Progressive Taxation and Macroeconomic (In)stability with Productive Government Spending*

Shu-Hua Chen†
National Taipei University

Jang-Ting Guo‡
University of California, Riverside

August 3, 2010

Abstract

This paper systematically examines the interrelations between a progressive income tax schedule and macroeconomic (in)stability in an otherwise standard one-sector real business model with productive government spending. We analytically show that the economy exhibits indeterminacy and sunspots only if the equilibrium wage-hours locus is positively sloped and steeper than the household's labor supply curve. Unlike in the framework with useless public expenditures, a less progressive tax policy may operate like an automatic stabilizer that mitigates belief-driven cyclical fluctuations. Our quantitative analysis shows that this result is able to provide a theoretically plausible explanation for the discernible reduction in U.S. output volatility after the Tax Reform Act of 1986 was implemented.

Keywords: Progressive Income Taxation, Equilibrium (In)determinacy, Productive Government Spending, Business Cycles.

JEL Classification: E32, E62.

---

*We thank Sharon Harrison, Kevin Lansing, Valerie Ramey, Richard Rogerson, Richard Suen, Yundong Tu, Stephen Turnvosky and seminar participants at UC San Diego, University of Washington, the 2008 Far Eastern and South Asian Meetings of the Econometric Society, the 2009 Chinese Economic Association in North America at the ASSA Meetings, the 2009 Annual Sympoisum of the Society for Nonlinear Dynamics and Econometrics, and the 2009 Annual Conference of the Asia-Pacific Economic Association for helpful comments and suggestions. We also thank Emre Yoldas for excellent research assistance. Part of this research was conducted while Guo was a visiting research fellow of economics at Academia Sinica, Taipei, Taiwan, whose hospitality is greatly appreciated. Of course, all remaining errors are our own.

†Department of Economics, National Taipei University, 151 University Rd., San Shia, Taipei, 237 Taiwan, 886-2-8674-7168, Fax: 886-2-2673-9727, E-mail: shchen@mail.ntpu.edu.tw.

‡Corresponding Author. Department of Economics, 4123 Sproul Hall, University of California, Riverside, CA, 92521 USA, 1-951-827-1588, Fax: 1-951-827-5685, E-mail: guojt@ucr.edu.

1 Introduction

Traditional Keynesian macroeconomics stipulates that progressive income taxation is an automatic stabilizer in mitigating the magnitude of fluctuations in disposable income and consumption. It follows that *ceteris paribus* the cyclical volatility of output is smaller when the economy is subject to a more progressive tax policy. As it turns out, this result continues to hold in the context of one-sector real business cycle (RBC) models. Schmitt-Grohé and Uribe (1997) show that standard one-sector RBC models with a constant returns-to-scale production technology may exhibit indeterminacy and sunspots under a balanced-budget rule where fixed public expenditures are financed by proportional taxation on labor or total income. Therefore, when agents' optimism leads to higher investment and hours worked, the government is forced to lower the tax rates as total output rises. This countercyclical fiscal formulation is qualitatively equivalent to regressive income taxation. By contrast, Guo and Lansing (1998) incorporate a progressive income tax schedule, whereby the household's marginal tax rate is increasing in its own level of taxable income, into Benhabib and Farmer's (1994) indeterminate one-sector no-sustained-growth RBC model with aggregate increasing returns-to-scale in production. These authors find that a sufficiently strong tax progressivity can stabilize the economy against business cycle fluctuations driven by agents' animal spirits.[1]

In the above-referenced work and other related previous studies, government purchases are postulated to generate no substitution effects in that they do not affect the marginal conditions for the household's consumption/savings or the firm's production decisions. However, the assumption of wasteful or useless public spending, although commonly adopted in the academic literature for analytical simplicity, is not necessarily the most realistic – at least for developed countries. In this paper, we systematically explore the stability effects of Guo and Lansing's (1998) progressive tax formulation in an otherwise standard one-sector RBC model with indivisible labor and productive public expenditures. Specifically, as in Barro (1990), government spending enters the firm's Cobb-Douglas production technology as an input that is complementary to private capital and labor services.

Our theoretical analysis demonstrates that the interrelations between the government's tax policy rule and macroeconomic (in)stability depend crucially on (i) the labor share of national income, (ii) the degree of positive external effects that public spending exerts on the firm's production process, (iii) the level parameter of the tax schedule, which also governs

[1] Christiano and Harrison (1999) obtain the same qualitative finding that progressive taxation on agents' labor effort is an automatic stabilizer within the endogenous growth version of the Benhabib and Farmer's (1994) one-sector representative-agent model.

the government size measured by the steady-state ratio of government purchases to GDP, and (iv) the slope parameter of the tax schedule that characterizes its progressivity feature. In particular, we first derive the analytical expression of the model's Jacobian matrix, and show that the *necessary* condition for our model economy to exhibit local indeterminacy is an upward-sloping equilibrium wage-hours locus which is steeper than the labor supply curve. It follows that endogenous belief-driven macroeconomic booms and downturns may occur as self-fulfilling equilibria. This turns out to be the same (necessary and sufficient) condition for indeterminacy and sunspot in Benhabib and Farmer's (1994) laissez-faire one-sector RBC model with a social technology that displays increasing returns in private capital and labor inputs.

Next, within the empirically plausible specification that capital's share of output is lower than that of labor, a comprehensive graphical investigation is undertaken to illustrate our model's local stability properties. Specifically, we are able to clearly divide the feasible parameter space into the regions of "saddle", "sink" and "source" under three different configurations. In the benchmark parameterization, the steady-state public expenditures to GDP ratio is postulated to be lower than the capital share of national income, which in turn is smaller than the level parameter of the tax scheme. In sharp contrast to Schmitt-Grohé and Uribe (1997) and Guo and Lansing (1998) with useless government purchases, raising the tax progressivity may turn our model's steady state from a saddle point to a sink provided public spending is sufficiently productive. This implies that a more progressive tax schedule can destabilize the economy by causing endogenous cyclical fluctuations. On the contrary, as in the existing studies, more progressive income taxation works like an automatic stabilizer within the other two parametric formulations. In these frameworks, the economy is more susceptible to equilibrium indeterminacy when the tax policy rule becomes flatter. Moreover, some recent findings in the RBC-based indeterminacy literature, such as those in Schmitt-Grohé and Uribe (1997) and Guo and Harrison (2004, 2008), can be shown as special cases of our analytical and graphical results.

To obtain further insights, we carry out a quantitative analysis of macroeconomic (in)stability within a calibrated version of our model economy. Our benchmark calibration, which is consistent with post-war U.S. data, turns out to be the empirically relevant formulation. With regard to calibrating the level and slope parameters of our postulated tax policy, we follow Cassou and Lansing's (2004) nonlinear least squares methodology and obtain year-by-year empirical estimates from the U.S. federal individual income tax schedule for the $1966 - 2005$ period. These estimations result in an average R-square of 0.867. We find that the estimated tax-level parameter displayed a slow downward trend during the $1966 - 1986$ sub-sample period and remained stable after 1987; and that the average values of this parameter turn out to be very similar across the two subperiods. In addition, our estimated tax-slope parameter exhibits a discernible structural break because of the implementation of the Tax Reform Act of 1986 (TRA-86). In particular, the U.S. tax code was more progressive prior to TRA-86, evidenced by the significant decrease of estimated progressivity between 1986 and 1987; and the average level of estimated tax progressivity has also shown a noticeable declining trend.

Given the estimated values of tax progressivity, together with the benchmark calibration of other model parameters, we find that the $1966 - 1986$ subperiod is characterized by equilibrium indeterminacy, whereas saddle-path stability prevails for the $1987 - 2005$ subperiod. It follows that the post-1986 economy *ceteris paribus* exhibits a lower cyclical volatility of output. This prediction turns out to be qualitatively consistent with the Great Moderation whereby the U.S. business cycle fluctuations have become less volatile since the mid 1980's. Our quantitative analysis thus provides a theoretically plausible explanation for the observed reduction in the magnitude of aggregate fluctuations after TRA-86, *i.e.* a less progressive tax schedule has operated like an automatic stabilizer that mitigates U.S. output volatility through eliminating belief-driven business cycles.[2]

The remainder of this paper is organized as follows. Section 2 describes the model and the tax schedule. Section 3 analytically and graphically examines the model's local dynamics. Section 4 undertakes a quantitative investigation of macroeconomic (in)stability in a calibrated version of our model economy. Section 5 concludes.

2 The Economy

We incorporate productive government purchases into a prototypical one-sector real business cycle (RBC) model under an income tax policy *a la* Guo and Lansing (1998). Households live forever, and derive utility from consumption and leisure. On the production side, each competitive firm produces output with a Cobb-Douglas technology that uses capital, labor and public spending as inputs. We assume that there are no fundamental uncertainties present in the economy.

[2] Interestingly, the moderation of cyclical fluctuations in U.K. (Stock and Watson, 2005) also coincided with fiscal changes that have considerably reduced its tax progressivity since 1985 (Giles and Johnson, 1994).

2.1 Firms

There is a continuum of identical competitive firms, with the total number normalized to one. Each firm produces output Y_t, using capital K_t, labor hours H_t and aggregate government spending G_t as complementary inputs, with the following Cobb-Douglas production function:[3]

$$Y_t = AK_t^{\alpha} H_t^{1-\alpha} G_t^{\chi}, \ A > 0, \quad 0 < \alpha < 1 \ \text{ and } \chi \geq 0. \tag{1}$$

Notice that the technology (1) exhibits constant returns-to-scale with respect to private capital and labor inputs; and χ captures the degree of positive external effects that public expenditures exert on the firm's production process. Moreover, we assume that $\chi < 1 - \alpha$ to rule out the possibility of sustained endogenous growth. Under the assumption that factor markets are perfectly competitive, the representative firm takes G_t as given and maximize its profits according to

$$r_t = \alpha \frac{Y_t}{K_t}, \tag{2}$$

$$w_t = (1 - \alpha) \frac{Y_t}{H_t}, \tag{3}$$

where r_t is the rental rate of capital and w_t is the real wage. In addition, α and $1-\alpha$ represent the capital and labor share of national income, respectively.

2.2 Households

The economy is populated by a unit measure of identical infinitely-lived households. Each household is endowed with one unit of time and maximizes

$$\int_0^{\infty} (\log C_t - BH_t) \, e^{-\rho t} dt, \ \ B > 0, \tag{4}$$

where $\rho > 0$ is the subjective discount rate, and C_t and H_t are the individual household's consumption and hours worked, respectively. The linearity of (4) in hours worked draws on the formulation of indivisible labor (Hansen, 1985; and Rogerson, 1988) that is commonly adopted in the RBC-based indeterminacy literature.[4]

The budget constraint faced by the representative household is given by

$$\dot{K}_t = (1 - \tau_t)(r_t K_t + w_t H_t) - \delta K_t - C_t, \quad K_0 > 0 \text{ given}, \tag{5}$$

[3] As in Barro (1990), G_t represents the flow of productive services that government spending yields. Alternatively, G_t can be interpreted as the stock of public capital with a depreciation rate of 100%. Allowing for partial depreciation of public capital will introduce another state variable to the model's dynamical system. This is an extension that is worth pursuing in future research.

[4] See Benhabib and Farmer (1999) for an excellent survey of this literature.

where K_t is the household's capital stock and $\delta \in (0,1)$ is the capital depreciation rate. Households derive income by supplying capital and labor services to firms, taking factor prices r_t and w_t as given. As in Guo and Lansing (1998), we postulate that the income tax rate τ_t takes the form

$$\tau_t = 1 - \eta \left(\frac{Y^*}{Y_t} \right)^{\phi}, \ \eta \in (0,1), \ \phi \in \left(\underline{\phi}, 1 \right), \tag{6}$$

where $Y_t = r_t K_t + w_t L_t$ represents the household's taxable income, and Y^* denotes the steady-state level of per capita income, which is taken as given by each agent. The parameters η and ϕ govern the level and slope of the tax schedule, respectively. When $\phi > (<)\, 0$, the tax rate τ_t increases (decreases) with the household's taxable income Y_t. When $\phi = 0$, all households face the constant tax rate $1 - \eta$ regardless of their taxable income.

Using (6), we obtain the expression for the marginal tax rate of income τ_{mt}, which is defined as the change in taxes paid by the household divided by the change in its taxable income, as follows:

$$\tau_{mt} \equiv \frac{\partial \left(\tau_t Y_t \right)}{\partial Y_t} = 1 - \eta \left(1 - \phi \right) \left(\frac{Y^*}{Y_t} \right)^{\phi}. \tag{7}$$

In this paper, our analyses are restricted to environments in which the government does not have access to lump-sum taxes or transfers, hence $\tau_t > 0$ and $\tau_{mt} > 0$ are imposed. We also require $\tau_t < 1$ to ensure that the government can not confiscate all productive resources, and $\tau_{mt} < 1$ so that households have an incentive to provide labor and capital services to firms. Finally, to guarantee the existence of an interior steady state, the economy's equilibrium after-tax interest rate $(1 - \tau_{mt})\, r_t$ must be a strictly decreasing function of K_t, which imposes another lower bound on τ_{mt}. In the steady state, the above considerations imply that $\eta \in (0,1)$ and $\phi < 1$. Moreover, the minimum possible level of ϕ is determined by the more restrictive lower-bound on the steady-state marginal income tax rate τ_m^*, hence

$$\underline{\phi} = \max \left\{ \frac{\eta - 1}{\eta}, \ \frac{(1-\eta)(1-\alpha-\chi)}{\chi\eta - \alpha(1-\eta)} \right\}. \tag{8}$$

Given these restrictions on η and ϕ, it is straightforward to show that when $\phi > 0$, the marginal tax rate is higher than the average tax rate given by (6). In this case, the tax schedule is said to be "progressive". When $\phi = 0$, the average and marginal tax rates coincide at the level of $1 - \eta$, thus the tax schedule is "flat". When $\phi < 0$, the tax schedule is said to be "regressive".

Households take into account the way in which the tax schedule affects their earnings when they decide how much to work, consume and invest over their lifetimes. Consequently, it is the marginal tax rate of income that governs the household's economic decisions. The first-order conditions for the representative household with respect to the indicated variables and the associated transversality condition (TVC) are

$$
\begin{aligned}
C_t \quad &: \quad \frac{1}{C_t} = \lambda_t, && (9)\\
H_t \quad &: \quad \frac{B}{\lambda_t} = \underbrace{\eta(1-\phi)\left(\frac{Y^*}{Y_t}\right)^{\phi}}_{(1-\tau_{mt})}\underbrace{(1-\alpha)\frac{Y_t}{H_t}}_{w_t}, && (10)\\
K_t \quad &: \quad \lambda_t\left[\underbrace{\eta(1-\phi)\left(\frac{Y^*}{Y_t}\right)^{\phi}}_{(1-\tau_{mt})}\underbrace{\alpha\frac{Y_t}{K_t}}_{r_t}-\delta\right] = \rho\lambda_t - \dot{\lambda}_t, && (11)\\
\text{TVC} \quad &: \quad \lim_{t\to\infty} e^{-\rho t}\lambda_t K_t = 0, && (12)
\end{aligned}
$$

where λ_t is the the Lagrange multiplier for the budget constraint (5), (10) equates the slope of the household's indifference curve to the after-tax real wage, (11) is the consumption Euler equation, and (12) is the transversality condition.

2.3 Government

The government sets the tax rate τ_t according to (6), and balances its budget each period. Hence, its period budget constraint is given by

$$G_t = \tau_t Y_t, \tag{13}$$

where government spending on goods and services G_t in turn contributes to the firms' production. With the government, the aggregate resource constraint for the economy is

$$C_t + \dot{K}_t + \delta K_t + G_t = Y_t. \tag{14}$$

3 Macroeconomic (In)stability

This section examines the local stability properties of competitive equilibria in the above model. First, we analytically derive the conditions under which the economy exhibits equilibrium (in)determinacy. Next, we undertake a systematic and comprehensive graphical analysis of our model's local dynamics. Finally, we point out that our theoretical results subsume some recent findings in the RBC-based indeterminacy literature as special cases.

3.1 Analytical Characterizations

To facilitate the analysis of the model's local stability properties, we make the following logarithmic transformation of variables: $k_t \equiv \log(K_t)$ and $c_t \equiv \log(C_t)$. It is straightforward to show that our model exhibits a unique interior steady state given by

$$k^* = \frac{\log\left\{A(1-\eta)^{\chi}\left[\frac{\eta(1-\alpha)(1-\phi)}{B}\right]^{1-\alpha}\right\} + (\chi-\alpha)\log x_1 - (1-\alpha)\log x_2}{1-\alpha-\chi}, \tag{15}$$

and

$$c^* = \frac{\log\left\{A(1-\eta)^{\chi}\left[\frac{\eta(1-\alpha)(1-\phi)}{B}\right]^{1-\alpha}\right\} + (\chi-\alpha)\log x_1 - \chi\log x_2}{1-\alpha-\chi}, \tag{16}$$

where

$$x_1 = \frac{\rho+\delta}{\alpha\eta(1-\phi)} > 0 \quad \text{and} \quad x_2 = \frac{\rho + [1-\alpha(1-\phi)]\,\delta}{(1-\phi)\,\alpha} > 0.$$[5]

The remaining endogenous variables at the economy's steady state can then be derived accordingly. Next, in the neighborhood of this steady state, the model's equilibrium conditions can be approximated by the following log-linear dynamical system:

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = \underbrace{\begin{bmatrix} \eta\left[\frac{\alpha(1-\phi)}{\Psi}-1\right]x_1 + x_2 & -\frac{\eta(1-\alpha)(1-\phi)}{\Psi}x_1 - x_2 \\ \alpha\eta(1-\phi)\left[\frac{\alpha(1-\phi)}{\Psi}-1\right]x_1 & -\frac{\alpha\eta(1-\alpha)(1-\phi)^2}{\Psi}x_1 \end{bmatrix}}_{\mathbf{J}} \begin{bmatrix} k_t - k^* \\ c_t - c^* \end{bmatrix}, \quad k_0 \text{ given}, \tag{17}$$

where

$$\Psi = 1 - \chi\underbrace{\left[\frac{1-\eta(1-\phi)}{1-\eta}\right]}_{\frac{\tau_m^*}{\tau^*}} - (1-\alpha)(1-\phi) \gtreqless 0, \tag{18}$$

and τ^* is the steady-state level of the average income tax rate. It follows that the determinant and trace of the model's Jacobian matrix $\mathbf{J}$ are

$$Det = \frac{\alpha\eta(1-\phi)\,\Omega}{\Psi}x_1x_2, \tag{19}$$

where

$$\Omega = \alpha(1-\phi) - \left\{1-\chi\left[\frac{1-\eta(1-\phi)}{1-\eta}\right]\right\} < 0;$$[6] (20)

and

$$Tr = \frac{Num}{\Psi}, \tag{21}$$

where

$$Num = \delta\chi\left[\frac{1-\eta(1-\phi)}{1-\eta}\right] + \rho\left[1-(1-\alpha)(1-\phi)\right] \gtreqless 0. \tag{22}$$

[5] $[1-\alpha(1-\phi)] > 0$ is ensured by the lower bound of ϕ given by (8).
[6] See the Appendix for the proof of $\Omega < 0$.

Since the first-order dynamical system (17) possesses one predetermined variable k_t, the economy displays saddle-path stability and equilibrium uniqueness if and only if the two eigenvalues of $\mathbf{J}$ are of opposite sign ($Det < 0$). When both eigenvalues have negative real parts ($Det > 0$ and $Tr < 0$), the steady state is a locally indeterminate sink that can be exploited to generate endogenous business cycle fluctuations driven by agents' self-fulfilling expectations or sunspots. The steady state becomes a source when both eigenvalues have positive real parts ($Det > 0$ and $Tr > 0$). In this case, any trajectory that diverges away from the completely unstable steady state may settle down to a limit cycle or to some more complicated attracting sets.

Given $0 < \alpha$, $\eta < 1$, x_1, x_2, $(1-\phi) > 0$ and $\Omega < 0$, the Jacobian's determinant (19) is positive when $\Psi < 0$, *i.e.*

$$\frac{(1-\alpha)(1-\phi)}{1-\chi\left[\frac{1-\eta(1-\phi)}{1-\eta}\right]} - 1 > 0. \tag{23}$$

This in turn constitutes a *necessary* condition for the steady state to be a sink as the positive determinant only guarantees that both eigenvalues have the same sign. The necessary and sufficient conditions for local indeterminacy also require the Jacobian's trace (21) to be negative. In particular, when condition (23) is satisfied, the model's Jacobian matrix exhibits a negative/positive trace (thus the resulting steady state is a sink/source) if its numerator (22) is positive/negative. When the inequality in (23) does not hold, the determinant of $\mathbf{J}$ (19) becomes negative, hence the economy's steady state is a (locally determinate) saddle point.

The intuition for above (in)determinacy result can be understood as follows. Substituting (1) and (13) into the logarithmic version of the labor-market equilibrium condition (10) yields that the slope of the aggregate labor demand schedule is given by $\frac{(1-\alpha)(1-\phi)}{1-\chi\left[\frac{1-\eta(1-\phi)}{1-\eta}\right]} - 1$, while the slope of the household's labor supply curve is 0 because of the specification of indivisible labor. Therefore, the condition that is necessarily needed to generate indeterminacy and sunspots in our model with productive government spending and progressive income taxation, as in (23), states that the equilibrium wage-hours locus is upward sloping and steeper than the labor supply curve. This turns out to be exactly the same (necessary and sufficient) condition for equilibrium indeterminacy in Benhabib and Farmer's (1994) laissez-faire one-sector RBC model with aggregate increasing returns-to-scale in production due to positive capital/labor externalities or monopolistic competition. In the context of our model economy, if the external impact of government purchases on the firm's production process together with the level and slope effects of the postulated tax policy are strong enough to make the equilibrium wage-hours locus intersect the household's labor supply curve from below, a positive sunspot shock will lead to simultaneous expansions in GDP, consumption, investment, hours worked and labor productivity. Consequently, agent's initial optimistic expectations about the economy's future become self-fulfilling in equilibrium.

3.2 Graphical Characterizations

Given the preceding analytical results, this subsection graphically examines the economy's local dynamics under different parameter configurations. Based on the empirical evidence that capital income accounts for a smaller percentage of GDP than labor income, our analyses are restricted to the specifications in which $\alpha < 1-\alpha$.[7] The level parameter of the tax schedule η is used to characterize the model's feasible parameter regions as well.

3.2.1 When $\alpha < \eta$ and $1-\eta < \alpha < 1-\alpha$

In this case, the most-binding constraint on $\underline{\phi}$ turns out to be a positive steady-state marginal tax rate of income, thus $\underline{\phi} = \frac{\eta-1}{\eta}$. Figure 1 plots the combinations of ϕ (the tax progressivity) and χ (the output elasticity of government spending) that lead to equilibrium (in)determinacy. The area above the downward-sloping curve $\Omega = 0$ is ruled out because it violates condition (20). Moreover, the negatively-sloped locus $\Psi = 0$, given by (18), divides the regions labeled "saddle" ($Det < 0$) and "sink" ($Det > 0$ and $Tr < 0$).[8] It can also be shown that the model's steady state cannot be a source, since it requires $\phi < \frac{\alpha}{\alpha-1}$, which is lower than the relevant $\underline{\phi}$ mentioned above. In sharp contrast to existing studies (*e.g.* Guo and Lansing, 1998; and Christiano and Harrison, 1999) with wasteful public expenditures, the arrows in Figure 1 demonstrate that when $1-\eta < \chi < 1-\alpha$, raising the tax progressivity ϕ eventually transforms the steady state from a saddle point into a sink. This implies that a more progressive tax schedule may destabilize the economy provided government purchases are sufficiently productive. By contrast, saddle-path stability and equilibrium uniqueness always prevail when $0 \leq \chi \leq 1-\eta$.

3.2.2 When $\alpha < \eta$ and $\alpha < 1-\eta < 1-\alpha$

As in the previous case, the most-binding constraint on $\underline{\phi}$ is $\tau_m^* > 0$, hence $\underline{\phi} = \frac{\eta-1}{\eta}$. Figure 2 shows that points below the locus $\Omega = 0$ are feasible as they satisfy condition (20), and

[7] For the sake of theoretical completeness, we have also studied the cases in which $\alpha > 1-\alpha$. It turns out that the Jacobian's determinant (19) is negative under all feasible parameter combinations. Hence, the economy always exhibits saddle-path stability and equilibrium uniqueness in this setting. These results are available upon request.

[8] It can be shown that $\frac{\partial \chi}{\partial \phi} < 0$ and $\frac{\partial^2 \chi}{\partial \phi^2} > 0$ along the loci of $\Omega = 0$ and $\Psi = 0$. Hence, both downward-sloping curves are convex in Figure 1.

that the area below the positively-sloped curve $\Psi = 0$ is the "saddle" region ($Det < 0$). Next, we derive the downward-sloping $Num = 0$, as in (22), that separates the zones of "sink" ($Det > 0$ and $Tr < 0$) and "source" ($Det > 0$ and $Tr > 0$).[9] Figure 2 also illustrates that for a given positive level of $\chi \in (0, 1-\eta)$, the model's local stability property switches from being a source to a sink, and then to a saddle point as the tax progressivity ϕ increases. Therefore, the economy is more susceptible to indeterminacy and sunspots when the tax scheme becomes flatter or less progressive (smaller ϕ). On the other hand, saddle-path stability cannot occur if $1 - \eta \leq \chi < 1 - \alpha$ because the steady state is either a sink or a source. It follows that within this particular parameter space, changing the tax progressivity ϕ will not eliminate the possibility of belief-driven business cycle fluctuations.

3.2.3 When $\alpha > \eta$ and $\alpha < 1 - \alpha < 1 - \eta$

In this case, the most-binding constraint on $\underline{\phi}$ is that the steady-state after-tax interest rate is monotonically decreasing in capital, thus $\underline{\phi} = \frac{(1-\eta)(1-\alpha-\chi)}{\chi\eta-\alpha(1-\eta)}$. Substituting $\chi = 0$ into this expression yields that $\phi > \frac{\alpha-1}{\alpha}$, which is depicted on the horizontal axis of Figure 3. From this figure, we see that the regions of "saddle", "sink" and "source" are separated by the dividing loci of $\Omega = 0$, below which condition (20) holds; $\Psi = 0$, below which the determinant of $\mathbf{J}$ (19) is negative; and $Num = 0$, below which the numerator of the Jacobian's trace (22) is positive.[10] Moreover, for all feasible values of $\chi \in [0, 1-\alpha)$, progressive income taxation works like an automatic stabilizer in that a higher ϕ changes the model's steady state from a sink/source to a saddle point.

3.3 Special Cases

Our analysis allows for a rich set of theoretical possibilities regarding the interrelations between the government's tax policy rule and macroeconomic (in)stability within a one-sector representative agent model, and helps bring together some recent results in the RBC-based indeterminacy literature. First, we recover the model of Guo and Harrison (2008, section 3.1) with productive government spending ($\chi > 0$) and a flat tax schedule ($\phi = 0$). Substituting

[9] It can be shown that $\frac{\partial \chi}{\partial \phi} < 0$ and $\frac{\partial^2 \chi}{\partial \phi^2} > 0$ along the loci of $\Omega = 0$ and $Num = 0$. Therefore, both negatively-sloped curves are convex in Figure 2. Moreover, we find that $\frac{\partial \chi}{\partial \phi} > 0$ and $\frac{\partial^2 \chi}{\partial \phi^2} < 0$ along the locus of $\Psi = 0$, thus it is upward-sloping and concave.

[10] It can be shown that $\frac{\partial \chi}{\partial \phi} > 0$ and $\frac{\partial^2 \chi}{\partial \phi^2} < 0$ along the loci of $\Omega = 0$ and $\Psi = 0$. Hence, both upward-sloping curves are concave in Figure 3. Moreover, we find that $\frac{\partial \chi}{\partial \phi} < 0$ and $\frac{\partial^2 \chi}{\partial \phi^2} > 0$ along the locus of $Num = 0$, thus it is negatively-sloped and convex. We also derive that at the intersection point of $\Omega = 0$ and $Num = 0$, $\phi = -\frac{\alpha\rho+(1-\alpha)\delta}{\alpha\delta+(1-\alpha)\rho} \in \left(\frac{\alpha-1}{\alpha}, \frac{\alpha}{\alpha-1}\right)$ and $\chi = \frac{\rho(1-2\alpha)(1-\eta)}{(\alpha-\eta)\delta+(1-\alpha-\eta)\rho} \in (0, 1-\alpha)$.

$\phi = 0$ into (23) yields that in this case, the steady state is an indeterminate sink when

$$\frac{1-\alpha}{1-\chi} - 1 > 0, \tag{24}$$

or $\alpha < \chi$ as shown in the vertical axes of Figures $1-3$. Notice that condition (24) is now not only necessary but also sufficient for the presence of indeterminacy and sunspots.

Moreover, Schmitt-Grohé and Uribe (1997) find that standard one-sector real business cycle models with a constant returns-to-scale production technology may exhibit local indeterminacy under a balanced-budget rule where fixed pubic expenditures are financed by proportional taxation on labor or total income. With this type of balanced-budget formulation, the government is forced to lower the tax rates as total output rises. This setting is qualitatively similar to our model with non-productive public expenditure ($\chi = 0$) and regressive income taxation ($\phi < 0$). The horizontal axes in Figures 2 and 3 confirm Schmitt-Grohé and Uribe's (1997) general point that a more regressive tax policy is destablizing the economy because the steady state changes from being a saddle point to a source as the tax progressivity ϕ falls.[11] However, the same result does not hold in Figure 1 since saddle-path stability always prevails when $\chi = 0$.

Finally, when $\chi = \phi = 0$, our model collapses to one with wasteful government purchases and a constant income tax rate, as in Guo and Harrison (2004). In this case, it is straightforward to show that the Jacobian's determinant (19) is negative, thus the eigenvalues of the log-linear dynamical system (17) are of opposite sign. It follows that the steady state is a saddle point, as can be seen at the origins of Figures $1-3$.

4 Quantitative Analysis

To obtain further insights of the above analytical and graphical results, this section undertakes a quantitative investigation of macroeconomic (in)stability within a calibrated version of our model economy. We are particularly interested in studying how tax policies affect the local stability properties of equilibria for combinations of parameters whose values are consistent with the post-war U.S. data. As is common in the real business cycle literature, the capital share of national income, α, is chosen to be 0.3; and the steady-state ratio of public expenditures to GDP (or the government size) is set equal to 20%, that is, $\frac{G^*}{Y^*} = 1-\eta = 0.2$. It follows that the empirically relevant formulation of our model corresponds to Figure 1 with $\alpha < \eta$

[11] Plugging $\chi = 0$ into (19) shows that the Jacobian's determinant is positive if and only if $\phi < \frac{\alpha}{\alpha-1}$, which in turn leads to a positive trace (21). As a result, the steady state is either a saddle point or a source along the horizontal axes of Figures 2 and 3.

and $1 - \eta < \alpha < 1 - \alpha$. Next, existing empirical estimates of the output elasticity of government spending, χ, exhibit a wide range from 0.03 (Eberts, 1986) to 0.39 (Aschauer, 1989). We adopt Li and Sarte's (2004) calibration and set $\chi = 0.25$ as a "consensus" benchmark. Substituting these parameter values into (23), the critical value of the tax progressivity that satisfies $\Psi = 0$ and separates the regions of "saddle" and "sink" in Figure 1 is $\phi_c = 0.1667$. In what follows, we examine whether the U.S. tax schedule has been more or less progressive than this threshold level, and then discuss the associated business-cycle implications.

The U.S. federal individual income tax schedule is progressive since it is characterized by several tax "brackets" (branches of income) that are taxed at progressively higher rates. For example, Figure 4 shows that there were fifteen marginal tax rates, ranging from zero to 50%, in 1986. Subsequently, the Tax Reform Act of 1986 (TRA-86) simplified the tax code and reduced the number of marginal tax rates to five in 1987, as shown in Figure 5. In addition, the top tax rate was lowered from 50% to 38.5%, whereas the bottom rate was raised from zero to 11%. These marginal tax rates can be used to construct the average tax schedules, which are also depicted in the figures.[12][13] In terms of calibrating the tax-schedule parameters η and ϕ according to (6), we follow Cassou and Lansing's (2004) methodology and postulate the empirical counterpart of $\frac{Y^*}{Y_t}$ to an income ratio that is defined as the inverse of the household's taxable income divided by its mean level. Moreover, the mean level of taxable income for married taxpayers who filed joint returns is adopted to represent Y^*.[14] Using $1,000$ data points with equal increments in taxable income, we carry out a year-by-year nonlinear least squares regression of the average tax rate τ_t on the corresponding income ratio $\frac{Y^*}{Y_t}$ for the $1966 - 2005$ period. These estimations result in an average R^2 of 0.867, and allow us to compare and contrast the U.S. tax code before and after TRA-86 quantitatively.

Figure 6 shows that the estimated tax "level" parameter $\hat{\eta}$ trended downward slowly during the $1966 - 1986$ sub-sample period and remained stable after 1987. Furthermore, the average

[12] For each level of income, the average tax rate is defined as total tax payments divided by total taxable income. For example, a married couple whose total taxable income was $80,000 in 1987 would pay 11 percent on the first $3,000 of their income, 15 percent on the next $25,000, 28 percent on the next $17,000, and 35 percent on the remaining $35,000. This adds up a total tax payment of $21,090, which corresponds to an average tax rate of 26.36 percent.

[13] According to NBER's TAXSIM model (http://www.nber.org/∼taxsim), the weighted average and marginal income tax rates were 15.1% and 28% in 1986; and fell respectively to 13.5% and 24.2% in 1987. See Feenberg and Coutts (1993) for an overview on how the TAXSIM model calculates average and marginal taxe rates from a stratified random sample of U.S. tax returns.

[14] For example, there were 42,377,012 joint returns filed in 1987, and the resulting total taxable income was $1,282,875,175,000. These figures imply that the mean taxable income across all joint returns was equal to $30,273. See U.S. Internal Revenue Service, *Statistics of Income Bulletin*, Volume 11, Summer 1991, Table 1, page 27.

values of $\hat{\eta}$ turn out to be very similar (around 0.8) across the two subperiods. It follows that the government spending to output ratio $1-\hat{\eta}$ over the full sample coincides closely with our benchmark parameterization ($= 0.2$). Figure 7 reports that the estimated tax "slope" parameter $\hat{\phi}$ exhibits a discernible structural break because of the implementation of TRA-86. Specifically, the tax schedule was more progressive prior to TRA-86, evidenced by the considerable decrease of $\hat{\phi}$ from 0.1335 in 1986 to 0.0745 in 1987. The average level of tax progressivity also displays a noticeable declining trend – it was 0.1679 between 1966 and 1986, and fell to 0.0634 for the $1987-2005$ sub-sample period. Hence, our estimation results provide strong empirical support for a significant progressivity reduction in the U.S. statutory income tax schedule associated with the Tax Reform Act of 1986.

Given the calibrated α, η and χ, together with the estimated values of tax progressivity $\hat{\phi}$, Figure 8 shows that the $1966-1986$ subperiod is characterized by a sink that exhibits indeterminacy and sunspots, whereas saddle-path stability prevails for the latter $(1987-2005)$ subperiod. Consequently, the cyclical volatility of output *ceteris paribus* is smaller after 1986 because equilibrium determinacy and uniqueness rule out the possibility of aggregate fluctuations caused by agents' animal spirits. This prediction turns out to be qualitatively consistent with the Great Moderation whereby the U.S. business cycle fluctuations have become less volatile since the mid 1980's. In particular, the standard deviation of HP-filtered real GDP per capita was 1.98% between 1966Q1 and 1986Q4, and decreased (by 51 percent) to 0.97% from 1987Q1 to 2005Q4. Therefore, our analysis offers a theoretically plausible explanation for the observed decline in the magnitude of macroeconomic fluctuations after TRA-86, *i.e.* a less progressive tax schedule operates like an automatic stabilizer that mitigates U.S. output volatility through eliminating belief-driven business cycles.[15] As it turns out, the moderation of cyclical fluctuations (Stock and Watson, 2005) and the significant reduction of tax progressivity (Giles and Johnson, 1994) have also simultaneously occurred in the U.K. economy since 1985.

5 Conclusion

This paper has systematically explored the relationships between progressive income taxation and equilibrium (in)determinacy in a prototypical one-sector real business cycle model

[15] Existing explanations for this reduction in U.S. output volatility include "active" monetary policy rules (Clarida, Galí and Gertler, 2000), improved information technology and inventory management (Khan, McConnell and Perez-Quiros, 2002), and lower variance of exogenous shocks (Stock and Watson, 2002), among others.

with productive government purchases. We analytically find that the economy exhibits an indeterminate steady state and thus a continuum of stationary sunspot equilibria only if the equilibrium wage-hours locus is upward sloping and intersects the household's labor supply curve from below. When the steady-state ratio of public spending to GDP is lower than the capital share of national income, which in turn is smaller than the level parameter of the tax schedule, a less progressive tax policy may stabilize the economy against business cycle fluctuations driven by changes in agents' non-fundamental expectations. Our quantitative analysis shows that this result is able to provide a theoretically plausible explanation for the moderation of U.S. output volatility after the Tax Reform Act of 1986 significantly reduced its tax progressivity.

This paper can be extended in several directions. For example, we can consider our model economy that allows for sustained endogenous growth (*i.e.* $\alpha + \chi = 1$ in equation 1) or with utility-generating government spending as a positive preference externality *a la* Guo and Harrison (2008, section 3.2). Moreover, in the context of Harrison's (2001) two-sector laissez-faire RBC model which possesses an indeterminate steady state under sufficiently strong investment externalities, Guo and Harrison (2001) show that a regressive tax schedule is needed to eliminate indeterminacy and sunspots, and that this economy with flat or progressive income taxation is more susceptible to belief-driven aggregate fluctuations. It would be worthwhile to examine the robustness of our results within a two-sector RBC framework. The above-mentioned extensions will further enhance our understanding of the qualitative and quantitative interrelations between a progressive/regressive tax policy and macroeconomic (in)stability in representative-agent models with useful public expenditures. We plan to pursue these research projects in the future.

6 Appendix

Proof of $\Omega < 0$. First, after substituting the tax schedule (6) and the government budget constraint (13) into the firm's production function (1), and then taking total differentiation, we obtain the output elasticities of capital and labor for the reduced-form social technology as follows:

$$\varepsilon_{K_t} = \frac{\alpha}{1 - \chi \frac{\tau_{mt}}{\tau_t}} \quad \text{and} \quad \varepsilon_{L_t} = \frac{1-\alpha}{1 - \chi \frac{\tau_{mt}}{\tau_t}}, \tag{A.1}$$

where $1 - \chi \frac{\tau_{mt}}{\tau_t} > 0$ to ensure that both ε_{K_t} and ε_{L_t} are strictly positive. Next, the existence of the model's interior steady state requires that the equilibrium after-tax interest rate, $(1 - \tau_{mt})\, r_t$ is a monotonically decreasing function in K_t, that is

$$\frac{\partial \left[(1 - \tau_{mt})\, r_t\right]}{\partial K_t} = \frac{\Omega_t}{1 - \chi \frac{\tau_{mt}}{\tau_t}} \underbrace{\frac{(1 - \tau_{mt})\, r_t}{K_t}}_{\text{positive}} < 0, \tag{A.2}$$

where $\Omega_t = \alpha\,(1 - \phi) - \left(1 - \chi \frac{\tau_{mt}}{\tau_t}\right)$. Since $1 - \chi \frac{\tau_{mt}}{\tau_t} > 0$, condition (A.2) implies that $\Omega_t < 0$ for all t. The expression of Ω in equation (20) of the text corresponds to the steady-state value of Ω_t, thus $\Omega < 0$.

References

[1] Aschauer, D.A. (1989), "Is Public Expenditure Productive?" *Journal of Monetary Economics* 23, 177-200.

[2] Barro, R.J. (1990), "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy* 98, 103-125.

[3] Benhabib, J. and R.E.A. Farmer (1994), "Indeterminacy and Increasing Returns," *Journal of Economic Theory* 73, 19-41.

[4] Benhabib, J. and R.E.A. Farmer (1999), "Indeterminacy and Sunspots in Macroeconomics," in J. Taylor and M. Woodford, eds., *Handbook of Macroeconomics,* Amsterdam: North Holland, 387-448.

[5] Cassou, S.P. and K.J. Lansing (2004), "Growth Effects of Shifting from a Graduated-Rate Tax System to a Flat Tax," *Economic Inquiry* 42, 194-213.

[6] Christiano, L.J. and S.G. Harrison (1999), "Chaos, Sunspots and Automatic Stabilizers," *Journal of Monetary Economics* 44, 3-31.

[7] Clarida, R., J. Galí and M. Gertler (2000), "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics* 115, 147-180.

[8] Eberts, R.W. (1986), "Estimating the Contribution of Urban Public Infrastructure to Regional Growth," Federal Reserve Bank of Cleveland Working Paper No. 8610.

[9] Feenberg, D. and E. Coutts (1993), "An Introduction to the TAXSIM Model," *Journal of Policy Analysis and Management* 12, 189-194.

[10] Giles, C. and P. Johnson (1994), "Tax Reform in the UK and Changes in the Progressivity of the Tax System, 1985-1995" *Fiscal Studies* 15, 64-86.

[11] Guo, J-T. and S.G. Harrison (2001), "Tax Policy and Stability in a Model with Sector-Specific Externalities," *Review of Economic Dynamics* 4, 75-89.

[12] Guo, J-T. and S.G. Harrison (2004), "Balanced-Budget Rules and Macroeconomic (In)stability," *Journal of Economic Theory* 119, 357-363.

[13] Guo, J-T. and S.G. Harrison (2008), "Useful Government Spending and Macroeconomic (In)stability under Balanced-Budget Rules," *Journal of Public Economic Theory* 10, 383-397.

[14] Guo, J-T. and K.J. Lansing (1998), "Indeterminacy and Stabilization Policy," *Journal of Economic Theory* 82, 481-490.

[15] Hansen, G.D. (1985), "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics* 16, 309-327.

[16] Harrison, S.G. (2001), "Indeterminacy in a Model with Sector-Specific Externalities," *Journal of Economic Dynamics and Control* 25, 747-764.

[17] Khan, J.A., M.M. McConnell and G. Perez-Quiros (2002), "On the Causes of the Increased Stability of the U.S. Economy," *Federal Reserve Bank of New York Economic Policy Review* May, 183-202.

[18] Li, W. and P-D. Sarte (2004), "Progressive Taxation and Long-Run Growth," *American Economic Review* 94, 1705-1716.

[19] Rogerson, R. (1988), "Indivisible Labor, Lotteries, and Equilibrium," *Journal of Monetary Economics* 21, 3-16.

[20] Schmitt-Grohé, S. and M. Uribe (1997), "Balanced-Budget Rules, Distortionary Taxes and Aggregate Instability," *Journal of Political Economy* 105, 976-1000.

[21] Stock, J.H. and M.W. Watson (2002), "Has the Business Cycle Changed and Why?" *NBER Macroeconomics Annual* 17, 159-218.

[22] Stock, J.H. and M.W. Watson (2005), "Understanding Changes in International Business Cycle Dynamics," *Journal of European Economic Association* 3, 968-1006.

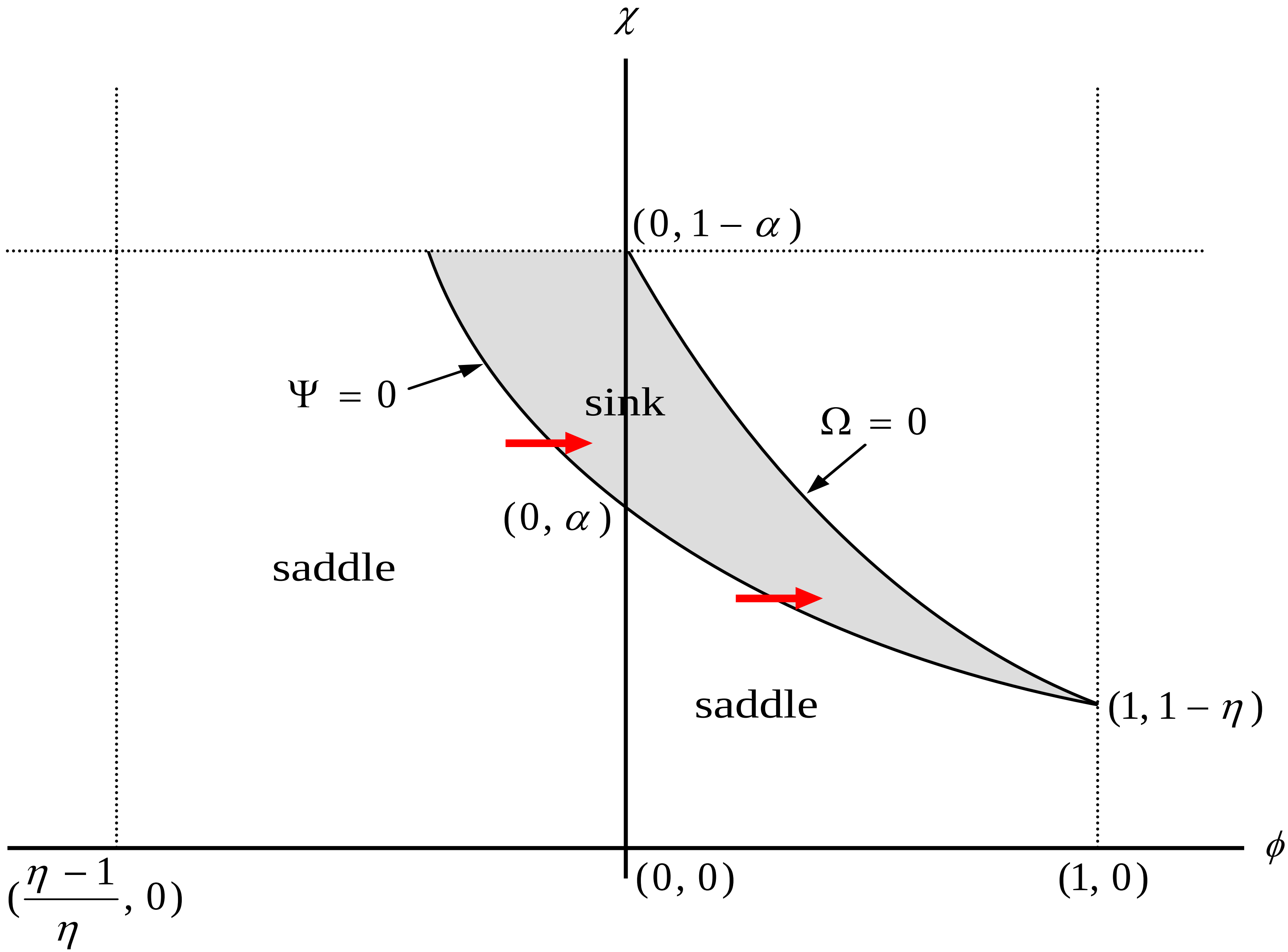


Figure 1: $\alpha < \eta$ and $1-\eta < \alpha < 1-\alpha$

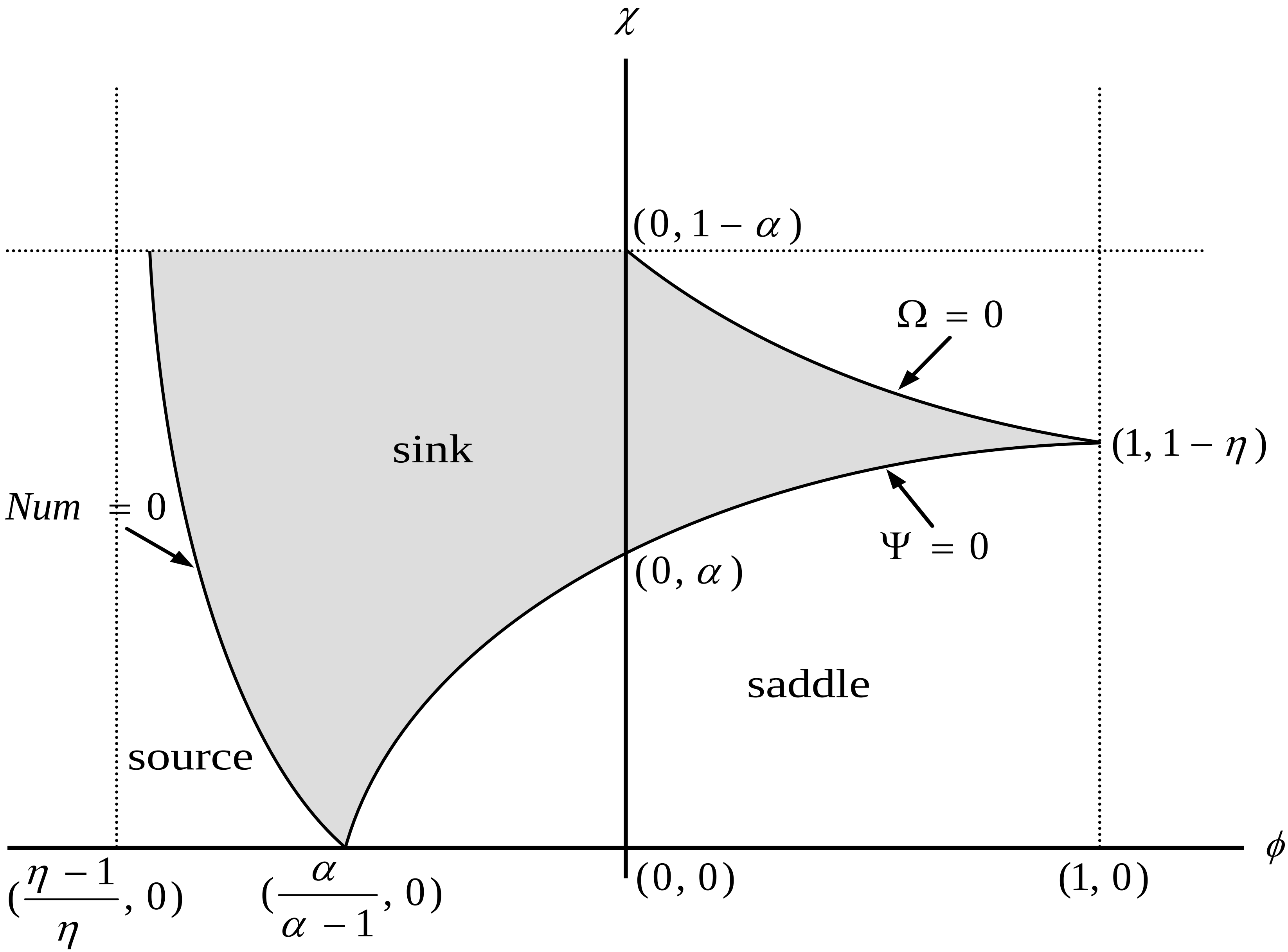


Figure 2: $\alpha < \eta$ and $\alpha < 1 - \eta < 1 - \alpha$

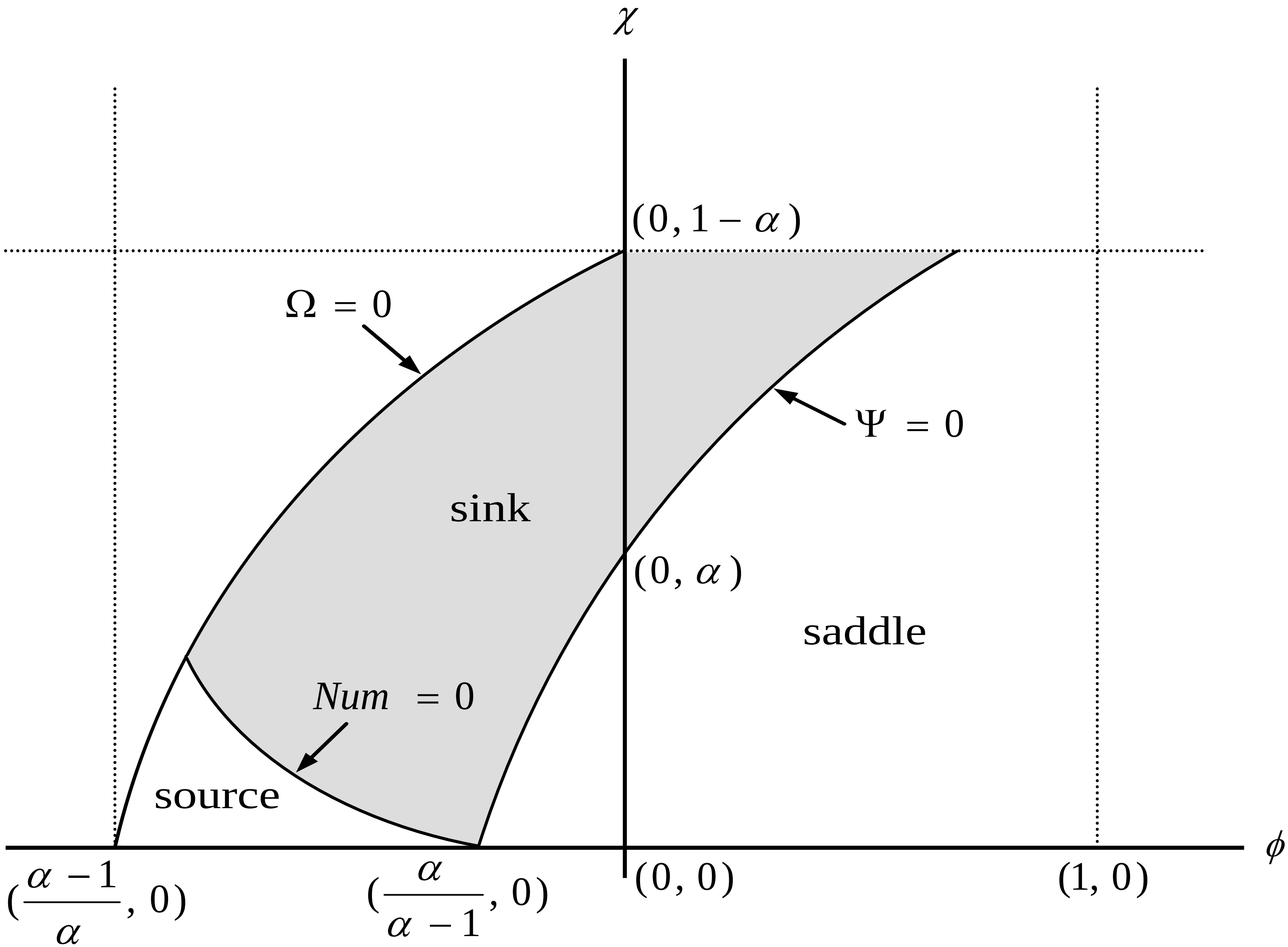
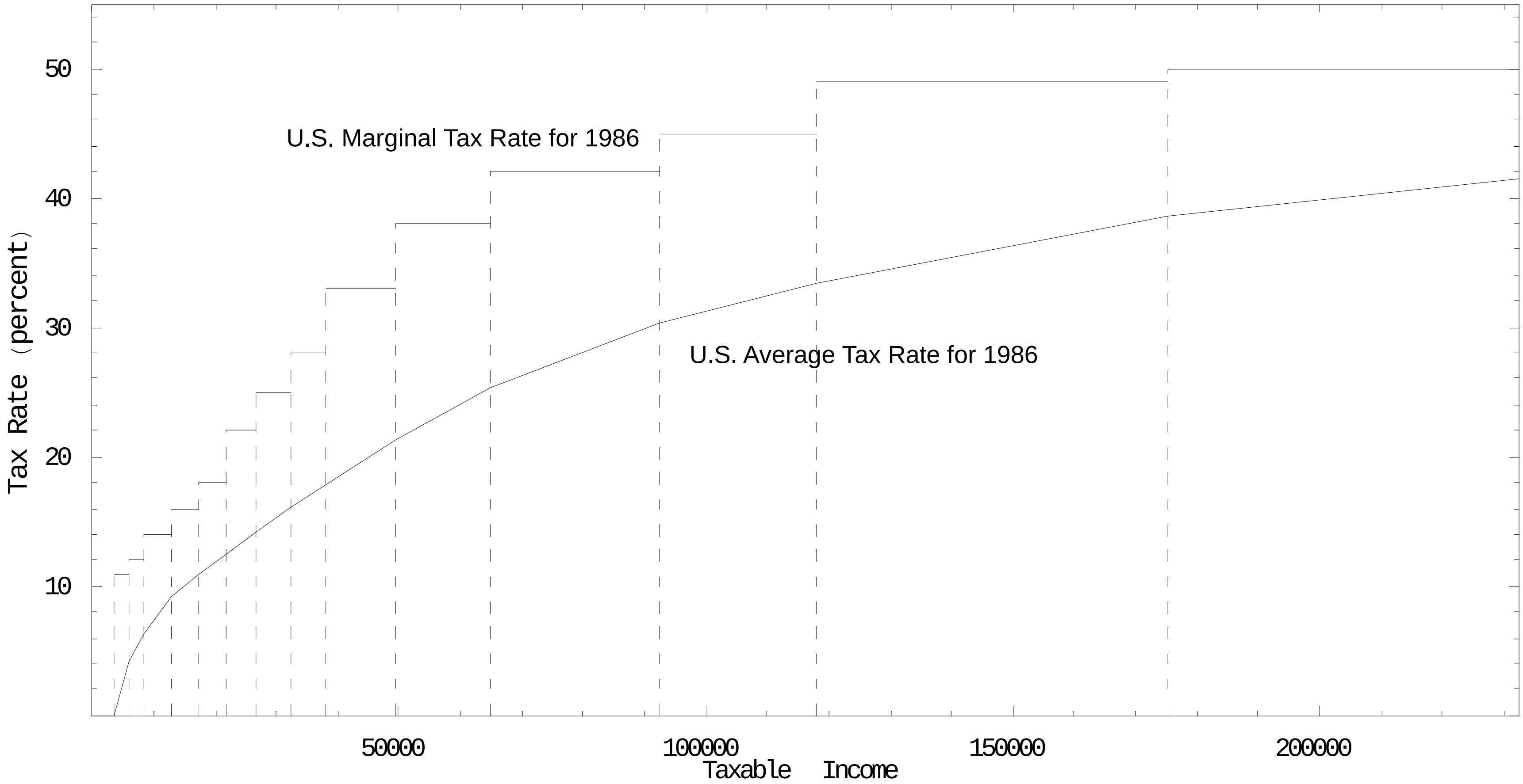


Figure 3: $\alpha > \eta$ and $\alpha < 1-\alpha < 1-\eta$

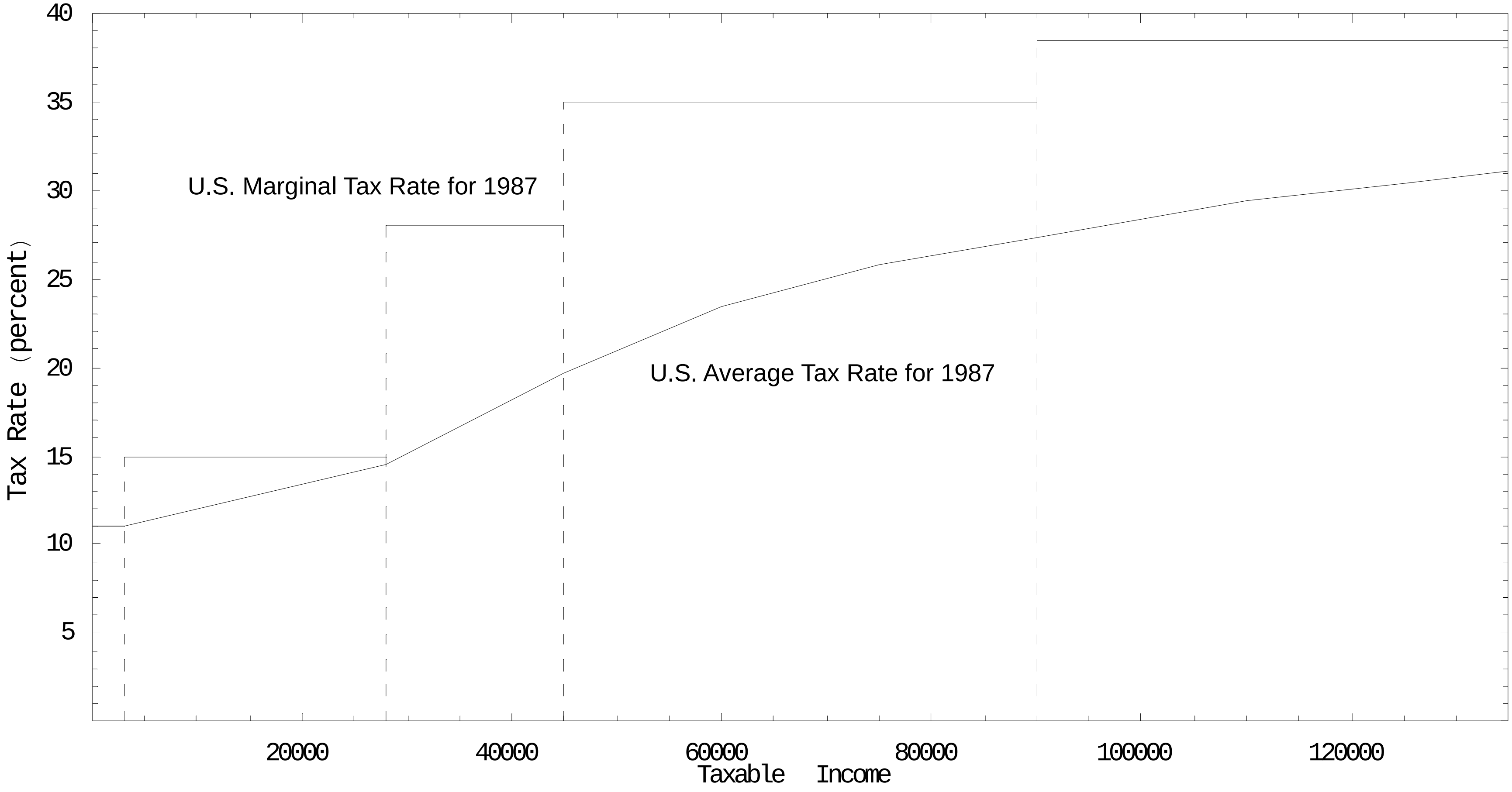
Figure 4: U.S. Tax Schedule for 1986



$$1-\tau_t = \eta(Y^*/Y_t)^{\phi}, \text{ where } Y^* = \text{ average taxable income = \$28,597;}$$

$$\hat{\eta} = 0.8058 \text{ (s.e. = 0.0013); } \hat{\phi} = 0.1335 \text{ (s.e. = 0.0111); } R^2 = 0.925$$

Figure 5: U.S. Tax Schedule for 1987



$$1-\tau_t = \eta(Y^*/Y_t)^{\phi}, \text{ where } Y^* = \text{average taxable income} = \$30{,}273;$$

$$\hat{\eta} = 0.7897 \text{ (s.e. = 0.0009)}; \ \hat{\phi} = 0.0745 \text{ (s.e. = 0.0011)}; \ R^2 = 0.823$$

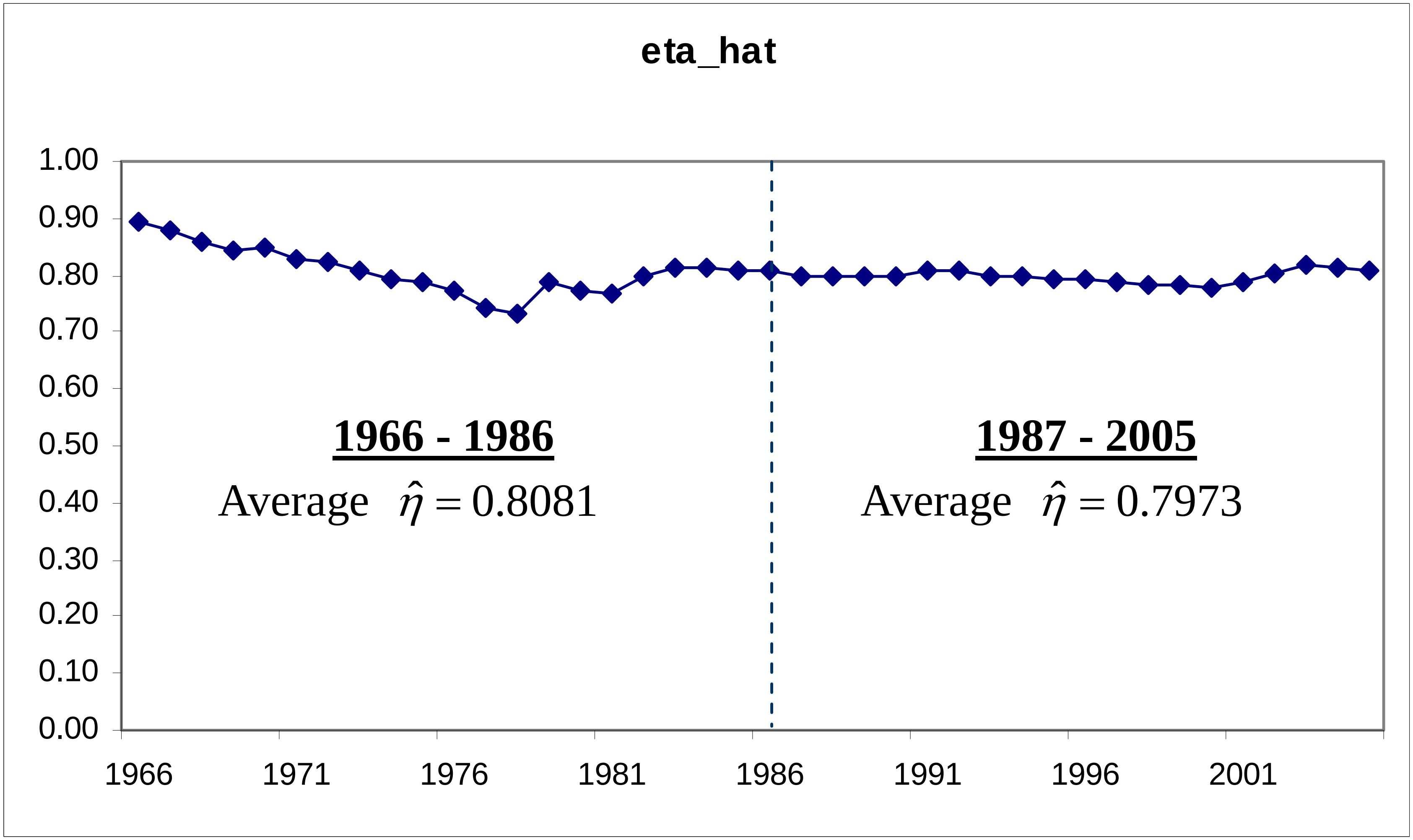


Figure 6: Estimated Tax “Level” Parameter η

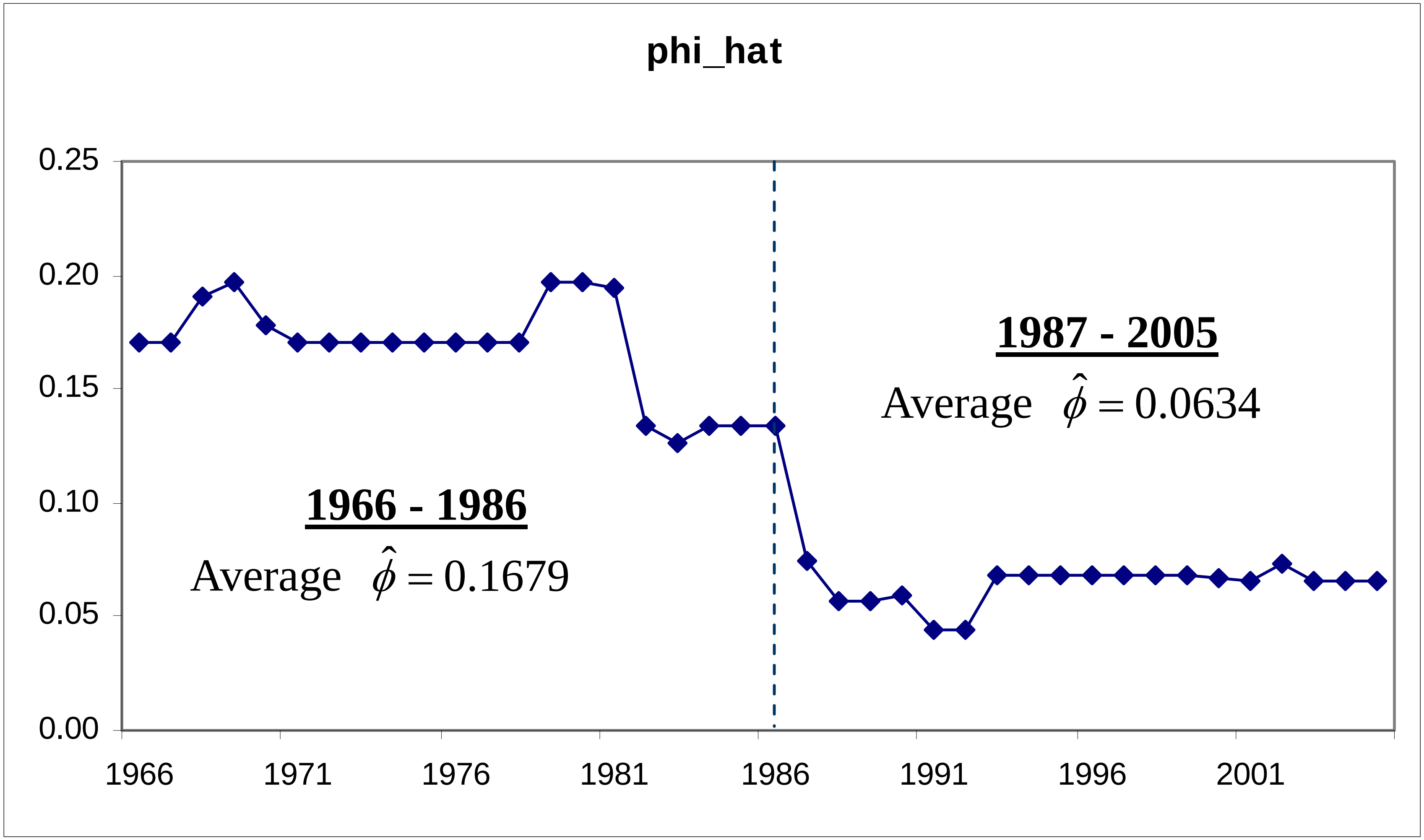


Figure 7: Estimated Tax "Slope" Parameter ϕ

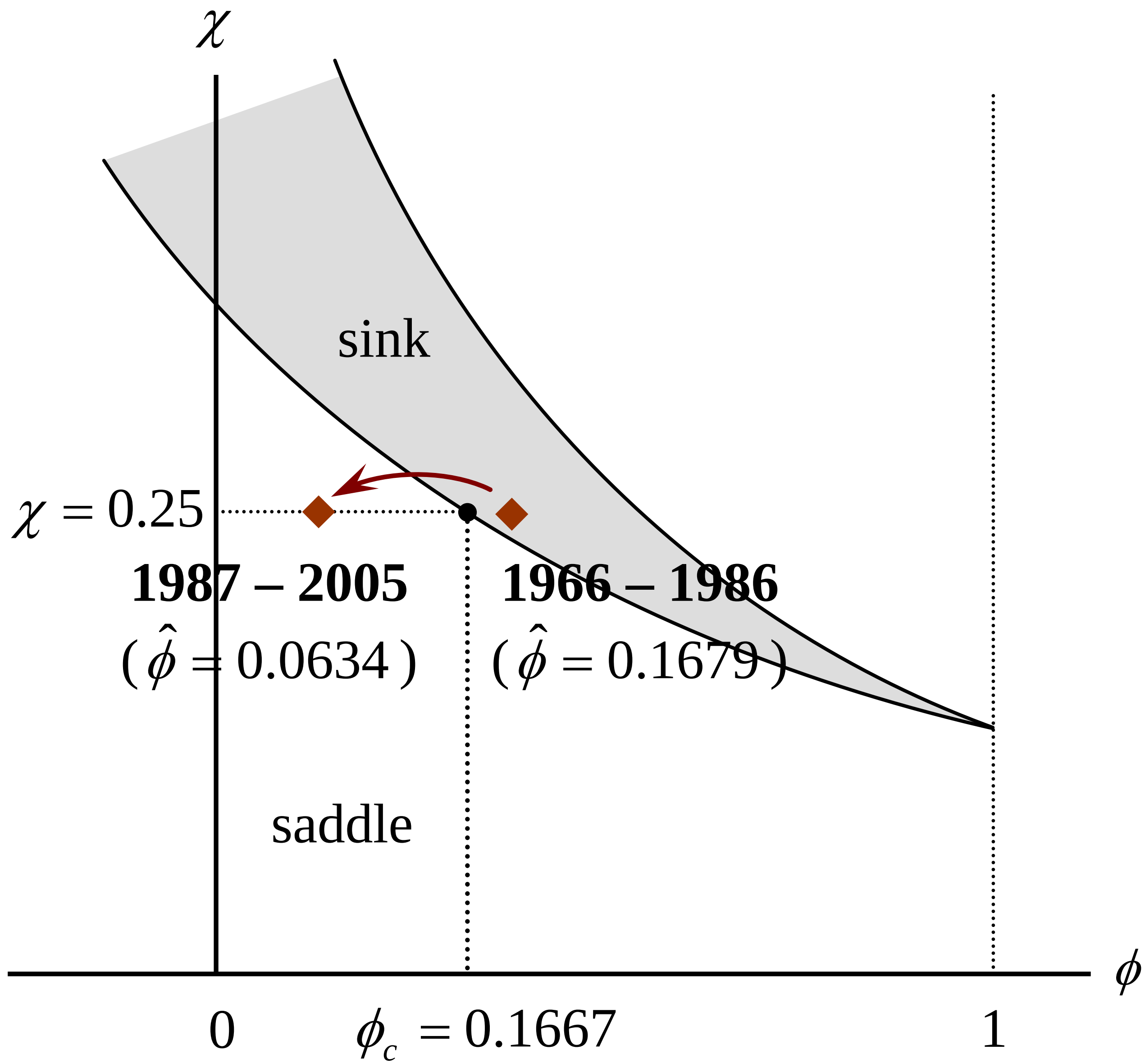


Figure 8: Great Moderation in U.S.