Nonparametric Estimation and Instrument Selection in the Conditional Capital Asset Pricing Model*

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October 14, 2009

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Abstract

This paper uses a functional coefficient regression to estimate time-varying betas and alphas in the conditional capital asset pricing model. Functional coefficient representation relaxes the strict assumptions on the structure of betas and alphas by combining the predictors into an index that best captures time variations in betas and alphas and estimates them nonparametrically. This index in betas and alphas helps us to determine which economic variables we should track and more importantly, in what combination. We select an appropriate index variable by using smoothly clipped absolute deviation penalty to functional coefficients. In such a way, estimation and variable selection can be done simultaneously. Our model performs better than the alternatives in explaining asset returns. Based on the empirical studies, we find no evidence to reject the conditional CAPM.

JEL classification: C13; C52; G12

Keywords: Conditional CAPM; Functional coefficient regression; Index model; Smoothly clipped absolute deviation penalty;

*Cai's research was supported, in part, by the National Science Foundation of China grant #70871003, and funds provided by the Cheung Kong Scholarship from Chinese Ministry of Education, the Minjiang Scholarship from Fujian Province, China and Xiamen University.

1 Introduction

The Capital Asset Pricing Model (CAPM) is a cornerstone in finance. It states the linear relation between excess return and the beta of the cash flow with respect to the market return. The betas in the CAPM are commonly assumed to be constant over time. However, recent empirical studies show ample evidences against this assumption in the literature since the relative risk of a firm's cash flow varies over the business cycle and the state economy or the economic variables. Alternatively, the CAPM can be believed to hold perfectly from period to period. In other words, it may be valid under the condition of current information set. If this is the case, the corresponding betas should be adjusted accordingly since the information sets are updated over time. This suggests the betas should be time-varying.

Time-varying betas have already been discussed extensively through two different channels in the literature. First, the betas are regarded as a function of time; for example, see the papers by Hall and Hart (1990), Johnstone and Silverman (1997), and Robinson (1997). This approach usually assumes there are threshold values for betas, either discretely, like the threshold CAPM proposed by Akdeniz, Altay-Salih and Caner (2003), or continuously, like smooth threshold transition (STR) developed by Lin and Teräsvirta (1994). This approach is criticized by uncovering the driving force of the betas. Namely, it does not show how and why betas are varying over time. Second, the betas are assumed to be affected by some instrument variables. These instrument variables can be the proxies of the latent variables (Ang and Chen (2007)), or some observable macro variables (Ferson and Harvey (1999)). This approach can give us more economic intuition on how and why the betas change. Since there is no overwhelming argument for either approach, this paper circumvents this debate and follows the latter one by assuming the betas are functions of some observable variables which are called financial instruments. Similar to Aït-Sahalia and Brandt (2001), the instrument variables are combined into an index that best captures time variations in the betas and the index can be explained as an economic state variable.

The index has a good interpretation as follows. First, from a statistical standpoint, the index avoids the curse of dimensionality because it allows us to reduce the multivariate problem to one where we can implement the nonparametric approach described above in a univariate setting (since the index is univariate). Second, from economic perspective, the index offers a convenient univariate summary statistic that describes the current state of the various time-varying economic indicators (investment opportunities for the portfolio investment). Finally, from a normative aspect, our results can help investors with any preference(s) to determine which economic variables they should track and, more importantly, in what single combination.

Different from the previous studies in the literature, we propose a method to solve the following problems which have not been fully advocated in the literature. The first problem is the strict assumption on the relation between the betas and the instrument variables. Ferson and Harvey (1999) impose the assumption that the betas are linear functions of the index, while the linearity assumption is really controversial. Wang (2002, 2003) finds s atrong evidence against it. The strong assumption will lead to model misspecification. As argued in Ghysels (1998), the inference and the estimation based on misspecification can be very misleading. So it is important to analyze the time-varying betas by relaxing this assumption.

Another problem, as pointed out by Aït-Sahalia and Brandt (2001), is the separation of instrument selection and model estimation. In the literature, it is common to choose the instruments by claiming the relationship between the instrument and the asset returns. But those evidences are found based on the models different with the one being explored. Since different objective functions place different emphases on the instruments, in order to select the instruments which best fit the model, the selection should be conducted directly when doing estimation.

To address the above two issues, we use functional coefficient regression (FCR) proposed by Cai, Fan and Yao (2000) to estimate the time-varying betas. Furthermore, by adding a penalty term (smoothly clipped absolute deviation penalty (SCAD) function introduced by Fan and Li (2001)), to the conventional FCR, we can estimate the state variable and betas simultaneously. This method has several advantages. First, FCR estimates the betas nonparametrically. It assumes the coefficients of the factors are some unknown functions of the state variable. The estimates are obtained by local linearly fitting (see Fan and Gijbels (1996)). Thus it can relax the strong assumption of the linearity, and avoid the model misspecification. Second, this method can choose the state variable which is most suitable for the regression model. It combines the variable selection and the parameter estimation together. Besides, SCAD is a data-driven method, and it can delete the "useless" instruments automatically. So we can put all the potential candidates into the model, and do not need to study the relation between individual variable with the asset returns.

The framework for us to talk about time-varying betas is the conditional CAPM. While, the conditional CAPM, as alternative to static CAPM, is quite controversial in the literature. Theoretically, the conditional CAPM can hold perfectly, period to period. And many models are derived based on it. For example, Jagannathan and Wang (1996) build the Premium-Labor model. However, Ferson and Harvey (1999) and Wang (2002, 2003) find the conditional CAPM is rejected from their empirical analysis. The results in Ferson and Harvey (1999) may be misleading due to the model misspecification. Wang (2002, 2003) uses four smoothing variables to do nonparametric estimations and tests. But the number of the observations in his data is around 600, which is too small to get reliable inferences. So the question on the validity of the conditional CAPM is still open.

Our results show that our model outperforms the alternative ones in fitting the data. And we can not find an evidence to reject the conditional CAPM model. At the same time, size effect and book-to-market effect disappear. This result questions the necessariness of the conditional Fama-French model although it has already been widely used in finance.

The rest of this paper is organized as follows: Section 2 describes our estimation model, while Section 3 presents some simulation results, and finally, Section 4 reports the empirical analysis. Section 5 concludes.

2 Econometric Model

2.1 Functional Coefficient CAPM

In a conditional CAPM, Ferson and Harvey (1999) assume the betas are linear function of the state variable. But Wang (2002, 2003) finds a strong evidence against this assumption. In order to circumvent a possible model misspecification, we adopt functional coefficient regression (FCR) model proposed by Cai, Fan and Yao (2000). This FCR representation does not impose any assumptions on the relation between the betas and the state variable. Also, pointed by Cai, Gu and Li (2009), a functional coefficient regression model might approximate a general nonparametric regression model well and has an ability to capture heteroscedasticity. Indeed, it fits the data locally by a linear function (see (3) later). Specifically, let $\{(z_t, x_t, y_t)\}_{t=1}^{+\infty}$ be jointly strictly stationary processes[1], where y_t is the excess return of the portfolios, and both z_t and x_t are the factors in the asset pricing model. In the conditional CAPM model, x_t is commonly the excess market return and z_t is a univariate state variable.

Define $m(z_0, x_0) = E(y_t | z_t = z_0, x_t = x_0)$. A functional coefficient model expresses $m(z, x)$ as

$$m(z_0, x_0) = \sum_{j=1}^{p} \beta_j(z_0) x_{j0}, \tag{1}$$

[1]In practice, financial data may not be exactly stationary. But we adopt this assumption for simplicity and it is a standard assumption in the literature. When z_t or x_t is nonstationary, the statistical inferences for this are totally different from those for the stationary case so that the econometric modeling for nonstationary case becomes complicated; see Cai, Li and Park (2009) for details.

so that the functional coefficient capital asset pricing model (FCCAPM) is given by

$$y_t = \sum_{j=1}^{p} \beta_j(z_t) x_{jt} + e_t, \tag{2}$$

where y_t is the excess return of portfolios, z_t is a state variable and x_t is a vector of the market excess returns. FCCAPM uses a local linear fitting (see Fan and Gijbels (1996)) to estimate $\beta_j(\cdot)$ in (2). For a given grid point z_0, we can approximate $\beta_j(z)$ locally by a linear function $\beta_j(z) = a_j + b_j(z - z_0)$ when z is in a neighborhood of z_0. The local linear estimator of $\beta_j(z_0)$ is defined as $\widehat{\beta}_j(z_0) = \widehat{a}_j$, where $\{\widehat{a}_j, \widehat{b}_j\}$ minimize the following locally weighted least squares

$$\sum_{t=1}^{n} \left[y_t - \sum_{j=1}^{p} \{a_j + b_j(z_i - z_0)\} x_{tj} \right]^2 K_h(z_t - z_0), \tag{3}$$

where $K_h(\cdot) = K(\cdot/h)/h$. $K(\cdot)$ is a kernel function on $\mathbb{R}^1$ and h is a bandwidth, which controls the amount of smoothing used in the estimation. Next, we write the minimization problem in (3) in a matrix form. Let

$$U = \begin{pmatrix} x_{11} & \cdots & x_{1p} & (z_1 - z_0)x_{11} & \cdots & (z_1 - z_0)x_{1p} \\ x_{21} & \cdots & x_{2p} & (z_2 - z_0)x_{21} & \cdots & (z_2 - z_0)x_{2p} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{n1} & \cdots & x_{np} & (z_n - z_0)x_{n1} & \cdots & (z_n - z_0)x_{np} \end{pmatrix}, \qquad W = \text{diag}\{K_h(z_t - z_0)\},$$

$B = (a_1, a_2, \cdots, a_p, b_1, b_2, \cdots, b_p)'$ and $Y = (y_1, y_2, \cdots, y_n)'$. Then, the minimization problem can be transformed to

$$\widehat{B} = (U'WU)^{-1}U'WY = \text{argmin}_B (Y - UB)'W(Y - UB), \tag{4}$$

which is the weighted least squares estimate. Clearly, (4) provides a formula for computational implementation, which can be carried out by any standard statistical package.

When z_0 is moved on the domain of z_t, the estimated curve of $\beta_j(z_0)$ is obtained.

2.2 Bandwidth Selection

It is well known that the bandwidth is important in nonparametric estimation. It balances the trade-off between the bias and the variance of the estimates. Following Cai and Tiwari (2000), Cai (2002) and Cai (2007), I find the optimal bandwidth h_{opt} by minimizing

$$\text{AIC}(h) = \log(\widehat{\sigma}^2) + 2(T_h + 1)/(n - T_h - 2), \tag{5}$$

where $\widehat{\sigma}^2 = \sum_{t=1}^n (\widehat{y}_t - y_t)^2/n$ and T_h is the trace of the hat matrix H_h which makes $\widehat{Y} = H_h Y$.

This selection criterion counteracts the over/under-fitting tendency of the generalized cross-validation and the classical AIC; see Cai and Tiwari (2000) and Cai (2002) for more details. Alternatively, one might use some existing methods in the time series literature although they may require more computing; see Fan and Gijbels (1996), Cai, Fan and Yao (2000) and Cai (2007).

2.3 Testing

In order to check the validity of the conditional CAPM, we have to test the significance of the intercept. In the conditional CAPM, we only consider one factor. In this setup, x_1 is constant of 1 and x_2 is the market excess return. To make the notation consistent, we use a_1 to denote the intercept, and a_2 to denote the coefficient of the market excess return. Hence, the testing problem can be formulated as

$$H_0 : a_1(z) = 0, \tag{6}$$

and

$$H_1 : a_1(z) \neq 0. \tag{7}$$

The sum square residuals (RSS) under the null hypothesis is

$$RSS_0 = n^{-1} \sum_{t=1}^{n} \{y_t - a_2(z_t) x_{t2}\}^2, \tag{8}$$

and the RSS under the alternative is

$$RSS_1 = n^{-1} \sum_{t=1}^{n} \{y_t - a_1(z_t) - a_2(z_t) x_{t2}\}^2. \tag{9}$$

The test statistic is defined as

$$T_n = \frac{RSS_0}{RSS_1} - 1. \tag{10}$$

We use the following nonparametric bootstrap approach to obtain the $p-$value of the statistic:

1. Collect the residuals $\{\widehat{e}_t\}$ by

$$\widehat{e}_t = y_t - \widehat{a}_1(z_t) x_{t1} - \widehat{a}_2(z_t) x_{t2}$$

2. Generate the bootstrap residuals $\{e_i^*\}$ from the empirical distribution of the centered residuals $\{\widehat{e}_t - \bar{\widehat{e}}_t\}$.

3. Define

$$y_t^* = \beta_1^* x_{t1} + \beta_2^* x_{t2} + e_t^*$$

4. Calculate the bootstrap test statistic J_T^* based on the sample $\{y_t^*, x_t, z_t\}$

5. Compute the p-value of the test based on the relative frequency of the event $\{J_T^* \geq J_T\}$ in the replications of the bootstrap sampling.

2.4 Instrument Variables Selection

Although it is common to model the betas as a function of observed macroeconomic variables, it is crucial to select them in practice (Cochrane (2001), Harvey (2001)). Usually, the instrument variables are chosen according to the specific studies on the relationship between that variable and asset returns. For example, spread between the returns of a three-month and a one-month Treasury bill (Ferson and Harvey (1991)), spread between Moody's Baa and Aaa corporate bond yield (Fama (1990)), spread between a ten-year and one-year Treasury bond yield (Fama and French (1989)) and one-month Treasury bill yield (Ferson (1989)) are empirically proved to be highly related with asset returns. Those instruments are proved to be useful in explaining the asset returns in the models different with the one we are studying. In order to select the instruments which best fit our model, we adopt the variable selection method proposed by Fan and Li (2001).

We define u as a $n \times k$ vector of instrument variables, and the linear combination of these variables $z = uc$ as the state variable for FCR, where c is a $k \times 1$ coefficient vector. The loss function of FCR can be expressed as

$$L(c) = \min_{a,b} \sum_{t=1}^{n} \left[y_t - \sum_{j=1}^{2} \{a_j + b_j(z_t(c) - z_0(c))\} x_{tj} \right]^2 K_h(c), \tag{11}$$

We add Smoothly Clipped Absolute Deviation penalty to the loss function $L(c)$ in equation

$$\min_{c} L(c) + n \sum_{i=1}^{k} P_{\lambda,v}(|c_i|), \tag{12}$$

where the first order derivative of $P_{\lambda,v}$ is

$$P'_{\lambda,v}(c_i) = \lambda I(c_i \leq \lambda) + \frac{(v\lambda - c_i)_+}{v-1} I(c_i > \lambda) \tag{13}$$

As shown in Fan and Li (2001), the SCAD penalty function leads to estimator with the three desired properties which can not be achieved by either L_p penalty function or hard penalty function. Three properties are: unbiasedness for large true coefficient to avoid unnecessary estimation bias, sparsity of estimating a small coefficient as 0 to reduce model complexity, and continuity of resulting estimator to avoid unnecessary variation in model prediction. More discussions about these properties can be found in the papers by Antoniadis and Fan (2001) and Fan and Li (2001).

The intuition is if the instrument variables are not helpful in explaining the time-varying betas, the estimated coefficient for this instrument will shrink to zero. Hence, in formatting the state variable, it will not be considered. This variable selection method is totally data-driven. It reveals the potential effects of the instrument variables at the same time keeps the analysis within the framework of FCR. Furthermore, the number of the instrument variables are not restricted anymore. SCAD will ignore those "useless" instruments automatically.

3 Monte Carlo Simulations

We illustrate the proposed method by one simulated data set. This data set mimics the actual portfolio returns and market returns in the conditional CAPM model. In addition, we generate three instrument variables, but assume only two of them formatting the state of the economy. We try to find whether our model can deliver better estimates in terms of the square root of average squared errors and pin down the actual state variable.

We generate z_{1t}, z_{2t} and z_{3t} from a uniform distribution between 0 and $\frac{2}{\sqrt{2}}$ as our instrument variables, and randomly draw x_t from a uniform distribution on [0, 1]. The data generating process is

$$y_t = (\frac{\sqrt{2}}{2} z_{1t} + \frac{\sqrt{2}}{2} z_{2t}) x_t + e_t, \tag{14}$$

where e_t is error term distributed normally with mean of 0 and standard deviation of 0.08. In this DGP, we can generate the mean value of y about 3%, which is equal to the excess return in the actual data roughly. Although only two instruments are involved in DGP and three are plugged into our estimation procedure, we expect our model can reveal this fact and decrease the effect of the irrelevant instrument somehow.

We repeat the simulation 1000 times, and report the estimates of c_1, c_2 and c_3, which are the coefficients for the three instruments z_{1t}, z_{2t} and z_{3t} in our model. The sample size is $n = 500$. Table (1) shows that c_3, the coefficient of the last instrument which is

Table 1: Estimation Results

	c_1	c_2	c_3
true value	0.7	0.7	0
mean	0.6989	0.7058	-0.0094
std	0.0408	0.0465	0.0982

We report the mean and the standard deviation for the estimates of the coefficients for the instruments based on 1000 simulations.

not in DGP, is insignificant. In addition, the estimates of c_1 and c_2 are very close their true values respectively. It seems that our model works well in this simple simulation.

4 Empirical Analysis

4.1 Data

We collect monthly returns on the Fama-French 25 portfolios for the period from July 1963 to December 2007. The instrument variables, following Ferson and Harvey (1999), are spread between the returns of a three-month and a one-month Treasury bill , spread between Moody's Baa and Aaa corporate bond yield, spread between a ten-year and one-year Treasury bond yield and one-month Treasury bill yield. In order to match the model, we obtain the one-month lagged data for the instrument variable.

We do not claim that these instrument variables are the set of all potential instruments. While, these are the most popular ones used in practice since interest rates and spreads are usually benchmark indexes for business cycle. Putting them into our model, we can get an idea on which variable plays more important role than the others. Certainly, it is feasible to incorporate more macro variables into the model since SCAD penalty criterion will shrink the coefficients of those "useless" ones to zero. Doing so will not hurt our estimation, but only needs more computation time.

4.2 Data Smoothing

We use our model to estimate the state variable and the asset returns. In order to compare the relative performances, we also employ the model in Ferson and Harvey (1999) (FH model) to fit the data. We report the mean square error in Table 2. The first column indicates the portfolios we study. "S1" and "B1" denote the order sorted by the size and the book-to-market ratio. The MSE delivered by our model and by the FH model are reported in the second and the third columns. We can see that the numbers in the second one are smaller than the last one for 24 out of 25 portfolios we consider, which suggests the better goodness-of-fitting of our model. The relative poor performance of the FH model may be due to the model misspecification. The linearity assumption imposed on the relationship between the state variable and the time-varying betas seems too strong. The fitting performance increases when this assumption is relaxed, which supports the findings in Wang (2002, 2003).

To explore further, we check how the choice for the state variable affects our results. Four different state variables are studied here. The first is the one selected by SCAD as our model describe. For the second one, we run a linear regression of the asset returns on the instrument variables, and treat the fitted value as the state variable. The last one is the state variables implied by the time-varying betas in the FH model[2]. All the state variables are used as the smoothing variable for FCR, and we report the MSE in Table 3. It can be seen that SCAD gives us the best state variable comparing these three choices. The majority of MSE in the second column is smaller than the ones in the

[2]The FH model allows the alpha and the beta to have different state variables.

Table 2: Mean Square Error

	FCR+SCAD	Ferson and Harvey
S1/B1	23.5013	23.6593
S1/B2	17.4739	17.9318
S1/B3	11.6561	12.0271
S1/B4	10.9146	11.1405
S1/B5	13.0242	13.0960
S2/B1	13.0710	13.4662
S2/B2	8.2584	8.6062
S2/B3	7.2401	7.3002
S2/B4	6.9146	7.2206
S2/B5	9.6246	10.1505
S3/B1	8.9532	9.4232
S3/B2	4.7495	4.9994
S3/B3	4.8994	5.3056
S3/B4	5.4290	5.7089
S3/B5	8.2024	8.8104
S4/B1	5.0565	5.3121
S4/B2	3.3605	3.4903
S4/B3	4.1210	4.4605
S4/B4	4.6244	5.1497
S4/B5	7.4822	8.2073
S5/B1	2.5501	2.7389
S5/B2	2.6179	2.6008
S5/B3	3.6954	3.9659
S5/B4	5.0903	5.4537
S5/B5	8.2831	8.7243

The first column indicates the portfolios we study. The second column reports the mean square error when our model is employed to smooth the data. The third column reports the mean square error delivered by the FH model.

other three columns. The reason is SCAD selects the variables to directly fit the asset returns and deletes the irrelevant instrument variables automatically. While, the other three state variables are chosen firstly, and then explored later their implications for the asset returns.

Table 3: Mean Square Error

	SCAD	OLS	Ferson and Harvey
S1/B1	23.5013	23.7455	23.5368
S1/B2	17.4739	17.8816	17.6533
S1/B3	11.6561	11.8707	11.7782
S1/B4	10.9146	10.9626	10.9384
S1/B5	13.0242	12.8156	13.0788
S2/B1	13.0710	13.0488	12.9443
S2/B2	8.2584	8.3966	8.4063
S2/B3	7.2401	7.2030	7.0263
S2/B4	6.9146	7.1082	6.9738
S2/B5	9.6246	9.5810	9.7361
S3/B1	8.9532	9.1437	9.1231
S3/B2	4.7495	4.9490	4.8437
S3/B3	4.8994	5.1125	5.0225
S3/B4	5.4290	5.4976	5.5340
S3/B5	8.2024	8.3260	8.4776
S4/B1	5.0565	5.0967	5.2376
S4/B2	3.3605	3.4608	3.4520
S4/B3	4.1210	4.2485	4.3299
S4/B4	4.6244	4.9007	4.7632
S4/B5	7.4822	7.7889	7.7664
S5/B1	2.5501	2.6510	2.5416
S5/B2	2.6179	2.6434	2.5448
S5/B3	3.6954	3.8493	3.8978
S5/B4	5.0903	5.1838	5.2623
S5/B5	8.2831	8.2256	8.2396

We use FCR to fit the data, but try different state variables. The second column reports the MSE when we use SCAD to find the state variable. The third column reports the MSE when we run a linear regression of the asset returns on the instrument variables and treat the fitted value as the state variable. The last one is the MSE when the state variables are chosen by the time-varying betas in the FH model.

When our model is employed, the estimated coefficients of the instruments can be summarized by Table 4. In order to avoid identification problem, we search the estimates over a unit circle. An interesting finding is that the spread between Moody's Baa and Aaa corporate bond yield plays the most important role in the state variables for most portfolios we analyze. This result is consistent with Bernanke (1990), who compares a number of interest rate variables and finds the best single variable is the spread between the commercial paper rate and Treasury bill rate. This spread is usually approximated by the spread between Moody's Baa and Aaa corporate bond yield. Jagannathan and Wang (1996) use this single state variable to describe the conditional CAPM model. While, obviously, the single variable is not enough to describe the state. Other variables, such as are spread between the returns of a three-month and a one-month Treasury bill, spread between a ten-year and one-year Treasury bond yield and one-month Treasury bill yield, can also contribute to revealing the economy state somehow. So Table 4 weakens the conditional model which just uses single macro variable as the state.

As aforementioned, we do not incorporate all the feasible instrument variables here because of computation burden. Instead, we try to argue that SCAD is a good method to pin down the state variable. It will shrink the coefficient of the instrument which is redundant to zero, like the portfolio labelled by S5/B2. Even if we add extra instrument variables into our model, it will never hurt.

4.3 Size and Book-to-market effects

Fama and French (1993) attack the statistic CAPM by illustrating that there are size and boo-to-market effects in asset returns. They argue that stock returns show strong tendencies that small stocks outperform large stocks, that firms with high book-to-market (B/M) ratios outperform those with low B/M ratios, and that stocks with high returns in the previous year continue to outperform those with low prior returns. These can not be explained by the statistic CAPM. Their findings are obtained by analyzing the intercept of the CAPM, which are called alpha.

The alphas stand for the abnormal returns of the assets. The abnormal returns occur

Table 4: Coefficients for Instrument Variables

Coefficient	BmA	r3m-r1m	r10y-r1y	r1m
S1/B1	0	0.4685	0.7080	0.5266
S1/B2	0.9030	0.3870	0.1864	0
S1/B3	0.9684	0.1709	-0.1533	0
S1/B4	0.9953	0	0	0
S1/B5	0.7838	-0.3686	0.1725	0.4692
S2/B1	0.1419	0	0.9854	0.0944
S2/B2	0.9654	0.1570	-0.1505	0.1441
S2/B3	0.7414	0.4695	0.3902	0.2785
S2/B4	0.8583	0.2622	0.3038	0.3199
S2/B5	0.9820	0.1267	0	0.1029
S3/B1	0.8474	0.5279	0	0
S3/B2	0.6013	-0.7517	-0.2654	0
S3/B3	0.3948	0.7905	0.3293	0.3329
S3/B4	0.9822	0.1193	0	0.1077
S3/B5	0.9750	0.1170	-0.1377	0.1290
S4/B1	0	0.3943	0.6864	0.6052
S4/B2	-0.7712	0.3660	0	-0.5199
S4/B3	0.7212	0.4265	0.3733	0.3981
S4/B4	0.5088	0.7612	0.2195	0.3371
S4/B5	0.5345	0.3359	0.7131	0.3051
S5/B1	-0.1695	-0.771	0.5592	0.2532
S5/B2	-0.9259	-0.377	0	0
S5/B3	-0.3312	0	-0.4451	0.831
S5/B4	0.4549	0.2759	0.7771	0.3362
S5/B5	-0.5204	0.5289	0.4539	0.4934

We use our method to estimate the coefficients of the instrument variables. The portfolios are the Fama-French 25 portfolios, and the instrument variables are the spread between Moody's Baa and Aaa corporate bond yield (BmA), the spread between the returns of a three-month and a one-month Treasury bill (r3m-r1m), spread between a ten-year and one-year Treasury bond yield (r10y-r1y) and one-month Treasury bill yield (r1m).

when there are other systematic risks, which are not captured by the model, in the market. So if the estimated alphas are significant, the model may be misspecified. In other words, the market return can not capture all the systematic risk. Ferson and Harvey (1999) follow this procedure studying the significance of the alphas in the conditional CAPM. In their setup, the linear structure is imposed in the time-varying alphas. And they find the alphas are significant, and hence, they conclude that the conditional CAPM is rejected.

Here, we explore the significance of the alphas based on our model. We estimate the time-varying alphas for the whole time period of the data, and report the mean value and the t-statistic of the alphas for each portfolio by Table 5. In addition, we test the hypothesis that the alpha is insignificant by the testing procedure mentioned before. The p-value is reported in the last row of each portfolio. 23 out of 25 portfolios have insignificant alphas. And we do not observe any size or book-to-market effect. Since the comparison of model performance suggests our model fitting the data more accurately, we believe Table 5 finds an evidence to support the conditional CAPM. Furthermore, it invokes the discussion on the necessity of the conditional Fama-French model.

Table 6 reports the mean and t-statistic of the estimated betas. As the alphas, they do not show any pattern related with unobservable latent factors.

4.4 Cross-sectional Regression

The conditional CAPM model can be expressed as

$$E[R_{it}|I_{t-1}] = \gamma_{1t-1}\beta_{it-1}, \tag{15}$$

where γ_{0t-1} is the conditional expected return on a "zero-beta" portfolio, and γ_{1t-1} is the conditional market risk premium. β_{it-1} can be estimated as our model suggests.

According to Jagannathan and Wang (1996), the unconditional version of Equation

Table 5: conditional Alphas

Portfolio	B1	B2	B3	B4	B5
S1	-0.4683	0.3403	0.3733	0.5821	0.6786
	(-0.9117)	(0.4502)	(0.5904)	(1.2953)	(1.6570)
p-value	0.2070	0.2310	0.3350	0.1670	0.0940
S2	-0.2681	0.1242	0.4111	0.4213	0.5298
	(-0.5916)	(0.5384)	(1.9106)	(1.8306)	(1.5042)
p-value	0.2530	0.9580	0.5200	0.3260	0.4730
S3	-0.1357	0.2390	0.3169	0.4033	0.5067
	(-0.2193)	(0.6913)	(1.0669)	(1.7922)	(1.5677)
p-value	0.0070	0.1160	0.2620	0.3230	0.3050
S4	-0.0277	0.0399	0.2151	0.3741	0.3853
	(-0.1107)	(0.2034)	(1.0109)	(1.4003)	(1.1756)
p-value	0.4980	0.9120	0.6130	0.1110	0.2580
S5	-0.1488	0.0281	0.0793	0.1601	0.1606
	(-0.1992)	(0.1259)	(0.1920)	(1.4413)	(0.3897)
p-value	0.0000	0.8780	0.0640	0.8880	0.3410

We use our model to estimate monthly time-varying alphas for July 1963 to December 2007. This table reports the average of the estimates. And the number in the bracket is t-statistic. p-values are obtained by testing the null hypothesis that the alpha is insignificant.

Table 6: conditional Betas

Portfolio	B1	B2	B3	B4	B5
S1	1.4686	1.2412	1.0678	1.0056	1.0246
	(11.7186)	(6.2297)	(7.6119)	(8.5138)	(6.4194)
S2	1.4669	1.1882	1.0649	1.0382	1.1178
	(12.6083)	(10.8024)	(9.0021)	(5.8642)	(5.1362)
S3	1.3806	1.1058	1.0021	0.9552	1.0460
	(7.4660)	(10.8426)	(7.6115)	(6.7841)	(5.6708)
S4	1.2459	1.0571	1.0170	0.9575	1.0248
	(13.9376)	(10.6607)	(7.6893)	(5.9702)	(5.6218)
S5	1.0247	0.9472	0.8732	0.8217	0.8642
	(7.5192)	(20.0417)	(10.7108)	(7.4150)	(5.1292)

We use our model to estimate monthly time-varying betas for July 1963 to December 2007. This table reports the average of the estimates. And the number in the bracket is t-statistic.

(15) is

$$E[R_{it}] = c_1\bar{\beta}_i + c_2\beta_i^{\gamma}, \tag{16}$$

where $\bar{\beta}_i = E(\beta_{it-1})$ and $\beta_i^{\gamma} \equiv Cov(R_{it}, R_{t-1}^{prem})/Var(R_{t-1}^{prem})$. R_{t-1}^{prem} can be approximated by the yield spread between BAA and AAA rated bonds. We run the cross-sectional regression:

$$E[R_{it}] = c_1\bar{\beta}_i + c_2\beta_i^{\gamma} + e_i, \quad i = 1, 2, \cdots, k \tag{17}$$

and test the hypothesis that the pricing errors ($e = (e_1, e_2, \cdots, e_k)'$) are jointly insignificant by following the steps in Shanken (1992). If our model successfully captures the systematic risks in the market, the pricing errors should be insignificant. The estimates and the p-value are summarized by Table 7.

Table 7: Cross-sectional Regression

Coefficient	c_1	c_2	$H_0 : e = 0$
Estimate	-0.0021	0.5556	
t-ratio	-0.0085	2.6125	
p-value	0.9932	0.0156	0.9999

It turns out that we can not reject the hypothesis that the pricing errors are insignificant. This supports the validity of the conditional CAPM from another asepct.

5 Conclusion

This paper uses a functional coefficient regression to estimate time-varying betas and alphas in the conditional capital asset pricing model. Functional coefficient representation relaxes the strict assumptions on the structure of betas and alphas by combining the predictors into an index that best captures time variations in betas and alphas and estimates them nonparametrically. This index in betas and alphas helps us to determine which economic variables we should track and more importantly, in what combination.

We select an appropriate index variable by using smoothly clipped absolute deviation penalty to functional coefficients. In such a way, estimation and variable selection can be done simultaneously.

Our findings are quite interesting. First, empirical results show that our model has the best goodness-of-fitting compared with the other ones in the literature. Second, our results support the conventional wisdom about the conditional CAPM that it holds from period to period. The conditional alphas are not significant for most portfolios we studies. At last, we do not observe any size or book-to-market effect in the conditional CAPM.

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