Misallocation Costs of Digging Deeper into the Central Bank Toolkit *

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Abstract

Central bank large-scale asset purchases, particularly the purchase of securities of nonfinancial firms, can induce a misallocation of resources through their heterogeneous effect on firms cost of capital. First, we analytically demonstrate the mechanism in a static model. We then evaluate the misallocation of resources induced by corporate security purchases and the associated output losses in a calibrated heterogeneous firm New Keynesian DSGE model. The calibrated model suggests misallocation effects from corporate security purchases can be large enough to make them less effective than government bond buys, which is not the case without accounting for misallocation effects.

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1 Introduction

Central banks, such as the Bank of Japan (BOJ), the Bank of England (BOE), the European Central Bank (ECB), and the Federal Reserve (the Fed), have used large-scale asset purchases (LSAPs) as a policy tool once they have reduced the short rate under their control to its effective lower bound.[1] While the BOJ, the ECB, and the Fed purchased government bonds as part of their LSAP programs, as they have chosen to buy other assets, their choices have differed: between them, the different central banks have purchased corporate securities, exchange-traded funds, mortgage-backed securities, and other assets. Yet there is little theory to guide central banks on whether there are aggregate costs to purchasing some private assets rather than others.

This paper addresses this gap in the literature, focusing specifically on the costs associated with large-scale purchases of nonfinancial corporate securities. Purchases of corporate securities may reduce financing costs by more for firms whose securities are purchased than those not purchased and potentially create distortions in the cost of capital among firms.[2] In standard models of firm financing and capital choice, differences in the cost of capital induce differences in firm investment decisions and thus the allocation of capital, which has consequences for the efficiency of the allocation and aggregate output. Our work builds a simple theory of LSAPs with this mechanism present to deliver analytical results regarding how large-scale purchases of corporate securities affect the allocation of capital among firms. We then introduce the key elements of our simple model into a New Keynesian DSGE model. With our calibrated DSGE model, we quantify the potential misallocative effects of large-scale purchases of nonfinancial firm corporate securities.

To demonstrate the economic mechanism at work within our DSGE model, the first part of the paper takes a two-period model of firm dynamics with a financial intermediary sector and demonstrates the conditions under which a large-scale purchase of nonfinancial firm corporate securities by the central bank affects the allocation of resources. The model allows us to separate

[1] Such LSAPs are often referred to as quantitative easing (QE) policies.

[2] Krishnamurthy and Vissing-Jorgensen (2011), among others, demonstrate that there are price effects of central bank bond buys that are more pronounced for the securities purchased than other securities.

two effects of a shock to interest rates that lowers rates for one set of firms (firms issuing corporate securities, which we denote as "large firms") more than for another set of firms (which we denote as "small firms"): (1) the effect on the allocation of capital (2) the effect on the aggregate capital stock. We isolate the key elements of our model that govern the size of each effect.[3]

In our model, following Gertler and Karadi (2011) and Gertler and Karadi (2013) (hereafter, GK11 and GK13), a financially constrained intermediary helps facilitate the financing of capital by firms. Intervention by the central bank in asset markets affects the quantity and distribution of assets which are intermediated and the constraint faced by the financial intermediary. Our simple model introduces the two key additional model elements we use to study the effects of misallocation.[4] The first is heterogeneity in production among multiple "groups" of firms. Specifically, we allow there to be two (or more) types of intermediate good firms that have similar production technologies. Their outputs are used in the production of the final good, and are imperfect substitutes.

Second, we introduce a collateral constraint that allows for richer heterogeneity in spreads than in the model of GK13. The constraint makes it more costly for banks to hold risky private-sector securities than government bonds. Additionally, the constraint discourages the concentration of certain classes of securities. In equilibrium, the constraint induces asymmetric effects in the credit spread response of purchasing securities of firms relative to that of the government.

The fact that the constraint discourages the concentration of assets of different types can be microfounded when assets have heterogeneous risk exposures and there is a simple limit on the variance of the returns to banks' equity holders.[5] We demonstrate that if bank debt holders monitor bank equity holders in an effort to avoid default, an optimal contract will naturally lead to the same relative spreads as those implied by such a constraint. In demonstrating this point, we show that when assets do not differ in their risk exposures, this leads to the linear constraints in GK11/GK13, where movements in spreads in response to LSAPs are solely functions of parameters and the inter-

[3] It is possible to use a large-scale purchase of corporate securities to either *reduce* a misallocation or increase a misallocation. However, we focus on the latter effect in explaining the mechanism, guided by our calibration.

[4] We build on the framework of GK13 which is widely used in studying QE policies across central banks.

[5] The fact that assets are differentially exposed to aggregate risk is an idea at the core of modern finance.

mediary's total weighted asset holdings. Further, we demonstrate what assumptions are necessary to obtain the functional form of the constraint we use to keep the model tractable.

Using our two-period model, we show that when the central bank buys the securities of one set of firms, it lowers the cost of capital for that set of firms by more than that for the other sets of firms, all else being equal. When the central bank buys government bonds, if spreads between loans to various types of firms (large and small firms, for instance) are small in steady state, the central bank reduces the cost of capital for all firms approximately evenly. Hence, in equilibrium, there is an additional effect on the allocation of resources from a corporate security purchase that does not occur to the same degree from a government bond purchase. Such a framework thus endogenizes the heterogeneous effect on borrowing costs from a large-scale purchase of nonfinancial firm corporate securities.

The second part of this paper quantifies the misallocation effects of LSAPs of securities issued by one set of firms (large firms) and not another set of firms (small firms) in a calibrated, DSGE model similar to our two-period model but with DSGE elements that we did not include in our two-period model: sticky prices, endogenous net worth of banks, and a representative household with habits that can hold securities facing a holding cost. In the DSGE model, the response of credit spreads to an asset purchase is not only heterogeneous, as in our two-period model, but also time-varying. We calibrate the new parameters we introduce using U.S. data.

In the calibrated model, a QE policy in which the central bank purchases public debt produces a positive, large effect on investment and output after a bad shock. However, a QE policy in which the central bank purchases the securities of large firms—although potentially inducing a similar effect on output—reduces the response of investment for small firms whose securities are not purchased by the central bank and induces a non-negligible misallocation of resources. In fact, away from the zero lower bound (ZLB), our calibration implies that the misallocation effect is large enough to make a government bond purchase *more effective* than a private security purchase in terms of increasing output, even though without misallocation a government bond purchase is *less effective* in increasing output than a large-scale corporate security purchase. The implied initial relative response to spreads is only about 0.12%, and yet we find this result. The misallocation effect is non-negligible away from the ZLB relative to movements in output; however, at the ZLB, the effect of LSAPs on output are amplified, while the misallocation effect is not. Therefore, the misallocation effect as a percentage of the potential output gain from large-scale corporate security purchases will be smaller at the ZLB.

The rest of the paper, after the literature review below, follows as such. Section 2 presents results from the simple model. Section 3 describes the full DSGE model. Section 4 discusses the calibration, and assesses the quantitative implications of large-scale purchases of nonfinancial firm corporate securities and the role of the ZLB. Section 5 concludes.

1.1 Related Literature

Our paper is closely related to the large literature on how misallocation affects the macroeconomy, built on the work of Hopenhayn and Rogerson (1993) and Hsieh and Klenow (2009), among many others. The closest papers to ours are Midrigan and Xu (2014) and Gilchrist et al. (2013), as they study how financial frictions that induce a misallocation (and result in a wider dispersion in credit spreads) affect the macroeconomy.[6]

This paper is also closely related to a literature that studies macroeconomic models with financial frictions to analyze the channels through which QE policies affect the economy. GK11 and GK13 study QE policies in a representative firm DSGE model with constrained financial intermediaries. Other papers in this literature, such as He and Krishnamurthy (2013) and Cúrdia and Woodford (2016), also emphasize the role of financial market imperfections in making QE effective.

The key theoretical contribution of our paper is to develop a tractable way to integrate non-proportional movements in spreads from large-scale asset purchases in a manner that can be mi-

[6] In Gilchrist et al. (2013), the financial frictions are not explicitly modeled but it is assumed that financial frictions induce the dispersion in borrowing. Further, in the robustness section of Midrigan and Xu (2014), there is an evaluation of how heterogeneity in borrowing rates induce a misallocation, although the focus of the paper is on how a worsening of financial frictions can induce a misallocation. Also, it is important to note that financial frictions can also distort entry and technology adoption in Midrigan and Xu (2014).

crofounded with an optimal contract. Such movements are necessary for matching the movements in credit spreads induced by QE programs (see Krishnamurthy and Vissing-Jorgensen (2013) or D'Amico and King (2013)). An alternative modeling approach for delivering non-proportional movements in spreads in New Keynesian DSGE models would be the "preferred habitat" approach (see Andrés et al. (2004), Vayanos and Vila (2009), or Chen et al. (2012)), although, to our knowledge, the exogenous segmentation that induces non-proportional movements in spreads has not been motivated with an optimal contract in such models. Non-proportional movements in spreads do not exist in GK11 or GK13, as the elasticity of financial intermediary demand for a security with respect to its own price is negative infinity. In our model, this elasticity is finite due to the collateral constraint being nonlinear in firm asset holdings, as we show in Section 2.1.

There is growing empirical evidence of there being heterogeneous real effects of LSAPs through various channels, exploiting such non-proportional movements in spreads from QE programs. Rodnyansky and Darmouni (2017) and Kurtzman, Luck, and Zimmermann (2018) show the Federal Reserve's purchases of MBS in QE1 and QE3 incentivized more lending and risk-taking, respectively, by banks with more MBS holdings. Di Maggio et al. (2016) identify an effect of QE on the volume of new mortgages originated and show that the type of mortgages originated were more likely to be those that could be securitized and sold to the Federal Reserve. Chakraborty et al. (2016) show banks that are more active in the MBS market reduce commercial lending subsequent to QE by the Fed, inducing the firms borrowing from these banks to reduce investment. Foley-Fisher et al. (2016) present evidence that the Federal Reserve's maturity extension program had a greater effect on the valuation, investment, and employment of firms which were more dependent on long-term debt.

In the models of GK11 and GK13, the central bank is less efficient at intermediating financial transactions than the private sector. The calibrated models suggest that QE policies by the central bank can reduce credit spreads and increase investment and output nonetheless. A related literature examines other indirect costs and benefits of QE policies. Hall and Reis (2015) assesses the potential risks to central bank solvency. Reis (2017) points out that central bank liabilities used to fund LSAPs are special in that they are free of default risk, and thus could prove as a useful policy tool in fighting inflation in a fiscal crisis. To keep to the economic point of interest, our paper does not incorporate such additional tradeoffs.

2 Demonstrating the Mechanism in a Simple Model

We begin by highlighting the main mechanism in our paper within a two-period model of firm capital choice and financing. The model details how the capital decisions of heterogeneous firms are affected by the financing environment and how central bank LSAPs change the allocation of capital and affect macroeconomic aggregates.

2.1 Model and Equilibrium

The two-period model consists of heterogeneous intermediate good firms, a representative final good firm, a representative financial intermediary, a representative capital producer, and a representative household. There is also a government and a central bank. The government has an exogenous policy to provide a positive net supply of risk-free bonds, B^s. The central bank has an exogenous policy that determines its government and corporate asset holdings of any given firm i, $S_{g,b}$ and $S_{g,i}$, respectively. There are J continuums of intermediate good firms, where intermediate good firms are indexed by i and the continuum they belong to by j. The mass of intermediate good firms is normalized to 1, so $\sum_{j=1}^{J}\int_{i\in j} di = 1$. Each intermediate good firm, i, produces a differentiated good using capital, k_i, with technology $y_i = A_i k_i^{\alpha}$, where A_i is the firm's exogenous productivity level. Firm productivity is assumed to be stochastic, with its realization occurring in period 2.

In terms of timing, prior to period 1, the household is born with an endowment good, E, and the financial intermediary is endowed with exogenous net worth, N. In period 1, capital and borrowing decisions are made. The household consumes some portion of the endowment good and sells the rest to the capital production sector. The household also makes loans to the financial intermediary. The capital producer then produces capital using the endowment good. Intermediate good firms each buy capital from the capital producing firm, which they must finance by borrowing from either the central bank or the financial intermediary. Meanwhile, the financial intermediary make its own borrowing and lending decisions. Afterwards, in period 2, production occurs. Once intermediate good firm productivity is realized, intermediate good firms produce intermediate output, y_i, sold at price, p_i, to the final good firm, repay their lenders, sell the post-depreciation capital to the household, and transfer their profits to the household. The final good firm produces output, Y, from $\left(\int y_i^{\rho} di\right)^{\frac{1}{\rho}}$ units of the production good, where $\rho < 1$. Households then consume the output of final good firms, as well as any remaining capital (after depreciation). We assume that capital may be converted back into the consumption good.

2.1.1 Final Good Firms, Capital Producers, and Households

The household chooses consumption in period 1 and period 2, C_1 and C_2, as well as its period 1 net lending to the intermediary, D_h, to maximize its utility function, $C_1 + \beta C_2$. The household receives gross interest rate R for lending its funds. When maximizing consumption, the household faces period 1 constraint, $C_1 + D_h \leq E$, and period 2 constraint, $C_2 + rD_h \leq T_2$, where T_2 are the profits of intermediate good firms, the capital good firm, and the financial intermediary in period 2 that are transferred to the household. First-order conditions from this problem imply that $R = \frac{1}{\beta}$.

The representative capital producing firm can produce K units of capital from $\phi(K)$ units of the endowment good in period 1. We assume $\phi^K(K)$ is a weakly convex function. Denote the price of capital as Q. The firm's maximization problem is: $\max_K QK - \phi(k)$, which implies $Q = \phi'(K)$. We assume any net profits of the representative capital producing good firm are rebated to households in period 2.

The final good sector, which operates in period 2, is competitive (and thus earns zero profits), where the price of the final good is set to be the numeraire. The representative final good firm chooses how much of each intermediate good to purchase by solving the maximization problem:

$$\max_{y_i} \left(\int y_i^{\rho} di\right)^{\frac{1}{\rho}} - \int_i p_i y_i di. \tag{1}$$

From (1), we can obtain the standard expression for the price of intermediate goods: $p_i = \left(\frac{y_i}{Y}\right)^{\rho-1}$.

Additionally, there are the following clearing conditions in period 1, $C_1 + \phi(K) = E$, and in period 2, $C_2 = Y + (1-\delta)K$.

2.1.2 Intermediate Good Firms

In period 1, intermediate good firms choose their capital stocks to maximize expected profits, defined as revenues from production plus undepreciated capital less expenditures on capital:

$$\max_{k_i} E\left[p_i y_i\right] \tau_i^k + (1-\delta)k_i - E\left[R_i\right] Q k_i,$$

where τ_i^k is an exogenous wedge that represents distortions or inefficiencies in capital allocation not included in the model. Notice, because firm productivity is stochastic, firms make capital and borrowing decisions under uncertainty. Capital expenditures by the firm must be financed in period 1 and then repaid in period 2, where R_i is the firm-specific gross interest rate at which a firm repays its borrowing. This maximization problem yields first-order condition for capital:

$$k_i = \left(\frac{\alpha\rho E\left[Y^{1-\rho} A_i^{\rho}\right] \tau_i^k}{\left(E\left[R_i\right] Q - (1-\delta)\right)}\right)^{\frac{1}{1-\alpha\rho}}. \tag{2}$$

Finally, intermediate good firms, in total, cannot use more of the capital good than is produced: $K \geq \int_i k_i di$.

2.1.3 Financial Intermediation

There is a representative financial intermediary with exogenous financial wealth, N, endowed to it prior to period 1. In period 1, the intermediary invests in financial assets, such as government securities and corporate securities. If the intermediary's holdings are not equal to its wealth, it either borrows or lends to the representative household at gross interest rate R. The market value of the financial intermediary's wealth is equal to the expected discounted present value of its wealth next period. Let $S_{p,i}$ denote intermediary holdings of securities issued by firm i to finance capital, and let $S_{p,b}$ index intermediary holdings of government securities. Let $S_{p,o}$ denote the market value of intermediary holding of all other securities. In period 1, we can therefore write the period 2 value of the intermediary as:

$$V = \beta \left(\sum_j \int_{i \in j} E\left[R_i - R\right] S_{p,i} di + S_{p,b} E\left[R_b - R\right] + S_{p,o} E\left[R_o - R\right] + RN \right). \qquad (3)$$

The financial intermediaries choose their holdings to maximize (3), its market value, subject to a collateral constraint that is *nonlinear* in its relative asset holdings:

$$V \geq \theta \sqrt{\sum_j \left(\int_{i \in j} S_{p,i} di \right)^2 \Delta_j + \left(S_{p,b}\right)^2 \Delta + \left(S_{p,o}\right)^2 \Delta_o}\,, \qquad (4)$$

where $\theta > 0$ is a parameter affecting the tightness of the collateral constraint, and $\Delta_j \forall j$, Δ, and Δ_o are positive parameters that affect the relative tightness of the constraint when holding securities of firms $i \in j$, government bonds, and other securities, respectively. In Proposition 1 below, we lay out the conditions under which (4) can be derived from a limit on the variance of the levered returns of the equity holders of financial intermediaries.

Proposition 1.

If assets held by the financial intermediary have risk profiles of the form $R_{i,t} = R_{j,t} + \epsilon_{i,t}$, for a security of firm i in continuum j, and we assume that $\epsilon_{i,t}, R_{j,t}, R_{b,t}, R_{o,t}$ are independent, then a limit on the variance of the returns to the intermediary's bank equity holders implies the intermediary faces a collateral constraint of the form in (4).

Proof. See Appendix A.

Essentially, when asset returns are heterogeneous in their risk exposures under our assumption of independence of returns, a limit on the variance of the returns to bank equity holders implies a constraint with the functional form of (4).[7] In the proof to Proposition 1, we show that a constraint

[7]Note that in this model, only the expectation of returns enters agents optimization problems. We can therefore make assumptions about the returns process and state-contingencies of securities to satisfy Proposition 1 without altering the agent's problem.

that is nonlinear in asset holdings arises even without the independence assumption. However, the independence assumption is required to obtain the tractable form of the constraint we observe in (4).[8] As part of the proof, we also show that in the case when firm assets are homogeneous in their risk exposures, our constraint becomes linear in asset holdings, as in GK13.

A condition on the variance of the returns to bank equity holders is a natural constraint when bank equity holders have limited liability (or there are externalities from the financial distress of banks) and assets are heterogeneous in their risk exposures. Specifically, in Appendix A, we prove that the relative credit spreads implied by (4) are identical to those from an optimal contracting problem in the face of costly default. We also show a limit on the variance of the returns to bank equity holders can be justified with existing policy, such as bank stress tests.

It is also useful to note that we can rewrite our constraint as a function of total bank asset holdings and their relative concentration:

$$V \geq \theta V_A \sqrt{\sum_j \left(\frac{\int_{i \in j} S_{p,i} di}{V_A} \right)^2 \Delta_j + \left(\frac{S_{p,b}}{V_A} \right)^2 \Delta + \left(\frac{S_{p,o}}{V_A} \right)^2 \Delta_o}\,, \tag{5}$$

where V_A is the market value of the financial assets held by the intermediary. The financial intermediary thus faces a collateral constraint which will depend on the assets purchased and their concentration.

The maximization problem of the intermediary yields the following first-order condition for the expected return on the securities issued by firm i of type j to finance capital:

$$E\left[R_i - R\right] = \frac{1}{\beta} \frac{\lambda \theta^2}{(1+\lambda)} \frac{(\int_{i \in j} S_{p,i} di)\Delta_j}{V}, \tag{6}$$

where λ is the Lagrangian multiplier on the collateral constraint. Note that since $S_{p,i} = Qk_i - S_{g,i}$, central bank purchases of corporate securities enter this condition. A similar condition results for

[8] The number of terms in the constraint without the independence assumption with n assets is equal to the n^{th} triangular number. With the independence assumption, we only have n terms for n classes of assets, which makes the constraint more tractable.

the interest rate on government bonds:

$$E\left[R_b\right] - R = \frac{\lambda\theta^2}{(1+\lambda)}\frac{S_{p,b}\Delta}{V}. \tag{7}$$

Now consider firms i of type j and h of type k, where $j \neq k$. We can then define relative spreads of the two firms as:

$$\frac{E\left[R_i\right] - R}{E\left[R_h\right] - R} = \frac{(\int_{i\in j} S_{p,i} di)\Delta_j}{(\int_{h\in k} S_{p,h} dh)\Delta_k}. \tag{8}$$

Given the capital expenditures of firm i must be equal to the sum of firm i securities held by financial intermediaries and the central bank, this leads to the following clearing condition: $Qk_i = S_{p,i} + S_{g,i}$. Similarly, clearing implies $B^S = S_{p,b} + S_{g,b}$ and $S_o = S_{p,o}$, where S_o is the exogenous stock of other financial assets.

2.1.4 Equilibrium

Given the household's endowment, E, the supply of government bonds, B^S, exogenous bank net worth, N, firm-level productivities and wedges, $\left\{A_i, \tau_i^k\right\}\forall i$, central bank purchases of corporate securities, $\left\{S_{g,i}\right\}\forall i$, and government bonds, $S_{g,b}$, an equilibrium in this model is a set of allocations, $\left\{C, Y, K, S_{p,b}, D_h\right\}$ and $\left\{k_i, y_i, S_{p,i}\right\}\forall i$, and prices, $\left\{Q, R, R_B, R_o\right\}$ and $\left\{R_i, p_i\right\}\forall i$, such that households maximize consumption subject to their budget constraint; intermediate good firms, final good firms, and capital producers maximize profits; financial intermediaries maximize profits subject to their collateral constraint; and resource constraints hold.

2.2 Effect of Central Bank Asset Purchases

Central bank purchases of either long-term government bonds ($S_{g,b}$) or corporate securities of firm i ($S_{g,i}$) will reduce the amount of that particular asset that has to be intermediated by the financial intermediary. LSAPs will directly affect credit spreads by changing: (1) the Lagrange multiplier on the collateral constraint, λ, and (2) the first-order condition (6), by reducing the concentration of firms of type j in the intermediary's balance sheet. Additionally, LSAPs will indirectly affect spreads through their effect on firm capital choices, as can be seen in the first-order condition for capital. Changing firm capital choices will also affect the price of capital, Q, and the amount of firm capital that needs to be intermediated, $Qk_i - S_{g,i}$, which enters directly into the collateral constraint. Proposition 2 states that we can analytically demonstrate the direct effect of LSAPs on bond spreads.

Proposition 2. *Holding firm capital choices, k_i, fixed, central bank LSAPs have the following effects.*

(i) A purchase of long-term government bonds, $S_{g,b}$,

(a) decreases λ, the Lagrangian multiplier on the collateral constraint.

(b) proportionately decreases firm spreads, that is:

$$\frac{\partial \frac{\int_{i\in j} E[R_i]-R}{\int_{h\in k} E[R_h]-R}}{\partial S_{g,b}} = 0, \tag{9}$$

where $j \neq k$.

(ii) A purchase of firm securities, $S_{g,i}$, for $i \in j$

(a) decreases λ, the Lagrangian multiplier on the collateral constraint.

(b) decreases spreads of the type purchased by more than those of other types, that is,

$$\frac{\partial \frac{\int_{i\in j} E[R_i]-R}{\int_{h\in k} E[R_h]-R}}{\partial S_{g,i}} < 0, \tag{10}$$

where $j \neq k$.

Proof. See Appendix B.

This proposition formalizes the direct effects of LSAPs on spreads. Directly buying the securities of only firms of type $i \in j$ will, holding capital choices constant, lower their spreads by more than of firms with $i \notin j$. This can be contrasted with the effect of purchases of long-term government debt, which will lower spreads proportionately. However, purchases of both long-term government bonds and corporate securities will, under some reasonable conditions, decrease spreads by loosening the collateral constraint faced by financial intermediaries (implying a lower multiplier on the constraint, λ).

Therefore, central bank LSAPs of corporate securities will induce asymmetric changes in spreads and therefore firm *cost of capital*. Through the first-order condition for capital, (2), these spreads will induce changes in firm capital choices and therefore both the allocation of capital and aggregate capital supply.

2.3 What Drives the Non-proportional Movements in Spreads?

One property of a nonlinear constraint such as the one we derive is that the intermediary's demand for a given type of security has a finite elasticity with respect to its own price. In contrast, in models with complete markets, the elasticity of demand for a security with respect to its own price (holding all other security prices fixed) is equal to negative infinity; similarly, in the model of GK2013, the elasticity of financial intermediary demand for a security with respect to its own price is negative infinity.[9] In our model, combining equations (3), (4), and (8) yields the following expression for the demand of the intermediary for securities of firms of type j:

$$S_{p,j} = E\left[R_j - R\right] \frac{V}{\Delta_j \left(1 - \frac{NR}{V}\right)}. \tag{11}$$

It can be easily verified that, holding all other prices (and therefore expected returns) fixed, that this has finite elasticity. Thus, our model delivers non-proportional effects in spreads from government bond purchases, as demonstrated in Proposition 2 in the subsection above.

[9] In GK2013, this is true from the arbitrage condition between different assets that the intermediary can intermediate.

2.4 Allocative Efficiency

Proposition 2 tells us how government and corporate security purchases directly affect borrowing rates of firms and the government, holding fixed the indirect effect of firm decisions changing in response. We cannot directly map asset purchases to aggregates analytically due to a nonlinearity in the equation linking firm credit spreads to central bank asset purchases (due to firm-capital choices reacting to the change in spreads, a second-order effect). However, to develop intuition as to how asset purchases affect allocative efficiency, it is useful to walk through how spreads affect the allocation and aggregates.

In equilibrium, we can express the relative holdings of firm capital as

$$\frac{k_i}{K} = \frac{\left(\frac{A_i^\rho \tau_i^k}{c_i}\right)^{\frac{1}{1-\alpha\rho}}}{\sum_j \int_{i\in j} \left(\frac{A_i^\rho \tau_i^k}{c_i}\right)^{\frac{1}{1-\alpha\rho}} di},$$

where c_i is firm cost of capital $(E[R_i] - R)Q + (1-\delta) + RQ$.

Therefore, the relative levels of firm cost of capital, c_i, have implications for the *relative* allocation of capital. Additionally, firm cost of capital affects the aggregate demand for capital from intermediate good firms:

$$K = \frac{\left(\int \left(A_i^\rho \left(\frac{\tau_i^k}{c_i}\right)^{\alpha\rho}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\frac{1-\rho}{\rho(1-\alpha+b_k)}} \left(\int \left(\frac{A_i^\rho \tau_i^k}{c_i}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\frac{1-\alpha}{1-\alpha+b_k}}}{\left(\frac{h_k}{\alpha\rho}\right)^{\frac{1}{1-\alpha+b_k}}}. \tag{12}$$

Building on (12), we can express aggregate output as

$$Y = \underbrace{K^\alpha}_{\text{Capital}} \underbrace{\frac{\left(\int \left(A_i^\rho \left(\frac{\tau_i^k}{c_i}\right)^{\alpha\rho}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\frac{1}{\rho}}}{\left(\int \left(\frac{A_i^\rho \tau_i^k}{c_i}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\alpha}}}_{\text{Allocation}}. \tag{13}$$

Note that (13) shows that output can be expressed as a function of aggregate capital, K^α, modified by a term that captures both the productivity of intermediate good firm production functions and the efficiency of the allocation of capital. For example, if there is no heterogeneity ($A_i = A, c_i = c_A, \tau_i^k = \tau^k$), then (13) reduces to $Y = AK^\alpha$.

To more clearly demonstrate the effect of spreads on output and allocative efficiency, we define c_A, the *weighted-average cost of capital faced by firms*:

$$\frac{1}{c_A} = \frac{\left(\int A_i^{\frac{\rho}{1-\alpha\rho}} \left(\frac{\tau_i^k}{c_i}\right)^{\frac{\alpha\rho}{1-\alpha\rho}} di\right)^{\frac{1-\rho}{\rho}} \left(\int A_i^{\frac{\rho}{1-\alpha\rho}} \left(\frac{\tau_i^k}{c_i}\right)^{\frac{1}{1-\alpha\rho}} di\right)^{1-\alpha}}{\left(\int A_i^{\frac{\rho}{1-\alpha\rho}} \left(\tau_i^k\right)^{\frac{\alpha\rho}{1-\alpha\rho}} di\right)^{\frac{1-\rho}{\rho}} \left(\int A_i^{\frac{\rho}{1-\alpha\rho}} \left(\tau_i^k\right)^{\frac{1}{1-\alpha\rho}} di\right)^{1-\alpha}}.$$

Note that aggregate capital depends only on c_A (and does not depend on heterogeneity in firm cost of capital). If all firms have the same cost of capital, $c_i = c_A \forall i$.

We can then define *cost of capital wedges*, $c_{\tau,i}$, between firm cost of capital and the weighted-average cost of capital as $c_{\tau,i} = \frac{c_A}{c_i}$. With these wedges, we can then express output as a function of aggregate capital, where the allocation depends only on firm productivities, A_i, cost of capital wedges, $c_{\tau,i}$, and other exogenous distortions, τ_i^k:

$$Y = \underbrace{K^\alpha}_{\text{Capital}} \underbrace{\frac{\left(\int \left(A_i^{\frac{\rho}{1-\alpha\rho}} \left(c_{\tau,i}\tau_i^k\right)^{\frac{\alpha\rho}{1-\alpha\rho}}\right) di\right)^{\frac{1}{\rho}\left(\frac{1-\alpha\rho}{1-\alpha}\right)}}{\left(\int A_i^{\frac{\rho}{1-\alpha\rho}} \left(\tau_i^k\right)^{\frac{\alpha\rho}{1-\alpha\rho}} di\right)^{\frac{(1-\rho)\alpha}{\rho(1-\alpha)}} \left(\int A_i^{\frac{\rho}{1-\alpha\rho}} \left(\tau_i^k\right)^{\frac{1}{1-\alpha\rho}} di\right)^{\alpha}}}_{\text{Allocation}}. \tag{14}$$

From (14), and building on proposition 2, there are two first-order consequences of central bank purchases of corporate securities that lower credit spreads heterogeneously. First, they will lower the weighted-average firm cost of capital, c_A, leading to a larger aggregate capital stock all else equal. However, they can also generate cost of capital wedges, $c_{\tau,i}$, which have consequences for the allocation of capital relative to the efficient level. The size (and direction) of these effects depend on how far the baseline allocation is from its efficient level and whether the cost of capital wedge changes induced by asset purchases exacerbate or undo distortions. The latter point can be further formalized by deriving output-maximizing cost of capital wedges, as we do in Proposition

3 below.

Proposition 3. *The output-maximizing allocation of firm cost of capital wedges satisfies* $c^*_{\tau,i} \propto \frac{1}{\tau_i^k}$ *Equivalently,* $c^*_i \propto c_A \tau_i^k$

Proof. See Appendix B.

Proposition 3 shows that the optimal cost of capital wedges are such that they exactly offset exogenous distortions τ_i^k. If there are no exogenous distortions then the optimal allocation arises when firms all have identical costs of capital $c_A = c_i$.

When there are only two types of firms ($j \in 1, 2$), where c_i and τ_i^k are symmetric for all $i \in j$, we can characterize all cost of capital wedges using just a single cost of capital wedge, $c_{\tau,1}$, and the weighted average cost of capital, c_A. The following corollary shows that in this case, output is monotonically decreasing as the cost of capital wedges move further from their output-maximizing values:

Corollary 3.1. *In the case with only two types of firms,* $\frac{\partial Y}{\partial |c_{\tau,1} - c^*_{\tau,1}|} < 0$.

Thus, given that large-scale corporate security purchases can induce heterogeneous movements in spreads, they can cause (reverse) a misallocation by moving cost of capital wedges away from (toward) the efficient allocation.[10] For example, if cost of capital wedges of type 1 (large) firms are greater (the cost of capital is lower) than those of type 2 (small) firms in steady state, central bank purchases of large firm assets increase large firm cost of capital wedges relatively further. In this setting, central bank asset purchases of large firm assets will distort the allocation further from its efficient level. Looking ahead, our New Keynesian DSGE model will be calibrated such that cost of capital wedges of large firms are greater than those of small firms in steady state, and central bank asset purchases of large firm assets will distort the allocation further from its efficient level. Thus, Corollary 3.1 provides the key intuition behind the results we will present in the calibrated New Keynesian DSGE model.

[10] A version of Corollary 3.1 can be derived for a case with more than two types of firms, but the measure of 'distance' from the efficient allocation will be a more complicated function of firm cost of capital wedges, exogenous distortions, and productivities.

3 New Keynesian DSGE Model

We evaluate the impact of the misallocative effect of LSAPs by the central bank in a richer environment where the effects of central bank asset purchases on firm borrowing rates are endogenized. We embed our simple model in a standard New Keynesian DSGE model following GK13. Along with explicitly modeling banks, the model has the key elements of a New Keynesian model: households, nonfinancial firms, capital goods producers, retail good firms, a central bank, and a government.

It is useful to start by outlining the changes we make to the nonfinancial firm sector and then discussing the changes made to households and the financial sector. Afterwards, we outline the remainder of the model and define an equilibrium.

3.1 Model Description

Nonfinancial and Capital Good Firms There are two continuums, indexed by $j \in (1, 2)$, of nonfinancial intermediate good firms. Each firm i in continuum j produces output with technology:

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{(1-\alpha)},$$

where $Y_{i,t}$ is the intermediate good output of firm i, $K_{i,t}$ is its capital stock, $L_{i,t}$ is its employment, $\alpha \in (0, 1)$ governs capital's share in production. Total intermediate good firm output, $Y_{m,t}$, is then computed using a CES aggregator:

$$Y_{m,t} = \left(\sum_j \omega_j \int_{i \in j} Y_{j,t}^{\rho} di \right)^{\frac{1}{\rho}},$$

where ω_j is a parameter greater than or equal to 0 that is a factor affecting the extent to which the output of intermediate good firms of type j enters total output, and ρ is the CES parameter. Note that if $\rho = 1$ then intermediate goods are perfectly substitutable.

All firms within each continuum face the same financing environment and production technology, as in Section 2. We can therefore represent each continuum of firms with a representative firm of type j. We can thus write the production technologies for representative firms as follows:

$$Y_{j,t} = A_{j,t} K_{j,t}^{\alpha} L_{j,t}^{(1-\alpha)},$$

where $Y_{j,t}$ is the intermediate good output of type j firms, $A_{j,t}$ is an index of type j TFP, $K_{j,t}$ is the type j firm capital stock, $L_{j,t}$ is the type j total employment.[11] We assume that $A_{j,t}$ follows an exogenous stochastic process and is equal to one in steady-state. Similarly, total intermediate good output is thus combined from type j intermediate good outputs with production function:

$$Y_{m,t} = \left(\sum_{j} \omega_j Y_{j,t}^{\rho} \right)^{\frac{1}{\rho}}.$$

Following the usual arguments from cost-minimization, the price of intermediate good j can be written as

$$P_{j,t} = \omega_j P_{m,t} Y_{m,t}^{1-\rho} Y_{j,t}^{\rho-1},$$

where $P_{m,t}$ is the relative price of intermediate goods. Firms choose labor to maximize revenues less labor expense, where W_t is the wage rate which is constant across firms. Then we can recover firm j's demand for labor from

$$W_t = P_{m,t} \frac{Y_{m,t}^{1-\rho} Y_{j,t}^{\rho}}{L_{j,t}} \omega_j \rho (1-\alpha), \tag{15}$$

which holds for each type j.

Capital is chosen by firms the period before it is effective, and capital depreciates at rate δ. Capital is exposed to capital quality shocks, ξ_t, where K units of capital chosen in advance become

[11] If we define $Y_{j,t} = \left(\int_{i \in j} Y_{i,t}^{\rho} di \right)^{\frac{1}{\rho}}$, $A_{j,t} = \left(\int_{i \in j} A_{i,t}^{\frac{\rho}{1-\rho}} di \right)^{\frac{1-\rho}{\rho}}$, $K_{j,t} = \int_{i \in j} K_{i,t} di$, $L_{j,t} = \int_{i \in j} L_{i,t} di$, it can be verified that all equilibrium conditions will be identical for representative firm j and all firms $i \in j$.

$\xi_t K$ units of capital in the next period. The capital stock of firm j therefore follows law of motion $K_{j,t+1} = \xi_{t+1}(I_{j,t} + (1-\delta)K_{j,t})$, where ξ_t are capital quality shocks, which are assumed to be common across firms. Firms require financing of their capital stocks, and they do so by issuing state-contingent claims that are perfectly monitored and enforced. Firms choose their capital stock the period before to maximize their expected next period profits:

$$\max_{K_{j,t-1}} E_{t-1}\left[(P_{j,t}Y_{j,t} - W_t L_{j,t})\,\tau_j^k + (1-\delta)Q_t\xi_t K_{j,t-1} - K_{j,t-1}Q_{t-1}R_{j,t}\right]$$

such that the choice of L_t satisfies (15), and where Q_t is the price of capital goods, τ_j^k is an exogenous wedge affecting the choice of capital, and $R_{j,t}$ the return on the securities firm j issues to finance marginal capital investment. The solution of this maximization problem yields that standard result that expected firm MPK (times the wedge) should be equal to the expected cost of capital:

$$\alpha\rho\frac{E_{t-1}\left[P_{j,t}Y_{j,t}\right]}{K_{j,t-1}}\tau_j^k = Q_{t-1}E_{t-1}\left[R_{j,t}\right] - (1-\delta)E_{t-1}\left[Q_t\xi_t\right]. \tag{16}$$

We can rearrange (16) to yield an expression for expected returns:

$$E_{t-1}\left[R_{j,t}\right] = \frac{E\left[(Z_{j,t+1} + (1-\delta)Q_t)\,\xi_t\right]}{Q_t}, \tag{17}$$

where $Z_{j,t+1} = \alpha\rho\frac{P_{j,t}Y_{j,t}}{\xi_t K_{j,t-1}}\tau_j^k$ is the realized marginal product of capital times the firm capital wedge. The expected rate of return therefore has a relationship with the marginal product of capital, which depends on $K_{j,t}$. Since Q_t is common to the two types of firms, differences in the rates of return imply differences in maginal products of capital between the two types.

We assume that only part of firm capital expenditures must be financed using external sources:

$$K_{j,t} = K_{j,I} + K_{j,E,t},$$

where $K_{j,I}$ is the amount of capital firms of type j do not need to finance externally. We assume that $K_{j,I}$ is low enough that firms will always use some external financing ($K_{j,E,t} > 0$) in equilibrium.[12] To keep $K_{j,I}$ constant, we impose that intermediate good firms make net transfers to the household each period, paying out earnings beyond those required to purchase $K_{j,I}$ units of capital.[13]

Following GK2013, we assume that the security of firm j has a realized rate of return of

$$R_{k,j,t+1} = \frac{Z_{j,t+1} + (1-\delta)Q_{t+1}}{Q_t}\xi_{t+1}. \tag{18}$$

Note that this is equal to (17) in expectation. The price of these firm securities is therefore Q_t.

We assume that capital is transferable between firms: thus, we have the capital accumulation equation:

$$K_{t+1} = \xi_{t+1}[I_t + (1-\delta)K_t], \tag{19}$$

where $K_t = \sum_j K_{j,t}$ and $I_t = \sum_j I_{j,t}$. When intermediate good firms make their investment decisions, they buy capital from each other or the competitive capital good producer.

The capital good producer solves the following maximization problem:

$$\max E_t \sum_{t=\tau}^{\infty} \Lambda_{t,\tau}\{Q_\tau I_\tau - [1 + f(\frac{I_\tau}{I_{\tau-1}})]I_\tau\},$$

where Λ_t is the household's stochastic discount factor. Thus, the price of capital goods can be determined from profit maximization as

$$Q_t = 1 + f(\frac{I_t}{I_{t-1}}) + \frac{I_t}{I_{t-1}}f'(\frac{I_t}{I_{t-1}}) - E_t\Lambda_{t,t+1}(\frac{I_{t+1}}{I_t})^2 f'(\frac{I_{t+1}}{I_t}). \tag{20}$$

[12] $K_{j,I}$ is incorporated to match the fact that firms in the U.S can finance much of their investment in the aggregate with internal funds.

[13] While positive in steady-state, in the case of a particularly bad shock, this transfer can be negative.

Imposing the functional form for f considered by GK13 in (20), we get

$$Q_t = 1 + \frac{\eta_i}{2}(\frac{I_t}{I_{t-1}} - 1)^2 + \eta_i(\frac{I_t}{I_{t-1}} - 1)\frac{I_t}{I_{t-1}} - E_t\Lambda_{t,t+1}(\frac{I_{t+1}}{I_t})^2\eta_i(\frac{I_{t+1}}{I_t} - 1),$$

where η_i is the inverse elasticity of net investment to the price of capital.

Retail ood Firm Problem The final good, Y_t, is produced using a mass one continuum of differentiated retail goods using CES production:

$$Y_t = [\int_0^1 Y_{ft}^{\frac{\epsilon-1}{\epsilon}} df]^{\frac{\epsilon}{\epsilon-1}}.$$

Retail good firms, however, just take intermediate output and repackage it. Thus, the marginal cost of production is P_{mt}, the price of the output of intermediate good firms. The retail good firm faces Calvo pricing. It can adjust its price with probability $1-\gamma$. The firms choose the same reset price P_t^*. Following the usual arguments, we can obtain the first-order condition:

$$\sum_{i=0}^{\infty} \gamma^i \Lambda_{t,t+i}[\frac{P_t^*}{P_{t+i}} - \mu P_{m,t+i}]Y_{f,t+i} = 0,$$

with $\mu = \frac{1}{1-1/\epsilon}$. We can thus recover the law of motion for prices:

$$P_t = [(1-\gamma)((P_t^*)^{(1-\epsilon)}) + \gamma(P_{t-1}^{1-\epsilon})]^{\frac{1}{1-\epsilon}}.$$

Bankers Bankers provide long-term financing to nonfinancial firms and the government, which are funded by their liabilities (short-term deposits of households). When bankers become workers, they bring back to the household their fraction of the net worth of the bank. Let $s_{p,j,t}$ denote the market value of a banker's holdings of private securities of type $j \in \{1,2\}$ firms, and let $s_{p,b,t}$ denote the market value of a given banker holdings of government bonds. Similarly, we assume that bankers can hold financial assets beyond firm and government securities. Let $s_{p,o,t}$ denote the market value of the banker's holdings of another perpetuity asset.[14] We assume that this asset is in positive and exogenous net supply B_o^S. The balance sheet of a given banker is thus:

$$s_{p,1,t} + s_{p,2,t} + s_{p,b,t} + s_{p,o,t} = n_t + d_t, \tag{21}$$

where n_t is banker's net worth and d_t the amount of deposits they hold (borrow from the household). Net worth is the difference between the gross return on assets and the cost of deposits:

$$n_t = \left(\sum_{j=1}^{j=2} R_{k,j,t} s_{p,j,t-1}\right) + R_{b,t} s_{p,b,t-1} + R_{o,t} s_{p,o,t-1} - R_t d_{t-1}. \tag{22}$$

Bankers will maximize their expected discounted value of net worth, where $\Lambda_{t,t+1}$ is the household's stochastic discount factor:

$$V_t = E_t \sum_{i=1}^{\infty} (1-\sigma)\sigma^{i-1} \Lambda_{t,t+1} n_{t+1}. \tag{23}$$

Each banker also faces a nonlinear collateral constraint on its asset holdings, identical to the one introduced in the simple model:

$$V_t \geq \theta \sqrt{\Delta_1 (s_{p,1,t})^2 + \Delta_2 (s_{p,2,t})^2 + \Delta (s_{p,b,t})^2 + \Delta_o (s_{p,o,t})^2}, \tag{24}$$

where the parameters $0 \leq \{\theta, \Delta, \Delta_1, \Delta_2, \Delta_o\}$ are parameters related to the risk exposures of assets. As before, this constraint penalizes the relative concentration of a banker's portfolio in one type of assets.[15]

Bankers choose $s_{p,1,t}$, $s_{p,2,t}$, $s_{p,b,t}$, and $s_{p,o,t}$ to maximize (23) subject to (21), (22), and (24). In addition, bankers are price takers, taking interest rates and spreads as given. We describe the

[14] These other assets are included to generate a more realistic financial intermediary balance sheet, as financial intermediaries hold assets other assets besides corporate securities and government bonds.

[15] As in the simple model, this can be justified as the consequence of a limit on the variance of levered bank portfolio returns, given some assumptions on the stochastic processes for firm TFP shocks. As we solve our model via log-linearization and consider only the impact of LSAP (purchase shocks), the parameterization of tfp shocks does not affect our quantitative results.

solution to the problem of the bank in Appendix C.

It is also useful to note that the problem of bankers is homogeneous of degree one in n_t. Thus, the bankers' problems can be treated as that of a representative financial intermediary. Let $S_{p,b,t}$, $S_{p,j,t}$, and $S_{p,o,t}$ be the aggregate amounts of government bonds, type j assets, and other financial assets bankers hold (obtained from the representative intermediary's problem), and letting N_t be bankers net worth, we can derive the law of motion for total net worth of all bankers as

$$N_t = \sigma\left(\sum_{j=1}^{j=2}(R_{k,j,t} - R_t)S_{p,j,t-1} + (R_{b,t} - R_t)S_{p,b,t-1} + (R_{o,t} - R_t)S_{p,o,t-1}\right) + N_e,$$

where N_e is the wealth of entering bankers.

Note that we defined the bankers choices in terms of the total market value of securities they choose to hold. Therefore the bankers intermediate $K_{p,1,t} = \frac{S_{p,1,t}}{Q_t}$ units of type 1 firm capital, and $K_{p,2,t} = \frac{S_{p,2,t}}{Q_t}$ units of type 2 firm capital. Similarly, bankers hold $\frac{S_{p,b,t}}{q_t}$ long-term government bonds, and $\frac{S_{p,o,t}}{q_{o,t}}$ units of other securities, where q_t is the price of government bonds and $q_{o,t}$ the price of other securities.

The long term government bonds are perpetuities and pay one dollar per period. If P_t is the price level, the real rate of return on the bond $R_{b,t+1}$ is

$$R_{b,t+1} = \frac{\frac{1}{P_t} + q_{t+1}}{q_t}.$$

Similarly, the real rate of return on the other security, also perpetuities that pay one dollar per period, is:

$$R_{o,t+1} = \frac{\frac{1}{P_t} + q_{o,t+1}}{q_{o,t}}.$$

Households There is a measure one continuum of households (all identical), each of which consumes the final good, saves by lending funds to banks and potentially the central bank and supplies labor.[16] Each household is composed of a fraction $1 - f$ workers and f bankers and has perfect consumption insurance. Workers are the members who supply labor to earn real wage, W_t, which the household shares among itself. Bankers also share any earnings with the household as a whole. In effect, the household owns the bank that its bankers manage. Define the overall transfers to households from firms and banks as Π_t. Households pay taxes, T_t. The household deposits funds in banks but only in banks the household's bankers do not manage. Workers can become bankers and vice versa over time. With probability σ, bankers stay bankers, and with probability $1-\sigma$, bankers become workers. Bankers face a finite horizon problem; in effect, they cannot retain earnings beyond the point at which they can fund all investment from their own capital. Workers are randomly selected to replace the bankers who switch to workers and receive a startup fund of $\frac{N_e}{(1-\sigma)f}$.

The household consumes C_t units of the final good. L_t is family labor supply. The household has habits in consumption, and the household's utility, u_t, is defined as follows:

$$u_t = E_t \sum_{t}^{\infty} \beta^i [\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+1+i}^{1+\varphi}], \tag{25}$$

where $0 < \beta < 1$, $0 < h < 1$, and $\chi, \phi > 0$.

Households are indifferent between deposits and government debt, as they both pay rate of return between periods $t-1$ and t of R_t, in equilibrium. Thus, we make this assumption throughout, calling both short-term debt, $D_{h,t}$.

Holding costs We also allow households to directly hold securities in the face of holdings costs. Define $K_{h,j,t}$ as the amount of type j private securities held by households, and let $B_{h,b,t}$ and $B_{h,o,t}$ denote the quantity of other government bonds and other securities held by households. Holding costs for security $x \in \{K_{h,j}, B_{h,b}, B_{h,o}\}$ are $\frac{\kappa}{2}\frac{(x_t-\bar{x})^2}{x_t}$ fraction of the market value of type x securities held by the household, if $x_t \geq \bar{x}$, where parameters κ and $\bar{x}$ are positive. With holding

[16] The economy we consider is the cashless limit.

costs, we rewrite budget constraint of the household:

$$C_t + D_{h,t} + \sum_{j=1}^{j=2} Q_{j,t}(K_{h,j,t} + \frac{1}{2}\kappa(K_{h,j,t} - \bar{K}_{h,j})^2) + q_t(B_{h,t} + \frac{1}{2}\kappa(B_{h,t} - \bar{B}_h)^2) + q_{o,t}(B_{h,o,t}$$
$$+\frac{1}{2}\kappa(B_{h,o,t} - \bar{B}_{h,o})^2) = W_t L_t + \Pi_t + T_t - N_e + R_t D_{h,t-1}$$
$$+\sum_{j=1}^{j=2} R_{k,j,t} Q_{j,t-1} K_{h,j,t-1} + R_{b,t} q_{t-1} B_{h,t-1} + R_{o,t} q_{o,t-1} B_{h,o,t-1}, \tag{26}$$

The household thus solves (25) subject to (26) choosing C_t, L_t, and D_{ht}, as well as its holdings of financial securities. Define u_{Ct} to be the marginal utility of consumption. We then have labor supply condition:

$$u_{Ct} W_t = \chi L_t^{\varphi},$$

and consumption-savings optimality condition:

$$E_t \beta \frac{u_{C,t+1}}{u_{C,t}} R_{t+1} = 1.$$

This can be use to define the household's stochastic discount factor:

$$\Lambda_{t,t+1} = E_t \beta \frac{u_{C,t+1}}{u_{C,t}}. \tag{27}$$

Central bank and government policy The central bank can purchase either government bonds (short or long term) or private securities. We assume the central bank is less efficient in intermediation than banks and thus pays ϱ_j per unit of type j securities intermediated and ϱ_b per unit of government bonds. As discussed in GK2013, these intermediation costs are likely to be small for government securities and rated liabilities of large firms on which there is extensive information, and likely to be large for loans to small firms which require considerable and costly monitoring. We therefore assume that ϱ_2 is large enough that these purchases are not conducted. In practice, purchase of corporate securities conducted by central banks have focused on investment-grade securities issued predominantly by very large firms.[17]

The central bank can issue riskless short-term debt $D_{g,t}$ which pay R_{t+1}.Thus, the central bank has balance sheet

$$\sum_j Q_t K_{g,j,t} + q_t B_{g,b,t} = D_{g,t},$$

where $K_{g,j,t}$ is central bank holdings of type j firm securities, and $B_{g,b,t}$ is central bank holdings of government bonds. The central bank costlessly transfers any profits to, or recovers any losses from, the government.

The central bank determines monetary policy using a Taylor rule. Define i_t as the net nominal interest rate, i as the steady-state nominal rate, π_t as the inflation rate P_{t+1}/P_t, and Y_t^* as the flexible-price equilibrium level of output. Then

$$i_t = i + \kappa_\pi \pi_t + \kappa_y(\log(Y_t) - \log(Y_t^*)) + \epsilon_t,$$

where ϵ_t is an exogenous shock. When we allow for a ZLB on interest rates:

$$i_t = \max\left\{0, i + \kappa_\pi \pi_t + \kappa_y(\log(Y_t) - \log(Y_t^*)) + \epsilon_t\right\}.$$

We can then determine the real interest rate with the standard Fisher relation:

$$1 + i_t = R_{t+1}\frac{P_{t+1}}{P_t}.$$

Government consumption, G, and the net interest payments from fixed amount of long-term bonds $\bar{B}$ are fixed. Revenues will include central bank earnings net costs plus collected taxes. We

[17]For example, the ECB's corporate security purchase program requires that purchased securities by rated investment grade by at least one rating agency.

thus have the consolidated government budget constraint:

$$G + (R_{b,t} - 1)\bar{B} \quad = \quad T_t + \sum_{j=1}^{j=2}(R_{k,j,t} - R_t - \varrho_j)Q_{t-1}K_{g,j,t-1} + (R_{b,t} - R_t - \varrho_b)q_{t-1}B_{g,b,t-1}.$$

Central bank LSAPs involve purchasing a fraction, $\varphi_{j,t}$ and $\varphi_{b,t}$, of outstanding type j private-sector securities or long-term government securities, respectively. To be precise, these policies are respectively modeled as

$$K_{g,j,t} \quad = \quad \varphi_{j,t}K_{j,E,t-1},$$

and

$$B_{g,b,t} \quad = \quad \varphi_{b,t}\bar{B},$$

where $\varphi_{s,j,t}$ and $\varphi_{b,t}$ are modeled as second-order auto-regressive policies.

Resource constraint, further clearing conditions, and equilibrium We have the resource constraint:

$$Y_t = C_t + [1 + f(\frac{I_t}{I_{t-1}})]I_t + G + \sum_{j=1}^{j=2} \varrho_j Q_{t-1}K_{g,j,t-1} + \varrho_g q_{t-1}B_{g,b,t-1}.$$

We then require that supply equals demand in our different markets. In the market for labor:

$$\omega_1(1-\alpha)\rho\frac{Y_{1,t}^{\rho}Y_{m,t}^{1-\rho}}{L_{1,t}}E_t u_{C,t} = \frac{1}{P_{m,t}}\chi L_t^{\phi},$$

and

$$\omega_2(1-\alpha)\rho\frac{Y_{2,t}^{\rho}Y_{m,t}^{1-\rho}}{L_{2,t}}E_t u_{C,t} = \frac{1}{P_{m,t}}\chi L_t^{\phi},$$

where $L_t = L_{1,t} + L_{2,t}$. In the market for capital, we have

$$K_{1,t+1} + K_{2,t+1} = I_t + (1-\delta)K_t,$$

where $K_t = K_{1,t} + K_{2,t}$. Clearing for each type j securities implies

$$K_{j,E,t} = K_{p,j,t} + K_{h,j,t} + K_{g,j,t}.$$

Government bonds and other securities must clear as well:

$$\begin{aligned} \bar{B} &= \frac{S_{p,b,t}}{q_t} + B_{h,b,t} + B_{g,b,t} \\ B_o^S &= \frac{S_{p,o,t}}{q_{o,t}} + B_{h,o,t} + B_{g,o,t}. \end{aligned} \tag{28}$$

Notice that with clearing in the markets for goods, labor, and all securities, by Walras' Law the market for riskless short-term debt also clears.

3.2 Misallocation Measure

We can construct a measure of misallocation by first constructing a counterfactual measure of output: the maximum output, $\hat{Y}$, which can be produced with a fixed amount of labor and capital. In our production environment, $\hat{Y}$ can be expressed as

$$\hat{Y}_t = A_t K_t^{\alpha} L_t^{(1-\alpha)} \left(\sum_{j=1}^{j=2} \omega_j^{\frac{1}{1-\rho}} \right)^{\frac{1-\rho}{\rho}}.$$

We therefore can define the losses from misallocation as $\hat{Y}_t - Y_t$.

4 Quantitative Results

In this section, we lay out our calibration strategy and then discuss the quantitative results generated from the New Keynesian DSGE model away from and then at the zero lower bound, along with a few extensions.

4.1 Calibration

We present the parameters used in our quantitative exercise in Table 1. In our calibration exercise, we follow the calibration strategy of GK13 for their parameters and calibrate the new parameters we introduced.[18] The new parameters, listed at the bottom of Table 1, concern firm heterogeneity and the collateral constraint. Our calibration of the parameters that govern the extent of misallocation in steady state and the response of the relative difference in spreads of type 1 and type 2 firms to a large-scale purchase of type 1 securities is meant to be conservative. Here, by conservative we mean the following: our calibration leads to lower estimates of the extent to which government bond security purchases lead to greater output gains than corporate sector security purchases, as compared to alternate potential calibrations in the literature.

We calibrate the collateral constraint parameters to match moments related to credit spreads and the leverage of financial intermediaries.[19] We normalize Δ to 1, and our relative spread constraint implies that in steady-state, $\frac{E[R_x-R]}{E[R_x-R]} = \frac{\Delta_x}{\Delta}\frac{S_{p,x}}{S_{p,g}}$, for $x \in \{1;2;o\}$. We can therefore calibrate Δ_1,Δ_2, and Δ_o to targeted relative intermediated credit spreads. We target credit spreads of 2% annually for both private securities, 1.5% annually for government securities, and 1.75% annually for other securities. Our model does not explicitly model default costs, so we set these numbers to recent data on US corporate AAA spreads and Agency MBS spreads, which have little default risk. As default risk accounts for an important part of the spreads on risky securities and bank

[18] The only two parameters whose target we change are related to our modified financial constraint. We lower σ to 0.92, as at the previous value of σ, intermediaries could save out of their constraint, because we have introduced new higher yielding assets. Second, the wealth of entering firms is re-calibrated along with the parameters in the financial constraint to match moments related to credit spreads and bank leverage.

[19] We discuss how our calibration is conservative, as compared to a calibration that would rely on evidence from the ECB's Corporate Sector Purchase Program in Appendix D.

Parameters	Value
From Gertler and Karadi (2013)	
Households	
Discount rate, β	0.995
Habit parameter, h	0.815
Relative utility weight of labor, χ	3.482
Steady-state Treasury supply, B/Y	0.450
Proportion of long-term Treasury holdings of the households, $\bar{B}^h/B$	0.750
Portfolio adjustment cost, κ	1.000
Inverse Frisch elasticity of labor supply, φ	0.276
Intermediate Good Firms	
Capital share, α	0.330
Depreciation rate, δ	0.025
Capital-Producing Firms	
Inverse elasticity of net investment to the price of capital, η_i	1.728
Retail Firms	
Elasticity of substitution, ϵ	4.167
Probability of keeping the price constant, γ	0.779
Government	
Steady-state proportion of government expenditures, G/Y	0.200
Inflation coefficient in the Taylor rule, κ_π	1.500
Markup coefficient in the Taylor rule, κ_X	-0.125
Financial Intermediaries	
Transfer to the entering bankers, N_e	0.0528
Survival rate of the bankers, σ	0.92
New Parameters	
Financial Intermediaries and Households	
Share of capital internally financed, $K_{1,I}/K_1$,	0.8
Share of capital internally financed, $K_{2,I}/K_2$,	0.2
Share of household capital intermediation, $\bar{K}_{h,1}/K_1$,	0.1
Share of household capital intermediation, $\bar{K}_{h,2}/K_2$,	0.1
Share of household other asset holdings, $\bar{B}_{h,o}/B_o^S$,	0.1
Stock of other assets, $B_o^S q_o$,	2.880
Regulatory constraint parameter on government bonds Δ	1
Regulatory constraint parameter on type 1 securities, Δ_1	2.888
Regulatory constraint parameter on type 1 securities, Δ_2	0.236
Regulatory constraint parameter on other securities, Δ_o	0.212
Common collateral constraint parameter, θ	0.883
Capital wedge τ_1^k,	1
Capital wedge τ_2^k,	3/4
Intermediate Good Firms	
CES parameter, ρ	0.9
Weight of type 1 firms in aggregation ω_1	1
Weight of type 2 firms in aggregation ω_2	1.1855

Table 1: Parameters

loans, we assume that frictions to financial intermediation contribute equally to spreads on financial securities. Other frictions in financing (unrelated to financial intermediation) are accounted for by the wedges in our model. The level of government bond spreads and the leverage of financial intermediaries are then used to pin down constraint parameter, θ, and the per-period transfer of entering bankers, N_e. We target a leverage ratio of banks of 5 following GK2013; our qualitative results on the relative effectiveness of LSAPs are unchanged for a wide range of leverage targets.[20]

For intermediate good firms, we set the CES parameter, ρ, to 0.9, implying a great deal of substitutability between the two types of firms. Consistent with our goal of being conservative with regard to the extent of misallocation that exists in steady state, our calibration is above the value considered in the benchmark model of Atkeson and Burstein (2010), a representative value in the literature.[21]

We calibrate our internal financing parameters to account for the following fact: Shourideh and Zetlin-Jones (2012) show that about 80% of investment by private firms is financed externally, compared to 20% for publicly traded firms. Chari (2013) notes that the fact that GK13 is not calibrated to be consistent with such aggregate facts should be resolved in future work. We set the proportion of steady-state capital that firms of type 1 finance internally at 80% and the amount that they finance externally directly from households at 10% (therefore 10% of their capital is financed via intermediaries in steady-state, which is consistent with half of their external financing being met by households). Firms of type 2 finance 20% of their capital internally, finance 10% directly from households (the same proportion as type 1 firms), and must rely on financial intermediaries to finance the remainder of their capital stock. However, internal financing is not crucial for our qualitative results; if we set the share of internal financing to zero, we still find that LSAPs of government bonds are more effective than corporate LSAPs away from the ZLB. We set the share of other assets held by the household to 0.1 (the same proportion as other private securities). Finally,

[20] We have verified that our results on the relative effectiveness of LSAPs hold for bank leverage ratios of between 4 (bank leverage in GK11) and 15 (approximate leverage of investment banks in US data).

[21] We use a slightly different notation for the CES aggregator, as compared to Atkeson and Burstein (2010). If we define the CES parameter in Atkeson and Burstein (2010) as $\tilde{\rho}$, then $\rho = \frac{\tilde{\rho}-1}{\tilde{\rho}}$. They set $\tilde{\rho}$ to 5 in their benchmark calibration, which implies $\rho = 0.8$ in our notation. Our results are qualitatively similar if we use that value.

we set the share of other assets to 0.45, the approximate fraction of commerical bank credit that is accounted for by mortgage-backed securities and real estate loans.

We calibrate firm capital wedges to match the relative marginal products of capital of small and large manufacturing plants in Kehrig and Vincent (2017), normalizing τ_1^k to 1. These wedges account for other firm-level frictions, such as default risk or agency frictions, which affect the level of firm investment.[22] We calibrate the weights of type j firms in the aggregate production function, ω_j, so that the labor share of type 1 firms is 30%, which is approximately the employment share of publicly traded firms in the U.S. Finally, we normalize the cost of intermediation, $\varrho_1 = \varrho_b = 0$, which does not matter quantitatively for the relative ordering of the output effects of different types of asset purchases, but matters for welfare.

4.2 Quantitative Results away from the ZLB

Figure 1 presents results from the impulse responses from type 1 (large) firm security purchases and government bond purchases, calibrated so that the central bank holdings of assets are equal to 2.5% of steady-state GDP at their peak.[23] In our case with firm heterogeneity and potential misallocative effects, government bond purchases are *more effective* in boosting output than corporate security purchases (of type 1 firms). This is the reverse of the result in the work GK13, which does not consider heterogeneity. The result of GK13 occurs in our model when the output of the two types of firms is perfectly substitutable ($\rho = 1$).

We see that following a large-scale type 1 corporate security purchase, the amount of type 1 firm capital that has to be intermediated, $S_{p,1}$, falls, reducing spreads on firms of type 1 and leading to a larger difference in spreads, $E[R_{K,2}] - E[R_{K,1}]$. This leads to a marked increase in the capital of type 1 firms, K_1. Due to GE price effects, there is a concomitant change in the capital of type

[22] Table 1 in Kehrig and Vincent (2017) reports value added and capital stock for single-pant firms and multi-plant firms of different sizes. Measuring the marginal product of capital using the average product of capital, as in the paper, yields the ratio of marginal products of capital between firms with at least 10 plants and single-plant firms is approximately $\frac{2}{3}$ (and also for higher firm cutoffs). The ratio of marginal products of capital between all multi-plant firms and single plant firms is approximately $\frac{3}{4}$. We use the more conservative number (implying less misallocation in steady-state), which implies that $\frac{\tau_2^k}{\tau_1^k} = \frac{3}{4}$. This implies TFP losses of 1% due to misallocation in steady-state.

[23] Following GK13, we use an AR(2) process to account for that fact that LSAPs are made gradually and then slowly phased out. We parameterize these process as $\varphi_{x,t} = 1.5\varphi_{x,t-1} - 0.575\varphi_{x,t-1} + \epsilon_{\varphi,x,t}$, for $x \in 1, b$.

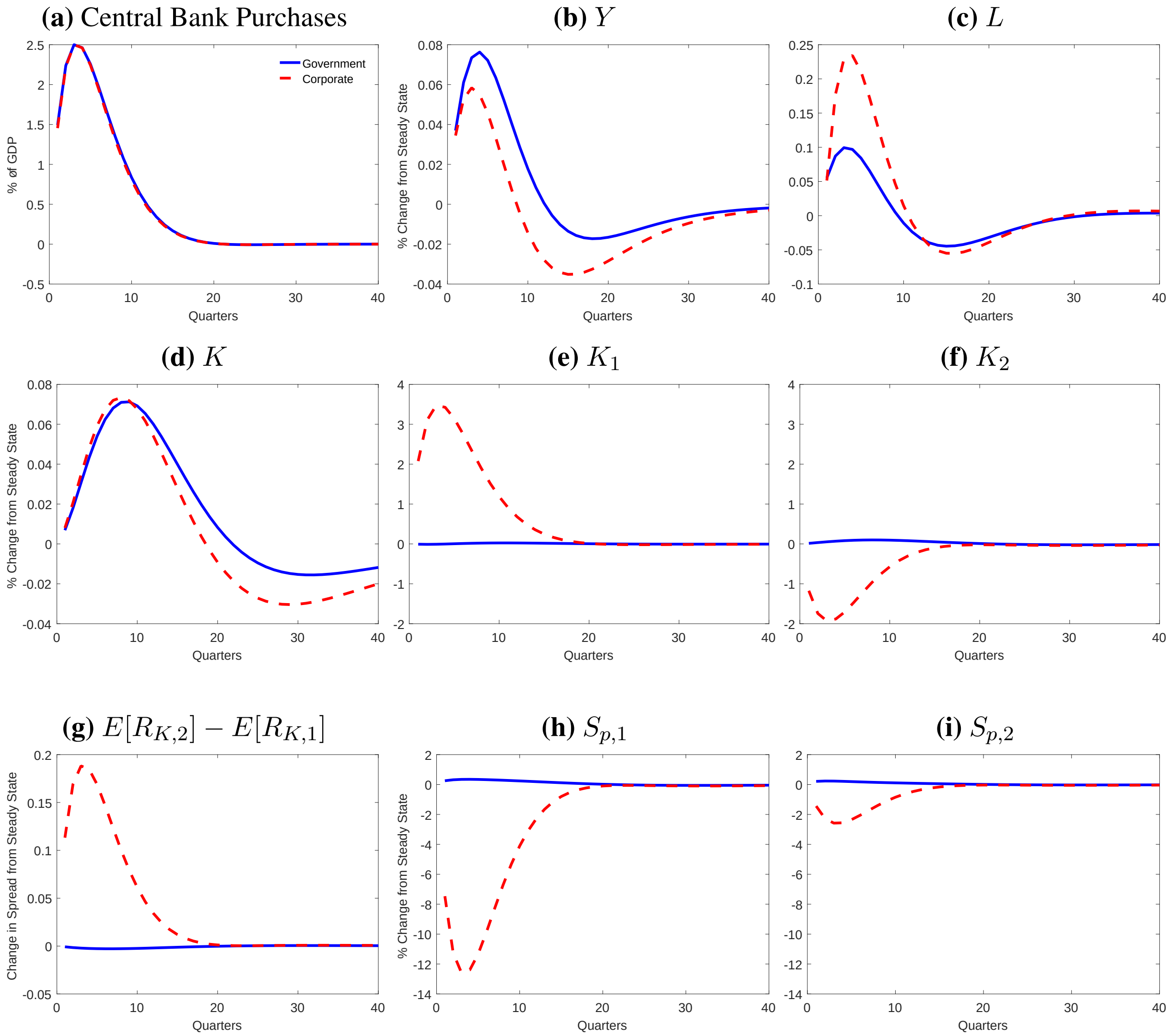

Figure 1: Government and Private Sector Asset Purchase Shocks

2 firms, K_2, as firms must finance part of their capital stocks. In our calibration, this change in the relative allocation of capital is inefficient, so there is a misallocation cost of corporate security purchases that reduces their effectiveness. There is still a positive effect on output, as lower average spreads lead to greater capital demand.

After a government bond purchase, we see different dynamics in terms of capital and spreads. A government bond purchase loosens the collateral constraint of the financial intermediary, which (all else equal) reduces spreads for all firms. Reducing the spreads for all firms reduces the extent of misallocation relative to the steady state (arising from the steady-state difference in spreads), and increases capital of type 2 firms (small firms) slightly more than type 1 firms. In this case, the

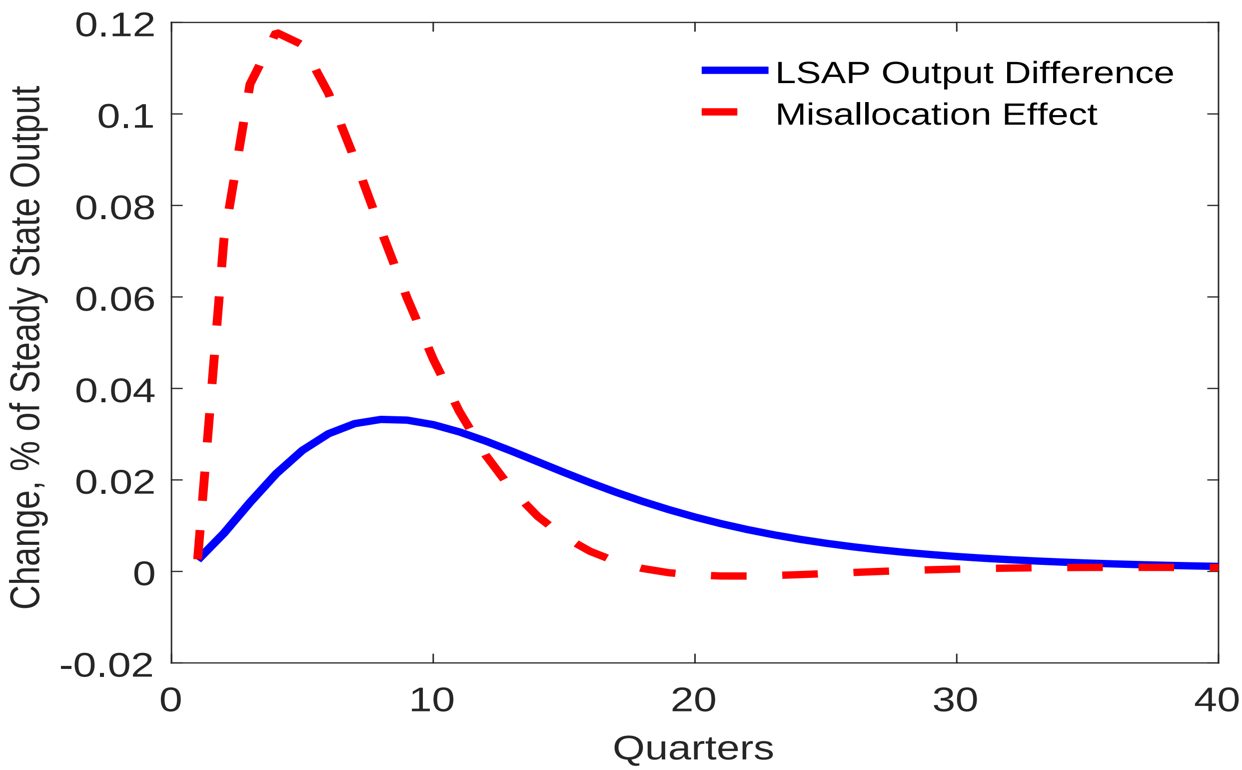

Figure 2: Misallocation Effect and Difference in LSAP Effectiveness

effectiveness of government bond purchases on increasing output is slightly amplified by its effect on the allocation of capital between firms.

The misallocation induced by large-scale corporate security purchases can be important when looking at the difference between the effectiveness of different types of LSAPs. The blue line in Figure 2 is the difference between the impulse response of output during the government security purchase and the impulse response of output during the large firm security purchase. The dashed red line is the output losses *directly* due to misallocation from the corporate security purchase. The direct misallocation effect is measured as the difference between the maximum output that could be produced with a given amount of capital and labor and what is actually produced.[24]

The losses due to the direct effects of misallocation in large firm corporate security purchases are quantitatively significant as compared to the difference in the effect of corporate versus government security purchases. In other words, when weighing different options for the types of

[24] There are also indirect effects of misallocation that our misallocation measure ignores. Capital and labor are taken as given in our misallocation measure, but, in fact, they are endogenously affected by misallocation as well.

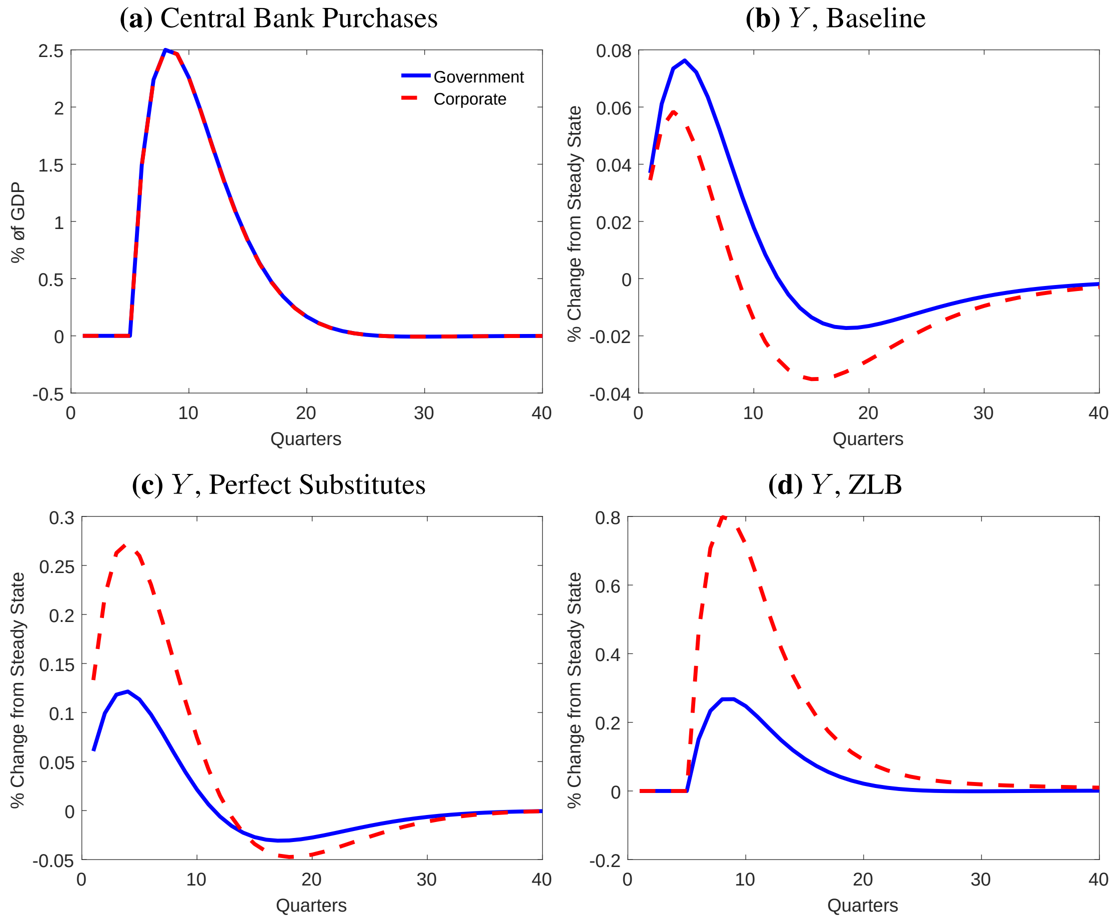

Figure 3: Effect of LSAPs on Output with Perfect Substitutes or ZLB

securities to buy as part of LSAPs, the misallocation effect of corporate security purchases should potentially be weighed as part of the trade-offs involved, as it can be quantitatively meaningful. This is the case with only an 0.12% implied initial relative difference in spreads caused by central bank corporate security purchases. Notice, this is a different result from that of GK13 who show that government security purchases induce smaller movements in output than private sector security purchases for a similarly sized purchase. Our model generates a similar result to GK13 when intermediate good firm products are perfect substitutes.[25] We show this result in panel (c) of Figure 3.

Overall, the calibrated impulse responses suggest that a large-scale corporate security pur-

[25]In our model, this is achieved when the CES parameter $\rho = 1$ and $\omega_j = 1$ for $j = 1, 2$. This result is due to purchases of private-sector securities having a larger effect on excess returns of private securities than purchases of government bonds have on excess returns of private securities, and this effect not being offset by a misallocation effect.

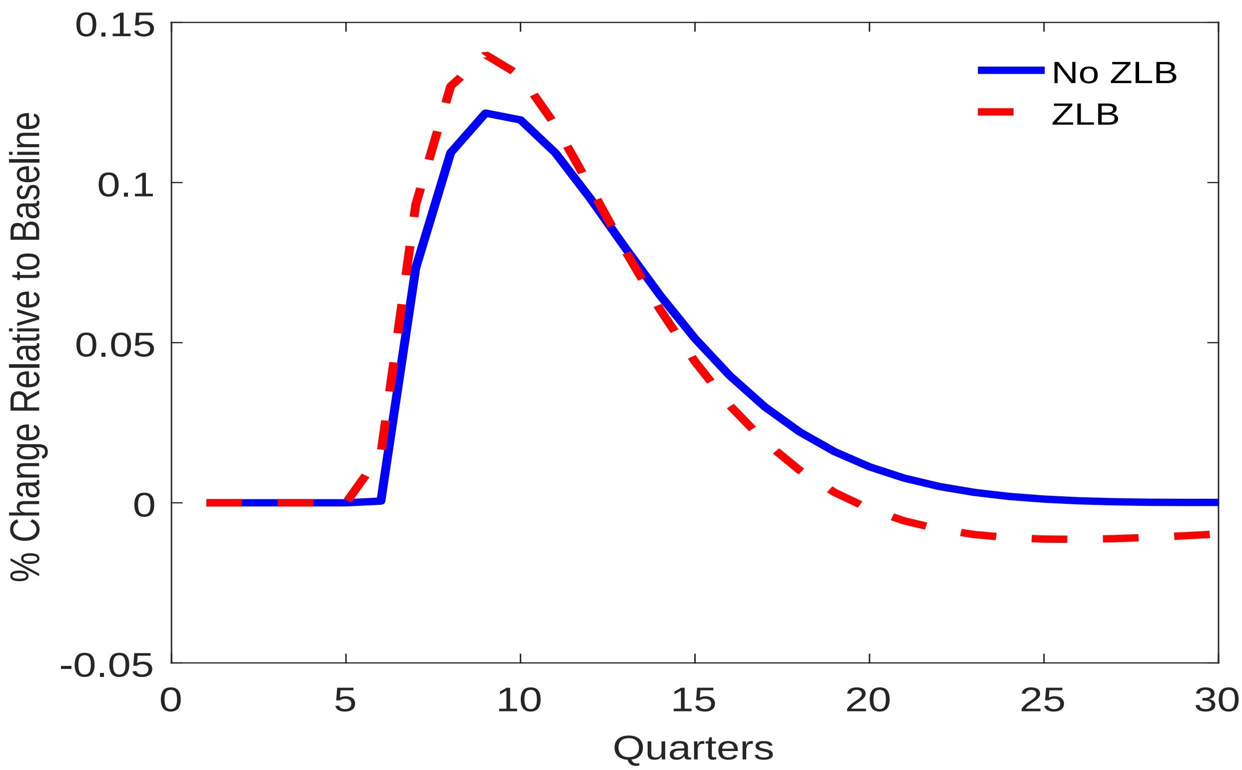

Figure 4: Misallocation Effect of Large-Scale Corporate Asset Purchases with and without the ZLB

chase induces a greater misallocation of resources than a large-scale government bond buy and the misallocation effect is a quantitatively significant fraction of the output gains from a large-scale corporate security purchase.

4.3 Quantitative Results at the ZLB

In our model, when at the ZLB, output losses from exogenous shocks, as well as the effectiveness of QE, are amplified.[26] To demonstrate this, we feed in capital quality shocks that force the economy to the ZLB. We then have the central bank perform similarly sized security purchases to our baseline case when the economy is at the ZLB. We show in panel (d) of Figure 3 that in this case, output gains from a QE program are indeed amplified relative to the baseline case where the ZLB does not bind.

We also compute our misallocation measure in response to asset purchases at the ZLB. We see from Figure 4 that our misallocation measure does not drastically change in response to corporate

[26] To incorporate the ZLB in our model, we follow the work of Guerrieri and Iacoviello (2015).

security purchases when we allow for a binding ZLB. This is because, accounting for level effects on excess returns, there is little change in the relative borrowing costs of type 1 and type 2 firms at the ZLB as compared to the baseline case. Hence, misallocation matters much more relative to movements in real output when the ZLB is not binding. There are arguments for the central bank to make QE part of its toolkit even away from the ZLB (for examples, see Quint and Rabanal (2017) or Gagnon (2016)). Our exercise sheds light on a potential counterargument to be considered when making such a claim, at least for large-scale corporate security purchases, as large-scale corporate security purchases can induce a quantitatively significant misallocation of resources.

4.4 Interest Rate Smoothing

A high degree of interest rate smoothing in the Taylor Rule is similar to moving to the ZLB in that it amplifies the extent to which LSAPs loosen the collateral constraint, while it does not amplify the misallocation effects of LSAPs. Consider a Taylor rule with interest rate smoothing of the form:

$$i_t = i + \rho_{ir}(i_{t-1} - i) + (1 - \rho_{ir}) \left(\kappa_\pi \pi_t + \kappa_y (\log(Y_t) - \log(Y_t^*))\right) + \epsilon_t,$$

where ρ_{ir} is a parameter that affects the degree of interest rate smoothing. Figure 5 presents the results of central bank purchases of corporate and government securities, with varying degrees of interest rate smoothing in the Taylor Rule.[27] We see that for lower values of interest rate smoothing, purchases of government bonds are more effective at stimulating output, but for higher values, corporate security purchases are more effective, as at the ZLB.

5 Conclusion

This paper studies an indirect effect of large-scale purchases of corporate securities: the misallocation of capital. Purchasing government bonds does not induce such misallocation on any

[27] Our baseline model is a special case where $\rho_{ir} = 0$. The degree of interest rate smoothing does not affect the steady-state of the model and therefore our calibration is unchanged.

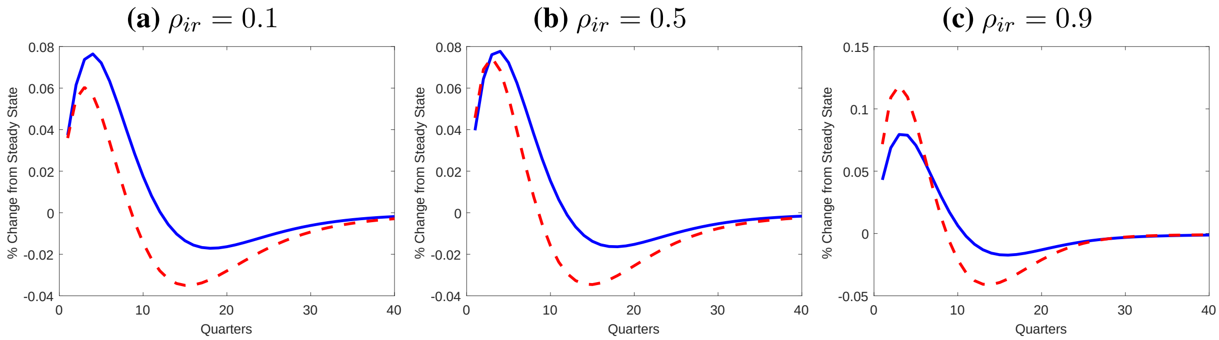

Figure 5: Effect of LSAPs on Output with Interest Rate Smoothing

comparable scale. In a calibrated DSGE model, we show the negative effect on output of a large-scale corporate security purchase through the misallocation of capital can be large enough to make government bond buys more effective than corporate security purchases, whereas this is not the case without accounting for such misallocation effects.

Key to obtaining this result in the model is that the collateral constraint is nonlinear. In equilibrium, the nonlinear constraint induces spreads to move conditional on intermediaries' holdings of assets, even in a log-linearized system. We show that our nonlinear constraint can be microfounded with an optimal contract when banks face the costs of default or there are costs of financial distress more generally. The fact that our constraint is both tractable for incorporation into dynamic, stochastic, general equilibrium models and is microfounded with an optimal contract should make it useful in other settings, especially in examining additional questions related to the heterogeneous effects of large-scale asset purchases.

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A A Collateral Constraint that is Nonlinear in Holdings

In the first section of this appendix, we prove the claim in Proposition 1 that given a constraint on the variance of returns to bank equity holders, when there are multiple aggregate shocks that are i.i.d. and assets are heterogeneous in their risk exposures, we obtain our collateral constraint, (4). For the proof, we first show that when assets are homogeneous in their risk exposures, the constraint implies the same functional form as the baseline collateral constraint in Gertler and Kiyotaki (2010). When assets are heterogeneous in their risk exposures but to only a single risk factor, we obtain the constraint in GK11 and GK13. We then derive (4) in the case where risk exposures are heterogeneous and there are multiple risk factors.

In the second section of this appendix, we show that a limit on the variance of returns to bank equity holders can be justified with an optimal contracting problem.

A.1 Proof to Proposition 1

Suppose the intermediary has market value, V_t, and holds a portfolio of securities, with $S_{i,t}$ indexing the market value of security i held by the intermediary at time t. The expected levered return at time t on the intermediary's portfolio of securities is then

$$\frac{\sum_i S_{i,t} E\left[R_{i,t}\right]}{V_t}, \tag{29}$$

where $R_{i,t}$ is the returns on each security i at time t.

A.1.1 Homogeneous Exposures

First, consider the case where all assets have identical exposure to aggregate shocks. Thus, the returns on each asset i at time t, $R_{i,t}$, have functional form $f\left(X_t\right) + \epsilon_{i,t}$, where $f(X_t)$ is some function of the aggregate shock vector, X_t, with variance $\sigma^2_{f(X)}$, and $\epsilon_{i,t}$ is idiosyncratic white noise. We can then derive the variance of bank equity returns due to aggregate shocks as:

$$\left(\frac{\sum_i S_{i,t} di}{V_t}\right)^2 \sigma^2_{f(X)}.$$

Note that this is equal to the total variance of bank equity if there is a continuum of assets and the intermediary can perfectly diversify aggregate risk. Now, if there is a requirement that the variance of bank equity returns due to aggregate shocks be below $\overline{\sigma^2}$, this leads to the constraint:

$$V_t \geq S_t \frac{\sigma_{f(X)}}{\overline{\sigma}},$$

where $S_t = \sum_i S_{i,t}$. Notice, since $\frac{\sigma_{f(X)}}{\overline{\sigma}}$ can be redefined as a constant, θ, we have obtained the baseline collateral constraint in Gertler and Kiyotaki (2010).

A.1.2 Heterogeneous Exposure to a Single Risk Factor

Now, consider the general case in which firm assets may have heterogeneous exposures to an aggregate shock vector. Returns on asset i at time t, $R_{i,t}$, are then a function of that single source of risk: $R_{i,t} = \Delta_i f(X_t) + \epsilon_{i,t}$ for each asset i.

The variance of bank equity returns due to aggregate shocks can thus be expressed as

$$\left(\frac{\sum_i \Delta_i S_{i,t}}{V_t}\right)^2 \sigma^2_{f(X)}.$$

Now, if there is a requirement that the variance of bank equity returns due to aggregate shocks is below $\overline{\sigma^2}$, this leads to the constraint:

$$V_t \geq \sum_i \Delta_i S_{i,t} \frac{\sigma_{f(X)}}{\overline{\sigma}}.$$

Notice, if we set $\frac{\sigma_{f(X)}}{\overline{\sigma}}$ equal to θ, we obtain the same collateral constraint as in GK11 or GK13.

A.1.3 Heterogeneous Exposures to Multiple Risk Factors

Finally, consider the case where there are multiple risk factors and returns have heterogeneous exposures. In this case, the returns on each asset i at time t, $R_{i,t}$, have functional form $f_i\left(X_t\right) + \epsilon_{i,t}$, where $f_i(X_t)$ is some function of the aggregate shock for asset i with variance $\sigma^2_{f_i(X_t)}$, and $\epsilon_{i,t}$ is idiosyncratic white noise. Also, define the covariance of type i returns with respect to asset k returns as $COV(f_i(X_t), f_k(X_t))$. For now, assume risk exposures are not necessarily independently distributed.

Notice, the expected levered return is still as in (29), but the variance of bank equity returns due to aggregate shocks becomes

$$\sum_i \frac{(S_{i,t})^2}{V_t^2}\sigma^2_{f_i(X_t)} + \sum_i \sum_{j\neq i} \frac{S_{i,t}S_{j,t}}{V_t^2} COV\left(f_i\left(X_t\right), f_j\left(X_t\right)\right). \tag{30}$$

Note that this is equal to the total variance of bank equity if there is a continuum of assets and the intermediary can perfectly diversify aggregate risk.

In turn, with the same limit on the variance of bank equity returns, $\overline{\sigma^2}$, we obtain:

$$V_t \geq \frac{\sqrt{\sum_i (S_{i,t})^2 \sigma^2_{f_i(X_t)} + \sum_i \sum_{j\neq i} S_{i,t}S_{j,t} COV\left(f_i\left(X_t\right), f_j\left(X_t\right)\right)}}{\overline{\sigma}}. \tag{31}$$

It is useful to point out a few properties of this constraint before demonstrating what assumptions are needed for it to reduce to the functional form of the constraint we use in the model. In particular, if we maximize $V_t = \sum_i S_{i,t} E\left[R_{i,t}\right] + (N - \sum_i S_{i,t})R$ subject to (31) with respect to $S_{i,t}$, we obtain:

$$E\left[R_{i,t}\right] - R = \frac{\lambda}{1-\lambda} \frac{(S_{i,t})\,\sigma^2_{f_i(X_t)} + \sum_{j\neq i} (S_{j,t})\, COV\left(f_i\left(X_t\right), f_j\left(X_t\right)\right)}{\sqrt{\sum_l (S_{l,t})^2 \sigma^2_{f_l(X_t)} + \sum_l \sum_{j\neq l} S_{l,t}S_{j,t} COV\left(f_l\left(X_t\right), f_j\left(X_t\right)\right)}}.$$

Now, notice that if we denote one asset as type b for government bonds, we can write:

$$\frac{E\left[R_{i,t}\right] - R}{E\left[R_{b,t}\right] - R} = \frac{(S_{i,t})\,\sigma^2_{f_i(X_t)} + \sum_{j\neq i} (S_{j,t})\, COV\left(f_i\left(X_t\right), f_j\left(X_t\right)\right)}{(S_{b,t})\,\sigma^2_{f_b(X_t)} + \sum_{j\neq i} (S_{j,t})\, COV\left(f_b\left(X_t\right), f_j\left(X_t\right)\right)}. \tag{32}$$

Our nonlinear constraint thus implies that the amount of a particular asset held will affect the shadow price of that asset if the constraint binds. We can contrast this equation with that of GK11 or GK13; in their case, $\frac{E[R_{i,t}]-R}{E\left[R_{b,t}\right]-R}$ is only a fraction of parameters and thus move proportionally no matter the asset purchased.

Recall, in Proposition 1, we assume that the aggregate shocks are independently distributed.

With this assumption, (31) becomes:

$$V_t \geq \sqrt{\sum_i (S_{i,t})^2 \frac{\sigma^2_{f_i(X_t)}}{\sigma^2}}. \tag{33}$$

Assuming risk exposures differ across government bonds, other assets, and each of type $j \in J$ firm securities and calibrating $\frac{\sigma^2_{f_i(X_t)}}{\sigma^2}$ as $\theta^2\Delta$, $\theta^2\Delta_o$, or $\theta^2\Delta_j$, respectively, we can then obtain (4) from (33).

A.2 Relation to Optimal Contracting Problem

We can justify the constraint in (4) by demonstrating that the implied relative spreads from this constraint are identical to those implied by an optimal contracting problem in the case where bank default (or financial distress) is costly and this cost is reflected in the bank's optimization problem.

Consider a risk-neutral bank with liquid net worth, N_t, that chooses the dollar quantity of assets, $S_{i,t}$ (over multiple assets $i \in I$), to hold. Net worth for the bank follows law of motion: $N_{t+1} = RN_t + \int_i (R_{i,t+1} - R) S_{i,t}$. Assume that asset returns are jointly normally distributed. Therefore, N_{t+1} is normally distributed as well.

The bank makes asset holding decisions in the face of a cost of the form $N_t c_D\left(\frac{N_{t+1}}{N_t}\right)$, where c_D is a function of the return on wealth. This cost function nests fixed default costs (proportional in pre-default wealth N_t), default costs proportional to the amount of debt the bank defaults on, costs due to nearing financial distress (but not defaulting), as well as social costs from a deterioration in net worth that are internalized.

In this setup, we can express the expected default cost of the intermediary, $E\left[N_t c_D\left(\frac{N_{t+1}}{N_t}\right)\right]$, as a function of the mean and variance of intermediary returns:

$$E\left[N_t c_D\left(\frac{N_{t+1}}{N_t}\right)\right] = N_t \vartheta\left(E\left[\frac{N_{t+1}}{N_t}\right], \text{Variance}\left[\frac{N_{t+1}}{N_t}\right]\right),$$

where ϑ is a continuous and differentiable function.

Define $s_{i,t}$ as the bank's holdings of asset i scaled by its net worth; so, $s_{i,t} = \frac{S_{i,t}}{N_t}$. We can therefore write the bank's maximization problem as

$$\max_{s_{i,t}} \quad N_t\left(R+\int_i s_{i,t}\left(E\left[R_{i,t}\right]-R\right)di-\vartheta\left(R+\int_i s_{i,t}\left(E_t\left[R_{i,t}\right]-r\right)di, \text{Variance}\left[\int_i s_{i,t}R_{i,t}di\right]\right)\right)$$

Using the notation in section A.1.2 of this appendix for the variances and covariances of returns to asset i, the first-order conditions of this problem yields:

$$E\left[R_{i,t}\right]-R=\frac{\vartheta_2}{1-\vartheta_1}\left(s_{i,t}\sigma^2_{f_i(X_t)}+\sum_{j\neq i}\left(s_{j,t}\right)COV\left(f_i\left(X_t\right),f_j\left(X_t\right)\right)\right), \tag{34}$$

where $\vartheta_1 < 0$ and $\vartheta_2 > 0$. Notice, it is straightforward that $\vartheta_1 < 0$ and $\vartheta_2 > 0$ when the intermediary faces fixed or proportional default costs, given that default is a left tail event.

Relative spreads generated from the bank's first-order conditions thus have an equivalence to those implied by the intermediary's problem with a constraint on the variance on the returns to bank equity holders, (32), as derived in section A.1.2 of this appendix.

B Proofs to Propositions 2 and 3

Here we present proofs to Propositions 2 and 3.

Proposition 2

Plugging in our expressions for spreads, (6) and (7), into our collateral constraint, (4), we can obtain an expression for $\frac{\lambda}{1+\lambda}$:

$$\frac{\lambda}{1+\lambda}=\frac{-NR+\nu_1}{\nu_1},$$

where

$$\nu_1 \quad = (\textstyle\sum_j \Delta_j(\int_{i\in j}(Qk_i-S_{g,i})di)^2+\Delta(B^S-S_{g,b})^2+\Delta_o S^2_{p,o})^{\frac{1}{2}}.$$

Holding capital choices (and thus Q) constant, notice, $\frac{\partial \frac{\lambda}{1+\lambda}}{\partial B_g} = (NR)(\nu_1^{-2})\frac{\partial \nu_1}{\partial B_g}$. This derivative is negative since $\nu_1 > 0$, $Nr > 0$, and

$$\frac{\partial \nu_1}{\partial B_g} = -\Delta(B^S - S_{g,b})(\sum_j \Delta_j(\int_{i\in j} Qk_i - S_{g,i}di)^2 + \Delta(B^s - S_{g,b})^2 + \Delta_o S_{p,o}^2)^{-\frac{1}{2}} < 0. \quad (35)$$

In turn, it must also be true that $\frac{\partial \lambda}{\partial S_{g,b}} < 0$, since $\frac{\lambda}{1+\lambda}$ is increasing in λ. Thus, we have proved (i)(a). Using an analogous approach, (ii)(a) follows from taking the derivative of $\frac{\lambda}{1+\lambda}$ with respect to $S_{g,i}$ for $i \in j$, holding capital choices constant. We can also obtain (i)(b) and (ii)(b) immediately from (8), taking the derivative of relative spreads with respect to $S_{g,b}$ and $S_{g,i}$, respectively.

Proposition 3

Note that output, defined in (14), is increasing in the term $\int \left(A_i^\rho \left(c_{\tau,i}\tau_i^k\right)^{\alpha\rho}\right)^{\frac{1}{1-\alpha\rho}} di$. The output-maximizing value for cost of capital wedges then can be found by solving the maximization problem:

$$\max_{c_{\tau,i}} \int_i \left(A_i^\rho \left(\frac{c_{\tau,i}}{\tau_i^k}\right)^{\alpha\rho}\right)^{\frac{1}{1-\alpha\rho}} di,$$

such that

$$\frac{\left(\int (A_i^\rho)^{\frac{1}{1-\alpha\rho}} \left(c_{\tau,i}\tau_i^k\right)^{\frac{\alpha\rho}{1-\alpha\rho}} di\right)^{\frac{1-\rho}{\rho}} \left(\int (A_i^\rho)^{\frac{1}{1-\alpha\rho}} \left(c_{\tau,i}\tau_i^k\right)^{\frac{1}{1-\alpha\rho}} di\right)^{1-\alpha}}{\left(\int (A_i^\rho)^{\frac{1}{1-\alpha\rho}} \left(\tau_i^k\right)^{\frac{\alpha\rho}{1-\alpha\rho}} di\right)^{\frac{1-\rho}{\rho}} \left(\int (A_i^\rho)^{\frac{1}{1-\alpha\rho}} \left(\tau_i^k\right)^{\frac{1}{1-\alpha\rho}} di\right)^{1-\alpha}} = 1.$$

This yields a first-order condition that can be simplified as $c_{\tau,i}\tau_i^k = \Xi \, \forall i$, where Ξ is a constant across firms. Proposition 3 follows.

Corollary 3.1

With only two groups of firms, holding the weighted-average rate of interest fixed, output is only affected by heterogeneous changes in spreads through the term:

$$\sum_{j=1,2} \left(A_j^{\rho} \left(c_{\tau,j}\tau_j^k\right)^{\alpha\rho}\right)^{\frac{1}{1-\alpha\rho}}, \tag{36}$$

where we define A_j such that $\left(A_j^{\rho}\right)^{\frac{1}{1-\alpha\rho}} = \int_{i\in j} \left(A_i^{\rho}\right)^{\frac{1}{1-\alpha\rho}} di$. Given cost of capital wedges, $c_{\tau,j}$, are defined between the cost of capitals facing firms and the weighted average cost of capital, we have clearing condition:

$$\frac{\left(\sum_{j=1,2} A_j^{\frac{\rho}{1-\alpha\rho}} \left(c_{\tau,j}\tau_j^k\right)^{\frac{\alpha\rho}{1-\alpha\rho}}\right)^{\frac{1-\rho}{\rho}} \left(\sum_{j=1,2} A_j^{\frac{\rho}{1-\alpha\rho}} \left(c_{\tau,j}\tau_j^k\right)^{\frac{1}{1-\alpha\rho}}\right)^{1-\alpha}}{\left(\sum_{j=1,2} A_j^{\frac{\rho}{1-\alpha\rho}} \left(\tau_j^k\right)^{\frac{\alpha\rho}{1-\alpha\rho}}\right)^{\frac{1-\rho}{\rho}} \left(\sum_{j=1,2} A_j^{\frac{\rho}{1-\alpha\rho}} \left(\tau_j^k\right)^{\frac{1}{1-\alpha\rho}}\right)^{1-\alpha}} = 0. \tag{37}$$

From (37), a shock that increases $c_{\tau,1}$ thus decreases $c_{\tau,2}$. Taking the derivative of (36) with respect to $c_{\tau,1}$, $\frac{\partial \sum_{j=1,2}\left(A_j^{\rho}\left(c_{\tau,j}\tau_j^k\right)^{\alpha\rho}\right)^{\frac{1}{1-\alpha\rho}}}{\partial c_{\tau,1}}$, yields:

$$\frac{\left(c_{\tau,2}\tau_2^k - c_{\tau,1}\tau_1^k\right)}{\left(\left(\frac{\alpha(1-\rho)}{1-\alpha}\right) \frac{\sum_j A_j^{\frac{\rho}{1-\alpha\rho}} \left(c_{\tau,j}\tau_j^k\right)^{\frac{1}{1-\alpha\rho}}}{\sum_j A_j^{\frac{\rho}{1-\alpha\rho}} \left(c_{\tau,j}\tau_j^k\right)^{\frac{\alpha\rho}{1-\alpha\rho}}} + \left(c_{\tau,2}\tau_2^k\right)\right)}. \tag{38}$$

The denominator of (38) is always positive, thus the sign of (38) is controlled by the numerator. If the level of the cost of capital wedge facing firms of type 1 is above (below) its optimal value, $c_{\tau,1}^{\star}$, then Proposition 3 together with (37) imply that $\left(c_{\tau,2}\tau_2^k - c_{\tau,1}\tau_1^k\right) < 0$ (> 0). Corollary 3.1 follows.

C Solution to the Problem of Bankers

Bankers choose $s_{p,1,t}$, $s_{p,2,t}$, $s_{p,b,t}$, and $s_{p,o,t}$ to maximize (23) subject to (21), (22), and (24). The Lagrangian of this problem is

$$
\begin{aligned}
\mathcal{L} \;=\; & E_t\left[\Lambda_{t,t+1}(\left(1-\sigma\right)n_{t+1}+\sigma V_{t+1}\right)\Big] + \lambda_t E_t\left[\Lambda_{t,t+1}\left(\left(1-\sigma\right)n_{t+1}+\sigma V_{t+1}\right]\right. \\
& -\lambda_t\left(\theta\sqrt{\Delta_1(s_{p,1,t})^2+\Delta_2(s_{p,2,t})^2+\Delta(s_{p,b,t})^2+\Delta_o(s_{p,o,t})^2}\right),
\end{aligned}
$$

where

$$
n_t = \left(\sum_{j=1}^{j=2} R_{k,j,t}s_{p,j,t-1}\right) + R_{b,t}s_{p,b,t-1} + R_{o,t}s_{p,o,t-1} - R_t\left(\sum_{x\in 1,2,b,o} s_{p,x,t-1} - n_{t-1}\right).
$$

Let λ_t be the Lagrange multiplier associated with the collateral constraint (24). Let $\tilde{\Lambda}_{t,t+1}$ be bankers' "augmented' stochastic discount factor, equal to the product of $\Lambda_{t,t+i}$, that is, the discount factor from the household's problem as defined in (27), and the multiplier $\left(\left(1-\sigma\right)+\sigma\frac{\partial V_{t+1}}{\partial N_{t+1}}\right)$. Taking first-order conditions and rearranging, we can obtain:

$$
E_t\left[\tilde{\Lambda}_{t,t+1}\left(R_{k,1,t+1}-R_{t+1}\right)\right] = \frac{\frac{\lambda_t}{(1+\lambda_t)}\theta\Delta_1 s_{p,1,t}}{\sqrt{\Delta_1(s_{p,1,t})^2+\Delta_2(s_{p,2,t})^2+\Delta(s_{p,b,t})^2+\Delta_o(s_{p,o,t})^2}},
$$

$$
E_t\left[\tilde{\Lambda}_{t,t+1}\left(R_{k,2,t+1}-R_{t+1}\right)\right] = \frac{\frac{\lambda_t}{(1+\lambda_t)}\theta\Delta_2 s_{p,2,t}}{\sqrt{\Delta_1(s_{p,1,t})^2+\Delta_2(s_{p,2,t})^2+\Delta(s_{p,b,t})^2+\Delta_o(s_{p,o,t})^2}},
$$

$$
E_t\left[\tilde{\Lambda}_{t,t+1}\left(R_{b,t+1}-R_{t+1}\right)\right] = \frac{\frac{\lambda_t}{(1+\lambda_t)}\theta\Delta s_{p,b,t}}{\sqrt{\Delta_1(s_{p,1,t})^2+\Delta_2(s_{p,2,t})^2+\Delta(s_{p,b,t})^2+\Delta_o(s_{p,o,t})^2}},
$$

and

$$E_t\left[\tilde{\Lambda}_{t,t+1}\Big(R_{o,t+1}-R_{t+1}\Big)\right]=\frac{\frac{\lambda_t}{(1+\lambda_t)}\theta\Delta_o s_{p,o,t}}{\sqrt{\Delta_1(s_{p,1,t})^2+\Delta_2(s_{p,2,t})^2+\Delta(s_{p,b,t})^2+\Delta_o(s_{p,o,t})^2}}.$$

These equations, when combined and rearranged, lead to the following expressions for relative spreads:

$$\frac{E_t\left[\tilde{\Lambda}_{t,t+1}\left(R_{k,2,t+1}-R_{t+1}\right)\right]}{E_t\left[\tilde{\Lambda}_{t,t+1}\left(R_{k,1,t+1}-R_{t+1}\right)\right]}=\frac{\Delta_2 s_{p,2,t}}{\Delta_1 s_{p,1,t}},$$

$$\frac{E_t\left[\tilde{\Lambda}_{t,t+1}\left(R_{b,t+1}-R_{t+1}\right)\right]}{E_t\left[\tilde{\Lambda}_{t,t+1}\left(R_{k,1,t+1}-R_{t+1}\right)\right]}=\frac{\Delta s_{p,b,t}}{\Delta_1 s_{p,1,t}},$$

and

$$\frac{E_t\left[\tilde{\Lambda}_{t,t+1}\left(R_{o,t+1}-R_{t+1}\right)\right]}{E_t\left[\tilde{\Lambda}_{t,t+1}\left(R_{k,1,t+1}-R_{t+1}\right)\right]}=\frac{\Delta_o s_{p,o,t}}{\Delta_1 s_{p,1,t}}.$$

Now, also note that we can write the value function as:

$$V_t=\sum_{x\in\{1,2,b,o\}}E\left[\tilde{\Lambda}_{t,t+1}\left(R_{x,t+1}-R_{t+1}\right)\right]s_{p,x,t}+R_{t+1}n_tE\left[\tilde{\Lambda}_{t,t+1}\right].$$

We can solve for the envelope condition by taking the derivative of this w.r.t n_t:

$$\frac{\partial V_t}{\partial n_t}=\sum_{x\in\{1,2,b,o\}}E\left[\tilde{\Lambda}_{t,t+1}\left(R_{x,t+1}-R_{t+1}\right)\right]\frac{\partial s_{p,x,t}}{\partial n_t}+R_{t+1}E\left[\tilde{\Lambda}_{t,t+1}\right].$$

If we assume that the constraint binds in equilibrium, taking the derivative of the constraint (24) and combining with the relative spread equations yields an expression for the shadow value of intermediary wealth:

$$\frac{\partial V_t}{\partial n_t} = \frac{R_{t+1} E\left[\tilde{\Lambda}_{t,t+1}\right]}{1 - E\left[\tilde{\Lambda}_{t,t+1}\left(R_{k,1,t+1} - R_{t+1}\right)\right] \frac{\frac{\partial V_t}{\partial n_t}\sqrt{\Delta_1(s_{p,1,t})^2 + \Delta_2(s_{p,2,t})^2 + \Delta(s_{p,b,t})^2 + \Delta_o(s_{p,o,t})^2}}{\theta \Delta_1 s_{p,1,t}}}. \tag{39}$$

And combining (24) and (39), we obtain the following leverage restriction:

$$n_t \frac{\partial V_t}{\partial n_t} \geq \theta \sqrt{\Delta_1(s_{p,1,t})^2 + \Delta_2(s_{p,2,t})^2 + \Delta(s_{p,b,t})^2 + \Delta_o(s_{p,o,t})^2}.$$

D Comparison of our Calibration to CSPP

There is limited empirical evidence on the effect of nonfinancial corporate security purchases on the relative spreads of nonfinancial firms. One potential way to understand this relative effect would be to exploit the recent experience of European firms to the ECB's Corporate Sector Purchase Program (CSPP). However, there are other ongoing initiatives by the European governments and the ECB, such as Targeted Long-Term Refinancing Operations, making it difficult to fully disentangle the direct and indirect effect of the ECB's CSPP on borrowing costs and no published paper has done so. Nonetheless, a recent working paper by Grosse-Rueschkamp et al. (2017) makes an attempt at this exercise, along with examining follow-on effects of the program on quantities and bank behavior. In an exercise where the authors examine the response of borrowing rates in corporate bond markets (for only public firms due to data limitations), they find that the program had an substantial and heterogeneous effect on firm bond spreads. In the [-3,7] day window around the announcement of CSPP, they find that spreads fell by 11% for CSPP eligible firms and by 6% for non-eligible firms. Given the level of credit spreads in our calibration, these changes in spreads imply a difference in spreads of 10 basis points. CSPP bond purchases have totaled approximately 166 billion Euros as of August 2018, approximately 1.5% of Eurozone annual GDP. Our calibrated model implies an initial relative difference in spreads due to central bank purchases of private securities of 12 basis points in response to a purchase of corporate securities equal to 2.5% of annual

GDP, as can be seen in panel (g) of Figure 1. In response to a purchase of corporate securities equal to 1.5%, our model implies an initial relative difference of 7-8 basis points. Changing our calibration to increase the relative spread effect would lead to greater misallocation and government bond purchases being even more effective than private bond purchases than our current calibration.