Endogenous TFP, Labor Market Policies and Loss of Skills*

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Abstract

This paper builds a model of endogenous TFP with search and matching frictions in the labor market with two features: production units are subject to idiosyncratic shocks and workers suffer skill loss during unemployment. I show that aggregating firms' micro-production decision leads to an aggregate production that is Cobb-Douglas in labor and capital. The endogenous TFP depends on two equilibrium characteristics of the labor market: the productivity of matches formed and active, and the aggregate skill distribution. In particular, the job destruction decision and the job finding rate are sufficient statistics that uniquely determine TFP. The labor market affects TFP through two channels. First, an increase in the reservation productivity raises the average match productivity and TFP. Second, the job finding rate and the job destruction decision shape the skill distribution, as it determines how long workers remain unemployed and the amount of skill loss. The paper then studies the effect of labor market policies on TFP. In contrast with previous studies, the effect of unemployment insurance on TFP depends on the relative size of the two channels. More generous unemployment insurance programs improve TFP only if the effect on the average productivity is larger than the compositional effect through to the skill channel.

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1. Introduction

Most of the large variation in income per capita across countries is accounted for by differences in TFP.[1] Even among OECD countries, a sample of relatively similar countries, TFP differences are surprisingly large and persistent. Therefore, a fundamental question in Economics is why some countries are so much more productive than others. With search frictions in the labor market, the individual decision by firms over which matches to form or keep active affects aggregate productivity. If firms form more productive matches, the average productivity of firms increases. At the aggregate level, this rise in firms' reservation productivity—the lowest productivity of an active job—leads to a higher TFP. In addition, there is abundant empirical evidence documenting that workers suffer large productivity losses during unemployment. When workers spend longer periods of time in unemployment, the economy suffers more human capital decay, which lowers the economy's TFP. Given that labor market characteristics determine what matches are formed or active, and how long workers remain unemployed, this paper investigates the effect of search frictions and skill loss during unemployment on TFP. Since labor market policy play an important role in shaping labor market outcomes, the paper further asks: what is the effect of labor market policies on TFP?

During the period 1960-1995, European countries (especially continental Europe) experienced a sharp increase in unemployment, whereas US unemployment remained relatively constant. In terms of productivity, TFP in Europe increased relative to the US. This was a period of time characterized by high unemployment insurance (UI) benefits in European countries. Therefore, theories that emphasize the role of labor market frictions and average firm productivity have been successful in accounting for these pre-1995 facts, see for example Lagos (2006) or Marimon and Zilibotti (1999). Intuitively, more generous UI benefits raise the outside option of workers and wage costs for firms—effectively, they act a search subsidies that allow workers to search for better matches. Overall, firms and workers find it optimal to raise their reservation productivity and form more productive matches, which raises the average productivity of active firms and as a result aggregate TFP. In terms of unemployment, a higher reservation productivity for acceptable matches leads to more job destruction and makes it harder for matches to form. Therefore, more generous benefits raise both TFP and unemployment. However, during the period 1995-2019, TFP

[1] See for example Caselli (2005), Hall and Jones (1999), Klenow and Rodriguez-Clare (1997) and Parente and Prescott (1994).

declined in Europe relative to the US. Contrary to the 1960-1995 period, this decline in relative TFP was not followed by a decline in unemployment in Europe relative to the US, even though UI benefits remained high in Europe. Therefore, an additional mechanism is required to account for these facts.

This paper develops a model of TFP à la Lagos (2006) in which workers lose skills during periods of unemployment. Search frictions in the labor market follow Mortensen and Pissarides (1994), where match-specific productivities are subject to idiosynchratic shocks and separations are endogenous. In addition, workers lose skills during unemployment. I show that aggregating the micro-level decision of firms leads to an aggregate production function that is Cobb-Douglas in capital and labor. TFP is derived in closed-form and is determined by the productivity of formed/active matches and the aggregate skills distribution. Further, the reservation productivity and the job finding rate are sufficient statistics and uniquely determine TFP, a property that makes the environment extremely tractable.

There are two channels through which the labor market affects TFP. The first channel, which I denote the *average productivity channel*, has been highlighted previously in the literature (Lagos (2006), Marimon and Zilibotti (1999)) and operates through the job destruction decision. As firms increase their reservation productivity, better matches are formed, which raises the average productivity of active matches and TFP. The second novel channel, which I denote the *skill channel*, operates through the compositional effect on the skill distribution. Changes in the reservation productivity and/or the job finding rate affect the skill distribution, given that they determine how long workers spend in unemployment and the amount of skill loss. If workers spend more time in unemployment, either because of a high reservation productivity or a low job finding rate, more productivity losses occur because of skill decay, which leads to lower TFP levels. In response to changes in UI benefits, these two channels work in opposite directions. Increases in UI benefits lead to both a higher reservation productivity and lower job finding rates. The increase in the reservation productivity raises TFP by improving the quality of the average active match. However, the higher reservation productivity combined with lower job finding rates lowers TFP, as they both lead to more skill decay. Therefore, contrary to theories that emphasize the average productivity channel (Lagos (2006), Marimon and Zilibotti (1999)), the effect on TFP is ambiguous. If the effect of UI benefits is quantitatively small on the skill distribution, but relatively large on the average productivity/job destruction decision, UI benefits raise TFP. By contrast, if the effect on the skill distribution is relatively large, raising UI benefits may lower TFP, contrary to previous studies, see Lagos (2006), Marimon and Zilibotti (1999).

Skill loss during unemployment introduces an additional channel that can rationalize the joint behavior of unemployment and TFP in the US relative to continental European countries such as France, Germany or Italy, as well as the observed negative correlation between trend unemployment and TFP, see for example Doppelt (2019). Theories that emphasize the job-destruction decision/average productivity channel predict that TFP and unemployment must move in the same direction in response to changes in UI benefits. Furthermore, for example in Lagos (2006), two countries with the same reservation productivity exhibit the same TFP levels, regardless of the job finding rate and how long workers stay in unemployment—i.e. the labor market tightness. With skill loss, the job finding rate (which depends on market tightness) matters for TFP because it affects unemployment duration and the skill distribution. Holding the reservation productivity constant, a country in which workers experience longer unemployment durations suffers more productivity losses due to skill decay, so at the aggregate its TFP will be lower. This additional skill channel leads to either negative or positive correlation between unemployment and TFP in response to changes in UI benefits, depending on the relative size of the two channels at work. Therefore, this paper provides a mechanism that can explain the joint behavior of unemployment, TFP and UI benefits for the entire period 1960-2019.

The property that the reservation productivity and the job finding rate are sufficient statistics and uniquely determine TFP makes the environment very tractable to study how the labor market and policy affect TFP. One need only derive the effect of the policy on the equilibrium reservation productivity and the job finding rate—i.e. labor market tightness—to evaluate the effect on TFP. In addition to UI benefits, this paper also studies the effect of an employment subsidy, a hiring subsidy and a firing tax on TFP. Hiring subsidies and firing taxes have unambiguous effects on TFP. A hiring subsidy stimulates job creation by firms and raises market tightness and the job finding rate. The higher job finding rate improves the outside option of workers, so in response workers increase their reservation productivity and accept better matches. Overall, a hiring subsidy raises both the reservation productivity and the job finding rate, both of which lead to higher TFP. Firing taxes work in the opposite direction and lower TFP, as they lower both the reservation productivity and the job finding rate. By contrast, and similar to UI benefits, employment subsidies have an ambiguous effect on TFP once one takes into account skill decay. Employment subsidies lead to a lower reservation productivity, which on the one hand lowers TFP due to the average productivity channel (the quality of the average match drops). On the other hand, employment subsidies lead to a higher job finding rate, as they give incentives for firms to create more jobs. The lower reservation productivity and higher job finding rate combined improve the skill distribution, which raises TFP. As with UI benefits, the overall effect on TFP depends on the relative size of each channel.

Related literature. Following Lagos (2006), this paper develops a model of endogenous TFP in the presence of search frictions in the labor market à la Mortensen and Pissarides (1994) and studies the effect of policies on TFP. To a lesser extent, this paper is also related to Marimon and Zilibotti (1999), who study productivity and its relation with UI in the presence of search frictions in the labor market. Both papers emphasize that UI benefits raise the average productivity of formed matches by raising the reservation productivity, a channel that explains the observed rise in unemployment and productivity in OECD countries relative to the US prior to 1996. Compared to these papers, I add human capital and loss of skill during unemployment. This introduces an additional mechanism (the skill channel) through which the job finding rate and the reservation productivity affect TFP. If changes in these two labor market equilibrium values raises unemployment duration, the economy suffers more productivity losses at the aggregate level due to skill decay, which leads to a lower TFP. The skill channel also affects the relationship between policy on TFP. In contrast to Lagos (2006) and Marimon and Zilibotti (1999), increases in UI benefits may lead to a lower TFP and raise unemployment if the skill channel is strong enough, consistent with the post-1996 facts.[2]

A number of papers incorporate skill loss during unemployment into models of the labor market à Diamond-Mortensen-Pissarides (DMP). Two important references in this literature are Ljungqvist and Sargent (1998) and Pissarides (1992). Pissarides (1992) shows that unemployment is more persistent when unemployed workers suffer skill decay during unemployment. Ljungqvist and Sargent (1998) provide a rationale for the high unemployment in Europe relative to the US due to the generous UI benefits in Europe. Their paper focuses on workers' labor supply and, unlike this paper, does not feature the decision of firms on how many vacancies to post. More importantly, there is no market tightness and the job finding rate depends on the reservation wage

[2]This paper is also related to Petrosky-Nadeau (2013), who studies TFP in the presence of credit and labor market frictions to explain the rise of TFP during the great recession, and to Ortego-Marti (2017c), who studies TFP differences across countries.

alone. By contrast, this paper shows that the skill and average productivity channels are important to explain the relationship between the labor market and TFP.[3] Doppelt (2019) focuses on the relationship between growth and unemployment and makes a contribution to the classical debate on the long-run relationship between growth and unemployment (see Mortensen and Pissarides (1998) and Aghion and Howitt (1994)). However, Doppelt (2019) results rely on a block-recursive equilibria concept to make the environment tractable. Given this block-recursive equilibrium, vacancies do not depend on the distribution of human capital, whereas this paper shows that this is an essential channel through which the labor market and policy affect TFP. In addition, there is no match-specific productivity, which is the driver of the average productivity channel in Lagos (2006) and in this paper, and what allows for an aggregate Cobb-Douglas production function.[4]

This paper is also motivated by the empirical findings from to the job displacement literature. This literature finds substantial and persistent wage losses due to unemployment. In particular, the quantitative work in this paper draws from the evidence in Ortego-Marti (2017c), Ortego-Marti (2017a) and Ortego-Marti (2017b).[5] To a lesser extent this paper is also related to the development accounting literature, which tries to account for the observed large TFP differences across countries, see Caselli (2005), Bils and Klenow (2000), Hall and Jones (1999), Klenow and Rodriguez-Clare (1997), Lagakos (2016) and Restuccia, Yang and Zhu (2008), and the references therein. However, none of these papers look at a frictional labor market or skill loss due to unemployment.

[3]In addition, their paper does not feature capital or an aggregate production function.

[4]In addition, Pavoni (2011) and Pavoni and Violante (2007) study unemployment insurance, and Ortego-Marti (2016) and Ortego-Marti (2017a) study the effect of skill loss on wage dispersion and unemployment fluctuations. However, none of these papers study TFP.

[5]See Fallick (1996) and Kletzer (1998) for a review of the early findings from this large literature, and Schmieder, von Wachter and Bender (2016) for more recent results. Some notable papers in this big literature include: Couch and Placzek (2010), Davis and von Wachter (2011), Jacobson, LaLonde and Sullivan (1993), Jarosch (2015), Schmieder *et al.* (2016) and von Wachter, Song and Manchester (2009), which use administrative data; Ortego-Marti (2016), Ruhm (1991) and Stevens (1997), which use the Panel Study of Income Dynamics (PSID); Addison and Portugal (1989), Carrington (1993), Farber (1997), Neal (1995) and Topel (1990) which use the Displaced Worker Survey (DWS) supplement of the Current Population Survey (CPS). In addition, Edin and Gustavsson (2008) find that one full year of non-employment is associated with a loss of the equivalent of 0.7 years of schooling using Swedish data on test scores assessing respondents' quantitative and analytical skills—even though respondents are mostly low skill workers. Other evidence of skill loss include Mincer and Polachek (1974) and Mincer and Ofek (1982), who study the effects of motherhood on women's earnings. For evidence of skill decay during employment breaks, see Beblo, Bender and Wolf (2008) and Gangl and Ziefle (2009) and the references therein. Further evidence on skill loss due to breaks in production are found in the provision of health services—David and Brachet (2011), Hockenberry, Lien and Chou (2008) and Hockenberry and Helmchen (2014)—, and in jobs involving routine tasks such as data entry—Globerson, Levin and Shtub (1989)—, mechanical assembly—Bailey (1989)—and car radio production—Shafer, Nembhard and Uzumeri (2001). This literature also finds that worker productivity depreciation increases with the duration of the break between tasks.

2. The labor market

This section describes the labor market, which builds on Mortensen and Pissarides (1994) and Pissarides (2000). Similar to Lagos (2006), aggregation leads to a Cobb-Douglas aggregate production function and an endogenous measure of TFP. Labor market outcomes affect TFP because when better matches are created, the overall productivity in the economy increases, as in the model of TFP in Lagos (2006). However, with loss of skills TFP also depends on how fast workers find jobs (which depends on the labor market tightness and on the number of posted vacancies), as this determines the share of workers with depreciated skills in the economy and the overall human capital. In particular, this section shows that while increases in the reservation productivity—the lowest productivity of a created job—raise TFP, drops in market tightness and the job finding rate lower TFP, as more skills depreciate when workers take longer to find a job. The tension between the effect of changes in the reservation productivity and the job finding rate provides a rationale for the reversal of TFP in continental European countries relative to the US and the negative relationship between changes in TFP and changes in the unemployment rate.

Time is continuous. There are two types of risk-neutral agents in the economy, workers and firms. They are infinitely lived and discount the future with the interest rate $r > 0$. Workers search for jobs and firms for applicants. The measure of workers is normalized to 1. In order to attract applicants, firms must post a vacancy. Firms are free to enter the vacancy market. This free entry decision determines the endogenous measure of firms. Following Pissarides (2000), the number of matches is determined by a matching function $m(u, v)$, where u is the number of unemployed workers and v the number of posted vacancies. Assume $m(u, v)$ satisfies the usual properties. In particular, it displays constant returns to scale, and is increasing and concave in both of its arguments. Matching is random, so workers find jobs at the Poisson rate $f(\theta) \equiv m(u, v)/u$, and firms meet candidates at a rate $q(\theta) \equiv m(u, v)/v$, where $\theta = v/u$ captures labor market tightness. Workers find jobs more quickly as market tightness θ increases and vacancies become more abundant. By contrast, when market tightness increases, the rate at which vacancies are filled $q(\theta)$ decreases, as firms must compete to attract workers. In other words, $f'(\theta) > 0$ and $q'(\theta) < 0$. In addition, the properties of the matching function imply that $f(\theta) = \theta q(\theta)$.

Firms have access to a production technology $f(x, n, k)$, where n is the number of hours, k is capital and x is the idiosyncratic match-specific productivity. In particular, following Lagos

(2006) and in the spirit of Houthakker (1955), the production function takes the form

$$f(x, n, k) = x \min\{n, k\}. \tag{1}$$

Intuitively, k captures the scale of production, where all projects are assumed to require the same scale of operation k. This Leontieff production function, together with aggregation and the distribution of idiosyncratic productivity gives a Cobb-Douglas production in the aggregate, as section 4 shows. In addition, assume that firms must rent k both when they are producing and posting a vacancy, i.e. capital must be installed before search begins. Similar to Lagos (2006), I abstract from capital accumulation and assume an exogenous rental rate c.[6]

Similar to Mortensen and Pissarides (1994), the idiosyncratic productivity x is a draw from a known distribution $G(.)$. Once a match is formed, the idiosyncratic productivity receives a shock at a Poisson rate λ, at which point the new idiosyncratic productivity is drawn from the same distribution $G(.)$. This Poisson process leads to some endogenous job destructions. In addition, separations also occur at an exogenous Poisson rate s.

Workers suffer some loss of skills or human capital during unemployment. To keep the environment and aggregation tractable, loss of skills takes the simplest form. There are two human capital levels, high (H) and low (L). Workers are initially born with a *high* human capital (H). At a Poisson rate σ, unemployed workers' skills depreciate and they become *low* skilled (L). Workers with a high human capital produce $f(x, n, k)$ when they are employed, while workers who have suffered a loss of skills produce a lower level $\delta f(x, n, k)$, with $0 < \delta < 1$. This is a simplified version of the skill process in Ljungqvist and Sargent (1998), and is sufficient to capture the effect of skill loss on TFP while making the environment tractable. The terminology of high (H) and low (L) skills simplifies the exposition, but these skills should not be confused with skills related to workers' educational level or experience. In other words, human capital in the model is net of other observables such as education, occupation or experience.[7] In addition, human capital losses due to unemployment are permanent. This assumption is well supported by the data, which shows that wage losses due to unemployment are very persistent and do not wash away over time.

[6]The focus of this paper is on developed countries, not on why TFP is so much lower in developing countries. As Parente and Prescott (1994) show, savings rates do not vary systematically across OECD countries. For papers that study TFP differences between developed and developing countries see the literature review in the introduction.

[7]See Caselli (2005) and the references therein for studies that focus on TFP differences due to the quality of human capital and other determinants of human capital, such as years of schooling. These sources of TFP differences are important, but not the focus of the paper.

Ljungqvist and Sargent (1998), Ortego-Marti (2016), Pavoni and Violante (2007), Pavoni (2011) and Pissarides (1992) make a similar assumption, among others. Section **??** reviews some of the findings from the empirical literature on skill loss during unemployment.

An important determinant of TFP is the endogenous distribution of workers by skills and aggregate human capital. Let u_H and u_L denote the number of high and low skill unemployed workers, and let u denote total unemployment, i.e. $u = u_H + u_L$. Define Δ_u as the fraction of *unemployed* workers with high skill H, i.e. $\Delta_u = u_H/u$. Similarly, e_H and e_L denote the number of employed workers of human capital H and L, and e denotes total employment. The fraction of *employed* workers with high human capital is given by $\Delta_e = e_H/e$. In order to ensure a stationary and non-degenerate distribution of workers by skill, assume that workers die or leave the labor force at a Poisson rate μ. Workers who leave the labor force are replaced by unemployed workers with high skills H.

High skill unemployed workers H receive income flow b, whereas low skill unemployed workers L receive $b\delta$. This value of non-market time includes unemployment insurance, home production and leisure. The assumption that non-market time is scaled by δ for low skilled workers, just as productivity, is common in the literature. This would be the outcome if unemployment insurance is proportional to wages, home production is higher for more skilled workers, and leisure is more valuable for high skilled workers.[8] Workers of type $i \in \{H, L\}$ earn wages $w_i(x)$ when they are employed.

Let U_H and U_L denote the value functions of unemployed workers with high and low skills. The Bellman equations for the unemployed are given by

$$(r + \mu)U_H = b + f(\theta) \int \max\{W_H(z) - U_H, 0\}dG(z) + \sigma(U_L - U_H), \tag{2}$$

$$(r + \mu)U_L = b\delta + f(\theta) \int \max\{W_L(z) - U_L, 0\}dG(z). \tag{3}$$

Intuitively, the Bellman equations satisfy that the return on the asset value must equal the flow payoffs and the expected changes in the asset value, using the effective discount rate $r+\mu$. Equation (2) captures that high skill workers receive b while unemployed. They find a job opportunity at a rate $f(\theta)$, which carries an expected net gain $\int \max\{W_H(z) - U_H, 0\}dG(z)$. This expected gain

[8] If non-market time is constant and equal to b for both types of workers, the distribution of unemployment shifts towards more low skilled workers, as their reservation productivity increases and their job finding rate drops. This alternative assumption would make the effect of skills depreciation on TFP stronger.

takes into account workers' optimal choice of whether to accept the job or not. In addition, while workers remain unemployed they lose some of their human capital and become a low skill worker at a rate σ. The same intuition applies for the Bellman equation for U_L, except that low skilled workers receive a lower non-market value $b\delta$ and do not suffer further human capital depreciation.

Let $W_i(x)$ denote the value function of a worker with skill level $i \in \{H, L\}$ employed in a job with match-specific productivity x. These value functions satisfy the Bellman equations

$$(r+\mu)W_i(x) = w_i(x) + \lambda \int \max\{W_i(z) - U_i, 0\} dG(z) - (\lambda + s)(W_i(x) - U_i). \tag{4}$$

Equation (4) captures that employed workers earn a wage $w_i(x)$, which depends on their human capital and match-specific productivity. At a rate λ, workers receive a shock to their match-specific productivity and a new match-specific productivity is drawn from $G(.)$. Given the new match-specific productivity, workers decide whether to stay on the job or become unemployed and search for a new job. In addition, the match is destroyed at an exogenous rate s.

Let $J_i(x)$ denote the value function of a job filled with a worker with human capital $i \in \{H, L\}$ and match-specific productivity x. These value functions satisfies the Bellman equation

$$(r+\mu)J_i(x) = \pi_i(x) + \lambda \int \max\{J_i(z) - V, 0\} dG(z) - (\lambda + s)(J_i(x) - V), \tag{5}$$

where $\pi_i(x)$ denotes firms' profits, for $i \in \{H, L\}$, which I discuss in more detail below. A filled position yields profits to the firm $\pi_i(x)$. A shock to the match-specific productivity arrives at a rate λ and a new productivity is drawn from $G(.)$. Upon arrival of the λ shock, the firm chooses whether to keep the match or destroy it. In addition, the match is exogenously destroyed at a rate s.

The structure of firms' costs and profits is similar to Lagos (2006). When the firm employs a worker with high human capital, profits are given by $\pi_H(x) = f(x, n, k) - w_H(x) - ck - \phi n - C(x, \phi)k$. There are three costs that affect firms' profits: the rental cost of capital ck, a variable cost ϕn and a fixed cost $C(x, \phi)k$. The cost ϕn captures variable costs that are proportional to the number of work hours, such as for example electricity costs. The cost $C(x, \phi)k$ is fixed in the sense that it can only be avoided if $k = 0$, i.e. if the firm shuts down. In addition, $C(x, \phi)$ is assumed to be decreasing in x. The assumption of variable and fixed costs ϕn and $C(x, \phi)$ allows for an equilibrium with labor hoarding depending on parameter values, i.e. an equilibrium where the firm keeps some low productivity matches but shuts down production ($n = 0$) until the match-specific productivity improves. Although labor hoarding affects capital utilization and the aggregate production function, the assumption of variable and fixed costs is not essential for the mechanism in the model and how skills depreciation affects TFP. Further, assume that $C(x, \phi) = \max\{\phi - x, 0\}$. This is a particularly convenient formulation that simplifies the derivation, but as Lagos (2006) shows, all that is needed for the results is that $C(x, \phi)$ is decreasing in x.

When the employed worker has low human capital, profits are given by $\pi_L(x) = \delta f(x, n, k) - w_L(x) - c\delta k - \phi\delta n - \delta C(x, \phi)k$. Alternatively, one could assume that costs are the same as when a high human capital worker is employed. This alternative specification complicates the algebra, but is not essential. In this alternative scenario, profits are even lower for firms employing a worker with low human capital. This raises the reservation productivity for low human capital workers. As a result, more endogenous separations occur for low human capital workers and their share in unemployment increases. A higher share of low human capital workers lowers the average human capital and TFP. In sum, this alternative specification for costs gives a larger role for depreciation of skills to TFP.

Using V to denote the value function of a posted vacancy, the Bellman equation is given by

$$rV = -ck + q(\theta)\left[\Delta_u \int \max\{J_H(z) - V, 0\}dG(z) + (1 - \Delta_u)\int \max\{J_L(z) - V, 0\}dG(z)\right]. \quad (6)$$

The firm must pay the capital rental rate ck while the vacancy is posted. At a rate $q(\theta)$ the firm meets a candidate from the unemployment pool. A fraction Δ_u of unemployed workers have a high human capital, while a fraction $1 - \Delta_u$ have a lower human capital due to depreciated skills. Once a firm meets a worker, it decides whether to produce or to continue searching depending on the match-specific productivity drawn from $G(.)$.

3. Equilibrium in the labor market

This section characterizes the equilibrium in the labor market, which is determined by two types of equilibrium conditions. First, a job creation condition captures firms' vacancy posting decision. Second, two job destruction conditions—one for each human capital level—capture firms' decision on which matches to keep or form. Overall, these conditions fully describe the equilibrium labor market tightness and the reservation productivity for high and low human capital workers, where the reservation productivity corresponds to the lowest match-specific productivity that firms and workers are willing to form or keep. Compared to the Mortensen and Pissarides (1994) model, the endogenous distribution of workers by human capital plays an important role in the equilibrium conditions, and will lead to novel results for aggregation and TFP compared to Lagos (2006). Aggregation and TFP, which are derived in section 4, further depend on the distribution of match qualities.

The optimal choice of hours $n = n(x)$ depends on the realization of the idiosyncratic productivity and whether it can cover the variable cost ϕ. If x is greater than ϕ, profits are strictly increasing in hours n up until $n = k$, and are strictly decreasing thereafter. By contrast, if x is lower than ϕ, profits are strictly decreasing in hours. As a result, the optimal choice of hours $n(x)$ that maximizes profits is

$$n(x) = \begin{cases} k, \text{ if } \phi < x, \\ 0, \text{ if } \phi \geq x. \end{cases} \tag{7}$$

However, note that the decision to keep or destroy the match is different than the choice of hours, which depends on the reservation productivity. Substituting optimal hours from (7) into the expression for profits gives

$$\pi^H(x) = (x - c - \phi)k - w^H(x). \tag{8}$$

$$\pi^L(x) = (x - c - \phi)\delta k - w^L(x). \tag{9}$$

There is free entry of firms in the vacancy market, which drives the value of a vacancy V to zero in equilibrium. Let $S^i(x)$ denote the surplus from a match with match-specific productivity x and employing a worker with human capital $i \in \{H, L\}$, i.e. $S^i(x) = J^i(x) + W^i(x) - U^i - V$. Using the Bellman equations and free entry gives

$$(r + \mu + \lambda + s)S^H(x) = (x - \phi - c)k + \lambda \int \max\{S^H(z), 0\}dG(z) - (r + \mu)U^H. \tag{10}$$

The surplus is clearly strictly increasing in x, so there exists a unique R_H such that $S(R_H) = 0$, and $S(x) > 0$ if and only if $x > R_H$. As a result, matches with high human capital workers are formed or maintained only if the idiosyncratic productivity x is greater or equal to R_H, i.e. $x \geq R_H$. If $x < R_H$, the match is either destroyed or not formed. Similarly, for low human capital workers we have

$$(r+\mu+\lambda+s)S^L(x) = (x-\phi-c)\delta k + \lambda \int \max\{S^L(z),0\}dG(z) - (r+\mu)U^L. \tag{11}$$

The surplus is increasing in x, so there exists a unique reservation productivity R_L such that $S^L(R_L) = 0$, and $S(x) > 0$ if and only if $x > R_L$. Substituting (10) evaluated at $x = R_H$ back into (10), and following the same procedure using (11) gives

$$S_H(x) = \frac{x - R_H}{r+\mu+\lambda+s}k, \text{ for all } x \geq R_H, \tag{12}$$

$$S_L(x) = \frac{x - R_L}{r+\mu+\lambda+s}\delta k, \text{ for all } x \geq R_L. \tag{13}$$

As is standard in the literature, I assume that the firm and the worker share the match surplus using Nash Bargaining, following Nash (1950). Let β denote the worker's bargaining strength. The wage solves the following Nash Bargaining problem

$$w_i(x) = \underset{w_i(x)}{\arg\max} \left[W_i(x) - U_i\right]^{\beta} \left[J_i(x) - V\right]^{1-\beta}. \tag{14}$$

After substituting the free entry condition $V = 0$, the solution to the above problem gives the surplus sharing condition for $i = H, L$

$$\beta J_i(x) = (1-\beta)(W_i(x) - U_i), \text{ for all } x \geq R_i. \tag{15}$$

In particular this implies that $J_i(x) = (1-\beta)S_i(x)$ and $W_i(x) - U_i = \beta S_i(x)$, for all $x \geq R_i$ and $i = H, L$. In other words, the firm and the worker get their outside option and a share $1-\beta$ and β of the surplus. Substituting the Bellman equations into the Nash Bargaining condition (15) gives the equilibrium wages as a function of workers' outside option U_i, $i \in \{H, L\}$

$$w_H(x) = \beta(x-c-\phi)k + (1-\beta)(r+\mu)U_H, \text{ for all } x \geq R_H, \tag{16}$$

$$w_L(x) = \beta(x-c-\phi)\delta k + (1-\beta)(r+\mu)U_L, \text{ for all } x \geq R_L. \tag{17}$$

To derive the value functions for unemployed workers U_i, for $i \in \{H, L\}$, substitute the Nash

Bargaining condition and the surpluses from (12) and (13) into (3) to get

$$(r+\mu)U_L = b\delta + \beta f(\theta) \int_{R_L} \frac{z - R_L}{r+\mu+s+\lambda} \delta k dG(z). \tag{18}$$

Subtracting the above expression from the Bellman equations for U_H in (2), and replacing the resulting difference back into (2) gives

$$\begin{aligned}(r+\mu)U_H =& \beta f(\theta) \left(\frac{r+\mu}{r+\mu+\sigma}\right) \int_{R_H} \frac{z-R_H}{r+\mu+s+\lambda} kdG(z) \\ &+ \frac{\sigma}{r+\mu+\sigma} \beta f(\theta) \int_{R_L} \frac{z-R_L}{r+\mu+s+\lambda} \delta k dG(z) + \left(\frac{r+\mu+\delta\sigma}{r+\mu+\sigma}\right) b.\end{aligned} \tag{19}$$

The following proposition shows that the reservation productivity is larger for low human capital workers, a result that will prove very useful in deriving aggregate TFP.

Proposition 1. *The reservation productivity is larger for workers with low human capital, i.e.* $R_H \leq R_L$.

A formal proof is included in the appendix. Intuitively, for a given match-specific productivity x a match with a high human capital worker produces a larger surplus. Therefore, both the firm and the worker are willing to form less productive matches if the worker has a high human capital. By contrast, better matches are required for the surplus to be positive with a low human capital worker, as these matches yield a lower surplus for the same match-specific productivity x. Setting $\delta = 1$, i.e. there is no skills depreciation, restores $R_L = R_H$. In this case the model corresponds to the baseline Mortensen and Pissarides (1994) model.

Using that $S^H(R_H) = S^L(R_L) = 0$ gives the job destruction conditions (JDH) and (JDL) for high and low human capital matches

$$\text{(JDH):} \quad (R_H - \phi - c)k + \lambda \int_{R_H} \frac{z-R_H}{r+\mu+\lambda+s} kdG(z) - (r+\mu)U_H = 0, \tag{20}$$

$$\text{(JDL):} \quad (R_L - \phi - c)\delta k + \lambda \int_{R_L} \frac{z-R_L}{r+\mu+\lambda+s} \delta k dG(z) - (r+\mu)U_L = 0, \tag{21}$$

where $(r+\mu)U_L$ and $(r+\mu)U_H$ are given by (18) and (19). Intuitively, the above conditions pin down the match specific productivities that satisfy that the surplus is zero, given market tightness θ. These conditions characterize firms' and workers' decision on which matches to form or keep.

Next, using the free entry condition $V = 0$ and the Bellman equation for vacancies (6) gives the job creation condition

$$\text{(JC):} \quad \frac{c}{q(\theta)} = \frac{(1-\beta)\left[\Delta_u \int_{R_H}(z-R_H)dG(z) + (1-\Delta_u)\delta \int_{R_L}(z-R_L)dG(z)\right]}{r+\mu+\lambda+s}. \tag{22}$$

Intuitively, the job creation destruction (JC) captures that firms enter the vacancy market until the expected cost from posting a vacancy—the left-hand side of (22)—equals the expected value of a filled job—the right-hand side of (22). The expected value captures that firms meet both high and low human capital workers from the unemployment pool, and that match-specific productivities are drawn from $G(.)$.

As (22) shows, the distribution of unemployment by skill Δ_u plays an important role in firms' job creation decision. This distribution is determined by labor market flows, together with a stationarity condition. Consider the pool of employed and unemployed workers with high human capital e_H and u_H. In steady-state, the flows in and out of these groups must be equal to ensure stationarity, i.e. the following flow equations hold

$$[f(\theta)(1-G(R_H))+\sigma+\mu]u_H = (s+\lambda G(R_H))e_H + \mu, \tag{23}$$

$$f(\theta)(1-G(R_H))u_H = (s+\lambda G(R_H)+\mu)e_H. \tag{24}$$

The left-hand side of the flow equation (23) equals the flows out of the pool u_H of unemployed workers with high human capital. Workers leave the pool u_H because they either find a job at a rate $f(\theta)(1-G(R_H))$, their skills depreciate at a rate σ, or die at a rate μ. The right-hand side corresponds to the flows into u_H. These flows include the employed workers with high human capital who lose their job, either from endogenous or exogenous separations, and new entrants. Similarly, the flow equation (24) captures that workers join the pool of employed workers e_H when they find a job at a rate $f(\theta)(1-G(R_H))$, and they leave the pool e_H when they either lose their job due to exogenous or endogenous separations at a rate $s+\lambda G(R_H)$, or when they leave the labor force at a rate μ. Combining the two flow equations (23) and (24) gives

$$u_H = \frac{\mu[s+\lambda G(R_H)+\mu]}{\mu f(\theta)(1-G(R_H))+(\sigma+\mu)(s+\lambda G(R_H)+\mu)}, \tag{25}$$

$$e_H = \frac{\mu f(\theta)(1-G(R_H))}{\mu f(\theta)(1-G(R_H))+(\sigma+\mu)(s+\lambda G(R_H)+\mu)}. \tag{26}$$

Similarly, the flow equations for workers with depreciated skills are given by

$$[f(\theta)(1-G(R_L))+\mu]u_H = (s+\lambda G(R_L))e_L + \sigma u_H, \tag{27}$$

$$f(\theta)(1-G(R_L))u_L = (s+\lambda G(R_L)+\mu)e_L. \tag{28}$$

Workers leave the pool u_L when they either find a job or leave the labor force, and join u_L when they lose their job or their skills depreciate. Workers join the pool e_L when they find a job, and leave e_L when they either lose their job or leave the labor force. Combining (27) and (28) gives

$$u_L = \frac{\sigma}{\mu}\left[\frac{s+\lambda G(R_L)+\mu}{f(\theta)(1-G(R_L))+s+\lambda G(R_L)+\mu}\right]u_H, \tag{29}$$

$$e_L = \frac{\sigma}{\mu}\left[\frac{f(\theta)(1-G(R_L))}{f(\theta)(1-G(R_L))+s+\lambda G(R_L)+\mu}\right]u_H, \tag{30}$$

where u_H is given by (25).[9] It is easy to verify that adding up the different pools of workers gives the total population, i.e. $u_H+e_H+u_L+e_L=1$. Combining the above expressions gives the share of unemployed and employed workers with high human capital

$$\Delta_u = \frac{\mu[f(\theta)(1-G(R_L))+s+\lambda G(R_L)+\mu]}{\mu[f(\theta)(1-G(R_L))+s+\lambda G(R_L)+\mu]+\sigma(s+\lambda G(R_L)+\mu)}, \tag{31}$$

$$\Delta_e = \frac{\mu(1-G(R_H))[f(\theta)(1-G(R_L))+s+\lambda G(R_L)+\mu]}{\mu(1-G(R_H))[f(\theta)(1-G(R_L))+s+\lambda G(R_L)+\mu]+\sigma(1-G(R_L))(s+\lambda G(R_H)+\mu)}. \tag{32}$$

The equilibrium consists of a tuple $\{R_H^*, R_L^*, \theta^*, U_H^*, U_L^*, \Delta_u^*, \Delta_e^*\}$ that satisfy: (1) the job destruction conditions (JDH) and (JDL) in (20) and (21); (2) the job creation condition (JC) in (22); (3) the value functions for unemployed workers (19) and (18); and (4) the stationary distributions (31) and (32). As in Pissarides (1992), the equilibrium exists but may not be unique, as the (JC) curve can be upward sloping under certain parameter combinations. However, quantitatively this happens under very extreme parameter values. For any standard calibration in the literature, the (JC) curve is clearly upwards sloping and the equilibrium is unique. The following proposition

[9] Substituting (25) gives

$$u_L = \sigma\left[\frac{s+\lambda G(R_L)+\mu}{f(\theta)(1-G(R_L))+s+\lambda G(R_L)+\mu}\right]\cdot\left[\frac{s+\lambda G(R_H)+\mu}{\mu f(\theta)(1-G(R_H))+(\sigma+\mu)(s+\lambda G(R_H)+\mu)}\right],$$

$$e_L = \sigma\left[\frac{f(\theta)(1-G(R_L))}{f(\theta)(1-G(R_L))+s+\lambda G(R_L)+\mu}\right]\cdot\left[\frac{s+\lambda G(R_H)+\mu}{\mu f(\theta)(1-G(R_H))+(\sigma+\mu)(s+\lambda G(R_H)+\mu)}\right].$$

provides some conditions for existence and uniqueness.

Proposition 2. *Let η denote the elasticity $\eta = -q'(\theta)\theta/q(\theta)$. Define $R_L(\theta)$ as the unique R_L as a function of θ that satisfies the (JDL) condition (21), using the Implicit Function Theorem. Let $\bar{\eta} = \sigma\sqrt{\frac{\mu+\sigma}{\mu}} \cdot \left\{\left(\sqrt{\frac{\mu+\sigma}{\mu}}+1\right)\left[\mu\left(\sqrt{\frac{\mu+\sigma}{\mu}}+1\right)+\sigma\right]+\sigma\sqrt{\frac{\mu+\sigma}{\mu}}\right\}^{-1}$ and $\varphi(x) = \int_x^\infty(1-G(z))dz$. Let θ_0 be defined by the market tightness that satisfies the (JDL) condition with $R_L = \varepsilon$, $\bar{\theta}$ be defined by the tightness θ that satisfies the (JC) condition (22) with $R_H = \varepsilon$, and $\underline{\theta}$ be defined by the tightness θ that satisfies the (JDH) condition (20) with $R_H = \varepsilon$. Assume that $\varepsilon - \phi - c + \varphi(\varepsilon)\left[\frac{\lambda}{r+\mu+s+\lambda} - \frac{f(\theta_0)[(r+\mu)\beta+\sigma\beta\delta]}{(r+\mu+\sigma)(r+\mu+s+\lambda)}\right] - \left(\frac{r+\mu+\delta\sigma}{r+\mu+\sigma}\right)\frac{b}{k} \geq 0$, $\eta > \bar{\eta}$ and $\underline{\theta} < \bar{\theta}$. Then the equilibrium exists and is unique.*

The appendix includes the proof. Intuitively, one can use the Implicit Function Theorem to express $R_L = R_L(\theta)$ as a function of market tightness θ using the (JDL) condition for L workers. This reduces the equilibrium to one job destruction condition and one job destruction condition in two variables, R_H and θ. The condition $\eta > \bar{\eta}$ ensures that the (JC) curve is downward sloping. Intuitively, if the elasticity of the matching rate η is "too large", the unemployment distribution improves too much when θ increases and may lead to more job posting, an issue present in other random matching models with skill decay, e.g. Pissarides (1992). The other two conditions ensure a crossing of the (JDH) and (JC) curves. Given the equilibrium R_H and θ, the equilibrium R_L follows from the $R_L = R_L(\theta)$ relationship given the (JDL) condition. The proposition provides sufficient conditions. Quantitatively, under any standard calibration the equilibrium exists and is unique.

4. Aggregation and TFP with loss of skills

An important feature of the model is that the equilibrium in the labor market, captured by the equilibrium market tightness θ and reservation productivities R_H and R_L, is effectively a sufficient statistic that determines the economy's TFP. Intuitively, the market tightness and reservation productivity determine both (1) the quality/productivity of formed matches; (2) the human capital distribution. This is in contrast with Lagos (2006), where the equilibrium reservation productivity *alone* pins down the economy's TFP, regardless of market tightness. In Lagos (2006), two economies with the same reservation productivity exhibit the same TFP, regardless of how long it takes for workers to find jobs—i.e. regardless of the actual equilibrium market tightness. In this environment, policies that increase the reservation productivity, such as increases in unemployment benefits, always lead to improvements in TFP, even if they raise unemployment and unemployment duration. However, once we take into account that long periods of unemployment have negative effects on workers' productivity, as this paper does, policies that reduce the job finding rate and increase unemployment duration lower TFP. This additional mechanism provides a rationale for the relationship between unemployment changes and TFP and between unemployment benefits and TFP, and may dampen the effect of unemployment insurance on TFP, while amplifying the effect of other policies such as hiring subsidies.

Let $\tilde{G}^H(.)$ denote the cumulative density function of the endogenous distribution of match specific productivities for high human capital workers, and let $\tilde{G}^L(.)$ denote the corresponding distribution for low human capital workers. The overall endogenous distribution of matches is denoted by $\tilde{G}(.)$. As with the distribution of workers by skill, these endogenous distribution are determined by flow equations and a stationarity condition. For each $i \in \{H, L\}$, the following flow equation holds

$$\begin{aligned}\frac{d[\tilde{G}^i(x)e^i]}{dt} =& \lambda e^H[1-\tilde{G}^i(x)][G(x)-G(R^i)] + f(\theta)u^i[G(x)-G(R^i)] \\ & - \lambda e^i\tilde{G}^i(x)[1-G(x)] - \lambda e^i\tilde{G}^i(x)G(R^i) - (s+\mu)e^i\tilde{G}^i(x), \text{ for all } x \geq R_i. \qquad (33)\end{aligned}$$

The right-hand side of (33) captures the flows in and out of the pool of workers employed at a job with match-specific productivity lower or equal than a given x, for $x \geq R_i$. The first term captures workers employed at a match with productivity higher than x who get a productivity shock that lowers their productivity below x, but not bleow R^i—i.e. the match is not destroyed. The next term corresponds to the flow of unemployed workers who find a job with match-specific productivity lower than x. The last three terms capture the outflows, which correspond to the number of employed workers who get a productivity shock that raises their productivity above x, and the jobs destroyed either due to endogenous separations, exogenous separations or workers leaving the labor force.

In steady-state, $d[\tilde{G}^i(x)e^i]/dt = 0$. Using the flow equation above gives, for $i = H.L$

$$\tilde{G}^i(x) = \left[\frac{\lambda}{\lambda+s+\mu} + \frac{f(\theta)}{s+\mu+\lambda}\frac{u^i}{e^i}\right][G(x)-G(R^i)], \text{ for } i \in \{H, L\}. \qquad (34)$$

Substituting u^i and e^i from (25), (29), (26) and (30) gives

$$\tilde{G}^i(x) = \frac{G(x) - G(R^i)}{1 - G(R^i)}, \text{ for } i \in \{H, L\}. \tag{35}$$

Intuitively, the distribution of observed match productivities $\tilde{G}^i(.)$ is a truncation of $G(.)$ above the reservation productivity R_i.

Let Y, K and N denote aggregate output, capital and labor—in this case total hours, as in Lagos (2006). The following derivations show the relationship between these aggregates. Aggregate capital is given by

$$\begin{aligned} K &= (1 - u + v)k \\ &= [1 - (1 - \theta)u]k. \end{aligned} \tag{36}$$

Intuitively, each employed worker and each vacancy requires an amount k of capital. Given that vacancies require capital but do not produce output, we can define effective capital K_e as the fraction of the aggregate capital that is operative and producing output, which is given by

$$K_e = \frac{1 - u}{1 - (1 - \theta)u} K. \tag{37}$$

A match with a worker with human capital $i \in \{H, L\}$ produces ($n > 0$) if the match specific-productivity: (1) yields a positive surplus (i.e. $x \geq R_i$); and (2) covers the variable cost (i.e. $x \geq \phi$). Therefore, for aggregation it is convenient to define μ_i as $\mu_i \equiv \max\{R_i, \phi\}$, for $i \in \{H, L\}$. Aggregate output Y is then given by

$$Y = (1 - \Delta_e)(1 - u) \int_{\mu_L} \delta f(x, n(x), k) d\tilde{G}^L(x) + \Delta_e(1 - u) \int_{\mu_H} f(x, n(x), k) d\tilde{G}^H(x). \tag{38}$$

Intuitively, the above aggregate adds the number of each type of employed worker and how much each of them produces depending on their match-specific productivity, taking into account the distribution by skill Δ_e and the distribution of match-specific productivities $\tilde{G}^i(.)$. Proceeding in a similar way, aggregate hours N are given by

$$N = (1 - u)\Delta_e \int_{\mu_H} n(x) d\tilde{G}^H(x) + (1 - u)(1 - \Delta_e) \int_{\mu_L} n(x) d\tilde{G}^L(x). \tag{39}$$

Substituting the number of hours from (7) and K_e from (37) into aggregate output and hours gives

$$Y = K_e\{(1-\Delta_e)[1-\tilde{G}^L(\mu_L)]\delta E(x|x \geq \mu_L) + \Delta_e[1-\tilde{G}^H(\mu_H)]E(x|x \geq \mu_H)\}. \tag{40}$$

$$N = K_e\{\Delta_e[1-\tilde{G}^H(\mu_H)] + (1-\Delta_e)[1-\tilde{G}^L(\mu_L)]\}. \tag{41}$$

Assume as in Lagos (2006) that $G(.)$ is Pareto, i.e. $G(x)$ is given by

$$G(x) = \begin{cases} 0 & \text{, if } x < \varepsilon. \\ 1 - \left(\frac{\varepsilon}{x}\right) & \text{, if } \varepsilon \leq x, \end{cases} \tag{42}$$

where $\varepsilon > 0$ and $\alpha > 1$. A Pareto distribution implies if $\mu_i \geq \varepsilon$, then $1 - G(\mu_i) = (\varepsilon/\mu_i)^\alpha$ and $E(x|x \geq \mu_i) = \alpha\mu_i/(\alpha-1)$. In addition, since (35) implies that $\tilde{G}^i(.)$ is a truncated distribution of $G(.)$ above R_i, $\tilde{G}^i(.)$ is also Pareto with parameters $R_i > 0$ and $\alpha > 1$, i.e. for $i \in \{H, L\}$

$$\tilde{G}^i(x) = \begin{cases} 0 & \text{, if } x < R_i. \\ 1 - \left(\frac{R_i}{x}\right) & \text{, if } R_i \leq x. \end{cases} \tag{43}$$

The above distribution implies that $1 - \tilde{G}^i(\mu_i) = (R_i/\mu_i)^\alpha$ and $E(x|x \geq \mu_i) = \alpha\mu_i/(\alpha-1)$. Substituting the distribution $\tilde{G}^i(.)$ into (41) and (40) gives

$$N = K_e\left[\Delta_e\left(\frac{R_H}{\mu_H}\right)^\alpha + (1-\Delta_e)\left(\frac{R_L}{\mu_L}\right)^\alpha\right], \tag{44}$$

$$Y = K_e\left[\Delta_e\left(\frac{R_H}{\mu_H}\right)^\alpha \mu_H + (1-\Delta_e)\left(\frac{R_L}{\mu_L}\right)^\alpha \delta\mu_L\right]\left(\frac{\alpha}{\alpha-1}\right). \tag{45}$$

Compared to Lagos (2006), aggregate output is a weighted average of the expected match-specific productivities, and scaled low human capital matches by δ to take into account their lower productivity. Importantly, the weights are given by the distribution of workers by skill Δ_e, which depends on both the reservation productivity and market tightness. When market tightness is high, workers find jobs quickly, are less likely to be unemployed and spend less time in unemployment. This reduces the amount of skill loss and improves the economy's TFP, as fewer workers have depreciated human capital. The novel results in this paper work through the compositional changes in Δ_e.

Next, I proceed to derive the economy's aggregate production function. TFP now depends on the relative size of each type's reservation productivity compared to the variable cost, as this will determined the amount of labor hoarding. Broadly speaking, given that $R_L \geq R_H$, there are three cases: (1) hoarding of both types of workers; (2) hoarding of only workers with high skills; (3) no hoarding.

Case 1: $R_H < R_L < \Phi$. In this case firms hoard labor for both types of workers, in the sense that matches with $x \in [R_i; \phi)$ are kept and formed, but hours are set to zero for these matches, i.e. $n(x) = 0$ for $x \in [R_i; \phi)$. Further, $\mu^L = \mu^H = \phi$. Substituting into aggregate labor N from (44) gives

$$N = \frac{K_e}{\mu^\alpha} \left[\Delta_e R_H^\alpha + (1 - \Delta_e) R_L^\alpha\right]. \tag{46}$$

Using the above expression to express μ as a function of K_e and N, and substituting into aggregate output Y from (45) gives

$$Y = \frac{\Delta_e R_H^{\frac{1}{\gamma}} + (1 - \Delta_e) R_L^{\frac{1}{\gamma}} \delta}{(1 - \gamma) \left[\Delta_e R_H^{\frac{1}{\gamma}} + (1 - \Delta_e) R_L^{\frac{1}{\gamma}}\right]^{1-\gamma}} \cdot K_e^\gamma N^{1-\gamma}, \tag{47}$$

where $\gamma \equiv 1/\alpha$.

Case 2: $\phi \leq R_H < R_L$. This corresponds to the no-hoarding case, and implies that $\mu_H = R_H$ and $\mu_L = R_L$. Substituting these into N and Y gives

$$N = K_e, \tag{48}$$

$$Y = \frac{\Delta_e R_H + (1 - \Delta_e) \delta R_L}{1 - \gamma} \cdot K_e^\gamma N^{1-\gamma}. \tag{49}$$

Case 3: $R_H < \Phi \leq R_L$. In this case there is hoarding of high human capital matches if $x \in [R_H, \phi)$, but no hoarding of low human capital matches. Intuitively, in this case only high human capital matches yield enough surplus to induce firms to hoard labor. Low human capital matches yield relative small surplus, and firms expect the match specific productivity to be high for a match to be formed—i.e. R_L is high. Using that $\mu_H = \phi$ and $\mu_L = R_L$ gives

$$N = K_e, \tag{50}$$

$$Y = \frac{\Delta_e R_H^{\frac{1}{\gamma}} + (1-\Delta_e) R_L \delta \phi^{\frac{1}{\gamma}-1}}{(1-\gamma)\left[\Delta_e R_H^{\frac{1}{\gamma}} + (1-\Delta_e)\phi^{\frac{1}{\gamma}}\right]^{1-\gamma}} \cdot K_e^{\gamma} N^{1-\gamma}. \tag{51}$$

The following proposition summarizes the above results and characterizes the economy's aggregate production function and TFP.

Proposition 3. *Let $\gamma \equiv 1/\alpha$ and K_e denote the operative capital. The economy's aggregate production function $Y = F(K_e, N)$ satisfies*

$$Y = F(K_e, N) = A K_e^{\gamma} N^{1-\gamma}, \tag{52}$$

where A is the economy's TFP and is given by

$$A = \begin{cases} A_l = \frac{\Delta_e R_H^{\frac{1}{\gamma}} + (1-\Delta_e) R_L^{\frac{1}{\gamma}} \delta}{\left[\Delta_e R_H^{\frac{1}{\gamma}} + (1-\Delta_e) R_L^{\frac{1}{\gamma}}\right]^{1-\gamma}} \cdot \frac{1}{1-\gamma} & , \text{ if } R_H < R_L \leq \phi. \\ A_m = \frac{\Delta_e R_H^{\frac{1}{\gamma}} + (1-\Delta_e) R_L \delta \phi^{\frac{1}{\gamma}-1}}{\left[\Delta_e R_H^{\frac{1}{\gamma}} + (1-\Delta_e)\phi^{\frac{1}{\gamma}}\right]^{1-\gamma}} \cdot \frac{1}{1-\gamma} & , \text{ if } R_H < \phi < R_L. \\ A_h = \left[\Delta_e R_H + (1-\Delta_e)\delta R_L\right] \cdot \frac{1}{1-\gamma} & , \text{ if } \phi \leq R_H < R_L. \end{cases} \tag{53}$$

When there are no loss of skills the above TFP equals the expression in Lagos (2006), i.e. with $\delta = 1$ (no skill loss), $R_H = R_L$ and $A_l = A_m = A_h = R/(1-\gamma)$. Interestingly, there are three *regimes* for TFP, whereby regimes I mean the regions in the (R_H, R_L) space defined in proposition 4 and over which TFP has a different functional form. The effect of changes in labor market outcomes $\{R_i, \theta\}$ depends on which regime the economy is. If the economy "switches" regimes due to a policy change, the effect of changes in labor market outcomes on TFP may be stronger in some regimes than in others. In particular, the effect of UI on TFP might be dampened as UI benefits move the economy from one regime to another. However, there are no jumps in TFP when the economy moves from one regime to the other, as the following proposition shows. The proof is included in the appendix.

Proposition 4. *The endogenous TFP A in (53) is continuous in (R_H, R_L).*

5. Discussion

One of the key insights in the Lagos (2006) model is that the reservation productivity is a sufficient statistic for TFP. This property captures how labor market characteristics, and therefore policy, affect TFP. More specifically, in Lagos (2006) TFP A is given by

$$A = \frac{R}{1-\gamma}, \tag{54}$$

where R is the equilibrium reservation productivity and $\gamma = 1/\alpha$ corresponds to the Pareto parameter of the distribution of idiosyncratic productivities, as in this paper. When $\delta = 1$, i.e. there is no skill loss, TFP in (53) coincides with TFP in (54) from Lagos (2006) . Two important results emerge from this model of TFP. First, without skill loss, any policy or shock that raises the reservation productivity unambiguously raises TFP, since all formed and active matches have now a higher productivity. This is what I denote the *average productivity channel*. Second, two economies with the same reservation productivity display the same TFP, regardless of their market tightness or job finding rates.

In this paper the model inherits the important sufficient statistic property in Lagos (2006). However, with skill loss *both* the reservation productivities (R_H, R_L) *and* the market tightness θ are sufficient statistics. This result captures an additional channel that operates through the skill distribution, which I denote the *skill channel*. The reservation productivities and the job finding rate now determine TFP, because they affect how often and for how long workers remain unemployed and therefore determine the skill distribution. As a result, even if two economies have the same reservation productivity, different market tightness will lead to different aggregate skills and different TFP levels.

More formally, changes in TFP can be decomposed in the following way

$$dA = \left(\frac{\partial A}{\partial R_H} + \frac{\partial A}{\partial \Delta_e}\frac{d\Delta_e}{dR_H}\right) dR_H + \left(\frac{\partial A}{\partial R_L} + \frac{\partial A}{\partial \Delta_e}\frac{d\Delta_e}{dR_L}\right) dR_L + \frac{\partial A}{\partial \Delta_e}\frac{\partial \Delta_e}{\partial f(\theta)} f'(\theta) d\theta. \tag{55}$$

An increase in Δ_e corresponds to a higher share of workers with high skills, i.e. an improvement in the skill distribution. It can easily be verified that $\partial A/\partial \Delta_e > 0$, that is, an improvement in the observed skill distribution raises TFP through the skill channel. Both the reservation productivities and market tightness determine the skill distribution. It can be easily verified that

$\partial\Delta_e/\partial f(\theta) > 0$, $\partial\Delta_e/\partial R_L > 0$ and $\partial\Delta_e/\partial R_H < 0$. Intuitively, fewer workers suffer skill decay if they find jobs faster ($\partial\Delta_e/\partial f(\theta) > 0$); fewer workers with low skill are employed when the reservation productivity R_L increases ($\partial\Delta_e/\partial R_L > 0$); and fewer workers with high skills are employed when the reservation productivity R_H increases ($\partial\Delta_e/\partial R_H < 0$). All of these effects are absent without skill loss.

The skill channel is particularly important to understand the effect of UI benefits on TFP. UI benefits raise the reservation productivity, but also lower the job finding rate. Therefore, the average productivity and the skill channels work in opposite directions. On the one hand UI benefits raise TFP because better matches are formed and active, due to the increase in the reservation productivities. This is the average productivity channel present in Lagos (2006) and Marimon and Zilibotti (1999). However, the increase in the reservation productivity and the drop in the job finding rate both deteriorate the skill distribution, as workers remained unemployed more often and for longer. This leads to more skill decay and lowers TFP, through the skill channel. Therefore, with skill loss the effect on TFP is now ambiguous. Overall, UI benefits raise TFP only if the policy affects the reservation productivity without affecting the skill distribution too much, i.e. if the average productivity channel dominates the skill channel.

6. The effect of labor market policies on TFP

Given the interaction between the labor market and TFP, and given that labor market policies affect equilibrium labor market characteristics, a natural question is how do labor market policies affect TFP? This section studies the effect of UI benefits, firing taxes, hiring subsidies and employment subsidies on TFP. These policies have been extensively studied in the literature, see for example Pissarides (2000). An important advantage of the sufficient statistic property of the endogenous TFP derived in (53) is that one need only look at how a policy affects the reservation productivity and labor market tightness to obtain the effect on TFP. In other words, evaluate the effect of the policy on the reservation productivity and market tightness (or the job finding rate), substitute into (53) and one readily gets the effect on TFP.

Labor market policies take the following expression. Let τ_e denote the employment subsidy, τ_h the hiring subsidy, τ_f the firing tax and τ_b benefits. As in Pissarides (2000), I assume that all the policies are proportional to the size of the firm k to remove scale effects. Using already the free entry condition $V = 0$, the Bellman equations for $i = H, L$ are given by

$$(r+\mu)U_H = b + f(\theta)\int \max\{W_H^0(z) - U_H, 0\}dG(z) + \sigma(U_L - U_H), \tag{56}$$

$$(r+\mu)U_L = b\delta + f(\theta)\int \max\{W_L^0(z) - U_L, 0\}dG(z), \tag{57}$$

$$(r+\mu)W_i(x) = w_i(x) + \lambda \int \max\{W_i(z) - U_i, 0\}dG(z) - (\lambda + s)(W_i(x) - U_i), \tag{58}$$

$$(r+\mu)J_i(x) = \pi_i(x) + \tau_e k + \lambda \int \max\{J_i(z) + \tau_f k, 0\}dG(z) - (\lambda + s)(J_i(x) + \tau_f k), \tag{59}$$

$$rV = -ck + q(\theta)\left[\Delta_u \int \max\{J_H^0(z), 0\}dG(z) + (1-\Delta_u)\int \max\{J_L^0(z), 0\}dG(z) + \tau_h k\right]. \tag{60}$$

The endogenous distributions Δ_u and Δ_e remain the same as without policy and are given by (31) and (32). The biggest difference compared to the model without policy is that with a hiring subsidy and a firing tax, the bargaining problem is different when the firm and the worker sign the contract than when the worker is taken on. At the time of signing the contract, the hiring subsidy is at stake but the firing tax does not apply yet. Therefore, the worker and the firm care about the hiring subsidy but not about the firing tax during bargaining. Once the match has been formed, the hiring subsidy is not applicable anymore, but the match is now subject to a firing tax if destroyed, so the worker and the firm take the firing tax into account during subsequent bargains. Under the assumption of continuous renegotiation, this leads to two different wages, a wage $w_i^0(x)$ upon signing the contract, and a continuation wage $w_i(x)$ once the match is formed—the "outside" and "inside" wages, to use Pissarides (2000) terminology. More generally, as in Pissarides (2000), variables with a nought superscript denote the time of signing stage, when only the hiring subsidy is at stake. Variables without a nought superscript denote that the match has already been formed and the firing tax applies. For example, $W_i^0(x)$ denotes the value function of an employed worker at the same stage, and so on.

The derivation of the equilibrium follows the same steps as with the model without policy, the details of the derivations are included in the appendix. The equilibrium is characterized by the following job creation condition (JC) and two job destruction conditions (JDH) and (JDL)

$$\text{(JDH):}\quad (R_H - \phi - c)k + \tau_e k + \lambda \int_{R_H} \frac{z - R_H}{r + \mu + \lambda + s} k dG(z) + (r+\mu)\tau_f k - (r+\mu)U_H = 0, \quad (61)$$

$$\text{(JDL):}\quad (R_L - \phi - c)\delta k + \tau_e k + \lambda \int_{R_L} \frac{z - R_L}{r + \mu + \lambda + s} k dG(z) + (r+\mu)\tau_f k - (r+\mu)U_L = 0, \quad (62)$$

$$\begin{aligned}\text{(JC):}\quad & \frac{\varepsilon^\alpha[\Delta_u R_H^{1-\alpha} + (1-\Delta_u)\delta R_L^{1-\alpha}]}{(\alpha - 1)(r + \mu + \lambda + s} + \\ & + (\tau_h - \tau_f)\left[\Delta_u \left(\frac{\varepsilon}{R_H}\right)^\alpha + (1-\Delta_u)\left(\frac{\varepsilon}{R_L}\right)^\alpha\right] - \frac{c}{(1-\beta)q(\theta)} = 0. \end{aligned} \quad (63)$$

where Δ_u is given by (31), and the value functions of unemployment U_H and U_L are given by

$$(r+\mu)U_L = \tau_b \delta k + \beta f(\theta)\left(\frac{\varepsilon}{R_L}\right)^\alpha \left[\frac{R_L \delta k}{(\alpha-1)(r+\mu+\lambda+s)} + (\tau_h - \tau_f)k\right], \quad (64)$$

$$(r+\mu)U_H = b\left(\frac{r+\mu+\delta\sigma}{r+\mu+\sigma}\right). \quad (65)$$

Sufficient conditions for existence and uniqueness are derived as in the model without policy. Quantitatively, under any standard calibration the equilibrium exists and is unique. Shutting down the loss of skill channel (i.e. $\delta = 1$ and/or $\sigma = 0$) yields the model in Lagos (2006).

The following proposition summarizes the effect of the policies on TFP.

Proposition 5. *(a) Hiring subsidies raise both the reservation productivities and the job finding rate, so they unambiguously raise TFP. Firing taxes lower both the reservation productivities and the job finding rate, so they unambiguously lower TFP.*

(b) Unemployment insurance benefits raise the reservation productivities but lower the job finding rate. Employment subsidies lower the reservation productivities but raise the job finding rate. Therefore, the effect of both policies on TFP is ambiguous.

Intuitively, with skill loss both the reservation productivities and the job finding rate are sufficient statistics for TFP. In Lagos (2006), all that matters is the effect on the reservation productivity, regardless of what happens to the market tightness (or the job finding rate). The reservation productivity is effectively a sufficient statistic for TFP. If the policy raises the reservation productivity, TFP increases independent of the effect on the job finding rate. With skill loss, the effect of labor market policies on TFP is different than in Lagos (2006) because of the skill channel. The market tightness matters now for TFP because it affects the skill distribution through its effect on the job finding rate. Both the reservation productivity and market tightness are sufficient statistics.

7. Conclusion

This paper develops a model of endogenous TFP with search and matching frictions and skill loss during unemployment. Aggregating the individual decisions of firms over which units they keep active and which matches they form leads to an aggregate production function that is Cobb-Douglas in capital in labor. TFP is determined by the equilibrium characteristics of the labor market. Further, the reservation productivity and the job finding rate are sufficient statistics and uniquely determine TFP, as they both capture the average productivity of active matches and the skill distribution. Contrary to previous studies that emphasize the role of the average match productivity, in the model there two channels that may affect TFP in opposite directions. In particular, the effect of UI benefits on TFP is ambiguous. UI benefits lead to both a higher reservation productivity and a lower job finding rate. Therefore, although UI benefits improve the quality of matches formed, they also worsen the skill distribution. This mechanism can account for the high unemployment rates and declining productivity in Europe relative to the US during the period 1996-2019 despite high UI benefits in European countries relative to the US, as well as the negative relationship between TFP and unemployment in the data. The paper then studies the effect of labor market policies on TFP.

Appendix

Proof of proposition 1

The proof proceeds by contradiction. Assume $R_L < R_H$. Define the function $h(x) = \int_x (z - x)dG(z)$. The function $h(x)$ is clearly strictly decreasing, as $h'(x) = -\int_x dG(z)$. Therefore, it follows that the assumption $R_L < R_H$ implies

$$\int_{R_L} (z - R_L)dG(z) > \int_{R_H} (z - R_H)dG(z). \tag{66}$$

Using the Bellman equations for U_L and U_H from (18) and (19) gives the following

$$\begin{aligned}
(r+\mu)\frac{U_L}{\delta} =& b+\beta f(\theta)\int_{R_L}\frac{z-R_L}{r+\mu+\lambda+s}kdG(z),\\
(r+\mu)U_H =& \left[\frac{r+\mu+\delta\sigma}{r+\mu+\sigma}\right]\\
&+\frac{\beta f(\theta)k}{r+\mu+\lambda+s}\left[\frac{r+\mu}{r+\mu+\sigma}\int_{R_H}(z-R_H)dG(z)+\frac{\sigma\delta}{r+\mu+\sigma}\int_{R_L}(z-R_L)dG(z)\right]\\
&< b+\frac{\beta f(\theta)k}{r+\mu+\lambda+s}\int_{R_L}(z-R_L)dG(z),
\end{aligned}$$

where the last inequality uses (66), $\delta<1$ and $(r+\mu+\delta\sigma)/(r+\mu+\sigma)<1$. As a result

$$(r+\mu)U_H < \frac{r+\mu}{\delta}U_L. \tag{67}$$

Using the expression for the surplus $S_H(x)$ and $S_L(x)$ from (10) and (11) gives

$$\begin{aligned}
(r+\mu+\lambda+s)\frac{S_L(R_L)}{\delta} &= (R_L-\phi-c)k+\lambda\int_{R_L}\frac{(z-R_L)k}{r+\mu+\lambda+s}dG(z)-\frac{(r+\mu)U_L}{\delta}\\
&< (R_L-\phi-c)k+\lambda\int_{R_L}\frac{(z-R_L)k}{r+\mu+\lambda+s}dG(z)-(r+\mu)U_H\\
&< (R_H-\phi-c)k+\lambda\int_{R_H}\frac{(z-R_H)k}{r+\mu+\lambda+s}dG(z)-(r+\mu)U_H\\
&= (r+\mu+\lambda+s)S_H(R_H)\\
&= 0,
\end{aligned}$$

where the above uses (67) and that $Rk+\lambda\int_R\frac{(z-R)k}{r+\mu+\lambda+s}dG(z)$ is strictly increasing in R. The above inequality implies that $S_L(R_L)<0$, which is a contradiction. $\square$

Proof of proposition 2, existence and uniqueness of equilibrium

The proof proceeds as follows. First, I express $R_L(\theta)$ as a function of θ alone using the (JDL) condition (21) and the Implicit Function Theorem. This allows me to express the equilibrium in terms of $\{\theta, R_H\}$ alone. I then show that the condition $\eta>\bar{\eta}$ is sufficient for the (JC) curve to be strictly downward-sloping. Next, I show that the condition $\underline{\theta}<\bar{\theta}$ ensures a crossing between the (JC) and (JDH) curves. Together these two results guarantee a unique intersection between the two curves. Therefore, the equilibrium exists and is unique. Given equilibrium $\{\theta, R_H\}$, a unique equilibrium R_L follows from the (JDL) condition (21).

Totally differentiating the (JDL) condition (21) gives

$$\left.\frac{dR_L}{d\theta}\right|_{JDL} = \frac{\beta(1-\eta)q(\theta)\int_{R_L}^{\infty}(z-R_L)dG(z)}{r+\mu+s+\lambda G(R_L)+\beta f(\theta)(1-G(R_L))}. \tag{68}$$

Using the above expression we get that conditional on the (JDL) condition (21), $dR_L/d\theta > 0$ for all θ. Given that the (JDL) condition (21) depends on R_L and θ only, applying the Implicit Function Theorem to the (JDL) condition defines $R_L = R_L(\theta)$ as a function of θ. Next, totally differentiating the (JDH) condition (20), substituting $dR_L/d\theta$ from above and rearranging gives

$$\begin{aligned}\left.\frac{dR_H}{d\theta}\right|_{JDH} &= \frac{\beta(1-\eta)q(\theta)}{r+\mu+\sigma}\left[r+s+\mu+\lambda G(R_H)+\beta f(\theta)(1-G(R_H))\left(\frac{r+\mu}{r+\mu+\sigma}\right)\right]^{-1}\cdot \qquad (69)\\ &\cdot\left[(r+\mu)\int_{R_H}^{\infty}(z-R_H)dG(z)+\frac{\sigma(r+\mu+s+\lambda G(R_L))}{r+\mu+s+\lambda G(R_L)+\beta f(\theta)(1-G(R_L))}\int_{R_L}^{\infty}(z-R_L)dG(z)\right].\end{aligned}$$

It follows that $dR_H/d\theta > 0$ for all θ, so the (JDH) curve is strictly upward-sloping.

Next, I prove that the (JC) curve (22) is downward-sloping given the parameter restriction $\eta > \bar{\eta}$. Compared to the DMP model without skill loss, the slope of the JC curve may be upward sloping because the distribution of workers across skills δ_u improves as firms post vacancies and θ increases. Differentiating the JC condition (22), and substituting the JC condition to replace $(1-\beta)[\varphi(R_H)-\delta\varphi(R_L)]$ gives

$$\begin{aligned}&-(1-\beta)\Delta_u(1-G(R_H))\frac{dR_H}{d\theta} =\\ &\quad (r+\mu+\lambda+s)\frac{ck\eta}{f(\theta)}-\left[(r+\mu+\lambda+s)\frac{ck}{q(\theta)}-(1-\beta)\delta\varphi(R_L)\right]\frac{\partial\Delta_u}{\partial\theta}\cdot\frac{\theta}{\Delta_u}\cdot\frac{1}{\theta}\\ &\quad +(1-\beta)\left[(1-G(R_L))(1-\Delta_u)\delta-[\varphi(R_H)-\delta\varphi(R_L)]\frac{\partial\Delta_u}{\partial R_L}\right]\frac{dR_L}{d\theta}.\end{aligned}$$

Except for the second term in the right-hand side of the above expression, all the remaining terms are positive, since $\partial\Delta_u/\partial R_L < 0$, $dR_L/d\theta > 0$ and $\partial\Delta_u/\partial\theta > 0$. In what follows, I find a sufficient condition for $(r+\mu+\lambda+s)\frac{ck}{f(\theta)}\left[\eta-\frac{\partial\Delta_u}{\partial\theta}\cdot\frac{\theta}{\Delta_u}\right]$ to be positive. Given this condition, the JC curve is downward-sloping. Clearly, this condition is overly restrictive. Quantitatively the JC may be downward sloping even if the condition does not hold, due to the remaining positive terms. First,

$\eta - \frac{\partial \Delta_u}{\partial \theta} \cdot \frac{\theta}{\Delta_u} > 0$ if and only if

$$\eta - (1-\eta)\frac{f(\theta)(1-G(R_L))(1-\Delta_u)}{f(\theta)(1-G(R_L)) + s + \lambda G(R_L) + \mu} > 0.$$

Let f^* be defined as $f^* = f(\theta)(1-G(R_L))$. Note that

$$\frac{f(\theta)(1-G(R_L))(1-\Delta_u)}{f(\theta)(1-G(R_L)) + s + \lambda G(R_L) + \mu} < \Phi(f^*),$$

where $\Phi(x)$ is defined as $\Phi(x) \equiv \frac{x}{x+s+\mu} \cdot \frac{\sigma(s+\mu)}{\mu(x+s+\mu)+\sigma(s+\mu)}$. Over $[0,\infty)$, the function $\Phi(x)$ is first strictly increasing, then strictly decreasing, with a maximum at x^*, with $x^* = (s+\mu)\sqrt{\frac{\mu+\sigma}{\mu}}$. Therefore,

$$\Phi(x^*) \leq \frac{\sqrt{\frac{\mu+\sigma}{\mu}}}{\sqrt{\frac{\mu+\sigma}{\mu}}+1} \cdot \frac{\sigma}{\mu\left(\sqrt{\frac{\mu+\sigma}{\mu}}+1\right)+\sigma}.$$

Therefore,

$$\eta - (1-\eta)\frac{f(\theta)(1-G(R_L))(1-\Delta_u)}{f(\theta)(1-G(R_L)) + s + \lambda G(R_L) + \mu} > \eta - \Phi(x^*).$$

Given that, after rearranging, $\eta - \Phi(x^*) > 0$ if and only if $\eta > \bar{\eta}$, it follows that $\eta > \bar{\eta}$ is a sufficient condition for $\eta - (1-\eta)\frac{f(\theta)(1-G(R_L))(1-\Delta_u)}{f(\theta)(1-G(R_L))+s+\lambda G(R_L)+\mu} > 0$. Therefore, it is also a condition for the JC condition to be downward-sloping, i.e. for $dR_H/d\theta < 0$ along the JC curve.

Having established sufficient conditions for the (JC) condition to be upward-sloping, I show next that the condition $\underline{\theta} < \bar{\theta}$ guarantees that there is a single crossing between the (JC) and (JDH) curves. Along the JDL curve (21), θ tends to infinity as R_L tends to infinity. As R_L tends to ε, θ tends to θ_0 in proposition 2, which is strictly positive given the assumptions in the proposition, since θ_0 is determined by

$$f(\theta_0) = (r+\mu+s+\lambda) \cdot \left[\frac{-\frac{b}{k} + (\varepsilon - \phi - c) + \lambda\frac{\varphi(\varepsilon)}{r+\mu+s+\lambda}}{\beta\varphi(\varepsilon)}\right].$$

In sum, $R_L(\theta)$ defined by the JDL condition (21) is a strictly increasing function that maps $[\theta_0, \infty) \to [\varepsilon, \infty)$. Using the $R_L(\theta)$ function, $\bar{\theta}$ is defined as the θ that satisfies the JC condition with $R_H = \varepsilon$, i.e. $\bar{\theta}$ satisfies

$$0 = -\frac{c}{q(\bar{\theta})} + \frac{1-\beta}{r+\mu+\lambda+s} \cdot \left[\bar{\Delta}_u \varphi(\varepsilon) + (1-\bar{\Delta}_u)\delta\varphi(R_L(\bar{\theta}))\right]$$

where $\bar{\Delta}_u$ is given by Δ_u evaluated at $R_H = \varepsilon$, $\theta = \bar{\theta}$ and $R_L = R_L(\bar{\theta})$. It is easy to verify that a unique $\bar{\theta}$ exists that satisfies the above condition, since the right-hand-side tends to a positive number as $\bar{\theta}$ tends to 0, and to minus infinity as $\bar{\theta}$ tends to infinity. Next, define $\underline{\theta}$ as the θ that satisfies the JDH condition (22) with $R_H = \varepsilon$, using $R_L(\theta)$, i.e. $\underline{\theta}$ satisfies

$$\begin{aligned} 0 =& \varepsilon - \phi - c + \frac{\lambda}{r+\mu+\lambda+s}\varphi(\varepsilon) - \frac{r+\mu+\delta\sigma}{r+\mu+\sigma}\frac{b}{k} \\ & - \beta\frac{f(\underline{\theta})}{(r+\mu+\sigma)(r+\mu+\lambda+s)}\left[(r+\mu)\varphi(\varepsilon) + \sigma\delta\varphi(R_L(\underline{\theta}))\right]. \end{aligned}$$

Again, it is easy to verify that a $\underline{\theta}$ that satisfies the above exists and is unique, given that JDH is strictly increasing. The right-hand-side of the above expression tends to minus infinity as $\underline{\theta}$ tends to infinity, and to a positive number (given the condition on θ_0 in proposition 2) as $\underline{\theta}$ tends to θ_0. Figure depicts the equilibrium in (θ, R_H). The equilibrium R_L follows from the relationship $R_L(\theta)$ defined by the JDL condition. Figure depicts the relationship $R_L(\theta)$ using the JDL condition (21).

Proof of proposition 4, continuity of TFP in (R_l, R_H)

First, let us prove continuity in R_L around ϕ, in other words, when we move from A_l to A_m in proposition 3. Make R_L tend to ϕ^-, i.e. $R_H < R_L < \phi$. TFP is given by A_l. Taking the limit as $R_L \to \phi^-$ gives

$$\lim_{R_L \to \phi^-} A_l = \frac{\Delta_e R_H^{1/\gamma} + (1-\Delta_e)\phi^{1/\gamma}\delta}{\left(\Delta_e R_H^{1/\gamma} + (1-\Delta_e)\phi^{1/\gamma}\right)^{1-\gamma}} \cdot \frac{1}{1-\gamma}.$$

Suppose now that $R_H < \phi < R_L$ and make R_L tend to ϕ^+. TFP is given by A_m and the limit is given by

$$\begin{aligned}\lim_{R_L \to \phi^+} A_m =& \frac{\Delta_e R_H^{1/\gamma} \phi^{1-1/\gamma} + (1-\Delta_e)\phi\delta}{\left(\Delta_e R_H^{1/\gamma} \phi^{-1/\gamma} + 1 - \Delta_e\right)^{1-\gamma}} \cdot \frac{1}{1-\gamma} \\ =& \frac{\Delta_e R_H^{1/\gamma} + (1-\Delta_e)\phi^{1/\gamma}\delta}{\left(\Delta_e R_H^{1/\gamma} + (1-\Delta_e)\phi^{1/\gamma}\right)^{1-\gamma}} \cdot \frac{1}{1-\gamma}.\end{aligned}$$

The two limits coincide, so TFP is continuous in R_L around ϕ.

Next, I prove continuity in R_H around ϕ, in other words, when we move from A_m to A_h in proposition 3. Suppose that $\phi < R_H < R_L$ (i.e. TFP is given by A_h)and that R_H tends to ϕ^+. The limit is given

$$\lim_{R_H \to \phi^+} A_h = \Delta_e \phi + (1-\Delta_e)\delta R_L.$$

Suppose now that $R_H < \phi < R_L$ (i.e. TFP is given by A_m)and that R_H tends to ϕ^-. The limit is given by

$$\begin{aligned}\lim_{R_H \to \phi^-} A_m =& \frac{\Delta_e R_H^{1/\gamma} \phi^{1-1/\gamma} + (1-\Delta_e)\delta R_L}{\left[\Delta_e \left(\frac{R_H}{\phi}\right)^{1/\gamma} + (1-\Delta_e)\right]^{1-\gamma}} \\ =& \Delta_e \phi + (1-\Delta_e)\delta R_L.\end{aligned}$$

The two limits coincide, so TFP is continuous in R_H around ϕ.

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