Balanced-Budget Rules and Macroeconomic Stability with Overlapping Generations*

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Abstract

In the context of a stylized two-period overlapping generations model, this paper examines the macroeconomic (in)stability effects of a balanced-budget rule whereby constant government spending is financed by endogenous taxation on agents' labor income or consumption expenditures. In sharp contrast to previous studies for a representative-agent framework, our baseline economy always exhibits local determinacy and equilibrium uniqueness, hence both labor and consumption taxes are stabilizing instruments against cyclical fluctuations driven by animal spirits. Moreover, this no-indeterminacy result and associated dynamic equivalence between endogenous labor versus consumption taxation remain qualitatively robust to various modifications in the household-preference or firm-production formulations.

Keywords*:* Balanced-Budget Rules; Indeterminacy; Overlapping Generations Model.

JEL Classification*:* E32; E62.

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1 Introduction

Understanding the business-cycle (de)stabilization effects of a balanced-budget rule has attracted much attention in the macroeconomics literature. This is an important research topic not only for its theoretical insights, but also for its broad implications for the design, implementation and evaluation of stabilization fiscal policies. In the context a standard one-sector real business cycle (RBC) model characterized by perfect competition and constant returns-to-scale in production, Schmitt-Grohé and Uribe (1997, section II) analytically show that when the balanced-budget policy rule is postulated to consist of constant government spending and proportional taxation on the household's labor income, a Laffer curve-type relationship between the labor tax rate and the resulting tax revenue will emerge, which in turn may lead to the existence of two interior stationary equilibria. These authors then derive the necessary and sufficient condition under which their model's low-tax steady state is an indeterminate sink that can be exploited to generate endogenous cyclical fluctuations caused by animal spirits.[1] Subsequently, Giannitsarou (2007, section 1) explores the equilibrium dynamics of an identical RBC macroeconomy, but with a slightly different tax scheme: a pre-set fixed level of public expenditures are financed by endogenous taxation on the household's consumption spending. In this environment, the economy is found to possess a unique interior steady state that is always a locally determinate saddle point; hence indeterminacy and sunspots will be completely ruled out. From a policy perspective, these earlier results altogether suggest that if the government intends to stabilize the macroeconomy against business cycles driven by agents' self-fulfilling expectations, a balanced-budget rule with endogenous consumption taxation (instead of labor income taxation) is called for.

Parallel to the above-referenced work and many previous pieces of related research on a representative-agent macroeconomy, we will examine the aggregate (in)stability effects of the same type of balanced-budget formulation within another "workhorse" analytical framework in modern macroeconomics – a stylized two-period overlapping generations (OLG) model *à la* Diamond (1965) – whereby constant government purchases are financed by endogenously determined labor or consumption taxes. Each agent is postulated to supply labor hours as well as accumulating physical capital when young; and consume in both time periods of her lifetime. As in the baseline setting of Schmitt-Grohé and Uribe (1997) and Giannitsarou (2007), our analysis begins with (i) an additively separable utility function that is logarithmic in consumption and convex in hours worked, and (ii) a Cobb-Douglas production technology

[1] See Benhabib and Farmer (1999) for other mechanisms that may yield equilibrium indeterminacy and belief-driven aggregate fluctuations within various real business cycle models.

that exhibits constant returns-to-scale in capital and labor inputs. These preference and technological specifications will facilitate the comparison of this paper's findings versus those from the existing RBC-based studies in a direct and transparent manner.

Under endogenous labor income taxation, we find that due to the opposite directions and identical strength of the income versus substitution effects, each young individual's optimal labor supply is a time-invariant constant that is independent of the labor tax rate. In addition, the model's equilibrium dynamics is governed by a scalar difference equation in capital that turns out to exhibit a negative vertical intercept, followed by an increasing concave curve which will intercept the 45-degree line twice provided the pre-specified level of public spending is lower than the revenue-maximizing counterpart. It is then straightforward to analytically show that the low-capital (high-tax) steady state is asymptotically unstable, whereas the high-capital (low-tax) steady state is asymptotically stable. When households become optimistic about the economy's future, they will choose to consume less and invest more today. While the resulting increase in the future capital stock raises next period's output and consumption (but no impact on hours worked), it also leads to a decrease in the return of today's investment spurt because of diminishing marginal product of capital.[2] It follows that agents' initial optimism cannot validate this alternative dynamic trajectory as a self-fulfilling equilibrium, and that both interior stationary equilibria will be locally isolated.[3] In sharp contrast to Schmitt-Grohé and Uribe (1997, section II), these results illustrate that macroeconomic instability caused by endogenous belief-driven cyclical fluctuations do not arise in the benchmark version of our two-period overlapping generations model with labor taxes.

When government purchases are financed by endogenous consumption taxation over both periods of an individual's lifetime, we derive that her optimal labor supply remains at the same constant level as that under labor income taxation, indicating the exact cancellation of intratemporal income and substitution effects as well.[4] In this case, the scalar difference equation in capital that characterizes this setting's equilibrium dynamics is analytically shown to exhibit a unique and asymptotically stable positive steady state. Upon the anticipation of a

[2] By contrast, the infinitely-lived representative household's labor supply is a monotonically decreasing function of the labor tax rate within Schmitt-Grohé and Uribe's (1997) one-sector RBC model. When optimistic individuals decide to work harder and invest more, the government is forced to lower the tax rate as total output rises. This countercyclical/regressive fiscal policy rule will help fulfill agents' initial rosy expectations, thus leading to indeterminacy of equilibria and sunspot-driven endogenous business cycles.

[3] As in Schmitt-Grohé and Uribe (1997) and Giannitsarou (2007), this paper's analysis is focused on the local (in)determinacy of equilibrium path(s) near each steady state. The global behavior of our model economy is a worthwhile topic for future research.

[4] Giannitsarou (2007, section 1) obtains the qualitatively identical result – a fixed labor supply in equilibrium that is independent of the consumption tax rate – within a prototypical one-sector representative-agent macroeconomy. As a result, the model's saddle-path stability and equilibrium uniqueness will remain unaffected by the postulated balanced-budget rule with endogenous consumption taxation.

decrease in the future consumption tax rate, the young agent chooses to consume less (thus save more) today and raise her old-age consumption due to the stronger intertemporal substitution effect. The associated increase in the next period's capital stock will reduce the corresponding real interest rate because the firms' production technology is strictly concave. We find that the overall effect turns out to be a lower rate of return on capital investment adjusted for consumption taxes, which in turn invalidates households' expectation of a subsequent tax cut. It follows that the economy's unique interior stationary state is locally isolated around which animal spirits cannot be a driving force of business cycle fluctuations, and that Giannitsarou's (2007, section 1) determinacy result will continue to hold within a standard two-period OLG macroeconomy.

From a comparative perspective, the proceeding analysis illustrates that under the same baseline preference and technological formulations, the aggregate (in)stability effects of an endogenous-tax policy rule within Diamond's (1965) two-period overlapping generations model are quite different from those of Schmitt-Grohé and Uribe (1997, section II) and Giannitsarou (2007, section 1) for a one-sector RBC macroeconomy. In particular, our OLG framework always exhibits local determinacy and equilibrium uniqueness, hence both labor and consumption taxation are operating as automatic stabilizers that will insulate the economy from cyclical fluctuations driven by agents' changing non-fundamental expectations. This finding in turn provides an interesting extension of Atkinson and Stiglitz's (1980, pp. 69-72) dynamic equivalence between proportional constant labor versus consumption tax rates to a macroeconomic context with time varying and endogenously determined taxation systems.

In terms of sensitivity analyses, we find that our no-indeterminacy result discussed above will continue to prevail with the following three separate variations to the benchmark utility or production setup: (i) when the social technology displays increasing returns-to-scale due to positive productive externalities from aggregate capital and labor inputs; (ii) when the household's separable preference formulation possesses a constant degree of relative risk aversion (*i.e.* the inverse for the intertemporal elasticity of substitution in consumption) that is not equal to one; and (iii) when each young agent's labor supply decision exhibits no income effect with respect to a change in the labor-income/consumption tax rate. Under endogenous labor income taxation, the economy's low-tax steady state is asymptotically stable in cases (i) and (iii), or a locally determinate saddle point in case (ii); whereas the high-tax stationary equilibrium is asymptotically unstable in cases (i) and (iii), or a totally unstable source in case (ii). Under endogenous consumption taxation, the economy's unique interior steady state is asymptotically stable in cases (i) and (iii), or a locally isolated saddle point in case (ii). For each modified setting, we intuitively show that the equality of the relevant consumption Euler equation will not hold upon an optimistic belief about the macroeconomy's future. It follows that our extended two-period overlapping generations model always exhibits local determinacy and equilibrium uniqueness, regardless of whether a constant level of government spending is financed by endogenous labor or consumption taxes.

The remainder of this paper is organized as follows. Section 2 describes our two-period overlapping generations model and analytically examines its equilibrium dynamics under endogenous labor income taxation. Section 3 studies the same baseline framework's local (in)stability properties under endogenous consumption taxation, and then discusses the linkage of our no-indeterminacy result with the Atkinson-Stiglitz dynamic equivalence. Section 4 considers three distinct extensions to re-examine the economy's equilibrium (in)determinacy attributes. Section 5 concludes.

2 The Benchmark Economy

Our analysis begins with incorporating Schmitt-Grohé and Uribe's (1997, section II) balanced-budget fiscal policy rule into a competitive and non-monetary stylized two-period overlapping generations (OLG) model *à la* Diamond (1965), whereby constant government purchases are financed by endogenous labor income taxation. Each agent supplies labor hours as well as undertaking capital accumulation when young; and consumes in both time periods of her lifetime. The economy's output is produced by an aggregate technology that exhibits constant returns-to-scale in capital and labor inputs. For the sake of directly comparing our findings versus those from previous RBC-based studies, we will follow Schmitt-Grohé and Uribe (1997) and consider the following baseline specifications: (i) an additively separable preference formulation that is logarithmic in consumption and convex in hours worked, together with (ii) a Cobb-Douglas production technology and (iii) useless public expenditures that do not affect the households' consumption/savings or the firms' factors demand decisions.[5]

2.1 Households

There is a single agent in each generation who lives for two periods and maximizes the following additively separable utility function:

$$u^t = \log c_t^t - A\frac{h_t^{1+\gamma}}{1+\gamma} + \beta \log c_{t+1}^t, \quad A > 0, \quad \gamma \geq 0, \quad 0 < \beta < 1, \quad t = 1,\ 2,\ \cdots, \tag{1}$$

where c_t^t (c_{t+1}^t) represents consumption, superscripts index generation or cohort, and subscripts index calendar time. The generation-t household supplies h_t units of labor hours in

[5] It is straightforward to show that all the results reported in this paper will be qualitatively robust to allowing for useful public spending that contributes to the firms' productivity or the households' utility.

youth that contributes to firms' production process, and does not work in its old (retirement) age. In addition, γ denotes the inverse for the wage elasticity of labor supply, and β is the subjective discount factor. We assume that there are no fundamental uncertainties present in the macroeconomy.

The period budget constraints faced by the cohort-t agent are

$$c_t^t + s_t^t = (1 - \tau_t) w_t h_t, \quad 0 < \tau_t < 1, \tag{2}$$

and

$$c_{t+1}^t = r_{t+1} s_t^t, \tag{3}$$

where s_t^t denotes the period-t savings of a young individual that are held in the form of additions to next period's capital stock k_{t+1}, w_t is the real wage rate, τ_t is the tax rate on labor income and r_{t+1} is the real gross interest rate. Under the commonly-adopted assumption (in the OLG literature) that physical capital fully depreciates after one period, it is immediately obvious that the associated market clearing condition is $s_t^t = k_{t+1}$, thus r_{t+1} can also be interpreted as the capital rental rate. In the first period of the economy, there exists an initial old agent (generation-0) with preferences given by $u^0 = c_1^0$, who is endowed with the exogenously-given capital stock $k_1 > 0$.

The first-order conditions for the cohort-t (≥ 1) household's dynamic optimization problem are

$$A c_t^t h_t^\gamma = (1 - \tau_t)\, w_t, \tag{4}$$

$$\frac{1}{c_t^t} = \beta \frac{r_{t+1}}{c_{t+1}^t}, \tag{5}$$

where (4) equates the slope of this individual's indifference curve to the after-tax real wage, and (5) is the standard Euler equation on her intertemporal consumption choices.

2.2 Firms

The production side of this one-sector macroeconomy is comprised of a unit measure of identical competitive firms. The representative firm produces output y_t with a constant returns-to-scale Cobb-Douglas production function

$$y_t = B k_t^\alpha\, h_t^{1-\alpha}, \;\; B > 0, \quad 0 < \alpha < 1, \tag{6}$$

where k_t is capital service provided by the generation-$(t-1)$ agent in her old age, and h_t is labor input supplied by the young individual of cohort-t. Under the assumption that factor markets are perfectly competitive, the firm's profit maximization conditions are given by

$$r_t = \alpha \frac{y_t}{k_t}, \tag{7}$$

$$w_t = (1-\alpha) \frac{y_t}{h_t}. \tag{8}$$

2.3 Government

As in Schmitt-Grohé and Uribe (1997, section II), the government endogenously sets the labor tax rate $\tau_t \in (0, 1)$ on young agents' wage income to finance a pre-specified constant amount of public expenditures, and balances its budget at each time period. Hence, the government's period budget constraint is

$$g = \tau_t w_t h_t, \tag{9}$$

where $g > 0$ denotes government spending on goods and services. Finally, it is straightforward to derive that the aggregate resource constraint for the economy, which turns out to be qualitatively identical to that under laissez faire, is given by

$$c_t^t + c_t^{t-1} + k_{t+1} + g = y_t, \tag{10}$$

where y_t represents total output or GDP. Equation (10) thus implies that the presence of endogenous labor taxation does not alter the economy-wide production possibility set because of the constancy of government purchases.

2.4 Macroeconomic Stability

This subsection analytically examines the number of interior stationary state(s) as well as their local stability properties within our two-period overlapping generations model under endogenous labor income taxation. We first substitute the generation-t household's budget constraints (2)-(3), together with the capital market clearing condition $s_t^t = k_{t+1}$, into equation (5) to obtain

$$k_{t+1} = \frac{\beta}{1+\beta} (1-\tau_t) w_t h_t. \tag{11}$$

Combining (2), (4) and (11) then yields that each young individual's labor supply in equilibrium is a fixed constant given by

$$h_t = \left(\frac{1+\beta}{A}\right)^{\frac{1}{1+\gamma}}, \quad \text{for all } t, \tag{12}$$

which is not influenced by the labor tax rate. Intuitively, we consider an increase in τ_t that will generate two intratemporal effects as follows. On the one hand, the resulting lower disposable income induces a young agent to reduce c_t^t and raise h_t – the income effect. On the other hand, the decrease in the after-tax real wage rate, while keeping consumption unchanged, leads to a fall in h_t – the substitution effect. Given our postulated utility function (1) which is logarithmic in consumption and additively separable from hours worked, the above-mentioned income versus substitution effects will be completely cancelled out each other. As a result, the equilibrium labor supply (12) is a time-invariant constant that is independent of τ_t. Next, after plugging (6), (8), (9) and (12) into (11), we find that the scalar difference equation in capital that characterizes the benchmark economy's dynamic equilibrium trajectories under perfect foresight is

$$k_{t+1} = D(k_t) \equiv \frac{\beta}{1+\beta}\left[B(1-\alpha)\left(\frac{1+\beta}{A}\right)^{\frac{1-\alpha}{1+\gamma}} k_t^{\alpha} - g\right], \quad k_1 > 0 \text{ given.} \tag{13}$$

As in Schmitt-Grohé and Uribe's (1997) representative-agent macroeconomy, it is straightforward to show that the number positive steady state(s) within our OLG model may be zero, one or two. Specifically, the government's tax revenue ($= g$) is equal to zero when the steady-state labor tax rate $\tau^{ss} = 0$ or 1; and it can be shown that the Laffer curve-type relationship between $g > 0$ and $\tau^{ss} \in (0, 1)$ is given by

$$g = (1-\alpha)^{\frac{1}{1-\alpha}}\left(\frac{\beta B}{1+\beta}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{1+\beta}{A}\right)^{\frac{1}{1+\gamma}} \tau^{ss}(1-\tau^{ss})^{\frac{\alpha}{1-\alpha}}. \tag{14}$$

Setting $\frac{\partial g}{\partial \tau^{ss}} = 0$ yields a unique steady-state income tax rate $\tau^* = 1-\alpha$ that maximizes the magnitude of public expenditures denoted as g^*. It follows that our baseline model possesses zero (two) interior stationary states(s) provided $g > (<)\ g^*$, as shown in Figure 1. Therefore, any small deviation from the revenue-maximizing steady state with τ^* and g^* will lead to its disappearance, or the emergence of dual stationary equilibria. This result implies that the economy undergoes a saddle-node bifurcation, which may cause the hard loss of equilibrium stability, as the government spending passes through the critical threshold g^*. Figure 1 also shows that when $g \in (0, g^*)$, the resulting steady states in the benchmark macroeconomy are characterized by τ_L^{ss} and τ_H^{ss}, where $\tau_L^{ss} < \tau^* < \tau_H^{ss}$. We can then derive that the corresponding stationary-state levels of capital are

$$k_L^{ss} = \Lambda \left(1 - \tau_H^{ss}\right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad k_H^{ss} = \Lambda \left(1 - \tau_L^{ss}\right)^{\frac{1}{1-\alpha}}, \tag{15}$$

where $\Lambda \equiv \left[\frac{\beta B(1-\alpha)}{1+\beta}\right]^{\frac{1}{1-\alpha}} \left(\frac{1+\beta}{A}\right)^{\frac{1}{1+\gamma}} > 0$ and $k_L^{ss} < k_H^{ss}$.

Using equations (13)-(15), the equilibrium dynamics for our baseline model with a positive level of $g < g^*$ can be illustrated by Figure 2 which presents the phase diagram that traces out how the capital stock evolves over time. It shows that the non-linear function $D(k_t)$ in (13) exhibits a negative intercept $(= -\frac{\beta g}{1+\beta})$ with the vertical axis, followed by an increasing concave curve that crosses the horizontal axis at

$$\hat{k} = \left[\frac{g\left(\frac{1+\beta}{A}\right)^{\frac{\alpha-1}{1+\gamma}}}{B(1-\alpha)}\right]^{\frac{1}{\alpha}} > 0. \tag{16}$$

Moreover, $D(k_t)$ will intersect the 45-degree line twice resulting in two positive stationary states $\{k_L^{ss}, k_H^{ss}\}$; and their associated local stability properties are discussed below.

Proposition 1. Under endogenous labor income taxation to finance a given level of public spending $g \in (0, g^*)$ within our two-period overlapping generations model, the low-capital (high-tax) steady state k_L^{ss} is asymptotically unstable, whereas the high-capital (low-tax) steady state k_H^{ss} is asymptotically stable. Therefore, the benchmark economy will not display equilibrium indeterminacy and endogenous business cycles driven by agents' animal spirits or sunspots for all feasible initial capital stock $k_1 > \hat{k}$.

Figure 2 shows that if the macroeconomy starts from either stationary equilibrium at $t = 1$, *i.e.* $k_1 = k_L^{ss}$ or $k_1 = k_H^{ss}$, it will stay there unperturbed afterwards. This figure further reveals that $0 < D'(k_H^{ss}) < 1 < D'(k_L^{ss})$, where $D'(\cdot)$ denotes the slope of the upward-sloping phaseline evaluated at a particular steady state. It follows that when the beginning capital stock does not coincide with k_L^{ss} or k_H^{ss}, the resulting nonstationary equilibria may take on five possible scenarios:

(i) $k_1 \in (0, \hat{k})$: as time progresses, the capital stock will continue to fall without bound yielding a negative value of GDP.

(ii) $k_1 = \hat{k}$: in this case, the economy will become stagnant with $k_2 = 0$ and thus zero output from $t = 2$ onwards.

(iii) $k_1 \in (\hat{k}, k_L^{ss})$: per the arrow of motion shown on the horizontal axis of Figure 2, the equilibrium sequence of capital stock will be decreasing and farther away from the lower steady state k_L^{ss} over time.

(iv) $k_1 \in (k_L^{ss}, k_H^{ss})$: in this region, the economy undertakes an expansion in capital accumulation along the transition path that will monotonically converge toward the high-capital steady state k_H^{ss}.

(v) $k_1 \in (k_H^{ss}, \infty)$: in this case, the capital-stock sequence will be monotonically declining on the convergent equilibrium trajectory toward the upper steady state k_H^{ss}.

Based on the preceding discussions, our analyses here will be restricted to cases (iii)-(v) as they are economically meaningful. We also note that the low-capital steady state is said to be locally unstable since the macroeconomy's equilibrium path will diverge away from it under all feasible initial conditions of capital stock, except for the special case with $k_1 = k_L^{ss}$. On the other hand, the high-capital steady state k_H^{ss} is said to be locally stable as the sequence of equilibrium capital stock will converge toward it for any beginning value of $k_1 > k_L^{ss}$. It follows that starting from the exogenously given $k_1 > \hat{k}$, the benchmark economy always exhibits local determinacy and equilibrium uniqueness; hence the neighborhood of either interior stationary state does not contain another dynamic trajectory of capital that may constitute a solution to the requisite difference equation (13). This finding implies that in sharp contrast to Schmitt-Grohé and Uribe (1997, section II), aggregate instability caused by endogenous belief-driven cyclical fluctuations will not occur within the baseline specification of our two-period overlapping generations model.

To understand the intuition behind the aforementioned no-indeterminacy result, we substitute (6), (7) and (12) into the generation-t household's intertemporal consumption Euler equation (5) to obtain

$$\frac{c_{t+1}^t}{c_t^t} = \beta \underbrace{\left[\alpha B \left(\frac{1+\beta}{A} \right)^{\frac{1-\alpha}{1+\gamma}} k_{t+1}^{\alpha-1} \right]}_{= r_{t+1}}. \tag{17}$$

Start the model with an arbitrary equilibrium path of consumption or investment, and suppose that the young agent at period t becomes optimistic about the economy's future.[6] Acting upon this change in non-fundamental expectations, the cohort-t household will consume less and save/invest more today, thus c_t^t falls while k_{t+1} rises. Due to the opposite directions and identical strength of the income versus substitution effects, the amount of labor hours provided by the young individual of generation-$(t+1)$, denoted as h_{t+1}, remains unchanged at the constant level given by (12). In addition, a higher k_{t+1} will exert two counteracting

[6] Schmitt-Grohé and Uribe's (1997, pp. 983-984) posits that the infinitely-lived representative household expects an increase in the future labor tax rate τ_{t+1} as the starting point to provide an intuitive explanation for their indeterminacy result. However, such a mechanism is not applicable to our two-period OLG model because the generation-t agent does not work in her old (retirement) age at period $t+1$.

effects at time $t+1$. First, it increases the aggregate output y_{t+1} as well as the old agent's consumption c_{t+1}^t, which (combined with a lower c_t^t) in turn raises the left-hand side of (17). Second, it leads to a decrease in r_{t+1} because of diminishing marginal product of capital, thus the right-hand side of (17) will fall. It follows that agents' initial optimism cannot justify this alternative dynamic trajectory as a self-fulfilling equilibrium, and that both interior steady states of our benchmark macroeconomy $\{k_L^{ss},\ k_H^{ss}\}$ are locally determinate or isolated. In sum, our analysis shows that under the same baseline utility and technological formulations, together with the postulated balanced-budget rule governed by countercyclical labor income taxation, Schmitt-Grohé and Uribe's (1997, section II) indeterminacy result is overturned within a two-period overlapping generations model *à la* Diamond (1965).

3 The Alternative Economy

This section examines the local stability properties of an identical competitive and non-monetary two-period overlapping generations model, but with a slightly different balanced-budget fiscal policy rule. As in Giannitsarou (2007), a pre-set fixed level of government purchases are financed by endogenous proportional taxation on agents' consumption spending. For the sake of analytical generality, we will begin with examining a framework in which an individual's consumption expenditures over both periods of her lifetime are subject to distortionary taxes.

3.1 Endogenous Consumption Taxation

In this environment, the budget constraints faced by the generation-t household are given by

$$(1+\tau_{ct})\,c_t^t + s_t^t = w_t h_t, \quad \tau_{ct} > 0, \tag{18}$$

and

$$(1+\tau_{ct+1})\,c_{t+1}^t = r_{t+1} s_t^t, \quad \tau_{ct+1} > 0, \tag{19}$$

where τ_{ct} (τ_{ct+1}) denotes the consumption tax rate levied on all agents who are alive at period t ($t+1$). The first-order conditions for this individual's dynamic optimization problem are then changed to

$$Ac_t^t h_t^{\gamma} = \frac{w_t}{1+\tau_{ct}}, \tag{20}$$

$$\frac{1}{\left(1+\tau_{ct}\right)c_t^t} = \beta\frac{r_{t+1}}{\left(1+\tau_{ct+1}\right)c_{t+1}^t}, \tag{21}$$

where (20) governs the labor supply decision and (21) is the consumption-tax adjusted intertemporal Euler equation. In addition, the government's period balanced-budget constraint now becomes

$$g = \tau_{ct}\left(c_t^t + c_t^{t-1}\right), \tag{22}$$

where c_t^{t-1} is consumption of the old agent from cohort-$(t-1)$.

Next, we follow the same solution procedure as in section 2 to find that (i) the relationship between $k_{t+1}\left(=s_t^t\right)$ and the period-t labor income $w_t h_t$ is modified to

$$k_{t+1} = \frac{\beta}{1+\beta}w_t h_t; \tag{23}$$

(ii) the constant level of each young individual's labor hours in equilibrium h_t, given by (12), remains unaffected – this indicates the exact cancellation of income and substitution effects induced by a change in the consumption tax rate τ_{ct}; and (iii) the scalar difference equation in capital that characterizes our alternative economy's equilibrium dynamics is

$$k_{t+1} = M\left(k_t\right) \equiv \left[\frac{\beta B\left(1-\alpha\right)}{1+\beta}\right]\left(\frac{1+\beta}{A}\right)^{\frac{1-\alpha}{1+\gamma}} k_t^{\alpha}, \quad k_1 > 0 \text{ given.} \tag{24}$$

It is then straightforward to derive that the modified model possesses a single interior stationary equilibrium of capital:

$$k^{ss} = \left[\frac{\beta B\left(1-\alpha\right)}{1+\beta}\right]^{\frac{1}{1-\alpha}}\left(\frac{1+\beta}{A}\right)^{\frac{1}{1+\gamma}} > 0. \tag{25}$$

Moreover, using the long-run versions of (6)-(8), (18)-(19) and (22)-(23), together with equations (12) and (25), it can be shown that the government's revenue g and the steady-state consumption tax rate τ_c^{ss} are related via

$$g = \left[\frac{\left(1+\alpha\beta\right)y^{ss}}{1+\beta}\right]\left(\frac{\tau_c^{ss}}{1+\tau_c^{ss}}\right), \quad \tau_c^{ss} > 0, \tag{26}$$

where $y^{ss} = B^{\frac{1}{1-\alpha}}\left[\frac{\beta(1-\alpha)}{1+\beta}\right]^{\frac{\alpha}{1-\alpha}}\left(\frac{1+\beta}{A}\right)^{\frac{1}{1+\gamma}}$ is total output at the economy's stationary state. Since $\frac{\partial g}{\partial \tau_c^{ss}} > 0$, equation (26) illustrates that as in Giannitsarou's (2007) representative-agent macroeconomy under endogenous consumption taxes, the Laffer curve does not arise in our two-period overlapping generations model with a unique positive steady state. Given the above discussions, we will obtain that

Proposition 2. Under any exogenously given initial capital stock $k_1 > 0$ and endogenous consumption taxation to finance a pre-specified constant level of government spending $g > 0$, our two-period overlapping generations model exhibits a unique and asymptotically stable interior steady state with $\tau_c^{ss} > 0$. Therefore, the alternative economy will not display equilibrium indeterminacy and endogenous belief-driven cyclical fluctuations.

Figure 3 depicts the phase diagram associated with the equilibrium difference equation (24). It shows that $M(k_t)$ is an increasing concave function through the origin, resulting in a single interior stationary state k^{ss} given by (25), which is asymptotically stable because of $M'(k^{ss}) = \alpha \in (0, 1)$. Hence, there exists a unique equilibrium trajectory that will converge toward it for all starting positive values of capital stock k_1. This result turns out to be qualitatively identical to that of Giannitsarou (2007) – endogenous consumption taxation does not lead to equilibrium indeterminacy and sunspot-driven macroeconomic fluctuations in the context of a standard one-sector RBC model or a prototypical two-period OLG model.

To explain the underlying intuition for this no-indeterminacy result, we note that the generation-t household's consumption Euler equation (21) can be rewritten as

$$\frac{c_{t+1}^t}{c_t^t} = \beta\left(\frac{1+\tau_{ct}}{1+\tau_{ct+1}}\right)\underbrace{\left[\alpha B\left(\frac{1+\beta}{A}\right)^{\frac{1-\alpha}{1+\gamma}} k_{t+1}^{\alpha-1}\right]}_{=r_{t+1}}. \tag{27}$$

Start from the economy's stationary state at period t, and suppose that the young agent anticipates the future consumption tax rate τ_{ct+1} to drop. Due to the stronger intertemporal substitution effect, the generation-t individual will consume less today and raise her old-age consumption, thus c_t^t falls and c_{t+1}^t rises. As a consequence, the left-hand side of (21) becomes higher. For this alternative dynamic path to be justified as a self-fulfilling equilibrium, the consumption-tax adjusted rate of return on capital at time $t+1$, *i.e.* the right-hand side of (21), needs to increase as well. However, the reduction of current consumption leads to a higher level of k_{t+1}, which (combined with the constant labor supply h_{t+1} per equation 12) in turn will lower the capital rental rate because of diminishing returns to productive inputs. It follows that the net effect is determined by the relative strength of counteracting period-$(t+1)$ decreases in the consumption tax rate versus the before-tax real interest rate. Based on Proposition 2 and Figure 3, we find that the decline in r_{t+1} dominates that in τ_{ct+1} to render a smaller right-hand side of equation (21), hence the cohort-t household's initial expectation of a future tax cut is invalidated. This finding implies that the model's unique interior steady state is locally determinate or isolated, and that agents' animal spirits cannot be a driving force of business cycle fluctuations. In sum, our analysis shows that under the same baseline preference and technological specifications, together with the postulated balanced-budget rule governed by endogenous consumption taxation, Giannitsarou's (2007) determinacy result will continue to hold within a two-period overlapping generations macroeconomy *à la* Diamond (1965).

Finally, it is straightforward to show that the preceding equilibrium-uniqueness result is robust to changes in the funding source of government revenue. In particular, the following two fiscal policy rules are considered: no taxation on old agents ($\tau_{ct+1} = 0$); and no taxation on young individuals ($\tau_{ct} = 0$). For each setting, we find that the scalar difference equation in capital that characterizes the model's equilibrium dynamics, as in (24), will remain unchanged. It follows that the macroeconomic stability feature of no indeterminacy and sunspots in our alternative OLG economy does not depend on which group(s) of households are paying the consumption taxes.

3.2 Dynamic Equivalence

Before proceeding with the sensitivity analysis, this subsection offers further insights into our no-indeterminacy result of sections 2.4 and 3.1 from a comparative perspective. In a basic intertemporal framework that excludes capital or savings taxation, Atkinson and Stiglitz (1980, pp. 69-72) show that a proportional constant consumption tax rate is equivalent to a flat tax schedule on labor/wage income because either fiscal formulation leads to an identical lifetime budget constraint, thereby yielding the same amount of tax revenue for the government; see also Salanie (2003, pp. 187-188). Subsequently, Renström (1997, pp. 32-33) points out that the Atkinson-Stiglitz dynamic equivalence will no longer hold when the consumption and labor tax rates are time varying and endogenously determined.

Giannitsarou (2007, p. 1425) refers to the above non-equivalence of endogenous-tax systems to help explain the differences between her saddle-path stability result under consumption taxes, versus the opposite aggregate instability per Schmitt-Grohé and Uribe (1997, section II) under labor taxes, within a standard one-sector real business cycle macroeconomy. When a pre-set fixed level of public expenditures are financed by consumption taxation, Giannitsarou (2007) finds that the representative agent's labor supply decision is independent of the tax rate. As a result, τ_c^{SS} does not enter the model's Jacobian matrix, thus it exerts no impact on the local dynamics around the unique interior steady state which is always a saddle point. It follows that Giannitsarou's (2007) RBC economy will not exhibit endogenous belief-driven business cycles. On the contrary, Schmitt-Grohé and Uribe (1997, section II) obtain a Laffer curve-type relationship between the labor tax rate and the resulting government revenue, indicating the possible existence of two interior stationary equilibria. Since the household's labor supply is monotonically decreasing in the tax rate due to a stronger substitution effect, τ_L^{ss} and τ_H^{ss} will affect the equilibrium dynamics of Schmitt-Grohé and Uribe's (1997, section II) RBC model. In particular, these authors analytically derive the necessary and sufficient condition under which the low-tax steady state is an indeterminate sink that can be exploited to generate cyclical fluctuations caused by animal spirits or sunspots. Therefore, these previous findings confirm Renström's (1997) claim that endogenous consumption and labor taxes are not equivalent in a prototypical one-sector representative-agent macroeconomy.

However, this paper shows that the RBC-based intertemporal non-equivalence of endogenous taxation does not continue to hold in Diamond's (1965) stylized two-period overlapping generations model. Under the postulated preference and technological specifications considered in sections 2 and 3, each young individual's labor supply is found to be independent of the time-varying tax rate on wage income or consumption spending (see equation 12). In either environment, the economy's equilibrium dynamics is governed by a scalar difference equation which is monotonically increasing and strictly concave in capital. As in the corresponding representative-agent counterpart, our OLG macroeconomy with a labor tax may possess two interior steady states, whereas a single positive stationary equilibrium will arise with a consumption tax. For all feasible values of exogenously given k_1, we find that every interior steady state is associated with local determinacy and equilibrium uniqueness; hence the likelihood of macroeconomic fluctuations driven by agents' self-fulfilling expectations is completely eliminated. It follows that the present study provides an intriguing extension of Atkinson and Stiglitz's (1980) dynamic equivalence to a standard two-period overlapping generations model under endogenous labor versus consumption taxation.

4 Sensitivity Analysis

For the sensitivity analysis, we will explore three distinct variations to re-examine the economy's local (in)stability properties when (i) the social technology displays increasing returns-to-scale due to positive productive externalities from aggregate capital and labor services; (ii) the household's intertemporal elasticity of substitution in consumption is not equal to one; and (iii) each young agent's labor supply decision exhibits no income effect with respect to a change in the labor or consumption tax rate. These extensions allow us to study the robustness of our theoretical findings obtained from sections 2 and 3, as well as further understanding the precise mechanisms through which local determinacy and equilibrium uniqueness may continue to occur within a stylized two-period overlapping-generations model under endogenous labor or consumption taxation that balances the government's budget.

4.1 Positive Productive Externalities

In this case, the representative firm's production technology (6) is changed to

$$y_t = Bk_t^{\alpha} h_t^{1-\alpha} \left[K_t^{\alpha\theta_K} H_t^{(1-\alpha)\theta_H} \right], \ B, \ \theta_K, \ \theta_H > 0, \quad 0 < \alpha < 1, \tag{28}$$

where θ_K and θ_H represent the degrees of positive productive externalities, generated from the economy-wide levels of capital K_t and labor H_t inputs, that are taken as given by each individual firm. In a symmetric equilibrium, all firms will make the same decisions with $k_t = K_t$ and $h_t = H_t$, for all t. It follows that the economy's aggregate production function that exhibits increasing returns-to-scale is given by

$$y_t = Bk_t^{\alpha(1+\theta_K)} h_t^{(1-\alpha)(1+\theta_H)}. \tag{29}$$

As in sections 2 and 3, our analysis below is restricted to an environment of $\alpha (1 + \theta_K) < 1$ such that sustained economic growth is not permitted.

It is then straightforward to derive that (i) the equilibrium prices of factor inputs (7)-(8) remain unaffected; (ii) equation (12) on the constant level of each young agent's optimal labor supply stays unchanged as well; and (iii) this modified model's equilibrium difference equation under endogenous labor income taxation becomes

$$k_{t+1} = \frac{\beta}{1+\beta} \left[B (1-\alpha) \left(\frac{1+\beta}{A} \right)^{\frac{(1-\alpha)(1+\theta_H)}{1+\gamma}} k_t^{\alpha(1+\theta_K)} - g \right], \quad k_1 > 0 \text{ given.} \tag{30}$$

Given the postulated parametric restriction $\alpha (1 + \theta_K) \in (0, 1)$, the phase diagram for (30) will be qualitatively identical to Figure 1 with an increasing concave curve in k_t that may intersect the 45-degree line twice. It follows that the no-indeterminacy result reported in Proposition 1 continues to prevail since the low-capital (high-tax) steady state k_L^{ss} is asymptotically unstable, and the high-capital (low-tax) steady state k_H^{ss} is asymptotically stable.

Similarly, it cane be shown that our modified model's equilibrium dynamics under endogenous consumption taxation is governed by

$$k_{t+1} = \left[\frac{\beta B (1-\alpha)}{1+\beta} \right] \left(\frac{1+\beta}{A} \right)^{\frac{(1-\alpha)(1+\theta_H)}{1+\gamma}} k_t^{\alpha(1+\theta_K)}, \quad k_1 > 0 \text{ given.} \tag{31}$$

As per Figure 2 and Proposition 2, the preceding equation also yields a unique interior stationary state k^{ss} that is asymptotically stable, hence the economy always displays local determinacy and equilibrium uniqueness. In sum, this subsection finds that our previous no-indeterminacy results are qualitatively robust to the inclusion of positive productive externalities, because young individuals' labor hours h_t are found to remain independent of the labor or consumption tax rate (see equation 12). Therefore, the dynamic equivalence between endogenous labor and consumption taxation as automatic stabilizers against belief-driven business cycles will be maintained within this extended two-period overlapping generations model.

4.2 Non-Unitary Intertemporal Elasticity of Substitution in Consumption

In this case, the generation-t household's utility function is postulated to exhibit a constant degree of relative risk aversion (CRRA) that is not equal to one:

$$u^t = \frac{\left(c_t^t\right)^{1-\sigma} - 1}{1-\sigma} - A\frac{h_t^{1+\gamma}}{1+\gamma} + \beta\frac{\left(c_{t+1}^t\right)^{1-\sigma} - 1}{1-\sigma}, \quad A,\ \sigma > 0, \quad \sigma \neq 1, \quad \gamma \geq 0, \quad 0 < \beta < 1, \tag{32}$$

where σ denotes the inverse for the intertemporal elasticity of substitution (IES) in consumption. Under endogenous labor income taxation, the associated first-order conditions are

$$A\left(c_t^t\right)^{\sigma} h_t^{\gamma} = (1-\tau_t)\, w_t \tag{33}$$

for the intratemporal labor supply decision, and

$$\left(\frac{c_{t+1}^t}{c_t^t}\right)^{\sigma} = \beta r_{t+1} \tag{34}$$

for the intertemporal consumption choices. In contrast to (12) with the constant level of hours worked under $\sigma = 1$, equation (33) yields that h_t is a strictly decreasing function of the labor tax rate τ_t when $\sigma \neq 1$ because the resulting substitution effect turns out to be stronger. Moreover, it is straightforward to show that the Laffer curve-type relationship between the stationary-state tax rate $\tau^{ss} \in (0,\, 1)$ and the government's tax revenue $g > 0$ is given by

$$g = (1-\alpha)B\tau^{ss}\Pi^{\frac{-\alpha\sigma}{1-2\alpha+\alpha\sigma}}\left(h^{ss}\right)^{1-\alpha+\Delta}, \tag{35}$$

where $\Pi \equiv \alpha^{\frac{\sigma-1}{\sigma}}B^{\frac{\sigma-2}{\sigma}}\left[\frac{A}{\beta(1-\alpha)(1-\tau^{ss})}\right]^{\frac{1}{\sigma}}$, $\Delta \equiv \frac{\alpha[(1-\alpha)(1-\sigma)-(\alpha+\gamma)]}{1-2\alpha+\alpha\sigma}$ and h^{ss} denotes the steady-state labor hours which are found to be a non-linear function of τ^{ss} and model parameters. We also analytically verify that substituting $\sigma = 1$ into (35) will recover equation (14). As in section 2, our sensitivity analysis here is focused on an environment that possesses two interior stationary equilibria characterized by τ_L^{ss} and τ_H^{ss}.

Next, we find that this general-CRRA economy's local (in)stability properties can be analyzed through the following log-linearized dynamical system:

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{h}_{t+1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \hat{k}_t \\ \hat{h}_t \end{bmatrix}, \ \ \hat{k}_1 \text{ given}, \tag{36}$$

where hat variables represent percentage deviations from their respective steady-state values, and $\mathbf{J}$ is the Jacobian matrix of partial derivatives on the model's equilibrium conditions in a neighborhood of each stationary state. The macroeconomy exhibits saddle-path stability and equilibrium uniqueness when one eigenvalue of $\mathbf{J}$ lies inside and the other outside the unit circle. When both eigenvalues are inside the unit circle, the steady state becomes an indeterminate sink around which there are a continuum of stationary equilibrium trajectories that display cyclical fluctuations driven by agents' animal spirits or sunspots. When both eigenvalues are outside the unit circle, the steady state becomes a totally unstable source.

Unlike our benchmark framework studied in section 2, we can not analytically derive the exact conditions that govern the equilibrium dynamics within this extended version of our macroeconomy while maintaining constant returns-to-scale in production. As a result, numerical experiments are conducted to quantitatively explore the economy's aggregate (in)stability attributes. Specifically, each period of an agent's lifetime is taken to be 30 calendar years. This feature, together with the annual real interest rate of 4%, leads to the calibrated discount factor $\beta = (0.96)^{30}$. The labor share of national income, $1 - \alpha$, is set to be 0.7; and the scaling parameters are normalized to $A = B = 1$ since they do not affect the model's local dynamics. In terms of the household's labor supply elasticity, we will explore the parametric specifications with $\gamma = 0$ (*i.e.* indivisible labor *à la* Hansen [1985] and Rogerson [1988]), 0.25 as in King, Plosser and Rebelo (1988) for their baseline RBC calibration, and 15 based on Altonji's (1986, Table 1) empirical estimates at the micro level. For the preference parameter σ, most previous studies have adopted the range of one to three in their quantitative analyses. However, some recent empirical research reports that $\sigma < 1$ thus the elasticity of intertemporal consumption substitution is higher than one; see Mulligan (2002), Vissing-Jørgensen and Attanasio (2003), and Gruber (2006), among others. Drawing upon these estimation results, our quantitative simulations will consider the interval of $\sigma \in [0.5, 3]$, where $\sigma = 0.5$ corresponds to the highest possible value of IES ($= 2$) that is regarded as empirically realistic.

Without loss of generality, we first impose $\tau_L^{ss} = \{0.1, 0.2, 0.3, 0.4\}$, together with the aforementioned parameter combinations, onto the right-hand side of equation (35) to obtain the resulting tax revenue g, and then numerically solve the corresponding τ_H^{ss} along the downward portion of the Laffer curve. For all the parametric configurations under consideration, we find that the model's low-tax steady state is a locally determinate saddle point and that the high-tax steady state is a source, which is surrounded by divergent or explosive trajectories that will eventually violate the economy's transversality condition. It follows that our no-indeterminacy result with the logarithmic utility function ($\sigma = 1$), as in Proposition 1, is qualitatively robust to the general CRRA preference formulation ($\sigma \neq 1$). Intuitively, a comparison of (17) versus (34) shows that changing the utility curvature will affect the magnitude, but not the upward direction of intertemporal consumption choices (*i.e.* $\frac{c_{t+1}^t}{c_t^t}$ rises) induced by the young agent's rosy anticipation, on the left-hand side of the relevant consumption Euler equation. It turns out that this optimism cannot be self-fulfilled, regardless of whether h_{t+1} is a fixed constant or not, since diminishing marginal product of capital (r_{t+1} falls) will decrease the right-hand side of both equations (17) and (34). Our analysis thus illustrates that equilibrium indeterminacy is completely eliminated within Diamond's (1965) two-period overlapping generations model under CRRA preferences and endogenous labor income taxation because neither interior steady state can be a sink.

On the other hand, the generation-t household's first-order conditions under endogenous consumption taxation are given by

$$A \left(c_t^t\right)^{\sigma} h_t^{\gamma} = \frac{w_t}{1+\tau_{ct}}, \tag{37}$$

which entails that h_t is no longer independent of τ_{ct} (*c.f.* section 3) and

$$\left(\frac{c_{t+1}^t}{c_t^t}\right)^{\sigma} = \beta r_{t+1}\left(\frac{1+\tau_{ct}}{1+\tau_{ct+1}}\right). \tag{38}$$

In addition, we find that the relationship between government spending g and the steady-state consumption tax rate τ_c^{ss} is now governed by a non-linear equation which may lead to multiple stationary equilibria, and that the model's equilibrium conditions can be approximated by the log-linearized dynamical system (36) as well. For each empirically plausible value of $\tau_c^{ss} = \{0.05,\ 0.1,\ 0.15,\ 0.2\}$, in conjunction with the calibrations on $\{\alpha,\ \beta,\ \gamma,\ \sigma,\ A,\ B\}$ discussed earlier, this macroeconomy always possesses a unique interior steady state that exhibits saddle-path stability. As for (27) and Proposition 2, while the cohort-t household's optimistic expectation raises the left-hand side of condition (38) with $\sigma \neq 1$, the consumption-tax adjusted rate of return on capital at period $t+1$, *i.e.* the right-hand side of (38), will fall. It follows that agents' initial optimism is invalidated, which in turn rules out the possibility of indeterminacy and sunspots. In sum, this subsection finds that both labor and consumption taxation will continue to serve as stabilizing instruments against endogenous cyclical fluctuations within our two-period overlapping generations model under the general CRRA utility formulation.

4.3 No-Income-Effect Preferences

This subsection incorporates a non-separable no-income-effect preference specification, as in Greenwood, Hercowitz and Huffman (GHH, 1988), into our model economy with overlapping two-period-lived agents and the constant-g balanced-budget policy rule. In particular, the cohort-t individual's utility function is postulated as

$$u^t = \log\left(c_t^t - A\frac{h_t^{1+\gamma}}{1+\gamma}\right) + \beta \log c_{t+1}^t, \quad A>0, \quad \gamma \geq 0, \quad 0<\beta<1, \tag{39}$$

and her first-order conditions under endogenous labor income taxation are

$$Ah_t^{\gamma} = (1-\tau_t)\, w_t, \tag{40}$$

$$\frac{c_{t+1}^t}{c_t^t - A\frac{h_t^{1+\gamma}}{1+\gamma}} = \beta r_{t+1}. \tag{41}$$

Since c_t^t is missing in equation (40), there is no income effect associated with the household's labor supply decision. It follows that the income elasticity of intertemporal substitution in hours worked (or leisure) is zero.

Next, it is straightforward to show that this economy's equilibrium conditions are characterized by the following pair of (one dynamic and one static) equations:

$$k_{t+1} = \frac{\beta\gamma A}{(1+\beta)(1+\gamma)}\left[\frac{(1-\alpha)B(1-\tau_t)}{A}\right]^{\frac{1+\gamma}{\alpha+\gamma}} k_t^{\frac{\alpha(1+\gamma)}{\alpha+\gamma}}, \quad k_1>0 \text{ given;} \tag{42}$$

and

$$g = \tau_t\left(\frac{1-\tau_t}{A}\right)^{\frac{1-\alpha}{\alpha+\gamma}}\left[(1-\alpha)B\right]^{\frac{1+\gamma}{\alpha+\gamma}} k_t^{\frac{\alpha(1+\gamma)}{\alpha+\gamma}}, \tag{43}$$

which can be used to express the labor tax rate τ_t as an implicit function of k_t and model parameters, given by $\tau_t = f(k_t;\, \alpha,\, \gamma,\, g,\, A,\, B)$. We then substitute $f(\cdot)$ into (42) to obtain the scalar difference equation in capital $k_{t+1} = \Phi(k_t)$ that cannot be explicitly written out either. Notice that when the young individual's labor hours are infinitely elastic ($\gamma = 0$), equation (42) becomes degenerate with $k_{t+1} = 0$. Therefore, our subsequent numerical experiments will consider other parameterizations that generate two positive steady states.

Under identical calibrated values of α, β, $\gamma = \{0.25,\, 15\}$, A and B as those in section 4.2, together with $\tau_L^{ss} = \{0.1,\, 0.2,\, 0.3,\, 0.4\}$ that yield the resulting levels of tax revenue g, we find that the low-tax (high-capital) stationary state is always asymptotically stable in that

$|\Phi'(k_H^{ss})| < 1$, and that the high-tax (low-capital) steady state is either asymptotically stable or asymptotically unstable with $|\Phi'(k_L^{ss})| > 1$. It follows that per Proposition 1, indeterminate dynamic trajectories will not arise within the vicinity of either interior stationary equilibrium. Upon an optimistic expectation about the economy's future, the cohort-t agent will consume less (c_t^t falls) and save more (k_{t+1} rises) today. Due to the lack of income effect, as seen in (40), h_t remains unchanged in response to the lower period-t consumption. In addition, a higher k_{t+1} leads to (i) an increase in h_{t+1} for the generation-$(t+1)$ household, via firms' labor demand function, which in turn raises the aggregate output y_{t+1} as well as the old individual's consumption c_{t+1}^t; and (ii) a decrease in the real interest rate r_{t+1} because of diminishing marginal product of capital. As a result, these belief-driven effects altogether will render the equality of the consumption Euler equation (41) impossible to hold, hence agents' initial optimism cannot be justified as a self-fulfilling equilibrium.

On the other hand, the generation-t household's first-order conditions under endogenous consumption taxation are given by

$$Ah_t^{\gamma} = \frac{w_t}{1+\tau_{ct}}, \tag{44}$$

which illustrates the absence of income effect on her labor supply decision, as c_t^t is missing; and

$$\frac{c_{t+1}^t}{c_t^t - A\frac{h_t^{1+\gamma}}{1+\gamma}} = \beta r_{t+1}\left(\frac{1+\tau_{ct}}{1+\tau_{ct+1}}\right). \tag{45}$$

In this case, the two (one dynamic and one static) equations that characterize the economy's equilibrium conditions are

$$k_{t+1} = \frac{\beta\gamma(1-\alpha)}{(1+\beta)(1+\gamma)}\left[\frac{(1-\alpha)B}{A(1+\tau_{ct})}\right]^{\frac{1-\alpha}{\alpha+\gamma}} k_t^{\frac{\alpha(1+\gamma)}{\alpha+\gamma}}, \quad k_1 > 0 \text{ given}, \tag{46}$$

which will collapse to $k_{t+1} = 0$ when labor hours are indivisible with $\gamma = 0$; and

$$g = \frac{\tau_{ct}}{1+\tau_{ct}}\left\{B^{\frac{1+\gamma}{\alpha+\gamma}}\left[\frac{1-\alpha}{A(1+\tau_{ct})}\right]^{\frac{1-\alpha}{\alpha+\gamma}} k_t^{\frac{\alpha(1+\gamma)}{\alpha+\gamma}} - k_{t+1}\right\}, \tag{47}$$

where the consumption tax rate τ_{ct} can then be implicitly expressed as a function of k_t, k_{t+1} and model parameters: $\tau_{ct} = z(k_t, k_{t+1}; \alpha, \gamma, g, A, B)$. Substituting $z(\cdot)$ into (46) results in the equilibrium difference equation $k_{t+1} = \Psi(k_t)$ that may possess multiple stationary equilibria.

Given $\tau_c^{ss} = \{0.05, 0.1, 0.15, 0.2\}$ along with the same calibrated values of α, β, $\gamma = \{0.25, 15\}$, A and B as those in the proceeding analyses, there exists a second stationary state whose consumption tax rate turns out to be higher than 100%. Since $\tau_c^{ss} > 1$ cannot be regarded as empirically plausible, this case will be abstracted from our quantitative simulations. For all the empirically realistic parametric configurations under consideration, we find that the resulting unique interior steady state is asymptotically stable, around which indeterminacy and sunspots do not occur. Upon the expectation of a decrease in the future consumption tax rate τ_{ct+1}, the cohort-t individual will (i) consume less while keeping her labor supply unaltered today, and (ii) raise her consumption expenditures and capital stock for the next period. It follows that the left-hand side of equation (45) rises, whereas the corresponding right-hand side falls because the decline in the before-tax capital rental rate r_{t+1} is quantitatively stronger than that in the period-$(t+1)$ consumption tax rate. In sum, this subsection shows that under the GHH formulation of non-separable preferences, our two-period overlapping generations model will always display local determinacy and equilibrium uniqueness, regardless of whether a constant level of public spending is financed by endogenous labor or consumption taxation.

5 Conclusion

In the context of a stylized two-period overlapping generations model, this paper examines the aggregate (in)stability effects of a balanced-budget rule whereby constant government purchases are financed by endogenous taxation on agents' labor income or consumption expenditures. The resulting policy implications turn out to be quite different from those of Schmitt-Grohé and Uribe (1997, section II) and Giannitsarou (2007, section 1) for the corresponding representative-agent macroeconomy. Under the same baseline preference and technological formulations as in previous RBC-based studies, our OLG economy will always exhibit local determinacy and equilibrium uniqueness, which in turn implies that both labor and consumption taxes are stabilizing instruments against belief-driven cyclical fluctuations. By contrast, a standard one-sector real business cycle model may display indeterminacy and sunspots when the pre-specified fixed level of public spending is financed by countercyclical labor income taxation. In addition, our no-indeterminacy result and associated dynamic equivalence between endogenously determined labor versus consumption taxes are found to remain qualitatively robust with respect to various modifications in the household-utility or firm-production configurations.

This paper can be extended in several directions. In particular, it would be worthwhile to incorporate additional features that have been shown to influence equilibrium (in)determinacy within intertemporal macroeconomic models, such as a two-period overlapping generations economy without first-period consumption *à la* Reichlin (1986), endogenous taxation on the household's capital or total income *à la* Schmitt-Grohé and Uribe (1997, section III) and Giannitsarou (2007, section 3), progressive income taxation *à la* Guo and Lansing (1998), and a general constant-elasticity-of-substitution production technology *à la* Guo and Lansing (2009), among others. These possible extensions will enhance our understanding of how model setup and/or fiscal policy rules govern the parametric region(s) of local (in)determinacy in a prototypical two-period overlapping generations macroeconomy. We plan to pursue these research projects in the near future.

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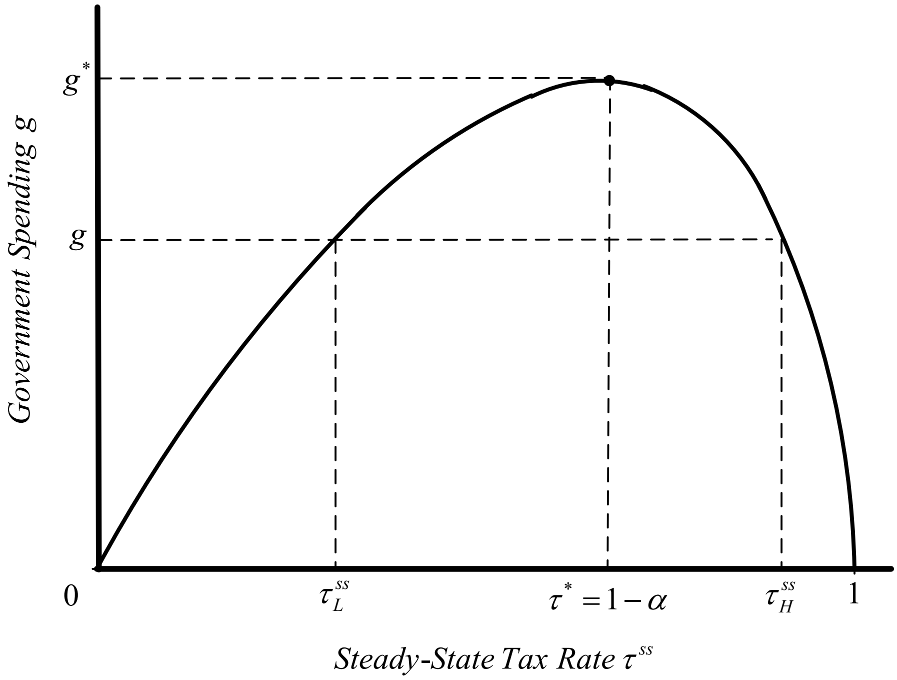

Figure 1. Steady-State Laffer Curve of the Benchmark Economy

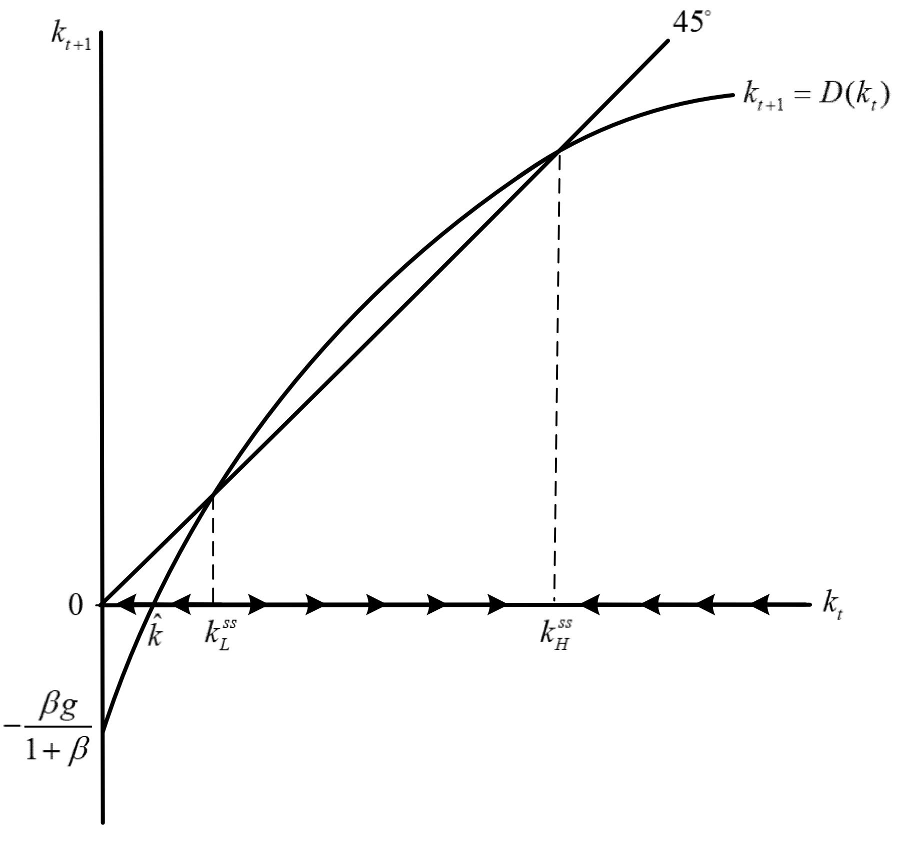

Figure 2. Phase Diagram under Endogenous Labor Taxation

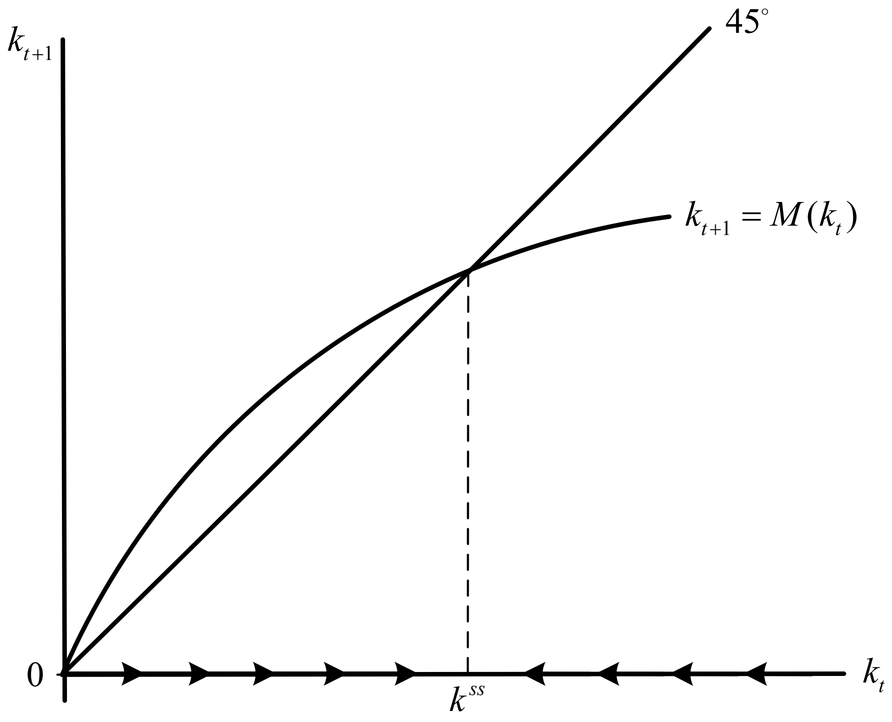

Figure 3. Phase Diagram under Endogenous Consumption Taxation