

Increasing Fundraising Success by Decreasing Donor Choice*

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Abstract

Suggested contributions, membership categories, and discrete, incremental thank-you gifts are devices often used by benevolent associations that provide public goods. Such devices focus donations into discrete levels, thereby effectively limiting the donors' freedom to give. We study the effects on overall donations of the tradeoff between rigid schemes that severely restrict the choices of contribution on the one hand, and flexible membership contracts on the other, taking into account the strategic response of contributors whose values for the public good are private information. We show flexibility dominates when i) the dispersion of donors' taste for the public good increases, ii) the number of potential donors increases, and iii) there is greater funding by an external authority. Using the number of default membership categories that National Public Radio stations offer as proxy for flexibility, we document the existence of empirical correlations consistent with our predictions: stations offer a larger number of suggested contribution levels as i) the incomes of the population served become more diverse, ii) the population of the coverage area increases, and iii) there is greater external support from the Corporation for Public Broadcasting.

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1 Introduction

Private provision of public goods plays an important role in the US economy.¹ Beyond familiar charities such as the Red Cross and environmental groups like the Sierra Club, many local associations support orchestras, zoos, community radio stations, and various other endeavors that can, at least in part, be thought of as public goods. It is therefore not surprising that the economics literature provides multiple answers to the question of why people give, including enlightened self-interest and altruism (e.g. Bergstrom *et al.*, 1986), warm-glow (e.g. Andreoni, 1989, 1990), prestige (Harbaugh, 1998a, 1998b), signaling (Glazer and Konrad, 1996), and selective incentives (Olson, 1965). The psychology, sociology, and marketing literatures add many other motivations, including the simple fact of being asked and the “even-a-penny-helps” technique.² Fundraising activities may well take into account all these motivations at various stages of a campaign.

We focus on one of the most common practices association managers and fund raisers use: accepting, recommending, recognizing, or otherwise rewarding donations according to endogenously designed bins or categories. This practice may take the form of a minimum suggested or accepted donation, of some level that must be reached to publicize a donation, of affixing nicknames to donation categories (e.g., in increasing order of donation, “member,” “supporter,” “benefactor”), or more generally of offering different combinations of selective incentives at various levels of contributions (for example, a bumper sticker for a \$20 donation, a bumper sticker and an audio cd for a \$50 donation, and so on).

A number of questions naturally arise about this practice. What response does it elicit from donors? How should levels that trigger benefits be chosen? Are there any observable characteristics of the donor population that push toward offering a membership scheme with many levels rather than only a few? We present a simple theoretical framework in which to analyze and answer these questions.

To provide further motivation and guidance, we have collected information about the number of “default” membership levels offered by National Public Radio (NPR) stations (that offer membership levels) and we have matched levels against three observable characteristics of potential donors: i) population in the coverage area, ii) external funding by the Corporation for Public Broadcasting (CPB), and iii) a measure of value dispersion in the coverage area.³ Figure 1 depicts the correlations among these variables. The relation between population and membership levels appears in the top panel. The middle panel contains the relation between CPB funding and membership levels. To construct the bottom panel, we have partialled-out median household income. The bottom panel contains the relation between the residuals of suggested membership

¹For example, Andreoni (2006) reports that private giving hovers between 1.5% and 2.1% of personal income in the US.

²Bekkers and Wiepking (2007), in their review of the literature on philanthropy, state that “Many people have developed cognitive strategies to reject responsibility for the welfare of others. One such strategy is the argument that ‘one cannot afford a donation.’ Legitimizing paltry contributions by adding the phrase ‘even a penny helps’ in a solicitation for contributions may neutralize these strategies [citations omitted].”

³A full description of the data, additional figures, and alternative regression specifications is available upon request.

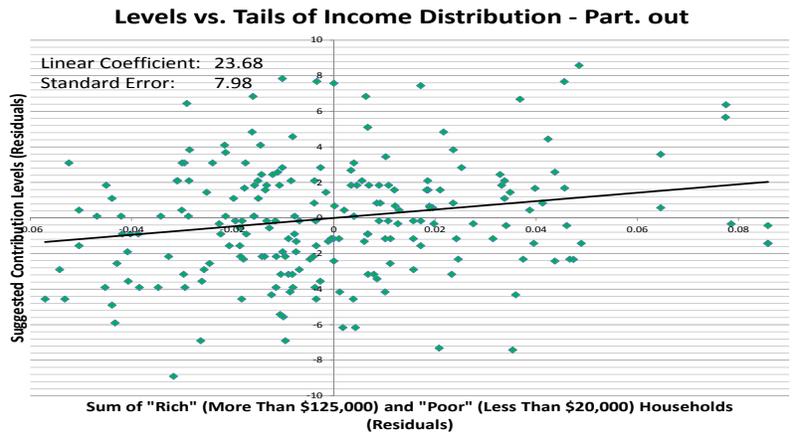
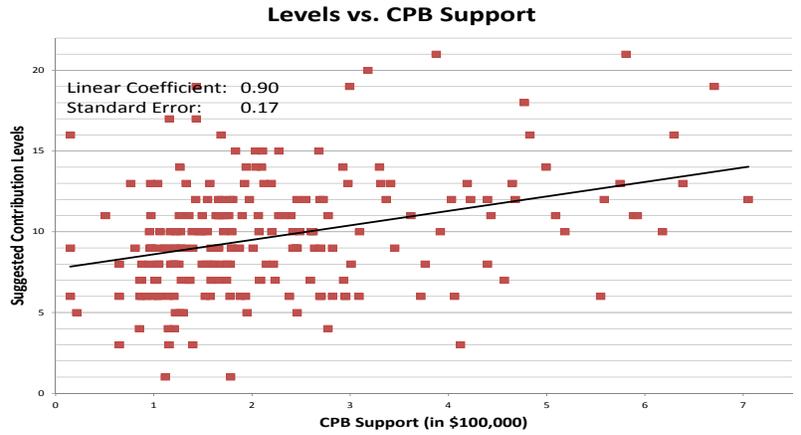
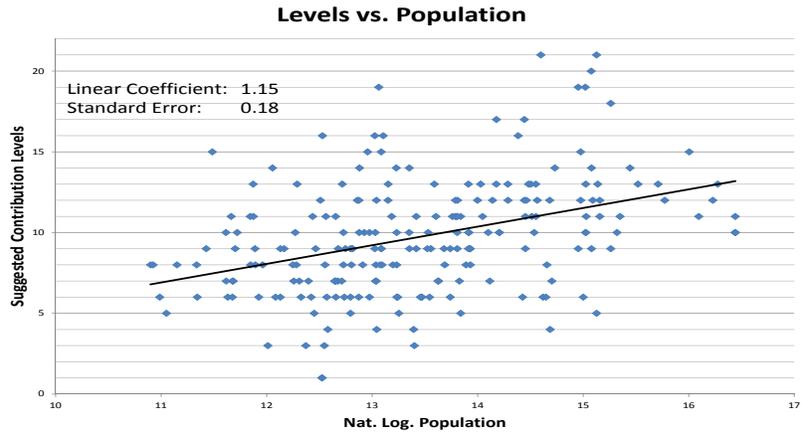


Figure 1: Correlations. Sources: NPR stations websites, Census, and annual report of the CPB.

Table 1: Dependent variable: number of default contribution levels proposed by an NPR network

i)	Logarithm of population over 18 years of age	1.3930**	(0.3331)
ii)	CPB support in 100,000's	0.5691**	(0.2118)
iii)	Logarithm of median income	3.2168	(4.4587)
	Proportion of households with income less than \$20,000	45.0427**	(14.7979)
	Proportion of households with income more than \$125,000	67.2287**	(24.2793)
	R squared	0.2735	
	<i>Number of observations</i>	210	

** Significant at the 1% level. Robust standard errors are in parenthesis. Controls here omitted include: voting percentage, average commute time, percentage of population with health insurance, demographic, education, and employment variables. Note: All independent variables are for the year 2000 with the exception of income variables, which are for the year 1999, and CPB support, which is for the year 2007. Sources: NPR stations websites, Census, and annual report of the CPB.

levels and the residuals of the weight of the tails of the income distribution. Table 1 analyzes simultaneously all three of these relations in a simple regression.

Figure 1 and Table 1 showcase these patterns: flexibility in the membership scheme (i.e., more default levels) is favored by i) larger population, ii) larger external contributions, and iii) larger heterogeneity. Our model rationalizes these patterns as the result of the fundraisers' desire to maximize donations.

We choose a “positive” theoretical approach for our model, similar to the one of Harbaugh (1998a), who directly targets the relationship between categories and prestige.⁴ Harbaugh (1998a) posits a pure warm-glow motivation for giving (donors receive a private benefit from their donations) and shows that, generically, creating one category donations have to fall into to be recognized—and thus rewarded with the additional private good “prestige”—dominates recognizing donations based on their exact amount. The force behind this result is “bunching at the low end” of a category, an effect empirically confirmed in Harbaugh (1998b), and experimentally observed by Andreoni and Petrie (2004) and by Li and Ryanto (2009). Transitioning from exact to categorical recognition, “bunching” refers to donations that end up being clustered at the cutoff value that triggers the beginning of a category, rather than falling in a neighborhood on either side of such cutoff.

Beyond prestige, “bunching at the low end” can be expected for other motivations for giving as well: from a theoretical point of view, the exact nature of the benefit, whether prestige or a more general selective incentive, is not fundamental to create bunching. Moreover, Croson and Marks (2001) experimentally observe bunching even in the case of simple suggestions of donation levels. Indeed, a very similar effect could be

⁴Another theoretical approach to answer these questions is mechanism design. Indeed, Cornelli (1996), albeit tangentially to her main goal of characterizing the optimal direct mechanism, suggests how implementation can occur through a scheme with categories. However, extending the results in Cornelli (1996) to answer the questions we are interested in appears complicated. Moreover, some authors describe the mechanism design approach as too complicated, too abstract, and too organizationally taxing to provide a realistic account of a situation with very many potential donors (see, e.g., Andreoni, 1998). Others, such as Martimort and Moreira (2010), comment on how a full-fledged mechanism design approach requires a degree of commitment power that may be excessive in a variety of situations.

reached, albeit simplistically, by restricting the agents' ability to donate to exactly either a pre-specified positive amount, or nothing at all: donors that without restrictions would have given a little less than the pre-specified amount find it best to up their contributions to this amount, rather than donating nothing, thus creating bunching.⁵ We find this is the easiest way to think about the effects of categories.

After reformulating the question in terms of restrictions in contributions, a natural place to look for an answer is the experimental literature comparing discrete-level contribution models with those with continuous contributions. There appears to be an expectation that continuous-level contribution schemes perform better. Cadsby and Maynes (1999), Hsu (2003), and Suleiman and Rapoport (1992) report finding in experimental situations that allowing continuous contribution possibilities significantly increases contributions over requiring that contributors either contribute nothing or their entire endowment. Authors explain this finding by noting that with continuous contributions there is a symmetric pure strategy equilibrium with provision while typically no such equilibrium exists when the contribution options are all or nothing (e.g., Cadsby and Maynes, 1999, p. 57). However, as Andreoni and Petrie (2004) point out, the theoretical comparison between total contributions when all donation amounts are possible and when agents' freedom to give is restricted depends on what contribution levels are available. No effort in this direction appears in this literature.

Therefore, at least three issues appear deserving of more study. First, while the warm-glow model of Harbaugh's is surely interesting, it is worth investigating the effects of restricting agents' freedom to donate in a pure-public-good model, thereby reintroducing strategic considerations and free-riding into the picture. (These considerations allow us to rationalize the positive correlation between flexibility in membership schemes and population size and external contributions.) Second, how is the optimal discrete contribution level chosen in this framework? Third, under what conditions does the restricted contribution-level membership scheme perform better than one with unrestricted levels? In particular, is one scheme always better than the other or does the choice reveal a true trade-off? And in this last case, which observable characteristics of the donor population are important for the trade-off?

Our basic theoretical model provides answers to these questions. We cast our analysis in a private-values subscription game framework. As our baseline case, we suppose all donations are welcome, as in Barbieri and Malueg (2010). Next, we consider the alternative policy in which the fundraisers specify a particular contribution that they will accept. This policy imposes bunching of types, and, as a representation of actual

⁵The equivalence, as far as bunching, of outright restrictions and categories is further discussed and illustrated in Section 5. Beyond creating bunching, there are of course other reasons to discourage small but positive donations. For example, extremely small donations may entail relatively large processing costs. Moreover, one may run the risk of legitimizing too small a donation. Indeed, in describing the limitation of the "even-a-penny-helps" technique, Bekkers and Wiepking (2007) state "...the phrase may even decrease the amount donated, exactly because it legitimizes paltry contributions [citation omitted]." Also see footnote 2 above.

membership schemes, it favors simplicity over realism. After characterizing the optimal level for the accepted contribution, we demonstrate the importance of the shape of the cumulative distribution function describing players’ private values for the discrete good: if it is convex (concave), the single contribution threshold (unrestricted contribution campaign) always raises greater contributions. While these cases are important for identifying the forces that make the continuous or discrete contribution framework preferred, it is more reasonable to expect the density of these values to be initially increasing and then decreasing if private values for the public good are correlated with income.

In this last case—that is, when the cumulative distribution of values is first convex and then concave—a true trade-off emerges. We showcase the basic forces underlying the decision by the fundraisers whether to restrict the freedom in choosing a contribution level by potential contributors, or to offer a flexible membership scheme in which choices are less constrained. Two such forces are and “extent of crowding-out” and “dispersion of values,” leading to the following predictions. A membership scheme that restricts contributors’ decisions becomes less attractive as

1. the number of potential contributors increases,
2. the amount provided by an external authority increases, or
3. the dispersion of values increases.

Moreover, we show graphically and by example how these predictions remain valid in two more realistic situations. First, we consider a comparison of 1-, 2-, and 3-level schemes, allowing for organizational or behavioral costs of complexity. Second, we enrich our basic model to cope with membership schemes in which agents are free to donate any amount they desire, but benefits kick in only for donations above a pre-specified amount.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3 we characterize the unique symmetric equilibrium for both unrestricted- and restricted-level schemes, and we calculate the optimal discrete contribution level. Section 4 explores the role of the shape of the distribution of values and presents the implications of “extent of crowding-out” and “dispersion of values” on the choice of restricting agents’ flexibility to donate. Section 5 contains the examples dealing with more realistic membership schemes and Section 6 concludes. All proofs are in the Appendix.

2 The Model

We study the problem of n players who simultaneously contribute to the funding of a binary public good. Player i ’s value for the good is v_i , $i = 1, \dots, n$. Players’ values are independently and identically distributed

random variables with cumulative distribution functions (cdf) F , which has support $[0, 1]$. A player's realized value is known only to that player. We suppose F is continuous with density function f . The cost of the public good is c , which we assume is a random variable *uniformly distributed* over the interval $[0, \bar{c}]$, where $\bar{c} \geq n$, and c is independent of players' values.⁶ The foregoing description is common knowledge.

In the terminology of Admati and Perry (1991), we consider the *subscription game*: players' contributions are refunded if they are insufficient to cover c . If the good is provided, then the payoff to player i is $v_i - (\text{player } i\text{'s contribution})$. If the good is not provided, then the payoff to player i is 0.

3 Equilibria

We look for a symmetric equilibrium strategy s . The expected utility of agent i with value v_i contributing x when other players use strategy $s(\cdot)$ is

$$U_i(x|v_i) \equiv (v_i - x) \Pr \left(c \leq x + \sum_{j \neq i} s(v_j) \right). \quad (1)$$

Because cost is distributed uniformly and independently of players' values, if agent i contributes x , then the probability that the good is provided is

$$\Pr \left(c \leq x + \sum_{j \neq i} s(v_j) \right) = \mathbb{E} \left[\Pr \left(c \leq x + \sum_{j \neq i} s(v_j) \mid v_j, j \neq i \right) \right] = \mathbb{E} \left[\frac{x + \sum_{j \neq i} s(v_j)}{\bar{c}} \right] = \frac{x + (n-1)K}{\bar{c}},$$

where $K \equiv \mathbb{E}[s(v_j)]$ is the expected contribution of agent j using strategy s . Now the expected utility of agent i , (1), becomes

$$U_i(x|v_i) = \frac{1}{\bar{c}}(v_i - x)(x + (n-1)K). \quad (2)$$

Note that if each player's expected contribution is K , then the probability of provision is $\mathbb{E}[\Pr(c \leq \sum s(v_i))] = nK/\bar{c}$. Thus, any change in the fundraising mechanism that changes the expected contribution will directly affect the probability that the good is provided. Moreover, note that the uncertainty in the cost threshold c makes our framework very close to the more traditional one of Bergstrom *et al.* (1986) in which the quantity of the public good is variable and contributions are sunk. Indeed, if agents have a simple multiplicative utility function over private and public good consumption, if v_i is reinterpreted as income so that private good consumption is $v_i - x$, and if the public good is available at constant marginal cost \bar{c} (so the total provided just equals total donations divided by \bar{c}), then the expected utility of agent i with income v_i that makes donation x is given in (2).

⁶Beyond Barbieri and Malueg (2010), uncertainty in the cost appears in Nitzan and Romano (1990) and McBride (2006).

3.1 Unrestricted contribution possibilities

Here we characterize the unique equilibrium when any nonnegative contributions are allowed. Since U_i in (2) is strictly concave in x , the first-derivative $\partial U_i(x|v_i)/\partial x = [v_i - (n-1)K - 2x]/\bar{c}$, along with the non-negativity constraint on x , yields the following “best-response” function for player i : $s(v_i) = \max\{0, [v_i - (n-1)K]/2\}$. Using this best-response function and the definition of K above, in the symmetric equilibrium the following equation must be satisfied by K :

$$\begin{aligned} K &= \mathbb{E} \left[\max \left\{ 0, \frac{1}{2}(v - (n-1)K) \right\} \right] = \frac{1}{2} \int_{(n-1)K}^1 (v - (n-1)K) f(v) dv \\ &= \frac{1}{2} \int_{(n-1)K}^1 (1 - F(v)) dv, \end{aligned} \quad (3)$$

where the final inequality follows from integration by parts. The right-hand side of (3) is continuous and strictly decreasing in K over $[0, 1/(n-1)]$, with value $\mathbb{E}[v]/2 > 0$ at $K = 0$ and value 0 at $K = 1/(n-1)$. Therefore, there is a unique value of K , which we denote by K^c , that solves (3). Consequently, with unrestricted contributions there is a unique symmetric equilibrium strategy, which is given by

$$s^c(v) = \begin{cases} \frac{1}{2}(v - (n-1)K^c) & \text{if } v \geq (n-1)K^c \\ 0 & \text{otherwise,} \end{cases} \quad (4)$$

where K^c solves (3) (it can be shown there are no asymmetric equilibria—see Barbieri and Malueg, 2010).

3.2 Binary contribution possibilities

Next we suppose players are restricted to contribution levels of 0 and x , where $x \in (0, 1)$. The equilibrium strategy will be of the form

$$s(v) = \begin{cases} x & \text{if } v \geq v^0 \\ 0 & \text{if } v < v^0, \end{cases} \quad (5)$$

for some value v^0 . Suppose all players but player 1 use such a strategy. If player 1 has value v , her expected payoff when not contributing is

$$U^{nc}(v) = v \Pr \left(\sum_{j \neq 1} s(v_j) \geq c \right) = v \times \frac{(n-1)(1 - F(v^0))x}{\bar{c}},$$

and her expected payoff when contributing x is

$$U^c(v) = (v - x) \Pr \left(x + \sum_{j \neq 1} s(v_j) \geq c \right) = (v - x) \times \frac{x + (n - 1)(1 - F(v^0))x}{\bar{c}}.$$

Solving the indifference condition $U^{nc}(v) = U^c(v)$ yields the threshold value

$$v^0 = x[1 + (n - 1)(1 - F(v^0))] = x + (n - 1)K, \quad (6)$$

where $K = x(1 - F(v^0))$ is a player's expected contribution. The middle expression in (6) is strictly decreasing in v^0 , with value nx when $v^0 = 0$ and value x when $v^0 = 1$, implying that for each $x \in (0, 1)$, there is a unique solution v^0 to (6). Hence, there is a unique symmetric equilibrium in the subscription game with binary contribution possibilities.

The following example applies the above analysis to show the common intuition favoring unrestricted contributions over discrete contribution possibilities may not be warranted.

Example 1 (The probability of provision: binary or continuous contribution possibilities).

Consider two players whose values are independently and identically distributed on $[0, 1]$ according to the cdf $F(v) = v^2$. If any contribution levels are allowed, then (3) reduces to

$$K = \int_K^1 \frac{1}{2}(1 - v^2) dv = \frac{1}{3} - \frac{K}{2} + \frac{K^3}{6},$$

the solution to which is $K^c \approx 0.223462$, which is also each player's expected contribution.

Next suppose players' contributions are restricted to be either 0 or x (we may assume $x \leq 1$). Equilibrium has players use a strategy of the form given in (5). Solution of the first equality of (6) yields the critical threshold $v^0(x) = (\sqrt{1 + 8x^2} - 1)/(2x)$. Each player's expected contribution is then $K(x) \equiv x \Pr(v \geq v^0(x)) = (\sqrt{1 + 8x^2} - 1 - 2x^2)/(2x)$. This expected contribution is strictly concave on $[0, 1]$, reaching its maximum at $x^* = (\sqrt{7 - \sqrt{17}})/4 \approx 0.424035$; the resulting expected contribution of each player is $K(x^*) \approx 0.238118$, which exceeds the expected contribution in the unrestricted-contribution model by about 6%. Identical conclusions hold as well for the probability of provision in the two settings. Obviously, though, for "poor" choices of x , the binary-contribution model yields strictly lower contributions than does the unrestricted model. \square

4 Continuous or discrete contributions?

If instead of allowing all contribution levels, the fundraisers restrict contributions to a finite set, then they face a tradeoff. On the one hand, some who might have preferred to give a positive amount now find themselves unwilling to give the minimum acceptable amount, which may reduce overall contributions. On the other hand, some who had planned to give an “intermediate” amount might now prefer to bump up their contributions to the minimum acceptable level, causing them to contribute more than they might otherwise have done, and this tends to raise contributions. Overall, the effect of setting a target contribution will balance these two effects, causing some potential contributors to drop out while encouraging others to give slightly more.

Example 1 clarifies how the choice of level for the restricted contribution scheme is crucial. For the rest of the analysis, we denote with K^d a player’s equilibrium expected contribution when the only contributions allowed are $\{0, x^d\}$, where x^d is the level that maximizes the equilibrium expected contribution. Thus, $K^d = [1 - F(v^0)]x^d$, where v^0 is the threshold value above which a player contributes, and, by (6), $v^0 = x^d + (n - 1)K^d$. At x^d , the first-order condition $dK^d/dx^d = 0$ implies

$$1 - F(v^0) = x^d f(v^0). \quad (7)$$

4.1 The cases of convex or concave F

Our first proposition shows that, when the density of players’ values is either increasing or decreasing, fundraisers have a clear preference for either the continuous or the binary contribution scheme.

Proposition 1 (Continuous versus binary contributions). *Let the common distribution of players’ independent values be F .*

1. *If F is convex, then $K^d \geq K^c$, with strict inequality if F is strictly convex.*
2. *If F is concave, then $K^d \leq K^c$, with strict inequality if F is strictly concave.*

Example 1 illustrates the first part Proposition 1. The intuition for Proposition 1 can be understood with reference to Figure 2, which depicts equilibrium strategies when players’ values are uniformly distributed over $[0, 1]$. In this case, $K^c = K^d$ and $x^d = s^c(1)$. The restriction to contributing either 0 or x^d leads types above v^0 to contribute more than in the continuous case, a benefit represented by region B . But this restriction causes types below v^0 to contribute nothing, and this cost is represented by region A . For the uniform distribution, v^0 is midway between $(n - 1)K^c$ and 1, so the areas of regions A and B are equal. And because the distribution of v is uniform, the weighted benefit of region B equals the weighted cost of region

A. Now suppose the distribution of values deviates from uniform by becoming slightly convex (i.e., the density is slightly increasing). Then, ignoring the induced change in strategies as a first-approximation, the weight on region B becomes larger than that on region A , so the binary-contribution setting yields greater contributions than the unrestricted setting. The comparison is reversed if the distribution becomes slightly concave, as then region A receives greater weight than region B . This accords with the general finding in Proposition 1.

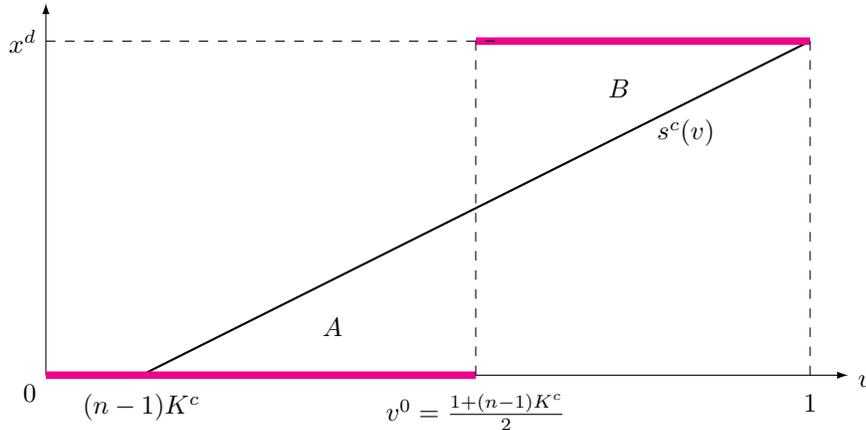


Figure 2: Comparison of binary and continuous strategies when F is uniform on $[0, 1]$: $K^c = K^d$.

4.2 The role of crowding-out and heterogeneity

When the cdf of players' values is neither concave nor convex, Proposition 1 does not yield a definitive comparison. It is however possible to obtain insights for the case of a distribution F that is first convex and then concave, whose mode equals the median, μ , and that deviates from symmetry in the direction of a thicker right-tail, i.e. $F(\mu - z) \leq 1 - F(\mu + z)$.

A first result, very useful for the rest of our analysis, is the following necessary condition for binary contributions to dominate continuous contributions.

Proposition 2 (Comparison of binary and continuous contributions). *Suppose the distribution F of players' values has mode and median μ and satisfies $F(\mu - z) \leq 1 - F(\mu + z)$. Furthermore, assume F is strictly convex for $v < \mu$ and F is strictly concave for $v > \mu$. If $K^d \geq K^c$, then $v^0 < \mu$.*

By Proposition 2, setting a fixed donation level such that $v^0 > \mu$ is counterproductive for the fundraiser. This result is intuitive, given the discussion preceding Proposition 1. There is a tradeoff in restricting agents' freedom to give. On the one hand, some who might have preferred to give a positive amount find themselves unwilling to give the minimum acceptable amount. On the other hand, some who had planned to give an

“intermediate” amount now prefer to increase their contributions. The cutoff between these two different responses to a restriction is v^0 : types immediately below v^0 become non-contributors. If the fundraiser’s choice puts v^0 in the concave part of the distribution, then types immediately below v^0 —those who reduce their contribution—outnumber types immediately above v^0 . We illustrate this reasoning for the all-important case $K^c = K^d$. Figure 3 depicts compares the equilibrium contribution functions, and we have superimposed a symmetric density function, labeled $f(v)$, for values. In Figure 3, because of the assumption $K^c = K^d$, triangles A and B are congruent since $v^0 - (n - 1)K^c = v^0 - (n - 1)K^d = x^d$. The continuous contribution scheme dominates in area C . Therefore, according to the density f , area B must be weighted more heavily than A to assure $K^c = K^d$, and that would be impossible if $v^0 > \mu$.

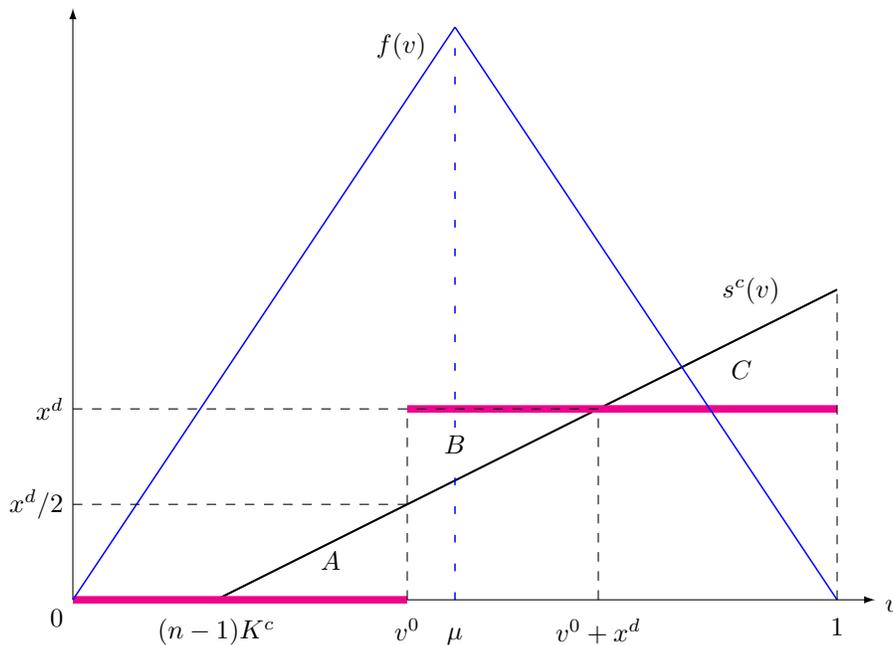


Figure 3: If f is symmetric and single-peaked and if $K^c = K^d$, then $v^0 < \mu$.

In the remainder of this section we study how expected contributions depend on the number of contributors, the level of funding from and external authority, and the degree of dispersion in the distribution of players’ values. The next proposition shows how changes in the number of potential contributors affect the relationship between K^d and K^c .

Proposition 3 (Number-of-player-induced ordering). *As the number of players n increases, at most one intersection between K^d and K^c can occur, at which K^c becomes larger than K^d . Moreover, for n sufficiently large, $K^c > K^d$.*

It turns out that the main force underlying our result on the number of agents is the same we identify

in the next proposition about crowding-out, so we postpone discussion until after that proposition. Let y denote the level of contributions that are exogenously provided by an external authority, and consider how players' contributions change as y increases. Replicating the steps leading to (3), we obtain that the equilibrium strategy is

$$s^c(v) = \max \left\{ 0, \frac{1}{2}(v - (n-1)K^c - y) \right\}, \quad (8)$$

where, in equilibrium, the expected contribution with unrestricted contributions, K^c , solves

$$K^c = \frac{1}{2} \int_{(n-1)K^c+y}^1 (1 - F(v)) dv. \quad (9)$$

Similarly, when contributions are restricted, the indifferent type v^0 , the optimally chosen level x^d , and the expected contribution amount K^d solve

$$v^0 = x^d + (n-1)K^d + y, \quad x^d = \frac{1 - F(v^0)}{f(v^0)}, \quad \text{and} \quad K^d = x^d(1 - F(v^0)). \quad (10)$$

We now hold constant all other parameters and consider how changes in the amount y , exogenously given by an external authority, affect the relationship between K^d and K^c .

Proposition 4 (Crowding-out-induced ordering). *As the amount y increases, at most one intersection between K^d and K^c can occur, at which K^c becomes larger than K^d .*

Using Figure 4, we here provide the intuition behind Proposition 4. Consider the situation where $K^d \geq K^c$ and suppose external funding y is increased by some small amount α . From (8) this shifts rightward the horizontal intercept of the strategy function, s^c , by α , which in turn causes s^c everywhere to shift down by no more than $\alpha/2$, because the slope of s^c is $1/2$. Consequently, this increase in external funding decreases K^c by less than $\alpha/2$. In the discrete-contribution regime, if we ignore any small adjustment in the optimal contribution level, then the switching point between contributors and noncontributors shifts rightward by α . So all types in the interval $(v^0, v^0 + \alpha)$ reduce their contributions by x^d , and the mass of this group is approximately $\alpha \times f(v^0)$. Consequently, the reduction in K^d is approximately $\alpha \times f(v^0) \times x^d$, which can be written as

$$\begin{aligned} \alpha \times f(v^0) \times x^d &= \alpha \times f(v^0) \times \left(\frac{1 - F(v^0)}{f(v^0)} \right) \\ &= \alpha(1 - F(v^0)) \\ &> \frac{\alpha}{2}, \end{aligned}$$

where the first equality follows from the optimal choice of x^d (see (7)) and the inequality follows because v^0 is less than the median (see Proposition 2). Thus, increasing external funding by α causes a greater reduction in K^d (more than $\alpha/2$) than in K^c (less than $\alpha/2$).

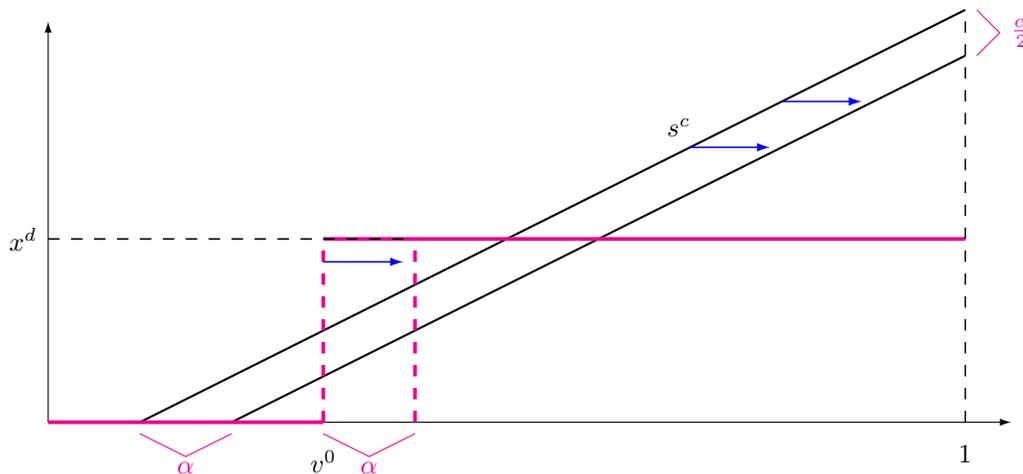


Figure 4: If $K^d \geq K^c$, then crowding out reduces K^d by more than it reduces K^c .

Propositions 3 and 4 are two manifestations of the same main force. Contributions from a membership scheme that restricts contributors' decisions are more responsive to changes in the environment (e.g., an increase in the number of potential contributors or an increase in external donations) than contributions from a more flexible scheme, *conditional on the fundraiser being indifferent between the two*, that is for $K^c = K^d$. The intuition for the result is that a flexible mechanism allows agents wishing to reduce their contributions to do so in a smooth, measured manner that is largely independent of their value. Indeed, in equilibrium, a given change in the expected donation of one player is achieved because (almost) all types who were contributing a positive amount end up reducing their donation by a common quantity. The adjustment is very different in a rigid membership scheme. The only possibility to reduce one's donation is to stop contributing at all. True to its characterization, a rigid membership scheme “breaks but does not bend” and forces a jerky response from agent types: types sufficiently far from v^0 do not change their behavior at all, while types sufficiently close to v^0 precipitously drop their contribution from x to nothing.⁷ Which of the the smooth or the jerky adjustments ends up being larger then depends on the relative importance of types near v^0 . As discussed earlier and as depicted in Figure 3, when the fundraiser is indifferent between the flexible or the rigid scheme, that is for $K^d = K^c$, it must be the case that types near v^0 are very important, and the jerky adjustment ends up being larger in expectation. Therefore, the larger the number of potential contributors or the amount provided by external sources, the larger the crowding-out for a rigid contribution

⁷In the words of Bergstrom *et al.* (1986), the jerky response happens only at the “extensive” margin, while the smooth response happens mostly at the “intensive” margin.

scheme, relative to a flexible one, up to the point in which flexibility becomes preferred by the fundraisers.

We next develop our last comparative statics result, which uses comparative peakedness of two distributions of players' values. Following Birnbaum (1948) and Proschan (1965), we say that cdf $F(v; a')$ is *more peaked about μ* than is $F(v; a)$ if $F(\mu + x; a') - F(\mu - x; a') \geq F(\mu + x; a) - F(\mu - x; a)$ for all $x > 0$. Propositions 5 and 6 below apply to all symmetric distributions $\{F(v; a)\}_a$ that are strictly convex for $v \in [0, \mu)$ and strictly concave for $v \in (\mu, 1]$, and for which increases in a imply increased peakedness about μ . For these propositions the assumption of symmetric distributions is sufficient, but not necessary, though assumptions alternative to symmetry require greater specification about the nature of increased peakedness.

Proposition 5 (Comparative statics for binary contributions). *Let $A \equiv (\underline{a}, \bar{a})$ be a nonempty interval and suppose the family of symmetric cdfs $\{F(\cdot; a)\}_{a \in A}$ exhibits increased peakedness as a increases. If $v^0 > \mu$, then K^d decreases in a . If $v^0 < \mu$, then K^d increases in a .*

Proposition 5 is especially interesting in comparison with the continuous-contribution case. From Barbieri and Malueg (2010) we know that K^c is always decreasing in a (this follows from the convexity of the contribution strategy in the continuous case and the fact that increased peakedness here implies decreased riskiness). In contrast, the relationship for the binary contribution possibilities case depends on the position of the threshold type v^0 relative to μ . This dependence is intuitive. If $v^0 < \mu$ and peakedness increases, then, even leaving x^d unchanged, the types above μ continue to contribute x^d and among those types below μ the number who are contributors increases, so overall contributions increase. Allowing for optimal adjustment of the contribution level x^d can further raise donations at the more peaked distribution. When $v^0 > \mu$, an analogous effect shows that a small increase in peakedness will decrease expected donations in the binary-contribution case.

A consequence of Propositions 2 and 5 is a ranking of total contributions K^d and K^c that depends on the peakedness of the distribution.

Proposition 6 (Peakedness-induced ordering). *Let $A \equiv (\underline{a}, \bar{a})$ be a nonempty interval and suppose the family of symmetric cdfs $\{F(\cdot; a)\}_{a \in A}$ exhibits increased peakedness as a increases. If $v^0 > \mu$, then K^d decreases in a . As peakedness of the distribution $F(v; a)$ increases, at most one intersection between K^d and K^c can occur, at which K^d becomes larger than K^c .*

The relationship between the binary vs. continuous comparison and peakedness, as just discussed, is intuitive. Offering only a limited number of alternative contribution levels—the binary contribution possibilities is an extreme case—is a way to target a subset of types (those in a right neighborhood of v^0) and induce them to contribute more than they otherwise would. Clearly, this effect obtains because agents have fewer

contribution options. The down side of this restriction of contribution possibilities is that some types may choose to contribute less than they otherwise would. Types smaller than the target may decide to contribute nothing at all while they would have contributed a smaller, but positive amount, if given the opportunity. Similarly, types larger than the target may be constrained to contribute less than they would have done if given more alternatives. The situation is illustrated in Figure 3 for $K^c = K^d$. Now increase peakedness slightly, and, as a first-approximation, suppose in the two scenarios players continue using the strategies depicted. An increase in peakedness of the distribution of values tends to reduce the significance of regions A and C while increasing that of region B , so that the equivalence of donations under the two contribution schemes breaks in favor of K^d as F becomes more peaked.

The following example uses a family of triangular distributions to illustrate our comparative statics findings.

Example 2 (Comparative statics for a family of triangular distributions).

Consider the density of v given by $f(v; a) = (1 - (1/4)a) + av$ for $v \in [0, 1/2]$ and $f(v; a) = (1 - (1/4)a) + a(1 - v)$ for $v \in (1/2, 1]$, where $a \in [0, 4]$ parameterizes the peakedness of the distribution. When a is zero, we have the usual uniform distribution on $[0, 1]$. As a increases, the weight on the tails of the distribution decreases and concentrates around the mean/median of $\frac{1}{2}$.

Table 2 illustrates Propositions 2–6. In accord with Proposition 5, for $n = 2$, $y = 0$, and $a = 0, 1, 2$, we have $v^0 \geq \mu$ and increases in peakedness reduce K^d ; but for $a = 2, 3, 4$, we have $v^0 \leq \mu$ and increases in a increase K^d . The data also reflect Proposition 6’s conclusion that as the distribution becomes more peaked, the binary scheme may come to dominate the continuous-contribution scheme (here the ranking switches for a value of a lying between 3 and 4). Note too that $K^d > K^c$ when $a = 4$, so Proposition 2 implies $v^0 < \mu$, which is indeed the case here ($v^0 = 0.46821 < .5 = \mu$). More generally, whether the graphs of K^d and K^c intersect depends on the available range for the peakedness. One may show that if the distribution $F(v, a)$ goes, in order of increasing peakedness, from uniform on $[0, \bar{v}]$, to a degenerate distribution on $\mu = \bar{v}/2$, then the intersection will happen.⁸

Next, given $y = 0$ and $a = 4$, Table 2 reveals that in moving from 2 to 3 players, the ranking of K^c and K^d switches, in accordance with Proposition 3.

Finally, returning to the case of two players and $a = 4$, Table 2 reports that the ranking of K^c and K^d switches as we increase y from 0 to 0.1, in accordance with Proposition 4. □

⁸When F is a degenerate distribution at μ , players contribute some common fixed amount s_0 in the Nash equilibrium with unrestricted contributions. In the best discrete contribution scheme, the fundraiser will generally choose a level $x^d \neq s_0$; for this reason, $K^d > K^c$ when F is a degenerate distribution.

Table 2: Results for optimal continuous or binary contribution schemes, depending on peakedness, $n = 2$

Parameter configuration	K^c	K^d	x^d	v^0
$n = 2, y = 0, a = 0$	0.17157	0.17157	0.41421	0.58578
$n = 2, y = 0, a = 1$	0.17058	0.16779	0.37242	0.54021
$n = 2, y = 0, a = 2$	0.16961	0.16667	0.33333	0.5
$n = 2, y = 0, a = 3$	0.16865	0.16718	0.31415	0.48133
$n = 2, y = 0, a = 4$	0.16772	0.16837	0.29984	0.46821
$n = 3, y = 0, a = 4$	0.12778	0.125	0.245	0.5
$n = 2, y = 0.1, a = 4$	0.13626	0.13341	0.256	0.4934

5 Extensions

The objective of this section is to gauge how well the intuitions we developed earlier fare when we consider richer, more realistic setups. We first consider a three-way comparison of 1-, 2-, and 3-level schemes. Then, we turn our attention to the situation where restrictions in contributions are not imposed at the outset, but they are “enforced” through the careful doling out of selective incentives.

5.1 A comparison of 1-, 2-, and 3-level membership schemes

From a technical point of view, few changes are needed with respect to the discrete-level model in Section 3.2. Indeed, given two possible donation levels x_l and x_h , the type indifferent between them is $v = (n - 1)K + y + x_l + x_h$, which encompasses equation (6). All other relevant considerations follow in a similar fashion. From a substantive point of view, the differences with the comparisons in Section 4 appear larger, at first glance. Indeed, while the 2-level scheme is more flexible than a 1-level scheme, it is straightforward to see how the 2-level scheme cannot possibly do worse than the 1-level scheme. Similar considerations hold true when comparing the 2- and 3-level schemes. Nevertheless, we argue that the results in Propositions 3, 4, and 6 remain relevant. For the comparisons in this section, it is the *degree* to which the more flexible schemes dominate the less flexible ones that is positively affected by population size, value heterogeneity, and extent of crowding-out. Thus, if there are costs to complexity, then these costs may overtake the benefits of greater flexibility, and this is more likely to occur when population size, value heterogeneity, and extent of crowding-out are small.

We can think of two reasonable sources of costs of complexity. First, on the operations side, setting up a scheme with more levels may well cost more (real-life instances include the cost of finding more sources of selective incentives, the cost of setting up more elaborate systems to receive payments and disburse benefits, and the like). Second, we can appeal to the behavioral literature on “choice aversion.” In particular, Iyengar and Lepper (2000) show how consumers facing a larger array of choices may actually decide to purchase less

Table 3: Comparison of 1-, 2-, and 3-level equilibrium contributions

Parameter Configuration	Total contributed, net of the complexity cost, in the 1-level scheme	Total contributed, net of the complexity cost, in the 2-level scheme	Total contributed, net of the complexity cost, in the 3-level scheme
(1) $n = 2, y = 0, a = 10$	0.365669	0.365661	0.364891
(2) $n = 2, y = 0, a = 9$	0.362793	0.362988	0.362257
(3) $n = 2, y = 0, a = 2$	0.335099	0.338812	0.338765
(4) $n = 2, y = 0, a = 1$	0.3325	0.336507	0.336523
(5) $n = 2, y = 0.1, a = 10$	0.282401	0.283408	0.282994
(6) $n = 2, y = 0.2, a = 10$	0.204029	0.206262	0.206275
(7) $n = 2, y = 0.3, a = 10$	0.133005	0.136296	0.136773
(8) $n = 3, y = 0, a = 10$	0.387884	0.389071	0.388481
(9) $n = 6, y = 0, a = 10$	0.427592	0.43105	0.430978
(10) $n = 7, y = 0, a = 10$	0.436578	0.440446	0.4405

often. In our framework, this may be modeled as a proportional reduction in value for the public good when agents are faced with many choices.

The next example describes a situation where the cost of complexity enters on the operations side, proportionally. Similar results obtain for the “choice aversion” interpretation (details available upon request).

Example 3 (Comparisons of 1-, 2-, and 3-level schemes). *Consider the density $f(v'; a) = \frac{[v'(1-v')]^a}{\int_0^1 [v(1-v)]^a dv}$ on $[0, 1]$. The values for the threshold membership levels are optimally chosen to maximize expected contributions. The cost of complexity for a q -level scheme is a percentage of total private donations: $0.0025qnK$, with $q = 1, 2, 3$. Table 3 summarizes the relevant quantities.*

As the table describes, we recover similar predictions to those in Propositions 3, 4, and 6 (maximal net contributions are shown in boldface). Value heterogeneity favors flexibility, as shown by configurations (1) and (2)–(4). Larger exogenous donations (not included in the total contributed above because they are the same for all contribution schemes) favor flexibility, as shown by configurations (1) and (5)–(7). Finally, a larger population favors flexibility as well, as configurations (1) and (8)–(10) show.

5.2 Benefit-induced restrictions

In this section we consider a more realistic membership scheme in which agents are free to donate any amount they desire, but “restrictions” in donations arise from the package of selective benefits. With respect to the model in Section 2, nothing changes about the way in which agents benefit from the public good. However, we now assume that contributors also enjoy a selective benefit $b(x)$, distributed by the association in exchange for a donation level x . We maintain the assumption, typical of the subscription game, that if the public good cannot be produced, then agents receive their contributions back and obtain a payoff of zero.

When the public good is produced, for simplicity, we assume $b(x)$ enters additively in the utility function, so that the expected utility of agent i in (2) now becomes

$$U_i(x|v_i) = \frac{1}{c}(v_i + b(x) - x)(x + (n - 1)K + y).$$

We consider two ways in which fundraisers allocate selective benefits. In the first, b equals an exogenously specified amount $q > 0$, but only if the donation x exceeds an endogenously chosen level x_b^d . Otherwise, $b = 0$. We label this the “discrete-benefit” scheme. In the second, b is a simple linear function of donations: $b(x) = \alpha x$, with $\alpha > 0$. These formulations resemble Harbaugh’s (1998a) introduction of “prestige” that results from contributions. If contributions are reported exactly, more prestige is “bought” with larger contributions, and we specify a proportional representation of this. If categories are introduced, as in Harbaugh, then after a donor contributes, the receiver simply reports publicly in which category that donor’s contribution fell.⁹ Alternatively, we introduce a single category, where anyone contributing at least x^d is reported to be a member of this category and thereby receives prestige benefit of q when the good is also provided. An important difference with Harbaugh’s setup remains: A contributors’ utility depends on other agents’ donations; thus, strategic considerations remain paramount. These strategic effects underlie our results on population size and crowding-out.

We begin the analysis with the “continuous-benefit” scheme. It is easy to retrace our steps leading to (4) and to show that in the only symmetric equilibrium donations are

$$s_b^c(v) = \begin{cases} \frac{1}{2} \left(\frac{v}{1-\alpha} - (n-1)K_b^c - y \right) & \text{if } v \geq ((n-1)K_b^c + y)(1-\alpha) \\ 0 & \text{otherwise,} \end{cases}$$

where K_b^c solves $K_b^c = E[s_b^c(v)]$, and y is the amount exogenously provided by an external authority—as in Proposition 4. The equilibrium expected value of selective benefits is

$$S_b^c = \int_{((n-1)K_b^c + y)(1-\alpha)}^1 \alpha s_b^c(v) (s_b^c(v) + (n-1)K_b^c + y) f(v) dv.$$

For the discrete-benefit scheme, types donating amounts smaller than x_b^d , that is, types for which $b(x) = 0$, either do not contribute, or if they do, they donate $\frac{1}{2}(v - (n-1)K_b^d - y)$, where K_b^d is the equilibrium expected contribution. Similarly, types that donate more than x_b^d (they receive benefit $b(x) = q$) contribute $\frac{1}{2}(v + q - (n-1)K_b^d - y)$. The previously mentioned agents have very low or very high values. For intermediate

⁹For example, the New Orleans Preservation Resource Center identifies major donors by their membership in categories designated as Italianate (\$25,000 and above), Greek Revival (\$15,000–\$24,999), Romanesque Revival (\$10,000–\$14,999), Steamboat Gothic (\$5,000–\$9,999), Queen Anne (\$2,500–\$4,999), Landmark (\$1,000–\$2,499), and Conservator (\$500–\$999).

values, the possibility of capturing the benefit $b(x) = q$ through an upwards departure in contributions may prove attractive. It is necessary to distinguish two cases. First, if there are types that contribute $x \in (0, x_b^d)$, then v_b^0 , which is the type indifferent between donating $(v - (n - 1)K_b^d - y)/2$ and x_b^d , equals

$$v_b^0 = (n - 1)K_b^d + y + 2x_b^d - 2\sqrt{q((n - 1)K_b^d + y + x_b^d)}.$$

In the second case there are no types contributing a positive amount smaller than x_b^d and v_b^{02} , which is the type indifferent between donating 0 and x_b^d , equals

$$v_b^{02} = ((n - 1)K_b^d + y + x_b^d)(x_b^d - q)/q.$$

In both cases, types larger than $(n - 1)K_b^d + y + 2x_b^d - q$ contribute more than x_b^d , as previously described.

Therefore, if $v_b^{02} \geq (n - 1)K_b^d + y$, then we are in the first case and equilibrium contributions are

$$s_b^d(v) = \begin{cases} 0 & \text{if } v \leq (n - 1)K_b^d + y, \\ \frac{1}{2}(v - (n - 1)K_b^d - y) & \text{if } (n - 1)K_b^d + y < v \leq v_b^0, \\ x_b^d & \text{if } v_b^0 < v \leq (n - 1)K_b^d + y + 2x_b^d - q, \\ \frac{1}{2}(v + q - (n - 1)K_b^d - y) & \text{if } (n - 1)K_b^d + y + 2x_b^d - q < v \leq 1, \end{cases}$$

with corresponding equilibrium value of selective benefits

$$S_b^d = \int_{v_b^0}^1 q (s_b^d(v) + (n - 1)K_b^d + y) f(v) dv.$$

In the second case, that is, if $v_b^{02} < (n - 1)K_b^d + y$, then there are no positive equilibrium contributions smaller than x_b^d and the equilibrium strategy is

$$s_b^{d2}(v) = \begin{cases} 0 & \text{if } v \leq v_b^{02}, \\ x_b^d & \text{if } v_b^{02} < v \leq (n - 1)K_b^d + y + 2x_b^d - q, \\ \frac{1}{2}(v + q - (n - 1)K_b^d - y) & \text{if } (n - 1)K_b^d + y + 2x_b^d - q < v \leq 1, \end{cases}$$

with corresponding equilibrium value of selective benefits

$$S_b^{d2} = \int_{v_b^{02}}^1 q (s_b^{d2}(v) + (n - 1)K_b^d + y) f(v) dv.$$

The following figure depicts $s_b^c(v)$ and $s_b^d(v)$ (which is the relevant contribution function for the parameters in the upcoming Example 4) and, for ease of notation, we indicate with \underline{v}_b^c (\underline{v}_b^d) the value at which the continuous-benefit (discrete-benefit) equilibrium strategy becomes positive. By the above descriptions, $\underline{v}_b^c = ((n-1)K_b^c + y)(1-\alpha)$ and $\underline{v}_b^d = (n-1)K_b^d + y$.

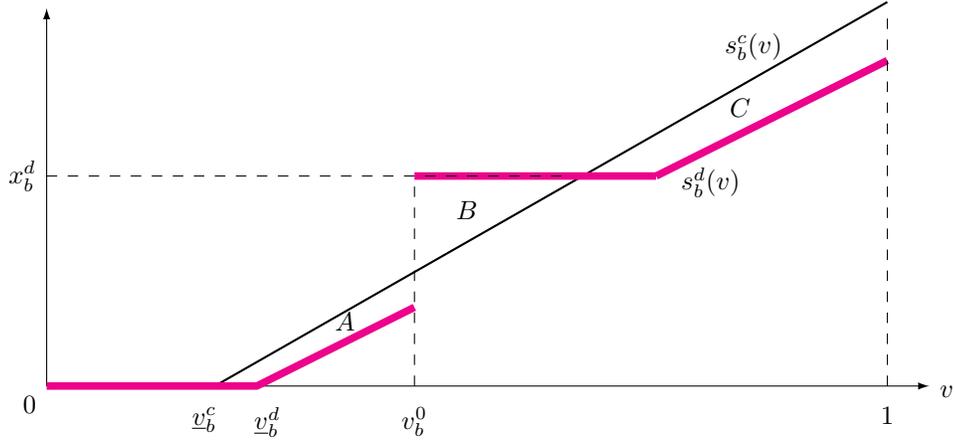


Figure 5: Comparison of discrete-benefit and continuous-benefit expected contributions.

Figure 5 shows that the comparison of discrete-benefit vs. continuous-benefit schemes is very similar to the comparison of discrete-level vs. continuous-level schemes in Figure 3: if $K_b^c = K_b^d$, then discrete does better in area B while continuous is superior in areas A and C . All earlier graphical intuitions about the advantages and pitfalls restricting agents' freedom to donate carry over to a scheme that restricts agents' rewards for a donation, because the relative position of areas A , B , and C is the same. Therefore, we may expect the comparative statics about K^d and K^c in Propositions 3, 4, and 6 to remain valid for their respective analogues K_b^d and K_b^c , as confirmed by the next example.

Example 4 (Discrete-benefit vs. continuous-benefit schemes). *Consider the same distribution the density $f(v;a)$ in Example 3. Let the value of selective benefits for the discrete-benefit scheme, q , be 0.1. Fix the expected value of selective benefits in equilibrium at 0.02, and let the marginal value of selective benefits for the continuous-benefit scheme, α , be endogenously determined to satisfy this restriction. Table 4 summarizes the relevant quantities.*

From Table 4 we see the switches in the order of K_b^c and K_b^d are all in accordance with Propositions 3, 4, and 6. Indeed, increasing peakedness, a , favors the discrete-benefit scheme, as the comparison of configurations (1) and (2) shows. Moreover, the continuous-benefit scheme is favored by an increase in the number of players, n , as configurations (2) and (3) show, and by an increase in the amount y exogenously

Table 4: Continuous-benefit vs. discrete-benefit equilibria, $q = 0.1$

Parameter Configuration	K_b^c	α	K_b^d	x_b^d
(1) $n = 2, y = 0, a = 1$	0.207968	0.191618	0.207766	0.480107
(2) $n = 2, y = 0, a = 6$	0.210818	0.209341	0.210893	0.431257
(3) $n = 3, y = 0, a = 6$	0.16292	0.231325	0.157974	0.398806
(4) $n = 2, y = 0.15, a = 6$	0.166548	0.229144	0.160568	0.400524

provided by an external authority, as configurations (2) and (4) show.¹⁰

The intuitive reason for why the relative position of areas A , B , and C is the one depicted in Figure 5 is straightforward. The position of area A is determined by the larger likelihood of positive contributions under the flexible scheme, which arises because even very small contributions are rewarded with selective benefits, in contrast with the rigid scheme. Area C reflects the fact that, under the flexible scheme, the marginal selective benefit is positive, while it is almost always zero under the rigid scheme. Thus, the incentive to contribute more than x_b^d in the rigid scheme comes only from the increase in the probability of provision. In contrast, under the flexible scheme, there arises the additional prize of a larger selective incentive. Therefore, it is not surprising that contributions larger than x_b^d are more frequent in the flexible scheme, as depicted in area C . Finally, since the flexible mechanism dominates for small and large values and since Figure 5 is drawn under the assumption $K_b^c = K_b^d$, it must be the case that the rigid mechanism dominates for intermediate values, as described by area B .

6 Conclusion

Fundraisers may profit from restricting donors' possible levels of contribution because such restrictions can induce some people to contribute more than they otherwise would. But this benefit must be weighed against the cost that these restrictions can also induce some people to give less than they otherwise would. The relative importance of these two effects determines whether such restrictions are indeed profitable.

Using a subscription game framework to study the private provision of a discrete public good, we have identified several factors militating in favor of greater flexibility for contributors. If the distribution of players' values is concave, then the flexible (continuous) contribution framework yields greater revenue. For symmetric distributions of players' values having a density that is first increasing and then decreasing, the flexible scheme is again preferred as i) the dispersion of donors' taste for the public good increases, ii) the

¹⁰We have also performed numerical comparisons between a scheme with one benefit level and a two-level scheme, and we find the same patterns as in Table 4. Indeed, the relatively more flexible two-level scheme generates larger expected contributions than the rigid one-level scheme when there are increases i) in the dispersion of values, ii) in the number of players, and iii) in the amount exogenously provided by an external authority. Full details are available upon request.

number of potential donors increases, and iii) there is greater funding by an external authority. These predictions of the model are consistent with patterns arising from fundraising practices of NPR stations in the US. We found that these stations offer a larger number of suggested contribution levels as i) the incomes of the population served become more diverse, ii) the population of the coverage area increases, and iii) there is greater external support from the Corporation for Public Broadcasting.

The more direct implications of our results concern fundraising: We identify easily obtainable characteristics of the target donor population that should be taken into account in the practical design of a campaign. Moreover, the forces behind our results appear to be relevant for more widely defined collective effort problems, such as team production. Our most general message is that the design of a campaign affects the responses of contributors in ways that at times are predictable. Exploiting such responses may prove valuable for further research. For instance, our model suggests that one may expect considerable variability in the extent of crowding-out effects across donors, when they are faced with a scheme with suggested membership levels. Donors' responses vary according to the position of their values with respect to critical cutoffs, and this position is at least in part revealed by their chosen donation. This observation may be of help to the large literature on crowding out.¹¹

Finally, it is worth pointing out our simple, but novel, empirical exercise in the Introduction only provides suggestive patterns, for which we offer a theoretical rationalization. Thus, more empirical research is needed in the directions that our model points out. Our results on the determination of the best discrete-contribution levels may also offer guidance to experimental analysis, making it possible to design experiments that have the potential to test whether offering membership categories actually raises total contributions.

¹¹See, e.g., Manzoor and Straub (2005) and references therein.

Appendix

The following lemma, building on (6) and (7), is useful for the rest of the analysis.

Lemma 1 (Bounding the best binary contribution). *Let F be the common distribution of the n players' independent values.*

1. *If F is convex on $[v^0, 1]$, then $x^d \geq [1 - (n - 1)K^d]/2$, with strict inequality if F is strictly convex on $[v^0, 1]$.*
2. *If F is concave on $[v^0, 1]$, then $x^d \leq [1 - (n - 1)K^d]/2$, with strict inequality if F is strictly concave on $[v^0, 1]$.*

Proof. If F is convex on $[v^0, 1]$, then $1 = F(1) \geq F(v^0) + f(v^0)(1 - v^0)$, so

$$\begin{aligned} x^d &= \frac{1 - F(x^d + (n - 1)K^d)}{f(x^d + (n - 1)K^d)} && \text{(by (6) and (7))} \\ &\geq 1 - (x^d + (n - 1)K^d), && \text{(by convexity of } F \text{ on } [v^0, 1]) \end{aligned} \quad (11)$$

implying

$$x^d \geq \frac{1}{2}(1 - (n - 1)K^d). \quad (12)$$

If F is strictly convex on $[v^0, 1]$, then the inequality in (11) holds strictly, and so too does (12). If instead F is concave on $[v^0, 1]$, then the inequality in (11) is reversed as is that in (12), with strict inequality holding in both (11) and (12) if F is strictly concave on $[v^0, 1]$. \square

Proof of Proposition 1. First suppose F is convex. The equilibrium contribution function in the continuous game is $s^c(v) = \max\{0, [v - (n - 1)K^c]/2\}$. Define $x^* = s^c(1) = [1 - (n - 1)K^c]/2$; in the equilibrium of the binary contribution game with $\{0, x^*\}$ denote by K^* a player's expected contribution. Because x^d maximizes a player's expected contribution, it must be that $K^d \geq K^*$. We will show $K^* \geq K^c$, with strict inequality if F is strictly convex, thereby proving part 1. Let φ denote a uniform probability distribution on the interval $[(n - 1)K^c, 1]$ (the use of this distribution will become clear below). The proof is by contradiction, so suppose the proposition is false, that is, $K^* < K^c$. Then

$$\begin{aligned} K^* &= x^*[1 - F(x^* + (n - 1)K^*)] \\ &> x^*[1 - F(x^* + (n - 1)K^c)] \end{aligned} \quad (13)$$

$$\begin{aligned}
&= \frac{1}{2}[1 - (n-1)K^c] \left[1 - F\left(\frac{1}{2}[1 + (n-1)K^c]\right) \right] \\
&= \frac{1}{2}[1 - (n-1)K^c][1 - F(E[v | \varphi])] \\
&\geq \frac{1}{2}[1 - (n-1)K^c][1 - E[F(v) | \varphi]] \\
&= \frac{1}{2}[1 - (n-1)K^c] \left[1 - \frac{1}{1 - (n-1)K^c} \int_{(n-1)K^c}^1 F(v) dv \right] \\
&= \frac{1}{2} \left[[1 - (n-1)K^c] - \int_{(n-1)K^c}^1 F(v) dv \right] \\
&= \frac{1}{2} \int_{(n-1)K^c}^1 (1 - F(v)) dv \\
&= K^c,
\end{aligned} \tag{14}$$

contradicting the assumption that $K^* < K^c$. (The last equality follows from (3).) If, further, F is strictly convex, then the contradiction hypothesis becomes $K^* \leq K^c$ and the inequality in (13) becomes weak while that in (14) becomes strict, again yielding a contradiction. This establishes part 1.

Next suppose F is concave, and suppose contrary to part 2 that $K^d > K^c$. Then Lemma 1 implies that $x^d < (1 - (n-1)K^c)/2 = s^c(1)$. We will show that the binary game with positive contribution x^d yields expected revenue less than K^c , contradicting the initial assumption that $K^d > K^c$.

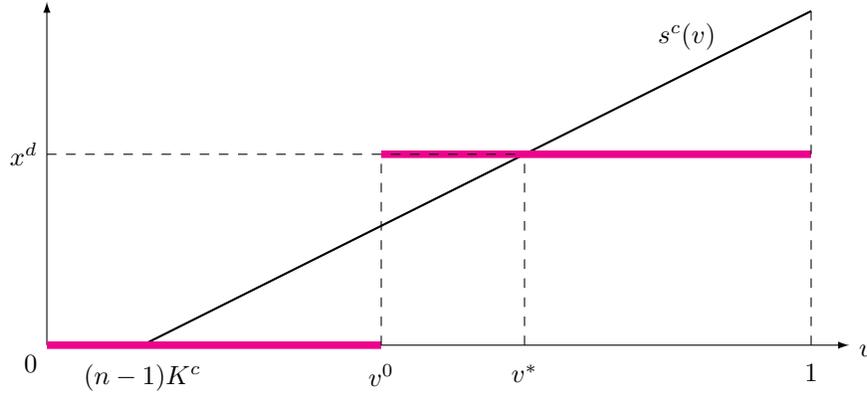


Figure 6: Comparison of binary and continuous contribution strategies when F is concave.

Figure 6 shows the comparison being made, where v^* solves $s^c(v) = x^d$. It is readily checked that $v^* = (n-1)K^c + 2x^d$. By (6), the associated binary-contribution game equilibrium has threshold value $v^0 = (n-1)K^d + x^d$. Let φ denote a uniform probability distribution on the interval $[(n-1)K^c, v^*]$. A

player's expected contribution in the continuous game is

$$\begin{aligned}
K^c &= \int_{(n-1)K^c}^{v^*} s^c(v)f(v) dv + \int_{v^*}^1 s^c(v)f(v) dv \\
&> \int_{(n-1)K^c}^{v^*} s^c(v)f(v) dv + s^c(v^*)[1 - F(v^*)] && \text{(by } v^* < 1) \tag{15}
\end{aligned}$$

$$\begin{aligned}
&= s^c(v)F(v) \Big|_{(n-1)K^c}^{v^*} - \frac{1}{2} \int_{(n-1)K^c}^{v^*} F(v) dv + s^c(v^*)[1 - F(v^*)] \\
&= s^c(v^*) - \frac{1}{2} \int_{(n-1)K^c}^{v^*} F(v) dv && \text{(by } s^c((n-1)K^c) = 0) \\
&= x^d \left[1 - \frac{1}{2x^d} \int_{(n-1)K^c}^{(n-1)K^c + 2x^d} F(v) dv \right] && \text{(by } s^c(v^*) = x^d) \tag{16}
\end{aligned}$$

$$\begin{aligned}
&= x^d [1 - \mathbb{E}[F(v) | \varphi]] \\
&\geq x^d [1 - F(\mathbb{E}[v | \varphi])] && \text{(by } F \text{ concave)} \tag{17} \\
&= x^d [1 - F((n-1)K^c + x^d)] \\
&\geq x^d [1 - F((n-1)K^d + x^d)] && \text{(by contradiction)} \\
&= x^d [1 - F(v^0)] && \text{(by (6))} \\
&= K^d,
\end{aligned}$$

contradicting the assumption that $K^d > K^c$. If, further, F is strictly concave, then the contradiction hypothesis becomes $K^d \geq K^c$ and the inequality in (15) becomes weak while that in (17) becomes strict, again yielding a contradiction. This establishes part 2. \square

Proof of Proposition 2. The proof is by contradiction: given $K^d \geq K^c$ we assume $v^0 \geq \mu$ and show this leads to the contradictory conclusion that $K^d < K^c$. Now suppose $K^d \geq K^c$ and $v^0 \geq \mu$. Because $v^0 \geq \mu$ and F is strictly concave $\forall v > v^0$, part 2 of Lemma 1 implies $x^d < [1 - (n-1)K^d]/2 \leq [1 - (n-1)K^c]/2$, where the second inequality follows from the assumption $K^d \geq K^c$. Proceeding as in the proof of Proposition 1 part 2, we obtain (15) and reach (16'):

$$K^c > x^d \left[1 - \frac{1}{2x^d} \int_{(n-1)K^c}^{(n-1)K^c + 2x^d} F(v) dv \right] = x^d (1 - \mathbb{E}[F(v) | \varphi]), \tag{16'}$$

where φ denotes a uniform probability distribution on the interval $[(n-1)K^c, (n-1)K^c + 2x^d]$.

We now separate the parameter space into three exhaustive regions and show that, in all three, (16') implies $K^c > K^d$. In the first region $\mu \leq (n-1)K^c$, so that F is strictly concave for the relevant range of

the integral in (16'), so that

$$\begin{aligned}
K^c &> x^d(1 - F(\mathbb{E}[v | \varphi])) = x^d[1 - F((n-1)K^c + x^d)] \\
&\geq x^d[1 - F((n-1)K^d + x^d)] \\
&= K^d.
\end{aligned}$$

In the second region $\mu \geq (n-1)K^c + 2x^d$, so from (16') we obtain

$$\begin{aligned}
K^c &> x^d(1 - \mathbb{E}[F(v) | \varphi]) \\
&\geq x^d[1 - F(v^0)] \\
&= K^d,
\end{aligned}$$

where the second inequality follows because $F(v) \leq F(v^0)$ for every v in the support of φ by the assumption $v^0 \geq \mu$.

We proceed to the analysis of the third and final region, $(n-1)K^c < \mu < (n-1)K^c + 2x^d$, with the help of Figure 7. We first define a new distribution function H . H agrees with F for $v < (n-1)K^c$. For larger values, H is the straight line passing through the points A and $(\mu, \frac{1}{2})$, up to the intersection between F and this straight line that occurs to the right of μ . In the figure, this intersection point is B . Therefore, between A and B , the equation that characterizes H is

$$H(v) = \frac{\frac{1}{2} - F((n-1)K^c)}{\mu - (n-1)K^c}(v - (n-1)K^c) + F((n-1)K^c).$$

We denote the horizontal coordinate of point B as b_x . To complete the definition of H , for values larger than b_x , H agrees with F . The curvature properties of F imply $H(\mu) = F(\mu)$, $H(v) \geq F(v)$ if $v < \mu$, $H(v) \leq F(v)$ if $v > \mu$, and H concave for $v \geq (n-1)K^c$. Moreover, in its linear part, H is symmetric. Indeed, it is easily verified that $H(\mu + z) + H(\mu - z) = 1$. Finally, note that $b_x \leq \mu + (\mu - (n-1)K^c)$, as depicted in the figure. Otherwise, for $z = \mu - (n-1)K^c$ (so that $H(\mu - z) = F(\mu - z)$) we would have $1 = H(\mu + z) + H(\mu - z) < F(\mu + z) + F(\mu - z)$, thus contradicting the assumption $F(\mu + z) + F(\mu - z) \leq 1$.

We now use these properties to show that

$$\int_{(n-1)K^c}^{(n-1)K^c + 2x^d} F(v) dv \leq \int_{(n-1)K^c}^{(n-1)K^c + 2x^d} H(v) dv. \tag{18}$$

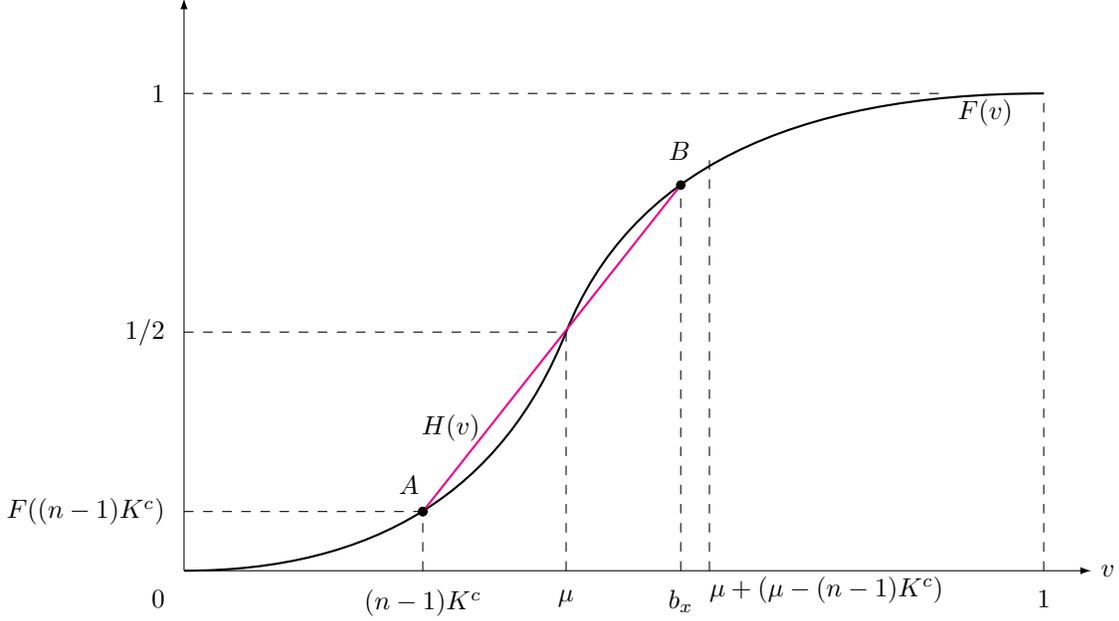


Figure 7: Linearization of F about its mode/median.

Indeed,

$$\begin{aligned} \int_{(n-1)K^c}^{(n-1)K^c+2x^d} (F(v) - H(v)) dv &= \int_{(n-1)K^c}^{\mu} (F(v) - H(v)) dv + \int_{\mu}^{(n-1)K^c+2x^d} (F(v) - H(v)) dv \\ &\leq \int_{(n-1)K^c}^{\mu} (F(v) - H(v)) dv + \int_{\mu}^{b_x} (F(v) - H(v)) dv, \end{aligned}$$

because F and H agree past b_x . Therefore, after the changes of variable $w_l = \mu - v$ and $w_r = v - \mu$, the expression above equals

$$\begin{aligned} &\int_0^{\mu-(n-1)K^c} (F(\mu - w_l) - H(\mu - w_l)) dw_l + \int_0^{b_x-\mu} (F(w_r + \mu) - H(w_r + \mu)) dw_r \\ &\leq \int_0^{\mu-(n-1)K^c} (F(\mu - w_l) - H(\mu - w_l)) dw_l + \int_0^{b_x-\mu} (1 - F(\mu - w_r) - 1 + H(\mu - w_r)) dw_r \\ &= \int_0^{\mu-(n-1)K^c} (F(\mu - w_l) - H(\mu - w_l)) dw_l + \int_0^{b_x-\mu} (H(\mu - w_r) - F(\mu - w_r)) dw_r. \end{aligned}$$

Now note that in the above expression the second integrand is positive, so that, using $b_x \leq \mu + (\mu - (n-1)K^c)$ and summarizing the above chain of inequalities, we obtain

$$\begin{aligned} &\int_{(n-1)K^c}^{(n-1)K^c+2x^d} (F(v) - H(v)) dv \\ &\leq \int_0^{\mu-(n-1)K^c} (F(\mu - w_l) - H(\mu - w_l)) dw_l + \int_0^{\mu-(n-1)K^c} (H(\mu - w_r) - F(\mu - w_r)) dw_r \leq 0, \end{aligned}$$

thus proving (18).

Using (16'), (18), and concavity of H on $[(n-1)K^c, 1]$, we now have

$$\begin{aligned}
K^c &> x^d \left[1 - \frac{1}{2x^d} \int_{(n-1)K^c}^{(n-1)K^c + 2x^d} H(v) dv \right] \\
&= x^d [1 - \mathbb{E}[H(v) | \varphi]] \\
&\geq x^d [1 - H(\mathbb{E}[v | \varphi])].
\end{aligned} \tag{19}$$

If $\mathbb{E}[v | \varphi] < \mu$, then (19) implies $K^c > x^d/2 \geq x^d(1 - F(v^0)) = K^d$. If $\mathbb{E}[v | \varphi] \geq \mu$, then, because $F(v) \geq H(v) \forall v \geq \mu$, (19) implies

$$K^c > x^d [1 - F(\mathbb{E}[v | \varphi])] = x^d [1 - F((n-1)K^c + x^d)] \geq x^d [1 - F((n-1)K^d + x^d)] = K^d.$$

Thus, in all cases, if $K^d \geq K^c$, then $v^0 \geq \mu$ is impossible, so it must be that $v^0 < \mu$. \square

Proof of Proposition 3. It is immediate to verify that both K^c and K^d are decreasing in n . Therefore, the first part of the proof is complete after we establish that if $K^d \geq K^c$, then $\left| \frac{dK^d}{dn} \right| > \left| \frac{dK^c}{dn} \right|$. From equations (3)–(7) and the definition of K^d we obtain

$$\begin{aligned}
\left| \frac{dK^d}{dn} \right| - \left| \frac{dK^c}{dn} \right| &= \frac{1 - F(v^0)}{1 + (n-1)(1 - F(v^0))} K^d - \frac{1 - F((n-1)K^c)}{2 + (n-1)(1 - F((n-1)K^c))} K^c \\
&\geq \frac{1 - F(v^0)}{1 + (n-1)(1 - F(v^0))} K^c - \frac{1 - F((n-1)K^c)}{2 + (n-1)(1 - F((n-1)K^c))} K^c \quad (K^d \geq K^c \text{ is assumed}) \\
&= \left(\frac{1 - 2F(v^0) + F((n-1)K^c)}{[1 + (n-1)(1 - F(v^0))][2 + (n-1)(1 - F((n-1)K^c))]} \right) K^c \\
&> 0,
\end{aligned}$$

where the final inequality follows because $v^0 < \mu$ (which follows from Proposition 2) implies $F(v^0) < 1/2$.

To show that for n sufficiently large $K^d < K^c$, proceed by contradiction; that is, suppose that there exists some N' such that for all $n > N'$, $K^d \geq K^c$ (the possibility of multiple intersections is excluded by the analysis of the previous paragraph). By Proposition 2, it must be that $v^0 < \mu$, for all $n > N'$. From the definition of K^d we have $K^d \geq x^d/2$, and by equation (7) $x^d \geq 1/(2f(v^0)) \geq 1/(2f(\mu))$, which together imply $\lim_{n \rightarrow \infty} K^d \geq 1/(4f(\mu)) > 0$; therefore $\lim_{n \rightarrow \infty} v^0 = \lim_{n \rightarrow \infty} (n-1)K^d + x^d = +\infty$, which contradicts $v^0 < \mu$. \square

Proof of Proposition 4. Making explicit our focus on the amount y , we denote with $K^c(y)$ the solution to (9) and with $K^d(y)$ the solution for K^d of (10). It is immediate to verify that both $K^c(y)$ and $K^d(y)$ are decreasing in y . Therefore, as in the proof of Proposition 3, the proof is complete if $K^d \geq K^c$ implies $\left| \frac{dK^d}{dy} \right| > \left| \frac{dK^c}{dy} \right|$. From equation (9) we obtain

$$\left| \frac{dK^c}{dy} \right| = \frac{1 - F((n-1)K^c) + y}{2 + (n-1)(1 - F((n-1)K^c + y))}, \quad (20)$$

while from (10) we have

$$\left| \frac{dK^d}{dn} \right| = \frac{1 - F(v^0)}{1 + (n-1)(1 - F(v^0))}. \quad (21)$$

Moreover, we can replicate the same steps leading to Proposition 2 for the case $y > 0$ and reach the same conclusion: $K^d \geq K^c$ implies $v^0 < \mu$. Simple algebra on equations (20) and (21) shows that the result follows if

$$1 - 2F(v^0) + F((n-1)K^c + y) > 0,$$

which is ensured by $v^0 < \mu$. □

Proof of Proposition 5. For distributions symmetric about μ , the condition of increased peakedness is equivalent to the condition that if $a' > a$ then $F(\mu + x; a') \geq F(\mu + x; a)$ and $F(\mu - x; a') \leq F(\mu - x; a)$ for all $x > 0$. Therefore, thinking of a marginal change in a , we have

$$(v - \mu) \frac{\partial F(v; a)}{\partial a} \geq 0. \quad (22)$$

Applying the implicit function theorem with respect to a to the system composed of equation (6), of equation (7), and of the definition of K^d yields, after rearrangement

$$(1 + (n-1)[1 - F(v^0(a); a)]) \frac{dK^d}{da} = -x^d \frac{\partial F(v^0(a); a)}{\partial a}.$$

Therefore, using (22), $\frac{dK^d}{da}$ has the opposite sign of $(v - \mu)$, thus establishing the proposition. □

Proof of Proposition 6. Assume first, by contradiction, that an intersection between K^d and K^c occurs at which K^c becomes larger than K^d . Then there exists a point at which $K^d = K^c$, but $dK^c/da \geq dK^d/da$. However, Proposition 2 implies $v^0 < \mu$, and Proposition 5 further implies $dK^d/da > 0$, thus yielding $dK^c/da > 0$, which contradicts the fact that $dK^c/da < 0$, as established in Barbieri and Malueg (2010, Proposition 5) (for symmetric distributions, it is readily verified that increased peakedness about μ implies decreased riskiness in the sense of second-order stochastic dominance). □

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