Minimal Relativism, Dominance, and Standard of Living Comparisons Based on Functionings*

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This version: 17 February 2005

* For helpful comments and discussions, we are grateful to Sudhir Anand, James Foster, Stephen Marglin, and Amartya Sen.

Abstract. This paper considers the functioning approach to the standard of living. It shows that, in evaluating standards of living, one is faced with deep-rooted tensions between the principle of minimal relativism, which requires some minimal respect for differences between the evaluations of different individuals and/or the differences between the norms of different communities, and the principle of dominance.

JEL Classification Numbers: D63, D71, I31

Introduction

The functioning approach to the notion of living standards is an important recent development in welfare economics. The core of the approach, initiated by Sen (1985, 1987) and Nussbaum (1988, 1993, 2000), consists of the idea that the notion of the standard of living should be formulated in terms of functionings (i.e., the 'doings' and 'beings' that have 'intrinsic value' for people). and capabilities rather than in terms of utility or commodities. Several related approaches stem from this central idea. First, we have the approach where a person's standard of living is assumed to be determined exclusively by his achieved functioning bundle. An alternative approach is based on the assumption that a person's standard of living depends exclusively on his capability set (i.e., the set of all mutually exclusive functioning bundles available to him), which reflects the opportunities available to the person. Lastly, we have a more general conceptual framework where a person's standard of living is determined by his capability set and / or the functioning bundle belonging to his capability set, which he actually achieves. It is clear that the first two frameworks are special cases of the third. The purpose of this paper is to demonstrate that, no matter which of these three routes one decides to follow, in comparing the living standards of different individuals one has to contend with a deep-rooted tension between two distinct compelling principles, namely, *minimal relativism*, which requires a minimal amount of respect for differences in social and cultural norms and evaluative standards, and *weak dominance*, which seems to have been widely accepted in the literature

Our analysis starts with the simplest, and, in some ways, the most operational, of the three frameworks, to wit, the framework in which a person's living standards are determined exclusively by her achieved functioning bundle. Here one is faced with the problem of comparing the living standards of a given person in different situations with different achieved functioning bundles, and the problem of comparing the living standards of different persons. First, consider the problem of comparing the living standards that a given individual enjoys with alternative functioning bundles. One of the issues that arises in this context is whether the ranking of an individual's living standards corresponding to different functioning bundles should be independent of who that individual happens to be. More specifically, the issue is whether the ranking should be invariant with respect to: (i) the specific individual's values, and (ii) the general cultural and social norms prevailing in the community to which the individual belongs. Of course, such invariance would not constitute a severe restriction if one could reasonably claim that individuals and communities did not differ in their evaluations of different functioning bundles. However, even if one accepts that, in general, the 'doings' and ‘beings’ that different people value are basically the same, very few people would be prepared to argue that different individuals and communities attach the same relative importance to different functionings. In the presence of such differences in relative valuations of individuals and communities, the requirement that the ranking of different functioning bundles in terms of an individual’s living standards should be invariant with respect to the valuations of the individual and the norms and standards of the community to which the individual belongs seems to be a very restrictive demand. It is plausible to say that there are some broad similarities in the values and norms of different individuals and/ or different communities. Also, in comparing an individual’s living standards in alternative situations, one may not always respect all the values of the individual; nor may one like to respect all the cultural and social norms prevailing in the individual’s community. But it seems to us that one would be taking an extreme position if either one claimed that there were no significant differences between the values, norms, and standards of different individuals and different communities or one claimed that such difference must not play any role whatsoever in standard of living comparisons for individuals. The denial of this extreme position leads us to what we call relativism in standard of living comparisons. How much of relativism one would be willing to accept depends on one’s judgement. However, it seems to us that most people would accept a minimal version of relativism that we introduce in this paper. This version just postulates the existence of two distinct individuals, i and j, and two functioning bundles x and y, such that we would be prepared to say that x offers a higher standard of living to i than y, but y offers a higher standard of living to j than x. In addition to minimal relativism, we postulate another property for standard of living comparisons based on functioning bundles. This is what we call weak dominance. The property of weak dominance requires that, for all functioning bundles x and y, if x is a bigger functioning bundle than y in the sense that x contains more of some functioning and no less of any functioning as compared to y, then the standard of living of any individual with the bundle x must be at least as great as the standard of living of any individual with the bundle y. Note that a stronger version of the weak dominance principle, which says that any individual with a bigger functioning bundle has a higher standard of living than any individual with a smaller functioning bundle, has been strongly endorsed in the literature (see, for example, Sen (1985, 1987)). Since functionings, as Sen (1985, 1987) and Nussbaum (1988, 1993, 2000) conceive them, are all desirable attributes, we find even this stronger version of the weak dominance principle highly appealing, but the weak dominance principle turns out to be sufficient for our purpose. We show that, if standard of living comparisons based on achieved functioning bundles satisfy a very mild continuity property, then they cannot simultaneously satisfy minimal relativism and the weak dominance principle. This is disturbing, since, as we have noted, both these requirements seem to be highly plausible.

After demonstrating the tension between minimal relativism and weak dominance when standard of living comparisons are based on achieved functioning bundles only, we extend the analysis to the more general case where living standards are judged by achieved functioning bundles or capability sets or both. We introduce generalized versions of minimal relativism, weak dominance, and continuity for this general framework. These generalized versions of the properties, which, like their original simpler counterparts, are highly plausible, turn out to be incompatible. Thus the tension between minimal relativism and the principle of weak dominance seems to be endemic in the functioning approach. However, as we note at the end of the paper, this is not a special feature of the functioning approach as such: an exactly similar problem arises even when the space under consideration is the usual commodity space rather than the functioning space.

The plan of the paper is as follows. In Section 2, we introduce some basic concepts and notation. In Section 3, we consider the approach where standard of living comparisons are based on functioning bundles only. In this section, we introduce the properties of minimal relativism, weak dominance, and continuity for standard of living comparisons, state our result showing the incompatibility of the three properties, and provide a diagrammatic illustration of the proof of this result (the formal proof is given in the appendix). In Section 4, we consider the most general framework that incorporates capability sets and/or functioning bundles. In this framework, we introduce generalized versions of minimal relativism, weak dominance, and continuity, state the result showing that these general versions are incompatible, and give a diagrammatic illustration of how the proof works. Again, the formal proof is given in the appendix. We conclude in Section 5 by drawing attention to certain similarities between our results and Sen's (1970, 1970a) famous theorem on the impossibility of the Paretian liberal, and the applicability of our formal results to commodity-based comparisons of living standards.

Some basic notation

Let $N = \{1,\ldots,n\}$ $(\infty > n > 1)$ be the set of all individuals in whose living standards we may be interested. The individuals in N may belong to different societies. Even if all the individuals in N belong to the same society, they may belong to distinct ethnic and cultural groups with significantly different norms and standards and they may have very different personal valuations of functioning bundles.

We assume that there are m $(\infty > m > 1)$ functionings. Each functioning $k(k = 1,\ldots,m)$ is measurable along an interval $[0, b(k)]$ where $0 < b(k)$ and $b(k)$ can be finite or ∞. While, formally it is easier to allow the quantity of every functioning to take any finite non-negative value, for some functionings (e.g., absence of illness), there may be an upper bound. Therefore, it seems reasonable to permit such an upper bound (without, however, making it mandatory). Let X be the set of all possible functioning bundles. We assume that $X = [0, b(1)] \times \cdots \times [0, b(m)]$. The functioning bundles will be denoted by $x, y, z,$ and so on. For all $x, y \in X$, $x > y$ if $x_k \geq y_k$ for all $k = 1,\ldots,m$ and $x \neq y$. For all $x \in X$, the norm of x is to be denoted by $\|x\|$.

The *capability set* of an individual is the (non-empty) set of all mutually exclusive functioning bundles available to the individual. Thus, given a capability set, the individual must choose exactly one functioning bundle in the capability set. Intuitively, the capability set reflects the opportunities open to the individual; the functioning bundle actually chosen by the individual from his capability set represents his achievement. We shall assume that the class of all non-empty, convex, compact, and comprehensive subsets of X constitutes the class of all possible capability sets for a person. Let Z denote this class. Let T denote the set of all (x, A) such that $x \in A \in Z$. Thus, T is the set of all ordered pairs (x, A) such that A is a capability set and x is a functioning bundle in A. We call an element (x, A) of T an *achievement – opportunity combination* (AOC). The intuitive interpretation of (x, A) is that it represents a situation where A is the capability set of a person and x is the functioning bundle chosen by the person from the set A.

Suppose we have an external evaluator (EE), who is comparing the standards of living of different individuals. As we noted in the introduction, we consider two distinct frameworks: a framework where the standard of living comparisons are based on achieved functioning bundles but not on capability sets, and a more general framework, where the standard of living comparisons can depend on either achieved functioning bundles or capability sets or both. To avoid possible confusion, we introduce somewhat different notation for the two frameworks. Consider first the case where the EE takes into account only achieved functioning bundles but not the capability sets. Note that, for every individual i and every $x \in X$, one can think of a set A in Z, such that $x \in A$ (for example, given x, the set of all $y \in X$ such that $y = x$ or $x > y$ belongs to Z and contains x.). Therefore, for every $i \in N$ and every $x \in X$, one can visualize i as choosing x from some capability set. Therefore, when the standard of living of every individual is assumed to depend only on her achieved functioning bundle, we can think of the EE's standard of living comparisons as being reflected in a binary relation $\succeq$ defined over $N \times X$. For all $i,j \in N$ and all $x,y \in X$, $(i,x) \succeq (j,y)$ denotes that i's standard of living, when i's achieved functioning bundle is x, is at least as high as j's standard of living, when j's achieved functioning bundle is y. We assume that $\succeq$ is reflexive and transitive but not necessarily complete (thus, some pairs in $N \times X$ may be non-comparable under $\succeq$). For all $i,j \in N$ and all $x,y \times X$, $(i,x) \succ (j,y)$ if and only if $(i,x) \succeq (j,y)$ and not $(j,y) \succeq (i,x)$. Intuitively, $(i,x) \succ (j,y)$ denotes that i's standard of living when i's achieved functioning bundle is x is higher than j's standard of living when j's achieved functioning bundle is y. At the risk of emphasizing the obvious, it may be worth clarifying that the EE's problem of ranking the elements of $N \times X$ is not the problem of ranking ' social states'. In our context, a natural interpretation of a social state is that a social state is an n-tuple of functioning bundles, with exactly one functioning bundle for each individual. It is then clear that the EE's problem of ranking $(i,x), (j,y)$, etc. is intuitively very different from the problem of how an outside ethical observer of the society should rank the different social states or n –tuples of functioning bundles.

Now consider the most general formulation where the standard of living of an individual is determined by her capability set or by her achieved functioning bundle or both. In this case, we shall represent the EE's comparisons of living standards by a binary relation $\trianglerighteq$ defined over $N \times T$. For all $(i,x,A), (j,y,B) \in N \times T$, $(i,x,A) \trianglerighteq (j,y,B)$ denotes that, in the EE's judgement, i's standard of living when i has the AOC (x,A) is at least as great as j's standard of living when j has the AOC (y,B). $\triangleright$ denotes the relation "offers a strictly higher standard of living" , i.e., for all $(i,x,A), (j,y,B) \in N \times T$, $(i,x,A) \triangleright (j,y,B)$ if and only if [$(i,x,A) \trianglerighteq (j,y,B)$ and not $(j,y,B) \trianglerighteq (i,x,A)$]. We assume that $\trianglerighteq$ is reflexive and transitive, but not necessarily complete (thus, $\trianglerighteq$ may fail to rank some pairs in $N \times T$). Again, note that the EE's task of ranking the elements of $N \times T$ is intuitively very different from the task of evaluating social states.

The case where standard of living comparisons depend on achieved functioning bundles but not capability sets

Minimal relativism, weak dominance, and continuity

Suppose we have an external evaluator (EE) who is comparing the standard of living of individuals in different possible situations on the basis of their achieved functioning bundles. We propose three restrictions on the EE's ranking $\succeq$ over $N \times X$.

Minimal Relativism:

In Section 1, we have argued that, though there may be general agreement about what functionings are relevant, it is difficult to claim that the evaluation of functioning bundles is the same for all individuals or that different communities share identical norms and cultural standards relating to the evaluation of functioning bundles. In the presence of such diversity, to require that, for all $i,j \in N$ and, for all $x,y \in X$, the external evaluator's ranking of (i,x) and (i,y) must be exactly analogous to his ranking of (j,x) and (j,y), seems to be an unacceptable form of 'universalism' (indeed, some may call it a form of cultural paternalism). Given that there seems to be considerable diversity in the values, norms, and standards by which individuals and communities assess alternative functioning bundles, the denial of such universalism intuitively leads us to the following property that we call minimal relativism.

Definition 3.1.

$\succeq$ satisfies *minimal relativism* if and only if, there exist $i,j \in N$ and $x,y \in X$, such that $(i,x) \succ (i,y)$ and $(j,y) \succ (j,x)$.

Minimal relativism may be viewed as an expression of one's desire to accord some respect to differences in the norms and standards with which different individuals or different communities evaluate functioning bundles. One can view it as a primitive property. Alternatively, if one likes, one can regard it as a consequence of other properties of $\succeq$ and assumptions about the real world. It may be worthwhile considering here one such property of $\succeq$, which has been discussed in the literature, and one such assumption about the real world. Suppose, for every $i \in N$, we have an ordering R_i over X, which reflects i's evaluation of alternative functioning bundles in terms of his own standard of living. For all $x,y \in X$, xR_iy then denotes that i regards x as offering him at least as high a standard of living as y. We write xP_iy if and only if xR_iy and not yR_ix (xP_iy thus indicates that i regards x as offering him a strictly higher standard of living than y). Consider now the property introduced in Definition 3.2 below.

Definition 3.2.

$\succeq$ satisfies the *principle of non-paternalistic comparisons* (PNPC) if and only if, for all $i \in N$ and all $x,y \in X$, $(i,x) \succeq (i,y)$ iff xR_iy.

PNPC requires that, in comparing the standards of living of a given individual in different situations with different functioning bundles, the EE should go by that individual's evaluation of his own living standards corresponding to those bundles. This, of course, is similar to the idea, common in classical welfare economics, of respecting the consumer's preferences over her own consumption bundles. The main difference is that here the 'respect' is for the individual's *evaluation* of functioning bundles rather than for his utility-based ranking when utility is interpreted in terms of choice or happiness or pleasure or desire fulfillment. A charge that is often levelled against the functioning approach is that it is paternalistic since it does not respect the individual's preferences. Sen (1987, p.32) has responded to this by emphasizing the difference between the individual's own evaluation of his functioning bundles in different situations and the individual's utility-based ranking of these situations under any of the usual interpretations of the term utility. Referring to this distinction, Sen (1987, p.32) writes,

> The distinction is of particular importance in dealing with the point ... that any departure from utility-based ranking must involve paternalism ... The problem is more complex than that since the person's own evaluation may involve differences from his own utility rankings ... The issue of paternalism, when it does arise, must relate to the rejection of the person's *self-evaluation* (rather than of utility).

Sen's distinction between a person's own evaluation of his standard of living and the person's utility-based ranking is a fundamental distinction. When a person's self-evaluation differs from his utility-based ranking, there does not seem to be anything particularly paternalistic about going by the person's self-evaluation as the appropriate evaluation of that person's living standards. However, if the EE wants to be non-paternalistic in the sense of accepting each individual's self-evaluation for the purpose of comparing that individual's living standards corresponding to different functioning bundles, then $\succeq$ would satisfy PNPC. Given PNPC, if we assume that there are at least two individuals i and j and two functioning bundles x and y in X, such that xP_iy and yP_jx, then it is clear that the EE's ranking $\succeq$ must satisfy minimal relativism. Thus, minimal relativism follows from PNPC together with the factual assumption of some (small) amount of diversity in the different individuals' evaluations of functioning bundles. If, in evaluating an individual's living standards, the EE wants to go not by that individual's personal evaluation, but by the norms generally prevailing in the community to which that individual belongs, then we can reinterpret R_i, R_j, etc. in terms of the standards prevailing in the communities of i, j, etc. In that case, PNPC will now reflect the EE's desire to respect the norms of the community to which the relevant individual belongs. Again, minimal relativism will follow from PNPC, reinterpreted in this way, together with the factual assumption of some diversity in the general values and standards prevailing in different communities. While it is possible to derive minimal relativism in this fashion from other explicitly stated properties of $\succeq$ and factual assumptions about the diversity of values to be found in the real world, we shall treat minimal relativism as having enough direct intuitive appeal to be treated as a primitive condition in its own right.

Weak Dominance:

Minimal relativism does not stipulate anything about interpersonal comparisons of living standards. In contrast, our next property, which uses the relation of dominance between functioning bundles, incorporates restrictions on such interpersonal comparisons

Definition 3.3.

(i) $\succeq$ satisfies *weak dominance* if and only if, for all $i,j \in N$ and all $x,y \in X$, $x > y$ implies $(i,x) \succeq (j,y)$.

(ii) $\succeq$ satisfies *dominance* if and only if, for all $i,j \in N$ and all $x,y \in X$, $x > y$ implies $(i,x) \succ (j,y)$.

Weak dominance is a compelling condition, given that the functionings are the doings and beings that people value and represent desirable attributes. It stipulates that a person with a bigger functioning bundle must have at least as high a standard of living as a person with a smaller functioning bundle. Though dominance is logically stronger than weak dominance, it is still an eminently acceptable condition, and it is not, therefore, surprising that it has been endorsed by various writers. For example, Sen (1987, pp. 29-30) writes,

> In comparing across class barriers, or in contrasting the living conditions of the rich with those of the very poor, ... the dominance partial ordering may indeed give many unequivocal judgements of the ranking of overall living standards. There is no reason for us to spurn what we can get in this way ...

The weaker property of weak dominance will suffice for our purpose in this paper.

Continuity:

Finally, we introduce a very plausible and familiar property of continuity of $\succeq$.

Definition 3.4.

$\succeq$ satisfies *continuity* if and only if, for all $i \in N$ and all $x,y,z \in X$, if $(i,x) \succ (i,y)$, then [there exists $\epsilon_1 > 0$, such that $\|x - z\| < \epsilon_1$ implies $(i,z) \succ (i,y)$] and [there exists $\epsilon_2 > 0$, such that $\|y - z\| < \epsilon_2$ implies $(i,x) \succ (i,z)$].

Continuity simply requires that, if the EE considers *i*'s standard of living from *x* to be higher than *i*'s standard of living from *y*, then the EE must consider *i*'s standard of living from *z* to be higher than *i*'s standard of living from *y*, where *z* is a bundle arbitrarily close to *x*, and the EE must consider *i*'s standard of living from *x* to be higher than *i*'s standard of living from *w*, where *w* is a bundle arbitrarily close to *y*. In a spirit similar to standard continuity properties in the theory of consumer's behaviour, our property of continuity just stipulates that the EE's ranking $\succeq$ does not show any 'big jumps'. Note that continuity is concerned with the comparisons of a given individual's living standards in different situations; it does not impose any restriction on interpersonal comparisons. We find the property compelling.

The incompatibility of weak dominance, minimal relativism, and continuity

We now prove one of our basic results, which shows the incompatibility of weak dominance, minimal relativism, and continuity.

Proposition 3.5. $\succeq$ cannot simultaneously satisfy weak dominance, minimal relativism, and continuity.

Proof: See the appendix.

It may be helpful to illustrate the proof of Proposition 3.5 with a diagram. Figure 1 has been drawn with the assumption that there are only two functionings. By minimal relativism, there exist $i,j \in N$ and $x,y \in X$, such that $(i,x) \succ (i,y)$ and $(j,y) \succ (j,x)$. For the sake of convenience in this diagrammatic exposition, we assume that, for each functioning *k*, $0 < x_k < b(k)$ and $0 < y_k < b(k)$; however, no such restrictive assumption is necessary for the formal proof given in the appendix. Given $(j,y) \succ (j,x)$, noting continuity, there exists $\epsilon_1 > 0$ such that, for all $x' \in X$, if $\|x - x'\| < \epsilon_1$, then $(j,y) \succ (j,x')$. Choose $w \in X$, such that $w > x$, and $\|x - w\| < \epsilon_1$. Then $(j,y) \succ (j,w)$. Since $(i,x) \succ (i,y)$, by continuity there exists $\epsilon_2 > 0$, such that, for all y' in X, if $\|y - y'\| < \epsilon_2$, then $(i,x) \succ (i,y')$. Take $z \in X$ such that $z > y$ and $\|z - y\| < \epsilon_2$. Then $(i,x) > (i,z)$. Thus we have, $(i,x) \succ (i,z)$ and $(j,y) \succ (j,w)$. However, by (WD), $(i,z) \succeq (j,y)$ and $(j,w) \succeq (i,x)$. Therefore, we have $(i,x) \succ (i,z), (i,z) \succeq (j,y), (j,y) \succ (j,w)$, and $(j,w) \succeq (i,x)$, which violates the transitivity of $\succeq$.

Insert Figure 1

Since continuity is a very appealing structural property of $\succeq$, we are inclined to interpret Proposition 3.5 as revealing a direct tension between minimal relativism and weak dominance.

The general case where standard of living comparisons depend on achieved

functioning bundles and capability sets

In Section 3, we have shown the difficulty of combining minimal relativism, weak dominance and continuity in comparing living standards in the framework where the standard of living is based exclusively on achieved functioning bundles. In this section, we extend the analysis to the framework in which the living standard is based on achieved functioning bundles or capability sets or both.

Some further notation

For any capability set A in Z, let $G(A)$ be the set of all x in A, such that x is not vectorially dominated by any y in A. We call $G(A)$ the *frontier* of A (since, by assumption, A is compact, $G(A)$ must be non-empty.) For all capability sets A and B in Z, let $A > B$ denote that: (i) B is a proper subset of A; and (ii) $G(A)$ dominates $G(B)$ in the sense that, for every y in $G(B)$ there is x in $G(A)$ such that $x > y$. For any capability set A in Z and any vector z in $\mathbb{R}^m_+$, let $A_{+z} = \{y \in \mathbb{R}^m : y_k \leq \min\{x_k + z_k, b(k)\}$ for some $x \in A, k = 1,\ldots,m\}$ and $A_{-z} = \{y \in X : y_k \leq \max\{x_k - z_k, 0\}$ for some $x \in A$, $k = 1,\ldots,m\}$. Clearly, A_{+z} and A_{-z} defined above are non-empty, compact, convex and comprehensive. Therefore, both A_{-z} and A_{+z} are in Z.

Given a capability set A in Z and given a positive number ϵ, let $A(\epsilon)$ be the set: $\{A' \in Z : A' = A_{+z}$ or $A' = A_{-z}$ for some $z \in \mathbb{R}^m_+$ with $\|z\| < \epsilon\}$. We note that $A(\epsilon)$ is a non-empty subset of Z.

Generalized minimal relativism, weak dominance and continuity

We propose three restrictions on the EE's ranking $\trianglerighteq$ over $N \times T$.

Minimal relativism:

In Section 3, we argued that evaluations of functioning bundles are not identical across individuals or communities. A similar argument may be made regarding the evaluations of achievement-opportunity combinations by individuals and communities. Therefore, the requirement that, for all $i, j \in N$, and for all $(x,A),(y,B) \in T$, the external evaluator's ranking of (i,x,A) and (i,y,B) must be exactly analogous to his ranking of (j,x,A) and (j,y,B), would again seems to be an intuitively unacceptable form of universalism. The denial of such universalism in the current framework leads us to the following property:

Definition 4.1.

$\trianglerighteq$ satisfies *minimal relativism* if and only if there exist $(x,A),(y,B) \in T$ and $i,j \in N$ such that, $(i,x,A) \triangleright (i,y,B)$, and $(j,y,B) \triangleright (j,x,A)$.

Weak dominance:

Consider two AOCs, (x,A) and (y,B), in T. Suppose that the achieved functioning bundle x is bigger than the achieved functioning bundle y, and that the capability set A is obtained by 'expanding' the capability set B so that B is a proper subset of A and the frontier of A is an expanded-out version of the frontier of B. In comparing living standards offered by (x,A) and (y,B) to individuals, we note that, in terms of both the achievement and the available opportunities, (x,A) dominates (y,B). Thus, it seems plausible to claim that whoever has the AOC (x,A) enjoys a standard of living at least as high as whoever has the AOC (y,B). This intuition leads us to introduce the following properties.

Definition 4.2.

(i) $\trianglerighteq$ satisfies *weak dominance* (WD) if and only if, for all $i,j \in N$, all $(x,A),(y,B) \in T,$ if $x > y$ and $A > B$ then $(i,x,A) \trianglerighteq (j,y,B)$.

(ii) $\trianglerighteq$ satisfies *dominance* (D) if and only if, for all $i,j \in N$ and all $(x,A),(y,B) \in T$, $[x > y$ and $A > B]$ implies $(i,x,A) \triangleright (j,y,B)$.

It should be noted that the spirit of (WD) and (D) is similar to that of 'dominance set-evaluation' rule discussed in Sen (1985, pp. 68), where he argues that the rule is "fairly non-controversial".

Continuity:

The last property to be introduced in this section is the following continuity property.

Definition 4.3.

$\trianglerighteq$ satisfies *continuity* if and only if, for all $i \in N$ and all $(x,A),(y,B) \in T$, if $(i,x,A) \triangleright (i,y,B)$, then [there exists $\epsilon_1 > 0$ such that, $(i,x,A) \triangleright (i,y',B')$ for all $(y',B') \in T$ with $\|y-y'\| < \epsilon_1$ and $B' \in B(\epsilon_1)$] and [there exists $\epsilon_2 > 0$ such that, $(i,x',A') \triangleright (i,y,B)$ for all $(x',A') \in T$ with $\|x-x'\| < \epsilon_2$ and $A' \in A(\epsilon_2)$].

Again, the continuity property introduced above is a very reasonable property. Suppose the EE considers the standard of living of i when she has the AOC (x,A) to be higher than when she has the AOC (y, B). Then continuity implies that: if A' is arbitrarily close to A, B' is arbitrarily close to B, x' is arbitrarily close to x, and y' is arbitrarily close to y, then (x,A) offers i a higher standard of living than (y',B') and (x',A') offers i a higher standard of living than (y,B).

The difficulty of combining generalized minimal relativism, weak dominance and continuity

We now prove a result similar to Proposition 3.5, which shows the incompatibility of generalized versions of minimal relativism, weak dominance and continuity for the general case.

Proposition 4.4. $\trianglerighteq$ cannot simultaneously satisfy minimal relativism, weak dominance and continuity.

Proof. See the appendix.

It may be helpful to illustrate the proof of Proposition 4.4 in terms of a diagram. We assume that there are only two functionings and that, for each functioning k, $b(k) = \infty$ (no such restrictive assumption is made in the formal proof given in the appendix). Consider Figure 2. Suppose $(i,x,A) \triangleright (i,y,B)$ and $(j,y,B) \triangleright (j,x,A)$, where A, B, x, and y are as shown in the diagram. Then, by continuity, we can find (x^+, A^+) such that: (i) A^+ is derived by 'expanding' A slightly and x^+ is derived from x by 'increasing' x along at least one direction; and (ii) $(j,y,B) \triangleright (j,x^+,A^+)$. Then, by dominance, we have $(j,x^+,A^+) \trianglerighteq (i,x,A)$. By continuity again, given that $(i,x,A) \triangleright (i,y,B)$, we can find (y^+, B^+) such that: (i) B^+ is derived by 'expanding' B

slightly and y^+ is derived from y by 'increasing' y along at least one direction; and (ii) $(i,x,A) \rhd (i,y^+,B^+)$. Then, by dominance, we have $(i,y^+,B^+) \unrhd (j,y,B)$. Therefore, we have obtained the chain: $(i,x,A) \rhd (i,y^+,B^+)$, $(i,y^+,B^+) \unrhd (j,y,B)$, $(j,y,B) \rhd (j,x^+,A^+)$ and $(j,x^+,A^+) \unrhd (i,x,A)$, which violates the transitivity of $\unrhd$.

Insert Figure 2

Concluding remarks

The main results of the paper show that, in comparing living standards in the functionings approach, if one satisfies the property of continuity, then either one must violate weak dominance or one must violate minimal relativism. This is unfortunate since continuity, minimal relativism and weak dominance are all highly attractive properties. We conclude by making two remarks.

First, it may be worth noting that, though our focus is on standard of living comparisons based on functionings, our formal results can be interpreted in other ways. For example, consider Proposition 3.5. We can reinterpret: (i) X as the m-dimensional commodity space, $\mathbf{R}^m_+$; and (ii) $\succeq$ as reflecting the external evaluator's comparisons of the living standards that different individuals enjoy with different commodity bundles. Re-interpreted in this fashion, Proposition 3.5 tells us that, if $\succeq$ satisfies continuity, then $\succeq$ cannot possibly satisfy simultaneously the following two criteria: (1) if one individual has a bigger commodity bundle than another individual, then the first individual's standard of living is at least as high as the second individual's standard of living; and (2) for some individuals, i and j, and some commodity bundles x and y, i's standard of living from x is higher than i's standard of living from y and j's standard of living from y is higher than j's standard of living from x.

Secondly, though, as we noted earlier, the EE's problem of ranking the elements of $N \times X$ or the elements of $N \times T$ is very different from the problem of ranking social states, Propositions 3.5 and 4.4 have some resemblance to Sen's (1970, 1970a) celebrated theorem on the impossibility of a Paretian liberal in the theory of social choice. Minimal relativism and the generalized version of minimal relativism are the counterparts of Sen's minimal liberalism. Weak dominance and generalized version of weak dominance, in their turn, are reminiscent of the weak Pareto principle, which was used by Sen. This structural similarity between Sen's theorem and our results is of interest, given that Sen's (1970, 1970a) problem and the problem that we discuss here are very different.

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Appendix

Proof of Proposition 3.5. Suppose that there is a reflexive and transitive $\succeq$ over $N \times X$ satisfying minimal relativism, weak dominance and continuity. Then we derive a contradiction.

By minimal relativism, there exist $i, j \in N$ and $x, y \in X$, such that

(A.1.) $(i,x) \succ (i,y)$ and $(j,y) \succ (j,x)$.

We first note that, there exists $y' \in X$ such that $y' > y$. For, otherwise, given that $X = [0, b(1)] \times \ldots \times [0, b(m)]$, we would have $y = (b(1), \ldots, b(m))$ where $b(1), \ldots, b(m)$ all must be finite. Since $(i,x) \succ (i,y)$, $x \neq y$. Hence, given $y = (b(1), \ldots, b(m))$ where $b(1), \ldots, b(m)$ are all finite, $y > x$. Then, by weak dominance, $(i,y) \succeq (i,x)$, which is in contradiction with $(i,x) \succ (i,y)$. Similarly, it can be shown that there exists $x' \in X$ such that $x' > x$.

By continuity, from $(i,x) \succ (i,y)$, there exists $\epsilon_1 > 0$ such that

(A.2.) $(i,x) \succ (i,y'')$ for all $y'' \in X$ with $\|y - y''\| < \epsilon_1$.

Note that there exists $y' \in X$ such that $y' > y$. Choose $y^+ = \lambda y + (1-\lambda)y'$ where λ is a positive number such that $(1-\lambda)\|y - y'\| < \min\{\epsilon_1, \|y - y'\|\}$. Clearly, $1 - \lambda > 0$, $y^+ \in X$, $y^+ > y$ and $\|y - y^+\| = (1-\lambda)\|y - y'\| < \min\{\epsilon_1, \|y - y'\|\}$. From (A.2.), it follows that

(A.3.) $(i,x) \succ (i,y^+)$ where $y^+ \in X$ with $y^+ > y$.

Similarly, by continuity, from $(j,y) \succ (j,x)$, we can find $x^+ \in X$ with $x^+ > x$ such that

(A.4.) $(j,y) \succ (j,x^+)$.

Noting that $x^+ > x$ and $y^+ > y$, by weak dominance, we then obtain

(A.5.) $(i,y^+) \succeq (j,y)$ and $(j,x^+) \succeq (i,x)$.

(A.3.), (A.4.), and (A.5.) constitute a contradiction with the transitivity of $\succeq$. This completes the proof. ■

To prove Proposition 4.4, we first prove the following lemma, Lemma A.1. Lemma A.1 essentially shows that, given the continuity property of $\trianglerighteq$, if, for some $i, j \in N$ and some $(x,A), (y,B) \in T$, we have $(i,x,A) \triangleright (i,y,B)$ and $(j,y,B) \triangleright (j,x,A)$, then, we can always find "interior" (x^*, A^*) and (y^*, B^*) in T such that $(i,x^*,A^*) \triangleright (i,y^*,B^*)$ and $(j,y^*,B^*) \triangleright (j,x^*,A^*)$.

Lemma A.1. Suppose $\trianglerighteq$ satisfies continuity. If, for some $i, j \in N$ and some $(x,A), (y,B) \in T$, $(i,x,A) \triangleright (i,y,B)$ and $(j,y,B) \triangleright (j,x,A)$, then there exist $(x^*,A^*), (y^*,B^*) \in T$ with $0 < x_k^* < b(k), 0 < y_k^* < b(k)$ for all $k = 1, \ldots, m$, $A^*_{-w} < A^* < A^*_{+w}$, and $B^*_{-z} < B^* < B^*_{+z}$ for some non-zero $w, z \in \mathbb{R}^m_+$ such that $(i,x^*,A^*) \triangleright (i,y^*,B^*)$ and $(j,y^*,B^*) \triangleright (j,x^*,A^*)$.

Proof. Suppose $\trianglerighteq$ satisfies continuity. Suppose, for some $i, j \in N$ and some $(x,A), (y,B) \in T$, $(i,x,A) \triangleright (i,y,B)$ and $(j,y,B) \triangleright (j,x,A)$. By continuity, from $(i,x,A) \triangleright (i,y,B)$, there exists $\delta_1 > 0$ such that

(A.6.) $(i,x,A) \triangleright (i,y',B')$ for all $(y',B') \in T$ with $\|y - y'\| < \delta_1$ and $B' \in B(\delta_1)$.

Similarly, by continuity, from $(j,y,B) \triangleright (j,x,A)$, there exists $\delta_2 > 0$ such that

(A.7.) $(j,y'',B'') \triangleright (j,x,A)$ for all $(y'',B'') \in T$ with $\|y - y''\| < \delta_2$ and $B'' \in B(\delta_2)$.

Let $\delta = \min\{\delta_1, \delta_2\}$. Then, from (A.6.) and (A.7.), we obtain

(A.8.) $(i,x,A) \rhd (i,y^{**},B^{**})$ and $(j,y^{**},B^{**}) \rhd (j,x,A)$ for all $(y^{**},B^{**}) \in T$ with $\|y - y^{**}\| < \delta$ and $B^{**} \in B(\delta)$.

Note that $X = [0,b(1)] \times \ldots \times [0,b(m)]$ and Z consists of all non-empty, compact, convex and comprehensive subsets of X. Therefore, we can find a $B^* \in B(\delta)$ and a non-zero vector $z \in \mathbb{R}^m_+$ with $\|z\| < \delta$ such that $B^*_{-z} < B^* < B^*_{+z}$. Noting that $B^* \in B(\delta)$ and $B^*_{-z} < B^* < B^*_{+z}$, we can then find $y^* \in B^*$ such that $0 < y^*_k < b(k)$ for all $k = 1,\ldots,m$ and $\|y - y^*\| < \delta$. Therefore, $(y^*,B^*) \in Z$. By (A.8.) and from the construction of such (y^*,B^*), we then obtain that $(i,x,A) \rhd (i,y^*,B^*)$ and $(j,y^*,B^*) \rhd (j,x,A)$.

Similarly, from $(i,x,A) \rhd (i,y^*,B^*)$ and $(j,y^*,B^*) \rhd (j,x,A)$, we can find $(x^*,A^*) \in T$ with $0 < x^*_k < b(k)$ for all $k = 1,\ldots,m$ and $A^*_{-w} < A^* < A^*_{+w}$ such that $(i,x^*,A^*) \rhd (i,y^*,B^*)$ and $(j,y^*,B^*) \rhd (j,x^*,A^*)$. ■

Proof of Proposition 4.4: Suppose that there is a reflexive and transitive $\trianglerighteq$ over $N \times T$ satisfying minimal relativism, weak dominance and continuity. Then we derive a contradiction.

By minimal relativism, there exist $i,j \in N$ and $(x,A), (y,B) \in T,$ such that

(A.9.) $(i,x,A) \rhd (i,y,B)$ and $(j,y,B) \rhd (j,x,A)$.

From Lemma A.1, without loss of generality, we can assume that

(A.10.) $(x,A),(y,B) \in T$ are such that $0 < x_k < b(k), 0 < y_k < b(k)$ for all $k = 1,\ldots,m$, $A_{-z} < A < A_{+z}$ for some non-zero $z \in \mathbb{R}^m_+$, and $B_{-w} < B < B_{+w}$ for some non-zero $w \in \mathbb{R}^m_+$.

By continuity, from $(i,x,A) \rhd (i,y,B)$, there exists $\epsilon_1 > 0$ such that

(A.11.) $(i,x,A) \rhd (i,y',B')$ for all $(y',B') \in T$ with $\|y - y'\| < \epsilon_1$, and $B' \in B(\epsilon_1)$.

Similarly, by continuity, from $(j,y,B) \rhd (j,x,A)$, there exists $\epsilon_2 > 0$ such that

(A.12.) $(j,y,B) \rhd (j,x',A')$ for all $(x',A') \in \Gamma$ with $\|x - x'\| < \epsilon_2$, and $A' \in A(\epsilon_2)$.

From (A.10.) and (A.11.), for some positive $\epsilon' \leq \epsilon_1$, we obtain

(A.13.) $(i,x,A) \rhd (i,y + w',B_{+w'})$ for some $(y + w',B_{+w'}) \in T$ with $w' \in \mathbb{R}^m_+, w' > 0, \|w'\| < \epsilon'$.

Similarly, from (A.10.) and (A.12.), for some positive $\epsilon'' \leq \epsilon_2$, we obtain

(A.14.) $(j,y,B) \rhd (j,x + z',A_{+z'})$ for some $(x + z',A_{+z'}) \in T$ with $z' \in \mathbb{R}^m_+, z' > 0, \|z'\| < \epsilon''$.

Since $z' > 0, w' > 0$, and given (A.10.), clearly, $x + z' > x, y + w' > y, A_{z'} > A, B_{+w'} > B$. By weak dominance, we then obtain

(A.15.) $(i,y + w',B_{+w'}) \trianglerighteq (j,y,B)$ and $(j,x + z',A_{+z'}) \trianglerighteq (i,x,A)$.

(A.13.), (A.14.), and (A.15.), together, contradict the transitivity of $\trianglerighteq$. This completes the proof. ■

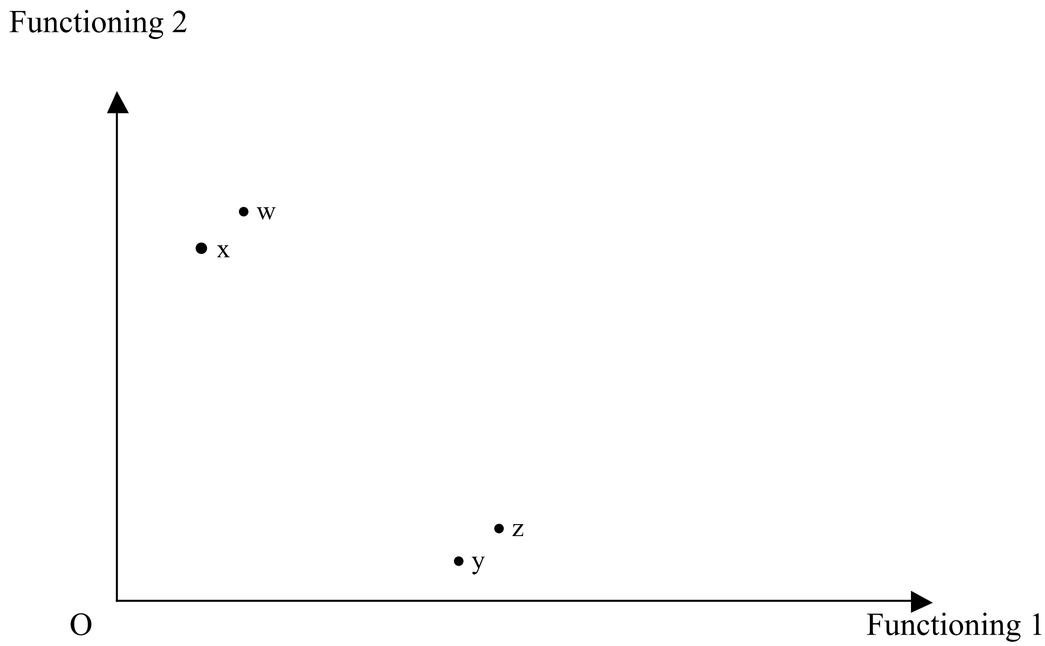

Figure 1

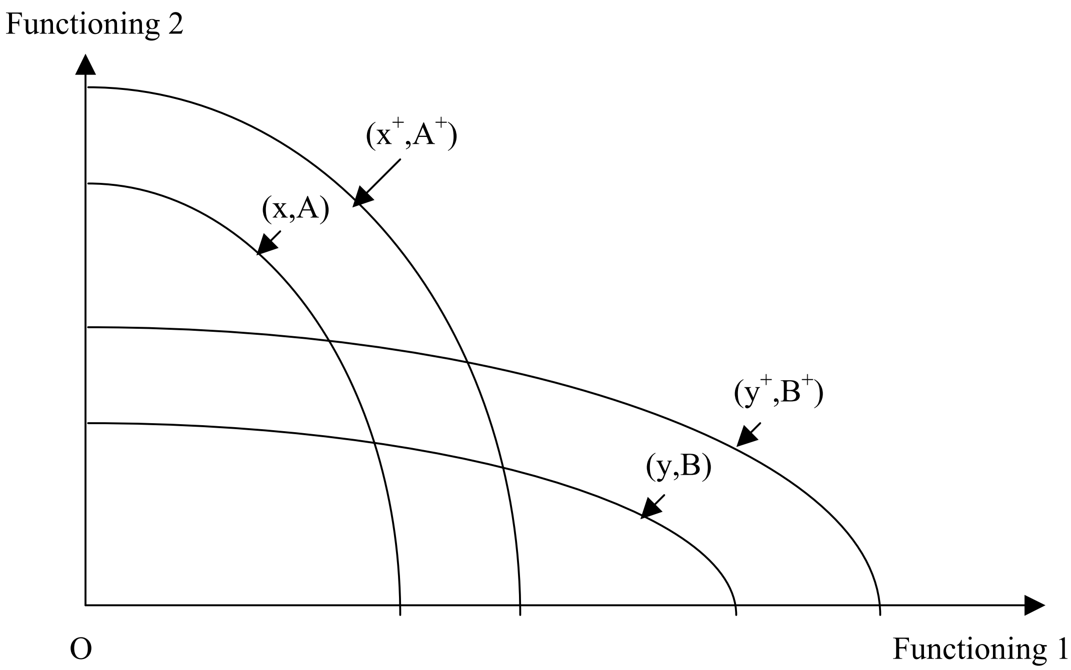

Figure 2