Monetary Policy and External Habit Formation in a Sticky Price Model

Yongseung Jung [*]

Abstract

This paper examines the effect of introducing external habit formation into a Calvo-type sticky price model. The paper shows that external habit formation improves the sticky price model in capturing the hump-shaped spectra of output and consumption as in the data. The paper also shows that the sticky price model with external habit formation make nominal interest rates to serve as countercylclical leading indicators over the business cycle.

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1. Introduction

In macroeconomics, the conventional wisdom is that an expansionary monetary policy generates a decrease in nominal interest rates and an increase in real activity, that is, long and persistent hump-shaped effects in real activities such as consumption and output. There is extensive empirical literature which supports this view. Bernanke and Blinder(1992), and Rotemberg and Woodford(1997) identify the innovations in federal fund rates as the money supply shock, contrary to the traditional econometric literature, while Christiano, Eichenbaum and Evans (CEE hereafter 1998), identify the innovations in nonborrowed reserves as the money supply shock, also contrary to the traditional econometric literature. According to their empirical analysis, the response of output to a positive monetary shock is hump-shaped in the sense that the maximum response of output to a positive monetary shock occurs 2 or 3 periods after the shock. They

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[*] Department of Economics, Catholic University of Korea and Department of Economics, UC Riverside. I would like to thank Stephen G. Cecchetti, Tack Yun, and participants at the Money and Banking seminar at the Bank of Korea, and Korean Macroeconomics seminar for helpful comments and conversations. This paper was completed while I was visiting the Department of Economics, UC Riverside. I also thank the department of economics and faculty, the UC Riverside for their hospitality. All errors are my own. E-mail: jungyongseung@hotmail.com.

also report that liquidity effects occur when the innovations in nonborrowed reserves are used as the monetary policy shock.

Some successful attempts have been made to explain these empirical facts. Gali (2000) discusses liquidity effects in a sticky price model, and concludes that the model can generate liquidity effects when there is a sufficient low money growth autocorrelation or a sufficient high risk aversion. However, his conclusion is not robust because it is based on a sticky price model without capital accumulation. Without capital accumulation, one needs only the dampened response of consumption to the monetary shock to generate the delayed hump shape response of output to the shock. Edge (2002) presents a sophisticated sticky price model with time-to-build, time-to-plan and habit persistence which is successful in generating a hump-shaped impulse response of output to monetary shock and liquidity effects.

In recent, some authors such as Watson (1993) suggest more sophisticated measures of fit to evaluate the performance of a dynamic stochastic general equilibrium model in frequency domain as well as in time domain. King and Watson (1996) and Stock and Watson (1999) document of important stylized facts of business cycles along the lines of Watson (1993). They demonstrate that the growth rate spectra of selected macroeconomic variables are relatively low at low frequencies, rise at middle frequencies, and then decline at high frequencies. This pattern is known as the typical spectral shape of growth rates. They also present empirical evidence that nominal interest rates are inverted leading indicators for real activity at business cycle frequencies. Moreover, the US data reveals that price level moves countercyclically.

However, these empirical findings pose an important and intriguing challenge to macroeconomics, because the existing sticky price models cannot generate the observed empirical facts, or the common hump shape of the power spectrum of output and the cross correlations between interest rates and output. For example, Ellison and Scott (2000) examine the performance of a Calvo-type sticky price model with exogenous money supply at both high frequencies and business cycle frequencies. They point out that the model fails because it generates insufficient output fluctuations at business cycle frequencies, as well as excessive output volatility at high frequencies. That is, the sticky price model cannot generate the hump-shaped spectra of the growth rates of the selected macroeconomic variables as in the data. King and Watson (1996) are also skeptical about the sticky price model in adequately accounting for the cross correlation between interest rates and real activity, notwithstanding their diverse successes.

In finance, it is well known that the common constant-relative-risk-averse, expected utility function is not satisfactory as the representative agent, expected additive utility model, sensibly restricted, cannot account for the puzzles in finance including the equity premium puzzle. Some authors have proposed alternative economic models to explain these puzzles in finance. An external habit formation model[1] is one of them. Although the external habit formation models have produced successes in generating the time-varying equity premium, they have been in economies without money.

This paper follows the proposal to incorporate the external habit in consumption into a sticky price model and explores the high and business cycle frequency implications along the lines of Watson (1993), and Ellison and Scott (2000). Because the external habit in consumption magnifies the persistence of consumption to the monetary shock, it can improve the performance of the sticky price model in generating the hump-shaped spectral density of output growth rates and the hump-shaped response of output to the monetary shock. To explore the role of external habit formation in a sticky price model, Abel's (1990, 1998) 'Catching up with the Joneses' formulation has been incorporated into the canonical Calvo-type sticky price model. In doing so, this paper investigates the following questions. First, the paper explores whether the model can generate the common hump-shaped growth rate spectra of the real variables such as output and consumption. Second, the paper addresses whether the external habit formation improves the performance of the models in explaining the selected variables at high frequencies. Third, the paper discusses whether the model can explain the observed fact that both nominal interest rates and prices are inverted leading indicators for real activity. Finally, the present paper discusses the extent to which the sticky price model with external habit generates the hump-shaped response of output to an exogenous monetary shock. In addition, the effects of an expenditure delays assumption in the sticky model, introduced by Rotemberg and Woodford (1997) and Bernanke, Gertler, and Gilchrist (1999, hereafter BGG(1999)) to generate the hump-shaped response of real spending to a monetary shock, are also examined. In this hybrid model, households that care about their neighborhood or have already precommitted themselves to a particular expenditure plan gradually adjust their consumption profile and labor supply to a monetary shock. Because households respond gradually to the shock, the volatilities of real variables at high frequencies tend to decrease.

The main findings of this paper can be summarized as follows. First, Abel's (1990, 1998) external habit formation improves the sticky price model in explaining the selected variables at nearly all frequencies in the case of exogenous monetary supply policy. External habit formation substantially decreases the volatility of output, consumption, investment and labor at nearly all frequencies in the Calvo-type sticky price model. Second, a canonical Calvo-type sticky price model with external habit in consumption can generate the hump-shaped growth rate spectra of consumption and the hump-shaped response of output to an exogenous monetary shock. Finally, the Calvo-type sticky price models with external habit formation partly succeed in making that nominal interest rates and prices serve as inverted leading predictors of real activity.

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[1] See Campbell(1999) for references of habit formations in economics and finance.

This paper is composed as follows. Section 2 presents of the features of the post-war U.S. business cycle. Section 3 specifies sticky price models with external habit formation in consumption as in Abel (1990, 1998). The properties of an equilibrium and discussions of the implications of the model related to monetary policy are in Section 4. Section 5 discusses the quantitative implications of the model. Section 6 contains concluding remarks.

2. Features of the Post-War U.S. Business Cycles

In this section, features of the post-war US business cycles are examined, focusing on the time series relationship between gross domestic product, interest rates, and prices. The statistical relationships presented in this section will be used to evaluate the performance of a sticky price model with external habit.

2.1. Power Spectrum of Selected Variables

The estimated spectra of the selected macroeconomic variables have the following characteristics in relation to business cycles. First, output, consumption, and investment show the typical spectral shape of growth rates: There is a common, hump-shaped spectrum of output, consumption, and investment. Because the height of the spectrum of each variable reflects the relative volatility, the average height of the spectrum of investment is the highest and that of consumption is the lowest. Rotemberg and Woodford (1995) stress that the hump shape of the power spectrum indicates that there is substantial predictability of the cyclical component of output growth. This interpretation of the power spectrum of output growth rates suggests the need for a business cycle model with highly persistent, but temporary, components. Second, the spectrum of money growth rate shows that variations in the monetary policy may be an important source of business cycles as noted by King and Watson (1996). Third, the spectra of inflation rates has a peak at a lower frequency than the peak in the spectrum of other variables, which suggests the role of nominal rigidity in the business cycles.

2.2. Business Cycle Comovements

Features of business cycles in terms of cross autocorrelations are also useful to examine. In Table 2, taken from Stock and Watson (1999), various moments of the selected variables calculated from the estimated spectral density matrix with only the business cycle (6-32 quarter) frequencies of the US over the sample period 1947:1 through 1996:4 are shown . Two key features are evident in Table 2. First, there are systematic movements of nominal interest rates in relation to output. The contemporaneous correlation between nominal interest rates (r_t) and output (y_t) is positive ($corr(r_t, y_t)$ = 0.41), and the correlations between nominal interest rates and future output are negative ($corr(r_t, y_{t+4}) = -0.58$). In other words, nominal interest rates serve as countercylclical leading indicators in the business cycle.[2]

Second, the correlation between prices (p_t) and output is negative ($corr(p_t, y_t) = -0.51$), and the correlations between prices and future output are more strongly negative ($corr(p_t, y_{t+2}) = -0.68$) than the contemporaneous correlation between output and prices. That is, prices are also leading countercylclical indicators in the business cycles.

The model's prediction for cross correlation properties of selected variables are evaluated in light of the evidence provided above.

3. The Sticky Price Model with Habit Formation

3.1. Habit Formation with Transaction Cost

To introduce the role of money in the model, it is assumed that money reduces the costs of consumption or investment transactions and the cost of time to shopping can be represented by a function of expenditure levels and real balances, as in Feenstra (1986). That is, when the household has real balance holdings equal to $m_t(\equiv \frac{M_t}{P_t})$, it must expend additional $\Psi(C_t, m_t)$ units of goods as transaction costs. Here M_t and P_t are the nominal money holdings and price level, respectively. As in Feenstra (1986), it is assumed that unit transaction cost function $\frac{\Psi(C_t, m_t)}{C_t} = \varphi(C_t, m_t)$ is homogenous of degree zero in both arguments with $\varphi_1 \geq 0$, $\varphi_2 < 0$, $\varphi_{11} \geq 0$, $\varphi_{22} \geq 0$, and $\varphi_{12} < 0$.

3.1.1. Households

The economy consists of a continuum of identical infinite-lived households. Following Abel (1990, 1998), and Campbell and Cochrane (1999), it is supposed that a representative household derives utility from the level of consumption relative to a time-varying subsistence or habit level. In particular, it is assumed that the habit is external in the sense that it is determined by the aggregate consumption of the nation as a whole, and not by the consumption of any individual household as in Abel (1990, 1998) and Campbell and Cochrane (1999). This greatly simplifies the analysis. The representative household chooses consumption, leisure, and portfolios to maximize its lifetime objective

$$E_t\left[\sum_{j=0} \beta^j u(C_{t+j}, L_{t+j}, H_{t+j})\right], \quad 0 < \beta < 1, \tag{3.1}$$

where C_t is a composite consumption index defined by

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\phi-1}{\phi}} dj\right]^{\frac{\phi}{\phi-1}}, \quad \phi > 1. \tag{3.2}$$

Here ϕ measures the elasticity of substitution among goods within each category, and β is the household's discount factor, and E_t denotes the conditional expectations operator on the information available in period t. L_{t+j} represents the domestic household's leisure in period $t+j$. The state of the economy, z_t evolves according to a Markov process described by a density function $f(z_{t+1}, z_t)$. H_t summarizes the influence of past consumption levels on today's utility.

There exists a complete asset market in the economy. In particular, it is assumed that there is a contingent one-period bond market as in Woodford (1996). Let $B(x_{t+1})$ denote the household's holdings of the bond, which pays one dollar if x_{t+1} state occurs next period and 0 otherwise. $P_B(x_{t+1}, x_t)$ denotes the price of such a bond at time t when the state of the economy at time t is x_t. The riskless one-period nominal interest rate at time t is given by $1 + r_t \equiv [\int P_B(x_{t+1}, x_t) dx_{t+1}]^{-1}$.

The household also faces a time constraint such that

$$L_t + N_t \leq \overline{N}, \tag{3.3}$$

where $\overline{N}$ denotes the time endowment of the household.

The optimal allocation for each differentiated good yields the demand functions:

$$C_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\phi} C_t, \tag{3.4}$$

for all $j \in [0,1]$, where $P_t = \left(\int_0^1 P_t(j)^{1-\phi} dj\right)^{\frac{1}{1-\phi}}$ is the price index for composite goods.

The timing of markets and the transactions facing the household need to be specified. When there is no investment expenditure delay, a household chooses its consumption and investment as well as asset holdings after the state of the money market is known. However, when investment expenditure delays are assumed as in Rotemberg and Woodford (1997) and BGG (1999), the household must choose its purchases of I_t at time $t-1$. This assumption means that the household decides its current investment expenditures at the beginning of the current period or at the end of the previous period before the current monetary shock is known.

Keeping this in mind, consider the household's problem. The household that has decided its current consumption and investment level starts with nominal wealth Θ_t, such as capital, money, and bonds, carried over from period $t-1$ and receives the lump-sum transfers of currencies T_t, before the market opens. For analytical simplicity, suppose that the household owns only capital stock to rent to firms and there is no firm-specific capital stock. Since we do not empirically observe large discrete capital stock adjustments, it is reasonable to introduce an adjustment cost in capital stock installments. If there are costs of installing capital, the capital stock will move more sluggishly. It is assumed that there are deadweight costs of installing capital stock. To preserve the simple model structure as far as possible, the Uzawa-Lucas-Prescott form of investment adjustment costs is adopted.

$$K_{t+1} = \psi(I_t/K_t)K_t + (1 - \delta_k)K_t, \tag{3.5}$$

where $\psi(I_t/K_t)$ is a positive, concave function, and I_t is the composite investment at period t, and K_t is the composite capital stock at period t. At the end of each period, the household receives wages, rents for capital, and dividends from each firm. Then the household faces the budget constraint given by

$$P_tC_t(1 + \varphi(C_t, m_t)) + P_tI_t + M_t + \int B(x_{t+1})P_B(x_{t+1}, x_t)dx_{t+1} \le \Omega_t + T_t. \tag{3.6}$$

The household's wealth at the beginning of the period t is

$$\Omega_t = M_{t-1} + B_{t-1} + W_{t-1}N_{t-1} + V_{t-1}K_{t-1} + \Pi_{t-1}, \tag{3.7}$$

where N_{t-1}, Π_{t-1}, W_{t-1}, and V_{t-1} denote the hours worked, the firm's nominal profits, nominal wages and nominal rental rate for capital stock given to the household at time $t-1$, respectively.

3.1.2. Firms

In the model, differentiated goods and monopolistic competition are introduced along the lines of Dixit and Stiglitz (1977). Suppose that there is a continuum of firms producing differentiated goods, and each firm indexed by j, $0 \le j \le 1$, produces its product with constant returns to scale, concave production technology. Each firm j takes P_t and the aggregate demand as given, and chooses its own product price $P_t(j)$. Since the input markets are perfectly competitive, the demands for labor and capital are determined by its cost minimization as follows,

$$\begin{array}{c} C(W_t, R_t, Y_t(j), N_o, z_t) \equiv \min_{N_t(j), K_t(j)}\{V_tK_t(j) + W_tN_t(j)\} \\ s.t. \quad Y_t(j) \le A_tF(K_t(j), z_t(N_t(j) - N_o)). \end{array} \tag{3.8}$$

Here N_o, z_t, and A_t are the household's fixed overhead cost in units of labor hours, labor augmenting permanent technology progress, and transitory technology process at period t, respectively. $Y_t(j)$ is the output of the jth firm. It is assumed that the technology shock follows an $AR(1)$ process, and the deterministic component of productivity, z_t is taken as growing deterministically, i.e. $\gamma = \frac{z_t}{z_{t-1}}$ for all t as in King, Plosser and Rebelo (1988, hereafter KPR (1988)).

$$\log A_t = \rho \log A_{t-1} + \xi_{At}, \quad -1 < \rho < 1. \tag{3.9}$$

where $E(\xi_{At}) = 0$ and ξ_{At} is i.i.d. over time.

From the firm's first order condition,

$$\begin{aligned} V_t &= MC_t(j)A_tF_1(K_t(j), z_t(N_t(j) - N_o(j))), \\ W_t &= MC_t(j)A_tF_2(K_t(j), z_t(N_t(j) - N_o(j))). \end{aligned} \tag{3.10}$$

The marginal cost of each firm is equal, i.e. $MC_t(j) = MC_t$ for each j as the production function is CRS. This also implies that $\frac{N_t(j) - N_0(j)}{K_t(j)} = \frac{N_t - N_0}{K_t}$ for all j, and thus the wage and rental rate can also be represented by (3.10).

3.1.3. Staggered Price Setting

The monopolistically competitive firms in the product markets set their own prices in advance by maximizing the present discounted value of profits. Suppose that only the fraction $(1 - \alpha)$ of the firms sets the new price, $P_{t,t}$, while the other fraction of firms, α sets its price by multiplying the average inflation rate or average monetary growth rate(ω) by their previous price level. The firms that are able to set new prices are chosen randomly each period, with each having an equal probability of being selected. The probability of an optimal price change is independent of both the time that has elapsed since the last optimal price change, and the degree to which costs and other market conditions have changed since. Let $D_{t,t+k}$ denote the demands at period $t + k$ facing firms that set their prices at time t, and $P_{t,t+k}$ the prices at period at $t + k$ that are predetermined at time t. As the price applies in period t with certainty, in period $t + 1$ with a probability α, in period $t + 2$ with α^2, and so on, the firm's maximization problem can be written as follows:

$$\max . E_t\{\sum_{k=0}^{\infty}(\alpha\beta)^k \lambda_{t+k} R_{t,t+k}[P_{t,t+k}D_{t,t+k}(P_{t,t+k}) - MC_{t+k}D_{t,t+k}(P_{t,t+k})]\}, \tag{3.11}$$

where λ_{t+k} is the marginal utility for the household of additional income at $t + k$, and $R_{t,t+k} = \frac{P_t}{\lambda_t P_{t+k}}$.

The newly determined price at time t is given by

$$P_{t,t} = \frac{E_t[\sum_{k=0}^{\infty}(\alpha\beta)^k \frac{\lambda_{t+k}}{P_{t+k}} D_{t,t+k} MC_{t+k}]}{(1 - \phi)E_t[\sum_{k=0}^{\infty}(\alpha\beta\omega)^k \frac{\lambda_{t+k}}{P_{t+k}} D_{t,t+k}]}, \tag{3.12}$$

given $P_{t,t+k} = \omega^k P_{t,t}$. As the price for composite goods at time t is determined by the aggregation of the predetermined prices $\{P_{t-j,t}\}_{j=0}^{s}$ according to $P_t = \left(\int_0^1 P_t(j)^{1-\phi} dj\right)^{\frac{1}{1-\phi}}$, P_t satisfies

$$P_t^{-\epsilon} = (1 - \alpha)P_{t,t}^{-\epsilon} + \alpha\omega^{-\epsilon}P_{t-1}^{-\epsilon}, \tag{3.13}$$

where $\epsilon = \frac{1-\phi}{\phi}$.

3.2. Monetary Policy

There has been extensive debate over the most appropriate way to model monetary policy in the U.S. It concerns whether the money supply rule is more appropriate than the interest rule to evaluate the effect of monetary policy in the actual economy. Recently, many leading macroeconomists follow Taylor's recommendation of a simple interest rule or a variant such as interest smoothing policy to evaluate the effect of monetary policy. An interest rule can be logically interpreted as a money supply rule or vice versa as CKM (1996) shows. In this paper, I follow CKM (1996) to employ the money supply rule to evaluate the model along the lines of Ellison and Scott (2000).

Assume that the central bank follows a simple money supply rule, namely, the growth rate of the money stocks follows an AR(1) process such as

$$\ln \omega_t = \rho \ln \omega_{t-1} + \varepsilon_{\omega t}, \tag{3.14}$$

where $\omega_t = \frac{M_t}{M_{t-1}}$, and ε_ω is a normally distributed shock with a mean, zero and a standard deviation, σ_ω.

4. Equilibrium

4.1. First Order Conditions

With this specification of utility, the first order conditions without investment delay are given by

$$u_1(C_t, L_t, H_t) = \Lambda_t(1 + \Phi_1(C_t, m_t)), \tag{4.1}$$

$$u_2(C_t, L_t, H_t) = \Lambda_t w_t, \tag{4.2}$$

$$\Lambda_t P_B(x_{t+1}, x_t) = \beta \Lambda_{t+1} f(x_{t+1}, x_t), \tag{4.3}$$

$$\Lambda_t(1 + \Phi_2(C_t, m_t)) = \beta E_t[\Lambda_{t+1}], \tag{4.4}$$

$$q_t = (\psi'(\frac{I_t}{K_t}))^{-1}, \tag{4.5}$$

$$K_{t+1} = \psi(I_t/K_t)K_t + (1 - \delta_k)K_t, \tag{4.6}$$

$$E_t[(1+v_{t+1})q_t] = E_t[q_{t+1}(1-\delta_k)A_{t+1}F_1(K_{t+1}, N_{t+1}-N_o))]. \tag{4.7}$$

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1}{\phi}} C_t, \;\; I_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1}{\phi}} I_t. \tag{4.8}$$

$$\begin{aligned} V_t &= MC_tA_tF_1(K_t, z_tN_t - N_o) \\ W_t &= MC_tA_tF_2(K_t, z_tN_t - N_o), \end{aligned} \tag{4.9}$$

$$P_{t,t} = \frac{E_t[\sum_{k=0}^{\infty}(\alpha\beta)^k \frac{\Lambda_{t+k}}{P_{t+k}} D_{t,t+k} MC_{t+k}]}{(1-\phi)E_t[\sum_{k=0}^{\infty}(\alpha\beta\omega)^k \frac{\Lambda_{t+k}}{P_{t+k}} D_{t,t+k}]}, \tag{4.10}$$

$$P_t^{-\epsilon} = (1-\alpha)P_{t,t}^{-\epsilon} + \alpha\omega^{-\epsilon}P_{t-1}^{-\epsilon}. \tag{4.11}$$

Here Λ_t is defined as $\beta E_t \bar{\Lambda}_{t+1} P_t$, where $\bar{\Lambda}$ is a Lagrange multiplier of the domestic household's budget constraint (3.6). u_i is a partial derivative of u with respect to a variable i and $v_t = \frac{V_t}{P_t}$, $w_t = \frac{W_t}{P_t}$. Equation (4.1), which is the first order condition for consumption goods, says that the marginal utility of consumption goods equals the sum of the marginal utility of wealth and that of the liquidity service of money. Equation (4.2) relates the marginal utility of leisure to the marginal utility of the real wage rate. Equations (4.3) and (4.4) refer to the intertemporal decisions of the household, that is, the decision of bond holdings and money holdings, respectively. In particular, the equations imply that the demand for real balance is a decreasing function of the nominal interest rate. The demand for real balance can be derived from

$$\Phi_2(C_t, \frac{M_t}{P_t}) = \frac{-r_t}{1+r_t}. \tag{4.12}$$

Suppose that $\Phi(C_t, M_t/P_t) = AC_t^{1+\zeta}(M_t/P_t)^{-\zeta}$, with $\zeta > 0$. Then the demand for real balance is given by

$$\frac{M_t}{P_t} = AC_t(\frac{r_t}{1+r_t})^{-\frac{1}{1+\zeta}}.$$

This is comparable to the textbook LM equation. Equation (4.5) says that Tobin's q equals the inverse of the investment/capital adjustment function derivative. Equation (4.7) represents the relationship between the rent paid to a unit of capital in $t+1$ and the expected return to holding a unit of capital from t to $t+1$. This first order condition shows the evolution of Tobin's q over time. Equation (4.8) says that the jth consumption goods and investment goods are determined by the cost minimization demands when the composite demands are given.

When there is a delay in investment expenditure, the first order condition with respect to the investment expenditure choice is replaced by

$$E_t[q_{t+1}] = E_t[(\psi'(\frac{I_{t+1}}{K_{t+1}}))^{-1}]. \tag{4.13}$$

4.2. Dynamics around Steady State

First, I will represent the economy system in a state space to explore the dynamics of the economy. Next I will analyze the response of the economy to shocks of technology and monetary policy using essentially the method of KPR(1988). That is, I restrict my attention to the case of small fluctuations of the endogenous variables around a steady state growth path. Since most of the following analysis will be done in stationary terms, it is more convenient to define a symmetric rational equilibrium in terms of a stationary one.

In this system, the state vector at period t, x_t consists of a technology shock (a_t), a monetary shock (ω_t), a predetermined capital stock (k_t), and a previous price level (p_{t-1}) (All in log forms). Since each firm in each group sets the same price in symmetric equilibrium, it is desirable to divide consumption and investment goods into groups on the basis of the staggered prices setting decisions times.

The stochastic symmetric stationary equilibrium consists of the bounded time invariant decision rules
$\{c(x_t),\ i(x_t),\ k(x_t),\ N(x_t)\}$ and prices $\{p_{t,t}(x_t),\ p(x_t),\ q(x_t),\ r(x_t),\ w(x_t),\ v(x_t),$
$\lambda(x_t)\}$ with the state of the economy x_t such that

1) The households decision rules, $\{c(x_t),\ i(x_t),\ N(x_t),\ \lambda(x_t)\}$ solve their optimization problem given the states and the prices.

2) The demands for labor and capital, $\{N(x_t),\ k(x_t)\}$ solve each firm's cost minimization problem and price setting rules $p_{t,t}$ solve its present value maximization problem given the states and the prices.

3) Each goods market, capital rental market, labor market, bond market, and money market are cleared at $\{p_{t,t}(x_t),\ q(x_t),\ v(x_t),\ w(x_t),\ r(x_t)\}$, respectively.

5. Quantitative Evaluation of the Models

5.1. preferences

Suppose that the household derives its utility from the power utility function of the ratio C_t/H_t as well as the leisure L_t,

$$u(C_t, L_t, H_t) = \begin{cases} \frac{(C_t/H_t)^{1-\sigma}}{1-\sigma} v(L_t), & \sigma > 0,\ \sigma \neq 1, \\ \log(C_t/H_t) + v(L_t), & \sigma = 1, \end{cases} \tag{5.1}$$

where σ^{-1} is the intertemporal elasticity of substitution in consumption. This subutility function satisfies the condition of balanced growth path as proved by KPR(1988). The stochastic sequence of habits $\{H_t\}_{t=0}^{\infty}$ is regarded as exogenous by the household and tied to the stochastic sequence of aggregate consumption $\{C_t\}_{t=0}^{\infty}$ as follows. For simplicity, I will specify H_t as an external habit depending on only aggregate consumption as in Abel (1990, 1998). That is,

$$H_t = \tilde{C}_{t-1}^{\kappa}, \tag{5.2}$$

where $\tilde{C}_{t-1}$ is aggregate past consumption and the parameter κ governs the degree of time-nonseparability. In this specification of habit formation, habit depends on one lag of consumption. Since there is a representative agent, aggregate consumption equals the household's consumption in equilibrium. That is, in equilibrium,

$$H_t = C_{t-1}^{\kappa}.$$

5.2. Parameter Values

Even though many RBC models assume the unit elasticity of intertemporal substitution ($\epsilon_C = \sigma^{-1} = 1$) taken from Hansen and Singleton (1982), many empirical studies on consumption tell us to be more cautious and conservative in choosing the value. Thus, this paper takes lower values of intertemporal elasticity of consumption, $\sigma = 2$, i.e. $\epsilon_C = 1/2$. I also choose a conservative intertemporal elasticity of labor supply, H_w equal to 1, which is smaller than that of KPR (1988), Rotemberg and Woodford (1992). With this temporal utility function, I can determine the parameter values which will be used in the simulation.

I adopt an estimate of M_1 growth rate of US over 1972 : 1-1996 : 4 which is given by

$$\log \omega_t = 0.00646 + 0.50569 \log \omega_{t-1} + \varepsilon_{\omega t}. \tag{5.3}$$

Suppose that $\Psi(C_t, M_t/P_t) = AC_t^{1+\zeta}(M_t/P_t)^{-\zeta}$, with $\zeta > 0$. Then the money demand elasticity to $\frac{r_t}{1+r_t}$ is $-\frac{1}{1+\zeta}$. Lucas (1988) found that the long-run income elasticity of money is 1 and the long-run interest rate semi-elasticity is -0.07 for 1958-1985 and -0.09 for 1900-1985 for M_1. However, King and Watson (1996) use a much smaller value of interest elasticity of money demand equal to -0.01 because the degree of money demand over the business cycle is much smaller than in the long run. Christiano, Eichenbaum, and Evans (1999 hereafter CEE (1999)) found that the estimate of this elasticity is much smaller than that of Lucas, i.e. -10^{-3}. For this reason, I will follow King and Watson (1996) and use -0.01 for this elasticity and determine the parameter values.

All parameter values used in this paper are reported in Table 1 and most of them are taken from KPR (1988), Lucas (1988), and Rotemberg and Woodford (1992). For the case of Abel's preference, $\kappa = 1$ is set as in Abel (1990, 1999). While the functional form for adjustment cost function, ψ, is not specified, three parameters describing the behavior around the steady state are specified. First, the steady state value of Tobin's q and the share of investment in the national product are specified. Since the steady state value of Tobin's q is 1.0, I also set the value of this variable to 1.0 in steady state. The same investment share in steady state is taken as in a model without adjustment cost. Next, the parameter determining the elasticity of the marginal adjustment cost function is specified. That is, the value of elasticity of i/k with respect to Tobin's q, η_q. This value reflects the volatility of investment. In the baseline model, 4 is chosen as in Bernanke, Gertler, and Gilchrist (1999 hereafter BGG(1999)). Finally, 1.1 is chosen as the benchmark average size of markup, μ. Though this value is much lower than the value that many sources of evidence suggest, it is consistent with the average markup estimates in Fernald and Basu (1993).

5.3. Relative Mean Square Approximation Error

To evaluate the goodness of fit of the models, the minimum approximation error representation developed in Watson (1993) is applied. Following Watson (1993), consider the error u_t defined by

$$u_t = y_t - x_t, \tag{5.4}$$

where x_t is the evolution of $n \times 1$ vector coming from the economic model, and y_t is the empirical counterparts of x_t. Suppose that x_t and y_t are transformed to be jointly covariance stationary. Then the autocovariance generating function (ACGF) of u_t, $A_u(z)$ is given by

$$A_u(z) = A_y(z) + A_x(z) - A_{xy}(z) - A_{yx}(z), \tag{5.5}$$

where $A_x(z)$ is the ACGF of x_t, $A_{xy}(z)$ is the cross ACGF between x_t and y_t and so forth.

Under certain assumptions, Watson (1993) suggested a bound on the relative mean square approximation error (RMSAE) for the economic model - the bound analogous to a lower bound on $1 - R^2$ from a regression - as following

$$R_j(\omega) = \frac{[A_u(z)]_{jj}}{[A_y(z)]_{jj}}, \; z = e^{-i\omega}, \tag{5.6}$$

where $[A_u(z)]_{jj}$, $[A_y(z)]_{jj}$ are the jth diagonal elements of $A_u(z)$, $A_y(z)$,respectively. Because $R_j(\omega)$ is the variance of the error relative to the variance of the data for each frequency, it tells us how well the economic model fits the data over different frequencies.

Because the spectrum of the data, y_t is not known, it must be estimated. In this paper, the spectrum of y was calculated by estimating a VAR with the imposition of a cointegration relationship among the interest variables as in King and Watson (1996). The VAR was specified as the regression of $s_t = (\Delta n_t, \Delta w_t, \Delta m_t, r_t, y_t - c_t, y_t - i_t, m_t - p_t - y_t, w_t - y_t + n_t)$ on to a constant and six lags of s_t.[3]

5.4. Implications of the Model

5.4.1. Power Spectrum and RMSAE

Can a sticky price model with a Calvo-type price setting rule explain the stylized facts about business cycles? In the sticky price model with a Calvo-type price setting rule, the price setting period of each firm is random as firms that get to set new prices are chosen randomly each period, with each having an equal probability of being selected.

The first issue addressed is whether the spectra of the growth rates of the selected variables calculated from the model correspond to the spectra implied by the data. For each selected variable, Figure 5.1 shows the spectrum of a sticky price model without habit (solid lines), the spectrum of the data (dotted lines), and the spectrum of the error required to reconcile the model and the data (long dashed lines) when $\alpha = 3/4$, i.e. one fourth of firms in the economy set their optimal prices per period.

Figure 5.1 shows that there are considerable differences between the model without habit and the data. The model has mass spectra at high frequencies, while the data has mass spectra around the business cycle frequencies. The power spectra of output and consumption of a Calvo-type sticky price model without habit increase with frequency. As Ellison and Scott(2000) point out, the Calvo-type sticky price model generates substantial volatility at high frequencies far in excess of that observed in the data. This failure is the weakness of the Calvo type sticky price model.

However, the sticky price model with external habit displays a hump-shaped spectrum for output with a noticeable business cycle peak as in the data. Moreover, the spectral density of consumption calculated from the sticky price model with external habit displays a peak at business frequencies as in the data. The spectrum of consumption of a Calvo type sticky price model with external habit displays a peak at around frequency 0.10 as in Figure 2, while the spectra of the real variables calculated from the data display a peak at about frequency 0.05. The volatility of output, consumption, investment and labor also substantially decrease at nearly all frequencies when external habit formation is introduced into the model. This can be seen more clearly in the spectrum of the error required to reconcile the model and the data. Figure 5.2 shows that the error required to reconcile the model and the data for output and consumption substantially decreases when external habit formation is introduced into the sticky price model. In particular, the error needed to reconcile the model and the data for output

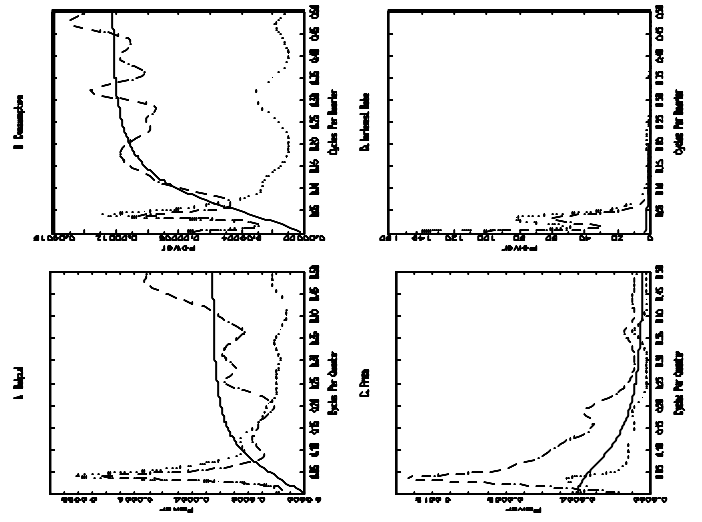

Figure 5.1:

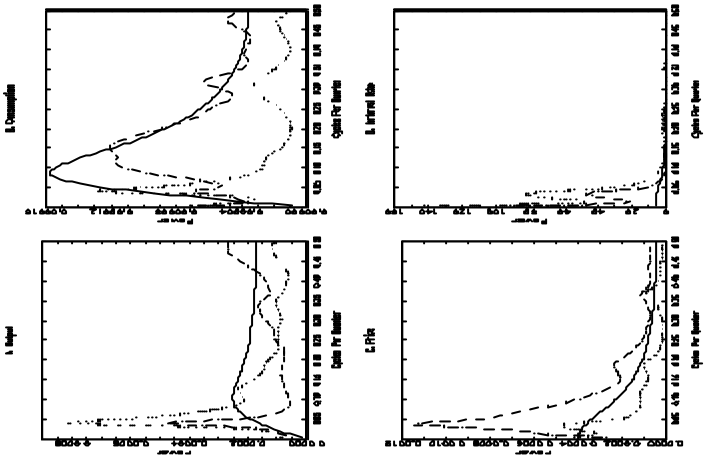

Figure 5.2:

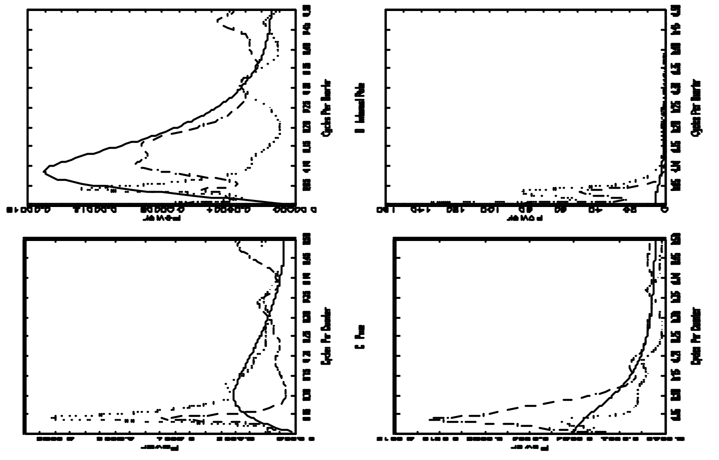

Figure 5.3:

and consumption at middle and high frequencies decreases sharply. These findings suggest that external habit formation plays an important role in the cyclical variability over the business cycle. That is, when external habit formation is incorporated into the basic sticky price model, its reaction in terms of consumption, labor, and output is gradual to the monetary shock. In the case of prices, the sticky price model has mass spectra at low frequencies irrespective of habit in consumption as in the data even though the model does not have a peak. In the case of nominal interest rates, the data has mass spectra at low frequencies irrespective of habit in consumption, while the sticky price model does not have.

The expenditure delays, introduced to generate the hump-shaped response of real spending to a monetary shock by BGG (1999) and Rotemberg and Woodford (1997), also improves the performance of the sticky price model in explaining the selected variables at high frequencies as well as at business cycle frequencies as in Figure 5.3. The power spectra of output and consumption display small value at nearly all frequencies and do not increase with frequency as in the data.

Table 3 provides a summary of the relative mean square approximation error (RMSAE) for the levels of the series integrated over business cycle frequencies (6-32 quarters) and those detrended by HP filter integrated across all frequencies when the unweighted trace of the spectrum is minimized as in Watson (1993). The figures in Table 3 confirms the findings in the spectral density. In terms of the RMSAE for the selected variables, a Calvo-type sticky price model with only external habit formation outperforms a Calvo-type model without habit formation. The RMSAEs for the selected variables substantially decrease either using only business cycle frequencies or HP filter integrated across all frequencies when habit formations are incorporated into the sticky price model. A Calvo-type sticky price model with external habit formation and investment expenditure delay also outperforms a Calvo-type model without habit formation. The RMSAEs for the selected variables substantially decrease either using only business cycle frequencies or HP filter integrated across all frequencies when both elements are incorporated into the sticky price model.

5.4.2. Serial Correlations and Impulse Response Functions

Next, I will consider whether the second moments and the impulse response function of the selected variables calculated from the model correspond to those implied by the data.

First, Table 4 provides the volatilities and serial correlations of the real variables in the model when one fourth of firms in the economy adjust their prices optimally per period. The relative volatility of consumption, output, and investment matches that of the data, though the absolute volatility is small. In regard to contemporaneous correlations of output and interest rates, nominal interest rates move procyclically irrespective of external habit formation in consumption. However, the correlations between current nominal interest rates and future output are positive when one does not incorporate the external habit into a sticky price model. If one incorporates external habit in consumption into the sticky price model, the correlations between current nominal interest rates and future output become negative ($corr(r_t, y_{t+4}) = -0.32$), which shows that nominal interest rates serve as countercylclical leading indicators over the business cycle. The introduction of the investment expenditure delay into the model also makes nominal interest rates to serve the role of the inverted leading indicators in the business cycle ($corr(r_t, y_{t+4}) = -0.39$). The correlations between prices (p_t) and future output are strongly negative ($corr(p_t, y_{t+2}) = -0.56$) as in the data when there is external habit in consumption.

Next, Figure 5.4 shows the dynamic effects of a positive monetary shock on consumption, output, investment, price, and interest rates when the degree of nominal rigidities is 3/4. The response of consumption to the monetary shock is hump-shaped when there is habit formation. This is because households that care about their neighborhood gradually adjust their consumption profile to the shock. However, the hump shape response of consumption to the monetary shock disappears when there is no external habit formation in consumption. The response of output to the shock is also hump-shaped when there is external habit or/ and investment expenditure delay. Moreover, the persistence of domestic consumption and output increases with the incorporation of habit into the

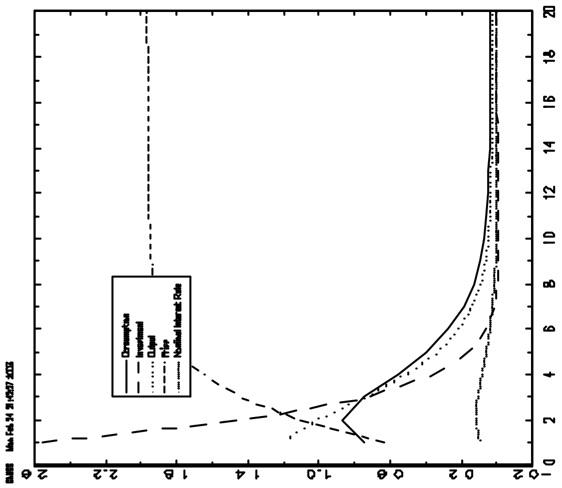

Figure 5.4:

sticky price model.

Overall, the Calvo-type sticky price model with external habit outperforms the Calvo-type sticky price model without habit. The spectral density of the selected variables calculated from the sticky price model with habit fits better than the spectral density calculated from the sticky price model without habit at nearly all frequencies.

6. Concluding Remarks

This paper specifies a sticky price model with external habit formation in consumption as in Abel (1990, 1998) and then investigates whether the model can generate the hump-shaped spectrum of selected real variables. Also, the relationship between interest rates, prices and real activity is discussed.

The introduction of external habit into the Calvo-type sticky price models improves the model's performance at nearly all frequencies. The habit formation substantially decreases the volatility of output, consumption, investment and labor at nearly all frequencies in the Calvo-type sticky price models. Moreover, the spectral density of consumption growth rates as well as output growth rates shows the hump shape, though milder than the spectral density calculated from the data.

In a future research agenda, it is desirable to explore more extensively the elements that allow replication of the typical hump-shaped spectral density of real variables in a sticky price model.

Table 1: The Calibrated Parameters

Parameter	Value	Description of Parameters
γ	1.004	steady state quarterly growth rate of technology
s_H	0.65	steady state labor share
δ	0.025	rate of depreciation of capital stock
r	0.016	steady state rate of return
$\epsilon_C(\sigma^{-1})$	1/3	intertemporal elasticity of consumption
M_i	-1	semi-elasticity of demand for money(percent)
μ	1.1	steady state markup
ϵ_{nk}	1	elasticity of substitution between capital and labor
N_w	1	intertemporal elasticity of labor supply
η_q	4	elasticity of i/k to Tobin's q

Table 2 : Moments of the Data

Variable	S.D.	Cross	Corr.	X_t	with	GDP	$(corr$	$(X_t,$	$Y_{t+k}))$	
		$k = -4$	-3	-2	-1	0	1	2	3	4
Y	1.66	0.03	0.33	0.66	0.91	1.00	0.91	0.66	0.33	0.03
C	0.78	0.05	0.29	0.55	0.75	0.87	0.85	0.71	0.49	0.25
I	4.97	0.04	0.32	0.61	0.82	0.89	0.83	0.65	0.41	0.18
N	1.61	0.37	0.63	0.85	0.94	0.88	0.67	0.36	0.03	-0.23
P	1.35	0.12	-0.04	-0.21	-0.38	-0.51	-0.62	-0.68	-0.67	-0.59
r	1.09	0.40	0.50	0.57	0.54	0.41	0.18	-0.10	-0.38	-0.58

Source : Stock and Watson (1999)

Note : r is 3 month Treasury bill rate.

Table 3 : Relative Mean Square Approximation Error

Variable	Basic Model	Model with Habit	Model with Habit and Delays
	Business Cycle	Frequencies:	6-32 Quarters
Y	0.83	0.42	0.38
C	1.40	1.11	0.81
I	0.44	0.48	0.56
N	1.04	0.68	0.61
P	3.53	3.05	2.79
r	0.52	0.33	0.38
	Hodrick-Prescott	Detrended Levels:	All Frequencies
Y	0.94	0.48	0.40
C	1.76	1.32	0.96
I	0.53	0.52	0.56
N	1.75	1.18	1.01
P	3.49	3.01	2.72
r	0.52	0.33	0.38

Note: Model with habit means a sticky price model with external habit formation, and model with habit and delays means a sticky price model with both external habit formation and expenditure delays.

Table 4 : Moments of the Model with External Habit ($\epsilon_C = 1/2$, $\alpha = 3/4$)

Variable	S.D.	Cross Corr. X_t with GDP ($corr(X_t, Y_{t+k})$)								
		$k = -4$	-3	-2	-1	0	1	2	3	4
Basic	*Model*									
Y	1.71	0.03	0.31	0.63	0.90	1.00	0.90	0.63	0.31	0.03
C	0.95	0.08	0.36	0.67	0.92	0.99	0.87	0.59	0.26	-0.03
I	3.51	0.00	0.28	0.61	0.88	0.99	0.91	0.66	0.34	0.07
N	2.24	0.06	0.29	0.54	0.72	0.74	0.59	0.33	0.05	-0.15
P	2.11	0.43	0.56	0.63	0.61	0.45	0.19	-0.10	-0.34	-0.47
r	1.45	-0.30	-0.18	0.09	0.40	0.64	0.70	0.56	0.30	0.05
Model	*with*	*Habit*								
Y	1.82	-0.06	0.23	0.58	0.88	1.00	0.88	0.58	0.23	-0.06
C	1.33	0.16	0.39	0.71	0.92	0.92	0.71	0.36	0.01	-0.22
I	3.33	-0.26	-0.03	0.33	0.71	0.95	0.95	0.73	0.39	0.09
N	2.54	-0.26	-0.04	0.19	0.49	0.72	0.77	0.61	0.30	-0.02
P	2.00	0.47	0.59	0.64	0.56	0.34	0.03	-0.27	-0.48	-0.56
r	0.78	0.16	0.40	0.62	0.71	0.63	0.38	0.07	-0.19	-0.32
Model	*with*	*Habit*	*and*	*Delay*						
Y	1.61	-0.03	0.25	0.60	0.89	1.00	0.89	0.60	0.25	-0.03
C	1.28	0.10	0.38	0.68	0.91	0.96	0.80	0.49	0.15	-0.13
I	2.65	-0.18	0.08	0.44	0.77	0.95	0.90	0.66	0.35	0.08
N	2.25	0.05	0.29	0.55	0.73	0.71	0.50	0.17	-0.14	-0.32
P	2.11	0.37	0.48	0.56	0.56	0.44	0.20	-0.09	-0.34	-0.49
r	0.75	0.23	0.45	0.62	0.67	0.54	0.27	-0.04	-0.28	-0.39

Note: α denotes the degree of nominal rigidities in the sticky price model.

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