Indeterminacy with Increasing Returns to Variety and Sector-Specific Externalities*

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Abstract

This paper examines the theoretical as well as quantitative interrelations between equilibrium indeterminacy, increasing returns to product variety and sector-specific productive externalities within a two-sector real business cycle model. We analytically derive the necessary and sufficient condition under which the model exhibits an indeterminate steady state. In a calibrated version of our model economy, the threshold level of investment externalities that leads to belief-driven cyclical fluctuations is shown to be monotonically increasing with respect to the degree of market competitiveness. We also show that compared to three previous studies, our two-sector macroeconomy requires the lowest, and therefore the most empirically plausible, magnitude of productive externalities to generate endogenous business cycles.

Keywords: Indeterminacy, Increasing Returns to Variety, Sector-Specific Externalities.

JEL Classification: E30, E32, O41.

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1 Introduction

Since the work of Benhabib and Farmer (1994) and Farmer and Guo (1994), considerable progress has been made over the last two decades in exploring the empirically-plausible conditions needed to generate equilibrium indeterminacy and sunspot-driven aggregate fluctuations within real business cycle (RBC) models.[1] In particular, the original Benhabib-Farmer-Guo one-sector model requires an implausibly high level of increasing returns-to-scale in production, *vis-à-vis* estimation results reported by Burnside (1996) and Basu and Fernald (1997), to exhibit a continuum of stationary equilibrium trajectories. Subsequent research, *e.g.* Benhabib and Farmer (1996), Weder (2000) and Harrison (2001), shows that a representative-agent macroeconomy with two distinct (consumption and investment) goods and sector-specific externalities from productive inputs is able to yield endogenous business cycles under a much less stringent circumstance. In addition, a recent piece by Chang, Hung and Huang (2011) obtain the qualitatively identical result in the context of a one-sector RBC model with increasing returns to product variety caused by monopolistic competition and free entry/exit of firms.

In this paper, we build upon Benhabib and Farmer's (1996) analyses and examine the theoretical as well as quantitative interrelations between equilibrium indeterminacy, sector-specific productive externalities and the degree of monopoly power within a continuous-time two-sector RBC model. Each production sector has an intermediate-good segment in which monopolistically competitive firms operate under fixed set-up costs and fully mobile capital and labor inputs. The equilibrium measure of these intermediate firms for consumption or investment goods is endogenously determined through the zero-profit condition. This in turn yields increasing returns to an expansion in product variety *à la* Chang, Hung and Huang (2011). A final output is produced within each sector from the set of available intermediate goods in a perfectly competitive setting.

Using the standard procedure of linearizing equilibrium conditions around the unique interior steady state, we first derive the analytical expression for the resulting Jacobian matrix of partial derivatives, and then find that the presence of consumption externalities exerts no effect on the model's local dynamics. When agents expect that the rate of return on investment will increase tomorrow, they need incentive to give up consumption today for more capital accumulation. It follows that no productive externality is needed in the consumption sector, from which agents move their capital and labor services, to fulfill their optimistic anticipation. Next, within the empirically realistic specification that capital's share of GDP is lower

[1] See Benhabib and Farmer (1999) for an early survey on this RBC-based indeterminacy literature.

than that of labor, we obtain the *necessary and sufficient* condition for our model economy to possess an indeterminate steady state, which turns out to depend on model parameters in a rather complicated way.

To gain further insights of our theoretical results, a quantitative investigation of macroeconomic (in)stability is undertaken within a calibrated version of our two-sector RBC model under parameter values that are consistent with post Korean-war U.S. time series data. Accordingly, a two-dimensional plot is constructed to divide the feasible parameter space into the regions of "Saddle", "Sink" and "Source" as functions of the size of investment externalities versus the magnitude of the price-cost markup ratio. We show that the threshold level for productive externalities in the investment sector that leads to endogenous cyclical fluctuations is monotonically increasing with respect to the degree of market competitiveness. In terms of the underlying economic intuition, consider the two opposing effects on the representative household's intertemporal consumption Euler equation as it becomes optimistic about the economy's future. On the one hand, an increase in today's investment expenditures will decrease next period's real interest rate because of diminishing marginal product of capital (*the MPK effect*). This channel in turn invalidates agents' initial optimism. On the other hand, due to market imperfection and productive investment externalities, the social production possibility frontier that traces out the trade-off between consumption and investment spending is downward-sloping and convex to the origin. As a consequence, the relative price of investment goods will fall (*the price effect*) upon the household's optimism that shifts capital and labor inputs out of producing consumption goods. This channel in turn justifies agents' spurt to invest more today. Our analysis shows that for a given level of intermediate firms' market power, the price effect quantitatively outweighs the *MPK* effect provided the degree of investment externalities is sufficiently high to exceed the lower bound associated with the requisite condition for indeterminacy and sunspots. In this case, the representative household's rosy expectation will be validated as a self-fulfilling equilibrium.

We also find that when the market competitiveness of intermediate firms falls, the curvature for the economy's convex social production possibility frontier becomes more pronounced. As a consequence, for the same magnitude of increases in capital and labor inputs that are moved into the investment sector because of agents' optimism, the resulting decrease in the relative price of investment goods will be larger, which in turn enhances the above-mentioned price effect. It follows that local indeterminacy may arise in our model economy without *any* investment externalities as long as increasing returns to product variety are sufficiently strong.

With regard to the empirical plausibility on the minimum level of productive externalities required for equilibrium indeterminacy, the RBC macroeconomy that we examine has the versatility to subsume three existing studies: (i) the Benhabib-Farmer two-sector model with perfectly-competitive production sectors; (ii) the Chang-Hung-Huang one-sector model with monopolistic competition and a social technology that exhibits positive externalities from capital and labor services; and (iii) the original Benhabib-Farmer-Guo one-sector model with perfect competition and aggregate increasing returns-to-scale in production. While keeping the calibrated values of other parameters unchanged, we find that the Benhabib-Farmer-Guo one-sector model needs the highest external effect, whereas our two-sector model requires the lowest, and therefore the most empirically plausible, level of productive externalities to generate belief-driven cyclical fluctuations. In sum, our quantitative analysis highlights two cooperating factors – increasing returns to product variety and sector-specific investment externalities – in producing multiple, indeterminate equilibria within a real business cycle model.

This paper is related to Weder (1998) who also explores equilibrium indeterminacy in a similar two-sector (consumption and investment) RBC model with monopolistic competition and costless entry/exit of firms. Our study differs from his in three aspects. First, in addition to fixed overhead costs, the sectoral production functions in Weder's model allow for increasing/constant/decreasing marginal costs, whereas sector-specific productive externalities are considered in our model. Second, we obtain the necessary and sufficient condition for local indeterminacy after analytically deriving the model's Jacobian matrix, whereas Weder's work does not. Third, Weder conducts numerical business-cycle simulations within his discrete-time macroeconomy driven by sectoral technology shocks and agents' animal spirits, whereas we focus on the occurrence of perfect-foresight competitive equilibria in our continuous-time setting.

The remainder of this paper is organized as follows. Section 2 describes the economy and analyzes its equilibrium conditions. Section 3 analytically and quantitatively examines our model's local stability properties. Section 4 concludes.

2 The Economy

Our analysis builds upon the continuous-time two-sector real business cycle (RBC) model *à la* Benhabib and Farmer (1996) that allows for market imperfection together with free entry and exit of firms. Households live forever, and derive utility from consumption and leisure. The production side of the economy consists of two distinct sectors for consumption and investment goods, respectively. Each sector has an intermediate-good segment in which monopolistically competitive firms operate under fixed set-up costs and sector-specific externalities from fully mobile capital and labor inputs. The equilibrium measure of these intermediate firms for consumption or investment goods is endogenously determined through the zero-profit condition. This in turn yields increasing returns to product variety as in Chang, Hung and Huang (2011). A final output is produced within each sector from the set of available differentiated intermediate goods in a perfectly competitive environment. We postulate that there are no fundamental uncertainties present in the economy.

2.1 Firms

Our model economy is comprised of two production sectors indexed by $m = c,\ I$, where c stands for consumption and I stands for investment. Since firms are solving a static profit maximization problem during each period, time subscripts will be suppressed for notational convenience throughout this subsection. The final good in each sector Y_m is produced from a continuum of intermediate inputs x_{mj}, where $j \in [0,\ N_m]$ and N_m represents the measure of (or the degree of variety for) intermediate goods available within sector m, through the following technology that exhibits constant returns-to-scale:

$$Y_m = \left(\int_0^{N_m} x_{mj}^{\lambda} dj \right)^{\frac{1}{\lambda}}, \quad m = c,\ I \ \text{ and } \ 0 < \lambda < 1. \tag{1}$$

Using p_{mj} to denote the dollar price of the jth intermediate input in sector m, and P_m to denote the dollar price of sectoral output Y_m, the first-order condition for final-good producers' profit maximization problem is given by

$$x_{mj} = \left(\frac{p_{mj}}{P_m} \right)^{\frac{1}{\lambda - 1}} Y_m, \quad m = c,\ I, \tag{2}$$

where the price elasticity of demand for x_{mj} is equal to $\frac{1}{1-\lambda}$. When $\lambda = 1$, the model collapses to one with perfectly competitive markets as in Benhabib and Farmer (1996) and Harrison (2001).

Each intermediate good is produced by a monopolist with the production function that allows for increasing returns-to-scale:

$$x_{mj} = \Psi_m K_{mj}^{\alpha} L_{mj}^{1-\alpha} - Z_m, \quad m = c,\ I,\ \ 0 < \alpha < 1 \ \text{ and } \ Z_m > 0, \tag{3}$$

where K_{mj} and L_{mj} are capital and labor inputs employed by the j'th intermediate producer in sector m; and Z_m represents a constant amount of intermediate goods that must be expended in sector m as fixed set-up costs of production before any sale is made. The presence of such overhead costs implies that the intermediate technology exhibits increasing returns-to-scale. Moreover, Ψ_m denotes the sectoral production externalities that each intermediate firm takes as given, and is postulated to take the following specification as in Chang, Hung and Huang (2011):

$$\Psi_m = (K_m^\alpha L_m^{1-\alpha})^{\theta_m}, \quad m = c,\ I,\ \text{ and } \ \theta_m \geq 0, \tag{4}$$

where K_m and L_m represent the within-sector aggregate levels of capital and labor services devoted to producing intermediate goods, *i.e.* $K_m = \int_0^{N_m} K_{mj} dj$ and $L_m = \int_0^{N_m} L_{mj} dj$; and θ_m measures the degree of sector-specific productive externalities. When $\theta_m > 0$, additional increasing returns will exist in (3) because of rising marginal productivities.

Using equations (2) and (3), together with the assumption that factor markets are perfectly competitive within each sector, it is straightforward to show that the first-order conditions for intermediate-good producers' profit maximization problem are given by

$$R_m = \frac{\lambda \alpha (x_{mj} + Z_m) p_{mj}}{K_{mj}}, \quad m = c,\ I, \tag{5}$$

$$W_m = \frac{\lambda (1-\alpha)(x_{mj} + Z_m) p_{mj}}{L_{mj}}, \quad m = c,\ I \tag{6}$$

where R_m is the nominal rental rate of capital and W_m is the nominal wage rate in sector m. Notice that intermediate firms in each sector will face the same factor prices, *i.e.* $R_c = R_I = R$ and $W_c = W_I = W$, since capital and labor inputs are postulated to be fully mobile across the two production sectors.

Under the maintained assumption of free entry and exit for intermediate-good producers in both sectors, their period profits will be equal to zero. This zero-profit condition in conjunction with (5) and (6) yield the (constant) equilibrium quantity of intermediate input x_{mj}:

$$x_{mj} = \frac{\lambda Z_m}{1-\lambda}, \quad m = c,\ I, \tag{7}$$

which also represents the size of an intermediate firm that turns out to be independent of any endogenous variable. In what follows, our analysis is restricted to the model's symmetric equilibria within each sector in which

$$p_{mj} = p_m,\ \ x_{mj} = x_m,\ \ K_{mj} = \frac{K_m}{N_m},\ \ L_{mj} = \frac{L_m}{N_m},\ \ m = c, I \text{ and for all } j \in [0, N_m]. \tag{8}$$

After substituting condition (8) into (1) and (2), we derive that the sectoral production functions are given by

$$Y_m = N_m^{\frac{1}{\lambda}} x_m, \quad m = c,\ I, \tag{9}$$

which will display increasing returns to an expansion in product variety since $0 < \lambda < 1$; and that the corresponding sectoral prices of intermediate goods are

$$p_m = P_m N_m^{\frac{1-\lambda}{\lambda}}, \quad m = c,\ I. \tag{10}$$

Finally, using equations (3), (4), (7) and (8) leads to the equilibrium measure of intermediate firms in sector m:

$$N_m = \left(\frac{1-\lambda}{Z_m}\right) K_m^{\alpha(1+\theta_m)} L_m^{(1-\alpha)(1+\theta_m)}, \quad m = c,\ I. \tag{11}$$

2.2 Households

The economy is populated by a unit measure of identical infinitely-lived households. Each household maximizes its present discounted lifetime utility

$$\int_{t=0}^{\infty} \left(\log C_t - \frac{L_t^{1+\gamma}}{1+\gamma}\right) e^{-\rho t}, \ \rho > 0, \text{ and } \gamma \geq 0, \tag{12}$$

where C_t and L_t are the household's consumption and hours worked, respectively; ρ is the subjective rate of time preference, and γ denotes the inverse of the wage elasticity for labor supply. Notice that the period utility function in (12) is consistent with long-run balanced growth, a feature that is commonly adopted in the real business cycle literature. Using the consumption good as the economy's numeraire, the real-valued budget constraint faced by the representative agent is given by

$$C_t + P_t I_t = \underbrace{r_t K_t + w_t L_t}_{=Y_t}, \tag{13}$$

where I_t is gross investment, $P_t \left(= \frac{P_{It}}{P_{ct}}\right)$ denotes the relative price of investment to consumption goods, $r_t \left(= \frac{R_t}{P_{ct}}\right)$ is the real rental rate, $w_t \left(= \frac{W_t}{P_{ct}}\right)$ is the real wage rate, Y_t is national income or GDP, and K_t is the household's capital stock that evolves according to the law of motion

$$\dot{K}_t = I_t - \delta K_t, \quad K_0 > 0 \text{ given}, \tag{14}$$

where $\delta \in (0,1)$ is the capital depreciation rate.

The first-order conditions for the representative household's dynamic optimization problem are

$$\frac{1}{C_t} = \frac{\phi_t}{P_t}, \tag{15}$$

$$C_t L_t^{\gamma} = w_t, \tag{16}$$

$$\frac{\dot{\phi}_t}{\phi_t} = \rho + \delta - \frac{r_t}{P_t}, \tag{17}$$

where ϕ_t is the co-state variable (measured in terms of utility) that can be interpreted as the shadow value of K_t; (16) equates the slope of household's indifference curve to the real wage rate, and (17) is the Keynes-Ramsey rule that governs the household's intertemporal choices of consumption.

2.3 Symmetric Equilibria and Steady State

Since firms use identical production technologies and face the same factor prices across the two sectors, the fractions of capital and labor inputs utilized in the consumption sector are equal,

$$\frac{K_{ct}}{K_t} = \frac{L_{ct}}{L_t} \equiv \mu_t \in (0,1). \tag{18}$$

We will focus on the model's symmetric equilibria in which the household's and firms' first-order conditions are all satisfied. Without loss of generality, the equilibrium price of the numeraire (consumption) good is normalized to unity, $P_{ct} = 1$. The equalities of demand by households and supply by firms in the consumption and investment sectors are given by $C_t = Y_{ct}$ and $I_t = Y_{It}$. Moreover, both the capital and labor markets will clear whereby $K_{ct} + K_{It} = K_t$ and $L_{ct} + L_{It} = L_t$. Using equations (5)-(11), (13) and (18), the equilibrium price of investment relative to consumption goods can be expressed as

$$P_t = \frac{\mu_t^{\frac{1-\lambda+\theta_c}{\lambda}} \left(\frac{Z_I}{Z_c}\right)^{\frac{1-\lambda}{\lambda}}}{(1-\mu_t)^{\frac{1-\lambda+\theta_I}{\lambda}}} \left[K_t^{\alpha} L_t^{1-\alpha}\right]^{\frac{\theta_c-\theta_I}{\lambda}}, \tag{19}$$

and the economy's aggregate production function or total output is given by

$$Y_t = \lambda \left(1-\alpha\right)^{\frac{1-\lambda+\theta_c}{\lambda}} \left(\frac{1-\lambda}{Z_c}\right)^{\frac{1-\lambda}{\lambda}} K_t^{\frac{\alpha(1+\theta_c)}{\lambda}} L_t^{1+\gamma-\frac{(\alpha+\gamma)(1+\theta_c)}{\lambda}}, \tag{20}$$

where $\frac{\alpha(1+\theta_c)}{\lambda} < 1$ to rule out the possibility of sustained endogenous growth.

It is straightforward to show that our model economy possesses a unique interior steady state. Specifically, the steady-state proportion of factor inputs allocated to the consumption sector, and the household's (aggregate) hour worked and capital stock are

$$\mu_{ss} = 1 - \frac{\alpha\delta}{\rho+\delta},\ L_{ss} = \left(\frac{1-\alpha}{\mu_{ss}}\right)^{\frac{1}{1+\gamma}} \text{ and } K_{ss} = \left[\frac{\lambda\left(1-\mu_{ss}\right)^{\frac{1+\theta_I}{\lambda}}\left(\frac{1-\lambda}{Z_I}\right)^{\frac{1-\lambda}{\lambda}} L_{ss}^{\frac{(1-\alpha)(1+\theta_I)}{\lambda}}}{\delta}\right]^{\frac{\lambda}{\lambda-\alpha(1+\theta_I)}}. \tag{21}$$

Given (21), the steady-state expressions of all the remaining endogenous variables can then be easily derived.

3 Macroeconomic (In)stability

In terms of the local stability properties of our model economy, we take linear approximations to its equilibrium conditions in a neighborhood of the steady state to obtain the following dynamical system:

$$\begin{bmatrix} \dot{K}_t \\ \dot{\phi}_t \end{bmatrix} = \mathbf{J} \begin{bmatrix} K_t - K_{ss} \\ \phi_t - \phi_{ss} \end{bmatrix}, \quad K_0 > 0 \text{ given,} \tag{22}$$

where $\mathbf{J}$ is the Jacobian matrix of partial derivatives for the transformed dynamical system. It can be shown that the Jacobian's determinant and trace are

$$Det = \frac{\alpha\mu_{ss}\left(1-\mu_{ss}\right)(1+\gamma)\left(\lambda\Pi L_{ss}\right)^2\left[\lambda-\alpha(1+\theta_I)\right]}{(1+\theta_I)[1-\alpha+(\alpha+\gamma)\mu_{ss}]-\lambda(1+\gamma)}, \tag{23}$$

$$Tr = \left(\frac{\Pi}{1-\mu_{ss}}\right)\left\{\left(1-\mu_{ss}\right)\left[\alpha(1+\theta_I)-\lambda\right] + \frac{\alpha\Omega}{(1+\theta_I)[1-\alpha+(\alpha+\gamma)\mu_{ss}]-\lambda(1+\gamma)}\right\}, \tag{24}$$

where

$$\Pi = \left(1-\mu_{ss}\right)^{\frac{1-\lambda+\theta_I}{\lambda}}\left(\frac{1-\lambda}{Z_I}\right)^{\frac{1-\lambda}{\lambda}} K_{ss}^{\frac{\alpha(1+\theta_I)-\lambda}{\lambda}} L_{ss}^{\frac{(1-\alpha)(1+\theta_I)-\lambda}{\lambda}} > 0, \tag{25}$$

$$\Omega = (1-\alpha)(1+\theta_I)(1-\mu_{ss})\left[\lambda-(1+\theta_I)(1-\mu_{ss})\right] + \mu_{ss}(1+\gamma)\left[\lambda(1+\theta_I-\lambda)-(1-\mu_{ss})(1+\theta_I)^2\right] \gtreqless 0, \tag{26}$$

and μ_{ss}, L_{ss} and K_{ss} are given by (21).

Since the dynamical system (22) possesses one predetermined variable K_t, the economy exhibits saddle-path stability and equilibrium uniqueness if and only if the two eigenvalues of $\mathbf{J}$ are of opposite sign ($Det < 0$). When both eigenvalues have negative real parts ($Det > 0$ and $Tr < 0$), the steady state is a locally indeterminate sink that can be exploited to generate endogenous cyclical fluctuations driven by agents' self-fulfilling expectations or sunspots. The steady state becomes a source when both eigenvalues have positive real parts ($Det > 0$ and $Tr > 0$).

3.1 Analytical Characterizations

Based on (23)-(24) and the subsequent discussion, this subsection analytically examines the condition(s) under which our two-sector RBC model exhibits equilibrium indeterminacy and belief-driven aggregate fluctuations ($Det > 0$ and $Tr < 0$). We first note that the degree of productive externalities in the consumption sector θ_c does not enter (23) or (24), hence it exerts no effect on the economy's local dynamics. This finding turns out to be reminiscent of Harrison (2001) under perfectly competitive markets. Intuitively, when agents expect the rate of return on investment to increase tomorrow, they will choose to give up today's consumption for more capital accumulation. It follows that no productive externality is needed in the consumption sector, from which agents move their capital and labor services, to fulfill their optimistic anticipation about the economy's future. This result thus allows us to set $\theta_c = 0$ from now on.

Second, in accordance with the observed evidence that capital income accounts for a smaller percentage of GDP than labor income, our analyses are restricted to empirically plausible specifications with $\alpha < 1 - \alpha$. We then find that under this assumption, the Jacobian's determinant (23) is positive when

$$\underbrace{\frac{\lambda(1+\gamma)(\rho+\delta)}{\alpha\delta(1-\alpha)+(1+\gamma)[\rho+(1-\alpha)\delta]} - 1}_{\equiv \theta_I^{\min}} < \theta_I < \frac{\lambda}{\alpha} - 1, \tag{27}$$

which in turn provides a necessary condition for indeterminacy and sunspots. Since $0 < \alpha$, λ, $\mu_{ss} < 1$, $\gamma \geq 0$ and L_{ss}, $\Pi > 0$, our model's Jacobian matrix possess a positive determinant if and only if

$$\frac{\lambda - \alpha(1+\theta_I)}{(1+\theta_I)[1-\alpha+(\alpha+\gamma)\mu_{ss}] - \lambda(1+\gamma)} > 0. \tag{28}$$

Condition (27) reports the feasible range of investment externalities $\theta_I \geq 0$ that leads to

$Det > 0$ when the numerator and denominator of (28) are both strictly positive.[2] Moreover, given the restriction of $\alpha < 1-\alpha$, it is straightforward to show that $\frac{\lambda(1+\gamma)(\rho+\delta)}{\alpha\delta(1-\alpha)+(1+\gamma)[\rho+(1-\alpha)\delta]} < \frac{\lambda}{\alpha}$.

Third, since $0 < \mu_{ss} < 1$ and $\Pi > 0$, the sign of the Jacobian's trace is determined by the terms inside the curly braces in equation (24). Using the expression of Ω as in (26), it can be shown that $Tr < 0$ if and only if

$$\theta_I < \frac{\lambda(1+\gamma)(\alpha\delta-\rho)}{\delta\left[\alpha(1+\gamma)+(1-\alpha)^2\right]-\left(1+\frac{\gamma\rho-\alpha\delta}{\rho+\delta}\right)[\rho+(1-\alpha)\delta]} - 1. \tag{29}$$

Finally, we cannot be certain whether (i) $\theta_I^{\min}$ given by (27) is strictly positive under all possible parameterizations; and (ii) the right-hand side of (27) or (29) is more demanding because α, γ, δ, λ and ρ enter these conditions in a rather complicated way. As a result, the necessary and sufficient condition for both eigenvalues of the Jacobian $\mathbf{J}$ to have negative real parts is given by[3]

$$\max\{\theta_I^{\min},\ 0\} < \theta_I < \min\left\{\frac{\lambda}{\alpha}-1,\ \frac{\lambda(1+\gamma)(\alpha\delta-\rho)}{\delta\left[\alpha(1+\gamma)+(1-\alpha)^2\right]-\left(1+\frac{\gamma\rho-\alpha\delta}{\rho+\delta}\right)[\rho+(1-\alpha)\delta]} - 1\right\}, \tag{30}$$

under which the model's steady state will become an indeterminate sink.

3.2 Quantitative Results

In this subsection, we undertake a quantitative investigation of macroeconomic (in)stability within a calibrated version of our two-sector RBC model for combinations of parameters whose values are selected based on empirically observed features of the post Korean-war U.S. economy. As in Benhabib and Farmer (1996) and Chang, Hung and Huang (2011), the labor share of national income, $1-\alpha$, is chosen to be 0.7; the subjective rate of time preference, ρ, is set to be 0.05; the labor supply elasticity, γ, is equal to 0 (*i.e.* indivisible labor, *à la* Hansen [1985] and Rogerson [1988], that is infinitely elastic); and the capital depreciation rate, δ, is fixed at 0.1.

Given the above benchmark parameterization, Figure 1 depicts the resulting local stability properties of our model economy as a function of the level of productive externalities in

[2] When the numerator and denominator of (28) are both negative, the resulting range of θ_I for $Det > 0$ is inconsistent with our maintained assumption $\alpha < 1-\alpha$.

[3] If $\frac{\lambda(1+\gamma)(\alpha\delta-\rho)}{\delta\left[\alpha(1+\gamma)+(1-\alpha)^2\right]-\left(1+\frac{\gamma\rho-\alpha\delta}{\rho+\delta}\right)[\rho+(1-\alpha)\delta]} - 1 < \theta_I^{\min}$, then there exists no feasible range of investment externalities over which our model exhibits an indeterminate steady state. Accordingly, we rule out this possibility.

investment versus the degree of intermediate firms' monopoly power. In particular, the $\phi_I - \lambda$ space is divided into regions of "Saddle", "Sink" and "Source". Using durables as a proxy for investment goods, we set the upper bound of investment externalities θ_I to be 0.33, which is Basu and Fernald's (1997) aggregation-corrected point estimate for returns-to-scale in the U.S. durables manufacturing industry, on the horizontal axis of Figure 1. In addition, we note that $\frac{1}{\lambda}$ is equal to the markup ratio of price over marginal cost, and that the range for its empirical estimates lies between 1 and 1.7; see Hall (1986), Domowitz, Hubbard and Petersen (1988), Morrison (1990), and Chirinko and Fazzari (1994), among others. Based on the midpoint of these estimation results, we set the lower bound of λ to be 0.71, as shown on the vertical axis of Figure 1.

Result 1. For a given level of $\lambda \in [0.95,\ 1]$, the economy's local dynamics changes from saddle-path stability to equilibrium indeterminacy, and then to complete instability as the degree of investment externalities increases. For a given level of $\lambda \in [0.85,\ 0.94]$, the model's steady state switches from being a sink to a source as θ_I rises. When $\lambda < 0.85$, the model's steady state is always a source.

As an illustrative example, when the price-cost markup is set to be 1.03 (or $\lambda = 0.97$) *à la* Basu and Fernald (1997) for the U.S. total private economy, our model exhibits a locally unique equilibrium (a saddle path) for $0 \leq \theta_I \leq 0.0318$. Local indeterminacy occurs for investment externalities in the range of $0.0319 \leq \theta_I \leq 0.1412$. The steady state turns into a source when θ_I is raised to the interval of $[0.1413, 0.33]$. In this case, any trajectory that diverges from this completely unstable steady state may settle down to a limit cycle or to some complicated attracting sets.

Result 2. When $0.85 \leq \lambda \leq 1$, the threshold level for productive externalities in the investment sector that leads to indeterminacy and sunspots, denoted as $\theta_I^{\min}$ given by (27), is monotonically increasing with respect to the degree of market competitiveness, *i.e.* $\frac{\partial \theta_I^{\min}}{\partial \lambda} > 0$.

To understand the intuition for this result, we note that the intertemporal consumption Euler equation in the discrete-time version of our model is given by

$$\frac{C_{t+1}}{C_t} = \beta \left[\frac{r_{t+1} + (1-\delta) P_{t+1}}{P_t} \right], \tag{31}$$

where β is the discount factor. Start from the model's steady state at period t, and suppose that agents become optimistic about the economy's future. Acting upon this change in non-fundamental expectations, the representative household will consume less (C_t falls) and invest more today, which in turn raises next period's capital stock (raising K_{t+1}), hours worked, output, and consumption (C_{t+1} rises). As a result, the left-hand side of (31) becomes higher. For this alternative dynamic path to be justified as a self-fulfilling equilibrium, the (price-weighted) rate of return on K_{t+1} net of depreciation, *i.e.* the right-hand side of (31), needs to increase as well.

As it turns out, the quantitative interdependence between ϕ_I and λ that governs our model's local stability properties depends crucially on the relative strength of two opposing forces. On the one hand, an increase in today's investment that raises K_{t+1} will lead to a lower real interest rate r_{t+1} because of diminishing marginal product of capital ($\frac{\alpha(1+\theta_c)}{\lambda} < 1$; see equation 20). Therefore, this *MPK effect* causes the right-hand side of (31) to fall. On the other hand, due to the presence of market imperfection as well as non-negative productive externalities in the investment sector, the economy's social production possibility frontier which traces out the trade-off between agents' consumption and investment expenditures is downward sloping and convex to the origin. It follows that its slope (or marginal rate of transformation), which is equal to the relative price of investment goods P_t, will decrease upon the household's optimism that shifts capital and labor inputs out of producing consumption goods ($\frac{dP_t}{d\mu_t} > 0$; see equation 19). Consequently, this *price effect* causes right-hand side of (31) to rise.

Results 1 and 2 together demonstrate that for a given level of intermediate firms' market power $\lambda \geq 0.85$, the price effect quantitatively outweighs the MPK effect provided the degree of investment externalities is sufficiently high to satisfy the lower bound associated with condition (30). In this case, indeterminacy and sunspot result as the right-hand side of (31) will rise to validate agents' initial anticipated increase in the return on capital. It follows that reducing the investment externalities to be lower than the critical $\theta_I^{\min}$ ($= 0.0319$ when $\lambda = 0.97$) is able to stabilize the economy against belief-driven business cycles because of a dominating MPK effect. Next, upon further examination of the vertical axis in Figure 1, we observe that

Result 3. When $0.85 \leq \lambda \leq 0.94$, the model's steady state is an indeterminate sink under no investment externalities ($\theta_I = 0$).

This result states that equilibrium indeterminacy will occur without any productive externality for investment when the degree of increasing returns to product variety (or the price-cost markup ratio) $\frac{1}{\lambda} \in [1.06,\, 1.18]$ generating a quantitatively stronger price effect. Using equations (7), (9), (11) and (19), together with $\theta_c = \theta_I = 0$, it can be shown that the economy's period-t production possibility set is

$$Z_c^{1-\lambda} C_t^{\lambda} + Z_I^{1-\lambda} I_t^{\lambda} = \lambda^{\lambda} (1-\lambda)^{1-\lambda} K_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \lambda < 1, \tag{32}$$

which depicts a curve that is strictly convex to the origin in the positive quadrant. Figure 2 shows that when the market power of intermediate firms rises (*i.e.* λ falls), the curvature for the social production possibility frontier becomes more pronounced. As a consequence, for the same magnitude of reductions in capital and labor inputs that are moved out of the consumption sector because of agents' optimism, the resulting decrease in the relative price of investment goods P_t will be larger. This implies that the aforementioned price effect is strengthened as the intermediate-good markets become less competitive, which in turns may yield indeterminacy and sunspots under no investment externalities.

Finally, it is worth noting that our model has the versatility to subsume three existing studies: (i) Benhabib and Farmer's (1996) two-sector model corresponds to the perfectly-competitive formulation with $\lambda = 1$ and $Z_c = Z_I = 0$; (ii) the one-sector model *à la* Chang, Hung and Huang (2011) corresponds to the imperfectly-competitive setting with $0 < \lambda < 1$, $P_t = 1$ (for all t) and an aggregate production function given by

$$Y_t = K_t^{\frac{\alpha(1+\theta_K)}{\lambda}} L_t^{\frac{(1-\alpha)(1+\theta_L)}{\lambda}}, \tag{33}$$

where θ_K and θ_L represent positive productive externalities from capital and labor services, respectively; and (iii) the Benhabib-Farmer-Guo one-sector model corresponds to the case with $\lambda = P_t = 1$, $Z_c = Z_I = 0$ and a social technology as in (33).

As discussed earlier under Result 1, our economy with $\lambda = 0.97$ possesses an indeterminate steady state when $0.0319 \leq \theta_I \leq 0.1412$. To place this quantitative result in perspective, Table 1 also reports the regions for local indeterminacy within the above-mentioned three nested models, while keeping the calibrated values of α, ρ, γ and δ unchanged. Notice that the minimum level of productive externalities needed for indeterminacy and sunspots is the smallest in our two-sector macroeconomy with increasing returns to product variety and investment externalities. Intuitively, Table 1 reflects two complementary factors in producing multiple, indeterminate equilibria within a real business cycle model. On the one hand, when production takes place in two distinct sectors, agents have the ability to reallocate resources between them to take advantage of increasing returns. Hence, by moving from the one-sector to the two-sector framework, the size of productive externalities required for equilibrium indeterminacy will fall. On the other hand, adding expansions to product variety raises the elasticity of output with respect to labor such that the resulting price effect becomes stronger upon agents' self-fulfilling expectations. As a result, the Benhabib-Farmer-Guo one-sector model requires the highest external effect, whereas our two-sector model needs the lowest, and therefore the most empirically plausible, degree of increasing returns-to-scale in production to generate belief-driven cyclical fluctuations.

Table 1: Regions of Local Indeterminacy	
Chang-Guo-Wang Two-Sector Model ($\lambda = 0.97$ and $P_t \neq 1$)	$0.0319 \leq \theta_I \leq 0.1412$
Benhabib-Farmer Two-Sector Model ($\lambda = 1$ and $P_t \neq 1$)	$0.0638 \leq \theta_I \leq 0.1764$
Chang-Hung-Huang One-Sector Model ($\lambda = 0.97$ and $P_t = 1$)	$0.3858 \leq \theta_L$
Benhabib-Farmer-Guo One-Sector Model ($\lambda = P_t = 1$)	$0.4286 \leq \theta_L$

4 Conclusion

This paper has examined how the theoretical as well as quantitative interrelations between (i) the level of increasing returns to product variety and (ii) the degree of sector-specific productive externalities affect the equilibrium dynamics of a real business cycle model with two distinct production sectors: consumption and investment. We analytically derive the necessary and sufficient condition for the economy to possess an indeterminate steady state and thus a continuum of stationary sunspot equilibria. Under a benchmark parameterization that is consistent with post Korean-war U.S. time series data, the minimum magnitude of investment externalities that leads to endogenous aggregate fluctuations is shown to be monotonically increasing with respect to the degree of market competitiveness. We also find that when intermediate firms' monopoly power is sufficiently strong, local indeterminacy may arise in our model without any externality for producing investment goods. Finally, in comparison with three predecessors that we consider, our two-sector macroeconomy requires the lowest, and therefore the most empirically plausible, degree of increasing returns-to-scale in production to generate belief-driven business cycles.

This paper can be extended in several directions. In particular, it would be worthwhile to incorporate additional features that have been shown to influence macroeconomic (in)stability properties in a two-sector RBC model, such as no-income-effect preferences *à la* Guo and Harrison (2010), government spending on goods and services *à la* Chang et al. (2015, 2019), and progressive income taxation *à la* Guo and Harrison (2015), among others. These possible extensions will allow us to examine the robustness of this paper's theoretical and quantitative results, as well as to further identify model parameters that govern the region of local indeterminacy within a multi-sector representative-agent macroeconomy. We plan to pursue these research projects in the near future.

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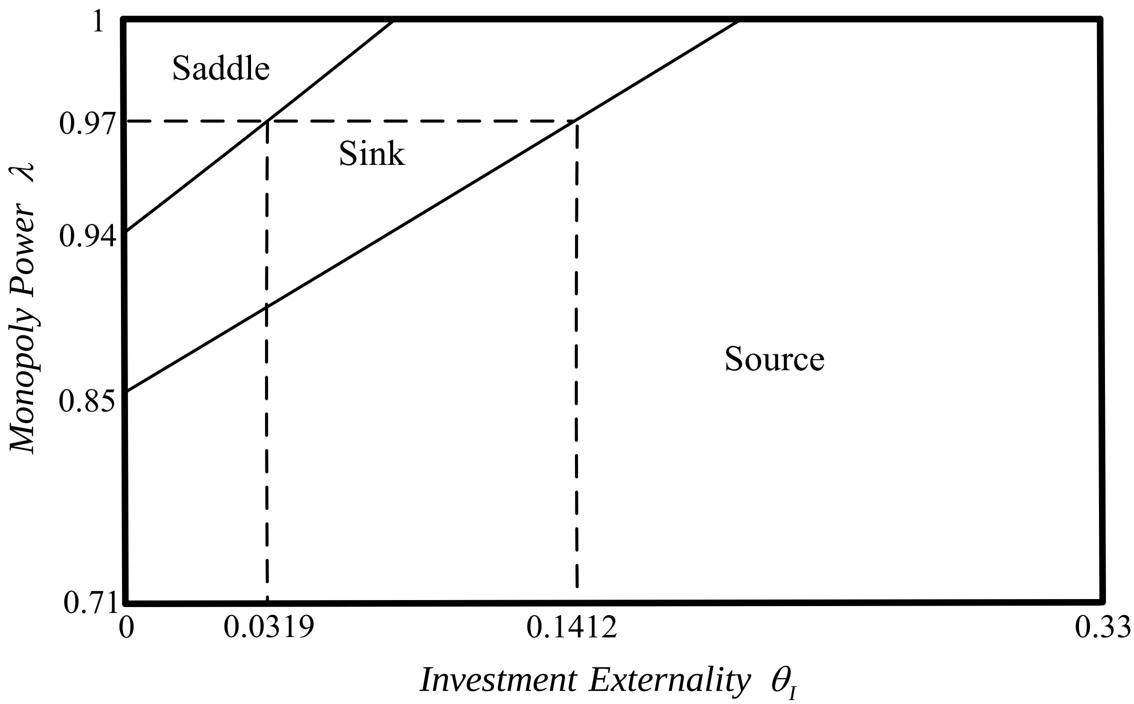

Figure 1. Local Stability Properties of the Steady State

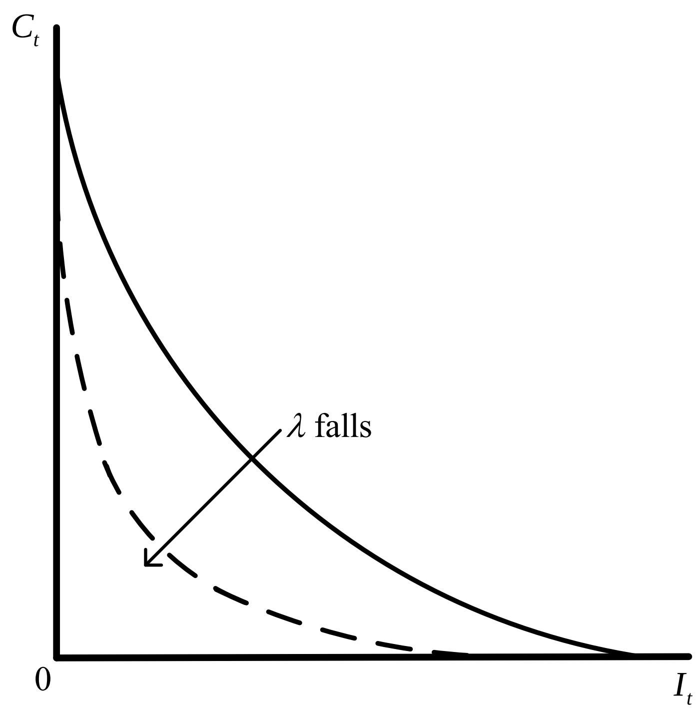

Figure 2. Social Production Possibility Frontier