A Dynamic Regional Model of Irrigated Perennial Crop Production

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Abstract

Perennial crop production is inherently dynamic due to several salient physical characteristics including an establishment period of several years, long lives in commercial production, and path-dependence of yields on input use and other exogenous factors such as weather. While perennial crop production is properly regarded as a dynamic investment under uncertainty, the literature on regional agricultural production is typically static, deterministic, and rarely are the dynamic biophysical elements of perennial crops represented. This paper seeks to address some of the shortcomings of the literature by developing a dynamic regional model of irrigated agriculture with representative perennial and annual crops. The model explicitly accounts for the age composition of perennial stocks including crop establishment period and age-dependent yields and input use.

The model is applied to wine grape production in the Riverland region of the Murray-Darling Basin (MDB) in Australia using a representative agricultural household to analyze joint consumption and investment decisions. Borrowing is allowed but the assumption of perfect capital markets is relaxed; the household faces an interest rate schedule that is increasing in the amount of debt held. We explore the dynamic properties of the model including the existence and uniqueness of a steady state and the conditions required for convergence to the steady state. Because the state-space required for an age-explicit regional model is too large for conventional dynamic programming methods, a running horizon algorithm is used to approximate an infinite horizon dynamic programming solution.

The effects of the age structure of initial perennial plantings are investigated. Starting with an initial age distribution of grape stocks different from the steady state levels leads to dampened oscillations in area planted by vintage with eventual convergence to a steady state with an equal age distribution. The impact of water entitlement reductions for several possible scenarios under the proposed MDB Plan are estimated under both deterministic and stochastic frameworks, the latter of which is based on Monte Carlo simulations that draw on the distribution of historical water diversions in the region. Also, the long-run water demand for perennial crops is identified by systematically running simulations over varying water allocation levels and capturing the farmer's marginal willingness to pay for water.

Contents

Introduction

The potential effects of climate variability and climate change on agricultural productivity is a well-researched subject as evidenced by Adams et al. (1990), Rosenzweig and Parry (1994), Rosegrant and Cline (2003), and Lobell et al. (2008) among others. The validity of the economic impacts derived from such studies depends crucially on the accuracy of the characterization of the underlying production processes. Accordingly, significant advances in the representation of the biophysical aspects of agricultural production have been made in studies such as Letey, Dinar, and Knapp (1985), and Kan, Schwabe, and Knapp (2002), and Schlenker and Roberts (2009). Unfortunately, similar advances in the representation of the complexities of perennial production are not well captured by the existing literature; consequently, the economics of perennial agriculture are poorly understood.[1]

Adequately modeling perennial crop production involves recognizing that it is inherently dynamic due to several salient physical traits including an establishment period of multiple years before marketable yields are produced, a long life in commercial production of up to 50 or more years, and the long-lasting impact of the pattern and timing of input use and other exogenous factors such as weather on the productivity of the crop over its life. Furthermore, the hump-shaped age-yield relationship characteristic of most perennial crops means that perennial production is essentially non-linear. Due to these factors, perennial crop production is best represented as an investment under uncertainty characterized by non-linear dynamics, a characterization not reflected by the current literature. Efforts to study perennial production by econometric methods are severely constrained by data limitations since the age distribution of the crop is rarely known. It is also very difficult, if not impossible, to determine this distribution from time series data on production levels due to the heterogeneous effects of weather and disease on perennial stocks, the possibility that it may be optimal to not harvest (i.e., mothball) or partially harvest in a given year, and the constantly changing mix of different varieties in production (Nerlove 1979). Given the difficulties inherent in an econometric approach, one would think that programming studies would be numerous but few such studies exist.

The motivation behind reconsidering how perennial crops have been represented is clear when one considers their value, especially in the role these crops play during drought and under climate change. While only approximately 9% of global crop area is devoted to perennial shrubs and trees (Monfreda, Ramankutty, and Foley 2008), regions with Mediterranean climates often devote a large amount of agricultural land to high-value perennial crops such as citrus, stone fruits, almonds, avocadoes, and grapes.

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[1]This critique of the literature, noted by Nerlove (1979) nearly 35 years ago, has been made as recently as 2001 by Just and Pope who make the claim, ".. the work of French, King, and Minami (1985) is essentially the last substantive work on perennial crops."

These perennial crops often constitute a large percentage of gross value of farm production in such regions. For example, perennials constituted over 1/3 of total crop value in California in 2011-12 (California Agricultural Statistics Service 2012) and perennial fruit crops, including grapes, represented approximately 41% of gross value of irrigated agriculture in the Murray-Darling Basin (MDB) of Australia in 2009-2010 (Australian Bureau of Statistics 2009). Additionally, for crops such as wine grapes there are often large associated industries including tourism that are vital to the local economy (MKF Research, 2007). Furthermore, when one considers what is driving the benefits of water markets among agricultural users and how they can mitigate the impacts of drought, it is clear that the markets allow water to move to its highest valued use, which in many circumstances means moving towards perennials (e.g. Connor, Schwabe, King, Kaczan, and Kirby 2009).

To better understand dynamic land use patterns and quantify water demand in the midst of drought and possible climate change, it is necessary to develop models that can both account for the benefits of re-allocating water between annuals and perennials as well as the costs of under-watering perennials. This paper attempts such an analysis by developing a dynamic regional model of irrigated agriculture with representative perennial and annual crops. The age composition of perennial stocks is accounted for explicitly as are the effects of the crop establishment period and age-dependent yields and input use. Because the state space required for an age-explicit regional model is too large for conventional dynamic programming methods (i.e., the curse of dimensionality), a running horizon algorithm (detailed in an appendix) is used to approximate an infinite horizon dynamic programming solution. The impacts from changes in economic and biophysical characteristics are estimated under both deterministic and stochastic frameworks, the latter of which is based on a time series of water diversions within the region. The long-run water demand for agriculture is identified, as are implications of changes in model parameters. For instance, results from the deterministic model suggest that starting with an age distribution of grapevine stocks outside the steady state leads to cycles in area planted by vintage and quantity supplied of wine grapes. Over very long time horizons the cycles in area planted are shown to be dampened oscillations which eventually converge to a steady-state with an equal age distribution.

This study is concerned with perennial production in arid and semi-arid regions where irrigation is common and, therefore focuses on the effects of water supply variability on farm management decisions which affect perennial stocks. The particular application we focus on is wine grape production in the Riverland region of South Australia in the MDB. The MDB is the main area of irrigated agriculture in Australia, using over 70% of the nation's irrigation water (Appels, Douglas, and Dwyer 2004). Recently, the MDB experienced the worst drought in recorded history, which put severe stress on the agricultural sector and perennial horticulture in particular (Grant, Knight, Nation, and Barratt 2007). The vast majority of perennial production depends on irrigation– 87% of fruit and over 96% of grape production was irrigated in 2009-10 (Australian Bureau of Statistics 2011, Australian Bureau of Statistics 2009).

Perennial farmers adapted to the drought in several ways including securing additional water through trade, deficit irrigation, mothballing, and removal of existing perennial stocks.

Since the vast majority of agricultural enterprises in Australia are family-owned (Barclay, Foskey, and Reeve 2007), we use a utility-maximizing representative agricultural household to analyze joint consumption and investment decisions. This is consistent with (Pope, LaFrance, and Just 2011) who note that the literature on risk in agricultural production lends credence to "a more integrative examination of the broader portfolio problem in agriculture that includes consumption, investment, and other risk sharing activities as well as production." Within this framework, borrowing is allowed but the assumption of perfect capital markets is relaxed; the household faces an interest rate schedule that is increasing in the amount of debt held. The representative household model is then used to analyze the effects of permanent reductions to agricultural water allocations as has been discussed in the policy outlined by the MDB Plan.

The next section provides a brief literature review. Section three details an analytical description of the model and the optimal steady state is characterized, followed by section four which explores, computationally, its dynamic properties. The conditions required for convergence to the steady state and length of time required to reach it are determined for various versions of the model. The effects of liquidity constraints and annual crop cultivation on the dynamics of the model are explored as well. The fifth section uses the model to analyze recent water reform policy in the MDB and the potential effects on the Riverland region where perennial crops are dominant. The sixth section draws conclusions and suggests possible directions for future research.

Literature Review

Much of the literature on perennial supply is comprised of extensions to the seminal model of annual crop supply by Nerlove (1956, 1958). The Nerlove model is a partial adjustment model with price expectations and is typically estimated as a reduced form econometric model in which area cultivated is regressed on past crop areas and prices (Askari and Cummings 1977). However, there are serious limitations to reduced form studies; chief among them is the fact that they do not allow for the recovery of the underlying structural parameters that drive perennial supply. While some papers such as Hartley, Nerlove, and Peters (1987) and Akiyama and Trivedi (1987) have used relatively more detailed datasets to estimate structural models, for perennial crops in most regions of the world detailed datasets large enough to support sophisticated econometric studies simply do not exist. A few studies use innovative methods to deal with data limitations; Kalaitzandonakes and Shonkwiler (1992) use a dynamic unobserved components model and, in the programming literature, Knapp and Konyar (1991) use a Kalman filter to recover the age distribution of perennial crops.

One theoretical paper used to validate the present study is Mitra, Ray, and Roy (1991) [MRR]. MRR explores an orchard problem as a class of *point input, flow outputs* vintage capital model. They characterize the age composition of the optimal stationary forest (OSF) and, given an arbitrary initial forest, determine if the optimal management program will converge to the OSF. Bellman and Hartley (1985) describe a theoretical dynamic programming (DP) model of perennial supply that accounts for all previous input decisions. However, their model would be difficult to implement in practice due to the large number of state variables required to capture the history of normally long-lived crops and no studies we know of attempt to follow their approach directly. Knapp (1987) is one applied study following Bellman and Hartley which uses explicitly defined age classes but does not consider the dynamic biophysical processes central to their paper.

Most programming studies of agricultural supply treat perennial crop production as tantamount to annual crop production, rarely attempting dynamic analyses. The regional agricultural literature often focuses on institutional or physical factors affecting agricultural land and water use and typically ignores any complications that might arise due to the presence of perennial crops. There is no study we know of which incorporates a detailed perennial production sector in a fully dynamic analysis of regional agricultural production. Among the models that at least recognize some of the characteristics of perennial crops, Rosegrant, Ringler, McKinney, Cai, Keller, and Donoso (2000) and Ward and Michelsen (2002) acknowledge that water demand for and the profitability of perennials may differ from other crops but no attempt is made to model perennial stocks. Howitt, MacEwan, Medellin-Azuara, and Lund (2010) put a lower bound on perennial crop area based on expected perennial crop retirements but otherwise treat annual and perennial crops as interchangeable. Marques, Lund, and Howitt (2005) model perennial crop production but the analysis is limited to a two-stage, Dantzig-style stochastic DP model with all permanent decisions on the level of investment in irrigation technology and perennial crops made in the first stage. In the second stage, annual crop choices are made as well as adjustments to irrigation techniques used. The model explicitly allows for deficit irrigation of perennials in years of drought and uses a linear penalty term to model the amount of the stock that is lost due to excessive stress. Similarly, Connor, Schwabe, King, Kaczan, and Kirby (2009) conduct a study of irrigated agriculture in the lower Murray-Darling Basin in which they test irrigator adaptation to different climate change scenarios. The model also employs a two-stage non-linear programming (NLP) approach and simple future yield penalties that account for deficit irrigation.

Analytical Model

Sequence Problem

The model as currently formulated assumes that costs accrue at the beginning of the time period and revenue at the end of the time period after harvesting. Below is a diagram of the model timing.
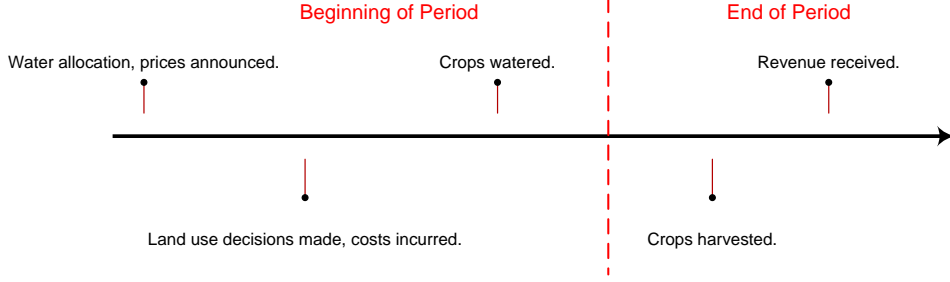

Figure 1: Model Timing

The functions which determine net benefits in the non-linear programming (NLP) model are specified below. $U(c_t)$ is the instantaneous utility function which is assumed to be one of the class of CRRA utility functions, c_t is current period consumption, $\underline{c}$ is the subsistence level of consumption, and α is the subjective discount factor. Agricultural households are assumed to maximize the discounted net present value of utility as follows:

$$\max_{c_t, s_{0,t}, x_t, z_{k,t}} \sum_{t=1}^{\infty} \alpha^{t-1} U(c_t) \tag{1}$$

$$U(c_t) = \frac{(c_t - \underline{c})^{1-\rho}}{1 - \rho} \tag{2}$$

$$c_t = (1 + r(a_t))a_t - a_{t+1} + \pi_t \tag{3}$$

The choice variables in the model are given by $\{c_t, s_{0,t}, x_t, z_{k,t}\}$ representing consumption, new perennial plantings, annual plantings, and perennial removals by age class respectively. Perennial crop age is indexed by $k = 0, 1, .., K$.

The budget constraint is given by (3) where a_t denotes the financial assets held by the household and the term π_t denotes profits from agricultural production. The standard assumption of perfect capital markets is relaxed as is reflected by the interest rate schedule $r(a_t)$ having a higher interest rate for borrowing than saving. To mimic increasing interest rates for borrowing as debt levels increase, the interest rate for borrowing and corresponding first derivative are expressed as

$$r_b(a_t) = \beta_0 + \beta_1 a_t^2 \tag{4}$$

$$r_b'(a_t) = 2\beta_1 a_t \tag{5}$$

where β_0 and $\beta_1 > 0$. The interest rate for saving (r_s) is constant and $\beta_0 > r_s > 0$. One possible parameterization of the interest rate function is shown below:
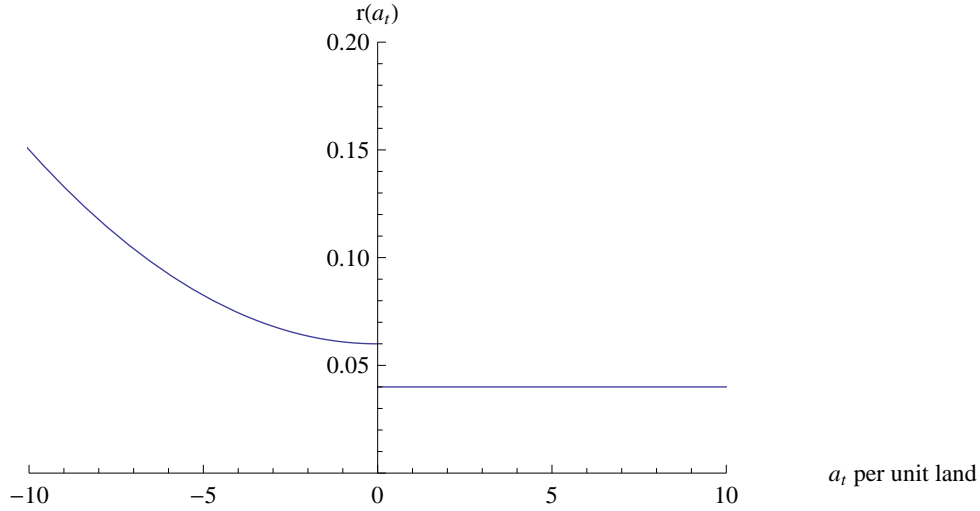

Figure 2: Interest rate schedule

Profits from agricultural production are given by (6)-(9) and are comprised of profits from existing perennials ($\pi_{s,t}$ derived from $s_{k,t}$ where $1 \leq k \leq K$) net of the cost of new plantings ($\pi_{0,t}$) plus annual crop profits ($\pi_{x,t}$). Note that annual and perennial cost terms are brought forward once within the respective profit sub-functions to reflect the timing of the model. Costs are incurred up front and profit is calculated at the end of the time period, which implies that consumption also occurs at the end of the period.

$$\pi_t = \pi_{0,t} + \pi_{s,t} + \pi_{x,t} \tag{6}$$

$$\pi_{0,t} = -\frac{1}{\alpha} \left(\gamma_w w_0 + \gamma_0 \right) s_{0,t} \tag{7}$$

$$\pi_{s,t} = \sum_{k=1}^{K} \frac{1}{\alpha} \left((\alpha p_s y_k - \gamma_w w_k - \gamma_k)(s_{k,t} - z_{k,t}) - \gamma_z z_{k,t} \right) \tag{8}$$

$$\pi_{x,t} = \frac{1}{\alpha} \left(\alpha p_x(x_t) y_x - \gamma_w w_x - \gamma_x \right) x_t \tag{9}$$

8

Cost terms include water costs per unit land (γ_w) and removal costs (γ_z) corresponding to the area of age k removed ($z_{k,t}$). For the purposes of this model, the water use coefficients per unit of land (w_k, w_x) and annual yield per unit of land (y_x) are exogenous. The price of perennial crops (p_s) is assumed to be time-invariant whereas the region exerts market power in the annual crop market and thus faces a downward-sloping demand curve with the price denoted ($p_x(x_t)$).

Perennial yields (y_k), water requirements (w_k), and non-water variable costs (γ_k) are all a function of crop age. The functional form used for each of the age-dependent functions is based on a review of the viticulture literature, regional production surveys, gross margin spreadsheets, and informal farmer interviews. In general terms, water requirements are monotonically increasing in age while non-water production costs are monotonically decreasing with age where planting costs are ascribed to the first period and removal costs are ascribed to age at removal. Yields are strictly monotonically increasing during establishment ($y_0 = 0 < y_1 < .. < y_{k'}$) then remain level before decreasing beyond a certain age ($y_{k'} = .. = y_{k''} > y_{k''+1} .. > y_K$) where k' and k'' represent age of maturity and age of decline respectively.

This specification is equivalent to the yield profile as specified in 2.1 of Mitra, Ray, and Roy (1991) (hereafter, MRR) with weak inequalities replaced by strict ones. Production costs, including those associated with irrigation, also vary with the age of the crop. It is assumed that yields, removal costs, and replanting costs dominate such that

$$\pi_0 < \pi_1 < .. < \pi_{k'} = .. = \pi_{k''} > \pi_{k''+1} .. > \pi_K \tag{10}$$

The yield and profit functions are illustrated below under certain parameter assumptions for the numerical model.
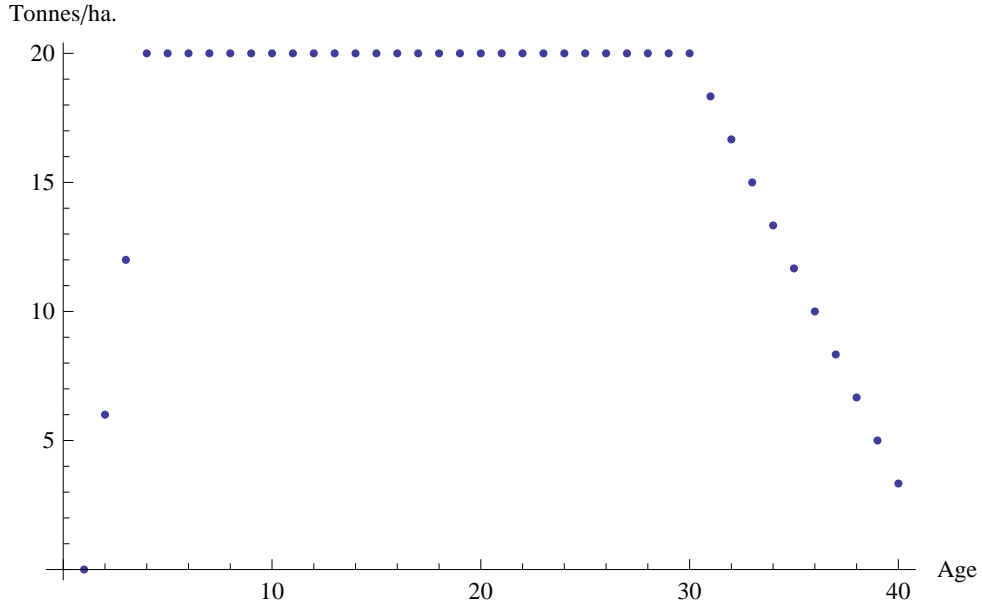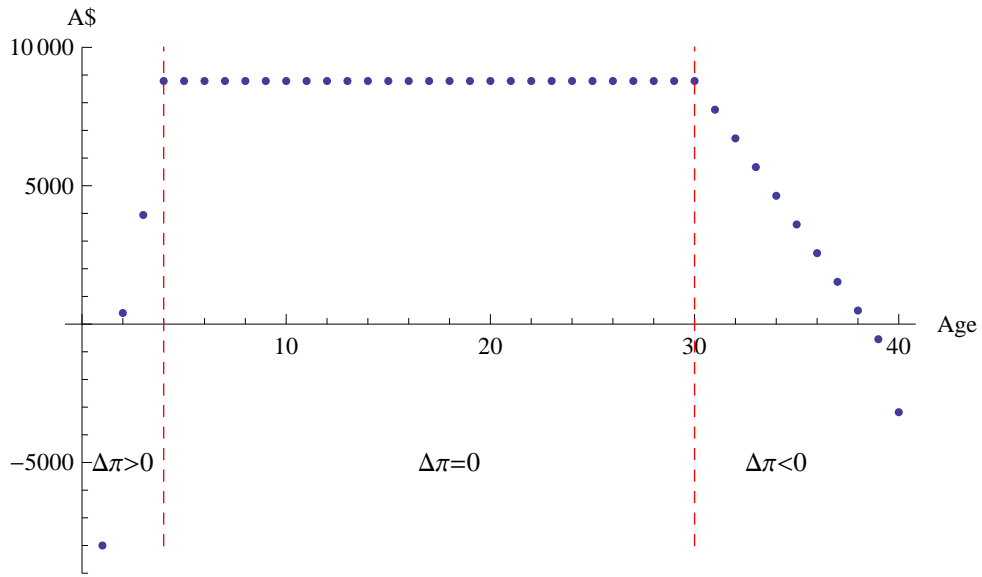
Figure 3: Yields by age

Figure 4: Profits by age

Removal are restricted such that $z_{0,t} = 0$, which implies that one cannot first plant and then remove the same plot in the same season. This choice could never be economically optimal since removals occur at the beginning of the time period and, even if this were not so, there are no yields in the first period. In addition, the final age class possible, if still in production, must be removed:

$$z_{K,t} = s_{K,t} \ \forall t \tag{11}$$

These restrictions on the first and last age class reduce the dimension of the control space by two variables; there are $K - 1$ removal variables in addition to consumption, perennial planting area, and annuals planting area, adding up to $K + 2$ control variables in total. The state variables are a_t and $s_{k,t}$ $\forall k > 0$, representing assets and area of land devoted to perennials by age class except for the first age class ($s_{0,t}$), which is a choice variable. The number of state variables is therefore $K + 1$. For a perennial such as wine grapes that may remain in production for 40 years[2] (Mullins, Bouquet, and Williams 1992), this implies 42 control variables and 41 state variables. Clearly, this leads to a high-dimensional optimization problem that cannot be analyzed using traditional DP methods. Instead, as is discussed later, a running horizon (RH) algorithm which approximates an infinite horizon DP algorithm is applied to make the problem computationally feasible.

The law of motion for financial assets is given by re-writing the budget constraint as (12) with the interpretation being that next period's assets are equal to current period assets with interest plus profits from agricultural production less total consumption. Equation (13) defines the law of motion for perennial stocks, indicating that the future area devoted to an age class is equal to previous period level of the next youngest age class less removals of that class.

$$a_{t+1} = (1 + r(a_t))a_t + \pi_t - c_t \ \forall t \tag{12}$$

$$s_{k+1,t+1} = s_{k,t} - z_{k,t} \ \forall k, t \tag{13}$$

Equations (14) and (15) are resource constraints for regional land use and regional water use respectively where regional land available is normalized to one unit and $\bar{q}$ represents total water volume available for irrigation.

$$\sum_{k=0}^{K} (s_{k,t} - z_{k,t}) + x_t \leq 1 \ \forall t \tag{14}$$

$$\sum_{k=0}^{K} w_k(s_{k,t} - z_{k,t}) + w_x x_t \leq \bar{q} \ \forall t \tag{15}$$

Note that, as previously mentioned, the timing of removals is such that they occur at the beginning of the time period rather than the end. This is captured by the term $s_{k,t} - z_{k,t}$ in (8), (14), and (15) and is required for running simulations in which low water allocation levels may necessitate perennial removals to meet the water constraint.

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[2] In principle, grapevines can stay in production for up to 100 years or more. However, yields decline significantly beyond a certain age, making older vines unsuitable for production except in the case of boutique wines that can command a premium price. This study seeks to understand mainstream wine production and the region the model is applied to is known for mid-priced, not fine, wines.

Aside from (13), several rotation constraints must be specified for a complete representation of the dynamics of perennial production. New plantings are constrained by (14) to be no greater than total land available given existing vintages net of removals and annual plantings. As Wan (1994) and others point out, theoretical models of forestry are closely related to standard growth theory models with a key difference being the so-called "cross-vintage constraint." This constraint ensures that the area devoted to a given vintage must be greater than or equal to the area subsequently devoted to the same vintage one period later. This point applies equally to perennial crop production and is formally stated here as

$$s_{k+1,t+1} \leq s_{k,t} \ \forall k < K, t \tag{16}$$

Mathematically, this is equivalent to putting the appropriate bounds on removals as follows:

$$0 \leq z_{k,t} \leq s_{k,t} \ \ \forall \ 1 \leq k \leq K-1, t \tag{17}$$

Consumption must be greater than or equal to some subsistence level $\underline{c}$ which is strictly positive. The initial stock of land devoted to perennial vintages and all the remaining variables except for assets are constrained to be non-negative as well.

$$c_t \geq \underline{c}, \ x_t \geq 0 \ \forall t \tag{18}$$

$$s_{k,t} \geq 0 \ \forall k, t \tag{19}$$

The debt collateral constraint or No-Ponzi-Game condition is

$$a_t + \sum_{n=1}^{K} \frac{\pi_{s,t+n}}{(1 + r(a_{t+n}))^n} \geq 0 \quad \forall t \tag{20}$$

This condition means that debt will never exceed the net present value of the profit stream which may be generated by the remaining perennial stock discounted at the interest rate corresponding to the debt level in each period. This ensures that if the representative farmer were to continue production over the economically productive life cycle of remaining perennials he or she would be able to repay the debt plus interest given that the interest rate will decrease as debt is paid off. Whereas many theoretical models require that debt be non-negative as a terminal condition, one may argue that allowing persistent, manageable levels of debt is much more realistic for models of small agricultural producers.

Dynamic Programming Problem

For the purposes of derivations, we will use the Dynamic Programming formulation of the model presented above. Rewriting (1) in Bellman form we have the following where $V(\theta_t)$ is the value function,

$\theta_{2,t} = \langle \theta_{1,t}, \theta_{2,t} \rangle$ is the vector of choice variables ($\theta_{1,t} = c_t, s_{0,t}, x_t, z_{1,t}, .., z_{K-1,t}$) and state variables ($\theta_{2,t} = a_t, s_{1,t}, .., s_{K,t}$), and $f(\theta_t) = \langle f_1(\theta_t), f_2(\theta_t) \rangle$ is a vector function given by (12) and (13) respectively.

$$V(\theta_{2,t}) = \underset{\theta_{1,t}}{\text{Max}}\, U(c_t) + \alpha V(f(\theta_t)) \tag{21}$$

The problem is subject to the constraints given by (14)-(19) which are denoted here as

$$g_j(\theta_t) \geq 0 \tag{22}$$

Given the above, the Lagrangian can be written compactly using j=1,..,6 constraints as

$$\mathcal{L}(\theta_t, \lambda_{j,t}) = U(c_t) + \alpha V(f(\theta_t)) + \sum_{j=1}^{6} \lambda_{j,t} g_j(\theta_t) \tag{23}$$

and expressed fully as

$$
\begin{aligned}
\mathcal{L}(\theta_t, \lambda_{j,t}) = &\ \frac{(\pi_t + (1 + r(a_t))a_t - a_{t+1} - \underline{c})^{1-\rho}}{1 - \rho} + \alpha V(f(\theta_t)) \\
&+ \lambda_{1,t}\left[1 - \sum_{k=0}^{K}(s_{k,t} - z_{k,t}) - x_t\right] + \lambda_{2,t}\left[\bar{q} - \sum_{k=0}^{K} w_k(s_{k,t} - z_{k,t}) - w_x x_t\right] \\
&+ \sum_{k=1}^{K-1} \lambda_{3,k,t} z_{k,t} + \sum_{k=1}^{K-1} \lambda_{4,k,t}(s_{k,t} - z_{k,t}) + \lambda_{5,t} x_t + \sum_{k=0}^{K} \lambda_{6,k,t} s_{k,t}
\end{aligned}
$$

The KKT conditions are specified as follows:

$$a_{t+1}: \qquad -(c_t^* - \underline{c})^{-\rho} + \alpha \frac{\partial V(\cdot)}{\partial f_1(\cdot)} \frac{\partial f_1(\cdot)}{\partial a_{t+1}^*} = 0 \tag{24}$$

$$s_{0,t}: \qquad -(c_t^* - \underline{c})^{-\rho}\left(\frac{\gamma_w w_0 + \gamma_0}{\alpha}\right) + \alpha \frac{\partial V(\cdot)}{\partial f_2(\cdot)} \frac{\partial f_2(\cdot)}{\partial s_{0,t}^*} - \lambda_{1,t}^* - \lambda_{2,t}^* w_0 + \lambda_{6,0,t}^* = 0 \tag{25}$$

$$x_t: \qquad \frac{1}{\alpha}(c_t^* - \underline{c})^{-\rho}\left(\alpha p_x(x_t^*)y_x + \alpha p_x'(x_t^*)y_x x_t^* - \gamma_w w_x - \gamma_x\right) - \lambda_{1,t}^* - \lambda_{2,t}^* w_x + \lambda_{5,t} = 0 \tag{26}$$

$$
z_{k,t}: \qquad
\left.
\begin{aligned}
&-\frac{1}{\alpha}(c_t^* - \underline{c})^{-\rho}(\alpha p_s y_1 - \gamma_w w_1 - \gamma_1 - \gamma_z) + \lambda_{1,t}^* + \lambda_{2,t}^* w_1 + \lambda_{3,1,t}^* - \lambda_{4,1,t}^* = 0 \\
&\qquad\qquad\qquad\qquad\qquad\qquad .. \\
&-\frac{1}{\alpha}(c_t^* - \underline{c})^{-\rho}(\alpha p_s y_{K-1} - \gamma_w w_{K-1} - \gamma_{K-1} - \gamma_z) + \lambda_{1,t}^* + \lambda_{2,t}^* w_{K-1} + \lambda_{3,K-1,t}^* - \lambda_{4,K-1,t}^* = 0
\end{aligned}
\right\}
\begin{aligned}
\text{K-1 equations}
\end{aligned}
\tag{27}
$$

$$\lambda_{1,t}^*\left(1 - \sum_{k=1}^{K}(s_{k,t}^* - z_{k,t}^*) - x_t^*\right) = 0 \,, \lambda_{1,t}^* \geq 0 \tag{28}$$

13

$$\lambda_{2,t}^* \left(\bar{q} - \sum_1^K w_k(s_{k,t} - z_{k,t}^*) - w_x x_t \right) = 0 \ , \lambda_{2,t}^* \geq 0 \tag{29}$$

$$\lambda_{3,k,t}^* z_{k,t}^* = 0 \ , \lambda_{3,k,t}^* \geq 0 \qquad \forall 1 \leq k \leq K-1 \tag{30}$$

$$\lambda_{4,k,t}^* (s_{k,t}^* - z_{k,t}^*) = 0 \ , \lambda_{4,k,t}^* \geq 0 \qquad \forall 1 \leq k \leq K-1 \tag{31}$$

$$\lambda_{5,t}^* x_t^* = 0 \ , \lambda_{5,t}^* \geq 0 \tag{32}$$

$$\lambda_{6,k,t}^* s_{k,t}^* = 0 \ , \lambda_{6,k,t}^* \geq 0 \qquad \forall k \tag{33}$$

$$g_j(\theta_t^*) \geq 0 \qquad j = 1, .., 6 \tag{34}$$

where (24)-(27) give the first order conditions, (28)-(33) specify the complementary slackness conditions, and (34) are the constraints evaluated at the optimal level of the choice variables.

Using the envelope condition, the standard Euler consumption equation corresponding to (24) is

$$(c_t^* - \underline{c})^{-\rho} = \alpha(c_{t+1}^* - \underline{c})^{-\rho}(1 + r(a_{t+1}^*) + r'(a_{t+1}^*)a_{t+1}^*) \tag{35}$$

Equation (35) simply means that on the optimal consumption path the marginal utility of current consumption must be equal to the discounted marginal utility of next period consumption adjusted for borrowing or saving between the two periods. Re-arranging terms and substituting for the interest rate terms, when borrowing is optimal we have

$$\left(\frac{c_{t+1}^* - \underline{c}}{c_t^* - \underline{c}} \right)^\rho = \alpha(1 + \beta_0 + 3\beta_1 a_{t+1}^{*2}) \tag{36}$$

and for the case of saving we have

$$\left(\frac{c_{t+1}^* - \underline{c}}{c_t^* - \underline{c}} \right)^\rho = \alpha(1 + r_s) \tag{37}$$

which indicates that the intertemporal marginal rate of substitution is equal to the discounted marginal cost (benefit) of borrowing (saving) between the two periods. Because of the vintage structure of the perennial, the Euler equation corresponding to (25) is more complicated. Knapp (1983) derives the analog of the envelope theorem for problems with inequality constraints. Applying equation (13) from Knapp to the area of the second perennial age class gives the envelope equation for this problem:

$$\frac{\partial V(\cdot)}{\partial s_{1,t}^*} = \frac{1}{\alpha}c_t^{*-\rho}\left(\alpha p_s y_1 - \gamma_w w_1 - \gamma_1\right) + \alpha\frac{\partial V(\cdot)}{\partial s_{2,t+1}^*} - \lambda_{1,t}^* - \lambda_{2,t}^* w_1 + \lambda_{4,t}^* + \lambda_{6,1,t}^* \tag{38}$$

Iterating forward one period and substituting into (25) gives the Euler equation

$$(c_t^* - \underline{c})^{-\rho}(\gamma_w w_0 + \gamma_0) + \lambda_{1,t}^* + \lambda_{2,t}^* w_0 - \lambda_{6,0,t}^* =$$
$$\alpha\left(\frac{1}{\alpha}(c_{t+1}^* - \underline{c})^{-\rho}\left(\alpha p_s y_1 - \gamma_w w_1 - \gamma_1\right) + \alpha\frac{\partial V(\cdot)}{\partial s_{2,t+2}^*} - \lambda_{1,t+1}^* - \lambda_{2,t+1}^* w_1 + \lambda_{4,1,t+1}^* + \lambda_{6,1,t+1}^*\right)$$

Notice that this equation still depends on the derivative of the unknown value function which represents the marginal value of the perennial stock. Due to the vintage structure, the value function cannot be substituted out except recursively by using the whole set of equations as follows:

$$\left.\begin{array}{l}
\dfrac{\partial V(\cdot)}{\partial s_{1,t+1}^*} = (c_{t+1}^* - \underline{c})^{-\rho}\pi_1 + \alpha\dfrac{\partial V(\cdot)}{\partial s_{2,t+2}^*} - \lambda_{1,t+1}^* - \lambda_{2,t+1}^* w_1 + \lambda_{4,1,t+1}^* + \lambda_{6,1,t+1}^* \\[2mm]
\qquad\qquad\qquad .. \\[2mm]
\dfrac{\partial V(\cdot)}{\partial s_{K-1,t+K}^*} = (c_{t+K}^* - \underline{c})^{-\rho}\pi_{K-1} + \alpha\dfrac{\partial V(\cdot)}{\partial s_{K,t+K+1}^*} - \lambda_{1,t+K}^* - \lambda_{2,t+K}^* w_1 + \lambda_{4,K-1,t+K}^* + \lambda_{6,K-1,t+K}^* \\[2mm]
\\[2mm]
\dfrac{\partial V(\cdot)}{\partial s_{K,t+K+1}^*} = 0
\end{array}\right\}\text{K-1 eqtns}$$
$$\tag{39}$$

where $\pi_k = p_s y_k - \frac{1}{\alpha}\left(\gamma_w w_k + \gamma_k\right)$ is average profit per unit of land by age class. This can be written more succinctly as

$$\frac{\partial V(\cdot)}{\partial s_{1,t+1}^*} = \sum_{n=1}^{K-1}\alpha^{n-1}\left[(c_{t+n}^* - \underline{c})^{-\rho}\pi_n - \lambda_{1,t+n}^* - \lambda_{2,t+n}^* w_n + \lambda_{4,t+n}^* + \lambda_{6,n,t+n}^*\right] \tag{40}$$

The recursive nature of the Euler equation is indicative of the vintage structure of the perennial crop and the stream of revenues that the initial investment may provide over the life of the crop. As MRR put it, this is a "point-input, flow-output" capital problem. However, the presence of future Lagrange multipliers also points out the dependence of future production levels on scarce and sometimes variable inputs such as irrigation water.

We can now substitute (40) into (25) to obtain

$$\sum_{n=1}^{K-1} \alpha^n \left[(c_{t+n}^* - \underline{c})^{-\rho} \pi_n - \lambda_{1,t+n}^* - \lambda_{2,t+n}^* w_n + \lambda_{4,t+n}^* + \lambda_{6,n,t+n}^* \right] = \frac{1}{\alpha}(c_t^* - \underline{c})^{-\rho}(\gamma_w w_0 + \gamma_0) + \lambda_{1,t}^* + \lambda_{2,t}^* w_0 - \lambda_{6,0,t}^*$$

(41)

which indicates that the discounted marginal benefit of the perennial plantings over its productive life must be equal to the full cost of planting as measured by the marginal utility of forgone consumption and the marginal opportunity costs of resources used.

Re-arranging (26), we have

$$p_x(x_t^*)y_x + p_x'(x_t^*)y_x x_t^* - \frac{1}{\alpha}(\gamma_w w_x + \gamma_x) = \frac{\lambda_{1,t}^* + \lambda_{2,t}^* w_x - \lambda_{5,t}}{(c_t^* - \underline{c})^{-\rho}}$$

(42)

which indicates that the marginal profit of the production of annuals must equal the marginal opportunity cost of resources used in dollar terms. Likewise for removals, the set of equations in (43) implies that the marginal cost of removal must equal the marginal benefit of relaxing the resource and removal constraints valued in dollar terms.

$$\left. \begin{array}{c} p_s y_1 - \dfrac{1}{\alpha}\left(\gamma_w w_1 + \gamma_1 + \gamma_z\right) = \dfrac{\lambda_{1,t}^* + \lambda_{2,t}^* w_1 - \lambda_{3,1,t}^* + \lambda_{4,1,t}^*}{(c_t^* - \underline{c})^{-\rho}} \\[2mm] .. \\[2mm] p_s y_{K-1} - \dfrac{1}{\alpha}(\gamma_w w_{K-1} + \gamma_{K-1} + \gamma_z) = \dfrac{\lambda_{1,t}^* + \lambda_{2,t}^* w_{K-1} - \lambda_{3,K-1,t}^* + \lambda_{4,K-1,t}^*}{(c_t^* - \underline{c})^{-\rho}} \end{array} \right\} \quad \text{K-1 equations} \quad (43)$$

Optimal Rotation and Steady State

Given the age-dependent yield function discussed above, the discrete time Faustmann rule for perennials can be expressed as

$$k^* = \underset{0 < k \leq K}{argmax} \frac{\sum_{k=0}^{k^*} \alpha^k \pi_k}{1 - \alpha^{k^*+1}}$$

(44)

where π_k is the profit from perennials of age class k only. Note that this is equivalent to (3.1) in MRR given the assumption of strict rather than weak inequalities for yields as discussed previously. This assumption also ensures that k^* will be unique rather than having two possible optimal removal ages as in MRR. Below is a plot showing the results of a simulation of the model using the parameters for wine grapes in the Riverland region, which shows an optimal removal age of 32 years old.
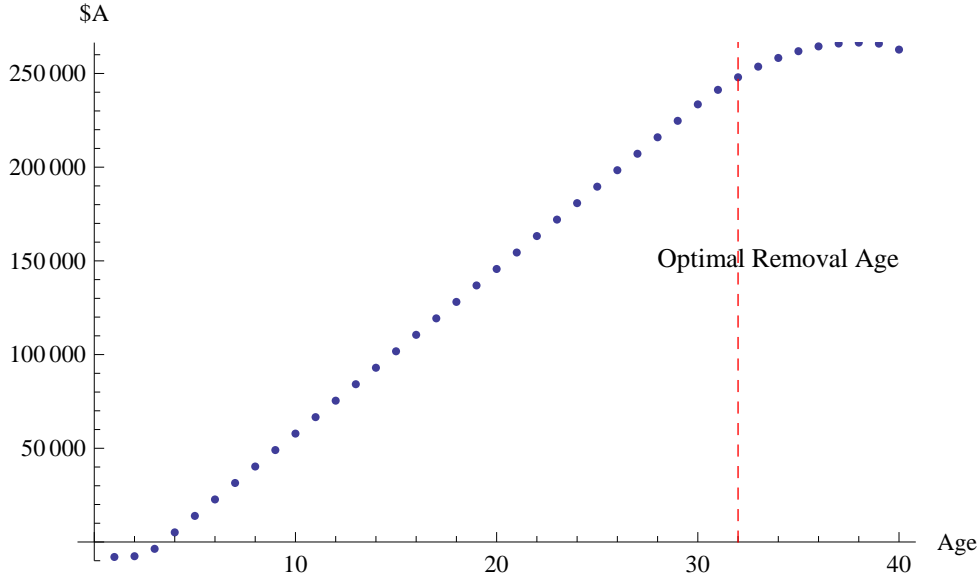
Figure 5: Total profits over life cycle

The steady state conditional on the optimal rotation age is given by evaluating the FOC's, envelope equations, laws of motion, constraints, and complementary slackness conditions at the steady-state values of all variables, i.e., where $\langle a^*, s^* \rangle = f(a^*, s^*)$ and x^*, z_k^* are constant. The constraints and complementary slackness follow exactly as in (28)-(34) when evaluated at the steady state levels of the variables. The steady state budget constraint and Euler equation of consumption respectively become

$$
c^* = \begin{cases}
(\beta_0 + \beta_1 a^{*2})a^* + \pi^* & a^* < 0 \\
\pi^* & a^* = 0 \\
r_s a^* + \pi^* & a^* > 0
\end{cases}
\tag{45}
$$

$$
1 + r(a^*) + r'(a^*)a^* = \frac{1}{\alpha}
\tag{46}
$$

The optimal steady-state asset level depends on the relationship between the subjective discount factor and the interest rate schedule. In general, the range of possible optimal steady-state asset levels is shown as a function of the subjective discount factor and interest rate schedule as
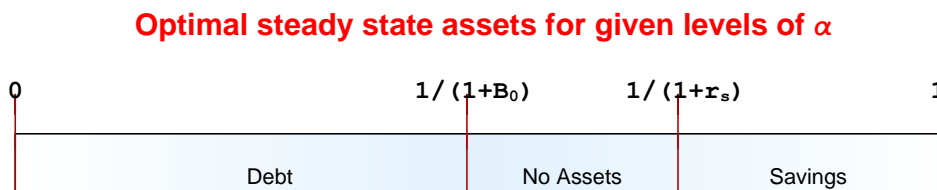

Figure 6: Steady-state assets for given levels of subjective discount factor

Whereas steady-state savings cannot be uniquely determined from (46) due to the flat interest rate for saving, given (4) and (5) the steady-state debt has two possible solutions:

$$a^* = \pm\sqrt{\frac{1 - \alpha - \alpha\beta_0}{3\alpha\beta_1}} \tag{47}$$

but since assets must be negative for debt only the negative solution is feasible. Note that for $\alpha > \frac{1}{1+\beta_0}$ this will give a complex root. However, such values of α mean that the representative farmer is either more patient than the market or would like to borrow at unavailable interest rates in the range (r_s, β_0) and hence will not be a borrower. The optimal steady-state asset debt level over the range of subjective discount factors for a given set of model parameters can be illustrated as:
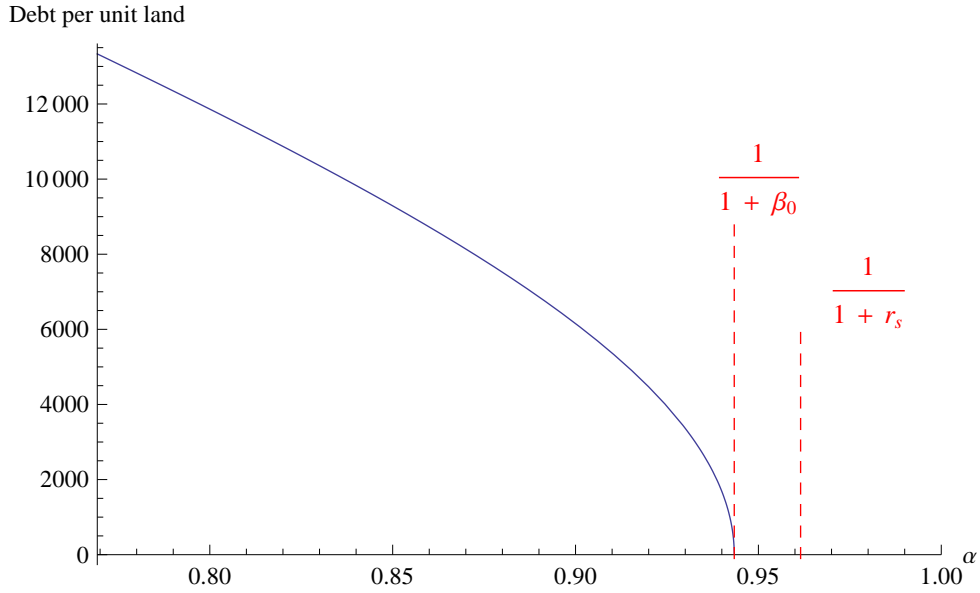

Figure 7: Steady state debt as function of subjective discount factor

$(\beta_0 = .06,\ \beta_1 = 4.5^-10\ ,\ r_s = .04)$

The steady-state area planted in annuals can be defined solely as a function of shadow values and model parameters using the derivations detailed in the appendices. The steady-state law of motion for perennials (13) is $s_{k+1}^* = s_k^* - z_k^*\ \forall k$ which given the existence of k^* implies that

$$s_{k+1}^* = \begin{cases} s_k^* & \forall k \leq k^* \\ 0 & \forall k > k^* \end{cases} \tag{48}$$

$$z_k^* = \begin{cases} 0 & \forall k \neq k^* \\ s_k^* & k = k^* \end{cases} \tag{49}$$

This means that at the optimal steady state the area devoted to different perennial vintages will be equal for new plantings through the optimal removal age k^* and zero for any older age. Also, removals will be zero for all age classes except k^*, which will be cleared in its entirety.

Computational Analysis

The table below depicts the main scenarios covered by the present model, ignoring the possibility of alternate specifications for the utility function. The two columns represent which resource is binding—land or water.[3] The top half of the table allows for the planting of annuals ($x \geq 0$) while the bottom half represents the pure perennial problem. Within the top and bottom halves there are separate scenarios which allow for borrowing and savings ($a \neq 0$) or not ($a = 0$). The MRR model is taken as a point of reference to validate the simplest version of the present model against and to compare to more detailed versions of the model. While this section explores the land-constrained versions of the model, the next section explores lowering water allocation levels permanently in deterministic and stochastic frameworks.

	Land Constrained	Water Constrained
No Annuals	I. No Assets (MRR)	V. No Assets
	II. With Assets	VI. With Assets
With Annuals	III. No Assets	VII. No Assets
	IV. With Assets	VIII. With Assets

Table 1: **Model Scenarios**

The MRR model approximately corresponds to a special case of the present model in which there are no annuals, no capital markets, and land is the only constrained resource (model I above). MRR's model assumes a unit of land and no other inputs with the total stock of trees held fixed. The only use for land is perennial production and any land cleared is automatically replanted with new crops. It is assumed that there are ages P and Q such that yields are monotonically increasing before age P, then constant through age Q, and monotonically decreasing thereafter. Amongst the key findings, the authors conclude that the OSF is invariant to the utility function used so long as it is twice continuously differentiable and concave. The growth pattern over the life of the crop and the discount factor determine the characterization of the OSF. Given a linear utility function, an arbitrary forest will not converge to the OSF whereas, given

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[3]In principle, one doesn't know a priori which resource is binding and it is possible that both resource constraints might bind. However, by setting the baseline water allocation to the mean water use level we have found that, for the parameters used, the water resource constraint is only binding when allocation levels drop below 90% of the baseline. Both resource constraints bind only as a special case.

some restrictions on the discount factor, the optimal program combined with a strictly concave utility function will always converge.

One difference between MRR and the present model is that the profits by age in the present model are more restrictive in the sense that the present model assumes strict inequalities of returns during establishment while MRR assume only that there must be one strict inequality. Also, the present model is less flexible with respect to age of maturity and decline in that the age of maturity and decline, P and Q respectively, are assumed to be distinct with both less than the oldest possible age class. Since the present model is less general in these ways, all proofs from MRR should apply a fortiori to the relevant case of the present model, which is characterized by pure perennial production with no capital markets. However, there is one major aspect in which MRR is very unrealistic; they normalize returns by age such that there are never negative returns to perennial plantings. High fixed costs and lags in production are salient characteristics of perennial crop production and ignoring these traits means that MRR's findings may not hold in the present model and especially in more general models which include borrowing and saving.

Initial Land Distribution

To easily generate a wide variety of initial land distributions, we have used the beta binomial discrete distribution as a data generating mechanism. The PDF is given as

$$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)} \tag{50}$$

where $B(\alpha, \beta)$ is the Beta function dependent on shape parameters α and β and the number of "trials" $n = 0, .., 30$. This results in 31 age classes for the initial land distributions; this number is chosen due to the fact that the model parameters cause it to be optimal to remove the vines in the 32nd period. Since removals are at the beginning of the period this means the vines actually remain in production for 31 years and hence 31 initial age classes is appropriate. The shape parameters can take any value in the range $(0, \infty)$ but for our purposes it should be sufficient to use all combinations of α and β in $\{0.25, 0.5, 0.75, 1, 1.5, 2, 3\}$. This gives 49 possible combinations to which we add the scenario of no initial perennials for a total of 50 initial land distributions. Below is a plot of some convenient land distributions, including the uniform land distribution which holds at the steady state, and the parameter values used to create them.
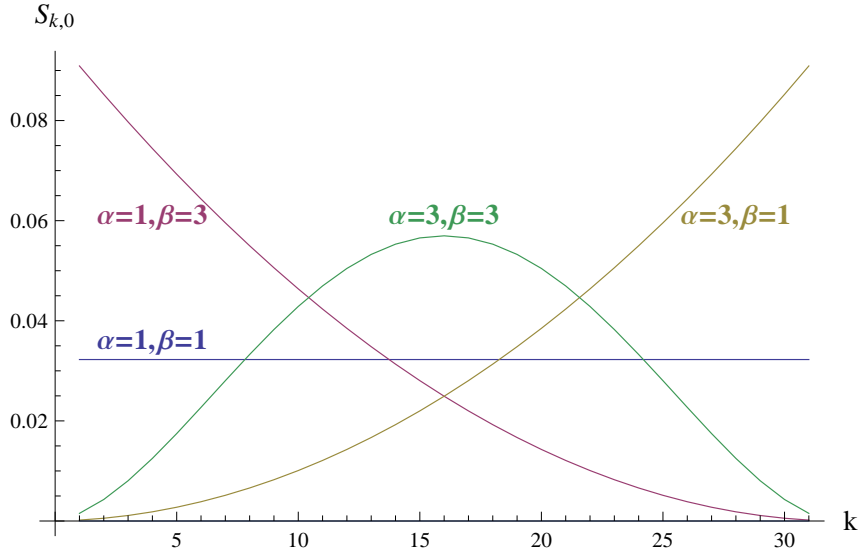
Figure 8: Land distributions generated using the beta binomial distribution

Difference from the Steady State

To describe the evolution of the perennial land holdings with respect to the difference from the steady state, we have chosen to use the L-1 norm as follows:

$$d_{ss} = \sum_{k=1}^{40} |s_{k,t} - s_k^*| \tag{51}$$

where $s_{k,t}$ is the land devoted to age k at any given time and s_k^* is the steady-state land area for each age $\left(\frac{1}{32} = .. = \frac{1}{32}; 0, .., 0\right)$ given the model parameters. Given the 50 initial land distributions generated, a plot of the L-1 norms relative to the steady state is shown below.
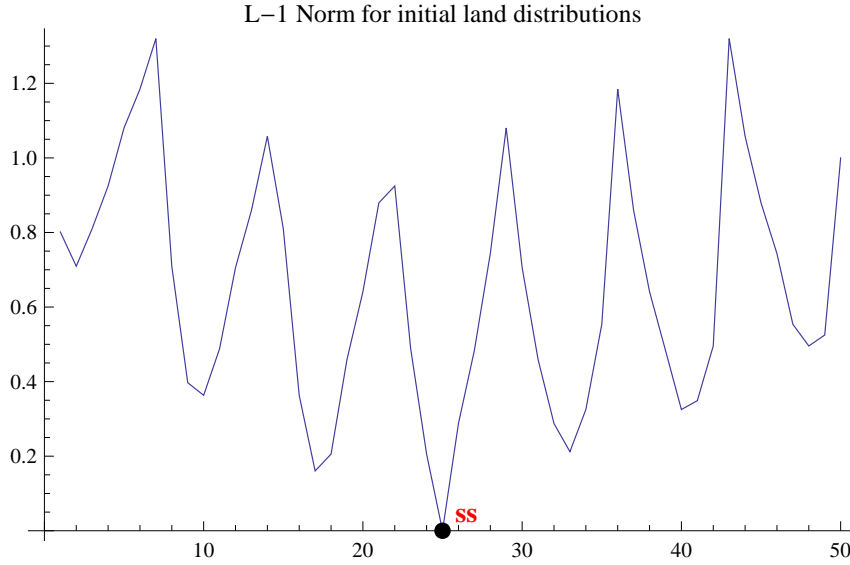
Figure 9: Deviations from steady state for initial land distributions

The symmetry of the plot results directly from the way in which the tuples of the shape parameters are used. Note that the 25th vector generated is the steady-state land distribution and hence has a norm of zero.

Model I. Pure Perennial Production

For the pure perennial problem, the only means of the household obtaining an income is by growing perennial crops. There are no annual crops and borrowing is not allowed. Given that consumption is constrained to be non-negative and that there are large fixed costs for new plantings, if the household starts with no perennial plantings it is effectively caught in a poverty trap and it is not possible to run the optimization model. To avoid this, it is necessary to provide an exogenous income stream c_0 which is constant over time. Also, to eliminate the possibility that the level of c_0 initially acts as a de facto credit constraint and therefore alters the household's planting sequence it has been set high enough to allow the household to plant all of the land available in the first period if it so chooses. Although artificial, in this way we can isolate the effects of initial land holdings on household behavior. The point of this exercise is not to be realistic but to systematically isolate and explore the features of the model.

A further important consideration is to test the degree to which the model behaves as predicted by MRR. The pure perennial model gives a very clean comparison to their model. The assumed age-yield relationship in this model is more restrictive than theirs and therefore their findings should hold here. However, there is one major aspect in which MRR is very unrealistic; they normalize returns by age such that there are never negative returns to perennial plantings. The fact that real-world perennial crops production is characterized in part by high fixed costs for new plantings is what causes the problem noted above and is the reason why an exogenous income stream must be specified. It is nonetheless possible to

22

compare the results from the model presented here to the salient findings in their paper, which are that perennial plantings will converge to an optimal stationary forest characterized by equal land area devoted to each age class assuming that 1) utility is strictly concave and 2) the discount rate (factor) is not too high (low). The findings here do, in fact, match these findings. The discount rate used throughout is $r_h = .05$ except where noted.

Linear Utility

Linear utility is time-additive separable with an infinite intertemporal elasticity of substitution and therefore effectively equivalent to the case of perfect capital markets. Only in the first period is it necessary to bound consumption away from zero via the exogenous income stream in order for the problem to be computationally feasible. After that point, the simulation may result in some periods in which consumption is lower than $c_0 = 9.0$. Note that consumption is given in $\$1,000$ per hectare and plantings are given as a fraction of the normalized land available; i.e. new plantings, and total land planted, are ≤ 1.0 hectares in each period.

100 Year Simulations

Simulations over a 100 year time horizon using a running horizon algorithm (detailed in Appendix 2) show that the household desires to plant all land in perennials as quickly as possible and to maintain full perennial stocks over time by replanting based on the Faustmann rule for perennials. Also, consumption and new plantings always move in opposite directions and there is no tendency for perennial plantings to smooth out over time. The L-1 norm is constant for each of the initial distributions shown here. The 3 initial distributions highlighted here are meant to give a sample which covers the cases of a young initial perennial stock ($\alpha = 1, \beta = 3$), a peaked/normally distributed one ($\alpha = 3, \beta = 3$), and the case in which there are no initial plantings.
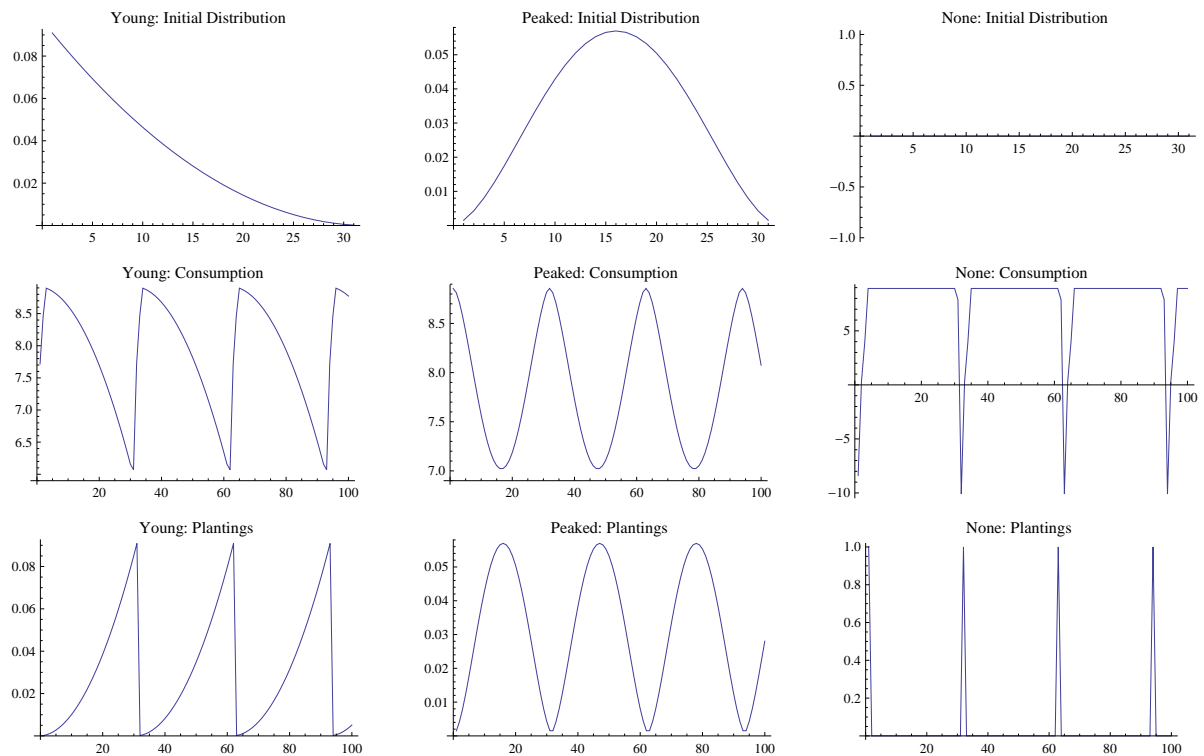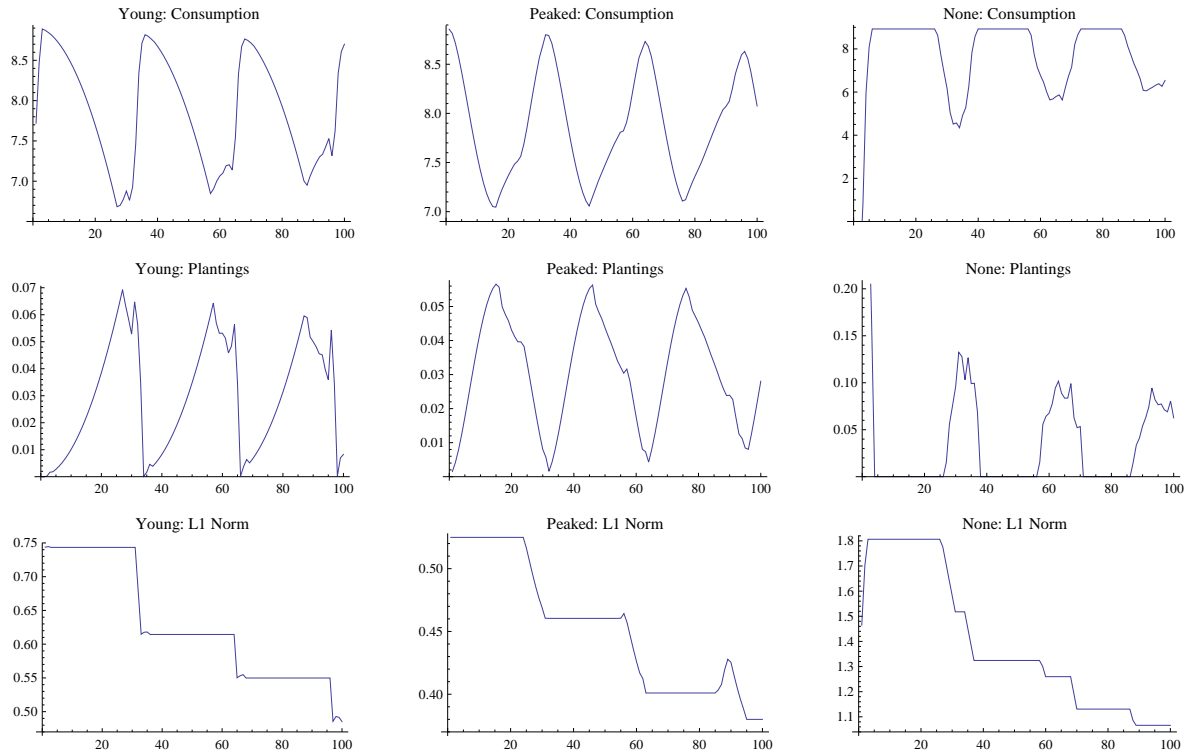
In order to be certain that the results shown above are not simply an artefact of using too short of a time horizon, results for a 1,000 year time horizon are given below. The results are exactly as above but on a longer time scale with a constant L-1 norm over the whole time horizon. Not only are these results compatible with MRR but, in the case of no initial plantings, they also agree with the findings of Mitra and Wan (1985) for the case of a regional forestry model with discounting. In that study, they find that the optimal program will lead to a periodic solution as is found here. Plots of the full 1,000 year simulation can be found in Appendix 3.

Log Utility

100 Year Simulations

Given strictly concave utility, the model should lead to convergence to the steady state in order to agree with MRR. The 100 year plots shown below do show substantial smoothing in consumption and planting over time but do not come close to converging to the steady state.

Young: Consumption   Peaked: Consumption   None: Consumption

Young: Plantings   Peaked: Plantings   None: Plantings

Young: L1 Norm   Peaked: L1 Norm   None: L1 Norm

The 1,000 year simulations exhibit continued movement toward the steady state. While convergence to the steady state does not occur, this, in itself, does not contradict MRR as they don't undertake any simulations or give any other indication of how long convergence might take in practice. In general, the time to convergence will vary with the discount rate used, the rotation length of the perennial crop, and the details of the age-yield relationship specified. Also, as all the plots shown thus far indicate, the closer the initial land distribution is to the steady state the quicker it will converge. It should be noted that in the case of both linear and log utility if the land distribution starts at the steady state it never leaves it as would be expected. Another point to note is that the young and old distributions are exactly symmetric ($\alpha = 1, \beta = 3$ and $\alpha = 3, \beta = 1$ respectively) and have the same constant L1-norm in the linear case. In the log utility case, the young distribution shows greater convergence toward the steady state. While log utility causes smoothing in both cases, this difference appears to be a result of the greater need to replant earlier in the case of the old distribution as opposed to the young. The delayed replanting schedule for the young distribution means that discounting will have a greater effect, diminishing the difference between the net present value of area replanted in successive years.

High Discount Rate

The optimization model was also run using high discount rates. For linear utility, using a discount rate of .20 (4 times higher than the base case) leads to slight oscillations in replantings but otherwise does not affect the results. In particular, the rank of L-1 norms remains fixed for the various scenarios and there is no trend to convergence. Only when raising the discount rate to extremely high values such as 0.5 does the model behavior change dramatically; specifically, there are no new plantings of perennials

in any scenario. The optimal removal age is extended from 32 years to 38 years and all scenarios remove the perennial plantings accordingly until they reach zero. For log utility, a discount rate of .20 similarly has small effects on the rate of convergence and a discount rate of .50 results in no new plantings just as in the linear utility case.

Model III. Annuals Included

Including annuals enables the model to converge to the steady state more quickly as can be seen below:
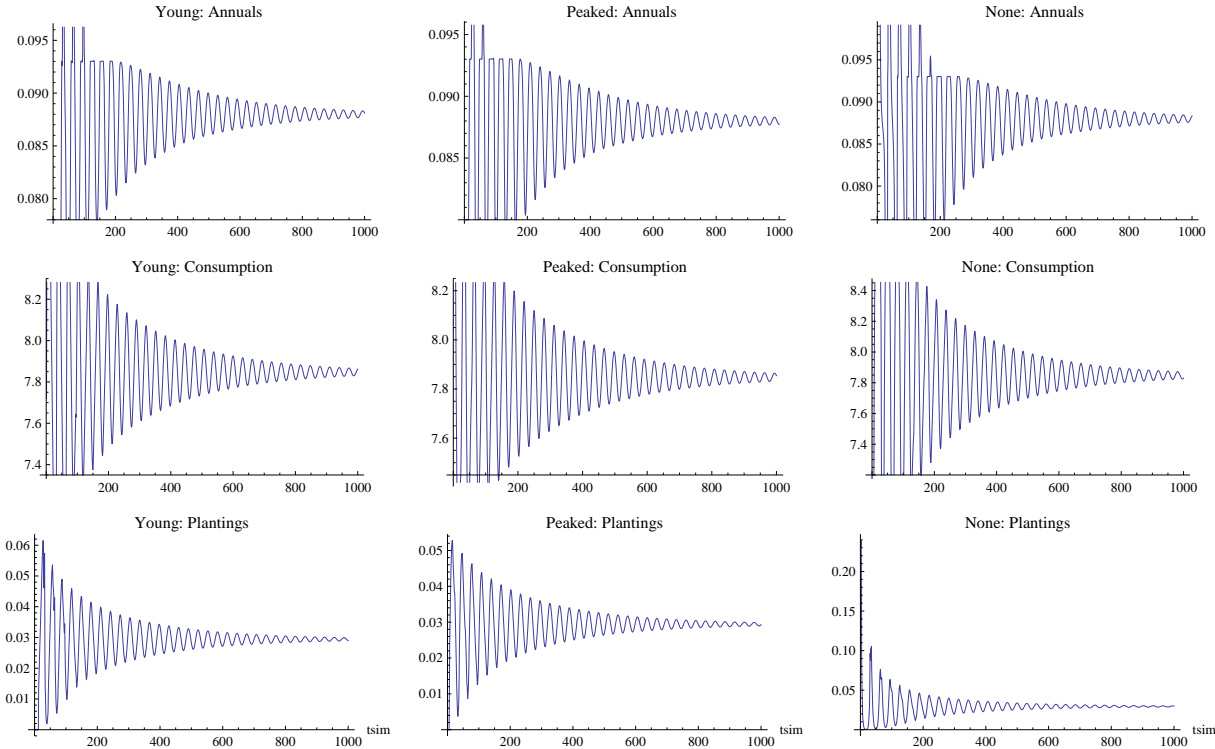
As a result of introducing annuals, the mean L1 norm decreases across the three scenarios between 37.5% to 53.6%.
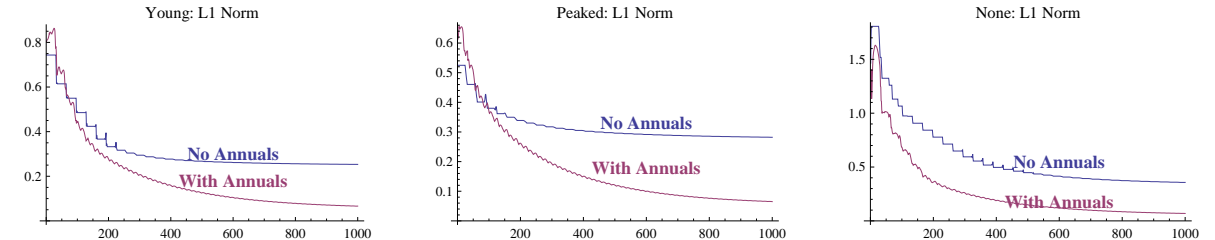
The effect of adding annuals adds little to overall consumption levels but does allow for significant reductions in the variance of consumption across the three scenarios. The table below shows the difference in endogenous consumption, i.e., consumption not including c_0, between models I and III. As you can see, the variance of endogenous consumptions decreases between approximately 61-73% across the three scenarios.

Table 2: **Differences in Endogenous Consumption ($1,000 per unit land)**

	Young	Peaked	None
Δ Median (Δ%)	-0.0136	-0.0063	-0.0327
	(-0.17)	(-0.08)	(-0.42)
Δ Variance (Δ%)	0.0957	0.111	0.521
	(60.85)	(72.73)	(61.72)
Δ Min	-0.0666	-0.0082	-4.441
Δ Max	-0.0311	-0.0431	0.0981

C_t from model I less C_t from model III

Model IV. Annuals and Borrowing/Saving

Recall that the budget constraint is defined as

$$c_t = (1 + r(a_t))a_t - a_{t+1} + \pi_t \tag{52}$$

The standard assumption of perfect capital markets is relaxed as is reflected by the interest rate schedule $r(a_t)$ having a higher interest rate for borrowing than saving. To mimic increasing interest rates for borrowing as debt levels increase, the interest rate for borrowing and corresponding first derivative are expressed as

$$r_d(a_t) = \beta_0 + \beta_1 a_t^2 \tag{53}$$

$$r_d'(a_t) = 2\beta_1 a_t \tag{54}$$

where β_0 and $\beta_1 > 0$ while the interest rate for saving (r_v) is constant and $\beta_0 > r_v > 0$.

Computational issues arise due to the discontinuous, non-linear interest rate schedule specified above. Specifically, because discontinuous functions are difficult for gradient-based solvers to handle we have been using a continuous non-linear approximation around the zero asset level. While this strategy works for solving the model, due to the extreme curvature it takes a long time to solve. Therefore, we have

re-formulated the problem by explicitly splitting assets into savings and debt with the relevant interest rate function specified for each. Assets are now defined as savings (v_t) less debt (d_t): $a_t = v_t - d_t$. The interest rate for savings, r_v, is given as a parameter and has a baseline value of 0.04. The interest rate for debt is a function defined as above with baseline values for the intercept and slope terms being 0.06 and 9×10^{-4} respectively. Given that both savings and debt are restricted to be non-negative, the budget constraint is re-written as:

$$c_t = (1 + r_v)v_t - (1 + r_d)d_t - a_{t+1} + \pi_t \qquad (55)$$

Specified this way, the model solution is much easier for the solver to compute and the interest rates used are exactly as specified above rather than an approximation for asset values near zero. The results below show that the introduction of borrowing and saving greatly decreases the time to convergence for the young and peaked land distributions while for no initial perennials the time to convergence is similar to model III except that the variance of the time paths for all variables is greatly decreased. Because the discount rate used below (0.05) lies in the kink of the interest rate schedule it is optimal to hold no assets. This leads to very quick convergence of assets to zero except in the case of no initial plantings.
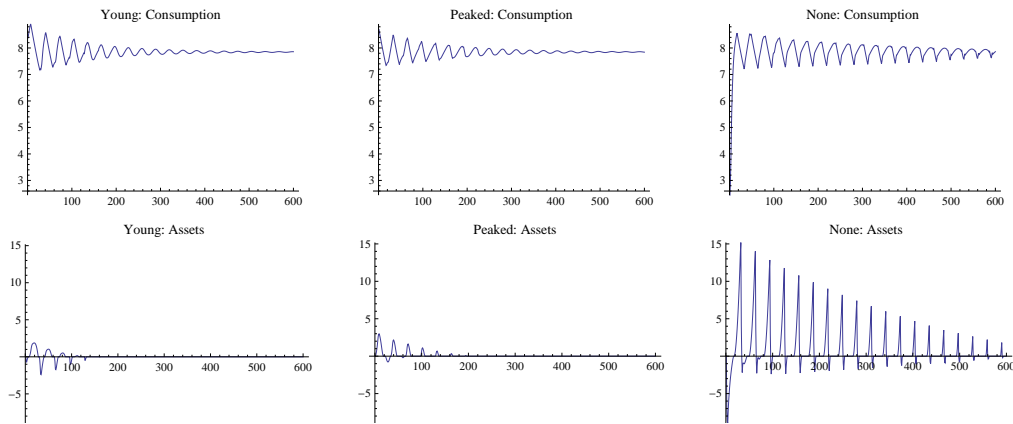

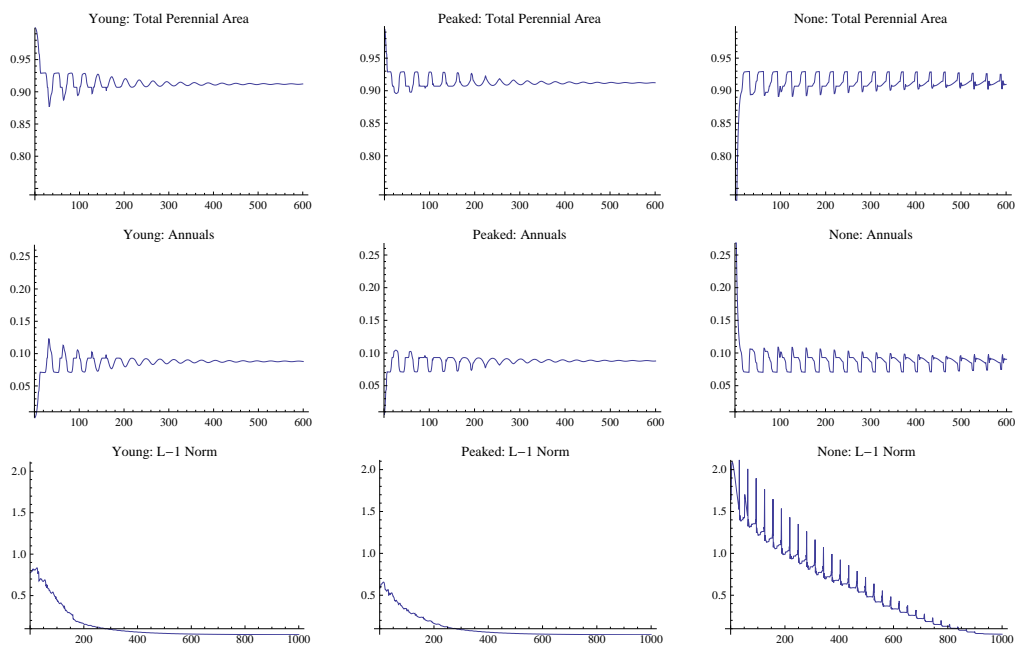Figure 10: Full model: Consumption and assets

28

Figure 11: Full model: Total perennial area, annual crop area, and L-1 norm

The table below shows results similar to that comparing models I and III with some reduction of variance due to the ability to borrow and save.

Table 3: **Differences in Endogenous Consumption ($1,000 per unit land)**

	Young	Peaked	None
Δ Median (Δ%)	-0.001	-0.002	-0.0336
	(-0.017)	(-0.027)	(-0.427)
Δ Variance (Δ%)	0.003	0.0003	0.092
	(5.48)	(0.699)	(28.575)
Δ Min	-0.073	-0.0156	-0.994
Δ Max	0.005	0	0.0358

C_t from model III less C_t from model IV

Policy Analysis

Policy Context

Aside from the MDB being the most important agricultural area in Australia, the cities of Adelaide and Canberra depend on its river flows as do the 16 Ramsar listed wetlands in the basin (Pittock and Connell 2010). The rivers of the MDB are among the most variable in the world when measured by the ratio of maximum to minimum annual flows. One way to manage this variability has been to build as much storage as possible; the MDB currently has enough storage to handle over twice the average annual river flow (Chartres and Williams 2006). Water rights have been over-allocated based on optimistic estimates of annual flows and irrigation has received higher priority than the environment for much of history of European settlement (Connell 2007). As a result, increasing environmental damage and threatened urban water supplies led to an agreement to cap water diversions across the basin at 1995 levels. An increase in the use of groundwater and surface water entitlements that were previously under-used have been seen after the cap, highlighting the need to manage groundwater and surface water together (Chartres and Williams 2006).

Recognition that the constituent parts of the basin are interdependent and therefore need to be managed as a whole (i.e. integrated catchment management) go back at least a century but attempts to act upon this realization have been consistently thwarted by the self-interested actions of local and state governments (Connell 2007). Or, in the words of Pittock and Connell (2010), ".. the Basin's management is a history of cultural resistance to the natural variability, with drought-induced crises triggering institutional reforms so far largely at the margins." The latest crisis was the extreme Millennium Drought from 1997-2009 and the latest attempts at reform are embodied in the National Water Initiative of 2004, the Water Act of 2007, and the proposed MDB Plan released in 2010. The MDB Plan has undergone intense public debate and several revisions; originally, basin-wide reductions of diversions of up to 4,000 gigaliters per year (GL/yr.) (approximately 31 % of current use) were called for but the most recent policy discussion proposes a reduction of only 2,750 GL/yr (22%). (Lamontagne, Aldridge, Holland, Jolly, Nicol, Oliver, Paton, Walker, Wallace, and Ye 2012). The final level of the Sustainable Diversion Limits (SDL's) set by the Plan remain under debate as the SDL's will not take effect until 2019 (Pittock and Connell 2010). Furthermore, no version of the Plan considers the potential impact of climate change even though the most thorough study of climate change in Australia suggests that under a median climate scenario there will be a 10% reduction in runoff across the basin by 2030 (CSIRO 2008).

Data

Wine Grapes

Wine grape production is very important to the economy of the Riverland. The Riverland produces more wine grapes than any other region in Australia and more than half of farm gate receipts in the region come from wine grapes. In addition, wine making is the biggest employer in the manufacturing sector of the region (Riverland Winegrape Grower's Association 2012). As can be seen in the figure below, many of the wine grape growers in the Riverland region have vineyards that are quite small. While most of the household farms have significant off-farm income, those less than 10 hectares in size are likely to be "hobbby farms." While there are many such farms, the second chart makes it clear that most of the area planted is on larger farms: 46% of farms are 50 hectares or larger and 61% are at least 25 hectares in area (Phylloxera and Grape Industry Board of South Autralia 2012)[Phylloxera Board]. To give some sense of the viability of relying on the proceeds of a farming operation with 25 hectares of vines, it is useful to note that average annual revenue over a 20 year period for this size farm assuming mature but relatively young vines would be $298,753.[4] While some of the grapes in a given year are grown by the wineries themselves, they typically constitute a small fraction of total production. In 2011, for example, independent growers produced 82.5% of the total grape crush (Phylloxera Board).
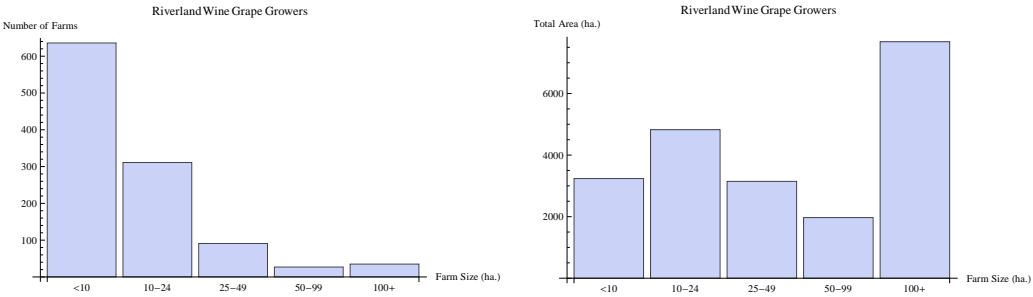

Figure 12: Riverland Wine Grape Farms (i) Size Distribution (ii) Total Area Planted by Farm Size

In order to determine average farm debt level per unit land for the representative farming household, it is necessary to determine average farm size. Given the presence of many smaller farms that may not rely on income from the farm itself, we exclude farms under 10 hectares in size and then use the average of the remaining farms. Doing so results in an average farm size of 38 hectares. Using data from Ashton, Hooper, and Oliver (2010), an estimate of debt held by farming households for the year 2007-08 can be determined. While the data is at the level of the whole basin it distinguishes between horticulture,

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[4]The revenue calculated based on Phylloxera Board data as 25 hectares ∗ 18.609 tonnes per hectare ∗ $661.37 per tonne = $298,753.

broadacre, and dairy farms. Horticultural operations held an average debt of just over $275,000. Given an average farm size of 38 hectares derived above, this implies and average debt per hectare of $7,250. This will serve as the initial debt level per hectare in the model.

The age distribution of the existing grape stocks in the Riverland were calculated as detailed in the appendix. The resulting estimated distribution is seen below.
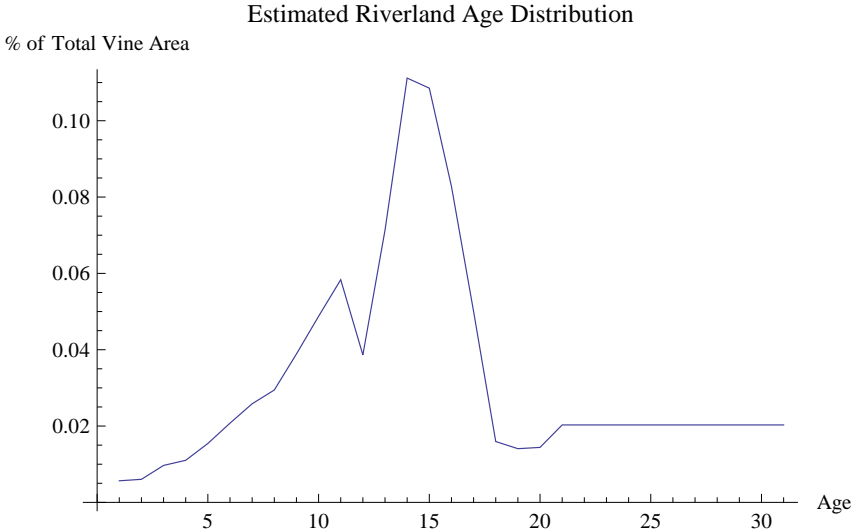
Estimated Riverland Age Distribution

Figure 13: Source: S.A. Winegrape Utilisation and Pricing Surveys 2000-2012 from the Phylloxera Board

Potatoes

Australia trades very few potatoes internationally and South Australia, including the Riverland, is the primary producer of potatoes for domestic consumption due to advantages of sandy soils and the availability of irrigation water. Additionally, potato cultivation is one of the biggest uses of irrigated land in the Riverland and is the most valuable irrigated horticulture crop next to wine grapes in the region (PIRSA Industry Structure and Strategy Team 2005). For that reason, using potatoes as a representative annual crop with a downward-sloping demand curve can be justified and may provide a solution to the bang-bang solutions which the model might otherwise produce. Data for prices and average area devoted to potatoes are drawn from data in Australian Bureau of Statistics (2009) and an estimate of the South Australian potato supply elasticity is taken from Mules and Jarrett (1966).

Water Use

Wine grapes use less water than potatoes; the model uses parameter values of 4.4 mL per hectare (mL/ha.) for mature wine grapes and 6.1 (mL/ha.) for potatoes. Over 86% of South Australian vineyards use high efficiency drip or micro spray irrigation systems (Australian Bureau of Statistics 2009) whereas center pivot is the dominant form of irrigation for potatoes (Department of Primary Industries 2011).

The irrigation system is accordingly assumed fixed for both crops with annualized costs of $4,000 and $2,000 per hectare for drip and center pivot systems respectively.

Diversions

Below is a graph of modeled historical annual irrigation diversions under current water entitlement rules and levels of development for a 110 year period in the South Australian MDB obtained from Connor, Banerjee, Kandulu, Bark, and King (2011). While the time series exhibits some volatility, it is much less variable than the underlying river flows with the main reason being that dam storage helps smooth water supply across years. Another reason is that in the past very little importance was given to maintaining sufficient river flows to maintain the health of ecosystems that depend on the river. Note that diversions are endogenous and ideally we would have a similar time series for allocations. However, due to the prioritization of irrigation over the environment irrigators have received 100% water allocations up until the recent drought, the vast majority of which has actually been diverted. The marked decrease at the end of the time series is due to an extremely severe drought which put severe strain on irrigators and ecosystems alike. The drought continued through 2009 and only ended in late 2010.
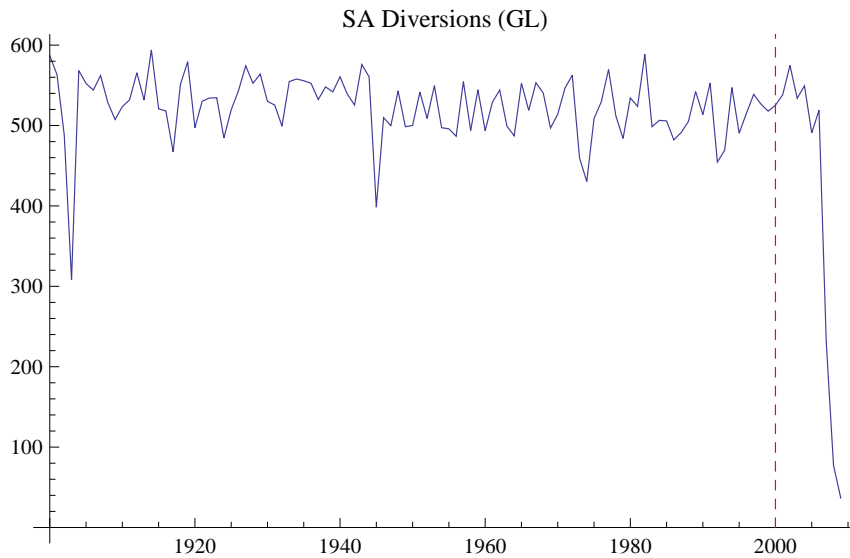
Figure 14: Modeled diversions in South Australia

The above data is historical and therefore provides a baseline to which other water availability scenarios may be compared. In addition, we have access to data on diversions modeled as if permanent water reductions had been in place over the same 110 year history. These time series maintain the same hydrological sequence but vary institutional rules in a way that results in nearly parallel downward shifts of the baseline time series as can be seen below:
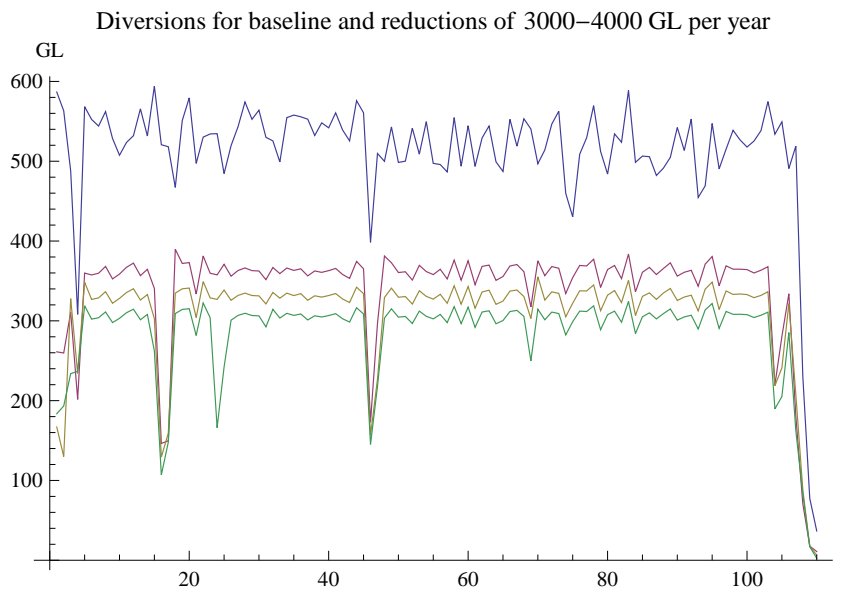

Figure 15: Diversions for South Australia under different scenarios

The result is that reductions by 3,000, 3,500, and 4,000 GL lead to median diversion levels that are 67%, 62%, and 57% of the baseline levels respectively.

34

Reductions in Irrigation Water

Given the parameters specific to the Riverland, the model can be used to analyze the effects of permanent reductions to agricultural water entitlements. Household consumption and debt as well changes in land use can be evaluated for alternate levels of reduction. We consider three scenarios: a baseline in which there are no changes to water entitlements and two levels of permanent reductions corresponding to 3,000 and 4,000 GL per year across the basin.

Water Demand

Given the estimated impacts of permanent water entitlement reductions on South Australian irrigators, which reside predominantly in the Riverland, we estimate the short-run and long-run agricultural water demand in the region for allocation reductions of 0-50%. This is done by running the optimization model at successively lower water allocation levels and capturing the shadow values from the water constraint. For each water level, the model is allowed to run for as many periods as needed to reach a steady state. The shadow value divided by the marginal utility of consumption for the first and last periods then define points on the short-run and long-run water demand curves respectively. The following results were obtained from the model without household saving (Model VII).
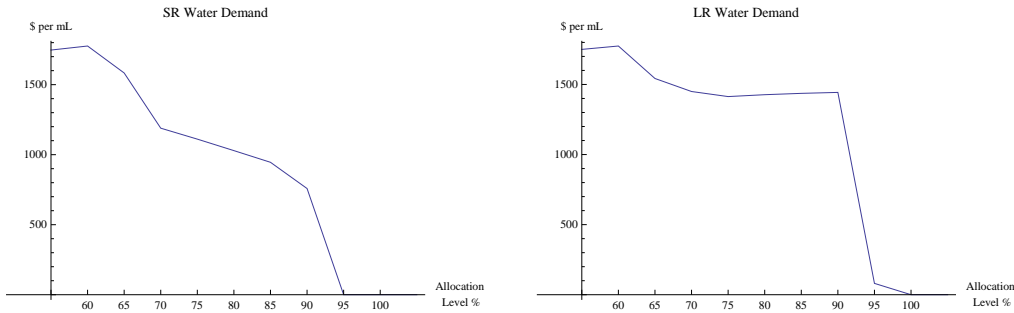

Figure 16: Water demand curves: Short run and long run

Note that, as expected, the long-run demand curve is more elastic than that of the short run but the long-run demand curve is generally higher. This reflects the fact that the initial conditions of the model are based on the situation that the region is in currently, which does not coincide with the steady state. Because this version of the model is not calibrated to constrain increased perennial plantings it devotes most of the land in the region to perennials in the long run, hence increasing the marginal value of water in agricultural production due to the relatively higher perennial prices. The model is constrained from a corner solution only because of the demand curve for annuals; annuals are removed up until the point that the marginal value of potatoes equals that of wine grapes. Now, consider the figure below that shows the shadow values for both land and water obtained from the model runs at varying levels of water allocations.
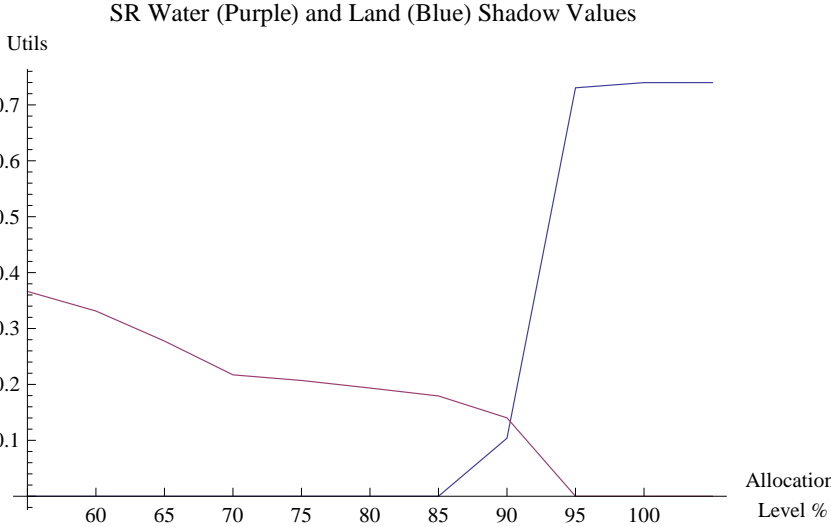

Figure 17: Water and land shadow values

The above indicates that the water resource constraint is binding for allocation levels up to around 90% of the baseline level. Above that level, land is the constrained resource and the constraint on water is slack.[5]

Below is a bar chart of long-run (steady state) land use for each water allocation level:

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[5]Note that irrigated land area is truly limited by the physical configuration of the irrigation infrastructure in the region. It would be neither economically nor politically feasible to extend the infrastructure.
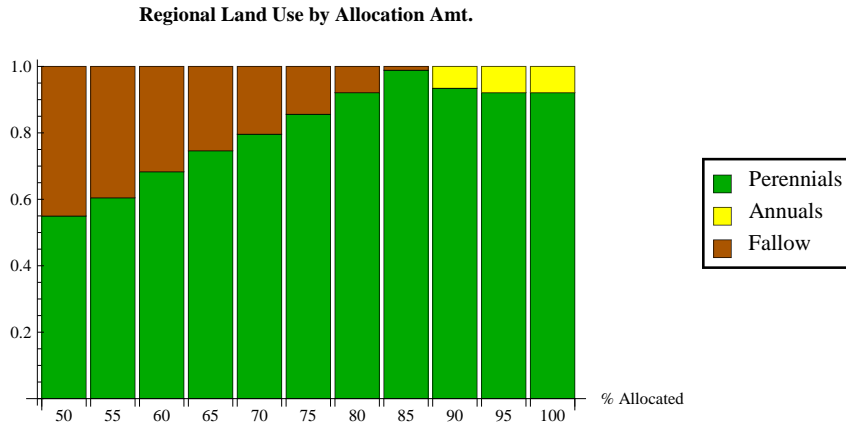
Figure 18: Land use by water allocation level

The interpretation of the above is that as water allocations are reduced first annuals are taken out of production and then perennial stocks are removed. Note that this overstates the likely impact of such reductions because deficit irrigation is not allowed by the model.

Stochastic Water Supply

Using the Monte Carlo procedure detailed in Appendix 5, we can look at the effect of stochastic water allocations on consumption, land use, and quantity supplied under the baseline and alternate water reduction levels.

Monte Carlo Simulation Results

Below is a histogram of water allocations pooled across all MC runs for each scenario considered, including the baseline plus scenarios where basin-wide water allocations have been reduced by 3,000 and 4,000 GL per year respectively. Since the water allocations are exogenous, there is no problem including such pooled histograms for allocations; however, for endogenous variables such plots would not make sense. This is because the time path for a given variable in a single model run reflects intertemporal trade-offs. Pooling together plots of endogenous variables across model runs would obliterate any economic content represented by the individual runs.

Below are some plots generated using model VIII which show the mean of a variable taken across all model runs for a specific point in time. We have also added lines which show the mean plus or minus 2 standard deviations. The following are such plots for the baseline water scenario in which allocations are not reduced:
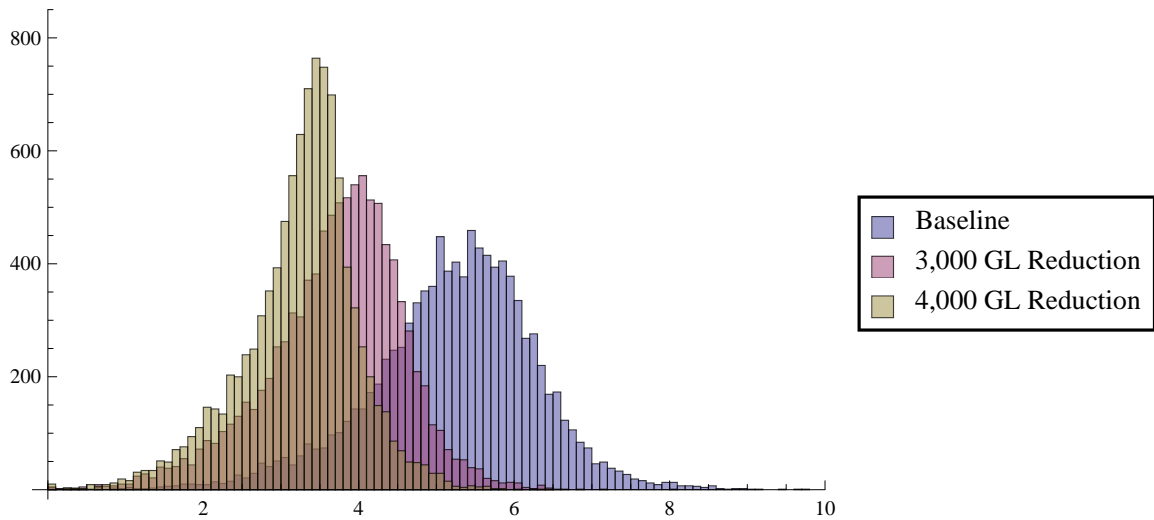
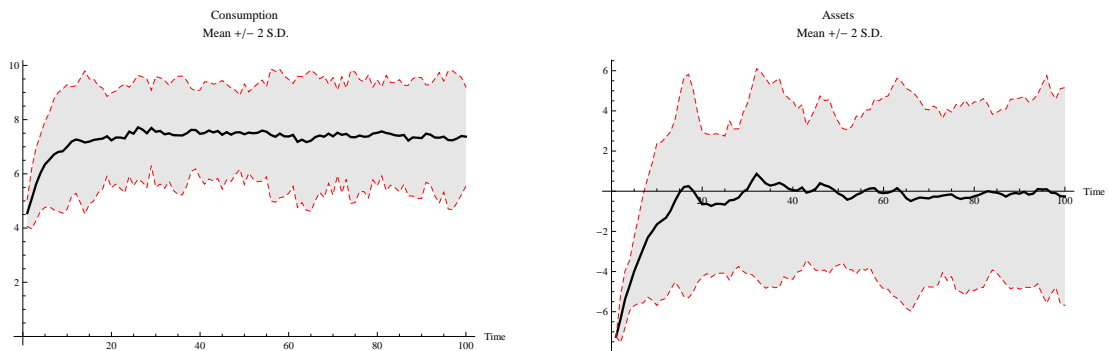Figure 19: Histogram of allocations pooled by scenario

Figure 20: Baseline consumption and assets under log utility
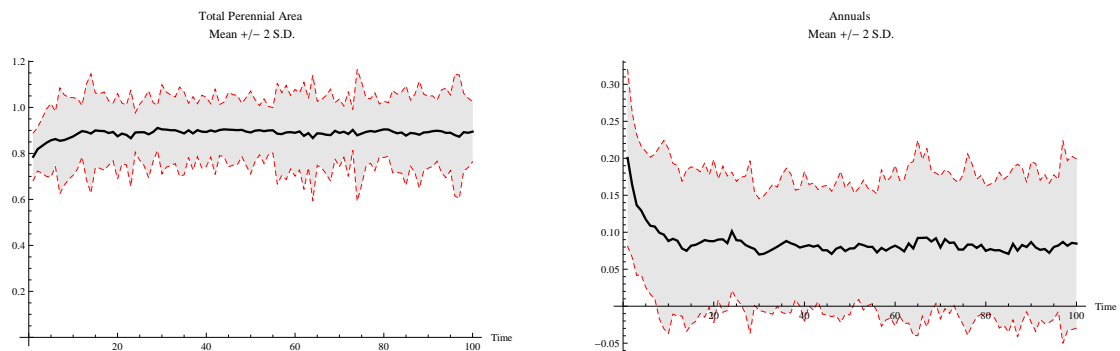
Figure 21: Baseline land use under log utility

Below are the mean values of key variables across the baseline, 3,000 GL reduction, and 4,000 GL reduction scenarios. Notice that consumption drops significantly as allocations are reduced although, as previously noted, this is an overestimate due to the lack of adaptation methods such as deficit irrigation in the model. Also, note that annuals increase in area for the 3,000 GL reduction but then drop below the original level at the 4,000 GL reduction level. Furthermore, rotation lengths shorten significantly in

response to uncertain water applies.

	Baseline	3000 GL	4000 GL
Consumption	7.36	5.56	4.42
Assets	-0.22	-1.58	-2.59
Total Perennials	0.899	0.755	0.646
Annuals	0.078	0.119	.066
Oldest Perennials	29.6	21.67	21.05

Table 4: **Mean values across Monte Carlo scenarios**

Recursive Utility

In order to better calibrate the model for policy analysis, we have created a version of the model which uses recursive utility. Specifically, the Epstein-Hynes recursive utility function is used:

$$U(C) = -\sum_{t=1}^{\infty} exp\left[-\sum_{\tau=1}^{t} u(c_\tau) \right] \tag{56}$$

where $u(c_\tau)$ represents the instantaneous utility function. Key features of this utility function are that utility depends on an aggregate of the future consumption path even for deterministic models and the discount rate is endogenous and decreasing in the index of future consumption. In practice, this allows for a model which can be calibrated to a given ratio of perennial to annuals land use as well as an optimal household debt level. This has been applied to models IV and VIII with some success. Results to follow. Here we consider only the stochastic version of model VIII. The plots of mean values from the Monte Carlo runs given above are shown here for the base allocation with Epstein-Hynes utility. Notice that the land use mix and asset levels are much different now:
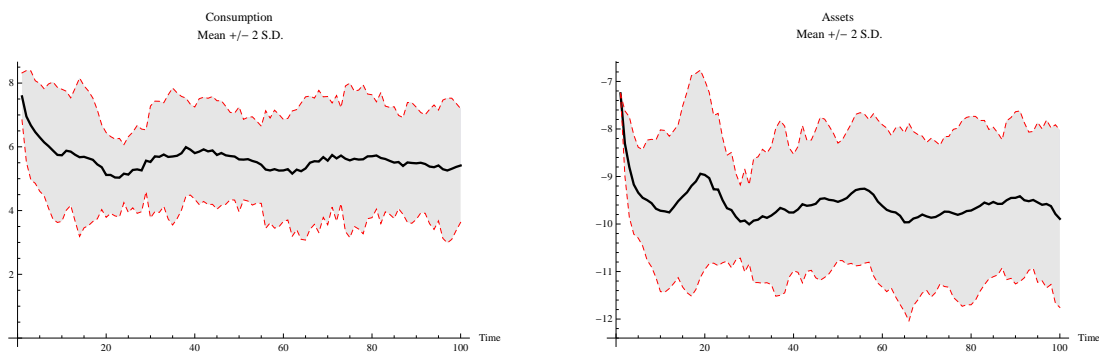

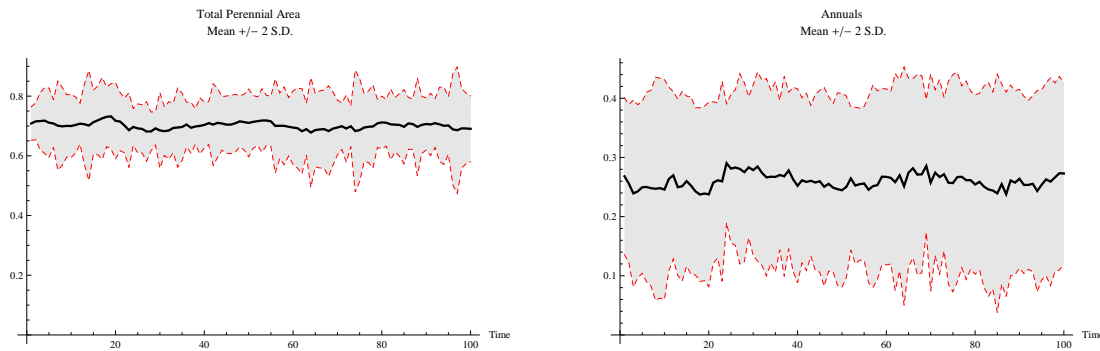Figure 22: Consumption and assets under Epstein-Hynes utility

Figure 23: Land use under Epstein-Hynes utility

Conclusions

This study develops a dynamic model of irrigated perennial production at a regional level and, in so doing, advances the literature on the economics of perennial supply as well as the literature on regional agricultural programming models. The model is explored analytically using a dynamic programming formulation and the optimal steady state conditional an optimal removal age is characterized. The model is then implemented in a dynamic optimization framework in GAMS. The most restricted form of the model is compared to the theoretical model of Mitra et al. (1992) and is found to be in agreement with several of their key results despite the more realistic representation of perennial production in the current model. The effects of including an annual crop and borrowing and saving under imperfect capital markets are then explored. Once the basic properties of the model are established, the model is used to investigate the effects of reduced water allocations on irrigators in the Riverland region of the MDB. This analysis allows for the derivation of agricultural water demand curves and the study of the additional impact of introducing stochastic water allocations.

The proposed MDB Plan does not address scientific findings indicating that climate change is likely to have a negative impact on water flows in the MDB. The official policy analysis only addresses long run average flows despite the fact that the rivers in the basin are among the most variable in the world. While others have made advances in addressing the potential impacts of water reform in the face of climate variability and climate change, the present study appears to be the only one to address such issues in the context of a fully dynamic model of perennial crop production. Although the emphasis in the current study on the vintage structure of perennial stocks introduces more realism than is commonly found in the literature, the model in its current form is limited in several respects and any quantitative results should be treated with caution. Perhaps the most obvious way in which the current model should be extended is by altering the crop production functions to allow for deficit irrigation. This fact combined with the absence of alternate crops, alternate water sources, and inter-regional water trade all tend to bias the results by limiting farmer flexibility. Hence, the quantitative findings presented here suggest an

upper bound on the likely impacts of water policy reform and stochastic water supply.

Aside from deficit irrigation, there are many other potentially fruitful extensions to the present study. The model could be readily integrated into a larger basin-wide hydro-economic optimization model. Including alternate water supplies such as groundwater and modelling water storage and carry-over would allow for more realistic adaptation to drought by perennial and annual crop producers and could allow for better estimates of agricultural water demand. While salinity is an important and ever-present concern in the MDB, it is not considered in the present model. Introducing salinity in a larger hydro-economic model with perennial production and groundwater would be very policy-relevant for the MDB and could also be adapted to analyze other regions where irrigated perennials and salinity are important such as California's Central Valley. In general, the model presented here could benefit from being incorporated into a more integrated hydro-economic model and, in turn, enrich the analysis from such a model.

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Appendix 1: Running Horizon Algorithm

Let us denote a time horizon which we are interested in optimizing over as $\mathbf{t} = \mathbf{1}, \mathbf{2}, ..., \mathbf{T}$. The previously described NLP model is defined over its own time horizon $\tau = \mathbf{1}, \mathbf{2}, ..., \mathbf{T(c)}$ which may be called *relative or artificial time*. Given the NLP model and an initial value of the variables for $t = 0$ denoted as $s_{k,0}$, we may solve the model repeatedly in a loop of length T using updated initial values each time. For the first iteration, the initial values of the NLP model are specified as $\hat{s}_{k,0}^{(1)} = s_{k,0}$ and after the first iteration the initial values for subsequent iterations are set to the value of the optimally chosen variables at $\tau = 1$ from the previous iteration of the NLP model. That is, if the optimal values of the variables for a given iteration t are denoted as $\hat{s}_{k,\tau}^{(t)}$ then we first make the assignment $s_{k,t}^* = \hat{s}_{k,1}^{(t)}$ and then update the initial value for the next iteration via the assignment $\hat{s}_{k,0}^{(t+1)} = s_{k,t}^*$. In this way, we may use the forward-looking NLP model to approximate the solution of an infinite horizon dynamic programming model.[6] The end result is a vector of variables corresponding to each period 1,2,..,T denoted as

$$< s_{k,1}^*, s_{k,2}^*, .., s_{k,T}^* > = < \hat{s}_{k,1}^{(1)}, \hat{s}_{k,1}^{(2)}, .., \hat{s}_{k,1}^{(T)} >, \forall \ k \text{ in } 1,2,..,K.$$

No-Ponzi-Game Conditions for RH Algorithm

Because the RH algorithm is an approximation to an infinite horizon DP algorithm it is not possible to implement the usual NPG conditions directly. Instead, the approach taken here it to have a traditional NPG condition for the last period of relative time (T(c)) and combine this with a check in the last period of the simulation model (T) to ensure that simulation does not allow debt to exceed the so-called natural debt limit which in this case is equal to the NPV of profits from perennial stocks at time T ($s_{k,T}$). The reason this is necessary is that because the model is effectively re-optimized after each period in the simulation it is possible for the economic agent to hold persistent debt so long as at the end of the planning horizon assets are non-negative. That is, for the planning horizon (relative, or artificial, time) we enforce

$$a_{T(c)} \geq 0 \tag{57}$$

and for the simulation in real time we check that the following natural debt limit condition holds:

$$a_T \geq \frac{-\sum_{n=1}^{K} \pi_{s,T+n}}{r(a_T)} \tag{58}$$

---

[6] Note that this approximation is possible due to the effect of discounting over a long time horizon; because periods in the distant horizon are so heavily discounted the effect of the terminal period on the first period decisions is negligible. However, a refinement of the algorithm that may merit inclusion in future research would be to assign a scrap value to the terminal period perennial stocks.

This condition means that debt in the last period must not be more than the net present value of profits from remaining perennial stocks discounted at the interest rate prevailing at that debt level. This ensures that if the representative farmer were to continue production over the economically productive life cycle of remaining perennial stocks he or she would be able to repay the debt plus interest at the rate prevailing in the last time period. Whereas many theoretical models disallow any debt in the last period, one may argue that allowing persistent, manageable levels of debt is much more realistic for models at any level of aggregation (consumer, household, industry, region, nation, etc.). In the case study under consideration here, persistent debt is certainly the norm for agricultural producers.

Appendix 2: Steady-State Derivations

The rest of the FOC's at the steady state are as follows:

$$\frac{1}{\alpha}(c^* - \underline{c})^{-\rho}(\gamma_w w_0 + \gamma_0) + \lambda_1^* + \lambda_2^* w_0 - \lambda_{6,0}^* = \alpha\frac{\partial V(f(a^*, s^*))}{\partial s_1^*} \tag{59}$$

$$p_x(x^*)y_x + p_x'(x^*)y_x x^* - \frac{1}{\alpha}(\gamma_w w_x + \gamma_x) = \frac{\lambda_1^* + \lambda_2^* w_x - \lambda_5^*}{(c^* - \underline{c})^{-\rho}} \tag{60}$$

$$\left.\begin{array}{c} p_s y_1 - \frac{1}{\alpha}(\gamma_w w_1 + \gamma_1 + \gamma_z) = \frac{\lambda_1^* + \lambda_2^* w_1 - \lambda_{3,1}^* + \lambda_{4,1}^*}{(c^* - \underline{c})^{-\rho}} \\ .. \\ p_s y_{K-1} - \frac{1}{\alpha}(\gamma_w w_{K-1} + \gamma_{K-1} + \gamma_z) = \frac{\lambda_1^* + \lambda_2^* w_{K-1} - \lambda_{3,K-1}^* + \lambda_{4,K-1}^*}{(c^* - \underline{c})^{-\rho}} \end{array}\right\} \quad \text{K-1 equations} \tag{61}$$

and the analog of the envelope theorem for perennial area at the steady state is given by the following

$$\frac{\partial V(\cdot)}{\partial s_1^*} = \sum_{n=1}^{k^*} \alpha^{n-1}\left[(c^* - \underline{c})^{-\rho}\pi_n - \lambda_1^* - \lambda_2^* w_n + \lambda_{4,n}^* + \lambda_{6,n}^*\right] \tag{62}$$

which substituted into (59) yields

$$\frac{1}{\alpha}(c^* - \underline{c})^{-\rho}(\gamma_w w_0 + \gamma_0) + \lambda_1^* + \lambda_2^* w_0 - \lambda_{6,0}^* = (c^* - \underline{c})^{-\rho}\sum_{n=1}^{k^*}\alpha^n\left[\pi_n - \lambda_1^* - \lambda_2^* w_n + \lambda_{4,n}^* + \lambda_{6,n}^*\right] \tag{63}$$

This simplifies to

$$c^* = \left(\frac{\sum_{n=1}^{k^*}\alpha^n\left[\pi_n - \lambda_1^* - \lambda_2^* w_n + \lambda_{4,n}^* + \lambda_{6,n}^*\right] - \frac{1}{\alpha}(\gamma_w w_0 + \gamma_0)}{\lambda_1^* + \lambda_2^* w_0 - \lambda_{6,0}^*}\right)^{\frac{1}{\rho}} + \underline{c} \tag{64}$$

which defines steady-state consumption as a function of the Lagrange multipliers and model parameters. Assuming a linear demand curve for annuals, (60) can also be solved for steady-state annuals crop area using the following:

$$p_x = \bar{p}_x + (x^* - \bar{x}) p_x'(x^*) \tag{65}$$

where $\bar{x}$ is the average area devoted to annuals, $\bar{p}_x$ is the average price of annuals, and $p_x'(x^*)$ is a scalar derived from the elasticity estimate. Using this, (60) simplifies to

$$x^* = 0.5 \left(\frac{1}{p_x'(x^*)} \left(\frac{\lambda_1^* + \lambda_2^* w_x - \lambda_5^*}{y_x(c^* - \underline{c})^{-\rho}} - \bar{p}_x + \frac{\gamma_w w_x + \gamma_x}{\alpha} \right) + \bar{x} \right) \tag{66}$$

and substituting in (64) gives

$$x^* = \frac{1}{2p_x'(x^*)} \left(\frac{(\lambda_1 + \lambda_2 w_x - \lambda_5) \left(\sum_{n=1}^{k^*} \alpha^n (\pi_n + \lambda_{6,n} - \lambda_1 + \lambda_{4,n} + \lambda_2 w_n) - \frac{\gamma_0 + w_0 \gamma_w}{\alpha} \right)}{y_x(\lambda_1 + \lambda_2 - \lambda_{6,0} w_0)} - \bar{p}_x + \frac{\gamma_w w_x + \gamma_x}{\alpha} \right) + \frac{\bar{x}}{2} \tag{67}$$

Let us now simplify the problem by assuming the land constraint is binding while the water constraint is not ($\lambda_1^* > 0$ and $\lambda_2^* = 0$) and furthermore that we do not have a corner solution in which either annuals or perennials are not cultivated. (The downward-sloping demand curve for annuals practically guarantees this in any case.) Then, given (48) and (49), the equivalent of the optimal stationary forest discussed in MRR adjusted for the alternate land use of annual production is the vector

$$s^* = \left\langle \frac{1 - x^*}{k^* + 1}, \frac{1 - x^*}{k^* + 1}, ..., \frac{1 - x^*}{k^* + 1}; 0, .., 0 \right\rangle \tag{68}$$

which implies that the vector of optimal removals is given by

$$z^* = \left\langle 0, .., 0, \frac{1 - x^*}{k^* + 1}, 0, .., 0 \right\rangle \tag{69}$$

where the only non-zero removal level occurs at k^*. For initial perennial stocks that include vintages older than k^*, convergence to the steady state would require that the oldest vintages, starting with k^*, be removed using the following rule:

$$z_t^* = \langle 0, .., 0; s_{k^*,t}, ..., s_{K,t} \rangle \tag{70}$$

Using (68) and (69), the steady-state profit equation is

$$\pi^* = \frac{1}{\alpha} \left(\frac{1 - x^*}{k^* + 1} \right) \left(\sum_{k=1}^{k^*} (\alpha p_s y_k - (\gamma_w w_k + \gamma_k)) - (\gamma_w w_0 + \gamma_0 + \gamma_z) \right) + (p_x(x^*) y_x - \frac{1}{\alpha} (\gamma_w w_x + \gamma_x)) x^* \tag{71}$$

which accounts for constant costs corresponding to equal plantings and removals each period.

The optimal rotation strategy has implications for the Lagrange multipliers. Because removals are all or nothing, exactly one of the two multipliers for removals ($\lambda_{3,k}^*$ and $\lambda_{4,k}^*$ $\forall 1 \leq k \leq K-1$) will be binding at the steady state. Furthermore, since the $\lambda_{6,k}^*$ terms correspond to the lower bound on the perennial area by age class, (68) implies that these must be equal to zero for all age classes included in the rotation and positive for older age classes. The two vectors of multipliers for removals can be written as

$$\lambda_3^* = \left\langle \lambda_{3,1}^*, .., \lambda_{3,k^*-1}^*, 0, \lambda_{3,k^*+1}^*, .., \lambda_{3,K-1}^* \right\rangle \tag{72}$$

$$\lambda_4^* = \left\langle 0, .., 0, \lambda_{4,k^*}^*, 0, .., 0 \right\rangle \tag{73}$$

and, using (43), the value of the multipliers can be expressed as

$$\lambda_{3,k}^* = \begin{cases} \lambda_1^* - (c^* - \underline{c})^{-\rho}(p_s y_k - \frac{1}{\alpha}(\gamma_w w_k + \gamma_k + \gamma_z)) & \forall k \neq k^* \\ 0 & k = k^* \end{cases} \tag{74}$$

$$\lambda_{4,k}^* = \begin{cases} 0 & \forall k \neq k^* \\ -\lambda_1^* + (c^* - \underline{c})^{-\rho}(p_s y_k - \frac{1}{\alpha}(\gamma_w w_k + \gamma_k + \gamma_z)) & k = k^* \end{cases} \tag{75}$$

while the vector of perennial stock multipliers is

$$\lambda_6^* = \left\langle 0, .., 0, \lambda_{6,k^*}^*, \lambda_{6,k^*+1}^*, .., \lambda_{6,K}^* \right\rangle \tag{76}$$

Given the restrictions on the multipliers, (64) and (67) can be re-written as

$$c^* = \left(\frac{\sum\limits_{n=1}^{k^*} \alpha^{n-1}\left[\pi_n - \lambda_1^* + \lambda_{4,n}^*\right] - \frac{1}{\alpha}(\gamma_w w_0 + \gamma_0)}{\lambda_1^*} \right)^{\frac{1}{\rho}} + \underline{c} \tag{77}$$

$$x^* = \frac{1}{2p_x'(x^*)} \left(\frac{\lambda_1 \left(\sum_{n=1}^{k^*} \alpha^{n-1}(\pi_n - \lambda_1 + \lambda_{4,n}) - \frac{\gamma_0 + w_0 \gamma_w}{\alpha} \right)}{y_x \lambda_1} - \bar{p}_x + \frac{\gamma_w w_x + \gamma_x}{\alpha} \right) + \frac{\bar{x}}{2} \tag{78}$$

While this description of the steady state is simplified, it still relies on the Lagrange multipliers for land and removals. It does not seem possible to eliminate them entirely and therefore it is not possible to give a simple numerical example here. However, we can look at the effect of perturbations of some key parameters at the steady state. For instance, area devoted to annuals displays the following properties

$$\frac{\partial x^*}{\partial p_s} = \frac{\sum_{n=1}^{k^*} \alpha^{n-1} y_n}{2p'_x(x^*)y_x} < 0 \tag{79}$$

$$\frac{\partial x^*}{\partial \bar{p_x}} = \frac{-1}{2p'_x(x^*)} > 0 \tag{80}$$

which indicates that area for annual crops will decrease in response to an increase in perennial prices and increase with respect to an outward shift in the demand curve. The effect of a change in the subjective discount factor $\left(\frac{\partial x^*}{\partial \alpha}\right)$ is a long expression that depends on the Lagrange multipliers but has a negative sign for any plausible values. Intuitively, this implies that if the farmer became more (less) patient, she will keep a longer (shorter) perennial rotation, thus devoting less (more) land to annuals.

Appendix 3: Additional Plots from the Computational Analysis

1,000 Year Simulations from Model I
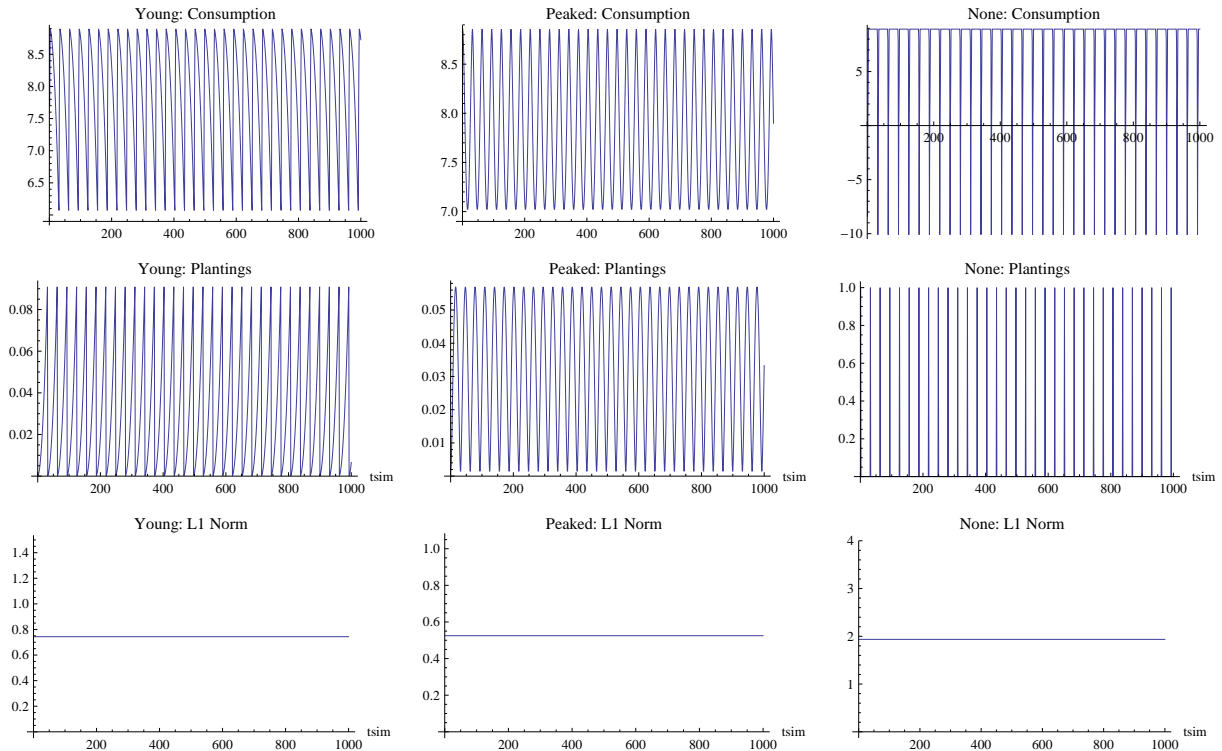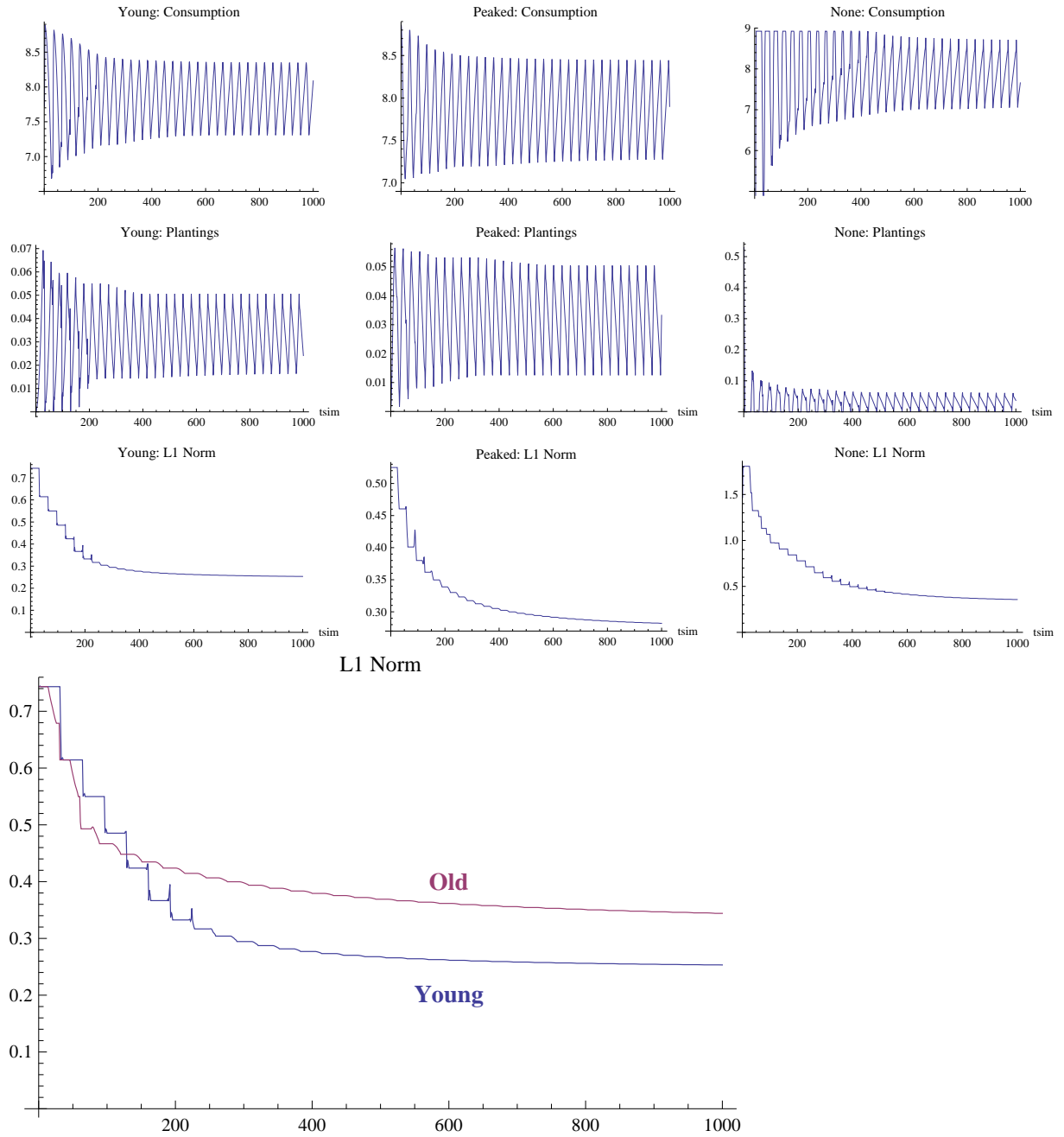
Appendix 4: Age Distribution Calculation

From the Phylloxera Board, we obtained data on total area planted and new plantings by year for 1996-2011 with partial data extending back to 1992. Using this data and assuming an initially even distribution across age classes up to 32 years old (the optimal removal age in my model), we attempted to derive an estimate of the age composition of vines using a simple mass balance equation:

$$s_t = s_{t-1} + s_{0,t} - z_t \tag{81}$$

where s_t is total vine area in year t, $s_{0,t}$ is new plantings, and z_t is total removals and all terms except removals are observed from 1996 onward. However, using this equation reveals serious questions about

the data as for numerous years this equation implies *negative* removals, which is logically impossible. After struggling to impose some consistency on the data we decided on a simple strategy for inferring the region's age composition. The steps are as follows:

- Assume the year 2011 total area data is correct.

- Assume all new plantings from previous years remain intact and constitute a fraction of the 2011 total.

- Subtract total new plantings from the previous step from total area in 2011 to determine total area of older vines planted in 1995 or earlier.

- Divide the old vine area equally across age classes 21-31.

- The resulting age distribution is then representative of the vine age composition as of the beginning of the 2012 growing year.

This is obviously a simplified picture of the true age composition given that removals regularly occur for younger age classes due to vine health problems or the need to change varieties due to market pressures. Also, there are clearly any number of plots with vines older than 32 years old. However, given the data available and the need to make some simplifying assumptions we believe that this estimated age distribution is adequate as it clearly captures the boom in new grape area during the period of 1996-2003. Furthermore, we know of no other estimated age distributions for the region.

Appendix 5: Monte Carlo Simulation

An analysis was conducted to characterize the variability of the time series and derive a distribution of water supply shocks for use in the dynamic simulations. After inspection of the auto-correlation and partial auto-correlation functions it was determined that the time series was an AR(1) process with no evidence of a unit root. A Jarque-Bera test was conducted, revealing that the residuals were most likely non-normal. We then fit a distribution to the residuals. To generate a synthetic time series with stochastic shocks drawn from the distribution of residuals, the following equation is used:

$$\tilde{d}_t = \hat{\beta}_0 + \hat{\beta}_1 d_{t-1} + \tilde{\epsilon}_t \tag{82}$$

where the β terms are the coefficients estimated in the regression and the term $\tilde{\epsilon}_t$ is a random shock drawn from the fitted distribution. The result is a time series which displays auto-correlation similar to the original time series. We then take a random draw from the values in the original time series and use it as the initial value in the equation above in combination with a random draw from the shock distribution. This gives a forecasted value with noise which is then used as the lagged term in the next

iteration of the equation. In this manner, we construct the data necessary for a Markov Chain Monte Carlo analysis. The data are then used to fix the water allocation in the NLP model, effectively making the water constraint stochastic.