Macroeconomic Stability under Balanced-Budget Rules and No-Income-Effect Preferences*

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Abstract

It has been shown that under an additively separable preference formulation between consumption and hours worked, indeterminacy and sunspots may arise in a standard one-sector real business cycle model when the labor tax rate is endogenously determined by a balanced-budget rule with a pre-specified constant level of government expenditures. This paper finds that local indeterminacy disappears if the period utility function is postulated to exhibit no income effect on the household's demand for leisure. In particular, the model's low-tax steady state always displays saddle-path stability and equilibrium uniqueness; whereas the high-tax steady state is either a source or a saddle point.

Keywords: Income Effect; Balanced-Budget Rules; Indeterminacy; Business Cycles.

JEL Classification: E32; E62.

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1 Introduction

Under the assumptions of perfect competition and constant returns-to-scale in production, the standard one-sector real business cycle (RBC) model exhibits an interior steady state that is a locally determinate or isolated saddle point around which there exists a unique convergent rational expectations equilibrium trajectory. In this economy with an additively separable utility function that is logarithmic in consumption and infinitely elastic in hours worked, Schmitt-Grohé and Uribe (1997, section II) analytically examine the macroeconomic (in)stability effects of a balanced-budget rule whereby constant government expenditures are financed by distortionary taxation on the household's labor income. Given the postulated fiscal specification, a perfect-foresight Laffer curve-type relationship between the labor tax rate and the resulting tax revenue ensues – the model possesses two interior steady states when the pre-specified level of public spending is lower than the revenue-maximizing counterpart. In this case, these authors derive the necessary and sufficient condition under which the low-tax steady state is an indeterminate sink that can be exploited to yield cyclical fluctuations driven by agents' animal spirits or sunspots. When the representative household becomes optimistic about the future of the economy and decides to work harder and invest more, the government is forced to decrease the labor tax rate as total output rises. This countercyclical tax policy helps fulfill agents' initial optimism, and thus destabilizes the macroeconomy by generating endogenous business cycles. On the other hand, the model's high-tax steady state is always a saddle point, hence no aggregate fluctuations will take place in its neighborhood.

In this paper, we extend Schmitt-Grohé and Uribe's theoretical analysis by considering a different preference formulation that is commonly adopted in the real business cycle literature. As in Greenwood, Hercowitz and Huffman (GHH, 1988), the period utility function is postulated to exhibit no income effect associated with the household's labor supply decision. With this specification of non-separable preferences, the relationship between the (fixed) level of government spending and the labor tax rate is also characterized by a Laffer curve that may possess two interior steady states. We show that in shape contrast to Schmitt-Grohé and Uribe (1997), equilibrium indeterminacy disappears in our model economy because neither steady state can be a sink. In particular, saddle-path stability arises when the steady-state tax rate is (i) lower than that maximizes the tax revenue or (ii) higher than a certain threshold value. Intuitively, in order for stationary sunspot equilibria to occur within a dynamic general equilibrium macroeconomic model, the consumption Euler equation must continue to hold in

¹See Benhabib and Farmer (1999) for other mechanisms that may yield indeterminacy and sunspots within various real business cycle models.

response to a change in non-fundamental expectations. Therefore, upon the anticipation of a higher rate of return on today's investment, agents will consume and work more next period. As it turns out, this optimism cannot be self-fulfilled under either circumstance since an increase in labor hours large enough to raise the after-tax marginal product of capital generates an unsustainable decrease in the household's intertemporal marginal rate of substitution between current versus future consumption expenditures. Furthermore, we find that the economy's high-tax steady state becomes a totally unstable source when the stationary-equilibrium tax rate falls within the remaining feasible range. In sum, our analysis illustrates the critical importance of income effect on the representative household's demand for leisure in generating Schmitt-Grohé and Uribe's instability result.

In the context of a one-sector RBC model with labor income taxation, Abad et al. (2017) investigate the interrelations between local stability of competitive equilibria and Schmitt-Grohé and Uribe's balanced-budget rule under a generalized constant returns-to-scale production technology and two classes of non-separable utility functions that subsume the GHH specification. Although these authors state the no-indeterminacy result with the no-income-effect preference formulation (Proposition 3, p. 267), they do not offer the underlying economic intuition. Moreover, unlike Schmitt-Grohé and Uribe (1997), they do not explore the possibility of multiple stationary equilibria caused by the presence of a Laffer curve. Here, we examine the equilibrium dynamics associated with each interior steady state, and also provide intuitive explanations.

The remainder of this paper is organized as follows. Section 2 presents the model and discusses its equilibrium conditions. Section 3 analyzes the economy's local dynamics under perfect foresight. Section 4 concludes.

2 The Economy

This paper incorporates a no-income-effect preference formulation, as in Greenwood, Hercowitz and Huffman (1988), into Schmitt-Grohé and Uribe's (1997, section II) one-sector real business cycle model with labor income taxation. Without loss of generality, we postulate that the economy's output is generated by a Cobb-Douglas production technology. This simplification streamlines our exposition without affecting any result of the paper.

2.1 Firms

The production side of the economy consists of a unit measure of identical competitive firms. The representative firm produces output Y_t , using capital and labor as inputs, with a constant returns-to-scale Cobb-Douglas production function

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}, \quad 0 < \alpha < 1. \tag{1}$$

Under the assumption that factor markets are perfectly competitive, the firm's profit maximization conditions are given by

$$r_t = \alpha \frac{Y_t}{K_t},\tag{2}$$

$$w_t = (1 - \alpha) \frac{Y_t}{H_t},\tag{3}$$

where r_t is the rental rate of capital and w_t is the real wage rate of labor.

2.2 Households

The economy is also populated by a unit measure of identical infinitely-lived households. Each household is endowed with one unit of time and maximizes

$$\int_0^\infty e^{-\rho t} \left[\log \left(C_t - A \frac{H_t^{1+\gamma}}{1+\gamma} \right) \right] dt, \quad A > 0, \tag{4}$$

where C_t and H_t are the individual household's consumption and hours worked, $\gamma \geq 0$ denotes the inverse of the intertemporal elasticity of substitution in labor supply, and $\rho \in (0,1)$ is the subjective discount rate. We assume that there are no fundamental uncertainties present in the economy.

The budget constraint faced by the representative household is given by

$$\dot{K}_t = (r_t - \delta)K_t + (1 - \tau_t)w_tH_t - C_t, \quad K_0 > 0 \text{ given},$$
 (5)

where K_t is the household's capital stock, $\delta \in (0,1)$ is the capital depreciation rate, τ_t is the labor-income tax rate. We require that $\tau_t \geq 0$ to rule out the possibility of income subsidies which could only be financed by lump-sum taxation, and that $\tau_t < 1$ such that households have incentive to provide labor services to firms.

The first-order conditions for the household's dynamic optimization problem under perfect foresight are

$$\left(C_t - A \frac{H_t^{1+\gamma}}{1+\gamma}\right)^{-1} = \Lambda_t,$$
(6)

$$AH_t^{\gamma} = (1 - \tau_t) w_t, \tag{7}$$

$$\frac{\dot{\Lambda}_t}{\Lambda_t} = \rho + \delta - r_t,\tag{8}$$

$$\lim_{t \to \infty} e^{-\rho t} \frac{K_t}{C_t} = 0, \tag{9}$$

where $\Lambda_t > 0$ is the Lagrange multiplier on the budget constraint (5), (7) equates the slope of the representative household's indifference curve to the after-tax real wage, (8) is the standard consumption Euler equation, and (9) is the transversality condition. Since C_t is missing in equation (7), there is no income effect associated with the household's labor supply decision. It follows that the income elasticity of intertemporal substitution in hours worked (or leisure) is zero.

2.3 Government

As in Schmitt-Grohé and Uribe (1997), the government endogenously sets the distortionary tax rate on labor income τ_t to finance a pre-specified constant level of public expenditures, and balances its budget at each point in time. Hence, the instantaneous government budget constraint is

$$G = \tau_t w_t H_t, \tag{10}$$

where $G \geq 0$ denotes government spending on goods and services. Finally, the aggregate resource constraint for the economy is given by

$$C_t + \dot{K}_t + \delta K_t + G = Y_t. \tag{11}$$

3 Analysis of Dynamics

Under Schmitt-Grohé and Uribe's fiscal policy rule with countercyclical labor income taxation, the number of our model's interior steady state(s) can be zero, one or two. Specifically, it is straightforward to show that the government's tax revenue (= G) is equal to zero when the steady-state tax rate $\tau^{ss} = 0$ or 1^2 ; and that the Laffer curve-type relationship between G > 0 and $\tau^{ss} \in (0, 1)$ is given by

$$G = \tau^{ss} \left(1 - \alpha \right) \left[\frac{\alpha}{\rho + \delta} \right]^{\left(\frac{\alpha}{1 - \alpha} \right) \left(\frac{1 + \gamma}{\gamma} \right)} \left[\frac{\left(1 - \alpha \right) \left(1 - \tau^{ss} \right)}{A} \right]^{\frac{1}{\gamma}}. \tag{12}$$

Setting $\frac{\partial G}{\partial \tau^{ss}} = 0$ yields a unique steady-state tax rate $\tau^* = \frac{\gamma}{1+\gamma}$ that maximizes the level of public expenditures denoted as G^* .³ It follows that our model possesses zero (two) interior

 $^{^{2}}$ When G=0, our model collapses to a standard one-sector RBC macroeconomy with no-income-effect preferences and constant returns-to-scale in production. As shown in Meng and Yip (2008) and Jaimovich (2008), this laissez-faire formulation always exhibits saddle-path stability and equilibrium uniqueness.

³When $\gamma = 0$, the revenue-maximizing steady state becomes degenerate with $\tau^* = G^* = 0$. Accordingly, our subsequent analyses of the model's equilibrium dynamics are restricted to cases under $\gamma > 0$.

steady states(s) provided G > (<) G^* , as shown in Figure 1. Therefore, any small deviation from the revenue-maximizing steady state with τ^* and G^* will lead to its disappearance, or the emergence of dual stationary equilibria. This result implies that the economy undergoes a saddle-node bifurcation which may cause the hard loss of equilibrium stability as the government spending passes through the critical level G^* . Figure 1 also shows that when $G \in (0, G^*)$, the resulting steady states in our model are characterized by τ_L^{ss} and τ_H^{ss} , where $\tau_L^{ss} < \tau^* < \tau_H^{ss}$. For a given steady-state labor tax rate, the analytical expressions of all remaining endogenous variables can then be easily derived.

Next, we take log-linear approximations to the model's equilibrium conditions in a neighborhood of each interior steady state to obtain the following dynamical system:

$$\begin{bmatrix} \dot{k}_t \\ \dot{\lambda}_t \end{bmatrix} = \mathbf{J} \begin{bmatrix} k_t \\ \lambda_t \end{bmatrix}, \quad k_0 \text{ given}, \tag{13}$$

where k_t and λ_t denote the log deviations of K_t and Λ_t from their respective steady-state values, and \mathbf{J} is the Jacobian matrix of partial derivatives for the transformed dynamical system. The trace and the determinant of the Jacobian are given by

$$Tr = \rho + \frac{(1 - \alpha)(\rho + \delta)\tau^{ss}}{\alpha - \tau^{ss} + \gamma(1 - \tau^{ss})},$$
(14)

and

$$Det = \left[\frac{\tau^{ss} - \gamma \left(1 - \tau^{ss}\right)}{\alpha - \tau^{ss} + \gamma \left(1 - \tau^{ss}\right)}\right] \underbrace{\left\{\frac{\delta \left(1 - \alpha\right) \left(\rho + \delta\right) \left[\left(1 - s_{i}\right) \left(1 + \gamma\right) - \left(1 - \alpha\right) \left(1 + \gamma \tau^{ss}\right)\right]}{s_{i} \left(1 + \gamma\right)}}_{\equiv \Psi(\tau^{ss}) > 0},$$

$$(15)$$

where s_i $\left(=\frac{\alpha\delta}{\rho+\delta}\right)$ is the steady-state ratio of investment to output.⁴ The local stability properties of our model's interior steady state(s) are determined by comparing the eigenvalues of **J** that have negative real parts to the number of initial conditions in the dynamical system (13), which is equal to one because k_t is a pre-determined state variable. As a result, the steady state exhibits saddle-path stability and equilibrium uniqueness when the two eigenvalues are of opposite signs (Det < 0). If both eigenvalues have negative real parts (Tr < 0 and Det > 0), then the steady state is an indeterminate sink around which there are a continuum of stationary equilibrium trajectories that display endogenous cyclical fluctuations driven by agents' animal spirits or sunspots. When both eigenvalues have positive real parts (Tr > 0 and Det > 0), the steady state becomes a totally unstable source.

⁴Using $\gamma > 0$ (see footnote 3), $s_i \in (0, \alpha)$ and α , $\tau^{ss} \in (0, 1)$, it can be shown that the bracket term in the numercator of $\Psi(\cdot)$, given by $(1 - s_i)(1 + \gamma) - (1 - \alpha)(1 + \gamma \tau^{ss}) > \gamma(1 - \alpha)(1 - \tau^{ss}) > 0$. This result, together with $0 < \delta < 1$ and $\rho > 0$, implies that $\Psi(\tau^{ss}) > 0$.

In sharp contrast to Schmitt-Grohé and Uribe (1997) with an additively separable household utility in consumption and labor hours, the following Proposition states that local indeterminacy does not arise within our model under non-separable no-income-effect preferences. That is, neither steady state (with τ_L^{ss} or τ_H^{ss}) can be a sink.

Proposition. For a given positive level of $G < G^*$, the economy's low-tax steady state $(0 < \tau_L^{ss} < \tau^*)$ is always a saddle point, whereas the high-tax steady state is either a source $\left(\tau^* < \tau_H^{ss} < \frac{\alpha + \gamma}{1 + \gamma}\right)$ or a saddle point $\left(\frac{\alpha + \gamma}{1 + \gamma} < \tau_H^{ss} < 1\right)$.

Proof. See the Appendix.

To understand the intuition behind our no-indeterminacy result, consider the consumption Euler equation (in discrete time for ease of interpretation) as follows:

$$\frac{C_{t+1} - A \frac{H_{t+\gamma}^{1+\gamma}}{1+\gamma}}{C_t - A \frac{H_{t+\gamma}^{1+\gamma}}{1+\gamma}} = \beta [1 - \delta + (1 - \tau_{t+1}) r_{t+1}],$$
(16)

where β denotes the discount factor. Start the model from an interior steady state at period t, and suppose that agents become optimistic about the economy's future. Acting upon this change in non-fundamental anticipation, the representative household will consume less and invest more today, thus C_t falls while K_{t+1} rises. Due to the lack of income effect, as seen in (7), H_t remains unchanged in response to the lower level of period-t consumption. In addition, a higher K_{t+1} leads to (i) a decrease in r_{t+1} because of diminishing marginal product of capital; and (ii) an increase in H_{t+1} via firms' labor demand function, which in turn raises the economy's output Y_{t+1} as well as the household's consumption C_{t+1} . Under the postulated balanced-budget constraint (10), the government is forced to reduce the labor tax rate τ_{t+1} as total income Y_{t+1} increases, thus $(1 - \tau_{t+1})$ rises. Consequently, the change in H_{t+1} will exert two counteracting effects on the intertemporal Euler equation. First, the smaller (bigger) the increase in H_{t+1} , the bigger (smaller, or a decrease may occur) the increase in the left-hand side of (16). Second, the bigger (smaller) the increase in H_{t+1} , the larger (smaller, or a decrease may occur) the rise in the after-tax equilibrium real interest rate $(1 - \tau_{t+1})r_{t+1}$.

For the above-mentioned alternative dynamic path to be justified as a self-fulfilling equilibrium, the household's consumption Euler equation must continue to hold in response to agents' rosy expectations. It turns out that the two offsetting effects, described in the previous paragraph, render the equality of (16) impossible within our model. When the economy begins at the low-tax steady state with $0 < \tau_L^{ss} < \tau^* \left(= \frac{\gamma}{1+\gamma} \right)$, a large increase in H_{t+1} is needed for $(1 - \tau_{t+1})r_{t+1}$ and the right-hand side to rise. With C_t falling and C_{t+1} rising, this would in turn decrease the left-hand side. On the other hand, when the starting steady-state tax rate is high over the interval $\frac{\alpha+\gamma}{1+\gamma} < \tau^{ss} < 1$, together with a small increase in H_{t+1} that

raises the left-hand side, the after-tax equilibrium return on capital investment $(1 - \tau_{t+1})r_{t+1}$ and the right-hand side cannot rise enough. As a result, agents' initial optimism will not be fulfilled under either tax specification, hence the economy exhibits saddle-path stability and equilibrium uniqueness. Finally, we find that the high-tax steady state with $\tau^* < \tau_H^{ss} < \frac{\alpha + \gamma}{1 + \gamma}$ is a source, which is surrounded by divergent or explosive trajectories that will eventually violate the transversality condition (9).⁵

As a side-by-side comparison, the consumption Euler equation in Schmitt-Grohé and Uribe's one-sector RBC model is given by

$$\frac{C_{t+1}}{C_t} = \beta [1 - \delta + (1 - \tau_{t+1})r_{t+1}]. \tag{17}$$

In this case, households' optimistic expectations that lead to higher investment today will unambiguously raise the left-hand side of this equation, and result in a lower before-tax real interest rate r_{t+1} due to diminishing returns to productive inputs. Under countercyclical labor income taxation $\left(\frac{\partial \tau_t}{\partial Y_t} < 0\right)$, these authors show that (i) the low-tax steady state may become an indeterminate sink when the right-hand side of (17) rises sufficiently; and (ii) the high-tax steady state is always a saddle point. Overall, our analysis illustrates that under perfect competition and constant returns-to-scale in production, Schmitt-Grohé and Uribe's (1997) indeterminacy result depends crucially on the presence of income effect associated with the household's labor supply decision.

4 Conclusion

Schmitt-Grohé and Uribe (1997, section II) analytically show that with an additively separable utility function between consumption and hours worked, a standard one-sector real business cycle model may possess an indeterminate stationary equilibrium when the labor tax rate is endogenously determined by a balanced-budget rule to finance a pre-specified fixed level of government spending. This paper complements their analysis by considering an alternative preference formulation that does not exhibit income effect associated with the household's labor supply decision. We find that local indeterminacy is no longer possible within this no-income-effect macroeconomy. In particular, the model's low-tax steady state always displays saddle-path stability and equilibrium uniqueness; whereas the high-tax steady state is either a source or a saddle point.

⁵As in Schmitt-Grohé and Uribe (1997) and Abad et al. (2017), we focus on the model's local stability properties, and leave its (nonlinear) global dynamics for future research.

5 Appendix

Proof of Proposition. Using (14), it is straightforward to show that Tr < 0 when $\tau^{ss} > \frac{\alpha + \gamma}{1 + \gamma}$, which is higher than $\tau^* = \frac{\gamma}{1 + \gamma}$; and that Tr > 0 when $\tau^{ss} < \frac{\alpha + \gamma}{1 + \gamma} \in (0, 1)$. Next, since $\Psi(\tau^{ss}) > 0$ (see footnote 4), the sign of Det as in (15) depends on whether $\frac{\tau^{ss} - \gamma(1 - \tau^{ss})}{\alpha - \tau^{ss} + \gamma(1 - \tau^{ss})}$ is positive or negative. In particular, Det > 0 if and only if $\tau^{ss} - \gamma(1 - \tau^{ss})$ and $\alpha - \tau^{ss} + \gamma(1 - \tau^{ss})$ have the same sign, which can happen when

- (a) $\tau^{ss} \gamma (1 \tau^{ss}) < 0$ and $\alpha \tau^{ss} + \gamma (1 \tau^{ss}) < 0$. This implies that $\tau^{ss} < \frac{\gamma}{1+\gamma}$ and $\tau^{ss} > \frac{\alpha+\gamma}{1+\gamma} > \frac{\gamma}{1+\gamma}$, thus generating a contradiction.
- (b) $\tau^{ss} \gamma (1 \tau^{ss}) > 0$ and $\alpha \tau^{ss} + \gamma (1 \tau^{ss}) > 0$. This implies that $\tau^{ss} > \frac{\gamma}{1+\gamma}$ and $\tau^{ss} < \frac{\alpha+\gamma}{1+\gamma}$.

On the other hand, Det < 0 if and only if $\tau^{ss} - \gamma (1 - \tau^{ss})$ and $\alpha - \tau^{ss} + \gamma (1 - \tau^{ss})$ have opposite signs, which can happen when

- (c) $\tau^{ss} \gamma (1 \tau^{ss}) > 0$ and $\alpha \tau^{ss} + \gamma (1 \tau^{ss}) < 0$. This implies that $\tau^{ss} > \frac{\gamma}{1+\gamma}$ and $\tau^{ss} > \frac{\alpha+\gamma}{1+\gamma}$, thus the more binding condition is $\tau^{ss} > \frac{\alpha+\gamma}{1+\gamma}$.
- (d) $\tau^{ss} \gamma (1 \tau^{ss}) < 0$ and $\alpha \tau^{ss} + \gamma (1 \tau^{ss}) > 0$. This implies that $\tau^{ss} < \frac{\gamma}{1+\gamma}$ and $\tau^{ss} < \frac{\alpha+\gamma}{1+\gamma}$, thus the more binding condition is $\tau^{ss} < \frac{\gamma}{1+\gamma}$.

At the low-tax steady state with $0 < \tau_L^{ss} < \frac{\gamma}{1+\gamma}$, the model's Jacobian matrix has Det < 0 as in case (d), hence it is a saddle point. In addition, at the high-tax steady state with $\frac{\gamma}{1+\gamma} < \tau_H^{ss} < \frac{\alpha+\gamma}{1+\gamma}$, we find that Tr > 0 and Det > 0 as in case (b), hence it is a source. Finally, when $\frac{\alpha+\gamma}{1+\gamma} < \tau_H^{ss} < 1$, the high-tax steady state is a saddle point because of Det < 0 as in case (c).⁶

⁶Local indeterminacy requires that both eigenvalues have negative real parts (Tr < 0 and Det > 0). However, under case (b) with $\frac{\gamma}{1+\gamma} < \tau^{ss} < \frac{a+\gamma}{1+\gamma}$ and Det > 0, the Jacobian's trace is also positive (Tr > 0). As a result, neither steady state can be a sink.

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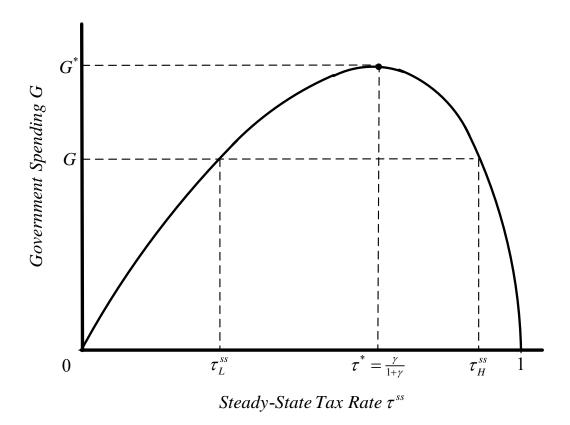


Figure 1. Steady-State Laffer Curve