

The Inverse Relationship between Farm Size and Productivity: Refocusing the Debate[†]

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Abstract

The relationship between farm size and productivity is a recurrent topic in development economics, almost as old as the discipline itself. This paper emphasizes the importance of choice of productivity measures in the inverse relationship literature. First, we seek to clarify the common measures, their relationships, and their advantages and limitations in empirical work. Second, we argue that much of the existing literature inappropriately uses partial measures such as land productivity. Third, we discuss the dynamic nature of the farm size – productivity relationship and show that the identification of these dynamics will in part depend upon the choice of productivity measure. Lastly, using a pseudo-panel of Brazilian farms that are aggregated at the municipality and farm size levels over the period 1985-2006, we provide new empirical evidence on the inverse relationship between farm size and both land productivity and total factor productivity. The empirical exercise highlights the importance of choice of measure when testing the inverse relationship. The inverse relationship between size and land productivity is alive and well. The relationship between total factor productivity and size, in contrast, has evolved with modernization during this period, becoming increasingly U-shaped or even positive.

Keywords: Inverse relationship, agriculture, farm size, total factor productivity (TFP), Brazil

JEL Codes: O13, Q12

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I. Introduction

The relationship between farm size and productivity is a recurrent topic in development economics, almost as old as the discipline itself. John Stuart Mill observed an inverse relationship as early as 1848, a relationship that he later posited had changed due to increasing capital intensity of farming (Lipton, 2009). The issue appeared in the works of Marx, resurfaced with Lenin and Chayanov in the early 20th century, and has captivated modern agricultural and development economists for over fifty years. Debate around the nature and causes of this relationship continues despite a mountain of empirical analysis, posing a puzzling question for 21st century researchers (Binswanger et al., 1995; Eastwood et al., 2010). Conventional economic wisdom expects resources to be allocated such that returns to land are equalized across farms; however, the empirical research on developing countries contradicts this and frequently identifies an inverse relationship. Policy-makers in developing countries have engaged the debate, as an inverse relationship between farm size and productivity indicates a role for small farms in development strategies and the potential for land reform to simultaneously generate improvements in equity and efficiency.

Harnessing such a relationship to inform policy requires correct interpretation of the empirical evidence as well as an understanding of its causes, the channels through which it operates, and the factors that condition its strength. Theoretical explanations have emerged to explain this phenomenon as resulting from household heterogeneity and/or market failure. Sen's (1966) formalization of the dual labor market hypothesis was among the first, positing that surplus labor generates a wage gap between capitalist and peasant farming, differences in the intensity of labor used, and an inverse relationship. An alternative approach highlights the role of endowments and credit constraints in generating distinct farmer classes, land-labor ratios, and an inverse relationship (Eswaran and Kotwal, 1986). These results are further strengthened with the consideration of moral hazard and the costs of monitoring hired labor (Feder, 1985). Risk aversion and the differing responses to price risk by net buyers and sellers provides yet another explanation (Barrett, 1996). Measurement error (Lamb, 2003; Carletto et al., 2013; Gourlay et al., 2017; Desiere and Jolliffe, 2018) and omitted variables such as soil quality (Bhalla and Roy, 1988; Benjamin 1995; Assunção and Braido, 2007) are two empirical issues that could lead to an

observed inverse relationship. Bevis and Barrett (2017) put forth a behavioral explanation, namely that increased productivity on the periphery of plots creates an “edge effect” that could generate the farm size – productivity relationship. The simplest explanation—decreasing returns to scale (DRS) in the production function—might also play a role, although the empirical literature typically assumes, or fails to reject, constant returns to scale.

Attempts to sort out the relative importance of these mechanisms have been mixed. Adding to the confusion is the variety of productivity measures and empirical approaches that have been used. As with Sen (1962), Deolalikar (1981), Bhalla and Roy (1988), Benjamin (1995), and Dillon et al. (2016), much of the early literature used land productivity (output per unit of land) as the measure of performance.¹ Conditioning land productivity on input use by estimating a production function, as done in the more recent work of Assunção and Braido (2007) and Barrett et al. (2010), among others, is a second commonly used approach that generates an alternative measure of performance. Still others employ value added per unit of land (Heltberg, 1998), profit per unit of land (Bardhan, 1973; Benjamin, 1995; Carletto et al., 2013; Foster and Rosenzweig, 2017), profit (Heltberg, 1998; Lamb, 2003), or technical efficiency (Carter, 1984; Helfand and Levine, 2004; Kagin et al., 2016). Despite the recognition that partial measures such as land productivity are flawed (Berry and Cline, 1979; Binswanger et al., 1995; Muyanga and Jayne, 2016), they continue to be used and highlighted, often alongside alternative measures. In some instances, there appears to be confusion regarding the distinction between partial and comprehensive productivity measures. Conceptual clarity is needed on how these measures relate to each other and to farm size.

Given that much of this empirical literature has worked with partial productivity measures, one must at least question the validity of the stylized fact that there is an inverse relationship between *productivity* and farm size in developing countries. This relationship may hold true when using some measures, such as land productivity, but not necessarily for other more preferable measures of productivity. Bardhan (1973) and Berry and Cline (1979) are two

¹ The literature often uses yield and land productivity interchangeably. We only use yield when talking about a physical measure of productivity for a single product (tons/hectare). Land productivity is more appropriate in a multiple-output context, requiring a method for aggregation.

examples where an alternative productivity measure leads to a direct relationship between farm size and productivity.

The lack of focus on total factor productivity and profit rate is a curious feature of the inverse relationship literature, especially given the early and widespread acknowledgement of their superiority over partial measures. As Barrett (1996) notes, this literature “habitually, perhaps cavalierly,” uses physical yields and productivity synonymously. In fact, empirical studies assessing the productivity-farm size relationship in the developed world, such as Garcia et al. (1982), Alvarez and Arias (2004), and Rasmussen (2010), almost exclusively use measures of technical efficiency or total factor productivity. Similarly, the literature estimating national level agricultural productivity is clear and explicit in their use of total factor productivity as a preferred measure (Fuglie, 2008; and Headey et al., 2010). Not only is a comprehensive productivity measure such as total factor productivity a theoretically preferred measure, it is also likely to be the measure most relevant to policy-makers formulating development strategies in relatively poor economies where poverty alleviation is a pressing need. In this light, we argue that the inverse relationship literature needs to explicitly shift its focus from land productivity to total factor productivity.

This paper emphasizes the importance of productivity measures in the inverse relationship literature. First, we seek to clarify the common measures, their relationships, and their advantages and limitations in empirical work. From this discussion, we lay out a framework for understanding how the farm size – productivity relationship may change over time, and how the choice of productivity measure matters for identifying those dynamics. Lastly, we provide new empirical evidence on the relationship between size and productivity in Brazil over the period 1985 to 2006. Using a pseudo-panel of farms aggregated at the municipality and farm size levels, we demonstrate the importance of choice of productivity measure. The analysis highlights how the relationship between total factor productivity and farm size has evolved with modernization during this period, shedding some light on the issues raised by Mill over 150 years ago.

II. Measures of Agricultural Productivity

Farm size may be related to a broad range of economic outcomes, such as employment, poverty, inequality, food security, and growth. While these are important issues connected to the role of farm size in a developing economy, here, as with the rest of the literature on the inverse relationship, we focus specifically on the concept of productivity.

The Unconditional Relationship between Land Productivity and Farm Size

Historically, land productivity is by far the most commonly used measure in the literature on the inverse relationship (IR). Even when alternative productivity measures are used, the relationship between gross land productivity and farm size is often a starting point because it is a benchmark for the expansive existing literature on the IR. Land productivity, q , is a partial measure of productivity:

$$\text{Land Productivity} = \frac{Q}{A} = q = \psi_u(A) \quad (1)$$

where A is the area of the farm, Q is an index of agricultural output, q is agricultural output per unit of land, and $\psi_u(A)$ connotes that land productivity may be a function of farm size. In a world where farm size and land productivity are unrelated we have $\frac{\partial \psi_u(A)}{\partial A} = 0$. However, the regularity with which empirical work has found $\frac{\partial \psi_u(A)}{\partial A} < 0$ has led to the stylized fact that they are inversely related, and is precisely what has led to the abundant interest in the relationship and its potential explanations. Figure 1 displays this relationship using data from Brazil for the years 1985, 1996, and 2006. While the relationship is potentially non-linear and may not be monotonic, for now we focus on the first order approximation.

The relationship captured by $\psi_u(A)$ is unconditional (u) in the sense that it is the simple bivariate relationship between land productivity and farm size. Factors that may be causing or influencing this relationship have not been controlled for. Using land productivity as a measure is inherently limited—as would be any partial measure of productivity—whenever there is more than one factor of production. If other factors vary systematically with farm size then we might expect to see an IR due to more intensive input use by small farms, implying that our focus on the relationship between land productivity and the size of the farm may be misplaced. Similarly,

analysis using different partial productivity measures may result in conflicting policy recommendations. Indeed, Sen's (1962) seminal contribution revealed precisely this type of systematic relationship between the intensity of labor use and farm size, leading to his formal exposition of the dual labor market hypothesis (Sen 1966). Figure 2 illustrates the problem in the case of Brazil. While there is an inverse relationship between land productivity and farm size, there is a direct relationship between labor productivity and size. Analysis of the farm size and productivity relationship using labor productivity suggests that larger farms are more productive than are their smaller counterparts., underscoring the need for a more comprehensive measure of productivity when attempting to identify any relationship with farm size.

The Conditional Relationship between Land Productivity and Farm Size

It is curious that the unconditional relationship between land productivity and farm size has remained such a focal point in the IR literature. A more appropriate approach is to use a conditional relationship, where the relationship is conditioned on a vector of controls, $X(A)$, that may be correlated with both land productivity and farm size:

$$q = \psi_u(A) = g(X(A), \psi_c(A)) \quad (2)$$

The conditional (c) relationship, $\psi_c(A)$, should differ from the unconditional relationship to the extent that the conditioning controls explain the unconditional IR. For example, the impact of varying input intensities can be controlled for by including the inputs as controls, household heterogeneity can be controlled for with household fixed effects, market failure controlled for with regional fixed effects, and omitted variables such as soil quality can be introduced. This is a useful approach for exploring the theoretical channels that explain the IR and is a strategy commonly used by researchers in recent empirical studies of the farm size – productivity relationship (Assunção and Braido, 2007; Barrett et al., 2010).

As discussed above, partial measures such as land productivity are potentially misleading when there are other factors of production. At the very least, understanding any relationship between productivity and farm size requires empirical analysis that controls for the intensity with which other factors of production are used. A natural way to handle this is to include those factor intensities as conditioning variables. For exposition, assume that land, labor (L), and capital (K)

are the only factors of production and that their intensities, labor per unit of land and capital per unit of land, are given by l and k , respectively. If these are the only controls then (2) becomes:

$$q = \psi_u(A) = g(k(A), l(A), \psi_c(A)) \quad (3)$$

From equation (3) we see that the IR as identified by the unconditional relationship between land productivity and farm size, $\frac{\partial \psi_u(A)}{\partial A}$, is composed of the relationship between capital intensity and farm size, labor intensity and farm size, and any conditional relationship between farm size and land productivity, $\frac{\partial \psi_c(A)}{\partial A}$. When differences in the use of other factors of production are controlled for, the conditional relationship between farm size and productivity captures a more comprehensive measure of productivity.

Exploring (3) highlights the ambiguity in how the land productivity and farm size relationship, as captured by $\psi_u(A)$, is related to the more general productivity and farm size relationship captured by $\psi_c(A)$. Differentiating (3) with respect to farm size shows:

$$\left(\frac{\partial \psi_u}{\partial A}\right) = \left(\frac{\partial g}{\partial k}\right) \left(\frac{\partial k}{\partial A}\right) + \left(\frac{\partial g}{\partial l}\right) \left(\frac{\partial l}{\partial A}\right) + \left(\frac{\partial g}{\partial \psi_c}\right) \left(\frac{\partial \psi_c}{\partial A}\right) \quad (4)$$

Assuming, quite reasonably, that output per unit of land is increasing in both capital and labor per unit of land, which is to say that $\frac{\partial g}{\partial k}$ and $\frac{\partial g}{\partial l}$ are both positive, it is plausible for the conditional relationship to be positive ($\frac{\partial \psi_c}{\partial A} > 0$) even if the unconditional relationship is negative ($\frac{\partial \psi_u}{\partial A} < 0$) if as is often the case $\frac{\partial k}{\partial A}$, $\frac{\partial l}{\partial A}$, or both are negative. In short, an unconditional IR is neither a necessary nor a sufficient condition for an inverse relationship between a broader measure of productivity and farm size as captured by $\psi_c(A)$.

When, as in (3), the conditional relationship includes all factors of production as controls, the approach is equivalent to estimating a production function. This conditional relationship is often interpreted as total factor productivity. Before returning to this issue let us introduce an alternative and commonly used approach to measuring the IR, one that uses variations of profit as a measure of productivity.

Profit and Value-Added per Unit of Land

There are two variants of this approach. Start with the measure of profit:

$$\Pi = Q - p_L L - p_K K - p_A A \quad (5)$$

where p_L is the price of labor, p_K the price of capital, and p_A the price of land. In expression (5) the output quantity index, Q , is constructed using prices in the aggregation process, making it interpretable as the value of output. It is natural to think about the level of profit as the product of output and profit per unit of output: $Q \frac{\Pi}{Q}$. Regardless of whether the profit per unit of output rises or falls with size, we would expect the level of output to dominate in the determination of the level of profit. A large farm that produces 1,000 units of output should generate more profit than a small farm that produces 10 units. The level of profit, then, is not a particularly good measure for comparing the productivity of farms of different sizes.

It is not profits per se that matter, but rather profitability. This requires the transformation of the profit level into a profit rate. Profit per unit of land, as used by Carletto et al. (2013), is one approach:

$$\text{Profit per unit of land} = \pi_A = \frac{\Pi}{A} = q - p_L l - p_K k - p_A = q - c = \phi(A) \quad (6)$$

where c is the cost per unit of land incurred in producing q . Profit per unit of land is a productivity measure that controls for the levels of other inputs additively, providing an improvement over land productivity. Measures of value added are similar, however they fall short of profit measures as they control only for intermediate inputs and not the complete set of factors of production. Despite being an improvement over land productivity and value-added, profit per unit of land is itself problematic because it is fundamentally a partial measure. The finding of a systematic inverse relationship with farm size, $\frac{\partial \phi(A)}{\partial A} < 0$, is of limited use because this partial measure does not control for the profitability with which other inputs are used:

$$\pi_A = \frac{\Pi}{A} = \frac{\Pi K}{K A} = \pi_K * k \quad (7)$$

Here we see that the profit per unit of land can be rewritten as the product of profit per unit of capital, π_K , and capital intensity. An observed inverse relationship between profit per unit of land and farm size could be associated with declining capital intensity as farm size increases, even if profit per unit of capital is increasing. If this were true, then the use of one partial measure or

the other would lead to conflicting policy recommendations. A more comprehensive measure is preferred.

TFP and Profit Rate

Comprehensive measures, of either productivity or profitability, are the appropriate means to measure the efficiency of resource use. Berry and Cline (1979), along with Binswanger et al. (1995), have argued that total factor productivity (TFP) or the profit rate are the preferable and correct choices for productivity measures when discussing farm size. We concur. Total factor productivity is a comprehensive productivity measure defined as the ratio of output to all inputs used. If, as before, land, labor, and capital are the only three factors of production and the quantity indices are constructed using prices, TFP can be written as:

$$TFP = \frac{Q}{\text{Input Quantity Index}} = \varphi(A) \quad (8)$$

TFP effectively captures the productivity with which all inputs are used in the production process. It measures units of output for every unit of input used, regardless of its type, and in this sense is a comprehensive measure of productivity. If this measure is a function of farm size, i.e. $\frac{\partial \varphi(A)}{\partial A} \neq 0$, then there is an unambiguous difference in how productively farms of different sizes utilize resources in agricultural production. An understanding of the determinants of $\varphi(A)$ would support effective policy design. Although this is widely acknowledged, an explicit focus on TFP is seldom the approach of empirical analyses of the IR in developing economies.

TFP can be written as a function of a profit rate, where profit is normalized by the total value of inputs used in production:

$$TFP = \frac{\pi}{p_L L + p_K K + p_A A} + 1 \quad (9)$$

The profit rate described in (9) is the rate most meaningful for assessing agricultural productivity in that it is consistent with TFP. Whereas TFP looks only at the gross value of production, the profit rate considers the net value of production. Any IR found using this profit rate should also be found using TFP as the productivity measure.

An alternative interpretation of this profit rate is that it is normalized by the total value of assets used in agricultural production. Consider, for instance, a potential farmer with financial

capital looking to invest in farming. If there are competitive factor markets then the value of the capital used to rent land, physical capital, and labor to run the operation is identically equal to the costs of production. Binswanger et al. (1995) advocate normalizing profit by “capital invested” or “assets.” This is appropriate as long as measured assets are restricted to those used in production and do not include all of household wealth (as is sometimes done in the literature). If other forms of wealth are erroneously included, and farm size and wealth are positively related, then an IR may appear to emerge even when profit per unit of invested capital is constant or increasing with farm size.

Linking TFP and Partial Measures

It is instructive to analyze the relationship between TFP and the other measures used. Consider first the relationship between TFP and the conditional relationship captured when analyzing a production function. Assume that the production function is homogenous of degree t so that $f(\lambda L, \lambda K, \lambda A) = \lambda^t f(L, K, A)$. Constant returns to scale (CRS) holds if $t = 1$, while decreasing returns (DRS) are observed if $t < 1$ and increasing returns (IRS) if $t > 1$. Setting $\lambda = \frac{1}{A}$ implies that the production function can be written as:

$$f(l, k, 1) = A^{-t} f(L, K, A) \quad (10)$$

and dividing output by farm size gives us land productivity:

$$q = \frac{f(L, K, A)}{A} = A^{t-1} f(l, k, 1) \quad (11)$$

As an example, assume a standard Cobb-Douglas production function where T is the unobserved measure of total factor productivity and production is a function of labor, capital and land:

$$f(L, K, A) = TL^\alpha K^\beta A^\gamma \quad (12)$$

If, as is often presumed, CRS holds, then farm size disappears from the right hand side of (12) after dividing through by farm size. If not, then the natural log of the production function takes the form:

$$\ln q = (t - 1)\ln A + \ln T + \alpha \ln l + \beta \ln k \quad (13)$$

If, as in (8), there exists a relationship between total factor productivity and size, $\varphi(A)$, we have:

$$\ln q = (t - 1)\ln A + \ln \varphi(A) + \alpha \ln l + \beta \ln k \quad (14)$$

From (14) it is clear that the conditional relationship identified in (3), $\psi_c(A)$, is composed of the relationship between TFP and farm size as well as any deviations from CRS in the production function.

Equation (14) highlights three features of the production function approach that are often overlooked in the literature. First, if CRS holds then the conditional relationship, $\psi_c(A)$, captures the relationship between TFP and farm size, $\varphi(A)$. Second, if CRS does not hold then it will be difficult to empirically differentiate whether a conditional relationship is driven by non-CRS, a relationship between TFP and farm size, or a combination of the two. This highlights the importance of explicitly investigating returns to scale and, if this is not done, to interpret any observed conditional relationship as potentially coming from non-CRS. Third, if the addition of any controls other than the factor intensities explain differences in the conditional relationship, which is composed of non-CRS and the TFP-farm size relationship, they should be interpreted as such.

Thus, when empirical researchers estimate a production function to explore the relationship between farm size and productivity they are, in effect, estimating the relationship between farm size and TFP, not farm size and land productivity. This point is almost always left undiscussed. Indeed, most empirical work that takes this approach shows the unconditional relationship first (often non-parametrically), followed by a series of regressions of production functions that include other controls, followed by interpretation of the two approaches as if they were exploring the same relationship. However, the conditional and unconditional relationships are by no means the same. They can plausibly take different signs, and will almost certainly have different magnitudes.

In a similar fashion, the relationship between TFP and both output per unit of land and profit per unit of land can be explored. TFP is a unit-less measure, but multiplying and dividing by $\frac{1}{A}$ allows the measure to be rewritten as:

$$TFP = \frac{q}{p_L l + p_K k + p_A} = \frac{q}{c} = \varphi(A) \quad (15)$$

From the expression above, TFP can be thought of as land productivity normalized by costs per unit of land, c . Whereas profit per unit of land is linear in q and c , and controls for costs per unit of land additively, TFP controls for costs per unit of land multiplicatively and is log-linear:

$$\ln TFP = \ln q - \ln c \quad (16)$$

How then does the relationship between TFP and farm size, $\varphi(A)$, relate to the relationship between profit per unit of land and farm size, $\phi(A)$? From (15), TFP can be rewritten as a function of profit per unit of land and cost per unit of land by adding and subtracting $\frac{c}{c}$:

$$TFP = \frac{q - p_L l - p_K k - p_A}{p_L l + p_K k + p_A} + 1 = \frac{\pi_A}{c} + 1 \quad (17)$$

Taking a derivative with respect to farm size:

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial \pi_A}{\partial A} \right) \left(\frac{1}{c} \right) - \frac{\left(\frac{\partial c}{\partial A} \right) \pi_A}{c^2} \quad (18)$$

In similar fashion, taking the derivative of (15) with respect to A yields:

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial q}{\partial A} \right) \left(\frac{1}{c} \right) - \frac{\left(\frac{\partial c}{\partial A} \right) q}{c^2} \quad (19)$$

Generalizing (18) and (19) by allowing $x \in \{ \pi_A, q \}$, and employing a little bit of algebra (see Appendix A):

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial x}{\partial A} \right) \left(\frac{1}{c} \right) \left[\frac{\varepsilon_{x,A} - \varepsilon_{c,A}}{\varepsilon_{x,A}} \right] \quad (20)$$

where $\varepsilon_{x,A}$ is the elasticity of partial productivity measure x (profit or output per unit of land) with respect to farm size, and $\varepsilon_{c,A}$ is the elasticity of costs per unit of area with respect to farm size. If there is an empirically observed inverse relationship between the partial measure and farm size such that $\frac{\partial x}{\partial A} < 0$, then we know $\varepsilon_{x,A}$ is negative. This implies that one of two possibilities must hold:

- (i) $\frac{\partial TFP}{\partial A} < 0$ and $\varepsilon_{x,A} < \varepsilon_{c,A}$
- (ii) $\frac{\partial TFP}{\partial A} > 0$ and $\varepsilon_{x,A} > \varepsilon_{c,A}$

If (i) is true then an IR between a partial measure and farm size reflects an IR between productivity and farm size as measured by TFP. When this is the case then either costs per unit of land are increasing in farm size or they are decreasing, but slower than the rate at which output per unit of land is decreasing. If (ii) is true then use of a partial measure is generating an incorrect indication about the productivity and farm size relationship, and TFP is actually directly related to farm size. However, this requires that $0 > \varepsilon_{x,A} > \varepsilon_{c,A}$. In such a case, costs per unit of land are negatively related to farm size and they are relatively more elastic than output per unit of land. This discussion highlights the conclusion that an IR between a partial measure of productivity and farm size is neither necessary nor sufficient for the existence of an IR between farm size and a comprehensive measure of productivity such as TFP.

Discussion of the Dynamics of TFP and Partial Measures

The conditions set out in (i) and (ii) provide a framework for considering how a modernizing agricultural sector can lead to a changing farm size – productivity relationship. Depending upon the stage of development and the institutional structure of the economy, partial measures of productivity may fail to capture the dynamics of the farm size – productivity relationship. We provide several examples below.

Consider first an economy at an early stage of development, where a traditional agricultural sector relies predominantly on land and labor as inputs to production and land, labor, and credit markets all exhibit imperfections. If an IR between land productivity and farm size exists in such a world, the underlying mechanisms are likely to be Sen’s dual labor market hypothesis and the monitoring costs associated with larger farms hiring wage labor. Larger farms must incur supervision costs to maintain a given labor intensity, implying that, even if labor intensity falls with farm size, $\varepsilon_{c,A}$ may be positive and (i) holds. Even if labor intensity is falling with farm size fast enough such that $\varepsilon_{c,A}$ is negative, the existence of supervision costs implies that $\varepsilon_{c,A}$ is likely to be less elastic than $\varepsilon_{q,A}$. Again, condition (i) holds and TFP, as with land productivity, will have an inverse relationship with farm size. In such an economy land productivity may provide an adequate proxy for TFP, even if the magnitudes of the two relationships differ.

In a second stage of agricultural development, advances in agricultural technology lead to mechanization, making capital a more important class of inputs for those farms that are able to adopt modern technologies. Such a stage may continue to be characterized by struggling institutions and imperfections in credit and labor markets. If larger farms have greater access to credit, or alternatively there is a minimum farm size required for the application of certain technologies, then large farms can more readily adopt the new technology and substitute away from labor. Smaller farms will continue to employ labor more intensively in the presence of labor market imperfections, implying the possibility that an inverse relationship between land productivity and farm size continues to exist even as improved access to capital for larger farms attenuates the relationship. The substitution away from labor towards capital avoids costly monitoring, moving large farms towards a more efficient mix of factors of production. Together with any economies of scale in the relatively capital-intensive technology, larger farms enjoy cost advantages and costs per unit of land fall as farm size increases. In this context, one would not be surprised if condition (ii) holds, with costs per unit of land ($\varepsilon_{c,A}$) falling more quickly than output per unit of land ($\varepsilon_{q,A}$). A direct relationship between TFP and farm size could emerge, even as an IR continues to exist for land productivity. Sadoulet and De Janvry (1995) discuss such a case, where a move to capital intensive agricultural production could generate a positive relationship between farm size and TFP. Unlike in the initial stage of development, mechanization and an increasingly important role for capital have rendered land productivity a misleading indicator of the relationship between farm size and TFP.

Alternatively, if the productivity gains of capital-intensive agriculture are large enough, a U-shape relationship may emerge. Compared to medium sized farms, small and large farms could have relatively high levels of output per hectare due to their respective labor- and capital-intensive production practices. In such a setting, the farm size – productivity relationship of the initial stage of development dominates the lower end of the farm size distribution while the farm size – productivity relationship of this second stage dominates the upper end of the distribution. This case captures that of Foster and Rosenzweig (2017), who show that transaction costs in the labor market and economies of scale for modern agricultural technology generate a U-shaped farm size – productivity relationship.

In a developed economy, a third stage of agricultural development may realign the relationships between TFP, land productivity and farm size. Institutions improve as economic development continues, and distortions in the labor and capital markets begin to disappear. While capital-intensive agriculture becomes widespread, sources of economies of scale other than mechanization—such as those that derive from information technologies—begin to come into play. Larger farms also have better access to managerial and technical expertise, and they have advantages in marketing and distribution. In such an environment, the inverse relationship between land productivity and farm size could disappear, implying $\varepsilon_{q,A} \geq 0$. Both land productivity and TFP could conceivably exhibit a direct relationship with farm size.

Once again, non-linearities in the productivity – farm size relationships may arise. Small farms may continue to find ways to remain relatively productive compared to medium sized farms. For example, specialized local knowledge from owner operators, specialization in high value, labor intensive niche agricultural products, institutional arrangements allowing small farms to take advantage of economies of scale in capital-intensive technologies, and targeted government programs may lead to small farms being as, if not more productive than their medium-sized counterparts. In such settings, a U-shape farm size – productivity relationship may emerge.

The discussion above suggests several important implications for the IR literature. First, it highlights how partial measures of productivity are unlikely to serve as appropriate proxies for comprehensive productivity measures such as TFP. Second, we should expect the farm size – productivity relationship to be dynamic, evolving with the economy. Third, a focus on the choice of productivity measure and the dynamics involved help to reconcile disparate empirical results. Empirical findings of an inverse farm size – land productivity relationship in economies at early and medium stages of development are fully consistent with empirical findings of a direct farm size – TFP relationship in more advanced economies. Thus, while a growing body of within-country studies has identified a diminishing inverse relationship over time (Deininger et al., 2016), or non-linearities in the farm size – productivity relationships (Foster and Rosenzweig, 2017; Muyanga and Jayne, 2016; Kimhi, 2006; Helfand and Levine, 2004), the multi-country study

led by Rada and Fuglie (2018) provides evidence for a U-shape across countries. These findings can be understood in the framework presented above.

III. Empirical Analysis

We now turn to an illustrative example using data on Brazilian agriculture. The intention here is not to explain the relationship between farm size and productivity by controlling for its potential determinants. Rather, we seek to use a regional analysis within Brazil to highlight how the choice of measure influences the observed relationship, potential non-linearities, and how the patterns change across stages of agricultural development.

Empirical Methodology

Consistent with the existing IR literature and the discussion above, we assume a Cobb-Douglas technology and estimate an average production function. Output and inputs for a farm in municipality m of size s in year t are normalized by area, A_{mst} . By estimating the model using intensities we effectively impose CRS, forcing any deviation from CRS into the estimated relationship between farm size and productivity. Survey year specific dummy variables for the five farm size classes, δ_{st} , are used to flexibly capture the relationship between farm size and TFP. The farm size class 0-5 ha in 1985 is excluded and used as a reference. While this structure allows the farm size and productivity relationship to change over time, the technology coefficients are assumed to remain constant, implying a time invariant agricultural technology. This assumption forces technical change into our measure of TFP. The estimated equation takes the form:

$$\ln y_{mst} = \beta_0 + \beta_1 \ln k_{mst} + \beta_2 \ln l_{mst} + \beta_3 \ln x_{mst} + \delta_{st} + \lambda_m + \varepsilon_{mst} \quad (21)$$

where y_{mst} is aggregate output per unit of land and k_{mst} , l_{mst} , and x_{mst} are the factors of production per unit of land (capital, family labor, and purchased inputs including hired labor, respectively), and λ_m are municipal fixed effects. Standard errors are clustered at the municipal level and, because the number of farms represented by each representative farm varies, each observation is weighted by the number of farms that it represents. With the above approach the TFP of a farm size bin in a given period is calculated by:

$$TFP_{st} = e^{\beta_0 + \delta_{st}} \quad (22)$$

A TFP index is then calculated for each size class in each period using the size class 0-5 ha in 1985 as a base level set to 100.

Data and Variables

The data come from the 1985, 1995/1996, and 2006 rounds of the Brazilian agricultural census, and are aggregated into five farm size classes for each municipality in Brazil.² The changing composition of Brazil's municipalities requires the construction of minimum comparable areas (AMCs) to generate a dataset of geographic units that is consistent over time. This process aggregates the 5,548 Brazilian municipalities in 2006 into 3,861 AMCs. For each farm size class in each AMC we generate a "representative farm" that characterizes the behavior of farms of that class in that AMC.

We begin with 47,365 representative farms for all of Brazil. Due to concern about the comparability of a small number (84) of extremely large farms, we remove all representative farms in the Northeast and South over 4,000 ha and all of those over 5,000 ha in the North, the Southeast, and the Center-West. We identify land productivity outliers in two stages. First, we cut the top and bottom 1% of the land productivity distribution from the sample. Second, using a quadratic specification we regress land productivity on farm size with AMC fixed effects and survey year dummy variables. From this regression we identify all representative farms with land productivity greater than four standard deviations from the conditional mean as outliers. Together, the data cleaning exercises remove 2.3% of the initial sample.³

Output is measured as the real value (R\$2006) of total production, valued using regional average prices for the given year and deflated with a price index developed from the data of Gasques et al. (2010). Farm size is measured in hectares (ha), and additional factors of production form quantity indices for family labor, purchased inputs, and capital. Counts of male, female, and child family members working on each farm are used to develop a family labor index measured

² The size classes are 0-5 ha, 5-20 ha, 20-100 ha, 100-500 ha, and 500+ ha. To protect the confidentiality of the farms, the Brazilian Institute of Geography and Statistics (IBGE) requires that each aggregate observation have at least 3 farms.

³ See Table A1 of Appendix B for the results of data cleaning from each stage of the process.

in adult male equivalents. The real value (R\$2006) of purchased inputs, including expenditure on fertilizer, seeds, hired labor, fuel, energy, soil amendments, and other inputs, are calculated with the price index used for output.

A measure of the total capital stock is calculated as a quantity index comprised of machine, animal, and tree capital stock sub-indices. The machine capital stock index values tractors, trucks, and other agricultural equipment using a constant set of sale prices developed by the Instituto de Economia Agrícola in São Paulo. The stock of animal capital is measured in cattle equivalents by valuing the annual service flows (estimated with output/stock ratios) of nine distinct animal stocks and aggregating with a set of time-invariant relative prices. The stock of tree capital is measured as the present value of expected future profits for thirteen different tree crops, using region-specific estimates of expected profits. The sub-indices are aggregated using region-specific weights estimated by regressing output on the capital stock sub-indices in 1985.

By focusing on a regional analysis, we are able to examine the relationship between farm size and productivity in light of each region's characteristics and stage of development. The five macro-regions differ in both the type of predominant agricultural activities and the degree of modernization, including the Amazon rainforest in the North, a large semi-arid region in the Northeast, a highly mechanized and commercial agriculture in the Southeast, a predominance of family farms in the South, and the Cerrado (savannahs) of the Center-West where grains have rapidly expanded and modernized in recent decades. We restrict attention to the North, Center-West, and Southeast, three macro-regions that adequately capture regional variation in Brazilian agriculture. Descriptive statistics for output and input intensities for these regions are shown in Table A2 of the appendix. Differences in input intensities reflect the heterogeneity in agricultural production across regions. The more traditional agricultural region in the North relies more heavily upon family labor, whereas the mechanized Southeast and Center-West use more capital and purchased input intensive.

Empirical Results

Figure 3 shows the unconditional relationship between land productivity and farm size class for the three regions under study. Despite considerable regional heterogeneity in their

agricultural activities, each region mirrors the country as a whole in displaying a strong inverse relationship between land productivity and farm size. If anything, the relationship appears to be growing stronger over time.

The results from region-specific estimates of equation (21) are shown in Table A3 of the appendix, which generate the TFP estimates presented in Figures 4-6. In the North (Figure 4), we estimate an inverse relationship between farm size and TFP. It is not, however, a linear relationship, but rather an emerging U-shaped inverse relationship with the farms over 500 ha becoming more productive than medium-sized farms. The significance tests in Table 1 confirm this, showing that while the productivity of farms between 20 ha and 500 ha is statistically less than the smallest farms in all periods, the largest farms are not statistically different than the smallest farms. Thus, while a strong negative relationship would be found in this region when using land productivity, a U-shaped relationship begins to emerge when TFP is used and linearity is not imposed.

The Center-West (Figure 5) demonstrates a more dynamic pattern. Table 1 shows that the farm size – TFP relationship in the Center-West in 1985 looked very similar to the inverse relationship in the North. However, by 2006 the inverse relationship had disappeared in the Center-West, with the TFP of all farm sizes being statistically indistinguishable from that of the smallest farms. The point estimates show that the largest farms in the Center-West were 45% less productive than the smallest farms in 1985, yet by 2006 they were 14% more productive, albeit statistically insignificantly so. Once again, a U-shape emerges, driven by rapid growth of the productivity of larger farms. This is the clearest case of a strong inverse relationship becoming reversed over the 21 year period. Using land productivity to measure the farm size – productivity relationship in a rapidly modernizing agricultural region such as the Center-West would completely miss this transformation.

The Southeast, in contrast, shows a positive non-linear relationship between farm size and TFP. The relationship was statistically flat in 1985, although the point estimates show that even in 1985 the largest farms were 26% more productive than the smallest. Rapidly rising TFP at the upper end of the farm size distribution makes the relationship more positive over time, and by 2006 the largest farms were 49% more productive than the smallest, and statistically so.

Once again, the relationship appears non-linear. This contrasts sharply to the persistent IR found in the Southeast when using land productivity as a measure.

In comparison to much of the development literature surrounding the IR, the Brazilian data used here represent a very heterogeneous group of farms and span a much greater range of farm sizes. A more accurate comparison group to the international literature might be farms less than 100 ha, which indeed make up approximately 90% of all Brazilian farms. Even when restricting our analysis to this subset of farms, the use of land productivity would still show a marked inverse relationship while the use of TFP would reveal a negative relationship that has disappeared in the more modernizing regions. Perhaps more importantly, our inclusion of the largest farm size class reveals that these farms have notably higher productivity in the more modern regions, and it is only when TFP is used that this becomes apparent. These are commercial farms that are unlikely to be included in most household surveys in developing countries.⁴

IV. Conclusion

We have sought to address an important weakness of the development economics literature on the inverse relationship between farm size and productivity. We argued that the most commonly used measure—land productivity—is problematic and potentially misleading. While this point has been noted in the literature, the use of land productivity or other partial measures of productivity remains widespread. Furthermore, where movements are made towards use of a comprehensive measure, such as TFP, the distinction is seldom made explicitly. The conceptual discussion provides a framework for assessing the implications of the choice of productivity measure. Theoretically it is clear that an inverse relationship between land productivity and farm size is neither necessary nor sufficient for an inverse relationship to exist between farm size and TFP.

⁴ The empirical results obtained here are comparable to those reported by Rada et al. (2018) using a similar dataset. The main difference is that they find faster TFP growth for the smallest farm size class, resulting in a more pronounced U-shape in 2006. The principal differences in empirical methodology are that they estimate TFP growth separately for each farm size class, and do not explore regional heterogeneity.

How much does this critique matter? We conduct an empirical analysis of three regions in Brazil using a pseudo-panel from 1985 to 2006 to contrast the land productivity – farm size relationship with the TFP – farm size relationship. In short, the choice of productivity measure matters greatly. As in many developing country contexts, there exists an inverse relationship between land productivity and farm size for Brazil, and within each of its macro-regions in every period. In contrast, the TFP and farm size relationship varies across time and space. The regional analysis of the TFP and farm size relationship shows that 1) land productivity is often inappropriate as a proxy for TFP; 2) the relationship is dynamic, changing with agricultural modernization; 3) the relationship is non-linear, often characterized by a U-shape; and 4) the very largest farms, such as those with more than 500 ha, are important to consider when assessing any relationship between farm size and productivity.

From a policy perspective, our findings have important implications for the debate about the future of small farms in developing countries. When using TFP, we see that superior productivity of small farms in traditional agricultural contexts is fully consistent with emergent productivity advantages for larger commercial farms in modernizing agricultural sectors. As economies develop, superior productivity is not likely a valid argument for the importance and future of small farms, as we expect larger farms to play an increasing role in driving agricultural productivity growth. As such, it is increasingly unlikely that redistributive land reform could positively impact both equity and efficiency. This does not, however, imply that small farms will disappear. We expect them to remain important for generating livelihoods for rural households, providing food security, and contributing to the development of rural economies. Productivity gains among small farmers will also continue to be essential for poverty alleviation. Policies targeting productivity gains for small farmers, such as those that build human capital, facilitate adoption of new technologies, and enhance access to markets via a reduction in transactions costs, will continue to be indispensable for reducing rural poverty in developing countries.

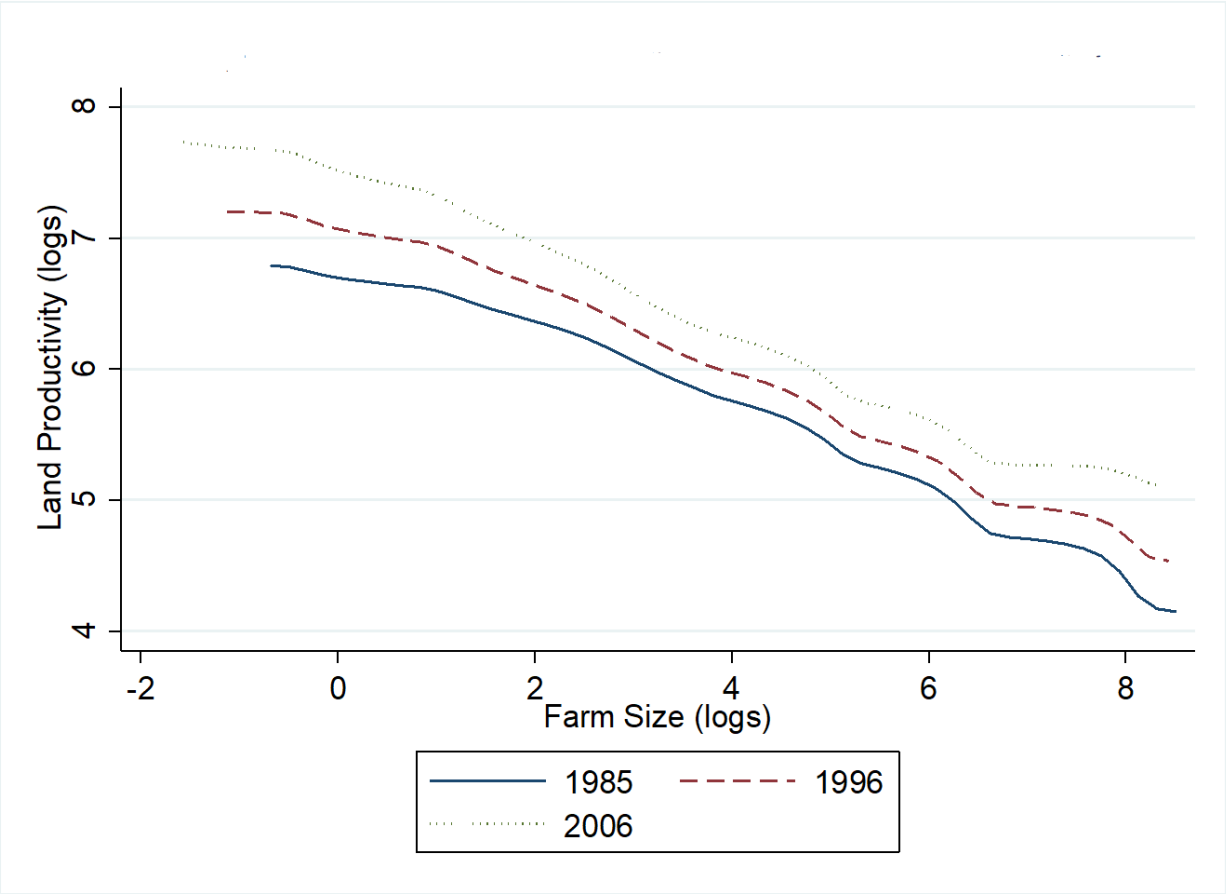
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Figure 1: Farm Size and Land Productivity, Brazil (logs)



Note: Smoothed as a local polynomial regression with bandwidth of 1.25 and Epanechnikov kernel.

Figure 2: Land and Labor Productivity, Brazil 2006

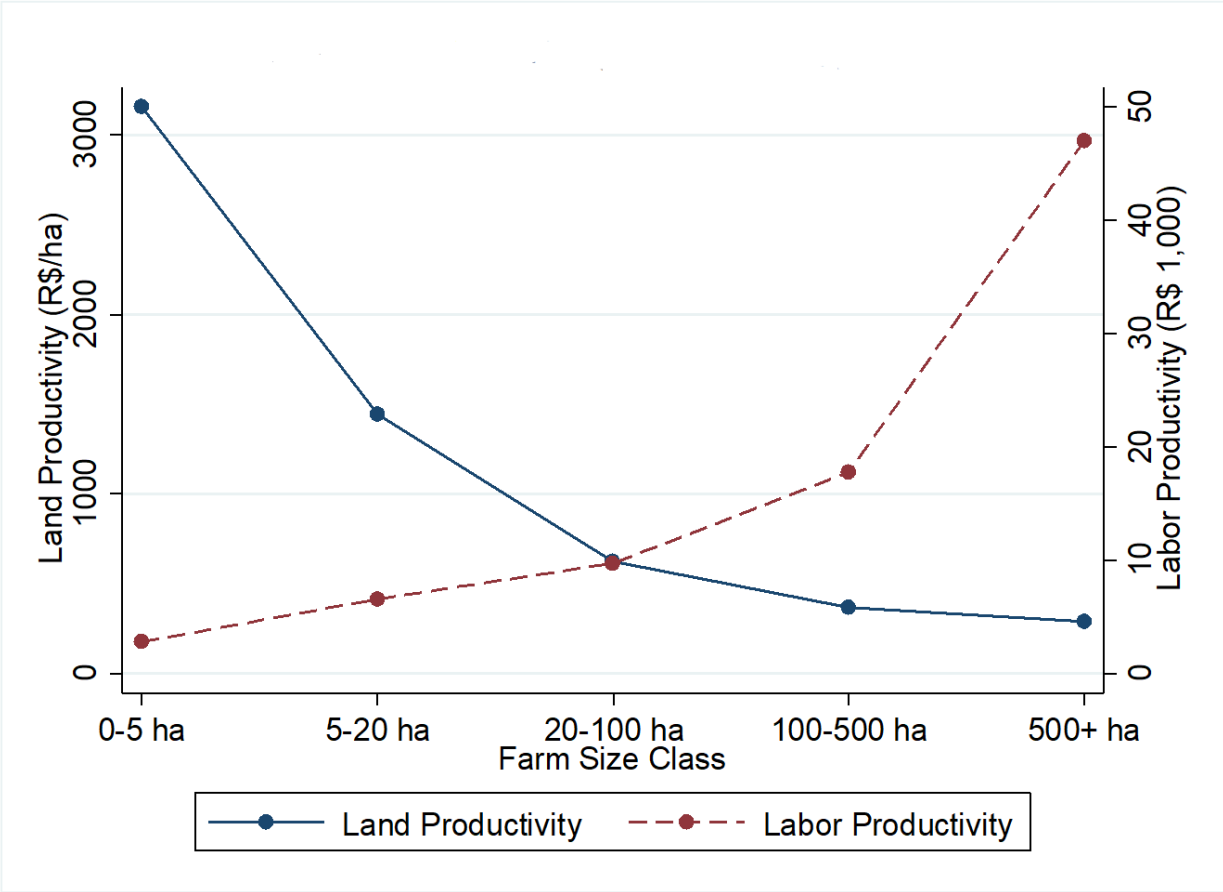


Figure 3: Land Productivity in Brazil by Region

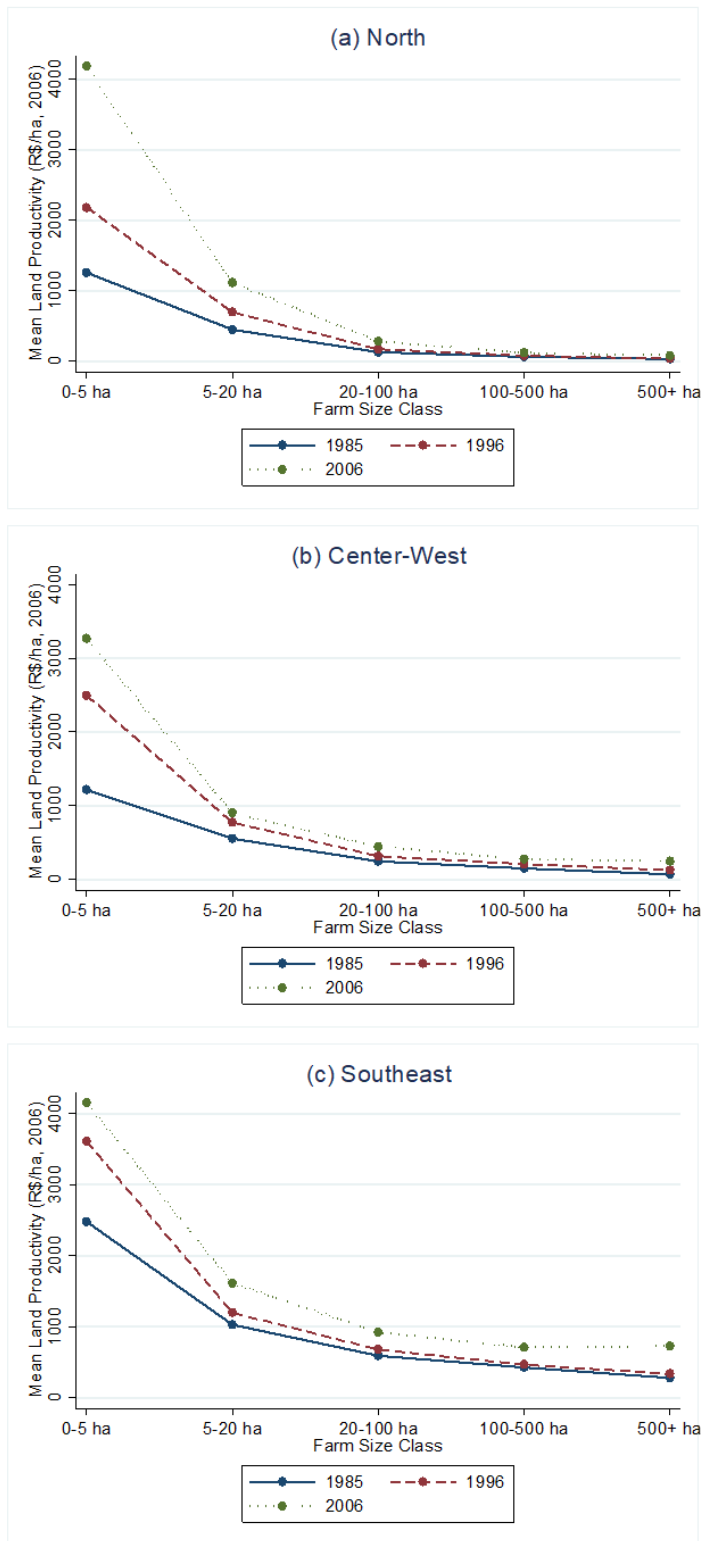


Figure 4: Total Factor Productivity in Brazil's North

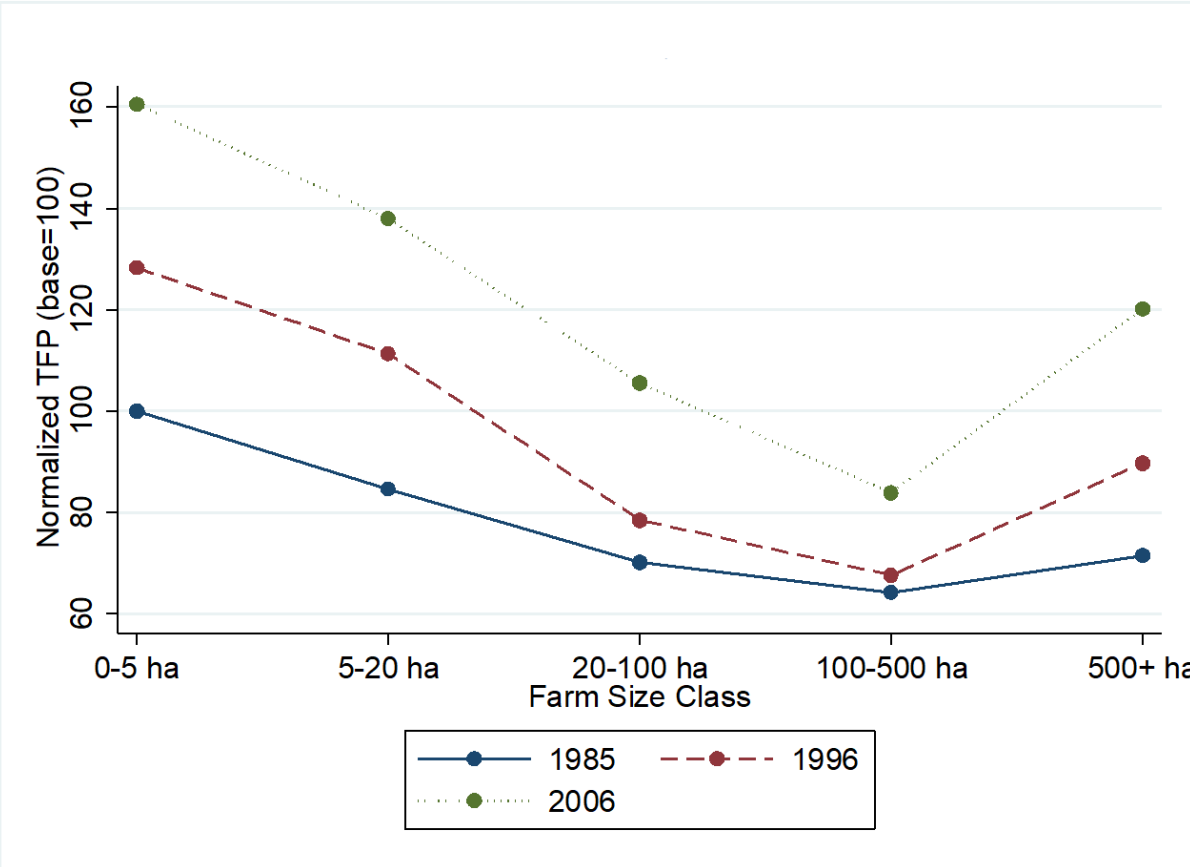


Figure 5: Total Factor Productivity in Brazil's Center-West

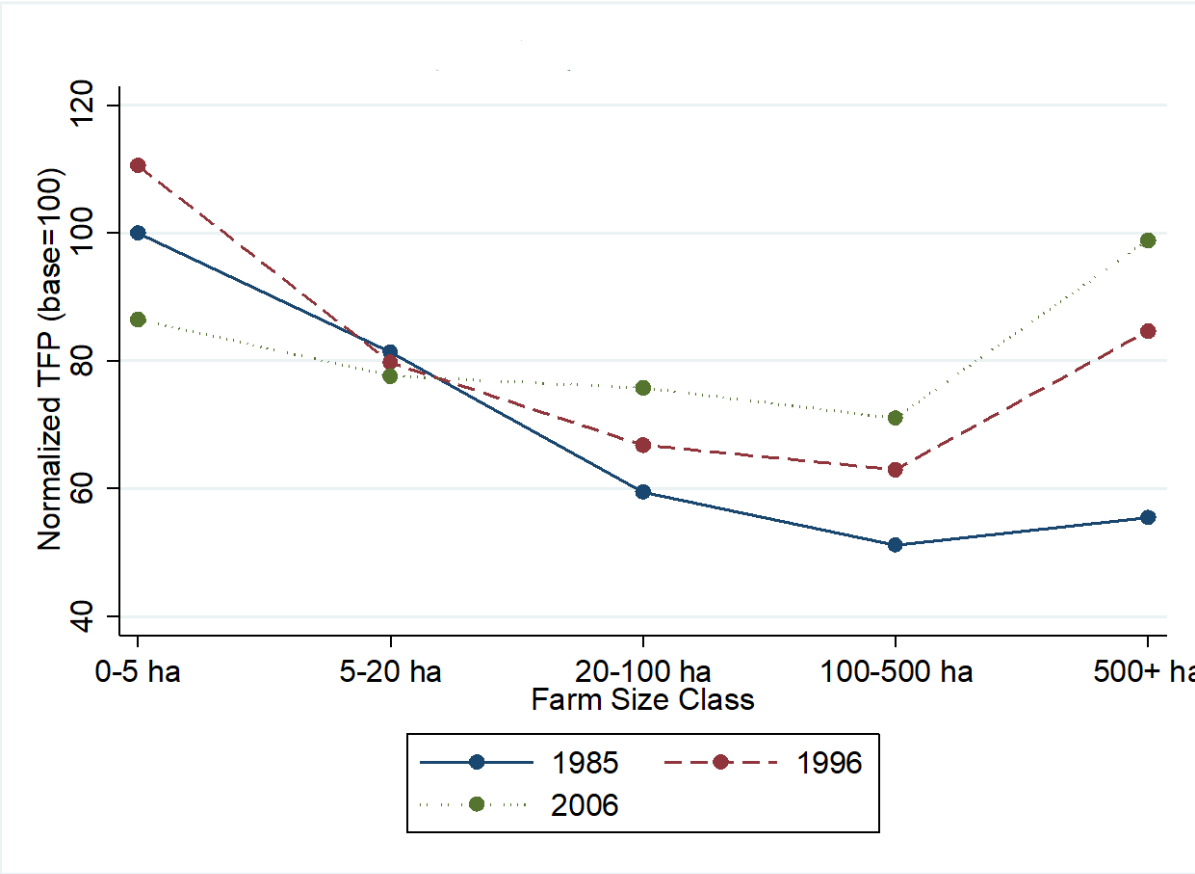


Figure 6: Total Factor Productivity in Brazil's Southeast

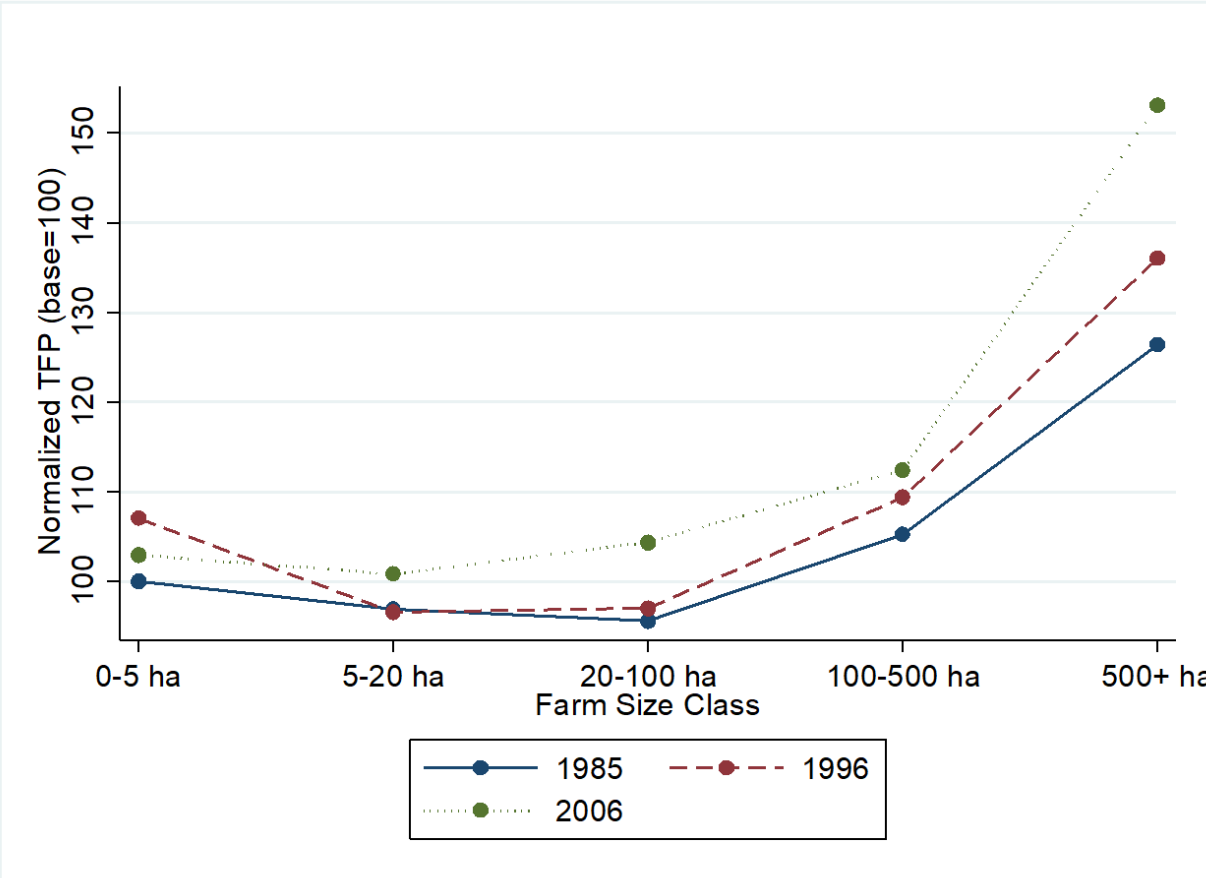


Table 1. Percentage Difference in TFP Relative to 0-5 ha Farms

	North			Center-West			Southeast		
	1985	1996	2006	1985	1996	2006	1985	1996	2006
5-20 ha	-15.39*	-13.22	-14.03	-18.63***	-27.90***	-10.20	-3.09	-9.76***	-2.06
	(0.086)	(0.131)	(0.293)	(0.004)	(0.003)	(0.440)	(0.383)	(0.009)	(0.623)
20-100 ha	-29.77**	-38.80***	-34.27**	-40.54***	-39.58***	-12.37	-4.41	-9.32	1.39
	(0.035)	(0.008)	(0.048)	(0.001)	(0.007)	(0.323)	(0.536)	(0.184)	(0.846)
100-500 ha	-35.87*	-47.27**	-47.70**	-48.88***	-43.02**	-17.86	5.26	2.19	9.21
	(0.053)	(0.013)	(0.038)	(0.005)	(0.034)	(0.405)	(0.641)	(0.850)	(0.442)
500 + ha	-28.61	-30.06	-25.13	-44.55*	-23.42	14.38	26.36	27.05	48.68**
	(0.382)	(0.383)	(0.547)	(0.094)	(0.471)	(0.695)	(0.150)	(0.147)	(0.018)
N	1,888	1,888	1,888	3,038	3,038	3,038	17,742	17,742	17,742

Base farm size bin, 0-5 ha. P-values from significance tests are in parentheses: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Appendix A

Proof of expression (20).

Allowing $x \in \{\pi_A, q\}$ to generalize equations (18) and (19), respectively, we have:

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial x}{\partial A}\right) \left(\frac{1}{c(q)}\right) - \frac{\left(\frac{\partial c(q)}{\partial A}\right) x}{c(q)^2}$$

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial x}{\partial A}\right) \left(\frac{1}{c(q)}\right) \left[1 - \frac{\left(\frac{\partial c(q)}{\partial A}\right) x}{c(q) \left(\frac{\partial x}{\partial A}\right)}\right]$$

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial x}{\partial A}\right) \left(\frac{1}{c(q)}\right) \left[1 - \frac{\left(\frac{\partial c(q)}{\partial A}\right) \left(\frac{1}{c(q)}\right)}{\left(\frac{\partial x}{\partial A}\right) \left(\frac{1}{x}\right)}\right] = \left(\frac{\partial x}{\partial A}\right) \left(\frac{1}{c(q)}\right) \left[1 - \frac{\left(\frac{\partial c(q)}{\partial A}\right) \left(\frac{A}{c(q)}\right)}{\left(\frac{\partial x}{\partial A}\right) \left(\frac{A}{x}\right)}\right]$$

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial x}{\partial A}\right) \left(\frac{1}{c(q)}\right) \left[\frac{\varepsilon_{x,A}}{\varepsilon_{x,A}} - \frac{\varepsilon_{c(q),A}}{\varepsilon_{x,A}}\right]$$

$$\frac{\partial x}{\partial A} = \left(\frac{\partial TFP}{\partial A}\right) c(q) \left[\frac{\varepsilon_{x,A}}{\varepsilon_{x,A} - \varepsilon_{c(q),A}}\right]$$

Appendix B

Table A1: Data Cleaning and Sample Size, by Region and Farm Size Class

	Farm Size Class (ha)	N	Less Farm Size Outliers	Less Tails of Land Productivity Distribution	Less Land Productivity Outliers	Percent Dropped from Cleaning
North	0-5	315	315	307	307	1.6%
	5-20	420	420	410	403	4.0%
	20-100	459	459	449	437	4.8%
	100-500	443	443	433	433	2.3%
	500 +	323	315	307	305	3.2%
Center-West	0-5	537	537	525	520	3.2%
	5-20	619	619	605	605	2.3%
	20-100	681	681	667	667	1.2%
	100-500	672	672	658	658	1.9%
	500 +	613	595	583	581	2.4%
Southeast	0-5	3,850	3,850	3,772	3,772	1.2%
	5-20	3,991	3,991	3,911	3,911	1.6%
	20-100	4,024	4,024	3,942	3,942	2.0%
	100-500	3,896	3,896	3,818	3,818	0.6%
	500 +	2,235	2,215	2,169	2,169	0.9%
<i>Brazil</i>		47,365	47,281	46,307	46,261	2.3%

Table A2: Descriptive Statistics, 2006

	Farm Size Class (ha)	Output/ha	Capital/ha	Family Labor/ha	Purchased Inputs/ha	Share of Farms	Share of Area	Share of Output
North	0-5	4,185.02	965.03	1.75	862.37	19.4%	<1.0%	0.69%
	5-20	1,110.46	333.92	0.24	214.43	17.4%	1.6%	12.0%
	20-100	282.24	129.06	0.05	73.54	43.0%	17.4%	32.2%
	100-500	120.65	88.47	0.01	58.18	16.8%	26.4%	20.9%
	500 +	78.98	54.90	<0.01	72.96	3.3%	54.3%	54.3%
Center-West	0-5	3,265.49	2,628.76	0.72	1,971.49	8.5%	<1.0%	1.0%
	5-20	902.03	851.41	0.16	506.95	20.2%	1.0%	2.4%
	20-100	444.53	378.38	0.04	231.87	37.8%	5.0%	8.3%
	100-500	276.48	279.76	<0.01	210.40	21.2%	13.8%	14.3%
	500 +	247.08	127.28	<0.01	246.53	12.3%	80.5%	74.2%
Southeast	0-5	4,152.85	3,903.90	0.85	1,892.53	28.8%	1.1%	5.5%
	5-20	1,611.97	1,797.21	0.18	1,061.63	32.1%	6.2%	11.7%
	20-100	923.89	906.85	0.04	554.65	28.5%	21.8%	23.5%
	100-500	711.38	555.44	<0.01	556.69	9.1%	32.9%	27.2%
	500 +	726.50	276.13	<0.01	715.38	1.5%	37.9%	32.1%

Table A3. Estimated Technology Coefficients

	North	Center-West	Southeast
Capital per ha	0.176*** (0.039)	0.175*** (0.039)	0.129*** (0.015)
Family Labor per ha	0.303*** (0.066)	0.149*** (0.045)	0.190*** (0.027)
Intermediate Inputs per ha	0.304*** (0.039)	0.426*** (0.040)	0.503*** (0.021)
Constant	4.093*** (0.244)	3.627*** (0.294)	3.285*** (0.104)
AMC FE	Yes	Yes	Yes
Farm Size Dummies	Yes	Yes	Yes
R-squared	0.95	0.90	0.91
N	1,888	3,038	17,742

Standard errors in parentheses: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Dependent variable is logged output; all independent variables are logged; all variables normalized by farm size.