

The Political Economy of Immigration and Population Ageing*

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Abstract

I propose a new theoretical framework to investigate the effects of population ageing on immigration policies. Voters' attitude towards immigrants depends on how the net gains from immigration are divided up in the society by the fiscal policy. In the theoretical literature this aspect is treated as exogenous to the political process because of technical constraints. This generates inconsistent predictions about the policy outcome. I propose a new equilibrium concept for voting models to analyse the endogenous relationship between immigration and fiscal policies and solve this apparent inconsistency. I show that the elderly and the poor have a common interest in limiting immigration and in increasing public spending. This exacerbates the effects of population ageing on public finances and results in a high tax burden on working age individuals and further worsens the age profile of the population. Moreover, I show that if the share of elderly population is sufficiently large, then a society is unambiguously harmed by the tightening in the immigration policy caused by the demographic change. The implications of the model are consistent with the patterns observed in UK attitudinal data and in line with the findings of the empirical literature about migration.

JEL classification: D72, C71, J610, H550.

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1 Introduction

What are the effects of population ageing on immigration policies? Do ageing societies tend to impose excessive restrictions on the inflow of foreign workers and if so, why? Should we expect an adjustment in immigration and spending policies to mitigate the impact of population ageing on public finances? This paper attempts to answer these questions using a theoretical model. In particular, I investigate why rapidly ageing countries - that arguably need more legal immigration - are imposing increasing restrictions to the inflow of immigrant workers and how this choice affects the tax burden faced by the working population. I also analyse the effects of these policy changes on the welfare of current and future generations. The importance of these questions is related to the vast fiscal effects of population ageing and immigration. The increase in longevity implies rising costs for the public sector, in particular the ones of public pensions and health care. The fall in the fertility rates causes an insufficient growth in the tax base. Both result in a pressure on public finances and tax rates. Several scholars and policy makers suggest that legal immigration can help in mitigating the effects of this problem, but this can happen only if there is political support for an increasingly open immigration policy. This analysis is therefore crucial to assess the fiscal soundness of ageing societies in the long run. Immigration also have demographic, social and cultural implications. Hence the study of immigration policies is also important to understand the evolution of the structure of our society in a broader sense.

1.1 Methods

In keeping with previous literature (Razin and Sadka, 1999), I analyse a political economy model with overlapping generations, in which voters differ in their income and in their age. In contrast with previous literature, however, I depart from a unidimensional policy space. Specifically, in each period the society chooses a two-dimensional policy consisting of an immigration quota and of the provision of an imperfect public good. The elderly receive an exogenous public pension that is financed by the tax revenues. The government budget is balanced, hence the political choice determines the tax rate on labour income. The bi-dimensionality of the policy allows one to model endogenously both the immigration policy and how the net fiscal benefits from immigration are divided up in the society. In detail, if immigrants generate a fiscal surplus, voters can employ it to increase public spending and/or to reduce taxes. The first choice mostly benefits the elderly and the low-income individuals, while the second favours the high earners. This implies that the way in which the net gains are divided up by the fiscal system is crucial to correctly assess the attitude towards immigration of different groups of voters. An endogenous analysis of both the immigration and the fiscal policy requires a bi-dimensional policy. Thus the standard tools in the Political Economy literature - based on unidimensionality - cannot be used to study this problem. This paper makes three key contributions towards addressing this problem. First, I propose a political process and a new equilibrium concept that are suitable for multidimensional problems. Second, I show that in this framework simple ordinal preference

restrictions are sufficient to deliver existence of equilibrium and sharp comparative static results on the policy outcome. Third, I apply this result to an overlapping generations model of immigration and public spending. This allows me to analyse how shocks on the longevity and on the fertility of the population affect immigration policy, public spending and the tax rate faced by the working population. The methodological contribution is based on a key restriction on the political process. Specifically, single politicians cannot commit to any platform other than their ideal policies, but they can form coalitions to enhance their ability to commit through internal agreements. Coalitions must be stable in equilibrium, in the sense that no subcoalition has a strict incentive to deviate and propose a different policy platform. Thus voters choose over a restricted set of platforms relative to standard voting models.

1.2 Summary of Results

I show that the elderly and the low income individuals have a common interest in reducing immigration and increasing public spending. Population ageing causes both an increase in the political power of these groups and a pressure on the government budget due to the rising cost of pensions. These two channels underpin the main results of this paper, which are as follows.

First, I show that, if the share of elderly is sufficiently large, a rise in the longevity and/or a fall in the natural growth rate of the population cause a tightening in the immigration policy and an increase in public spending. The reduced inflow of immigrant workers implies a reduction in the tax base. This, together with the rise in public spending in public goods and pensions, causes a sharp rise in the tax rate. Hence the political process tends to exacerbate the effects of population ageing on public finances.

Second, the effects of demographic shocks tend to worsen with time. In detail, a reduction in the immigration quota in the current period implies a change in the future age profile of the population because immigrants are mostly young and have weakly higher fertility rates relative to the natives. This causes further population ageing in the following periods and reinforces the effects.

Third, if the share of retired population is sufficiently large, then the tightening in the immigration policy generates a welfare loss for the society as a whole and harms the future generations.

These results suggest that ageing countries, that arguably need more immigration, tend to reduce it instead. This causes vast and persistent welfare and demographic effects and can affect the fiscal sustainability of the public sector in these countries.

1.3 Related Literature

Population ageing has been significant since the mid-twentieth century and it is expected to have dramatic demographic consequences in the next decades (see Figure 1). On one hand there are strong theoretical and empirical arguments in support of legal immigration as an instrument to ensure the financial soundness of a rapidly ageing society (Razin and Sadka,

1999 and Dustmann and Frattini, 2014). On the other hand the recent political debate in many countries is dominated by the discussion about how to limit the inflow of foreigners by introducing increasingly restrictive immigration policies. In many European countries this political agenda has led to a substantial tightening of immigration restrictions from 1994 (Boeri and Brucker, 2005) as shown in Figure 2. About the USA, Ortega and Peri (2009) provide evidence of an increase in the restrictiveness of immigration policy in the period 1994-2005. These trends in the implemented policies are consistent with a widespread and increasing aversion to immigration in those countries. Attitudinal data show that in the UK the share of citizens that would like immigration into their country to decrease has risen from 72.8% to a staggering 79.1% during the last 10 years (British Social Attitude Survey, 2003-2013). Moreover, the elderly are consistently more averse to immigration relative to the young. In the UK 85.7% of the individuals aged 60 or over would like less immigration while 71.2% of the individuals under 40 years old share the same opinion (British Social Attitude Survey, 2013). In the USA, the corresponding values are 47.3% and 39.2% (General Social Survey, 2014). These statistics suggest that population ageing may play an important role in the collective choice about immigration policies.

The empirical studies of the determinants of immigration policy are mostly based on attitudinal data and provide two main consistent facts that are relevant for this paper. The first fact is that age, education and income have a significant impact on the disapproval of further immigration and that in particular the elderly tend to have stronger preferences against further immigration in comparison with the young. Dustmann and Preston (2007), Facchini and Mayda (2007) and Card *et al.* (2011), using respectively data from the British Social Attitude Survey, the International Social Survey Programme and the European Social Survey, all support this finding. The latter paper also provides evidence that this result is mainly due to the perceived effect of immigration on the composition of the community in which the respondents live (or “compositional amenities”) and to its economic effects. The second important fact is that economic hostility to immigration is driven by concern about effects on public finances at least as much as by effects on labour market outcomes (Dustmann and Preston, 2006, 2007; Boeri, 2010). Consistently with this finding, Milner and Tingley (2009) show that public finance aspects play a major role in shaping the immigration policy in the US. This is somewhat surprising given that there is not convincing empirical evidence about negative net effects of legal immigration on public finances (Preston, 2014), and that on the contrary some studies suggest that legal immigrants may be net contributors to the fiscal system in several countries (Dustmann *et al.* 2010, Dustmann and Frattini 2014).

Lastly, the empirical literature about public spending provides an important result for this analysis. That is, population ageing affects fiscal policies in two key ways. On one hand there are direct effects - largely exogenous to the political process - due to changes in the cost of pensions, health care and education (Banks and Emmerson, 2003). On the other hand there is evidence that indirect political effects play an important role in shaping spending policies (Persson and Tabellini, 1999; Galasso and Profeta, 2004). Accounting for these two aspects is

crucial to understand how demographic shocks affect the tax rates.

These three empirical findings justify some of the modelling choices of this paper. In particular: (i) the choice of an overlapping generation model with a crucial role for the elderly in shaping the equilibrium policy, (ii) the main role played by the political determination of tax rates and public spending in shaping the attitudes of different individuals towards immigration, (iii) the explicit account for the “compositional amenities” in the preferences of native individuals, (iv) the inclusion of both exogenous and endogenous effects of ageing on the size of public spending.

The theoretical literature has analyzed the effects of population ageing on three political outcomes that are crucial for this paper, namely: (i) the immigration policies, (ii) the public spending policies, and how these two affect (iii) the tax policy (Razin and Sadka, 1999, 2000, 2002). The use of unidimensional models to study this problem (that is largely prevalent in the literature) has constrained the analysis to a unique endogenous outcome variable. The implication is that fiscal and immigration policies have been studied separately. This resulted in two complementary streams of literature whose key trade-offs are going to be relevant in the model proposed in this paper.

The first analyzes the political effects of ageing on public spending and intergenerational redistribution. Persson and Tabellini (1999) show that in a simple overlapping generation model the extent of intergenerational redistribution towards the elderly is increasing in the share of elderly population, and Tabellini (1990), Lindert (1996) and Perotti (1996) provide a partial empirical support to this hypothesis. Razin *et al.* (2002) propose a second channel: a larger share of elderly implies a higher tax burden on the median voter, because it corresponds to a lower share of taxpayers relative to the share of net benefit receivers. These two channels imply opposite effects of ageing on the level of public spending in equilibrium: the pro-tax coalition becomes larger but each taxpayer is relative less supportive of public spending.

The second stream of literature analyzes the determinants of immigration policy. If on one hand some papers focus on immigration policies related to the *quality* of immigrants, such as skill requirements (Benhabib, 1996 and Ortega, 2005), on the other hand the prevalent approach - of which this paper is an example - analyses policies that restrict the *number* of immigrants such as immigration quotas (see Preston, 2014 for a survey). These papers (Kemnitz, 2003; Krieger, 2003; Ben-Gad, 2012) emphasize the importance of intergenerational aspects such as the pension system and the investment in education in explaining the determinants of the political choice about immigration policies.

A crucial finding in this literature is that the unidimensionality assumption has important consequences on the predictive power of these models. In particular, it generates inconsistent predictions about the comparative statics of the outcome variable depending on the specific restrictions that are imposed in order to satisfy the required condition. An example of these paradoxical effects is described in Facchini and Mayda (2008, 2009) and Haupt and Peters (1998). They study a simple economy characterized by a linear income tax and assume that revenues

are lump-sum rebated to all citizen. In this setting one may choose to meet the requirement of unidimensionality by imposing the exogeneity of either (i) the level of public spending in benefits or of (ii) the income tax rate. These two assumptions corresponds respectively to the classes of (i) “Tax adjustment models” (TAM, e.g. Scholten and Thum, 1996) and “Benefit adjustment models” (BAM, e.g Razin and Sadka, 1999, 2000) and imply opposite predictions about the relationship between pre-tax income, age and attitude towards immigration (Figure 1-2-3-4). Specifically, the first model implies that the elderly and the low income individuals are more hostile to immigration than the young and high income, while the opposite is true in the second model. The intuition that underpins these two apparently contradictory results lies in the consequence of an increase in the legal inflow of immigrants. Consider for instance the case in which immigrants are net contributors to the fiscal system. If publicly provided benefits are set exogenously, then the effect of an increase in immigration is a fall in the tax rate. Conversely, if the exogenous variable is the tax rate, then the effect is a rise in public spending per capita. As a result, in the former case immigration benefits mostly the young and high income voters, while in the latter the elderly and the low income individuals enjoy the largest share of the gains. In a recent paper Preston (2014) clarifies that the source of this inconsistency lies in how the social gains generated by immigration are divided up among different groups. This division is an output of the political process, but existing models treat it as an input.

The issue is even more relevant for the purposes of this paper because I aim not only to understand the patterns of immigration policy but more generally to address how a democratic society responds to population ageing in terms of immigration and fiscal policy, and the overall consequences on the public finances. These questions can be addressed only in a framework that allows immigration, spending and tax policy to be endogenously determined.

The theoretical literature has recognized the crucial importance of multidimensionality of the policy space in order to study the determinants of immigration policies, but all the existing studies are based on unidimensional models because of technical reasons. The early papers by Plott (1967), Tullock (1967) and Devis *et al.* (1972) have established rather restrictive conditions for the existence of a *Condorcet Winner* - a platform that is preferred to any alternative by a majority of voters - if the policy space is multidimensional. Grandmont (1978) has elegantly generalized these conditions with the concept of *Intermediate Relations*. The use of Grandmont’s result in Political Economy applications is restricted to simple problems of redistribution (e.g. Borge, Rattsø, 2004) because of the extreme constraints that it imposes on preferences’ heterogeneity. These requirements are way too restrictive for applications in which different subgroups of the voting population (such as the working age and the retired individuals in this paper) have sufficiently heterogeneous preferences over the set of available policies¹.

Alternatives to unidimensional voting models are popular in the literature, but they are not generally useful to answer questions about the comparative statics of the equilibrium policy

¹In the supplementary material I provide an example of why the Grandmont conditions usually fail to apply in this framework, and in particular to the model that I present in section 3 of this paper.

outcomes because they do not deliver sharp analytical predictions about the policy response to a shock to the voters' distribution. This can be due either to a large multiplicity of equilibria, like in the Citizen-Candidate models (Besley and Coate, 1997) and in the Party Unanimity Nash Equilibrium (Roemer, 1999), or to the lack of analytical comparative statics results, like in Probabilistic Voting models (Lindbeck et al. 1987, Banks et al. 2003).

This paper is based on another stream of literature (Levy 2004, 2005) which exploits the role of coalitions and political parties in ensuring stability in a multidimensional deterministic voting model. I contribute to this literature by proposing a voting process that, under appropriate preferences restrictions, delivers sharp predictions about the equilibrium policy outcome, and it is therefore suitable to answer the questions of the paper.

1.4 Organization of the paper

The paper is organized as follows. In the next section I introduce the main model and an equilibrium concept that allows me to answer the questions. Section 3 presents the main results of the paper, which are stated in Theorems 7-8. In section 4 I propose four extensions of the basic framework and section 5 I analyze the welfare implications of the results in section 3. In section 6 I discuss some limitations of this work and future directions of research. Appendix A provides an analysis of the determinants of the attitude towards immigration in the UK based on the British Social Attitude Survey and show that they are consistent with the one implied by the model proposed in this paper.

2 A Political Model of Immigration and Spending Policy

This section is constituted by two parts. In the first I present the political process and the equilibrium concept. In the second I describe the economic model of immigration and public spending and I show that it satisfies the conditions required by the political process. These two theoretical tools are then used to derive the main results of this paper, which are stated in section 3.

2.1 The Political Process

I define a political process that translates individual preferences into a policy outcome x_t in each period t . The elements of the vector x_t represent the relevant policy outcomes, namely the immigration quota (M_t) and the uniform provision of an imperfect public good (Y_t). I adopt a political model of coalitions which extends the one introduced in a companion working paper

(Dotti 2015). It is a general tool with a potentially large range of applicability, some of which are mentioned in the concluding section of this paper. The closest example in the literature is in Levy (2004, 2005). A formal description of the equilibrium concept is provided in Appendix B.

The political process is based on the assumption that voters can form coalitions in order to enhance their capacity to influence the policy outcome. Each individual can be the member of only one coalition, thus a *coalition structure* is defined as a partition of the set of voters. As in Levy (2005), a coalition can only offer credible policies, that is, policies in the Pareto set of its members. Thus, when a voter runs as an individual candidate, he can only offer his ideal policy, as in the “citizen-candidate” model. On the other hand, when heterogeneous individuals join together in a coalition, their Pareto set is larger than the set of their ideal policies. This assumption captures the idea that within a coalition individuals can commit to policies that represents a compromise among the members, and that these internal agreements are credible for the voting population provided that not all the members have an incentive to renegotiate the terms of the deal.

Individuals play a two stage game: in the first stage they form coalitions in support of a certain proposed policy platform (or no policy) and in the second stage a voting game is played over the set of policies that are proposed by at least one coalition in the previous stage. Coalitions are required to be stable in equilibrium, in the sense that each coalition must possess at least one policy vector in its Pareto set such that - if the policy is proposed - there is no subcoalition that have a strict incentive to deviate and propose a different platform (named a *deviator* in this case)². If the deviation occurs the policy initially proposed by the coalition may become unfeasible. Therefore the profitability of a deviation depends on the behavior of the remaining part of the coalition that did not participate in the deviation. I assume that this subgroup responds to the deviation by proposing a policy (if any) that is capable of reducing the final payoff of some (or of all) the deviating players and therefore to prevent the deviation, and no policy if such platform does not exist. It can be shown that the main result of this section are robust to different assumptions about such behaviour (see Appendix B). Moreover, I assume that the profitability of a deviation is determined by the final outcome of the voting process³. I assume a tie-breaking rule for the case in which, given the other platforms that are offered in equilibrium, all members of a given coalition are indifferent between offering a platform and running at all. Specifically, I impose that in equilibrium a coalition facing such a situation does not propose any platform. The same restriction is assumed in Levy (2005) and it is justified if one considers some small costs of running for elections which are not explicitly assumed in the model.

If there is at least one policy in the Pareto set of a certain coalition that does not face any

²One can also allow for mergers between coalitions with no effects on the results in Theorems 1-2-3.

³ Alternatively one can assume that the equilibrium choices of other coalitions do not affect the behavior of potential deviators (in such case the stability is purely *internal* to the coalition), with no effects on the comparative statics results.

deviator, then this policy is *feasible* and the coalition is *stable*. A *stable coalition structure* is a partition of the set of voters in which all coalitions that are part of such partition are stable in the sense described above.

Before observing the coalition structure each coalition (including one-member coalitions) propose either a feasible policy platform or no policy. Then the coalition structure and the proposed platforms are observed by all the players. Voters (the whole population) vote one of the available policy platforms and the election's outcome is a weak *Condorcet Winner*, which I name a *winning policy*. If no policy is offered or no weak *Condorcet Winner* exists, a default policy is implemented which is worse for all players than any other outcome⁴. A set of platforms (named a *policy profile*), a stable coalition structure and a winning policy constitute a *coalitional equilibrium* of the game if one of the coalition is a (weak) *Condorcet Winner* of the voting game at the second stage (see appendix B.4. for a formal definition).

Notice that, differently from Levy (2005), I do not assume sincere voting: the existence of a *Condorcet Winner* at the second stage of the voting game implies a result that is robust to a fully sophisticated voting behavior and to a number of different voting protocols.

2.1.1 Preferences and Policy Space

The policy space X_t must be a subset of the the d -dimensional real space R^d with typical element x_t , such that the partially ordered set (X_t, \leq) is a convex and complete sublattice of R^d and can be different across different periods. In the two-dimensional application of this paper these requirements are satisfied if both the immigration policy M_t and the spending policy Y_t lie between zero and an upper bound, i.e. $0 \leq M_t \leq \bar{M}$ and $0 \leq Y_t \leq \bar{Y}$. A typical platform is given by $x_t = (x_{1t}, x_{2t})$ with $x_{1t} = M_t$ and $x_{2t} = -Y_t$.

Each individual i is endowed with a reflexive, complete and transitive preference ordering \succeq^i that can be represented by a x_t -continuous and θ_t -concave utility function⁵ $V : X_t \times \Theta_t \times \Phi \rightarrow R$, where $\theta_t^i \in \Theta_t$ is the parameter that identifies the preference of a voter i and the parameter space Θ_t is a totally ordered set. This is equivalent to say that all voters can be ordered along a single preference dimension. Lastly, $\varphi \in \Phi$ is a h -dimensional vector of parameters that do not differ across voters. Denote with \wedge and \vee the meet and joint operators in the lattice X_t . Lastly, suppose that individual preferences are such that the function V satisfies:

1. *Supermodularity (SM)* in x_t : $V(x_t' \vee x_t'', \theta_t, \varphi) - V(x_t', \theta_t, \varphi) \geq V(x_t'', \theta_t, \varphi) - V(x_t' \wedge x_t'', \theta_t, \varphi)$
for all $\theta_t \in \Theta_t$, for all $\varphi \in \Phi$ and for all $x_t', x_t'' \in X_t$.

⁴The comparative statics results apply even if the default policy is the platform implemented in the previous period, see Appendix C.

⁵For any function f defined on the convex subset X_t of R^d , we say that f is concave in direction $v \neq 0$ if, for all x , the map from the scalar s to $f(x + sv)$ is concave. (The domain of this map is taken to be the largest interval such that $x + sv$ lies in X_t .) We say that f is i -concave if it is concave in direction v for any $v > 0$ with $v_i = 0$. See Quah (2007).

2. *Strict Single Crossing Property (SSCP)* in (x_t, θ_t) : $V(x'_t, \bar{\theta}_t, \varphi) - V(x''_t, \bar{\theta}_t, \varphi) > V(x'_t, \underline{\theta}_t, \varphi) - V(x''_t, \underline{\theta}_t, \varphi)$ for all $x'_t, x''_t \in X_t$ such that $x'_t \geq x''_t$ and $x'_t \neq x''_t$, for all $\varphi \in \Phi$ and for all $\bar{\theta}_t, \underline{\theta}_t \in \Theta_t$ such that $\bar{\theta}_t > \underline{\theta}_t$.

These assumptions are common in many fields of Economic Theory. Notice that these properties are stated in a very general form, but in the case of a C^2 objective function one can simply use the sufficient conditions in Milgrom and Shannon (1994) in order to verify that the function satisfies *SM* and *SSCP*, namely $\frac{\partial^2 V}{\partial x_{it} \partial x_{jt}} \geq 0$, $\forall x_t \in X_t$, $\forall i \neq j$ and $\frac{\partial^2 V}{\partial x_{it} \partial \theta_t} > 0$ $\forall x_t \in X_t$, $\forall \theta_t \in \Theta_t$, $\forall i$. These sufficient conditions are usually easier to verify in comparison with the one implied by the definitions of *SM* and *SSCP*.

2.1.2 Equilibrium and Monotone Comparative Statics

Define the set of ideal policies $I_t(i) \equiv \{x_t | x_t \in \arg \max_{y \in X_t} V(y, \theta_t^i, \varphi)\}$,⁶ and the set of *equilibrium policies* as the union of all the policies that are winning policies in some coalitional equilibrium of the game. Notice that because Θ_t is a totally ordered set, one can identify a median element θ_t^v . The individual characterized by this value of the parameter is the *median voter* denoted by the index v .⁷

The main results that are relevant for this analysis are stated in the following theorems:

Theorem 1. (*Median Voter Theorem*). (i) *A coalitional equilibrium of the voting game exists.* (ii) *In any equilibrium the set of winning policies is a subset of the set of ideal points of the median voter v .* (iii) *If the median voter has a unique ideal policy, this policy is going to be the one chosen in any equilibrium.*

Proof. See Appendix C.1.

Theorem 2. (*Monotone Comparative Statics*). *The set of equilibrium policies of the voting game is (i) a sublattice of X_t which is (ii) monotonic nondecreasing in θ_t^v .*

Proof. See Appendix C.1.

Lastly, consider a totally ordered subset $\Phi' \subseteq \Phi$ and suppose that the objective function $V(x_t, \theta_t, \varphi)$ satisfies the *Single Crossing Property (SCP)* in (x_t, φ) , namely $V(x'_t, \theta_t, \bar{\varphi}) - V(x''_t, \theta_t, \bar{\varphi}) \geq V(x'_t, \theta_t, \underline{\varphi}) - V(x''_t, \theta_t, \underline{\varphi})$ for all $x'_t \geq x''_t$, and for all $\bar{\varphi}, \underline{\varphi} \in \Phi'$ such that $\bar{\varphi} \geq \underline{\varphi}$. Then I can state the following result:

⁶Notice that the completeness of X_t implies compactness in the order-interval topology. On bounded sets in R^d , the order-interval topology coincides with the Euclidean topology (Birkhoff 1967). Hence $I_t(i) \neq \emptyset$ for all i .

⁷In the case of a discrete even number of voters I assume that the ties are broken in favor of the individual with the lower index. Different assumptions would not affect the results in the next paragraphs.

Theorem 3. (*Monotone Comparative Statics 2*). *The set of equilibrium policies of the voting game is monotonic nondecreasing in φ .*

Proof. See Appendix C.1.

An interpretation of this generalized Median Voter Theorem is given in Appendix B. The results in this sections provide a tool to analyze the effects of a shock on the distribution of voters or on a preference parameter on the policy outcome that emerges in a political equilibrium. One only has to verify that an economic model satisfies the conditions stated in this section and then use Theorem 2-3 to formulate the predictions about the comparative statics of the platform that is implemented in equilibrium.

2.2 The Economic Environment

In this section I introduce an economic model of immigration and public spending in the spirit of the ones in the literature, in particular of Razin and Sadka (1999). Differently from the latter, I allow for the endogeneity of both the spending variable (an imperfect Public Good) and the immigration policy (in the form of a quota in each period t).

2.2.1 Demographic structure

Consider an overlapping generation model with three generations in each period t : the children (ch), the working age population (y) and the elderly (o). In each period only the native individuals of working age and the elderly (which include both the native and immigrants of the previous period) have voting rights (highlighted in capital letters in Fig. 7).

| time | $t - 1$ | | | t | | | $t + 1$ | |
|-----------------|-------------------|---------|---|-------------------|------|---|-------------------|------|
| born $t - 3$ | OLD | (o) | → | × | | | | |
| born $t - 2$ | NATIVE (n) | (y) | → | OLD | (o) | → | × | |
| | Immigrant (m) | | | | | | | |
| born $t - 1$ | Children | (ch) | → | NATIVE (n) | (y) | → | OLD | (o) |
| | | | | Immigrant (m) | | | | |
| born t | | | | Children | (ch) | → | NATIVE (n) | (y) |
| | | | | | | | Immigrant (m) | |
| born $t + 1$ | | | | | | | Children | (ch) |

Fig. 7. Structure of Overlapping Generations

Each period has length normalized to 1 and it is characterized by a native working age population of size n_t and a number of immigrants m_t in their working age. Natives and immigrants have potentially different exogenous expected fertility rates denoted by σ_t^n and σ_t^m respectively. An elderly individual at time t has life expectancy $l_{t-1} \leq 1$. At the end of each period immigrants and their children are fully assimilated to the native population in terms of costs and fertility behavior. The size of each part of the population is summarized in Fig. 8.

| $t - 1$ | | | t | | | $t + 1$ | |
|---|---------------|------|-----------------------------------|---------------|------|---|---------------|
| $l_{t-2}(n_{t-2} + m_{t-2})$ | (<i>o</i>) | → | × | | | | |
| $n_{t-1} + m_{t-1}$ | (<i>y</i>) | → | $l_{t-1}(n_{t-1} + m_{t-1})$ | (<i>o</i>) | → | × | |
| $\sigma_{t-1}^n n_{t-1} + \sigma_{t+1}^m m_{t-1}$ | (<i>ch</i>) | → | $n_t + m_t$ | (<i>y</i>) | → | $l_t(n_t + m_t)$ | (<i>o</i>) |
| | | born | $\sigma_t^n n_t + \sigma_t^m m_t$ | (<i>ch</i>) | → | $n_{t+1} + m_{t+1}$ | (<i>y</i>) |
| | | | | | born | $\sigma_{t+1}^n n_{t+1} + \sigma_{t+1}^m m_{t+1}$ | (<i>ch</i>) |

Fig. 8. Size of each generation

Denote with o_t the size of the elderly population, i.e. $o_t = l_{t-1}(n_{t-1} + m_{t-1})$. Notice that o_t is an increasing function of longevity. This assumption captures in a simple way the implications of a more realistic model⁸. Thus, the total number of individuals that possess voting rights at time t is $N_t = n_t + o_t$. Also notice that the way in which I define the size of different groups in the population implies a number of voters that is not necessarily a natural number, while in reality (and in the voting model proposed in the previous subsection) that must be the case. Given that the object of this study are policies that are typically decided at country level, and that the effects of this approximation tend to disappear as the number of individuals grows large, these assumptions are reasonable and commonly used in the literature (e.g. Razin and Sadka, 1999).

2.2.2 Individual preferences

An individual i of working age (y) at time t has preferences that are represented by a utility function whose arguments are consumption of private goods C_s and the imperfect Public Good Y_s , and the share of immigrants in the total population of working age M_s in the form:

$$U_t^{i,y} \left(C_t^{i,y}, C_{t+1}^{i,o}, M_t, M_{t+1}, Y_t, Y_{t+1} \right) = C_t^{i,y} + b(Y_t) - c(M_t) + \beta l_t \left[C_{t+1}^{i,o} + d(Y_{t+1}) - \hat{c}(M_{t+1}) \right]$$

where β is a parameter capturing how an individual discounts future utility. For retired individuals $U_t^{i,o}$ is constructed in a similar way, except that it only includes consumption and share of immigrants in the current period of life:

⁸In a continuous time model the number of elderly in each moment in time t is given by $\int_{s=1}^{1+l} n_{t-s}(s) + m_{t-s}(s) ds$ which is also linearly increasing in the longevity l and in the size of the oldest generation of elderly $n_{t-1-l}(1+l) + m_{t-1-l}(1+l)$.

$$U_t^{i,o} \left(C_t^{i,o}, M_t, Y_t \right) = l_{t-1} \left[C_t^{i,o} + d(Y_t) - \hat{c}(M_t) \right]$$

The functions d , c and \hat{c} are restricted to take only weakly positive values. Moreover, b and d are strictly concave while c and \hat{c} are strictly convex.

2.2.3 Production

Individual productivity is given by ϵ_t^i and has average $\bar{\epsilon}_t$. The distribution of ϵ_t^i is perfectly observed by all agents, it does not change over time and there is at least one native individual with $\epsilon_t^i = 0$. Immigrants have the same expected productivity as the natives. Individuals are endowed with 1 unit of time and their labour supply is perfectly inelastic. I assume a linear production function $F_t(L_t) = \xi_t L_t$ in which the total supply of effective labour is given by $L_t = (m_t + n_t)\bar{\epsilon}_t$. Perfect competition on the labour market implies a wage rate per unit of effective labour $w_t = \xi_t$. Therefore individual pre-tax income is given by:

$$y_t^i = w_t \epsilon_t^i$$

and has average \bar{y}_t . The assumption of inelastic labour supply simplifies the results and it is not crucial for driving the pay-offs of the model (in the additional material I show that the results are identical if all individuals have the same tax elasticity of labour supply). The assumption of a linear production function rules out the effects of changes in the aggregate labour supply on wages and it is common in the literature (e.g. Razin and Sadka, 2000). It is justified if one considers that in a more complex economy these effects tend to be offset by the adjustment of the capital stock of the economy - not explicitly assumed in this analysis - in the relatively long time framework of a generation, and this adjustment is particularly strong if firms have access to international capital markets (see Ben-Gad, 2012). In the additional material I show that the main results of this paper are mostly unaffected in the case of a strictly concave production function.

2.2.4 Public finances

The public sector raises revenues through a linear tax τ_t on labour income and spend them in the publicly provided good Y_t and in pensions for the elderly. In section 4.3 I introduce an extension of the model in which the government also provides public education. The government faces an exogenous amount of forgone tax revenues $\lambda_t = \lambda(w_t)$ per immigrant. This assumption captures the idea that the certain skills may be country-specific and therefore the immigrant may earn less than native individuals with similar productivity levels. Alternatively one can assume that immigrants and natives have different average productivities $\bar{\epsilon}_t^m, \bar{\epsilon}_t^n$, and λ_t to be a function

$\lambda(w_t, \bar{c}_t^m, \bar{c}_t^n)$ that captures the net forgone government revenue due to the difference in income⁹.

I assume a Pay-As-You-Go pension system (in section 4.1 I present an extension in which I allow for a partially funded system). The state pension paid to an individual i at time t is denoted by p_{t-1}^i and has average \bar{p}_{t-1} . It is promised to a working age individual at time $t-1$ and it is predetermined at time t . It is a constant flow, such that the total transfer is $l_{t-1}p_{t-1}^i$ (the flow amount times the time the pension is going to be paid for). It is a function of the relative income of the pensioner in the previous period y_{t-1}^i/\bar{y}_{t-1} and of the growth rate of working age population. At time $t-1$, when the promise is made, m_t is not yet determined, because it is a function of the immigration policy at time t . Thus the promised pension is a function of an exogenously fixed amount of immigrants \hat{m}_t (can be equal to zero). This assumption allows voters to ease the burden of pension on the working age population by choosing an immigration quota larger than \hat{m}_t . The assumptions on the pension system ensure that a certain positive amount of pensions is provided even if the pivotal voter typically prefers no pensions at all. Although not explicitly modeled in this paper, the assumption of an exogenous positive provision of public pensions in an overlapping generation model is justified in a game theoretical framework like the one in Rangel and Zeckhauser (2001). Lastly, two parameters $\alpha \geq 0$, $\gamma \geq 0$ determine the features of public pension system. In detail, the pension system can be either Beveridgean (if $\gamma = 0$), Bismarckian (if $\alpha = 0$) or a combination of the two. The state pension p_{t-1}^i is given by the formula:

$$p_{t-1}^i = \left(\alpha + \gamma \frac{y_{t-1}^i}{\bar{y}_{t-1}} \right) \frac{n_t + \hat{m}_t}{n_{t-1} + m_{t-1}} = \left(\alpha + \gamma \frac{y_{t-1}^i}{\bar{y}_{t-1}} \right) \frac{\bar{\sigma}_{t-1}}{(1 - \widehat{M}_t)}$$

where $\bar{\sigma}_{t-1} = \frac{n_t}{(m_{t-1} + n_{t-1})}$ is the natural growth factor of the working population between period $t-1$ and t and $\widehat{M}_t = \frac{\hat{m}_t}{n_t + \hat{m}_t}$ is the share of immigrants implied by the default level of immigration \hat{m}_t . Notice that if native and immigrants have different birth rates, i.e. $\sigma_t^n \neq \sigma_t^m$, then the natural growth rate of the population $\bar{\sigma}_t$ is itself endogenous in the immigration policy, and in particular: $\bar{\sigma}_t = \frac{\sigma_t^n n_t + \sigma_t^m m_t}{n_t + m_t} = \sigma_t^m M_t + \sigma_t^n (1 - M_t)$. Lastly, notice that the total cost of the pension system per taxpayer is decreasing in the number of immigrant workers that are allowed to enter the country in period t , while p_t^i is increasing in M_{t-1} if $\sigma_{t-1}^m > \sigma_{t-1}^n$.

I assume that the government budget is balanced in every period. The choice of not allowing for public debt simplifies the analysis and does not affect the trade-offs of the model. The government budget constraint ensure that the total public spending in public goods, pensions and the costs of immigration do not exceed the total tax revenue, and has form:

$$Y_t(m_t + n_t) + l_{t-1}\bar{p}_{t-1}(m_{t-1} + n_{t-1}) + \lambda_t m_t \leq \tau_t(m_t + n_t)\bar{y}_t$$

Assume that the governmental budget constraint is satisfied with equality (it must be true at any equilibrium of the voting game¹⁰). Using the formula for the pensions the governmental

⁹Notice that these two assumptions have consequences on the post-tax income of the immigrants

¹⁰In the case in which the pivotal voter is retired or has zero income one has to rule out Pareto inferior outcomes to ensure this result.

budget constraint can be rewritten as follows:

$$\tau_t = \tau(M_t, Y_t, \bar{y}_t) = \bar{y}_t^{-1} \left(\lambda_t M_t + (\alpha + \gamma) l_{t-1} \frac{(1 - M_t)}{(1 - \widehat{M}_t)} + Y_t \right)$$

Notice that this formulation implies that working age voters can ease the tax burden on their income by voting for a more open immigration policy, intuitively because as a consequence of an increase in the number of immigrants the expenditure in pensions is going to be shared among a larger working age population resulting in lower income taxes. I can use this formula to state the feasibility condition of the policy space:

$$0 \leq \tau_t(M_t, Y_t, \bar{y}_t) \leq k$$

for some $k < 1$. This restriction ensures that the implied tax rate on income will not exceed 1 or becomes negative. Notice that this restriction is crucial for the results in the next section to apply: if the tax rate hits the upper bound then the model and its predictions become similar to the ones of a standard Benefit Adjustment Model (See Appendix C.7).

Lastly, it is easy to show that the consumption of private goods of a young individual is given by post-tax income such that $C_t^{i,y} = (1 - \tau_t)y_t^i$ and the consumption of old people at time t depends only on the amount of pensions provided by the government, i.e. $C_t^{i,o} = p_{t-1}^i$.

2.2.5 Voters' objective function

Substituting the formulas for C_t^y and C_{t+1}^o into the utility function of a young voter one gets the indirect utility function $V_t^{i,y} = V^y(M_t, Y_t; y_t^i)$:

$$V_t^{i,y} = (1 - \tau_t)y_t^i + b(Y_t) - c(M_t) + \beta l_t \left[\left(\alpha + \gamma \frac{y_t^i}{\bar{y}_t} \right) \frac{\bar{\sigma}_t}{(1 - \widehat{M}_{t+1})} + d(Y_{t+1}) - c(M_{t+1}) \right]$$

The next step is to state the objective function of the elderly. Using the formula for $C_t^{i,o}$ into the utility function of an elderly voter I get $V_t^{i,o} = V^o(M_t, Y_t; y_{t-1}^i)$:

$$V_t^{i,o} = l_{t-1} \left[\left(\alpha + \gamma \frac{y_{t-1}^i}{\bar{y}_{t-1}} \right) \frac{\bar{\sigma}_{t-1}}{(1 - \widehat{M}_t)} + d(Y_t) - \hat{c}(M_t) \right]$$

The formula above delivers the main intuition that underpins the results in this paper. The key is to notice that retired individuals internalize (indirectly) the positive effects of immigration through the level of public spending in the imperfect Public Good, but given that the tax rate on income is also an endogenous variable in the model they always prefer, given a certain level of public spending, a policy that finances it with high taxes on the income of native workers rather than with a larger number of immigrants. This result follows from the fact that in this model the elderly dislike immigration as much as the young but, differently from the latter, they do

not internalize the negative effects of high taxes on the working age population. Hence the same preferences are represented by the function $V_t^o = d(Y_t) - \hat{c}(M_t)$ for all the elderly at time t . This objective function implies that the attitude of the elderly towards immigration is always more negative than the one of any working age individual even if immigrants are net contributors in financing the public spending of which the elderly are net beneficiaries. This is consistent with the empirical findings outlined in section 1 and it is crucial in order to understand the comparative statics of the equilibrium outcomes of the model that we will present in the next sections of this paper. Define $\theta_t^{i,y}$ as the ratio of i 's income to mean income at time t :

$$\theta_t^{i,y} = \frac{\epsilon_t^i}{\bar{\epsilon}_t} = \frac{y_t^i}{\bar{y}_t}$$

The preferences of each young native individual i are uniquely identified by the parameter $\theta_t^{i,y} \in \Theta_t^y$ with $\Theta_t^y = \{\theta_t^{1,y}, \theta_t^{2,y}, \dots, \theta_t^{n_t,y}\}$. Notice that the function $V_t^{i,y}$ can be written as function of one exogenous parameter $\theta_t^{i,y}$ and of the choice variables (M_t, Y_t) at time t plus the parameters $\varphi = (\{\alpha, \beta, \gamma, \sigma_s^n, \sigma_s^m, l_s\}_{s=0}^\infty)$. Moreover, the definition of $\theta_t^{i,y}$ implies that the cumulative distribution of $\theta_t^{i,y}$ is the same as the one of ϵ_t^i . I denote the c.d.f. of this distribution with Q . The value of y_{t-1}^i/\bar{y}_{t-1} does not affect the preferences of an elderly individual j over x_t , therefore all the elderly have the same preferences. This means that we can set a unique parameter $\theta_t^{j,o} = \theta_t^o \in \Theta_t^o$ which identifies the preferences of each elderly individual j at time t such that $\Theta_t^o = \{\theta_t^o, \theta_t^o, \dots, \theta_t^o\}$. I assign to all the elderly a parameter $\theta_t^o = -1$. I can now define the parameter set:

$$\Theta_t := \{\Theta_t^y \cup \Theta_t^o\}$$

which is a totally ordered set. In order to show that the preferences described in this section satisfy the conditions for the existence of a coalitional equilibrium I define a new objective function that includes both $V_t^{i,y}$ and $V_t^{i,o}$ and has the following form:

$$V_t^i = V(x_t; \theta_t^i, \varphi) = \begin{cases} V_t^{i,y} & \text{if } \text{age} = y \\ \kappa V_t^o & \text{if } \text{age} = o \end{cases}$$

With $x_{1t} = M_t$ and $x_{2t} = -Y_t$ and for an arbitrarily large $\kappa > 0$. Notice that κ represents a strictly increasing transformation of the original objective function of the elderly therefore κV_t^o implies the same preferences as V_t^o . Also notice that the state of the economy at the beginning of period $t+1$ is fully summarized by the share of elderly to young natives $g_{t+1} = o_{t+1}/n_{t+1}$, which is therefore the unique endogenous state in the dynamic process. Denote with $h_t = \{x_s, g_s\}_{s=0}^t$ the full history of policy choices and states observed by all agents up to time t , with $h_t \in H_t$. Denote with x_{t+1}^{**} the expected equilibrium policies at time $t+1$ given the information available at time t . I assume rational expectation, therefore, given the structure of information, the expectation is the same for all the voters. I also assume that such expectations only depend on the state of

the economy in that period, namely the endogenous state $g_{t+1}(x_t, \varphi)$ and the parameters φ^{11} , such that:

$$x_{t+1}^{**}(g_{t+1}, \varphi, h_t) = x_{t+1}^{**}(g_{t+1}, \varphi, h'_t)$$

for all histories $h_t, h'_t \in H_t$. Notice that in equilibrium the expectations must be correct, such that $x_{t+1}^{**}(g_{t+1}, \varphi, h_t) = x_{t+1}^*(g_{t+1}, \varphi, h'_t)$. This assumption - which implies that the endogenous state satisfies the Markov property - eliminates those equilibria in which different histories corresponds to different equilibrium choices even if the economic environment is identical. Notice that g_{t+1} is perfectly known at time t because there is no uncertainty about the distribution of future productivity¹². I can state the following Lemma:

Lemma 4. *In a coalitional equilibrium, conditional on g_t , (i) each individual's ideal policy x_t^i and (ii) the equilibrium policy x_t^* at time t are invariant to the equilibrium policy and to the value of the endogenous state at each time $t - s$, i.e. $x_t^i(g_t, \varphi, h_{t-1}) = x_t^i(g_t, \varphi, h'_{t-1})$ and $x_t^*(g_t, \varphi, h_{t-1}) = x_t^*(g_t, \varphi, h'_{t-1}) \forall t$ and $\forall h_{t-1}, h'_{t-1} \in H_{t-1}$.*

Proof. See Appendix C.3.

Summarizing, g_t is the unique endogenous state variable of this dynamic system and a coalitional equilibrium in this model (if it exists) is a temporary equilibrium that depends only on the value of the state variable g_t at time t and is independent of the previous history conditional on g_t . Notice that g_t is the ratio of elderly relative to native individuals of working age, and therefore it represents the crucial variable in order to determine the identity of the pivotal voter.

Lemma 4 allows one to disregard the effects of current policy choices (other than the effects on g_t) on future equilibrium outcomes when calculating the optimality conditions for each voter. This implies that future equilibrium policy outcomes affect the individual objective functions at time t only through their effects on g_{t+1} . The consequence is that I can rewrite a working age voter's objective function as follows:

$$V_t^{i,y} = (1 - \tau_t)\omega_t^i + b(Y_t) - c(M_t) +$$

$$\beta l_t \left[\frac{(\alpha + \gamma \theta_t^i) \bar{\sigma}_t}{(1 - \widehat{M}_{t+1})} + d(Y_{t+1}^{**}(g_{t+1}, \varphi)) - \hat{c}(M_{t+1}^{**}(g_{t+1}, \varphi)) \right]$$

where $Y_{t+1}^{**}(g_{t+1}, \varphi)$ and $M_{t+1}^{**}(g_{t+1}, \varphi)$ are the expected equilibrium policies at time $t + 1$ which are a function solely of g_{t+1} and φ . Notice that this objective function implies that an

¹¹The value of x_{t+1}^{**} is also a function of the distribution of productivity Q , but this is omitted in the formula.

¹² The result is the same if one allows for uncertainty and the size of the population is very large, because the law of large numbers implies that the identity of the median voter in the next period is known with probability equal to 1.

interior solution for the optimal policy of individual i with a partially open immigration policy $M_t > 0$ may exist even if immigrants “contribute less than what they take out” in the current period, or more precisely if at a given policy $(M_t, -Y_t)$ a marginal increase in the number of immigrants at constant Y_t implies, *ceteris paribus*, a rise in the income tax rate τ_t . This is true because if immigrants have higher fertility rates in comparison with the natives ($\sigma_t^m > \sigma_t^n$), then a native individual of working age will have a future benefit from immigration. Specifically, higher immigration today implies a lower dependency ratio tomorrow and, as a consequence, a more generous state pension system. This aspect of the model also solves the dichotomy between “skilled immigration” and “unskilled immigration” in the patterns of attitude towards immigrants and income that are described in the traditional models such as Facchini and Mayda (2008); in the model proposed here the attitude towards immigration may improve with income even if the immigrants are a net burden for the society in the short run, because preferences accounts for the future positive effect of immigration, and these future benefits are increasing with income if the Bismarckian component of the pension system is positive ($\gamma > 0$). Using the previously defined V_t^i function I can state the following result:

Lemma 5. *The strictly concave function $V(x_t; \theta_t^i, \varphi)$ satisfies SM and SSCP in $(x_t; \theta_t^i)$ for all $\theta_t^i \in \Theta_t$ and all $\varphi \in \Phi$.*

Proof. See Appendix C.4.

This lemma is crucial in order to establish existence of a coalitional equilibrium, and therefore to derive all the results in the next section of this paper.

3 Results

In this section I present the main results of the paper, namely the existence and characterization of the voting equilibrium, the analytical comparative statics results, the dynamics of the equilibrium outcome and the simulation of the other long-run implications of the model. Notice that all the results described in this section - except for the cases in which the opposite is explicitly stated - are also valid for the extended version of the model with endogenous public education presented in section 4.3. The proofs in Appendix C include both the basic model and the extended one (the objects that refer to the extended model are denoted with a tilda in the proofs).

3.1 Coalitional Equilibrium: Existence and Characterization.

Using the results in the previous sections we get that (i) the Policy Space (X_t, \leq) is a convex and complete sublattice of R^2 ; (ii) the parameter set Θ_t is a totally ordered set; (iii) the objective function $V(x_t; \theta_t^i, \varphi)$ satisfies Supermodularity and the Strict Single Crossing Property in $(x_t; \theta_t^i)$. Therefore all the conditions for the existence of Coalitional Equilibrium are satisfied. Moreover,

the objective function of each working age individual is strictly concave. I can state the following results:

Lemma 6. *(i) a coalitional equilibrium for the voting game exists. Moreover, (ii) in any coalitional equilibrium at time t the equilibrium policy is the unique ideal point of the median voter $x_t^v = x_t^* \in I_t(v)$. (iii) The parameter θ_t^v that identifies the median voter is weakly decreasing in g_t .*

Proof. See Appendix C.4.

Having established existence of an equilibrium and uniqueness of the policy outcome, I can use the result of the Monotone Comparative Statics of the equilibrium outcome in order to study the effects of shocks on the voters' distribution on the equilibrium policy outcome.

3.2 Main Result: Comparative Statics

In this section I analyse the short-run effects of shocks on the parameters that are related to population ageing on the equilibrium policy outcome. That is, how the equilibrium policy vector changes as a consequence of a shock - in the period in which the shock is observed - relative to the equilibrium level in absence of any shock. One has to account for four aspects: (i) how the direct preferences over policies of each native individual of working age are affected by the shock (“preference effect”), (ii) how the indirect preferences changes because of the effects of the shocks on the governmental budget constraint (“budget effect”) and (iii) how the identity of the pivotal voter changes as a consequence of the changes in the demographic composition of the population induced by the shock (“political effect”). Lastly, one has to keep into account the ability of a fully rational agent to anticipate that if $\sigma_t^m \neq \sigma_t^n$, then the choice of the immigration policy at time t affects the demographic structure of the voting population in the following periods and can therefore change the political equilibrium in the future. One may think that voters are unlikely to really anticipate this (iv) “sophisticated effect”, therefore whenever this aspect is relevant in this section I will distinguish between the predictions that emerge with “naive” agents - i.e. if voters expectations do not account for future political effects of current policies - and the ones implied by fully “sophisticated” agents. The approach used is the following. First I verify if there is any effects of type (i), (ii) and (iv). In detail, if V_t^i satisfies the condition of Theorem 3 for a given value of g_t , then the theorem can be used to establish the sign of these effects. Then I study the effects of type (iii). If g_t is affected by the shock, then Lemma 6 (iii) implies a change in the parameter that identifies the pivotal voter and therefore Theorem 2 can be used to formulate the predictions. The results about the tax rate τ_t stated in this section refer to the case in which immigrants provide, on average, a contribution to public finances sufficient to ensure that τ_t is weakly decreasing in M_t . This is true whenever the average cost per pensioner

is sufficiently large, namely if $l_{t-1}\bar{p}_{t-1} \geq \lambda_t$. The results about M_t and Y_t are valid even if the latter condition does not hold.

3.2.1 Unanticipated rise in the longevity of the retired population

I analyse the effects of an increase in l_{t-1} keeping other parameters constant.

For effects of type (i)-(ii)-(iv) one can verify that V_t^i satisfies the *SCP* by studying the cross derivatives of $V_t^{i,y}$ with respect to each policy dimension and l_{t-1} . Denote with $V_{M_t}^{i,y}$ ($V_{Y_t}^{i,y}$) the partial derivative of $V_t^{i,y}$ with respect to the policy dimension M_t (Y_t) and with $V_{M_t\varphi_j}^{i,y}$ ($V_{Y_t\varphi_j}^{i,y}$) the cross derivative of $V_t^{i,y}$ with respect to M_t (Y_t) and a parameter φ_j . In this case we have:

$$V_{M_t l_{t-1}}^{i,y} = \frac{\theta_t^i (\alpha + \gamma)}{(1 - \widehat{M}_t)} \geq 0$$

$$V_{Y_t l_{t-1}}^{i,y} = 0$$

Consider a vector of parameters $\tilde{\varphi} \in \Phi$. Define a subset $\Phi_j \subseteq \Phi$ as follows: $\Phi_j := \{\varphi \in \Phi | \varphi_k = \tilde{\varphi}_k \forall k \neq j\}$, where j is the position of the longevity of the elderly l_{t-1} at time t in vector φ . Notice that Φ_j is a totally ordered set. Moreover, the signs of the cross derivatives imply that $V_t(x_t; \theta_t^v, \varphi)$ satisfies *SM* and *SCP* in $(x_t; \varphi)$, it also satisfies *SM* and *SSCP* in $(z_t; \varphi)$ where $z_t = (x_{1t}, -x_{2t})$. The conditions of Theorem 3 are satisfied, therefore at constant g_t the effect is a weak rise in M_t and no changes in Y_t . Moreover, $V_{M_t}^{i,y}$ is solely affected through the budget constraint hence the effect is of type (ii) (“budget effect”).

For effects of type (iii) notice that $g_t = \frac{l_{t-1}}{\bar{\sigma}_{t-1}}$ is increasing in l_{t-1} . Lemma 6 (iii) implies that θ_t^v is decreasing in g_t . Hence Theorem 2 implies a weak increase in the public spending variable Y_t and a weakly more restrictive immigration policy M_t .

The total effect of an increase in l_{t-1} is therefore weakly positive on the public spending variables Y_t and ambiguous on the immigration policy M_t . There are cases in which one effects dominates and therefore the comparative statics result for the immigration policy is also sharp. In particular, I can state the following results:

Theorem 7. *The effect of an increase in the life expectancy l_{t-1} is weakly positive on the spending policy and ambiguous on the immigration policy. Moreover, there exists a threshold $\hat{g}_t \in [0, 1]$ such that if $g_t \geq \hat{g}_t$ then the effect on immigration policy is unambiguously (weakly) negative and the effect on the tax rate is strictly positive.*

Proof. See Appendix C.5.1

In order to get an intuition of what drives this results, consider the following cases. If $g_t = 1$, then the pivotal voter has $\theta_t^v = 0$, which implies that $V_{M_t l_{t-1}}^{v,y} = 0$. Thus, there is no “budget

effect” and the “political effect” weakly dominates. On the other hand, consider the case in which the variance of the income distribution is arbitrarily close to zero (e.g. $y_t^i = y_t$ for all i). In this case, as long as $g_t \neq 1$, the θ_t^v of the pivotal voter is unaffected by changes in the share of elderly, which implies that the “political effect” is zero and that the “budget effect” weakly dominates. Theorem 7 is a consequence of the negative relationship between age and attitude towards immigration and the positive one between age and attitude towards public spending implied by the model. This result suggests a link between the size of the two effects and two characteristics on the voting population: the share of elderly and the degree of income inequality. Moreover, it implies that the sign of the effect of an increase in longevity on the equilibrium level of the immigration quota is the one implied by the Tax Adjustment Model if the share of elderly is large enough and there is sufficient income inequality, and the one implied by the Benefit Adjustment Model in societies characterized by opposite features.

3.2.2 Unanticipated fall in the natural growth rate of the working age population

The natural growth rate of the native population is $\frac{n_t}{n_{t-1}+m_{t-1}} - 1 = \bar{\sigma}_{t-1} - 1$. In this model the effect of an unanticipated fall in such rate has same sign as the one of a decrease in the lagged birth rate of the natives σ_{t-1}^n . This is true because one can show that $\bar{\sigma}_{t-1} = \sigma_{t-1}^n M_{t-1} + \sigma_{t-1}^n (1 - M_{t-1})$, which implies that $\bar{\sigma}_{t-1}$ is predetermined at time t . This kind of shock corresponds for instance to the case in which the birth rate actually experienced during the period $t - 1$ is smaller than the one expected at the beginning of that period. Therefore I analyse the effects of a shock on σ_{t-1}^n . I can state the following:

Theorem 8. *The effect of a decrease in the growth rate of the working age population is a weak decrease in the openness of the immigration policy and a weak increase in spending in the imperfect Public Good and in the tax rate.*

Proof. See Appendix C.5.2

Notice that conditional on g_t the shock has no effect on the equilibrium policy outcome (i.e. there are no effects of type (i), (ii), (iv)). The reason is that the pension system adjusts its size to changes in the birth rate for the reasons described in section 2. Nevertheless a fall in σ_{t-1}^n implies a rise in g_t , which corresponds to a “political effect”. Using Theorem 2 one gets the result stated above.

3.2.3 Rise in life expectancy of the working age population

I analyse the effects of a shock on the life expectancy of the current working age population l_t . First of all notice that g_t is unaffected by changes in l_t , which means that there is no “political

effect”. The results of this paragraph are summarized in Theorem 9.

Theorem 9. *The effect of an increase in the life expectancy l_t is ambiguous on the immigration policy. If voters are “naive” then the effect is weakly positive. If the birth rate of the native is the same as the one of the immigrants, then there is no effect.*

Proof. See Appendix C.5.3

In order to understand this result it is useful to analyze the cross derivative of $V_t^{i,y}$ with respect to the immigration policy M_t and the parameter l_t .

$$V_{M_t l_t}^{i,y} = \underbrace{\frac{\beta(\alpha + \gamma\theta_t^i)}{(1 - \widehat{M}_{t+1})}(\sigma_t^m - \sigma_t^n)}_{\text{preferences effect}} - \underbrace{\frac{d}{dl_t} \left\{ \frac{\beta l_t^2}{\bar{\sigma}_t^2} \left[d'(Y_{t+1}^{**}) \frac{dY_{t+1}^{**}}{d\theta_{t+1}^v} - \check{c}'(M_{t+1}^{**}) \frac{dM_{t+1}^{**}}{d\theta_{t+1}^v} \right] \frac{d\theta_{t+1}^v}{dg_{t+1}} (\sigma_t^m - \sigma_t^n) \right\}}_{\text{sophisticated effect}}$$

First of all notice that if $\sigma_t^m = \sigma_t^n$, then the cross derivatives are equal to zero and g_{t+1} is unaffected by changes in l_t , therefore a shock on l_t has no effects on the equilibrium outcome.

If $\sigma_t^m \geq \sigma_t^n$ the sign of $V_{M_t l_t}^{i,y}$ is ambiguous. The reason is that two different effects enter the formula. On one hand a rise in the life expectancy makes consumption after retirement more attractive. This increases the desirability of better future pensions and therefore implies a more favorable attitude towards immigration (“preferences effect”). On the other hand more immigration today reduces the value of g_{t+1} . This changes the expected equilibrium policy in the next period in a way that harms a retired individual (“sophisticated effect”). In particular, a decrease in g_{t+1} causes a weak rise in M_{t+1}^{**} and a weak fall in Y_{t+1}^{**} , because of the future political effect. Which of the two effects dominates depends on many aspects, including the income distribution at time $t + 1$ and the values of g_t and g_{t+1} . In particular notice that if the variance of the income distribution of the working age population tends to zero, then $\frac{d\theta_{t+1}^v}{dg_{t+1}} = 0$ and therefore the “preferences effect” dominates. Finally, if agents are “naive” then there is no “sophisticated effect” and therefore an increase in l_t has a weakly positive effect on the openness of the immigration policy.

3.2.4 Fall in the birth rate of the native population

I analyse the effects of a fall in σ_t^n . The results are summarized in the following theorem.

Theorem 10. *The effect of a decrease in the birth rate of the native population σ_t^n is ambiguous on the immigration policy and on the tax rate. If voters are “naive”, then the effect is weakly positive on the immigration policy and weakly negative on the tax rate.*

Proof. See Appendix C.5.4

Similarly to the previous case, the presence of a “sophisticated effect” and of a “preferences effect” that can have opposite sign implies that the sign of the comparative statics is ambiguous.

If voters are “naive”, then the preferences effect implies a weakly less restrictive immigration policy M_t . Moreover if the immigrants are net contributors to the fiscal system, this also implies a weak fall in the tax rate τ_t . The intuition is that a fall in the birth rate of the natives implies a stronger positive impact of immigration of future pensions and no fiscal effects in the short run.

Theorem 10 implies that a fall in the birth rate can have positive effects on public finances and cause a fall in the tax rate because of an increasingly liberal immigration policy in the short run. If this result may seem paradoxical, section 3.4 clarifies that this effect is true only in the current period, while in the long run a fall in the birth rate may have strong negative effects on public finances and tax rates.

3.2.5 Shocks on the income distribution of the working age population

Given the state g_t , a shock on the income distribution of the working age population affects the equilibrium outcome if and only if it implies a change in the pivotal voter θ_t^v . If this is the case, it represents a shock of type (iii), if it is not, it has no effects. For instance a shock that results in a median preserving spread of such distribution does not imply any change in the identity of the median voter and therefore it does not affect the policy outcome. I can state the following result.

Theorem 11. *An increase in the median to mean income ratio implies in equilibrium (i) a weak increase in the openness of the immigration policy M_t and (ii) a weak decrease in the public spending in the imperfect Public Good Y_t . Moreover, (iii) if the immigrants are net contributors to the fiscal system then it also implies a weak fall in the tax rate τ_t .*

Proof. Results (i), (ii), follows directly from Theorem 3. Result (iii) follows directly from the governmental budget constraint and results (i), (ii). Q.E.D.

Corollary 12. *The equilibrium levels of Y_t and M_t respond in opposite directions to shocks to the voters’ distribution.*

Proof. Straightforward from Theorem 11.

Notice that this results implies a positive correlation between the tightness of the immigration policy and the spending in the imperfect public good. This suggests that the concerns about the relationship between an open immigration policy and cuts to public benefits, which are documented in all attitudinal studies, may have some ground in the observed policy outcomes even if immigrants are net contributors to the tax system.

3.3 Steady-state Equilibrium

I define a long-run equilibrium of the overlapping generation model as a sequence of coalitional equilibria. An equilibrium at time t is a steady state if, in absence of shocks on the parameters,

we have $x_s^* = x_s^{**} = x^{ss}$ and $g_s = g^{ss}$ for all $s \geq t$, where the superscript ss denote the steady state value of a state or a control variable. In other words, in a steady state the equilibrium policy and the natural growth rate of the population are constant over time. Recall that g_t is the only states that evolves endogenously in the dynamic system and that the temporary coalitional equilibrium is unique (conditional on g_t) in each period. Therefore in order to show that the economy is at a steady state it is sufficient to show that $g_s = g^{ss}$ for all $s \geq t$. Uniqueness also implies that if $g_t = g_{t+1}$ at t then in absence of shocks it must be true that $g_{t+s} = g_t = g^{ss}$ for all $s > t$. Hence if $g_t = g_{t+1}$ then the economy is at a steady state.

Lemma 13. *An equilibrium for the OGM exists and the economy always converges to a steady-state. Moreover, if $\sigma_t^m = \sigma_t^n = \sigma_t$ then the political equilibrium at time t is independent of the previous political choices and the economy converges immediately to the steady state after a shock.*

Proof. See Appendix C.5.6.

3.4 Dynamics

The analysis of the dynamics of the OGM is a complex exercise because of the number of different short-run effects described in the previous sections. There are anyways interesting results that can be stated about the long-run effects of shocks in this framework. In particular, I present two analytical results: (i) the long-run effects of an unanticipated permanent shock on the longevity of the elderly l_{t-1} and/or on the natural growth rate of the native population $\bar{\sigma}_{t-1}$ on the sequence of political equilibria from the period after the shock until the economy converges to a new steady state (keeping other parameters constant); (ii) the long-run effects of an unanticipated permanent shock in the life expectancy (l_t) and/or on the expected birth rate of the native population (σ_t^n) in the case in which immigration does not cause changes in the age profile of the society (i.e. $\sigma_t^m - \sigma_t^n \leq \eta$ for arbitrarily small η). For the other cases which I cannot address analytically I propose a simulation in section 3.5 which show that the results are not qualitatively different from the one presented in the following paragraphs.

3.4.1 Long-run effects of a permanent shock on the longevity of the retired population and on the natural growth rate of the working age population

The sign of the long-run effects of a positive shock on the longevity l_{t-1} or on the natural growth rate of working age population $\bar{\sigma}_{t-1} - 1$ at time t depend on the ambiguous short-run effects on the immigration policy stated in Theorem 7-8. In order to address the effects at period $t+1$ and the following ones it is sufficient to notice that, given that the shock is permanent, the collective

choice problem at time $t + 1$ is identical to the one at time t except for the value of g . Thus, all the changes in the equilibrium choices at time $t + 1$ must be due to the evolution of the endogenous state g_{t+1} . In particular notice that g_{t+1} is strictly decreasing in M_t , and therefore if $M_t \geq M_{t-1}$ ($M_t \leq M_{t-1}$) then $g_{t+1} \leq g_t$ ($g_{t+1} \geq g_t$). Lemma 6 (iii) ensures that the parameter that identifies the pivotal voter changes accordingly $\theta_{t+1}^v \geq \theta_t^v$ ($\theta_{t+1}^v \leq \theta_t^v$). Therefore I can state the following results.

Theorem 7b. *The long-run effect of an increase in l_{t-1} on the immigration policy has same sign as the short-run effect and a weakly larger magnitude. If $g_t \geq \hat{g}_t$ then the effect on immigration policy is (weakly) negative and the effect on the public spending and the tax rate is strictly positive.*

Proof. See Appendix C.5.5.

Similarly one can analyse the long-run effects of a fall in the natural growth rate of the native population of working age (or equivalently of σ_{t-1}^n , see section 3.2.2). The result is the following.

Theorem 8b. *The long-run effect of a decrease in the natural growth rate of the native population is a weak decrease in the openness of the immigration policy and a weak increase in spending in the imperfect Public Good and in the tax rate. All the effects have weakly larger magnitude relative to the short-run effects.*

Proof. See Appendix C.5.5.

These results imply that the effects of population ageing are persistent and tend to increase in magnitude in the periods after the shock. The reason is that - if immigrants have higher fertility rate relative to the natives - then a change in the size of the immigration flow affects the distribution of voters in the following periods. In particular, a more restrictive immigration policy in the current period implies further population ageing in the future and therefore an increase in magnitude of the initial effects.

3.4.2 Long-run effects of a permanent rise in life expectancy

I analyse the long run effects of changes in l_t . This shock generates a number of effects that affect the temporary equilibrium as described in the previous sections. Moreover, the specific path of policies depends on the timing of the different shocks (for instance shocks on l_t and l_{t-1} may occur simultaneously). I study the case in which σ_t^m is arbitrarily close to σ_t^n and the shock is permanent, i.e. l_{t+s} is equal to the new value of l_t for all $s > t$. This case is simple to analyze because the long run effects at time t and $t + 1$ after the shock correspond, respectively, to the temporary effects of a rise in l_t and l_{t-1} described in the previous sections. Moreover, given that I assume $\sigma_t^m - \sigma_t^n$ to be arbitrarily small, then the “preference effect” and the “sophisticated

effect” can be disregarded and the economy converges to the new steady state one period after the shock. Under the proposed restrictions I can state a sharper result:

Theorem 14. *The long-run effects of an increase in the life expectancy is a weak rise in public spending and, if $g_t \geq \hat{g}_t$, a weak decrease in the openness of the immigration policy.*

Proof. In the case of $\sigma_t^m - \sigma_t^n \leq \eta$ the sign of the long-run effect corresponds to the short-run effect of an increase in l_{t-1} . Q.E.D.

3.4.3 Long-run effects of a permanent fall in the birth rate of the natives

I study the long-run effects of a fall in σ_t^n , in the case in which $\sigma_t^m - \sigma_t^n$ is arbitrarily small and the shock is permanent, i.e. the rise of σ_t^n implies that σ_{t+s}^n will be equal to the new value of σ_t^n for all $s > t$.

Theorem 15. *The long-run effects of a decrease in the birth rate of the native population is a weak rise in public spending. The effect on the openness of the immigration policy is ambiguous at time t and weakly negative in the following periods. If voters are “naive” the effect on the openness of the immigration policy is weakly positive at time t and weakly negative in the following periods.*

Proof. In the case of $\sigma_t^m - \sigma_t^n \leq \eta$ the long-run effect corresponds to the short-run effect of an decrease in σ_t followed by a decrease in σ_{t-1} . Q.E.D.

The results in this section suggest that if immigrants are not too different from the natives in terms of fertility rates, then the long run effects of population ageing follow the patterns of the corresponding short-run effects.

3.5 Simulation

Some interesting cases cannot be fully described analytical, in particular the long-run effects of permanent shocks on the parameters in the case in which the birth rate of immigrant is different from the one of the natives. In order to study these cases I run a simulation of the model whose results are extensively presented in the supplementary material of this paper. This exercise shows that the effects due to the sophistication of voters may be substantial in terms of levels of the equilibrium policy, but they do not generally imply qualitatively different predictions about the shape of the curves describing the policy response to shocks on the parameters. I find that for several different parametrizations - if the difference in the birth rate of immigrants and natives

$(\sigma_t^m - \sigma_t^n)$ is not too large¹³ - then the predictions of Theorems 14 and 15 are valid even if voters are “sophisticated” (Figures 9-10-11-12). Figures 13 and 14 show the response of the immigration policy M_t and of the spending policy Y_t to a permanent rise in the life expectancy of the retired population, both for the case of “naive” voters (dashed lines) and of sophisticated voters (solid lines). Although the shape of the two lines is very similar, the equilibrium level is different. Sophisticated individuals fully internalize the effect of current immigration on the composition of the society in the following period. In particular, they anticipate that more immigration in the current period would imply a higher share of young individuals in the next period, and therefore an equilibrium policy that is less favorable to them when they will be retired. Therefore the equilibrium with “sophisticated” voters features a more restrictive immigration policy and a higher public spending in comparison with the case of “naive” voters.

The simulation exercise can also help to understand the factors that determine the speed of convergence to the steady state after a shock. The crucial aspect is that the speed is decreasing in the size of the “sophisticated effect”, specifically in the value of $\sigma_t^m - \sigma_t^n$. Figures 15-16 show the path of convergence of the immigration policy M_t after a positive (solid line) and a negative (dashed line) shock on the endogenous state g_t , in the case of high difference (Fig. 15, $\sigma_t^m - \sigma_t^n = 1$) and low difference (Fig. 16, $\sigma_t^m - \sigma_t^n = 0.2$) in the birth rates of immigrants and natives.

This exercise suggests that the key results in the previous sections still apply even to the cases in which the long run effects of shocks in the model cannot be characterized analytically. Thus, I can conclude that - in a society characterized by a very large share of retired individuals - population ageing leads to a policy that is closer to the needs of the elderly. In particular, high public spending and increasingly restrictive immigration policies are going to be implemented. These policy changes imply an increasing tax burden on the working age generation and may affect the fiscal sustainability of public spending in the long run.

4 Extensions

In this section I propose three extensions in which I address if and how the equilibrium political choices differ from the one presented in section 3 in the case in which one introduces alternative forms of public intervention in social spending and a different legal status of the immigrants. Specifically, I analyse the implication of the model if (i) the pension system is partially funded, if (ii) immigrants do not acquire voting rights and if (iii) the government provides public education. On one hand the main comparative statics results of this paper remain generally valid. On the other hand these exercises deliver some understanding of how different rules in the public sector may affect the attitude towards immigration of the voting population. Lastly, in section 4.4, I

¹³For large value of $\sigma_t^m - \sigma_t^n$ the steady-state may not be unique and a shock may cause a transition to a different equilibrium path.

describe an extension of the model in which the labour market is segmented. In particular, I study the case in which the elderly demand specific services, such as home care, and only immigrants possess the skills to provide such services. In this case the results may differ substantially from the ones of the baseline model.

4.1 Partially funded pension system

The assumption of a pure Pay-As-You-Go pension system is a very stylized description of how the social security for the elderly is organized in most developed countries. In particular partially funded pension schemes are becoming increasingly common. There is empirical evidence of an increasing importance of the funded part of the pension relative to the “state pension” in European countries (Galasso and Profeta, 2004). The theoretical analysis proposed by Rangel and Zeckhauser (2001) suggests that this phenomenon may also be related to the increase in the number of elderly relative to the working age population.

In the model proposed in this paper I did not explicitly account for savings. One simple possibility is to model the funded part of the pension system as a form of compulsory savings. Under this assumption each individual has to save an amount $\psi(\rho)s(y_t^i)$ when young and she will receive $(1+r)\psi(\alpha+\gamma)s(y_t^i)$ when retired, where r is the exogenous interest rate and ψ is a strictly decreasing function. The total pension received by i at time $t+1$ becomes:

$$p_t^i = \left(\alpha + \gamma \frac{y_t^i}{\bar{y}_t} \right) \frac{\bar{\sigma}_t}{(1 - \widehat{M}_{t+1})} + (1+r)\psi(\alpha+\gamma)s(y_t^i)$$

This formulation implies that if the state-pension component falls (e.g. if γ decreases) then the funded pension part rises. Notice that - because the utility function is linear in consumption - the size of the compulsory saving does not affect the preferences of voters over policies. Thus, the effect of a marginal transition towards a fully funded pension system simply corresponds to the effect of a fall in the Beveridgean part of the state pension α or of a fall in the Bismarckian part γ (or both). Hence I can state the following result:

Theorem 16. *The effect of a marginal decrease in the size of the public pension system in the short run is an increase in the restrictiveness of the immigration policy. In the long run the increase in restrictions to immigration and an increase in public spending in the imperfect Public Good. The total effect on the tax rate is ambiguous.*

Proof. See Appendix C.5.6.

The intuition of this result is simple. If the share of the Pay-As-You-Go component of the pension system decreases in favor of a fully funded scheme, then the fiscal gains from immigration for a worker decrease because the size of pension costs to be shared among the working age

population is smaller. Moreover, after retirement gains also decrease, because the soundness of the public finances has a lower impact on the overall pension received. Therefore all voters become more averse to immigration and ask for a more restrictive policy. In the long run, if the immigrants have higher fertility rates relative to the natives, this political choice causes an increase in the share of elderly individuals with consequences that are similar to the ones described in section 3.4.1 for the case of increasing life expectancy, namely a further tightening in the immigration policy and an increase in the endogenous part of public spending.

An important aspect of this analysis is that if the size of the state pension system becomes too small (e.g. small $\alpha + \gamma$) then the total gains from immigration for a working age individuals may become negative, which implies an equilibrium in which the most restrictive immigration policy is implemented.

4.2 Voting Rights: *Ius Soli* vs. *Ius Sanguinis*

So far I have assumed that the children of immigrants that are born in the guesting country are awarded the voting right when they become adults. Moreover, in the model voting rights can be also acquired after a sufficiently long period of legal residency. These assumptions are consistent with the legal procedures to obtain citizenship - and consequently voting rights - in several countries such as the US, Canada and France (*Ius Soli*). In many other countries - such as the UK, Japan, Germany and Italy - the legal requirements are often quite different and they do not typically imply an automatic award of the citizenship based of the place of birth only. The most common case is that at least one of the parents must possess the citizenship in order for the children to obtain the same status (*Ius Sanguinis*). It is out of the scope of this paper to formulate assumptions that would correctly describe the law of different countries. Nevertheless, in order to understand the possible effects of different legal requirements, it is useful to analyze the consequences of the opposite assumption in comparison with the one in section 3.2 of this paper. Namely, assume an extreme form of *Ius Sanguinis* in which neither the immigrants nor their children ever obtain the nationality. This assumption is clearly extreme it only serves as a term of comparison.

The main implications of the model are unaffected by this modification, except that for one aspect. Specifically, immigrants and their children do not become members of the voting population at any point in time. Therefore the choice of the immigration policy does not affect the future composition of the voting population. This implies that there is no “sophisticated effect” in this case, and therefore some of the results in section 4 are sharper. Namely for any σ_t^n, σ_t^m such that $\sigma_t^m \geq \sigma_t^n$ one gets:

Theorem 17. (i) the short-run effect of a rise in l_t is unambiguously (weakly) positive on the immigration policy M_t and weakly negative on the tax rate τ_t ; the long-run effects of (ii) an

increase in the life expectancy and of (iii) a decrease in the birth rate of the native population is a weak rise in public spending and, if $g_t \geq \hat{g}$, a weak increase in the openness of the immigration policy at time t followed by a weak fall in the following periods.

Proof. The relevant variable for determining the pivotal voter is in this case $\tilde{g}_t = \frac{l_{t-1}\tilde{n}_{t-1}}{\tilde{n}_t} = \frac{l_{t-1}}{\sigma_{t-1}^n}$ where $\tilde{n}_{t-1} \leq n_{t-1}$ is the number of individuals that possess voting rights at time $t-1$ and it is smaller or equal to the number of individuals that are born in the country. Notice that \tilde{g}_t is independent of M_{t-1} . The rest of the analysis is unaffected. All the proofs are identical to the ones for Theorems 9-10 except that no ‘‘sophisticated effect’’ occurs.

Theorem 17 suggests that the analytical predictions of the model are not strongly affected by the cross-country differences in the law that regulates the acquisition of the citizenship, and that on the contrary some results tend to become sharper and more general in the case of an extreme version of the *Ius Sanguinis*.

4.3 Endogenous public education

I analyse an extension of the model in which the income of an individual depends not only on the wage rate and on her productivity, but also on the amount of education she received when she was a child. I assume that education is uniformly provided by the government and has decreasing returns given by the strictly concave function f . Individual fertility of natives is given by the random variable k_t^i that is i.i.d. and with $E[k_t^i] = \sigma_t^n$. I get that the income of an individual i at time t can be written as follows:

$$y_t^i = f(e_{t-1})w_t\epsilon_t^i$$

and the total supply of effective labour at time t becomes $L_t = f(e_{t-1})\bar{\epsilon}_t(n_t + m_t)$. The budget constraint accounts for the public spending in education, such that the formula for the tax rate on labour income becomes:

$$\tau_t = \tau(e_t, M_t, Y_t, \bar{y}_t) = \bar{y}_t^{-1} \left[\bar{\sigma}_t e_t + \lambda_t M_t + (\alpha + \gamma) l_{t-1} \frac{(1 - M_t)}{(1 - \widehat{M}_t)} + Y_t \right]$$

Moreover, I assume that working age individuals (retired individuals) care about the utility of their children (grandchildren) such that the utility function of an individual of generation a can be written as follows:

$$\tilde{U}_t^{i,a} = U_t^{i,a} \left(\{C_s^i, M_s, Y_s\}_{s=t}^{t+1} \right) + \delta^a E \left[k_t^i U_t^{j,y} \left(\{C_s^j, M_s, Y_s\}_{s=t+1}^{t+2} \right) \right]$$

Lastly, I assume that the number of voters is large, such that the uncertainty about the size of the future generation does not affect the result. Notice that given the assumptions about k_t^i the preferences shown above can also represent individuals that care about the *next generation*

rather than about their children and grandchildren. The structure of the overlapping generations model in the same as in the baseline model, except for the presence of an additional endogenous state e_{t-1} which affects the average income at time t . A coalitional equilibrium exists and most results of this augmented model about the comparative statics of shock on life expectancy are the same as the ones described in the previous section. (See Appendix C).

The interesting aspect of this analysis is the counterintuitive effect of population ageing on public investment in education (per child). These effects are stated in the following theorem.

Theorem 18. *The effects of an increase in the longevity of the retired population l_{t-1} and /or of a decrease in the growth rate of native population σ_{t-1}^n is a weak increase in the public spending in education per child e_t .*

Proof. See Appendix C.5.6.

The intuition that underpins this result is that if an elderly individual cares about her grandchildren (i.e. $\delta^o > 0$), then she will always support any policy that increases the spending in education through a rise in the taxes on the working age population because she is not affected by this rise in the tax rate.

The consequence of Theorem 18 is that the next generation may enjoy a better education and a higher pre-tax income as a consequence of population ageing. But the change in welfare is not necessarily positive for these individuals. The negative side for future generations may come from the results in Theorems 7 and 8, which hold also in the augmented model (see Appendix C). In particular if $\sigma_t^m > \sigma_t^n$ and at period t a more restrictive immigration policy is implemented, then the future generations may have to face an society with a larger share of elderly which implies, *ceteris paribus*, higher tax rates on labour income and more public spending, which can be harmful for the most productive individuals of the new generation. The second result is the following.

Theorem 19. *If voters are “naive” and $\frac{l_t p_{t+1}^v}{e_t} \geq \frac{\theta_t^v}{\beta}$, then the effects of a decrease in the birth rate of the native population σ_t^n is a weak fall in the public spending in education per child e_t and a weak increase in the openness of the immigration policy. Else both effect have an ambiguous sign.*

Proof. See Appendix C.5.3.

Theorem 19 suggests that the cost of public education may play a role in shaping the effects shocks on fertility rates on the immigration policy. On one hand immigration tends to reduce the pressure of the pensions system on public finances, on the other hand it causes an increase in the total costs of public education. If the latter effect is sufficiently strong, the predictions are going to be different from the one implied by the baseline model. This is particularly relevant if one considers that several countries are implementing reforms in order to reduce the Pay-As-You-Go share of the pensions received by the elderly in favor of a fully funded system (see section 4.1).

Nevertheless the public expenditures for the elderly represents a large share of the governmental budget in most western countries and, more importantly, they consistently exceed the ones in education and childcare (OECD 2015, 2015b), therefore the assumption $\frac{l_t p_{t+1}^v}{e_t} \geq \frac{\theta_t^v}{\beta}$ appears to be the one that better describes the facts in most OECD countries.

4.4 Services for the Elderly (“Elderly Goods”)

In this section I analyse an extension of the model in which the labour market is segmented. In particular, I study the case in which immigrants possess the skills to provide those services that are needed only by the elderly, such as home care, while the natives workers do not. This may be the case if immigrants are selected by the firms in the receiving country on the basis of their qualifications and previous work experience. Suppose that the elderly consume a different private good denoted by O_t . This good is produced with the same technology as the consumption good C_t and the imperfect public good Y_t , but only the immigrant workers are capable of producing it. Immigrants can also be employed in the production of the other goods. For simplicity I assume that there is no difference in the average tax payments of immigrants and natives, i.e. $\lambda_t = 0$, that the default immigration is $\widehat{M}_t = 0$ and I analyse the case in which $\sigma_t^m - \sigma_t^n$ is arbitrarily small. There are two possibilities. If at the equilibrium there are enough immigrant workers to satisfy the demand for “elderly goods” at a sufficiently low price, then the segmentation of the labour market is irrelevant and the results are identical to the baseline model. The perfect substitutability in production and the perfect competition ensure that all prices are unaffected by immigration choices. The implications change dramatically if in the proximity of an equilibrium there are not enough immigrant workers to satisfy the demand for the “elderly good” at the constant price¹⁴. I can state the following result.

Theorem 20. *If $g_t \leq 1$ then the equilibrium immigration policy is $M_t = 0$, else a positive level of immigration is possible.*

Proof. See supplementary material, SM.5.2.

This result implies that, as long as the majority of voters is of working age, the society always chooses the most restrictive immigration policy. Moreover, a shock on the longevity or the fertility of the native population does not affect the immigration policy in equilibrium. The channel that underpins this result is the effect of immigration on equilibrium prices. Specifically, immigrants are endogenously hired in the sector that produces the “elderly good” O_t , but they consume only the other two goods C_t and Y_t . As a result, immigration in equilibrium implies a rise in the relative prices faced by the young natives, offsetting the fiscal benefits generated by immigrants and making working age voters extremely hostile to immigration. The conclusion one can derive from this section is that some implications of the analysis presented in section 3 of this paper are true for this extended case only if in the proximity of the equilibrium the immigration policy

¹⁴Notice that multiplicity of equilibria is possible in this case.

is not too restrictive. If the number of immigrants is too low to satisfy the demand of services for the elderly, then some predictions in section 3 of the paper are no longer valid. The result in this case is somewhat paradoxical: a society that is in great need of immigrants to satisfy the demand of services for the elderly tend to be very averse to any positive level of immigration of specialized workers. Additional details and results about this extension of the model are available in the supplementary material.

5 Welfare analysis

In the previous section I have proved that a rise in the longevity or a fall in the birth rate of the native population generates a political pressure towards more restrictive immigration policy. This does not necessarily imply that this change is desirable on the point of view of the society as a whole. In this section I present a welfare analysis which shows that, if a society has certain demographic characteristics, a marginal increase in the restrictions to immigration is unambiguously harmful for the society.

I define a measure of the wellbeing of the society in the form of a *Social Welfare Function* (*SWF*). The idea that is exploited in this section is that if at an equilibrium point the marginal effect of an increase in a policy dimension x_{jt} on the *SWF* is greater than than the one of the median voter (and at the equilibrium $x_{jt}^* < \bar{x}_{jt}$), then there exists a policy with $x'_{jt} > x_{jt}^*$ which is welfare improving. This implies in turn that if as a consequence of a shock a certain policy dimension j is such that $x_{jt-1}^* > x_{jt}^*$, then x_{jt} has moved in the “wrong direction” on a social welfare point of view and that the society would benefit, *ceteris paribus*, from a marginal change in the direction of x_{jt-1}^* . In other words, the society is harmed by the change in policy at the margin. Consider a *SWF* that is a weighted average of the utility of each individuals of the working age generation (y), of the retired generation (o) at time t and the expected future utility of the children (ch), where $\mu_t^a(\theta_s^i)$ represents the Pareto weight assigned to an individual i of generation a at time t . Notice that I am not ruling out either the possibility that the *SWF* attributes zero weight to the immigrants or the possibility that some or all the immigrants have positive weight¹⁵. The *SWF* has form:

$$SWF(M_t, Y_t; \varphi) = \int_0^{\bar{\theta}_t} \mu_t^y(\theta_t^i) V^y(M_t, Y_t; \theta_t^i, \varphi) q(\theta_t^i) d\theta_t^i + \\ + \int_0^{\bar{\theta}_{t-1}} \mu_t^o(\theta_{t-1}^i) V^o(M_t, Y_t; \theta_{t-1}^i, \varphi) q(\theta_{t-1}^i) d\theta_{t-1}^i + \int_0^{\bar{\theta}_{t+1}} \mu_{t+1}^y(\theta_{t+1}^i) E_t \left[V^y(M_{t+1}^{**}, Y_{t+1}^{**}; \theta_{t+1}^i, \varphi) \right] q(\theta_{t+1}^i) d\theta_{t+1}^i$$

Most welfare implications of this analysis depend on the Pareto weights assigned to each individual in the *SWF*. For instance, some results that can be obtained using a specific *SWF* (e.g. Utilitarian or Rawlsian) are presented in the supplementary material. Nevertheless an interesting general result can be stated under relative weak restrictions on the *SWF*. Specifically, I analyse

¹⁵One has to specify the objective function of an immigrant in this case.

the welfare effects of changes in the immigration policy keeping the other policy dimension constant at the equilibrium level. This analysis is also consistent with the extended model presented in section 4.3.

5.1 Welfare effects of a marginal opening in the Immigration Policy

Assume that $c'(M_t) < \infty$ for all $x_t \in X_t$ and that at the equilibrium $0 < M_t < \bar{M}_t$, i.e. the solution is internal for the immigration policy. Then I can state the following result.

Theorem 21. *For any Social Welfare Function $SWF(M_t, Y_t; \varphi)$ that assigns a strictly positive weight to each native individual of working age, there exist a threshold $\check{g}_t \in [0, 1]$ such that if $g_t \geq \check{g}_t$ then a marginal tightening in the immigration policy caused by a change in the equilibrium outcome reduces the Social Welfare.*

Proof. See Appendix C.6.

The intuition that underpins this result is that as the parameter θ_t^v that identifies the pivotal voter get close to 0, the benefits for the other individuals of working age from a marginal opening of the immigration policy increase at an increasing rate. Also notice that the converse of the statement in Theorem 21 is not always true. This means that a threshold $\check{g}_t \in [0, 1]$ such that if $g_t \leq \check{g}_t$ then the society would benefit from a marginally more restrictive immigration policy may not exist for all the SWF s with the features stated above. Nevertheless, the statement is true for Utilitarian and Rawlsian SWF .

The result in Theorem 21 suggests that societies characterized by high income inequality and/or by a high share of elderly on the total population (which have a g_t close to 1 or larger) are likely to adopt excessively restrictive immigration policies. Moreover, it implies that a tightening in the immigration law - for instance the one caused by population ageing - reduces the Social Welfare. In other words, the policy adjustment of the immigration quota is harmful for the society. This result is suggestive in the light of the increasingly and rather controversial restrictions to immigrations that have been progressively introduced in countries characterized by a rapidly ageing population and by a high degree of income inequality, such as the UK and the USA, or in countries that feature by a very large elderly population, such as Japan or Italy.

In the supplementary material I propose a welfare analysis about the effects of a change in the public spending in the imperfect Public Good and in education. These results are less general because they rely on more restrictive assumptions about the SWF (e.g. Utilitarianism). Nevertheless, they suggest that the allocation of public spending may be too generous for the imperfect Public Good and perhaps insufficient for education in society characterized by high income inequality and by a large share of elderly.

6 Concluding Remarks

This paper investigates the interaction between two crucial demographic, economic and social processes in our society - ageing and immigration - and how these two shape - and are themselves shaped to some extent - by the political choices of the affected countries. In particular, I study the effects on immigration policies of two major demographic changes that have caused population ageing in western societies, namely increasing life expectancy and decreasing birth rates. The main finding concerns the fiscal consequences of population ageing. That is, if the share of elderly population is large enough, population ageing increases the political pressure to restrict the inflow of immigrant workers into the country and to rise public spending. This result implies that the negative effects of population ageing on public finances - due to increasing costs for public pensions - may be exacerbated by the endogenous political effects on immigration and public spending policies. Direct and indirect effects of the ageing phenomenon may affect the overall fiscal soundness of the public sector in the long run. The second result looks at the demographic consequences of ageing. In particular, I show that the effects of a demographic shock on the age profile of the population tend to worsen with time because of the endogenous political effects on the immigration policies. The third finding is about social welfare. I show that the changes in the immigration policy induced by population ageing tend to harm the society, in particular the young individuals and future generations.

One element that emerges from this analysis is that the way in which costs and benefits generated by immigration are divided up in the society is crucial to determine the attitudes towards immigration of different demographic groups. This implies that an analysis of the political processes that lead to the division of these net gains is essential in order to assess the political effects of ageing on immigration policies. Thus, the study of the latter cannot abstract from how fiscal policies are determined.

There are anyway some limitations in this analysis that one has to consider. First, in this study the endogenous adjustment of wages has no effect on the equilibrium policy choices. This is due to the assumption that the individual labour supply is perfectly inelastic both at the extensive and at the intensive margin. This modelling choice is justified by theoretical (Ben-Gad, 2004) and empirical considerations (Dustmann and Preston, 2006, 2007; Boeri, 2010) and can be relaxed to some extent (see additional material). Nevertheless this aspect is likely to play a role in shaping immigration policies. Thus, this is a topic that calls for further research. Secondly, I do not fully investigate the effects of the heterogeneity in the productivity of immigrants. This aspect may be particularly relevant given that this heterogeneity may be - at least to some extent - endogenous in the political process. For instance simple theoretical models suggest that countries with a generous welfare systems may attract relatively low skilled immigrants (Borjas, 1999) and that the attitude towards different types of immigration may vary with the composition of skills of the native population (Benhabib, 1996). Even if the empirical literature provide limited support for these channels (see Preston, 2014), they represents important elements to enrich the study

of the determinants of immigration policy. Finally, a deeper analysis of the determinants of the aversion to immigration due to concerns related to the effects on the “compositional amenities” of the society is needed in order to better understand what really shapes immigration policies. This aspect has been shown to play a major role in attitudinal studies (Card *et al.*, 2011) and it is an active field of research in other disciplines (see Brettell and Hollifield, 2007), but it has not been sufficiently analyzed with the tools of economic theory.

A more general remark should be made about the model of political interaction and the equilibrium concept used in this paper. This framework represents a tool that does not only serves for the purposes of this analysis, but it is sufficiently general to be used in many other applications in Political Economy. In the companion paper (Dotti, 2015) I show how this theoretical framework can be useful to analyze problems of redistribution in the spirit of Meltzer and Richards (1981) and to explain how the controversial predictions about the relationship between income inequality and size of the government that are typical of that literature are simply a consequence of the unidimensionality restriction. Opposite implications - more consistent with the empirical evidence - can be derived if a slightly richer policy space is assumed. There are many other questions in Political Economy for which the multidimensionality of the policy space represents a major obstacle in the analysis, and that therefore represent a promising field of application for the voting model presented in this paper. An example is the literature about the political determination of tax rates on different types of income (e.g. labour vs capital income). The intrinsic bi-dimensional nature of the problem have often constrained the analysis to a very restricted policy space (e.g. Benhabib and Bassetto, 1999). Moreover, the choice of an equilibrium concept that is essentially based on the very flexible theoretical framework of the Generalized Monotone Comparative Statics (Milgrom and Shannon, 1994; Quah, 2007) implies that the range of applicability of this tool may be potentially further extended to analyze a wider range of Political Economy questions (see Dotti, 2015).

Lastly, I emphasize that this analysis delivers an essentially pessimistic message about the evolution of our society in the immediate future and its consequences for the young generations. If population ageing means an increasing power for the elderly to shape public policies according to their needs, the main victims of this process are going to be the young, both the ones born in rich countries and the ones native of poorer regions. On one hand the former will have to support the fiscal burden of an increasingly large and long-living elderly population through high tax rates on their income. On the other hand the latter are going to be prevented from searching for better employment opportunities by the excessively restrictive immigration policies that are going to be implemented in the high income countries.

Appendix

A Facts (British Social Attitudes Survey)

In this section I investigate the determinants of the attitudes towards immigration and public spending of adult residents in Great Britain using data from the British Social Attitude Survey, and in particular from the rounds of data 2009 - 2011 - 2013 that includes a specific section about immigration. The dataset accounts for a total of 6639 observations. The explanatory variables are the age of the respondent, the income decile of the household and the highest educational qualification attained by the respondent, on a scale from 1 (postgraduate degree) to 8 (no qualification). Observations of individuals with foreign qualifications have been omitted. Dummy variables capture whether the household includes children, and if the respondent is a woman, if she lives in rural areas, if she is born abroad and if she is not part of any religion. Characteristics related to the employment status and type are captured by dummies. In particular, I include the effects of being employed in a manual job, unemployed or retired.

A.1 Determinants of Attitude towards Immigration

The outcome variable LETIN captures the attitude towards further immigration in the country. The question is “Do you think the number of immigrants to Britain nowadays should be increased a lot, increased a little, remain the same as it is, reduced a little or reduced a lot?” and the respondents must choose a value on a discrete scale from 1 (“increased a lot”) to 5 (“reduced a lot”). The variable LETIN measures therefore the degree of aversion towards further immigration. I use an ordered Logit model because of the discrete and ordered nature of the outcome variable. Table 1 presents the results of this analysis. In line with what observed in the literature and with what is implied by the model proposed in this paper, the age of the respondent has a significant effect on the hostility towards immigration. Similarly low level of education tend to be associated with a stronger aversion to immigrants. Notice that even if the parameter on household income is not significant, the strong positive correlation between income and education imply (unconditional of the education attainment) a negative relationship between income and hostility towards immigration, as implied by the model. Finally the presence of children in the household, the location in a urban area and the birth of the respondent outside of the UK are all significantly related to a more positive attitude towards immigrants.

| Ordered Logit - Dependent Variable: LETIN | | | | | |
|---|-------------------------|-------------------------|------------------------|-------------------------|-------------------------|
| VARIABLES | (1) Robust SE | (2) Cluster SE | (3) Robust SE | (4) Cluster SE | (5) Cluster SE |
| Age | 0.00949*** (0.00196) | 0.00949*** (0.00202) | 0.0117*** (0.00269) | 0.0117*** (0.00310) | |
| Income | -0.0258** (0.0114) | -0.0258*** (0.00917) | -0.0235* (0.0120) | -0.0235*** (0.00655) | -0.0241*** (0.00884) |
| LowEdu | 0.248*** (0.0177) | 0.248*** (0.0206) | 0.238*** (0.0185) | 0.238*** (0.0220) | 0.257*** (0.0195) |
| BornAbroad | -0.790*** (0.154) | -0.790*** (0.162) | -0.768*** (0.157) | -0.768*** (0.169) | -0.806*** (0.164) |
| Children | -0.267*** (0.0650) | -0.267*** (0.0822) | -0.247*** (0.0668) | -0.247*** (0.0888) | -0.225*** (0.0785) |
| Rural | 0.172*** (0.0323) | 0.172*** (0.0624) | 0.167*** (0.0324) | 0.167*** (0.0620) | 0.182*** (0.0615) |
| Sex | | | 0.0778 (0.0625) | 0.0778 (0.0744) | |
| Unemployed | | | -0.0793 (0.143) | -0.0793 (0.110) | |
| ManualWork | | | 0.139* (0.0720) | 0.139 (0.0893) | |
| Retired | | | -0.0813 (0.112) | -0.0813 (0.0943) | |
| Religion | | | 0.0779 (0.0635) | 0.0779 (0.0516) | |
| year | -0.0635** (0.0298) | -0.0635** (0.0314) | -0.0615** (0.0298) | -0.0615** (0.0308) | -0.0668** (0.0323) |
| Age65 | | | | | 0.209** (0.0857) |
| Observations | 4,421 | 4,421 | 4,421 | 4,421 | 4,421 |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

A.2 Determinants of Attitude towards Public Spending

The outcome variable TaxSpend is a measure of the attitude towards public spending financed through taxation. This variable capture the fundamental trade-off that drives the results in section 3, namely the degree of aversion to higher taxes in exchange of more social spending. The question is “Suppose the government had to choose between the three options on this card: reduce taxes and spend less on health, education and social benefits, Keep taxes and spending on these services at the same level as now, Increase taxes and spend more on health, education and social benefits. Which do you think it should choose?” and the respondents must choose a value on a discrete scale from 1 (“spend less”) to 3 (“spend more”). I use an ordered Logit model for the same reasons explained in the previous section. Table 2 shows the results of this analysis. The relationship between the outcome variable and the age and the income of the respondent

are both significant and the signs are consistent with the implication of the model. In line with the previous literature, unemployment is also related with a more favorable attitude towards public spending. It is somewhat surprising that low levels of education are associated with a stronger aversion to taxes and public spending. This may be due to factors that are not considered in the theoretical analysis and that are likely to vary across different education level, such as knowledge of the structure of the fiscal system, awareness of the demographic and economic structure of the country and degree of altruism.

| Ordered Logit - Dependent Variable: TaxSpend | | | | | |
|--|-------------------------|-------------------------|------------------------|------------------------|------------------------|
| VARIABLES | (1) Robust SE | (2) Cluster SE | (3) Robust SE | (4) Cluster SE | (5) Cluster SE |
| Age | 0.0102*** (0.00161) | 0.0102*** (0.00178) | 0.0124*** (0.00270) | 0.0124*** (0.00282) | |
| Income | -0.0320*** (0.00983) | -0.0320*** (0.00996) | -0.0422*** (0.0126) | -0.0422*** (0.0123) | -0.0385*** (0.0129) |
| LowEdu | -0.0451*** (0.0134) | -0.0451*** (0.00997) | -0.0669*** (0.0175) | -0.0669*** (0.0145) | -0.0473*** (0.0150) |
| BornAbroad | | | -0.0424 (0.111) | -0.0424 (0.0702) | -0.0760 (0.0733) |
| Children | | | -0.0765 (0.0677) | -0.0765 (0.0828) | -0.0262 (0.0816) |
| Rural | | | -0.0103 (0.0323) | -0.0103 (0.0310) | 0.00376 (0.0281) |
| Sex | 0.180*** (0.0504) | 0.180*** (0.0476) | 0.188*** (0.0637) | 0.188*** (0.0492) | 0.161*** (0.0490) |
| Unemployed | 0.251** (0.116) | 0.251** (0.125) | 0.202 (0.142) | 0.202 (0.138) | 0.144 (0.127) |
| ManualWork | | | 0.202*** (0.0691) | 0.202*** (0.0506) | 0.185*** (0.0483) |
| Retired | | | -0.173 (0.106) | -0.173* (0.0946) | 0.230** (0.117) |
| Religion | 0.218*** (0.0517) | 0.218*** (0.0486) | 0.235*** (0.0645) | 0.235*** (0.0598) | 0.167*** (0.0594) |
| year | 0.00639 (0.0298) | 0.00639 (0.0314) | 0.00614 (0.0300) | 0.00614 (0.0310) | 0.000110 (0.0310) |
| Age65 | | | | | -0.136 (0.112) |
| Observations | 6,639 | 6,639 | 4,421 | 4,421 | 4,421 |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Appendix B. Coalitional Equilibrium: A Formal Description

B.1 Setting

In the model described in section 3 of this paper a voting game is played in every period t . As the features of the game in each period are identical, I simplify the notation by suppressing the time index t in this section and in the proofs to Theorems 1-2-3 (sections B and C.1). Consider a voting game with N voters such that each voter $i \in \mathcal{N}$ is denoted by a vector of parameters $\theta^i \in \Theta$. Assume (Θ, \leq) is a totally ordered set for some transitive, reflexive, antisymmetric order relation \leq . This allows me to establish a total order in the set of players \mathcal{N} , such that for all $i, j \in \mathcal{N}$, one gets $i \leq j$ if and only if $\theta^i \leq \theta^j$. For instance suppose θ^i is individual i 's income, then $\theta^i \in [\underline{\theta}, \bar{\theta}]$ and Θ is a totally ordered set under the order relation \leq .

Each individual $i \in \mathcal{N}$ is endowed with a reflexive, complete and transitive preference ordering \succeq^i that can be represented by a x -continuous and θ -concave utility function¹⁶ $V : X \times \Theta \times \Phi \rightarrow R$, where $\varphi \in \Phi$ is a vector of parameters that do not differ across individuals.

The policy space X_t is a subset of the the d -dimensional real space R^d . In order to characterize X_t it is useful to recall some definitions.

Let (L, \leq) be a partially ordered set, with the transitive, reflexive, antisymmetric order relation \leq . For x' and x'' elements of L , let $x' \vee x''$ denote the least upper bound, or join, of x' and x'' in L , if it exists, and let $x' \wedge x''$ denote the greatest lower bound, or meet of x' and x'' in L , if it exists. The set L is a lattice if for every pair of elements x' and x'' in L , the join $x' \vee x''$ and meet $x' \wedge x''$ do exist as elements of L . Similarly, a subset X of L is a sublattice of L if X is closed under the operations meet and join. A sublattice X of a lattice L is a convex sublattice of L , if $x' \leq x'' \leq x'''$ and x', x''' in X implies that x'' belongs to X , for all elements x', x'', x''' in L . Finally, a sublattice X of L is complete if for every nonempty subset X' of X , $\inf(X')$ and $\sup(X')$ both exist and are elements of X .

Recall the d -dimensional real space R^d is a partially ordered set under the transitive, reflexive, antisymmetric order relation \leq ¹⁷. Moreover (R^d, \leq) is a lattice given the definition above. Now we have all the elements to characterize the policy space X . Let $X \subseteq R^d$ be a convex sublattice of (R^d, \leq) , then (X, \leq) is a partially ordered set with order relation \leq . An example of a policy space that satisfies my assumption is given by the family of sets $Y = \{x \in R^d | x_i \in [\underline{x}_j, \bar{x}_j], j = 1, 2, \dots, d\}$ where $\underline{x}_i, \bar{x}_i \in R$ for all j .

Subset of voters can form coalitions $\mathcal{A} \subseteq \mathcal{N}$. Each voter can be member of only one coalition, i.e. $\mathcal{A}^j \cap \mathcal{A}^k = \emptyset \forall \mathcal{A}^j \neq \mathcal{A}^k$. The role of coalitions in this model is to increase the commitment capacity of individuals over policies. Define $p_{X, \mathcal{A}}(x') \equiv \{x \in X : x \succeq^i x' \forall i \in \mathcal{A}, x \not\succeq^i x' \forall i \in \mathcal{A}^c\}$ to be the set of allocation in X that are Pareto superior to some vector $x' \in X$ for coalition \mathcal{A} . Denote with a^j the platform proposed by coalition \mathcal{A}^j . I assume that a coalition can propose any

¹⁶For any function f defined on the convex subset X of R^d , we say that f is concave in direction $v \neq 0$ if, for all x_t , the map from the scalar s to $f(x + sv)$ is concave. (The domain of this map is taken to be the largest interval such that $x + sv$ lies in X .) We say that f is i -concave if it is concave in direction v for any $v > 0$ with $v_i = 0$. See Quah (2007).

¹⁷For $x', x'' \in R^d$ $x' \leq x''$ if and only if $x'_i \leq x''_i$ for all $i = 1, 2, \dots, d$.

policy in the Pareto set of its members, i.e. $a^j \in P(\mathcal{A}^j)$ where $P(\mathcal{A}^j) \equiv \{x \in X : p_{X, \mathcal{A}^j}(x) = \emptyset\}$, or no policy. If a coalition is a singleton then the Pareto set reduces to the set of ideal policies of its unique member (as in a citizen-candidate model).

B.2 Stability

In this section I describe a stability concept that is fully “internal”, in the sense that the profitability of a deviation for a subgroup of members of a given coalition is independent of the strategies of other coalitions. In section B.5 I describe the alternative concept in which the latter aspect is kept into account. The main results are unaffected.

In order to define stability in this model we need to characterize a coalition structure and the preferences of each coalition. A coalition structure is defined as a partition \mathbb{P} of \mathcal{N} , i.e. a set of subsets of \mathcal{N} such that $\emptyset \notin \mathbb{P}$, $\cup_{\mathcal{A} \in \mathbb{P}(\mathcal{N})} \mathcal{A} = \mathcal{N}$ and if $\mathcal{A}^j, \mathcal{A}^k \in \mathbb{P}(\mathcal{N})$ with $\mathcal{A}^j \neq \mathcal{A}^k$, then $\mathcal{A}^j \cap \mathcal{A}^k = \emptyset$.

I define a complete social preference relation \succ and \succeq such that \succ is irreflexive i.e. $x \not\succeq x$ and \succeq is reflexive i.e. $x \succeq x$ and the weak and strong relations are dual, i.e. $x' \succeq x'' \Leftrightarrow \neg x' \succ x''$ (\succeq is not necessarily transitive). In particular, I am assuming *Majority Voting*, which is the most common and widely used criterion in order to establish a social preference relation. Formally $x' \succeq x''$ if and only if $\sum_{i=1}^N 1[V(x', \theta^i, \varphi) \geq V(x'', \theta^i, \varphi)] \geq N/2$ which implies that $x' \succ x''$ if and only if $\sum_{i=1}^N 1[V(x', \theta^i, \varphi) \geq V(x'', \theta^i, \varphi)] \geq N/2$ and $\sum_{i=1}^N 1[V(x'', \theta^i, \varphi) \geq V(x', \theta^i, \varphi)] < N/2$. Consider the definition of *tournament* adapted from Dutta [1988]. A *tournament* on X is a complete and asymmetric binary relation \succ^T : for each pair x', x'' of distinct outcomes, one (and only one) of $x' \succ^T x''$ and $x'' \succ^T x'$ holds. Notice that the social preference relation \succeq does not necessarily imply a *tournament*, i.e. it is possible that $x' \neq x''$ and $x' \sim x''$.

Given this preference relation we can define $SP_A(x') \equiv \{x \in A : x \succ x'\}$ where $A \subseteq X$, to be the strictly preferred set of x' in A and $K(A) \equiv \{x \in A : SP_A(x) = \emptyset\}$ to be the set of *SP-maximal* alternatives in A , or the Core. The crucial aspect of the concept of stability relies on the idea of “deviator” or “credible threat”.

Definition 1. Define the relation $S_{\mathcal{A}}$ as follow: $xS_{\mathcal{A}}x'$ if and only if (i) $x \in P(\mathcal{A}')$ for some $\mathcal{A}' \subseteq \mathcal{A}$, $\mathcal{A}' \neq \emptyset$, (ii) $x \succ^i x'$ for all $i \in \mathcal{A}'$, (iii) $x \succeq x''$ for all $x'' \in P(\mathcal{A} \setminus \mathcal{A}')$. Consider the set $S_{\mathcal{A}}(x') \equiv \{x \in X : xS_{\mathcal{A}}x'\}$. The set $S_{\mathcal{A}}(x')$ is the set of deviators to policy x' for a coalition \mathcal{A} .

$S_{\mathcal{A}}(x')$ corresponds to the set of policies that are strictly preferred to x' by each member of any subcoalition $\mathcal{A}' \subseteq \mathcal{A}$ and that are preferred by the society to any policy that can be proposed by the residual coalition $\mathcal{A} \setminus \mathcal{A}'$. Using this concept we can define the *S-Core* (SK) to be the set of policies that do not face any “credible threat” from any subcoalition of \mathcal{A} , or more formally:

Definition 2. The set $SK(\mathcal{A}) = \{x \in P(\mathcal{A}) : S_{\mathcal{A}}(x) = \emptyset\}$, or *S-Core*, is the set of $S_{\mathcal{A}}$ -maximal alternatives in $P(\mathcal{A})$.

With this structure I can now define a concept of stability for a coalition structure in this game:

Definition 3. A coalition \mathcal{A} is stable if and only if $SK(\mathcal{A})$ is nonempty.

Example. It is useful to give an example of why a coalition that does not satisfy the definition above is unlikely to survive. Suppose $SK(\mathcal{A}) = \emptyset$. Then for any $x \in P(\mathcal{A})$, $\exists x' \in P(\mathcal{A}')$ and $\mathcal{A}' \subseteq \mathcal{A}$ such that $x' \succ^i x \forall i \in \mathcal{A}'$ and $x' \succeq x'' \forall x'' \in P(\mathcal{A} \setminus \mathcal{A}')$, i.e. there exists a subset of the coalition \mathcal{A} and a policy $x' \in P(\mathcal{A}')$ such that x' is strictly preferred to x by all members of the subcoalition \mathcal{A}' and x' is also preferred by the society as a whole to any policy x'' that the remaining part of the original coalition $\mathcal{A} \setminus \mathcal{A}'$ can propose.

It is natural to consider this coalition structure unstable because for any policy chosen by this coalition in its Pareto Set (e.g. through some form of bargaining), the choice of this policy would not be self-enforcing because a subcoalition \mathcal{A}' can deviate and propose a different policy that makes each member of the subcoalition strictly better off, that is preferred by the society as a whole, and such that the remaining part of the original coalition $\mathcal{A} \setminus \mathcal{A}'$ cannot prevent this deviation because there is no feasible “punishment” policy that can represent a credible threat for the deviating players.

Definition 4. A stable coalition structure is a partition \mathbb{P} of \mathcal{N} such that all the coalitions $\mathcal{A} \in \mathbb{P}$ are stable.

B.3 Preferences

In order to establish the result I need to restrict individual and social preferences. The kind of restrictions I am going to use are very common in the many fields of Economic Theory. About individual preferences the assumptions are *Supermodularity (SM)* and *Strict Single Crossing Property (SSCP)*. Recall that individual preferences can be represented by a function $V : X \times \Theta \times \Phi \rightarrow R$. A function V satisfies:

1. *SM* in x if and only if $V(x' \vee x'', \theta^i, \varphi) - V(x', \theta^i, \varphi) \geq V(x'', \theta^i, \varphi) - V(x' \wedge x'', \theta_t^i, \varphi)$ for all $\theta^i \in \Theta$, for all $\varphi \in \Phi$ and for all $x', x'' \in X$.
2. *SSCP* in (x, θ^i) if and only if $V(x', \bar{\theta}, \varphi) - V(x'', \bar{\theta}, \varphi) > V(x', \underline{\theta}, \varphi) - V(x'', \underline{\theta}, \varphi)$ for all $x' \geq x''$ such that $x', x'' \in X$, $x' \neq x''$, for all $\varphi \in \Phi$ and for all $\bar{\theta}, \underline{\theta} \in \Theta$ such that $\bar{\theta} > \underline{\theta}$.

Given the individual preferences described above, I define $I(i)$ to be set of ideal points of an individual i , i.e. $I(i) \equiv \{x | x \in \arg \max_{y \in X_i} V(y, \theta^i, \varphi)\}$ ¹⁸.

B.4 Equilibrium

Define the set of stable coalition structures: $\mathcal{P}(\mathcal{N}) := \{\mathbb{P} | SK(\mathcal{A}) \neq \emptyset \forall \mathcal{A} \in \mathbb{P}\}$ with typical element \mathbb{P}^* . Denote with $\nu(\mathbb{P})$ the number of coalitions that are part of \mathbb{P} (including singletons). Denote with a^j the policy proposed by coalition \mathcal{A}^j (if any). Finally define a policy profile $A_{\mathbb{P}} := \{a^1, a^2, \dots, a^j, \dots, a^{\nu(\mathbb{P})}\}$ with $a^j \in P(\mathcal{A}^j)$.

Definition 5. (*Winning policy*) A policy vector $a^j \in A_{\mathbb{P}}$ is a winning policy if and only if it is in the Core of $A_{\mathbb{P}}$, i.e. $a^j \in W(A_{\mathbb{P}})$ if and only if $a^j \in K(A_{\mathbb{P}})$.

Given that the Social preference relation is given by Majority Voting this is equivalent to say that, if a^j is a Condorcet Winner¹⁹ then $a^j \in W(A_{\mathbb{P}})$ implies $a^j \in K(A_{\mathbb{P}})$; if not, then a^j is a “weak” form of *Condorcet winner*, namely a policy vector that is undominated over the set of alternatives $A_{\mathbb{P}}$ (I will call it a *weak Condorcet winner*). If no *weak Condorcet winner* exists over $A_{\mathbb{P}}$, or $A_{\mathbb{P}} = \emptyset$, a default (neutral) policy x^N is implemented such that $x \succ^i x^N$ for all $i \in \mathcal{N}$ and all $x \in X$ and thus $x \succeq x^N$ for all $x \in X$.

I can now define an equilibrium for the voting game as follows.

Definition 6. A pure strategy equilibrium is a coalition structure \mathbb{P} , a policy profile $A_{\mathbb{P}}$ and a winning policy $w^* \in W(A_{\mathbb{P}}) \subseteq A_{\mathbb{P}}$ such that: (i) $\mathbb{P} \in \mathcal{P}(\mathcal{N})$ is a stable coalition structure; (ii) $a^j \in SK(\mathcal{A}^j)$ for all $\mathcal{A}^j \in \mathbb{P}$; (iii) the set of winning policies $W(A_{\mathbb{P}})$ is nonempty.

In other words in an equilibrium each coalition is stable and is represented by one of the policy vectors that makes it stable, and the winning policy is a (weak) Condorcet Winner of the reduced games in which the policy space is reduced to $A_{\mathbb{P}} \subseteq X$. Finally, I define the set that is the object of a the comparative statics exercises of the paper.

Definition 7. The set of equilibrium policies $E(\mathcal{N})$ is the union of the sets of policies that are chosen in any equilibrium of the voting game, i.e. $E(\mathcal{N}) := \cup_{\mathbb{P} \in \mathcal{P}(\mathcal{N})} W(A_{\mathbb{P}})$.

This last definition completes the description of the voting process and of the equilibrium concept. In the next paragraph I describe some implications in terms of coalition structures that can emerge in an equilibrium of the voting game.

¹⁸Notice that the completeness of X implies compactness in the order-interval topology. On bounded sets in R^d , the order-interval topology coincides with the Euclidean topology (Birkhoff 1967). Hence $I(i) \neq \emptyset$ for all $i \in \mathcal{N}$.

¹⁹The relationship between the concepts of (strong) *Core* and *Condorcet Winner* is described in Ordershook, 1986, pp. 347-349.

B.4.1 Alternative stability concept

If voters are sufficiently sophisticated, they evaluate the profitability of a deviation keeping into account not only the choices that are internal to the coalition but also the strategies of other coalitions. This is true if deviating players calculate their payoffs anticipating the final outcome in terms of policy that is implemented after the elections. A crucial difference with respect to the stability concept described in the previous section is that coalitions are now allowed to be inactive, i.e. not propose any platform at all. In this case, the inactive coalition proposes the neutral platform x^N and $a^j \in [P(\mathcal{A}^j) \cup \{x^N\}]$.

In this case, define the relation $T_{\mathcal{A}^j, A_{\mathbb{P}}}$ as follow: $xT_{\mathcal{A}^j, A_{\mathbb{P}}}x'$ if and only if (i) $x \in P(\mathcal{A}')$ for some $\mathcal{A}' \subseteq \mathcal{A}^j, \mathcal{A}' \neq \emptyset$, (ii) $x \succ^i w^*$ for all $i \in \mathcal{A}', \forall w^* \in W(\{a^1, a^2, \dots, x', \dots, a^{\nu(\mathbb{P})}\})$, (iii) $x \succeq x''$ for all $x'' \in P(\mathcal{A}^j \setminus \mathcal{A}') \cup (A_{\mathbb{P}} \setminus \{a^j\})$. Consider the set $T_{\mathcal{A}^j, A_{\mathbb{P}}}(x') \equiv \{x \in X : xT_{\mathcal{A}^j, A_{\mathbb{P}}}x'\}$. The set $T_{\mathcal{A}^j, A_{\mathbb{P}}}(x')$ is the set of deviators to policy x' for a coalition \mathcal{A}^j . This means that a deviation occurs if x cannot be defeated not only by any policy x'' that the remaining part of the coalition can credibly commit to propose, but also by all the policy proposed by other coalitions. Moreover, x must make all the deviators better off in comparison with the platform that wins if no deviation occur (which is not necessarily the one proposed by coalition \mathcal{A}^j)²⁰. This second stability concept uses a slightly different set of supporting policies: $TK(\mathcal{A}, A_{\mathbb{P}}) = \{x \in [P(\mathcal{A}^j) \cup \{x^N\}] : T_{\mathcal{A}, A_{\mathbb{P}}}(x) = \emptyset\}$, or T -Core, is the set of $T_{\mathcal{A}}$ – maximal alternatives in $P(\mathcal{A})$ given partition \mathbb{P} and a policy profile $A_{\mathbb{P}}$. Similarly to the first stability concept, a coalition \mathcal{A} is stable if and only if $TK(\mathcal{A}, A_{\mathbb{P}})$ is nonempty. The set of stable coalition structures is now defined as $TP(\mathcal{N}) := \{\mathbb{P} | TK(\mathcal{A}, A_{\mathbb{P}}) \neq \emptyset \forall \mathcal{A} \in \mathbb{P}\}$. A pure strategy equilibrium is a coalition structure \mathbb{P} , a policy profile $A_{\mathbb{P}}$ and a winning policy $w^* \in W(A_{\mathbb{P}})$ such that: (i) $\mathbb{P} \in P(\mathcal{N})$ is a stable coalition structure; (ii) $a^j \in TK(\mathcal{A}^j, A_{\mathbb{P}})$ for all active $\mathcal{A}^j \in \mathbb{P}$; (iii) the set of winning policies $W(A_{\mathbb{P}})$ is nonempty.

In this case the results presented in the next pages do not differ, provided that one chooses an appropriate rules to establish if a coalition decide to be active or not given other coalitions' strategies. In particular I assume that a coalition \mathcal{A}^j in equilibrium does not propose any policy platform if all its members are weakly worse off if they propose a policy platform rather than not running at all, i.e. $(\mathbb{P}, A_{\mathbb{P}}, w)$ is not an equilibrium if $\exists \mathcal{A}^j \in \mathbb{P}$ and $a^j \in A_{\mathbb{P}}$ such that $w \succeq^i w^*$ for all $i \in \mathcal{A}^j$ and for all $w \in W(A_{\mathbb{P}} \setminus \{a^j\})$. This assumption is equivalent to the one in Levy (2005) and implies that at any equilibrium only one policy platform is proposed, i.e. $A_{\mathbb{P}} = \{a^j\}$ for some $j \in \{1, 2, \dots, \nu(\mathbb{P})\}$. Notice that this is true because the equilibrium (if it exists) is a (weak) *Condorcet Winner*, and this implies that $W(A_{\mathbb{P}}) \subseteq W(A_{\mathbb{P}} \setminus \{a^j\})$ for all $a^j \notin W(A_{\mathbb{P}})$. The intuition is that the additional condition for a policy to be a deviator, namely $x \succeq a^k \forall a^k \in A_{\mathbb{P}} \setminus \{a^j\}$ only matters at an equilibrium for partitions in which \mathcal{A} is a not winning

²⁰ Notice that in this case one may allow a coalition to commit on no policy, with no changes in the main results.

coalition because in all other cases (in equilibrium) $a^k = x^n$ for all $k \neq j$. The latter cases do not matter on the equilibrium policy outcome, unless the deviation generates a winning coalition. Hence, even if there may be additional coalition structures that are stable using $TK(\mathcal{A}, A_{\mathbb{P}})$ instead of $SK(\mathcal{A})$ (and vice versa), the implications about the equilibrium policy described in Theorems 1-2-3 are unaffected (see Appendix C.1).

B.5 Stable Coalition Structures

In this section I provide some results about stable and unstable coalition structures in this framework. This is interesting in order to understand the political content of the analytical results in Theorem 1, 2 and 3 and are valid under both stability concepts. Assume that individual preferences satisfy SM and $SSCP$ and recall that v is the index that denotes the median voter, i.e. the player with median $\theta^i \in \Theta$. I can state the following results.

Lemma 22. (*Lateral Coalitions*). *Any coalition \mathcal{A}^j that includes either (a) only individuals with index ($i \leq v$) or (b) only individuals with index ($j \geq v$) is always stable. Therefore a coalition structure \mathbb{P} is stable if each coalition $\mathcal{A}^i \in \mathbb{P}$ satisfies either (a) or (b).*

Proof. See additional material.

Lemma 23. (*Central Coalitions*). *(i) Any coalition \mathcal{A}^j that include both (a) individuals with index ($i < v$) and (b) individuals with index ($j > v$) plus at least one individual with index v is stable if at least one policy $x^v \in I(v)$ is in the Core of a game $(\mathcal{N}, P(\mathcal{A}^j) \cup \{x^v\}, V)$ for all $\mathcal{A}^j \subseteq \mathcal{A}$. (ii) If the Core of the full game (\mathcal{N}, X, V) is non-empty, then any “Central Coalition” is stable, including the Grand Coalition of all voters.*

Proof. See additional material.

Following Levy (2004), I define a “partisan” equilibrium as follows.

Definition 8. *A partisan equilibrium is an equilibrium in which all party members vote for their party’s platform, if it offers one (party members are not restricted in their votes if their party is not offering a platform).*

This definition implies that in a Partisan Equilibrium no voter has a strict incentive to vote for a policy different from the one proposed by the coalition she is part of.

Lemma 24. (*Ends-Against-the-Middle Coalitions*). *(i) Any coalition \mathcal{A}^j that include both (a) individuals with index ($i < v$) and (b) individuals with index ($j > v$) but it does not include any individual with index v is stable only if either of the following is true: (1) $a^j \in I(v)$; (2) $a^j \geq x^v$ for all $x^v \in I(v)$; (3) $a^j \leq x^v$ for all $x^v \in I(v)$ (4) $a^j = \emptyset$. Therefore (ii) if $I(v) \cap P(\mathcal{A}^j) = \emptyset$, then there is no Partisan Equilibrium in which \mathcal{A}^j is stable and $a^j \neq \emptyset$.*

Proof. See additional material.

Results in Lemmas 22-23-24 provide an intuitive understanding of why I obtain a median voter theorem result: with preferences that satisfy *SM* and *SSCP* and coalitions are constrained to propose credible policies the only way to get any policy outcome different from the ideal policy of the median voter is a coalition in the Ends-Against-the-Middle type. This kind of coalition is not stable in this framework (unless a losing platform or no platform is offered) and therefore there is no equilibrium that violates the generalized median voter theorem. This is coherent with the idea that coalitions of individuals with opposite political views are less likely to occur. In the additional material I show an example of the coalition structures that can be stable in a two-dimensional space and how the equilibrium concept relates to the citizen-candidate model (Besley and Coate, 1997).

C Proofs

C.1 Theorem 1

Theorem 1. (*Median Voter Theorem*). (i) A coalitional equilibrium of the voting game exists. (ii) In any equilibrium the set of winning policies is a subset of the set of ideal points of the median voter v . (iii) If the median voter has a unique ideal policy, this policy is going to be the one chosen in any equilibrium.

In order to prove this result we need to introduce some additional notation. Define $\theta_{\mathcal{A}} := \{\theta^i | i \in \mathcal{A}\}$ and $\theta_{\mathcal{A}} \in \Theta_{PS}$ where Θ_{PS} is the power set of Θ . Suppose the coalition \mathcal{A} has k members. Consider a set of $k \times 1$ weighting vectors $\Lambda_{\mathcal{A}} \equiv \{\lambda : \sum_{i \in \mathcal{A}} \lambda^i = 1\}$ for each coalition \mathcal{A} and a function $G : X \times \Lambda_{\mathcal{A}} \times \Theta_{PS} \times \Phi \rightarrow R$ defined as follows: $G(x, \lambda_{\mathcal{A}}, \theta_{\mathcal{A}}, \varphi) = \sum_{i \in \mathcal{A}} \lambda_{\mathcal{A}}^i V(x, \theta^i, \varphi)$. Lemma 24, 25, 26 and 27 are results that will be used as part of the proofs of Lemma 28 and 29, which constitute the main part of the proof of Theorem 1.

Lemma 24. *If V is a continuous function of x and X is a convex set then any point \tilde{x} in the Pareto set of \mathcal{A} is a solution to $\max_{x \in X} G(x, \lambda, \theta_{\mathcal{A}}, \varphi)$ for some vector $\lambda_{\mathcal{A}}(\tilde{x}) \in \Lambda_{\mathcal{A}}$.*

Proof. M.W.G., Proposition 16.E.2.

We need to define four additional objects. For each $\tilde{x} \in P(\mathcal{A})$ define:

- (i) a vector $\underline{\lambda}_{\mathcal{A}}(\tilde{x}, j)$ such that $\underline{\lambda}_{\mathcal{A}}^i(\tilde{x}, j) = \lambda_{\mathcal{A}}^i(\tilde{x}) \forall i \in \mathcal{A} : \theta_i < \theta_j$, $\underline{\lambda}_{\mathcal{A}}^i(\tilde{x}, j) = 0 \forall i \in \mathcal{A} : \theta_i > \theta_j$, $\underline{\lambda}_{\mathcal{A}}^i(\tilde{x}, j) = \sum_{i \in \mathcal{A}, i \geq j} \lambda^i$;
- (ii) a vector $\bar{\lambda}_{\mathcal{A}}(\tilde{x}, j)$ such that $\bar{\lambda}_{\mathcal{A}}^i(\tilde{x}, j) = \lambda^i \forall i \in \mathcal{A} : \theta_i > \theta_m$, $\bar{\lambda}_{\mathcal{A}}^i(\tilde{x}, j) = 0 \forall i \in \mathcal{A} : \theta_i < \theta_j$, $\bar{\lambda}_{\mathcal{A}}^i(\tilde{x}, j) = \sum_{i \in \mathcal{A}, i \geq j} \lambda^i$;
- (iii) the set $\Lambda_{\mathcal{A}}(\tilde{x}, j) = \{\underline{\lambda}_{\mathcal{A}}(\tilde{x}, j), \lambda_{\mathcal{A}}(\tilde{x}), \bar{\lambda}_{\mathcal{A}}(\tilde{x}, j)\}$;

(iv) an order relation \leq^λ given by: $\lambda_1 \leq^\lambda \lambda_2$ iff $\lambda_1^i \geq \lambda_2^i \forall i \leq m$ and $\lambda_1^i \leq \lambda_2^i \forall i > m$. It follows that $(\Lambda_{\mathcal{A}}(\tilde{x}, j), \leq^\lambda)$ is a totally ordered set.

Lemma 25. *If V satisfies SM and SSCP then the Pareto Set $P(\mathcal{A})$ of a coalition of players $\mathcal{A} \subseteq \mathcal{N}$ is such that $y \in P(\mathcal{A})$ only if $y \geq \sup \{I(l)\}$ and $y \leq \inf \{I(h)\}$ where $\min(\mathcal{A})$ and $h = \max(\mathcal{A})$.*

Proof. Denote $\bar{x}^l = \sup \{I(l)\}$ and $\underline{x}^h = \inf \{I(h)\}$. Suppose $y \not\geq \bar{x}^l$ but $y \in P(\mathcal{A})$. Because of the optimality of \bar{x}^k and because X is a lattice, it must be true that $V(\bar{x}^l, \theta^l) \geq V(y \wedge \bar{x}^l, \theta^l)$. Supermodularity implies $V(y \vee \bar{x}^l, \theta^l) \geq V(y, \theta^l)$. Notice that $y \not\geq \bar{x}^l$ implies $y \vee \bar{x}^l \neq y$. Hence the Strict Single Crossing Property implies $V(y \vee \bar{x}^l, \theta^i, \varphi) > V(y, \theta^i, \varphi) \forall \theta^i > \theta^l$. Given that $\theta^i > \theta^l$ is true for all $(\theta \in \mathcal{A}) \cap (\theta \neq \theta^l)$ we have that $\exists x \in X$ such that $V(x, \theta, \varphi) \geq V(y, \theta, \varphi) \forall \theta \in \mathcal{A}$ and $V(x, \theta, \varphi) > V(y, \theta, \varphi)$ for at least one $\theta \in \mathcal{A}$, i.e $p_{X, \mathcal{A}}(y) \neq \emptyset$. Hence $y \notin P(\mathcal{A})$. Similarly one can show that $y \in P(\mathcal{A})$ only if $y \leq \underline{x}^h$. Q.E.D.

Lemma 26. *The function $G(x, \lambda, \theta_{\mathcal{A}}, \varphi)$ satisfies (i) SM in x and (ii) SCP in $(x, \lambda) \forall \lambda \in \Lambda_{\mathcal{A}}(\tilde{x}, j)$ for all $\tilde{x} \in P(\mathcal{A})$.*

Proof. (i) SM. G is the sum of SM functions so it is supermodular (proof in Milgrom, Shannon, 1994). (ii) SCP. Using the definition of SCP, G satisfies the SCP if and only if: $G(\bar{x}, \lambda, \theta_{\mathcal{A}}, \varphi) - G(\underline{x}, \lambda, \theta_{\mathcal{A}}, \varphi) \geq G(\bar{x}, \lambda, \theta_{\mathcal{A}}, \varphi) - G(\underline{x}, \lambda, \theta_{\mathcal{A}}, \varphi) \forall \bar{x} \geq \underline{x}, \lambda \in \Lambda_{\mathcal{A}}(\tilde{x}, j)$. Use the definitions of G and $\lambda_{\mathcal{A}}(\tilde{x})$ and $\underline{\lambda}_{\mathcal{A}}(\tilde{x}, j)$:

$$\begin{aligned} & [G(\bar{x}, \lambda_{\mathcal{A}}(\tilde{x}), \theta_{\mathcal{A}}, \varphi) - G(\underline{x}, \lambda_{\mathcal{A}}(\tilde{x}), \theta_{\mathcal{A}}, \varphi)] - [G(\bar{x}, \underline{\lambda}_{\mathcal{A}}(\tilde{x}, j), \theta_{\mathcal{A}}, \varphi) - G(\underline{x}, \underline{\lambda}_{\mathcal{A}}(\tilde{x}, j), \theta_{\mathcal{A}}, \varphi)] = \\ & = \left(\sum_{\substack{i \in \mathcal{A} \\ i \geq j}} \lambda^i [V(\bar{x}, \theta^i, \varphi) - V(\underline{x}, \theta^i, \varphi)] \right) - \left(\sum_{\substack{i \in \mathcal{A} \\ i \geq j}} \lambda^i [V(\bar{x}, \theta^j, \varphi) - V(\underline{x}, \theta^j, \varphi)] \right) = \\ & = \sum_{\substack{i \in \mathcal{A} \\ i \geq j}} \lambda^i ([V(\bar{x}, \theta^i, \varphi) - V(\underline{x}, \theta^i, \varphi)] - [V(\bar{x}, \theta^j, \varphi) - V(\underline{x}, \theta^j, \varphi)]) \end{aligned}$$

Notice that $[V(\bar{x}, \theta^i, \varphi) - V(\underline{x}, \theta^i, \varphi)] - [V(\bar{x}, \theta^j, \varphi) - V(\underline{x}, \theta^j, \varphi)] \geq 0$ for $\forall i \geq j$ and $\lambda^i \geq 0 \forall i$ hence the sum above is also weakly positive, which implies $[G(\bar{x}, \lambda_{\mathcal{A}}(\tilde{x}), \theta_{\mathcal{A}}, \varphi) - G(\underline{x}, \lambda_{\mathcal{A}}(\tilde{x}), \theta_{\mathcal{A}}, \varphi)] - [G(\bar{x}, \underline{\lambda}_{\mathcal{A}}(\tilde{x}, j), \theta_{\mathcal{A}}, \varphi) - G(\underline{x}, \underline{\lambda}_{\mathcal{A}}(\tilde{x}, j), \theta_{\mathcal{A}}, \varphi)] \geq 0$. Similarly one can show that this is also true for $(\lambda_{\mathcal{A}}(\tilde{x}), \bar{\lambda}_{\mathcal{A}}(\tilde{x}, j))$ and $(\underline{\lambda}_{\mathcal{A}}(\tilde{x}, j), \bar{\lambda}_{\mathcal{A}}(\tilde{x}, j))$. Q.E.D.

Define $\widetilde{M}(\mathcal{A}, \lambda) := \arg \max_{x \in X, \lambda \in \Lambda} G(x, \lambda, \theta_{\mathcal{A}}, \varphi)$ and the sets $\mathcal{A}^{\leq j} := \{i | (i \in \mathcal{A}) \cap (i \leq j)\}$ and $\mathcal{A}^{\geq j} := \{i | (i \in \mathcal{A}) \cap (i \geq j)\}$ for some $j \in \mathcal{A}$.

Lemma 27. *(i) If $x' \in \widetilde{M}(\mathcal{A}, \underline{\lambda}_{\mathcal{A}}(\tilde{x}, j))$ and $x \in \widetilde{M}(\mathcal{A}, \lambda_{\mathcal{A}}(\tilde{x}))$, then either $x' = x$, or if $x' \neq x$, then $V(x', \theta^j, \varphi) \geq V(x, \theta^j, \varphi)$ and $V(x', \theta^i, \varphi) > V(x, \theta^i, \varphi) \forall i < j$ and $x' \leq x$. Moreover,*

(ii) if $x \in \widetilde{M}(\mathcal{A}, \lambda_{\mathcal{A}}(\tilde{x}))$ and $x'' \in \widetilde{M}(\mathcal{A}, \bar{\lambda}_{\mathcal{A}}(\tilde{x}, j))$, then either $x'' = x$, or if $x'' \neq x$, then $V(x'', \theta^j, \varphi) \geq V(x, \theta^j, \varphi)$ and $V(x'', \theta^i, \varphi) > V(x, \theta^i, \varphi) \forall i > j$ and $x'' \geq x$.

Proof. We know $x' \leq x$ and $G(x', \lambda_{\mathcal{A}}(\tilde{x}, j), \theta_{\mathcal{A}}, \varphi) \geq G(x, \lambda_{\mathcal{A}}(\tilde{x}, j), \theta_{\mathcal{A}}, \varphi)$ from Monotone comparative statics. Suppose $V(x', \theta^j, \varphi) < V(x, \theta^j, \varphi)$ and $x' \neq x$. Then it must be true that $\sum_{i \in \mathcal{A}} \lambda_{\mathcal{A}}^i(\tilde{x}, j) [V(x', \theta^i, \varphi) - V(x, \theta^i, \varphi)] > V(x', \theta^j, \varphi) - V(x, \theta^j, \varphi)$. Using $\sum_{i \in \mathcal{A}} \lambda_{\mathcal{A}}^i(\tilde{x}, j) = 1$ the above can be rearranged as follows:

$$\sum_{i \in \mathcal{A}} \lambda_{\mathcal{A}}^i(a, j) ([V(x', \theta^i, \varphi) - V(x, \theta^i, \varphi)] - [V(x', \theta^j, \varphi) - V(x, \theta^j, \varphi)]) > 0$$

Notice that $x' \leq x$ and $i \leq j \forall i \in \mathcal{A}$, hence *SSCP* implies $[V(x', \theta^i, \varphi) - V(x, \theta^i, \varphi)] - [V(x', \theta^j, \varphi) - V(x, \theta^j, \varphi)] \leq 0 \forall i \in \mathcal{A}$ and hence

$$\sum_{i \in \mathcal{A}} \lambda_{\mathcal{A}}^i(a, j) ([V(x', \theta^i, \varphi) - V(x, \theta^i, \varphi)] - [V(x', \theta^j, \varphi) - V(x, \theta^j, \varphi)]) \leq 0$$

which leads to a contradiction. Hence it must be true that $V(x', \theta^j, \varphi) \geq V(x, \theta^j, \varphi)$. Given that $x' \leq x$, $x' \neq x$, *SSCP* implies $V(x', \theta^i, \varphi) > V(x, \theta^i, \varphi) \forall i < j$. This statement implies that x is not part of the Pareto set of $\mathcal{A}^{\leq j}$ if $x' \neq x$. In the same way it is easy to show that the statement (ii) is also true. Q.E.D.

Lemma 28. *The coalition \mathcal{A}^v (could be a singleton) that includes the median voter v is stable only if $a^v \in I(v)$.*

Proof. Suppose $a^v \notin I(v)$. This cannot be the case if $\theta^i = \theta^v$ for all members of \mathcal{A}^v . This situation is illustrated in Fig. 17. Hence consider the case in which there is at least one $i \in \mathcal{A}^v$ such that either $i > v$ or $i < v$ (or both).

(i) If $a^v \geq \underline{x}^v (\leq)$ for any $\underline{x}^v = \inf\{I(v)\}$ and $a^v \wedge x^v \in P(\mathcal{A}^v)$. $a^v \notin I(v)$ implies $V(x^v, \theta^m, \varphi) > V(a^v, \theta^v, \varphi)$. *SSCP* implies $V(x^v, \theta^i, \varphi) > V(a^v, \theta^i, \varphi)$ and $x^v \succ^i a^v \forall i \in \mathcal{N} : \theta^i \leq \theta^v (\geq)$. Recall that any $c \in P(\mathcal{A}^v \setminus \mathcal{A}^{\leq v})$ is $c \geq \underline{x}^v$ (because of Lemma 25). Hence either $c \in I(v)$ or $\sum_{i=1}^n 1[V(\underline{x}^v, \theta^i, \varphi) > V(c, \theta^i, \varphi)] > n/2 \forall c \in P(\mathcal{A}^v \setminus \mathcal{A}^{\leq v})$ which implies $x^v \succ c \rightarrow a^v \notin SK(\mathcal{A}^v)$.

(ii) If $a^v \not\geq x^v, a^v \not\leq x^v$. Consider $a^v \vee x^v (a^v \wedge x^v)$. Revealed preferences and $a^v \notin I(v)$ imply $V(x^v, \theta^v, \varphi) > V(a^v \vee x^v, \theta^v, \varphi)$. *SM* implies $V(a^v \wedge x^v, \theta^v, \varphi) > V(a^v, \theta^v, \varphi)$. *SSCP* implies $V(a^v \wedge x^v, \theta^i, \varphi) > V(a^v, \theta^i, \varphi) \forall i \in \mathcal{N} : \theta^i \leq \theta^v$. Recall that any $c \in P(\mathcal{A}^v \setminus \mathcal{A}^{\leq v})$ is $c \geq \bar{x}^v \geq a^v \wedge x^v$ because of Lemma 25. Hence either $c \in I(v)$ or $\sum_{i=1}^n 1[V(a^v \wedge x^v, \theta^i, \varphi) \geq V(c, \theta^i, \varphi)] > n/2$ which implies $a^v \wedge x^v \succ c$.

If $a^v \wedge x^v$ is part of the Pareto set of $\mathcal{A}^{\leq v}$ this constitute a feasible and profitable deviation.

If and $a^v \wedge x^v \notin P(\mathcal{A}^{\leq v})$ (case described in Fig. 18 in the additional material), recall that X is a convex set and $V(x, \theta, \varphi)$ is θ -concave, hence as $a^v \in \mathcal{A}^v$ it has to be the solution to a problem in the form $a^v \in \arg \max_{x \in X} G(x, \lambda_{\mathcal{A}^v}(a^v), \theta_{\mathcal{A}^v}, \varphi)$. Consider the following alternative:

$\tilde{x} \in \widetilde{M}(\mathcal{A}, \underline{\lambda}_{\mathcal{A}^v}(a^v, v))$ (see Lemma 27). We know from Lemma 27 that $\tilde{x} \leq a^v$. First of all notice that $\widetilde{M}(\mathcal{A}, \underline{\lambda}_{\mathcal{A}^v}(a^v, v)) = \widetilde{M}(\mathcal{A}', \lambda')$ for some λ' , which implies that $\tilde{x} \in P(\mathcal{A}')$, i.e. it is in the Pareto set of \mathcal{A}' . One needs to show that $\tilde{x} \neq a^v$ and that $\tilde{x} \succeq^i a^v \forall i \in \mathcal{A}'$. Suppose $\tilde{x} = a^v \rightarrow a^v \in P(\mathcal{A}')$. But from point (ii) we know that $V(a^v \wedge x^v, \theta_i, \varphi) > V(a^v, \theta^i, \varphi) \forall i \in \mathcal{N} : \theta^i \leq \theta^v \rightarrow a^v \notin P(\mathcal{A}')$ → Contradiction. Hence $\tilde{x} \neq a^v$ and $\tilde{x} \leq a^v$. Moreover, Lemma 25 implies $\tilde{x} \succeq^v a^v$. This means that $V(\tilde{x}, \theta^v, \varphi) \geq V(a^v, \theta^v, \varphi)$ and because $\tilde{x} \neq a^v$, the *SSCP* implies $V(\tilde{x}, \theta^i, \varphi) > V(a^v, \theta^i, \varphi) \forall i \in \mathcal{N} : \theta^i \leq \theta^v$. Recall that any $c \in P(\mathcal{A}^v \setminus \mathcal{A}^{\leq v})$ is $c \geq \bar{x}^v \geq \tilde{x}$. Hence either $c \in M(v)$ or $\sum_{i=1}^n 1[V(\tilde{x}, \theta_i, \varphi) \geq V(c, \theta_i, \varphi)] > n/2$ which implies $\tilde{x} \succ c \rightarrow a^v \notin SK(\mathcal{A}^v)$.

One can also show that some kind of coalitions containing v are stable. Suppose $a^v \in I(v)$, and in particular say $a^v = \bar{x}^v = \sup\{I(v)\}$ ($a^v = \underline{x}^v = \inf\{I(v)\}$). Consider any coalition $\mathcal{A}^{\leq v}$ ($\mathcal{A}^{\geq v}$) such that $\theta^i \leq \theta^v (\geq) \forall i \in \mathcal{A}^{\leq v}$. From Lemma 22 we know that any $b \in P(\mathcal{A}^{\leq v})$ it must be true that $b \leq x^v (\geq)$. Moreover, a deviation implies Optimality implies $V(x^v, \theta^v, \varphi) > V(b, \theta^v, \varphi)$. *SSCP* implies $V(x^v, \theta^i, \varphi) > V(b, \theta^i, \varphi)$ and $x^v \succeq^i b \forall i \in \mathcal{N} : \theta^i \geq \theta^v (\leq)$. Hence $\sum_{i=1}^n 1[V(x^v, \theta^i, \varphi) > V(b, \theta^i, \varphi)] \geq n/2 \forall b \in P(\mathcal{A}^{\leq v})$ which implies $x^v \succ b \forall b \in P(\mathcal{A}^v) \rightarrow a^v \in SK(\mathcal{A}^v)$.

Finally Consider any coalition \mathcal{A}^v such that $\theta^i \leq \theta^v (\geq) \forall i \in \mathcal{A}^v$ and $a^v = \bar{x}^v (\underline{x}^v)$. From Lemma 24 we know that any $b \in P(\mathcal{A}^v)$ it must be true that $b \leq \bar{x}^v (\geq \underline{x}^v)$. This implies $V(\bar{x}^v, \theta^v, \varphi) > V(b, \theta^v, \varphi)$. *SSCP* implies $V(\bar{x}^v, \theta^i, \varphi) > V(b, \theta^i, \varphi)$ and $\bar{x}^v \succ^i b \forall i \in \mathcal{N} : \theta^i \geq \theta^v (\leq)$. Hence $\sum_{i=1}^n 1[V(\bar{x}^v, \theta^i, \varphi) > V(b, \theta^i, \varphi)] > n/2 \forall b \in P(\mathcal{A}^v)$ which implies $\bar{x}^v \succ a^v \forall a \in P(\mathcal{A}^v) \rightarrow SP_{P(\mathcal{A}^v)}(a^v) = \emptyset \leftrightarrow a^v \in K(\mathcal{A}^v)$. Q.E.D.

Lemma 29. *Any coalition \mathcal{A}^j that does not contain the median voter m is stable only if $\exists a^j$ such that either of the following is true: (i) $a^j \in I(m)$; (ii) $a^j \geq x_m$ for all $x_m \in I(m)$; (iii) $a^j \leq x_m$ for all $x_m \in I(m)$.*

Proof. Suppose $a^j \notin I(m)$ and $a^j \not\geq x^m, a^j \not\leq x^m$ for some $x^m \in I(m)$. There are two possible cases.

(i) say $x^k \in I(k)$ and $\forall k \in \mathcal{A}^j$ it is true either $x^k \geq a^j$ or $x^k \leq a^j$. This case is illustrated in Fig. 19 in the additional material. Consider x_j such that $x_j \in \arg \max_{x \in \{\bar{x}, \underline{x}^{v-1}\}} V(x, \theta^v, \varphi)$. Consider $\underline{x}^w (\bar{x}^z)$ where $w = \max_{i < v, i \in \mathcal{A}^j} i$ ($z = \min_{i > v, i \in \mathcal{A}^j} i$). Suppose $\underline{x}^w \succeq^v \bar{x}^z$ ($\underline{x}^w \prec^v \bar{x}^z$) Notice that $\underline{x}^w \neq a^j$, optimality implies $V(\underline{x}^w, \theta^w, \varphi) > V(a^j, \theta^w, \varphi)$ (strict because $a^j \not\geq x^v, a^j \not\leq x^v$ *SM* and *SSCP* imply $x^v \wedge a^j \succ^w a^j$ and optimality implies $\underline{x}^w \succeq^w x^v \wedge a^j$). Notice that because $\underline{x}^w \neq a^j$ *SSCP* implies $V(\underline{x}^w, \theta^i, \varphi) > V(a^j, \theta^i, \varphi) \forall i \in \mathcal{N} : \theta^i < \theta^v (>)$. Also notice that $\underline{x}^w (\bar{x}^z)$ is in the Pareto set $P(\mathcal{A}^{<v}) = P(\{i \in \mathcal{A} : i < v\})$ ($P(\mathcal{A}^{>v}) = P(\{i \in \mathcal{A} : i > v\})$) because it is the lowest (highest) ideal point of the highest (lower) member of the subcoalition $\mathcal{A}^{<v}$ (see Lemma 25). Finally notice that any policy $b \in P(\mathcal{A}^j \setminus \mathcal{A}^{<v})$ must be $b \geq \bar{x}^z (\leq \underline{x}^w)$ (because of Lemma 25). Hence given that $x^w \neq a^j$ (see above), then $\sum_{i=1}^n 1[V(x^w, \theta^i, \varphi) \geq V(b, \theta^i, \varphi)] > n/2 \forall b \in P(\mathcal{A}^j \setminus \mathcal{A}^{<v})$ which implies $x^w \succeq b \forall b \in P(\mathcal{A}^j \setminus \mathcal{A}^{<v}) \rightarrow a^j \notin SK(\mathcal{A}^j)$.

(ii) $\exists x^k \in I(k), k \in \mathcal{A}^j, \theta^k > \theta^m (<)$ such that $x^k \not\geq a^j$ and $x^k \not\leq a^j$. Consider $x^k \wedge a^j$. Notice

that $x^k \not\geq a^j$ and $x^k \not\leq a^j$ imply $x^k \wedge a^j \neq a^j$. Optimality implies $V(x^k, \theta^k) \geq V(x^k \vee a^j, \theta^k)$. *SM* implies $V(x^k \wedge a^j, \theta^k, \varphi) \geq V(a^j, \theta^k, \varphi)$. *SSCP* implies $V(x^k \wedge a^j, \theta^i, \varphi) > V(a^j, \theta^i, \varphi) \forall i \in \mathcal{N} : \theta^i < \theta^k$. Hence $\sum_{i=1}^n 1[V(x^k \wedge a^j, \theta^i, \varphi) > V(a^j, \theta^i, \varphi)] > n/2$. which implies $x^k \wedge a^j \succ a^j$.

If $x^k \wedge a^j$ is part of the Pareto set of $\mathcal{A}^{<k}$ this constitute a feasible and profitable deviation. If not, recall that X is a convex set and $V(x, \theta, \varphi)$ is θ -concave, hence as $a^j \in P(\mathcal{A}^j)$ it has to be the solution to a problem in the form $a^j \in \arg \max_{x \in X} G(x, \lambda_{\mathcal{A}^j}(a^j), \theta_{\mathcal{A}^j}, \varphi)$. This case is illustrated in Fig. 20 in the additional material. If $x^k \wedge a^j$ is not part of the Pareto set of \mathcal{A}^j , consider the following alternative: $\tilde{x} \in \widetilde{M}(\mathcal{A}, \underline{\lambda}_{\mathcal{A}^j}(a^j, k))$ (see Lemma 27). We know from Lemma 27 that $\tilde{x} \leq a^j$. First of all notice that $\widetilde{M}(\mathcal{A}, \underline{\lambda}_{\mathcal{A}^j}(a^j, k)) = \widetilde{M}(\mathcal{A}', \lambda')$ for some λ' , which implies that $\tilde{x} \in P(\mathcal{A}')$, i.e. it is in the Pareto set of $\mathcal{A}^{\leq k}$. We need to show that $\tilde{x} \neq a^j$ and that $\tilde{x} \succ^i a^j \forall i \in \mathcal{A}^{\leq k}$. Suppose $\tilde{x} = a^j \rightarrow a^j \in P(\mathcal{A}^{\leq k})$. From point (ii) we know that $V(x^k \wedge a^j, \theta^i, \varphi) > V(a^j, \theta^i, \varphi) \forall i \in \mathcal{N} : \theta_i \leq \theta^v \rightarrow a^j \notin P(\mathcal{A}^{\leq k}) \rightarrow$ Contradiction. Hence $\tilde{x} \neq a^j$ and $\tilde{x} \leq a^j$. Moreover, Lemma 27 implies $\tilde{x} \succ^k a^j$. This means that $V(\tilde{x}, \theta^k, \varphi) > V(a^j, \theta^k, \varphi)$ and because $\tilde{x} \neq a^j$ *SSCP* implies $V(\tilde{x}, \theta^i, \varphi) > V(a^j, \theta^i, \varphi) \forall i \in \mathcal{N} : \theta_i \leq \theta_j$. Recall that any $c \in P(\mathcal{A} \setminus \mathcal{A}^{\leq k})$ is $c \geq \bar{x}_k \geq \tilde{x}$. Hence either $c \in I(v)$ or $\sum_{i=1}^n 1[V(\tilde{x}, \theta^i, \varphi) \geq V(c, \theta^i, \varphi)] > n/2$ which implies $\tilde{x} \succ c \rightarrow a^j \notin SK(\mathcal{A}^j)$. Q.E.D.

Now consider the same analysis but in the case in which the stability concept is given by the *TK* instead of the *SK*. The crucial intuition of this case is that at any equilibrium only one coalition propose a policy.

Lemma 30. *In any TK-stable equilibrium only one coalition proposes a policy.*

Proof. Suppose that more than one coalition propose a policy. Then either there is a weak *Condorcet Winner*, in which case all the other coalition have an incentive to withdraw without a change in their payoff (because of the tie-break rule). If there is no *Condorcet Winner*, given that the default policy is a platform that is strictly worse than any other, then a withdraw implies a weakly better outcome for everybody.

I can now state two Lemmas that are equivalent to Lemma 28-29 for the case of *TK* stability. They are given by:

Lemma 28b. *The coalition \mathcal{A}^v (could be a singleton) that includes the median voter v is stable at an equilibrium only if $a^v \in I(v)$ or $a^v = \emptyset$.*

Proof. At an equilibrium either $A_{\mathbb{P}(\mathcal{N})} \setminus \{a^v\} = \emptyset$, in which case the analysis of stability is totally equivalent to the one in lemma 28, or $A_{\mathbb{P}(\mathcal{N})} \setminus \{a^v\} \neq \emptyset$, such that $A_{\mathbb{P}(\mathcal{N})}$ can be an equilibrium policy profile only if $a^v = \emptyset$ (because it is optimal that only the winning coalition runs with a platform given the tie-break rule).

Lemma 29b. *Any coalition \mathcal{A}^j that does not contain the median voter v is stable at an equilibrium only if $\exists a^j$ such that either of the following is true: (i) $a^j \in I(v)$, (ii) $a^j = \emptyset$.*

Proof. At an equilibrium either $A_{\mathbb{P}(\mathcal{N})} \setminus a^j = \emptyset$, in which case the analysis of stability is totally equivalent to the one in lemma 28, or $A_{\mathbb{P}(\mathcal{N})} \setminus a^v \neq \emptyset$, such that $A_{\mathbb{P}(\mathcal{N})}$ can be an equilibrium policy profile only if $a^j = \emptyset$ (because it is optimal that only the winning coalition runs with a platform given the tie-break rule).

I can now use Lemmas 24-29 to prove Theorem 1. Recall that the theorem states the following.

Theorem 1. (*Median Voter Theorem*). (i) A coalitional equilibrium of the voting game exists. (ii) In any equilibrium the set of winning policies is a subset of the set of ideal points of the median voter v . (iii) If the median voter has a unique ideal policy, this policy is going to be the one chosen in any equilibrium.

Proof. The results in Lemma 28 (28b) and Lemma 29(29b) imply that the only policies that can be proposed by stable coalitions in equilibrium are either $a^v = x^v \in I(v)$ or $a^l \leq x^v$ or $a^h \geq x^v$ for all $x^v \in I(v)$. Recall optimality implies $V(a^v, \theta^v, \varphi) > V(a^l, \theta^v, \varphi)$ and *SSCP* implies $V(a^v, \theta^i, \varphi) > V(a^l, \theta^i, \varphi) \forall i \in \mathcal{N} : \theta^i \geq \theta^v$. Similarly $V(a^v, \theta^v, \varphi) > V(a^h, \theta^v, \varphi)$ and *SSCP* implies $V(a^v, \theta^i, \varphi) > V(a^h, \theta^i, \varphi) \forall i \in \mathcal{N} : \theta^i \leq \theta^v$. Also, the coalition structure in which every coalition is a singleton is always stable. Hence a (weak) Condorcet winner among the proposed policies exists, which is also the policy chosen in an equilibrium of the coalitional game (i). The total order in the policy space effectively available in all reduced games generated by a stable coalition structure implies the weak Condorcet winner must be always some $a^v \in I(v)$ (ii). The proof of (iii) is straightforward from (i) and (ii). In the case of the *TK* stability concept, there is no equilibrium in which a coalition other than \mathcal{A}^v wins proposing a policy $a^j \notin I(v)$, because in such case the fact that either $a^j \leq x^v$ or $a^j \geq x^v$ for all $x^v \in I(v)$ implies that the median voter can change his strategy (e.g. leaving his coalition) and being strictly better off. For instance suppose $a^j \geq \bar{x}^v$ and $a^j \notin I(v)$. Then optimality implies $V(x^v, \theta^v) \geq V(x^v \vee a^j, \theta^v)$ and by *SM* one gets $V(x^v \wedge a^j, \theta^v, \varphi) > V(a^j, \theta^v, \varphi)$. *SSCP* implies $V(x^v \wedge a^j, \theta^i, \varphi) > V(a^j, \theta^i, \varphi)$ for all $i \leq v$. Hence either $x^v \wedge a^j$ is in $P(\mathcal{A}^{\leq v})$ or one can use part (ii) of the proof of Lemma 28 to show that there is a policy $\tilde{x} \in P(\mathcal{A}^{\leq v})$ such that $V(\tilde{x}, \theta^i, \varphi) > V(a^j, \theta^i, \varphi)$ for all $i \leq v$, which means that the subcoalition $\mathcal{A}^{\leq v} \subseteq \mathcal{A}^v$ possess a profitable deviation. Q.E.D.

Corollary 31. (i) The equilibrium policy is in the Core of a winning coalition, i.e. $x \in W(A_{\mathbb{P}(\mathcal{N})^*}) \rightarrow x \in K_{\mathcal{A}^v}(X)$ for some winning coalition \mathcal{A}^v . Moreover, (ii) the equilibrium policy is in the Core of the reduced game, i.e. $x \in W(A_{\mathbb{P}(\mathcal{N})^*}) \rightarrow x \in K(A_{\mathbb{P}(\mathcal{N})^*})$ for any equilibrium policy profile $A_{\mathbb{P}^*}$.

Proof. Straightforward from Theorem 1.

Theorem 2. (*Monotone Comparative Statics*). The set of equilibrium policies of the voting game is (i) a sublattice of X_t which is (ii) monotonic nondecreasing in θ_t^v .

Proof. Notice that the coalition structure in which the winning coalition includes all the individuals $\theta^i = \theta^v$ and all other coalitions are singleton is always stable if *SM* and *SSCP* are satisfied, because no individual in \mathcal{A}^v has strict incentive to deviate and all the other coalition do not admit any deviation. Given the definition of $E(\mathcal{N})$ and that $I(v)$ is a superset of the unions of subsets of $I(v)$, all I need to show is that all elements of $I(v)$ are equilibria in that particular coalition structure, and this implies $E(\mathcal{N}) = I(v)$. Suppose this is not true. Then $\exists i \in \mathcal{N}$ with $\theta^i \neq \theta^v$ and $x \in I(i)$, such that $V(x, \theta^i, \varphi) > V(a^v, \theta^i, \varphi)$ for a majority on individuals. If (i) $x \geq a^v$ (\leq) then optimality implies $V(x, \theta^v, \varphi) \leq V(a^v, \theta^v, \varphi)$ and by the *SSCP* $V(x, \theta^v, \varphi) < V(a^v, \theta^v, \varphi)$ for all $i < j$ ($>$) which means that there is no strict majority that supports x against a^v . Contradiction. If (ii) $x \not\geq a^v$ and $x \not\leq a^v$ then optimality implies $V(a^v, \theta^v, \varphi) \geq V(x \wedge a^v, \theta^v, \varphi)$ and $V(a^v, \theta^v, \varphi) \geq V(x \vee a^v, \theta^v, \varphi)$. Using *SM* one gets $V(x \vee a^v, \theta^v, \varphi) \geq V(x, \theta^v, \varphi)$ and $V(x \wedge a^v, \theta^v, \varphi) \geq V(x, \theta^v, \varphi)$. Finally the *SSCP* implies $V(x \vee a^v, \theta^i, \varphi) > V(x, \theta^i, \varphi)$ for all $i > v$ and $V(x \wedge a^v, \theta^j, \varphi) > V(x, \theta^j, \varphi)$ for all $j < v$, which means that $x \notin I(i)$ for all $i \in \mathcal{N}$ such that $i \notin \mathcal{A}^v$, and this means that x cannot be proposed by any other coalition, and therefore it cannot defeat a^v . Contradiction. This means that $E(\mathcal{N}) = I(v)$, and $I(v)$ is monotone nondecreasing in θ^v by Theorem 4 in Milgrom and Shannon, 1994. Q.E.D.

Theorem 3. (*Monotone Comparative Statics 2*). *The set of equilibrium policies of the voting game is monotonic nondecreasing in φ .*

Proof. (i) In the proof of Theorem 2 I have shown that $E(\mathcal{N}) = I(v)$. *SM* and *SCP* imply that $I(v)$ is monotone nondecreasing in φ (Theorem 4 in Milgrom and Shannon, 1994). Q.E.D.

C.2.1 Alternative assumption about the default policy

One may want to assume that the default platform is the policy implemented in the previous period (if feasible).

Notice that in this case all the equilibria described by Theorem 1 in which the coalitions are “Lateral” in the sense defined in section B Lemma 22 are unaffected, because no losing coalition (or subcoalition) can change the equilibrium outcome by proposing a platform or by deviating. Therefore at least some equilibria with the characteristics described in Theorem 1 is unaffected. Nevertheless, there may additional equilibria in which some coalitions (at least 3) propose a policy only to ensure that no *Condorcet Winner* exists and therefore they can achieve the default policy. Suppose that is the case. The characterizations of all the equilibria given in Theorem 1 is no longer valid. Nevertheless the comparative statics results in Theorem 2-3 still apply. The reason is that the only equilibria that do not satisfy Theorem 1 are the ones in which the default option is implemented, which means that the comparative statics is null, i.e. $x_t^* = x_{t-1}^*$ and therefore the outcome is nondecreasing in θ_t^v and φ as stated in the two Theorems.

C.3 Lemma 4

Lemma 4. *In a coalitional equilibrium, conditional on g_t , (i) each individual's ideal policy x_t^i and (ii) the equilibrium policy x_t^* at time t are invariant to the equilibrium policy and to the value of the endogenous state at each time $t - s$, i.e. $x_t^i(g_t, \varphi, h_{t-1}) = x_t^i(g_t, \varphi, h'_{t-1})$ and $x_t^*(g_t, \varphi, h_{t-1}) = x_t^*(g_t, \varphi, h'_{t-1}) \forall t$ and $h_{t-1}, h'_{t-1} \in H_{t-1}$.*

Proof. Part (ii) is simply a consequence of rational beliefs. Part (i) follows the F.O.C.s for each individual i at time t :

$$\begin{aligned}\tilde{V}_{M_t}^{i,y} &= -c'(M_t) + \theta_t^i (\alpha + \gamma) \frac{l_{t-1}}{(1 - \widehat{M}_t)} - \theta_t^i \lambda_t + \left(\frac{\beta l_t}{(1 - \widehat{M}_{t+1})} (\alpha / \theta_t^i + \gamma) - e_t \right) (\sigma_t^m - \sigma_t^n) \theta_t^i + \\ &\quad + \beta l_t \sum_{j=1}^3 \frac{\partial V_{t+1}^{i,o}}{\partial x_j} \frac{dx_{j,t+1}^{**}}{dM_t} \\ \tilde{V}_{Y_t}^{i,y} &= -\theta^i + b'(Y_t) + \beta l_t \sum_{j=1}^3 \frac{\partial V_{t+1}^{i,o}}{\partial x_j} \frac{dx_{j,t+1}^{**}}{dY_t} \\ \tilde{V}_{e_t}^{i,y} &= -E(\sigma_t) \theta_t^i + \delta \sigma_t^n f'(e_t) E(\bar{\omega}_{t+1}) + \beta l_t \sum_{j=1}^3 \frac{\partial V_{t+1}^{i,o}}{\partial x_j} \frac{dx_{j,t+1}^{**}}{de_t}\end{aligned}$$

as x_{t+1}^{**} only depends upon $g_{t+1} = \frac{l_t}{\bar{\sigma}_t(M_t)}$, (and in the case of endogenous education, the pivotal voter is unaffected by e_t) then the above reduces to:

$$\begin{aligned}\tilde{V}_{M_t}^{i,y} &= -c'(M_t) + \theta_t^i (\alpha + \gamma) \frac{l_{t-1}}{(1 - \widehat{M}_t)} - \theta_t^i \lambda_t + \left(\frac{\beta l_t}{(1 - \widehat{M}_{t+1})} (\alpha / \theta_t^i + \gamma) - e_t \right) (\sigma_t^m - \sigma_t^n) \theta_t^i + \\ &\quad + \beta l_t \sum_{j=1}^3 \frac{\partial V_{t+1}^{i,o}}{\partial x_j} \frac{dx_{j,t+1}^{**}}{dg_{t+1}} \frac{l_t (\sigma_t^m - \sigma_t^n)}{\bar{\sigma}_t^2} \\ \tilde{V}_{Y_t}^{i,y} &= -\theta^i + b'(Y_t) \\ \tilde{V}_{e_t}^{i,y} &= -E(\sigma_t) \theta_t^i + \delta \sigma_t^n f'(e_t) E(\bar{\omega}_{t+1})\end{aligned}$$

Given that the F.O.C.s are invariant to h_s for all $s < t$ and that the optimum is unique, then the the individual ideal policy must be invariant to h_s for all $s < t$ as well. Q.E.D.

C.4 Lemma 5-6

Lemma 5. *The strictly concave function $V(x_t, ; \theta_t^i, \varphi)$ satisfies SM and SSCP in $(x_t; \theta_t^i)$ for all $\theta_t^i \in \Theta$ and for all $\varphi \in \Phi$.*

Proof. Given the definition of V_t^i (\tilde{V}_t^i for the full model):

$$V_t^i = V(x_t; \theta_t^i, \varphi) = \begin{cases} V^{i,y} & \text{if young then } \theta = \theta_t^i \\ V^o & \text{if old then } \theta = -1 \end{cases}$$

With $x_{1t} = M_t$, $x_{2t} = -Y_t$, $x_{3t} = -e_t$ and for an arbitrarily large $\kappa > 0$. Notice that κV^o implies the same preferences as \tilde{V}^o . First I need to show that each component $V_t^{i,y}$, V_t^o ($\tilde{V}_t^{i,y}$, \tilde{V}_t^o) satisfies the required property and then I will show that it also holds for the overall function V_t^i (\tilde{V}_t^i). Recall the objective function of a young individual in the baseline model is:

$$V_t^{i,y} = (1 - \tau_t)\omega_t^i + b(Y_t) - c(M_t) + \beta l_t \left(\frac{(\alpha + \gamma\theta_t^{i,y})\bar{\sigma}_t}{(1 - \widehat{M}_{t+1})} + d(Y_{t+1}^{**}) - c(M_{t+1}^{**}) \right)$$

and in the full model is:

$$\begin{aligned} \tilde{V}_t^{i,y} &= (1 - \tau_t)f(e_{t-1})\omega_t^i + b(Y_t) - c(M_t) + \\ &\beta l_t \left((\alpha + \gamma\theta_t^{i,y}) \frac{\bar{\sigma}_t}{(1 - \widehat{M}_{t+1})} + d(Y_{t+1}^{**}) - c(M_{t+1}^{**}) \right) + \delta^y \sigma_t^n f(e_t)\bar{\omega}_{t+1} \end{aligned}$$

Below I derive the conditions for the full model with endogenous education. Given these conditions, the ones for the baseline model are straightforward. Given that the function $\tilde{V}_t^{i,y}$ is C^2 sufficient conditions for *SM* and *SSCP* are simply related to the sign of the cross derivatives and in particular: $\tilde{V}_{e_t M_t}^{i,y}, \tilde{V}_{e_t Y_t}^{i,y} \leq 0$, $\tilde{V}_{M_t Y_t}^{i,y} \geq 0$ for all $x \in X$ and all $\theta_t^i \in \Theta$ and $\tilde{V}_{e_t \theta_t^i}^{i,y}, \tilde{V}_{Y_t \theta_t^i}^{i,y} < 0$, $\tilde{V}_{M_t \theta_t^i}^{i,y} > 0$ or all $x \in X$ and all $\theta_t^i \in \Theta$. The first derivatives are:

$$\begin{aligned} \tilde{V}_{M_t}^{i,y} &= -c'(M_t) + \theta_t^i (\alpha + \gamma) \frac{l_{t-1}}{(1 - \widehat{M}_t)} - \theta_t^i \lambda_t + \left(\frac{\beta l_t (\alpha / \theta_t^i + \gamma)}{(1 - \widehat{M}_{t+1})} (\alpha / \theta_t^i + \gamma) - e_t \right) (\sigma_t^m - \sigma_t^n) \theta_t^i + \\ &+ \beta l_t \left(d(Y_{t+1}^{**}) \frac{\partial Y_{t+1}^{**}}{\partial M_t} - c(M_{t+1}^{**}) \frac{\partial M_{t+1}^{**}}{\partial M_t} \right) \\ \tilde{V}_{Y_t}^{i,y} &= -\theta_t^i + b'(Y_t) \end{aligned}$$

$$\tilde{V}_{e_t}^{i,y} = -\bar{\sigma}_t \theta_t^i + \delta \sigma_t^n f'(e_t) \bar{\omega}_{t+1}$$

Calculate the cross derivatives of $\tilde{V}_t^{i,y}$ with respect to each two policy dimensions:

$$\tilde{V}_{e_t M_t}^{i,y} = -\theta_t^i (\sigma_t^m - \sigma_t^n) \leq 0$$

$$\tilde{V}_{e_t Y_t}^{i,y} = V_{Y_t M_t}^{i,y} = 0$$

And with respect to each policy dimension and the parameter θ_t^i (recall that x_t^{**} is a function of solely g_{t+1} and it is therefore invariant to θ_t^i):

$$\begin{aligned}\tilde{V}_{e_t \theta_t^i}^{i,y} &= -E(\sigma_t) < 0 \\ \tilde{V}_{M_t \theta_t^i}^{i,y} &= \frac{(\alpha + \gamma) l_{t-1}}{(1 - \widehat{M}_t)} - \lambda_t - \left(e_t - \frac{\beta l_t \gamma}{(1 - \widehat{M}_{t+1})} \right) (\sigma_t^m - \sigma_t^n) > 0 \\ \tilde{V}_{Y_t \theta_t^i}^{i,y} &= -1 < 0\end{aligned}$$

Notice that the FOCs with respect to M_t imply that an interior solution with a partially open migration policy $M_t > 0$ can exist even if immigrants “contribute less than what they take out” in the current period, or more precisely if at a given policy (e_t, Y_t, M_t) a marginal increase in the number of migrants at constant e_t, Y_t implies a rise in the income tax rate. This is true because a native individual of working age will have a future benefit from immigration $\frac{\beta l_t (\sigma_t^m - \sigma_t^n)}{(1 - \widehat{M}_{t+1})} (\alpha + \gamma \theta_t^i)$ which incorporates the fact that he will partially internalize the positive effect of immigration today on the governmental budget constraint in the following period through the adjustment in the pension system. This finding solves in this model the dichotomy between “skilled migration” and “unskilled migration” in the patterns of attitude towards immigration and income that are described in the traditional models such as Facchini and Mayda (2008); in my model the attitude towards migration improve with income even if the immigrants are a net burden for the society in the short run, because as long as the Bismarkian component of the pension system is positive ($\gamma > 0$) this imply that the future benefits of immigration are increasing with income.

The next step is to state the elderly’s objective function and calculate its first derivatives. Using the formulas for C_{t+1}^o we get:

$$\tilde{V}^o = l_{t-1} [d(Y_t) - c(M_t)] + \delta^o E(k_t^i) f(e_t) \bar{\omega}_{t+1}$$

First derivatives are:

$$\tilde{V}_{e_t}^o = E(k_t^i) f'(e_t) E(\omega_{t+1}) > 0$$

$$\tilde{V}_{M_t}^o = -l_{t-1} c'(M_t) < 0$$

$$\tilde{V}_{Y_t}^o = l_{t-1} d'(Y_t) > 0$$

Cross derivatives:

$$\tilde{V}_{e_t M_t}^o = \tilde{V}_{e_t Y_t}^o = \tilde{V}_{Y_t M_t}^o = 0$$

Notice that the preferences for (M_t, Y_t, e_t) are the same for all elderly individuals. Now I can show that the function $\tilde{V}(x_t; \theta_t^i, \varphi)$ satisfies (i) *SM* and (ii) *SSCP* in $(x_t; \theta_t^i)$.

(i) *SM*. It follows from *SM* of $\tilde{V}_t^{i,y}$ and \tilde{V}_t^o . (ii) *SSCP*. I need to show that if $x_t' \geq x_t''$, $x_t' \neq x_t''$ and $\theta_t' > \theta_t''$ then

$$\tilde{V}(x_t'; \theta_t'', \varphi) - \tilde{V}(x_t''; \theta_t'', \varphi) > \tilde{V}(x_t'; \theta_t', \varphi) - \tilde{V}(x_t''; \theta_t', \varphi)$$

(ii) (a) $\theta_t', \theta_t'' \neq -1$. *SSCP* follows from *SSCP* of $V_t^{i,y}$ and V_t^o . (ii) (b) $\theta_t' \neq -1$, $\theta_t'' = -1$. Notice that $\tilde{V}(x_t'; \theta_t'', \varphi) - \tilde{V}(x_t''; \theta_t'', \varphi) > 0$ is always true under the assumption previously stated so it is sufficient to choose κ large enough such that *SSCP* holds trivially. (ii) (c) $\theta_t', \theta_t'' = -1$. Straightforward.

Also notice that under the restriction the parameter set $\Theta_t := \left\{ \theta_t^i | \theta_t^i = \begin{array}{ll} \theta_t^{i,y} & \text{if } \text{age} = y \\ -1 & \text{if } \text{age} = o \end{array} \right\}$ is a totally ordered set. Q.E.D.

Lemma 6. (i) a coalitional equilibrium for the voting game exists. Moreover, (ii) in any coalitional equilibrium at time t the equilibrium policy is the unique ideal point of the median voter $x_t^v = x_t^* \in I(v)$. (iii) The parameter that identifies the pivotal voter is weakly increasing in g_t .

Proof. (i) Lemma 5 and the definitions of the policy space X_t and of the parameter space Θ_t imply that all the conditions for the existence of a coalitional equilibrium in Theorem 1 are satisfied. (ii) The strict concavity of the objective function of each working age individual and the convexity of X imply that the pivotal voter has a unique ideal policy, and therefore that is the only policy vector that can be implemented in any coalitional equilibrium of the voting game. (iii) If $g_t \leq 1$, then the median individual in the totally ordered set Θ_t solves $Q(\theta_t^v)n_t + l_{t-1}(m_{t-1} + n_{t-1}) = [1 - Q(\theta_t^v)]n_t^{21}$. Rearranging and solving for θ_t^v one gets $\theta_t^v = Q^{-1}\left(\frac{1-g_t}{2}\right)$ which is weakly positive and weakly decreasing in g_t . If $g_t > 1$, then the parameter of the pivotal voter is fixed at $\theta_t^i = -1$. Q.E.D.

C.5 Comparative statics

C.5.1 Unanticipated rise in the longevity of the retired population

Theorem 7. The effects of an increase in the life expectancy l_{t-1} is weakly positive on the spending policy and ambiguous on the immigration policy. Moreover, there exists a threshold $\hat{g} \in [0, 1]$ such that if $g_t \geq \hat{g}$ then the effect on immigration policy is unambiguously (weakly) negative and the effect on the tax rate is strictly positive.

²¹The tie-breaking rule assumed in section 2.1.2 ensures that this formula is correct even if the number of voters is even.

Proof. Calculate the cross derivatives of $V_t^{i,y}$ ($\tilde{V}_t^{i,y}$) with respect to each policy dimension M_t, Y_t, e_t and the parameter l_{t-1} .

$$\tilde{V}_{M_t l_{t-1}}^{i,y} = \frac{\theta_t^v (\alpha + \gamma)}{(1 - M_t^*)} > 0$$

$$\tilde{V}_{Y_t l_{t-1}}^{i,y} = 0$$

$$\tilde{V}_{e_t l_{t-1}}^{i,y} = 0$$

(i) Effects at fixed g_t . Consider a totally ordered subset $\Phi^j := \{\varphi \in \Phi \mid \varphi_i = \hat{\varphi}_i \forall i \neq j\}$ where j is the position of the longevity parameter in the vector φ , i.e. $\varphi_j = l_{t-1}$. Notice that $\tilde{V}(x; \theta_t^i, \varphi)$ in Φ^j satisfies *SM* in (x_t) and *SSCP* in $(x_t; \varphi)$, it also satisfies *SM* in (z_t) and *SSCP* in $(z_t; \varphi)$ for $z_t = (x_{1t}, -x_{2t}, -x_{3t})$. Using Theorem 3, one gets $\Delta M_t \geq 0$, $\Delta Y_t = 0$, $\Delta e_t = 0$, $\Delta \tau_t \leq 0$.

(ii) Recall that

$$g_t = \frac{l_{t-1}}{\bar{\sigma}_{t-1}}$$

which is increasing in l_{t-1} . Hence a rise in l_{t-1} corresponds to a change in the voter distribution such that the new median voter is lower than before. Hence $\Delta M_t \leq 0$, $\Delta Y_t \geq 0$, $\Delta e_t \geq 0$, $\Delta \tau_t \geq 0$. Total effect: ambiguous for M_t . But $\Delta e_t \geq 0$, $\Delta Y_t \geq 0$. Finally notice that if $g_t = 1$ then $\theta_t^v = 0$ and $\tilde{V}_{M_t l_{t-1}}^{i,y} = 0$, which means that the “budget effect” is equal to zero and therefore the the political effect (weakly) dominates. Hence there exists a threshold $\hat{g} \in [0, 1]$ (possibly $\hat{g} = 1$) such that if $g_t \geq \hat{g}$ then the effect on immigration policy is unambiguously (weakly) negative. Q.E.D.

C.5.2 Unanticipated fall in the natural growth rate of the native population

Theorem 8. *The effects of a decrease in the growth rate of the working age population is a weak decrease in the openness of the immigration policy and and a weak increase in spending in the imperfect Public Good and in the tax rate.*

Proof. Calculate the cross derivatives of $V_t^{i,y}$ ($\tilde{V}_t^{i,y}$) with respect to each policy dimension M_t, Y_t, e_t and the parameter σ_{t-1}^n .

$$\tilde{V}_{e_t \sigma_{t-1}^n}^{v,y} = \tilde{V}_{Y_t \sigma_{t-1}^n}^{v,y} = \tilde{V}_{M_t \sigma_{t-1}^n}^{v,y} = 0$$

Recall that, but the share of “old” voters decreases at each point in time:

$$g_t = \frac{l_{t-1}}{\bar{\sigma}_{t-1}}$$

which is decreasing in σ_{t-1}^n . Using Theorem 3, a fall in σ_{t-1}^n implies $\Delta M_t \leq 0$, $\Delta Y_t \geq 0$, $\Delta e_t \geq 0$, $\Delta \tau_t \geq 0$. Q.E.D.

C.5.3 Rise in the life expectancy of the working age population

Theorem 9. *The effects of an increase in the life expectancy l_t is ambiguous on the immigration policy. If voters are “naive” then the effect is weakly positive. If the birth rate of the native is the same as the one of the immigrants, then there is no effect.*

Proof. One needs to analyze the cross derivative of $\tilde{V}_t^{i,y}$ with respect to M_t , Y_t , e_t and the parameter l_t . Define $\tilde{\pi}_{t+1}^{i,o} = (\alpha + \gamma\theta_t^i) \frac{\bar{\sigma}_t}{1 - \widehat{M}_{t+1}} + d(Y_{t+1}) - c(M_{t+1})$ (this is only relevant for the case of endogenous public education).

$$\begin{aligned} \tilde{V}_{M_t l_t}^{i,y} &= \underbrace{\frac{\beta(\alpha + \gamma\theta_t^i)}{(1 - \widehat{M}_{t+1})} (\sigma_t^m - \sigma_t^n)}_{\text{preferences effect}} - \underbrace{\frac{\beta 2l_t}{\bar{\sigma}_t^2} \left[\sum_{j=1}^3 \frac{d\tilde{\pi}_{t+1}^{i,o}}{dx_{j,t+1}^{**}} \frac{\partial x_{j,t+1}^{**}}{\partial \theta_{t+1}^v} \right] \frac{d\theta_{t+1}^v}{dg_{t+1}} (\sigma_t^m - \sigma_t^n)}_{\text{sophisticated effect}} \\ &\quad - \underbrace{\frac{\beta l_t^2}{\bar{\sigma}_t^2} \left[\sum_{j=1}^3 \frac{d^2 \tilde{V}_{t+1}^{i,o}}{dx_{j,t+1}^{**}} \frac{\partial x_{j,t+1}^{**}}{\partial \theta_{t+1}^v} \frac{dx_{j,t+1}^{**}}{dl_t} + \frac{d\tilde{\pi}_{t+1}^{i,o}}{dx_{j,t+1}^{**}} \frac{d}{dl_t} \left(\frac{\partial x_{t+1}^{**}}{\partial \theta_{t+1}^v} \right) \right] \frac{d\theta_{t+1}^v}{dg_{t+1}} (\sigma_t^m - \sigma_t^n)}_{\text{sophisticated effect}} \end{aligned}$$

$$\tilde{V}_{Y_t l_t}^{i,y} = 0$$

$$\tilde{V}_{e_t l_t}^{i,y} = 0$$

First of all notice that if $\sigma_t^m = \sigma_t^n$, then the cross derivatives are equal to zero and g_{t+1} is unaffected by changes in l_t , therefore a shock on l_t has no effects on the equilibrium outcome.

If $\sigma_t^m \geq \sigma_t^n$ the sign of $\tilde{V}_{M_t l_t}^{i,y}$ is ambiguous. The reason is that two different effects enter the formula. On one hand an increase in life expectancy increase the relative weight of consumption after retirement in the utility function of a working age individual, increasing the desirability of better future pensions and therefore of an increase in the number of immigrants at time t (“preferences effect”). On the other hand there is a “sophisticated effect” that concerns the effect of current political choices on future outcomes. If the “preferences” effect dominates, then using the same procedure as in C.5.1 I can show that $\tilde{V}(x_t; \theta_t^i, \varphi)$ satisfies *SM* in (x_t) and *SSCP* in $(x_t; \varphi)$ in Φ^j where $\varphi_j = l_t$, it also satisfies *SM* in (z_t) and *SSCP* in $(z_t; l_t)$, for $z_t = (x_{1t}, -x_{2t}, -x_{3t})$, which by Theorem 3 implies $\Delta M_t \geq 0, \Delta \tau_t \leq 0$ and no effect on the other variables. If the “sophisticated” effect dominates in a similar way one can show that $\Delta M_t \leq 0, \Delta \tau_t \geq 0$. . If agents are “naive” then there is no “sophisticated effect” because $\frac{d\theta_{t+1}^v}{dg_{t+1}} = 0$ and therefore an increase in l_t has a weakly positive effect on the openness of the immigration policy. Q.E.D.

C.5.4 Effects of an increase in σ_t^n .

Theorem 10. *The effects of a decrease in the birth rate of the native population σ_t^n is a weak increase in the openness of the immigration policy and a fall in the tax rate. The effects of a decrease in the birth rate of the native population σ_t^n is ambiguous on the immigration policy. If voters are “naive”, then the effect is weakly positive.*

Proof. Calculate the cross derivatives of $V_t^{i,y}$ ($\tilde{V}_t^{i,y}$) with respect to each policy dimension M_t, Y_t, e_t and the parameter σ_t^n . $\tilde{v}_{t+1}^{i,o}$ is defined as in C.5.3.

$$\begin{aligned} \tilde{V}_{M_t \sigma_t^n}^v &= \underbrace{e_t \theta_t^v}_{b.e.} - \underbrace{\frac{\beta l_t (\alpha + \gamma \theta_t^v)}{(1 - \widehat{M}_{t+1})}}_{\text{preferences effect}} + \\ &+ \underbrace{\frac{\beta l_t}{\bar{\sigma}_t^2} \left[d'(Y_{t+1}^{**}) \frac{\partial Y_{t+1}^{**}}{\partial \theta_{t+1}^v} - \tilde{c}'(M_{t+1}^{**}) \frac{\partial M_{t+1}^{**}}{\partial \theta_{t+1}^v} \right] \frac{d\theta_{t+1}^v}{dg_{t+1}} \left[1 + \frac{2(1 - M_t)(\sigma_t^m - \sigma_t^n)}{\bar{\sigma}_t} \right]}_{\text{sophisticated effect}} + \\ &+ \underbrace{\frac{\beta l_t (\sigma_t^m - \sigma_t^n)}{\bar{\sigma}_t^2} \left\{ \left[\sum_{j=1}^3 \frac{d^2 \tilde{\pi}_{t+1}^{i,o}}{dx_{j,t+1}^{**}} \left(\frac{\partial x_{j,t+1}^{**}}{\partial \theta_{t+1}^v} \right)^2 \frac{dx_{j,t+1}^{**}}{d\sigma_t} + \frac{d\tilde{\pi}_{t+1}^{i,o}}{dx_{j,t+1}^{**}} \frac{\partial x_{t+1}^{2**}}{\partial (\theta_{t+1}^v)^2} \right] \left(\frac{d\theta_{t+1}^v}{dg_{t+1}} \right)^2 \right\}}_{\text{sophisticated effect}} + \\ &+ \underbrace{\left[\sum_{j=1}^3 \frac{d\tilde{\pi}_{t+1}^{i,o}}{dx_{j,t+1}^{**}} \frac{\partial x_{j,t+1}^{**}}{\partial \theta_{t+1}^v} \right] \frac{d^2 \theta_{t+1}^v}{d(g_{t+1})^2}}_{\text{sophisticated effect}} \end{aligned}$$

Notice that in this case the effect of σ_t^n on future outcomes reduces to the effects on the pivotal voter (see Lemma 9). Also notice that $d'(Y_{t+1}^{**}) \frac{\partial Y_{t+1}^{**}}{\partial \theta_{t+1}^v} - \tilde{c}'(M_{t+1}^{**}) \frac{\partial M_{t+1}^{**}}{\partial \theta_{t+1}^v} \leq 0$ because of Lemma 12.

Hence for $\sigma_t^m = \sigma_t^n$ the sophisticated effect is weakly positive hence the overall effect is ambiguous. If agents are naive then $\frac{d\theta_{t+1}^v}{dg_{t+1}} = 0$ and the overall sign is negative if and only if:

$$\frac{l_t p_{t+1}^v}{e_t} \geq \frac{\theta_t^v}{\beta}$$

i.e. the total transfer in pensions to the median voter at time $t + 1$ is sufficiently large in comparison with his tax expenditure in education per pupil (notice that this is always true in the basic model with no public education).

$$\tilde{V}_{Y_t \sigma_t^n}^v = 0$$

$$\tilde{V}_{e_t \sigma_t^n}^v = -\theta_t^v(1 - M_t) + \delta f'(e_t)\bar{\omega}_{t+1} > 0$$

as long as $e_t > 0$ at the equilibrium (this condition is only relevant for the extended model). $\tilde{V}(x_t; \theta_t^i, \varphi)$ satisfies *SM* in (z_t') and *SSCP* in $(z_t'; \varphi)$ in Φ^j ($\varphi_j = \sigma_t^n$), where $z_t' = (-x_{1t}, -x_{2t}, -x_{3t})$, it also satisfies *SM* in (z_t'') and *SSCP* in $(z_t''; \varphi)$ where $z_t'' = (-x_{1t}, -x_{2t}, x_{3t})$. By Theorem 3 a fall in σ_t^n implies: $\Delta M_t \geq 0$, $\Delta Y_t = 0$, $\Delta e_t \leq 0$, $\Delta \tau_t \leq 0$. Q.E.D.

C.5.5 Steady-state

Lemma 13. *An equilibrium for the OGM exists and the economy always converges to a steady-state. Moreover, if $\sigma_t^m = \sigma_t^n = \sigma_t$ then the political equilibrium at time t is independent of the previous political choices and the economy converges immediately to the unique steady state after a shock.*

Proof. Fix the value of the parameters. Notice that a temporary coalitional equilibrium always exists because of Lemma 6, therefore an equilibrium of the OGM also exists because it is simply a sequence of such temporary equilibria. The equilibrium political choice at time t depends uniquely on the value of the state g_t . Notice that g_t depends on the parameters l_{t-1} , σ_t^m , σ_t^n and on the choice variable M_{t-1} but is independent of anything else. This implies that the evolution of g depends uniquely on the evolution of M . Notice that if $\sigma_t^m = \sigma_t^n = \sigma_t$ then the political equilibrium at time t is independent of the previous political choices because the state g_t is independent of history: $g_t = \frac{l_{t-1}}{\sigma_t} = g^*$, which implies in turn that the economy converges immediately to the steady state after a shock. Also notice that in this case the equilibrium is independent of the lagged value M_{t-1} , hence the steady-state is unique. If $\sigma_t^m > \sigma_t^n$ then this is no longer true and the convergence may take several periods. Finally notice that at constant parameters if $g_t = g_{t+1}$ for some t , then $g_{t+s} = g_t$ for all $s > 0$, i.e. $g_t = g_{t+1}$ is sufficient for a steady state. Suppose a steady state does not exist, i.e. $g_t \neq g_{t+1}$ for all t . If $g_{t+1} > g_t$ ($<$) then the pivotal voter $\theta_{t+1}^v \leq \theta_t^v$ (\geq) which using Lemma 9 implies $M_{t+1}^* \leq M_t^*$ (\geq). This implies in turn that $g_{t+2} \geq g_{t+1}$ (\leq). If $g_{t+2} = g_{t+1}$ then we have reached a steady state. If instead $g_{t+2} > g_{t+1}$ ($<$) the process continues recursively. There are two possibilities. Either the process stops because $g_{t+s} = g_{t+s+1}$ and a steady state is achieved or we have that $|g_{t+s+1} - g_{t+s}| < |g_{t+s} - g_{t+s-1}|$. Suppose this is not true, then it must be true that $|M_{t+s+1}^* - M_{t+s}^*| \geq |M_{t+s}^* - M_{t+s-1}^*|$. The process above if no steady state exists implies $M_{t+s+1}^* < M_{t+s}^*$ ($>$) for all $s \geq 0$ which implies in turn that the previous conditions simplifies to $M_{t+s+1}^* - M_{t+s}^* \leq M_{t+s}^* - M_{t+s-1}^*$ (\geq). This implies that if M is unbounded it will diverge to $-\infty$ ($+\infty$). But $M_t \in [\underline{M}, \bar{M}]$ by assumption, hence the process will stop at $M_{t+z}^* = \underline{M}$ (\bar{M}) for some $z \geq 0$. The generalized monotone comparative statics implies that the political equilibrium in the next period will also display $M_{t+z+1}^* = \underline{M}$ (\bar{M}), which means $M_{t+z+1}^* = M_{t+z}^*$ and implies $g_{t+z+1} = g_{t+z}$. Hence the system has achieved a steady state, and this is a contradiction to the initial statement. Q.E.D.

Theorem 7b. *The long-run effect of an increase in l_{t-1} on the immigration policy has same sign as the short-run effect and a weakly larger magnitude. If $g_t \geq \hat{g}$ then the effect on immigration policy is (weakly) negative and the effect on the public spending and the tax rate is strictly positive.*

Proof. If at time t the “Budget Effect” prevails at time t , i.e. $M_t \geq M_{t-1}$, then $g_{t+1} \leq g_t$ and $\theta_{t+1}^v \geq \theta_t^v$ by Lemma 6. Using Theorem 2 one gets $M_{t+1} \geq M_t$ and $Y_{t+1} \leq Y_t$. Notice that this implies recursively $\theta_{t+s+1}^v \geq \theta_{t+s}^v$ and therefore $M_{t+s+1} \geq M_{t+s}$ and $Y_{t+s+1} \leq Y_{t+s}$ for all $s > 0$. Hence I can conclude that at the new steady state $M^{ss} \geq M_t \geq M_{t-1}$ and $Y^{ss} \leq Y_t$ but $Y^{ss} \geq Y_{t-1}$, which means that the long run effect of an increase in l_{t-1} is positive on the openness of the immigration policy and ambiguous on the public spending variable, which increases at the time in which the shock occurs and falls in the following periods. Similarly one can show that if at time t the “Political Effect” dominates, then at the new steady state $M^{ss} \leq M_t \leq M_{t-1}$ and $Y^{ss} \geq Y_t \geq Y_{t-1}$.

Theorem 8b. *The long-run effect of a decrease in the growth rate of the native population is a weak decrease in the openness of the immigration policy and a weak increase in spending in the imperfect Public Good and in the tax rate. All the effects have weakly larger magnitude relative to the short-run effects.*

Proof. Similar to the previous case.

C.5.6 Partially funded pension system

Theorem 16. *The effect of a marginal decrease in the size of the public pension system in the short run is an increase in the restrictiveness of the immigration policy. In the long run the increase in restrictions to immigration and an increase in public spending in the imperfect Public Good. The total effect on the tax rate is ambiguous.*

Proof. It is sufficient to show that the objective function $V_t^{i,y} (\tilde{V}_t^{i,y})$ satisfies the SCP in $\alpha (\gamma)$. Calculate the cross derivatives of $V_t^{i,y} (\tilde{V}_t^{i,y})$ with respect to $M_t, Y_t (e_t)$ and the parameter $\alpha (\gamma)$.

$$\begin{aligned}\tilde{V}_{M_t \alpha}^{i,y} &= \frac{\theta_t^i l_{t-1}}{(1 - M_t^*)} + \frac{\beta l_t (\sigma_t^m - \sigma_t^n)}{(1 - M_{t+1}^*)} \geq 0 \\ \tilde{V}_{Y_t \alpha}^{i,y} &= 0 \\ \tilde{V}_{e_t \alpha}^{i,y} &= 0\end{aligned}$$

Hence given a subset Φ^j defined as in C.5.1 with $\varphi_j = \alpha (\gamma)$, one can show that \tilde{V}_t^i satisfies the SCP with respect to $(x_t; \varphi)$ and to $(z_t; \varphi)$ with $z_t = (x_{1t}, -x_{2t} - x_{3t})$. Using Theorem 3 this implies that the short-run effect of a fall in $\alpha (\gamma)$ is $\Delta M_t \leq 0$. In the long run the effect of a weak fall in M_t is a rise in g_t , which implies in turn a “political effect” at time $t + 1$ with

$\Delta M_t \leq 0 \Delta Y_t \geq 0$, which implies recursively the same effect for all the periods after $t + 1$ until the economy converges to a new steady state. Notice that the effect on the tax rate is ambiguous at time t because of a simultaneous reduction of the total cost of pension (as α falls) and of the workforce (because of the fall in M_t), while from time $t + 1$ the tax rate increases until a new steady state is achieved, because of the fall in the workforce and the rise in public spending. Therefore the overall long-run effect is ambiguous. Q.E.D.

Theorem 18. *The effects of an increase in the longevity of the retired population l_{t-1} and /or of a decrease in the growth rate of native population σ_{t-1}^n is a weak increase in the public spending in education per child e_t .*

Proof. Straightforward from C.5.1 and C.5.2.

C.6 Welfare Analysis: Immigration Policy

Theorem 22. *For any Social Welfare Function $SWF(M_t, Y_t; \varphi)$ that assigns a strictly positive weight to each native individual of working age, there exist $\check{g}_t \in [0, 1]$ such that if $g_t \geq \check{g}_t$ then a marginal tightening in the immigration policy caused by a change in the equilibrium outcome reduces the Social Welfare.*

Proof. Notice that the theorem above is stated for the baseline model without endogenous education. Here I show the proof for the full model with SWF denoted by $\widetilde{SWF}(M_t, Y_t, e_t; \varphi)$. The proof of the baseline model is straightforward. Define the overall weight of each generation as follows:

$$\begin{aligned} \int_0^{\bar{\theta}_t} \mu_t^y(\theta^i) q_t(\theta^i) d\theta^i &= \mu^y \\ \int_0^{\bar{\theta}_{t-1}} \mu_t^o(\theta^i) q_{t-1}(\theta^i) d\theta^i &= \mu^o \\ \int_0^{\bar{\theta}_{t+1}} \mu_{t+1}^y(\theta^i) q_{t+1}(\theta^i) d\theta^i &= \mu^c \end{aligned}$$

Normalize $\mu^y = 1$ and assume $\mu^y + \mu^o + \mu^c = \mu$ with $0 < \mu < \infty$. This can be done without loss of generality under the assumption that $\mu_t^y(\theta^i) > 0$ for each native individual of working age. Suppose the equilibrium policy x_t^* is such that $\underline{M}_t < M_t < \overline{M}_t$, which implies that a marginal opening in the immigration policy is feasible. If the difference between the marginal social benefit for the society from an increase in M_t and the marginal utility of M_t for the pivotal

voter evaluated at the equilibrium policy vector is strictly positive, i.e.

$$\widetilde{WD}_{M_t}(M_t^*, Y_t^*, e_t^*; \varphi) = \widetilde{SWF}_{M_t}(M_t^*, Y_t^*, e_t^*; \varphi) - V_{M_t}^{v,y}(M_t^*, Y_t^*, e_t^*; \varphi) > 0$$

then a marginal increase in the openness of the immigration policy M_t is, ceteris paribus, beneficial for the society. Notice that if $\underline{M}_t < M_t < \overline{M}_t$, then $V_{M_t}^{v,y}(M_t^*, Y_t^*, e_t^*; \varphi) = 0$ from the F.O.C. The social benefit for the society from an increase in M_t is given by:

$$\begin{aligned} \widetilde{SWF}_{M_t} &= \int_0^{\bar{\theta}_t} \mu_t^y(\theta_t^i) V_{M_t}^y(M_t^*, Y_t^*, e_t^*; \theta_t^i, \varphi) q_t(\theta_t^i) d\theta_t^i + \int_0^{\bar{\theta}_{t-1}} \mu_t^o(\theta_{t-1}^i) V_{M_t}^o(e_t, M_t, Y_t; \theta_{t-1}^i, \varphi) q_{t-1}(\theta_{t-1}^i) d\theta_{t-1}^i + \\ &\quad + \int_0^{\bar{\theta}_{t+1}} \mu_{t+1}^y(\theta_{t+1}^i) E[V_{M_t}^y(M_{t+1}, Y_{t+1}, e_{t+1}; \theta_{t+1}^i, \varphi)] q_{t+1}(\theta_{t+1}^i) d\theta_{t+1}^i = \end{aligned}$$

First of all notice that the linearity in consumption of the utility function implies

$$E[V_{M_t}^{i,y}(M_{t+1}, Y_{t+1}, e_{t+1}; \theta_{t+1}^i, \varphi)] = V_{M_t}^{i,y}(M_{t+1}, Y_{t+1}, e_{t+1}; \bar{\theta}_{t+1}, \varphi) \text{ hence}$$

$$\int_0^{\bar{\theta}_{t+1}} \mu_{t+1}^y(\theta_{t+1}^i) E[V_{M_t}^{i,y}(M_{t+1}, Y_{t+1}, e_{t+1}; \theta_{t+1}^i, \varphi)] q_{t+1}(\theta_{t+1}^i) d\theta_{t+1}^i =$$

$E[\mu_{t+1}^y(\theta_{t+1}^i)] V_{M_t}^{i,y}(M_{t+1}, Y_{t+1}, e_{t+1}; \bar{\theta}_{t+1}, \varphi)$. Moreover, notice that a change in x_t only affects the future generation through a fall in g_{t+1} , which has no effects neither on the budget constraint at time $t+1$ nor on the preferences of an individual (it only affects the political equilibrium at time $t+1$). Therefore $V_{M_t}^{i,y}(M_{t+1}, Y_{t+1}, e_{t+1}; \theta_{t+1}^i, \varphi)$ is independent of M_t and therefore *SSCP* implies: $V_{M_t}^{i,y}(M_{t+1}, Y_{t+1}, e_{t+1}; \bar{\theta}_{t+1}, \varphi) \geq$

$V_{M_t}^{i,y}(M_{t+1}, Y_{t+1}, e_{t+1}; \theta_{t+1}^v, \varphi)$ as long as $\theta_{t+1}^v \leq \bar{\theta}_{t+1}$. I use the latter result and I substitute the formulas for $V_{M_t}^{i,y}$, $V_{M_t}^{i,o}$ into \widetilde{WD}_{M_t} , and I can write the following inequality:

$$\begin{aligned} \widetilde{WD}_{M_t} &\geq \left[(\alpha + \gamma) \frac{l_{t-1}}{(1 - \widehat{M}_t^*)} - \lambda_t + \left(\frac{\beta l_t \gamma}{(1 - \widehat{M}_{t+1}^*)} - e_t \right) (\sigma_t^m - \sigma_t^n) \right] \left(\int_0^{\bar{\theta}_t} \theta_t^i \mu_t^y(\theta_t^i) q_t(\theta_t^i) d\theta_t^i - \theta_t^v \right) + \\ &\quad - c'(M_t) \int_0^{\bar{\theta}_{t-1}} \mu_t^o(\theta_{t-1}^i) g(\theta_{t-1}^i) d\theta_{t-1}^i = \end{aligned}$$

Notice that:

$$V_{M_t}^{v,y} = -c'(M_t) + \theta_t^v (\alpha + \gamma) \frac{l_{t-1}}{(1 - \widehat{M}_t)} - \theta_t^v \lambda_t + \left(\frac{\beta l_t}{(1 - \widehat{M}_{t+1})} (\alpha / \theta_t^v + \gamma) - e_t \right) (\sigma_t^m - \sigma_t^n) \theta_t^v$$

also represent the FOC of the optimization problem of the pivotal individual. This implies that if at the equilibrium $\underline{M}_t < M_t$ then:

$$(\alpha + \gamma) \frac{l_{t-1}}{(1 - \widehat{M}_t)} - \lambda_t + \left(\frac{\beta l_t \gamma}{(1 - \widehat{M}_{t+1})} - e_t \right) (\sigma_t^m - \sigma_t^n) \geq \frac{1}{\theta_t^v} \left(c'(M_t) - \frac{\alpha \beta l_t (\sigma_t^m - \sigma_t^n)}{(1 - \widehat{M}_{t+1})} \right)$$

Define the weighted average

$$E_{g_t}(\mu_t^y \theta^i) = \int_0^{\bar{\theta}_t} \lambda_t^y(\theta_t^i) \theta_t^i g_t(\theta_t^i) d\theta_t^i = h_{g_t} \int_0^{\bar{\theta}_t} \theta_t^i \dot{g}_t(\theta_t^i) d\theta_t^i = h_{g_t} E_{\dot{g}_t}(\theta_t)$$

for some p.d.f \dot{g}_t . Notice that $h_{g_t} E_{\dot{g}_t}(\theta_t) > 0$ under the assumption that $\mu_t^y(\theta_t^i) > 0$ for each native individual of working age. Therefore we can state the following inequality:

$$\widetilde{WD}_{M_t} \geq \left(c'(M_t) - \frac{\alpha \beta l_t (\sigma_t^m - \sigma_t^n)}{(1 - \widetilde{M}_{t+1})} \right) \frac{h_{g_t} E_{\dot{g}_t}(\theta_t) - \theta_t^v}{\theta_t^v} - c'(M_t) \mu^o$$

The F.O.C.s of the pivotal individual plus the assumption that immigrants are not net beneficiaries (in expectation) of the fiscal system imply $c'(M_t) - \frac{\alpha \beta l_t (\sigma_t^m - \sigma_t^n)}{(1 - \widetilde{M}_{t+1})} > 0$ for $\underline{M}_t < M_t < \bar{M}_t$. Finally notice that because of a previous assumption $c'(M_t) < \infty$ and that $\mu^o < 0$ imply:

$$\lim_{\theta_t^v \rightarrow 0^+} \left(c'(M_t) - \frac{\alpha \beta l_t (\sigma_t^m - \sigma_t^n)}{(1 - \widetilde{M}_{t+1})} \right) \frac{h_{g_t} E_{\dot{g}_t}(\theta) - \theta_t^v}{\theta_t^v} - c'(M_t) \mu^o = +\infty$$

Therefore, given a certain distribution of weights, either $\widetilde{WD}_{M_t}(M_t^*, Y_t^*, e_t^*; \varphi) > 0$ for all $\theta_t^v > 0$, else the Intermediate Value Theorem implies the existence of a threshold $0 < \check{\theta}_t < \bar{\theta}$ such that $\widetilde{WD}_{M_t}(M_t^*, Y_t^*, e_t^*; g_t, \varphi) = 0$. This threshold is always meaningful because I have previously assumed that there is at least one individual j with $e_t^j = 0$ and therefore $\theta_t^j = 0$. Moreover, $\widetilde{WD}_{M_t}(M_t^*, Y_t^*, e_t^*; \varphi)$ is strictly decreasing in θ_t^v because \widetilde{WD}_{M_t} is independent of θ_t^v and $V_{M_t}^{v,y}$ is strictly decreasing in θ_t^v because of the *SSCP*. Therefore if the wage distribution is such that $\theta_t^v < \check{\theta}_t$ then $\widetilde{WD}_{M_t}(M_t^*, Y_t^*, e_t^*; \varphi) > 0$ which implies that it would be welfare improving to increase M_t . Lastly, because of Lemma 6 (iii), a threshold $\check{g}_t \in [0, 1]$ exists, such that if $g_t \geq \check{g}_t$ iff $\theta_t^v < \check{\theta}_t$, which implies the result stated. Q.E.D.

C.7

In section 3 we have restricted the policy space in such a way that for all $x_t \in X$ the tax rate is internal $0 < \tau_t < k < 1$. Suppose that this assumption fails and at an equilibrium $\tau_t = k$. In this case it is hard to study the full model but something can be said about the baseline model with $x_t = (M_t, -Y_t)$. If $\tau_t = k$ the policy space is unidimensional, thus the traditional Median Voter Theorem applies. Consider the slope of the indifference curve of an individual i :

$$MRS_{M_t, Y_t}^{i,y} = - \frac{\left(\frac{\beta l_t}{(1 - \widetilde{M}_{t+1})} (\alpha / \theta_t^i + \gamma) \right) (\sigma_t^m - \sigma_t^n) \theta_t^i - c'(M_t)}{b'(Y_t)}$$

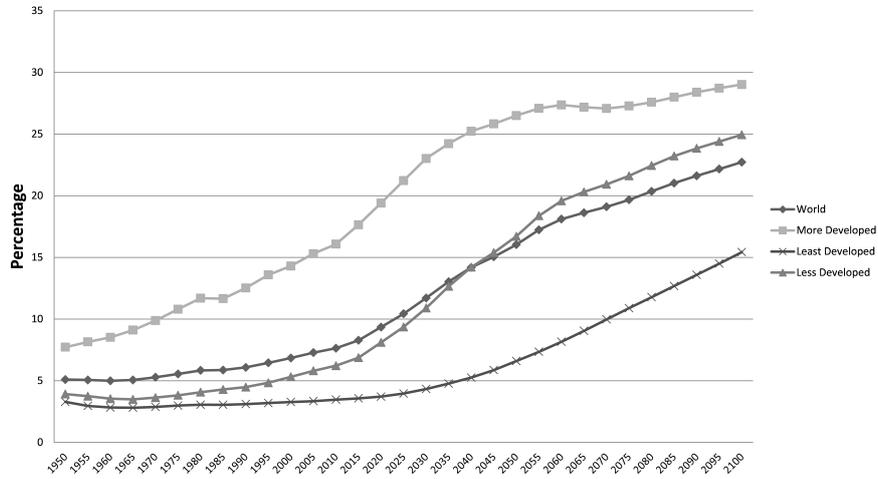
and

$$\frac{\partial MRS_{M_t, Y_t}^{i, y}}{\partial \theta_t^i} = - \frac{\left(\frac{\beta l_t}{(1 - M_{t+1})} (\alpha / \theta_t^i + \gamma) \right) (\sigma_t^m - \sigma_t^n)}{b'(Y_t)} \leq 0$$

One can use the *Spence-Mirrlees* condition to make predictions about the effects of changes in the pivotal voter on the equilibrium outcome. The result above is in line with the ones of most Benefit Adjustment Models. Specifically, an increase in the relative share of the elderly implies, *ceteris paribus*, an increase in public spending and a reduction of the immigration quota. In this framework I cannot derive analytical results about the effects of a rise in life expectancy, because this kind of shock typically involves not only a change in the pivotal voter but also in the position and slope of the budget constraint, such that the sign of the overall effect cannot be determined using the *Spence-Mirrlees* condition only.

Figures

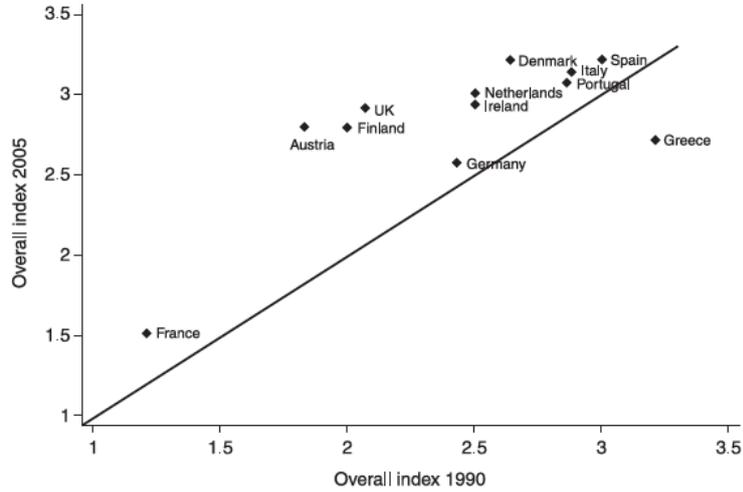
Figure 1: Share of Population of Age 65 or Older



Source: United Nations, 2015

Evolution of the share of population of age above 65 from 1950 to 2015 and the forecast for the next decades (source: United Nations, 2015).

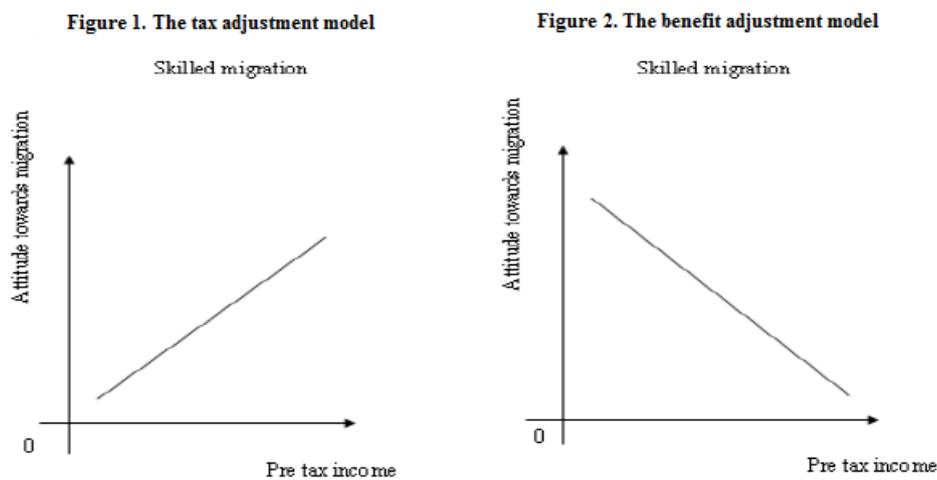
Figure 2: Trends in Migration Policies



Source: Boeri and Brucker, 2005

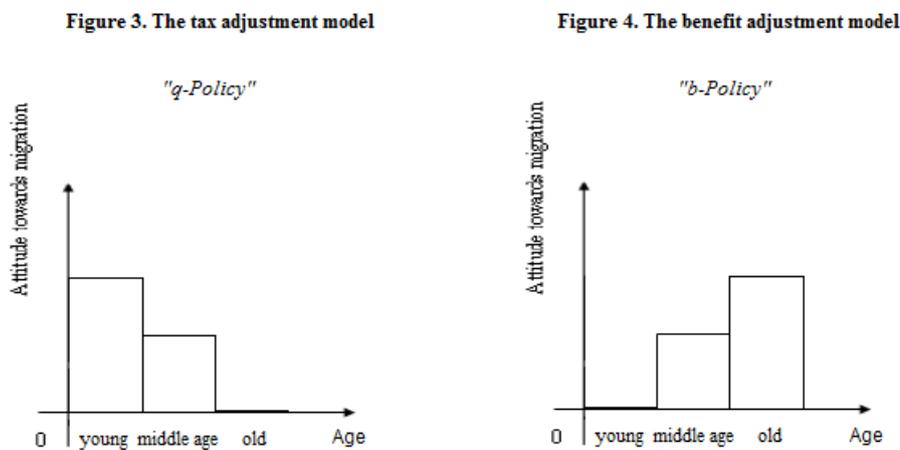
Comparison of the value of the index of tightness of immigration policies proposed by Boeri and Brucker (2005) in 1990 and 2005 for 12 European countries.

Figure 3-4: Effects of income on the attitudes towards immigration



Relationship between income and attitude towards immigration (preferred number of immigrants) in a Tax Adjustment Model (Fig. 1) and in a Benefit Adjustment Model (Fig. 2). Based on Facchini and Mayda (2008).

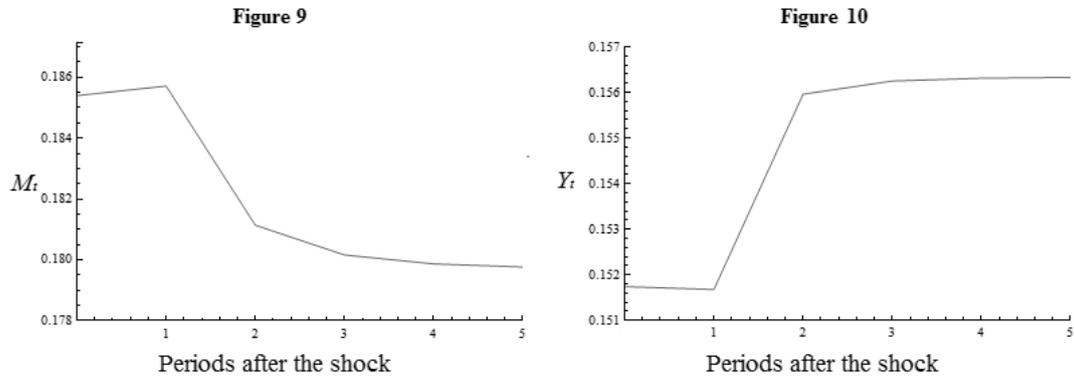
Figure 5-6: Effects of age on the attitudes towards immigration



Attitude towards immigration (preferred number of immigrants) of different generations of voters in a Tax Adjustment Model (Fig. 3) and in a Benefit Adjustment Model (Fig. 4). Based on Haupt and Peters (1998).

Figure 9-10: Long-Run Effects of an Increase in Life Expectancy

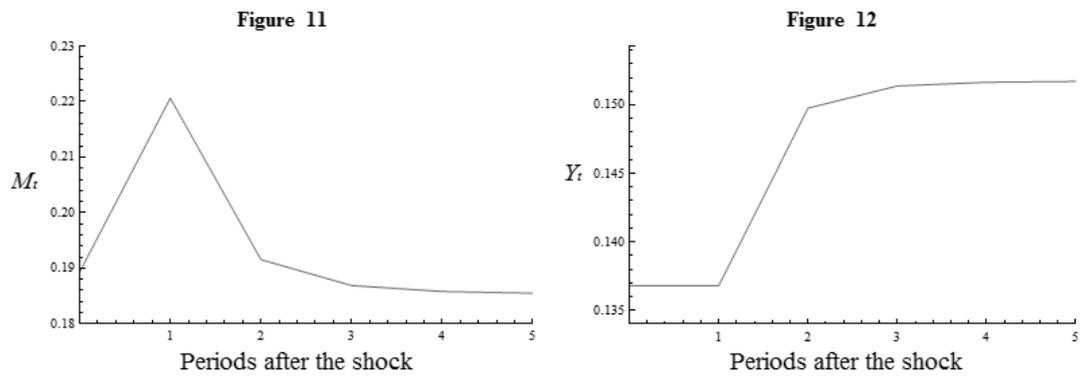
Parameters: $\sigma^n = 1$, $\sigma^m = 1.5$, before shock $l = 0.6$, after shock: $l = 0.62$.



Effects of a positive shock on the life expectancy of the elderly on the immigration quota M_t (Fig. 9) and on public spending per worker Y_t (Fig. 10).

Figure 11-12: Long-Run Effects of a Decrease in the Birth Rate of the Natives

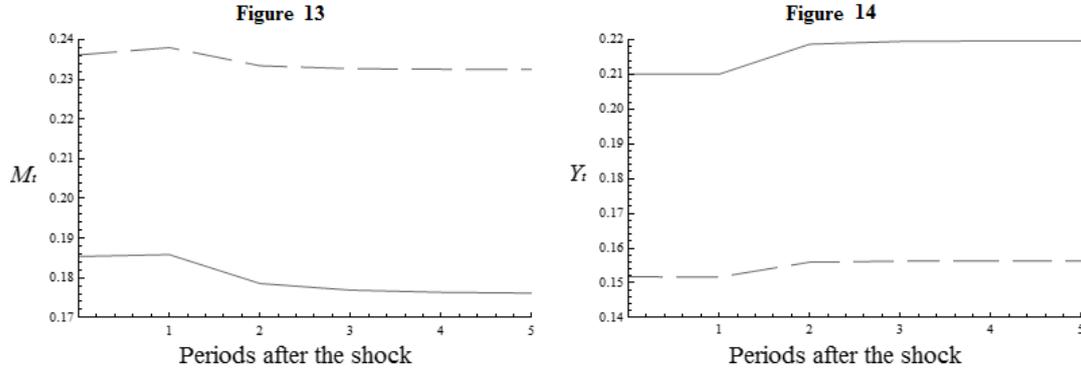
Parameters: $\sigma^n = 1.2$, $\sigma^m = 1.5$, $l = 0.6$, after shock: $\sigma^n = 1$.



Effects of a negative shock on the birth rate of the native population on the immigration quota M_t (Fig. 11) and on public spending per worker Y_t (Fig. 12).

Figure 13-14: “Naive” vs. “Sophisticated” agents

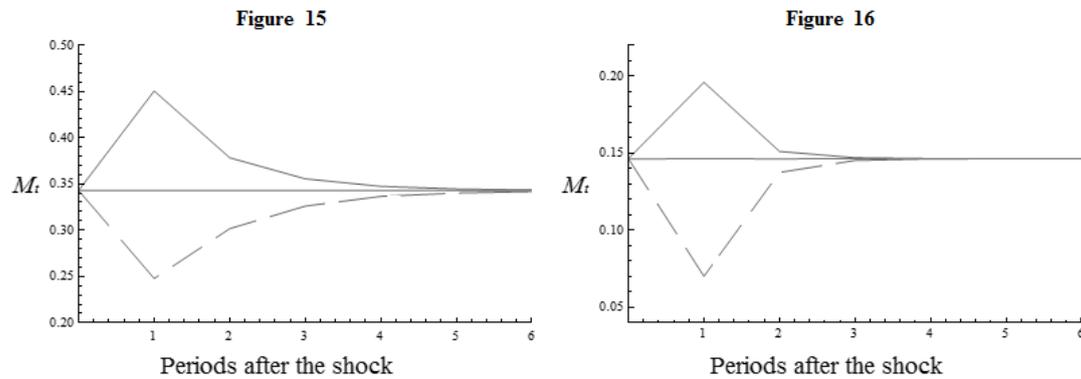
Parameters: $\sigma^n = 1$, $\sigma^m = 1.5$, before shock $l = 0.6$, after shock: $l = 0.62$.



Effects of a positive shock on the life expectancy of the elderly on the immigration quota M_t (Fig. 13) and on public spending per worker Y_t (Fig. 14) for “naive” (dashed line) and “sophisticated” voters (solid line).

Figure 15-16: Convergence to the Steady-State

Parameters: $\sigma^n = 1$, $l = 0.6$.



Effects of a temporary negative shock on g_t (solid lines) and of a temporary negative shock on g_t (dashed line) for $\sigma^m = 1.5$ (Fig.15) and $\sigma^m = 2$ (Fig.16).

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