Choice, internal consistency, and rationality*

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Abstract.

Classical rational choice theory is built on several important internal consistency conditions. In recent years, the reasonableness of those internal consistency conditions have been questioned and criticized, and several responses to accommodate such criticisms have been proposed in the literature. This paper develops a general framework to accommodate the issues raised by criticisms of classical rational choice theory, and examines the broad impact of the criticisms on rational choice theory from both a normative and positive point of view.

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1. Introduction

The literature on the theory of choice and preference contains a large number of "internal consistency conditions", such as Chernoff's condition (see Chernoff 1954), the weak axiom of revealed preference (see Samuelson 1938, 1947, 1948, and Arrow 1959), the strong axiom of revealed preference (see Houthakker 1950, and Arrow 1959), and the congruence axiom (see Richter 1966, 1971). Typically, these conditions take the following general form: if an agent chooses (or does not choose) certain options from sets $A, B, \ldots$, of feasible options, then the agent will (or, alternatively, will not) choose certain options from sets $A', B', \ldots$, of feasible options.[1] The conditions play dual conceptual roles in the standard theory of choice. First, they are treated as properties of rational choice, the (often implicit) claim being that, if the agent is 'rational', then her choices must satisfy these conditions. Second, they are also treated as testable hypotheses regarding the agent's choice behavior. The focus of this paper is on the former interpretation though we also comment briefly on the latter interpretation.

While the internal consistency conditions have been widely accepted as conditions that a rational agent should satisfy, from time to time examples have appeared in the literature to question that position. The earliest examples that we know of are to be found in Luce and Raiffa (1957). More recently, Sen (1993) has introduced some further examples in the same spirit and has argued that the reasonableness or intuitive appeal of these conditions cannot be judged without referring to the motives and objectives of the agent making choices. The examples of Luce and Raiffa (1957) and Sen (1993) pose a challenge to the standard theory of rational choice. In particular, they raise the following two questions. First, is there a general reformulation of the conventional theory of rational choice that can accommodate the examples of Luce and Raiffa and Sen? Second, if at all one can find such a reformulation, will it constitute an adequate response to Sen's argument that internal consistency conditions by themselves cannot constitute adequate intuitive criteria for assessing whether the agent's choices are rational?

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[1] For example, the weak axiom of revealed preference, one of the most well-known of such conditions, says that, if, given a set of options that contains both *x* and *y*, the agent chooses *x* and rejects *y*, then, given any other set *B* that contains x, the agent does not choose *y*.

The main purpose of this paper is to explore these two issues. We first develop a general framework, which is a reformulation of the conventional theory, to accommodate the examples of Luce and Raiffa and Sen. Next we argue that, though our reformulation of the conventional theory, as well as other less general reformulations[2] in the existing literature, can take care of the problem of internal inconsistency of choice in the examples of Luce and Raiffa (1957) and Sen (1993), this does not in any way detract from Sen's basic argument that the reasonableness of internal consistency conditions as conditions for rational choice cannot be judged without going into the agent's motives and objectives.

The plan of the paper is as follows. In Section 2, we introduce some basic notation and definitions. In section 3, we present the examples of Luce and Raiffa and Sen and discuss their structural features. In Section 4, we develop a general framework that can accommodate the examples due to Luce and Raiffa and Sen. In Section 5, we comment on several existing formulations that deal with issues raised by the examples showing violations of internal consistency conditions. In Section 6, we provide an assessment of what we believe to be the central point of Sen's analysis, namely, that the reasonableness of internal consistency conditions for choice cannot be decided without referring to the objectives and motives of the agent making the choices. Section 7 contains brief remarks on our model and on some broader issues relating to the theory of rational choice.

2. The basic notation and definition

An agent *i* is characterized by a triple $<Y, \mathcal{Y}, F>$, where Y is a given non-empty set of options, $\mathcal{Y}$ is a non-empty class of non-empty subsets of Y, and F is a function, which, for every $A \in \mathcal{Y}$, specifies exactly one non-empty subset of A, to be denoted by $F(A)$. $\mathcal{Y}$ is to be interpreted as the different (non-empty) sets of feasible alternatives (*menus* or *opportunity sets*) with which the agent under consideration may be confronted, and, given a menu, $A, F(A)$ is to be interpreted as the set of options that the agent chooses from A. We call F the *choice function* of the agent.

[2] See Section 5 for a discussion of some of these reformulations.

For all $A \in \mathcal{Y}$, a binary relation $R(A)$ over A is defined as follows: for all $x, y \in Y$, $[xR(A)y$ iff $x \in F(A)$ and $y \in A]$. The asymmetric part of $R(A)$ is denoted by $P(A)$ and is defined as: $[xP(A)y$ iff $xR(A)y$ and not $yR(A)x]$. Let R be a binary relation over Y defined as follows: for all $x, y \in Y$, $[xRy$ iff there exists $A \in \mathcal{Y}$, such that $xR(A)y]$ and $[xPy$ iff $(xRy$ and not $yRx)]$. $\mathcal{R}$ is the transitive closure of R, i.e. , for all $x, y \in X, x\mathcal{R}y$ iff there exist $z_1, z_2, \ldots, z_n \in Y$, such that ($x = z_1$ and $y = z_n$) and ($z_1 R\, z_2$, $z_2 R z_3$, … , and $z_{n-1} R z_n$).

<u>Definition 2.1.</u> Consider an agent characterized by $<Y, \mathcal{Y}, F>$. The choice function F (or, equivalently, the agent) satisfies:

(i) *Chernoff's condition* iff , for all $A, B \in \mathcal{Y}$ and all $x, y \in Y$, if $A \subseteq B$ and $xP(A)y$, then not $yR(B)x$;

(ii) the *weak axiom of revealed preference* iff, for all $A, B \in \mathcal{Y}$ and all $x, y \in A \cap B$, $xR(A)y$ implies not $yP(B)x$;

(iii) the *congruence axiom* iff for all $x, y \in Y$, if $x\mathcal{R}y$, then not yPx;

(iv) *rationalizability in terms of an ordering* iff there exists an ordering, $\succcurlyeq$, defined over Y, such that, for all $A \in \mathcal{Y}$, $F(A) is\ the\ set\ of$ $\succcurlyeq$-greatest elements in A.

The properties introduced in Definition 2.1 are very familiar in the literature[3]. The first three of these properties, Chernoff's condition, the weak axiom of revealed preference, and the congruence axiom are in ascending order of logical strength, and the congruence axiom is a necessary and sufficient condition for the choice function F to be rationalizable in terms of an ordering (see Richter 1966). Chernoff's condition, which requires that, if an option x is revealed to be strictly better than an option y in a set A, then when the set A is enlarged to a set B, y cannot be revealed to be at least as good as x in the set B, is the weakest of the four properties and seems highly plausible. As we shall, however, see in Examples 3.1 to 3.4 below, when choices are menu-dependent, this condition becomes immediately questionable. The weak axiom of revealed preference requires that, whenever an option x is revealed to be at least as

[3] See Chernoff (1954) for Chernoff's condition; Samuelson (1938, 1947) for the weak axiom of revealed preference, and Richter (1966) for the congruence axiom.

good as an option y in a set A, then y cannot be revealed to be better than x in another set B. The congruence axiom says that if an option x is revealed to be at least as good as an option y through possibly a chain of feasible sets, then y cannot be revealed to be better than x in any set. Rationalizability of F in terms of an ordering, which is logically equivalent to the congruence axiom for F, requires that the agent's choices should be compatible with the notion of choice on the basis of a preference ordering.

3. Examples

Example 3.1 (Luce and Raiffa 1957). Consider two different Nash-bargaining problems illustrated in Figure 3.1 below. The utility functions of the two individuals are the same in

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Insert Figure 3.1

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both problems. The first bargaining problem, where the threat point is *o* and the set of feasible pairs of utility numbers is *oab*, is presented to the arbitrator first and he declares *s* to be the solution. The second bargaining problem has the same threat point as the first, but the set of feasible pairs of utility numbers is *ocsb*. For the second bargaining problem, the arbitrator declares the solution to be *s'*. These (ethical) choices of the arbitrator violate Chernoff's condition (insofar as *s'* is chosen and *s* is rejected for the smaller set *ocsb*, while *s* is chosen and *s'* is rejected in the larger set *oab*. The intuitive reason for switching the choice corresponding to *ocsb* from *s* to *s'* that Luce and Raiffa consider is as follows. In declaring the solution for a given bargaining problem, the arbitrator takes into account the extent to which, at the solution point, each party will fall short of the maximum utility that she can possibly get in the feasible set (intuitively, this may be thought of as the degree of her 'disappointment' at the solution point). For the bargaining problem where the feasible set is *oab*, the arbitrator gives the solution *s* where the pair of shortfalls (from the maximum possible utilities) of the two parties are given by (*ac, bd*). If, for the second problem the arbitrator keeps the solution unchanged at *s*, then the shortfalls of the two parties will be (0, *bd*) and, in that case, 2 will not have any disappointment at all. Guided by this consideration, the arbitrator shifts the solution for the second problem to *s′* where each of 1 and 2 feels some disappointment.

Example 3.2 (Sen 1993). In a party, when the fruit tray comes to an individual, there are several pears and one apple in the tray. He chooses a pear. If, however, the tray had an additional apple, he would have chosen an apple. The individual's choices violate Chernoff's condition. The explanation lies in the fact that, when the tray contains only one apple, choosing it violates social norms ("a polite person does not pick up a fruit if it happens to be the single fruit of its type in the tray").

Note that, in Examples 3.1 and 3.2, in judging an option, the agent is using a criterion (possibly, with other criteria) that depends on the set of feasible alternatives from which the option is being chosen. In Example 3.1, such a criterion is provided by the shortfalls from the maximum possible utilities in the feasible set, and, in Example 3.2, the criterion of fulfilling social norms depends on whether the chosen fruit is the single fruit of its type in the feasible set. These two examples illustrate what we shall call the agent's *menu-dependent criteria* for judging options.

Example 3.3 (Luce and Raiffa 1957). The waiter in a restaurant gives a customer a menu for the day's dishes, which has two items: steak and fish. The customer orders fish. The waiter subsequently reports that, because of a mistake, frog's legs and fried snails have been omitted from the day's menu but they are available. The customer then orders steak. Again, this is a violation of Chernoff's condition. The intuition is that the customer would choose steak rather than fish or fried snails or frog's legs if he has some assurance that the restaurant is good ("an indifferent restaurant would not know how to handle steak") and the customer's experience tells him that fried snails and frog's legs are served only by good restaurants.

Example 3.4 (Sen 1993). A man and a woman have met only a few hours ago. The man asks the woman whether she would like a cup of coffee with him. The woman accepts the suggestion rather than going home. The man next tells her that there is also another possibility: she could have coffee with him or, if she would like to do so, she could also smoke marijuana with him. The woman now declines both the options and chooses to go home. Such choices, of course, violate Chernoff's condition. The additional option of smoking marijuana that the man offers gives the woman more information about the man, and, not only does she care about whether or not she takes coffee with someone, but she also cares about what sort of person she takes her coffee with.

In Examples 3.3 and 3.4, the menu from which the agent makes her choice gives her more information about the options. They constitute examples of what we shall call *menu-based information*.[4] Note that, Examples 3.3 and 3.4 share one feature of Examples 3.1 and 3.2. In Examples 3.3 and 3.4, as in Examples 3.1 and 3.2, the agent has concerns that are not captured in the original descriptions of the options: the description "steak", in itself, does not capture the quality of the steak with which the agent is concerned in Example 3.3; nor does the description, "taking tea with Mr. j" say anything about the type of person one is having tea with - a criterion or concern of the agent in example 3.4. There is, however, one difference between Examples 3.1 and 3.2 on the one hand and Examples 3.3 and 3.4 on the other. Neither the criterion of who is making how much sacrifice compared to the maximum feasible utility (see Example 3.1) nor the criterion represented by the social norm of not picking up a last fruit of its type (see Example 3.2) can be articulated without referring to the menu under consideration. In Example 3.3, however, the criterion of quality can, in principle, be articulated without any reference to the menu where "steak" figures; similarly, the criterion, in Example 3.4, of not mixing with drug addicts also can be stated without any reference to the menu where the option "having tea with Mr. j" figures. Thus, the criteria under consideration in Examples 3.3 and 3.4 are definable without any reference to the menu; what seems to be menu-dependent in Example 3.3 (resp. example 3.4) is the agent's (imperfect) information, at the time of choosing, about the fulfillment of the relevant criterion when he chooses option "steak" (resp.

[4] In this paper, we focus on some conceptual issues that arise in the presence of menu-dependent criteria and menu-dependent information. Similar issues, however, can arise even in the absence of menu-dependent criteria/ information when the agent's choice from a given menu is influenced by the state of the world and the state of the world is not a part of the description of the options. A consumer, who chooses cold salad over hot soup if the weather is very warm but chooses hot soup over cold salad if the weather is cold, manifests such 'state-dependence' (see Bandyopadhyay, Dasgupta, and Pattanaik 1999, 2004).

the option “having tea with Mr. j”)[5]. This difference provides the basis of our distinction between situations of “menu-dependent criteria” and situations of “menu-dependent information”.

Examples 3.1 through 3.4 show that, in certain fairly plausible situations, the agent’s choice function can violate Chernoff’s conditions, which constitutes one of the weakest of consistency conditions discussed in the literature. [6] Yet, in none of these examples, the agent acts in a way that one can reasonably call ‘irrational’. While Luce and Raiffa presented their examples to show how reasonable agents may violate internal consistency conditions for choice, Sen used his counterexamples to make a broader methodological point, namely that, the reasonableness of the internal consistency properties of choice cannot be determined without considering the criteria or motives behind the agent’s choices. In Examples 3.2 and 3.4, as well as in Examples 3.1 and 3.3 due to Luce and Raiffa, the agent’s violation of Chernoff’s condition does not seem irrational at all once we know the reasons behind the agent’s choices. The conclusion that Sen sought to draw from his examples was that it is not possible to formulate the theory of rational choice exclusively in terms of internal consistency of choices without going beyond choice as such to explore the criteria or motives guiding the agent’s choices.

Examples 3.1 through 3.4 naturally raise the following issues.

Issue 1. Can the phenomena described in these examples be accommodated in the standard framework of revealed preference theory by plausibly reformulating the model so that the consistency conditions will not be violated in the reformulated model?

Issue 2. If at all it is possible to reformulate the model so as to accommodate the phenomena described in the examples without any violation of internal consistency conditions in the reformulated model, then what are the implications of such reformulation for the basic

[5] As we explain in Section 4, in Examples 3.3 and 3.4 the agent can be seen as choosing in a situation of uncertainty.

[6] Note that, not only do the agent’s choices violate Chernoff’s condition in each of the examples, they also violate the following consistency condition, which is even weaker:

for all $A, B \in \mathcal{Y}$ and all $x, y \in Y$, if $A \subseteq B$ and $xP(A)y$, then not $yP(B)x$.

methodological point raised by Sen, to wit, whether one can discuss the reasonableness of internal consistency conditions without going beyond the concept of choice to look into the objectives guiding the agent's choices?

Issue 3**.** The problems raised by Examples 3.1 through 3.4 have been typically discussed in the context of the theory of revealed preference where one starts with the primitive notion of choice rather than with preferences of the agent. Do similar problems arise in models where one starts with the primitive notion of preference?

Issue 4**.** Finally, what happens if we treat the consistency conditions for choice as empirically testable hypotheses rather than as normative conditions for the agent's 'rationality'?

In Section 4, we take up the first issue. We show that it is indeed possible to adapt the basic model so that the agent's behavior, when considered in the adapted version of the original model, will not violate the relevant internal consistency condition for choice. In Section 6, we take up the other three issues noted above.

4. A general framework to handle the phenomena of menu-dependent criteria and menu-dependent information

Faced with the violation of internal consistency conditions in situations involving menu-dependent criteria, one of the typical responses of theorists has been to reformulate some or all the characterizing features (the universal set of alternatives, the set of potential menus, and the choice function) of the agent in a way that gets rid of such violation. In this section, we present a reformulation of the conventional theory to accommodate Examples 3.1 and 3.2 discussed earlier. Though, for the purpose of our discussion, we focus on the case of menu-dependent criteria, at the end of this section we indicate how similar reformulations can deal with menu-dependent information (Examples 3.3 and 3.4).

In Example 3.1, as well as in Example 3.2, the agent uses a menu-dependent criterion to assess the options available in alternative menus. Every such criterion refers to some features of the choice of a given alternative from a given menu. Thus, in Example 3.1, the criterion for possible distinction between the choice of an option, say, x, from the set, A, of feasible options and the choice of the same option from another set, B, of feasible options is based on the shortfalls of the two individuals' utilities from the maximum possible utilities for them in the sets A and B. If, given x, the shortfalls in the case of A are the same as the shortfalls in the case of B, then there is no basis for distinguishing between the choice of x from A and the choice of x from the set B so far as the given criterion is concerned. In Example 3.2, the criterion refers to choosing the single fruit of its type. Of course, the criterion could have been different: it could have been choosing an apple when there is only apple in the fruit tray without any reference to the choice of any other fruit; it could have been choosing a mango from a fruit tray containing only one mango when there are older persons at the table (social convention may dictate that, in the case of mangoes only, one has to defer to older persons at the table!), and so on. The analytical strategy that we adopt is to introduce the notion of *indistinguishability/ distinguishability*, in terms of the relevant features, between the choice of an option from one opportunity set and the choice of the same option from another opportunity set.

Consider a case of menu-dependent criteria, where $\Omega \equiv <X, \mathcal{X}, C>$ is the 'initial' characterization of the agent ($X, \mathcal{X}$, and C being, respectively, the universal set of options, the class of potential menus, and the agent's choice function), and C violates some internal consistency condition, say, Chernoff's condition. In Example 3.1, X is the set of all feasible pairs of utility payoffs, and, in Example 3.2, it is the set of all possible different fruits. Let X' be the set of all x in X, such that, for some $A \in \mathcal{X}$, $x \in A$. For all $x \in X'$, let $\sim_x$ be a reflexive, symmetric, and transitive binary relation defined over the set Z_x, where Z_x is defined as the set of all $ordered$ pairs (x, A) such that $x \in A \in \mathcal{X}$; thus, $\sim_x$ is an equivalence relation defined over Z_x). Our intended intuitive interpretation of $(x, A) \sim_x (x, B)$ is that, in terms of the descriptive features referred to by the menu-dependent criteria (or, criterion) under consideration, choosing x from the set A is *indistinguishable* from choosing x from the set B. For every $x \in X^{'}$, $\sim_x$ is assumed to be exogenously given. Note that, for distinct alternatives $x, y \in X'$, we have not introduced any notion of distinguishability or otherwise between choosing x from a set $A \in \mathcal{X}$ and choosing y from a set $B \in \mathcal{X}$ (irrespective of whether or not A and B are identical) because we do not need this additional information for our purpose.

It may be worth clarifying the intuitive content of the equivalence relations $\sim_x$, $\sim_y$, etc. Our notion of equivalence relations such as $\sim_x$, $\sim_y$, etc., is different from the concept of an ordering over $\{(a, A): a \in A \in \mathcal{X}\}$, that makes comparisons of the type "the choice of x from the menu A fulfills the menu-based criteria at least much as the choice of b from the menu B". Not only is it true that our equivalence relations, $\sim_x$, $\sim_y$, …, do not presuppose, either formally or intuitively, any *ordering* over $\{(a, A): a \in A \in \mathcal{X}\}$, but the intuitive interpretation that we have for these equivalence relations is also very different from the notion of fulfilling the menu-based criteria to the same extent. An example may help to make this clear. Suppose that the menu-based criteria consist of: (i) a social norm of politeness, which forbid the choice of a single pear in the fruit tray in a party; and (ii) another social norm of politeness, which requires that, even when there are many apples and many pears in the fruit tray, a guest should not choose a pear if the hostess has already mentioned that, while she purchased the pears in the market, the apples come from her trees, which she has grown with much care and fondness. Further, suppose picking up the single pear in the tray is generally regarded as just as much of a lapse in manners as choosing a pear when there are many pears and many apples but the apples are known to have come from the hostess's favorite apple trees. In this case, we would not consider the choice of the single pear in the tray to be indistinguishable (in terms of the descriptive features relevant for the purpose of social norms) from the choice of a pear when the apples are known to have come from the hostess's apple trees, though it may be generally accepted that the two choices violate social norms "to the same degree". Given the features relevant for the social norms, our notion of indistinguishability between (x, A) and (x, B) in Z_x refers to indistinguishability in terms of these descriptive features rather than in terms of identical degrees of violation of social norms. Indistinguishability in terms of the relevant descriptive features intuitively entails identical degrees of violation of social norms, but the converse is not necessarily true. Note that, though we have chosen to interpret our equivalence relations as relations indicating indistinguishability in terms of the descriptive features relevant for social norms and not as relations indicating identical degrees of violation of social norms, it may be noted that much of what we say below applies to both these interpretations.

For all $x \in X'$ and all $(x, A) \in Z_x$, let $E[x, A]$ be the equivalence class of (x, A) defined by $\sim_x$,i.e., $E[x, A]$ is the class of all $(x, B) \in Z_x$, such that $(x, B) \sim_x (x, A)$. Having introduced the relevant equivalence relations and the corresponding equivalence classes, we can now transform the initial characterization, $\Omega \equiv < X, \mathcal{X}, C >$, of the agent in our example of menu-dependent criteria into a new characterization $\Omega_* \equiv < X_*, \mathcal{X}_*, C_* >$ specified as follows:

X_* is the set of all ordered pairs $(x, E[x, A])$ such that $x \in A \in \mathcal{X}$; (4.1)

$\mathcal{X}_*$ is the class of all A_* such that for some $A \in \mathcal{X}$, $A_* = \{(x, E[x, A]) : x \in A\}$; (4.2)

C_* is a function, which, for every $A_* \in \mathcal{X}_*$, specifies the unique set $\{(z, E[z, A]) : A \in \mathcal{X};\ A_* = \{(x, E[x, A]) : x \in A\};$ and $z \in C(A)\}$. (4.3)

(Note that, for every $A_* \in \mathcal{X}_*$, there must exist a unique $A \in \mathcal{X}$ such that $A_* = \{(x, E[x, A]) : x \in A\}$).

It is clear that, given the equivalence relations, there exists a unique triple $< X_*, \mathcal{X}_*, C_* >$ satisfying (4.1), (4.2) and (4.3). Essentially, what is involved in the transition from the characterization of the agent in terms of $X, \mathcal{X}, C$ to her characterization in terms of $< X_*, \mathcal{X}_*, C_* >$ is the replacement of the original notion of an option by an 'extended' notion of an option to capture an aspect that is relevant for the agent's choices but does not figure in the original description of an option. The concept of the class of potential menus, as well as the concept of options chosen from a menu, is then adjusted accordingly.

How does our reformulated framework tackle the violation of Chernoff's condition considered in Examples 3.1 and 3.2? To answer this question, we first note the following result. Its proof can be found in the appendix.

Proposition 4.1. Let $< X, \mathcal{X}, C >$ be the initial characterization of the agent, and let $< X_*, \mathcal{X}_*, C_* >$ be the modified characterization satisfying (4.1), (4.2), and (4.3).

(i) If C satisfies any of the properties (Chernoff's condition, the weak axiom of revealed preference, the congruence axiom, and rationalizability in terms of an ordering) introduced in Definition 2.1, then C_* satisfies the same condition.

(ii) It is, however, possible for C_* to satisfy all the properties introduced in Definition 2.1 while C violates Chernoff's condition, the weakest of these properties.

Proposition 4.1 shows that the requirement of any of the properties, Chernoff's condition, the weak axiom of revealed preference, the congruence axiom, and rationalizability in terms of an ordering, for the choice function C_* figuring in the new characterization $\Omega_* \equiv < X_*, \mathcal{X}_*, C_* >$ is weaker than the requirement of the same property for the choice function C figuring in the initial characterization $\Omega \equiv < X, \mathcal{X}, C >$. Therefore, when C_* satisfies, say Chernoff's condition but C does not, we can say that, though the agent, characterized by $< X, \mathcal{X}, C >$, violates Chernoff's condition, the violation is really due to the fact that he regards the choice of an option x from a feasible set A to be distinguishable from the choice of the same option x from a subset B of A, and when the problem is re-formulated to take into account such "distinguishibility" of acts of choosing in addition to the features of the initial options, the violation of Chernoff's condition disappears in the re-formulated choice problem.

Having reformulated the initial characterization, $\Omega \equiv < X, \mathcal{X}, C >$, of the agent so as to derive the new characterization, $\Omega_* \equiv < X_*, \mathcal{X}_*, C_* >$, which satisfies (4.1), (4.2), and (4.3), we can now define a modified notion of rationalizability of the initial choice function C in terms of an ordering. The choice function C is said to satisfy Ω_*-based *rationalizability in terms of an ordering* iff C_* is rationalizable in terms of an ordering.

The following proposition follows immediately from the definition of Ω_*-based rationalizability in terms of an ordering and the fact that, for every choice function, the congruence axiom is equivalent to rationalizability of that choice function in terms of an ordering (see Richter 1966).

Proposition 4.2. Let $< X, \mathcal{X}, C >$ be the initial characterization of the agent, and let $< X_*, \mathcal{X}_*, C_* >$ be the modified characterization satisfying (4.1), (4.2), and (4.3). C satisfies Ω_*-based rationalizability in terms of an ordering iff C_* satisfies the congruence axiom.

It may be noted that, when, for all $x \in X'$ and all $(x, A), (x, B) \in Z_x$, $(x, B) \sim_x (x, A)$, the Ω_*-based rationalizability of C becomes equivalent to the standard rationalizability of C.

On the other hand, when, for all $x \in X'$ and all $(x, A), (x, B) \in Z_x$, $(x, B) \sim_x (x, A) \Rightarrow (x, B) = (x, A)$, *any choice function C is* Ω_**-based rationalizable (by an ordering)*.

Before concluding this section, we indicate briefly how the case of menu-dependent information can be handled by a variant of the modeling strategy discussed above. For convenience in exposition, we concentrate on Example 3.3. Here the choice of a dish can be intuitively thought of as being associated with an uncertain prospect; for some dishes, the uncertain prospects may be trivial (i.e., they may really be certain prospects), but for others, such as steak, the uncertain prospect is non-trivial. The uncertainty involved may be probabilistic (for example, the uncertain prospect may be a lottery with probability p for getting high quality steak and probability $1 - p$ for getting low quality steak) or non-probabilistic. The important point is that the uncertain prospect, however conceived, that is associated with the choice of a dish, such as steak, can change, depending on the menu from which the option is chosen. Thus, assuming for the moment that the uncertainty involved is probabilistic, the agent's (subjective) probability for high quality steak can go up when he learns that the menu also includes frogs' legs and fried snail. Given this, for all $x \in X'$, we can now introduce an equivalence relation $\sim_x$ over Z_x with the following interpretation: $(x, A) \sim_x (x, B)$ means that the uncertain prospect that the agent associates with x when the menu is A is the same as the uncertain prospect that the agent associates with x when the menu is B. We can then suitably transform the original characterization of the agent into a new characterization, where the equivalence classes, $E[x, A], E[y, B]$, etc., constitute the re-specified options (one can think of a different interpretation of $\sim_x$, under which the specification of the new options as $(x, E[x, A]), E(y, E[y, B])$, etc., would make sense, but we find the interpretation of $\sim_x$ given above more direct and natural).

5. Some alternative reformulations

As we mentioned earlier, faced with the violation of internal consistency conditions in situations involving menu-dependent criteria, a typical response by theorists is to reformulate some or all the characterizing features, including the universal set of alternatives, the set of potential menus, and the choice function, of the agent in such a way that, such violation no longer occurs in the new formulation. In what follows, we consider a few alternative strategies for such reformulation to be found in the existing literature. [7]

5.1. Re-specification of the set of potential menus

In an interesting paper, Bossert and Suzumura (2009a) present a formulation that re-constructs the set of potential menus. They focus on Example 3.2 of Sen where the 'anomalous' choice behavior of the agent arises from the agent's concern about social norms, and suggest a formal device through which such social norms can be incorporated in the model.[8] Without going into the details of their formal analysis, we outline here the intuition underlying their framework. In Example 3.2, the violation of Chernoff's condition occurs in a framework where the agent i is characterized by $< X, \mathcal{X}, C >$ such that the universal set of options, X, is the set of all possible fruits, and the class of potential menus, $\mathcal{X}$, are alternative sets of fruits that the agent may have to choose from. The agent's choice behavior embodied in the choice function is such that, for all $A \in \mathcal{X}$ and all $x \in A$, if x is the only fruit of its type in A and A contains several fruits of some other type, then $x \notin C(A)$. Essentially, the analysis of Bossert and Suzumura can be interpreted as a transformation of the original characterization, $< X, \mathcal{X}, C >$, of the agent into another characterization where the universal set of options continues to be X, but the set of potential menus and the choice function are re-specified. There are several intuitive assumptions underlying this re-specification.

(i) First, it is assumed that, for every $A \in \mathcal{X}$, the social norms specify a unique (possibly empty) subset A' of A, the interpretation of A' being that, given the menu A, A' is the set of all options in A that the social norms forbid the agent to choose from A (if A' is empty, then the norms do not forbid the choice of any option in A). Thus, social norms constitute a function, g, which, for every $A \in$

[7] We focus on responses that are based on various notions of rationalizability in terms of a single ordering. See Kalai, Rubinstein and Spiegler (2002) for a response based on rationalizability by multiple orderings.

[8] In a related paper, Bossert and Suzumura (2009b) discuss a framework dealing with issues raised by Examples 3.3 and 3.4. The essence of their framework is to re-specify the options as uncertain prospects in a setting of non-probabilistic uncertainty and then to reconstruct the menus in terms of the newly specified options. Given such re-speccified options and menus, the violation of Chernoff's condition disappears.

$\mathcal{X}$, specifies exactly one subset $g(A)$ of A, $g(A)$ being the set of options in A, which the agent is forbidden to choose.

(ii) It is assumed that, for all $A \in \mathcal{X}$, $A - g(A)$ is non-empty so that every potential menu contains some options that the agent can choose from that menu without violating the social norms.

(iii) The agent characterized by $< X, \mathcal{X}, C >$ always conforms to social norms, so that, given a menu A in $\mathcal{X}$, $C(A) \subseteq A - g(A)$.

Given these assumptions, and given any internal consistency condition, σ, one can say that C *satisfies* σ *subject to the constraint imposed by the norm* g iff there exists a triple $< X_+, \mathcal{X}_{+,} C_+ >$ such that

X_+=X; (5.1)

$\mathcal{X}_+$ is the set of all non-empty subsets A_+ of X_+ such that, for some $A \in \mathcal{X}, A_+$=$A - g(A)$; (5.2)

C_+ is a function, which, for every $A_+ \in \mathcal{X}_+$, specifies a non-empty subset $C_+(A_+)$ of A_+, such that $C_+(A_+) = C(A)$; (5.3)

and

C_+ satisfies the internal consistency condition σ. (5.4)

It is easy to see that the following condition, (5.5), is a necessary and sufficient condition for the existence of a triple $< X_+, \mathcal{X}_{+,} C_+ >$ satisfying (5.1), (5.2), and (5.3):

for all $A, B \in \mathcal{X}$, if $A - g(A) = B - g(B)$, then $C(A) = C(B)$, (5.5)

and that, if there exists a triple $< X_+, \mathcal{X}_{+,} C_+ >$ satisfying (5.1), (5.2), and (5.3), then it must be the unique triple satisfying (5.1), (5.2), and (5.3). What this means is that, if (5.5) is satisfied, then one can canonically construct another characterization of the agent such that, for every possible menu A, in the initial characterization, the set of options that the agent chooses from A in the initial characterization is the set of options that the agent chooses from the menu $A - g(A)$ in the new charaterization. Thus, when the choice function in the new characterization satisfies the internal consistency condition σ, we can say that, though i , characterized by $< X, \mathcal{X}, C >$, may be violating σ, the violation is due to the fact that, given a menu, he treats the options forbidden by the norms as "socially infeasible", and, when the menus are re-specified to take into account such "social infeasibility" in addition to physical infeasibility, the violation of σ disappears in the re-specified choice problem.

It may be noted the framework proposed by Bossert and Suzumura (2009a) is a special case of our general framework. To see this, we note the following. Consider the equivalence relation $\sim_x$ as follows: for all $x \in X'$ and all $(x, A), (x, B) \in Z_x$, $x \in \big(A - g(A)\big) \cap \big(B - g(B)\big) \Rightarrow (x, B) \sim_x (x, A)$. It is then straightforward to see that Ω_*-based rationalizability of C becomes equivalent to the rationalizability of C subject to the constraints imposed by social norms.

While the analytical strategy used by Bossert and Suzumura (2009a) to tackle the violation of internal consistency conditions in Example 3.2 is of considerable interest, it is based on several strong intuitive assumptions, which severely restrict its applicability. First, consider the case where the menu-based criteria happen to be social norms. The notion of social norms simply forbidding the choice of certain options from a given menu may be plausible, but the assumption that the agent never chooses an option forbidden by social norms is highly restrictive outside the limited context of Sen's specific example. Even if we leave aside the case of extreme non-conformists who may make it a point to violate social norms, the assumption is implausible. After all, conforming to social norms is only one of many different considerations that an individual takes into account in making a choice from a given menu and there is no compelling reason why an individual would give overwhelming priority to this consideration. If the choice of an alternative x from a menu A is sufficiently attractive in terms of other considerations, the individual may decide to choose x from A at the cost of violating certain social norms. If i is extraordinarily fond of mangoes and i has not eaten a mango for the last five years, then he may choose a mango from a fruit tray in a party even when it is the only mango in the tray, though he would not choose a single pear left in a fruit tray, given that he does not hanker for a pear as much as he hankers for a mango. In reconstructing the theory of rational choice to accommodate menu-based criteria such as social norms, one should not have to depend on the restrictive assumption that the agent never violates social norms.

A somewhat less serious objection to the analytical strategy of Bossert and Suzumura (2009a) is that, while, in Example 3.2, there is some intuitive plausibility in the notion that the social norms involved forbid the choice of certain options from some menus, the notion of certain choices from a given menu being forbidden is much less natural in other examples of menu-dependent criteria. Thus, in Example 3.1, the shortfalls from the maximum possible utilities may constitute one of the relevant considerations for the arbitrator, but there does not seem to be an intuitively natural way in which this menu-dependent criterion can be conceived as absolutely forbidding certain solutions in Nash-bargaining problems.

5.2 Re-specification of options to make the menu a part of the description of an option

Another way of handling the phenomenon of menu-dependent criteria and menu-dependent information may be to redefine the options so as to make the menu itself a part of the description of the newly defined options or alternatives[9]. For example, in Example 3.2, one can redefine the options so that the newly defined option is the act of choosing a particular fruit from a given fruit tray. Once this is done, the options are "choosing an apple from a fruit tray that has one apple and ten pears", "choosing an apple from a fruit tray that has ten pears and two apples", and so on. Thus, if the original characterization of the agent is $\Omega \equiv <X, \mathcal{X}, C>$, the respecified characterization will be $\Omega_+ \equiv < X_+, \mathcal{X}_+, C_+>$, where

X_+ is the set of all (x, A) such that $A \in \mathcal{X}$ and $x \in A$;

$\mathcal{X}_+$ is the class of all non-empty subsets A_+ of X_+ such that, for some $A \in \mathcal{X}$, $A_+ = A \times \{A\}$; and

C_+ is a choice function, such that, for every $A_+ \in \mathcal{X}_+$ $C_+(A_+)=C(A) \times \{A\}$, where A is the element of $\mathcal{X}$ such that $A_+ = A \times \{A\}$.

[9] One of us vaguely recalls having read a long time ago an interesting unpublished paper of the late Stig Kanger that discussed such a reformulation of the problem. We have not been able to locate the paper to check the details. See also Suzumura and Xu (2001) for defining an option in a similar fashion.

It can be checked that, in this reformulation, of the original characterization of the agent, the choice function C_+ cannot possibly violate any of the consistency conditions introduced in Definition 2.1. In the case of Cheronff's condition, this is obvious because there do not exist distinct A_+,$B_+ \in \mathcal{X}_+$ such that A_+ is a subset of B_+. C_+ also cannot violate the weak axiom of revealed preference or the congruence axiom or rationalizability in terms of an ordering, though the reason here is a little less obvious than the reason in the case of Chernoff's condition. Thus, if we follow this method of transforming the original characterization of the agent, then we will eliminate the problem of the violation of the internal consistency condition, but then the conditions will cease to be of interest in the reformulated version since their violation will become a logical impossibility. The formulation discussed in this subsection can be seen to be, formally, a special case of our general formulation of Section 4, when for all $x \in X'$, we define $\sim_x$ in the following way: for all distinct $A, B \in \mathcal{X}$ such that $x \in A$ and $x \in B$, $(x, A) \sim_x (x, A)$ and not $[(x, A) \sim_x (x, B)]$.

5.3 Re-specification of the notion of rationalizability of a choice function

Another possible response to the examples by Luce and Raiffa, and Sen is to retain the original choice problem but to use different notions of rationalizability to accommodate the behavior illustrated by the examples. An early contribution that modifies the standard notion of rationalizability to accommodate Sen's example is by Baigent and Gaertner (1996). In their approach, they develop a notion of rationalizability according to which there exists an ordering $\succcurlyeq$ over the universal set of options, such that, if there are several $\succcurlyeq -$greatest elements in a given menu, then the choice set for the given menu is given by the set of all $\succcurlyeq -$ greatest elements in the menu, and if there is exactly one $\succcurlyeq -$greatest element in the menu, then the choice set is given by the set of all $\succcurlyeq -$ greatest elements in the set of options that is left after excluding from the given menu the unique $\succcurlyeq -$greatest element there. In their approach, the external reference /motivation/social norm is taken into account in the formulation explicitly. As a consequence, the framework developed by Baigent and Gaertner (1996) is specific to the example due to Sen (1993). In a latter contribution along the line of retaining the original choice problem but with a different notion of rationalizability, Gaertner and Xu (1999a) consider the choice of the median option(s) according to a linear ordering over the universal set. Gaertner and Xu (1997, 1999b), Baigent (2007) and Xu (2007) consider variants of different notions of (non-standard) rationalizability introduced in Baigent and Gaertner (1996), and Gaertner and Xu (1999a) for the original choice problem. Again, these non-standard notions of rationalizability are specific to particular choice behaviors as the axiomatic structures in their framework are designed to handle the respective choice behaviors. In a related contribution, Gaertner and Xu (2004) develop a notion of rationalizability of choice functions based on the idea that, sometimes, the agent may refuse to choose any option even if this is the only option available. Thus, in their framework, the choice set of a non-empty feasible set can be empty. This emptiness of the choice set is due to the agent's concern about the procedure of bringing out the feasible set under consideration: from the agent's perspective, the procedure that brought about the feasible set is so "undesirable" that only a show of protest in the form of the refusal to choose any option from the feasible set is justifiable. For example, when there are several newspapers available in a country, an agent is observed to choose the one that is published by the government (the official newspaper); however, when the government bans all other newspapers except the official one and the other which is fairly pro government, the agent is observed to choose not to read any newspaper. The agent's behavior clearly violates Chernoff's condition, and yet is quite reasonable under the circumstances.

The above notions of non-standard rationalizability of choice functions can be reframed under the formulation presented in Section 4 so that they all become special cases of Ω_*-based rationalizability for suitable choice problems with properly chosen equivalence relations. For example, for the choice of the median, one can introduce the notion of indistinguishability as follows: (x, A) is indistinguishable from (x, B) if and only if, according to some criteria, the agent regards x as the median element of each of the two sets, A and B.

6. A reassessment of Sen's argument

In Section 4, we have developed a rather general framework in which the choice problems are reformulated to encompass issues relating to menu-based criteria without making many of the assumptions that are typically made in the existing approaches discussed in Section 5.

What, however, is the relevance of our reconstruction in Section 4, as well as the reconstructions discussed in Section 5, for Sen's criticism of the interpretation of internal consistency conditions as rationality properties that can stand on their own without any reference to the objectives of the agent under consideration? We now take up this issue and we reach a conclusion very different from that often reached in the existing literature.

It seems to us that, while our reformulation of the type of choice problems illustrated in Examples 3.1 through 3.4 as well as other reformulations including that of Bossert and Suzumura (2009a, b) are of interest, they do not address the central problem that Sen raised; nor do they reduce in any way the impact of Sen's criticism of the status the internal consistency conditions have sometimes been accorded in the literature on the theory of revealed preference. To see this, consider again the contention of Sen in this context. For convenience, we concentrate on Example 3.2, though the discussion can be readily extended to all the other examples in Section 3. At the risk of being over-elaborate, let us spell out explicitly the different strands of Sen's reasoning.

Let $<X, \mathcal{X}, C>$ be the initial characterization of the agent, where the universal set of options, X, is simply the set of all possible fruits. Sen's argument then seems to proceed in three distinct steps. The first step consists of the observation that, when we characterize the agent in terms of $<X, \mathcal{X}, C>$, the agent's choice function C violates Chernoff's condition. The second step consists of the observation that, when we know that, not only does the agent care about the fruit that he eats, but he also cares about the social norms under consideration, his choice behavior with respect to fruits does not seem irrational or bizarre at all. The third step consists of the conclusion that Sen draws from the two observations. The conclusion is that the appeal of Chernoff's condition as a property of rational choice in the choice problem characterized by $<X, \mathcal{X}, C>$ depends on our information about the objectives or motives of the agent: if we are not aware that, not only does the agent care about what fruit he eats, but he also cares about conforming to certain social norms about the choice of fruits in a party, then, given the information that we have, the violation of Chernoff's condition would seem to us to be an indication of "irrational" choice; on the other hand, if we know that the agent cares about the social norms under consideration in addition to caring about what fruit he eats, then the agent would seem reasonable to us despite the observed violation of Chernoff's condition by the choice function C.

Now consider what our analysis in Section 4 above shows. What it shows is that, if the theorist modeling the agent's choice behavior knows that the agent cares about the social norms, in addition to caring about what fruit he (i.e. the agent) eats, then the theorist can plausibly transform the characterization given by $<X, \mathcal{X}, C>$ into another characterization $<X_*, \mathcal{X}_*, C_*>$ where C_* satisfies Chernoff's condition. While this is of interest, does it intuitively contradict in any way Sen's conclusion? We do not think so. Indeed, it seems to us that our analysis only serves to reinforce the point that Sen is making. It is true that, if the theorist knows about the agent's concern about social norms, he can plausibly transform the original characterization of the agent so as to get rid of the problem of violation of Chernoff's condition in the reformulated characterization. But such a plausible formal transformation of the original characterization will be possible only if the theorist knows that the agent cares about certain social norms and also knows what these norms are (this latter piece of information is necessary for the theorist to decide whether the choice of an option x from a set A is, in terms of the social norms the agent cares about, distinguishable from the choice of x from another set B). The decision between the alternative formulations, $<X, \mathcal{X}, C>$ and $<X_*, \mathcal{X}_*, C_*>$, itself will depend on the theorist's information and beliefs about the objectives or motivations of the agent. If we do not know anything about the relevance of social norms for the situation described in Example 3.2, then there is no way of formulating the choice problem in terms of $<X_*, \mathcal{X}_*, C_*>$. Then we would presumably formulate the choice problem in terms $<X, \mathcal{X}, C>$, and, in that case, the agent's behavior would seem to violate Chernoff's condition. What Sen's analysis emphasizes is the importance of our information about the agent's concerns in assessing the intuitive appeal of internal consistency condition as criteria for 'rational choice' in any given model of choice. What our analysis demonstrates is that, given suitable information about the agent's concerns, it may be possible to reformulate the choice problem plausibly in such a way that an internal consistency condition will be satisfied in the reformulated version though it was violated in the original formulation. But, of course, the very possibility of such reformulation will depend on our information and/ or belief about the agent's concerns.

The issue can be stated slightly differently. One can distinguish between two distinct aspects of the intuitive notion of rationality. The first is the rationality of an agent's goals (*goal rationality*). For example, it is possible to argue that the agent has irrational goals if, other things remaining the same, the agent would choose to torture more of the animals around him. In general, positive economics has scrupulously avoided the issue of goal rationality. It has exclusively focused on what may be called *structural rationality*, i.e., the issue of whether the choices that the agent makes are coherent given the goals that the agent has. It is this notion of coherence of choice, given the goals of the agent that the internal consistency conditions are intended to capture. What Sen's argument shows is that, even when we identify the notion of rational choice with this limited notion of structural rationality or coherence of choices given whatever goals the agent may have, what constitutes coherent choice given one set of goals may not be coherent for a different set of goals. Therefore, whether or not the agent is being coherent in his choices would depend crucially on our intuition about what constitute the goals of the agent. Where do the agent's goals enter into our formal models of an agent's choices? Note that before we can even formally define the internal consistency conditions, we have to describe the choice situation, namely the universal set of options, the class of opportunity sets that the agents may choose from, and the notion of the agent's choice function. The very first step here is, of course, to specify the universal set of options which embodies our conception of the type of objects that the agent is really concerned with[10]. If the universal set of options is specified as eating a mango, eating an apple, …. , then implicitly we are taking the view that the agent is concerned only with what fruit he eats. If in a model of choice specified in this fashion, the agent's choices violate Chernoff's condition, then it may be because the options as we have specified them (in the course of specifying the universal set which reflects the goals of the agent) completely and correctly capture what the agent cares about, but the choices of the agent are incoherent given the agent's concerns and goals, or it may be because we have specified the options in a way that does not reflect the concerns of the agent appropriately. Therefore, even when we limit ourselves, as we typically do in positive economics, to the notion

[10] Cf. Dasgupta, Kumar, and Pattanaik (2000).

of structural rationality as the only conception of rationality, whether the choice behavior of an agent is structurally irrational (i.e., incoherent, given the agent's goals) cannot be decided independently of what we consider to be the goals or objectives of the agent. It seems to us that Sen's criticism of the view of internal consistency conditions as properties of rational choice that can stand on their own without any reference to the agent's concerns is valid in a fundamental sense and no formal reconstruction of the agent's characterization detracts from its impact in any way. Thus, our assessment of Sen's arguments regarding the status of internal consistency conditions as properties of rational choice is very different from that of Bossert and Suzumura (2009a) who, after demonstrating that some of Sen's examples can be accommodated in their modified version of the conventional framework[11], conclude that their analysis "builds a bridge between rationalizability theory and Sen's criticism" and that what emerges from their analysis is "the possibility of a peaceful coexistence of a norm-conditional rationalizability theory and Sen's elaborate criticism against the internal consistency of choice."

We would like to clarify two other related points. First, note that so far we have taken a normative interpretation of the internal consistency conditions by treating them as conditions that a *rational* individual's choice behavior will satisfy. Sen focused on this interpretation which is often adopted in the literature. What if we treat the internal consistency conditions as testable empirical hypotheses regarding an agent's choice behaviour? It is easy to see that the problem to which Sen drew our attention in the context of the normative interpretation of internal consistency conditions arises again, though in a somewhat different form, when we treat those conditions as empirically testable hypotheses. We can introduce the internal consistency conditions only after we specify the universal set of options, i.e., only after we commit ourselves to a particular view of the alternatives that the agent chooses. If, given a particular specification of the choice problem, the choice function of the agent turns out to violate some internal consistency condition, say Chernoff's condition, then such violation falsifies the conjunction of two hypotheses. The first hypothesis is that our specification of the universal set of options correctly embodies what the agent is concerned with in making his

[11] To be more specific, Bossert and Suzumura (2009a) are concerned with the conventional framework of the theory of revealed preference.

choices and the second hypothesis is that the agent's choice function satisfies Chernoff's condition when the options are specified in a way that correctly captures the agent's goals. Given the falsification of the conjunction of these two hypotheses (i.e., given that the choice function figuring in a given characterization of the agent violates Chernoff's condition), we cannot decide whether to reject the second hypothesis without committing ourselves to a position about whether or not the options, as we have specified them in our formal model, capture the objectives of the agent. [12]

The second point is this. Sen's original argument was formulated with reference to the theory of revealed preference where we start with the primitive concept of choice rather than preference. What happens if we start with the primitive notion of preference and formulate, as usual, the notion of structural rationality in terms of the requirement that the agent's preferences be an ordering, that is, the requirements that the agent's preferences satisfy reflexivity, connectedness, and transitivity? It is easy to see that this would make little difference to the validity or impact of the basic point of Sen. Again, before we can even introduce the notion of the agent's preferences, we need to specify the universal set of options over which the preferences are to be defined. If the preferences violate, say, transitivity[13], then again we have to face the problem of deciding whether, given our chosen formulation of the choice problem, the requirement of transitivity has much appeal in light of what we know about the agent's goals.

7. Conclusions

In this paper, we have formulated a more general framework than those suggested in the literature to accommodate the counter-examples due to Luce and Raiffa and Sen. We have also argued that, though our formulation, as well as those in the existing literature, is of interest for

[12] See Dasgupta, Kumar, and Pattanaik (2000). See Quine (1953) on methodological issues relating to the testing of joint hypotheses.

[13] The question may arise how we conclude that transitivity of preferences are violated. If one believes that preferences are non-observable, then the violation of transitivity is to be inferred by : (i) postulating some relation between the agent's preferences and the choice(s) of the agent from different opportunity sets; and (ii) asking whether, given the postulated relation between the agent's preferences and his choices from opportunity sets, the observed choices are compatible with the requirement that preferences be transitive.

certain purposes, it does not in any way affect either the validity or the conceptual impact of Sen's contention that the reasonableness of internal consistency conditions as conditions for rational choice cannot be judged without going into the agent's motives and objectives. Our conclusion here differs significantly from the position taken by Bossert and Suzumura (2009a, b) vis-à-vis Sen's analysis in their important recent contribution.

We believe that Sen's basic point has important implications for the classical economic theory of rational choice. The classical economic theory of rational choice has focused almost exclusively on structural rationality as distinct from goal rationality. Structural rationality itself, however, embodies the notion of coherent choice, *given the agent's goals and concerns*. It is, therefore, not possible to conclude whether or not the agent satisfies structural rationality simply on the basis of our observations of the agent's choice behavior without referring to the concerns of the agent. Choice behavior that may appear incoherent for some set of concerns, may be perfectly coherent for another set of concerns. In one sense, it is not even possible to construct a formal model of rational choice without introducing some presupposition, whether explicit or implicit, about the agent's concerns. In formal models of rational choice, the specification of the options captures our assumption, often implicit, regarding the concerns of the agent. The examples of menu-dependent criteria and information given by Luce and Raiffa and Sen are important reminders that, when, in the framework of our formal model, the agent's choices violate internal consistency conditions, such violation cannot be taken as a definite indication of structurally incoherent choice; instead, it may be simply due to the fact that the specifications of options in our formal model does not capture certain concerns that the agent has. This, of course, raises the question whether an outside observer, say, an economist, observing an agent's choice behavior, can ever be certain whether, given the goals of the agent, he is behaving in an incoherent fashion. The answer to this question would seem to be in the negative for the following reason. In general, there can be an infinite number of different goals and concerns guiding the agent's choices. No matter how carefully the economist may specify the options, there will still remain the possibility that his specification of the options does not capture some concerns of the agent and the seeming incoherence is due to that. The best that the economist can say is that, if the information that he has about the agent's concerns and that he has put into his conception of an option is correct and complete, then the agent's choice behavior is incoherent. This tentative position would seem to be more justifiable than the position that internal inconsistency of the agent's observed choice behavior (choice being seen in terms of the observer's conception of the agent's 'options') is a conclusive indicator of "irrationality", as well as the position that, if the observed choice behavior of the agent violates internal consistency, then there must be some concerns of the agent that are not captured by the specification of the options, and the internal inconsistency will disappear once such concerns are incorporated in the specification of options.

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Appendix

Proof of Proposition 4.1. Let $<X, \mathcal{X}, C>$ be the initial characterization of the agent, and, given $\sim_x$ for every $x \in X'$, let $< X_*, \mathcal{X}_*, C_*>$ be the modified characterization satisfying (4.1), (4.2), and (4.3).

(i)

Chernoff's condition. Suppose the choice function C satisfies Chernoff's condition. Let $A_*, B_* \in \mathcal{X}_*$ and $x_*, y_* \in X_*$ be such that $x_*, y_* \in A_* \subseteq B_*$ and $x_* \in C_*(A_*)$ but $y_* \notin C_*(A_*)$. We need to show that $y_* \notin C_*(B_*)$. From the definition of $\mathcal{X}_*$, there exists $B \in \mathcal{X}$, such that $B_* = \{(b, E[b, B]): b \in B\}$. Since $A_* \subseteq B_*$, it must be the case that, for some $A \in \mathcal{X}$ with $A \subseteq B$, $A_* = \{(a, E[a, A]): a \in A\}$. Since $x_*, y_* \in A_*$, there must be $a^{'}, b' \in A$ such that $x_* = (a^{'}, E[a', A])$ and $y_* = (b^{'}, E[a', A])$. Note that $x_* \in C_*(A_*)$ but $y_* \notin C_*(A_*)$. It must be true that $a^{'} \in C(A)$ and $b' \notin C(A)$. Since C satisfies Chernoff's condition, from $A \subseteq B$, and $a^{'} \in C(A)$ and $b' \notin C(A)$, we must have $b' \notin C(B)$. Therefore, by the definition of C_*, we obtain $y_* = (b^{'}, E[a', A]) \notin C_*(B_*)$.

The weak axiom of revealed preference. Suppose the choice function C satisfies the weak axiom of revealed preference. Let $A_*, B_* \in \mathcal{X}_*$ and $x_*, y_* \in X_*$ be such that $x_*, y_* \in A_* \cap B_*$ and $x_* \in C_*(A_*)$. We need to show not($y_* \in C_*(B_*)$ and $x_* \notin C_*(B_*)$). From the definition of $\mathcal{X}_*$, there must be some $A, B \in \mathcal{X}$, such that $A_* = \{(a, E[a, A]): a \in A\}$ and $B_* = \{(b, E[b, B]): b \in B\}$. Since $x_*, y_* \in A_* \cap B_*$, there must be $x, y \in A \cap B$ such that $x_* = (x, E[x, A]) = (x, E[x, B])$ and $y_* = (y, E[y, A]) = (y, E[y, B])$. Suppose $x_* \in C_*(A_*)$, and suppose to the contrary that $y_* \in C_*(B_*)$ and $x_* \notin C_*(B_*)$. Then, we must have $x \in C(A)$, $y \in C(B)$ and $x \notin C(B)$. This contradicts our assumption that the choice function C satisfies the weak axiom of revealed preference. Therefore, not($y_* \in C_*(B_*)$ and $x_* \notin C_*(B_*)$) holds, showing that the choice function C_* satisfies the weak axiom of revealed preference.

The congruence axiom and rationalizability in terms of an ordering. It is fairly easy to see that, when the choice function C is rationalizable in terms of an ordering, the choice function C_* must be rationalizable in terms of an ordering as well. Given that the congruence axiom for C is logically equivalent to rationalizability of C in terms of an ordering, and that the congruence axiom for C_* is logically equivalent to rationalizability of C_* in terms of an ordering, it follows that if C satisfies the congruence axiom, then C_* satisfies the congruence axiom.

(ii)

An example will suffice to prove Proposition 4.1 (ii). Let $X = \{x, y, z, w\}$; $\mathcal{X} = \{A, X\}$, where $A = \{x, y, z\}$; $C(A) = \{x, y\}$; $C(X) = \{z, w\}$; $[(x, A) \sim_x (x, X)$; $(y, A) \sim_y (y, X)$; and not $(z, A) \sim_z (z, X)]$. (If one likes, one can think of x and y as two apples, z and w as two pears, and the equivalence relations as reflecting the norms of not choosing a fruit that is the only one of its type in the menu.) It is clear that C violates Chernoff's condition. Now consider an ordering R_* (with asymmetric factor P_* and symmetric factor I_*) over X_*= $\{(x, A), (y, A), (z, A), (x, X), (y, X), (z, X), (w, X)\}$, such that $(z, X) I_* (w, X) P_* (x, A) I_* (y, A) P_* (x, X) I_* (y, X) P_* (z, A)$. It can be checked that C_* is rationalizable in terms of the ordering R_*. ■

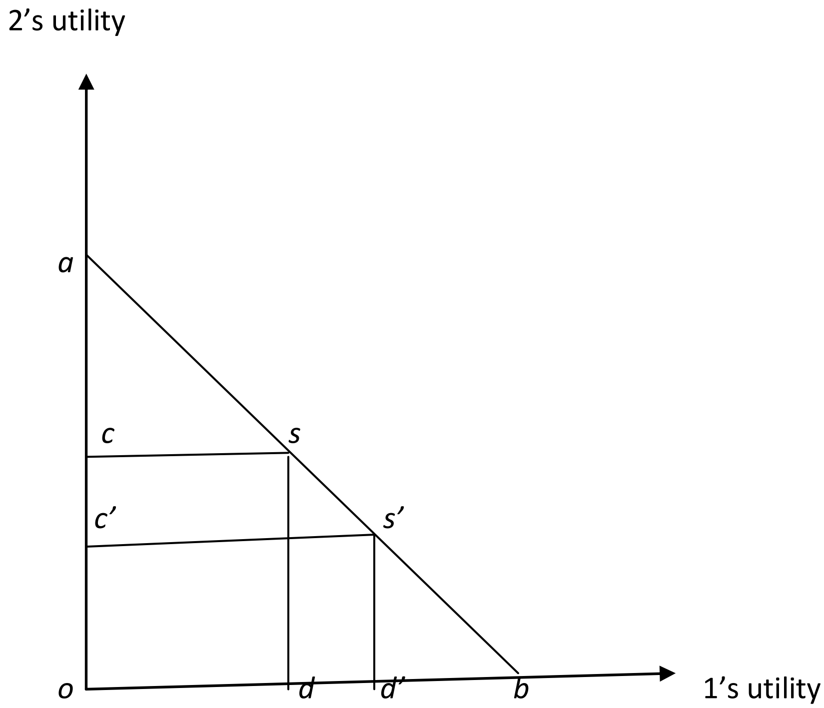

Figure 3.1