Tax Policy and Aggregate Stability in an Overlapping Generations Model*

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Abstract

In the context of a two-period non-monetary overlapping generations model with Cobb-Douglas preference and technological specifications, this paper explores the quantitative interrelations between equilibrium (in)determinacy versus (i) a progressive tax schedule on wage income and (ii) a balanced-budget rule with endogenous labor taxation. In sharp contrast to previous studies on a one-sector representative-agent macroeconomy, we find that both fiscal formulations are stabilizing instruments against cyclical fluctuations driven by agents' self-fulfilling beliefs. The key policy implication of our no-indeterminacy result is that depending on what is the underlying analytical environment, countercyclical income taxation may stabilize or destabilize the business cycle.

Keywords: Tax Policy; Equilibrium Indeterminacy; Overlapping Generations Model.

JEL Classification: E32; E62.

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1 Introduction

Since the late 1990's, considerable progress has been made in exploring the interrelations between local stability properties of competitive equilibria and tractable fiscal policy rules within a prototypical one-sector real business cycle (RBC) macroeconomy. Given the assumptions of perfect competition and constant returns-to-scale in production, this model's interior steady state is a locally determinate or isolated saddle point around which there exists a unique convergent equilibrium trajectory, as well as macroeconomic fluctuations driven by exogenous shocks to economic fundamentals such as endowments, preferences and technology. Under a balanced-budget rule that consists of constant government spending and proportional taxation on the infinitely-lived representative agent's wage income, Schmitt-Grohé and Uribe (1997, section II; SU hereafter) find that a Laffer curve-type relationship between the labor tax rate and the resulting tax revenue will emerge, which in turn may lead to the existence of two interior stationary equilibria. These authors then analytically show that the low-tax steady state can display equilibrium indeterminacy and sunspot-induced endogenous business cycles. On the other hand, Guo and Lansing (1998; GL hereafter) incorporate a progressive income tax schedule, whereby the household's average and marginal tax rates are increasing functions of its total factor income, into an otherwise standard one-sector laissez-faire RBC model with an indeterminate steady state under aggregate increasing returns to productive inputs. These authors find that a sufficiently high degree of tax progressivity is able to stabilize the economy against cyclical fluctuations generated by changes in agents' animal spirits.[1] The aforementioned results thus illustrate that in the context of a one-sector representative-agent macroeconomy, the business-cycle destabilization effect of Schmitt-Grohé and Uribe's (1997, section II) countercyclical balanced-budget arrangement are qualitatively opposite to that of the macroeconomic stabilization impact of Guo and Lansing's (1998) procyclical progressive tax scheme.

Motivated by the above RBC-based research on the aggregate (in)stability effects of the SU versus the GL fiscal policy rule, we will examine the robustness of their contrasting consequences within a two-period non-monetary overlapping generations (OLG) model *à la* Diamond (1965). For the sake of analytical simplicity, Lloyd-Braga, Nourry and Venditti's (2007, section 6; LNV hereafter) parsimonious framework with Cobb-Douglas preference and technological formulations is adopted here as our baseline setting. Each agent's lifetime utility function exhibits linear homogeneity in her young and old consumption expenditures combined,

[1] By contrast, Guo and Harrison (2001, 2011) find that sufficiently regressive income taxation is needed to insulate the economy from belief-driven aggregate fluctuations in a two-sector RBC model with equilibrium indeterminacy under laissez faire.

which are then postulated to be additively separable from hours worked in youth. Moreover, all the savings of young individuals are held in the form of additions to future capital stock that will be fully depreciated after one period. On the production side, the economy's social technology displays increasing returns-to-scale due to the presence of positive productive externalities from aggregate labor hours.[2] In this environment (without government intervention), its interior stationary state is found to be a locally indeterminate sink when the level of labor externalities is sufficiently strong and/or each young household's labor supply is sufficiently elastic. In what follows, our analyses will take LNV's local (in)determinacy results as the focal point of departure.[3]

Starting with LNV's two-period Cobb-Douglas overlapping generations model that exhibits multiple equilibria under laissez faire, a slightly modified Guo-Lansing tax schedule, characterized by monotonically increasing average and marginal tax rates on young individuals' wage earnings, is first incorporated into.[4] In this case, the economy's equilibrium conditions can be expressed as a first-order nonlinear dynamical system in capital and labor. We then quantitatively examine the resulting local stability properties for calibrated parameter values that are consistent with post Korean-war U.S. time series data. As it turns out, Guo and Lansing's (1998) determinacy result – saddle-path stability and equilibrium uniqueness will take place when the postulated policy rule is sufficiently progressive – continues to hold within our OLG macroeconomy. Intuitively, start the no-government LNV model from its steady state, and suppose that the young generation-t agent becomes optimistic about the economy's future. Acting upon this belief, she will consume less and invest more today, which in turn generate increases in the next period's capital stock, hours worked, total output and real wage rate. If the productive externalities from aggregate labor inputs are strong enough, the rate of return on capital investment will rise to validate the initial optimism as a self-fulfilling equilibrium. Under the GL progressive taxation scheme, the higher marginal tax rate at period $t+1$ leads to a decrease in labor supply, thus "taxing away" the augmented marginal product of capital. It follows that the equality of the relevant consumption Euler equation will no longer

[2] Within a general analytical framework which does not specify the preference and technology functional forms, Lloyd-Braga, Nourry and Venditti (2007, footnote 10) find that local indeterminacy cannot occur under positive capital, combined with no labor, externalities in the firms' production process. Without loss of generality, these authors then assume zero capital externality in their local stability analyses of the general model (sections 4-5, pp. 521-528) as well the Cobb-Douglas economy (section 6).

[3] As in Schmitt-Grohé and Uribe (1997), Guo and Lansing (1998) and many previous studies, this paper is restricted to analyzing the local dynamics of equilibrium path(s) around each steady state. The global behavior of our model economy is a worthwhile topic for future research.

[4] Since Lloyd-Braga, Nourry and Venditti's (2007) focus is on indeterminacy and sunspots, our analysis will begin with introducing the Guo-Lansing progressive tax schedule to LNV's indeterminate macroeconomy inhabited by two-period-lived overlapping generations.

hold, hence this procyclical fiscal formulation operates like a classic automatic stabilizer which may eliminate the possibility of sunspot-driven business cycle fluctuations in a two-period Cobb-Douglas overlapping generations model.

We also find that the minimum level of the Guo-Lansing tax progressivity required to stabilize our OLG macroeconomy against endogenous business cycles will *ceteris paribus* increase when (i) the degree of labor externalities in production is stronger; or (ii) the share of first-period consumption over a young individual's wage income falls; or (iii) the labor supply elasticity becomes higher. The intuitions behind these numerical findings are straightforward. While keeping the values of other parameters unchanged, the economy is more susceptible to indeterminacy and sunspots in cases (i) and (iii) because the young cohort-t agent's optimistic expectation yields a higher increment in hours worked at period $t+1$; or in case (ii) since the average/marginal propensity to consume out of a young household's disposable wage earnings is lower, which in turn raises the amount of additions to the next period's capital stock as well as labor hours. As a result, more progressive labor income taxation is needed for suppressing belief-induced macroeconomic fluctuations. Interestingly, these OLG-based comparative statics outcomes associated with the GL tax schedule turn out to be qualitatively identical to those in the context of a one-sector RBC model.

Next, we incorporate the SU balanced-budget policy rule into LNV's heterogeneous-agent model with zero or "relatively small" labor externalities such that the no-government economy possesses a locally determinate steady state. Quite surprisingly, Schmitt-Grohé and Uribe's (1997, section II) instability result – aggregate fluctuations caused by equilibrium indeterminacy may arise under endogenous labor taxation – does *not* remain robust within a two-period Cobb-Douglas overlapping generations macroeconomy. For all the admissible parametric configurations with two interior stationary equilibria, we show that the low-tax steady state is a saddle point and the high-tax steady state is a totally unstable source. Upon an expected expansion in future economic activity, the young generation-t agent chooses to sacrifice today's consumption for more savings, hence two counteracting effects on the dynamic equation that governs her labor supply decision ensue. On the one hand, its left-hand side will rise because higher levels of capital stock, hours worked and total output at period $t+1$ lead to increases in this individual's old-age consumption as well as the rate of return to investment. On the other hand, a lower current consumption shifts her labor supply curve to the right, generating a fall in the real wage rate and an increase in hours worked at period t. Under SU's countercyclical fiscal formulation, the government's tax revenue will stay the same since the labor tax rate moves in the opposite direction and by the same proportion as the young household's wage income. It follows that the overall effects of a higher period-t labor hours on the right-hand-side of the optimal labor-supply condition are theoretically ambiguous. Our numerical simulations find that starting at the model's low-tax steady state, its right-hand-side as a whole may decrease or does not rise enough to match with the left-hand-side increase in the marginal product of capital. As a result, the macroeconomy will display saddle-path stability and equilibrium uniqueness in that the beginning optimism is not justified. When the model starts at the high-tax stationary equilibrium, the hike in the right-hand-side expression is found to quantitatively dominate that of the corresponding left-hand-side increase. Consequently, this steady state is an unstable source surrounded by divergent or explosive trajectories that will eventually violate the economy's transversality condition.

In terms of policy implications, this paper illustrates that the aggregate (in)stability effects of our postulated tax policy rules within a two-period Cobb-Douglas overlapping generations model are rather different from those of Schmitt-Grohé and Uribe (1997, section II) and Guo and Lansing (1998) for a one-sector representative-agent framework. Specifically, we show that both SU's countercyclical and GL's procyclical fiscal formulations are stabilizing instruments in a calibrated OLG macroeconomy against cyclical fluctuations driven by agents' self-fulfilling beliefs. This is a valuable finding not only for its theoretical insight, but also for its broad implications for the design, implementation and evaluation of macroeconomic stabilization policies. Of particular relevance here is whether endogenous labor income taxation, *à la* Schmitt-Grohé and Uribe (1997, section II), raises the magnitude of business cycles or not will depend critically on what is the primitive analytical environment: a one-sector real business cycle model or a two-period overlapping generations model that exhibits saddle-path stability under laissez faire.

The remainder of this paper is organized as follows. Section 2 describes our two-period Cobb-Douglas overlapping generations model and quantitatively explores its local stability properties under progressive labor income taxation. Section 3 examines the same economy's equilibrium dynamics under endogenous labor taxation. Section 4 discusses our no-indeterminacy results *vis-à-vis* previous findings obtained in a one-sector representative-agent macroeconomy from a comparative perspective. Section 5 concludes.

2 The Economy

Our model economy is comprised of heterogeneous households, competitive firms and the government. In particular, we incorporate two analytically-tractable fiscal policy rules into Lloyd-Braga, Nourry and Venditti's (2007, section 6) two-period non-monetary overlapping generations (OLG) model that exhibits Cobb-Douglas preference and technological formulations along with a locally (in)determinate interior steady state under laissez faire. First, this section examines a progressive tax schedule *à la* Guo and Lansing (1998) with monotonically increasing (or procyclical) average and marginal tax rates on young individuals' wage income. In the next section, the government endogenously chooses a countercyclical labor tax rate to finance constant public expenditures on goods and services, as in Schmitt-Grohé and Uribe (1997, section II). Each agent is postulated to supply labor hours as well as accumulating physical capital in youth; and consume in both time periods of her lifetime. The economy's production side consists of a social technology that displays increasing returns-to-scale due to positive productive externalities from aggregate labor inputs. To facilitate comparison with previous work, government spending is posited to be useless in that it does not contribute to utility or production. We assume that there are no fundamental uncertainties present in the macroeconomy.

2.1 Households

As in Lloyd-Braga, Nourry and Venditti (2007, section 6), there is a single agent in each generation who lives for two periods and maximizes the following utility function that is Cobb-Douglas in consumption and separably convex in hours worked:

$$u^t = \left(c_t^t\right)^{\alpha}\left(c_{t+1}^t\right)^{1-\alpha} - A\frac{h_t^{1+\gamma}}{1+\gamma}, \quad A>0, \quad \gamma \geq 0, \quad 0<\alpha<1, \quad t=1,\ 2,\ \cdots, \tag{1}$$

where c_t^t (c_{t+1}^t) represents consumption, superscripts index generation or cohort, and subscripts index calendar time. This generation-t agent supplies h_t units of labor hours when young that contribute to firms' production process, and does not work in her old (retirement) age. In addition, A is a preference parameter and γ denotes the inverse for the wage elasticity of labor supply.

The period budget constraints faced by the cohort-t household are

$$c_t^t + s_t^t = (1-\tau_t)w_t h_t, \quad 0<\tau_t<1, \tag{2}$$

and

$$c_{t+1}^t = r_{t+1}s_t^t, \tag{3}$$

where s_t^t represents the period-t savings of a young agent that are held in the form of additions to next period's capital stock k_{t+1}, w_t is the real wage rate, τ_t is the tax rate on labor income which will be specified below and r_{t+1} is the real gross interest rate. Under the commonly-adopted assumption (in the OLG literature) that the depreciation rate of physical capital is

100% after one period, the associated market clearing condition is $s_t^t = k_{t+1}$. It follows that r_{t+1} can also be interpreted as the rental rate that the old individual of generation-t receives from providing capital services to firms. In the first period of the economy, an existing old agent of cohort-0 is endowed with the exogenously-given capital stock $k_1 > 0$, and a preference formulation given by $u^0 = c_1^0$.

Similar to Guo and Lansing (1998), the labor tax rate τ_t is specified as taking on the functional form that is continuously differentiable in the cohort-t household's wage income $y_{ht} = w_t h_t$:

$$\tau_t = 1 - \eta \left(\frac{\bar{y}_h}{y_{ht}} \right)^{\phi}, \quad 0 < \eta < 1, \quad 0 \leq \phi < 1, \tag{4}$$

where $\bar{y}_h = \bar{w}\bar{h}$ denotes the steady-state level of per capita labor income that is taken as given by each agent; and the parameters η and ϕ govern the level and slope (or elasticity) of the tax scheme, respectively. Using (4), we find that the marginal tax rate τ_t^m, defined as the change in taxes paid by a young generation-t individual divided by the change in her wage earnings, is given by

$$\tau_t^m = \frac{\partial \left(\tau_t y_{ht} \right)}{\partial y_{ht}} = 1 - \eta \left(1 - \phi \right) \left(\frac{\bar{y}_h}{y_{ht}} \right)^{\phi}. \tag{5}$$

In this section, our analyses are restricted to an environment in which households have an incentive to provide labor services and the government cannot confiscate productive resources, thus $0 < \tau_t,\ \tau_t^m < 1$ is imposed. At the model's stationary state ($y_{ht} = \bar{y}_h$), these conditions imply that $\eta \in (0,\ 1)$ and $\phi \in (\frac{\eta-1}{\eta},\ 1)$, where $\frac{\eta-1}{\eta} < 0$. Per the observed U.S. federal individual income tax schedule, we note that the listed statutory marginal tax rate τ_t^m is an increasing function with respect to taxable-income ($w_t h_t$) brackets. This empirical feature thus implies a progressive fiscal policy rule with $\phi > 0$, whereby the marginal tax rate is higher than the corresponding average tax rate given by (4). As a useful baseline formulation, the macroeconomy under flat labor taxation with $\phi = 0$ and a constant $\tau_t = \tau_t^m = 1 - \eta$ is considered. It follows that the elasticity parameter which also governs the degree of tax progressivity is further restricted to the interval $0 \leq \phi < 1$.

As in many previous studies, we postulate that individuals are able to rationally anticipate the way in which the tax schedule affects their net earnings when they decide how much to work, consume and invest over their lifetimes. Consequently, it is the marginal tax rate of labor income τ_t^m that will govern each agent's economic decisions. Hence, the first-order conditions for the cohort-t (≥ 1) household's dynamic optimization problem are

$$\frac{Ah_t^{\gamma}}{\alpha\left(c_t^t\right)^{\alpha-1}\left(c_{t+1}^t\right)^{1-\alpha}} = \left(1-\tau_t^m\right)w_t, \tag{6}$$

$$\frac{c_{t+1}^t}{c_t^t} = \left(\frac{1-\alpha}{\alpha}\right)r_{t+1}, \tag{7}$$

where (6) equates the slope of this individual's indifference curve to the after-tax real wage, and (7) is the standard Euler equation on her intertemporal consumption choices. Since the marginal rate of substitution between current consumption and leisure involves c_{t+1}^t under our postulated non-separable utility function (1), equation (6) that governs the period-t labor supply decision will become dynamic across two consecutive periods (*c.f.* intratemporal under a separable preference formulation).

2.2 Firms

The technological side of this one-sector macroeconomy is comprised of a continuum of identical competitive firms that are indexed by i and distributed uniformly over [0, 1]. The representative firm i produces output y_{it} according to a Cobb-Douglas production function

$$y_{it} = x_t k_{it}^s h_{it}^{1-s}, \;\; 0 < s < 1, \tag{8}$$

where k_{it} and h_{it} are capital and labor inputs, respectively, and x_t represents productive externalities that are taken as given by each individual firm. As in Lloyd-Braga, Nourry and Venditti (2007, section 6), we postulate that externalities take the form

$$x_t = h_t^{\theta}, \;\; \theta \geq 0, \tag{9}$$

where $h_t \equiv \int_{i=0}^{1} h_{it}di$ denotes the economy-wide level of labor services. In a symmetric equilibrium, all firms make the same decisions such that $k_{it} = k_t \left(\equiv \int_{i=0}^{1} k_{it}di\right)$ and $h_{it} = h_t$, for all i and t. As a result, (9) can be substituted into (8) to obtain the following social technology for total output y_t that will display increasing returns to productive inputs:

$$y_t = k_t^s h_t^{1-s+\theta}, \tag{10}$$

where the level of aggregate returns-to-scale in production is equal to $1+\theta$. Under the assumption that factor markets are perfectly competitive, the first-order conditions for the representative firm's profit maximization problem are

$$r_t = s\frac{y_t}{k_t}, \tag{11}$$

$$w_t = (1-s)\frac{y_t}{h_t}, \tag{12}$$

where s and $1-s$ represent the capital and labor share of national income, respectively.

2.3 Government

The government sets the labor tax rate τ_t according to (4), and balances its budget each period. Hence, its period budget constraint is given by

$$g_t = \tau_t w_t h_t, \tag{13}$$

where g_t is public spending on goods and services. With the government, the aggregate resource constraint for our model economy is

$$c_t^t + c_t^{t-1} + k_{t+1} + g_t = y_t, \tag{14}$$

where y_t represents total output or GDP.

2.4 Equilibrium Dynamics

We first substitute the generation-t household's budget constraints (2)-(3), together with (i) the capital market clearing condition $s_t^t = k_{t+1}$, (ii) the marginal tax rate τ_t^m given by (5) and (iii) the factor prices $\{r_{t+1}, w_t\}$ as in conditions (11)-(12), into the dynamic labor-supply equation (6) to obtain

$$\left(sk_{t+1}^{s-1}h_{t+1}^{1-s+\theta}\right)^{\alpha-1} = \frac{\alpha\eta\,(1-\phi)\,\bar{y}_h^{\phi}(1-s)^{1-\phi}}{A}\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} k_t^{s(1-\phi)}h_t^{(\theta-s)(1-\phi)-\phi-\gamma}, \tag{15}$$

where $\bar{y}_h = (1-s)\bar{k}^s\bar{h}^{1-s+\theta}$ denotes the stationary equilibrium level of per capita labor income that is a Cobb-Douglas function of the steady-state capital $\bar{k}$ and labor $\bar{h}$ inputs. We also follow the same derivation procedure, in conjunction with the average tax rate τ_t given by (4), to rewrite the consumption Euler equation (7) as

$$k_{t+1} = \eta\,(1-\alpha)\,\bar{y}_h^{\phi}\left[(1-s)\,k_t^s h_t^{1-s+\theta}\right]^{1-\phi}. \tag{16}$$

As a result, our model's equilibrium conditions can be summarized by the above pair of first-order non-linear difference equations in k_t and h_t.

It is straightforward to show that the simultaneous equations (15)-(16) possess a unique interior steady state given by

$$\bar{k} = \left[\frac{\Pi^{\frac{1-s+\theta}{\Lambda}}}{\eta\left(1-\alpha\right)\left(1-s\right)} \right]^{\frac{1}{\Omega}}, \tag{17}$$

where $\Pi \equiv \frac{\alpha\eta(1-s)(1-\phi)[\frac{s(1-\alpha)}{\alpha}]^{1-\alpha}}{A}$, $\Lambda \equiv (1-\alpha)(1-s+\theta) - \gamma - s + \theta$, and $\Omega = s - 1 - \frac{(1-s+\theta)[s+(1-\alpha)(s-1)]}{\Lambda}$; and

$$\bar{h} = \left[\Pi \bar{k}^{s+(1-\alpha)(s-1)} \right]^{-\frac{1}{\Lambda}}. \tag{18}$$

The remaining endogenous variables at the economy's stationary state can then be derived accordingly. Next, we find that the local (in)stability properties of our two-period OLG model under progressive labor income taxation can be analyzed through the following log-linearized dynamical system:

$$\begin{bmatrix} \tilde{k}_{t+1} \\ \tilde{h}_{t+1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \tilde{k}_t \\ \tilde{h}_t \end{bmatrix}, \;\; \tilde{k}_1 \text{ given}, \tag{19}$$

where tilde variables represent percentage deviations from their respective steady-state values, and $\mathbf{J}$ is the Jacobian matrix of partial derivatives on the equilibrium difference equations (15)-(16) in a neighborhood of the model's unique stationary state. Since the dynamical system (19) possesses one predetermined variable k_t, the macroeconomy exhibits saddle-path stability and equilibrium uniqueness if and only if one eigenvalue of $\mathbf{J}$ lies inside and the other outside the unit circle. When both eigenvalues are inside the unit circle, the steady state becomes an indeterminate sink around which there are a continuum of stationary equilibrium paths that will display cyclical fluctuations driven by agents' animal spirits or sunspots. When both eigenvalues are outside the unit circle, the steady state becomes a totally unstable source that is surrounded by divergent or explosive trajectories.

2.5 Quantitative Results

Since the trace and determinant of the Jacobian matrix $\mathbf{J}$ as in (19) are complicated functions of model parameters, we cannot analytically derive the exact (in)stability conditions that govern the equilibrium evolutions of capital stock and hours worked within our macroeconomy. As a result, numerical experiments are undertaken to quantitatively explore the economy's aggregate (in)determinacy properties under calibrated parameterizations which are consistent with empirically observed features of the post Korean-war U.S. data. As in Lloyd-Braga, Nourry and Venditti (2007, p. 527), the labor share of national income, $1-s$, is chosen to be $\frac{2}{3}$; and the share of first-period consumption over a young agent's (before-tax) wage income,

α, is set to be 0.51. On the other hand, the labor supply elasticity parameter, γ, is equal to 0 (*i.e.* indivisible labor, *à la* Hansen [1985] and Rogerson [1988], that is infinitely elastic); and the preference parameter is normalized to $A = 1$ because it does not affect the model's local dynamics. Given the preceding baseline combination of parameter values, we first find that the no-government version of our model economy possesses an indeterminate steady state when the level of labor externality in production is higher than a critical threshold, denoted as θ^c, of 0.016.

In accordance with Guo and Lansing's (1998) analysis of an indeterminate one-sector real business cycle model under laissez faire, the quantitative simulations below will consider $\theta \geq 0.02 > \theta^c$, together with the above-mentioned benchmark calibration on $\{\alpha,\ s,\ \gamma,\ A\}$, such that multiple equilibria will arise in our OLG macroeconomy without income taxation; and then numerically examine the business-cycle stabilization effects of the Guo-Lansing nonlinear fiscal formulation (4). In particular, we calibrate the tax-level parameter η to be 0.8, which in turn implies that the steady-state government spending to GDP ratio is 0.2. Figure 1 depicts the resulting local stability properties of our model as a function of the size of productive externalities in labor versus the degree of tax progressivity, whereby the $\theta - \phi$ space is divided into regions of "Saddle" and "Sink". Per the parametric restriction $\theta < s$ that yields a standard downward-sloping aggregate labor demand schedule, we also set the upper bound of labor externalities on its horizontal axis to be 0.3. Our first finding is

Proposition 1. For a given level of labor externalities with $\theta \in [0.02, 0.3]$, our two-period overlapping generations model exhibits saddle-path stability and equilibrium uniqueness when the tax progressivity associated with the postulated fiscal policy rule (4) is sufficiently high.

As an example, when the labor externality takes on an empirically realistic value of $\theta = 0.1$ (see Laitner and Stolyarov, 2004), the macroeconomy's unique interior stationary equilibrium turns into a locally isolated saddle point as the degree of tax progressivity is raised to $\phi \geq 0.14$. Intuitively, start the laissez-faire model ($\eta = 1$ and $\phi = 0$) from its steady state, and suppose that the young agent at period t becomes optimistic about the economy's future. Acting upon this change in non-fundamental expectations, the cohort-t individual will consume less and invest more today, thus c_t^t falls while k_{t+1} rises. In addition, a higher k_{t+1} leads to increases in h_{t+1} and w_{t+1} for the generation-$(t + 1)$ household, through firms' labor demand function (12), which in turn raises the aggregate output y_{t+1} as well as the old agent's consumption c_{t+1}^t. If the external effects from aggregate labor are sufficiently strong (specifically $\theta > 0.016$ within our parameterized setting), the rate of return on capital investment r_{t+1} will rise to justify such an alternative dynamic trajectory as a self-fulfilling equilibrium.

In the aforementioned environment, the key to the success of any policy which intends to insulate the macroeconomy from belief-induced cyclical fluctuations is to dampen the mechanism that makes for multiple equilibria. Under Guo and Lansing's (1998) progressive taxation scheme, the government will raise the marginal tax rate τ_{t+1}^{m} on the wage income for the young agent of generation-$(t+1)$, which in turn diminishes her willingness to work harder. Therefore, this procyclical budgetary arrangement can prevent the cohort-t household's optimistic expectations from becoming fulfilled, provided the slope of the fiscal policy rule (which also indexes the degree of progressivity) ϕ is set high enough to suppress the sunspot-driven spurt in h_{t+1} and thus "tax away" the augmented marginal product of capital $\left(= sk_{t+1}^{s-1}h_{t+1}^{1-s+\theta}\right)$ at period $t+1$. It follows that the equality of the relevant consumption Euler equation (7) will no longer hold; thus our postulated progressive tax schedule (4) operates like a classic automatic stabilizer which may eliminate the possibility of endogenous business cycles, and render the equilibrium unique as well as determinate within our model economy. Next, Figure 1 shows that

Proposition 2. Under the Guo-Lansing progressive tax schedule over $0.02 \leq \theta \leq 0.3$, the minimum level for the tax progressivity that leads to saddle-path stability (denoted as $\phi_{\min}$) is monotonically increasing with respect to the degree of labor externalities, *i.e.* $\frac{\partial \phi_{\min}}{\partial \theta} > 0$.

The intuition for this Proposition is straightforward. When the generation-t individual anticipates an increase in the future return of today's investment, she needs incentive to give up current consumption in exchange for more capital accumulation. If the productive externalities from aggregate labor hours become stronger with a higher θ, it will be easier to fulfill her optimistic expectation; hence our model economy is more susceptible to indeterminacy and sunspots. In this case, the degree of tax progressivity required for saddle-path stability will rise such that the increase in the young generation-$(t+1)$ agent's labor supply does not validate the initial optimism on a higher marginal product of capital. It follows that *ceteris paribus* as the size of labor externalities increases, more progressive labor income taxation is called for "leaning against the wind" to stabilize our OLG macroeconomy against business cycle fluctuations driven by animal spirits $\left(\frac{\partial \phi_{\min}}{\partial \theta} > 0\right)$. In addition, our sensitivity analysis finds that

Proposition 3. A lower threshold degree of the Guo-Lansing tax progressivity $\phi_{\min}$ is *ceteris paribus* needed to attain local determinacy when the share of first-period consumption over a young household's wage income α rises or the labor supply elasticity $\frac{1}{\gamma}$ falls.

While keeping the values of other parameters unchanged, it can be shown that an increase in α will raise the average/marginal propensity to consume out of a young individual's disposable wage income. This in turn reduces the amount of additions to the next period's capital stock as well as labor hours. As a result, the generation-t individual's rosy anticipation of a higher rate of return from belief-driven investment expansion is less likely to be fulfilled. Therefore, a lower degree of the tax progressivity is needed for local determinacy and equilibrium uniqueness, *i.e.* $\frac{\partial \phi_{\min}}{\partial \alpha} < 0$.

When the labor supply elasticity becomes smaller (a higher γ), households are less willing to move out of leisure into labor. It follows that an optimistic expectation about the economy's future will yield a lower increment in hours worked at period $t+1$, thus decreasing the likelihood of validating the cohort-t agent's beginning optimism of a higher return on capital. Consequently, as compared to our benchmark parameterization with infinitely elastic labor hours ($\gamma = 0$), a less progressive tax policy is required to suppress sunspot-induced macroeconomic fluctuations, *i.e.* $\frac{\partial \phi_{\min}}{\partial \gamma} < 0$. In sum, this section shows that under the postulated progressive tax schedule with procyclical labor income taxation, Guo and Lansing's (1998) local determinacy and comparative statics results will continue to hold within a two-period Cobb-Douglas overlapping generations model *à la* Lloyd-Braga, Nourry and Venditti (2007, section 6).

3 Endogenous Labor Income Taxation

This section examines the aggregate (in)stability attributes of an identical competitive and non-monetary two-period overlapping generations model, but with a slightly different balanced-budget fiscal policy rule. As in Schmitt-Grohé and Uribe (1997, section II), a pre-set fixed level of government purchases are financed by endogenously-determined income taxation on each young agent's wage earnings. It follows that the government's period budget constraint is changed to

$$g = \mu_t w_t h_t, \tag{20}$$

where $g > 0$ denotes public spending on goods and services and $\mu_t \in (0, 1)$ is the labor tax rate. In this case, the first-order conditions for the generation-t (≥ 1) agent's dynamic optimization problem are

$$\frac{A h_t^{\gamma}}{\alpha \left(c_t^t\right)^{\alpha-1} \left(c_{t+1}^t\right)^{1-\alpha}} = (1 - \mu_t)\, w_t \tag{21}$$

and equation (7) for her labor supply decision and intertemporal consumption choices, respectively.

3.1 Analysis of Dynamics

Under the same Cobb-Douglas preference and technological formulations as those in section 2, together with the countercyclical labor taxation given by (20), it is straightforward to show that the pair of non-linear difference equations in capital and labor which characterize our modified economy's equilibrium dynamics are

$$\left(sk_{t+1}^{s-1}h_{t+1}^{1-s+\theta}\right)^{\alpha-1}=\frac{\alpha}{A}\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}h_t^{-(1+\gamma)}\left[(1-s)\,k_t^s h_t^{1-s+\theta}-g\right],\tag{22}$$

and

$$k_{t+1}=(1-\alpha)\left[(1-s)\,k_t^s h_t^{1-s+\theta}-g\right].\tag{23}$$

Next, we use the long-run versions of equations (10), (12) and (20)-(23) to find that the exogenously-given public spending g and the steady-state tax rate on labor income μ^{ss} exhibit the following Laffer curve-type relationship:

$$g=\left(\frac{\mu^{ss}\Phi}{1-\alpha}\right)(1-\mu^{ss})^{-1+\frac{1+\frac{\Psi}{1-s+\theta}}{\Delta}},\tag{24}$$

where $\Psi\equiv\gamma+s-\theta-(1-\alpha)(1-s+\theta)$, $\Delta\equiv(1-\alpha)(1-s)-s+\frac{\Psi(1-s)}{1-s+\theta}$ and $\Phi\equiv\left\{\frac{\alpha(1-s)}{A}\left[\frac{(1-\alpha)s}{\alpha}\right]^{1-\alpha}[(1-\alpha)(1-s)]^{\frac{\Psi}{1-s+\theta}}\right\}^{\frac{1}{\Delta}}$. It is immediately clear that the government's tax revenue is equal to zero when μ^{ss} takes on the extreme value of 0 or 1. Setting $\frac{\partial g}{\partial\mu^{ss}}=0$, it can be shown that the stationary level of the labor tax rate $\mu^*\in(0,\,1)$ which maximizes the amount of public expenditures (denoted as g^*) is given by

$$\mu^*=\frac{\gamma(1-s)-\theta}{\alpha+\gamma+(1-\alpha)(s-\theta)}.\tag{25}$$

It follows that our model possesses zero (two) interior steady states(s) provided $g>(<)\,g^*$, as shown in Figure 2. As a result, any small deviation from the revenue-maximizing steady state with g^* and μ^* will lead to its disappearance, or the emergence of dual stationary equilibria. This finding implies that the macroeconomy undergoes a saddle-node bifurcation, which may cause the hard loss of equilibrium stability, as the government spending passes through the critical value g^*. Figure 2 also shows that when $g\in(0,\,g^*)$, the economy's stationary states are characterized by μ_L^{ss} and μ_H^{ss}, where $\mu_L^{ss}<\mu^*<\mu_H^{ss}$. We can then derive that the corresponding stationary-state levels of capital stock and labor hours are

$$k_L^{ss}=\frac{(1-\alpha)(1-\mu_H^{ss})g}{\mu_H^{ss}}\quad\text{and}\quad k_H^{ss}=\frac{(1-\alpha)(1-\mu_L^{ss})g}{\mu_L^{ss}};\tag{26}$$

together with

$$h_L^{ss} = \left[\frac{(k_L^{ss})^{1-s}}{(1-\alpha)(1-s)(1-\mu_H^{ss})} \right]^{\frac{1}{1-s+\theta}} \quad \text{and} \quad h_H^{ss} = \left[\frac{(k_H^{ss})^{1-s}}{(1-\alpha)(1-s)(1-\mu_L^{ss})} \right]^{\frac{1}{1-s+\theta}} \tag{27}$$

where $k_L^{ss} < k_H^{ss}$ and $h_L^{ss} < h_H^{ss}$. After taking log-linear approximations to this model's equilibrium conditions (22)-(23) in a neighborhood of each interior steady state, the associated local stability properties will then be determined by the dynamical system (19) as well.

3.2 Calibrated Economies

With endogenous labor income taxation, this subsection quantitatively examines the local dynamics of our two-period overlapping generations model under identical calibrated values of α, s and A as those in section 2.5. Using equation (25), it is straightforward to find that when each young agent's labor supply is infinitely elastic ($\gamma = 0$), the resulting revenue-maximizing steady-state tax rate will not be feasible because of $\mu^* \leq 0$. Therefore, our subsequent numerical experiments will explore parameterized environments that possess two interior stationary equilibria characterized by μ_L^{ss} and μ_H^{ss} under $\gamma = 0.25$ (see King, Plosser and Rebelo, 1988), 0.5 and 1. In addition, the requirement $\mu^* \in (0,\ 1)$ leads to the following inequality constraint on the level of labor externality in production: $\theta < \hat{\theta} \equiv \gamma(1-s)$.

In accordance with Schmitt-Grohé and Uribe's (1997, section II) analysis of a prototypical and determinate one-sector real business cycle model without the government, calibrated OLG settings with $0 \leq \theta < \theta^c$ are analyzed here such that our laissez-faire macroeconomy ($g = 0$) does not exhibit indeterminacy and sunspots. Moreover, the parametric restriction $\theta < s$ which generates a standard negatively-sloped aggregate labor demand schedule needs to be maintained as well. Given all the above considerations, the most-binding upper bound of labor externalities is given by

$$\theta < \theta^{\max} \equiv \min\left(\hat{\theta},\ \theta^c,\ s\right). \tag{28}$$

As a result, we derive that $\theta^{\max} = 0.1667$ when $\gamma = 0.25$; and that $\theta^{\max} = \frac{1}{3}$ when $\gamma = 0.5$ or 1.

Proposition 4. Under all feasible combinations of $\gamma = \{0.25,\ 0.5,\ 1\}$ and $\theta \in [0,\ \theta^{\max})$ discussed above, together with endogenous labor income taxation to finance a pre-set constant level of government spending $g \in (0,\ g^*)$ within our two-period overlapping generations model, the low-tax (high-capital) steady state μ_L^{ss} is a saddle point and the high-tax (low-capital) steady state μ_H^{ss} is a source. Consequently, this modified economy will not display equilibrium indeterminacy and endogenous business cycles driven by agents' animal spirits or sunspots for any initial capital stock $k_1 > 0$.

We first substitute $\alpha = 0.51$, $s = \frac{1}{3}$ and a specific pair of calibrated $\{\gamma, \theta\}$ into equation (25) to find the highest possible steady-state labor tax rate μ^*. Without loss of generality, arbitrarily-selected values of μ_L^{ss} $(< \mu^*)$ are imposed onto the right-hand side of (24), along with the aforementioned parameter combinations, to obtain the resulting amount of tax revenue g. We then numerically solve the corresponding μ_H^{ss} from the downward-sloping portion of the economy's Laffer curve, as well as the stationary equilibrium levels of capital and labor through (26)-(27). It turns out that for all admissible parametric configurations under consideration, our model's low-tax steady state exhibits saddle-path stability and local uniqueness; whereas the high-tax steady state turns out to be a totally unstable source.

To understand the economic intuition behind this no-indeterminacy result, we substitute (10), (12) and (13) into the generation-t household's optimal labor-supply condition (21) to arrive at

$$\left(\frac{c_{t+1}^t}{c_t^t}\right)^{1-\alpha} = \frac{A}{\alpha}\left[\frac{h_t^{1+\gamma}}{(1-s)\,k_t^s h_t^{1-s+\theta} - g}\right], \tag{29}$$

where $\frac{c_{t+1}^t}{c_t^t} = \left(\frac{1-\alpha}{\alpha}\right) r_{t+1}$ from the consumption Euler equation (7).

Start the model with an arbitrary equilibrium trajectory of consumption or investment, and suppose that the young individual at period t anticipates a higher rate of return from her savings. In light of this optimistic belief, the cohort-t household will reduce c_t^t and raise s_t^t $(= k_{t+1})$ today. Moreover, a higher k_{t+1} generates increase in h_{t+1} and w_{t+1} for the generation-$(t+1)$ agent, via the representative firm's labor demand function, which in turn expands the economy's total output y_{t+1} and the old individual's consumption c_{t+1}^t at period $t+1$. It follows that the left-hand side of (29), as well as the marginal product of next period's capital stock r_{t+1}, will become higher.[5] On the other hand, a lower c_t^t decreases the marginal rate of substitution between consumption and hours worked, which will then cause a rightward shift of the labor supply curve yielding a fall in w_t and an increase in h_t. Under the balanced-budget policy rule given by (20), the government's tax revenue remains unchanged $(= g)$ because the labor tax rate μ_t moves in the opposite direction and by the same proportion as the young household's wage income. For a given capital stock k_t together with the calibrated parameters $\{\alpha, \gamma, \theta, s, A, g\}$, we find that the overall impacts of a higher level of the period-t labor hours

[5] Under the SU balanced-budget formulation as in (20), higher h_{t+1} and w_{t+1} will force the government to cut the labor tax rate μ_{t+1} at period $t+1$. This in turn yields further increments in hours worked for the young cohort-$(t+1)$ individual as well as the rate of return to capital investment r_{t+1}. As a result, the relevant Euler equation (7) for intertemporal consumption choices will continue to be satisfied.

on the right-hand side of (29) are theoretically ambiguous; and that they will be determined by the relative strength of how this change in h_t quantitatively affects the sign and magnitude of the numerator versus the denominator inside the square bracket.

For the aforementioned alternative dynamic path to be justified as a self-fulfilling equilibrium, the optimum condition which governs the young agent's labor supply decision must continue to hold in response to her rosy expectations. It turns out that the two offsetting right-hand-side effects, described in the previous paragraph, render the equality of (29) impossible under endogenous labor income taxation. When the model begins at the low-tax steady state with μ_L^{ss} and h_H^{ss}, our numerical experiments show that for each feasible parameterization, the right-hand-side expression as a whole may decrease or does not rise enough to match with the left-hand-side increase in the intertemporal consumption ratio. As a result, the macroeconomy will display saddle-path stability and equilibrium uniqueness since the young generation-t household's initial optimism cannot be fulfilled. We also find that when the model starts at the high-tax stationary equilibrium with μ_H^{ss} and h_L^{ss}, the hike in the right-hand-side of (29) quantitatively dominates that of the corresponding left-hand-side increase. Consequently, this steady state is an unstable source surrounded by divergent or explosive trajectories that will eventually violate the economy's transversality condition. In sum, this section shows that under the postulated balanced-budget rule with countercyclical labor income taxation, Schmitt-Grohé and Uribe's (1997, section II) local indeterminacy result will *not* remain robust within Lloyd-Braga, Nourry and Venditti's (2007, section 6) two-period Cobb-Douglas overlapping generations model.

4 Discussion

Before proceeding to the conclusion, this section provides further insights on our no-indeterminacy result of sections 2.5 and 3.2 from a comparative perspective. In the context of a standard one-sector no-government real business cycle model characterized by perfect competition and constant returns-to-scale in production, Schmitt-Grohé and Uribe (1997, section II) show that indeterminacy and sunspots may arise under their balanced-budget policy rule with endogenous labor income taxation. When the infinitely-lived representative household's optimistic expectations lead to more capital accumulation and higher hours worked, the government is forced to decrease the labor tax rate thereby helping raise the rate of return to investment. Consequently, such a budgetary arrangement is able to validate agents' initial optimism, which will then yield multiple equilibria and belief-driven cyclical fluctuations. This type of countercyclical fiscal formulation is qualitatively equivalent to regressive income taxation. On the other hand, Guo and Lansing (1998) incorporate an income tax schedule, whereby the household's marginal tax rate is monotonically increasing in its total factor income, into an indeterminate one-sector RBC model under laissez faire and aggregate increasing returns in productive inputs. Upon the sunspot-induced labor and investment spurts in this environment, the representative agent will face a higher tax rate and thus the likelihood of a lower net marginal product of future capital stock. Therefore, such a budgetary provision can prevent the beginning non-fundamental expectations from becoming self-fulfilling, which in turn renders saddle-path stability and equilibrium uniqueness. This form of procyclical fiscal specification is qualitatively equivalent to progressive income taxation. In sum, the GL progressive tax scheme operates like a traditional Keynesian-type automatic stabilizer within a one-sector representative-agent setting; whereas the SU balanced-budget rule will destabilize a one-sector RBC macroeconomy by generating endogenous business cycles caused by shocks to agents' animal spirits.

Quite interestingly, this paper illustrates that these RBC-based opposite business-cycle stabilization effects of the GL versus the SU policy rules do not continue to prevail in a parsimonious two-period Cobb-Douglas overlapping generations model *à la* Lloyd-Braga, Nourry and Venditti (2007, section 6). Acting upon an expected expansion in future economic activity, the young generation-t individual will consume less and invest more today, which in turn raise the next period's capital stock, labor hours, real wage rate, total output and consumption (for her old age). As in the corresponding representative-agent counterpart, our OLG macroeconomy possesses a single positive steady state under the GL progressive tax schedule, whereas two interior stationary equilibria may occur under the SU balanced-budget specification. For all feasible combinations of calibrated parameters, we find that every interior steady state is associated with local determinacy and equilibrium uniqueness; hence both fiscal formulations are stabilizing instruments against the business cycle driven by agents' self-fulfilling beliefs. It follows that whether endogenous labor taxation, as in Schmitt-Grohé and Uribe (1997, section II), amplifies the magnitude of aggregate fluctuations or not depends crucially on what is the underlying analytical framework: a one-sector real business cycle model or a two-period overlapping generations model which exhibits saddle-path stability under laissez faire.

5 Conclusion

In the context of Lloyd-Braga, Nourry and Venditti's (2007, section 6) two-period non-monetary overlapping generations model with Cobb-Douglas preference and technological specifications, this paper quantitatively examines the interrelations between aggregate (in)stability versus two tractable fiscal policy rules that have been previously studied in the real business cycle literature: (i) a progressive tax schedule on wage income and (ii) a balanced-budget scheme with countercyclically endogenous labor taxation. We find that the resulting macroeconomic (de)stabilization effects are quite different from those of Schmitt-Grohé and Uribe (1997, section II) and Guo and Lansing (1998) in a one-sector representative-agent framework. In particular, both fiscal formulations turn out to be stabilizing instruments that will suppress sunspot-driven cyclical fluctuations for all admissible parametric configurations under consideration, hence our calibrated OLG economy always exhibits local determinacy and equilibrium uniqueness. From a policy perspective, this paper's saddle-stability finding *vis-à-vis* Schmitt-Grohé and Uribe's (1997, section II) RBC-based indeterminacy result implies that depending on what is the primitive analytical environment, countercyclical income taxation may operate like an automatic stabilizer or destabilizer within a macroeconomy.

This paper can be extended in several directions. For example, it would be worthwhile to incorporate additional features which have been shown to be important in generating multiple equilibria in dynamic general equilibrium macroeconomic models, such as an overlapping generations economy with two production sectors *à la* Galor (1992); endogenous capital income taxation and/or the presence of public debt *à la* Schmitt-Grohé and Uribe (1997, section III); a general constant-elasticity-of-substitution production technology *à la* Guo and Lansing (2009); and a balanced-budget rule with countercyclical consumption taxation *à la* Nourry, Seegmuller and Venditti (2013), among others. These possible extensions will further enhance our understanding of how various modelling setups and/or different fiscal policy rules affect the likelihood of equilibrium indeterminacy within a two-period overlapping generations macroeconomy. We plan to pursue these research projects in the near future.

References

[1] Diamond, P.A. (1965), "National Debt in a Neoclassical Growth Model," *American Economic Review* 55, 1126-1150.

[2] Galor, O. (1992), "A Two-Sector Overlapping-Generations Model: A Global Characterization of the Dynamical System," *Econometrica* 60, 1351-1386.

[3] Guo, J.-T. and K.J. Lansing (1998), "Indeterminacy and Stabilization Policy," *Journal of Economic Theory* 82, 481-490.

[4] Guo, J.-T. and S.G. Harrison (2001), "Tax Policy and Stability in a Model with Sector-Specific Externalities," *Review of Economic Dynamics* 4, 75-89.

[5] Guo, J.-T. and K.J. Lansing (2009), "Capital-Labor Substitution and Equilibrium Indeterminacy," *Journal of Economic Dynamics & Control* 33, 1991-2000.

[6] Guo, J.-T. and S.G. Harrison (2011), "Corrigendum: Tax Policy and Stability in a Model with Sector-Specific Externalities," http://faculty.ucr.edu/~guojt/Corrigendum.pdf.

[7] Hansen, G.D. (1985), "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics* 16, 309-327.

[8] King, R.G., C.I. Plosser and S.T. Rebelo (1988), "Growth and Business Cycles I. The Basic Neoclassical Model," *Journal of Monetary Economics* 21, 195-232.

[9] Laitner, J. and D. Stolyarov (2004), "Aggregate Returns to Scale and Embodied Technical Change: Theory and Measurement using Stock Market Data," *Journal of Monetary Economics* 51, 191-233.

[10] Lloyd-Braga, T., C. Nourry and A. Venditti (2007), "Indeterminacy in Dynamic Models: When Diamond meets Ramsey," *Journal of Economic Theory* 134, 513-536.

[11] Nourry, Carine, T. Seegmuller and A. Venditti (2013), "Aggregate Instability under Balanced-Budget Consumption Taxes: A Re-examination," *Journal of Economic Theory* 148, 1977-2006.

[12] Rogerson, R. (1988), "Indivisible Labor, Lotteries, and Equilibrium," *Journal of Monetary Economics* 21, 3-16.

[13] Schmitt-Grohé, S. and M. Uribe (1997), "Balanced-Budget Rules, Distortionary Taxes and Aggregate Instability," *Journal of Political Economy* 105, 976-1000.

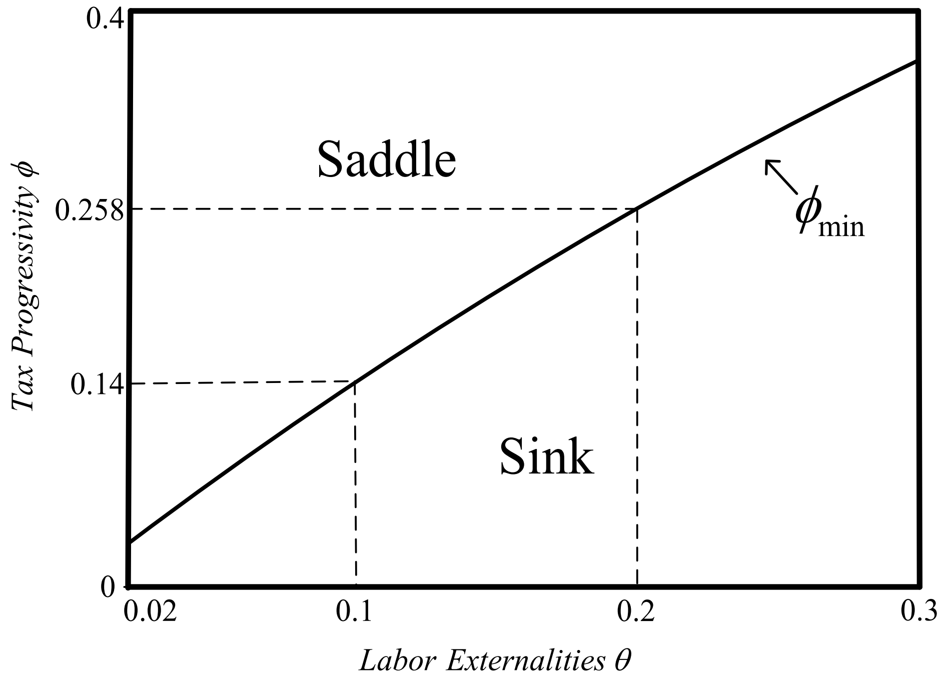

Figure 1. Local Stabilities under Progressive Labor Income Taxation

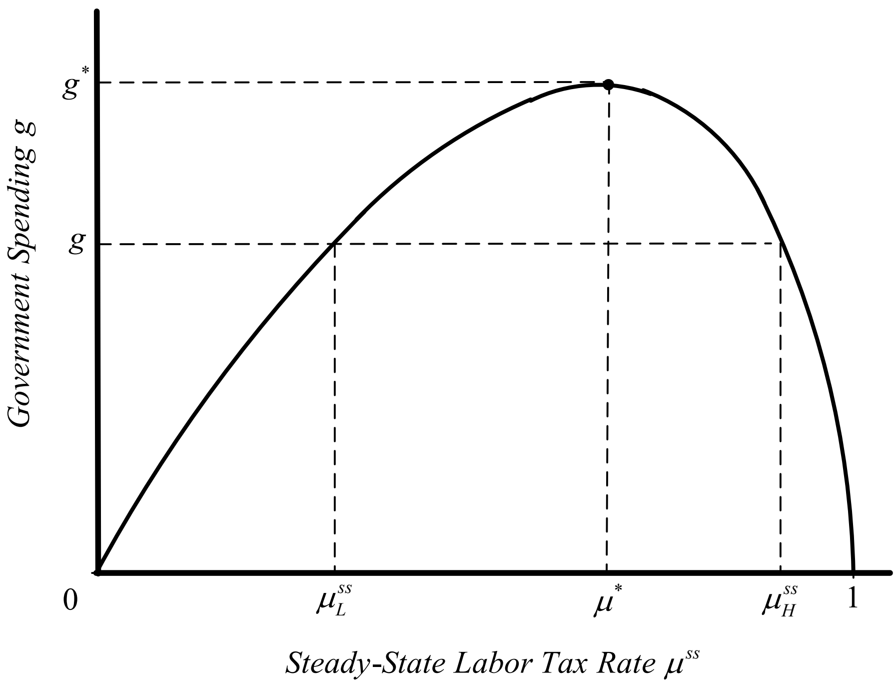

Figure 2. Steady-State Laffer Curve under Endogenous Labor Taxation