Dynamic Small Open Economy Model with Involuntary Unemployment

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Abstract

It is well known that real business cycle small open economy models rely on Greenwood, Hercowitz and Huffman (1988) preferences to match the countercyclical trade balance observed in open economies, as well as other second moments. In contrast, standard preferences a la King, Plosser and Rebelo (1988) are abandoned in this literature and are commonly labeled as ineffective due to their inability to yield a countercyclical trade balance. In this paper, I re-examined standard preferences in a small open economy (SOE) model calibrated to Canada. Contrary to prior claims I show that a SOE model with standard preferences and 'involuntary' unemployment with efficient risk sharing obtains a countercyclical trade balance and well matches main empirical regularities that emerge in open economies.

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Keywords: small open economy, indivisible labor, trade balance, preferences.

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1. INTRODUCTION

In a seminal paper, Mendoza (1991) builds a real business cycle small open economy (SOE) model that is consistent with main empirical business cycle regularities of Canada. In particular, the model correctly predicts a countercyclical trade balance and a positive correlation between savings and investment.[1] However, a number of anomalies arise, such as a low standard deviation of total hours and a perfect positive correlation between hours and output. These irregularities emerge due to a Greenwood, Hercowitz and Huffman (1988) momentary utility specification, where the elasticity of intertemporal substitution associated with leisure is zero.[2] Subsequently, Correia, Neves and Rebelo (1995) and Schmitt-Grohe and Uribe (2003) naturally consider the standard preferences of King, Plosser and Rebelo (1988) to overcome the above-mentioned anomalies, however they find these preferences to produce adverse results.[3] First, in contrast to empirical open economy observations, the correlation between output and the trade balance is strongly positive instead of negative.[4] Furthermore, the model understates volatilities of a number of real variables, including consumption and total hours. In light of these findings, it is a common practice in the SOE literature to label standard

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[1] Backus *et al*. (1992) find that the relationship between trade balance and output is negative when looking at quarterly data for 12 developed (OECD) countries. Similar results obtain in Backus and Kehoe (1992), who look at a hundred years of annual data for 10 developed countries, and in the empirical work of Krugman and Baldwin (1987).

[2] Henceforth referred to as GHH preferences.

[3] Schmitt-Grohe and Uribe (2003) calibrate the model to Canadian data, while Correia, Neven and Rebelo (1995) use data for Portugal.

[4] In the small open economy model the response of the trade balance to a positive technology shock depends on two opposing effects, a pro-investment effect and a pro-borrowing effect. The pro-investment effect implies that individuals increase investment abroad due to the spread of the rise in income over future periods, while the pro-borrowing effect induces individuals to increase foreign borrowing to increase domestic capital stock and take advantage of the rise in productivity. In the GHH framework, the latter effect dominates, leading to a countercyclical trade balance, while in the standard preference framework, consumption is very smooth, and hence the pro-investment effect dominates, which yields a positive correlation between output and the trade balance.

preferences as inept in matching business cycle fluctuations and to rely on GHH preferences instead.

Recently, a number of modifications have been added to the model of Mendoza (1991). Letendre (2004) finds that introducing endogenous capital utilization improves the standard deviations of output and hours, while adding habit in consumption improves the dynamic properties of consumption and the trade balance.[5] Letendre and Luo (2007) show the impact of combining productivity shocks and investment specific shocks. They find that business cycle dynamics improve with respect to output and the trade balance dynamics, but at the expense of consumption and total hours fluctuations. Guo and Janko (2008) introduce intertemporally non-separable labor supply and endogenous capital utilization and find that the model accounts for key business cycle properties of Canadian data.[6] However, common to all three papers is the necessary use of GHH preferences. To date, little attempt has been made to model non-GHH preferences in a technology driven SOE model.[7]

In this paper, I re-examine standard preferences of King, Plosser and Rebello (1988, henceforth KPR) in a SOE model, preferences that incorporate a positive wealth effect. The model is modified to include indivisible labor, following Rogerson and Wright (1988) and is calibrated to Canadian data. Given this specification, it is shown

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[5] In contrast to Mendoza (1991) who uses annual data, Letendre (2004) calibrates the model to quarterly Canadian data. In addition, Mendoza calibrates the AR(1) technology process by matching the volatility of output in the model to the one observed in the data, while Letendre (2004) estimates the AR(1) process using Canadian data. Given this parameterization, he finds that a base model without endogenous capital utilization and habit in consumption leads to a low volatility in output and hours.

[6] Theirs is the only model that obtains a correlation between output and hours that closely matches the data. All other papers have a correlation of 1.

[7] In a recent paper Jaimovich and Rebelo (2008) consider a specification of preferences that allow for a varying wealth effect in a SOE model, preferences that nest standard preferences of King et al. (1988) and the GHH preferences as special cases. However, the objective of their work is to analyze the impact of unanticipated news about future total factor productivity.

that the model overcomes the claim common to the SOE literature that GHH preferences are necessary for the model to be consistent with main business cycle regularities of open economies and outperforms the GHH model specification along several dimensions.

The model in this paper differs from the non-GHH model considered in Correia, Neves and Rebelo (1995) and Schmitt-Grohe and Uribe (2003) in that it assumes that labor is indivisible. Hence, agents either work a fixed number of hours or they do not work at all. This non-convexity in preferences is eliminated using employment lotteries as in Rogerson and Wright (1988). Individual agents enter a lottery that with some positive probability determines whether they work or remain unemployed.[8] Moreover, efficient risk sharing implies that the consumption levels of the employed and unemployed differ, that is the employed consume more.[9] Consequently, when the economy is hit with a positive technology shock, the number of employed rises and so does their individual consumption level. Hence, the rise in consumption for those that become employed causes a jump in aggregate consumption. Moreover, the value of the risk aversion parameter plays an important role, since risk sharing implies that the higher the value of risk aversion the greater the consumption of those that are employed. Consequently, while individual agents smooth consumption, aggregate consumption is much more volatile with involuntary unemployment. The high jump in consumption in the presence of a positive technology shock ultimately gives rise to a negative response of the trade balance for a sufficiently high risk aversion parameter.

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[8] Individuals enter into a contract that allows for random layoffs and the lottery itself determines whether one is employed or is unemployed.

[9] Hansen (1985) shows that with the use of log-log utility the consumption of employed and unemployed agents is identical. Here, the log-log utility case is not considered. Hence, in our model consumption is higher for employed as compared to unemployed, which implies that unemployed are worse off. Rogerson and Wright (1988) is labeled refer to the unemployment in this model as involuntary unemployment

It is important to note that indivisible labor alone is not enough to obtain a countercyclical trade balance in a SOE with an operating wealth effect. A sensitivity analysis reveals that the value of the risk aversion parameter has to be above 3 in order to obtain a negative correlation between output and the trade balance. However, even with a lower risk aversion value, the model improves upon the divisible labor model, although the correlation is still positive. Specifically, in the 'involuntary' unemployment model, with a risk aversion coefficient of 2 the correlation between output and the trade balance is 0.37, while the standard KPR model yields a correlation of 0.99. Although, this is an improvement, it does not match the data (= –0.29), nor is the correlation negative as in the basic GHH model (= –0.89).[10] However, increasing the risk aversion parameter to for example 3(4) yields a negative correlation of –0.20(–0.36). Thus, the indivisible labor KPR model can yield a countercyclical trade balance given reasonable parameterization.[11] With respect to other moments, the 'involuntary' unemployment model yields higher output, consumption and labor volatilities, which rise in the risk aversion parameter. Overall, the KPR model with indivisible labor can well match a number of empirical regularities that emerge in open economies. However, as the GHH model, it fails to account for the correlation of hours and consumption with output.

The paper is organized as follows. The model is presented in section 2. In section 3 the equilibrium, the solution method and the calibration are discussed. Section

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[10] These findings are based on a basic SOE model in that it does not incorporate habit formation, variable capital utilization or intertemporally non-separable labor supply. Moreover, the comparisons are made given a risk aversion parameter of 2 in all models.

[11] Note that considering higher values of the risk aversion parameter in the standard KPR model does not yield a negative correlation between output and the trade balance, in fact the correlation remains positive and high.

4 presents quantitative results that include a sensitivity analysis to the relative risk aversion parameter. Section 5 concludes the paper.

2. THE ECONOMY

The economy consists of a unit measure of identical, infinitely-lived individuals who maximize expected discounted lifetime utility

$$\mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \mathrm{U}(\mathrm{c}_t, l_t), \tag{1}$$

where $\beta \in (0,1)$, c_t is consumption, and l_t is leisure.

The functional form of the momentary utility function exploited here follows King, Plosser and Rebelo (1998) and is the one considered by Correia, Neven and Rebelo (1995) and Schmitt-Grohe and Uribe (2003). It is given by

$$\mathrm{U}(\mathrm{c}_t, l_t) = \left\{ \frac{\left[\mathrm{c}_t^{\omega} l_t^{1-\omega}\right]^{1-\sigma} - 1}{1-\sigma} \right\}, \qquad 0 < \omega < 1, \sigma > 0, \tag{2}$$

where σ is the relative risk aversion parameter. In contrast, typical preferences utilized in the SOE literature are of GHH form and are given by

$$\mathrm{U}\left(\mathrm{c}_t, l_t\right) = \frac{\left\{ \left[\mathrm{c}_t - \frac{\left(1-l_t\right)^{\omega}}{\omega} \right]^{1-\sigma} - 1 \right\}}{1-\sigma}, \qquad \omega > 1, \sigma > 0. \tag{3}$$

It is easy to show that the wealth effect in (3) is zero.[12]

The utility function in (2) is nonseparable in consumption and leisure, concave and twice continuously differentiable. Agents in the economy are endowed with one unit

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[12] Although I provide results for a model with GHH preferences in section 4 to facilitate comparisons, the reader is referred to Mendoza (1991), Correia *et al.* (1995), etc. for a detailed exposition of these preferences.

of time that they allocate between leisure and labor. In addition, following Hansen (1985), individuals are assumed to either work $h_0 > 0$ fixed hours or not at all. This indivisibility of labor introduces non-convexities in preferences. Here, employment lotteries are utilized to convexify the choice set as in Hansen (1985), Rogerson (1988) Rogerson and Wright (1988) and Hansen and Sargent (1988).[13] At the beginning of period t individuals enter a lottery that, with probability p_t determines whether they work and with probability 1- p_t remain unemployed. The lottery provides a mechanism for dividing the continuum of agents into two subsets, one where each individual is employed (and receives an income that allows him to consume c_{1t}) and another where each agent is unemployed (receiving an income that allows for the consumption of c_{2t}).[14] Agents therefore trade contracts with the firm, a contract that allows for random layoffs. Prior to the draw of the lottery the expected utility for an individual is

$$p_t u(c_{1t}, 1-h_0) + (1-p_t) u(c_{2t}, 1).^{15} \tag{4}$$

Total consumption allocation across both types of individuals must satisfy the feasibility condition

$$p_t c_{1t} + (1-p_t) c_{2t} = c_t. \tag{5}$$

Maximizing (4) subject to the constraint in (5) yields the risk-sharing condition

$$c_{1t} = c_{2t} \left(\frac{1-h_0}{1} \right)^{\frac{(1-\omega)(1-\sigma)}{1-\omega(1-\sigma)}}, \tag{6}$$

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[13] Convexifying the set via lotteries allows agents to be better off ex ante and thus is welfare improving. Rogerson and Wright (1988) provide a thorough welfare analysis in economies with involuntary unemployment via indivisibilities in labor.

[14] Following Hansen (1985), one can think of each agent drawing a realization of a random variable z_t from a uniform distribution on [0,1]. Each individual i is then identified as x(i,z) according to the following rule:

$$x_t(i,z) \equiv i+z_t \text{ if } i+z_t \le 1 \text{ and } x_t(i,z) \equiv i+z_t-1 \text{ if } i+z_t > 1.$$

If $x_t(i,z) \le 1-p_t$ then the agent is unemployed, and if $x_t(i,z) > 1-p_t$ then the agent is employed.

[15] Subscript 1 is used for employed workers and subscript 2 for unemployed workers.

which states that employed agents consume more than unemployed agents as long as $\sigma > 1$.[16] Now when the economy is hit with a positive technology shock, the number of employed rises and so does their individual consumption level, a jump from c_{1t} to c_{2t}. Hence, as consumption for those that become employed rises, so does aggregate consumption. Furthermore, with a higher value of σ a given rise in employed causes the rise in their consumption to be higher, thus further increasing aggregate consumption. In addition, equation (6) implies that for $\sigma > 1$ aggregate consumption smoothing is not as prominent as individual consumption smoothing. Consequently, the volatility of consumption in the indivisible labor economy with $\sigma > 1$ is greater, as compared to the divisible economies of Correia, Neves, and Rebelo (1995) and Schmitt-Grohe and Uribe (2003).[17]

Using Eq. (6) and (5) in (4) yields, after some algebra, the utility function of a 'representative' consumer:

$$u(c_t, l_t) = \frac{\{c_t^{\gamma_1} \Phi(l_t)^{\gamma_1} - 1\}}{1 - \sigma} \tag{7}$$

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[16] With log-log preferences as in Hansen (1985), the risk sharing condition together with the feasibility condition implies $c_{1t}=c_{2t}=c_t$, thus all agents consume the same level of consumption. This latter specification is known to only slightly increase aggregate consumption and is only considered in the sensitivity analysis section of this paper. See Greenwood and Huffman (1988) and Rogerson and Wright (1988) for a discussion of the $\sigma > 1$ case.

[17] As in the standard real business cycle literature, a positive technology shock in this model is followed by an immediate rise in total hours worked (due here to the extensive margin). A large literature questions this prediction. In particular the work by Gali (1999), Shea (1998), Kiley (1998) and Francis and Ramey (2005), among others argue that this prediction is inconsistent with the empirical evidence, which they argue shows technology shocks to lead to an immediate fall in total hours followed by a rise. Nevertheless, this line of thought has come under critique as well Christiano *et al.* (2004) argue that what determines whether a positive technology shock has an immediate positive or negative impact on hours is how low-frequency component of hours worked is treated; they argue the response it positive. Moreover, in their (2003) paper they show that Canadian hours worked increase following a positive technology shock. Chang and Hong (2003) use disaggregated data and find the results to be mixed. Uhlig (2003) argues that the initial response is zero, followed by a positive response. In light of the inconsistent findings I consider the proposition of a positive response of hours to a positive technology shock to be reasonable. Furthermore, relying on this proposition allows for a comparison to other papers in the SOE literature.

$$\Phi(l_t) = \left[\left(\frac{1-l_t}{h_0} \right) (1-h_0)^{\frac{\gamma_1}{\gamma_2}} + \left(1 - \frac{1-l_t}{h_0} \right) (1)^{\frac{\gamma_1}{\gamma_2}} \right]^{\frac{\gamma_2}{\gamma_1}}$$

where $l_t=1-p_t h_0$, $\gamma_1 = (1-\omega)(1-\sigma)$ and $\gamma_2 = 1-\omega(1-\sigma)$. Total hours (=$h_t$) in the economy are given by $p_t h_0$ and fluctuate with movements in the employment rate, p_t.[18]

The budget constraint of the representative consumer is given by

$$c_t + i_t + (1+r_{t-1})d_{t-1} + \psi(d_t - \bar{d})^2 = y_t + d_t, \qquad \psi > 0, \tag{8}$$

where i_t is investment and r_{t-1} is the international real interest rate on foreign debt d_t. Agents are assumed to incur positive portfolio adjustment costs whenever foreign debt differs from its long run level $\bar{d}$.[19] This assumption is made for purely technical reasons, as without portfolio adjustment costs the equilibrium is nonstationary.[20]

Output in this economy is produced using a constant returns to scale Cobb-Douglas production function:

$$y_t = z_t k_t^{\theta} h_t^{1-\theta} \tag{9}$$

where k_t is capital and z_t represents the technology shock. The production function is increasing in both its arguments and is twice continuously differentiable. The technology shock is assumed to evolve according to

$$z_t = \rho z_{t-1} + \varepsilon_t, \tag{10}$$

where the random variable ε_t is i.i.d. with mean zero and a standard deviation of σ_z.

Capital stock in the economy evolves according to the following law of motion

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[18] Ex-post the lottery, the representative consumer chooses h_t.

[19] The assumption that $\psi>0$ guarantees that $\lim_{j\to\infty} E_t d_{t+j} = \bar{d}$.

[20] Schmitt-Grohe and Uribe (2003) consider alternative stationarity-inducing mechanisms, such as the endogeneous discount factor utilized by Mendoza (1991) and show the quantitative results along business cycle frequencies to be insensitive to other stationarity-inducing techniques.

$$k_{t+1} = (1-\delta)k_t + i_t - \phi\left(k_{t+1} - k_t\right)^2, \qquad \phi > 0, \tag{11}$$

where $\delta \in (0,1)$ represents the capital depreciation rate. The last term in (11) gives the capital adjustment costs of net investment.[21]

The first-order conditions of the representative household with respect to h_t, c_t, k_{t+1}, and d_t, and the associated transversality conditions (TVC) are

$$c_t \quad : \quad \lambda_t = \left(\frac{1-\gamma_1}{1-\sigma}\right) c_t^{-\gamma_1} \Phi\left(1-h_t\right)^{\gamma_1} \tag{12}$$

$$l_t \quad : \quad \lambda_t \frac{\partial y_t}{\partial h_t} = \left(\frac{\gamma_1}{1-\sigma}\right) c_t^{1-\gamma_1} \Phi\left(1-h_t\right)^{\gamma_1 - 1} \frac{\partial \Phi\left(1-h_t\right)}{\partial h_t} \tag{13}$$

$$k_{t+1} \quad : \quad \lambda_t \left[1 + 2\phi(k_{t+1} - k_t)\right] = \beta \underset{t}{E}\left\{\lambda_{t+1}\left[\theta e^{z_t} k_t^{\theta-1} h_t^{1-\theta} + 1 - \delta + 2\phi(k_{t+1} - k_{t+2})\right]\right\}, \tag{14}$$

$$d_t \quad : \quad \lambda_t\left[1 - 2\psi(d_t - \bar{d})\right] = \beta\left(1 + r_t\right) \underset{t}{E} \lambda_{t+1}, \tag{15}$$

$$\text{TVC}_1 : \quad \lim_{j\to\infty} \underset{t}{E}\left[\theta_{t+j}\lambda_{t+j}k_{t+1+j}\right] \leq 0, \tag{16}$$

where λ_t is the marginal utility of consumption given in (12). Equation (13) is the intra-temporal condition that equates the real value of the marginal product of labor to the representative consumer's marginal rate of substitution between consumption and leisure and equation (14) gives the Euler equation for intertemporal consumption choices. The representative consumer's intertemporal choices of foreign bonds are governed by (15), where the left hand side is the benefit in utility terms of increasing debt by one unit, thus increasing consumption by one unit minus the cost of adjusting debt, and the right hand side is the cost in today's utility terms of taking on this debt plus the interest on the debt.

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[21] Mendoza (1991) finds that without capital adjustment costs the volatility of investment is much too high, due to the household's ability to borrow from abroad to finance domestic investment in the presence of a positive technology shock.

3. EQUILIBRIUM, SOLUTION METHOD AND CALIBRATION

The competitive equilibrium is a set of stochastic processes $\{c_t, h_t, k_{t+1}, y_t, d_t, \lambda_t\}$ that satisfy equations (8)-(9), the law of motion for capital (11), the first-order conditions (12) -(15) together with the productivity disturbance (10), the initial conditions z_0, k_0, and d_{-1}, and the transversality condition in (16). The world interest rate is assumed to be constant and equal to r. Finally, the model is solved using the method of undetermined coefficient (Campbell, 1994).

The parameter values are calibrated for Canada. Following Mendoza (1991), the discount factor β is set at 0.993 and the capital's share in total income θ is set at 0.32. The depreciation rate δ is set equal to 0.02 corresponding to an 8% annual depreciating rate. The value of ω is set to 0.22 as in Schmitt-Grohe and Uribe (2003) so that the steady state total hours equal 0.2 when labor is divisible. In the 'involuntary' unemployment model, h_0 and p are chosen, ensuring that the steady state level of total hours h is again equal to 0.2, and ω is left at 0.22. The portfolio adjustment costs parameter ψ is set to match the volatility of the current account to output ratio in the data (=0.29) and the steady state of the trade balance to output ratio is set at 0.02.[22] The capital adjustment costs parameter ϕ is set to ensure that the volatility of investment to the volatility of output in the model matches the one in the data (=2.97). Finally, the values of ρ and σ_z are obtained from Letendre (2004) and are set equal to 0.944 and 0.006 respectively.

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[22] The common practice is to match the current account to output ratio when pinning down ψ. However, in the standard KPR model it is not possible to match this ratio, and hence the trade balance to output ratio is matched instead.

Lastly the value of σ is chosen. The empirical literature estimating the relative risk aversion parameter yields a wide range of parameter values. For example, Hansen and Singleton (1982) argue that σ is between 0.35 and 1, Constantinides (1990) obtains values in the range of [10,15], and Lucas (1994) finds the values to be in the midrange of [2,3], as do Constantinides, Donaldson and Mehra (2002) who obtain estimates in the range of [2,5].[23] In addition, in recent studies Meyer and Meyer (2005, 2006) assert that differences across estimates stem from differences in outcome variable and that the range in the estimates is not as large as it may at first appear. Specifically, they consider four previous studies and obtain a comparable estimate of the relative risk aversion for wealth, which they find to be in between 0.8 and 5.[24] In light of these findings, pining down a specific value for σ is difficult. Hence, the starting point in this paper is to follow the SOE literature and set the coefficient of relative risk aversion to 2 in all models.[25] This is then followed by a sensitivity analysis, where-in the values in the range of [1,5] are considered.[26]

4. QUANTITATIVE RESULTS

Tables 1-3 report second moments obtained from Canadian data, the GHH model, the standard "KPR-divisible" labor model, and the "KPR-indivisible" labor model; all

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[23] Other literature that argues for a low value (less than 1) for σ includes the work of Blake (1996) who obtains a value of 0.23 and Szpizo (1986) who obtains values in the range of [0.5-0.8]. Studies that yield double-digit estimates include Obstfeld (1994) who obtains the value of 18 and Kandel and Stambaugh (1991) with 30. Lastly, for estimates in the midrange, see also Barsky, Juster, Kimball and Shapiro (1997) who find the range of [4,8] and Zeldes (1989) who obtains estimates between 2 and 3.

[24] Moreover, their findings suggest that for consumption, relative risk aversion is about 5 times as high as the relative risk aversion for wealth.

[25] This is the value used in the work of Mendoza (1991), Letendre (2004), Guo and Janko (2009), etc.

[26] This range takes into account a reasonable set of values and shows the impact on the statistical moments as σ rises.

models use the value of σ=2.[27] The models are simulated for 84 periods, the same number as in the Canadian sample, 1981.1-2001.4, where all series are passed through the Hodrick-Prescott filter.[28] Statistics that display the superscript * and ** indicate that a moment is not statistically different from its simulated counterpart at the 5% and 1% level of significance, respectively.[29] Tables 4-6 give the sensitivity analysis results. Tables 1 and 4 shows the standard deviations, Tables 2 and 5 show the autocorrelations, and Tables 3 and 6 give the correlations with output. Lastly, Figures 1 and 2 show the quarterly impulse response functions of output, hours, consumption, investment, and the trade balance to output ratio in % terms to a 1% deviation in the technology shock.[30]

4.1 Simulation findings

I start by confirming the findings of Correia *et al.* (1995) and Schmitt-Grohe and Uribe (2003), reported in column three. With standard preferences, the KPR-divisible model yields a low volatility of consumption as agents excessively smooth consumption over time. Furthermore, the volatility of output and total hours is understated, and the correlation between output and the trade balance is highly procyclical (=0.99). It is for these reasons that standard preferences of King, Plosser and Rebelo (1988) are in general

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[27] Note that although the GHH model is identical to that of Mendoza (1991), here quarterly data are used as compared to the annual data used by Mendoza.

[28] The data are collected following Letendre (2004) from the Canadian Socio-economic Information and Management (CANSIM) data base. Output is the real gross domestic product (D100126); consumption is the personal expenditure on non-durable goods and services (D100107); investment includes investment in machinery and equipment, non-residential structures, and residential structures (D100114). The Current Account is obtained by using the nominal current account balance (D59832) and deflating by the GDP deflator (ratio of nominal (D14816) to real GDP), and the trade balance is calculated using exports (D100119) and imports for goods and services (D100122). Lastly, employment of age 15 and above (D980595) is used to obain employment.

[29] Under the null hypothesis that our theoretical model is true, we examine whether the historical sample statistics obtained from the data lie within the 95% or 99% confidence interval based on the distribution of 1,000 realizations of simulated moments. All reported moments are averages of the 1,000 realizations. This method was proposed by Gregory and Smith (1991) and utilized in Letendre (2004), Letendre and Luo (2007), and Guo and Janko (2009).

[30] Figure 1 reports the results for all three models given σ=2 and Figure 2 gives impulse responses for σ=2,3,4 and 5.

deemed unsuitable in the SOE literature and abandoned in favor of the GHH preferences, which fairly well match main empirical regularities for Canada (see column 2).

The effect of adding 'involuntary' unemployment to the model of Correia *et al.* (1995) and Schmitt-Grohe and Uribe (2003) is reported in the last column (KPR-indivisible) of Table 1 through 3. When labor is indivisible, the level of employed workers increases in response to a positive technology shock. Specifically, Figure 1 shows that the response of total hours is twice as high in a model with 'involuntary' unemployment as compared to the model with standard KPR preferences. This is consistent with the findings of Hansen (1985) and Hansen and Wright (1985), who find 'involuntary' unemployment to increase the elasticity of labor supply. Overall, the impact is a standard deviation in hours of 1.24% when labor is indivisible, as compared to 0.58% when it is divisible, thus leading to a better match the 1.25% volatility found in the data (see Table 1).[31] The greater response in employment translates to a greater response in output. Specifically, the volatility of output in the KPR-indivisible model is 1.60%, which closely matches 1.72% found in the data, in contrast to 1.14% obtained in the KPR-divisible model.[32] In addition, Figure 1 shows the response of output in the 'involuntary' unemployment economy to be qualitatively similar to that of hours, where the persistence of both hours and output are higher than in the KPR-divisible model. The impact on consumption is similar, in that again the volatility rises, now from 0.43% to 0.54% when indivisible labor is added, however this rise is not enough to match the volatility found in the data (=0.93%). Hence, along this dimension the GHH model does

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[31] Here, the KPR-indivisible model is the only model that yields a moment that is not statistically different from the value found in the data at the 5% level of significance.

[32] Note that the value of 1.60% is not statistically different from the 1.72% found in the data at the 5% level.

better, with a standard deviation of 0.92%. However, in light of equation (6), higher values of σ will yield higher consumption volatility, as shown in the sensitivity analysis below.

Next, adding indivisible labor to the standard KPR model drastically decreases the correlation between output and the trade balance (see Table 3). While the correlation is still positive (=0.37), it is much lower than what is found in the KPR-divisible model (=0.99). Since consumption is no longer excessively smoothed out with 'involuntary' unemployment, the response of the trade balance is stronger when the economy is subject to a positive productivity shock. Specifically, Figure 1 shows that the qualitative response of the trade balance in the KPR-indivisible labor model more closely resembles that in the GHH model, both exhibiting a hump-shape response, while the response in the KPR-divisible model is minimal. Although overall, the correlation is positive given σ=2, the next section shows that a slightly higher σ results in a countercyclical trade balance.

There are a number of dimensions along which the indivisible KPR model does poorly. First, the correlation between output and hours remains high, as does the correlation of consumption and investment with output as reported in Table 3. Second, Table 2 shows that the autocorrelation of the trade balance is too high. Lastly, the autocorrelations of output, hours and investment are too low. Similar drawbacks persist in the GHH model as well.

4.2 Sensitivity Analysis

The effects of altering the relative risk aversion parameter in the KPR-indivisible model are reported in Tables 4 through 6 and shown in Figure 2. With indivisible labor, as σ rises, the volatility of hours, output and consumption rises (see Table 4). When σ=4.5 for example, the volatility of output is matched to that found in the data (=1.72%), the volatility of hours is too high (=1.42% vs. 1.25%) and the volatility of consumption is only slightly lower (=0.85% vs. 0.93%). Nevertheless, all values of σ considered in Table 2, give rise to standard deviations of output and hours that are not statistically different from their empirical counterparts at the 5% level of significance. This is an improvement over the GHH model, where this holds only for output. Figure 2 shows that the impulse responses of output and hours are only slightly quantitatively different as σ rises, however the difference between the GHH model and the KPR-indivisible model is striking, with hours being more responsive in the initial period. In the case of consumption, Figure 2 shows that consumption is more volatile as σ rises, with the volatility in the data being matched at the 5% level of significance for value of σ greater than 2.5. As σ rises, the ratio of c_{1t} to c_{2t} increases, hence consumption of employed increases more than that of unemployed, thus increasing aggregate consumption. The impact on the trade balance to output ratio from a rise in σ is shown in the last panel of Figure 2. For a risk aversion parameter greater than 2, the initial response of the trade balance is negative, as it is in the GHH model. In subsequent periods, the trade balance rises, reaching a peak in approximately 13-17 quarters, and then falls back to zero. Notice that the response of investment is very limited, thus it is the trade balance that moves in the opposite direction as consumption rises. Hence, in the SOE model, separation between the saving-investment decision causes "consumption-smoothing" via the trade-balance.[33] Moreover, as agents become more risk averse they rely more on the

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[33] This point is emphasized in Mendoza (1991, pp.807). The lack of a wealth effect reinforces this mechanism, as the pro-borrowing effect dominates the pro-lending effect. However, when standard

foreign asset, which acts as an insurance against consumption volatility. However, when preferences are standard, and consumption smoothing operates via the wealth effect, less "consumption-smoothing" occurs via the trade balance.[34] Consequently, as shown in Table 6, for $\sigma = 3.5$ and above the trade balance is countercyclical, with the correlations not being statistically different from their empirical counterpart at the 5% level of significance. Hence, the KPR-indivisible model can yield a countercyclical trade balance given reasonable σ parameterization.

Lastly, a rising σ does yield a lower autocorrelation of the trade balance, however the impact of changing σ is minimal with respect to the autocorrelations of output, consumption, investment and total hours (see Table 5). In addition, increasing σ does not improve upon the correlations of consumption, investment, and hours with output (Table 6). Clearly, while the model is able to improve upon certain moments, the KPR-indivisible model has an inability to match the data along some dimensions, especially with respect to the autocorrelations and the correlation of output and hours remain problematic.

5. CONCLUSION

In this paper, a small open economy model with standard King, Plosser and Rebello (1988) preferences is re-examined. The model is modified by incorporating indivisible labor, specifically focusing on 'involuntary' unemployment, in which case employed agents consume higher levels of consumption from unemployed agents. The model is

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preferences of KPR are considered, the wealth effect implies the pro-lending effect dominates and hence consumers do not make use of the trade balance for "consumption-smoothing".

[34] The correlation in the standard KPR continues to be high and positive for high values of the risk aversion parameter, such as $\sigma = 30$.

found to yield higher volatility of output, hours, and consumption, as well as a countercyclical trade balance given reasonable parameter values. Of course, I do find the results to depend on the calibration of the risk aversion parameter.

While the GHH model considered in this paper followed Mendoza (1991), a number of modifications have been made recently to the SOE model to improve upon his findings. Specifically, Letendre (2004) incorporates consumption habit and variable capital utilization, and Guo and Janko (2008) introduce intertemporal labor supply into a SOE model with variable capital utilization. The impact of variable capital utilization in the two papers, is a rise in volatility of hours, output and consumption. I added variable capital utilization (using the parameterization used by Guo and Janko) into the KPR-indivisible model and found it to have similar results. The volatility of hours, output, investment, and consumption increased beyond the levels found in the Canadian data, this being due to the additional amplification mechanism that variable capital utilization provides. Moreover, the value of the risk aversion parameter slightly increased to 5.5 in order to obtain a countercyclical trade balance as found in the data. Again, in light of the findings of Letendre (2004) and Guo and Janko (2008), it is likely that consumption habit would reduce the volatility of consumption in the KPR-indivisible model, and similarly leisure habit (or labor adjustment costs) would likely reduce the volatility of hours and output. However, a detailed analysis of the role of these additional mechanisms is not conducted here, as the objective of the paper is not to obtain a model with best performing second moments relative to the data, but to show that standard preferences can to some extent match important business cycle regularities of open economies contrary to claims made to date. The simplicity of the SOE model considered in this paper allows for a clear exposition of this finding, and hence shows that standard preferences may have been prematurely abandoned in the literature.

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FIGURE 1. Impulse Response Functions

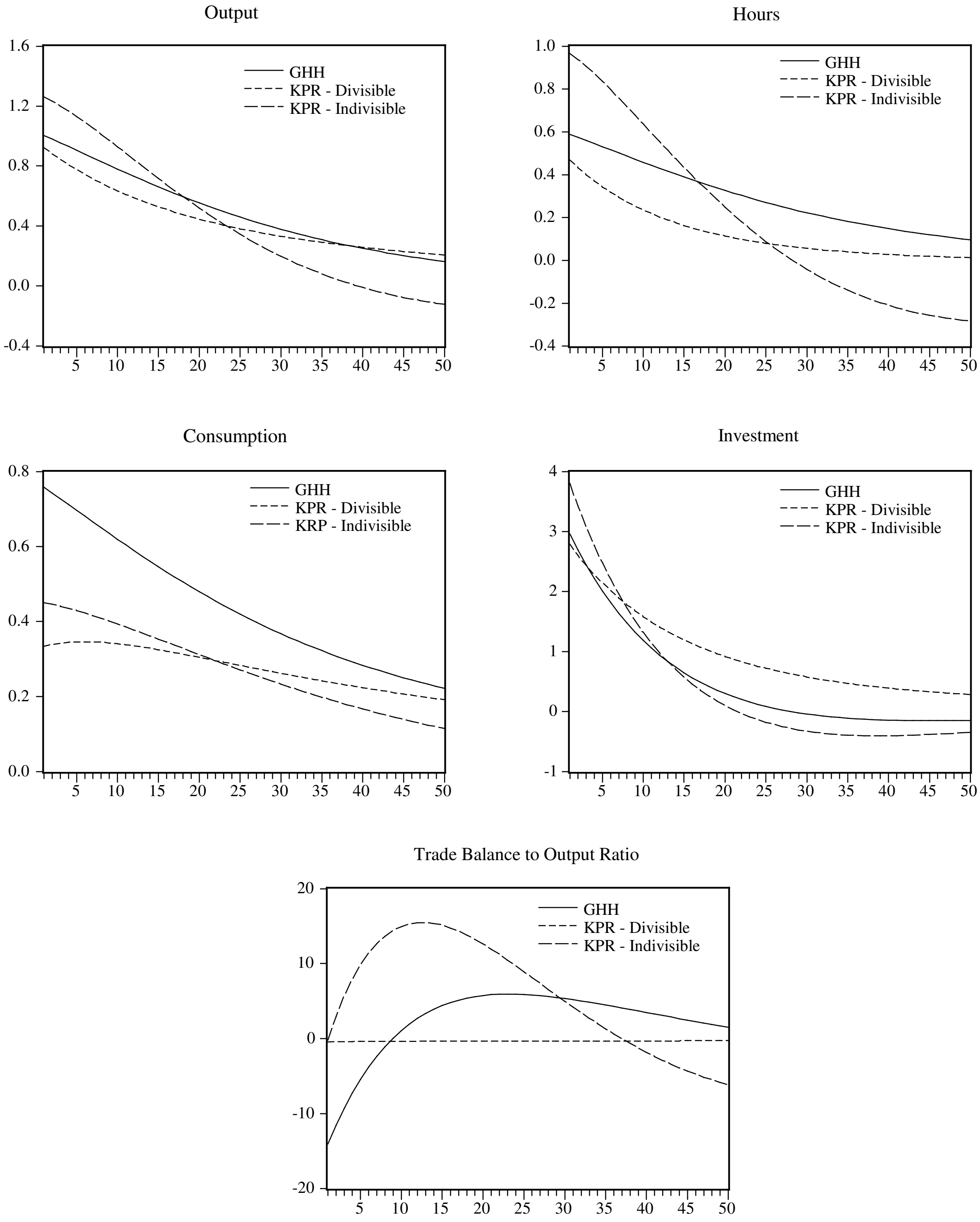

FIGURE 2. Impulse Reseponse Functions: Sensitivity Analysis

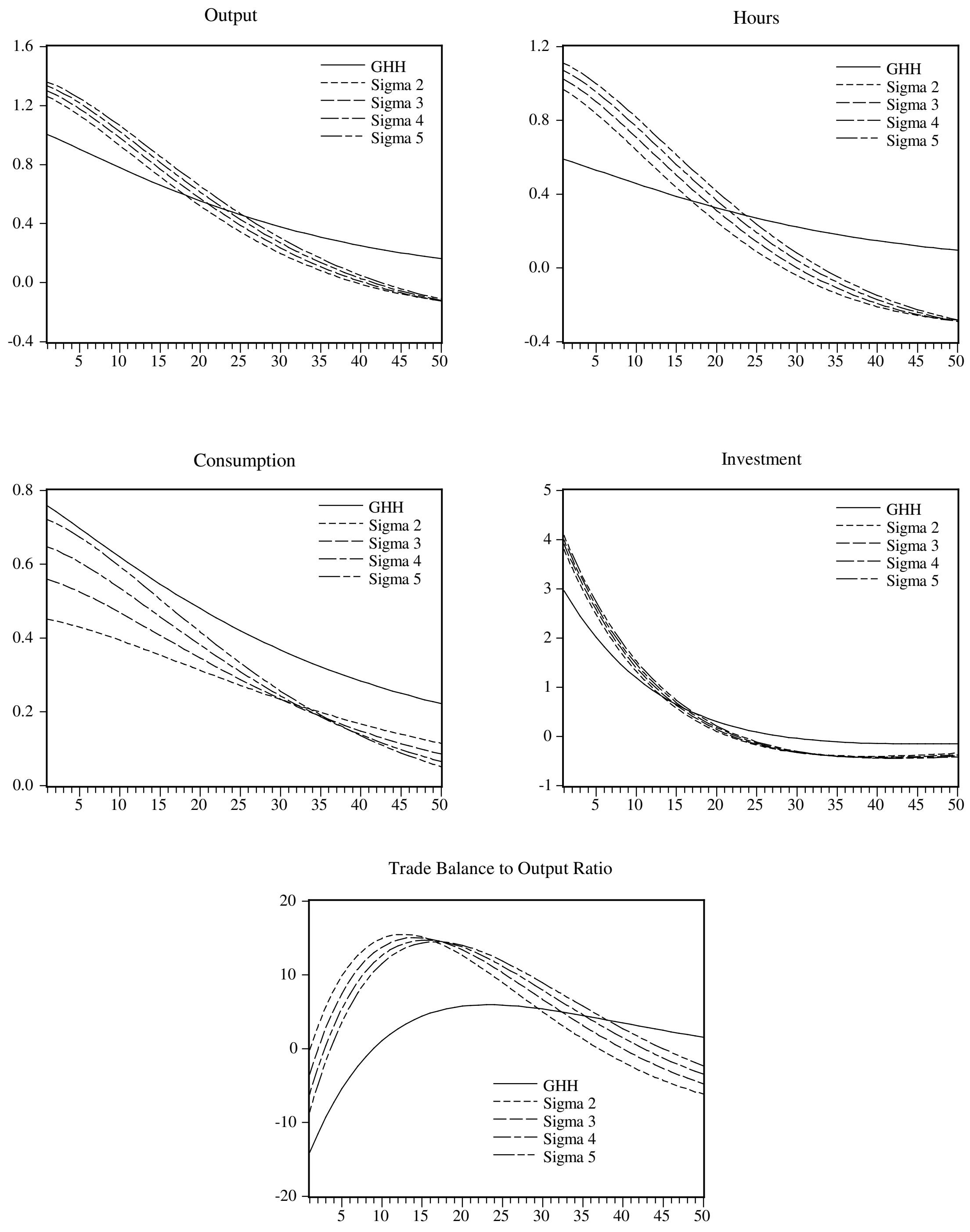

TABLE 1. Standard Deviations

	Data	GHH	KPR-divisible	KPR-indivisible
Output	1.72(0.19)	1.26*	1.14	1.60**
Consumption	0.93(0.12)	0.92**	0.43	0.54
relative volatility	*0.54*	*0.73*	*0.37*	*0.34*
Investment	5.13(0.49)	3.75	3.40	4.77*
Total Hours	1.25(0.160	0.74	0.58	1.24**
relative volatility	*0.73*	*0.59*	*0.51*	*0.78*
TBY	0.87(0.09)	0.28	0.87*	0.28
CAY	0.28(0.03)	0.28*	0.85	0.28*

TABLE 2. Autocorrelations

	Data	GHH	KPR-divisible	KPR-indivisible
Output	0.91(0.21)	0.69	0.67	0.70
Consumption	0.83(0.24)	0.69*	0.70*	0.69*
Investment	0.88(0.18)	0.67	0.67	0.67
Total Hours	0.92(0.24)	0.68	0.66	0.70
TBY	0.67(0.17)	0.68**	0.69**	0.95
CAY	0.68(0.18)	0.68**	0.69**	0.95

TABLE 3. Correlations with Output

	Data	GHH	KPR-divisible	KPR-indivisible
Consumption	0.80(0.23)	0.99	0.98	0.99
Investment	0.77(0.21)	0.98	1.00	0.97
Total Hours	0.91(0.24)	1.00	0.99	0.99
TBY	-0.29(0.21)	-0.87	0.99	0.37
CAY	-0.26(0.21)	-0.85	0.99	0.34

Notes for all tables: Numbers in parentheses are the Newey-West heteroskedasticity and autocorrelation consistent standard errors of the moments. In columns 2-4, the superscripts ** and * show that a data moment is not statistically different from its simulated counterpart at the 5% and 1% level of significants, respectively.

TABLE 4. Sensitivity Analysis - Standard Deviations

	Data	σ=1	σ=1.5	σ=2	σ=2.5	σ=3	σ=3.5	σ=4	σ=4.5	σ=5
Output	1.72	1.56**	1.60**	1.60**	1.65**	1.67**	1.69**	1.70**	1.72**	1.75**
Consumption	0.93	0.34	0.48	0.54	0.63	0.69	0.75	0.80	0.85	0.90
relative volatility	*0.54*	*0.22*	*0.30*	*0.34*	*0.38*	*0.41*	*0.44*	*0.47*	*0.49*	*0.51*
Investment	5.13	4.64*	4.76*	4.77*	4.92**	4.99**	5.04**	5.07**	5.13**	5.22**
Total Hours	1.25	1.17**	1.22**	1.24**	1.30**	1.34**	1.37**	1.38**	1.42**	1.45**
relative volatility	*0.73*	*0.75*	*0.77*	*0.78*	*0.79*	*0.80*	*0.81*	*0.81*	*0.83*	*0.83*
TBY	0.87	0.29	0.28	0.29	0.29	0.28	0.28	0.29	0.28	0.28
CAY	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28

TABLE 5. Sensitivity Analysis - Autocorrelations

	Data	σ=1	σ=1.5	σ=2	σ=2.5	σ=3	σ=3.5	σ=4	σ=4.5	σ=5
Output	0.91	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
Consumption	0.83	0.69*	0.69*	0.69*	0.70*	0.70*	0.70*	0.70*	0.70*	0.71*
Investment	0.88	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67
Total Hours	0.92	0.70	0.70	0.70	0.71	0.71	0.70	0.70	0.70	0.71
TBY	0.67	0.92	0.94	0.95	0.94	0.98	0.89	0.86	0.84	0.82
CAY	0.68	0.92	0.94	0.95	0.94	0.92	0.89	0.86*	0.84*	0.82*

TABLE 6. Sensitivity Analysis - Correlati0ns with Output

	Data	σ=1	σ=1.5	σ=2	σ=2.5	σ=3	σ=3.5	σ=4	σ=4.5	σ=5
Consumption	0.8	0.98	0.99	0.99	0.99	0.99	0.99	0.99	1.00	0.99
Investment	0.77	0.97	0.97	0.97	0.96	0.97	0.97	0.97	0.97	0.97
Total Hours	0.91	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00	0.99
TBY	-0.29	0.66	0.53	0.37	0.19	0.05*	-0.08**	-0.20**	-0.28**	–0.36**
CAY	-0.26	0.64	0.50	0.34	0.15	0.01*	-0.12**	-0.23**	-0.32**	-0.39**