Skill Loss during Unemployment and the Scarring Effects of the COVID-19 Pandemic

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Abstract

We integrate the SIR epidemiology model into a search and matching framework in which workers lose human capital during unemployment. As the number of infections rises, fewer jobs are created, the unemployment rate increases and the composition of skills among the unemployed deteriorates, thereby reducing TFP. We calibrate the model to quantify the effect of a three month lockdown on TFP through loss of skill during unemployment. Sixty-two weeks after the pandemic begins, TFP reaches its lowest value with a decline of 0.56%, which is nearly 50% of the productivity losses typically seen in recessions.

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Moreover, the longer the downturn lasts, the greater the potential for longer-term damage from permanent job loss and business closures. Long periods of unemployment can erode workers’ skills and hurt their future job prospects.” – Jerome H. Powell before the U.S. Senate on June 16, 2020

1 Introduction

As of September 2020, it has been six months since the World Health Organization declared COVID-19 a pandemic. The cost in terms of lives lost has been substantial, as nearly 850,000 people have died. The economic costs have also been extraordinary. In the period between March-April 2020, nearly twenty million jobs were lost in the U.S. and the unemployment rate remains at levels not observed since the early 1980s. As the number of infections continues to grow and workers remain displaced from their job, there is potential for the pandemic to cause long-lasting economic costs, effectively scarring the economy for years to come. As seen in the above statement from Federal Reserve Chairman Jerome H. Powell, policymakers are already concerned about this possibility as it is well documented that workers lose human capital during long periods of unemployment. When workers lose skills during unemployment, longer unemployment spells worsen the skill composition of the work force, which in turn decreases TFP.

We integrate the canonical SIR framework (Kermack and McKendrick, 1927) with a search and matching model in which workers lose human capital while unemployed to study the effects of the COVID-19 pandemic on unemployment, the skill composition of unemployed workers and TFP. Our integration of the SIR framework with a search and matching model follows Kapička and Rupert (2020) by assuming employed workers have more opportunities to become infected than unemployed workers. When employed workers become infected, they are not productive and face the possibility of dying. As a pandemic evolves, firms create less jobs due to the increased risk that their employee becomes infected and the match no longer produces output. We extend this framework by assuming workers are exposed to skill loss shocks when they are unemployed or are employed and not working due to being infected.

Through the addition of skill loss shocks, our model allows us to study the dynamics of the skill composition of workers and TFP following the outbreak of a pandemic. As infections rise and less jobs are created, the probability of finding a job decreases, and the unemployment rate increases. As workers face longer unemployment durations, they are more likely to lose skills and the skill composition of the labor force deteriorates over the course of the pandemic. Following the worsening of the skill composition, average labor productivity, and hence TFP decreases.

We calibrate the model to quantify the effect of the COVID-19 pandemic on unemployment, the skill composition of the unemployed and TFP through workers losing their skills during unemployment. In our baseline exercise without any policy intervention, the unemployment rate increases by nearly 3.8 percentage points, the skill composition of unemployed workers worsens, and TFP decreases by 0.44%. Given that the typical decline in TFP during recessions is 1.13%, the baseline results generate a decline in TFP close to 39% of the typical productivity losses seen in past recessions. Moreover, the effects of the pandemic on TFP are long lasting; TFP reaches its lowest point only 55 weeks after the onset of the pandemic and remains far below its pre-pandemic value even 100 weeks after the pandemic started.

To study the effect of a lockdown, we increase the job separation probability for three months at the onset of the pandemic. By increasing job separations, fewer firms create jobs, fewer workers are employed and infections drop. Our quantitative results show that this policy saves nearly 65,000 lives. However, there is a substantial cost in terms of increased unemployment. The increased separations combined with reduced
job creation increases the unemployment rate by nearly 7.6 percentage points in a period of three months. There is also a long-term economic cost associated with the lockdown; as the increased unemployment rate causes more workers to be exposed to human capital depreciation, further worsening the skill composition of job seekers. We find that, sixty-two weeks after the pandemic began, TFP reaches its lowest point and has decreased by 0.56%. Conducting the same calculation as with our baseline results, a 0.56% decline in TFP due to loss of skill during unemployment corresponds to nearly 50% of the productivity losses in previous recessions, indicating that the COVID-19 pandemic and the recession it has caused will leave significant scarring effects on the economy for years to come.

2 Related Literature

There is a burgeoning economic literature on the COVID-19 pandemic. Here, we briefly review the most related literature. Our paper is most closely related to Kapička and Rupert (2020) who integrate the SIR framework of Kermack and McKendrick (1927) into the Mortensen-Pissarides (Pissarides (1985), Mortensen and Pissarides (1994)) model of equilibrium unemployment. They assume employed workers have more interactions than unemployed workers, and hence have a higher probability of becoming infected. The model is used to study the dynamics of wages and the unemployment rate throughout the pandemic and optimal quarantine policies. We employ a similar framework, but also add loss of skill during unemployment, which allows us to study the long-run effects of a pandemic and lockdowns on the skill composition of the unemployed and TFP. In addition, skill loss improves the performance of the Mortensen-Pissarides model in response to shocks (Ortego-Marti (2017a)).

Other studies discussing the impact of COVID-19 on the labor market include Gregory et al. (2020), who develop a framework with both permanent and temporary layoffs to forecast labor market dynamics following a lockdown shock. Their framework does not model the pandemic and does not study loss of skill during unemployment. Petrosky-Nadeau and Valletta (2020) and Sahin et al. (2020) forecast unemployment dynamics following the initial spike in unemployment following the onset of the Covid-19 pandemic, while Coibion et al. (2020) document large flows into non-participation and that initial job losses were larger than implied by initial unemployment insurance claims.

Many other papers have introduced the SIR framework of Kermack and McKendrick (1927) into the economic models and applied them to the COVID-19 pandemic. Atkeson (2020) provided an early introduction into SIR models and how they could be applied to the current pandemic while Fernández-Villaverde and Jones (2020) developed an SIRD model to forecast the COVID-19 pandemic under lockdowns and changes to social distancing behaviour. Eichenbaum et al. (2020) extended the SIR framework to study the relationship between economic decisions and epidemics and optimal containment policies. Garibaldi et al. (2020) extended the SIR framework to include search frictions and explicitly model interactions between agents. There are many papers that have characterized optimal policy responses to the COVID-19 pandemic. Hall et al. (2020) develop a framework to study the optimal tradeoff between consumption and deaths while Alvarez et al. (2020) study optimal lockdown policies. Berger et al. (2020) develop a SEIR framework to study optimal quarantine and testing. Guerrieri et al. (2020) demonstrate how the initial supply shock associated with the Covid-19 pandemic can lead to a subsequent aggregate demand shock and optimal fiscal and monetary policy response. Both Bethune and Korinek (2020) and Farboodi et al. (2020) study in detail the externalities present in an economic environment with a pandemic and characterize optimal policy responses.

Finally, our paper is closely related to previous work on loss of skill during unemployment. Two seminal
papers in this literature are Pissarides (1992) and Ljungqvist and Sargent (1998). Pissarides (1992) shows that unemployment is more persistent when unemployed workers suffer skill decay during unemployment, whereas Ljungqvist and Sargent (1998) provide a rationale for the high unemployment in Europe relative to the US due to the generous UI benefits in Europe. Ortego-Marti (2017c, 2020) show how loss of skill during unemployment impacts TFP while Doppelt (2019) focuses on the classical debate over the long-run relationship between growth and unemployment. Laureys (2020) discusses the externalities caused by loss of skill during unemployment and the implications for optimal policy. Our project is also related to Ortego-Marti (2016) who studies wage dispersion in the presence of skill loss, and to Heathcote et al. (2020) who study how loss of skill during unemployment can increase inequality in the long-run. However, none of these papers study consider an epidemiological SIR model to study the effect of a pandemic. We contribute to this literature by developing a framework which can be used to model the effect of a pandemic on TFP through loss of skill during unemployment, and by providing quantitative results regarding the long-run effect of the COVID-19 pandemic on both unemployment and TFP in the United States.

3 Environment

Time, agents, and preferences. Time is discrete and indexed by \( t \in \mathbb{N}_0 \). There are two types of agents: a large measure of firms and workers whose initial population is normalized to one. All agents are risk-neutral and have a discount factor \( \beta \in (0, 1) \). Workers are categorized by their employment status (employed or unemployed), skill level (high or low skill), and health status (susceptible, infected, or recovered). In each period, a measure \( \mu \) of workers enter the labor force as unemployed who are highly skilled and susceptible.

Health statuses. Workers can be susceptible to the infection but not yet infected (S), infected but not yet recovered or deceased (I), or recovered and immune from further infection (R). The probability that a susceptible person becomes infected depends on their employment status. Employed workers become infected with probability \( \pi^E I_t = \pi^E I_t \) where \( I_t \) is the stock of infected workers at time \( t \). Unemployed workers become infected with probability \( \pi^U I_t = \pi^U I_t \). Following Kapička and Rupert (2020), we assume that employed workers have more interactions than the unemployed and hence have more opportunities to become infected, i.e. \( \pi^E > \pi^U \). Infected workers recover with probability \( \pi_R \) and die from the infection with probability \( \pi_D \).

Skills and technology. Workers are heterogenous in their skill due to skill loss during unemployment. There are two levels of skill indexed by \( \chi \in \{ L, H \} \): low (L) and high (H). Employed high skill workers produce \( y \) units of output per period, while low skill workers produce \( \delta y \) with \( \delta \in (0, 1) \). If a susceptible employed worker becomes infected, they remain employed and do not produce output. Unemployed workers receive utility \( b \) while unemployed, representing the value of leisure, home-production, and unemployment benefits. Skill loss occurs as follows. In each period, high skill workers who are either unemployed or employed and infected permanently become low skilled with probability \( \sigma \).

The labor market. Workers search for jobs while firms search for applicants in a frictional labor market. Unemployed infected workers can not look for a job and remain unemployed until either they recover or die. Firms with a vacancy incur a vacancy posting cost \( k > 0 \) each period. The labor market is unsegmented, i.e. firms posting a vacancy can meet unemployed workers of either skill level. The number of meetings between
firms and workers, $M_t$, is given by the aggregate meeting function $M_t = m(U_t, V_t)$, where $U_t$ is the stock of unemployed workers who are not infected at the beginning to period $t$ and $V_t$ is the stock of vacancies. The meeting function exhibits constant returns to scale and is increasing and concave in both of its arguments. Workers meet firms with probability $f(\theta_t) = m(U_t, V_t)/U_t$ where $\theta_t = V_t/U_t$ is labor market tightness. We assume $f(\theta)$ is strictly increasing in $\theta$ with $\lim_{\theta \to 0} f(\theta) = 0$ and $\lim_{\theta \to \infty} f(\theta) = 1$. Firms meet workers with probability $q(\theta_t) = m(U_t, V_t)/V_t$ where $q(\theta)$ is strictly decreasing in $\theta$, $\lim_{\theta \to 0} q(\theta) = 1$, and $\lim_{\theta \to \infty} q(\theta) = 0$. An unemployed workers’ skill level and health status is observable upon meeting the firm. Filled jobs are destroyed with an exogenous probability $s$.

**Timing.** At the beginning of each period, firms post vacancies and hire workers. After hiring takes place, high skill workers who remain unemployed or employed and infected then experience skill depreciation shocks. Workers then experience infection, recovery, and death shocks. A fraction $\mu$ of the remaining workers then leave the labor force. Finally, all remaining filled jobs are hit with separation shocks.

### 4 Accounting

In this section, we characterize the flows of workers across employment statuses, skill levels, and health statuses. Let $N_t^{\chi S}$, $N_t^{\chi I}$, and $N_t^{\chi R}$ be the measure of unemployed workers at time $t$ of skill level $\chi$ and respective health status. Further, let $E_t^{\chi S}$, $E_t^{\chi I}$, and $E_t^{\chi R}$ denote the respective measures of employed workers. The aggregate measure of unemployed and employed workers of each respective skill type is given by

$$N_t^\chi = N_t^{\chi S} + N_t^{\chi I} + N_t^{\chi R}, \quad E_t^\chi = E_t^{\chi S} + E_t^{\chi I} + E_t^{\chi R},$$

while the aggregate stocks of unemployed and employed workers across health statuses are given by

$$N_t^\chi S = N_t^{\chi S} + N_t^{H S}, \quad E_t^\chi S = E_t^{\chi S} + E_t^{H S},$$

$$N_t^\chi I = N_t^{\chi I} + N_t^{H I}, \quad E_t^\chi I = E_t^{\chi I} + E_t^{H I},$$

$$N_t^\chi R = N_t^{\chi R} + N_t^{H R}, \quad E_t^\chi R = E_t^{\chi R} + E_t^{H R},$$

where the aggregate measure of unemployed and employed workers are given by $N_t = N_t^L + N_t^H = N_t^S + N_t^I + N_t^R$ and $E_t = E_t^L + E_t^H = E_t^S + E_t^I + E_t^R$. The measures of workers of skill level $\chi$ who are susceptible ($S_t^\chi$), infected ($I_t^\chi$), and recovered ($R_t^\chi$) are given by

$$S_t^\chi = N_t^{\chi S} + E_t^{\chi S},$$

$$I_t^\chi = N_t^{\chi I} + E_t^{\chi I},$$

$$R_t^\chi = N_t^{\chi R} + E_t^{\chi R}.$$

The aggregate measures of susceptible, infected, and recovered workers are given by $S_t = S_t^L + S_t^H$, $I_t = I_t^L + I_t^H$, and $R_t = R_t^L + R_t^H$, respectively. The population at time $t$, $Pop_t$, is given by

$$Pop_t = N_t + E_t = S_t + I_t + R_t.$$
With these identities in hand, we characterize the laws of motion for unemployment and employment. Beginning with unemployment among low skill workers, we have

\[ N_{t+1}^{LS} = (1 - \mu)[(1 - f(\theta_t))(1 - \pi_t^{UI})][N_t^{LS} + \sigma N_t^{HS}] + s(1 - \pi_t^{EI})E_t^{LS}], \]  
\[ N_{t+1}^{LI} = (1 - \mu)[(1 - \pi_R - \pi_D)][N_t^{LI} + sE_t^{LI} + \sigma N_t^{HI} + \sigma sE_t^{HI}] + \pi_t^{UI}[N_t^{LS} + \sigma N_t^{HS}] + s\pi_t^{EI}E_t^{LS}], \]  
\[ N_{t+1}^{LR} = (1 - \mu)[(1 - f(\theta_t))][N_t^{LR} + \pi_R][N_t^{LI} + sE_t^{LI} + \sigma N_t^{HI} + \sigma sE_t^{HI}] + sE_t^{LR}], \]  
\[ N_{t+1}^L = (1 - \mu)[(1 - f(\theta_t))(1 - \pi_t^{UI})][N_t^{LS} + \sigma N_t^{HS}] + (1 - f(\theta_t))[N_t^{LR} + \pi_R][N_t^{HI} + \sigma sE_t^{HI}] + \theta_t^UIE_t^{L}(1 - \pi_D)]. \]

As seen in (1), the stock of susceptible unemployed low skill workers will contain a fraction $(1 - f(\theta_t))(1 - \pi_t^{UI})$ of those susceptible low skill workers who did not find a job or become infected, a fraction $(1 - f(\theta_t))(1 - \pi_t^{UI})\sigma$ of the susceptible high skill workers who remain unemployed, susceptible, and became low skilled. Additionally, the stock of susceptible unemployed low skill workers contains a fraction $s(1 - \pi_t^{EI})$ of the low skill susceptible workers who were employed, lost their job, and did not get infected. Equation (2) shows that next period’s stock of infected low skill unemployed workers is composed of a fraction $(1 - \pi_R - \pi_D)$ of those who began the period infected and remain infected, a fraction $\pi_t^{UI}$ of susceptible unemployed workers who become infected, and a fraction $s\pi_t^{EI}$ of employed susceptible workers who lose their job and become infected. From equation (3), the stock of low skill unemployed workers who are recovered contains a fraction $1 - f(\theta_t)$ of unemployed recovered workers who did not find a job, a fraction $\pi_R$ of infected workers who recover and are unemployed, and a fraction $s$ of recovered workers who are employed that lose their job. Finally, equation (4) aggregates across health statuses to describe the evolution of the aggregate stock of unemployed low skill workers.

The flows of unemployed high skill workers are given by

\[ N_{t+1}^{HS} = \mu + (1 - \mu)[(1 - \sigma)(1 - f(\theta_t))(1 - \pi_t^{UI})N_t^{HS} + s(1 - \pi_t^{EI})E_t^{HS}], \]
\[ N_{t+1}^{HI} = (1 - \mu)[(1 - \pi_R - \pi_D)(1 - \sigma)][N_t^{HI} + sE_t^{HI}] + \pi_t^{UI}[(1 - \sigma)N_t^{HS} + s\pi_t^{EI}E_t^{HS}], \]
\[ N_{t+1}^{HR} = (1 - \mu)[(1 - \sigma)((1 - f(\theta_t))N_t^{HR} + \pi_RN_t^{HI} + \pi_RS_E_t^{HI}) + sE_t^{HR}], \]
\[ N_{t+1}^H = \mu + (1 - \mu)[(1 - \sigma)((1 - f(\theta_t)(1 - \pi_t^{UI}))N_t^{HS} + (1 - f(\theta_t))N_t^{HR} + \pi_RN_t^{HI} + sE_t^{HR}],  
\[ + (1 - \pi_D)(1 - \sigma)E_t^{HR}] + sE_t^{HR} - (\pi_D + \sigma(1 - \pi_D))E_t^{HR}]. \]

Equations (5)-(8) have a similar interpretation to (1)-(3) with a few notable differences. First, as seen in (5), there is an additional flow into the stock of unemployed high skill susceptible workers, $\mu$, from new workers entering the labor force. Additionally, the stocks of unemployed high skill workers account for the possibility of skill loss among high skill workers who are either unemployed or employed and infected. From (4) and (8), aggregating across skill levels gives the aggregate flows of unemployed workers

\[ N_{t+1} = \mu + (1 - \mu)[(1 - f(\theta_t)(1 - \pi_t^{UI}))N_t^S + (1 - f(\theta_t))N_t^R + (1 - \pi_D)N_t^I + s[E_t - \pi_D E_t^I]]. \]
Next, we focus on the flows of employed workers. The flows of low skill employed workers satisfy

\[
E_{t+1}^{LS} = (1 - \mu)\left[(1 - \pi_t^{EI})(1 - s)E_t^{LS} + f(\theta_t)(1 - \pi_t^{UI})N_t^{LS}\right],
\]

(10)

\[
E_{t+1}^{LI} = (1 - \mu)(1 - s)\left[(1 - \pi_R - \pi_D)[E_t^{LI} + \sigma E_t^{HI}] + \pi_t^{EI}E_t^{LS}\right],
\]

(11)

\[
E_{t+1}^{LR} = (1 - \mu)(1 - s)\left[E_t^{LR} + \pi_R E_t^{LI} + \pi_R E_t^{HI} + f(\theta_t)N_t^{LR}\right],
\]

(12)

\[
E_{t+1}^{L} = (1 - \mu)\left[(1 - s)[E_t^{L} - \pi_D E_t^{LI} + \sigma E_t^{HI}(1 - \pi_D)] + f(\theta_t)(1 - \pi_t^{UI})N_t^{LS} + N_t^{LR}\right].
\]

(13)

Equation (10) illustrates that the stock of employed susceptible workers contains a fraction \(1 - \pi_t^{EI}(1 - s)\) of the employed susceptible workers who did not become infected and did not lose their job and a fraction \(f(\theta_t)(1 - \pi_t^{UI})\) of the unemployed susceptible workers who found a job and did not become infected. From (11), workers will remain infected and employed with probability \((1 - \pi_R - \pi_D)(1 - s)\) and susceptible employed workers enter next period’s stock of employed infected workers with probability \(\pi_t^{EI}\). Equation (12) shows that next period’s stock of employed recovered workers contains a fraction \(1 - s\) of the employed workers who have recovered and did not lose their job and a fraction \(f(\theta_t)\) of the unemployed recovered workers who find a job. Equation (13) aggregates across health statuses to illustrate the aggregate flows of employment among low skill workers.

The flow equations for employment among high skill workers are given by

\[
E_{t+1}^{HS} = (1 - \mu)\left[(1 - \pi_t^{EI})(1 - s)E_t^{HS} + f(\theta_t)(1 - \pi_t^{UI})N_t^{HS}\right],
\]

(14)

\[
E_{t+1}^{HI} = (1 - \mu)(1 - s)\left[(1 - \sigma)(1 - \pi_R - \pi_D)E_t^{HI} + \pi_t^{EI}E_t^{HS}\right],
\]

(15)

\[
E_{t+1}^{HR} = (1 - \mu)(1 - s)\left[E_t^{HR} + (1 - \sigma)\pi_R E_t^{HI} + f(\theta_t)N_t^{HR}\right],
\]

(16)

\[
E_{t+1}^{H} = (1 - \mu)(1 - s)\left[E_t^{H} - (\sigma(1 - \pi_D) + \pi_D)E_t^{HI} + f(\theta_t)(1 - \pi_t^{UI})N_t^{HS} + N_t^{HR}\right].
\]

(17)

where the main difference to equations (10)-(13) is that high skill workers can experience skill loss while they are employed and infected.

From (13) and (17), the aggregate flows of employment are given by

\[
E_{t+1} = (1 - \mu)\left[(1 - s)[E_t - \pi_D E_t^I] + f(\theta_t)(1 - \pi_t^{UI})N_t^{I} + N_t^{R}]\right].
\]

(18)

Using \(S_t^{x} = N_t^{xS} + E_t^{xS}\), the flows of susceptible workers by skill level and in aggregate are given by

\[
S_{t+1}^{L} = (1 - \mu)\left[(1 - \pi_t^{UI})N_t^{LS} + (1 - \pi_t^{ EI})E_t^{LS} + \sigma(1 - f(\theta_t))(1 - \pi_t^{UI})N_t^{HS}\right],
\]

(19)

\[
S_{t+1}^{H} = \mu + (1 - \mu)\left[(1 - \pi_t^{UI})N_t^{HS} + (1 - \pi_t^{EI})E_t^{HS} - \sigma(1 - f(\theta_t))(1 - \pi_t^{HS})N_t^{HS}\right],
\]

(20)

\[
S_{t+1} = \mu + (1 - \mu)\left[(1 - \pi_t^{UI})S_t - (\pi_t^{EI} - \pi_t^{UI})E_t^{S}\right],
\]

(21)
while the dynamics for infections are given by

\[ I_{t+1}^L = (1 - \mu)[(1 - \pi_R - \pi_D)I_t^L + \sigma I_t^H + \pi_t^U [N_t^{LS} + \sigma N_t^{HS}] + \pi_t^{EI} E_t^{LS}], \]

\[ I_{t+1}^H = (1 - \mu)[(1 - \sigma)(1 - \pi_R - \pi_D)I_t^H + \pi_t^U N_t^{HS} + \pi_t^{EI} E_t^{HS}], \]

\[ I_{t+1} = (1 - \mu)[(1 - \pi_R - \pi_D)I_t + \pi_t^U S_t + (\pi_t^{EI} - \pi_t^U) E_t^S], \]

and the dynamics for recoveries are given by

\[ R_{t+1}^L = (1 - \mu)[R_t^L + \pi_R I_t^L + \sigma[(1 - f(\theta_t))N_t^{HR} + \pi_R N_t^{HI} + \pi_R s E_t^{HI}]], \]

\[ R_{t+1}^H = (1 - \mu)[R_t^H + \pi_R I_t^H - \sigma[(1 - f(\theta_t))N_t^{HR} + \pi_R N_t^{HI} + \pi_R s E_t^{HI}]], \]

\[ R_{t+1} = (1 - \mu)[R_t + \pi_R I_t]. \]

Letting \( D_t \) denote the total number of deaths from the pandemic at time \( t \), it follows that

\[ D_{t+1} = D_t + \pi_D I_t. \]

Finally, the population evolves according to

\[ Pop_{t+1} = Pop_t - (D_{t+1} - D_t). \]

5 Equilibrium

5.1 Bellman Equations

Let \( U_t^{LS}, U_t^{LI}, \) and \( U_t^{LR} \) denote the lifetime discounted utility of an unemployed worker with skill level \( \chi \) who is susceptible, infected, and recovered. Further, let \( W_t^{LS}, W_t^{LI}, \) and \( W_t^{LR} \) denote the lifetime discounted utility of an employed worker with skill level \( \chi \) and respective health status. We normalize the value of death to 0. The value functions for low skill unemployed workers are given by

\[ U_t^{LS} = b + \beta \{ f(\theta_t)(1 - \pi_t^U) W_t^{LS} + \pi_t^U U_{t+1}^{LI} + (1 - f(\theta_t))(1 - \pi_t^U) U_{t+1}^{LS} \}, \]

\[ U_t^{LI} = b + \beta \{ (1 - \pi_R - \pi_D) U_{t+1}^{LI} + \pi_R U_{t+1}^{LR} \}, \]

\[ U_t^{LR} = b + \beta \{ f(\theta_t) W_t^{LR} + (1 - f(\theta_t)) U_{t+1}^{LR} \}, \]

where \( \beta \equiv \beta(1 - \mu) \). From (30), unemployed low skill workers who are susceptible enjoy utility \( b \). With probability \( f(\theta_t)(1 - \pi_t^U) \) they find a job and do not become infected. They become infected and remain unemployed with probability \( \pi_t^U \). With probability \( (1 - f(\theta_t))(1 - \pi_t^U) \) they remain unemployed and susceptible. Equation (31) shows that a low skill unemployed worker who is infected has utility \( b \). With probability \( (1 - \pi_R - \pi_D) \) they remain infected and recover with probability \( \pi_R \). Recall that infected workers can not search for jobs, so they remain unemployed even if they recover. As for recovered workers, (32) shows that they face a standard labor search problem where they either find a job with probability \( f(\theta_t) \) or
do not with complementary probability. Recovered workers do not face the probability of infection as they have gained immunity.

The value functions of high skill unemployed workers are given by

\begin{equation}
U_t^{HS} = b + \bar{\theta} \{ f(\theta_t) (1 - \pi_t^{UI}) W_{t+1}^{HS} + \sigma [\pi_t^{UI} W_{t+1}^{LI} + (1 - f(\theta_t)) (1 - \pi_t^{UI}) U_{t+1}^{LS}] + (1 - \sigma) [\pi_t^{UI} W_{t+1}^{HI} + (1 - f(\theta_t)) (1 - \pi_t^{UI}) U_{t+1}^{HS}] \},
\end{equation}

(33)

\begin{equation}
U_t^{HI} = b + \bar{\theta} \{ \sigma [(1 - \pi_R - \pi_D) U_{t+1}^{LI} + \pi_R U_{t+1}^{LR}] + (1 - \sigma) [(1 - \pi_R - \pi_D) U_{t+1}^{HI} + \pi_R U_{t+1}^{HR}] \},
\end{equation}

(34)

\begin{equation}
U_t^{HR} = b + \bar{\theta} \{ f(\theta_t) W_{t+1}^{HR} + (1 - f(\theta_t)) [\sigma U_{t+1}^{LR} + (1 - \sigma) U_{t+1}^{HR}] \}.
\end{equation}

(35)

Equations (33)-(35) have a very similar interpretation as (30)-(32) in terms of the transitions between health statuses and employment statuses. However, an important difference is the possibility of skill loss. Equation (33) shows that if the worker does not find a job, then with probability \(\sigma\) they become low skilled and face the possibility of becoming infected. Equations (34) and (35) illustrate that unemployed high skill workers who are infected or recovered continue to face the possibility of skill loss.

Turning to the value functions for employed low skill workers, they are given by

\begin{equation}
W_t^{LS} = w_t^{LS} + \bar{\theta} \{ \pi_t^{EI} [(1 - s) W_{t+1}^{LI} + u_t^{LI} + (1 - \pi_t^{EI}) [(1 - s) W_{t+1}^{LS} + u_t^{LS}] \},
\end{equation}

(36)

\begin{equation}
W_t^{LI} = w_t^{LI} + \bar{\theta} \{ (1 - \pi_R - \pi_D) [(1 - s) W_{t+1}^{LI} + u_t^{LI} + \pi_R [(1 - s) W_{t+1}^{LR} + u_t^{LR}] \},
\end{equation}

(37)

\begin{equation}
W_t^{LR} = w_t^{LR} + \bar{\theta} \{ (1 - s) W_{t+1}^{LR} + u_t^{LR} \}.
\end{equation}

(38)

Equation (36) details that employed low skill workers who are susceptible earn a wage \(w_t^{LS}\) and with probability \(\pi_t^{EI}\) become infected while working. Conditional on getting infected, they remain employed with probability \(1 - s\). If the worker does not become infected, they still face the possibility of losing their job and transitioning to unemployment. From (37), employed low skill workers earn their wage, \(w_t^{LI}\). The worker remains infected with probability \((1 - \pi_R - \pi_D)\) and recovers with probability \(\pi_R\). Conditional on surviving, they remain employed with probability \(1 - s\). Equation (38) shows that recovered workers face a standard problem, as they only face the possibility of losing their job.

The value functions for high skill employed workers are given by

\begin{equation}
W_t^{HS} = w_t^{HS} + \bar{\theta} \{ \pi_t^{EI} [(1 - s) W_{t+1}^{HI} + u_t^{HI} + (1 - \pi_t^{EI}) [(1 - s) W_{t+1}^{HS} + u_t^{HS}] \},
\end{equation}

(39)

\begin{equation}
W_t^{HI} = w_t^{HI} + \bar{\theta} \{ \sigma [(1 - \pi_R - \pi_D) [(1 - s) W_{t+1}^{HI} + u_t^{HI} + \pi_R [(1 - s) W_{t+1}^{HR} + u_t^{HR}] \} + (1 - \sigma) [(1 - \pi_R - \pi_D) [(1 - s) W_{t+1}^{HI} + u_t^{HI} + \pi_R [(1 - s) W_{t+1}^{HR} + u_t^{HR}] \},
\end{equation}

(40)

\begin{equation}
W_t^{HR} = w_t^{HR} + \bar{\theta} \{ (1 - s) W_{t+1}^{HR} + u_t^{HR} \}.
\end{equation}

(41)

Equations (39)-(41) have the same interpretation as (36)-(38) except that high skill workers face the risk of skill loss while they are employed and infected.

We now shift our attention to the firms’ value functions. Let \(V_t\) denote the value of a vacancy and \(J_t^{S}, J_t^{I},\) and \(J_t^{R}\) the value a filled job with a worker of skill level \(\chi\) and respective health status. Additionally,
we introduce some notation to describe the composition of job seekers. Let \( \varphi_t \) denote the share of job seekers with low skills and \( \phi_t^\chi \) the share of job seekers with skill level \( \chi \) who are susceptible. The value of a vacancy satisfies

\[
V_t = -k + \bar{\beta} \{ q(\theta_t) \left[ \varphi_t \left[ \phi_t^L (1 - \pi_t^{UI}) J_{t+1}^{LS} + (1 - \phi_t^L) J_{t+1}^{LR} \right] + (1 - \varphi_t) \left[ \phi_t^H (1 - \pi_t^{UI}) J_{t+1}^{HS} + (1 - \phi_t^H) J_{t+1}^{HR} \right] \right] \\
+ \left( 1 - q(\theta_t) \right) \left[ \varphi_t \left( 1 - \phi_t^L \pi_t^{UI} \right) + (1 - \varphi_t) \left( 1 - \phi_t^H \pi_t^{UI} \right) \right] \} V_{t+1} + \beta \mu V_{t+1}. \tag{42}
\]

Equation (42) shows that vacant firms incur the vacancy posting cost \( k \) and meet a worker with probability \( q(\theta_t) \). Conditional on meeting a worker, the firm meets a low skill worker with probability \( \varphi_t \) and high skill worker with probability \( 1 - \varphi_t \). Among meetings with a worker of skill type \( \chi \), firms match with a susceptible worker with probability \( \phi_t^\chi (1 - \pi_t^{UI}) \), which accounts for the risk that a susceptible worker they meet becomes infected, and a recovered worker with probability \( 1 - \phi_t^\chi \). The firm continues to have a vacancy either if it does not meet a worker or meet a worker who became infected in the same time period.

The value functions for filled jobs with low skill workers are given by

\[
J_{t}^{LS} = \delta y - w_t^{LS} + \bar{\beta} \left\{ \pi_t^{EI} (1 - s) J_{t+1}^{LI} + (1 - \pi_t^{EI}) (1 - s) J_{t+1}^{LS} + s V_{t+1} \right\} + \beta \mu V_{t+1}, \tag{43}
\]

\[
J_{t}^{LI} = -w_t^{LI} + \bar{\beta} \left\{ (1 - \pi_t - \pi_D) (1 - s) J_{t+1}^{LI} + \pi_D (1 - s) J_{t+1}^{LR} + (\pi_D + s (1 - \pi_D)) V_{t+1} \right\} + \beta \mu V_{t+1}, \tag{44}
\]

\[
J_{t}^{LR} = \delta y - w_t^{LR} + \bar{\beta} \left\{ (1 - s) J_{t+1}^{LR} + s V_{t+1} \right\} + \beta \mu V_{t+1}. \tag{45}
\]

From (43), a filled job with a low skill susceptible worker generates a profit of output net of the worker’s wage, \( \delta y - w_t^{LS} \). The worker becomes infected and the job is not destroyed with probability \( \pi_t^{EI} (1 - s) \). The probability that the worker remains susceptible and the job is not destroyed is given by \( (1 - \pi_t^{EI}) (1 - s) \). Equation (44) illustrates that employed workers who are infected do not generate output and earn a wage \( w_t^{LI} \). If the job is not destroyed, the worker remains infected with probability \( 1 - \pi_t - \pi_D \) or recovers with probability \( \pi_D \) and the employment relationship continues with probability \( 1 - s \). With probability \( \pi_D + s (1 - \pi_D) \), either the infected worker dies or the worker survives and the job is destroyed. In either case, the firm returns to having a vacancy. Finally, (45) represents that a filled job with a recovered worker is standard, as the worker’s health status no longer changes.

The value functions for filled jobs with high skill workers are given by

\[
J_{t}^{HS} = y - w_t^{HS} + \bar{\beta} \left\{ \pi_t^{EI} (1 - s) J_{t+1}^{HI} + (1 - \pi_t^{EI}) (1 - s) J_{t+1}^{HS} + s V_{t+1} \right\} + \beta \mu V_{t+1}, \tag{46}
\]

\[
J_{t}^{HI} = -w_t^{HI} + \bar{\beta} \left\{ \sigma [(1 - \pi_t - \pi_D) (1 - s) J_{t+1}^{LI} + \pi_D (1 - s) J_{t+1}^{LR}] + (1 - \sigma) [(1 - \pi_t - \pi_D) (1 - s) J_{t+1}^{HI} + \pi_D (1 - s) J_{t+1}^{HR}] + (\pi_D + s (1 - \pi_D)) V_{t+1} \right\} + \beta \mu V_{t+1}, \tag{47}
\]

\[
J_{t}^{HR} = y - w_t^{HR} + \bar{\beta} \left\{ (1 - s) J_{t+1}^{HR} + s V_{t+1} \right\} + \beta \mu V_{t+1}. \tag{48}
\]

Equations (46)-(48) are the same as (43)-(45) with the exception of the possibility of high skill workers suffering a loss of skill while they are employed and infected.
5.2 Pre-pandemic Steady-State

In this section we study the steady-state equilibrium in the labor market before the onset of the pandemic. We start by introducing the free-entry condition for vacancy creation, wage determination, and the steady-state distribution of workers. The equilibrium is then defined and characterized.

There is free entry of firms, which drives the value of a vacancy to zero in equilibrium. As is standard in the literature, wages are determined by Nash bargaining. Denoting $\eta \in [0, 1]$ as the worker’s bargaining power, wages solve:

$$w^\chi = \arg \max \left[ W^\chi - U^\chi \right] \eta \left[ J^\chi \right]^{1-\eta}. \quad (49)$$

Letting $F^\chi = J^\chi + W^\chi - U^\chi$ denote the total surplus of a match between a firm and a worker of skill level $\chi$, the solution to (49) gives the following surplus sharing rules

$$W^\chi - U^\chi = \eta F^\chi; \quad J^\chi = (1 - \eta)F^\chi. \quad (50)$$

Using the Bellman equations, surplus sharing rules, and letting $\Delta^{H,L} \equiv U^H - U^L$ denote the cost of skill-loss, we have

$$F^L = \frac{\delta y - b}{1 - \beta (1 - s - \eta f(\theta))}, \quad (51)$$

$$F^H = \frac{\gamma y - b + \bar{\beta}(1 - f(\theta))\sigma\Delta^{H,L}}{1 - \beta (1 - s - \eta f(\theta))}. \quad (52)$$

From (52), the surplus in a match with a high skill worker is increasing in the probability of skill loss, $\sigma$, as the cost of skill loss, $\Delta^{H,L}$, reduces the worker’s reservation wage. Substituting (51)-(52) into (50) and solving for the wages gives

$$w^L = \frac{\eta \delta y [1 - \bar{\beta}(1 - s - f(\theta))] + (1 - \eta) b [1 - \bar{\beta}(1 - s)]}{1 - \beta (1 - s - \eta f(\theta))}, \quad (53)$$

$$w^H = \frac{\eta \gamma [1 - \bar{\beta}(1 - s - f(\theta))] + (1 - \eta) b [1 - \bar{\beta}(1 - s - f(\theta))\sigma\Delta^{H,L}] [1 - \bar{\beta}(1 - s)]}{1 - \beta (1 - s - \eta f(\theta))}. \quad (54)$$

Using the free entry condition, $V = 0$, and substituting (50)-(52) into the Bellman for vacancies, (42), we have the job creation condition

$$\frac{k}{q(\theta)} = \frac{\bar{\beta} (1 - \eta) \left[ \varphi (\delta y - b) + (1 - \varphi) \left( y - b + \bar{\beta} \sigma (1 - f(\theta))\Delta^{H,L} \right) \right]}{1 - \beta (1 - s - \eta f(\theta))}, \quad (55)$$

which illustrates that firms create jobs until the expected cost from posting a vacancy, the left hand side of (55), equals the expected value of filling a vacancy, the right hand side of (55). In the pre-pandemic steady-state, the expected value of a filled job captures the heterogeneous skills among unemployed workers.

From the flow equations in Section 4, the fraction of unemployed workers who are less-skilled, $\varphi$, is given by

$$\varphi = \frac{\sigma (1 - \mu)(1 - f(\theta))(1 - (1 - \mu)(1 - s)]}{\mu (1 - \mu) f(\theta) + [\mu + (1 - \mu)(1 - f(\theta))\sigma][1 - (1 - \mu)(1 - s)]}, \quad (56)$$

From (56), $\varphi$ is increasing in the probability of skill loss, $\sigma$, as an increase in the risk of skill loss increases the flow of high skill unemployed workers to low skill unemployed workers. The composition, $\varphi$, is also
increasing in the separation probability, \( s \), as having more workers entering unemployment from employment exposes more high skill workers to the risk of skill loss. Also, \( \varphi \) is decreasing in market tightness, \( \theta \). If firms create more jobs, then high-skill workers are more likely to exit unemployment and avoid the risk of skill loss while unemployed. The opposite is also true: if there is a downturn and less jobs are created, then high skill workers face more opportunities for skill loss, leading to a higher fraction among the pool of unemployed who are less-skilled.

We close the model with the steady-state unemployment rate:

\[
    u = \frac{\mu + (1 - \mu)s}{\mu + (1 - \mu)(s + f(\theta))}.
\]

**Definition 1.** A steady-state equilibrium is a tuple \( \{\theta, \varphi, u\} \) such that market tightness, \( \theta \), satisfies (55), the fraction of unemployed workers who are less-skilled, \( \varphi \), is given by (56), and the unemployment rate, \( u \), is given by (57).

**Proposition 1.** Assume that \( \delta y > b \) and

\[
    k < \frac{\bar{\beta}(1-\eta)\sigma(1-\mu)(\delta y - b) + \mu(y - b)}{(1 - \beta(1-s))\mu + (1 - \mu)\sigma}.
\]

There exists an active steady-state equilibrium with \( \theta > 0 \).

As in Pissarides (1992), the equilibrium with loss of skill during unemployment may not be unique. This is due to the fact that as firms create more jobs, the skill composition of the unemployed improves, which means the right hand side of the job creation condition can be upward sloping. This occurs, quantitatively, only under extreme and unrealistic parameter values. With a characterization of the pre-pandemic economy in hand, we turn to the equilibrium during a pandemic.

### 5.3 Equilibrium during a Pandemic

In this section, we describe the equilibrium in the labor market after the onset of a pandemic. As before, wages continued to be determined through Nash bargaining. Additionally, we assume that wages are renegotiated each period. Letting \( \Omega \in \{LS, LI, LR, HS, HI, HR\} \) denote the worker’s skill and health status, wages solve

\[
    w_\Omega^t = \arg \max \left[ W_\Omega^t - U_\Omega^t \right] \frac{\eta}{1 - \eta} \left[ J_\Omega^t \right]^{1 - \eta}.
\]

The solution to (59) gives the surplus sharing rules

\[
    W_\Omega^t - U_\Omega^t = \eta F_\Omega^t; \quad J_\Omega^t = (1 - \eta)F_\Omega^t.
\]

where \( F_\Omega^t = J_\Omega^t + W_\Omega^t - U_\Omega^t \) is the total surplus of a match. To characterize the entry of firms, it will be useful to describe the evolution of match surpluses over time. Combining the surplus sharing rules with the
Bellman equations, we can write the law of motions for the total surpluses of each match:

\[ F_t^{LS} = \delta y - b + \bar{\beta}\{\pi_t^{EI}(1-s)F_{t+1}^{LL} + [(1-s)(1-\pi_t^{EI}) - \eta f(\theta_t)(1-\pi_t^{UI})]F_{t+1}^{LS} \]
\[ + (\pi_t^{EI} - \pi_t^{UI})\Delta_{t+1}^{LL,LS}\}, \]  

\[ F_t^{HS} = y - b + \bar{\beta}\{\pi_t^{EI}(1-s)F_{t+1}^{HI} + [(1-s)(1-\pi_t^{EI}) - \eta f(\theta_t)(1-\pi_t^{UI})]F_{t+1}^{HS} \]
\[ + (\pi_t^{EI} - \pi_t^{UI})\Delta_{t+1}^{HI,HS} + \sigma[\pi_t^{UI}\Delta_{t+1}^{HI,LI} + (1-\pi_t^{UI})(1-f(\theta_t))\Delta_{t+1}^{HS,LS}\}, \]  

\[ F_t^{LI} = -b + \bar{\beta}(1-s)\{(1-\pi_R - \pi_D)F_{t+1}^{LI} + \pi_R F_{t+1}^{LR}\}, \]

\[ F_t^{HR} = -b + \bar{\beta}(1-s)\{\sigma[(1-\pi_R - \pi_D)F_{t+1}^{HI} + \pi_R F_{t+1}^{HR}] + (1-\sigma)[(1-\pi_R - \pi_D)F_{t+1}^{HI} + \pi_R F_{t+1}^{HR}]\}, \]

\[ F_t^{LR} = \delta y - b + \bar{\beta}[1-\sigma f(\theta_t)]F_{t+1}^{LR}, \]

\[ F_t^{HR} = y - b + \bar{\beta}\{(1-s - \eta f(\theta_t))F_{t+1}^{HR} + \sigma(1-f(\theta_t))\Delta_{t+1}^{HR,LR}\}, \]

where \( \Delta_t^{\Omega,\Omega'} \equiv U_t^{\Omega'} - U_t^{\Omega} \) represent the difference in lifetime utility between state \( \Omega' \) and state \( \Omega \). From the Bellman equations, they satisfy

\[ \Delta_t^{H,LS} = \bar{\beta}\{\eta f(\theta_t)(1-\pi_t^{UI})[F_t^{HS} - F_t^{LS}] + (1-\sigma)\pi_t^{UI}\Delta_{t+1}^{HI,LI} + \]
\[ (1-\pi_t^{UI})[1-(1-f(\theta_t))\sigma]\Delta_{t+1}^{HS,LS}\}, \]

\[ \Delta_t^{LI,LS} = \bar{\beta}\{(1-\pi_t^{UI})[\Delta_{t+1}^{LI,LS} - \eta f(\theta_t)F_t^{LS}] + \pi_R \Delta_{t+1}^{LR,LI} - \pi_D U_t^{LI}\}, \]

\[ \Delta_t^{HI,HS} = \bar{\beta}\{(1-\pi_t^{UI})[\Delta_{t+1}^{HI,HS} - \eta f(\theta_t)F_t^{HS}] + \pi_R \Delta_{t+1}^{HR,HI} - \pi_D U_t^{HI}\]
\[ + \sigma[\pi_R \Delta_{t+1}^{HR,LR} + (1-f(\theta_t))(1-\pi_t^{UI})\Delta_{t+1}^{HS,LS} + (1-\pi_t^{UI} - \pi_R - \pi_D)\Delta_{t+1}^{LI,HI}\}], \]

\[ \Delta_t^{HI,LI} = \bar{\beta}(1-\sigma)\{(1-\pi_R - \pi_D)\Delta_{t+1}^{HI,LI} + \pi_R \Delta_{t+1}^{HR,LR}\}, \]

\[ \Delta_t^{LR,LI} = \bar{\beta}\{\eta f(\theta_t)F_{t+1}^{LR} + (1-\pi_R)\Delta_{t+1}^{LR,LI} + \pi_D U_t^{LI}\}, \]

\[ \Delta_t^{HR,HI} = \bar{\beta}\{\eta f(\theta_t)F_{t+1}^{HR} + (1-\pi_R)\Delta_{t+1}^{HR,HI} + \pi_D U_t^{HI}\]
\[ + \sigma[(1-f(\theta_t) - \pi_R)\Delta_{t+1}^{LR,HR} - (1-\pi_R - \pi_D)\Delta_{t+1}^{HI,LI}\}], \]

\[ \Delta_t^{HR,LR} = \bar{\beta}\{\eta f(\theta_t)[F_{t+1}^{HR} - F_{t+1}^{LR}] + (1-\sigma(1-f(\theta_t)))\Delta_{t+1}^{HR,LR}\} \]

The last step before arriving at the job creation condition is to describe the composition of job seekers by skill level and health status. The fraction of job seekers with low-skills is given by

\[ \varphi_t = \frac{S_t^L + R_t^L}{S_t + R_t} \quad 1 - \varphi_t = \frac{S_t^H + R_t^H}{S_t + R_t}, \]

where \( S_t + R_t \) is the total measure of job seekers as infected workers do not search. The fraction of job
seekers of skill level $\chi$ who are susceptible is given by

$$
\phi^\chi_t = \frac{N^\chi_S}{N^\chi_S + N^\chi_R}.
$$

(75)

Under the free-entry condition, the value of a vacancy is zero at all time periods, i.e. $V_t = 0$, $\forall t \in \mathbb{N}_0$. This gives the following job creation condition to relate the expected cost of a vacancy to the expected surplus of a filled job:

$$
\frac{k}{q(\theta_t)} = \beta(1-\eta)\{\varphi_t[\phi^L_t(1-\pi^U_t)F^L_{t+1} + (1-\phi^L_t)F^L_{t+1}] + (1-\varphi_t)[\phi^H_t(1-\pi^U_t)F^H_{t+1} + (1-\phi^H_t)F^H_{t+1}]\}.
$$

(76)

From (76), firms do not only consider the skill composition of unemployed workers, but also the composition of job seekers and the probability a susceptible worker they meet becomes infected.

**Definition 2.** An equilibrium is a sequence of worker allocations across labor market and health statuses \{\(N^l_t, E^l_t, N^\chi_t, E^\chi_t, N_t, E_t, S_t, I_t, R_t, D_t\}\}_{t=0}^\infty, composition of job seekers \{\(\varphi_t, \phi^\chi_t\)\}_{t=0}^\infty, match surpluses \{F^\Omega_t\}_{t=0}^\infty, and market tightness \{\theta_t\}_{t=0}^\infty for \(\chi \in \{L, H\}\) and \(\Omega \in \{LS, LI, LR, HS, HI, HR\}\) such that the allocation of workers across labor market statuses evolve according to (1)-(18), the allocation of workers across health statuses evolves according to (19)-(28), the composition of job seekers is given by (74)-(75), match surpluses satisfy (61)-(66), and market tightness satisfies (76).

We assume the labor market is initially in the Pre-Pandemic steady-state, where market tightness solves (55), the composition of skills is given by (56), and the unemployment rate is given by (57). To introduce a pandemic, the initial allocation across health statuses is given by \{\(N^\chi_0, E^\chi_0, N^\chi_0, E^\chi_0\)\} and \(\{N^\chi_R, E^\chi_R\} = \{0, 0\}\) for \(\chi \in \{L, H\}\) where the initial number of infected, \(\sum_\chi [N^\chi_I + E^\chi_I]\), is a small fraction of the population.

### 6 Quantitative Analysis

#### 6.1 Calibration Strategy

A unit of time is one week. The discount factor is $\beta = 0.99^{1/52}$. The weekly separation probability is set to $s = 0.035/(52/12)$. The weekly probability of leaving the labor force is $\mu = 1/2080$, which corresponds to being in the labor force on average for 40 years. We normalize the output produced by high-skill workers to one, i.e. $y = 1$. Following Hall and Milgrom (2008), the value of unemployment, $b$, is set so that the ratio of $b$ to average wages is equal to 0.71. With this strategy, we find $b = 0.5203$. The matching function is Cobb-Douglas

$$
M_t = AU_t^\alpha V_t^{1-\alpha},
$$

(77)

where we set the matching efficiency, $A$, to target a weekly job-finding probability of 0.45/(52/12). Combined with normalizing steady-state market tightness to one as in Shimer (2005), we have $A = 0.1038$ and $k = 0.3047$. Based on Petrongolo and Pissarides (2001) and Pissarides (2009), the elasticity of the matching function, $\alpha$, is set to 0.5 and we subsequently assume $\eta = 0.5$ to implement the Hosios (1990) condition.

The remaining labor market parameters are the probability of skill loss, $\sigma$, and the output produced by low-skill workers, $\delta$. Following the previous literature on skill loss (e.g., Laureys (2020) and Ortego-Marti (2016)), we calibrate these parameters to match the empirical evidence on the effect of unemployment.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9998</td>
</tr>
<tr>
<td>$y$</td>
<td>Productivity of high-skill workers</td>
<td>1.0000</td>
</tr>
<tr>
<td>$s$</td>
<td>Separation probability</td>
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<td>$\mu$</td>
<td>Probability of exiting the labor force</td>
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<td>$A$</td>
<td>Matching efficiency</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>Elasticity of the matching function</td>
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</tr>
<tr>
<td>$\eta$</td>
<td>Worker’s bargaining power</td>
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</tr>
<tr>
<td>$\sigma$</td>
<td>Probability of skill loss</td>
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<tr>
<td>$\delta$</td>
<td>Productivity of low-skill workers</td>
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</tr>
<tr>
<td>$k$</td>
<td>Vacancy posting cost</td>
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<tr>
<td>$b$</td>
<td>Value of unemployment</td>
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</tr>
<tr>
<td>$\pi_D$</td>
<td>Probability of death from infection</td>
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</tr>
<tr>
<td>$\pi_R$</td>
<td>Probability of recovery</td>
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<tr>
<td>$\pi_U$</td>
<td>Infection exposure of unemployed workers</td>
<td>0.1953</td>
</tr>
<tr>
<td>$\pi_E$</td>
<td>Infection exposure of employed workers</td>
<td>0.6783</td>
</tr>
</tbody>
</table>

Table 1: Parameter values

duration on wages. However, as discussed in Laureys (2020), this empirical evidence can not be used to choose a unique value of both $\sigma$ and $\delta$. Thus, we set $\sigma = 1/13$, which corresponds to skill loss taking 3 months on average and is well supported by the empirical evidence on how quickly skill loss occurs.\(^1\) Given this value of $\sigma$, we then choose $\delta$ to match the estimated effects of unemployment duration on wages. That is, we choose a value of $\delta$, and given the pre-pandemic steady-state wages across skill levels and transition probabilities between employment and unemployment, we simulate 10,000 employment histories and estimate the following regression:

$$\ln(wage) = \beta_0 + \beta_1 \times Unhis + \epsilon,$$

(78)

where $Unhis$ is the length of the unemployment spell in months and $\ln(wage)$ are log wages. For each simulated employment histories, we compute $\beta_1$ and repeat this process 100 times where then have an average estimate of $\beta_1$. We vary $\delta$ and repeat this exercise until our average estimate of $\beta_1$ is $-0.012$, which is well in line with empirical estimates of the effect of unemployment history on wages (Ortego-Marti, 2016; Schmieder et al., 2016). Through this procedure, we find $\delta = 0.725$.

There are four health parameters to calibrate. We follow Eichenbaum et al. (2020) and set the recovery probability as $\pi_R = 0.3850$ and the death probability to be $\pi_D = 0.0039$. Finally, we follow Kapička and Rupert (2020) and set $\pi_U = 0.1953$ and $\pi_E = 0.6783$ who calibrate the ratio $\pi_U / \pi_E$ to match data on the relative amount of social interactions unemployed and employed workers have and to target a steady-state value of infected and recovered to be two-thirds.\(^2\) Table 1 summarizes the parameter values.

\(^1\)See Ortego-Marti (2016, 2017b,c) for evidence from the PSID regarding how quickly human capital depreciates during unemployment and how losses vary across occupations and sectors. In the Appendix, we recalibrate the model for two different cases: one where skill loss occurs on average in 1 month and a second case where it takes an average of 6 months for loss of skill to occur. We perform the same quantitative exercises under these alternative calibrations and show the quantitative results are robust to the choice of $\sigma$.

\(^2\)The data on number of social interactions across unemployed and employed workers is based on a Gallup survey after the onset of the COVID-19 pandemic to take into account social distancing. See Kapička and Rupert (2020) for more details.
6.2 Baseline Results

We assume the economy is in the pre-pandemic steady-state and the population is normalized to one. We then introduce the onset of a pandemic by assuming $0.001\%$ of the population becomes infected.

Figure 1(a) demonstrates the spread of the infection by showing the fraction of the population that is infected in each week. Infections peak in weeks 27-28 where $8.72\%$ of the population is infected. After one year, the fraction of the population infected is well below $1\%$ and approaches $0\%$ thereafter. Figure 1(b) illustrates the cumulative amount of deaths throughout the pandemic. The amount of deaths levels off after one year, at $0.65\%$ of the population. With the U.S. population estimated at 328.2 million, this corresponds to a tragic death toll of 2,133,300.

![Figure 1: Total Infections and Deaths](image)

Next, Figure 2 shows the connection between the pandemic and the labor market by presenting the infection probabilities across employment statuses. It is not surprising, given that $\pi^E > \pi^U$, the probability of becoming infected is larger for employed workers than those who are unemployed. At the peak of the pandemic, the probability of becoming infected for employed workers is $5.92\%$, whereas it is $1.70\%$ for unemployed workers.

Figure 3(a) demonstrates the dynamics of market tightness throughout the pandemic. As employed workers have a higher chance of becoming infected, and not producing output while infected, market tightness immediately declines at the onset of the pandemic from 1 to 0.7416. As the pandemic becomes worse and infections increase, market tightness further decreases until it reaches its lowest value of 0.3161 after 22 weeks. As the pandemic starts to recede and the number of infections decreases, job creation slowly recovers. Figure 3(b) presents the corresponding dynamics of unemployment. Given the effect of the pandemic on market tightness, the job-finding probability decreases the unemployment increases. The unemployment rate peaks at $11.4\%$ after 30 weeks and slowly declines thereafter.

From Figure 3 there are long-lasting effects of the pandemic on market tightness and unemployment for many months even after the number of infections is essentially zero. Figure 4 examines this in further detail by studying the effect of the pandemic on the fraction of the unemployed who are low-skill ($\varphi$). As seen in Figure 4(a), the composition deteriorates over the course of the pandemic until the fraction of unemployed workers who are low-skilled peaks after 39 weeks. Moreover, the composition is very slow to recover and remains at an elevated level 100 weeks after the onset of the pandemic.
We conclude our baseline results by showing the effect of the pandemic on TFP, which we define as average labor productivity. That is,

\[
TFP = \frac{y[\delta(E^{LS} + E^{LR}) + (E^{HS} + E^{HR})]}{E^{LS} + E^{LR} + E^{HS} + E^{HR}}.
\]  

(79)

As the skill level of the unemployed worsens over the pandemic, as seen in Figure 4(a), the composition of employed workers shifts to more low-skill workers whose productivity is \(\delta y\), causing TFP to decrease. Figure 4(b) illustrates the scarring effects of a pandemic on TFP. We see that TFP slowly declines through the pandemic and follows closely the dynamics of the composition of unemployed. TFP reaches its lowest value after 55 weeks, where it is 0.44% below the pre-pandemic steady-state value. We also see that TFP is slow to recover, as it is still 0.4% below the pre-pandemic steady-state value after 100 weeks.

How does a 0.44% decline in TFP compare with previous recessions? To investigate, we calculate the
average decline in TFP in U.S. recessions between 1954-2017 and find that TFP typically decreases by 1.13% in recessions.\textsuperscript{3} Thus, our baseline results generate a decline in TFP that is nearly 39\% of the typical productivity losses seen in past recessions.

6.3 Separation shock

To simulate a lockdown, we increase the separation probability from $s = 0.0081$ to $s = 0.0173$ (a monthly separation probability of 0.075) at the onset of the pandemic.\textsuperscript{4} We study a three month lockdown by assuming the separation probability remains at the elevated level for three months before returning to $s = 0.0081$.

Figures 5-6 illustrate the effect of imposing a three month lockdown on the evolution of the pandemic. Beginning with Figure 5(a), increasing job separations “flattens the curve” as the fraction of the population that is infected peaks at 7.82\% in week 29, as opposed to a peak of 8.72\% a few weeks earlier in the baseline results. Figure 5(b) shows that lower infections results in less total deaths, as cumulative amount of deaths decreases from 0.65\% of the population to 0.63\%, saving 65,640 lives. From Figure 6, the lockdown reduces the peak infection probability among employed workers from 5.92\% to 5.30\%, while the peak infection probability among unemployed workers decreases 1.7\% to 1.53\%.

Figure 7 demonstrates the impact of the separation shock on market tightness and the unemployment rate. Starting with Figure 7(a), the initial decline in market tightness is slightly larger with the lockdown. As the pandemic evolves, however, the rate of decline in market tightness is slower than the baseline results. This is due to the fact that the lockdown slows down the onset of the pandemic and employed workers have a lower probability of becoming infected. After the lockdown ends, market tightness declines further as the infections pick up. It is in week 23 that market tightness reaches its lowest value of 0.2930 and begins to slowly recover.

Figure 7(b) shows the corresponding dynamics of the unemployment rate. As expected, the imposition of

\textsuperscript{3}We use the series “Total Factor Productivity at Constant National Prices for United States” developed by Feenstra et al. (2015) and downloadable at https://fred.stlouisfed.org/series/RTFPNAUSA632NRUG. We de-trend the series with a linear time trend and then calculate the average percentage deviations from the trend in NBER recession years.

\textsuperscript{4}According to the Job Openings and Labor Turnover Survey, the average monthly separation probability between March and May 2020 was 6.8%. As discussed by Colhion et al. (2020), initial job losses were likely undercounted, hence we impose a slightly larger separation probability of 7.5\%. 

a lockdown through increased separations causes the unemployment rate to substantially increase within a short amount of time. The unemployment rate peaks at 15.23% in week 13, directly after the lockdown ends. As the separation rate returns to its pre-pandemic level, the unemployment rate initially declines at a fast past. However, as the pandemic and number of infections worsens and market tightness continues to decrease, the recovery in the unemployment slows down. Between weeks 13-20, the unemployment rate decreases from 15.23% to 13.8%, or 1.43 percentage points. However, in the next twelve weeks, the unemployment rate declines by 0.80 percentage points. It is only after the number of infections substantially declines that the recovery in market tightness, and thus the unemployment rate speeds up and approaches the pre-pandemic unemployment rate.

Finally, Figure 8 illustrates the long-term consequences of the separation shock on the composition of unemployed workers and TFP. Figure 8(b) shows that the average skill level among unemployed workers deteriorates at a faster pace under the lockdown. Moreover, as the amount of job creation decreases further
after the lockdown ends, the composition of unemployed further worsens after the lockdown ends. Under the separation shock, the fraction of unemployed who are low-skill peaks at 94.04%, whereas the composition peaks at 93.31% in the baseline results. Additionally, the fraction of unemployed who are low-skill remains higher relative to the baseline results even after 100 weeks.

Figure 8(b) demonstrates the effect of the lockdown on TFP. Given that the skill composition of the unemployed is worse with the lockdown, it is not surprising that TFP declines even further with the lockdown. TFP reaches its lowest value of 0.9944 after 62 weeks, which is 0.12% lower than the lowest point in the baseline scenario. Given that TFP typically declines by 1.13% in recessions, the decline under the three month lockdown accounts for nearly 50% of the usual productivity losses in recessions. Further, the decline in TFP relative to the baseline scenario does not close between weeks 60-100, illustrating the additional decline in productivity due to the lockdown persists for many months after the pandemic has ended.
7 Conclusion

The health and economic costs caused by the COVID-19 pandemic have already been substantial. If workers lose skills during unemployment, the economic costs of the pandemic are likely to be long-lasting, potentially scarring the economy for years to come. To study this, we have integrated a frictional labor market with loss of skill during unemployment with the Kermack and McKendrick (1927) SIR framework. The model shows that the onset of a pandemic reduces job creation, which in turn exposes unemployed workers to loss of skill. As the skill composition of unemployed workers worsens over the pandemic, average labor productivity, or TFP, decreases. Our model suggests that the scarring effects of the COVID-19 pandemic on the economy through loss of skill during unemployment will be substantial as the decline in TFP following a three month lockdown accounts for nearly 50% of the productivity losses typically observed in recessions.

Much more work remains to be done. As discussed by many economists, there are externalities present in an environment where an agent’s actions impact the probability of others becoming infected. Moreover, there are inefficiencies associated with skill loss during unemployment, as firms do not internalize the effect of their job creation decision on the skill composition of unemployed. Thus, there is much to be learned by characterizing optimal allocations and the role of labor market policies in our environment.
References


Appendix

Proof of Proposition 1

We begin by deriving the closed-form job creation condition. From the Bellman equations for unemployed workers, it is simple to show

\[ U^H - U^L = \frac{\bar{\beta} \eta f(\theta)[F^H - F^L]}{1 - \bar{\beta}(1 - (1 - f(\theta))\sigma)}. \]  

(80)

Substituting (80) into (52), we have that \( F^H \) satisfies

\[ F^H = y - b + \bar{\beta}\left( (1 - s - \eta f(\theta))F^H + \bar{\beta}(1 - f(\theta))\sigma \eta f(\theta)\frac{F^H - F^L}{1 - \bar{\beta}(1 - (1 - f(\theta))\sigma)} \right). \]

(81)

Substituting for \( F^L \) using (51) and solving for \( F^H \) yields

\[ F^H = \frac{(y - b)[1 - \bar{\beta}(1 - (1 - f(\theta))\sigma)](1 - \bar{\beta}(1 - s - \eta f(\theta))) - \bar{\beta}^2(1 - f(\theta))\sigma \eta f(\theta)(\delta y - b)}{[1 - \bar{\beta}(1 - (1 - f(\theta))\sigma)][1 - \bar{\beta}(1 - s - \eta f(\theta))]^2 - \bar{\beta}^2(1 - f(\theta))\sigma \eta f(\theta)[1 - \bar{\beta}(1 - s - \eta f(\theta))].} \]

(82)

With equations (51) and (82), we can write the job creation condition as

\[ \frac{k[1 - \bar{\beta}(1 - s - \eta f(\theta))]}{\beta(1 - \eta)q(\theta)} = \varphi(\delta y - b) + \\
(1 - \varphi) \frac{(y - b)[1 - \bar{\beta}(1 - (1 - f(\theta))\sigma)](1 - \bar{\beta}(1 - s - \eta f(\theta))) - \bar{\beta}^2(1 - f(\theta))\sigma \eta f(\theta)(\delta y - b)}{[1 - \bar{\beta}(1 - (1 - f(\theta))\sigma)][1 - \bar{\beta}(1 - s - \eta f(\theta))]^2 - \bar{\beta}^2(1 - f(\theta))\sigma \eta f(\theta)}, \]

(83)

where \( \varphi \) is given by (56). A sufficient condition for an equilibrium to exist will ensure that the left hand side and right hand side of (83) cross at least once. It is easy to verify that as \( \theta \to \infty \), the left hand side approaches \( \infty \) while the right hand side converges to \( y - b \). Thus, a sufficient condition for at least one crossing is that the value of the left hand side is below that of the right hand side at \( \theta = 0 \). It is straightforward to verify that this is true when (58) holds.

\[ \blacksquare \]

Computation Procedure

We assume that the economy has reached its post-pandemic steady-state at a date, \( T \), that is sufficiently far into the future and compute the equilibrium as follows.

1. Guess a sequence \( \{\theta_t\}_{t=0}^{T-1} \).

2. Given the sequence of market tightness and initial values, \( I_0 \) and \( \text{Pop}_0 \), compute \( \{I_t, \varphi_t, \phi^\chi_t\}_{t=0}^{T-1} \).

3. Using output from step 2, iterate backwards from \( T \) to compute the sequence \( \{F_t^{LS}, F_t^{LR}, F_t^{HS}, F_t^{HR}\}_{t=0}^{T-1} \).

4. Using output from both steps 2 and 3, compute a new sequence \( \{\theta_t^*\}_{t=0}^{T-1} \) using the job creation condition.

5. Adjust the initial guess in step 1 using a gradient-based method until the sum of squared differences between \( \{\theta_t\}_{t=0}^{T-1} \) and \( \{\theta_t^*\}_{t=0}^{T-1} \) is arbitrarily small.
Alternative Calibrations

In the baseline calibration, we take the assumption that it takes workers on average 3 months to experience loss of skill during unemployment. We then chose $\delta$ to match the empirical evidence on the effect of length of unemployment duration on wages. Here, we present two alternative calibrations: one where it takes on average 1 month for workers to experience loss of skill and a second where it takes on average six months for workers’ human capital to depreciate. Table 2 shows how the alternative strategies change the calibrated parameters.\(^5\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Baseline</th>
<th>1 month skill loss</th>
<th>6 month skill loss</th>
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<tr>
<td>$\sigma$</td>
<td>Probability of skill loss</td>
<td>0.0769</td>
<td>0.2308</td>
<td>0.0385</td>
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<tr>
<td>$\delta$</td>
<td>Productivity of low-skill workers</td>
<td>0.7250</td>
<td>0.5725</td>
<td>0.7475</td>
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<td>$k$</td>
<td>Vacancy posting cost</td>
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<td>0.2158</td>
<td>0.3796</td>
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<tr>
<td>$b$</td>
<td>Value of unemployment</td>
<td>0.5203</td>
<td>0.4065</td>
<td>0.5474</td>
</tr>
</tbody>
</table>

Table 2: Parameter values under alternative values of $\sigma$

Quantitative Results: Skill Loss in 1 Month

We carry out the same quantitative exercises as in Sections 6.2-6.3, with the only difference being we use parameter values under the assumption skill loss occurs on average within one month. Figures 9-12 present the results.

![Graph showing fraction infected over weeks](image1)

![Graph showing total deaths over weeks](image2)

(a) Fraction infected

(b) Total deaths

Figure 9: Total infections and deaths - Skill loss in 1 month

\(^5\)Parameters not listed in Table 2 take the same values as in Table 1.
Figure 10: Probability of infection - Skill loss in 1 month

(a) Baseline

(b) Separation shock

Figure 11: Market tightness and unemployment - Skill loss in 1 month

(a) Market tightness

(b) Unemployment rate

Figure 12: Composition of the unemployed and TFP - Skill loss in 1 month

(a) Composition of unemployed workers

(b) Total Factor Productivity
Quantitative Results: Skill Loss in 6 Months

Finally, we simulate the effect of a pandemic under the calibration where skill loss occurs on average in 6 months. Figures 13-16 present the results.

Figure 13: Total infections and deaths - Skill loss in 6 months

(a) Fraction infected

(b) Total deaths

Figure 14: Probability of infection - Skill loss in 6 months

(a) Baseline

(b) Separation shock
Figure 15: Market tightness and unemployment - Skill loss in 6 months

Figure 16: Composition of the unemployed and TFP - Skill loss in 6 months