t+1}), \end{aligned}$$

where $\underline{q}$, $\overline{q}$ *and* $\mathcal{B} \equiv \frac{1}{1-\beta}$ *are constant.*

Although the functional forms for the outside values seem arbitrary at this stage, we will see that in the extension to a general equilibrium they do in fact take these forms. Also notice that Assumption 1 guarantees that the functions for the outside values are differentiable and strictly increasing as we assumed earlier, which in turn implies $D_{2,3} > 0$. We are now ready to normalize all growing variables, starting with the contract values.

The value of the contract for the investor can be expressed recursively as $V_t = y(\lambda_t)h_t - C_t + \beta E_t V_{t+1}$ and normalized to

$$v_t = y(\lambda_t) - c_t + \beta E_t g(\lambda_t, \varepsilon_{t+1}) v_{t+1}, \tag{12}$$

where $v_t = V_t/h_t$ and $c_t = C_t/h_t$.

The value of the contract for a manager can be expressed recursively as $Q_t = \ln(C_t) + \alpha \ln(1-\lambda_t) + \beta E_t Q_{t+1}$. If we subtract $\mathcal{B}\ln(h_t)$ on both sides, and add and subtract $\beta \mathcal{B} E_t \ln(h_{t+1})$ on the right hand side, we obtain

$$Q_t - \mathcal{B}\ln(h_t) = \ln(c_t) + \alpha \ln(1-\lambda_t) + \beta \mathcal{B} E_t \ln\left(\frac{h_{t+1}}{h_t}\right) + \beta E_t \Big[Q_{t+1} - \mathcal{B}\ln(h_{t+1}) \Big].$$

Defining $q_t = Q_t - \mathcal{B}\ln(h_t)$, we can rewrite the above expression more compactly as

$$q_t = \ln(c_t) + \alpha \ln(1-\lambda_t) + \beta E_t \Big[\mathcal{B} \ln\Big(g(\lambda_t, \varepsilon_{t+1}) \Big) + q_{t+1} \Big]. \tag{13}$$

The enforcement constraint for the manager after the realization ε_{t+1} is

$$Q_{t+1}(h_{t+1}) \geq (1-\rho) \cdot \underline{Q}_{t+1}(h_t) + \rho \cdot \overline{Q}_{t+1}(h_{t+1}).$$

Using $q_{t+1} = Q_{t+1}(h_{t+1}) - \mathcal{B}\ln(h_{t+1})$ and the functional form for the outside values from Assumption 1, the enforcement constraint (6) can be written in normalized form as

$$q_{t+1} \geq (1-\rho)\underline{q} + \rho\bar{q} - (1-\rho)\mathcal{B}\ln\Big(g(\lambda_t, \varepsilon_{t+1})\Big). \tag{14}$$

The right-hand-side of the normalized enforcement constraint depends on λ_t (provided that $\rho < 1$). Thus, investment affects the outside value of the manager and, when the enforcement constraint is binding, it affects the manager's compensation. This property is a direct consequence of the assumption that the outside value of the manager *without* an external offer depends on h_t, while the outside value *with* an external offer depends on h_{t+1}. If both values depended on the embedded human capital h_{t+1}, then the term $(1-\rho)\mathcal{B}\ln(g(\lambda_t, \varepsilon_{t+1}))$ would disappear. The value of quitting would still depend on ρ but it would not affect the optimal investment decision λ_t.

The constraint that insures that the manager chooses the optimal investment is

$$\alpha\ln(1-\lambda_t) + \beta E_t Q_{t+1}\Big(g(\lambda_t, \varepsilon_{t+1})h_t\Big) \geq$$
$$\alpha\ln(1-\hat{\lambda}_t) + \beta E_t\Big[(1-\rho)\cdot \underline{Q}_{t+1}(h_t) + \rho\cdot\overline{Q}_{t+1}\Big(g(\hat{\lambda}_t, \varepsilon_{t+1})h_t\Big)\Big],$$

where λ_t is the investment recommended by the optimal contract and $\hat{\lambda}_t$ is the investment chosen by the manager (deviation). Normalizing, we can rewrite the incentive-compatibility constraint as

$$\alpha\ln(1-\lambda_t) + \beta E_t\Big[q_{t+1} + \mathcal{B}\ln\Big(g(\lambda_t, \varepsilon_{t+1})\Big)\Big] \geq$$
$$\alpha\ln(1-\hat{\lambda}_t) + \beta E_t\Big[(1-\rho)\underline{q} + \rho\bar{q} + \rho\mathcal{B}\ln\Big(g(\hat{\lambda}_t, \varepsilon_{t+1})\Big)\Big]. \tag{15}$$

We can now provide a more explicit characterization of the manager's optimal investment deviation $\hat{\lambda}_t$. This maximizes the expected value of quitting net of the effort cost, that is, the right-hand-side of (15). The optimality condition (5) takes now the form

$$\frac{\alpha}{1-\hat{\lambda}_t} \geq \frac{\rho\beta\mathcal{B}p\bar{\varepsilon}}{1+\hat{\lambda}_t\bar{\varepsilon}}, \tag{16}$$

which is satisfied with equality if $\hat{\lambda}_t > 0$. As implied by Proposition 2, we can now see more explicitly that $\hat{\lambda}$ is increasing in the probability ρ.

The original contractual problem (7) with one-sided limited commitment can be reformulated in normalized form using the 'promised utility' approach. This maximizes the normalized investors' value subject to the normalized promise-keeping, limited enforcement and incentive-compatibility constraints, that is,

$$v(q) = \max_{\lambda, c, q(\varepsilon')}\Big\{y(\lambda) - c + \beta E g(\lambda, \varepsilon')v\Big(q(\varepsilon')\Big)\Big\} \tag{17}$$

subject to (13), (14), (15).

The solution to this problem provides the investment policy $\lambda = \varphi^{\lambda}(q)$, the consumption policy $c = \varphi^{c}(q)$, and the continuation utilities $q(\varepsilon) = \varphi^{q}(q, \varepsilon)$. Because of the normalization, these policies are independent of h. However, once we know h, we can reconstruct the original, non-normalized values, that is, $C = ch$ and $Q = q + \mathcal{B}\ln(h)$. Also, once we know the investment policy λ and the realization of the shock ε, we can determine the next period human capital as $h(1 + \lambda\varepsilon)$. In this way we can construct the whole sequence of human capital. Therefore, to characterize the optimal contract we can focus on the normalized policies.

The policies $\varphi^{\lambda}(q)$, $\varphi^{c}(q)$, and $\varphi^{q}(q, \varepsilon)$ satisfy the first order conditions derived in Appendix A, which we can write as

$$c = \mu, \tag{18}$$

$$\kappa + \frac{\alpha(\mu + \chi)}{1 - \lambda} = \beta p \bar{\varepsilon} \left[v\Big(q(\bar{\varepsilon})\Big) + \frac{\mathcal{B}[\mu + \chi + (1 - \rho)\gamma(\bar{\varepsilon})]}{1 + \lambda\bar{\varepsilon}} \right], \tag{19}$$

$$\mu(\varepsilon) = \frac{\mu + \chi + \gamma(\varepsilon)}{1 + \lambda\varepsilon}. \tag{20}$$

The variables μ, $\gamma(\varepsilon)$ and χ are the Lagrange multipliers for constraints (13)-(15). The envelope condition $v'(q) = -\mu$ shows the equivalence between the normalized problem (17) and the original problem (7).[8]

For the case with double-sided limited commitment, we can reformulate problem (11) in normalized form in a similar fashion. Using the 'promised utility' approach, the partnership contract with double-sided limited commitment can be written as

$$v(q) = \max_{c, q(\varepsilon)} \left\{ y(\hat{\lambda}) - c + \beta E g(\hat{\lambda}, \varepsilon) v\Big(q(\varepsilon)\Big) \right\} \tag{21}$$

subject to

$$q = \ln(c) + \alpha \ln(1 - \hat{\lambda}) + \beta E \left[\mathcal{B} \ln\Big(g(\hat{\lambda}, \varepsilon)\Big) + q(\varepsilon) \right]$$

$$q(\varepsilon) = (1 - \rho)\underline{q} + \rho\bar{q} - (1 - \rho)\mathcal{B}\ln\Big(g(\hat{\lambda}, \varepsilon)\Big), \qquad \text{for all } \varepsilon,$$

where $\hat{\lambda}$ is determined by condition (16).

In this case, the optimal-deviation investment $\hat{\lambda}$ is independent of q. As a result, $\hat{\lambda}$ determines a lower bound on the normalized utility, denoted by q_{min}, which satisfies the

[8]Appendix B shows another way of writing the optimization problem recursively, starting directly from the original problem (7). From the recursive problem we obtain the same first-order conditions (18) and (19) while condition (20) is simply the law of motion of the co-state variable in (7).

condition

$$q_{min} = \ln\Big(c(q_{min})\Big) + \alpha \ln(1-\hat{\lambda}) + \beta E_t \Big[(1-\rho)\underline{q} + \rho\bar{q} + \rho\mathcal{B}\ln\Big(g(\hat{\lambda},\varepsilon)\Big)\Big]. \tag{22}$$

Problem (21) can be seen as a special case of problem (17) where we have replaced the incentive-compatibility constraint (15) with $\lambda = \hat{\lambda}$. Furthermore, we have imposed that the enforcement constraint (14) is always satisfied with equality. This is because any promise that exceeds the outside value of the manager will be renegotiated ex-post. Notice that in this problem the decision variables, c and $q(\varepsilon)$, are fully determined by the promise-keeping and incentive-compatibility constraints. Therefore, the problem can be solved without performing any optimization, besides solving for $\hat{\lambda}$.

2.4 Contract properties

In this subsection we show the properties of the optimal contract numerically for different contractual environments. The goal is to illustrate the qualitative features of the optimal contract rather than its quantitative properties. The specific parameter values will be described in Section 4.1 when we conduct a numerical exercise with the general model. The computational procedure is described in Appendix D.

As we have seen, the solution to the contractual problem (17) with one-sided limited commitment provides the investment policy $\lambda = \varphi^\lambda(q)$, the consumption policy $c = \varphi^c(q)$, and the continuation utilities $q(\varepsilon) = \varphi^q(q,\varepsilon)$. Because of the normalization, these policies are independent of h. However, once we know these policies and the initial h, we can construct the whole sequence of human capital as well as the non-normalized values of consumption, $C = ch$, and lifetime utility, $Q = q + \mathcal{B}\ln(h)$. Therefore, to characterize the optimal contract we can just focus on the normalized policies and their characterization provided by the first order conditions (18)-(20). This is also the case for the solution to the double-sided limited commitment problem (21).

Figure 4 plots the values of next period normalized continuation utilities, $q(\varepsilon) = \varphi^q(q,\varepsilon)$, which are functions of current normalized utility, q. The left panel is for the environment with one-sided limited commitment and the right panel is for the environment with double-sided limited commitment. We discuss first the case with one-sidel limited commitment.

One-sided limited Commitment. For relatively high values of q, limited commitment constraints are not binding and the manager's value evolves as if the contract was fully enforceable: normalized utility promised when the investment succeeds, $\varepsilon = \bar{\varepsilon}$, is lower than the normalized utility when the investment fails, $\varepsilon = 0$. Remember, however, that these are normalized utilities. In terms of non-normalized utility, $Q = q + \mathcal{B}\ln(h)$, we have that the manager's utility Q stays the same for both realizations of ε. Of course, when $\varepsilon = \bar{\varepsilon}$, the constancy of Q implies that $q = Q - \mathcal{B}\ln(h)$ declines. Therefore, the value of the contract for the manager, relatively to his human capital, declines. However, as q declines, the limited commitment constraints become binding and, eventually,

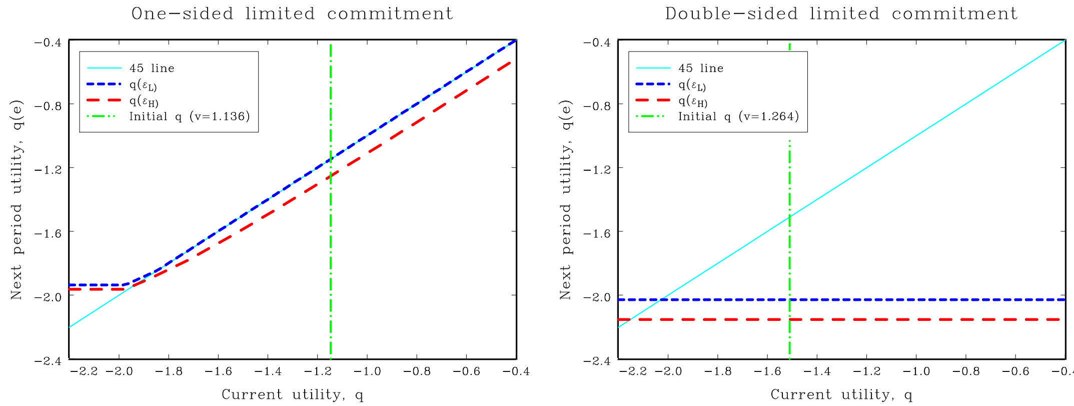

Figure 4: Continuation utilities with one-sided and double-sided limited commitment.

the normalized utility reaches the lower bound q_{min} as defined in (22). At this point consumption grows at the same rate as h and q remains constant at q_{min}.

The initial value of the contract for the manager, $\bar{q}$, is shown by the vertical line.[9] In terms of overall dynamics, we observe that the initial normalized utility q starts at a relatively high value, where limited commitment constraints are not binding. Then, if $\varepsilon = 0$, the next period value of q does not change. If $\varepsilon = \bar{\varepsilon}$, the next period q declines (again, this means that the non-normalized utility grows less than human capital). Therefore, q declines in probability until it reaches the lower bound $q \leq q_{min}$.

Double-sided limited commitment. In the environment with double-sided limited commitment, the investor does not commit to the contract and renegotiates any promises that exceed the outside value for the manager. As a result, the manager always receives the outside value. The only exception is in the first period when the manager receives the value indicated in the figure by the vertical line. After the initial period, q jumps immediately to the outside value and fluctuates between two values. The fact that the initial q is bigger than subsequent values implies that in the first period the manager receives a higher payment (consumption).

Investment. Figure 5 plots the investment policy λ. For high values of the q, the enforcement constraint is not binding and the choice of λ is only determined by the investment cost, part of which is given by the effort dis-utility. For lower values of q, however, the enforcement constraint for the manager is either binding or close to be binding. Consequently, a higher value of λ increases the outside value for the manager. This implies that a higher λ must be associated to a higher promised utility for the manager, which is costly for the investor. This discourages investment and explains

[9]The determination of this value requires an iterative procedure in which we guess $\bar{q}$ and solve for the optimal contract. We can then use the solution for the optimal contract (which is based on the guess for $\bar{q}$) to update the guess until convergence.

why λ decreases as q declines until it reaches the lower bound q_{min}. After that it stays constant.

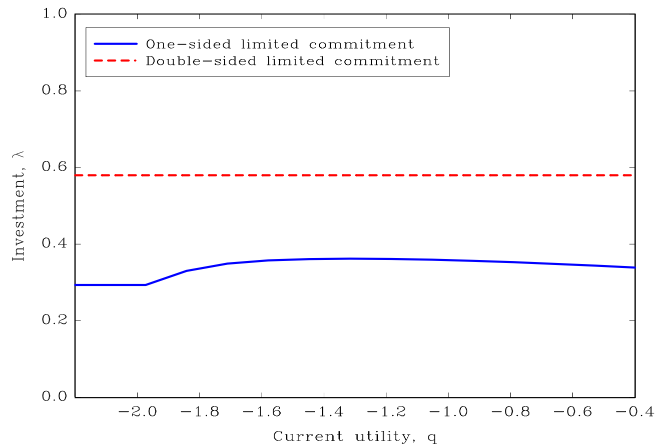

Figure 5: Investment λ as a function of q.

We now turn to the environment with double-sided limited commitment. In this case λ is independent of q since the manager always choose $\lambda_t = \hat{\lambda}$. Given the limited commitment from the investor, the manager knows that her value is always equal to the outside value. Thus, the objective of the manager is to choose the investment that maximizes the outside value net of the utility cost of effort. But in doing so, the manager does not take into account that investment also generates the pecuniary cost $\kappa\lambda h$ to the firm. Thus, the investment level is inefficient in general.

For the particular parametrization considered in this numerical example, the investment chosen in the double-sided limited commitment is bigger than in the environment with one-sided limited commitment. However, this does not apply to any set of parameters. In fact, there are two contrasting effects that determine λ. One the one-hand, with double-side limited commitment, the manager does not take into account the pecuniary cost $\kappa\lambda h$. This leads to a higher λ. On the other, when the manager chooses λ, she maximizes the outside value, which is the value of finding occupation in a new firm. But the investment has value only if the manager finds a new occupation and this happens only with probability $\rho < 1$. Instead, the investment made within the existing firm has value with probability 1. This leads to a lower $\hat{\lambda}$.

To summarize, to have that the the investment in the double-sided limited commitment is bigger than the investment in the one-sided limited commitment, we need that the cost parameter κ and the probability ρ are sufficiently large.

3 General model

We now embed the the financial sector in a general equilibrium framework. This allows us to endogenize the competition parameter ρ and the outside values for managers, $\underline{Q}_{t+1}(h_t)$

and $\overline{Q}_{t+1}(h_{t+1})$.

There are two sectors in the model—financial and nonfinancial—and three types of agents—a unit mass of *investors*, a unit mass of *skilled workers*, and a mass $N > 1$ of *unskilled workers*. Unskilled workers are only employed in the nonfinancial sector while skilled workers can be employed in either sectors.[10] Investors are the owners of firms and are risk neutral. Alternatively, investors can have the same preferences for consumption as workers but be able to fully diversify their portfolios.

Workers, skilled and unskilled, have the same utility $\ln(c_t) + \alpha \ln(1 - \lambda_t)$. However, only managerial occupations in the financial sector requires effort λ_t and, therefore, the utility of unskilled workers and skilled workers employed in the nonfinancial sector reduces to $\ln(c_t)$.

All agents discount future utility by the factor $\hat{\beta}$ and survive with probability $1 - \omega$. In every period there are newborn agents of each type so that the population size and composition remain constant over time. Newborn skilled workers are endowed with initial human capital h_0 while the human capital of unskilled workers is normalized to 1. The motivation for adding this particular demographic structure is to prevent the distribution of h_t to become degenerate. The assumption of a constant h_0 together with the finite lives of skilled workers guarantee that the distribution of h_t across financial managers converges to an invariant distribution and the model is stationary in level. Taking into account the survival probability, the 'effective' discount factor is $\beta = \hat{\beta}(1-\omega)$. Using the effective discount factor β, the previous characterization of the optimal contract between managers and investors applies here without any modification.

Only skilled workers can become managers in the financial sector. We denote by S the mass of skilled workers that are employed in the nonfinancial sector and $1 - S$ is the mass of skilled workers employed in the financial sector (financial managers).

The nonfinancial sector is competitive and produces output with the technology $F(N, H)$, where N is the number of unskilled workers and H is the aggregate efficiency-units of labor supplied by skilled workers who are employed in the nonfinancial sector. This results from the aggregation of human capital of all skilled workers employed in the nonfinancial sector. As we will see, in equilibrium, the human capital of skilled workers employed in the nonfinancial sector is h_0. Therefore, $H = h_0 S$. For simplicity, we abstract from capital accumulation.

The production function is strictly increasing and concave in both arguments and homogeneous of degree one (constant returns). Thus, the wages earned in the nonfinancial sector by unskilled and skilled workers per efficiency-unit of labor are, respectively,

$$w^N = \frac{\partial F(N, H)}{\partial N}, \qquad w^S = \frac{\partial F(N, H)}{\partial H}.$$

While the nonfinancial sector is perfectly competitive, the hiring process in the financial sector is characterized by matching frictions. More specifically, skilled workers

[10] An alternative interpretation of the model is that the financial sector encompasses all the 'innovative segments' of the economy, financial and nonfinancial, where similar organizational changes have taken place. However, in this paper we prefer to focus on the financial sector where the changes described in the introduction are more evident.

find occupation in the financial sector when matched with vacancies funded by investors. Denoting by ρ_{t+1} the matching probability, the lifetime utility of a skilled worker with human capital h currently employed in the nonfinancial sector is

$$\underline{Q}_t(h) = \ln(w^S \cdot h) + \beta\Big[(1-\rho_{t+1}) \cdot \underline{Q}_{t+1}(h) + \rho_{t+1} \cdot \overline{Q}_{t+1}(h)\Big]. \tag{23}$$

The skilled worker consumes income $w^S h$ in the current period. In the next period, with probability ρ_{t+1} she finds an occupation in the financial sector. In this case the lifetime utility is $\overline{Q}_{t+1}(h)$. With probability $1-\rho_{t+1}$ she remains employed in the nonfinancial sector and the lifetime utility is $\underline{Q}_{t+1}(h)$. In this extended model, the value for a skilled worker (manager) of not finding an occupation in the financial sector is the value of being employed in the nonfinancial sector. The function $\overline{Q}_{t+1}(h)$ is the value of a new contract for the financial manager. Therefore, the probability ρ_{t+1} and the outside values $\underline{Q}_{t+1}(h)$ and $\overline{Q}_{t+1}(h)$ are now endogenous and determined in the general equilibrium.

3.1 Matching and general equilibrium

In the financial sector, investors post vacancies for skilled workers that specify the level of human capital h and the value of the contract for the worker $\overline{Q}_t(h)$. This is the value of the long-term contract signed between the firm and the manager. The cost of posting a vacancy is τh.

Let $I_t(h, \overline{Q}_t)$ be the number of vacancies posted for managers with human capital h offering $\overline{Q}_t(h)$. Furthermore, denote by $S_t(h, \overline{Q}_t)$ the number of skilled workers with human capital h in search of an occupation in the financial sector with posted value $\overline{Q}_t(h)$. The number of matches is determined by the matching function $m\Big(I_t(h, \overline{Q}_t), S_t(h, \overline{Q}_t)\Big)$. The probabilities that a vacancy is filled and a worker finds occupation are, respectively,

$$\phi_t(h, \overline{Q}_t) = \frac{m\Big(I_t(h, \overline{Q}_t), S_t(h, \overline{Q}_t)\Big)}{I_t(h, \overline{Q}_t)} \qquad \text{and} \qquad \rho_t(h, \overline{Q}_t) = \frac{m\Big(I_t(h, \overline{Q}_t), S_t(h, \overline{Q}_t)\Big)}{S_t(h, \overline{Q}_t)}.$$

Investors can freely enter in the financial sector and post vacancies. Therefore, the following free-entry condition will be satisfied in equilibrium:

$$\phi_t(h, \overline{Q}_t) V_t(h, \overline{Q}_t) = \tau h.$$

We can now take advantage of the properties of the optimal contract characterized in the previous section where we have shown that the value of the contract for the investor is linear in h; that is, $V_t(h, \overline{Q}_t) = v_t(\bar{q}_t)h$. The variable $\bar{q}_t$ is the normalized value of the contract for a newly hired manager. Therefore, it is enough to determine $\bar{q}_t$ to define a menu of posted contracts for all possible levels of human capital h. More precisely, once $\overline{q}_t$ is decided, the investor offers $\overline{Q}_t = \overline{q}_t + \mathcal{B}\ln(h)$ to the manager with human capital h. Then, focusing on a symmetric equilibrium in which the probability of filling a vacancy is independent of h, the free-entry condition can be rewritten in normalized form as

$$\phi_t(\bar{q}_t) \cdot v_t(\bar{q}_t) = \tau. \tag{24}$$

Appendix C discusses the equilibrium conditions in more detail and shows that in equilibrium the the manager receives a fraction $1-\eta$ of the surplus created by the match. The same outcome would arise if we assume Nash bargaining with the bargaining power of managers equal to $1-\eta$ (Hosios (1990) condition).

Next we normalize the employment value for skilled workers in the nonfinancial sector, equation (23). In normalized terms this can be rewritten as

$$\underline{q}_t = \ln(w_t^N) + \beta\Big[(1-\rho_{t+1})\cdot \underline{q}_{t+1} + \rho_{t+1}\cdot \bar{q}_{t+1}\Big]. \tag{25}$$

The values $\underline{q}_t$ and $\bar{q}_t$ correspond to the normalized outside values used in the previous characterization of the optimal contract. The only difference is that in a general equilibrium these values could be time dependent. In a steady state, however, they are constant. We now have all the ingredients to define a steady state general equilibrium.

Definition 2 (Steady state) *Given a contractual regime (one-sided or double-sided limited commitment), a steady state general equilibrium is defined by*

1. *Policies $\lambda = \varphi^\lambda(q)$, $c = \varphi^c(q)$, $q(\varepsilon) = \varphi^q(q,\varepsilon)$ for contracts in the financial sector;*
2. *Normalized utilities for skilled workers in the nonfinancial sector, $\underline{q}$, skilled workers newly hired in the financial sector, $\bar{q}$, and initial normalized value for investors, $\bar{v}$;*
3. *Skilled workers employed in the nonfinancial sector, S, posted vacancies for skilled workers I, filling probability, ϕ, and finding probability, ρ;*
4. *Distribution of skilled workers employed in the financial sector $\mathcal{M}(h,q)$;*
5. *Law of motion for the distribution of financial managers, $\mathcal{M}_{t+1} = \Phi(\mathcal{M}_t)$;*

Such that

1. *The policy rules $\varphi^\lambda(q)$, $\varphi^c(q)$, $\varphi^q(q,\varepsilon)$ solve the optimal contract;*
2. *The normalized utilities $\underline{q}$ and $\bar{q}$ and investor value $\bar{v}$ solve (24), (25) and (33);*
3. *Filling and finding probabilities satisfy $\phi = m(I,S)/I$ and $\rho = m(I,S)/S$.*
4. *The law of motion $\Phi(\mathcal{M})$ is consistent with contract policies $\varphi^\lambda(q)$ and $\varphi^q(q,\varepsilon)$.*
5. *The distribution of managers is constant, that is, $\mathcal{M} = \Phi(\mathcal{M})$.*

3.2 Inequality

The general model features three types of workers: 1) unskilled workers employed in the nonfinancial sector; 2) skilled workers employed in the nonfinancial sector; 3) skilled workers employed in the financial sector. Therefore, we can study inequality between skilled and unskilled workers; between skilled workers employed in the financial and nonfinancial sectors; across skilled workers employed in the financial sector. Here we focus on the distribution of income across skilled workers employed in the financial sector.

Since the income of workers employed in the financial sector is proportional to human capital, we can use h as a proxy for the distribution of income. As a specific index of inequality we use the square of the coefficient of variation in human capital, that is,

$$\text{Inequality index} \equiv \frac{\text{Var}(h)}{\text{Ave}(h)^2}.$$

In a steady state equilibrium with double-sided limited commitment, the inequality index can be calculated exactly. Let's first derive the steady state employment in the financial sector, $1-S$. The number of skilled workers searching for a financial occupation is S. Using the formula for the flow of skilled workers into financial occupations (at rate ρ) and out of financial occupations (at rate ω), we obtain

$$1-S = \frac{\rho(1-\omega)}{\rho+\omega-\rho\omega}.$$

Next we compute the average human capital for the mass $1-S$ of workers employed in the financial sector,

$$\text{Ave}(h) = \omega\sum_{j=0}^{\infty}(1-\omega)^j E_j h_j.$$

The index j denotes the employment tenure for active managers (employment periods). Therefore, $j=0$ identifies newly hired workers of mass $\omega(1-S)$ (in a steady state the mass of newly hired workers is equal to the mass of workers who die). Since managers survive with probability $1-\omega$, the fraction of workers who have been active for j periods is $\omega(1-\omega)^j$.

The variance of h across the $1-S$ workers is calculated as

$$\text{Var}(h) = \omega\sum_{j=0}^{\infty}(1-\omega)^j E_j\Big(h_j - \text{Ave}(h)\Big)^2,$$

which has a similar interpretation as the formula used to compute the average h.

Using the property of the model with double-sided limited commitment where all firms choose the same λ, and therefore, all managers experience the same expected growth in human capital, Appendix E shows that the average human capital and the inequality index take the forms

$$\text{Ave}(h) = h_0\left[\frac{\omega}{1-(1-\omega)Eg(\hat{\lambda},\varepsilon)}\right], \quad (26)$$

$$\text{Inequality index} = \frac{[1-(1-\omega)Eg(\hat{\lambda},\varepsilon)]^2}{\omega[1-(1-\omega)Eg(\hat{\lambda},\varepsilon)^2]} - 1. \quad (27)$$

Therefore, the average human capital and the inequality index are simple functions of the investment $\hat{\lambda}$. We then have the following proposition.

Proposition 3 *The average human capital and the inequality index for financial managers are strictly increasing in $\hat{\lambda}$.*

That the average human capital increases with investment is obvious. The dependence of the inequality index on $\hat{\lambda}$, instead, can be explained as follows. If $\hat{\lambda} = 0$, the human capital of all managers will be equal to h_0 and the inequality index is zero. As $\hat{\lambda}$ becomes positive, inequality increases for two reasons. First, since the growth rate $g(\hat{\lambda}, \varepsilon)$ is stochastic, human capital will differ *within* the same cohort of managers (managers with the same employment tenure). Second, since each cohort experiences growth, the average human capital differs *between* cohorts of managers. More importantly, the cross sectional dispersion in human capital induced by these two mechanisms (the numerator of the inequality index) dominates the increase in average human capital (the denominator of the inequality index). Thus, the inequality index increases in $\hat{\lambda}$.

We can compute explicitly the *within* and *between* cohort inequality by decomposing the variance of h as follows:

$$\text{Var}(h) = \omega \sum_{j=0}^{\infty} (1-\omega)^j E_j \Big(h_j - \text{Ave}_j(h) \Big)^2 + \omega \sum_{j=0}^{\infty} (1-\omega)^j \Big(\bar{h}_j - \text{Ave}(h) \Big)^2,$$

where $\text{Ave}_j(h)$ is the average human capital for the j cohort (managers employed for j periods). The first term sums the variances of each cohort while the second term sums the squared deviation of each cohort from the overall average. Using the above decomposition, the appendix shows that the *within* and *between* cohort inequality indices have simple analytical expressions and they are both strictly increasing in $\hat{\lambda}$.

Proposition 4 *Both within and between cohort inequalities are strictly increasing in $\hat{\lambda}$.*

4 The impact of organizational changes

We now explore the core issue addressed in this paper, that is, how the organizational changes described in the introduction affect risk taking, sectoral income and inequality. We identified two important effects that followed the organizational change in the financial sector:

1. ***Facilitated entry and increased competition***: The separation between investors and managers enlarged the base of potential investors who could fund new investment projects. In the context of our model this is captured, parsimoniously, by a reduction in the vacancy cost τ. A lower value of τ generates more entry and, therefore, more competition for managers.

2. ***Weakened the commitment of investors***: While the limited commitment of managers was also a feature of the traditional partnership (managers were not

prevented from leaving the partnership), the commitment of investors was much stronger since there was not a sharp distinction between investors and managers. Even from a legal stand point, it was difficult for a partnership to replace a partner without a consensual agreement. A feature of a corporation, instead, is a clearer separation between investors and managers. In the context of our model, this is captured by a shift to the environment with double-sided limited commitment.

To summarize, we formalize the demise of traditional partnership as a shift to an environment where there is more competition for managers and contracts have limited enforceability also for investors. We start exploring the consequences of the higher competition for managers in the environment with double-sided limited commitment.

Proposition 5 *In the environment with double-sided limited commitment, a steady state equilibrium with a lower value of τ features:*

1. *Greater risk-taking, that is, higher $\hat{\lambda}$.*
2. *Larger share and higher relative productivity of the financial sector.*
3. *Lower stock market valuation of financial institutions.*
4. *Greater income inequality within and between sectors (financial and nonfinancial).*

The first property is an immediate consequence of Proposition 2: the lower value of τ increases the probability of a match and, consequently, it raises the incentive of the manager to exert more effort to increase the outside value.

The second property, that is, the increase in the size of the financial sector derives in part from higher employment and in part from higher investment. The increase in the share of employment, however, would arise even if there were no contractual frictions. The higher investment, instead, is a direct consequence of the contractual frictions. This is a novel feature of our model which is key to capture the 'productivity' increase in the financial sector relatively to other sectors. This is consistent with the pattern shown in Figure 1 according to which the share of value added of the financial sector has increased more than the share of employment.

The third property is a direct consequence of the property of the model for which the initial value of the contract for the manager, $\bar{q}$, increases with the probability of a match ρ, which is higher in the steady state with a lower value of τ (as already mentioned above). This effect of increased competition for managers is common across organizational forms where there is a division between investors and managers. However, the effect is likely to be stronger in environments in which there is limited commitment also for investors. We show below that this is true in our simulations.

Finally, the fourth property follows from the first property, that is, from the higher investment $\hat{\lambda}$. As we have seen in Proposition 3, a higher value of $\hat{\lambda}$ increases human capital accumulation and inequality in the financial sector. At the same time, since more skilled workers will be employed in the financial sector, fewer skilled workers will be employed in the nonfinancial sector. Because of the complementarity between skilled and unskilled workers in the nonfinancial sector, the wage of skilled workers increases while the wage of unskilled workers declines. Thus, the model generates greater inequality also between sectors and between skills.

The next question is how the equilibrium properties are affected by the second implication of the structural change, that is, a shift from an environment with one-sided limited commitment to an environment with double-sided limited commitment. For this type of change, however, we are unable to derive analytical results. Therefore, we explore them numerically.

4.1 Numerical example

The numerical example is only meant to illustrate the qualitative properties of the model and the choice of parameters is not based on specific calibration targets.

Parameter values. We normalize the number of investors and skilled workers to 1. The number of unskilled workers is set to $N = 3$ so that the fraction of skilled workers is 25% of the total labor force. The production function in the nonfinancial sector is specified as $F(K, N) = N^{\nu} H^{1-\nu}$ with $\nu = 0.9$. The death probability is $\omega = 0.05$ and the discount factor is set to $\beta = (1 - \omega)\hat{\beta} = 0.92$. The dis-utility parameter is set to $\alpha = 0.03$ and the innovation cost to $\kappa = 0.013$. The high realization of the innovation shock is $\bar{\varepsilon} = 0.03$ with probability $\bar{p} = 0.5$ (the low realization of the shock is zero). The matching function takes the form $m(I, S) = A\, I^{\eta} U^{1-\eta}$, with $A = 0.5$ and $\eta = 0.5$. The cost of posting a vacancy is set to $\tau = 0.75$. In the model with double-sided limited commitment, these parameters imply that the steady state probability of receiving an offer is $\rho = 0.42$, the fraction of skilled workers employed in the nonfinancial sector is $S = 0.11$ and the fraction employed in the financial sector are $1 - S = 0.89$.

Results. To show how competition affects innovation policies in the two environments with one-sided and double-sided limited commitment, Figure 6 plots the policy $\lambda = \varphi^{\lambda}(q)$ in two steady state equilibria, each characterized by a different value of τ. In the first steady state $\tau = 0.75$ (low competition) and in the second $\tau = 0.25$ (high competition). Table 1 reports the steady state values for some key variables.

In the environments with one-sided limited commitment (left panel), more competition for managers is associated with lower investment λ, which is consistent with Proposition 1. As can be seen from Table 1, the probability of receiving offers increases with more competition. Since this increases the outside value for managers, a larger share of the return from innovations must be shared with managers. Therefore, innovations become less attractive for investors.

In contrast, when neither managers nor investors can commitment (right panel), more competition leads to more innovation, as Proposition 2 predicts. Also in this environment the probability of external offers increases, which raises the external value of managers and makes investment less attractive for investors. In order to implement the optimal λ, investors would need to promise adequate future compensation. The problem is that

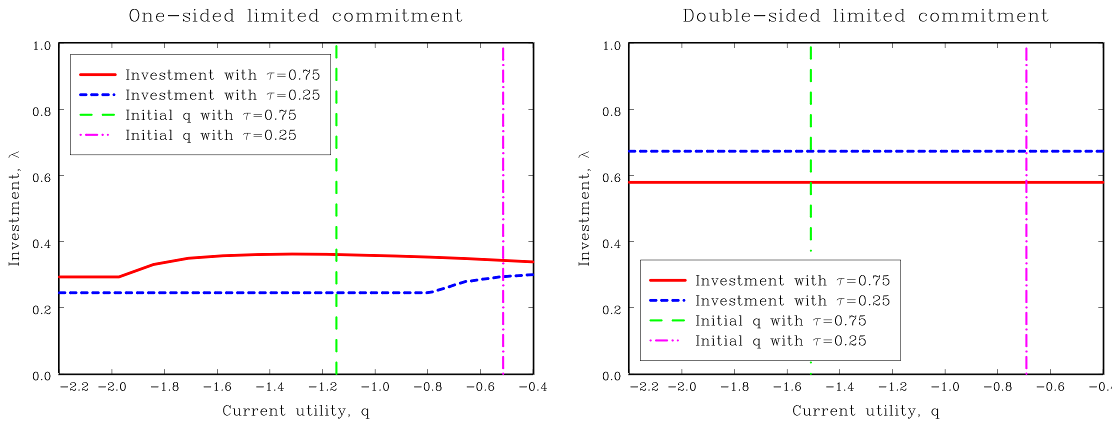

Figure 6: Steady state investment policies for different τ in the environments with one-sided and double-sided limited commitment.

future promises are not credible with double-sided limited commitment and the only way managers can increase their contract values is by raising their outside value. This is achieved by choosing higher λ, which is true for any value of τ. With a lower τ, however, the probability of an external offer ρ increases. Since the manager benefits from higher innovation only if she receives an external offer, the higher probability ρ raises the manager's incentive to raise λ.

So far we have shown that the environment with double-sided limited commitment can generate higher risk-taking as a result of greater competition. We now show that this environment also captures other important changes observed in the US economy. As shown in Table 1, a lower τ is associated with a larger fraction of skilled workers employed in the financial sector. More importantly, the model generates an even larger increase in the share of value added. Also, a lower τ is associated with a reduction in the value of investors (relative to human capital). In particular, Table 1 shows how a reduction of τ translates into a reduction of investors' *interim* value of the contract and a reduction of the probability of filling a vacancy, satisfying $\phi(\bar{q}) \cdot v(\bar{q}) = \tau$. Furthermore, it also shows how the decrease in $v(\bar{q})$ is larger in the environment with double-sided limited commitment[11].

These two properties are consistent with the observed expansion of the financial sector and the decline in market valuation of financial institutions. The last feature we want to emphasize is that inequality also increases. As can be seen in Table 1, higher competition leads to greater inequality both *within* and *between* cohorts of skilled workers employed in the financial sector as Proposition 4 predicts.

[11]The value at $\tau = 0.25$ with respect to $\tau = 0.75$ is 42.9% with double-sided limited commitment and 46.6% with one-sided limited commitment.

Table 1: Steady state properties of equilibria associated with different values of τ in the environments with one-sided and double-sided limited commitment.

	One-sided limited commitment	*Double-sided limited commitment*
Low competition ($\tau = 0.75$)		
Average value of λ	0.334	0.580
Offer probability, ρ	0.379	0.412
Filling probability, ϕ	0.660	0.593
Number of skilled workers financial sector	0.878	0.889
Share of value added from financial sector	0.598	0.622
Earnings unskilled workers	0.196	0.194
Earnings skilled workers nonfinancial sector	0.535	0.583
Investors' financial contract (norm.) value $v(\bar{q})$	1.136	1.265
Average earnings skilled workers financial sector	0.948	0.976
Average investor value	1.749	2.226
Within inequality fin sector ($\times$100)		0.225
Between inequality fin sector ($\times$100)		4.301
Coefficient of variation		0.213
High competition ($\tau = 0.25$)		
Average value of λ	0.267	0.673
Offer probability, ρ	0.528	0.543
Filling probability, ϕ	0.473	0.460
Number of skilled workers financial sector	0.909	0.912
Share of value added from financial sector	0.608	0.641
Earnings unskilled workers	0.190	0.190
Earnings skilled workers nonfinancial sector	0.700	0.716
Investors' financial contract (norm.) value $v(\bar{q})$	0.529	0.543
Average earnings skilled workers financial sector	0.995	1.072
Average investor value	0.753	0.995
Within inequality ($\times$100)		0.336
Between inequality ($\times$100)		6.301
Coefficient of variation		0.258

5 Conclusion

The financial crisis of 2007-2009 has brought attention to the growth in size and importance of the financial sector over the past few decades as well as the increase in risk taking. Much attention has also been focused on the extremely high compensation of financial professionals. Why did these trends emerge over this period of time? In this paper we have argued that changes in the organizational structure of financial firms have opened up competition for managerial skills and that this enhanced competition has further deepened the changes in the organisational forms by decreasing commitment. Specifically, the move away from the traditional partnership favoured competition for managers with weak commitment to the partnership, while increased competition weakened the commitment of investors toward their managers. Furthermore, these changes could have played an important role in another widely documented trend that occurred during the same period—the increase in income inequality.

The fact that inequality has increased over time, especially in anglo saxon countries, is well documented (e.g. Saez and Piketty (2003)). The increase in inequality has been particularly steep for managerial occupations in financial industries (e.g. Bell and Van Reenen (2010)). In this paper we propose one possible explanation for this change. We emphasize the increase in competition for human talent that followed the organizational changes in the financial sector. In an industry where the enforcement of contractual relations is limited, the increase in competition raises the managerial incentives to undertake risky investments. Although risky innovations may have a positive effect on aggregate production, the equilibrium outcome may not be efficient and generates greater income inequality. The higher competition for managerial talent seems consistent with the evidence that managerial turnover, although not explicitly modelled in the paper, has also increased during the last thirty years.

We have shown these effects through a dynamic general equilibrium model with long-term contracts, subject to different levels of commitment and enforcement. The model features two sectors—financial and nonfinancial—with innovations taking place only in the financial sector. Of course, the assumption that only the financial sector innovates is a simplification that we made to keep the model tractable and the analysis focused. An alternative interpretation of the model is that the financial sector is actually the collection of the most 'innovative segments' of the economy, financial and nonfinancial, where similar contractual frictions emerge and the type of organizational changes described in the paper could have similar effects. We decided to focus on the financial sector because this is where the organizational changes have been more evident and leave the analysis of other sectors for future research.

Appendix

A First order conditions for Problem (17)

Let μ and $\gamma(\varepsilon)$ be the lagrange multipliers associated with the promise-keeping constraint and the enforcement constraint. Then the lagrangian can be written as

$$\begin{aligned}
v(q) &= y(\lambda) - c + \sum_{\varepsilon}\Big[\beta g(\lambda,\varepsilon) v\Big(q(\varepsilon)\Big)\Big] p(\varepsilon) \\
&+ \mu\left\{\ln(c) + \alpha\ln(1-\lambda) + \beta\sum_{\varepsilon}\Big[\mathcal{B}\ln\Big(g(\lambda,\varepsilon)\Big) + q(\varepsilon)\Big]p(\varepsilon) - q\right\} \\
&+ \chi\left\{\alpha\ln(1-\lambda) + \beta\sum_{\varepsilon}\Big[\mathcal{B}\ln\Big(g(\lambda,\varepsilon)\Big) + q(\varepsilon)\Big]p(\varepsilon) - D\right\} \\
&+ \beta\sum_{\varepsilon}\Big[q(\varepsilon) - d + (1-\rho)\mathcal{B}\ln\Big(g(\lambda,\varepsilon)\Big)\Big]\gamma(\varepsilon)p(\varepsilon)
\end{aligned}$$

The first order conditions with respect to λ, c and $q(\varepsilon)$ are, respectively,

$$y_\lambda(\lambda) + \beta\sum_{\varepsilon}\left[g_\lambda(\lambda,\varepsilon)v\Big(q(\varepsilon)\Big) + \mathcal{B}\left(\frac{g_\lambda(\lambda,\varepsilon)}{g(\lambda,\varepsilon)}\right)\Big(\mu + \chi + (1-\rho)\gamma(\varepsilon)\Big)\right]p(\varepsilon) - \frac{\alpha(\mu+\chi)}{1-\lambda} = 0$$

$$-1 + \frac{\mu}{c} = 0$$

$$g(\lambda,\varepsilon)v_q\Big(q(\varepsilon)\Big) + \Big(\mu + \chi + \gamma(\varepsilon)\Big) = 0$$

Substituting the envelope condition $v_q(q) = -\mu$ and using the functional forms of $y(\lambda)$ and $g(\lambda,\varepsilon)$ we obtain equations (19)-(20).

B Alternative formulation of the normalized problem

The contractual Problem (8) for one-sided limited commitment can be normalized as

$$\begin{aligned}
w(\mu) = \min_{\chi,\gamma(\varepsilon')}\max_{c,\lambda}&\left\{y(\lambda) - c + \mu\Big(\ln(c) + \alpha\ln(1-\lambda)\Big) - \chi\alpha\ln\left(\frac{1-\lambda}{1-\hat{\lambda}}\right) + \right. \\
&\left.\beta E\Big[\mu'\mathcal{B}\ln\Big(g(\lambda,\varepsilon')\Big) + g(\lambda,\varepsilon')w(\mu') - \chi d(\hat{\lambda},\varepsilon',\rho) - \gamma(\varepsilon')d(\lambda,\varepsilon',\rho)\Big]\right\}
\end{aligned} \quad (28)$$

$$\textbf{s.t.} \quad \mu' = \frac{\mu + \chi + \gamma(\varepsilon')}{g(\lambda,\varepsilon')},$$

where $d(\lambda, \varepsilon', \rho) = (1-\rho)\underline{q} + \rho\bar{q} + \rho\mathcal{B}\ln(g(\lambda, \varepsilon))$ and $w(\mu) = v^P(\mu) + \mu q^P(\mu)$, with v^P and q^P being the normalized values (as a function of the Pareto weight μ) of the investor and the manager, respectively. That is, $q^P(\mu) = Q(h, \tilde{\mu}) - \mathcal{B}\ln(h)$ and $w(\mu) = W(h, \tilde{\mu})/h - \mu\mathcal{B}\ln(h)$. It is easy to verify that the first order conditions for this problem are given by equations (18)-(20). The contractual problem with double-sided limited commitment can be seen as a special case of (28) with $\lambda = \hat{\lambda}$ and $\chi = 0$, resulting in the law of motion $\mu' = \hat{\gamma}(\varepsilon')/g(\hat{\lambda}, \varepsilon')$.

C The posted contract

As it is well known, with directed search there is an indeterminacy of rational expectations equilibria based on agents coordinating on arbitrary beliefs. Following the literature on directed search, we restrict beliefs by assuming that searching managers believe that small variations in matching value are compensated by small variations in matching probabilities so that the expected application value remains constant. See Shi (2006). More specifically, if $\overline{Q}^*_t(h)$ is the value of the equilibrium contract, then for any $\overline{Q}_t(h)$ in a neighbourhood $\mathcal{N}(\overline{Q}^*)$ of $\overline{Q}^*_t(h)$, the following condition is satisfied,

$$\rho_t\Big(h, \overline{Q}_t(h)\Big) \cdot \Big[\overline{Q}_t(h) - \underline{Q}_t(h)\Big] = \rho_t\Big(h, \overline{Q}^*_t(h)\Big) \cdot \Big[\overline{Q}^*_t(h) - \underline{Q}_t(h)\Big], \tag{29}$$

where we have made explicit that the probability of a match depends on the value received by the manager. This condition says that managers are indifferent in applying to different employers who offer similar contracts since lower values are associated with higher probabilities of matching. In a competitive equilibrium with directed search, investors take $\overline{Q}^*_t(h)$ as given and choose the contract by solving the problem

$$\max_{\overline{Q}_t(h)} \quad \Big\{\phi_t\Big(h, \overline{Q}_t(h)\Big) \cdot V\Big(h, \overline{Q}_t(h)\Big)\Big\} \tag{30}$$

$$\text{subject to (29)},$$

where $V_t(h, Q)$ is the value for the investor. The analysis of the optimal contract after matching have shown that the investor's value is a function of the value promised to the manager. The equilibrium solution also provides the initial value of the contract for the investor $V_t(h, \overline{Q}_t(h))$.

For any h, if $\overline{Q}'_t(h)$ is also the value of an equilibrium contract, the investor must be indifferent: $\phi_t\Big(h, \overline{Q}'_t(h)\Big) \cdot V_t\Big(h, \overline{Q}'_t(h)\Big) = \phi_t\Big(h, \overline{Q}^*_t(h)\Big) \cdot V_t\Big(h, \overline{Q}^*_t(h)\Big)$. Therefore, we will only consider symmetric equilibria where investors offer the same contract $(h, \overline{Q}_t)$.

Furthermore, competition in posting vacancies implies that, for any level of human capital h, the following free entry condition must be satisfied in equilibrium,

$$\phi_t\Big(h, \overline{Q}_t(h)\Big) \cdot V_t\Big(h, \overline{Q}_t(h)\Big) = \tau h. \tag{31}$$

We can take advantage of the of the linear property of the model and normalize the above equations. We have shown that the value of a contract for the investor is linear in h, that is, $V_t\Big(h, Q_t(h)\Big) = v_t(q_t)h_t$. Therefore, the free entry condition can be rewritten in normalized form as

$$\phi_t(\overline{q}_t) \cdot v_t(\bar{q}_t) = \tau. \tag{32}$$

This takes also into account that we focus on a symmetric equilibrium in which the probability of filling a vacancy is independent of h (which justifies the omission of h as an explicit argument in the probability ϕ_t)[12].

The investor's problem (30) can be rewritten as

$$\begin{aligned} \overline{q}_t &= \arg\max_{q} \Big\{\phi_t(q) \cdot v_t(q)\Big\} \\ & \text{subject to} \\ & \rho_t(q)(q - \underline{q}_t) = \rho_t(\bar{q}^*_t)(\bar{q}^*_t - \underline{q}_t), \; \forall q \in \mathcal{N}(\bar{q}^*_t) \end{aligned}$$

We can solve for the normalized initial utility $\bar{q}_t$ by deriving the first order condition which can be rearranged as

$$1 - \eta = \frac{-v'_t(\bar{q}_t)(\bar{q}_t - \underline{q}_t)}{v_t(\bar{q}_t) - v'_t(\bar{q}_t)(\bar{q}_t - \underline{q}_t)}. \tag{33}$$

The right-hand side is the share of the surplus (in utility terms) going to the manager. Thus, the manager receives the fraction $1 - \eta$ of the surplus created by the match.

D Numerical solution

We describe first the numerical procedure used to solve Problem (17) for exogenous outside values $\underline{q}$ and $\overline{q}$ and for exogenous probability of offers ρ. We will then describe how they are determined in the steady state equilibrium.

Solving the optimal contract. The iterative procedure is based on the guesses for two functions

$$\begin{aligned} \gamma &= \psi(q) \\ v &= \Psi(q). \end{aligned}$$

The first function returns the multiplier γ (derivative of investor's value) as a function of the promised utility. The second function gives us the investor value v also as a function of the promised utility.

[12] In equilibrium only skilled workers who have never been employed in the financial sector will be actively searching. Since they have never been employed in the financial sector, they all have human capital h_0. For determining the probability of a match when a financial manager decides to quit, we incur the problem that the number of posted vacancies is discrete. In this case we assume that investors randomize over the posting of a vacancy that is targeted at a manager with human capital h.

Given the functions $\psi(q)$ and $\Psi(q)$, we can solve the system

$$\left[\beta v\Big(q(\bar{\varepsilon})\Big) + \left(\frac{\beta \mathcal{B}}{1+\lambda\bar{\varepsilon}}\right)\Big(\gamma + \chi + (1-\rho)\mu(\bar{\varepsilon})\Big)\right] p\bar{\varepsilon} = \kappa + \frac{\alpha(\gamma+\chi)}{1-\lambda} \tag{34}$$

$$c = \gamma \tag{35}$$

$$\left(\frac{\beta}{\beta}\right) g(\lambda,\varepsilon)\psi(q(\varepsilon)) = \gamma + \chi + \mu(\varepsilon) \tag{36}$$

$$v = y(\lambda) - c + \beta \sum_{\varepsilon} g(\lambda,\varepsilon)\Psi\Big(q(\varepsilon)\Big)p(\varepsilon) \tag{37}$$

$$q = \ln(c) + \alpha\ln(1-\lambda) + \beta\sum_{\varepsilon}\left(\mathcal{B}\ln\Big(g(\lambda,\varepsilon)\Big) + q(\varepsilon)\right)p(\varepsilon) \tag{38}$$

$$\chi\left\{\alpha\ln(1-\lambda) + \beta\sum_{\varepsilon}\left[q(\varepsilon) + \mathcal{B}\ln\Big(g(\lambda,\varepsilon)\Big)\right]p(\varepsilon) - \alpha\ln(1-\hat{\lambda}) - \beta\sum_{\varepsilon}\left[(1-\rho)\underline{q} + \rho\bar{q} + \rho\mathcal{B}\ln\Big(g(\hat{\lambda},\varepsilon)\Big)\right]p(\varepsilon)\right\} = 0 \tag{39}$$

$$\mu(\varepsilon)\left[q(\varepsilon) - (1-\rho)\underline{q} - \rho\bar{q} + (1-\rho)\mathcal{B}\ln\Big(g(\lambda,\varepsilon)\Big)\right] = 0 \tag{40}$$

The first three equations are the first order conditions with respect to λ, c, $q(\varepsilon)$, respectively. Equation (37) defines the value for the investor and equation (38) is the promise-keeping constraint. Equations (39) and (40) formalize the Kuhn-Tucker conditions for the incentive-compatibility and enforcement constraints.

Notice that equations (39) and (40) must be satisfied for all values of ε which can take two values. Therefore, we have a system of 9 equations in 9 unknowns: λ, c, v, γ, χ, $q(\varepsilon)$, $\mu(\varepsilon)$. Once we have solved for the unknowns we can update the functions $\Psi(q)$ and $\psi(q)$ using the solutions for v and γ.

Solving for the steady state equilibrium. The iterates starts by guessing the steady state values of $\bar{q}$ and ρ. Given these two values, we can determine $\underline{q}$ using equation (25). With these guesses we can solve for the optimal contract as described above. This returns the functions $v = \Psi(q)$ and $\gamma = \psi(q)$ in addition to $\lambda = \varphi^{\lambda}(q)$ and $q(\varepsilon) = \varphi^{q}(q,\varepsilon)$.

Once we have these functions we determine the new values of $\bar{q}$ and ρ using the free-entry condition (32) and the bargaining condition (33). We keep iterating until convergence, that is, the guessed values of $\bar{q}$ and ρ are equal to the computed values (up to a small approximation error).

E Derivation of inequality index

In each period there are different cohorts of active managers who have been employed for j periods. Because managers die with probability ω, the fraction of active managers in the j cohort (composed of managers employed for j periods) is equal to $\omega(1-\omega)^j$. Denote by h_j the human capital of a manager who have been employed for j periods. Since human capital grows at the gross rate $g(\hat{\lambda},\varepsilon)$, we have that $h_j = h_0 \Pi_{t=1}^{j} g(\hat{\lambda},\varepsilon_t)$. Of course, this differ across mangers of the same cohort because the growth rate is stochastic. The average human capital is then computed as

$$\bar{h} = \omega \sum_{j=0}^{\infty} (1-\omega)^j E_j h_j, \tag{41}$$

where E_j averages the human capital of all agents in the j-cohort. Because growth rates are serially independent, we have that $E_j h_j = h_0 E g(\hat{\lambda},\varepsilon)^j$. Substituting in the above expression and solving we get

$$\bar{h} = \frac{h_0 \omega}{1-(1-\omega)Eg(\hat{\lambda},\varepsilon)}.$$

We now turn to the variance which is calculated as

$$\text{Var}(h) = \omega \sum_{j=0}^{\infty} (1-\omega)^j E_j (h_j - \bar{h})^2.$$

This can be rewritten as

$$\text{Var}(h) = \omega \sum_{j=0}^{\infty} (1-\omega)^j \Big(E_j h_j^2 - \bar{h}^2 \Big).$$

Using the serial independence of the growth rates, we have that $E_j h_j^2 = h_0^2 [Eg(\hat{\lambda},\varepsilon)^2]^j$. Substituting and solving we get

$$\text{Var}(h) = \frac{h_0^2 \omega}{1-(1-\omega)Eg(\hat{\lambda},\varepsilon)^2} - \bar{h}^2$$

To compute the inequality index we simply divide the variance by $\bar{h}^2$, where $\bar{h}$ is given by (41). This returns the inequality index (26).

To separate the *within* and *between* components of the inequality index, let's first rewrite the formula for the variance of h as follows:

$$\text{Var}(h) = \omega \sum_{j=0}^{\infty} (1-\omega)^j \Big[(E_j h_j^2 - \bar{h}_j^2) - (\bar{h}_j^2 - \bar{h}^2) \Big],$$

where $\bar{h}_j = E_j h_j = h_0 Eg(\hat{\lambda}, \varepsilon)^j$ is the average human capital for the j cohort. Substituting the expression for h_j and $\bar{h}_j$ and solving we get

$$\begin{aligned}\text{Var}(h) \quad &= \quad \left(\frac{h^2\omega}{1-(1-\omega)Eg(\hat{\lambda},\varepsilon)^2} - \frac{h^2\omega}{1-(1-\omega)(Eg(\hat{\lambda},\varepsilon))^2} \right) + \\ &\quad \left(\frac{h^2\omega}{1-(1-\omega)(Eg(\hat{\lambda},\varepsilon))^2} - \bar{h}^2 \right)\end{aligned}$$

Dividing by $\bar{h}^2$ using the expression for $\bar{h}$ derived in (41), we are able to write the inequality index as

$$\begin{aligned}\text{Inequality index} \quad &= \quad \left(\frac{[1-(1-\omega)Eg(\hat{\lambda},\varepsilon)]^2}{\omega[1-(1-\omega)Eg(\hat{\lambda},\varepsilon)^2]} - \frac{[1-(1-\omega)Eg(\hat{\lambda},\varepsilon)]^2}{\omega[1-(1-\omega)(Eg(\hat{\lambda},\varepsilon))^2]} \right) + \\ &\quad \left(\frac{[1-(1-\omega)Eg(\hat{\lambda},\varepsilon)]^2}{\omega[1-(1-\omega)(Eg(\hat{\lambda},\varepsilon))^2]} - 1 \right) \qquad (42)\end{aligned}$$

The first term is the *within* cohorts inequality while the second term is the *between* cohorts inequality. Both terms are strictly increasing in $\hat{\lambda}$.

■

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