Downtown Curbside Parking Capacity*

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Abstract

In downtown areas, what proportion of curbside should be allocated to parking? In contrast to most previous work on the economics of parking, this paper focuses on optimal curbside parking capacity in both first-best (where pricing is efficient) and second-best (where pricing is inefficient) environments. It first considers the situation where there is only curbside parking, and then the situation where there may be both curbside and private garage parking. For each situation, it examines the short run in which curbside parking capacity is fixed and the long run in which it is a policy variable.

Keywords: cruising for parking, curbside parking, curbside parking capacity, garage parking, traffic congestion

JEL Classification: D04, L91, R41, R48

1 Introduction

When asked what policies should be applied to deal with downtown traffic congestion, most American economists at least would respond, "(auto) congestion pricing". But there is much more to the design of an efficient downtown transportation system than getting the price of downtown auto travel right. While no one, to our knowledge, has attempted a precise estimate, the figure is bandied about that, on average, one-half of the full price (time and money cost) of a downtown auto trip is the full price of parking, which includes the parking fee and the time cost searching for parking and walking between the parking location and the destination. This figure is likely too high, but without doubt downtown parking policy has been neglected relative to its importance.

This paper considers a particular facet of downtown parking – downtown curbside parking. In keeping with the method of transportation economics, this paper will conceptualize curbside parking policy in terms of a two-by-two matrix, with the short run (pricing) and long run (capacity) in one dimension,

and first best (no distortions) and second best (distortions) in the other. The principles of first-best transportation pricing and capacity are well understood, and apply straightforwardly to curbside parking. While the theory of the second best too is well understood, its application is less straightforward since results depend on the nature of the distortions. In the context of curbside parking, there are two principal distortions. The first is the distortion that congestion pricing is designed to address, the underpricing of auto travel. The second is the underpricing of curbside parking, which seems to be ubiquitous in the US at least.[1] In Boston for example, the curbside meter rate was recently raised to \$1.25/hr, whereas the price of garage parking downtown for an hour or any fraction thereof averages over \$10.[2]

Since the literature has given considerable attention to second-best policy responses to underpriced urban auto travel, the focal question of the paper is "What is the second-best level of curbside parking capacity when both urban auto travel and curbside parking are underpriced?" To address this question, this paper investigates a stylized model of downtown parking and traffic congestion that builds on previous work by the authors (Arnott and Inci, 2006, 2010; and Arnott and Rowse, 2009). To simplify, the model assumes that individuals are identical, all travel is by car, the downtown street network is fixed and isotropic (spatially homogeneous), and public curbside (on-street) parking is supplemented by private garage (off-street) parking supplied elastically at unit cost, considers only the steady state, and provides an aggregative (or, in the terminology of transportation science, macroscopic) treatment of traffic congestion. The paper develops the analysis diagrammatically through

[1] The Netherlands is the only country we know of where curbside parking fees are on average about same as garage parking fees (van Ommeren *et al.*, 2012).

[2] There are four other distortions, which this paper abstracts from. The first is the underpricing of employer-provided parking; the second is the preferential treatment accorded residents through resident parking; the third is the market power conferred on private off-street parking operators (parking lots and parking garages) through a combination of the friction of space and discrete spacing of parking garages deriving from scale economies in their construction; and the fourth is minimum parking requirements, which require new developments to provide a minimum number of parking spaces per unit area of floor space (which vary according to the land use).

an extended numerical example. Thus, its emphasis is on elucidating basic principles and providing economic intuition rather than on obtaining general analytical results.

Underpricing curbside parking gives rise to excess demand for curbside parking. The rationing mechanism is cruising for parking, which is costly in itself but also exacerbates urban auto congestion. When only curbside parking is provided in equilibrium, equilibrium occurs at the point of intersection of the demand curve for urban auto trips and the curbside parking capacity constraint, with the density of cars cruising for parking adjusting so that the equilibrium price is attained. When there is both curbside parking and garage parking in equilibrium, the density of cars cruising for parking adjusts so that the full price (time and money cost) of curbside parking equals the full price of garage parking. The complexity of the second-best curbside parking capacity problem is hinted at by observing that there are two different ways to eliminate cruising for parking. The first is to increase curbside parking capacity to the point where excess demand for curbside parking is eliminated. The second is to eliminate curbside parking. Consistent with this observation, the main results of the paper are that, with underpriced parking: (*i*) when demand is low (relative to the density of road space and curbside), only curbside parking should be provided and at the level that just eliminates cruising for parking; (*ii*) when demand is high, curbside parking should be eliminated, with all parking taking place in private parking garages; and (*iii*) when demand is moderate, depending on parameter values, it may or may not be desirable to have a mix of curbside and garage parking, and hence cruising for parking.

While this paper focuses sharply on downtown parking, some of the insights it generates may apply to other economic situations where there are two alternative methods of acquiring a good, one high priced with less congestion, the other low priced with more congestion. One is the simultaneous provision of medical care by the public sector, at subsidized rates, and by the private sector, at market-determined rates.

The paper is organized as follows. Section 2 provides a brief review of the

literature on the economics of parking. Section 3 sets the stage by adapting Walters' (1961) model of highway traffic congestion to downtown traffic congestion without parking. Section 4 adds curbside parking but not garage parking, considering both the first best and the second best, and both the short run and the long run. Section 5 extends the analysis to the simultaneous provision of curbside parking and private garage parking. Section 6 concludes. An (online) appendix investigates the stability of the various equilibria.

2 Literature Review

Donald Shoup deserves much of the credit for raising awareness of the many policy issues related to parking. Since the 1980's, he has been crusading for cashing out free and heavily subsidized employer-provided and curbside parking, and for eliminating minimum parking requirements. The best point of entry into the literature on parking is his magisterial book, *The High Cost of Free Parking* (Shoup, 2005), which not only advocates policy changes, but also provides a wealth of information and data related to parking. *Lots of Parking: Land Use in a Car Culture* (Jakle and Sculle, 2004) provides an informative history of parking in the US.

The economic study of parking has been hampered by a lack of systematic data. There is plenty of fragmentary data from parking studies of downtown neighborhoods or small downtown areas, but only very recently have data started to be collected systematically on parking turnover and occupancy rates over the course of the day and over an entire downtown area. Prompted by Shoup's advocacy, the City of San Francisco has undertaken a large experiment, SF*park* (http://sfpark.org), to ascertain the effects of gradually raising downtown curbside meter rates, differentiated by neighborhood and time of day, so as to achieve a common target curbside occupancy rate of 85%. Detailed data are being collected and are publicly available, but, to our knowledge, no academic studies analyzing them have yet been published. Also, Jos van Ommeren has been entrepreneurial in his collection of parking data in the

Netherlands. Kobus *et al.* (forthcoming) used comprehensive parking data for the city of Almere in the Netherlands, where the price of all parking in the central business district is regulated, to study car drivers' choice between curbside and garage parking; van Ommeren *et al.* (2012) used data from the Dutch National Travel Survey to estimate the determinants of the time spent cruising for parking; and van Ommeren *et al.* (2011) and van Ommeren and Wentink (2012) employ two other databases to study other empirical aspects of parking in the Netherlands.

With few empirical regularities to guide the economic modeling of downtown parking, there has tended to be a proliferation of models, each addressing a different subset of features of downtown parking. These features include: (1) parking and rush-hour traffic dynamics (Arnott, de Palma, and Lindsey, 1991; Qian, Xiao, and Zhang, 2012), (2) curbside parking (Arnott and Inci, 2006, 2010; Arnott and Rowse, 2009), (3) off-street/garage parking, including spatial competition between parking garages (Anderson and de Palma, 2004, 2007; Calthrop and Proost, 2006; Arnott and Rowse, 2009), (4) parking and land use, including minimum parking requirements (Shoup, 1999; Anderson and de Palma, 2007; Cutter and Franco, 2012; Hasker and Inci, 2012), (5) cruising/searching for parking (Arnott and Rowse, 1999; Calthrop, 2001; Anderson and de Palma, 2004; Shoup, 2005, Part II; Arnott and Inci, 2006, 2010), (6) parking and traffic congestion (Arnott and Inci, 2006, 2010; Arnott and Rowse, 2009), (7) the subsidization of parking, including employer-provided parking, validated parking, and resident parking, (8) parking and modal choice, including the treatment of mass transit (Arnott and Rowse, 2012), (9) parking as a source of local public revenue (Shoup, 2005, Ch. 19), (10) parking and visit duration (Glazer and Niskanen, 1992; Calthrop and Proost, 2006), (11) parking for freight delivery, (12) curbside parking time limits (Calthrop and Proost, 2006; Arnott and Rowse, 2011), and (13) the political economy of downtown parking.[3]

[3]Parking for freight delivery and the political economy of downtown parking have been discussed in passing in the economics literature, but, to our knowledge, no paper has been published in the economics literature that focuses on either. van Ommeren and Wentink

Following Vickrey (1954), the primary theme of the literature on parking economics has been that efficiency requires that parking be priced at its social opportunity cost, just like any other commodity. A secondary and related theme has been that, with efficient pricing, standard investment rules should be applied to determine optimal parking capacity. Most models have assumed identical individuals, though Arnott and Rowse (2012) considered some complications arising from heterogeneity. Some models have considered an isotropic downtown area (*e.g.,* Calthrop and Proost, 2006; Arnott and Inci, 2006); others have considered parking relative to a central location (*e.g.,* Arnott, de Palma, and Lindsey, 1991; Anderson and de Palma, 2004; Qian, Xiao, and Zhang, 2012); and none to date has considered parking on an explicit street network. All models have ignored aggregate uncertainty, and only papers by the authors have considered parking capacity. Among those papers that treat traffic congestion, some have assumed bottleneck congestion (*e.g.,* Arnott, de Palma, and Lindsey, 1991; Qian, Xiao, and Zhang, 2012), and others classic flow congestion (Arnott and Inci, 2006, 2009; Arnott and Rowse, 2009, 2011, 2012). There is, of course, a substantial literature in transportation on parking, most of which considers engineering aspects, such as the design of curbside parking meters, parking garages, and parking information systems.

3 Downtown Traffic Congestion with No Parking

To set the stage for further analysis, we start by adapting Walters' (1961) familiar diagrammatic analysis of highway congestion to downtown traffic. For the moment, we ignore downtown parking, essentially assuming that parking is costless. We assume that downtown is isotropic; one can imagine a boundless Manhattan network of one-way streets. We also assume that drivers are

(2012) provides an empirical estimation, based on social surplus analysis, of the deadweight loss due to resident parking.

identical and that the demand for trips initiated per unit area-time, D, is stationary and is a function of the full price of a trip, F:

$$D = D(F) \quad . \tag{1}$$

To simplify we ignore the money costs of travel. Thus, the user cost of a trip, UC, equals the travel time cost of a trip, which equals the trip length, m, times travel time per mile, t, times the value of time, ρ:

$$UC = \rho m t \quad . \tag{2}$$

Travel time per mile, t, is an increasing function of V, the density of traffic per unit area, namely, $t = t(V)$, with $t' > 0$, $t'' > 0$, and with $t(0) > 0$ being free-flow travel time. In order to distinguish the full price of a trip and the user cost, we assume that a toll of size τ is applied, so that the full price of a trip equals the user cost plus the toll:

$$F = UC + \tau \quad . \tag{3}$$

In steady state, the number of trips initiated per unit area-time equals the number of trips terminated per unit area-time. We refer to this as the *steady-state condition*, and the steady-state number of trips per unit area-time as *throughput*,[4] and denote it by r. The steady-state number of trips initiated per unit area-time is given by the demand function. The steady-state number of trips terminated per unit area-time equals traffic density divided by the length of time each car spends in traffic, mt. Thus, the steady-state condition

[4]Throughput has units of cars per unit area-time. In steady state, throughput is the same as the entry flow and exit flow per unit area. We avoid the term *flow* to avoid confusion. The fundamental identity of traffic flow states that flow, f, equals density times velocity. Applying that identity in the current context gives $f = V/t(V) = mr$. Throughput is the exit rate (= entry rate) from the flow of traffic per unit area-time, which equals flow divided by trip length.

is

$$r = D\left(\rho mt(V) + \tau\right) = \frac{V}{mt\left(V\right)} \quad . \tag{4}$$

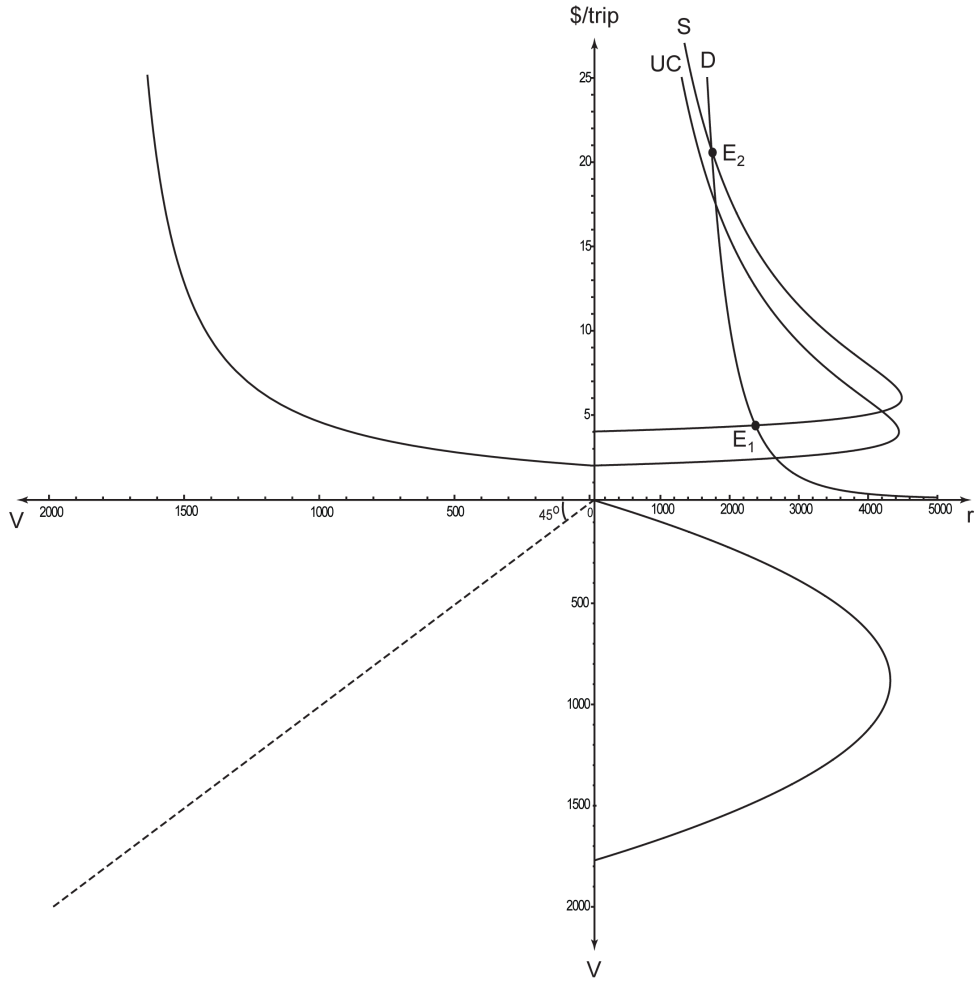


Figure 1: The Fundamental Traffic Diagram applied to downtown traffic

The equilibrium can be derived geometrically using the four-quadrant diagram of Figure 1. Quadrant II plots the relationship between user cost and traffic density ($UC = \rho mt(V)$). Quadrant III shows the 45-degree line. Quadrant IV depicts the steady-state relationship between throughput and density, $r = V/(mt(V))$ or $V = V(r)$. Quadrant I displays three curves, the user cost curve labeled UC, which relates user cost to throughput ($UC = \rho mt(V(r))$),

the supply curve labeled S, which relates full price of a trip to throughput ($F = UC + \tau = \rho m t(V(r)) + \tau$) and is obtained by a vertical shift of the user cost curve by τ, and the inverse demand function, which relates willingness to pay to throughput ($D^{-1}(F)$). Equilibrium is given by the point of intersection of the demand and supply curves.

Following Vickrey, travel on the upward-sloping portion of the user cost curve is termed congested travel, and travel on the backward-bending portion is termed hypercongested travel. With congested travel, travel time and user cost increase with throughput. With hypercongested travel, travel time and user cost decrease with throughput. Congested travel corresponds to normal travel, and hypercongested travel to traffic jam situations.

Throughout the paper, the following specific functional forms are maintained:

$$D(F) = D_0 F^{-a} \tag{5}$$

$$t(V) = \frac{t_0}{1 - \frac{V}{V_j}} \tag{6}$$

with parameter values

$$a = 0.2, \quad t_0 = 0.05, \quad V_j = 1778.17, \quad m = 2.0, \quad \rho = 20.0 \quad . \tag{7}$$

The units of measurement are miles, hours, and dollars. The functional forms and parameters are the same as those assumed in Arnott and Inci (2006, 2010), and the rationale for their choices is given in Arnott and Inci (2006). Trip demand is assumed to be iso-elastic, with demand elasticity, a, equal to 0.2 and demand intensity D_0. Travel congestion is described by Greenshields' Relation, which specifies a negative linear relationship between velocity and density. In (6), t_0 is free-flow travel time, which is the inverse of free-flow velocity, and V_j is jam density, which is the maximum possible density of traffic per unit area.[5] The demand intensity parameter is allowed to vary, in

[5] Greenshields' Relation has the property that flow, $f = Vv(D) = V/t(V)$, is max-

order to examine how equilibrium changes with demand. Figure 1 is drawn with the base case demand intensity of $D_0 = 3190.94$. Here and throughput the paper, the space allocated to downtown streets, and hence jam density, is taken as exogenous.

Figure 1 shows two equilibria. At E_1 traffic flow is congested and at E_2 it is hypercongested. There is also an equilibrium, E_3, corresponding to gridlock – zero flow and an infinite trip price – which cannot be displayed in the figure. Appendix A examines the stability of equilibria in detail. Out of equilibrium, per unit area the change in effective density equals the entry flow minus the exit flow. According to the intuitive adjustment dynamics that we assume, which are the same as those in Arnott and Inci (2010), the equilibrium corresponding to the lowest point of intersection of the demand and supply curves is stable, that corresponding to the next lowest point of intersection is unstable, and higher points of intersection alternate between stable and unstable. Applying, this rule, E_1 is stable, E_2 unstable, and E_3 stable.

Figure 2 focuses on the upward-sloping portion of the user cost curve. On this portion of the user cost curve, marginal social cost is defined as the derivative of aggregate user cost with respect to throughput, and equals user cost plus the congestion externality cost (the cost to inframarginal users due to the increase in throughput slowing them down). The figure is simply the standard textbook diagram of traffic congestion in the context of our model. The optimal level of throughput is that for which marginal social benefit (which coincides with the marginal willingness to pay or inverse demand) equals marginal social cost, and is labeled O. Because congestion is unpriced in the equilibrium, the equilibrium flow exceeds the optimum flow. The optimum can be decentralized by imposing a toll τ^* equal to the congestion externality cost, evaluated at the social optimum, and the deadweight loss from congestion

imized when density equals one-half jam density. With no on-street parking, maximum flow (or capacity flow) equals $f = V_j/(2t(V_j/2))$, so that maximum throughput equals $V_j/(2mt(V_j/2)) = 8890.8$. At maximum flow, velocity is one-half of free-flow velocity. Travel is congested when velocity exceeds one-half free-flow velocity, and is hypercongested otherwise.

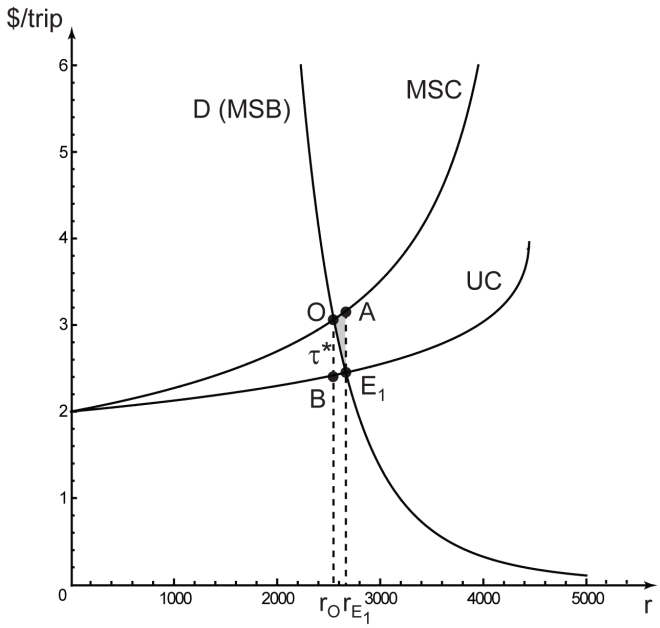


Figure 2: Equilibrium and social optimum with no parking

being unpriced is drawn as the shaded area AE_1O.

4 Downtown Traffic Congestion with Only Curbside Parking

We now modify the model to take into account that drivers must park. In this section, we rule out garage parking and consider only curbside parking. Curbside parking affects the analysis in four ways. First, increasing the amount of curbside allocated to parking reduces road space available for traffic flow, which reduces jam density.[6] Second, the amount of curbside parking constrains the throughput of the downtown traffic network to be no more than

[6] We assume that curbside allocated to parking reduces jam density by the same amount whatever the occupancy rate of curbside parking. The rationale is that, under at least moderately congested traffic conditions, even if only one curbside parking space is occupied on one side of the block, traffic flow is effectively excluded from that lane for the entire block.

the curbside parking turnover rate, which we term *curbside parking capacity.* With P curbside parking spaces per unit area and a visit duration of l, curbside parking capacity is P/l. Third, drivers pay a curbside parking fee per unit time[7] (the 'meter rate') of f. Fourth, if there is insufficient curbside parking to ration the demand with the curbside parking fee, cruising for parking occurs, and the travel time cost, which includes cruising-for-parking time cost, adjusts to clear the market.

To simplify, we provide a crude treatment of parking search. We assume that each driver travels to his destination block, and if a parking space is available he takes it, and if one is not he drives around the destination block until a space opens up. Thus, curbside parking involves no walking. Furthermore, we ignore the random variation that occurs due to the small number of parking spaces on each block, and assume that curbside parking is either *saturated* (fully occupied) everywhere, or *unsaturated* everywhere.

We shall first consider optimal curbside parking pricing. We shall then examine optimal curbside parking capacity, conditional on curbside parking being efficiently priced (first-best capacity) and inefficiently priced (second-best capacity). In all our analysis, we assume that no congestion tolling is employed. Because the distance traveled and the visit duration are fixed, optimal pricing can be achieved by efficiently pricing curbside parking even when congestion pricing is not employed, which is why we refer to the optimal capacity with efficient curbside parking pricing as first best.[8]

We have already distinguished between throughput and flow. Steady-state throughput is the rate at which trips are initiated and terminated per unit area-time. Steady-state flow is the number of car-mls traveled per unit area-time. When cruising for parking occurs, there is a further distinction between throughput and flow – flow includes cars that are cruising for parking but throughput does not.

[7] To keep the analysis simple, we consider only linear curbside parking payment schedules.

[8] See Verhoef, Nijkamp, and Rietveld (1995), which discusses the use of parking fees as a substitute for road pricing.

Two adjustments need to be made to the specification of the congestion technology to accommodate curbside parking. First, it is necessary to account for the reduction in roadside capacity due to parking. We assume that *effective jam density* is related to the amount of street space allocated to traffic flow. In particular, where Ω is the jam density with no curbside parking, effective jam density, V_j, equals jam density times the proportion of street space allocated to traffic flow, $1 - P/P_{\max}$, where P is the density of curbside parking spaces per unit area and $P_{\max}$ its maximum possible value. Thus:

$$V_j = \Omega \left(1 - \frac{P}{P_{\max}}\right) \quad . \tag{8}$$

Second, the specification of the congestion technology needs to account for cars cruising for parking and for the congestion interaction between cars in transit and cars cruising for parking. We make the simple assumption that a car cruising for parking generates θ times as much congestion as a car in transit. Thus, where T is the density of cars per unit area that are in transit and C is the density of cars per unit area that are cruising for parking, the travel time function is

$$t\left(T, C, P\right) = \frac{t_0}{1 - \frac{T+\theta C}{V_j}} \quad . \tag{9}$$

We maintain the following parameters for the rest of the paper, which are the same as those used in Arnott and Inci (2006, 2010) and are justified there:

$$\theta = 1.5, \quad \Omega = 2667.36, \quad P_{\max} = 11136, \quad l = 2 \quad . \tag{10}$$

For the base case, we also assume that the curbside parking fee is \$1.00/hr, so that the parking fee for the trip is \$2.00, and that curbside parking is permitted on one side of the street everywhere, so that $P = 3712$ and $P/l = 1856$.

4.1 The short run with only curbside parking

4.1.1 First-best optimum in the short run with only curbside parking

Consider a benevolent planner who has direct control over the transportation system and its users. She would never choose to have cruising for parking because the same throughput (and hence the same social benefit) can be achieved at lower cost without it. Since the amount of curbside parking is fixed, her problem is to choose throughput and in-transit traffic density to maximize social surplus subject to the steady-state condition and the curbside parking capacity constraint. Where $X(r)$ is the social benefit from throughput r (the area under the inverse demand curve up to this level of throughput), her maximization problem is[9]

$$\max_{r,T} X(r) - \rho T \tag{11}$$

$$s.t.$$

$$r = \frac{T}{mt(T, 0, P)} \tag{i}$$

$$r \leq \frac{P}{l} \quad . \tag{ii}$$

Figure 3 displays the solutions with the assumed congestion and demand functions, and parameter values, for two demand curves, D_1 and D_2, corresponding to different levels of demand intensity. The curbside parking capacity constraint is labeled CPC. We define the (unconstrained) *short-run social marginal cost of throughput* as $\rho[\partial T/\partial r]$, where $\partial T/\partial r$ is the derivative of the smaller root of T in the steady-state condition, holding P fixed and ignoring the curbside parking capacity constraint, and label the corresponding locus as $SRMSC(r; P)$.

[9] In writing out the various maximization problems presented in the paper, we omit the obvious non-negativity constraints on r, T, C, P, and the density of garage parking spaces, $rl - P$.

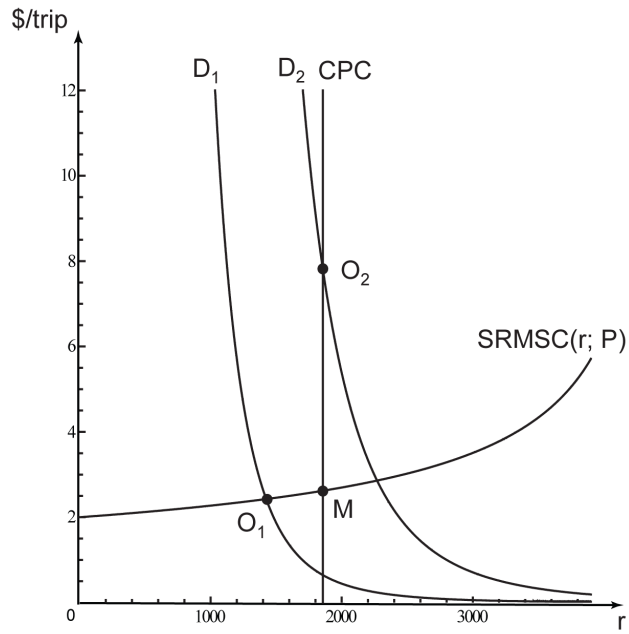


Figure 3: First-best optimum in the short run with only curbside parking

With demand curve D_1, the curbside parking capacity constraint does not bind, and the first-best optimum, O_1, is at the point of intersection of the demand and $SRMSC$ curves. Since we have assumed that the reduction in roadside capacity caused by curbside parking depends on the amount of curbside allocated to parking, independent of its occupancy rate, the marginal driver generates no parking externality. Short-run marginal social cost therefore equals user cost plus the congestion externality cost, so that the social optimum can be decentralized by setting the parking fee over the duration of the visit equal to the congestion externality cost, which is calculated per Figure 2.

With demand curve D_2, the curbside parking capacity constraint binds, and the first-best optimum, O_2, is at the point of intersection of the demand curve and the curbside parking capacity constraint. The short-run marginal social cost now equals the user cost plus the congestion externality cost plus a *parking scarcity rent*, which is given by the vertical distance between O_2 and M, the point of intersection of $SRMSC(r; P)$ and CPC. The social optimum

can be decentralized by setting the parking fee over the duration of the visit equal to the congestion externality cost plus the parking scarcity rent.

We could have proceeded alternatively by defining the *capacity-constrained short-run marginal social cost curve* as the short-run marginal social cost curve for levels of throughput below the capacity constraint, combined with that portion of the capacity constraint above its point of intersection with the (unconstrained) short-run marginal social cost curve. The short-run, first-best optimum then lies at the point of intersection of the demand curve and the capacity-constrained short-run marginal social cost curve.

4.1.2 Second-best optimum in the short run with only curbside parking

There are two distortions in the second-best problem. No congestion toll can be charged, and the parking fee is not optimal.[10] The second-best optimization problem in the short run is degenerate in that the constraints determine the solution because both the parking fee and capacity are given. The second-best optimum is therefore the equilibrium that generates the highest social surplus.

An equilibrium may entail unsaturated or saturated parking. Consider first equilibria with unsaturated parking. Since parking is unsaturated, there is no cruising for parking. The user cost is $UC = \rho mt(T, 0, P)$ and the full price is

[10] A caveat is in order. In the paper, the underpricing of curbside parking is taken as an exogenous distortion, whereas in a broader model it would be treated as endogenous. The common explanation for the underpricing of curbside parking is that downtown merchant associations lobby hard to keep curbside parking meter rates low so that downtown shops be more competitive with suburban shopping centers, most of which provide free parking. This in turn raises the question of why suburban shopping centers provide free parking. One explanation is that minimum parking requirements at suburban shopping centers are so excessive that there is an excess supply of parking there, even at a zero price. This in turn raises the question of why minimum parking requirements at suburban shopping centers are so excessive. And so on. The paper's conclusions might be altered if such considerations were taken into account. One might say that we are committing the functionalist fallacy in reverse. The functionalist fallacy in this context is to assume that, since the underpricing of curbside parking is so widespread, it must be for good reason. The functionalist fallacy in reverse is to overlook that curbside parking might be underpriced for good reason.

$F = UC + fl$, where T satisfies the steady-state condition. From these results, the *unsaturated user cost curve* for the exogenous level of P, $UC(r; P)$, can be derived, which differs from the user cost curve of the previous section only in that curbside parking reduces road capacity. At levels of throughput where the curbside parking capacity constraint does not bind, the supply curve is obtained as the unsaturated user cost curve shifted up by fl, and any point of intersection of the demand curve and this portion of the supply curve is an unsaturated equilibrium.

Now consider equilibrium with saturated parking. Parking is saturated because the curbside parking capacity constraint binds, and except in the situation where it just binds there is cruising for parking. Equilibrium therefore entails two density variables, the density of cars in transit and the density of cars cruising for parking, C. They are determined by two equilibrium conditions. The first is the familiar steady-state condition modified to take into account cruising for parking:

$$D(F) = \frac{T}{mt\,(T, C, P)} \quad , \tag{12}$$

where the full price equals the in-transit travel time cost, plus the expected cruising-for- parking time cost, plus the parking fee:

$$F = \rho mt\,(T, C, P) + \frac{\rho Cl}{P} + fl \quad . \tag{13}$$

Since C cars are cruising for parking per unit area and since the turnover rate of curbside parking spaces is P/l per unit area, the probability that a car cruising for parking gets a space per unit time is $P/(Cl)$, so that expected cruising-for-parking time is Cl/P.

The second equilibrium condition, the *cruising-for-parking equilibrium condition*, is that the rate at which cars enter cruising for parking, which equals the rate at which they exit the in-transit pool, equals the rate at which cars

exit cruising for parking, which equals the parking turnover rate:

$$\frac{T}{mt\left(T,C,P\right)}=\frac{P}{l} \quad . \tag{14}$$

The steady-state condition and the cruising-for-parking condition provide two non-linear equations in two unknowns, T and C. Their analysis is complex. Arnott and Inci (2006) derive the conditions under which the two curves intersect in T-C space, and for which therefore there exists a saturated equilibrium. Furthermore, they prove that (with $\theta \geq 1$), if a saturated equilibrium exists, it is unique. Here, we derive the properties we need for our diagrammatic analysis through heuristic argument. We ask: What are the minimum and maximum full prices consistent with saturated parking, and therefore with (14) being satisfied? For a given level of P and with $C = 0$, (14) has two roots for T. The smaller root, for which travel is congested, corresponds to the minimum full price, and the larger root, for which travel is hypercongested, corresponds to the maximum full price (higher prices correspond to traffic jams that yield a level of throughput less than P/l).

Turn to Figure 4. First, plot the unsaturated user cost curve for the level of P corresponding to the curbside parking capacity constraint. Second, shift this curve up by fl, yielding the *unsaturated full price curve*. Third, draw in the curbside parking capacity constraint. The portion of the unsaturated full price curve to the right of the curbside parking capacity constraint is not relevant to the analysis. The supply curve has three parts, the upward-sloping and backward-bending portions of the unsaturated full price curve to the left of the curbside parking capacity constraint, and the portion of the curbside parking constraint between these two portions of the unsaturated full price curve. One may alternatively obtain the supply curve as the capacity-constrained user cost curve (defined analogously to the capacity-constrained marginal social cost curve), shifted up by fl.

Figure 4 shows three demand curves, each corresponding to a different level of demand intensity. While not obvious from the diagram, for all three demand

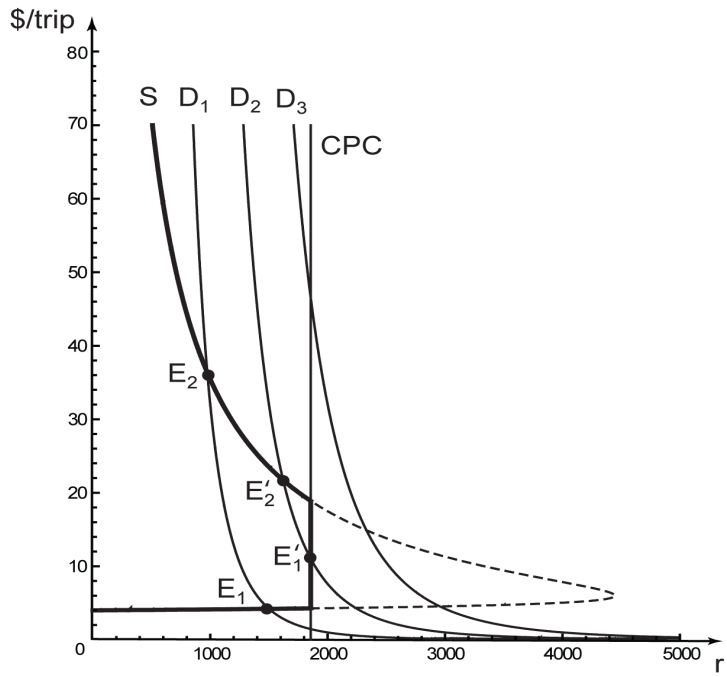


Figure 4: Equilibria in the short run with only curbside parking

curves, gridlock is an equilibrium. With gridlock, the steady-state condition is satisfied since the entry and exit flows are both zero, and parking is unsaturated. With low demand intensity (demand curve D_1 with $D_0 = 2000$), there are three equilibria: E_1, which is unsaturated, congested, and stable; E_2, which is unsaturated, hypercongested, and unstable; and the gridlock equilibrium. With medium demand intensity (demand curve D_2 with $D_0 = 3000$), there are again three equilibria: E_1', which is saturated and stable, and may be either congested or hypercongested; E_2', which is unsaturated, hypercongested, and unstable; and the gridlock equilibrium. With high demand intensity (demand curve D_3 with $D_0 = 4000$), the equilibria corresponding to E_1 and E_2 disappear, with only the gridlock equilibrium remaining. Social surplus equals consumer surplus plus curbside meter revenue. For each of the three demand curves, both consumer surplus and curbside meter revenue are highest at the equilibrium corresponding to the lowest full price. Thus, for the three demand curves, the short-run second-best optima are E_1, E_1', and the gridlock

equilibrium, respectively.

Consider the equilibrium E_1' in more detail. In this saturated equilibrium, the stocks of cars in transit and cruising for parking adjust to clear the market, such that the full price is at the point of intersection of the demand curve and the curbside parking capacity constraint. The equilibrium values of T and C are 444 and 392, which imply a velocity of 8.36 mph and hence hypercongested travel. The full price of a trip is \$11.03, of which \$4.78 is in-transit travel time cost, \$4.24 is expected cruising-for-parking time cost, and \$2.00 is the curbside parking fee. The deadweight loss associated with inefficient pricing in this equilibrium equals social surplus at the first-best optimum minus social surplus in the equilibrium. The first-best optimum is also at E_1', and has $T = 211$ and $C = 0$, and in-transit travel time cost of \$2.27. Thus, the deadweight loss is \$6.75 per driver and \$12500 per ml^2-hr. The first-best optimum can be decentralized by charging each driver \$8.75 for curbside parking for the two hours, which is achieved with a curbside parking fee of \$4.37/hr.

Raising the parking fee causes the supply curve to shift up by the increase in the parking payment. When the parking fee is raised to a level between \$1.00/hr and \$4.37/hr, the equilibrium remains at E_1'. Raising the parking fee over this range has no effect on the equilibrium full price but results in increased parking fee revenue. Thus, the extra revenue is raised with zero burden. An obvious question is then why local governments choose to forgo such an efficient source of revenue.

4.2 The long run with only curbside parking

4.2.1 First-best optimal curbside parking capacity with only curbside parking

When curbside parking is saturated, increasing curbside parking capacity by a small amount has two effects, one positive and one negative. The positive effect is to raise throughput and hence the social benefit from travel, the area under

the demand curve and to the left of the curbside parking capacity constraint. The negative effect is to reduce the amount of road space available to traffic flow, which causes the unsaturated user cost curve to rise. At first-best optimal capacity, the two effects balance at the margin.

We now develop a geometric construct to determine first-best curbside parking capacity. Return to Figure 3. What is the cheapest way of achieving throughput of 1856? M indicates the point of intersection of $SRMSC(r; P)$ and the curbside parking capacity constraint $r = P/l$. It therefore corresponds to the point $SRMSC(P/l; P)$. It is also at the lower kink point of the capacity-constrained short-run marginal social cost curve corresponding to $r = P/l$. If parking capacity is reduced slightly, the throughput of 1856 cannot be achieved. If parking capacity is increased slightly to $P+dP$, the throughput of 1856 can be achieved but at a higher social cost, since $SRMSC(r; P)$ lies below $SRMSC(r; P + dP)$. Thus, M gives the minimum marginal social cost associated with the throughput of 1856. At M, curbside parking is saturated and there is no parking scarcity rent.

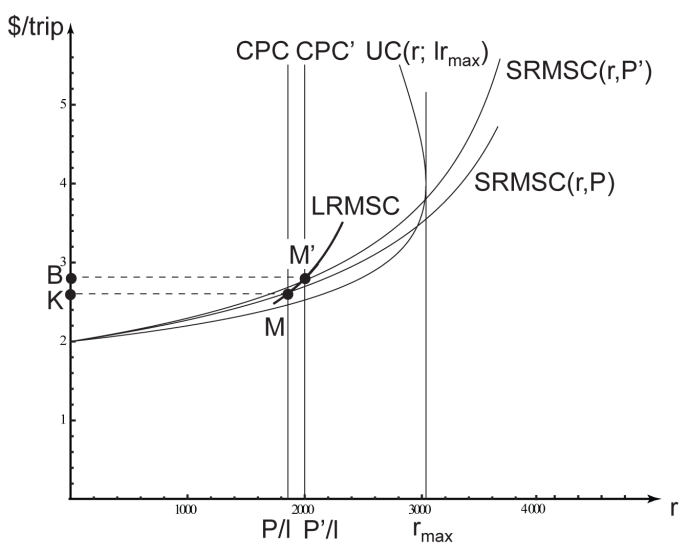


Figure 5: Construction of the long-run marginal social cost curve with only curbside parking

There is a point corresponding to M for every level of throughput, up to

some maximum level of throughput, $r_{\max}$. Joining these points gives the long-run marginal social cost curve,[11] $LRMSC$. Here, $r_{\max}$ is the maximum level of throughput that can be accommodated on downtown streets, and is that level of throughput for which the curbside parking capacity constraint is tangent to the corresponding user cost curve. Figure 5 indicates how two points on the $LRMSC$ curve are obtained, the point M corresponding to $r = P/l = 1856$ and the point M' corresponding to $r = P'/l = 2008$, as well as how $r_{\max}$ is determined.

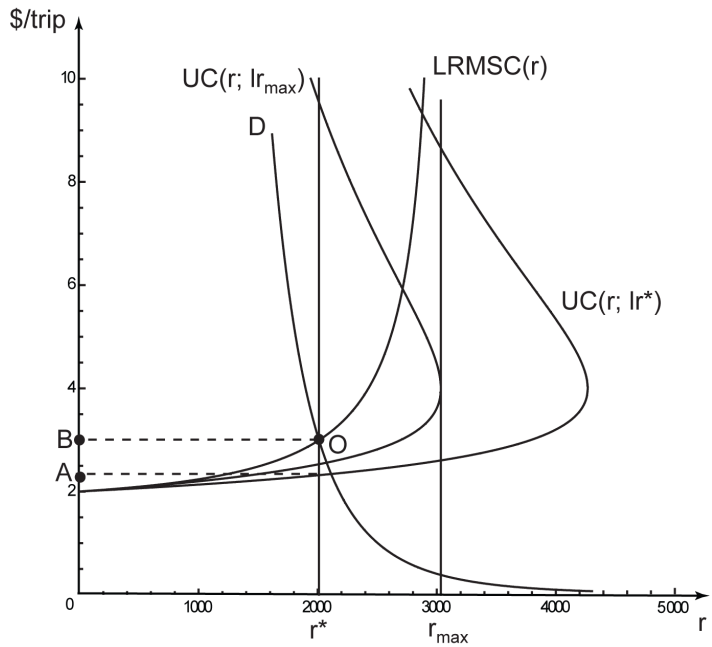


Figure 6: The first-best optimal curbside parking capacity with only curbside parking

[11] Long-run social cost is ρT. Thus, long-run marginal social cost equals $\rho(dT/dr)$, where dT/dr is the change in T induced by a change in r such that: (i) the steady-state condition is satisfied; and (ii) parking capacity is increased along with r such that the curbside parking capacity constraint just binds. Let $T(r, P)$ denote the lower value of T satisfying $r = T/(mt(T, 0, P)$. Then, $LRMSC(r) = \rho dT/dr = \rho(\partial T/\partial r) + \rho(\partial T/\partial P)(dP/dr) = \rho(\partial T/\partial r) + \rho l(\partial T/\partial P)$. Here, $\rho(\partial T/\partial r) - UC(r, P)$ is the congestion externality cost, and $\rho l(\partial T/\partial P)$ is the parking externality cost. When throughput is increased by one unit, the number of curbside parking spaces increases by l units. This reduces the road space available for traffic flow and hence increases congestion. The parking externality cost is the increase in aggregate in-transit travel time costs associated with this increased congestion.

Figure 6 displays the social optimum corresponding to a demand intensity of 2500. The socially optimal level of throughput, $r^* = 2008$, is given by the point of intersection of the corresponding demand curve and $LRMSC(r)$, O ($= M'$ in Figure 5) in the diagram, first-best optimal capacity is $P^* = r^*l = 4015$, and $LRMSC(r^*) = \$2.99$. Decentralization of the social optimum entails charging a parking fee that solves $f^*(r^*)l = LRMSC(r^*) - UC(r^*; r^*l)$.

4.2.2 Second-best optimal curbside parking capacity with only curbside parking

The distortions in the second best are the absence of congestion tolling and the underpricing of curbside parking. The way we shall proceed is to determine whether, at a short-run second-best optimum, social surplus is increased or decreased with an increase in curbside parking capacity. Recall that social surplus equals the sum of consumer surplus and curbside parking fee revenue. Turn back to Figure 4. The short-run second-best optimum occurs at the lowest point of intersection of the supply curve and the demand curve. For the moment, assume that the price associated with this point of intersection is finite. If the short-run second-best optimum is saturated and has cruising for parking, social surplus is increased by increasing curbside parking capacity. The vertical portion of the supply curve shifts to the right, resulting in an increase in throughput, and hence an increase in both consumer surplus and parking fee revenue. If the short-run second-best optimum is unsaturated, social surplus is increased by decreasing curbside parking capacity. There are two cases to consider, that where the short-run second-best optimum lies on the upward-sloping portion of the supply curve, and that where it lies on the backward-bending portion. Consider the former case first. Decreasing curbside parking capacity shifts the upward-sloping portion of the supply curve down, resulting in an increase in throughput, and hence an increase in both consumer surplus and parking fee revenue. Now consider the latter case, where the short-run second-best optimum lies on the backward-bending portion of the supply curve. Reducing curbside parking capacity causes the unsaturated

portion of the supply curve to shift to the right. Since, at the short-run second-best optimum, the demand curve is flatter than the backward-bending portion of the supply curve, the equilibrium moves down the demand curve, again increasing social surplus. Thus, long-run second-best optima have the property that the capacity constraint just binds.

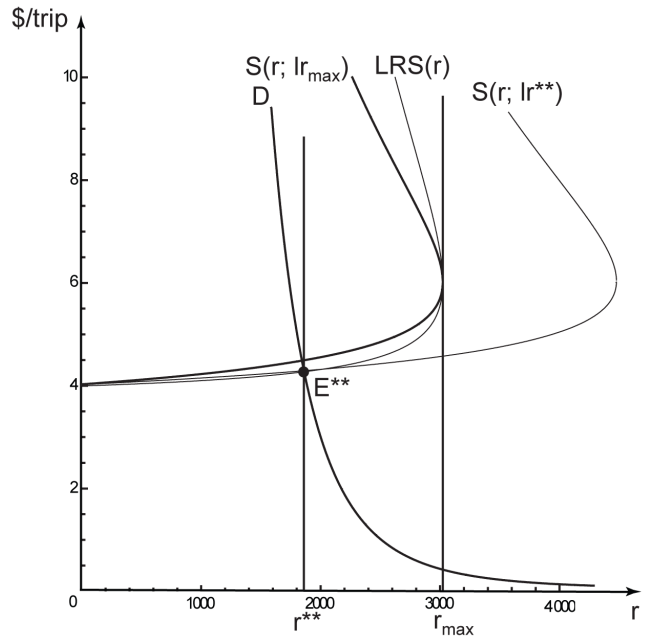


Figure 7: Second-best optimal curbside parking capacity in the long run with only curbside parking

This result points to a method for determining second-best throughput and capacity, analogous to that employed in the previous subsection. Plot the $UC(r; rl)$ curve, along which the capacity constraint just binds. Shifting the curve up by fl generates the long-run supply curve, $LRS(r)$. The long-run second-best optimal throughput, r^{**}, corresponds to the point of intersection of the demand curve and the long-run supply curve with the highest level of throughput. And second-best optimal capacity equals $P^{**} = r^{**}l$. Figure 7 displays this construction, and second-best optimal capacity when $D_0 = 2500$. Here, $r^{**} = 1870$, $P^{**} = r^{**}l = 3740$, and $LRS(r^{**}) = \$4.27$.

The relationship between the first- and second-best optimal capacities can

be inferred from Figure 8, which plots the $LRMSC(r)$, $LRS(r)$, and the demand curve. With both first- and second-best optimal capacities, the curbside parking capacity constraint just binds. Thus, parking capacity equals throughput times visit length, so that the analysis can be conducted in terms of throughput. The long-run first-best optimum is at the point of intersection of $LRMSC(r)$ and the demand curve, and the long-run second-best optimum is at the point of intersection of $LRS(r)$ and the supply curve.

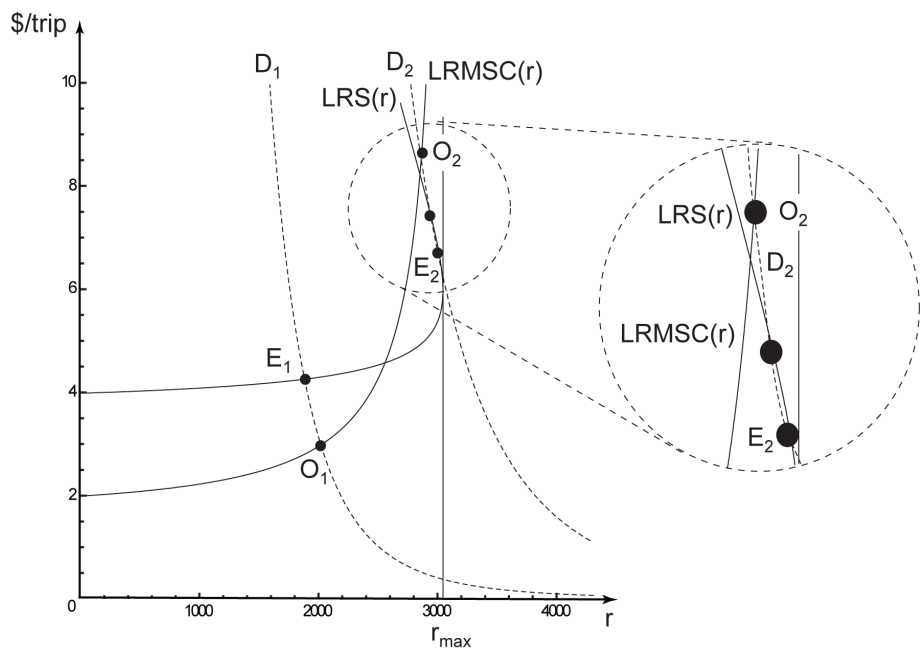


Figure 8: The relationship between first- and second-best optimal curbside parking capacity in the long run with only curbside parking

Consider first the case where the demand curve intersects the long-run supply curve on its upward-sloping portion. There are two sub-cases. In the first ("low" demand), which corresponds to D_1, E_1, and O_1 in the figure, at a price of \$1/hr curbside parking is overpriced, so that the LRS curve lies above the $LRMSC$ curve, and $P^* > P^{**}$. In the second sub-case ("moderate" demand), which is not shown in the figure, at a price of \$1/hr curbside parking is underpriced, so that the LRS curve lies below the $LRMSC$ curve, and $P^* < P^{**}$.

Consider next the case ("high" demand) where the demand curve intersects the long-run supply curve on its backward-bending portion. The second-best optimum lies at the lower point of intersection of the two curves, E_2 in the figure. In the numerical example, at the second-best optimal level of throughput, $LRMSC$ is greater than LRS, implying that curbside parking is underpriced and $P^* < P^{**}$. It is possible that at the second-best optimal level of throughput $LRMSC$ is less than LRS, implying that curbside parking is overpriced and $P^* > P^{**}$. Note that over the range of demand intensities for which this case applies, as demand intensity increases, second-best parking capacity falls.

At "very high" levels of demand intensity, the demand curve intersects the long-run supply only with gridlock. The second-best level of curbside parking capacity is zero and $P^* > P^{**}$.

This discussion leads to two central results. First, *at any long-run second-best optimum, parking is saturated but there is no cruising for parking.* And second, *second-best curbside parking capacity exceeds first-best curbside parking capacity when curbside parking is underpriced at the long-run second-best optimum, and falls short of first-best curbside parking capacity when curbside parking is overpriced at the second-best optimum.*

5 Downtown Traffic Congestion with Both Curbside and Garage Parking

In the downtowns of towns and small cities, there is typically enough parking space curbside to accommodate demand without severely impeding traffic flow. But in most locations where traffic congestion is significant, curbside parking needs to be supplemented by off-street parking, whether in a parking lot or garage.

We shall treat off-street parking – which we shall refer to generically as garage parking – in the simplest possible way, by assuming that it is provided

continuously over space by the private sector at a cost of $c = \$2.5/\text{hr}$ per space. In fact, in the downtowns of major metropolitan areas, there is typically an irregular grid of parking garages, some public, some private, which engage in spatial competition with one another. Arnott and Rowse (2009) model this spatial competition, taking into account the technology of garage construction. But, here, we provide a simpler treatment,[12] in order to simplify analysis.

5.1 First-best optimal curbside parking capacity with both curbside and garage parking

The full first-best problem is to maximize social surplus subject to the steady-state condition, and with respect to r, T, and P. Social surplus equals social benefit minus social cost. For a particular level of throughput, social benefit equals the area under the demand curve up to that level of throughput, and social cost equals aggregate in-transit travel time cost and aggregate garage parking cost. In the first best, efficient supply can be analyzed independently of the level of demand. In the first stage, for each level of throughput, the planner decides on the combination of curbside and garage parking that minimizes aggregate cost. In the second stage, she decides on the surplus-maximizing level of throughput.

We start by treating the first stage. Three marginal social cost curves can be obtained at this stage: MSC_1 is the marginal social cost curve for régime 1, where only curbside parking is provided; MSC_2 is the marginal social cost curve for régime 2, where both curbside and garage parking are provided; and MSC_3 is the marginal social cost curve for régime 3 in which only garage parking is provided. Régime 1 is defined only for the interval of throughputs that are achievable when only curbside parking is provided. Régime 2 is defined only for that interval of throughputs for which a mix

[12] We could alternatively assume that there is an upward-sloping supply schedule for garage parking. If the supply schedule starts at a price below a zero price, then some garage parking is always provided, and the dichotomy between low-demand situations where garage parking is not provided and higher-demand situations in which it is disappears.

of curbside and garage parking is efficient. And régime 3 is defined only for that interval of throughputs that are achievable when only garage parking is provided.

The maximum level of throughput consistent with only curbside parking, which was labeled $r_{\max}$ in Section 4.2.1, is the largest value of r satisfying the steady-state condition, and is therefore solved as $\max_{r,T} r$ subject to the constraint $r = T/(mt(T, 0, rl)$. In the numerical example, this maximum value is 3034. Any level of throughput up to this maximum value is consistent with only curbside parking. The maximum level of throughput consistent with only garage parking is the largest value of r satisfying the steady-state condition, and is therefore solved as $\max_{r,T} r$ subject to the constraint $r = T/(mt(T, 0, 0)$. In the numerical example, this maximum value is 6668. Any level of throughput up to this maximum value is consistent with only garage parking.

The analysis of régime 2 is more complex. Resource costs with throughput r are $RC_2(r) = \rho T + c(rl - P)$. These resource costs are minimized subject to the steady-state condition, with respect to T, r, and P. Régime 2 applies only in the interval of throughputs for which the first best entails both curbside and garage parking (*i.e.* for which the maximizing P is strictly greater than zero and strictly less than rl). The minimum throughput consistent with this requirement is 2936, and the maximum throughput consistent with this condition is 6212. For low levels of throughput, below 2936, providing only curbside parking is efficient. For intermediate levels of throughput, above 2936 and below 6212, providing a mix of curbside and garage parking is efficient. And for high levels of throughput, above 6212, providing only garage parking is efficient.

Figure 9, Panel A displays the average and marginal cost curves for each of the three régimes. The average cost curve, ASC, is the lower envelope of the régime-specific average cost curves. Each régime-specific average and marginal cost curve is drawn as a solid line for the interval of throughputs over which the régime is average-cost minimizing. The régime 1 and 3 mar-

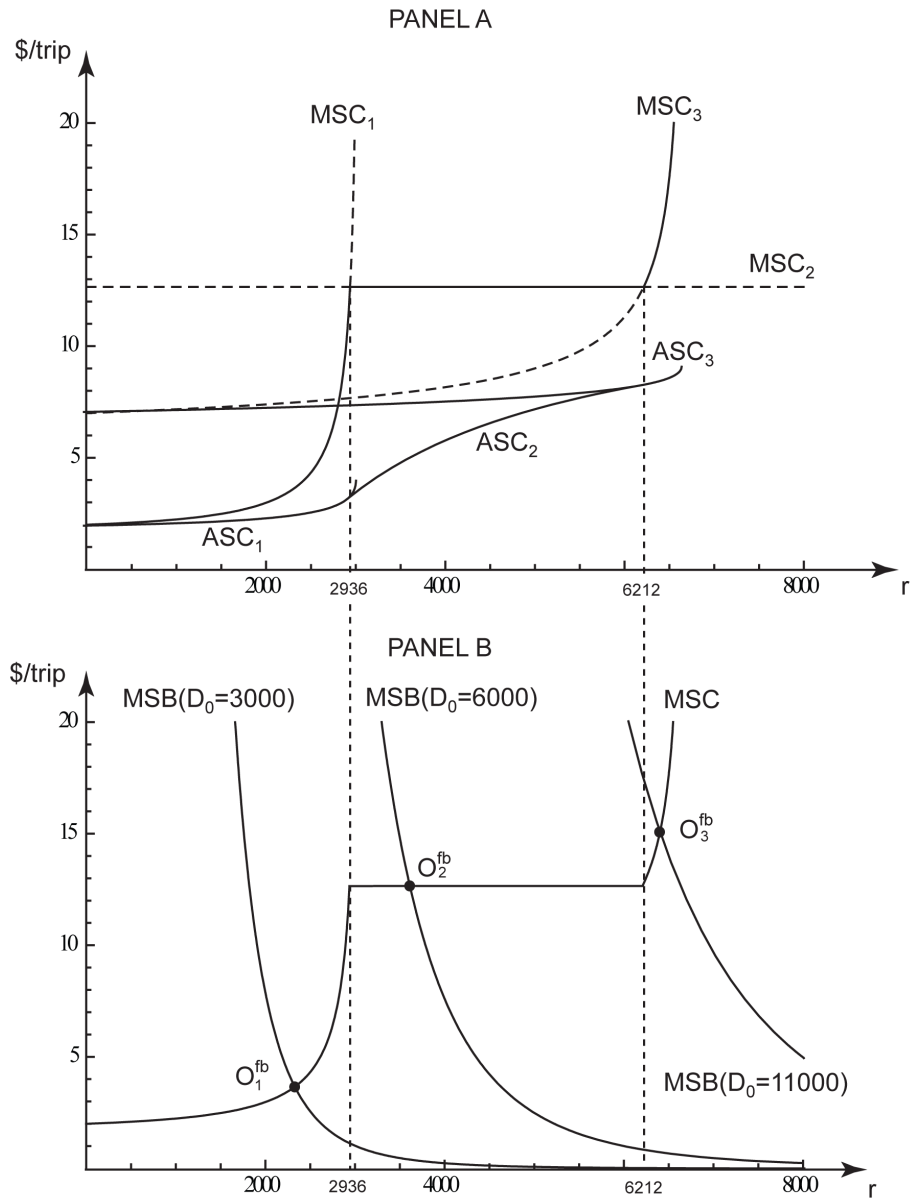


Figure 9: Marginal cost, average cost, and marginal benefit curves in the first-best optimum with both curbside and garage parking

ginal cost curves over the interval of throughput where they are defined but not average-cost minimizing are drawn as dashed lines; régime 2, meanwhile, is defined only over the interval of throughput where it is average-cost minimizing.

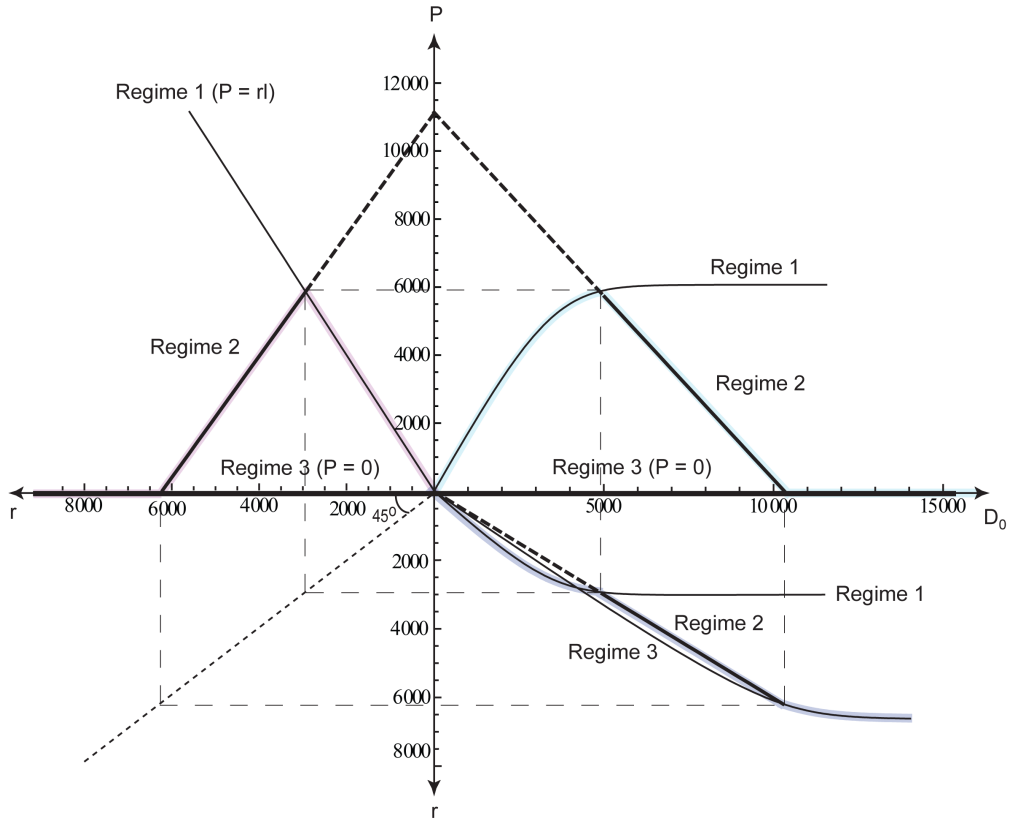
$ASC_1(r)$ is $\rho T/r$, where T is the smaller root satisfying $r = T/(mt(T, 0, rl))$. $MSC_1(r)$ is the same as $LRMSC(r)$ in Figure 6. Both the régime 1 average and marginal social cost curves are upward sloping since an increase in r is associated with both higher traffic density and reduced road space for travel. $ASC_2(r)$ is $\rho T/r + c(rl - P(r))/r$, where T is the smaller root satisfying $r = T/(mt(T, 0, P(r))$ and $P(r)$ is the average-cost minimizing curbside parking capacity as a function of throughput. The régime 2 marginal social cost curve is horizontal.[13] $ASC_3(r)$ is $\rho T/r + cl$, where T is the smaller root satisfying $r = T/(mt(T, 0, 0))$, and $MSC_3(r)$ is upward sloping since an increase in r is associated with higher traffic density.

Figure 9, Panel B displays the social optimum at three levels of demand intensity, a low demand intensity ($D_0 = 3000$) social optimum where only curbside parking is optimal, O_1^{fb}, a medium demand intensity ($D_0 = 6000$) social optimum where a mix of curbside and garage parking is optimal, O_2^{fb}, and a high demand intensity ($D_0 = 11000$) social optimum where only garage parking is optimal, O_3^{fb}. Figure 10 shows the relationship between P and D_0 (quadrant I), P and r (quadrant II), and r and D_0 (quadrant IV), as D_0 changes, for each of the three régimes and for the full optimum (shown as

[13] This result is due to a property of the travel time (congestion) function, that travel time is increasing in the ratio of traffic density to effective jam density, $T/(V_j(1 - P/P_{\max}))$ or $TP_{\max}/V_j(P_{\max} - P) \equiv \xi$. With this property, $t(T, 0, P) = \hat{t}(\xi(T, P))$, and $t_T = \hat{t}'\xi/T$ and $t_P = \hat{t}'\xi/(P_{\max} - P)$. In this régime, the cost minimization problem is $\min_{T,P} \rho T + c(rl - P)$ subject to $r = T/(mt(T, 0, P)$. Letting λ denote the Lagrange multiplier on the constraint, the first-order conditions for T is $\rho - \lambda r(1/T - t_T/t) = 0$ and for P is $-c + \lambda r t_P/t = 0$. It turns out that in this régime, it is cost-minimizing to increase r by increasing T in direct proportion to r, which implies that t and hence ξ remains unchanged, which in turn implies that $P_{\max} - P$ increases in direct proportion to r. Rewriting the first-order condition for T as $\rho - \lambda(r/T)(1 - t_T T/t) = \rho - \lambda(r/T)(1 - \hat{t}'\xi/t) = 0$ and that for P as $-c + \lambda r t_P/t = -c + \lambda(r/(P_{\max} - P)(\hat{t}'\xi/t) = 0$, it can be seen that the first-order conditions continue to be satisfied when r is increased in the manner indicated, and furthermore that λ, marginal social cost, remains constant.

In the example, a unit increase in throughput is accompanied by a reduction in the number of curbside parking spaces of 1.79, and hence results in an increase in the number of garage parking spaces of 3.79, and an increase in the cost of garage spaces of \$9.48. Furthermore, since T increases in direct proportion to r, the increase in total in-transit travel cost equals the increase in in-transit cost of the added driver, which has a cost of \$3.17. Thus, the marginal social cost equals \$12.65.

highlighted curves). Note that all the functions are continuous.



Note: Highlighted paths denote optimal régimes.

Figure 10: Régimes in the first-best optimum with both curbside and garage parking

We now discuss decentralization of the first best. One scheme that works is for the planner to set a congestion toll equal to the congestion externality cost and the curbside parking fee equal to the parking externality cost, both evaluated at the social optimum, and to choose the optimal curbside parking capacity, leaving it to individuals to decide how frequently to travel and to the private sector to decide on the garage parking fee and the number of garage spaces. Since garage parking is continuously produced over space at constant unit cost, competition between garage operators results in the equilibrium garage price equaling this unit cost. When the optimum entails only curbside

parking, the full price of an auto trip equals its social cost, so that individuals choose the optimal number of trips. Furthermore, since only curbside parking is provided, the full price of curbside parking falls short of the full price of garage parking, so that there is no demand for garage parking and hence no garage parking supplied by the private sector. When the optimum entails both curbside and garage parking, their shadow prices are the same, as are the corresponding full prices that individuals face when the government sets the congestion toll and curbside parking fee at the appropriate level in the absence of cruising for parking. If explicit congestion tolling is infeasible, the social optimum can still be achieved by raising the curbside fee and imposing a tax on garage parking, both by the amount of the optimal congestion toll. Thus, the full price of curbside parking equals the full price of garage parking, and drivers face the marginal social cost of a trip, evaluated at the social optimum, and so choose the optimal number of trips. Furthermore, garage parking operators choose to provide the socially optimal amount of garage parking; if they provide less, there is excess demand for parking, which induces an increase in the quantity supplied, and if they provide more, there is excess supply, which induces a decrease in the quantity supplied. Finally, since the full prices of curbside and garage parking are equal, and since the quantity of parking supplied equals the quantity of parking demanded, curbside parking is saturated and there is no cruising for parking.

5.2 Second-best optimal curbside parking capacity with both curbside and garage parking

In the second best, the government decides on curbside parking capacity, with the curbside parking fee set exogenously, with no congestion toll applied, and with the private sector deciding the garage parking fee and garage parking capacity. The analysis is considerably more difficult than for the first best. First, when curbside and garage parking are simultaneously provided in equilibrium, if the curbside parking fee is less than the garage parking fee, then the stock

of cars cruising for parking adjusts to equilibrate the full prices of curbside and garage parking. Second, the second-best analysis does not permit the neat separation of supply and demand that occurred in the first-best analysis. The first-best analysis proceeded in two stages: first, for every level of throughput, the mix of curbside and garage parking that minimizes social cost was calculated (Figure 9, Panel A); and second, adding a demand function, the full optimum was determined by the intersection of the demand curve and the relevant marginal social cost curve (Figure 9, Panel B). In the second-best analysis, in contrast, the private sector provides garage parking when and only when it is profitable for it to do so, which depends on the level of demand.

We shall proceed by first examining equilibrium in the short run, in which curbside parking, though not garage parking, is fixed. This will provide insight into how to formulate the government's second-best choice of curbside parking capacity, as a function of the level of demand intensity.

5.2.1 The short run with both curbside and garage parking

We define a short-run equilibrium to be an equilibrium in which curbside parking is fixed but in which the amount of garage parking is variable, being decided by the private sector. We assume that curbside parking is priced below the unit cost of garage parking (which is the normal case in the US though not in all of Western Europe), and that the government does not tax or subsidize garage parking. As in the short-run equilibrium analysis of the previous section, the government has no policy instruments at its disposal, so that the second-best optimization problem is degenerate, with the short-run second-best optimum coinciding with one of the short-run equilibria.

When there are both curbside and garage parking, drivers choose whichever is cheaper. Thus, when the curbside parking fee is less than the garage parking fee, and when the private sector chooses to provide garage parking, the stock of cars cruising for parking adjusts so that the full prices of curbside and garage parking are equalized. Since the full price of curbside parking is $fl + \rho Cl/P$

and that of garage parking is cl, the full prices are equalized when

$$fl + \rho \frac{Cl}{P} = cl \quad . \tag{15}$$

We term this the *full price equalization condition.* Rearranging gives

$$C = (c - f)\,\frac{P}{\rho} \equiv \hat{C} \quad . \tag{16}$$

Thus, when both curbside and garage parking are provided in equilibrium, the stock of cars cruising for parking increases in proportion to the differential between the garage and curbside parking fee and to the density of curbside parking spaces. This yields the obvious but important point that cruising for parking does not occur when no curbside parking is provided.

We start by defining three different short-run full price curves, as functions of the level of throughput. The first corresponds to the situation where there is no garage parking and no cruising for parking. We have defined régime 1 to be the régime in which there is no garage parking, whether or not there is cruising for parking. We define régime 1A to correspond to situations in which there is no garage parking and there is no cruising for parking, and régime 1B to correspond to situations in which there is no garage parking and there is cruising for parking. The full price in régime 1A is $F_{1A} = \rho mt(T, 0, P) + fl$. Applying the steady-state condition per the construction of Figure 4 generates a locus relating r to F_{1A}, which we refer to as the *régime 1A short-run full price curve.* The full price in régime 2 is $F_2 = \rho mt(T, \hat{C}, P) + cl = \rho mt(T, \hat{C}, P) + \rho \hat{C} l/P + fl$, where $\hat{C}$ is given by (16), from which the *régime 2 short-run full price curve* is constructed. The full price in régime 3 is $F_3 = \rho mt(T, 0, 0) + cl$, from which the *régime 3 short-run full price curve* is constructed.

Figure 11 is like Figure 4, but adds garage parking. Since curbside parking is provided, the figure does not treat the situation where only garage parking is provided. The figure displays three loci, the régime 1A full price curve, the régime 2 full price curve, and the curbside parking capacity constraint, $P/l = 1856$. Because there are $\hat{C}$ cars cruising for parking in régime 2, but none

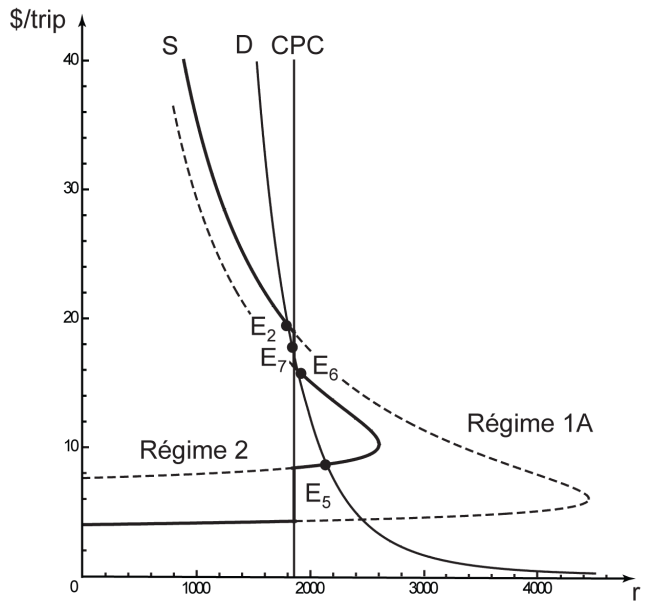


Figure 11: Equilibria with both curbside and garage parking

in régime 1A, the régime 2 full price curve has a higher F-intercept than the régime 1A full price curve and a lower maximum throughput. The two full price curves may or may not intersect; if they do, at an intersection point, travel in régime 1A is hypercongested while that in régime 2 may be congested or hypercongested. For levels of throughput below the curbside parking capacity constraint, curbside parking is unsaturated, so that it is unprofitable for the private sector to provide garage parking, and the régime 2 full price curve is drawn as a dashed curve since it is inapplicable. For levels of throughput above the curbside parking capacity constraint, both curbside and garage parking are provided, and the régime 1A full price line is drawn as a dashed curve since it is inapplicable.

The supply curve, shown as the bold curve in the figure, contains five portions: the portion of the régime 2 full price curve to the right of CPC; the two portions of the régime 1A full price curve to the left of the CPC, one upward-sloping, the other backward-bending; and the two segments of the CPC, each joining the régime 1A and régime 2 full price curves. The last

two segments corresponds to situations in which the curbside parking rent is positive, but not sufficiently high to make private sector provision of garage parking possible, and in which therefore there is only curbside parking and there is cruising for parking (these two segments therefore correspond to the régime 1B full price curve).

The demand curve in the figure is drawn for $D_0 = 3300$. Equilibria correspond to points of intersection of the demand and supply curves. With this level of demand intensity, there are five equilibria, one of which, the gridlock equilibrium, is not displayed on the diagram. These equilibria alternate between stable and unstable. There are three stable equilibria. One is the gridlock equilibrium; the second, E_7, is a hypercongested equilibrium with saturated curbside parking, a stock of cars cruising for parking below $\hat{C}$, and no garage parking; and the third, E_5, is a congested equilibrium with saturated parking, a stock of cars cruising for parking equal to $\hat{C}$, and garage parking.[14] Among the equilibria, E_5 has the highest social surplus, and is therefore the short-run second-best optimum. It has the highest level of throughput, hence the lowest full price, and hence the highest consumer surplus. Furthermore, curbside parking is saturated, so that curbside parking fee revenue is maximized.

Figure 11 illustrates an important point. Starting at E_5, raising the curbside meter rate does not change the full price of garage parking, cl, but via (16) causes the stock of cars cruising for parking to fall. Cruising-for-parking time costs fall dollar for dollar with the increase in the curbside meter revenue. The decrease in the stock of cars has the added benefit that in-transit travel costs fall. Thus, raising the curbside meter rate generates an increase in social

[14] It can be shown that with $D_0 = 3300$, the short-run, first-best social optimum conditional on $P/l = 1856$ entails only curbside parking. Per capita resource cost at this social optimum is \$2.36, calculated as $\rho mt(T, 0, P)$ where T is the smaller root solving $r = T/(mt(T, 0, P))$. With this information, Figure 11 can be applied to calculate the deadweight losses associated with the various short-run equilibria. Since the level of throughput is the same in E_7 as in the first-best, short-run social optimum, the deadweight loss associated with it equals the amount per driver resource cost minus \$2.36, times 1856 drivers per ml^2-hr, which can be shown to equal \$26474 per ml^2-hr. The deadweight loss associated with E_5 can be shown to equal \$8488 per ml^2-hr.

surplus exceeding the curbside meter revenue collected. The increased revenue is therefore raised with negative burden! Raising the meter rate from \$1.00/hr to \$2.50/hr generates \$5568 per ml^2-hr in extra curbside meter revenue and an increase in social surplus of \$8488 per ml^2-hr.

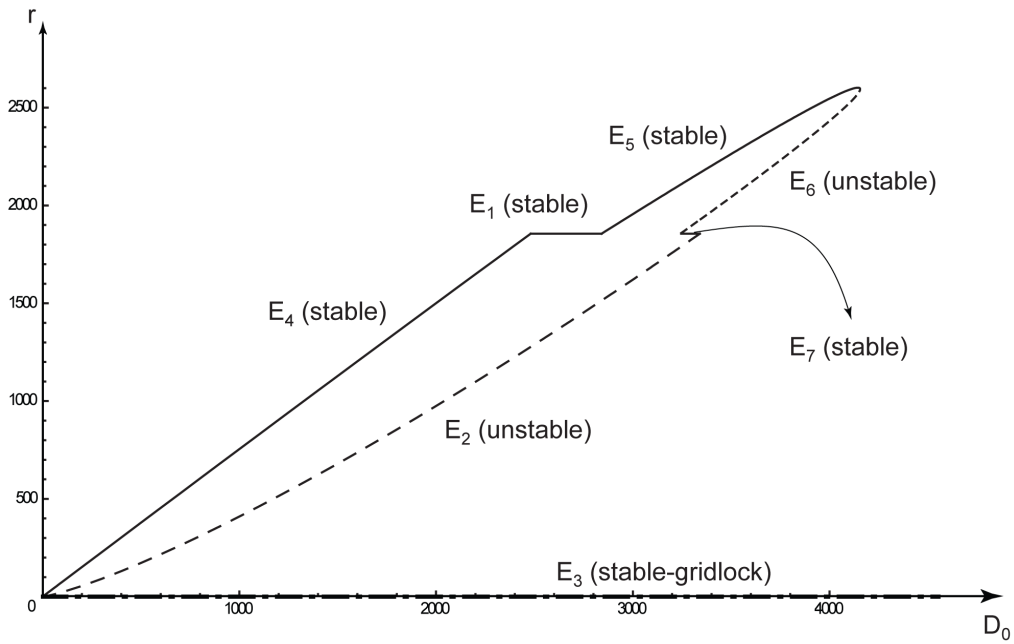


Figure 12: Bifurcation diagram

Figure 12 displays a bifurcation diagram, relating the level of throughput to demand intensity for each of the various equilibrium types, with $P/l = 1856$. The subscripts on the Es denote the type of equilibrium. Equilibria of type 4 are unsaturated and lie on the upward-sloping portion of the supply curve; an example is shown as E_1 in Figure 4 (not to be confused with equilibria of type 1, denoted by E_1 in Figure 12). Equilibria of type 1 are saturated, and lie on CPC; an example is shown as E_1' in Figure 4. Equilibria of type 3 are gridlock equilibria. All the other equilibrium types are illustrated in Figure 11. Observe that at all levels of demand intensity, the short-run second-best optimum is the equilibrium with the highest level of throughput. This equilibrium has the highest consumer surplus, and its curbside meter revenue is always at least as high as that of the other equilibria.

When the government provides no curbside parking, the supply function is the régime 3 full price curve, and the short-run second-best optimum is the equilibrium (corresponding to points of intersection of the demand and supply curves) with the highest level of throughput (since it has higher consumer surplus than the other equilibria, and since curbside parking revenue is zero in all the equilibria). The above constructive procedure generates the short-run second-best social surplus function for all levels of P.

5.2.2 The long run with both curbside and garage parking

The previous subsection solved for the short-run (conditional on P), second-best social surplus as a function of demand intensity: $SRSS^{SB}(D_0; P)$. In this section, we solve for long-run second-best social surplus and curbside parking capacity as functions of demand intensity. One procedure is to solve for the long-run social surplus function as the upper envelope of the corresponding short-run functions: $LRSS^{SB}(D_0) = \max_P SRSS^{SB}(D_0; P)$, from which second-best optimal curbside parking capacity as a function of D_0 can be solved for. This procedure is sound but inefficient. Another procedure is to solve a grand social surplus maximization problem. There are two difficulties. The first is how to impose the constraint that the full price equalization condition, $C = (c - f)P/l$, applies in régimes 2 and but not in régime 1. One can do so by writing $(rl - P)(C - (c - f)P/l) = 0$, but existing optimization packages are not designed to deal with constraints in this form. The second difficulty is how to impose the constraint that the private sector rather than the planner chooses how much garage parking to provide.

We proceed by adapting the second procedure. To deal with the second difficulty, we shall ignore the constraint that the private sector rather than the planner chooses how much garage parking to provide, and then argue that the constraint is not binding at the social optimum. To deal with the first difficulty, we shall solve separately for long-run social surplus functions specific to each of the three régimes, imposing the full price equilibrium condition only on

the régime 2 social surplus maximization problem, and then obtain the long-run social surplus function as the upper envelope of the régime-specific social surplus functions.

Régime 1 obtains when there is only curbside parking. Second-best social surplus for régime 1, as a function of demand intensity, is obtained from the following constrained optimization problem:

$$\max_{r,T,C,P} \; X(r) - \rho(C+T) \tag{17}$$

$$s.t.$$

$$r = \frac{T}{mt(T,C,P)} \tag{i}$$

$$r = D\left(\rho mt(T,C,P) + \rho\frac{Cl}{P} + fl\right) \tag{ii}$$

$$r \leq \frac{P}{l} \quad . \tag{iii}$$

$X(r)$ is the social benefit function and $\rho(C+T)$ the social cost function, so that the maximand is social surplus. Constraints (i) and (ii) together are the steady-state condition, and constraint (iii) is the curbside parking capacity constraint. Since unoccupied curbside parking spaces generate a social cost but no social benefit, and since therefore curbside parking is always saturated at an optimum, constraint (iii) may be written as an equality constraint. From earlier reasoning, we know that an interior maximum exists when the long-run supply function intersects the demand function at a finite price. If the demand function lies everywhere below the long-run supply function, the maximization problem is solved by $r = T = C = P = 0$. If the demand function lies above the long-run supply function at all finite prices, the maximization problem is solved by $r = C = P = 0$ and $T = V_j$. With our specific functional forms and parameters, an interior maximum exists for demand intensities from 0 to 4395.

Régime 2 obtains when there is both curbside and garage parking. Second-best social surplus for régime 2, as a function of demand intensity, is obtained

as the solution to the following maximization problem:

$$\max_{r,T,C,P} \; X(r) - \rho(C+T) - c(rl - P) \tag{18}$$

$$s.t.$$

$$r = \frac{T}{mt(T,C,P)} \tag{i}$$

$$r = D\left(\rho mt(T,C,P) + \rho\frac{Cl}{P} + fl\right) \tag{ii}$$

$$0 < P < rl \tag{iii}$$

$$C - (c-f)\frac{P}{\rho} = 0 \quad . \tag{iv}$$

Constraint (iii) guarantees that there is both curbside and garage parking. From the economics of the problem, we know that there is a connected interval of demand intensities over which it does not bind and over which there is an interior maximum. With our functional forms and parameters, an interior maximum exists for demand intensities from 2969 to 8311.

Régime 3 obtains when there is only garage parking. Second-best social surplus for régime 3, as a function of demand intensity, is obtained as the solution to the following maximization problem:

$$\max_{r,T,C} \; X(r) - \rho T - crl \tag{19}$$

$$s.t.$$

$$r = \frac{T}{mt(T,0,0)} \tag{i}$$

$$r = D(\rho mt(T,0,0) + cl) \quad . \tag{ii}$$

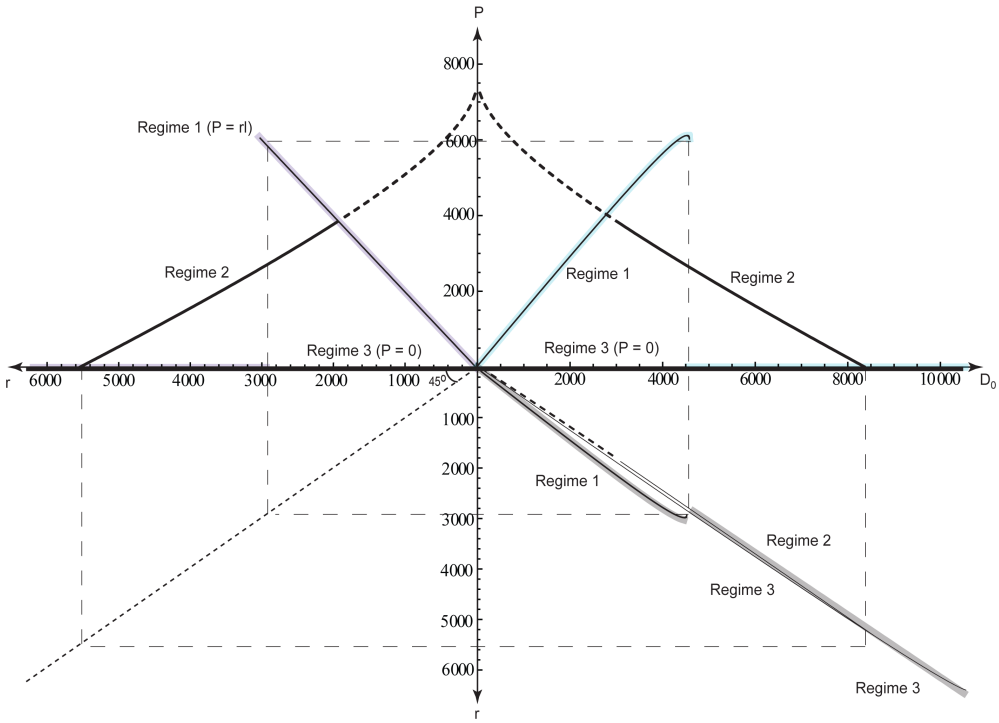
From the economics of the problem, we know that there is a connected interval of demand intensities over which this régime has an interior optimum. With our specific functional forms and parameters, an interior maximum exists for demand intensities from 0 to 10346.

Define the long-run second-best best optimum, conditional on demand in-

tensity, to be whichever of the solutions to (17), (18), and (19) yields the highest social surplus.

Finally, we need to establish that the constraint that the market rather than the planner chooses garage capacity does not bind at the social optimum. Suppose that the long-run second-best optimum entails no garage parking. It must then be the case in the corresponding decentralized social optimum that the curbside parking fee is less than or equal to the garage parking unit cost. Since the garage parking market is competitive, the garage parking fee equals the garage parking unit cost, so that the curbside parking fee is less than or equal to the garage parking fee, in which case garage operators cannot make positive profits by providing parking spaces. Suppose that the social optimum entails both curbside and garage parking. It must then be the case in the corresponding decentralized social optimum that the curbside parking fee equals the garage parking fee. Individuals have no incentive to change where the planner assigned them to park, and garage operators, since they are making zero profit on each parking space, have no incentive to change the number of garage parking spaces they provide. Suppose that the social optimum entails only garage parking. It must then be the case in the corresponding decentralized social optimum that the curbside parking fee is greater than or equal to the garage parking unit cost. Since the garage parking market is competitive, the garage parking fee equals the garage parking unit cost, so that the curbside parking fee is greater than or equal to the garage parking fee. Individuals have no choice since there are no curbside parking spaces available, and garage operators have no incentive to increase or decrease the number of garage spaces, since they are making zero profit on each space.

Figure 13 displays the results, highlighting the optimal régime for each level of demand intensity. The most striking result is that, with the functional forms and parameter values assumed in the example, the long-run second-best optimum never entails having both curbside and garage parking; that is, for no level of demand intensity is social surplus maximized in régime 2. Only curbside parking is provided up to $D_0 = 4395$, and only garage parking above



Note: Highlighted paths denote optimal régimes.

Figure 13: Régimes in the second-best optimum with both curbside and garage parking

this demand intensity level. The broad intuition is that cruising for parking occurs only in régime 2, and that the cruising for parking generated by the fee differential between garage and curbside parking would be so costly that the régime should be avoided.

A more precise intuition is as follows. Start with a situation where operating in régime 1 is second-best optimal. Recall that, with only curbside parking, second-best capacity is that for which curbside parking is just saturated and therefore has no scarcity rent. Since the curbside parking scarcity rent is zero, the private sector provides no garage parking. Now gradually raise demand intensity. At each level of demand intensity, the planner has the option of either increasing curbside parking capacity such that it remains just saturated, or of discontinuously reducing the level of curbside parking, thereby forcing

the parking scarcity rent up so it becomes profitable for the private sector to provide garage parking, or of eliminating curbside parking completely. With the specific functional forms and parameters of the numerical example, the planner chooses to remain in régime 1 up to the demand intensity at which the demand function is tangent to the long-run supply function with only curbside parking (recall Figure 8). At an incrementally higher level of demand intensity, demand cannot be accommodated with only curbside parking except with gridlock, and the planner has the choice between reducing the amount of curbside parking discontinuously or eliminating it completely, and in the example chooses to eliminate it completely.

Now look at the result from a different perspective, starting from a situation where operating in régime 3 is second-best optimal, and gradually reduce demand intensity. Traffic congestion diminishes to the point where providing some curbside parking is second-best optimal. The planner then has the choice between introducing curbside parking at an incremental level, which induces a small amount of cruising for parking, or of discontinuously increasing the stock of curbside parking by so much that the provision of garage parking becomes unprofitable, thereby forcing the equilibrium into régime 1. In the example, the planner will choose the latter option.

Normally, an increase in demand intensity leads to an increase in throughput and hence an increase in parking capacity. There is, however, one interesting exception. For D_0 slightly below 4395, where the second best entails only curbside parking, increased demand intensity leads to decreased curbside parking capacity. This curiosum can be explained with reference to Figure 11. The demand curve intersects only the backward-being portion of the supply curve, and when this occurs second-best throughput (which corresponds to the equilibrium E_2 in that figure) and hence second-best capacity fall as demand intensity rises.

5.3 Comparison of first- and second-best optimal capacities

In the previous section, we showed that, when there is only curbside parking and except for very high demand intensities for which gridlock occurs, second-best capacity exceeds first-best capacity when curbside parking is underpriced, and falls short of it when parking is overpriced. Furthermore, the second best always entails curbside parking being just saturated – saturated with no cruising for parking. When both curbside and garage parking may be provided, the results are considerably more complex. Since obtaining general results appears difficult, we focus on the numerical example.[15]

Figure 14 plots first- and second-best curbside parking capacity, as functions of demand intensity. All three régimes are present in the first best. For low levels of demand intensity, up to $D_0 = 4878$, it is efficient to provide only curbside parking; for intermediate levels, between $D_0 = 4878$ and $D_0 = 10321$, it is efficient to have both curbside and garage parking, with the amount of curbside parking declining monotonically with demand intensity; and for high levels of demand intensity, above $D_0 = 10321$, it is efficient to have only

[15] Wheaton (1978) and Wilson (1983) were the first papers to compare first-best with second-best optimal road capacity, where the distortion in the second best is the underpricing of road travel. There are two effects, which operate in different directions. Start at the decentralized social optimum (with the optimal congestion toll) and reduce the toll. The reduced full price of travel stimulates demand, which causes traffic to become more congested, which by itself increases the marginal benefit of capacity, but increasing capacity would lower the full price of travel even further, which would generate latent demand, increasing the deadweight loss due to underpriced congestion and reducing the marginal benefit. The former effect is first order, the latter is second order. Since Wheaton considered only a small reduction in the toll below its optimal value, the first-order effect dominated the second-order effect, so that he found second-best capacity to exceed first-best capacity. Wilson undertook a global analysis, and found that, with a zero toll, second-best capacity exceeds first-best capacity for demand elasticity below 1.0 but falls short of it for sufficiently elastic demand. The economics of the optimal curbside parking capacity problem differs substantially from that of the optimal road capacity problem. Unlike the optimal road capacity problem, in the optimal curbside parking capacity problem, an expansion of curbside capacity *increases* road congestion while relaxing the curbside parking capacity constraint. Furthermore, there are two alternative technologies for parking, curbside parking and garage parking, and two non-price rationing mechanisms, traffic congestion and cruising for parking.

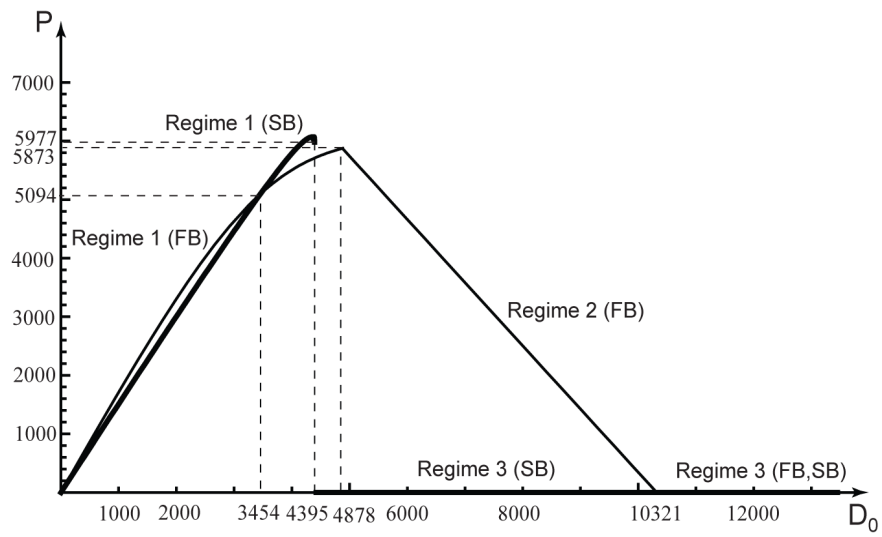
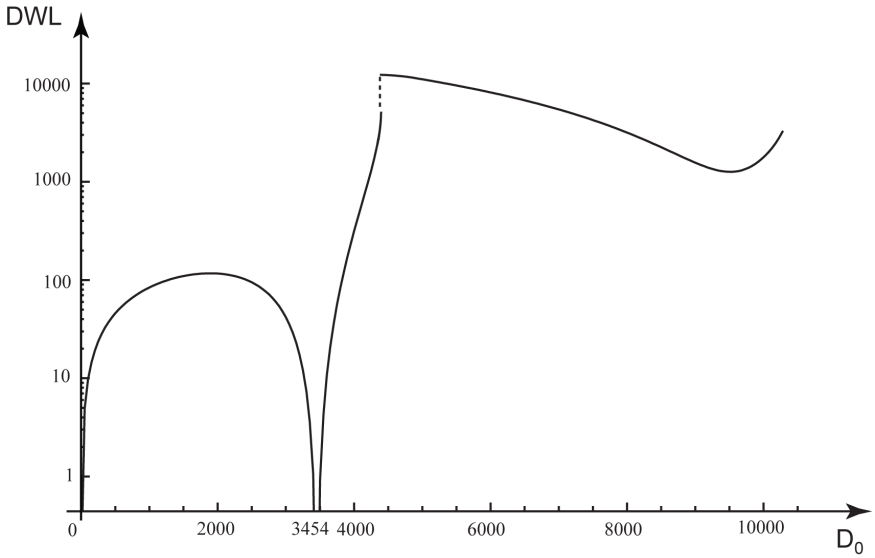


Figure 14: Comparison of the first-best and the second-best optima

garage parking. In the second best in contrast, régime 2, with both curbside and garage parking, is second-best efficient for no interval of demand intensity. Only curbside parking is provided for demand intensities up to $D_0 = 4395$, and beyond that level of demand intensity only garage parking is provided.

Several other points bear note. First, over that range of demand intensities where only curbside parking is provided in both the first best and the second best (up to $D_0 = 4395$) a result from the previous section, where only curbside parking is considered, carries through: for levels of demand intensity where curbside parking is underpriced (below $D_0 = 3454$), second-best curbside parking capacity falls short of first-best capacity; and when curbside parking is overpriced (for D_0 between 3454 and 4395), second-best curbside parking capacity exceeds first-best capacity. Second, in the interval of demand intensities between 4395 and 4878, only curbside parking is provided in the first best and only garage parking in the second best. This result can be understood with reference to Figure 8. Over this range of demand intensities, with only curbside parking, traffic is only congested at the first-best optimum but would be gridlocked at the second-best optimum. Third, it is efficient to eliminate curbside parking for a larger interval of demand intensities in the

second best than in the first best.



Note: A log 10 scale is employed on the DWL-axis, such that each tick increase corresponds to a ten-fold increase in deadweight loss.

Figure 15: Deadweight loss from inefficient pricing of curbside parking as a function of demand intensity

At each level of demand intensity, the deadweight loss deriving from the combination of the underpricing of urban auto travel and the mispricing (overpricing or underpricing) of curbside parking equals social surplus at the first-best optimum minus social surplus in the second-best optimum. Figure 15 displays the dollar deadweight loss per ml^2-hr as a function of demand intensity, using a log scale on the y-axis. There are three distinct intervals of demand intensity. For low levels of demand intensity (up to 3454), there is only curbside parking in both the first and second best, and in the second best the curbside parking fee of \$1.00/hr results in curbside parking being *overpriced*. As demand intensity increases, deadweight loss rises to a maximum of about \$100/ml^2-hr and then falls to zero (so that log DWL equals negative infinity) at that level of demand intensity for which \$1.00/hr is the efficient curbside parking fee. For moderate levels of demand intensity (between 3454 and 4395), there is only curbside parking in both the first and second best, and

in the second best curbside parking becomes increasingly *underpriced* as demand intensity increases. Over this interval, deadweight loss increases rapidly with demand intensity as the road space in the second-best equilibrium becomes increasingly congested, until at $D_0 = 4395$ the efficient equilibrium with only curbside parking disappears. There is then a discontinuous increase in deadweight loss as the planner switches from providing only curbside parking to only garage parking in the second best. Deadweight loss attains a local maximum at this point, with a value of about \$12300/ml^2-hr, since the mix of parking technologies employed in the first and second best is as different as possible, with all curbside parking in the first best and all garage parking in the second best. As demand intensity increases, there are two offsetting effects on deadweight loss. On one hand, the deadweight loss associated with the inefficient mix of parking technologies falls, since it becomes optimal to provide an increasing proportion of parking in garages in the first best. On the other hand, the deadweight loss due to traffic congestion being unpriced in the second best increases. The former effect dominates up to a demand intensity of around 9500, and the latter for the range of demand intensities from 9500 to 10321. At $D_0 = 10321$, in the second best the efficient equilibrium with only garage parking disappears and is replaced by the gridlock equilibrium, which entails an infinite deadweight loss compared to the first best.

It would be imprudent to generalize from a specific numerical example. Nevertheless, the numerical example does illustrate some general policy insights. First, for high levels of demand intensity, it is inefficient to provide curbside parking, whether or not it is underpriced. The simple reason is that the social value of road space is higher for traffic flow than for curbside parking, which explains why curbside parking is rarely provided along major arterials during peak periods. Second, underpricing curbside parking can introduce considerable distortion, even when the amount of curbside parking is optimized, lowering not only social surplus but consumer surplus as well. From an alternative perspective, raising the curbside meter rate may generate efficiency gains that are several times the increased meter revenue generated. Third, when cashing out curbside parking is not politically attractive, and when pro-

viding garage parking is not cost effective, it is second-best efficient to expand curbside parking to the point where cruising for parking is eliminated.

Even in our simple model, determining optimal curbside parking capacity when curbside parking is underpriced is difficult. Determining optimal curbside parking capacity in realistic situations in which demand varies over time and space, users are heterogeneous in terms of trip distance, value of time, and parking duration, curbside parking limits and local government operation of some parking garages are additional policy tools, parking garages are provided discretely rather than continuously over space, with private parking garages having market power, and both demand and supply contains stochastic elements, will be even more difficult. The appropriate way to address this complexity in policy practice will be to simulate policies using downtown traffic network microsimulation models,[16] with enriched parking modules.

6 Concluding Remarks

Parking is an intrinsic element of the downtown transportation problem, and enlightened downtown parking policy can do much to relieve downtown traffic congestion. This paper focused on a particular downtown parking issue: How much curbside should be allocated to parking when the private sector can provide garage parking at constant unit cost? It addressed this question in the context of a steady-state, macroscopic model of downtown parking and traffic congestion, with both curbside (on-street) and garage (off-street) parking, and for both first- and second-best environments. In the first-best environment, optimal congestion pricing is in force and curbside parking is efficiently priced. In the second-best environment, there are two distortions, the underpricing of urban auto travel and of curbside parking, both of which are ubiquitous in North American cities during peak periods. The underpricing

[16] Traffic network microsimulation models, such as VISSIM, Paramics, and TransModeler, are now routinely used to simulate the effects of proposed improvements to the network of streets and to the system of traffic lights, but in most the treatment of parking is primitive.

of curbside parking leads to excess demand, manifest as cruising for parking, which is not only wasteful in itself but also exacerbates traffic congestion. The paper developed the analysis through the diagrammatic exposition of an extended numerical example, with the aims of elucidating general principles and of developing economic intuition.

The choice of first-best curbside parking capacity is a fairly routine application of first-best investment rules. The price of curbside parking is set to clear the market, so that parking is "just saturated" (fully occupied but with no cruising for parking). Curbside capacity should be expanded to the point where the marginal cost of additional capacity equals the marginal benefit. The marginal cost derives from the increase in congestion due to allocating less street space to traffic. The nature of the marginal benefit depends on whether demand relative to road capacity is sufficiently high to make the provision of garage parking profitable. If demand is lower than this level, so that all parking is curbside, increasing curbside parking permits more trips downtown. If demand is higher than this level, so that the private sector provides garage parking, increasing curbside parking reduces the resource cost of parking garages. At low levels of demand intensity, it is efficient to have only curbside parking, with the amount of curbside parking increasing in demand intensity. At intermediate levels, it is efficient to have both curbside and garage parking, with the amount of curbside parking decreasing as demand intensity increases. And at high levels of demand intensity, it is efficient to have only garage parking.

Cruising for parking complicates the choice of second-best curbside parking capacity. When demand intensity is low, it is second-best efficient to have only curbside parking, with the curbside parking capacity set such that parking is "just saturated", and with curbside parking capacity increasing in demand intensity. As demand intensity rises, a critical level of demand intensity is reached at which it is second-best efficient for the policy maker to discontinuously reduce the amount of curbside parking, which generates the parking scarcity rent needed to make private provision of garage parking prof-

itable. It may be second-best efficient for the policy maker to continue to provide some curbside parking. In this case, there is cruising for parking, with the stock of cars cruising for parking adjusting such that the full price (including cruising-for-parking time cost) of curbside parking equals the unit cost of garage parking. But it may instead be second-best efficient for the policy maker to eliminate all curbside parking. The latter option is more efficient when curbside parking is severely underpriced, since the simultaneous provision of curbside and garage parking then gives rise to more cruising for parking.

The relationship between the first- and second-best curbside parking capacities is complex. We note only two results. First, when demand intensity is sufficiently low that only curbside parking is provided in both the first and second best, second-best curbside parking capacity exceeds first-best parking capacity. Second, there may be a range of demand intensities in which only curbside parking is provided in the first best and only garage parking in the second best.

We modeled curbside parking as being either unsaturated with no cruising for parking, just saturated with no cruising for parking, or saturated with cruising for parking. Realistically, at the level of the downtown area, there is a gradual transition from unsaturated to saturated parking (Levy, Martens, and Berenson, 2012). As the demand for curbside parking increases, curbside parking becomes saturated on an increasingly high proportion of blocks, so that there is cruising for parking even when curbside parking is not everywhere saturated, and the mean curbside parking occupancy rate increases. Extending the model of this paper to treat this and other realistic complications should not undermine its economic logic.

A Online Appendix: Stability Analysis (not intended for publication)

Arnott and Inci (2010) provided a thorough stability analysis of equilibria in a variant of the model presented above with only curbside parking. In this appendix, we extend their analysis to investigate the stability of equilibria when both curbside and garage parking are present, and when only garage parking is present. Stability analysis of traffic congestion has proved difficult since it requires solving for the out-of-equilibrium dynamics of traffic flow over time and space. The treatment of downtown as isotropic simplifies the analysis considerably since at any point in time traffic flow is the same throughout the downtown area; the analysis then entails solving ordinary rather than partial differential equations. Arnott and Inci further simplified the problem by making some special assumptions[17] that render the differential equation system autonomous (time does not enter the analysis explicitly), which permits phase-plane/state-space analysis. The arrows give the direction of motion, under the assumption that drivers decide whether to travel based on myopic expectations (more precisely, the entry rate at time t is assumed to depend on the *perceived* full price of a trip, which depends only on traffic conditions at time t).

We first introduce a new piece of notation to facilitate geometric presentation of the stability analysis in 2D space. We define

$$R = \begin{cases} C & \text{for } R \geq 0 \\ Q - P & \text{for } R \leq 0 \end{cases}, \tag{A.1}$$

where Q is the stock of occupied curbside parking spaces. In words, when R is positive, which corresponds to saturated curbside parking, it equals the

[17] They assume that trip lengths are negative exponentially distributed, which implies that the exit rate from the in-transit pool at time t depends only on the stock of cars in transit and cruising for parking at that point in time, and not on the history of congestion. They also assume that visit durations are negative exponentially distributed, which implies that the exit rate from curbside parking depends only on the amount of curbside parking.

stock of cars cruising for parking, and when R is negative, which corresponds to unsaturated curbside parking, it equals minus the stock of unoccupied curbside parking spaces. This allows us to depict the transition between saturated and unsaturated parking in a single phase plane. As R increases from being negative to being positive, the stock of unoccupied curbside parking spaces shrinks, until at $R = 0$ parking is saturated with no cruising for parking, and then remains saturated with the stock of cars cruising for parking increasing.

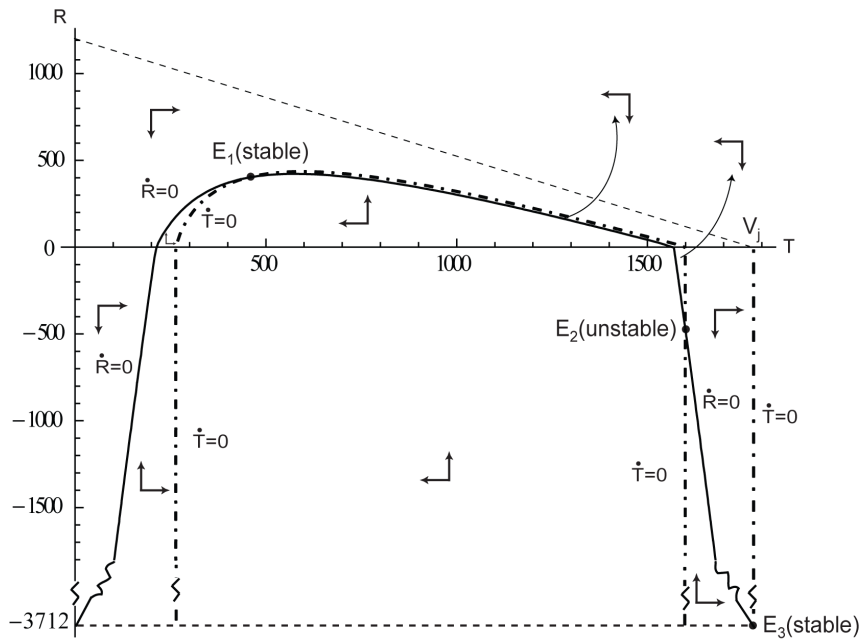


Figure 16: Transient dynamics of downtown traffic when there is only curbside parking

Figure 16 displays the stability analysis with only curbside parking for the base case level of curbside parking, $P = 3712$, and for the demand intensity indicated by $D_2 = 3000$ in Figure 4. The state of the system is characterized by T and R, with T on the horizontal axis and R on the vertical axis. Above $R = 0$, parking is saturated and there is cruising for parking, and below $R = 0$ parking is unsaturated and there are unoccupied curbside parking spaces. The arrows indicate the direction of motion of the two state variables. Three loci are displayed in T-R space. The first, the dashed line, is the jam density line;

combinations of T and R to the right of the line are infeasible.

The second locus is the $\dot{R} = 0$ locus. For $R \geq 0$, the locus corresponds to the cruising-for-parking equilibrium condition $\dot{C} = 0 = T/(mt(T, C, P)) - P/l$, along which the stock of cars cruising for parking remains unchanged; below this locus, the stock of cars cruising for parking is increasing, and above it the stock is decreasing. For $R \leq 0$, the locus corresponds to $\dot{Q} = 0 = T/(mt(T, C, P)) - Q/l$; below this locus, the stock of occupied curbside parking spaces is increasing, and above it the stock is decreasing.

The third locus is the steady-state condition that $\dot{T} = 0 = D(F) - T/(mt(T, C, P)) = D\left(\rho mt(T, C, P)\right) + \rho Cl/P + fl) - T/(mt(T, C, P))$. When curbside parking is saturated, the $\dot{T} = 0$ locus is a curve in T-C space, above which the stock of cars in-transit is increasing and below which it is decreasing. When curbside parking is unsaturated, $C = 0$, and the $\dot{T} = 0$ locus corresponds to those levels of T for which the stock of cars in transit remains unchanged. There are three such levels of T, all corresponding to points of intersection of the unsaturated user cost curve, shifted up by the curbside parking fee, and the demand curve. The one furthest to the left corresponds to the upward-sloping portion of the user cost curve, the middle one to the backward-bending portion of the curve, and the one on the right to gridlock. The stock of cars in transit is increasing for T lower than the T furthest to the left and between the middle T and the gridlock T, and is decreasing between the T furthest to the left and the middle T.

Consistent with Figure 4, there are three equilibria. The equilibrium E_1 in Figure 16 corresponds to the equilibrium E_1' in Figure 4, and is saturated, stable, and congested. The equilibrium E_2 in Figure 16 corresponds to the equilibrium E_2' in Figure 4, and is unsaturated, stable, and hypercongested. The equilibrium E_3 in Figure 16 corresponds to the gridlock equilibrium, which cannot be displayed in Figure 4.

In the remainder of the appendix, we show how the stability analysis can be adapted to the situation with both curbside and garage parking, and then

apply the adapted stability analysis to determine the stability of the equilibria analyzed in section 5.

In the analysis of Section 5, since the curbside parking fee is lower than the garage parking fee, garage parking occurs only when curbside parking is saturated. Thus, allowing for garage parking does not affect the stability analysis when curbside parking is unsaturated, and hence the portion of the phase plane with negative R. The addition of garage parking adds the full price equalization condition that $R \leq \hat{C} = (c-f)P/\rho$. When $R < \hat{C}$, curbside parking is cheaper than garage parking so that no one parks in a garage, and the stability analysis of Figure 16 continues to apply. When $R > \hat{C}$, however, the stability analysis of Figure 16 needs to be modified. If $R > \hat{C}$, garage parking is cheaper than curbside parking. We assume that when this occurs the number of cars cruising for parking falls instantaneously such that $R = \hat{C}$ is satisfied. Thus, above $C = \hat{C}$, the direction of motion is vertically downward. Otherwise, the direction of motion in the phase plane is unchanged.

Figure 17 portrays the phase plane for six different levels of demand intensity. Recall that an increase in demand intensity has no effect on the $\dot{R} = 0$ locus but causes the $\dot{T} = 0$ locus to shift downward. Start with Panel A, which has the lowest level of demand intensity. Qualitatively, this corresponds to the situation shown in Figure 4 with demand level D_1. All three curbside parking equilibria are unsaturated, so that there is no demand for garage parking. Turn next to Panel B, with the next lowest level of demand intensity. Qualitatively, this panel corresponds to the situation shown in Figure 4 with demand level D_2. The equilibrium corresponding to E_1' in that figure is saturated. Cruising for parking occurs, but the stock of cars cruising for parking is not sufficient to make the provision of garage parking profitable. In Panel B, this corresponds to the equilibrium E_1 lying below the $R = \hat{C}$ locus.

Now turn to Panel C. The demand intensity is the same as that used in the construction of Figure 16. Thus, comparison of Panel C, Figure 17, and Figure 16 shows how admitting garage parking alters the equilibria of Figure 16. Now, the stock of cars cruising for parking in equilibrium E_1' in Figure

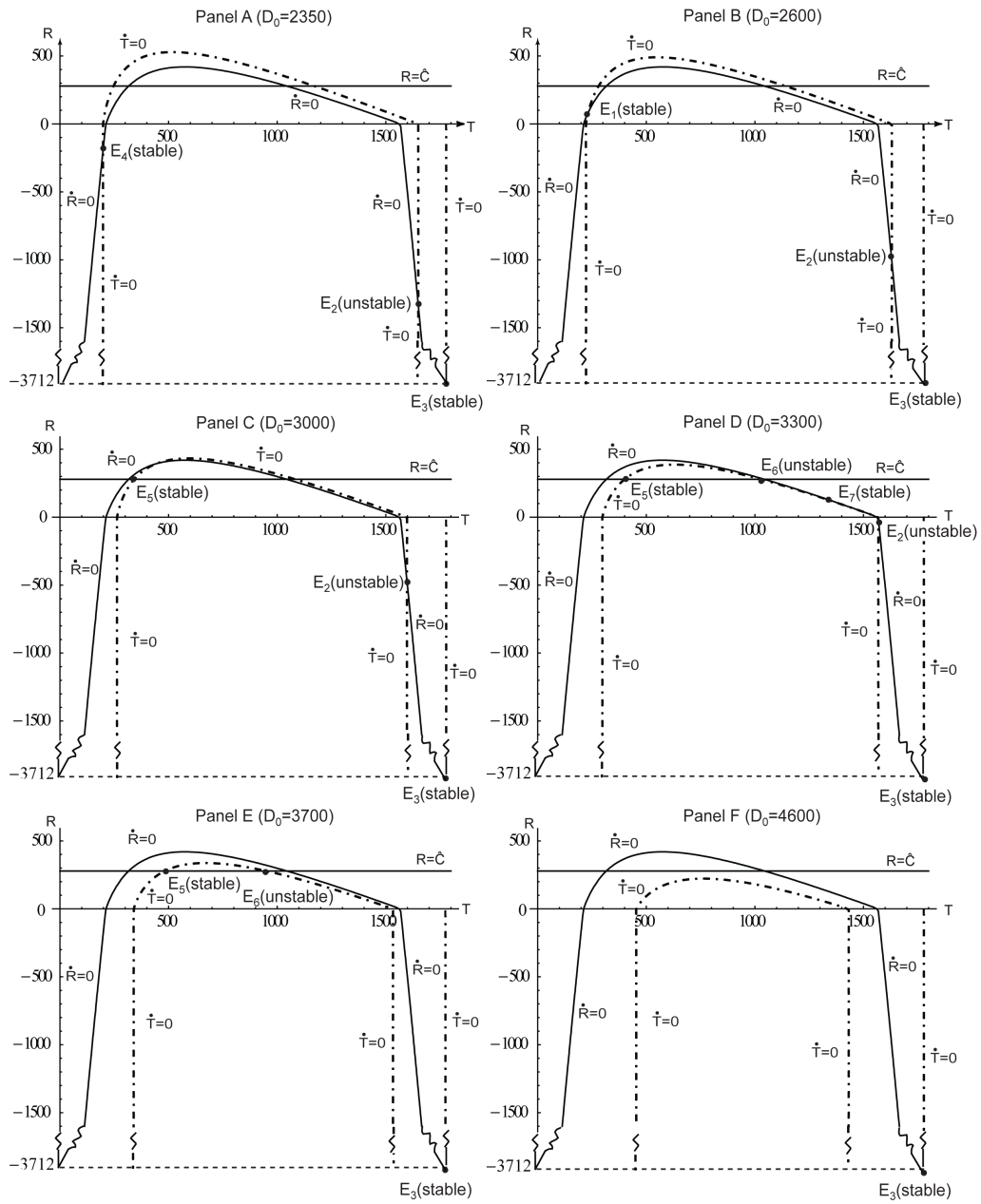


Figure 17: Transient dynamics of downtown traffic when there are both curbside and garage parking

4 is sufficiently high to make garage parking profitable. Garage parking is provided, and the equilibrium E_1' in Figure 4 is replaced by the equilibrium

with the same qualitative properties as E_5 in Figure 11, which is saturated, stable, and congested. The other two equilibria remain unsaturated.

In Panel D, demand intensity is close to that for the demand curve drawn in Figure 11 so that the equilibria are qualitatively the same. There are now five equilibria. In the stable, congested equilibrium E_5, garage parking is provided and curbside parking is saturated. In the unstable, hypercongested equilibrium E_6, garage parking is provided and curbside parking is saturated. In the stable, hypercongested equilibrium E_7, curbside parking is saturated but the stock of cars cruising for parking is insufficient for garage parking to be profitable. In the unstable, hypercongested equilibrium E_2, curbside parking is unsaturated. Finally, there is the gridlock equilibrium. Panel E corresponds to Figure 11 but with a higher level of demand intensity such that the equilibria E_7 and E_2 disappear. Panel F corresponds to Figure 11 with an even higher level of demand intensity such that only the gridlock equilibrium remains. Thus, the stability analysis of Figure 17 confirms the stability properties of the various equilibria asserted in the bifurcation diagram of Figure 12.

The above discussion has been mechanical. It will be useful to provide some intuition, which can be done by describing the process of adjustment along three sample trajectories in Panel D. Start with a situation in which downtown is empty. Then, the demand is turned on at the demand intensity $D_0 = 3300$, and remains at that level forever. Cars start entering the city streets, traffic density builds, and an increasing number of curbside parking spaces become occupied.[18] With unsaturated parking, the trajectory lies between the $\dot{R} = 0$ and $\dot{T} = 0$ loci. In due course, parking becomes saturated and cruising for parking commences. The stock of cars in-transit and cruising for parking continue to increase, which corresponds to the trajectory continuing to lie between the $\dot{R} = 0$ and $\dot{T} = 0$ loci, but now with saturated parking and cruising for parking. In due course, the stock of cars cruising for parking

[18] Recall that the adjustment process assumes, first, that trip lengths are negative exponentially distributed with mean m, so that parking spaces start becoming occupied as soon as there is traffic on the road, and, second, that the entry rate at time t depends upon the stock of cars in transit and cruising for parking at that point in time.

becomes sufficiently large that it becomes profitable for garage parking to be provided. The stock of cars in transit continues to increase and the stock of garage parking spaces to be expanded until the equilibrium E_5 is reached.

Consider the unstable equilibrium E_6. Since the equilibrium is saddle-path stable, its stable arms are the boundary between E_5 and E_7's zones of attraction. Start slightly to the left of E_6 on $R = \hat{C}$. There is both curbside and garage parking, and cruising for parking satisfies the full price equalization condition, which continues to be satisfied throughout the adjustment process. The stock of cars in transit is slightly lower than at E_6. Turn to Figure 11, which describes the same situation as Panel D, but in another space. On the demand side, a stock of cars in transit slightly below that at E_6 results in the trip price being somewhat lower than at E_6 and the entry rate therefore being somewhat higher. On the supply side, because traffic is hypercongested, the lower stock of cars in transit implies a higher exit rate (throughput). Because the demand curve is steeper than the supply curve at E_6, the quantity of trips supplied is higher than at E_6 by more than the quantity of trips demanded, which results in a fall in the stock of cars in transit. In due course, the reduction in the stock of cars in transit becomes sufficiently large that travel becomes congested, and continued reductions in the stock of cars in transit causes throughput to fall, while the quantity of trips demanded continues to rise. This eventually results in achievement of the stable equilibrium at E_5.

The story is similar if the starting point is slightly to the right of E_6 in Figure 17, Panel D. The initial stock of cars in transit is slightly higher than at E_6. On the demand side, a stock of cars in transit slightly above that at E_6 results in the trip price being somewhat higher than at E_6, and the entry rate therefore being somewhat lower. On the supply side, because traffic is hypercongested, the higher stock of cars in transit implies a lower exit rate. Because the demand curve is steeper than the supply curve, the quantity of trips supplied is lower than at E_6 by more than the quantity of trips demanded, which results in an increase in the stock of cars in transit, and traffic become increasingly hypercongested. The reduced throughput causes a

reduced demand for garage parking and eventually zero demand.

Increasing curbside parking capacity would alter the stability analysis in three ways. First, it would shift the whole $\dot{R} = 0$ locus down. Second, it shifts only the upper part of the $\dot{T} = 0$ locus down without changing its T-intercepts. And third, it shifts the full price equalization condition down. Otherwise the analysis remains qualitatively the same.

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