# DOES GERRYMANDERING VIOLATE THE FOURTEENTH AMENDMENT?: INSIGHT FROM THE MEDIAN VOTER THEOREM 

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#### Abstract

This paper argues that "gerrymandering" understood here to mean the intentional redrawing of legislative district boundaries to benefit a given party, robs opposition voters of implicit bargaining power.. Using the Median Voter Theorem and statistical examples, this paper argues that the presence of minority voters in a legislative district influences the majority party's choice of candidate, whenever minority voters are present in sufficient number to pose a credible challenge. When, through gerrymandering, lawmakers insure that minority voters cannot mount such a challenge, they deny such voters equal protection under the law.


NOTE: This paper is written in a pedagogical style to facilitate its use in classroom discussion.

## Key Words

Election, Election Law, Voter Protection, Voting, Voting Law, Gill vs. Whitford, gerrymandering, median voter theorem

## JEL Classification Codes

K160

## 1) Introduction

There has in recent years been increased journalistic outrage over gerrymandering, or the redrawing of legislative district boundaries to benefit a given candidate or group. The US Supreme court has ruled that such redistricting is unconstitutional when it discriminates on racial or ethnicity grounds. It has also ruled that gerrymandering may be unconstitutional if it is done to discriminate based on political beliefs, however no constitutional challenge has prevailed as yet on those grounds. The 2015 case Gill vs. Whitford ${ }^{1}$, filed in US court for the Western District of Wisconsin argues that gerrymandering: "is unconstitutional because it treats voters unequally, diluting their voting power based on their political beliefs, in violation of the Fourteenth Amendment's guarantee of equal protection. ${ }^{22}$.

The plaintiffs argued that the district plan, adopted in 2012 by the Wisconsin State Assembly allowed Republican candidates to win sixty of the state's ninety-nine assembly seats even though Democrats won the majority of the statewide assembly vote ${ }^{3}$. The plaintiffs conclude that: "[Gerrymandering] causes policies to be enacted that do not accurately reflect the public will. In the end, a political minority is able to rule the majority..." ${ }^{4}$

In November 2016 The US District Court ruled the Wisconsin plan unconstitutional, based upon a three-pronged test. This test asked if the redistricting plan:
(1) "intended to place a severe impediment on the effectiveness of the votes of individual citizens on the basis of their political affiliation,
(2) has that effect, and,
(3) cannot be justified on other, legitimate legislative grounds. ${ }^{5}$

The State of Wisconsin has appealed the case, and it is pending before the US supreme court at the time of this writing.

This article looks at the district court's test, and from a standpoint of Political Economic theory. We will consider a hypothetical case where the power of minority voters is unduly impeded even when there is no apparent discrepancy between the relative number of seats a party wins and the relative number of votes it receives. This article argues that whenever minority voters within a legislative district maintain sufficient numbers to mount a credible challenge, they implicitly bargain with the majority party, motivating it to moderate its choice of candidate at the primary stage. When such voter numbers are not maintained, the majority party is free to choose a more extreme candidate who is less able to represent the interests of district constituents as a whole. While such districts may occur naturally as the result of demographics, the matter becomes legally problematic when district elections are rendered noncompetitive through redistricting. This article argues that polarization of the legislature logically follows from widespread gerrymandering as result of the loss of implicit bargaining power by the minority.

1 Whitford v. Gill in United States District Court for the Western District of Wisconsin, (2017)
${ }^{2}$ Whitford vs. Nichol Complaint and Exhibits p. 2
${ }^{3}$ Whitford vs. Nichol Complaint and Exhibits p. 2
${ }^{4}$ Whitford vs. Nichol Complaint and Exhibits p. 2
5 "Whitford v. Gill". Harvard Law Review. 2017

This article presents a theoretical thought experiment, designed to show how statistical manipulation of district populations "impedes the effectiveness of the votes of individual citizens on the basis of their political affiliation." The model will assume an electorate made up of individuals concerned with a single issue, whose preference for such (either for or against) can be quantified and placed on a scale. We will assume the population's preferences are distributed normally about a mean $\mu_{P}$ and with standard deviation $\sigma_{P}$. The electorate will be divided evenly between two parties, call them Democrat and Republican, based on whether their preferences place them to the right or left of $\mu_{P}$. The population will then assigned to legislative districts, first randomly and then in a fashion simulating the effect of gerrymandering. We will consider three cases: The first establishes a baseline, predicting the preference distribution of elected representatives assuming no strategic action on the part of either party. We then consider the case where parties select candidates strategically in districts that are not gerrymandered. Finally, we consider the case where parties select candidates strategically when all districts are gerrymandered.

## 2) A Review of The Median Voter Theorem

There have been several game-theoretic treatments of the bargaining process between candidates in an election ${ }^{6}$. For our purposes, there are refinements on the so-called Median Voter Theorem, first proposed by Harold Hotelling in $1929^{7}$, and mathematically formalized by Duncan Black in $1948^{8}$. The Median Voter Theorem predicts that: Given two candidates, and one issue of concern to voters, the candidate elected will be one espousing the preferences of the median voter. In the following illustration adapted from Gruber ${ }^{9}$, we consider voters concerned only with the level of spending on a public good such as national defense. Furthermore, we assume that the voters' preferences are single peaked or "Goldilocks", in that there is one level of spending that each considers "just right" as opposed to "too little" or "too much". To keep matters simple, we assume that this voting population consists of only eleven individuals, In Figure 2-1 the voters $X_{1}$ through $X_{11}$ are placed on a spending scale ranging from zero to fifty percent of the federal budget, according to their "peak" or most preferred level of spending.

We introduce two candidates; call them "Left" ( L ) and "Right" (R). Candidate $L$ initially prefers to spend only $15 \%$ on defense while candidate $R$ prefers to spend $45 \%$. Candidate $L$ will capture all the voters whose preferences lie to his left, while Candidate R captures all the voters to his right. The voters with preferences lying between those of the candidates will gravitate towards the closest candidate.

[^0]

(b)

(c)

(d)

Figure 2-1 IIlustration of the Mean Voter Theorem ${ }^{10}$

For the sake of the theorem, it is assumed that each's candidate's primary concern is with willing the election. The candidates compete by adjusting their "preferred" spending level to capture the largest number of votes. As shown in Figure 2-1b Candidate $R$ announces that he now prefers to spend only $35 \%$ on defense. He thus captures voter $X_{6}$ away from Candidate L.. Candidate L responds by advocating a spending level of $20 \%$, recapturing voter $\mathrm{X}_{6}$. Such competition continues until both candidates advocate the same spending level, a level that matches the preference of the median voter $\left(\mathrm{X}_{6}\right)$ as shown in Figure 2-1d.

## 3) The Baseline Case: No Strategic Nomination of Candidates and no Gerrymandering of Legislative districts

As mentioned in Section 1, we assume the electorate's preferences are normally distributed $N\left(\mu_{P}, \sigma_{P}\right)$ as shown in Figure 3-1. Voters with preference higher, or to the right of $\mu_{P}$ we will call "Republican" while voters with preference lower, or to the left of $\mu_{P}$ we will call "Democrat" At this point though party affiliation at this point no more than a label.

[^1]

Figure 3-1: (b) Preference distribution of legislative districts established from random samples of the US population. (a) Preference distribution of elected representatives predicted by the Median Voter Theorem (not drawn to scale).

If we assign voters randomly to the legislative districts, the standard deviation $\sigma_{i}$ of voter preferences within each district $i$ would be the same $\sigma_{P}$ as for the entire population. The district preference means $\mu_{i}$ will deviate from the population mean due to sampling error. These will be normally distributed about $\mu_{P}$ with standard deviation $\sigma_{L}=\sigma_{P} / \sqrt{n}$ where $n$ is the average voter turnout in the districts. For our purposes it is sufficient to assume that $n$ will always be larger than ten thousand so that $\sigma_{L}$ will be less than one percent of $\sigma_{P}$.

Normality of the preference distributions, and the mean voter theorem imply that each district $i$ will elect a representative with preference $\mu_{i}$. The distribution of the representatives within the legislature will thus be $N\left(\mu_{P}, \sigma_{L}\right)$. Since the representatives of all representatives are tightly distributed around $\mu_{P}$, it is evident that this "baseline" process is completely neutral, in that the actual preference distribution of legislators will entirely be the product of other factors.

## 4) Case 2: Strategic Nomination of Candidates by Parties and no Gerrymandering of Legislative districts

In this second case, we will assume the usual two-stage election process where candidates are chosen strategically by their parties during the primary stage. We will see that, if races remain at least somewhat competitive, the implicit bargaining power of the minority will cause the strategic selection process to produce the same result as the baseline.

In the two stage election process, each candidate faces a "double constituency" The candidate must satisfy the preferences of members his or her party to win the primary, and the preferences of the entire electorate to win the general election. The double constituency problem has been studied empirically by Brady et. al ${ }^{11}$. and several others, the results of which will be discussed shortly. Presently we will model the problem candidates whose positions (preferences) are fixed, and the double constituency problem is solved game-theoretically by party members.

As in the baseline case, we assume that all voters in the population are randomly distributed to legislative districts. Within each district, the primary constituencies are the registered voters of each party, while the general constituency is the population of the entire district. Primary voters (or caucus members) prefer a candidate whose preferences match the median voter of their own party. To win the general election however, they will need to choose an e"lectable" candidate close to the preferences of the median voter of the district. This traps primary voters in the Prisoner's Dilemma illustrated in Figure 4-1 ${ }^{12}$


Figure 4-1 Prisoner's Dilemma game illustrating partisan nomination strategy when districts are not gerrymandered

[^2]To find the preferences of a median party member, we consider a "typical" Republican district shown in Figure 4-1. The preference distribution of all voters in the district is $N\left(\mu_{R}, \sigma_{P}\right)=$ $N\left(\left\{\mu_{P}+\sigma_{L}\right\}, \sigma_{P}\right)$. The preference distribution of Republican voters in the district consists of the right tail of this distribution plus area under the curve between $\mu_{P}$ and $\mu_{R}$. Since this distance is very small we can ignore it for the moment. Thus, the median Republican voter's preference $m_{R}$ solves:

$$
\begin{equation*}
\frac{1}{4}=\frac{1}{\sigma_{U S} \sqrt{2 \pi}} \int_{0}^{z} e^{\left(\frac{z^{2}}{2}\right)} d z \quad \text { where } \quad z=\left|\frac{m_{R}-\mu_{P}}{\sigma_{P}}\right| \tag{4-1}
\end{equation*}
$$



Figure 4-2: (a) Preference distribution of district medians as compared to total population (not drawn to scale) (b) Preference distribution of typical Republican district established from a random sample of the total population. (Typical Democrat district is mirror image.)

The preference of the median Republican voter is approximately:

$$
\begin{equation*}
m_{R} \approx \mu_{P}+0.7 \sigma_{P} \tag{4-2}
\end{equation*}
$$

This places him or her to the right of three fourths of the total population

By similar reasoning it is apparent that in a district where Democrats are in the majority, the median registered Democrat's preference $m_{D}$ would be given by:

$$
\begin{equation*}
m_{D} \approx \mu_{P}-0.7 \sigma_{P} \tag{4-3}
\end{equation*}
$$

This is where the implicit bargaining power of the minority party voter plays a key role. Were the Democrats in this case to nominate a candidate with preferences to the right end of their distribution (i.e. at $\mu_{P}$ ) they prevail in the general election. Republicans will expect this and instead will nominate a more "electable" candidate with preferences positioned at $\mu_{R}$. Even though the Democrats would lose in this scenario, they have negotiated with Republicans for a more moderate candidate by their very presence in the district. Applying this result to all districts, the resulting distribution of elected representatives would again be $N\left(\mu_{P}, \sigma_{L}\right)$ as it was in the Baseline case.

There is considerable empirical support for the assumptions made here. Studies have shown that primary voters tend to be more extreme than general election voters ${ }^{13}$, as are those who attend nominating conventions ${ }^{14}$. Presidential candidates respond by courting these "base" voters ${ }^{15}$ during the primaries, following Richard Nixon's "Run to the right, then run to the center" strategy ${ }^{16}$. Regarding congressional elections, candidates with more extreme ideologies tend to do better in closed primaries than they do in open primaries ${ }^{17}$, while open primaries tend to produce more competitive results in the succeeding general elections ${ }^{18}$. Brady e. al. demonstrate the empirical evidence of the "double constituency" hypothesis by showing that:
(1) the dynamics of candidate ideology and election outcomes are opposite in primary elections from what they are in general elections.
(2) Ideologically moderate candidates are more likely to draw challengers.
(3) Primary challengers position themselves to the left of Democratic incumbents and to the right of Republican incumbents

And finally, that:
(1) Primary losses are more likely to occur among ideologically moderate candidates.

[^3]
## 5) Case 3: Strategic Nomination of Candidates by Parties with all Legislative Districts Gerrymandered

To simulate the impact of gerrymandering, we assign the population to districts so that there is a decisive majority of one or the other party in every district. The district's party members will be assigned by random draw from the respective parties in the general population. We will assume that the majority-minority ration is the same in every district, and that it is just large enough to guarantee the races to be non-competitive. Given that the total population is still evenly divided between Republicans and Democrats, this will as before, result in an equal number of seats won by each party. The elected body however will be highly polarized due to destruction of the minority's party's implicit bargaining power within each district.

The district's voter preference distribution, shown in Figure 5-1b, will not be normal, hence the district median will be shifted in the majority party's favor. This median however will have no impact. Since the majority party never faces a credible threat, it is free to choose the more extreme candidate preferred by its base. Within each district therefore, preferences of the elected candidate will be determined by those of its majority party's median voter. Using earlier assumptions regarding sample size, we see that the distribution of representatives from Republican districts will have distribution $N\left(\mu_{R}, \sigma_{P}\right)=N\left(\left\{\mu_{P}+\sigma_{L}\right\}, \sigma_{P}\right)$, while the distribution of representatives from Democratic districts will be distributed $N\left(\mu_{D}, \sigma_{P}\right)=N\left(\left\{\mu_{P}-\sigma_{L}\right\}, \sigma_{P}\right)$ as shown in Figure 5-1a.

It should be remembered that legislative representatives have a fiduciary responsibility to represent the interests of all constituents once elected. With minority voters essentially rendered impotent, there is no incentive for district representatives to treat their interests with the same regard as those of majority voters. Lack of equal protection is thus manifest.

The polarized makeup of the legislature simulated here presents an additional problem. The farther apart representatives' preferences are, the more difficult it will be for them to come to any kind of agreement ${ }^{19}$. This presents a strong incentive for the legislature's majority party to enact legislation on its own. To the extent that a majority party can do so, minority legislators lose their implicit bargaining power. This would deny equal protection to all voters in districts carried by the minority party.

Empirical evidence that gerrymandering increases the polarization is modest though clear ${ }^{20}$. Following the 1990's redistricting, polarization in the US House of Representatives increased faster than it did in the Senate ${ }^{21}$. Additionally, the distribution of presidential voting tends to be more bimodal in congressional districts than in other geographic units not affected by political districting ${ }^{22}$. . Finally, using simulations, McCarty et. al. demonstrate that "blind" redistricting

[^4]would produce more competitive and heterogeneous districts that we currently have ${ }^{23}$, and that redistricting, and that polarization "is primarily a function of how Democrats and Republicans represent the same districts" ${ }^{24}$


Figure 5-1 (b) Preference distribution of typical Republican district established by separate random samples of Republicans and Democrats with Republicans outnumbering Democrats. (Typical Democrat district is mirror image.)
(a) Preference distribution of Republican legislators as compared to total population.

## 6) Conclusion

Ultimately, the question: "Has gerrymandering placed a severe impediment on the effectiveness of the votes of individual citizens on the basis of their political affiliation?" is one the courts must answer. From a theoretical stand point it certainly may, given the behavioral assumptions required by the prisoner's dilemma game and the Median Voter Theorem. When used as the sole predictor of election results, the median voter theorem may be inadequate due to the restrictive nature of the required assumptions. When used to compare results between

[^5]gerrymandered and non-gerrymandered districts however, most of these assumptions become irrelevant. There is no reason to expect that availability of information or the intensities of individual's preferences would depend on the makeup of a districts population. Strictly speaking though, one may argue that voter decisions are not generally based on a single issue as the theorem assumes. It is generally believed however that the issues that typically define a party's platform are sufficiently correlated in voter's minds to be considered a single issue. The studies cited here tacitly assume voters can be ranked according to a single parameter. McCarty et. all. expressly define their independent variable as an aggregate of partisan issues.

Given the demographics of the region, maximizing the number of competitive districts would maximize the number of voters whose actions are effective. Guaranteeing that voters do lot lose their implicit bargaining power through redistricting, would, in my view, improve their equal protection under the law.

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[^0]:    ${ }^{6}$ See Aldrich, (1983) Alesina (1988), Calvert (1985), and Whittman (1977)
    ${ }_{8}^{7}$ Hotelling (1929)
    ${ }^{8}$ Black (1948)
    ${ }^{9}$ Gruber (2009)

[^1]:    ${ }^{10}$ See Gruber (2009)

[^2]:    ${ }^{11}$ Brady et. al. (2007)
    ${ }^{12}$ The more complex variants of this model that appear in the literature [ see Alesina (1988)] do not regard party preferences as determined by its registered voters, but do consider preferences intensities (which the median voter teorem does not). Similar results are obtained. A party's chance of winning a general election is greater, the closer its announced preferences are to the median voter, hence the announced policies of both parties will tend to converge. This occurs whether or not the candidate is obliged to follow his announced preferences after the election, provided the game is repeating (voters know the candidate or party's past behavior.)

[^3]:    ${ }^{13}$ Norrander (1989)
    ${ }^{14}$ McClosky, et. al (1960), Soule and McGrath (1975), and Stone and Rapoport (1994)
    ${ }^{15}$ Aldrich (1995), Polsby and Wildavski (2004), and Schlesinger (1991)
    ${ }^{16}$ Brady et. al (2007) p. 82
    ${ }^{17}$ Gerber and Morton (1998)
    ${ }^{18}$ Brady et. al. (2007)

[^4]:    ${ }^{19}$ See Brady et. al. pp. 799-800 for a formal discussion.
    ${ }^{20}$ Stonecash et. al. (2003) see also Carson et. al. 2007
    ${ }^{21}$ Groseclose et. al. (1999)
    ${ }^{22}$ McCarty, et. al. (2009) p. 668

[^5]:    ${ }^{23}$ McCarty et. al. (2009) p.678-79
    ${ }^{24}$ McCarty et. al. (2009) p. 666

