

Density Forecast Evaluation in Unstable Environments ¹

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April 18, 2016

¹We are grateful to the participants at the International Symposium in Forecasting, Rotterdam (Holland), and to the seminar participants at University of Southern California, University of California, San Diego, Universidad Carlos III de Madrid (UC3M), Universidad Autónoma de Madrid, Universidad Complutense de Madrid, and Bank of Spain for useful comments. Gloria González-Rivera acknowledges the financial support of the 2015 Chair of Excellence UC3M/Banco de Santander as well as the support of the Spanish Ministerio de Economía y Competitividad (grant ECO2012-32854) and the UC-Riverside Academic Senate grants.

Abstract

We propose a density forecast evaluation method in the presence of instabilities, which are defined as breaks in any conditional moment of interest and/or in the functional form of the conditional density of the process. Within the framework of the autocontour-based tests proposed in González-Rivera *et al.* (2011, 2015), we construct Sup- and Ave-type tests calculated over a collection of subsamples in the evaluation period. These tests enjoy asymptotic distributions that are nuisance-parameter free, they are correctly sized and very powerful on detecting breaks in the parameters of the conditional mean and conditional variance. A power comparison with the tests of Rossi and Sekhposyan (2013) shows that our tests are more powerful across the models considered. We analyze the stability of a dynamic Phillips curve and find that the best one-step-ahead density forecast of changes in inflation is generated by a Markov switching model that allows state shifts in the mean and variance of inflation changes as well as in the coefficient that links inflation and unemployment.

Key Words: Generalized Autocontour-based Testing, Structural breaks, Phillips Curve.

JEL Classification: C01, C22, C53.

1 Introduction

Generally, instability is understood as changes in the parameters of a proposed forecasting model over the forecasting horizon. For clarification purposes, consider a simple model $y_{t+1} = \beta'x_t + \sigma\varepsilon_{t+1}$ with $\varepsilon_t \sim i.i.d.N(0, 1)$. The model is unstable over time if the slope coefficients β can change over the forecasting sample, either smoothly or abruptly to contain one or multiple breaks. We may also entertain a time varying variance such that σ may be also subject to breaks, and we may have different conditional probability density functions, e.g. more or less thick tails, over different periods of time. This definition is general enough to account for most types of instability discussed in the current applied econometric literature. Up to today, the most comprehensive survey in the subject is Rossi (2014) in the *Handbook of Economic Forecasting* that reports extensive empirical evidence of instabilities in macroeconomic and financial data. Some examples follow.

The instability of predictive regressions, in which the significance of predictive regressors varies over different subsamples, has been documented in studies of predictability of stock returns (see Goyal and Welch, 2003; Paye and Timmermann, 2006; Rapach and Zhou, 2014), in exchange rate predictions (see Rossi, 2006; Rogoff and Stavrakeva, 2008) and in output growth and inflation forecasts (see Stock and Watson, 2003; Rossi and Sekhposyan, 2010). Naturally linked to this evidence is the econometric question on testing for parameter stability and structural breaks in the data, which has an illustrious history. From Chow (1960) test to most recent works such as Andrews (1993), Andrews and Ploberger (1994), Pesaran and Timmermann (2002), among others, testing for breaks has mainly focused on the behavior of the conditional mean. Our contribution aims to extend testing for instabilities to the *full* conditional density forecast that includes not only any conditional measure of interest, e.g. mean, variance, duration, etc. but also the functional form of the assumed conditional density function. To our knowledge, the literature on this question is very thin. We only know of the work by Rossi and Sekhposyan (2013) who are also testing for the instability of the density model but with a different methodology from ours. In the forthcoming

sections, we will explain the differences and will provide a comparison between both approaches.

The testing methodology that we propose is based on the AutoContouR (ACR) device introduced by González-Rivera *et al.* (2011, 2012) and generalized later on in González-Rivera and Sun (2015). In these works, the null hypothesis of our tests is a correctly specified density forecast (joint hypothesis of correct dynamics in the moments of interest and correct functional form of the density). Following Diebold's (1998) work, we calculate Rosenblatt (1952) probability integral transforms (PIT) associated with the point forecasts. Under the null, the PITs must be i.i.d uniformly distributed $U[0,1]$. The Generalized AutoContouR (G-ACR) is a device (set of points) that is very sensitive to departures from the null in either direction and consequently, it provides the basis for very powerful tests. More specifically, for a time series of PITs, we construct the G-ACRs as squares (in the univariate case) of different probability areas within the maximum square (area of 1) or as hyper-cubes (in the multivariate case) of different probability volumes within the maximum hyper-cube formed by a multidimensional uniform density $[0, 1]^n$ for $n \geq 2$. By statistically comparing the location of the empirical PITs and the volume of the empirical G-ACRs with the location and volume of the population G-ACRs, we are able to construct a variety of tests for correct density forecast. Since the shapes of the G-ACRs can be visualized, we can extract information about where and how the rejection of the null hypothesis comes from. This testing framework is the foundation for the new stability tests, Sup- and Ave-type statistics, proposed in this manuscript. In a potentially unstable data environment, we will form rolling subsamples within the forecasting sample. For every subsample, we apply a battery of G-ACR tests, and to detect instabilities, we construct a Sup- and an Ave-type statistics. Though the limiting distribution of these tests is a function of Brownian motions, the tests are nuisance-parameter free and their distribution can be tabulated.¹

The paper is organized as follows. In section 2, we review the G-ACR approach to make the

¹Though we focus on out-of-sample density forecasts, the methodology proposed in this paper can also be applied to in-sample specification testing.

exposition self-contained and we introduce the new statistics with their asymptotic distributions. In section 3, we assess the finite sample properties (size and power) of the tests. We offer an extensive assessment by considering (i) fixed, rolling, and recursive estimation schemes; (ii) different ratios of prediction sample size to estimation sample size; and (iii) break points that occur in different periods of the prediction sample. We also present a comparison of power of our tests with those of Rossi and Sekhposyan (2013). In section 4, we apply the tests to assess the stability of the Phillips curve from 1958 onwards by evaluating the models proposed in Amisano and Giacomini (2007). We conclude in section 5. Appendix A contains mathematical proofs and Appendix B a description of the parametric bootstrap to correct the size of the tests. We also provide a supplementary file with additional simulation material.

2 Statistics and Asymptotic Distributions

2.1 Construction of the Statistics

The test statistics are based on the autocontour (ACR) and generalized autocontour (G-ACR) methodology proposed by González-Rivera *et al.* (2011, 2012, 2014) that provides powerful tests for dynamic specification of the conditional density model either in-sample or out-of-sample environments. In the present context, we adapt these tests to instances where instabilities may be present in the data so that, beyond the evaluation of the density model, we will also be able to detect unstable periods.

Let Y_t denote the random process of interest with conditional density function $f(y_t|\Omega_{t-1})$, where Ω_{t-1} is the information set available up to time $t - 1$. Observe that the random process Y_t could enjoy very general statistical properties, e.g. heterogeneity, dependence, etc. The researcher will construct the conditional model by specifying a conditional mean, conditional variance or other conditional moments of interest, and making distributional assumption on the functional form of

$f(\cdot)$. Based on the conditional model, she will proceed to construct a density forecast. If the proposed predictive density model for Y_t , i.e. $\{f_t^*(y_t|\Omega_{t-1})\}_{t=1}^T$ coincides with the true conditional density $\{f_t(y_t|\Omega_{t-1})\}_{t=1}^T$, then the sequence of probability integral transforms (PIT) of $\{Y_t\}_{t=1}^T$ w.r.t $\{f_t^*(y_t|\Omega_{t-1})\}_{t=1}^T$ i.e. $\{u_t\}_{t=1}^T$ must be *i.i.d* $U(0, 1)$ where $u_t = \int_{-\infty}^{y_t} f_t^*(v_t|\Omega_{t-1})dv_t$. Thus, the null hypothesis $H_0 : f_t^*(y_t|\Omega_{t-1}) = f_t(y_t|\Omega_{t-1})$ is equivalent to the null hypothesis $H'_0 : \{u_t\}_{t=1}^T$ is *i.i.d* $U(0, 1)$ (see Rosenblatt (1952), Diebold *et al.*, 1998).

In order to provide a self-contained exposition, we briefly review the approach in González-Rivera and Sun (2014). We start by constructing the generalized autocontours (G-ACR) under *i.i.d.* uniform PITs. Under $H'_0 : \{u_t\}_{t=1}^T$ *i.i.d* $U(0, 1)$, the $\text{G-ACR}_{\alpha_i, k}$ is defined as the set $B(\cdot)$ of points in the plane (u_t, u_{t-k}) such that the square with $\sqrt{\alpha_i}$ -side contains at most $\alpha_i\%$ of observations, i.e.,

$$\text{G-ACR}_{\alpha_i, k} = \{B(u_t, u_{t-k}) \subset \mathfrak{R}^2 \mid 0 \leq u_t \leq \sqrt{\alpha_i} \text{ and } 0 \leq u_{t-k} \leq \sqrt{\alpha_i}, \text{ s.t. } : u_t \times u_{t-k} \leq \alpha_i\}$$

We construct an indicator series I_t^{k, α_i} as follows

$$I_t^{k, \alpha_i} = \mathbf{1}((u_t, u_{t-k}) \in \text{G-ACR}_{\alpha_i, k}) = \mathbf{1}(0 \leq u_t \leq \sqrt{\alpha_i}, 0 \leq u_{t-k} \leq \sqrt{\alpha_i})$$

Based on this indicator, González-Rivera and Sun (2014) proposed the following t-tests and chi-square statistics to test the null hypothesis $H'_0 : \{u_t\}_{t=1}^T$ *i.i.d* $U(0, 1)$, which is equivalent to testing for correct specification of the full conditional density model.

(1) t-ratio testing: for fixed autocontour α_i and fixed lag k ,

$$z \equiv \frac{\sqrt{T-k}(\hat{\alpha}_i - \alpha_i)}{\sigma_{k,i}} \xrightarrow{d} N(0, 1)$$

where $\hat{\alpha}_i = \frac{\sum_{t=k+1}^T I_t^{k, \alpha_i}}{T-k}$, and $\sigma_{k,i}^2$ is the asymptotic variance of $\hat{\alpha}_i$.

(2) chi-squared testing:

(2.1) For a fixed lag k , $\mathbf{C}'_k \Omega_k^{-1} \mathbf{C}_k \xrightarrow{d} \chi_C^2$ where $\mathbf{C}_k = (c_{k,1}, \dots, c_{k,C})'$ is a $C \times 1$ stacked vector with element $c_{k,i} = \sqrt{T-k}(\widehat{\alpha}_i - \alpha_i)$, and Ω_k the asymptotic variance-covariance matrix of the vector \mathbf{C}_k .

(2.2) For a fixed autocontour α_i , $\mathbf{L}'_{\alpha_i} \Lambda_{\alpha_i}^{-1} \mathbf{L}_{\alpha_i} \xrightarrow{d} \chi_K^2$ where $\mathbf{L}_{\alpha_i} = (\ell_{1,\alpha_i}, \dots, \ell_{K,\alpha_i})'$ is a $K \times 1$ stacked vector with element $\ell_{k,\alpha_i} = \sqrt{T-k}(\widehat{\alpha}_i - \alpha_i)$, and Λ_{α_i} is the asymptotic variance-covariance matrix of the vector \mathbf{L}_{α_i} .

In a potentially unstable environment, we will construct the tests within the following rolling sample scheme. The total sample size T is divided into two parts: in-sample observations (R) and out-of-sample observations (P). We form subsamples of size r from $t - r + 1$ up to t , where $t = R + r, \dots, T$. In each subsample, we evaluate the proposed predictive density by calculating the three statistics reviewed in (1), (2.1), and (2.2), which we call z , C and L . As a result, we obtain three sets of $n \equiv T - r - R + 1$ tests each i.e., $\{z_j\}_{j=1}^n$, $\{C_j\}_{j=1}^n$ and $\{L_j\}_{j=1}^n$. Finally, to detect instabilities, we construct Sup-type and Avg-type statistics by taking the supremum (S) and the average (A) respectively over each set $\{|z_j|\}_{j=1}^n$, $\{C_j\}_{j=1}^n$ and $\{L_j\}_{j=1}^n$ so that we obtain the following six statistics: $S_{|z|}$, S_C , S_L and $A_{|z|}$, A_C , A_L .

2.2 Asymptotic Properties of the Statistics

Under the following set of assumptions, we provide three propositions, which proofs are provided in the Appendix A.

- A1: For $T \rightarrow \infty$, $R \rightarrow \infty$, $P \rightarrow \infty$, $\lim_{T \rightarrow \infty} \frac{P}{R} = 0$; and for $r, P \rightarrow \infty$, $\lim_{T \rightarrow \infty} \frac{r-k}{P} = m$, where r is the size of the rolling subsample in the evaluation set, $m \in (0, 1)$, and k is the lag in the I_t^{k,α_i} indicator. ²

²According to several Monte Carlo studies (West, 2006, and the references herein), the ratio P/R is considered sufficiently small and parameter uncertainty will be almost negligible when $P/R \leq 10\%$. When $\lim_{T \rightarrow \infty} \frac{P}{R} \neq 0$,

- A2: $E|I_t^{k,\alpha_i}|^q < \Delta < \infty$ for some $q \geq 2$. This assumption is trivial as the second moments of the indicator are well defined as we will see next.
- A3: The data $\{y_t\}$ comes from a stationary and ergodic process.³

Proposition 1 Let J be the index that identifies a particular subsample in the evaluation period, i.e. $J = [Ps]$, $s \in [m, 1]$, $[Pm] = r - k$, and let $\hat{\alpha}_i(J) = \frac{\sum_{t=R+1+J-r+k}^{R+J} I_t^{k,\alpha_i}}{r-k}$ be the corresponding subsample proportion based on the indicator. The Sup- and Avg-tests are

$$S_{|z|} = \sup_J \left| \frac{\sqrt{r-k}(\hat{\alpha}_i(J) - \alpha_i)}{\sigma_{k,\alpha_i}} \right|$$

$$A_{|z|} = \frac{1}{P-r+1} \sum_J^{\bar{J}} \left| \frac{\sqrt{r-k}(\hat{\alpha}_i(J) - \alpha_i)}{\sigma_{k,\alpha_i}} \right|$$

where $\sigma_{k,\alpha_i}^2 = \alpha_i(1 - \alpha_i) + 2\alpha_i^{3/2}(1 - \alpha_i^{1/2})$

Given assumptions A1-A3, and under the null hypothesis of *i.i.d* $U(0, 1)$ PITs, the asymptotic distribution of the tests, for $P \rightarrow \infty$, is the following

$$S_{|z|} \xrightarrow{d} \sup_{s \in [m, 1]} \frac{|W(s) - W(s - m)|}{\sqrt{m}}$$

$$A_{|z|} \xrightarrow{d} \int_{\underline{s}}^{\bar{s}} \frac{|W(s) - W(s - m)|}{\sqrt{m}} ds$$

where $W(\cdot)$ is a standard univariate Brownian motion.

parameter uncertainty will affect the asymptotic variance σ_{k,α_i}^2 of the statistic. By taking a mean value expansion around the true parameter values θ_0 and applying the Slutsky's theorem we obtain

$$\sqrt{P}(\hat{\alpha}_i(\hat{\theta}) - \alpha_i) = \sqrt{P}(\hat{\alpha}_i(\theta_0) - \alpha_i) + \frac{\sqrt{P}}{\sqrt{R}} \sqrt{R}(\hat{\theta} - \theta_0)' \lim_{R \rightarrow \infty} E \left\{ \frac{\partial \hat{\alpha}_i}{\partial \theta} \Big|_{\theta = \theta_0} \right\} + o_p(1)$$

The variance of the second term of the expansion as well as the covariance between the first and second terms will contribute to the asymptotic variance of the statistics. For calculation of these terms, see González-Rivera, Senyuz, and Yoldas (2011). In practice, these terms will be calculated by a fully parametric bootstrap procedure.

³This assumption can be relaxed to include more general mixing processes because the relevant conditions to invoke limiting theorems as the FCLT are those of the indicator process.

Proposition 2 For a fixed lag k , write $c_{k,i}(J) = \sqrt{r-k}(\widehat{\alpha}_i(J) - \alpha_i)$ and stack $c_{k,i}(J)$ for different autocontours levels $i = 1, 2, \dots, C$ such that $\mathbf{C}_k(J) = (c_{k,1}(J), \dots, c_{k,C}(J))'$ is the $C \times 1$ stacked vector. The Sup- and Avg-tests are

$$S_C = \sup_J \mathbf{C}_k(J)' \Omega_k^{-1} \mathbf{C}_k(J)$$

$$A_C = \frac{1}{P-r+1} \sum_{\underline{J}}^{\bar{J}} \mathbf{C}_k(J)' \Omega_k^{-1} \mathbf{C}_k(J)$$

where Ω_k is the asymptotic variance and covariance matrix of the random vector $\mathbf{C}_k(J)$. Given assumptions A1-A3, and under the null hypothesis of *i.i.d* $U(0, 1)$ PITs, the asymptotic distribution of the tests, for $P \rightarrow \infty$, is the following

$$S_C \xrightarrow{d} \sup_{s \in [m, 1]} \frac{(\mathbf{W}(s) - \mathbf{W}(s-m))' (\mathbf{W}(s) - \mathbf{W}(s-m))}{m}$$

$$A_C \xrightarrow{d} \int_{\underline{s}}^{\bar{s}} \frac{(\mathbf{W}(s) - \mathbf{W}(s-m))' (\mathbf{W}(s) - \mathbf{W}(s-m))}{m} ds$$

where $\mathbf{W}(\cdot)$ is a standard C -variate Brownian motion.

Proposition 3 For a fixed autocontour α_i , write $\ell_{k,\alpha_i}(J) = \sqrt{r-k}(\widehat{\alpha}_i(J) - \alpha_i)$ and stack ℓ_{k,α_i} for $k = 1, \dots, K$. Let $\mathbf{L}_{\alpha_i}(J) = (\ell_{1,\alpha_i}(J), \dots, \ell_{K,\alpha_i}(J))'$ be the $K \times 1$ stacked vector. The Sup- and Avg-tests are

$$S_L = \sup_J \mathbf{L}_{\alpha_i}(J)' \Lambda_{\alpha_i}^{-1} \mathbf{L}_{\alpha_i}(J)$$

$$A_L = \frac{1}{P-r+1} \sum_{\underline{J}}^{\bar{J}} |\mathbf{L}_{\alpha_i}(J)' \Lambda_{\alpha_i}^{-1} \mathbf{L}_{\alpha_i}(J)|$$

where Λ_{α_i} is the asymptotic variance and covariance matrix for the random vector \mathbf{L}_{α_i} . Given assumptions A1-A3, and under the null hypothesis of *i.i.d* $U(0, 1)$ PITs, the asymptotic distribution

of the tests, for $P \rightarrow \infty$, is the following

$$S_L \xrightarrow{d} \sup_{s \in [m, 1]} \frac{(\mathbf{W}(s) - \mathbf{W}(s - m))'(\mathbf{W}(s) - \mathbf{W}(s - m))}{m}$$

$$A_L \xrightarrow{d} \int_{\underline{s}}^{\bar{s}} \frac{(\mathbf{W}(s) - \mathbf{W}(s - m))'(\mathbf{W}(s) - \mathbf{W}(s - m))}{m} ds$$

where $\mathbf{W}(\cdot)$ is a standard K -variate Brownian motion.

We tabulate the percentiles of the asymptotic distributions of the tests provided in Propositions 1 to 3. Since the distributions depend on m , which is proportion of the rolling sample to the total evaluation sample, we consider the following values of $m \in [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]$. In Table 1, we report the percentiles of the distributions of the $S_{|z|}$ and $A_{|z|}$ statistics; in Table 2, those for the S_C and A_C considering the 13 autocontour $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$; and in Table 3 those for the S_L and A_L tests considering 5 lags.

[TABLES 1-3 ABOUT HERE]

Since a model can be considered misspecified whenever does not take into account breaks in the data, the proposed stability tests can be also viewed as statistics assessing jointly the specification and the time stability of the model. In practice, the tests will be most useful when models that are estimated with R observations, need to be validated over a new set of observations, in our case, over the prediction sample of P observations, and in doing so, we assess whether the model is stable over different time periods. In the following sections, Monte Carlo simulations and the evaluation of the Phillips curve, we show how to use the stability tests.

3 Monte Carlo Simulations

We perform extensive Monte Carlo simulations to assess the finite sample properties (size and power) of the proposed statistics. For \sqrt{R} -consistent estimators of the parameters of the model,

i.e. $(\widehat{\theta} - \theta_0) = O_p(R^{-1/2})$ with a well-defined asymptotic distribution, and under assumption A1, parameter uncertainty is asymptotically negligible. In this case, the critical values tabulated in Tables 1 to 3 can be used directly. Whenever the condition $P/R \rightarrow 0$ is violated, we need to adjust the asymptotic variance of the tests, otherwise the size of the tests will be distorted. As we mention in the previous section, the adjustments to the variance will be difficult to calculate analytically because they are model dependent. For practical purposes, we propose to implement a fully parametric bootstrap procedure because the density model is fully known under the null hypothesis of correct dynamic specification and correct functional form of the conditional density, i.e., stability of the density model over the prediction sample. The bootstrap procedure entails either to bootstrap the asymptotic variance and use the tabulated critical values or bootstrap directly the distribution of the proposed Sup- and Ave- tests to obtain their critical values. In the Appendix B, we explain how we implement the parametric bootstrap to obtain bootstrapped critical values. Nevertheless, in the following simulations, we also explore the effect of different values of the ratio P/R on the size of the tests. We consider fixed, rolling, and recursive forecasting schemes.

3.1 Size of the Tests

Under the null hypothesis of a stable density model, we consider the following data generating process: $y_t = \alpha_1 + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sigma \epsilon_t$ where $x_t = \phi_1 + \phi_2 x_{t-1} + \nu_t$, and ϵ_t and $\nu_t \sim N(0, 1)$, $\phi_1 = 1.38$, $\phi_2 = 0.77$, $\alpha_1 = 1.5$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $\sigma = 1$. We consider sample sizes of $T = 150$ (with evaluation sample $P=60$), $T = 375$ (with $P=150$), and $T = 750$ (with $P=300$) observations, and for each sample size, we consider the proportion $m = r/P$ equal to $m = 1/3$, $m = 1/2$ and $m = 2/3$. We maintain the ratio P/R constant and equal to $2/3$. In total, we run nine experiments, of which we present here the two most extremes: small sample size with small subsample window ($T = 150, m = 1/3$) and large sample size with large subsample window ($T = 750, m = 2/3$). The

size results of these two cases are presented in Tables 4 and 5 respectively. The remaining seven cases are available in a “supplementary material” file. We work with 13 autocontour coverage levels $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$. The number of Monte Carlo replications is 1,000, the number of bootstrap samples is 500, and the nominal size is 5%.

[TABLES 4-5 ABOUT HERE]

The overall size of the tests is very good in the nine experiments considered. There are not substantial differences among the fixed, rolling, and recursive estimation schemes. We find some small size distortions (under-sized) when the sample is small ($T = 150$) and the autocontour levels are extreme (1% and 99%) but as the sample size increases, the distortion disappears. For the individual tests $S_{|z|}^{k,\alpha_i}$ and $A_{|z|}^{k,\alpha_i}$ (k and α_i fixed), the Ave-test tends to have better size than the Sup-test. The tests S_C , A_C , S_L , and A_L have very good size even in small samples.

For the same generating process explained in the above paragraphs, with a small estimation sample $R = 96$ and a small subsample window $m = 1/3$, we have considered three increasing values for the ratio P/R , i.e., $1/4$ ($T = 120$), $1/2$ ($T = 144$), and $3/4$ ($T = 168$). The size results for these three cases are presented in Tables 6 to 8. For the case of $P/R = 1/4$, we observe that the individual tests $S_{|z|}^{k,\alpha_i}$ are more undersized than the tests $A_{|z|}^{k,\alpha_i}$, for which the average size is about 4%. The portmanteau tests S_C , A_C , S_L , and A_L have a size between 3 and 5%. Note that for this case we have a very small prediction sample $P = 24$ and the subsample window includes only 8 observations. For the case of $P/R = 1/2$, there is an overall improvement of the size for all tests, and for the case $P/R = 3/4$, most tests have a size around the nominal size of 5%. It seems that the small distortions that we observe are due to the small number of observations in the subsample window. As we increase the ratio P/R , the parametric bootstrap is properly taking care of the effect of parameter uncertainty in the variance of the tests and delivering tests with the correct size.

[TABLES 6-8 ABOUT HERE]

3.2 Power of the Tests

To assess the power of the tests, we generate data from four different processes, all of them containing a break point. The model that we maintain under the null hypothesis is the same as the one considered in the study of the size properties: $y_t = \alpha_1 + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sigma \epsilon_t$ with $x_t = \phi_1 + \phi_2 x_{t-1} + \nu_t$, ν_t and $\epsilon_t \sim N(0, 1)$. The total sample size (T) is 650, $R = 350$, $P = 300$, and $m = 1/3$. The break point happens at at $R + \tau P$, where $\tau = 1/3, 1/2$, and $2/3$. In the following experiments, the number of Monte Carlo replications is 1000 and bootstrapped samples 500. We maintain a nominal test size of 5%.

The four data generating mechanisms are the following:

DGP1: Break in the intercept of $y_t = \alpha_t + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sigma \epsilon_t$, $\epsilon_t \sim N(0, 1)$:

$$\alpha_t = \begin{cases} \alpha_1 = 1.5 & \text{if } t < \text{break} \\ \alpha_2 = 2 & \text{otherwise} \end{cases}$$

with $\beta_1 = 0.5$, $\beta_2 = 0.6$, $\sigma = 1$.

DGP2: Break in the variance of $y_t = \alpha + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sigma_t \epsilon_t$, $\epsilon_t \sim N(0, 1)$:

$$\sigma_t = \begin{cases} \sigma_1 = 1.5 & \text{if } t < \text{break} \\ \sigma_2 = 1.8 & \text{otherwise} \end{cases}$$

with $\alpha = 1.5$, $\beta_1 = 0.5$, $\beta_2 = 0.6$.

DGP3: Breaks in the slope coefficients of $y_t = \alpha + \beta_{1,t} y_{t-1} + \beta_{2,t} x_{t-1} + \sigma \epsilon_t$, $\epsilon_t \sim N(0, 1)$:

$$\beta_{1,t} = \begin{cases} \beta_{1,1} = 0.5 & \text{if } t < \text{break} \\ \beta_{1,2} = 0.3 & \text{otherwise} \end{cases}$$

$$\beta_{2,t} = \begin{cases} \beta_{2,1} = 0.6 & \text{if } t < \text{break} \\ \beta_{2,2} = 0.4 & \text{otherwise} \end{cases}$$

with $\alpha = 1.5$, $\sigma = 1$.

DGP4: Breaks in the intercept, variance and slope coefficients of $y_t = \alpha_t + \beta_{1,t}y_{t-1} + \beta_{2,t}x_{t-1} + \sigma_t\epsilon_t$, $\epsilon_t \sim N(0, 1)$:

$$\alpha_t = \begin{cases} \alpha_1 = 1.5 & \text{if } t < \text{break} \\ \alpha_2 = 2 & \text{otherwise} \end{cases}$$

$$\sigma_t = \begin{cases} \sigma_1 = 1.5 & \text{if } t < \text{break} \\ \sigma_2 = 1.8 & \text{otherwise} \end{cases}$$

$$\beta_{1,t} = \begin{cases} \beta_{1,1} = 0.5 & \text{if } t < \text{break} \\ \beta_{1,2} = 0.3 & \text{otherwise} \end{cases}$$

$$\beta_{2,t} = \begin{cases} \beta_{2,1} = 0.6 & \text{if } t < \text{break} \\ \beta_{2,2} = 0.4 & \text{otherwise} \end{cases}$$

Note that the breaks considered are not very extreme. We perform all the simulations under fixed, rolling, and recursive estimation schemes. We report the power results for the fixed and rolling schemes for the four DGPs with $\tau = 1/3$ in Tables 9 to 12 and those for the recursive scheme in the supplementary material.

[TABLES 9-12 ABOUT HERE]

Under the fixed scheme, the tests are most powerful (power of about 90%) to detect breaks in intercept and slope coefficients (DGP1, DGP3, and DGP4). Both Ave- and Sup- tests enjoy similar performance, either for single hypothesis ($S_{|z|}$ and $A_{|z|}$) or for joint hypothesis (S_C , A_C , S_L , A_L). The power drops when testing for breaks in the variance (DGP2) with values of 10-60% for the single hypothesis tests and of 60% for the joint hypothesis tests. Under the rolling scheme,

as we expected the tests are overall less powerful because by rolling the estimation sample, the model adjusts slowly to the new parameters values. Nevertheless, the power of S_C , A_C , S_L and A_L is still very high (around 50-80%) for DGP1, DGP3, and DGP4, and around 40% for DGP2. For all DGPs, the Ave-test is more powerful than the Sup. Under the recursive scheme, the tests performance is slightly worse than in the fixed scheme but slightly better than in the rolling scheme.

We move the location of the break, i.e., $\tau = 1/2$ and $2/3$ and we present the power results for DGP3 with a rolling scheme in Table 13. The overall finding is that the power of the tests decreases as the break point moves towards the end of the prediction sample, which is expected. However, the decrease in power is not very abrupt. For instance, for the A_C test the power moves from about 80% ($\tau = 1/3$) to 65% ($\tau = 1/2$) to 55% ($\tau = 2/3$)

[TABLE 13 ABOUT HERE]

3.3 Comparison with Rossi & Sekhposyan tests

We compare our tests with those of Rossi & Sekhposyan (R&S) (2013). In both instances, we entertain the same null hypothesis, i.e. the constancy of the density model (specification of the model and the functional form of the conditional density) over the prediction sample. R&S tests follow the set up of Corradi and Swanson (2006) so that their tests, also based on the PITs of the proposed model, are a function of the distance between the empirical cumulative distribution function and that of the uniform distribution, which is a 45 degree line. The κ_P test is a Kolmogorov-Smirnov-type statistics and the C_P is a Cramer-von Mises-type statistic. Our G-ACR tests are based on the object "autocontour" and measure the dense-ness and uniform distribution of the PITs over a collection of squares in a two-dimensional space (u_{t-k}, u_t) . By construction, our tests exploit the independence properties of the PITs directly. The asymptotic distribution of the R&S tests is based on the statistical properties of an empirical process. Our tests are simpler in that they rely on the properties of a binary indicator with well-defined moments. When parameter

uncertainty is non-negligible, the critical values of the R&S tests and the G-ACR tests are obtained by simulation.

We follow the same experimental design explained in section 4.2 of R&S article. Their tests κ_P and C_P are directly comparable with our tests S_C and A_C stacking all the 13 autocontour levels. We consider the same DGPs as in Table 2 of R&S article: DGP1 considers the case of misspecification in the functional form of the density, DGP2 considers instability in the mean and variance parameters, and DGP3 considers time-varying distortions in the shape of the assumed conditional density. We run the experiments with $R = 500$, $P = 100$ and a rolling ratio $m = 0.8$ in the prediction sample. The models are estimated recursively. The number of Monte Carlo replications is 1000 with 500 bootstrap samples.

When $c = 0$ (DGP1 and DGP2) and $T_1/T = 600$ (DGP3), the respective DGPs coincide with the model under the null hypothesis, so the rejection frequency is equal to the size of the tests. The larger the value of c of the lowest the ratio T_1/T , the furthest the DGP model is from the model under the null. All tests have good sizes but the $S_C^{1,13}$ and $A_C^{1,13}$ tests are more powerful across DGPs, in particular for the cases of DGP2 ($c = 0.5$ and $c = 1$) and DGP3 ($T_1/T = 580$ and 560).

Empirical Rejection Frequencies

DGP1				
c	κ_P	C_P	$S_C^{1,13}$	$A_C^{1,13}$
0	0.04	0.04	0.05	0.04
0.5	0.04	0.05	0.07	0.07
1	0.13	0.14	0.21	0.20
1.5	0.30	0.34	0.48	0.40
2	0.50	0.60	0.71	0.63
2.5	0.73	0.82	0.87	0.84
DGP2				
c	κ_P	C_P	$S_C^{1,13}$	$A_C^{1,13}$
0	0.04	0.04	0.05	0.05
0.5	0.16	0.14	0.53	0.50
1	0.66	0.47	0.99	0.97
1.5	0.97	0.92	1.00	1.00
2	1.00	1.00	1.00	1.00
2.5	1.00	1.00	1.00	1.00
DGP3				
T_1/T	κ_P	C_P	$S_C^{1,13}$	$A_C^{1,13}$
600	0.04	0.04	0.05	0.05
580	0.27	0.29	0.73	0.50
560	0.78	0.78	1.00	0.99
540	0.96	0.97	1.00	1.00

4 Density Forecast Evaluation of the Phillips Curve

We apply the proposed tests to analyze the stability of the Phillips curve. Stock and Watson (1999) found some empirical evidence in favor of the Phillips curve as a forecasting tool, they showed that inflation forecasts produced by the Phillips curve were more accurate than forecasts based on simple autoregressive or multivariate models but they also found parameter instabilities across different subsamples. Rossi and Skehposyan (2010) showed that the predictive power of the Phillips curve disappeared around the time of the Great Moderation. Based on scoring rules, Amisano and Giacomini (2007) compared the density forecast accuracy of several models of the

Phillips curve and concluded that the best density forecast is produced by a Markov-switching model. Since their comparisons are based on the *average* forecasting performance of competing models over time, they cannot directly address the instabilities widely documented in the literature. In this section, we consider the models in Amisano and Giacomini (2007) and we focus on their absolute density forecast performance in the presence of instabilities. Our starting model is a linear Phillips curve (Stock and Watson, 1999), in which changes of the inflation rate depend on their lags and on lags of the unemployment rate i.e.,

$$\Delta\pi_t = \alpha_1 + \beta_1\Delta\pi_{t-1} + \beta_2\Delta\pi_{t-2} + \beta_{12}\Delta\pi_{t-12} + \gamma u_{t-1} + \sigma\epsilon_t$$

where $\pi_t = 100 \times \ln(CPI_t/CPI_{t-12})$; CPI_t is the consumer price index for all urban consumers and all items; u_{t-1} is the civilian unemployment rate; and $\epsilon_t \sim N(0, 1)$. The data is collected from the FRED database; monthly CPI and unemployment series are both seasonally adjusted. The time series run from 1958M01 to 2012M01 (updated sample from 1958M01-2004M07 in Amisano and Giacomini). Standard tests on $\Delta\pi_t$ and u_t show that they do not have a unit root. On implementing our tests, we consider the same estimation sample as in Amisano and Giacomini, from 1958M01 to 1987M12 (360 observations). The evaluation sample runs from 1988M01 to 2012M01 (289 observations) with subsamples of size $r = 200$.

4.1 Evaluation of the Linear Phillips Curve

We present the evaluation results for the linear Phillips Curve in Table 14 under fixed and rolling estimation schemes. The recursive case is in the supplementary material.

[TABLE 14 ABOUT HERE]

The portmanteau tests S_C , A_C , S_L , and A_L indicate a clear rejection of the linear model. On examining the individual tests $S_{|z|}$ and $A_{|z|}$, the rejection comes from the middle autocontours,

between 40% and 70% coverage, and from the 95-99% levels (large changes in inflation). In Figure 1, we plot the t - and C -statistics over the evaluation period in a sequential fashion. The tests break through their corresponding critical values around the 60th statistic, and the values of the tests keep on increasing reaching two local maxima, which points to two potential breaks: the first in the 64th statistic for the t -tests (1993M03 to 2009M11) and in the 71th statistic for the C tests (1993M11 to 2010M06), and the second local maximum in the 78th statistic that corresponds to the period 1994M06 to 2011M01. All these periods include the years 1993-2007 in the so-called Great Moderation and the years after the deep financial crisis of 2008.

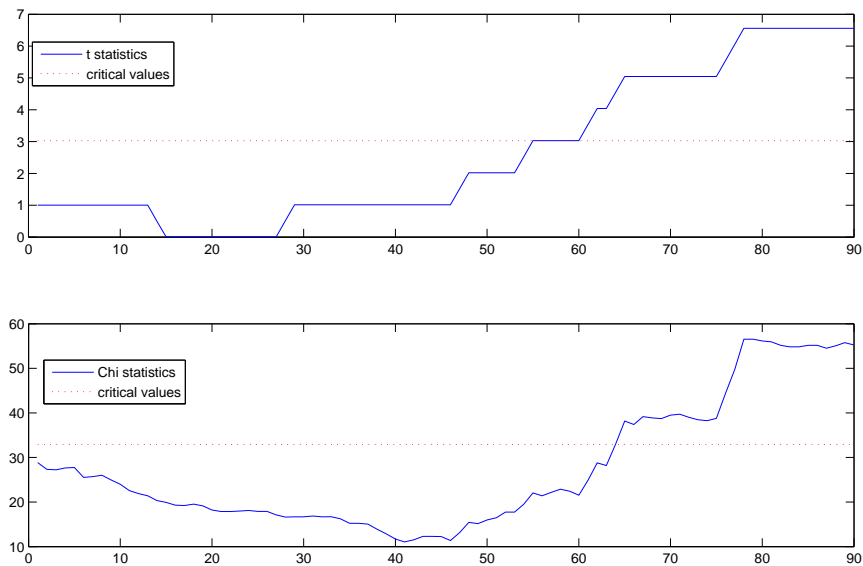


Figure 1: **Plots of t -ratio (99% autocontour) and C Statistics for Linear Phillips Curve (fixed scheme)**

4.2 Evaluation of Non-Linear Phillips Curve

Given the rejection of the linear Phillips curve, we proceed with a flexible specification by assuming that the coefficients in the linear model vary according to a Markov switching mechanism. We

consider a two-state Markov switching model, i.e.,

$$\Delta\pi_t = \alpha^{s_t} + \beta_1^{s_t} \Delta\pi_{t-1} + \beta_2^{s_t} \Delta\pi_{t-2} + \beta_{12}^{s_t} \Delta\pi_{t-12} + \gamma^{s_t} u_{t-1} + \sigma^{s_t} \epsilon_t$$

where the unobserved state variable s_t switches between two states, 1 or 2, with transition probabilities $Pr(s_t = j | s_{t-1} = i) = p_{ij}$ for $i, j = 1, 2$; and ϵ_t is assumed to be a standard normal variate. Thus, this model allows for non-Gaussian density forecasts. Since all the coefficients depend on the state variable (Model 1), the model is extremely flexible and it will adapt to potential breaks or instabilities that may occur over time. We have run our test statistics and we fail to reject the null hypothesis of correct specification. However, we would like to investigate what coefficients are key to understand where the nonlinear behavior comes from. We consider six additional specifications,

Model 2: $\Delta\pi_t = \alpha + \beta_1^{s_t} \Delta\pi_{t-1} + \beta_2^{s_t} \Delta\pi_{t-2} + \beta_{12}^{s_t} \Delta\pi_{t-12} + \gamma u_{t-1} + \sigma \epsilon_t$, (intercept and unemployment coefficient do not depend on s_t).

Model 3: $\Delta\pi_t = \alpha^{s_t} + \beta_1 \Delta\pi_{t-1} + \beta_2 \Delta\pi_{t-2} + \beta_{12} \Delta\pi_{t-12} + \gamma^{s_t} u_{t-1} + \sigma^{s_t} \epsilon_t$, (inflation coefficients do not depend on s_t).

Model 4: $\Delta\pi_t = \alpha + \beta_1 \Delta\pi_{t-1} + \beta_2 \Delta\pi_{t-2} + \beta_{12} \Delta\pi_{t-12} + \gamma u_{t-1} + \sigma^{s_t} \epsilon_t$, (only the standard deviation depends on s_t).

Model 5: $\Delta\pi_t = \alpha^{s_t} + \beta_1 \Delta\pi_{t-1} + \beta_2 \Delta\pi_{t-2} + \beta_{12} \Delta\pi_{t-12} + \gamma^{s_t} u_{t-1} + \sigma \epsilon_t$, (only the intercept and unemployment coefficient depend on s_t).

Model 6: $\Delta\pi_t = \alpha^{s_t} + \beta_1 \Delta\pi_{t-1} + \beta_2 \Delta\pi_{t-2} + \beta_{12} \Delta\pi_{t-12} + \gamma u_{t-1} + \sigma^{s_t} \epsilon_t$, (only the intercept and the standard deviation depend on s_t).

Model 7: $\Delta\pi_t = \alpha + \beta_1 \Delta\pi_{t-1} + \beta_2 \Delta\pi_{t-2} + \beta_{12} \Delta\pi_{t-12} + \gamma^{s_t} u_{t-1} + \sigma \epsilon_t$, (only the unemployment coefficient depends on s_t).

Out of these six specifications, we observe outright rejections at the 5% significance level for

Model 2 (rejection in the 90-99% autocontours), Model 4 (rejection in the 95-99% autocontours), and Model 7 (rejection in the 30-50% autocontours). We only fail to reject Model 3 in which we allow the intercept, the unemployment coefficient, and the standard deviation of the error to be state-dependent while the rest of the parameters are constant. In this case, all statistics (joint and individual) enjoy p-values larger than 5%. Models 5 and 6 are close contenders to Model 3. In Model 5, where only the intercept and the unemployment coefficient are state-dependent, we observe a marginal rejection (p-values around 4%) with the $S_{|z|}$ and $A_{|z|}$ tests for the 1 and 5% autocontours. In Model 6, where only the intercept and the standard deviation of the error are state-dependent, we observe a marginal rejection (p-values around 4%) with the $S_{|z|}$ test for the 1% autocontour. The overall message of these testing results is that a shift in the level α^{st} and a shift in the standard deviation σ^{st} (Model 6) may be enough to understand the instability of the Phillips curve but if, in addition, we let the coefficient that links inflation and unemployment to be state-dependent (Model 3), the density forecast of inflation changes will be more robust to unstable times. We report the testing results for Models 3 and 6 in Table 15.

[TABLE 15 ABOUT HERE]

For the preferred model, Model 3, we compute the natural rate of unemployment (NAIRU), which is the ratio between (minus) the intercept term and the coefficient on unemployment. In recession times (state 1), the estimated NAIRU is 7.52% and in expansions (state 2), 5.53%. The persistence of state 1 is $p_{11} = 0.96$ and that of state 2 is $p_{22} = 0.99$ so that recessions are shorter than expansions. The estimated variance of state 1 is 0.20 and that of state 2 is 0.04 so that recessions are more volatile than expansions.

5 Conclusion

We have provided a battery of tests to assess the stability of the density forecast over time, which offer important advantages for the empirical researcher. These tests are nuisance-parameter free and their asymptotic distributions can be tabulated. If the tests reject the null hypothesis of a stable density forecast, the shapes of the empirical generalized autocontours can be visualized to extract information regarding the direction of the rejection. Regardless of the estimation scheme (fixed, rolling, or recursive), their finite sample properties are superior. In some instances, the Ave-tests tend to have a slightly better size than the Sup-tests. Both types are more powerful on detecting breaks in the intercept and slope coefficients than on detecting breaks in the variance. The tests can also be easily applied to multivariate random processes of any dimension. We have compared our tests with those of Rossi and Sekhposyan (2013) and found that our G-ACR tests are more powerful across the DGPs considered. As an application of the tests, we have analyzed the stability of the Phillips curve. A linear model is strongly rejected in favor of a non-linear specification that allows shifts in the mean and in the variance of the inflation changes as well as in the coefficient linking inflation and unemployment. The break of the linear model occurs during the Great Moderation years.

Appendix A

Proof of Proposition 1:

The indicator function I_t^{k,α_i} is a Bernoulli random variable with the following moments: $E(I_t^{k,\alpha_i}) = \alpha_i$, $Var(I_t^{k,\alpha_i}) = \alpha_i(1 - \alpha_i)$ and covariance

$$r_h^{\alpha_i} \equiv cov(I_t^{k,\alpha_i}, I_{t-h}^{k,\alpha_i}) = \begin{cases} 0 & \text{if } h \neq k \\ \alpha_i^{3/2}(1 - \alpha_i^{1/2}) & \text{if } h = k \end{cases}$$

Since the indicator process is stationary and ergodic, $\hat{\alpha}_i(J)$ satisfies the condition of global covariance stationarity required for the FLCT to apply (Theorem 7.17) in White (2001). Since $J = [Ps]$, $s \in [m, 1]$, and $r - k = [Pm]$, we write as $P \rightarrow \infty$

$$\begin{aligned} W_P(s) &\equiv \frac{\sqrt{(r-k)}(\hat{\alpha}_i(J) - \alpha_i)}{\sigma_{k,i}} = \frac{\sum_{t=R+1+J-r+k}^{R+J} (I_t^{k,\alpha_i} - \alpha_i)}{\sqrt{r-k}\sigma_{k,i}} \\ &= \frac{\sum_{t=R+1}^{R+J} (I_t^{k,\alpha_i} - \alpha_i)}{\sqrt{(r-k)}\sigma_{k,i}} - \frac{\sum_{t=R+1}^{R+J-(r-k)} (I_t^{k,\alpha_i} - \alpha_i)}{\sqrt{(r-k)}\sigma_{k,i}} \\ &= \frac{\sqrt{P}}{\sqrt{Pm}} \frac{\sum_{t=R+1}^{R+[Ps]} (I_t^{k,\alpha_i} - \alpha_i)}{\sqrt{P}\sigma_{k,i}} - \frac{\sqrt{P}}{\sqrt{Pm}} \frac{\sum_{t=R+1}^{R+[P(s-m)]} (I_t^{k,\alpha_i} - \alpha_i)}{\sqrt{P}\sigma_{k,i}} \\ &\xrightarrow{d} \frac{1}{\sqrt{m}}(W(s) - W(s-m)) \end{aligned}$$

where $W(\cdot)$ is the standard Brownian motion and the limiting distribution, in the last line, is a direct consequence of the FCLT. Finally, by the Continuous Mapping Theorem, we have:

$$\begin{aligned} S_{|t|} &= \sup_J \left| \frac{\sqrt{r-k}\hat{\alpha}_i(J)}{\sigma_{k,\alpha_i}} \right| \xrightarrow{d} \sup \frac{|W(s) - W(s-m)|}{\sqrt{m}} \\ A_{|t|} &= \frac{1}{P-r+1} \sum_{\underline{J}}^{\bar{J}} \left| \frac{\sqrt{r-k}\hat{\alpha}_i(J)}{\sigma_{k,\alpha_i}} \right| \xrightarrow{d} \int_{\underline{s}}^{\bar{s}} \frac{|W(s) - W(s-m)|}{\sqrt{m}} \end{aligned}$$

where $\underline{J} = [P\underline{s}]$ and $\bar{J} = [P\bar{s}]$.

Proof of Proposition 2

Let Ω_k be the variance-covariance matrix of $\mathbf{C}_k(J)$ whose typical element $\omega_{i,j}$ is calculated as follows

$$\text{cov}(c_{k,i}, c_{k,j}) = \text{cov}(I_t^{k,\alpha_i}, I_t^{k,\alpha_j}) + \text{cov}(I_t^{k,\alpha_i}, I_{t-k}^{k,\alpha_j}) + \text{cov}(I_{t-k}^{k,\alpha_i}, I_t^{k,\alpha_j}) + o(1)$$

If $i = j$, by Proposition 1, $\omega_{i,i} = \text{var}(\sqrt{T-k}(\hat{\alpha}_i - \alpha_i)) = \alpha_i(1 - \alpha_i) + 2\alpha_i^{3/2}(1 - \alpha_i^{1/2})$. If $i < j$, $\alpha_i < \alpha_j$, and we have

$$\begin{aligned} \text{cov}(I_t^{k,\alpha_i}, I_t^{k,\alpha_j}) &= E(I_t^{k,\alpha_i} \times I_t^{k,\alpha_j}) - \alpha_i \times \alpha_j = \alpha_i(1 - \alpha_j) \\ \text{cov}(I_t^{k,\alpha_i}, I_{t-k}^{k,\alpha_j}) &= E(I_t^{k,\alpha_i} \times I_{t-k}^{k,\alpha_j}) - \alpha_i \times \alpha_j = \alpha_i \times \alpha_j^{1/2} - \alpha_i \times \alpha_j \\ \text{cov}(I_{t-k}^{k,\alpha_i}, I_t^{k,\alpha_j}) &= E(I_{t-k}^{k,\alpha_i} \times I_t^{k,\alpha_j}) - \alpha_i \times \alpha_j = \alpha_i \times \alpha_j^{1/2} - \alpha_i \times \alpha_j \end{aligned}$$

If $i > j$, the above expressions hold by just switching the subindexes i and j .

Since the vector $\mathbf{C}_k(J)$ is globally stationary, we can invoke a multivariate FLCT (see Theorem 7.29 and 7.30 in White (2001)). By following the same arguments as in Proposition 1, we have as $P \rightarrow \infty$,

$$\begin{aligned} \mathbf{W}_P(s) &\equiv \Omega_k^{-1/2} \mathbf{C}_k(J) \\ &\xrightarrow{d} \frac{1}{\sqrt{m}} (\mathbf{W}(s) - \mathbf{W}(s-m)) \end{aligned}$$

where $\mathbf{W}(s)$ is a C -variate Brownian process. By the Continuous Mapping Theorem, we have

$$\begin{aligned} S_C &= \sup_J \mathbf{C}_k(J)' \Omega_k^{-1} \mathbf{C}_k(J) \\ &\xrightarrow{d} \sup_{s \in [m,1]} \frac{(\mathbf{W}(s) - \mathbf{W}(s-m))' (\mathbf{W}(s) - \mathbf{W}(s-m))}{m} \end{aligned}$$

and

$$\begin{aligned}
A_C &= \frac{1}{P-r+1} \sum_{\underline{J}}^{\bar{J}} |\mathbf{C}_k(J)' \Omega_k^{-1} \mathbf{C}_k(J)| \\
&\xrightarrow{d} \int_{\underline{s}}^{\bar{s}} \frac{(\mathbf{W}(r) - \mathbf{W}(r-m))' (\mathbf{W}(r) - \mathbf{W}(r-m))}{m}
\end{aligned}$$

Proof of Proposition 3: Let Λ_{α_i} be the variance-covariance matrix of $\mathbf{L}_{\alpha_i}(J)$ whose typical element $\lambda_{j,k}$ is calculated as

$$\lambda_{j,k} = \begin{cases} \alpha_i(1 - \alpha_i) + 2\alpha_i^{3/2}(1 - \alpha_i^{1/2}) & \text{if } j = k \\ 4\alpha_i^{3/2}(1 - \alpha_i^{1/2}) & \text{if } j \neq k \end{cases}$$

Therefore, the vector $\mathbf{L}_{\alpha_i}(J)$ is globally stationary, and we can invoke a multivariate FLCT (see Theorems 7.29 and 7.30 in White (2001)). By following the same arguments as in Proposition 1, we have as $P \rightarrow \infty$,

$$\begin{aligned}
\mathbf{W}_P(s) &\equiv \Lambda_{\alpha_i}^{-1/2} \mathbf{L}_{\alpha_i}(J) \\
&\xrightarrow{d} \frac{1}{\sqrt{m}} (\mathbf{W}(s) - \mathbf{W}(s-m))
\end{aligned}$$

where $\mathbf{W}(s)$ is a L -variate Brownian process. By the Continuous Mapping Theorem, we have

$$\begin{aligned}
S_L &= \sup_J \mathbf{L}_{\alpha_i}(J)' \Lambda_{\alpha_i}^{-1} \mathbf{L}_{\alpha_i}(J) \\
&\xrightarrow{d} \sup_{s \in [m,1]} \frac{(\mathbf{W}(s) - \mathbf{W}(s-m))' (\mathbf{W}(s) - \mathbf{W}(s-m))}{m}
\end{aligned}$$

$$\begin{aligned}
A_L &= \frac{1}{P-r+1} \sum_{\underline{J}}^{\bar{J}} \mathbf{L}_{\alpha_i}(J)' \Lambda_{\alpha_i}^{-1} \mathbf{L}_{\alpha_i}(J) \\
&\xrightarrow{d} \int_{\underline{s}}^{\bar{s}} \frac{(\mathbf{W}(s) - \mathbf{W}(s-m))' (\mathbf{W}(s) - \mathbf{W}(s-m))}{m}
\end{aligned}$$

Appendix B

Description of the Parametric Bootstrap

Suppose that we are interesting in calculating the size of the test. Consider a DGP, e.g, $y_t = \alpha + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sigma \varepsilon_t$ with population parameters $\alpha = 1.5$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $\sigma = 1$ and $\varepsilon_t \rightarrow N(0, 1)$. The information on x_t is predetermined, so it is known in advance. Assume a sample size, e.g., $T = 750$, that is split as $R = 450$ (estimation sample) and $P = 300$ (prediction sample), and $r = 100$ ($m = 1/3$) is the rolling sample over the prediction sample. We run 1,000 Monte Carlo replications, and 500 bootstrap samples within each Monte Carlo replication. The Monte Carlo/bootstrap procedure consists of the following step:

1. One Monte Carlo replication: draw from the conditional density of ε_t , e.g. from $N(0, 1)$. With the population parameters inserted into the DGP, generate one Monte Carlo sample of observations $\{y_t\}$.
2. With the first replication of observations $\{y_t\}$, estimate the model parameters, e.g., $\hat{\theta} = (\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2, \hat{\sigma})$. Obtain the sequence of one-step-ahead density forecasts, i.e. $f_t(y_t | \Omega_{t-1}, \hat{\theta})$ and the corresponding PITs, i.e. $u_t = \int_{-\infty}^{y_t} f_t(\nu_t | \Omega_{t-1}, \hat{\theta}) d\nu_t$ for the first subsample of size 100 in the prediction sample. With these PITs, construct the tests $|z_1|$, C_1 , and L_1 . By rolling one observation at the time over the prediction sample, we have now a second subsample of size 100 and we construct again the one-step-ahead density forecasts, the corresponding PITs, and tests $|z_2|$, C_2 , and L_2 . If we keep rolling the observations, for $r = 100$ over $P = 300$, we will have 201 statistics, that is, $\{|z_j|\}_{j=1}^{201}$, $\{C_j\}_{j=1}^{201}$, and $\{L_j\}_{j=1}^{201}$. Over these collections of tests, we calculate the supremum (S) and the average (A) to obtain the following six statistics $S_{|z|}$, S_C , S_L and $A_{|z|}$, A_C , A_L .

In order to take care of parameter uncertainty, we implement a bootstrap procedure within each of the Monte Carlo replications. We draw 500 bootstrap samples as follows:

3. We fixed the parameters to the estimated values in step 2, e.g., $\hat{\theta} = (\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2, \hat{\sigma})$ and draw from

the conditional density of ε_t , e.g. from $N(0, 1)$ to generate bootstrap samples of $\{y_t^b\}$. For each bootstrap sample, we proceed to estimate the parameters again $\hat{\theta}^b = (\hat{\alpha}^b, \hat{\beta}_1^b, \hat{\beta}_2^b, \hat{\sigma}^b)$ and, as in step 2, we construct density forecasts, PITs, and the collection of tests $\{|z_j|^b\}_{j=1}^{201}$, $\{C_j^b\}_{j=1}^{201}$, and $\{L_j^b\}_{j=1}^{201}$ to finally obtain the statistics $S_{|z|}^b, S_C^b, S_L^b$ and $A_{|z|}^b, A_C^b, A_L^b$. We repeat 500 times and we obtain the empirical distribution (quantiles) of each of the six statistics. For instance, we could compute the bootstrap critical value or 95% percentile for each test over the 500 samples. Suppose that we call the 95% percentile $S_{|z|}^{b(95\%)}$ for the tests $\{S_{|z|}^b\}_{b=1}^{500}$, and similarly for the other statistics.

4. Given the empirical distribution (over the 500 bootstrap samples) of each statistic, we bring the values of the tests computed in step 2, i.e. $S_{|z|}, S_C, S_L$ and $A_{|z|}, A_C, A_L$, and compare them with the bootstrapped critical values obtained in step 3. For instance, if $S_{|z|}^{b(95\%)} \leq S_{|z|}$, we count it as one, otherwise as a zero.

Observe that steps 2, 3, and 4 are carried out for one Monte Carlo replication. Thus,

5. Repeat steps 1, 2, 3, and 4 for 1000 Monte Carlo replications and record the number of ones (step 4) across the 1000 replications. The size of the test will be the percentage number of ones over 1000.

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Tables: Asymptotic Distributions

In the following six tables, the percentiles are obtained from 2000 replications with a sample size of 20,000 observations in each replication.

Asymptotic Distribution of $S_{ z }$ Statistic									
<i>m</i>									
Percentile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
99%	3.950	3.713	3.510	3.396	3.428	3.213	3.089	3.112	2.951
95%	3.502	3.267	3.066	2.926	2.866	2.679	2.537	2.439	2.406
90%	3.292	2.987	2.845	2.642	2.571	2.361	2.240	2.121	2.015
80%	3.020	2.684	2.522	2.372	2.217	2.053	1.935	1.849	1.655
70%	2.843	2.489	2.315	2.156	1.999	1.834	1.697	1.582	1.392
60%	2.702	2.343	2.145	1.983	1.819	1.653	1.529	1.383	1.197
50%	2.569	2.203	2.015	1.830	1.664	1.515	1.365	1.221	1.021
40%	2.457	2.090	1.884	1.684	1.515	1.373	1.212	1.066	0.867
30%	2.339	1.975	1.747	1.540	1.379	1.228	1.071	0.915	0.730
20%	2.201	1.822	1.586	1.384	1.226	1.074	0.923	0.791	0.606
10%	2.033	1.629	1.420	1.199	1.055	0.901	0.761	0.631	0.479
5%	1.922	1.492	1.290	1.065	0.943	0.813	0.663	0.535	0.394
1%	1.730	1.265	1.072	0.829	0.739	0.631	0.498	0.409	0.287

Asymptotic Distribution of $A_{ z }$ Statistic									
<i>m</i>									
Percentile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
99%	1.241	1.435	1.656	1.824	2.204	2.181	2.360	2.542	2.583
95%	1.078	1.199	1.355	1.487	1.697	1.694	1.774	1.854	1.998
90%	1.004	1.088	1.206	1.300	1.418	1.406	1.490	1.586	1.666
80%	0.918	0.970	1.035	1.101	1.115	1.128	1.183	1.264	1.256
70%	0.870	0.891	0.922	0.948	0.939	0.928	0.966	1.017	1.019
60%	0.825	0.825	0.827	0.837	0.818	0.796	0.807	0.824	0.825
50%	0.789	0.760	0.760	0.741	0.710	0.685	0.682	0.675	0.661
40%	0.752	0.706	0.690	0.653	0.615	0.581	0.551	0.534	0.507
30%	0.711	0.649	0.617	0.568	0.524	0.487	0.453	0.417	0.378
20%	0.673	0.589	0.544	0.497	0.451	0.409	0.358	0.320	0.274
10%	0.608	0.517	0.465	0.409	0.359	0.319	0.269	0.234	0.186
5%	0.567	0.458	0.413	0.342	0.310	0.260	0.219	0.188	0.140
1%	0.478	0.381	0.325	0.251	0.235	0.200	0.152	0.135	0.096

Table 1: Asymptotic Distributions of $S_{|z|}$ and $A_{|z|}$ Statistics

Asymptotic Distribution of S_C Statistic									
m									
Percentile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
99%	42.488	39.592	38.534	37.694	37.361	35.925	36.051	34.591	31.804
95%	37.159	34.956	32.983	32.277	30.902	29.597	29.008	27.773	25.962
90%	35.155	32.562	30.722	29.378	28.069	27.176	26.181	24.685	23.356
80%	32.624	29.964	28.071	26.673	25.329	24.182	22.935	21.621	20.175
70%	30.932	28.330	26.359	24.906	23.421	22.132	20.932	19.423	18.125
60%	29.689	26.991	24.908	23.416	21.858	20.579	19.179	17.901	16.550
50%	28.540	25.798	23.648	22.111	20.626	19.136	17.708	16.501	15.241
40%	27.482	24.522	22.468	20.900	19.254	17.672	16.440	15.111	13.829
30%	26.508	23.385	21.197	19.510	17.873	16.413	15.058	13.685	12.477
20%	25.351	22.174	19.836	18.193	16.514	14.927	13.582	12.250	10.898
10%	23.731	20.400	18.067	16.333	14.568	13.061	11.741	10.480	8.987
5%	22.376	18.902	16.821	14.879	13.338	11.876	10.383	9.037	7.852
1%	20.394	16.833	14.737	12.807	11.208	9.884	8.275	6.929	5.905

Asymptotic Distribution of A_C Statistic									
m									
Percentile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
99%	16.342	18.610	20.265	22.301	24.658	26.534	27.422	28.202	27.987
95%	15.298	16.576	17.794	18.927	19.924	20.838	21.866	22.084	22.533
90%	14.762	15.734	16.456	17.287	18.022	18.629	18.996	19.612	19.965
80%	14.099	14.648	15.143	15.477	16.014	16.449	16.761	16.975	17.096
70%	13.672	14.029	14.209	14.439	14.705	14.937	15.107	15.111	15.290
60%	13.295	13.393	13.478	13.566	13.597	13.552	13.629	13.790	13.772
50%	12.955	12.873	12.847	12.820	12.666	12.591	12.534	12.514	12.547
40%	12.623	12.388	12.178	12.014	11.722	11.471	11.290	11.362	11.311
30%	12.262	11.851	11.535	11.227	10.909	10.576	10.364	10.166	10.073
20%	11.828	11.264	10.837	10.423	9.991	9.555	9.288	8.965	8.786
10%	11.298	10.483	9.871	9.330	8.746	8.297	7.860	7.558	7.143
5%	10.840	9.895	9.120	8.436	7.849	7.358	6.750	6.380	5.967
1%	10.030	8.791	7.993	7.279	6.363	5.659	5.069	4.629	4.382

Table 2: Asymptotic Distribution of S_C and A_C Statistics (13 autocontours)

Asymptotic Distribution of S_L Statistic									
m									
percentile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
99%	26.825	25.069	23.492	23.010	22.057	20.621	19.801	19.370	17.756
95%	22.719	21.121	19.941	18.372	17.199	16.042	15.645	14.470	13.631
90%	20.867	19.109	17.489	16.318	15.203	14.241	13.301	12.552	11.636
80%	18.859	16.979	15.273	14.045	12.859	11.973	11.213	10.267	9.325
70%	17.452	15.505	13.917	12.671	11.665	10.585	9.708	8.788	7.949
60%	16.389	14.279	12.920	11.587	10.492	9.537	8.610	7.765	6.927
50%	14.810	12.425	11.052	9.796	8.827	7.916	6.830	6.071	5.192
40%	15.534	13.300	11.910	10.616	9.576	8.678	7.698	6.917	6.068
30%	13.970	11.675	10.164	8.891	7.999	6.948	6.048	5.213	4.423
20%	13.066	10.697	9.249	8.023	7.039	6.054	5.113	4.388	3.646
10%	11.896	9.535	8.158	7.011	5.863	4.966	4.213	3.484	2.743
5%	11.145	8.544	7.070	6.093	5.099	4.286	3.492	2.786	2.187
1%	9.583	7.267	5.857	4.821	3.950	3.173	2.585	1.907	1.510

Asymptotic Distribution of A_L Statistic									
m									
percentile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
99%	7.260	8.419	9.620	10.837	12.021	12.943	13.509	14.458	14.629
95%	6.507	7.336	7.948	8.396	9.045	9.636	10.116	10.569	10.913
90%	6.122	6.673	7.034	7.503	7.961	8.313	8.562	8.929	9.220
80%	5.705	6.038	6.259	6.452	6.643	6.822	6.995	7.104	7.309
70%	5.383	5.583	5.669	5.733	5.820	5.868	5.929	5.977	6.040
60%	5.164	5.208	5.200	5.198	5.212	5.158	5.191	5.122	5.128
50%	4.742	4.604	4.468	4.361	4.224	4.021	3.885	3.817	3.767
40%	4.951	4.890	4.831	4.770	4.663	4.570	4.506	4.452	4.438
30%	4.535	4.316	4.123	3.922	3.717	3.476	3.324	3.174	3.088
20%	4.294	3.988	3.712	3.418	3.174	2.973	2.715	2.584	2.480
10%	4.017	3.539	3.189	2.922	2.602	2.320	2.086	1.927	1.746
5%	3.752	3.237	2.867	2.508	2.246	1.921	1.695	1.459	1.292
1%	3.337	2.681	2.222	1.875	1.621	1.388	1.150	0.960	0.777

Table 3: Asymptotic Distribution of S_L and A_L Statistics (five lags)

Tables: Size and Power of the Tests

I. Small Sample (T) and Small Subsample Window (m)

T=150, R=90,P=60	$S_{ z }^{1,1}$	$S_{ z }^{1,2}$	$S_{ z }^{1,3}$	$S_{ z }^{1,4}$	$S_{ z }^{1,5}$	$S_{ z }^{1,6}$	$S_{ z }^{1,7}$	$S_{ z }^{1,8}$	$S_{ z }^{1,9}$	$S_{ z }^{1,10}$	$S_{ z }^{1,11}$	$S_{ z }^{1,12}$	$S_{ z }^{1,13}$
fixed	0.042	0.037	0.033	0.046	0.049	0.037	0.04	0.051	0.045	0.045	0.048	0.036	0.039
rolling	0.028	0.04	0.041	0.041	0.038	0.042	0.04	0.04	0.041	0.046	0.044	0.036	0.038
recursive	0.029	0.041	0.042	0.04	0.039	0.039	0.038	0.039	0.044	0.042	0.039	0.035	0.032
	$A_{ z }^{1,1}$	$A_{ z }^{1,2}$	$A_{ z }^{1,3}$	$A_{ z }^{1,4}$	$A_{ z }^{1,5}$	$A_{ z }^{1,6}$	$A_{ z }^{1,7}$	$A_{ z }^{1,8}$	$A_{ z }^{1,9}$	$A_{ z }^{1,10}$	$A_{ z }^{1,11}$	$A_{ z }^{1,12}$	$A_{ z }^{1,13}$
fixed	0.047	0.048	0.049	0.056	0.055	0.058	0.049	0.059	0.049	0.057	0.052	0.055	0.057
rolling	0.032	0.038	0.048	0.043	0.041	0.046	0.046	0.045	0.04	0.046	0.048	0.046	0.031
recursive	0.024	0.037	0.037	0.043	0.043	0.038	0.043	0.052	0.043	0.044	0.042	0.035	0.033
	$S_C^{1,13}$	$A_C^{1,13}$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$			
fixed	0.059	0.051	0.06	0.06	0.052	0.045	0.063	0.063	0.058	0.055			
rolling	0.038	0.043	0.045	0.052	0.042	0.042	0.051	0.05	0.045	0.045			
recursive	0.033	0.043	0.044	0.055	0.053	0.052	0.052	0.058	0.039	0.033			
	$S_{ z }^{1,7}$	$S_{ z }^{2,7}$	$S_{ z }^{3,7}$	$S_{ z }^{4,7}$	$S_{ z }^{5,7}$	$A_{ z }^{1,7}$	$A_{ z }^{2,7}$	$A_{ z }^{3,7}$	$A_{ z }^{4,7}$	$A_{ z }^{5,7}$			
fixed	0.04	0.038	0.042	0.04	0.038	0.049	0.053	0.054	0.051	0.039			
rolling	0.04	0.033	0.032	0.033	0.034	0.036	0.038	0.039	0.04	0.032			
recursive	0.038	0.037	0.031	0.035	0.034	0.043	0.039	0.04	0.05	0.037			

Notation: 13 autocontours $\mathcal{C} = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

In $S_{|z|}^{k,\alpha_i}$ and $A_{|z|}^{k,\alpha_i}$, lag k and autocontour α_i are fixed.

In $S_C^{k,\alpha}$ and $A_C^{k,\alpha}$, k is fixed and α is the total number of autocontours considered.

In S_L^{k,α_i} and A_L^{k,α_i} , up to k lags are considered and α_i is a fixed autocontour.

Table 4: **Size of the statistics:T=150 R=90 P=T-R=60 $m = 1/3$ (nominal size 5%)**

II. Large Sample (T) and Large Subsample Window (m)

T=750, R=450, P=300	$S_{ z }^{1,1}$	$S_{ z }^{1,2}$	$S_{ z }^{1,3}$	$S_{ z }^{1,4}$	$S_{ z }^{1,5}$	$S_{ z }^{1,6}$	$S_{ z }^{1,7}$	$S_{ z }^{1,8}$	$S_{ z }^{1,9}$	$S_{ z }^{1,10}$	$S_{ z }^{1,11}$	$S_{ z }^{1,12}$	$S_{ z }^{1,13}$
fixed	0.048	0.040	0.044	0.049	0.046	0.051	0.052	0.048	0.052	0.04	0.042	0.05	0.032
rolling	0.049	0.041	0.056	0.051	0.048	0.041	0.041	0.045	0.057	0.056	0.049	0.038	0.039
recursive	0.046	0.048	0.045	0.044	0.051	0.048	0.044	0.045	0.047	0.057	0.058	0.039	0.04
	$A_{ z }^{1,1}$	$A_{ z }^{1,2}$	$A_{ z }^{1,3}$	$A_{ z }^{1,4}$	$A_{ z }^{1,5}$	$A_{ z }^{1,6}$	$A_{ z }^{1,7}$	$A_{ z }^{1,8}$	$A_{ z }^{1,9}$	$A_{ z }^{1,10}$	$A_{ z }^{1,11}$	$A_{ z }^{1,12}$	$A_{ z }^{1,13}$
fixed	0.049	0.045	0.049	0.051	0.046	0.049	0.051	0.043	0.049	0.052	0.043	0.047	0.049
rolling	0.048	0.042	0.046	0.055	0.044	0.047	0.043	0.044	0.044	0.043	0.049	0.044	0.048
recursive	0.04	0.043	0.041	0.063	0.065	0.045	0.053	0.045	0.044	0.051	0.042	0.044	0.043
	$S_C^{1,13}$	$A_C^{1,13}$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$			
fixed	0.045	0.049	0.047	0.044	0.053	0.05	0.049	0.042	0.046	0.051			
rolling	0.045	0.044	0.043	0.064	0.063	0.044	0.048	0.041	0.044	0.05			
recursive	0.043	0.047	0.045	0.057	0.052	0.048	0.048	0.048	0.051	0.048			
	$S_{ z }^{1,7}$	$S_{ z }^{2,7}$	$S_{ z }^{3,7}$	$S_{ z }^{4,7}$	$S_{ z }^{5,7}$	$A_{ z }^{1,7}$	$A_{ z }^{2,7}$	$A_{ z }^{3,7}$	$A_{ z }^{4,7}$	$A_{ z }^{5,7}$			
fixed	0.052	0.053	0.042	0.042	0.043	0.051	0.048	0.042	0.054	0.049			
rolling	0.041	0.048	0.043	0.039	0.04	0.043	0.043	0.047	0.049	0.043			
recursive	0.044	0.051	0.041	0.038	0.041	0.053	0.054	0.043	0.045	0.048			

Notation: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

In $S_{|z|}^{k,\alpha_i}$ and $A_{|z|}^{k,\alpha_i}$, lag k and autocontour α_i are fixed.

In $S_C^{k,\alpha}$ and $A_C^{k,\alpha}$, k is fixed and α is the total number of autocontours considered.

In S_L^{k,α_i} and A_L^{k,α_i} , up to k lags are considered and α_i is a fixed autocontour.

Table 5: **Size of the statistics: T=750 R=450 P=T-R=300 $m = 2/3$ (nominal size 5%)**

III. Small Sample (T), Small Subsample Window (m), and $P/R = 1/4$

T=120, R=96, $P/R = 1/4$	$S_{ z }^{1,1}$	$S_{ z }^{1,2}$	$S_{ z }^{1,3}$	$S_{ z }^{1,4}$	$S_{ z }^{1,5}$	$S_{ z }^{1,6}$	$S_{ z }^{1,7}$	$S_{ z }^{1,8}$	$S_{ z }^{1,9}$	$S_{ z }^{1,10}$	$S_{ z }^{1,11}$	$S_{ z }^{1,12}$	$S_{ z }^{1,13}$
fixed	0.015	0.023	0.024	0.032	0.038	0.034	0.032	0.033	0.037	0.031	0.025	0.023	0.016
rolling	0.012	0.019	0.016	0.028	0.031	0.035	0.033	0.026	0.034	0.032	0.018	0.018	0.013
recursive	0.014	0.021	0.017	0.03	0.029	0.032	0.031	0.028	0.03	0.03	0.015	0.016	0.015
	$A_{ z }^{1,1}$	$A_{ z }^{1,2}$	$A_{ z }^{1,3}$	$A_{ z }^{1,4}$	$A_{ z }^{1,5}$	$A_{ z }^{1,6}$	$A_{ z }^{1,7}$	$A_{ z }^{1,8}$	$A_{ z }^{1,9}$	$A_{ z }^{1,10}$	$A_{ z }^{1,11}$	$A_{ z }^{1,12}$	$A_{ z }^{1,13}$
fixed	0.029	0.032	0.04	0.047	0.043	0.043	0.044	0.044	0.046	0.052	0.029	0.032	0.021
rolling	0.023	0.025	0.025	0.035	0.045	0.045	0.04	0.045	0.045	0.041	0.027	0.025	0.021
recursive	0.021	0.028	0.024	0.031	0.043	0.046	0.033	0.042	0.045	0.042	0.027	0.027	0.023
	$S_C^{1,13}$	$A_C^{1,13}$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$			
fixed	0.038	0.032	0.03	0.034	0.032	0.03	0.045	0.041	0.042	0.042			
rolling	0.032	0.033	0.032	0.037	0.035	0.036	0.044	0.042	0.051	0.046			
recursive	0.029	0.031	0.035	0.033	0.034	0.032	0.045	0.044	0.039	0.042			
	$S_{ z }^{1,7}$	$S_{ z }^{2,7}$	$S_{ z }^{3,7}$	$S_{ z }^{4,7}$	$S_{ z }^{5,7}$	$A_{ z }^{1,7}$	$A_{ z }^{2,7}$	$A_{ z }^{3,7}$	$A_{ z }^{4,7}$	$A_{ z }^{5,7}$			
fixed	0.032	0.033	0.037	0.032	0.034	0.044	0.043	0.044	0.042	0.039			
rolling	0.033	0.032	0.033	0.034	0.031	0.041	0.042	0.04	0.039	0.038			
recursive	0.031	0.03	0.029	0.032	0.029	0.033	0.032	0.031	0.034	0.031			

Notation: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

In $S_{|z|}^{k,\alpha_i}$ and $A_{|z|}^{k,\alpha_i}$, lag k and autocontour α_i are fixed.

In $S_C^{k,\alpha}$ and $A_C^{k,\alpha}$, k is fixed and α is the total number of autocontours considered.

In S_L^{k,α_i} and A_L^{k,α_i} , up to k lags are considered and α_i is a fixed autocontour.

Table 6: **Size of the statistics: T=120, R=96, P=24, $P/R = 1/4$ $m = 1/3$ (nominal size 5%)**

IV. Small Sample (T), Small Subsample Window (m), and $P/R = 1/2$

T=144, R=96, $P/R = 1/2$	$S_{ z }^{1,1}$	$S_{ z }^{1,2}$	$S_{ z }^{1,3}$	$S_{ z }^{1,4}$	$S_{ z }^{1,5}$	$S_{ z }^{1,6}$	$S_{ z }^{1,7}$	$S_{ z }^{1,8}$	$S_{ z }^{1,9}$	$S_{ z }^{1,10}$	$S_{ z }^{1,11}$	$S_{ z }^{1,12}$	$S_{ z }^{1,13}$
fixed	0.022	0.029	0.032	0.037	0.041	0.042	0.034	0.039	0.04	0.044	0.027	0.026	0.023
rolling	0.019	0.026	0.027	0.035	0.039	0.041	0.039	0.032	0.041	0.046	0.038	0.032	0.02
recursive	0.021	0.029	0.026	0.032	0.034	0.035	0.035	0.033	0.042	0.043	0.034	0.021	0.023
	$A_{ z }^{1,1}$	$A_{ z }^{1,2}$	$A_{ z }^{1,3}$	$A_{ z }^{1,4}$	$A_{ z }^{1,5}$	$A_{ z }^{1,6}$	$A_{ z }^{1,7}$	$A_{ z }^{1,8}$	$A_{ z }^{1,9}$	$A_{ z }^{1,10}$	$A_{ z }^{1,11}$	$A_{ z }^{1,12}$	$A_{ z }^{1,13}$
fixed	0.034	0.035	0.04	0.043	0.048	0.056	0.043	0.044	0.04	0.047	0.036	0.033	0.027
rolling	0.032	0.034	0.033	0.045	0.043	0.041	0.041	0.046	0.043	0.044	0.033	0.032	0.028
recursive	0.031	0.032	0.03	0.046	0.042	0.042	0.04	0.045	0.047	0.045	0.034	0.03	0.032
	$S_C^{1,13}$	$A_C^{1,13}$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$			
fixed	0.037	0.044	0.037	0.036	0.038	0.038	0.043	0.042	0.046	0.046			
rolling	0.034	0.042	0.039	0.037	0.037	0.036	0.041	0.045	0.042	0.044			
recursive	0.031	0.039	0.032	0.033	0.034	0.034	0.042	0.043	0.044	0.042			
	$S_{ z }^{1,7}$	$S_{ z }^{2,7}$	$S_{ z }^{3,7}$	$S_{ z }^{4,7}$	$S_{ z }^{5,7}$	$A_{ z }^{1,7}$	$A_{ z }^{2,7}$	$A_{ z }^{3,7}$	$A_{ z }^{4,7}$	$A_{ z }^{5,7}$			
fixed	0.035	0.04	0.038	0.039	0.038	0.043	0.05	0.044	0.049	0.043			
rolling	0.039	0.036	0.037	0.037	0.039	0.041	0.048	0.041	0.039	0.04			
recursive	0.035	0.039	0.033	0.038	0.036	0.04	0.041	0.039	0.042	0.039			

Notation: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

In $S_{|z|}^{k,\alpha_i}$ and $A_{|z|}^{k,\alpha_i}$, lag k and autocontour α_i are fixed.

In $S_C^{k,\alpha}$ and $A_C^{k,\alpha}$, k is fixed and α is the total number of autocontours considered.

In S_L^{k,α_i} and A_L^{k,α_i} , up to k lags are considered and α_i is a fixed autocontour.

Table 7: **Size of the statistics: T=144, R=96, P=48, $P/R = 1/2$ $m = 1/3$ (nominal size 5%)**

V. Small Sample (T), Small Subsample Window (m), and $P/R = 3/4$

T=168, R=96, $P/R = 3/4$	$S_{ z }^{1,1}$	$S_{ z }^{1,2}$	$S_{ z }^{1,3}$	$S_{ z }^{1,4}$	$S_{ z }^{1,5}$	$S_{ z }^{1,6}$	$S_{ z }^{1,7}$	$S_{ z }^{1,8}$	$S_{ z }^{1,9}$	$S_{ z }^{1,10}$	$S_{ z }^{1,11}$	$S_{ z }^{1,12}$	$S_{ z }^{1,13}$
fixed	0.032	0.041	0.04	0.042	0.043	0.042	0.041	0.046	0.046	0.046	0.043	0.032	0.039
rolling	0.031	0.04	0.044	0.04	0.04	0.046	0.046	0.049	0.051	0.046	0.051	0.031	0.038
recursive	0.029	0.042	0.043	0.053	0.052	0.049	0.043	0.043	0.042	0.042	0.039	0.036	0.035
	$A_{ z }^{1,1}$	$A_{ z }^{1,2}$	$A_{ z }^{1,3}$	$A_{ z }^{1,4}$	$A_{ z }^{1,5}$	$A_{ z }^{1,6}$	$A_{ z }^{1,7}$	$A_{ z }^{1,8}$	$A_{ z }^{1,9}$	$A_{ z }^{1,10}$	$A_{ z }^{1,11}$	$A_{ z }^{1,12}$	$A_{ z }^{1,13}$
fixed	0.042	0.046	0.047	0.043	0.042	0.043	0.041	0.48	0.052	0.045	0.041	0.038	0.038
rolling	0.037	0.039	0.04	0.045	0.045	0.047	0.046	0.047	0.051	0.042	0.043	0.037	0.039
recursive	0.038	0.042	0.037	0.048	0.049	0.047	0.042	0.045	0.049	0.44	0.039	0.036	0.038
	$S_C^{1,13}$	$A_C^{1,13}$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$			
fixed	0.04	0.044	0.043	0.044	0.043	0.045	0.046	0.046	0.045	0.047			
rolling	0.04	0.052	0.043	0.05	0.044	0.049	0.049	0.044	0.045	0.45			
recursive	0.042	0.052	0.046	0.043	0.042	0.04	0.043	0.045	0.044	0.045			
	$S_{ z }^{1,7}$	$S_{ z }^{2,7}$	$S_{ z }^{3,7}$	$S_{ z }^{4,7}$	$S_{ z }^{5,7}$	$A_{ z }^{1,7}$	$A_{ z }^{2,7}$	$A_{ z }^{3,7}$	$A_{ z }^{4,7}$	$A_{ z }^{5,7}$			
fixed	0.041	0.047	0.046	0.044	0.045	0.041	0.041	0.045	0.043	0.045			
rolling	0.046	0.04	0.042	0.043	0.044	0.046	0.048	0.046	0.045	0.043			
recursive	0.043	0.041	0.042	0.046	0.044	0.042	0.043	0.045	0.044	0.042			

Notation: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

In $S_{|z|}^{k,\alpha_i}$ and $A_{|z|}^{k,\alpha_i}$, lag k and autocontour α_i are fixed.

In $S_C^{k,\alpha}$ and $A_C^{k,\alpha}$, k is fixed and α is the total number of autocontours considered.

In S_L^{k,α_i} and A_L^{k,α_i} , up to k lags are considered and α_i is a fixed autocontour.

Table 8: **Size of the statistics: T=168, R=96, P=72, $P/R = 3/4$ $m = 1/3$ (nominal size 5%)**

Fixed Scheme	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.14	0.45	0.765	0.956	0.979	0.987	0.995	0.992	0.987	0.976	0.938	0.867	0.589
$l = 2$	0.14	0.464	0.789	0.955	0.981	0.99	0.993	0.993	0.988	0.974	0.943	0.864	0.593
$l = 3$	0.14	0.419	0.792	0.965	0.985	0.997	0.997	0.994	0.985	0.971	0.937	0.871	0.594
$l = 4$	0.11	0.439	0.797	0.953	0.986	0.993	0.994	0.994	0.99	0.979	0.938	0.866	0.595
$l = 5$	0.12	0.453	0.796	0.952	0.984	0.992	0.994	0.993	0.989	0.971	0.938	0.867	0.598
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.08	0.244	0.698	0.898	0.948	0.969	0.981	0.978	0.968	0.945	0.904	0.812	0.588
$l = 2$	0.09	0.277	0.741	0.896	0.952	0.965	0.979	0.983	0.972	0.947	0.898	0.815	0.585
$l = 3$	0.07	0.276	0.72	0.893	0.941	0.969	0.98	0.984	0.967	0.943	0.906	0.815	0.588
$l = 4$	0.07	0.319	0.702	0.887	0.957	0.971	0.979	0.981	0.971	0.944	0.904	0.817	0.586
$l = 5$	0.08	0.339	0.722	0.906	0.951	0.968	0.985	0.976	0.972	0.944	0.903	0.816	0.584
	$S_C^{l,13}$	$A_C^{l,13}$											
$l = 1$	0.961	0.921											
$l = 2$	0.965	0.924											
$l = 3$	0.967	0.923											
$l = 4$	0.963	0.921											
$l = 5$	0.965	0.921											
$C = 7$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
	0.989	0.985	0.976	0.964	0.98	0.969	0.956	0.942					

Rolling Scheme	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.18	0.12	0.308	0.416	0.459	0.394	0.42	0.264	0.21	0.08	0.119	0.092	0.08
$l = 2$	0.11	0.11	0.295	0.324	0.323	0.431	0.433	0.333	0.193	0.084	0.123	0.093	0.08
$l = 3$	0.11	0.11	0.258	0.402	0.386	0.39	0.472	0.325	0.207	0.096	0.123	0.094	0.09
$l = 4$	0.13	0.11	0.289	0.439	0.41	0.413	0.378	0.325	0.173	0.093	0.121	0.092	0.082
$l = 5$	0.16	0.11	0.262	0.445	0.418	0.423	0.438	0.318	0.171	0.091	0.121	0.092	0.083
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.25	0.318	0.559	0.783	0.738	0.710	0.534	0.634	0.418	0.18	0.21	0.102	0.091
$l = 2$	0.22	0.292	0.527	0.733	0.71	0.714	0.56	0.621	0.438	0.17	0.216	0.100	0.092
$l = 3$	0.24	0.275	0.521	0.768	0.756	0.744	0.499	0.638	0.442	0.155	0.224	0.103	0.092
$l = 4$	0.25	0.28	0.519	0.806	0.775	0.759	0.611	0.638	0.421	0.133	0.21	0.101	0.092
$l = 5$	0.25	0.305	0.516	0.797	0.781	0.769	0.533	0.626	0.392	0.121	0.24	0.101	0.093
	$S_C^{l,13}$	$A_C^{l,13}$											
$l = 1$	0.638	0.749											
$l = 2$	0.521	0.733											
$l = 3$	0.619	0.801											
$l = 4$	0.641	0.822											
$l = 5$	0.637	0.803											
$C = 7$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
	0.414	0.419	0.408	0.432	0.563	0.59	0.578	0.532					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

$S_{|z|}^{l,7}, A_{|z|}^{l,7}$ for $l = 1, 2, \dots, 5$; 7 refers to the 50% autocontour.

$S_L^{l,7}, A_L^{l,7}$ stacking lags up to $l = 2, \dots, 5$ and considering the 50% autocontour.

$S_C^{l,13}$ and $A_C^{l,13}$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.

1000 Monte Carlo replications and 500 bootstrap samples.

T=650, R=350, P=300, $m = 1/3$, and break point at $R + \tau P$ for $\tau = 1/3$

Table 9: Power for DGP1 under Fixed and Rolling Schemes

Fixed Scheme	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.292	0.271	0.191	0.084	0.073	0.109	0.281	0.278	0.391	0.512	0.638	0.638	0.518
$l = 2$	0.26	0.239	0.171	0.079	0.075	0.106	0.282	0.292	0.418	0.518	0.637	0.639	0.52
$l = 3$	0.285	0.265	0.175	0.089	0.075	0.12	0.283	0.29	0.404	0.53	0.635	0.639	0.523
$l = 4$	0.311	0.26	0.162	0.088	0.063	0.096	0.282	0.284	0.399	0.512	0.624	0.636	0.526
$l = 5$	0.273	0.253	0.158	0.068	0.068	0.108	0.282	0.288	0.408	0.539	0.645	0.638	0.531
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.345	0.242	0.162	0.09	0.078	0.127	0.275	0.271	0.361	0.472	0.561	0.604	0.527
$l = 2$	0.327	0.225	0.16	0.081	0.077	0.12	0.269	0.277	0.377	0.467	0.56	0.606	0.522
$l = 3$	0.347	0.239	0.167	0.089	0.087	0.123	0.272	0.286	0.374	0.468	0.558	0.6	0.527
$l = 4$	0.368	0.246	0.159	0.09	0.069	0.111	0.272	0.277	0.345	0.455	0.55	0.592	0.523
$l = 5$	0.317	0.222	0.146	0.086	0.072	0.12	0.273	0.288	0.37	0.469	0.564	0.6	0.523
	$S_C^{l,13}$	$A_C^{l,13}$											
$l = 1$	0.659	0.595											
$l = 2$	0.651	0.604											
$l = 3$	0.666	0.625											
$l = 4$	0.644	0.625											
$l = 5$	0.652	0.611											
$C = 7$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
	0.286	0.285	0.281	0.285	0.271	0.273	0.276	0.275					
Rolling Scheme	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.116	0.104	0.088	0.063	0.05	0.056	0.112	0.11	0.171	0.341	0.302	0.264	0.261
$l = 2$	0.114	0.09	0.086	0.063	0.054	0.051	0.118	0.113	0.172	0.318	0.296	0.264	0.262
$l = 3$	0.107	0.094	0.076	0.065	0.06	0.053	0.108	0.126	0.189	0.335	0.28	0.273	0.266
$l = 4$	0.113	0.104	0.072	0.054	0.058	0.049	0.175	0.114	0.167	0.329	0.287	0.266	0.267
$l = 5$	0.109	0.093	0.08	0.053	0.05	0.053	0.16	0.121	0.184	0.337	0.274	0.265	0.267
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.179	0.129	0.099	0.078	0.066	0.075	0.19	0.151	0.204	0.36	0.344	0.35	0.378
$l = 2$	0.169	0.112	0.082	0.075	0.066	0.076	0.18	0.145	0.205	0.386	0.345	0.343	0.379
$l = 3$	0.175	0.139	0.092	0.07	0.067	0.077	0.196	0.159	0.228	0.378	0.352	0.348	0.378
$l = 4$	0.183	0.143	0.083	0.063	0.066	0.077	0.192	0.144	0.201	0.358	0.342	0.352	0.375
$l = 5$	0.159	0.129	0.09	0.063	0.066	0.065	0.191	0.151	0.208	0.371	0.347	0.35	0.377
	$S_C^{l,13}$	$A_C^{l,13}$											
$l = 1$	0.409	0.414											
$l = 2$	0.363	0.394											
$l = 3$	0.381	0.427											
$l = 4$	0.398	0.431											
$l = 5$	0.306	0.318											
$C = 7$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
	0.168	0.115	0.113	0.112	0.129	0.189	0.191	0.182					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

$S_{|z|}^{l,7}, A_{|z|}^{l,7}$ for $l = 1, 2, \dots, 5$; 7 refers to the 50% autocontour.

$S_L^{l,7}, A_L^{l,7}$ stacking lags up to $l = 2, \dots, 5$ and considering the 50% autocontour.

$S_C^{l,13}$ and $A_C^{l,13}$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.

1000 Monte Carlo replications and 500 bootstrap samples.

T=650, R=350, P=300, $m = 1/3$, and break point at $R + \tau P$ for $\tau = 1/3$

Table 10: Power for DGP2 under Fixed and Rolling Schemes

Fixed Scheme	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.95	0.998	0.999	0.999	0.999	1	0.97	0.985	0.961	0.849	0.449	0.33	0.29
$l = 2$	0.96	0.998	0.999	1	0.999	0.997	0.982	0.981	0.96	0.859	0.444	0.16	0.19
$l = 3$	0.97	0.997	0.998	1	0.999	0.998	0.979	0.982	0.956	0.85	0.436	0.35	0.31
$l = 4$	0.96	0.997	0.998	1	0.998	0.998	0.98	0.984	0.965	0.853	0.446	0.36	0.13
$l = 5$	0.96	0.997	1	0.999	1	0.999	0.982	0.983	0.96	0.854	0.447	0.2	0.23
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.93	0.951	1	0.999	0.999	0.995	0.95	0.973	0.914	0.475	0.38	0.25	0.16
$l = 2$	0.931	0.967	0.998	1	0.999	0.994	0.952	0.974	0.896	0.518	0.34	0.22	0.17
$l = 3$	0.937	0.98	0.998	0.999	0.997	0.995	0.95	0.973	0.883	0.555	0.36	0.24	0.19
$l = 4$	0.935	0.96	1	0.998	0.997	0.994	0.95	0.968	0.908	0.487	0.31	0.32	0.22
$l = 5$	0.939	0.95	0.999	0.998	0.997	0.995	0.95	0.962	0.875	0.545	0.31	0.12	0.23
	$S_C^{l,13}$	$A_C^{l,13}$											
$l = 1$	1	0.97											
$l = 2$	1	0.98											
$l = 3$	0.999	0.97											
$l = 4$	1	0.999											
$l = 5$	1	1											
$C = 7$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
	0.98	0.98	0.98	0.98	0.97	0.969	0.97	0.97					

Rolling Scheme	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.46	0.461	0.395	0.184	0.26	0.329	0.396	0.457	0.401	0.134	0.212	0.092	0.08
$l = 2$	0.358	0.32	0.338	0.381	0.421	0.475	0.511	0.508	0.39	0.121	0.241	0.093	0.08
$l = 3$	0.437	0.398	0.389	0.442	0.465	0.524	0.524	0.518	0.385	0.099	0.223	0.094	0.09
$l = 4$	0.445	0.425	0.444	0.481	0.502	0.544	0.54	0.512	0.345	0.095	0.232	0.092	0.082
$l = 5$	0.448	0.409	0.425	0.468	0.511	0.54	0.546	0.506	0.361	0.075	0.232	0.092	0.083
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.189	0.219	0.332	0.473	0.584	0.678	0.726	0.739	0.706	0.395	0.163	0.082	0.06
$l = 2$	0.585	0.628	0.686	0.788	0.808	0.834	0.826	0.815	0.716	0.356	0.171	0.081	0.06
$l = 3$	0.68	0.711	0.728	0.796	0.832	0.839	0.832	0.811	0.7	0.334	0.163	0.081	0.07
$l = 4$	0.684	0.733	0.765	0.832	0.851	0.861	0.846	0.815	0.698	0.308	0.162	0.081	0.062
$l = 5$	0.674	0.717	0.762	0.82	0.846	0.86	0.848	0.814	0.669	0.291	0.161	0.081	0.063
	$S_C^{l,13}$	$A_C^{l,13}$											
$l = 1$	0.57	0.817											
$l = 2$	0.539	0.845											
$l = 3$	0.53	0.814											
$l = 4$	0.532	0.78											
$l = 5$	0.534	0.782											
	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
	0.477	0.496	0.48	0.421	0.728	0.75	0.749	0.703					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

$S_{|z|}^{l,7}, A_{|z|}^{l,7}$ for $l = 1, 2, \dots, 5$; 7 refers to the 50% autocontour.

$S_L^{l,7}, A_L^{l,7}$ stacking lags up to $l = 2, \dots, 5$ and considering the 50% autocontour.

$S_C^{l,13}$ and $A_C^{l,13}$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.

1000 Monte Carlo replications and 500 bootstrap samples.

T=650, R=350, P=300, $m = 1/3$, and break point at $R + \tau P$ for $\tau = 1/3$

Table 11: Power for DGP3 under Fixed and Rolling Schemes

Fixed Scheme	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	1	1	0.999	1	1	1	0.999	0.999	0.989	0.984	0.969	0.93	0.79
$l = 2$	0.999	1	0.999	0.996	1	1	1	1	0.98	0.983	0.967	0.901	0.78
$l = 3$	0.998	1	1	1	1	0.999	1	1	0.996	0.98	0.965	0.912	0.781
$l = 4$	1	0.999	1	1	0.998	1	0.999	1	0.99	0.98	0.962	0.923	0.744
$l = 5$	1	0.996	1	1	1	1	1	0.998	1	0.98	0.966	0.917	0.775
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.989	1	0.995	1	1	1	0.98	0.998	0.989	0.96	0.914	0.812	0.639
$l = 2$	0.989	1	0.995	0.995	0.997	1	0.988	0.989	0.986	0.967	0.934	0.815	0.633
$l = 3$	0.99	1	1	1	1	0.997	0.985	0.99	0.986	0.966	0.914	0.815	0.622
$l = 4$	1	0.996	1	1	0.996	0..	0.989	0.991	0.986	0.967	0.954	0.817	0.644
$l = 5$	1	0.996	1	1	1	1	0.989	0.996	0.986	0.97	0.914	0.816	0.675
	$S_C^{l,13}$	$A_C^{l,13}$											
$l = 1$	1	1											
$l = 2$	1	1											
$l = 3$	1	1											
$l = 4$	1	1											
$l = 5$	1	1											
$C = 7$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
	1	1	1	1	1	1	1	1					

Rolling Scheme	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.569	0.461	0.486	0.589	0.696	0.746	0.76	0.729	0.69	0.643	0.481	0.331	0.123
$l = 2$	0.44	0.387	0.358	0.577	0.664	0.73	0.744	0.733	0.703	0.629	0.464	0.332	0.12
$l = 3$	0.541	0.465	0.421	0.528	0.649	0.695	0.705	0.71	0.691	0.624	0.445	0.332	0.127
$l = 4$	0.586	0.514	0.48	0.525	0.639	0.698	0.703	0.694	0.663	0.618	0.447	0.319	0.131
$l = 5$	0.571	0.495	0.484	0.525	0.643	0.724	0.722	0.723	0.678	0.627	0.448	0.318	0.135
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.25	0.318	0.559	0.761	0.831	0.861	0.858	0.839	0.819	0.763	0.606	0.461	0.236
$l = 2$	0.22	0.292	0.527	0.739	0.794	0.835	0.857	0.852	0.82	0.767	0.601	0.46	0.242
$l = 3$	0.24	0.275	0.521	0.705	0.789	0.835	0.85	0.854	0.819	0.767	0.609	0.45	0.237
$l = 4$	0.25	0.28	0.519	0.699	0.795	0.833	0.86	0.842	0.8	0.736	0.606	0.443	0.24
$l = 5$	0.25	0.305	0.516	0.704	0.789	0.837	0.846	0.848	0.823	0.75	0.588	0.446	0.242
	$S_C^{l,13}$	$A_C^{l,13}$											
$l = 1$	0.75	0.85											
$l = 2$	0.74	0.85											
$l = 3$	0.76	0.87											
$l = 4$	0.76	0.83											
$l = 5$	0.771	0.81											
$C = 7$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
	0.656	0.558	0.558	0.56	0.801	0.724	0.656	0.605					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

$S_{|z|}^{l,7}, A_{|z|}^{l,7}$ for $l = 1, 2, \dots, 5$; 7 refers to the 50% autocontour.

$S_L^{l,7}, A_L^{l,7}$ stacking lags up to $l = 2, \dots, 5$ and considering the 50% autocontour.

$S_C^{l,13}$ and $A_C^{l,13}$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.

1000 Monte Carlo replications and 500 bootstrap samples.

T=650, R=350, P=300, $m = 1/3$, and break point at $R + \tau P$ for $\tau = 1/3$

Table 12: Power for DGP4 under Fixed and Rolling Schemes

Rolling Scheme $\tau = 1/2$	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.326	0.322	0.315	0.132	0.198	0.278	0.352	0.425	0.368	0.109	0.183	0.081	0.069
$l = 2$	0.305	0.246	0.286	0.304	0.294	0.413	0.442	0.456	0.348	0.113	0.193	0.079	0.072
$l = 3$	0.318	0.322	0.301	0.365	0.348	0.401	0.463	0.427	0.372	0.078	0.191	0.083	0.079
$l = 4$	0.322	0.316	0.325	0.392	0.456	0.487	0.459	0.464	0.301	0.083	0.192	0.087	0.069
$l = 5$	0.341	0.301	0.297	0.387	0.432	0.445	0.476	0.447	0.351	0.069	0.177	0.072	0.069
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.143	0.187	0.216	0.342	0.403	0.503	0.631	0.645	0.614	0.314	0.133	0.064	0.052
$l = 2$	0.273	0.613	0.632	0.723	0.765	0.728	0.668	0.727	0.625	0.322	0.124	0.062	0.046
$l = 3$	0.404	0.647	0.687	0.745	0.794	0.716	0.589	0.724	0.653	0.301	0.116	0.067	0.064
$l = 4$	0.382	0.587	0.618	0.742	0.783	0.786	0.586	0.722	0.622	0.246	0.125	0.065	0.053
$l = 5$	0.313	0.578	0.628	0.765	0.784	0.785	0.658	0.729	0.635	0.204	0.119	0.065	0.052
	$S_C^{13,l}$	$A_C^{13,1}$											
$l = 1$	0.513	0.653											
$l = 2$	0.452	0.654											
$l = 3$	0.447	0.649											
$l = 4$	0.501	0.639											
$l = 5$	0.472	0.624											
$C = 7$	$S_C^{2,7}$	$S_C^{3,7}$	$S_C^{4,7}$	$S_C^{5,7}$	$A_C^{2,7}$	$A_C^{3,7}$	$A_C^{4,7}$	$A_C^{5,7}$					
	0.358	0.442	0.436	0.397	0.694	0.672	0.576	0.542					

Rolling Scheme $\tau = 2/3$	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.221	0.206	0.221	0.106	0.148	0.214	0.268	0.397	0.261	0.096	0.132	0.067	0.056
$l = 2$	0.215	0.194	0.214	0.228	0.226	0.331	0.345	0.402	0.279	0.101	0.152	0.071	0.061
$l = 3$	0.242	0.247	0.205	0.204	0.215	0.347	0.344	0.401	0.244	0.069	0.144	0.068	0.053
$l = 4$	0.212	0.272	0.232	0.224	0.222	0.362	0.353	0.398	0.235	0.073	0.132	0.068	0.058
$l = 5$	0.253	0.265	0.247	0.252	0.253	0.343	0.385	0.408	0.245	0.062	0.147	0.063	0.057
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.112	0.154	0.164	0.213	0.361	0.453	0.538	0.597	0.526	0.223	0.103	0.056	0.042
$l = 2$	0.214	0.527	0.487	0.583	0.644	0.682	0.523	0.623	0.535	0.213	0.102	0.054	0.041
$l = 3$	0.253	0.562	0.524	0.572	0.634	0.672	0.474	0.643	0.534	0.223	0.109	0.057	0.042
$l = 4$	0.302	0.475	0.516	0.602	0.657	0.685	0.454	0.638	0.547	0.156	0.105	0.058	0.042
$l = 5$	0.268	0.467	0.537	0.586	0.635	0.674	0.464	0.648	0.518	0.157	0.103	0.057	0.043
	$S_C^{13,l}$	$A_C^{13,1}$											
$l = 1$	0.423	0.544											
$l = 2$	0.413	0.538											
$l = 3$	0.409	0.563											
$l = 4$	0.427	0.557											
$l = 5$	0.418	0.556											
$C = 7$	$S_C^{2,7}$	$S_C^{3,7}$	$S_C^{4,7}$	$S_C^{5,7}$	$A_C^{2,7}$	$A_C^{3,7}$	$A_C^{4,7}$	$A_C^{5,7}$					
	0.305	0.334	0.354	0.375	0.514	0.523	0.517	0.498					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

$S_{|z|}^{l,7}, A_{|z|}^{l,7}$ for $l = 1, 2, \dots, 5$; 7 refers to the 50% autocontour.

$S_L^{l,7}, A_L^{l,7}$ stacking lags up to $l = 2, \dots, 5$ and considering the 50% autocontour.

$S_C^{l,13}$ and $A_C^{l,13}$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.

1000 Monte Carlo replications and 500 bootstrap samples.

T=650, R=350, P=300, $m = 1/3$, and break point at $R + \tau P$ for $\tau = 1/2, 2/3$

Table 13: **Power for DGP3 under Rolling Scheme $\tau = 1/2$ and $2/3$**

Tables: Phillips Curve

Fixed Scheme	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.017	0.529	0.523	0.471	0.268	0.002	0.008	0.045	0.136	0.091	0.085	0.031	0.0001
$l = 2$	0.015	0.285	0.55	0.434	0.481	0.002	0.005	0.041	0.157	0.086	0.09	0.027	0.0001
$l = 3$	0.028	0.584	0.614	0.651	0.419	0.006	0.005	0.03	0.049	0.088	0.073	0.013	0.0001
$l = 4$	0.013	0.104	0.807	0.285	0.209	0.009	0.003	0.027	0.098	0.088	0.069	0.012	0.0001
$l = 5$	0.031	0.221	0.212	0.302	0.206	0.009	0.006	0.034	0.103	0.088	0.057	0.006	0.0001
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.342	0.671	0.759	0.405	0.163	0.042	0.013	0.248	0.243	0.243	0.412	0.199	0.042
$l = 2$	0.335	0.453	0.621	0.352	0.972	0.043	0.022	0.336	0.297	0.223	0.332	0.25	0.04
$l = 3$	0.494	0.579	0.316	0.567	0.309	0.041	0.016	0.216	0.225	0.235	0.359	0.197	0.049
$l = 4$	0.338	0.348	0.764	0.319	0.152	0.033	0.024	0.144	0.264	0.223	0.327	0.226	0.042
$l = 5$	0.138	0.595	0.637	0.238	0.142	0.032	0.024	0.218	0.273	0.222	0.308	0.184	0.042
	$S_C^{l,13}$	$A_C^{l,13}$											
$l = 1$	0.004	0.022											
$l = 2$	0.001	0.019											
$l = 3$	0.003	0.016											
$l = 4$	0.001	0.025											
$l = 5$	0.001	0.025											
$C = 7$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
	0.006	0.009	0.012	0.01	0.027	0.018	0.027	0.029					

Rolling Scheme	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.528	0.46	0.843	0.472	0.142	0.002	0.001	0.007	0.008	0.093	0.281	0.026	0.0002
$l = 2$	0.258	0.35	0.58	0.251	0.229	0.007	0.004	0.009	0.002	0.081	0.24	0.024	0.0003
$l = 3$	0.559	0.31	0.741	0.437	0.16	0.002	0.001	0.002	0.002	0.066	0.288	0.013	0.0003
$l = 4$	0.38	0.31	0.479	0.463	0.17	0.001	0.001	0.003	0.008	0.088	0.275	0.021	0.0002
$l = 5$	0.285	0.249	0.297	0.225	0.12	0.007	0.001	0.003	0.003	0.063	0.327	0.019	0.0002
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.789	0.801	0.853	0.656	0.143	0.001	0.023	0.033	0.032	0.145	0.297	0.271	0.024
$l = 2$	0.267	0.55	0.812	0.244	0.568	0.002	0.018	0.024	0.045	0.127	0.283	0.337	0.037
$l = 3$	0.751	0.918	0.923	0.723	0.144	0.007	0.015	0.032	0.047	0.114	0.288	0.24	0.023
$l = 4$	0.358	0.476	0.575	0.448	0.108	0.001	0.015	0.031	0.031	0.142	0.313	0.271	0.034
$l = 5$	0.305	0.655	0.576	0.251	0.129	0.009	0.018	0.031	0.039	0.13	0.286	0.288	0.033
	$S_C^{l,13}$	$A_C^{l,13}$											
$l = 1$	0.001	0.009											
$l = 2$	0.0003	0.004											
$l = 3$	0.0003	0.01											
$l = 4$	0.0003	0.005											
$l = 5$	0.002	0.004											
$C = 7$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
	0.008	0.008	0.006	0.009	0.038	0.025	0.019	0.031					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

$S_{|z|}^{l,7}, A_{|z|}^{l,7}$ for $l = 1, 2, \dots, 5$; 7 refers to the 50% autocontour.

$S_L^{l,7}, A_L^{l,7}$ stacking lags up to $l = 2, \dots, 5$ and considering the 50% autocontour.

$S_C^{l,13}$ and $A_C^{l,13}$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.

500 bootstrap samples. T=649, R=360, P=289, $m = 0.69$

Table 14: **Bootstrapped P-values: Linear Phillips Curve (Fixed and Rolling Schemes)**

Model 3	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.452	0.662	0.454	0.406	0.466	0.694	0.702	0.814	0.788	0.57	0.41	0.254	0.296
$l = 2$	0.428	0.552	0.218	0.464	0.436	0.678	0.652	0.816	0.684	0.55	0.41	0.276	0.296
$l = 3$	0.476	0.784	0.454	0.424	0.466	0.66	0.602	0.874	0.782	0.582	0.43	0.206	0.296
$l = 4$	0.656	0.738	0.676	0.55	0.584	0.72	0.807	0.884	0.82	0.584	0.386	0.224	0.296
$l = 5$	0.652	0.538	0.422	0.444	0.536	0.78	0.836	0.844	0.842	0.578	0.416	0.244	0.296
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.442	0.644	0.43	0.39	0.568	0.682	0.856	0.822	0.818	0.596	0.45	0.278	0.294
$l = 2$	0.402	0.446	0.318	0.362	0.326	0.488	0.674	0.834	0.72	0.57	0.446	0.308	0.296
$l = 3$	0.606	0.748	0.446	0.388	0.454	0.558	0.804	0.868	0.79	0.606	0.434	0.23	0.294
$l = 4$	0.638	0.768	0.666	0.348	0.58	0.722	0.86	0.874	0.824	0.584	0.4	0.226	0.296
$l = 5$	0.632	0.514	0.404	0.432	0.522	0.722	0.83	0.852	0.86	0.582	0.412	0.246	0.39
	$S_C^{l,13}$	$A_C^{l,13}$											
$l = 1$	0.614	0.596											
$l = 2$	0.48	0.454											
$l = 3$	0.494	0.482											
$l = 4$	0.51	0.478											
$l = 5$	0.488	0.472											
$C = 7$	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
	0.518	0.566	0.548	0.598	0.534	0.578	0.564	0.618					
Model 6	$S_{ z }^{l,1}$	$S_{ z }^{l,2}$	$S_{ z }^{l,3}$	$S_{ z }^{l,4}$	$S_{ z }^{l,5}$	$S_{ z }^{l,6}$	$S_{ z }^{l,7}$	$S_{ z }^{l,8}$	$S_{ z }^{l,9}$	$S_{ z }^{l,10}$	$S_{ z }^{l,11}$	$S_{ z }^{l,12}$	$S_{ z }^{l,13}$
$l = 1$	0.044	0.238	0.335	0.141	0.401	0.433	0.474	0.559	0.478	0.577	0.299	0.254	0.042
$l = 2$	0.042	0.345	0.346	0.168	0.392	0.514	0.521	0.442	0.418	0.552	0.260	0.215	0.053
$l = 3$	0.042	0.242	0.369	0.110	0.375	0.512	0.477	0.498	0.457	0.595	0.301	0.210	0.044
$l = 4$	0.043	0.316	0.378	0.154	0.411	0.566	0.457	0.423	0.485	0.506	0.297	0.239	0.052
$l = 5$	0.049	0.321	0.403	0.196	0.434	0.492	0.479	0.470	0.482	0.598	0.303	0.198	0.052
	$A_{ z }^{l,1}$	$A_{ z }^{l,2}$	$A_{ z }^{l,3}$	$A_{ z }^{l,4}$	$A_{ z }^{l,5}$	$A_{ z }^{l,6}$	$A_{ z }^{l,7}$	$A_{ z }^{l,8}$	$A_{ z }^{l,9}$	$A_{ z }^{l,10}$	$A_{ z }^{l,11}$	$A_{ z }^{l,12}$	$A_{ z }^{l,13}$
$l = 1$	0.076	0.375	0.431	0.221	0.397	0.383	0.545	0.495	0.572	0.493	0.319	0.264	0.115
$l = 2$	0.092	0.231	0.403	0.207	0.381	0.387	0.560	0.449	0.562	0.404	0.351	0.263	0.102
$l = 3$	0.072	0.191	0.352	0.220	0.390	0.335	0.583	0.621	0.553	0.448	0.425	0.263	0.102
$l = 4$	0.066	0.230	0.402	0.255	0.391	0.336	0.518	0.418	0.554	0.493	0.320	0.264	0.113
$l = 5$	0.072	0.338	0.429	0.212	0.416	0.382	0.581	0.523	0.570	0.511	0.315	0.264	0.104
	$S_C^{13,l}$	$A_C^{13,l}$											
$l = 1$	0.366	0.480											
$l = 2$	0.299	0.433											
$l = 3$	0.323	0.449											
$l = 4$	0.326	0.436											
$l = 5$	0.351	0.446											
	$S_C^{2,7}$	$S_C^{3,7}$	$S_C^{4,7}$	$S_C^{5,7}$	$A_C^{2,7}$	$A_C^{3,7}$	$A_C^{4,7}$	$A_C^{5,7}$					
	0.488	0.475	0.483	0.464	0.464	0.419	0.461	0.404					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

$S_{|z|}^{l,7}, A_{|z|}^{l,7}$ for $l = 1, 2, \dots, 5$; 7 refers to the 50% autocontour.

$S_L^{l,7}, A_L^{l,7}$ stacking lags up to $l = 2, \dots, 5$ and considering the 50% autocontour.

$S_C^{l,13}$ and $A_C^{l,13}$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.

500 bootstrap samples.

T=649, R=360, P=289, $m = 0.69$

Table 15: **Bootstrapped P-values: Non-Linear Phillips Curve (Fixed Scheme)**