

Education as Unemployment Insurance: A Model with Endogenous Educational Requirement for Job Application and Its Policy Implications

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Abstract

This paper combines a search model and signaling game to analyze the interrelationship between labor market outcome and educational choices as well as relevant policy implications, featuring endogenous educational requirement for job application. It explains more than 60% of the unemployment rate difference between college and high school graduates. It predicts that higher unemployment benefit encourages individual to take education. It is also predicted that aggregate unemployment rate decreases if higher educational subsidy is provided, even though both skill-specific unemployment rates increase. Furthermore, according to this model, skill-biased technological change will increase the unemployment rate and lower the wages of uneducated workers.

JEL classification: E24, H52, I21, J24, J64, J68

Key words: search model, signaling game, endogenous educational requirement, unemployment rate gap, unemployment benefit, educational subsidy, skill-biased technological change

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1 Introduction

When explaining educational choices, traditional wisdom, for example, Spence (1973), Stiglitz (1975) and Riley (1979), emphasizes more on the higher wages brought by education, ignoring its effects on lowering unemployment. However, as pointed out by Pissarides (1981), unemployment would reduce the return from labor market participation and increase the demand for education. Kiefer (1985) argues that education could alleviate the problem of unemployment by reducing the duration and possibility of it. These observations are supported by US data. Figure 1 shows the unemployment rate of high school graduates (u_n), that of workers with Bachelor's degrees but no graduate degrees (u_e), and aggregate unemployment rate of the two types combined. This figure clearly exhibits that college education affects the probability of being unemployed. Hence, a second incentive of taking education is to lower the cost of unemployment.

Another very important observation about labor market and education is that employers often requires a certain level of educational achievement to apply for a job. Existing literatures, for example, Makenna (1996) explains this phenomenon by assuming that uneducated workers cannot produce anything when a job requires education. Similar assumption is used by Albrecht and Vroman (2002). However, under a lot of scenarios, it is the case that uneducated workers can do the same job as educated workers, except that they are not that efficient.

This paper, therefore, tries to answer the following questions under the framework of the search and matching model, endogenizing educational requirement of job creation and emphasizing the interdependence of education and unemployment: (1) Why people take education; (2) Why educated workers are less likely to be unemployed compared to the uneducated workers; (3) How unemployment benefit may affect educational choices; (4) How educational subsidy would affect social welfare; (5) What are the effects of a skill-biased technological shock..

There are several existing studies taking into consideration of the interdependence of education and unemployment. Laing, Palivos and Wang (1995) integrated search and matching into a two sector (education and production) growth model in which educational effort is endogenously determined. As a result, equilibrium unemployment exists. However, this paper still focuses on the effect of human capital accumulation rather than unemployment. Makenna

(1996) provides a model with heterogenous labor and sequential labor search. In that paper, educated workers are less likely to be unemployed because they are adaptable to more types of jobs while uneducated workers are suitable for only one type. Eggert, Krieger and Meier (2010) developed a model in which unemployment was a result of efficiency wage, discussing how unemployment gap stimulated education and migration. Charlot, Decreuse and Granier (2005) argue that education can increase adaptability and productivity which leads to a lower unemployment possibility. Since it is a representative agent model, there is no differences in educational choice.

In this paper, a signaling game and search model is combined to explain the educational choices. Vesala (2004) developed a similar model with signalling and labor market search. The key difference is that education as a signal will lead to a higher wage but the same unemployment rate in that model, whereas workers with different educational achievements are going to experience different unemployment rates in this paper.

The intuitions of my model is as the following. Suppose people are different in term of their endowed ability and it takes less effort for a “smarter” individual to take education, i.e., the cost of taking education for “smarter” ones is lower. People can observe that educated workers tend to earn higher wages and are less likely to be unemployed. Hence, they take these facts as the benefits. Only the smarter workers will find it attractive to take education, as the benefits of taking education will outweigh the cost of it for them.

Hence, education itself is a signal of higher ability and firms will have the belief that educated workers are the more productive ones. Suppose all workers have the same preference and their unemployment benefit are the same. Consequently, the reservation wages required by the two types of workers are the same.¹ If firms and workers split the joint benefit from matching according to Nash Bargaining, firms can expect higher profits if a job vacancy is filled with an educated worker. Since educational achievements are observable, entrepreneurs are more willing to post vacancies requiring education, which makes the market less “tight” for educated workers. This is why firms specify educational requirements, even though une-

¹In reality, the unemployment benefits are not the same among different workers. They are positively related to wage income previously earned. However, since each state sets its minimum and maximum unemployment benefit, the replacement ratio is negatively related to previous wage income. A constant unemployment benefit will catch this point and dramatically simplify the analysis without changing qualitative results.

educated workers can do the same job as their educated peers. It will be, therefore, easier for educated unemployed workers to find a job, which results in a lower unemployment rate for them. The realized equilibrium coincides with the ex-ante belief about the benefits of taking education. These results explain why the observed unemployment rate of educated workers is significantly lower than the unemployment rate of uneducated workers, as well as the incentive of taking education.

Keeping this in mind, I will argue that education serves as an alternative insurance against unemployment as it reduces the probability of being unemployed before unemployment actually happens. Hence, the larger the difference between the two unemployment rates, the higher the value of education, which leads to a strong policy implication of unemployment benefit on educational choices. As the unemployment benefit increases, so will the reservation wage. As a result, firms' profits from existing jobs will decrease, which causes firms to post less vacancies. Consequently, unemployment rates will rise. Since unemployment benefit takes a smaller proportion of the wage of educated workers, they relatively suffer less from the tightened labor market. As the unemployment rate of uneducated workers increases faster, more people would be willing to take education. Hence, an increase in unemployment benefit tends to improve the quality of workers.² Traditional literature would overestimate the cost of unemployment benefit without taking into account its impact on educational choices. Through the mechanism stated above, a change in unemployment benefit will also affect wage inequality. These aspects are also analyzed in this paper.

Another policy that can be considered is educational subsidy, which can directly affect people's educational choices. The effects of a change in educational subsidy on educational choice, skill-specific unemployment rates, aggregate unemployment rate and social welfare will be analyzed. It shows that an increase in educational subsidy will lead to a large welfare gain in the long run and reduce aggregate unemployment rate.

Furthermore, I will also use the model to analyze the impacts of a skill-biased technological change. Existing literature and economic data show that such a change will lead to stronger

²This argument holds when unemployment benefit is around the current level. Obviously, when unemployment benefit is very high, people will not care about unemployment rate because they can still enjoy very high level of consumption when unemployed. Hence, if unemployment benefit is very high, a further increase of it will lead to a decrease in educational effort.

inequality - uneducated workers tend to be absolutely worse-off. The model presented in this paper can offer an alternative explanation on this phenomenon. When a skill-biased technological change hits the economy, the demand for educated workers will increase, leading to higher wage and lower unemployment rate for educated workers. The higher benefit of education will encourage more people to take education. Hence, the people who still do not want to take education are the least productive ones. Consequently, the absolute quality of uneducated workers will decrease, leading to a tighter labor market for them. This change will reduce their wages and raise the probability of unemployment.

The rest of the paper is organized as the following. Section 2 presents the theoretical model. Parameterization of this model is discussed in Section 3. Section 4 analyzes the impacts of changes in unemployment benefit and educational subsidy. The consequences of a skill-biased technological change is presented in section 5. Section 6 concludes the paper.

2 Model

Consider an economy without aggregate uncertainty. It is inhabited by N residents who are different only in terms of their ability when they are born. Call the residents born in period τ generation τ . The ability of resident i of generation τ , z_i^τ , is a continuous variable and is bounded by $[z_l, z_h]$. Denote the unconditional cdf and pdf of z_i^τ by $F(\cdot)$ and $f(\cdot)$. In each period, each resident may die with a probability of δ , where $\delta \in (0, 1)$. Meanwhile, δN new residents are born. Assume the death and birth of residents are independent from ability. As a result, the population of the model economy and the overall ability of the population remain constant over time. Denote the subjective discount rate of a resident by $\beta \in [0, 1]$. Hence, after considering the probability of death, the effective discount rate is $\rho = \beta(1 - \delta)$.

When a resident is born, she has a one-time chance to determine whether to take education or not. This assumption would make the theoretical analysis much simpler. Since this paper mainly focuses on the steady states of the model economy, this assumption is not unrealistic. Note that a resident will only take education if the life-time utility of being educated is larger than the life-time utility of not being educated. At the steady state, if taking education is better for a resident in the second period, it must also be better for her in the first period. Hence, there is no reason for a resident to postpone the decision.

Each resident at each period is endowed with one unit of time, which has three mutually exclusive alternative uses: taking education, searching for a job or working for wage. If taking education, this resident will stay in school for k periods and her labor productivity will become $p_i = (1 + e)z_i^T$ as she graduates, where $e > 0$. She then starts to search for a job in the $k + 1^{th}$ period. If not taking education, her labor productivity is $p_i = z_i^T$ and she starts to search for a job immediately after her birth. Call residents actively searching for a job unemployed workers. In each period, an unemployed worker can search for a job only once. The society has an ex-ante belief on the probabilities of uneducated and educated unemployed workers for successfully finding a job each period. If the resident manages to find a job in the current period, she will start to work in the next period. If the resident is currently working, there is a probability x that the job will be destructed. Call x separation rate.³

In the model economy, there exists several "small" firms, which are owned and created by entrepreneurs with the same subjective discount rate β and death probability δ . Each firm consists of only one job position. Call unfilled job positions vacancies. Some vacancies require educated workers while the others do not. Denote the number of vacancies requiring and not requiring education in period t by $v_{e,t}$ and $v_{n,t}$ respectively. In each period, the society has ex-ante beliefs on the probabilities for each type of vacancies being filled. If a job position (firm) is filled by a resident i in the current period, the firm would start to produce p_i from the next period until the (job) firm is destructed. Before matching, firms cannot observe each job candidate's ability or productivity. Nevertheless, they can observe each job candidate's educational status. After matching and before the two parties start to negotiate the wage, a firm can immediately observe its worker's productivity without any uncertainty. This assumption can greatly simplify the model without changing the qualitative result. A very similar assumption is adopted by Pries (2008).

³In a search and matching model, steady state unemployment rate depends on per period job separation rate and the probability of finding a job. To highlight the mechanism presented in this paper, I assume that educated and uneducated workers are subject to the same job separation rate. Fallick and Fleischman (2004) show that the monthly total separation rates for college graduates and high school graduates are 0.042 and 0.064. The relatively small difference between the two separation rates are not enough to explain the large difference between the two unemployment rates.

2.1 The Entrepreneur's Problem

First, consider a firm hiring resident i of generation τ , given that the resident chose not to take education. The realized asset value of this firm in period t , $J_{i,n,t}^\tau$, therefore, can be written recursively as

$$J_{i,n,t}^\tau = z_i^\tau - w_{i,n,t}^\tau + \rho(1-x)J_{i,n,t+1}^\tau, \quad (1)$$

where $w_{i,n,t}^\tau$ is the negotiated wage if resident i is not educated. $z_i - w_{i,n,t}^\tau$ in (1) is the flow profit of this firm. In the next period, the firm may still exist with a probability of $1-x$. After discounting, the continuation value of the firm is $\rho(1-x)J_{i,n,t+1}^\tau$.

Now, however, suppose this resident i of generation τ chose to be educated rather than uneducated. Then, the asset value of the firm hiring her in period t , $J_{i,e,t}^\tau$, can be written as

$$J_{i,e,t}^\tau = (1+e)z_i^\tau - w_{i,e,t}^\tau + \rho(1-x)J_{i,e,t+1}^\tau, \quad (2)$$

where $w_{i,e,t}^\tau$ is the negotiated wage if resident i is educated.

Entrepreneurs have an ex-ante belief that there is a threshold level of ability \bar{z}_τ , such that any resident of generation τ would choose to take education if and only if her ability is greater than \bar{z}_τ .

Define the expected value of firms in period t whose employees are uneducated residents of generation τ as $J_{n,t}^\tau$. It follows that

$$J_{n,t}^\tau = E_{z_i^\tau}(J_{i,n,t}^\tau | z_i^\tau \leq \bar{z}^\tau). \quad (3)$$

Similarly, define the expected value of firms in period t whose employees are educated residents of generation τ as $J_{e,t}^\tau$. It follows that

$$J_{e,t}^\tau = E_{z_i^\tau}(J_{i,e,t}^\tau | z_i^\tau > \bar{z}^\tau). \quad (4)$$

Denote the numbers of unemployed uneducated and educated residents in period t by $u_{n,t}$ and $u_{e,t}$, respectively. Denote the numbers of generation τ unemployed uneducated and educated residents in period t by $u_{n,t}^\tau$ and $u_{e,t}^\tau$, respectively. It is obvious that $u_{n,t} = \sum_{\tau=-\infty}^t u_{n,t}^\tau$ and $u_{e,t} = \sum_{\tau=-\infty}^t u_{e,t}^\tau$.⁴ Denote the number of employed uneducated and

⁴Since education takes k periods to complete. We must have $u_{e,t} = \sum_{\tau=-\infty}^{t-k} u_{e,t}^\tau$. However, by definition, for any $\tau > t-k$, $u_{e,t}^\tau = 0$. Hence, $u_{e,t} = \sum_{\tau=-\infty}^t u_{e,t}^\tau$.

educated workers of generation τ in period t by $n_{n,t}^\tau$ and $n_{e,t}^\tau$. It must follow that

$$n_{n,t}^\tau = \delta N(1 - \delta)^{t-\tau} F(\bar{z}^\tau) - u_{n,t}^\tau, \quad (5)$$

and

$$n_{e,t}^\tau = \begin{cases} \delta N(1 - \delta)^{t-\tau} [1 - F(\bar{z}^\tau)] - u_{e,t}^\tau, & \text{if } t > \tau + k \\ 0, & \text{if otherwise} \end{cases}. \quad (6)$$

Denote the numbers of employed uneducated and educated residents in period t by $u_{n,t}$ and $u_{e,t}$, respectively. Obviously, $n_{n,t} = \sum_{\tau=-\infty}^t n_{n,t}^\tau$ and $n_{e,t} = \sum_{\tau=-\infty}^t n_{e,t}^\tau$. Note that workers recruited in period t will start working in period $t + 1$. Define the expected value of firms in period $t + 1$ with newly recruited uneducated and educated residents as $J_{n,t+1}$ and $J_{e,t+1}$.

Hence, we have

$$J_{n,t+1} = \sum_{\tau=-\infty}^t \frac{u_{n,t}^\tau}{u_{n,t}} J_{n,t+1}^\tau, \quad (7)$$

and

$$J_{e,t+1} = \sum_{\tau=-\infty}^t \frac{u_{e,t}^\tau}{u_{e,t}} J_{e,t+1}^\tau. \quad (8)$$

Denote the cost of posting a vacancy or creating a new firm by q . By free entry condition, it must be the case that the value of a vacancy is zero. That is, the cost of creating a firm should be equal to the discounted expected value of the firm in the next period if it is filled with a worker, multiplied by the probability of successfully filling the vacancy. Hence,

$$q = \rho g_{n,t} J_{n,t+1}, \quad (9)$$

and

$$q = \rho g_{e,t} J_{e,t+1}, \quad (10)$$

where $g_{n,t}$ and $g_{e,t}$ are the ex-ante probability that a vacancy requiring and not requiring education are filled, respectively.

2.2 The Resident's Problem

In period t , the flow utility of the i^{th} resident of generation τ is

$$\phi_{i,t}^\tau = \begin{cases} c_{i,t}^\tau - \gamma(z_h - z_i^\tau) & \text{if in school} \\ c_{i,t}^\tau & \text{if unemployed} \\ c_{i,t}^\tau - \eta & \text{if working for wage} \end{cases}, \quad (11)$$

where $c_{i,t}^\tau$ is the consumption of resident i of generation τ and $\gamma, \eta > 0$. The flow utility function implies that one's disutility derived from education depends on her ability, which is a straightforward and commonly used assumption. If a resident is endowed with a low level of ability, it can be painful to try to understand the knowledge taught in the class.

Assume that all goods are perishable and there is no financial market. Hence, residents consume all their income in each period. Denote the labor income of resident i in period t by $w_{i,t}^\tau > 0$. $w_{i,t}^\tau$ either equals to $w_{i,n,t}^\tau$ or $w_{i,e,t}^\tau$, depending resident i 's educational status. If a resident is unemployed, regardless of her educational achievement, she will receive $b > 0$ as unemployment benefit from the government. If a resident is taking education, her consumption would be $s > 0$, which is the difference between the educational subsidy the agent receives and the tuition she pays. Hence, the flow utility can be rewritten as

$$\phi_{i,t}^\tau = \begin{cases} s - \gamma(z_h - z_i^\tau) & \text{if taking education} \\ b & \text{if unemployed} \\ w_{i,t}^\tau - \eta & \text{if working for wage} \end{cases}. \quad (12)$$

Consider resident i of generation τ . If she chooses not to take education, then her life-time utility in period τ can be written recursively as

$$\Phi_{i,n,\tau}^\tau = b + \rho[\mu_{n,\tau}\Psi_{i,n,\tau+1}^\tau + (1 - \mu_{n,t})\Phi_{i,n,\tau+1}^\tau], \quad (13)$$

where $\Phi_{i,n,\tau}^\tau$ is the life-time utility of resident i of generation τ measured in period τ if she is uneducated and unemployed, $\Psi_{i,n,\tau+1}^\tau$ is her life-time utility in period $\tau+1$ if she is uneducated but employed, $\mu_{n,\tau}$ is the ex-ante job finding probability for an uneducated worker in period τ . The first term of (13) is the flow utility. Since the resident searches for a job in period t , there is a probability of $\mu_{n,t}$ that she can find a job and start working next period. Under this scenario, her life-time utility in period $\tau+1$ would be $\Psi_{i,n,\tau+1}^\tau$. However, there is a probability of $1 - \mu_{n,\tau}$ that she unfortunately fails to find a job, in which case her life-time utility would be $\Phi_{i,n,\tau+1}^\tau$. After discounting, the continuation value of being unemployed in period τ would be $\rho[\mu_{n,\tau}\Psi_{i,n,\tau+1}^\tau + (1 - \mu_{n,t})\Phi_{i,n,\tau+1}^\tau]$.

For any $t > \tau$, $\Psi_{i,n,t}^\tau$ can be written recursively as

$$\Psi_{i,n,t}^\tau = w_{i,n,t}^\tau - \eta + \rho[(1 - x)\Psi_{i,n,t+1}^\tau + x\Phi_{i,n,t+1}^\tau]. \quad (14)$$

The term $w_{i,n,t}^\tau - \eta$ in (14) is the flow utility. As the resident is working in period t , the firm hiring her may still exist with a probability of $1 - x$ in period $t + 1$, under which scenario her life-time utility in period $t + 1$ would be $\Psi_{i,n,t+1}^\tau$. However, there is also a chance that the job is destructed in period $t + 1$ with a probability of x . If this happens, her life-time utility in period $t + 1$ would become $\Phi_{i,n,t+1}^\tau$. After discounting, the continuation value of being employed in period t is $\rho[(1 - x)\Psi_{i,n,t+1}^\tau + x\Phi_{i,n,t+1}^\tau]$.

Now, consider the same resident i of generation τ . If she chooses to take education, then her life-time utility when she is born can be written as

$$\begin{aligned}\Omega_i^\tau &= \sum_{t=\tau}^{\tau+k} \rho^{t-\tau} [s - \gamma(z_h - z_i^\tau)] + \rho^{k+1} \Phi_{i,e,\tau+k+1}^\tau \\ &= \frac{1 - \rho^k}{1 - \rho} [s - \gamma(z_h - z_i^\tau)] + \rho^{k+1} \Phi_{i,e,\tau+k+1}^\tau,\end{aligned}\tag{15}$$

where Ω_i^τ is the life-time utility of resident i in period τ if the worker is educated. For any $t \geq \tau + k + 1$ and $\Phi_{i,e,t}^\tau$ is resident i 's life-time utility in period t if she is unemployed and educated. Similar to (13), $\Phi_{i,e,t}^\tau$ can be written recursively as

$$\Phi_{i,e,t}^\tau = b + \rho[\mu_{e,t} \Psi_{i,e,t+1}^\tau + (1 - \mu_{e,t}) \Phi_{i,e,t+1}^\tau],\tag{16}$$

where $\Psi_{i,e,t}^\tau$ is the life-time utility of resident i of generation τ in period t if she is educated and employed and $\mu_{e,\tau}$ is the ex-ante job finding probability for an educated worker in period t . Following the logic of (14), $\Psi_{i,e,t}^\tau$ can be written recursively as

$$\Psi_{i,e,t}^\tau = w_{i,e,t}^\tau - \eta + \rho[(1 - x)\Psi_{i,e,t+1}^\tau + x\Phi_{i,e,t+1}^\tau].\tag{17}$$

Hence, resident i who is born in period τ will take education if $\Omega_i^\tau > \Phi_{i,n,\tau}^\tau$.

2.3 Matching Technology and The Determination of Wages

Since the job market is less tight for educated workers and a worker's productivity is assumed to be revealed immediately after the match. There is no reason for an unemployed educated worker to search for a job which does not require education, as the job finding rate would be smaller if they do so. For each type of workers and vacancies, there is a constant-return-to-scale matching technology. Assume the matching functions exhibit a Cobb-Douglas form.

Denote the numbers of matches made in period t for uneducated and educated workers as $m_{n,t}$ and $m_{e,t}$, respectively. Hence, it follows that

$$m_{n,t} = \varphi v_{n,t}^\alpha u_{n,t}^{1-\alpha}, \quad (18)$$

and

$$m_{e,t} = \varphi v_{e,t}^\alpha u_{e,t}^{1-\alpha}, \quad (19)$$

where φ is the matching efficiency and α is the elasticity of matches with respect to vacancies. Denote the market tightness for uneducated and educated workers as $\theta_{n,t} = v_{n,t}/u_{n,t}$ and $\theta_{e,t} = v_{e,t}/u_{e,t}$, respectively.

Following standard literature, assume that the wage is determined by a Nash Bargaining process. In each period, after the matches are made, workers and firms start to negotiate the wage. Consider resident i of generation τ and the firm hiring her, they negotiate a wage as if they maximize $(\Psi_{i,n,t}^\tau - \Phi_{i,n,t}^\tau)^\lambda (J_{i,n,t}^\tau)^{1-\lambda}$ or $(\Psi_{i,e,t}^\tau - \Phi_{i,e,t}^\tau)^\lambda (J_{i,e,t}^\tau)^{1-\lambda}$, depending on whether resident i chose to take education or not. The first-order condition is

$$(1 - \lambda)(\Psi_{i,n,t}^\tau - \Phi_{i,n,t}^\tau) = \lambda J_{i,n,t}^\tau, 0 < \lambda < 1 \quad (20)$$

or

$$(1 - \lambda)(\Psi_{i,e,t}^\tau - \Phi_{i,e,t}^\tau) = \lambda J_{i,e,t}^\tau. \quad (21)$$

Define $S_{i,n,t}^\tau = J_{i,n,t}^\tau + \Psi_{i,n,t}^\tau - \Phi_{i,n,t}^\tau$ or $S_{i,e,t}^\tau = J_{i,e,t}^\tau + \Psi_{i,e,t}^\tau - \Phi_{i,e,t}^\tau$ as the joint surplus when the match is made, given the worker is uneducated or educated. By (20) and (21), it follows that

$$\Psi_{i,n,t}^\tau - \Phi_{i,n,t}^\tau = (1 - \lambda)S_{i,n,t}^\tau, \quad (22)$$

and

$$\Psi_{i,e,t}^\tau - \Phi_{i,e,t}^\tau = (1 - \lambda)S_{i,e,t}^\tau. \quad (23)$$

Hence, λ is the bargaining power of workers.

2.4 The Existence of Separating Equilibrium

The intuition of the existence of a separating equilibrium is straightforward. It is obvious that for all residents in all periods, no matter educated or not, the reservation wage should

be $b + \eta$. Since firms believe that only workers with an ability greater than a threshold level would take education and education itself could enhance the a worker's productivity, they must also believe that the joint surplus obtained through a match with an educated worker must be greater than the joint surplus obtained through a match with an uneducated worker. Since firms can get a constant proportion of the joint surplus through the bargaining process, the value of firms hiring educated workers would also be higher. Hence, the expected value of a firm hiring an educated would be larger. By (7) and (8), it means that entrepreneurs are more willing to post vacancies requiring education, which in turn implies that educated workers enjoy a higher job finding rate and are less likely to be unemployed.

Meanwhile, as a result of the enhancement of productivity due to education, by taking education, residents can earn higher wages. From the resident's point of view, the benefit of education is a higher wage and a lower unemployment rate, whereas the major cost of education is the disutility from education. Given that the enhancement of productivity is propotional to the endowed ability of the resident, the benefit of taking education is positively related to ability. As the disutility from education decreases as the ability increases, the cost of education is negatively related to the ability. These two results imply that there exists a threshold level of ability for the residents, if a resident's ability is greater than the threshold, she would like to take the education since the benefits of education would exceed the costs of education. As a result, the ex-post belief would be consistent with the ex-ante belief of the firms, which results in a separating equilibrium.

2.5 The Steady State

2.5.1 Equilibrium at the Steady State

At the steady state, variables such as z , w , θ , J , Ψ , Φ , Ω and S , are no longer sensitive to time period or generation. Hence, I use z_i and Ω_i to represent the ability and the steady state life-time utility if taking education for residents of type i . Note that the type of residents is solely defined by their endowed ability. $X_{i,j}$ is used to represent the steady state values of the corresponding variables, where $X = \{w, J, \Psi, \Phi, S\}$ and $j = \{n, e\}$. Furthermore, let μ_j and g_j denote the job finding rates and vacancy filling rates at the steady state. Denote the steady state level of $\theta_{j,t}$ as θ_j .

Consider a type i worker who chooses not to take education. Based on the steady state version of (13) and (14), it follows that

$$\Phi_{i,n} = \frac{b[1 - \rho(1 - x)] + \rho\mu_n(w_{i,n} - \eta)}{(1 - \rho)[1 - \rho(1 - x - \mu_n)]}, \quad (24)$$

and

$$\Psi_{i,n} - \Phi_{i,n} = \frac{w_{i,n} - \eta - b}{1 - \rho(1 - x - \mu_n)}. \quad (25)$$

(25) exhibits the net benefit of being employed for an uneducated type i worker at the steady state.

Now, consider a type i resident who chooses to take education instead. A simple manipulation of the steady state version of (16) and (17) yields

$$\Phi_{i,e} = \frac{b[1 - \rho(1 - x)] + \rho\mu_e(w_{i,e} - \eta)}{(1 - \rho)[1 - \rho(1 - x - \mu_n)]}, \quad (26)$$

and

$$\Psi_{i,e} - \Phi_{i,e} = \frac{w_{i,e} - \eta - b}{1 - \rho(1 - x - \mu_e)}. \quad (27)$$

(27) shows the net benefit of being employed for an educated type i worker at the steady state.

Plugging (26) into the steady state version of (15) yields

$$\Omega_i = \frac{1 - \rho^k}{1 - \rho} [s - \gamma(z_h - z_i)] + \rho^k \frac{b[1 - \rho(1 - x)] + \rho\mu_e(w_{i,e} - \eta)}{(1 - \rho)[1 - \rho(1 - x - \mu_n)]}. \quad (28)$$

Therefore, a type i worker would like to take education if and only if

$$\frac{1 - \rho^k}{1 - \rho} [s - \gamma(z_h - z_i)] + \rho^k \frac{b[1 - \rho(1 - x)] + \rho\mu_e(w_{i,e} - \eta)}{(1 - \rho)[1 - \rho(1 - x - \mu_n)]} > \frac{b[1 - \rho(1 - x)] + \rho\mu_n(w_{i,n} - \eta)}{(1 - \rho)[1 - \rho(1 - x - \mu_n)]}.$$

Any type i resident knows that $w_{i,e}$ and $w_{i,n}$ are functions of z_i . Hence, when considering the threshold level of ability to take education, explicit form of $w_{i,e}$ and $w_{i,n}$ are needed.

Now, consider firms' behavior. Based on the steady state version of (1) and (2), simple algebra shows that

$$J_{i,n} = \frac{z_i - w_{i,n}}{1 - \rho(1 - x)}, \quad (29)$$

and

$$J_{i,e} = \frac{(1 + e)z_i - w_{i,e}}{1 - \rho(1 - x)}. \quad (30)$$

Plugging (25), (27), (29) and (30) into the steady state version of (20) and (21) yields

$$\begin{aligned} w_{i,n} &= \frac{\lambda[1 - \rho(1 - x - \mu_n)]z_i + (1 - \lambda)[1 - \rho(1 - x)](\eta + b)}{1 - \rho(1 - x) + \lambda\rho\mu_n} \\ &= z_{i,n} - \frac{(1 - \lambda)[1 - \rho(1 - x)](z_{i,n} - \eta - b)}{1 - \rho(1 - x) + \lambda\mu_n}, \end{aligned} \quad (31)$$

and

$$\begin{aligned} w_{i,e} &= \frac{\lambda[1 - \rho(1 - x - \mu_e)](1 + e)z_i + (1 - \lambda)[1 - \rho(1 - x)](\eta + b)}{1 - \rho(1 - x) + \lambda\rho\mu_e} \\ &= (1 + e)z_{i,e} - \frac{(1 - \lambda)[1 - \rho(1 - x)][(1 + e)z_{i,e} - \eta - b]}{1 - \rho(1 - x) + \lambda\mu_e}. \end{aligned} \quad (32)$$

Hence, the steady state equilibrium wages are positively related to the job finding rates. This relationship also confirms the result that educated workers will not search for a job with no educational requirements. Not only will the job finding rate be lower, but the wage they can get will also fall short. Plugging (31) and (32) into (24) and (26), we have

$$\Phi_{i,n} = A_n z_i + B_n + C_n, \quad (33)$$

and

$$\Phi_{i,e} = A_e z_i + B_e + C_e, \quad (34)$$

where

$$\begin{aligned} A_n &= \frac{\lambda\rho\mu_n}{(1 - \rho)[1 - \rho(1 - x) + \lambda\rho\mu_n]}, \\ B_n &= \frac{\rho\mu_n(1 - \lambda)[1 - \rho(1 - x)](\eta + b)}{(1 - \rho)[1 - \rho(1 - x - \mu_n)][1 - \rho(1 - x) + \lambda\rho\mu_n]}, \\ C_n &= \frac{b[1 - \rho(1 - x)] - \rho\mu_n\eta}{(1 - \rho)[1 - \rho(1 - x - \mu_n)]}, \\ A_e &= \frac{\lambda\rho\mu_e(1 + e)}{(1 - \rho)[1 - \rho(1 - x) + \lambda\rho\mu_e]}, \\ B_e &= \frac{\rho\mu_e(1 - \lambda)[1 - \rho(1 - x)](\eta + b)}{(1 - \rho)[1 - \rho(1 - x - \mu_e)][1 - \rho(1 - x) + \lambda\rho\mu_e]}, \end{aligned}$$

and

$$C_e = \frac{b[1 - \rho(1 - x)] - \rho\mu_e\eta}{(1 - \rho)[1 - \rho(1 - x - \mu_e)]}.$$

Plugging (34) into (28) yields

$$\Omega_i = D_e z_i + G_e, \quad (35)$$

where

$$D_e = \frac{1 - \rho^k}{1 - \rho} \gamma + \rho^k A_e,$$

and

$$G_e = \frac{1 - \rho^k}{1 - \rho} (s - \gamma z_h) + \rho^k (B_e + C_e).$$

As long as $D_e - A_n > 0$, there exists a

$$\bar{z} = \frac{B_n + C_n - G_e}{D_e - A_n}, \quad (36)$$

such that whenever $z_i > \bar{z}$, a type i worker would choose to take education and vice versa. Note that $D_e - A_n$ is not necessarily greater than zero. If the subjective discount factor is too small, or the probability of death and the time education takes is too large, then agents may not be patient enough to take advantage of higher future wage and lower unemployment rate. However, if the effectiveness of education, e , is large enough, then workers will have incentives to take education. Generally speaking, \bar{z} is negatively related to e , γ , s , and ρ . \bar{z} , however, is positively related to k and b . I am going to show the relationship between \bar{z} and b as well as s in detail in section 4.

If the economy is in equilibrium, it must be the case that the ex-ante and ex-post beliefs on job finding rates match each other. Hence, by definition, $\mu_n = \varphi \theta_n^\alpha$ and $\mu_e = \varphi \theta_e^\alpha$. Similarly, in the equilibrium, $g_n = \varphi \theta_n^{\alpha-1}$ and $g_e = \varphi \theta_e^{\alpha-1}$. As a result, \bar{z} is a function of θ_n and θ_e in the equilibrium. Therefore, the free-entry conditions become

$$q = \rho g_n(\theta_n) E_{z_i}(J_{i,n} | z_i \leq \bar{z}(\theta_n, \theta_e)), \quad (37)$$

and

$$q = \rho g_e(\theta_e) E_{z_i}(J_{i,e} | z_i > \bar{z}(\theta_n, \theta_e)), \quad (38)$$

which can be used to solve for θ_n and θ_e in the equilibrium. All other endogenous variables can be solved using equilibrium θ_n and θ_e .

Proposition 1 *Holding ρ , η , b , x , λ , e , q , φ and α constant, θ_e is an increasing function of \bar{z} , given the existence of $\bar{z} \in [z_l, z_h]$.*

Proof. Suppose the proposition does not hold.

Hence, there exist $\bar{z}_0, \bar{z}_1 \in [z_l, z_h]$ such that $\bar{z}_0 < \bar{z}_1$, $q = \rho g_e(\theta_{e,0}) E_{z_i}(J_{i,e} | z_i > \bar{z}_0)$, $q = \rho g_e(\theta_{e,1}) E_{z_i}(J_{i,e} | z_i > \bar{z}_1)$ and $\theta_{e,0} \geq \theta_{e,1}$. Now, consider $J_{i,e}$. The value of $J_{i,e}$ is determined by z_i, \bar{z}, θ_e . Specifically, according to (32) and (30)

$$J_{i,e}(z_i; \theta_e) = \frac{(1 - \lambda)[(1 + e)z_{i,e} - \eta - b]}{1 - \rho(1 - x) + \lambda\mu_e}. \quad (39)$$

Hence,

$$E_{z_i}(J_{i,e} | z_i > \bar{z}_0) = \frac{(1 - \lambda)[(1 + e)(\bar{z}_0 + z_h)/2 - \eta - b]}{1 - \rho(1 - x) + \lambda\mu_e(\theta_{e,0})},$$

and

$$E_{z_i}(J_{i,e} | z_i > \bar{z}_1) = \frac{(1 - \lambda)[(1 + e)(\bar{z}_1 + z_h)/2 - \eta - b]}{1 - \rho(1 - x) + \lambda\mu_e(\theta_{e,1})}.$$

Since μ_e is an increasing function of θ_e , $\mu_e(\theta_{e,0}) \geq \mu_e(\theta_{e,1})$. As is assumed, $\bar{z}_0 < \bar{z}_1$. Hence, $E_{z_i}(J_{i,e} | z_i > \bar{z}_1) > E_{z_i}(J_{i,e} | z_i > \bar{z}_0)$. Therefore, $g_e(\theta_{e,1}) < g_e(\theta_{e,0})$. Since g_e is a decreasing function of θ_e , $\theta_{e,1} > \theta_{e,0}$. Contradiction. ■

Proposition 2 *Holding $\rho, \eta, b, x, \lambda, q, \varphi$ and α constant, θ_n is an increasing function of \bar{z} , given the existence of $\bar{z} \in [z_l, z_h]$.*

Proof. Suppose the proposition does not hold.

Hence, there exist $\bar{z}_0, \bar{z}_1 \in [z_l, z_h]$ such that $\bar{z}_0 < \bar{z}_1$, $q = \rho g_n(\theta_{n,0}) E_{z_i}(J_{i,n} | z_i \leq \bar{z}_0)$, $q = \rho g_n(\theta_{n,1}) E_{z_i}(J_{i,n} | z_i \leq \bar{z}_1)$ and $\theta_{n,0} \geq \theta_{n,1}$. Now, consider $J_{i,n}$. The value of $J_{i,n}$ is determined by z_i, \bar{z}, θ_n . Specifically, according to (32) and (30)

$$J_{i,n}(z_i; \theta_n) = \frac{(1 - \lambda)(z_{i,n} - \eta - b)}{1 - \rho(1 - x) + \lambda\mu_n}. \quad (40)$$

Hence,

$$E_{z_i}(J_{i,n} | z_i \leq \bar{z}_0) = \frac{(1 - \lambda)[(\bar{z}_0 + z_l)/2 - \eta - b]}{1 - \rho(1 - x) + \lambda\mu_n(\theta_{n,0})},$$

and

$$E_{z_i}(J_{i,n} | z_i \leq \bar{z}_1) = \frac{(1 - \lambda)[(\bar{z}_1 + z_l)/2 - \eta - b]}{1 - \rho(1 - x) + \lambda\mu_n(\theta_{n,1})}.$$

Since μ_n is an increasing function of θ_n , $\mu_n(\theta_{n,0}) \geq \mu_n(\theta_{n,1})$. As is assumed, $\bar{z}_0 < \bar{z}_1$. Hence, $E_{z_i}(J_{i,n} | z_i \leq \bar{z}_1) > E_{z_i}(J_{i,n} | z_i \leq \bar{z}_0)$. Therefore, $g_n(\theta_{n,1}) < g_n(\theta_{n,0})$. Since g_n is a decreasing function of θ_n , $\theta_{n,1} > \theta_{n,0}$. Contradiction. ■

These propositions will be very useful when used to analyze the labor market outcomes caused by changes in educational subsidy or skill-biased technological shocks. The intuitions

of the propositions are very simple. As \bar{z} increases, less people are taking education. The average quality of both educated and uneducated workers will increase. Hence, firms are more willing to post vacancies for both types of workers. The labor markets will be less tight for both types of workers, which is represented by larger θ_n and θ_e . However, these results do not necessarily mean that the aggregate unemployment rate will be higher when \bar{z} decreases. This is because the unemployment rate for educated workers is smaller than that of uneducated workers. As \bar{z} decreases, more people take education, the weight on the smaller unemployment rate will be larger, which tends to lower the aggregate unemployment rate. This topic will be discussed in detail in Section 4.

2.5.2 Unemployment Rates in the Steady State

Consider uneducated workers. Define N_n as the steady state population of uneducated labor force. Obviously, $N_n = NF(\bar{z})$. Based on the dynamics of unemployment of uneducated workers, we must have

$$u_n = u_n(1 - \mu_n(\theta_n))(1 - \delta) + \delta N_n + x(1 - \delta)(N_n - u_n). \quad (41)$$

The first term on the right hand side of (41) measures the number of uneducated workers unemployed in the last period who are still alive and unemployed in the current period. The second term characterizes the number of newly born people who choose not to take education and start to search for a job immediately. The third term measures the number of currently alive uneducated workers who lost their jobs in the last period. Hence, the steady state unemployment rate for uneducated workers is

$$\frac{u_n}{N_n} = \frac{\delta + x(1 - \delta)}{1 - (1 - x - \mu_n(\theta_n))(1 - \delta)}. \quad (42)$$

If $\delta = 0$, then (42) will collapse to $x/(x + \mu_n)$, which is the steady state unemployment rate in a standard search and matching model.

Consider educated workers. Denote the steady state population of educated labor force by N_e . It follows that

$$N_e = N[1 - F(\bar{z})] - N[1 - F(\bar{z})]\delta \sum_{j=0}^{k-1} (1 - \delta)^j. \quad (43)$$

The first term on the right hand side of (43) measures the total amount of people whose ability is greater than the threshold ability. The second term measures the amount of people who are currently taking education. The steady state unemployment rate of educated workers is characterized by

$$\frac{u_e}{N_e} = \frac{\delta + x(1 - \delta)}{1 - (1 - x - \mu_e(\theta_e))(1 - \delta)}. \quad (44)$$

One purpose of this paper is to explain the unemployment rate gap between educated and uneducated workers. Proposition 3 shows the existence of the gap.

Proposition 3 *The steady state unemployment rate of uneducated workers is higher than the steady state unemployment rate of educated workers.*

Proof. According to (42) and (44), I need to show that $\mu_e(\theta_e) > \mu_n(\theta_n)$. Suppose not, that is $\mu_e(\theta_e) \leq \mu_n(\theta_n)$. Hence, $\theta_e \leq \theta_n$. According to (39) and (40), it can be shown that

$$E_{z_i}(J_{i,e}|z_i > \bar{z}) = \frac{(1 - \lambda)[(1 + e)(\bar{z} + z_h)/2 - \eta - b]}{1 - \rho(1 - x) + \lambda\mu_e(\theta_e)},$$

and

$$E_{z_i}(J_{i,n}|z_i \leq \bar{z}) = \frac{(1 - \lambda)[(\bar{z} + z_l)/2 - \eta - b]}{1 - \rho(1 - x) + \lambda\mu_n(\theta_n)}.$$

Therefore, $E_{z_i}(J_{i,e}|z_i > \bar{z}) > E_{z_i}(J_{i,n}|z_i \leq \bar{z})$. To satisfy (37) and (38), $g_n(\theta_n)$ must be greater than $g_e(\theta_e)$. Hence, $\theta_e > \theta_n$. Contradiction. Hence, $\mu_e(\theta_e) > \mu_n(\theta_n)$ and $\frac{u_e}{N_e} > \frac{u_n}{N_n}$. ■

3 Parameterization

Since in a search model, a worker searching for a job will be unemployed for at least one period. The length of one period should be small enough. Following Hall and Milgrom (2008), one period in this model is set to be 10 days, or 1/3 month. Hence, the discount factor β is set to be 0.999, such that the annual discount factor is 0.96. Assume the life of a person satisfies the Poisson distribution. Then δ is set to be 4.42×10^{-4} , such that the life expectancy is 80 years.⁵ Since higher education is to be discussed, k is set to be 146, implying that a student stays in a college for four years. I normalize the lower bound of ability, z_l , to 1. Normalize the total population, N , to 1.

⁵Economic agents enter into the model at the age of 18. Hence, the probability corresponds to an average existence of 57 years in the model.

Following conventional settings, I set the bargaining power λ for both types of workers as 0.5. According to Hosios (1990), a search economy reaches Pareto Optimality when the sum of bargaining power of workers and the vacancy elasticity of matching is 1. Following this condition, I set α to be 0.5. The job separation rate is obtained from Job Openings and Labor Turnover Survey (JOLTS). The average job separation rate from December 2000 to March 2011, is 0.036. Hence, x is calibrated to 0.012.

In addition to the parameters stated above, there are still eight of them to be pinned down. Therefore, eight calibration targets are needed. Among them, six are based on the data of the Current Population Survey (CPS) from year 1992 to year 2011. In order to focus on the choices of whether to take college education, all the observations with educational achievements less than a high school diploma or higher than a bachelor's degree are dropped. The first calibration target is the aggregate unemployment rate. During this period, the average unemployment rate is 5.4%. The second calibration target is the proportion of people with high school diploma but without a college degree, which is 62.2%. The third target is the ratio of average weekly wages between high school graduates and workers with bachelor's degrees. The weekly wage is calculated as the last year's wage income divided by the number of weeks employed last year.⁶ It turns out that the wage ratio given by CPS is 0.57. The fourth target is the ratio between average weekly unemployment benefit and average weekly wage of high school graduates, which is 0.52 over the period from 1992 to 2011.⁷ The fifth calibration target is the ratio between average weekly educational subsidy and average weekly unemployment benefit, which turns out to be 0.33.⁸ The sixth calibration target is ratio between the standard deviation of weekly wage income and average weekly wage of high school graduates. I found this ratio to be 1.37.

The seventh calibration target is the average market tightness. According to the data from

⁶In the original dataset, only the number of weeks unemployed was provided. Number of weeks employed last year is calculated as the difference between 52 and number of weeks unemployed last year. This is an approximation because the calculation rule out the possibility that a worker was not in the labor force during a certain period last year.

⁷To calculate the average weekly unemployment benefit, I dropped the observations without any unemployment benefit income first, and then divided unemployment benefit income by the number of weeks unemployed.

⁸To calculate the average weekly educational subsidy, I dropped all observations with zero income from educational assistance, then divided educational assistance by 52.

JOLTS, the average market tightness from December 2000 to March 2011 is 0.44. According to Silva and Toledo (2009) and Milgrom and Hall (2008), the average cost of posting one vacancy is about 9 days of the average wage. Hence, I set q as 90% of the average wage as the last calibration target, since one period is set to be 10 days.

Table I summarizes the value of the parameters. To show the predicting power of this model, the steady state unemployment rates of the two groups of workers generated by the model are compared with those from data. This comparison is summarized in Table II. Since the two unemployment rates are not calibration targets, it shows that the mechanism presented in this paper can explain 64% of the difference between the two unemployment rates. The overestimate of u_e and underestimate of u_n might be attributed to the assumption that the matching functions and job separation rates for the two types of workers are the same.

4 Policy Experiments

4.1 Unemployment Benefit

The first policy to be examined is a change in unemployment benefit, while holding all other variables constant. Specifically, consequences of a change in unemployment benefit on educational choices and wage inequality are scrutinized. For the purpose of this section, I calculate the values of the variables at the steady state for each value of unemployment benefit between 0.556 to 0.876.

4.1.1 Educational Choice

Figure 2 exhibits the proportion of people choosing to be educated at the steady state for a given value of unemployment benefit. It is clear that as unemployment benefit increases, more people will take education.

With a more generous unemployment benefit, workers will ask for higher wages when bargaining for the wage. Hence, firms will earn fewer profits and have less incentives to post vacancies, which will result in higher unemployment rate. However, since unemployment benefit takes up a larger proportion of wage income for uneducated workers than that of educated workers, the former ones will suffer from a larger increase in unemployment rate.

Since one goal of getting more education is to decrease the probability of unemployment, the value of education increases when the difference between the two unemployment rates increases. As a result, more workers opt to take education.

To clarify this, figure 3 presents the unemployment rate of uneducated workers subtracted by that of educated workers for each value of unemployment benefit. The upward slope of the curve clearly shows that it is the unemployment gap that motivates more people to take education when unemployment benefit increases. These results show that standard search and matching model, in which educational choices are not considered, will overestimate the cost of an increase in unemployment benefit. Actual data seems to exhibit the same style provided in figure 3. During the past decades, the real unemployment benefit per month per person remained stable, except for the periods of recessions during which the real monthly unemployment benefit per person is much higher. Simultaneously, the unemployment rate gap between the two groups of workers also tends to increase.

As a result of the change in composition of the labor force, the cost of increasing unemployment benefit is smaller than that predicted in the traditional search model.

4.1.2 Wage Inequality

Several empirical studies, for example, Koeniger, Leonardi and Nunziata (2007) and Mooi-Reci (2011), show that a more generous unemployment benefit tends to lower wage inequality. This result is also confirmed in this paper. Figure 4 exhibits the negative relationship between wage Gini coefficient and unemployment benefit.

Note that the Gini coefficient generated by the model is about 0.18 in the benchmark steady state. Nevertheless, according to United Nations Development Programme (2006), the wage Gini coefficient in US is 0.4.⁹ The large difference between the two numbers might be due to at least two reasons. First, the target groups of workers are high school graduates and college graduates, which are in the middle of the spectrum of educational achievements. Hence, extreme values of wage income are not generated by the model. If this model is further expanded to allow more than two levels of education, the Gini coefficient should be magnified. Second, the model in this paper does not consider frictional wage inequality, in addition to

⁹See "United Nations Development Programme. (2006). Human Development Report 2006: Beyond Scarcity: Power, Poverty and the Global Water Crisis."

ability inequality. Nevertheless, Hornstein, Krusell and Violante (2011) argues that frictional wage inequality is relatively large.

4.2 Educational Subsidy

4.2.1 Unemployment Rates and Wages

The following proposition discusses what happens when educational subsidy changes.

Proposition 4 *An increase in educational subsidy leads to higher steady state unemployment rates for both educated and uneducated workers. For any newly born individual that would have the same educational choice given the original and new level of educational subsidy, they will earn lower wages in the steady state.*

Proof. According to (36), when s increases, \bar{z} will decrease, that is, more people are going to take education. According Proposition 1 and 2, both θ_e and θ_n are increasing functions of \bar{z} . Hence, θ_e and θ_n will decrease due to the increase of s . As characterized by (42) and (44), $\frac{u_e}{N_e}$ and $\frac{u_n}{N_n}$ are negatively related to corresponding market tightness. Hence, $\frac{u_e}{N_e}$ and $\frac{u_n}{N_n}$ will increase due to the increase of s . Based on (31) and (32), a worker's wage is negatively related to market tightness. Unless, a newly born worker would have different educational choices, the decrease of θ_e and θ_n will lead to lower wages. ■

The intuition of Proposition 4 is straightforward. Unlike unemployment benefit that affects educational choices by changing the unemployment rate gap, educational subsidy can directly change the incentive of taking education. An increase in the subsidy certainly encourages more people to take education. Therefore, the threshold level ability of taking education decreases. Such a dip in the threshold ability means that the expected productivity of educated workers will decrease, as the additional workers who take education are less productive compared to those who decided to take education before the change in educational subsidy. As a result, entrepreneurs will expect less profits from educated workers and a lower $E_{z_i}(J_{i,e}|z_i > \bar{z}(\theta_n, \theta_e))$. Hence, the free entry condition predicts that the g_e must increase, or the labor market for educated workers will be tighter. Due to the tighter labor market for educated workers, the unemployment rate will increase. It has been shown in (32) that the equilibrium wage of an educated worker is positively related to job finding rate and market tightness. As

a result of a tighter labor market, the equilibrium wage of an originally educated worker will decrease.

Now, consider the changes in the labor market requiring no education. A decrease in the threshold ability of taking education is also a bad news for uneducated workers. Those who switch from being uneducated to educated are the most productive ones in the initial uneducated group. Hence, entrepreneurs will also expect a lower $E_{z_i}(J_{i,e}|z_i > \bar{z}(\theta_n, \theta_e))$ as a result of an increase in educational subsidy, which means that g_n becomes larger and the market is tighter for uneducated workers. Similar to educated workers, the uneducated workers will also experience an increase in unemployment rate and a decrease in wages. The changes of skill specific unemployment rates are summarized in figure 5 and figure 6.

Given the arguments stated above, all those types of workers that have the same educational choices before and after the increase in educational subsidy are worse-off. However, the types of workers who originally wouldn't take education but choose to be educated after the increase in subsidy will be better-off, as they suffer less from unemployment and earn higher wages.

Though both skill-specific unemployment rates increase, the aggregate unemployment rate does not necessarily rise. This is because the fraction of workers choosing to take education increases. As a result, the weight of the group of workers with lower unemployment rate becomes larger. This compositional change of labor force may reduce, neutralize or even reverse the effects of the increase in skill-specific unemployment rates, depending on the choice of parameter values. Figure 7 presents the aggregate unemployment rate for each value of educational subsidy. It turns out that, based on the calibrated parameters, the aggregate unemployment rate decreases as educational subsidy increases.

4.2.2 Social Welfare

In this part, I show that an increase in educational subsidy will lead to an improvement of social welfare in the long run, according to the simulation results shown below. In the previous sections, the goods market clearing condition is not explicitly stated. To proceed, I need to specify the budget constraint of the government and use it to construct the resource constraint.

Assume that each resident in the economy has to pay a lump sum tax τ_t in period t

regardless of her labor force status or employment status. The taxes collected are used to pay for the unemployment benefit and educational subsidy and the government runs a balanced budget in each period. Obviously, this tax scheme will not distort any incentive, and it will not change any educational choice. Hence, the labor market outcomes obtained from sections 2 and 3 can be transferred to this part directly. Assume that an entrepreneur's flow utility function equals her consumption. Since all profits are obtained by entrepreneurs. It is thus obvious that the goods market clearing condition is

$$tc = y - q(v_e + v_n), \quad (45)$$

where tc is steady state total consumption, y is steady state output, v_e and v_n are steady state vacancies for educated and uneducated workers. Provided with the production function and distribution of ability, I can show that

$$y = \frac{1}{2}(z_l + \bar{z})(N_n - u_n) + \frac{1}{2}(1 + e)(\bar{z} + z_h)(N_e - u_e). \quad (46)$$

Since the utility functions of all agents (residents and entrepreneurs) in the economy are linear in consumption, the summation of the flow utility of all agents at the steady state, χ , can be written as

$$\chi = tc - \eta(N_n + N_e - u_n - u_e) - \frac{1}{2}\gamma(z_h - \bar{z})N[1 - F(\bar{z})]\delta \sum_{j=0}^{k-1} (1 - \delta)^j. \quad (47)$$

The second term of (47) is the total disutility from working. $\frac{1}{2}\gamma(z_h - \bar{z})$ is the average disutility from education and $N[1 - F(\bar{z})]\delta \sum_{j=0}^{k-1} (1 - \delta)^j$ is the number of people currently taking education. Hence, the third term of (47) shows the total disutility from education. Hence, the social welfare, Δ , at the steady state can be characterized as

$$\Delta = \frac{\chi}{1 - \rho}. \quad (48)$$

Since Δ is a monotonic transformation of χ , I will concentrate my analysis on effect of a change in educational subsidy on χ . As stated above, when s increases, more people choose to take education and aggregate unemployment rate decreases, both of which lead to more total output. As a result, there is a utility gain from more consumption (the first term of χ). However, as unemployment decreases, disutility from working increases (the second term of χ). Since \bar{z} goes down due to the increase of s , the average disutility from education increases.

Together with an increase in the number of people taking education, total disutility from education should also increase (the third term of χ).

According to the simulation results, the positive impact from the first term of (47) as a result of a higher educational subsidy outweighs the negative impact from the last two terms. Hence, based on this model, a higher educational subsidy actually improves the welfare of the society as a whole. Figure 8 shows the positive relationship between unemployment benefit and social welfare.

4.2.3 A Related Issue

In recent years, newly graduated college students from China often find it harder to find a job and their starting wages are lower than their predecessors. Along with this phenomenon is the increase of the number of college students. These observations can also be explained by the model and the intuitions are very similar to those introduced in section 4.2.1. Since 1998, China's supply of higher education have been continuously increasing. As a result, the required college entrance examination score to qualify for higher education decreased dramatically. Within the framework of this model, it means that the disutility from education decreases as it takes less time and effort to prepare for the exam. Based on (36), it means that \bar{z} falls and more people would like to take education. Hence, employers expect that the aggregate quality of educated workers decreases, posting less jobs for them. Suffering from the tightened labor market, educated workers earn lower wages and are more likely to be unemployed

5 Skill-biased Technological Change

Since 1970s, uneducated workers' real wage has decreased both absolutely and relatively compared to the real wage of educated workers even though the supply of educated labor increased dramatically. One explanation is that technology improvements are biased to educated workers such that the demand for uneducated workers decreased relatively. The model presented in this paper can also be used to analyze the effect of a skill-biased technological change. In addition to the mechanism introduced by traditional literature on skill-biased technological change, the model provides a new explanation on the effect of such technological shocks,

specifically, the quality of uneducated workers can be affected.

To model skill-biased technological change, suppose there is a productivity bonus for educated workers. That is, $p_i = (1 + e)(1 + \varepsilon)z_i$ if type i workers take education, where $\varepsilon > 0$ is the size of a one time technological shock. Technically, a skill-biased shock is equivalent to an increase in the effect of education.

The following proposition analyzes the effect of such a skill-biased technological change on uneducated workers.

Proposition 5 *When there is a positive skill-biased technology shock, in the steady state, more people will take education. The steady state unemployment rate of uneducated workers will increase, the steady state wage of each uneducated worker and the steady state average wage of uneducated workers will decrease.*

Proof. Technically, a skill-biased shock is equivalent to an increase in the effect of education. Hence, consider an increase of e in (36). This change will lead to a smaller \bar{z} , which means more people will take education as $1 - F(\bar{z})$ is the steady state proportion of people taking education. According to Proposition 2, θ_n is an increasing function of \bar{z} . Hence, θ_n will decrease. Based on (42), it can be concluded that the unemployment rate of uneducated workers will increase. According to (31), $w_{i,n}$ is an increasing function of θ_n . Hence, the wage of each type of individuals, who still choose not to be educated after the change in e , decreases. The average wage of uneducated worker can be written as

$$\begin{aligned} w_n &= \frac{\lambda[1 - \rho(1 - x - \mu_n)](z_l + \bar{z})/2 + (1 - \lambda)[1 - \rho(1 - x)](\eta + b)}{1 - \rho(1 - x) + \lambda\rho\mu_n} \\ &= (z_l + \bar{z})/2 - \frac{(1 - \lambda)[1 - \rho(1 - x)][(z_l + \bar{z})/2 - \eta - b]}{1 - \rho(1 - x) + \lambda\mu_n}. \end{aligned} \quad (49)$$

The first line of (49) shows that $\partial w_n / \partial \bar{z} > 0$. The second line of (49) shows that $\partial w_n / \partial \theta_e > 0$. Since both \bar{z} and θ_e decreases as the result of an increase in e , the average wage of uneducated workers decreases. ■

The intuition of Proposition 5 is as the following. When ε increases, the productivity of educated workers will increase. Holding all other variables constant, the profits a firm can earn from an educated worker becomes larger, making entrepreneurs more willing to post vacancies for educated workers. Based on (32) and (44), an educated worker will earn higher

wage and suffer less from unemployment. Hence, the incentive of taking education would be stronger under the presence of skill-biased technological change.

As a result, more people are willing to take education. Similar to the analysis in the previous subsection, as the number of educated workers increases, the average quality of uneducated worker deteriorates. Hence, entrepreneurs will post less vacancies for uneducated workers, leading to a tighter labor market and higher unemployment rate. According to (31), the tightened labor market causes the real wage of an uneducated worker to decrease. However, this is not the only reason why the average real wage of uneducated workers decreases. Therefore, the decrease in \bar{z} will also contribute to the decrease in the average wage, in addition to the drop of wage caused by weak demand for uneducated workers.

The average real wage and unemployment rate of uneducated workers given various level of skill-biased technological change are presented in figure 9 and figure 10.

6 Conclusion

This paper considers a combination of search model and a signaling game. Based on this model, by taking education, workers send a positive signal to entrepreneurs. As a result, firms are more willing to post vacancies and hire educated workers, which means educational achievement for job application is required by entrepreneurs to distinguish high-productivity workers from their low-productivity peers. Benefiting from a less tight labor market, educated workers enjoy higher wages and suffer less from unemployment, which motivate residents to take education. This model can explain about 64% of unemployment rate gap between educated and uneducated workers.

If unemployment benefit is increased, both skill specific unemployment rates will increase. The model predicts that, however, the unemployment gap will increase as the consequence. Hence, the higher unemployment benefit stimulates more people to take education.

Higher educational subsidy encourages more workers to take education, lowering the expected ability of both types of workers. Both skill-specific unemployment rates, therefore, will increase. However, as the weight of educated workers becomes larger, aggregate unemployment rate will decrease. The additional utility brought by extra consumption will outweigh the additional disutility from extra working and education, which means the welfare effect of

a higher educational subsidy is positive.

When the economy encounters a skill-specific technology change, firms will post more vacancies for educated workers, creating a stronger incentive to take education. There are two consequences due to this change. First, the average quality of uneducated workers is lower. Second, the labor market for uneducated workers becomes tighter. Both of them tend to increase the unemployment rate of uneducated workers and lower their real wage, which is consistent with economic data since 1970s.

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Parameter	Description	Value
N	total population	1.00
β	discount factor	.999
δ	probability of death	4.42×10^{-4}
k	periods needed for education	146
λ	workers' bargaining power	.500
α	elasticity of matching	.500
x	job separation rate	.012
η	disutility from working	.091
b	unemployment benenefit	.726
z_l	lower bound of ability	1.00
z_h	upper bound of ability	2.40
γ	rate of disutility from education	.374
e	productivity return of education	.203
φ	matching efficiency	.338
q	vacancy posting cost	1.86
s	educational subsidy	.242

Table I: Values of Model Parameters

Unemployment Rate	Model	Data (92-11)
educated workers	3.80%	2.91%
uneducated workers	6.40%	7.00%

Table II: Unemployment Rates

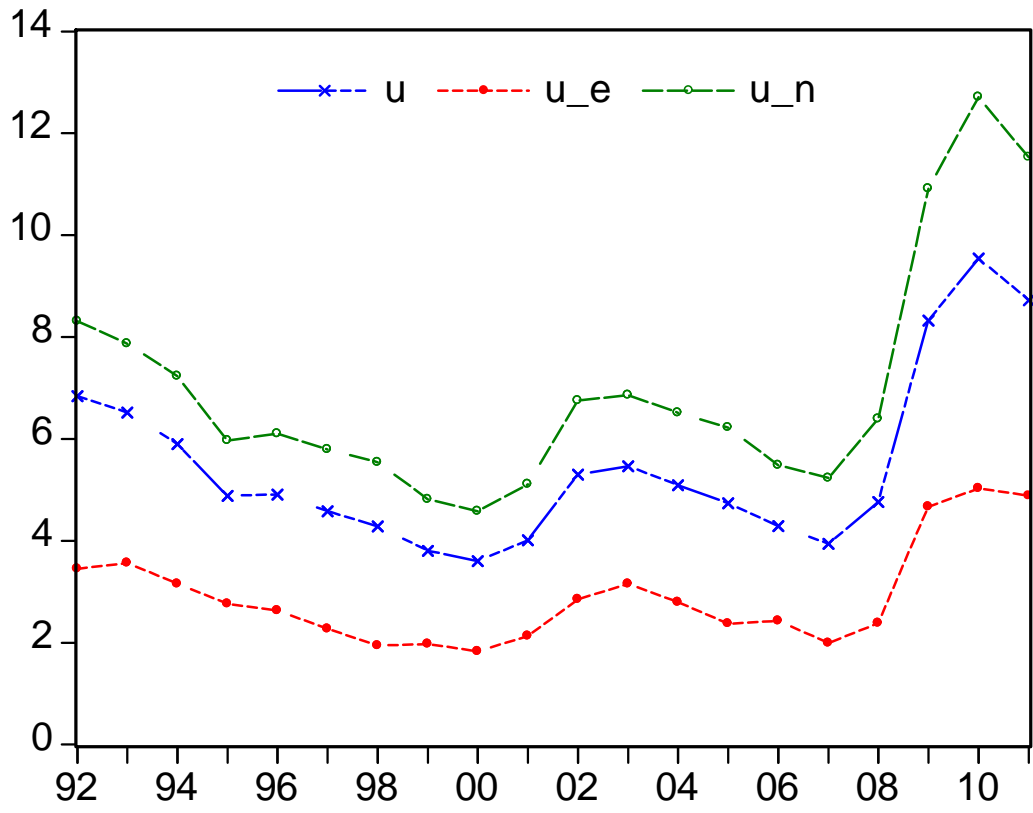


Figure 1: Unemployment Rates

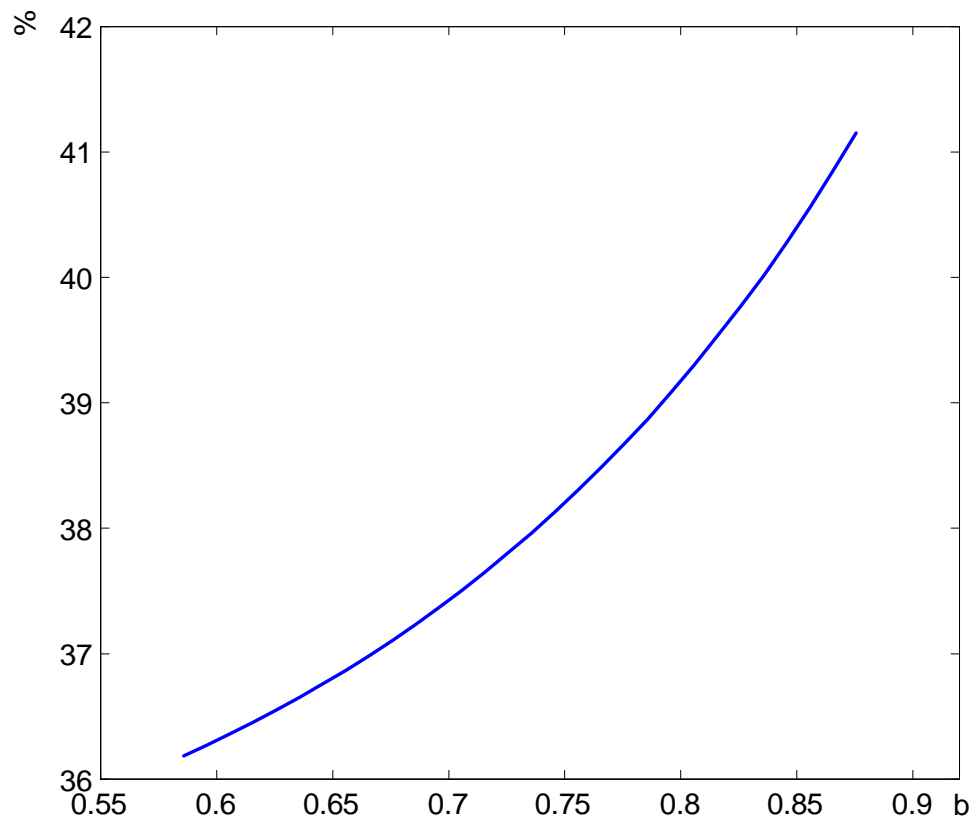


Figure 2: Proportion of Educated Workers

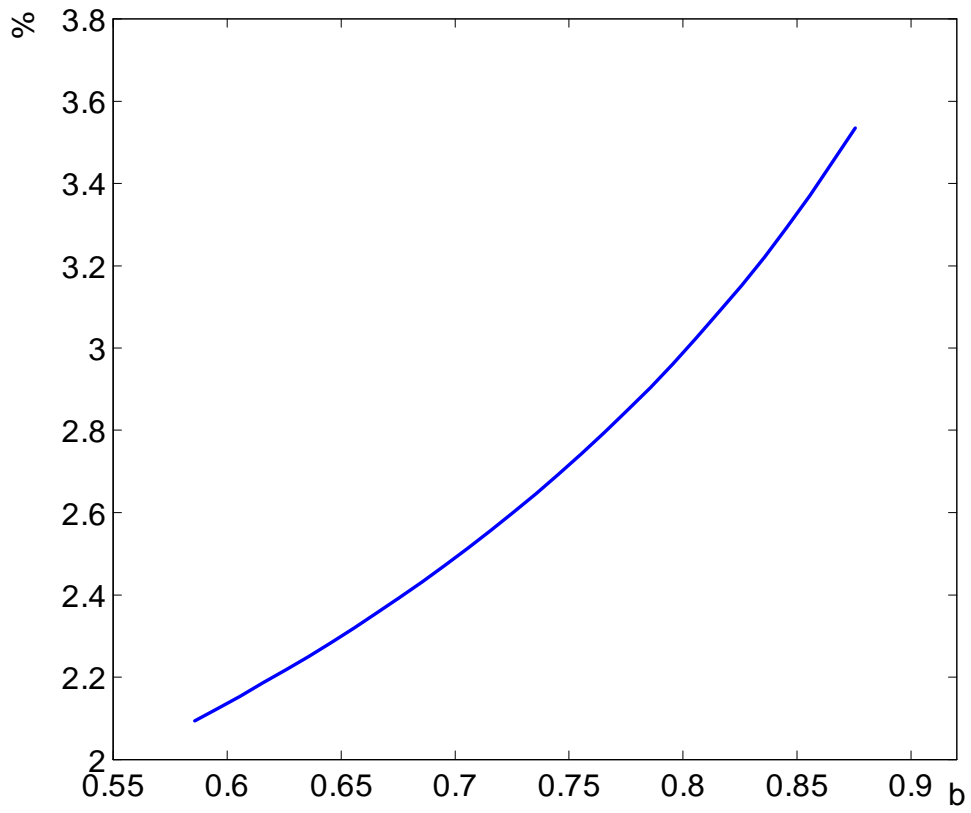


Figure 3: Unemployment Rate Gap ($u_n - u_e$)

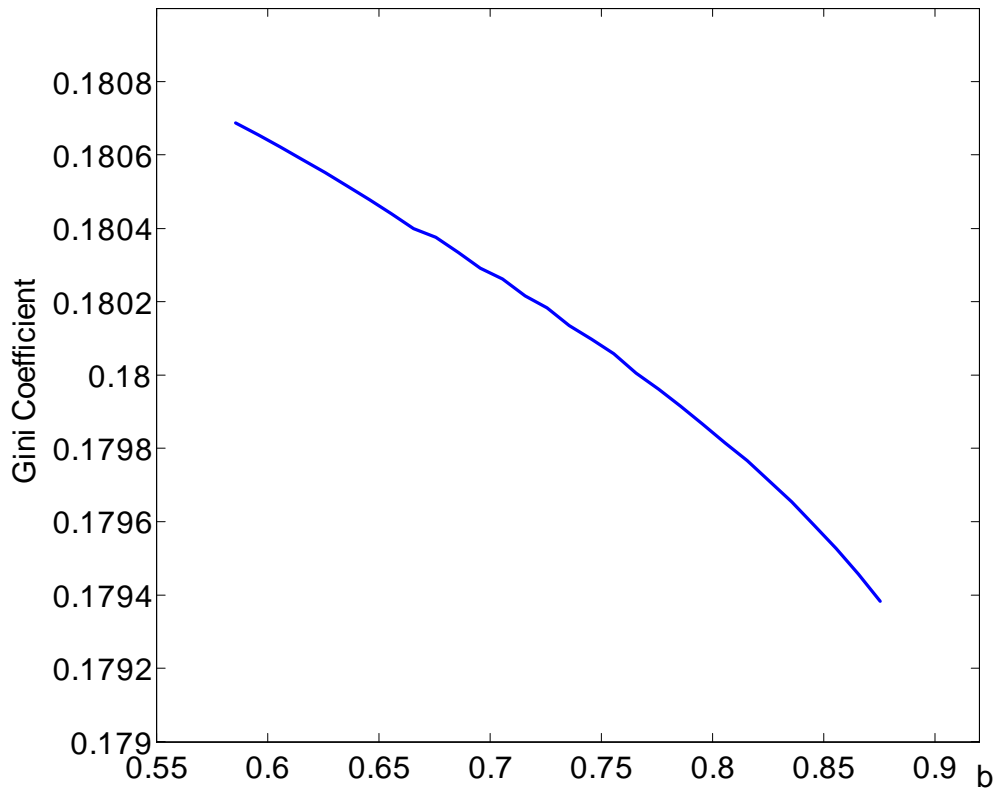


Figure 4: Gini Coefficient

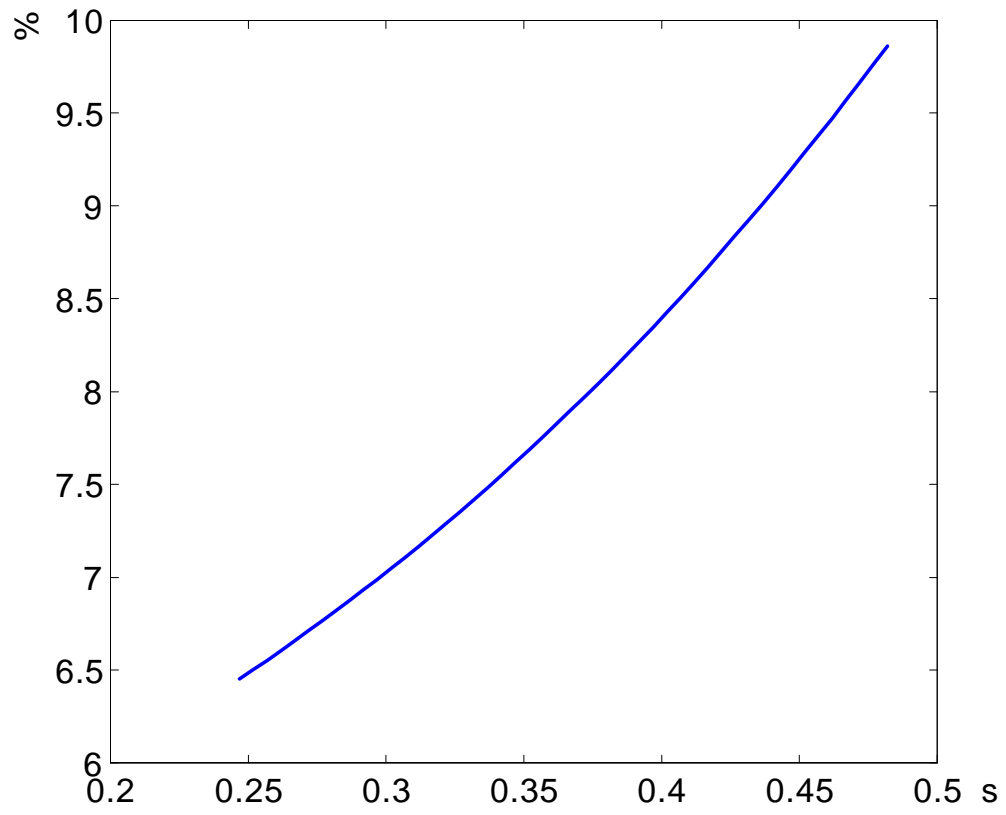


Figure 5: Unemployment Rate of Uneducated Workers

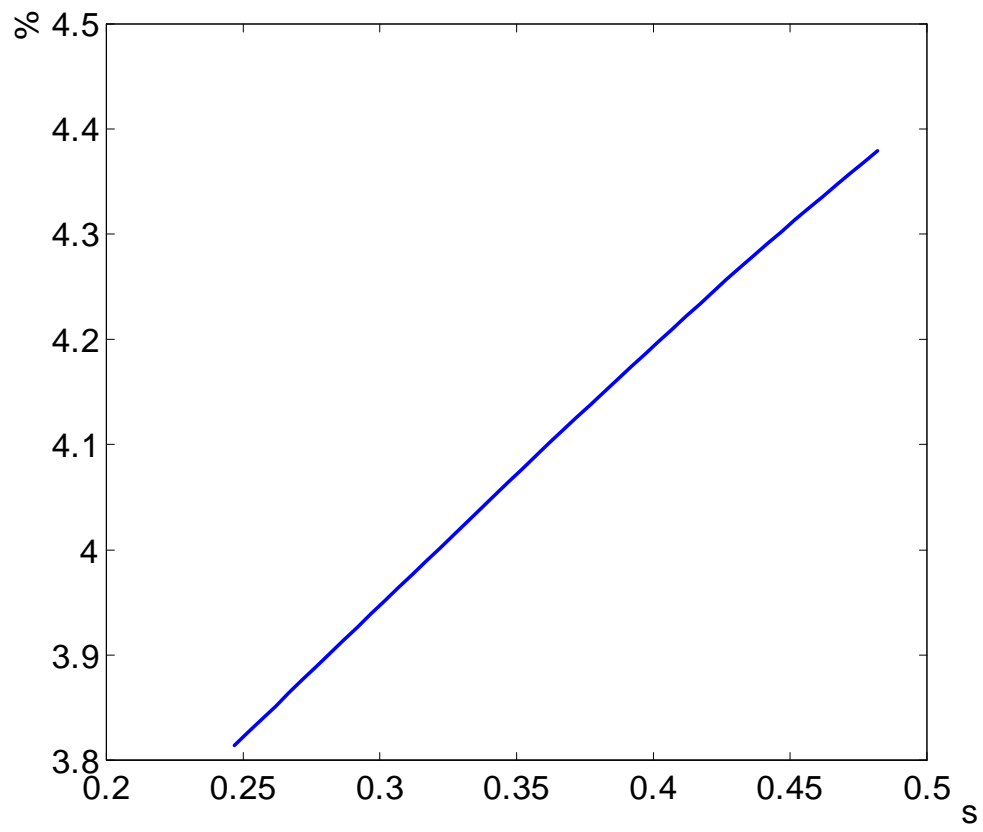


Figure 6: Unemployment Rate of Educated Workers

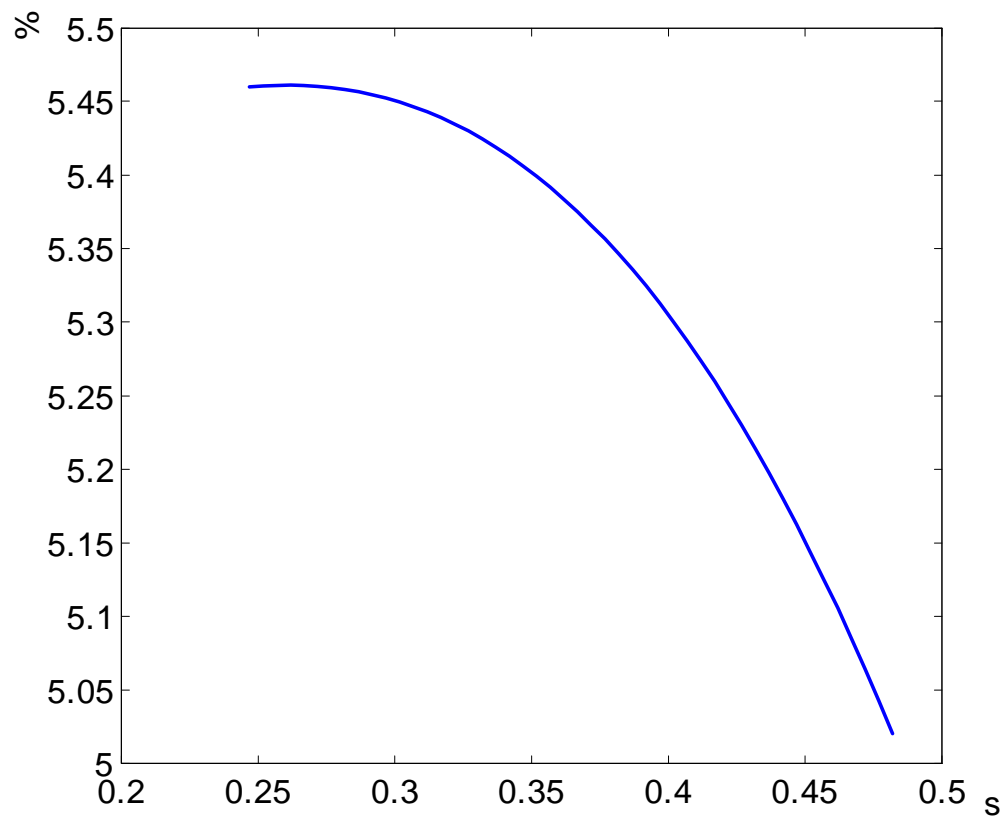


Figure 7: Aggregate Unemployment Rate

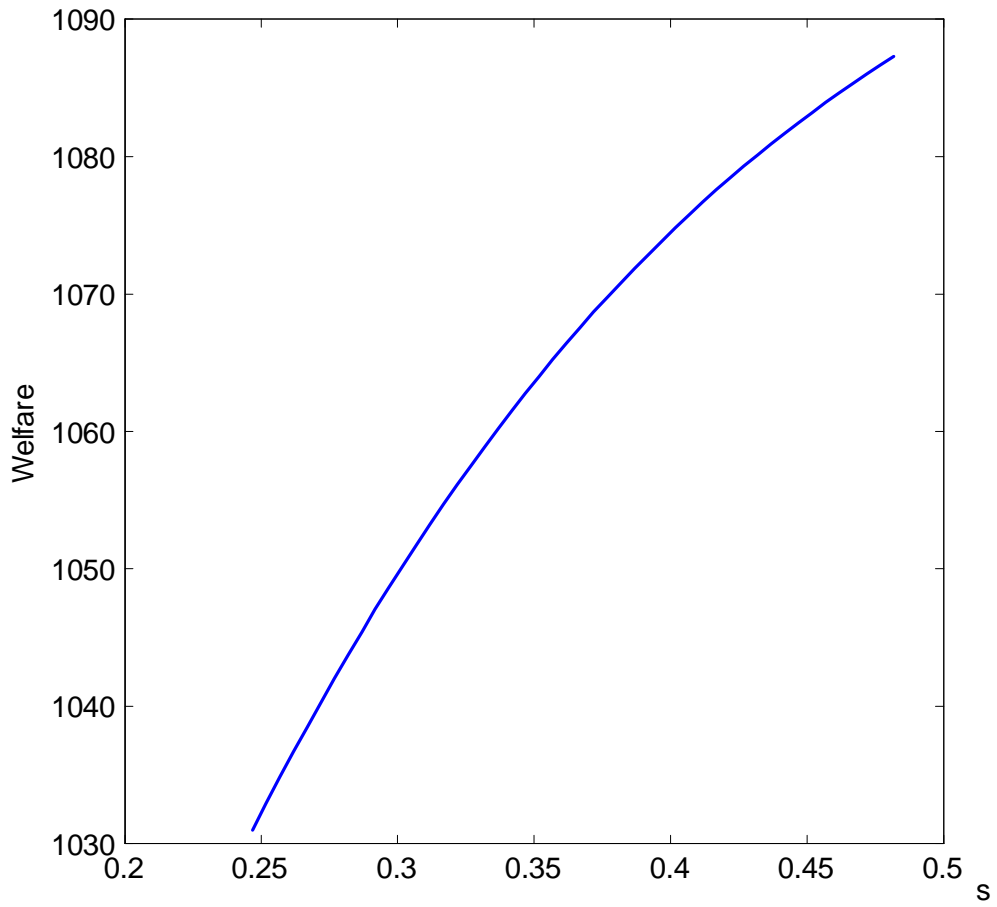


Figure 8: Social Welfare

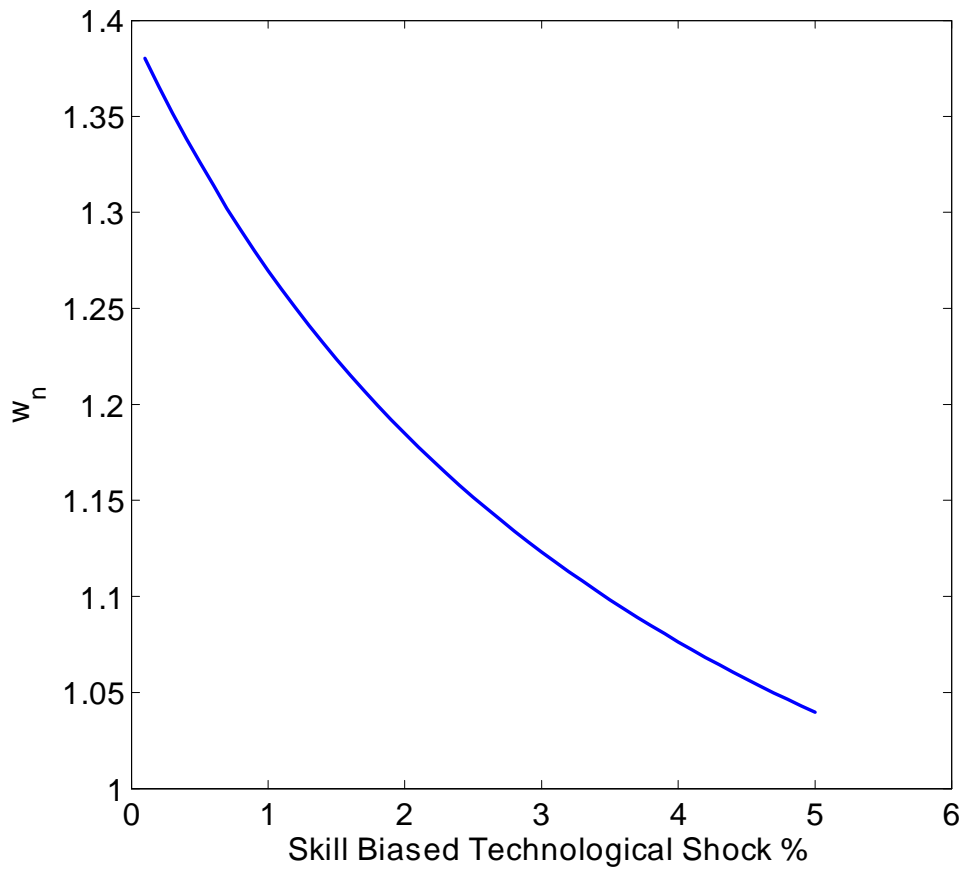


Figure 9: Average Wage of Uneducated Workers

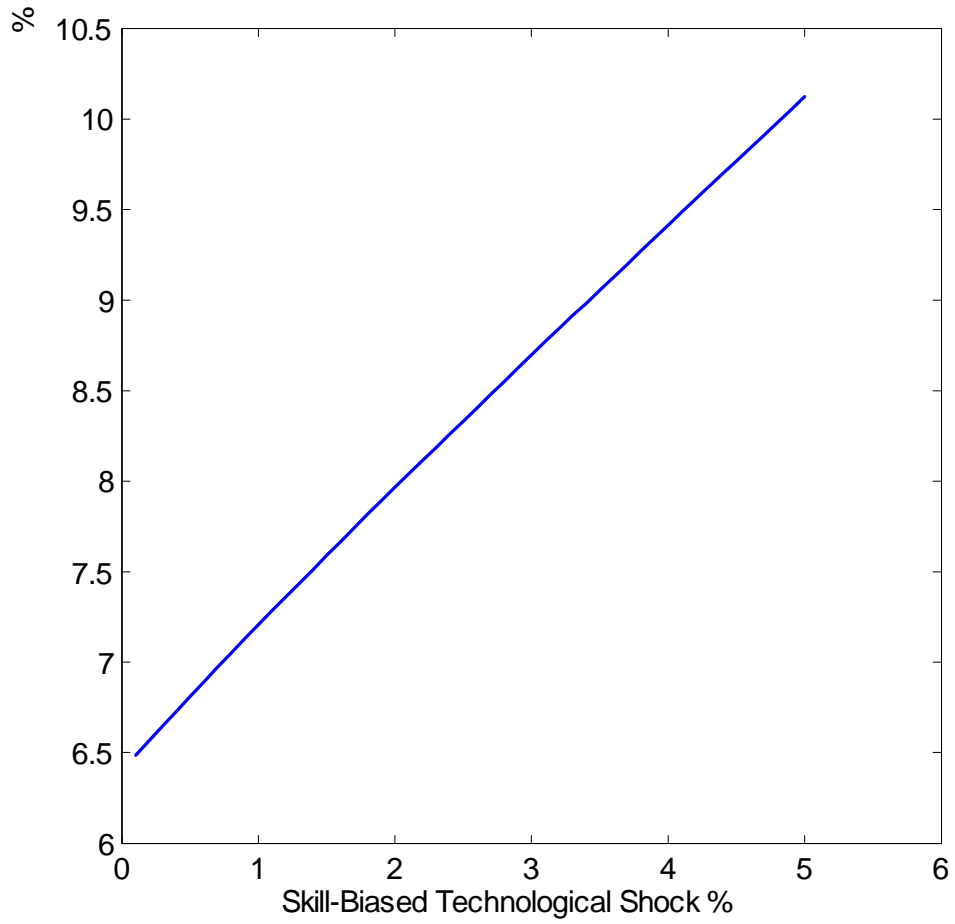


Figure 10: Unemployment Rate of Uneducated Workers