Progressive Taxation, Endogenous Growth, and Macroeconomic (In)stability*

Shu-Hua Chen†
National Taipei University

Jang-Ting Guo‡
University of California, Riverside

September 15, 2014

Abstract

In the context of a standard one-sector AK model of endogenous growth, we show that the economy exhibits equilibrium indeterminacy and belief-driven aggregate fluctuations under progressive taxation of income. When the tax schedule is regressive or flat, the economy's balanced growth path displays saddle-path stability and equilibrium uniqueness. These results imply that in sharp contrast to a conventional automatic stabilizer, progressive income taxation may destabilize an endogenously growing macroeconomy by generating cyclical fluctuations driven by agents' self-fulfilling expectations or sunspots.

Keywords: Progressive Income Taxation, Endogenous Growth, Equilibrium (In)determinacy.

JEL Classification: E62, O41.

*We thank Been-Lon Chen, Juin-Jen Chang, Yiting Li, David Malueg, Kazuo Mino, Victor Ortego-Marti and Yan Zhang for helpful comments and suggestions. Part of this research was conducted while Guo was a visiting research fellow of economics at Academia Sinica, Taipei, Taiwan, whose hospitality is greatly appreciated. Of course, all remaining errors are our own.

†Department of Economics, National Taipei University, 151 University Rd., San Shia District, New Taipei City, 23741 Taiwan, 886-2-8674-1111 ext. 67165, Fax: 886-2-2673-9880, E-mail: shchen@mail.ntpu.edu.tw.

‡Corresponding Author. Department of Economics, 3133 Sproul Hall, University of California, Riverside, CA, 92521 USA, 1-951-827-1588, Fax: 1-951-827-5685, E-mail: guojt@ucr.edu.

1 Introduction

Following the resurgent interest in long-run economic growth that began in the mid-1980's, there has been an extensive literature analyzing the macroeconomic impacts of income taxation within various formulations of endogenously growing dynamic general equilibrium models.[1] As it turns out, the vast majority of previous theoretical studies postulate a constant tax rate on the households' taxable income.[2] While this assumption is commonly adopted for the sake of analytical simplicity, it is not consistent with the U.S. federal individual income tax schedule that is characterized by several "tax brackets" (branches of income) taxed at progressively higher rates. Motivated by this gap in the existing literature, we examine the (in)stability effects of Guo and Lansing's (1998) nonlinear taxation structure, whereby the associated tax progressivity is governed by a single parameter, in the simplest one-sector AK model of endogenous growth: a representative-agent economy with fixed labor supply and useless government spending that does not contribute to utility or production.[3] Not only does our work provide valuable theoretical insights, it also yields important implications about the (de)stabilization role of tax policies in a macroeconomy with sustained economic growth.

In sharp contrast to traditional Keynesian-type stabilization policies, progressive income taxation does not operate as an automatic stabilizer in our model because it leads to equilibrium indeterminacy and endogenous belief-driven growth fluctuations.[4] Start from a particular balanced growth path, and suppose that agents become optimistic about the economy's future. Acting upon this expectation, the representative household will reduce consumption and raise investment today, which in turn generates another dynamic trajectory. When the tax progressivity is positive, we show that the equilibrium after-tax marginal product of capital is monotonically increasing along the downward-sloping transitional path as the consumption-to-capital ratio rises. As a consequence, agents' initial optimistic anticipation is validated and the alternative path becomes a self-fulfilling equilibrium. On the contrary, our model exhibits

[1] Early examples in this area include Barro (1990), King and Rebelo (1990), Rebelo (1991), and Barro and Sala-i-Martin (1992), among many others.

[2] Noted exceptions are Yamarik (2001), Li and Sarte (2004), and Greiner (2006), among others. However, none of these papers focus on the (de)stabilization effects of a nonlinear tax policy rule.

[3] In an earlier article, Chen and Guo (2013) explore the interrelations between progressive income taxation and macroeconomic (in)stability in a one-sector endogenous growth model with productive flow of public expenditures.

[4] By contrast, Guo and Lansing (1998) find that sufficiently progressive taxation can stabilize a one-sector real business cycle model, which possesses an indeterminate steady state under laissez-faire, against cyclical fluctuations driven by agents' animal spirits.

saddle-path stability and equilibrium uniqueness under regressive[5] or flat income taxation. These findings altogether imply that in the context of a standard one-sector AK model of endogenous growth, changing the tax schedule from being flat or regressive to progressive will magnify the magnitude of aggregate fluctuations and thus destabilize the macroeconomy.

The remainder of this paper is organized as follows. Section 2 describes our model and derives the economy's unique balanced-growth equilibrium path. Section 3 analytically examines the interrelations between tax progressivity and equilibrium (in)determinacy. Section 4 concludes.

2 The Economy

We incorporate a progressive/regressive income tax schedule *á la* Guo and Lansing (1998) into the one-sector AK model of endogenous growth under perfect foresight. The economy is populated by a unit measure of identical infinitely-lived households. Each household provides fixed labor supply and maximizes its discounted lifetime utility

$$U = \int_0^\infty \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad \sigma \geq 1, \tag{1}$$

where c_t is consumption, $\rho > 0$ denotes the subjective rate of time preference, and σ represents the inverse of the intertemporal elasticity of substitution in consumption. Based on the empirical evidence for this preference parameter in the mainstream macroeconomics literature, our analysis is restricted to the cases with $\sigma \geqslant 1$. The budget constraint faced by the representative household is

$$c_t + \dot{k}_t + \delta k_t = (1-\tau_t) y_t, \quad k_0 > 0 \text{ given}, \tag{2}$$

where k_t is the household's capital stock, $\delta \in (0,1)$ is the capital depreciation rate, y_t is GDP, and τ_t represents a proportional income tax rate. Output y_t is produced by a unit measure of identical competitive firms with the Cobb-Douglas production function

$$y_t = A k_t^\alpha \bar{k}_t^{1-\alpha}, \quad A > 0, \quad 0 < \alpha < 1, \tag{3}$$

[5] By contrast, Schmitt-Grohé and Uribe (1997) show that a prototypical one-sector real business cycle model, coupled with a balanced-budget rule that qualitatively resembles regressive income taxation, may exhibit indeterminacy and sunspots.

where $\bar{k}_t$ is the economy-wide average level of capital services. In a symmetric equilibrium, all firms make the same decisions such that $k_t = \bar{k}_t$, which in turn yields the following social technology that allows for sustained economic growth:

$$y_t = Ak_t. \tag{4}$$

With regard to the income tax rate, we adopt the sustained-growth version of Guo and Lansing's (1998, p.485, footnote 4) nonlinear tax structure and postulate τ_t as

$$\tau_t = 1 - \eta \left(\frac{y_t^*}{y_t} \right)^{\phi}, \ \eta \in (0,1), \ \phi \in \left(\underline{\phi}, 1 \right), \tag{5}$$

where y_t^* denotes a benchmark level of income that is taken as given by the representative household. In our model with endogenous growth, y_t^* is set equal to the level of per capita output on the economy's balanced growth path (BGP) whereby $\frac{\dot{y}_t^*}{y_t^*} = \theta > 0$ for all t.[6] In addition, the marginal tax rate τ_{mt}, defined as the change in taxes paid by the household divided by the change in its taxable income, is given by

$$\tau_{mt} = \frac{\partial(\tau_t y_t)}{\partial y_t} = \tau_t + \eta\phi \left(\frac{y_t^*}{y_t} \right)^{\phi}. \tag{6}$$

Our analyses are restricted to an environment with $0 < \tau_t, \, \tau_{mt} < 1$ such that (i) the government does not have access to lump-sum taxes or transfers, (ii) the government cannot confiscate all productive resources, and (iii) households have incentive to provide factor services to firm's production process. Along the economy's balanced-growth equilibrium path where $y_t = y_t^*$, these considerations imply that $0 < \eta < 1$ and $\frac{\eta-1}{\eta} < \phi < 1$. On the other hand, the convexity of the household's budget set requires that the after-tax marginal product of capital $(1 - \tau_{mt})MPK_t$ must be strictly decreasing with respect to k_t, which in turn implies that $\phi > \frac{\alpha-1}{\alpha}$ on the balanced growth path. It follows that the lower bound on the parameter ϕ of the tax schedule (5) is determined by

$$\underline{\phi} = \max \left\{ \frac{\alpha - 1}{\alpha}, \ \frac{\eta - 1}{\eta} \right\}. \tag{7}$$

Given the postulated restrictions on η and ϕ, equation (6) shows that the marginal tax rate τ_{mt} is higher than the average tax rate τ_t when $\phi > 0$. In this case, the tax schedule is said to

[6] To guarantee the existence of a balanced growth path in our subsequent analyses, the household's taxable income y_t in equilibrium needs to grow at the same rate as the baseline level of income y_t^*. Moreover, the constant (positive) growth rate θ for y_t^* will be endogenously determined through the model's equilibrium conditions (see equation 16).

be "progressive". When $\phi = 0$, the average and marginal tax rates coincide at the value $1 - \eta$ and the tax schedule is said to be "flat". When $\phi < 0$, the tax schedule is "regressive".

In making decisions about how much to consume and invest over their lifetimes, agents take into account the effect in which the tax schedule influences their net earnings. As a result, it is the marginal tax rate of income τ_{mt} that will govern the household's economic decisions. The first-order conditions for the representative agent with respect to the indicated variables and the associated transversality conditions (TVC) are

$$c_t \quad : \quad c_t^{-\sigma} = \lambda_t, \tag{8}$$

$$k_t \quad : \quad \lambda_t \left[\underbrace{\eta(1-\phi)\left(\frac{y_t^*}{y_t}\right)^{\phi}}_{(1-\tau_{mt})} \underbrace{\alpha\frac{y_t}{k_t}}_{MPK_t} - \delta \right] = \rho\lambda_t - \dot{\lambda}_t, \tag{9}$$

$$\text{TVC} \quad : \quad \lim_{t\to\infty} e^{-\rho t}\lambda_t k_t = 0, \tag{10}$$

where (8) states that the marginal utility of consumption is equal to the Lagrange multiplier λ_t on the household's budget constraint (2). Equation (9) is the Keynes-Ramsey condition that describes how the stock of physical capital evolves over time, and (10) is the transversality condition. Finally, the government sets the income tax rate τ_t according to (5), and balances its budget at each point in time. Hence, the instantaneous government budget constraint is given by

$$g_t = \tau_t y_t, \tag{11}$$

where g_t is public spending on goods and services that does not contribute to the household's utility or firms' production.

We focus on the economy's balanced growth path along which output, consumption and physical capital exhibit a common, positive constant growth rate θ. To facilitate the subsequent dynamic analyses, we adopt the following variable transformations: $x_t \equiv \frac{g_t}{k_t}$ and $z_t \equiv \frac{c_t}{k_t}$. Per these variable transformations, the model's equilibrium conditions (with $\frac{\dot{y}_t^*}{y_t^*} = \theta$ imposed) can be collapsed into the following autonomous dynamical system:

$$\dot{x}_t = -\phi(A - x_t)(\theta - A + \delta + x_t + z_t), \tag{12}$$

$$\frac{\dot{z}_t}{z_t} = \frac{1}{\sigma}\left[\alpha(1-\phi)(A - x_t) - \delta - \rho\right] - A + \delta + x_t + z_t. \tag{13}$$

A balanced-growth equilibrium is characterized by a pair of positive real numbers (x^*, z^*) that satisfy $\dot{x}_t = \dot{z}_t = 0$. It is straightforward to show that our model economy possesses a unique BGP with

$$x^* = A(1-\eta) \tag{14}$$

and

$$z^* = \frac{A\eta[\sigma - \alpha(1-\phi)] + (1-\sigma)\delta + \rho}{\sigma}; \tag{15}$$

and that the common (positive) rate of economic growth is given by

$$\theta = \frac{\alpha A\eta(1-\phi) - \delta - \rho}{\sigma}. \tag{16}$$

3 Macroeconomic (In)stability

In terms of the BGP's local dynamics, we analytically derive the Jacobian matrix J of the dynamical system (12)-(13) evaluated at $(x^*,\ z^*)$, and find that its determinant and trace are

$$Det = -\frac{\alpha A\eta\phi(1-\phi)z^*}{\sigma}, \tag{17}$$

$$Tr = -A\eta\phi + z^*. \tag{18}$$

Proposition. The economy's balanced-growth equilibrium exhibits local indeterminacy (*i.e.* a sink) and belief-driven aggregate fluctuations under progressive income taxation with $0 < \phi < 1$; whereas saddle-path stability and equilibrium uniqueness take place under regressive taxation with $\underline{\phi} < \phi < 0$, where $\underline{\phi}$ is given by (7).

Proof. The BGP's local stability property is determined by comparing the eigenvalues of J that have negative real parts with the number of initial conditions in the dynamical system

(12)-(13), which is zero because x_t and z_t are both non-predetermined jump variables in our model.[7] Since $\sigma \geq 1$, $0 < \alpha$, $\eta < 1$ and A, $z^* > 0$, the Jacobian's determinant (17) is negative when $0 < \phi < 1$, indicating that the two eigenvalues are of opposite signs in their real parts. In this case, the economy exhibits endogenous growth fluctuations driven by agents' self-fulfilling expectations or sunspots. When $\underline{\phi} < \phi < 0$, the BGP displays local determinacy in that both eigenvalues of the Jacobian matrix J have positive real parts ($Det > 0$ and $Tr > 0$). ■

To help understand the above (in)determinacy results, we use (12) and (13) to construct the model's phase diagram under progressive taxation as depicted in Figure 1. It is straightforward to show that the equilibrium loci $\dot{x}_t = 0$ and $\dot{z}_t = 0$ are downward sloping, and that the associated negatively-sloped stable arm (denoted as SS) is flatter than the $\dot{z}_t = 0$ locus, followed by $\dot{x}_t = 0$. Next, start from a particular balanced growth path characterized by (x^*, z^*), and suppose that agents become optimistic about the future of the economy. Acting upon this belief, households will invest more and consume less today, which in turn lead to another dynamic trajectory $\left\{x_t^{'}, z_t^{'}\right\}$ that begins at (x_0', z_0') with $x_0' > x^*$ and $z_0' < z^*$. Figure 1 shows that for this alternative path to become a self-fulfilling equilibrium, the after-tax return on investment $(1 - \tau_{mt})MPK_t$ must be monotonically increasing along the transitional path SS as the consumption-to-capital ratio $z_t \equiv \frac{c_t}{k_t}$ rises. From (3), (5), (6) and (11), together with the chain rule, it can be shown that

$$\left.\frac{d\left[(1-\tau_{mt})MPK_t\right]}{dz_t}\right|_{SS} = \underbrace{\frac{d\left[(1-\tau_{mt})MPK_t\right]}{dx_t}}_{=-\alpha(1-\phi)<0} \underbrace{\left.\frac{dx_t}{dz_t}\right|_{SS}}_{\text{negative}} > 0. \tag{19}$$

As a consequence, agents' initial rosy anticipation is validated.

Under regressive income taxation, when households become optimistic and decide to raise their investment expenditures today, the preceding mechanism that makes for multiple equilibria, *i.e.* an increase in the equilibrium after-tax marginal product of capital, will generate divergent trajectories away from the original balanced growth path. This implies that given the initial capital stock k_0, the period-0 levels of the household's consumption c_0 as well as the government's spending g_0 are uniquely determined such that the economy immediately jumps onto its balanced-growth equilibrium (x^*, z^*), and always stays there without any possibility of deviating transitional dynamics. It follows that equilibrium indeterminacy and endogenous

[7] As for the initial condition of consumption c_0, the period-0 level of government spending g_0 (a flow variable) will be endogenously determined. Using the model's equilibrium conditions, it can be shown that $x_t = A[1 - \eta e^{\phi(\theta+\delta-A+x_t+z_t)}]$, thus both $x_0 = \frac{g_0}{k_0}$ and $z_0 = \frac{c_0}{k_0}$ are not predetermined.

growth fluctuations can never occur in this setting.

Finally, when the tax schedule is flat with $\tau_t = \tau_{mt} = 1 - \eta$, we substitute $\phi = 0$ into (12) and find that the ratio of public expenditures to physical capital x_t remains unchanged over time. This implies that the dynamical system (12)-(13) now becomes degenerate. Resolving our model with $\phi = 0$ leads to the following single differential equation in z_t that describes the equilibrium dynamics:

$$\frac{\dot{z}_t}{z_t} = \frac{\alpha A\eta - \delta - \rho}{\sigma} - A\eta + z_t + \delta, \tag{20}$$

which has a unique interior solution z^* that satisfies $\dot{z}_t = 0$ along the balanced-growth equilibrium path. We then linearize (20) around the BGP and find that its local stability property is governed by the positive eigenvalue $z^* > 0$. Consequently, our endogenously growing economy exhibits saddle-path stability and equilibrium uniqueness under flat income taxation since there is no initial condition associated with (20).

4 Conclusion

This paper examines the theoretical interrelations between progressive taxation of income and equilibrium (in)determinacy in a one-sector AK model of endogenous growth with fixed labor supply and useless public spending. We find that the economy exhibits an indeterminate balanced-growth equilibrium path when the tax progressivity is positive, and that saddle-path stability and equilibrium uniqueness take place under regressive or flat taxation of income. These results imply that in sharp contrast to a conventional automatic stabilizer, moving the government's fiscal policy rule toward progressive taxation may destabilize an endogenously growing macroeconomy with aggregate fluctuations driven by agents' self-fulfilling expectations or sunspots. In terms of possible extensions, it would be worthwhile to explore alternative mechanisms for generating sustained economic growth (*e.g.* human capital accumulation) and/or an economy with multiple production sectors.[8] We plan to pursue these research projects in the near future.

[8] Chen and Guo (2014) show that the results reported in this paper remain qualitatively unchanged when variable labor supply is incorporated into a one-sector endogenous growth model.

References

[1] Barro, R.J. (1990), "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy* 98, S103-S125.

[2] Barro, R.J. and X. Sala-i-Martin (1992), "Public Finance in Models of Economic Growth," *Review of Economic Studies* 59, 645-661.

[3] Chen, S-H. and J-T. Guo (2013), "On Indeterminacy and Growth under Progressive Taxation and Productive Government Spending," *Canadian Journal of Economics* 46, 865-880.

[4] Chen, S-H. and J-T. Guo (2014), "Progressive Taxation as an Automatic Destabilizer under Endogenous Growth," unpublished manuscript.

[5] Greiner, A. (2006), "Progressive Taxation, Public Capital, and Endogenous Growth," *FinazArchiv* 62, 353-366.

[6] Guo, J-T. and K.J. Lansing (1998), "Indeterminacy and Stabilization Policy," *Journal of Economic Theory* 82, 481-490.

[7] King, R.G. and S. Rebelo (1990), "Public Policy and Economic Growth: Developing Neoclassical Implications," *Journal of Political Economy* 98, S126-S150.

[8] Li, W. and P-D. Sarte (2004), "Progressive Taxation and Long-Run Growth," *American Economic Review* 94, 1705-1716.

[9] Rebelo, S. (1991), "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy* 99, 500-521.

[10] Schmitt-Grohé, S. and M. Uribe (1997), "Balanced-Budget Rules, Distortionary Taxes and Aggregate Instability," *Journal of Political Economy* 105, 976-1000.

[11] Yamarik, S. (2001), "Nonlinear Tax Structures and Endogenous Growth," *Manchester School* 69, 16-30.

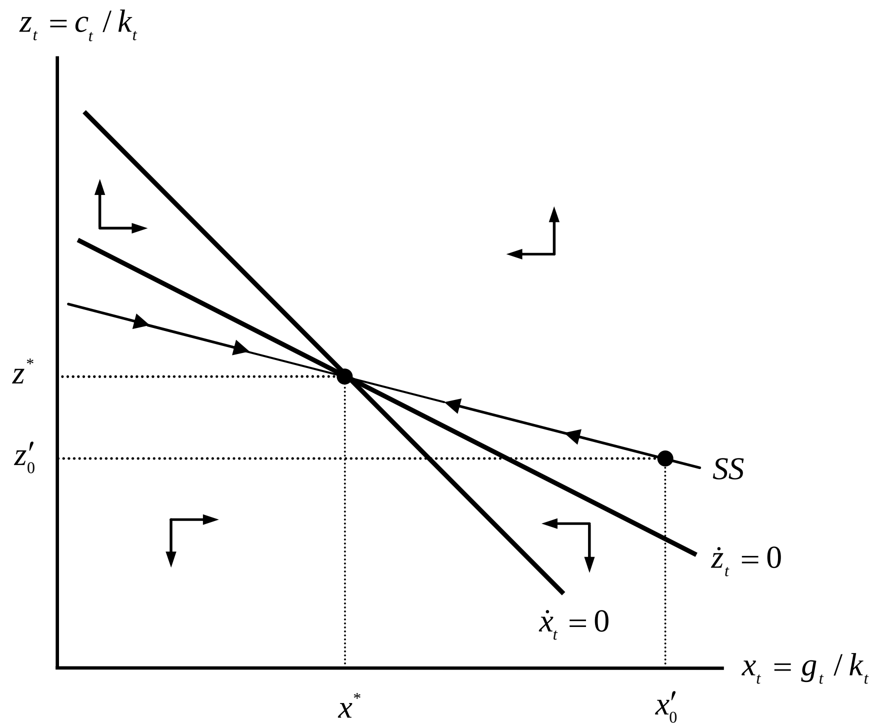

Figure 1. When 0 < ϕ < 1: Indeterminacy