On the Growth and Velocity Effects of Money*

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May 3, 2007

Abstract

We show that a one-sector AK model of endogenous growth with the most generalized cash-in-advance constraint is able to account for (i) the observed long-run negative relationship between the nominal growth rate of money and the income velocity of money, (ii) the empirically ambiguous effect of changing inflation on the economy's output growth, and (iii) the divergent growth experience of countries that start with similar macroeconomic conditions.

Keywords: Cash-in-Advance Constraint, Endogenous Growth.

JEL Classification: E52, O42.

*Discussions with Richard Suen which led to this paper are gratefully acknowledged. We also thank Sharon Harrison for helpful comments and suggestions. Part of this research was conducted while Guo was a visiting research fellow at Academia Sinica, Taipei, Taiwan, whose hospitality is greatly appreciated. Of course, all remaining errors are our own.

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1 Introduction

This paper is concerned with the following three macroeconomic stylized facts: (i) there exists a discernible long-run negative relationship, both over time and across countries, between the growth rate of nominal money supply and the income velocity of money (Palivos et. al., 1993); (ii) the empirical evidence on the output-growth effect of money/inflation is mixed as this correlation can be negative (Roubini and Sala-i-Martin, 1992; De Gregorio, 1993; and Barro, 1995), positive (Gomme, 1993; and Bullard and Keating, 1995), or zero (Levine and Renelt, 1992; and Clark, 1997); and (iii) countries like the Philippines and South Korea, which had similar macroeconomic conditions in 1960, exhibited divergent growth experience in the next 30 years (Lucas, 1993). We show that a one-sector AK model of endogenous growth with the most generalized cash-in-advance (CIA) constraint is able to provide a theoretical explanation for these stylized facts simultaneously.

In an earlier work, Chen and Guo (2007) examine a similar monetary endogenous growth model in which the entire consumption purchases and a non-negative proportion of gross investment must be financed by the household's real balances (*a la* Wang and Yip, 1992). It turns out that the growth and velocity effects of money are closely linked with the number and location of the economy's balanced growth paths (BGP). In particular, when there are two BGP equilibria, the low-growth equilibrium path displays negative growth and velocity effects of money, whereas the high-growth BGP equilibrium exhibits positive effects of money/inflation on output growth and velocity. However, the positive velocity effect of money is not qualitatively consistent with the empirical findings documented in Palivos et. al. (1993).

Motivated by this inconsistency with international data, we extend Chen and Guo's study by considering the most generalized liquidity constraint. Specifically, in addition to investment, the fraction of the household's consumption expenditures that are financed by its money holdings is allowed to take a positive value of smaller than one. Moreover, in order to explain stylized fact (iii) regarding growth divergence from the same initial condition, our analysis is restricted to parametric specifications that possesses dual balanced growth paths. In this case, the velocity effect of money/inflation depends not only on the relative strength of two opposing forces dubbed the *portfolio substitution effect* and the *intertemporal substitution effect*, but also on whether the CIA-constrained proportion of consumption purchases is higher or lower than that of gross investment. We find that in contrast to Chen and Guo (2007), the high-growth equilibrium path, along which the intertemporal substitution effect dominates, exhibits a negative comovement between the money velocity and the nominal growth rate of money provided the fraction of consumption spending subject to the liquidity constraint is smaller.

2 The Model

We consider a monetary endogenous growth model in which the representative household's dynamic optimization problem is to maximize its discounted lifetime utility

$$U = \int_0^\infty \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \tag{1}$$

subject to the budget constraint

$$c_t + i_t + \dot{m}_t = y_t - \pi_t m_t + \tau_t\,, \tag{2}$$

where c_t is consumption, $\rho \in (0,1)$ is the subjective discount rate, and $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution in consumption. In addition, i_t is gross investment, π_t is the inflation rate, m_t denotes the real money balances, and τ_t represents the real lump-sum transfers that households receive from the monetary authority. Output y_t is produced by

$$y_t = Ak_t, \;\; A > 0\,, \tag{3}$$

where k_t is the household's capital stock. Investment adds to the stock of physical capital according to the law of motion

$$\dot{k}_t = i_t - \delta k_t, \;\; k_0 > 0 \text{ given}, \tag{4}$$

where $\delta \in [0,1]$ is the capital depreciation rate.

The representative household also faces the most generalized cash-in-advance (CIA) or liquidity constraint as follows:

$$\phi_c c_t + \phi_i i_t \leq m_t, \;\; 0 < \phi_c \leq 1 \text{ and } 0 \leq \phi_i \leq 1, \tag{5}$$

where ϕ_c and ϕ_i represent the fractions of consumption and investment expenditures that must be financed by the household's real balances m_t. Notice that when $\phi_c = 1$, we recover the model of Chen and Guo (2007).

The first-order conditions for the representative household with respect to the indicated variables and the associated transversality conditions (TVC) are

$$
\begin{aligned}
c_t &: \quad c_t^{-\sigma} = \lambda_{mt} + \phi_c \psi_t, && (6)\\
i_t &: \quad \lambda_{kt} = \lambda_{mt} + \phi_i \psi_t, && (7)\\
k_t &: \quad \dot{\lambda}_{kt} = (\rho + \delta)\lambda_{kt} - A\lambda_{mt}, && (8)\\
m_t &: \quad \dot{\lambda}_{mt} = (\rho + \pi_t)\lambda_{mt} - \psi_t, && (9)\\
\text{TVC}_1 &: \quad \lim_{t\to\infty} e^{-\rho t}\lambda_{kt}k_t = 0, && (10)\\
\text{TVC}_2 &: \quad \lim_{t\to\infty} e^{-\rho t}\lambda_{mt}m_t = 0, && (11)
\end{aligned}
$$

where λ_{mt} and λ_{kt} are the utility values (or shadow prices) of real money balances and physical capital, respectively, and ψ_t represents the Lagrange multiplier for the CIA constraint (5), which is postulated to be strictly binding in equilibrium.

We assume that the nominal money supply is growing at a constant rate $\mu > 0$, hence the resulting seigniorage returned to households as lump-sum transfers are $\tau_t = \mu m_t$. Moreover, clearing in the goods and money markets imply that

$$c_t + i_t = y_t, \tag{12}$$

and

$$\dot{m}_t = (\mu - \pi_t)\, m_t. \tag{13}$$

On a balanced growth path (BGP) of the economy, output, consumption, capital and real money balances exhibit a common positive growth rate denoted by θ. Using the transformed variables $p_t \equiv \frac{\lambda_{kt}}{\lambda_{mt}}$ and $z_t \equiv \frac{c_t}{k_t}$, our model's equilibrium conditions can be expressed as the following autonomous dynamical system:

$$\frac{\dot{p}_t}{p_t} = \frac{\frac{\sigma(p_t-1)}{\phi_i} + [\sigma - g_2(z_t)]\left(\delta - \frac{A}{p_t}\right) + \sigma\,[1 - g_2(z_t)]\,(A - \delta - z_t) - \sigma\mu - \rho g_2(z_t)}{\sigma - g_1(p_t)g_2(z_t)}, \tag{14}$$

$$\frac{\dot{z}_t}{z_t} = \frac{1}{\sigma}\left[g_1(p_t)\frac{\dot{p}_t}{p_t} + \frac{A}{p_t} - \rho - \delta\right] - A + \delta + z_t, \tag{15}$$

where $g_1(p_t) \equiv \frac{\phi_i - \phi_c}{\phi_c(p_t-1)+\phi_i}$ and $g_2(z_t) \equiv \frac{(\phi_c-\phi_i)z_t}{(\phi_c-\phi_i)z_t+\phi_i A}$.

A BGP equilibrium is characterized by a pair of positive real numbers (p^*, z^*) such that $\dot{p}_t = \dot{z}_t = 0$. It is straightforward to derive from (14) and (15) that p^* is the solution to the quadratic equation

$$p^* = 1 + \phi_i \left[\left(1 - \frac{1}{\sigma}\right) \left(\frac{A}{p^*} - \delta\right) + \mu + \frac{\rho}{\sigma} \right] \equiv f(p^*), \tag{16}$$

and that the expression for z^* is

$$z^* = \frac{1}{\sigma} \left(\rho + \delta - \frac{A}{p^*}\right) + A - \delta. \tag{17}$$

With (16) and (17), it follows that the common rate of economic growth θ and the corresponding income velocity of money V^* are given by

$$\theta = \frac{1}{\sigma} \left(\frac{A}{p^*} - \rho - \delta\right) \quad \text{or} \quad \theta = A - \delta - z^*, \tag{18}$$

$$V^* = \frac{A}{\phi_i A + (\phi_c - \phi_i) z^*}. \tag{19}$$

3 Growth and Velocity Effects of Money

As mentioned in the Introduction, we focus on configurations in which the economy possesses multiple balanced growth paths so that stylized fact (iii) can be accounted for. As in Suen and Yip (2005) and Chen and Guo (2007), this requires that (i) the intertemporal-elasticity parameter $\sigma < 1$, and (ii) a non-zero fraction of gross investment is subject to the liquidity constraint $\phi_i \neq 0$.[1] These two parametric restrictions are thus maintained throughout the subsequent analyses.[2] To examine the existence and number of BGP equilibria, we first note that equilibrium p^* can be derived from the intersection(s) of $f(p^*)$ in (16) and the 45-degree line. Figure 1 shows that when $\sigma < 1$ and $0 < \phi_i \leq 1$, $f(p^*)$ is a upward-sloping concave curve. Therefore, depending on the model's structural parameter values, the number of BGP equilibria can be zero, one or two. When $f(p^*)$ is tangent to the 45-degree line, the economy exhibits a unique BGP equilibrium characterized by $\hat{\sigma}$ and $\hat{p}$.[3] Moreover, it is straightforward to show that $\frac{\partial f(p^*)}{\partial \sigma} = \frac{\phi_i \theta}{\sigma} > 0$, hence there exists no (two) balanced growth path(s) provided $\sigma <$ ($>$) $\hat{\sigma}$. In the case of two BGP equilibria, the equilibrium path with a lower relative

[1] Notice that $\sigma < 1$ is not inconsistent with empirical evidence from the U.S. aggregate time series, as found in Eichenbaum, Hansen and Singleton (1988), among others.

[2] The results for the cases with $\sigma \geq 1$ and/or $\phi_i = 0$, where the model exhibits a unique BGP equilibrium, are available upon request.

[3] Using $f'(\hat{p}) = 1$ and (16) evaluated at $\hat{p}$, it can be shown that that $\hat{\sigma}$ and $\hat{p}$ are jointly determined by $\hat{\sigma} = \frac{\phi_i A}{\phi_i A + (\hat{p})^2}$ and $\hat{p} = \frac{1}{2} \left\{ 1 + \phi_i \left[\left(\frac{1}{\hat{\sigma}} - 1 \right) \delta + \mu + \frac{\rho}{\hat{\sigma}} \right] \right\}$.

shadow price of capital, denoted as p_1^*, will grow faster than the other that is associated with p_2^*, that is $\theta(p_1^*) > \theta(p_2^*)$ (see equation 18 where $\frac{\partial\theta}{\partial p^*} < 0$).

3.1 Growth Effect

Figure 1 shows that a higher nominal money growth shifts the locus of $f(p^*)$ upward. This leads to a theoretical ambiguity regarding the sign for the growth effect of money. Intuitively, how money affects the economy's output growth depends crucially on the relative strength of two opposing forces. On the one hand, a rise in the money growth rate μ generates a higher inflation, which in turn increases the cost of money holdings. As a result, the representative household substitutes out of real balances and into capital (the portfolio substitution effect), raising the relative shadow price of capital p^* because of a higher demand $\left(\frac{dp^*}{d\mu} > 0\right)$. This will reduce its net rate of return and thus the BGP's growth rate. On the other hand, a higher inflation *ceteris paribus* induces the representative household to consume less and invest more today in exchange for higher future consumption (the intertemporal substitution effect). This expands the supply of capital, thereby lowering its relative shadow price $\left(\frac{dp^*}{d\mu} < 0\right)$. It follows that the economy's output growth rate will rise.[4]

As shown in Figure 1, starting from the low-growth BGP equilibrium associated with $\theta(p_2^*)$, a higher nominal money growth raises the relative shadow price of capital because of a dominating portfolio substitution effect. Consequently, the economy displays a negative relationship between output growth and money/inflation $\left(\frac{d\theta(p_2^*)}{d\mu} < 0\right)$. On the contrary, due to a stronger intertemporal substitution effect, the BGP's growth rate and money are positively correlated along the high-growth equilibrium path $\left(\frac{d\theta(p_1^*)}{d\mu} > 0\right)$. Notice that both theoretical results are consistent with stylized fact (ii) where the existing empirical evidence on the output-growth effect of money/inflation is mixed.

3.2 Velocity Effect

To analyze the velocity effect of money, we first note that the BGP's consumption-capital ratio z^* is positively correlated with the relative shadow price of capital p^* (see equation 17). As

[4] The portfolio substitution effect (PSE) and the intertemporal substitution effect (ISE) can be separately identified by decomposing $\frac{dp^*}{d\mu}$ as

$$\frac{dp^*}{d\mu} = \frac{\phi_i}{\underbrace{[1 + \frac{\phi_i A}{(p^*)^2}]}_{PSE > 0} + \underbrace{[-\frac{\phi_i A}{\sigma (p^*)^2}]}_{ISE < 0}}.$$

discussed earlier, an increase in the money growth rate μ yields a higher relative shadow price of capital p^* when the portfolio substitution effect dominates along the low-growth equilibrium path. This will lead to decreases in investment, output, and consumption because of a lower net rate of return on capital. Since the marginal propensity to consume is smaller than 1, the decline in consumption will be smaller than that in output. Hence, the consumption-output ratio rises. Due to the AK production function, the consumption-capital ratio z^* will rise as well, thus $\frac{dz^*}{d\mu} > 0$. Notice that this result is reversed $\left(\frac{dz^*}{d\mu} < 0\right)$ in the high-growth BGP equilibrium as the intertemporal substitution effect is stronger .

Next, the BGP's income velocity of money V^* is negatively (positively) related to z^* when $\phi_c > (<)\ \phi_i$ (see equation 19). The intuition for this finding can be understood by using (3) and (12) to rewrite the money velocity as

$$V^* = \frac{A}{\phi_c z^* + \phi_i(\frac{i^*}{k^*})}. \tag{20}$$

In light of the AK technology, the goods-market clearing condition (12) implies that changing z^* generates the same magnitude of opposite movement in the investment-capital ratio $\frac{i^*}{k^*}$, that is $\Delta z^* = -\Delta\left(\frac{i^*}{k^*}\right)$. It follows that in response to a rise of z^*, the denominator of (20) will increase (decrease) if $\phi_c > (<)\ \phi_i$, which in turn produces a lower (higher) money velocity V^*.

Based on the previous discussions, we find that the impact of changing μ on V^* (through z^*) depends not only on the location of the BGP equilibrium, but also on whether the CIA-constrained proportion of consumption expenditures ϕ_c exceeds or falls below the investment fraction ϕ_i. Specifically, on the low-growth equilibrium path associated with $\theta(p_2^*)$ and a stronger portfolio substitution effect, the velocity effect of money is negative (positive) when $\phi_c > (<)\ \phi_i$. On the contrary, since the intertemporal substitution effect dominates along the high-growth BGP with $\theta(p_1^*)$, the velocity effect of money is positive (negative) when $\phi_c > (<)$ ϕ_i.[5] The above results imply that stylized fact (i), where the rate of nominal money growth and the income velocity of money are negatively correlated, can be obtained in either BGP equilibrium.

[5]Notice that $V^* = \frac{1}{\phi_i}$ when $\phi_c = \phi_i$ (see equation 19). In this case, the income velocity of money is independent of changes in the nominal money growth rate $\frac{dV^*}{d\mu} = 0$.

4 Conclusion

We have shown that in a one-sector monetary AK model of endogenous growth with the most generalized cash-in-advance constraint, the presence of two balanced-growth-path equilibria offers a plausible explanation for the variations in growth experience between (for example) the Philippines and South Korea. Moreover, we find that when the portfolio substitution effect dominates and the liquidity constraint is applicable more to consumption purchases, the effects of changing inflation on the economy's output growth and velocity are both negative along the low-growth equilibrium path. By contrast, when the opposing intertemporal substitution effect is stronger and the CIA-constrained fraction of gross investment is higher, the economy's high-growth BGP equilibrium exhibits a positive growth effect and a negative velocity effect of money. As a result, each of the two BGP equilibrium paths in our model is able to account for the three macroeconomic stylized facts discussed at the beginning of this paper.

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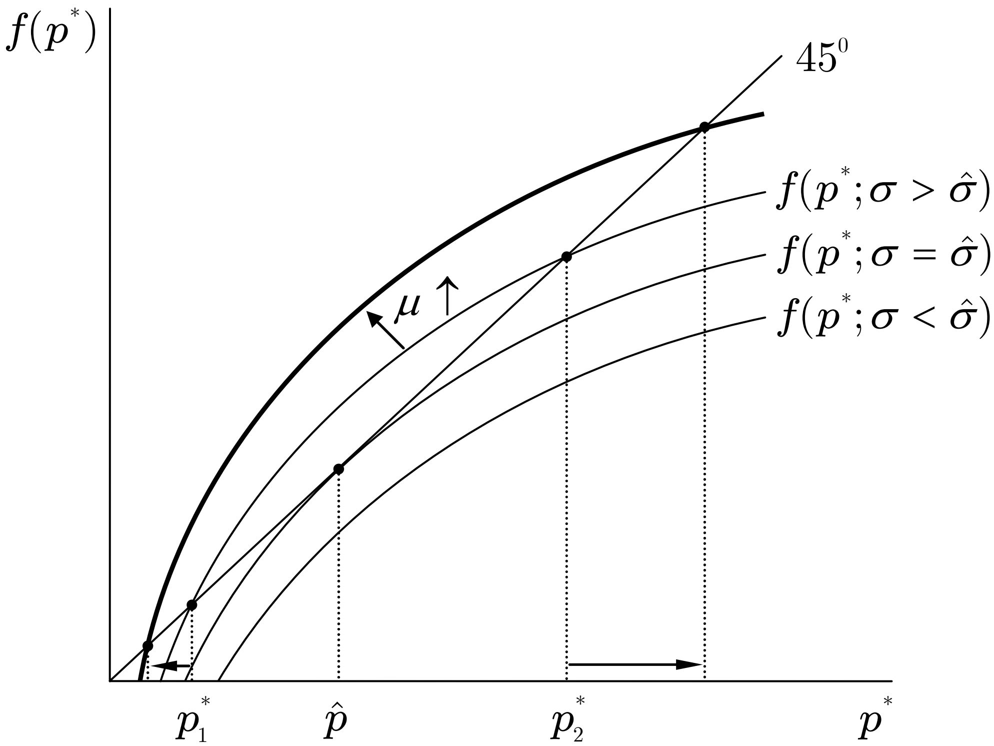

Figure 1: $\sigma < 1$ and $0 < \phi_i \leq 1$