Common Property Resources in Private Property Regime with Inequality[¤]

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June 2002 (1st draft)

Abstract

The economic heterogeneity among the members of a society plays a signi¯cant role in determining the level of extraction of common property resources (CPR). This paper has considered one particular form of economic heterogeneity, i.e., inequality in distribution of private property resources (PPR) which is used with CPR units to produce a private good. The inequality in distribution of PPR has led to a constrained optimization problem for allocation of e®ort to the individual in the CPR ¯eld. Two possible constraints or restrictions on the choice of individual's action in common property ¯eld have been focussed here; viz., e®ort endowment restriction (in terms of PPR), and complementarity restriction on e®ort (i.e., restriction in terms of marginal productivity of e®ort given the size of PPR). For di®erent types of indiividuals with di®erent size of PPR, the binding restrictions on their choice of optimal allocation of e®ort in the CPR ¯eld may be di®erent. In a two-player-two-stage backward induction CPR game with inequality in private property resources the present paper has shown that in the subgame perfect Nash equilibrium it is for the `poor' private property owner the complementarity restriction and for the `rich' the e®ort endowment restriction will be the binding constraint resulting unequal payo® or output from unequal action.These results in the theoretical exercises coin some ideas to indicate how those binding restrictions can play the deterministic role in individual's voluntary participation in collective action in common property ¯eld.

key words: common property, inequality in private property , e®ort endowment restriction, complementarity restrictionon on e®ort, subgame perfect Nash equilibrium

JEL classi¯cation codes: D63, D71, H41, O12, O13, O23

[¤]I express my deepest gratitude to Prasanta Pattanaik for his sincere guidance, discussion and comments on this paper. This paper is a part of the research supported by the Ministry of Environment and Forest, Government of India, through Overseas Post-doctoral Fellowship in University of California, Riverside, under World Bank India Capacity Building Programme in Environmental Economics.

Introduction

The economic heterogeneity among the members of a society plays a significant role in determining the level of extraction of common property resources (CPR).This paper has considered one particular form of economic heterogeneity, i.e., inequality in distribution of private property resources (PPR) which is used with CPR units to produce a private good. In the context of inequality in the distribution of PPR, the motivation of this paper is to build up the analytical constructs to answer the following set of questions related to common property field. 1) Does the equal access to all the members of the community always lead to equal level of action in the common property field? 2) Does the equal access always lead to equal benefit to all members of the community? And finally, 3) how can the linkage between PPR inequality and CPR be articulated into the issue of efficient management of common property resources and into the issue of evolving an institution for collective action?

In economic literature, at micro and macro level analysis of common property resources we can identify at least two distinct but related areas where the issue of economic inequality has been focussed. Firstly it has been highlighted in intertemporal welfare economics in the context of intergenerational equity and efficient allocation of resources which is basically guided by Rawlsian rule of distributive justice (Rawls, 1971). The problem of identifying an efficient and equitable growth path for an economy (in the sense of ensuring a constant per capita level of consumption in each intertemporal time period (Solow, 1974; Stiglitz,1974)) has been found to be sensitive to a number of different issues viz., the technological substitutability of natural resources by reproducible capital (Solow, 1974; Stiglitz,1974; Heal and Dasgupta,1974; Dasgupta and Mitra,1983), the substitutability among the exhaustible resources themselves (Hartwick,1978), intertemporal preferences (Burton, 1993), and the inter and intragenerational competition among the resource users (Jorgensen and Yeung, 2000).

The present paper however belongs to the other interesting area which is particularly concerned with the effect of economic inequality on the behavioural aspect of the resource users. Some of the analyses in this field has been motivated to describe the survival strategy of the poor rural folk with a recourse to some degree of substitutability of capital by common natural resources (Dasgupta,2000; Dasgupta, 1987; Jodha,1986;1990) The common property resources (CPR) in that context are playing some remissive role on inequality. In the last fifteen years most of the analyses in this area however have been articulated to explain the effect of economic inequality on the potential of the success of collective actions in common property resources (Ostrom,1995; Baland and Platteau, 1996;Johnson and Bardhan,2002; Bardhan,2001; Bardhan and Mukherjee;2001). The agenda has been flashed out from the very powerful and controversial hypothesis of Olson (Olson,1965) contending that inequality might be favourable for provision of public goods which has been sometimes kept aligned with the contention that, the inequality might be favourable for collective action in common property field. Various field studies like the case of Texas shrimp fishery (Johnson and Libecap;1982) the case studies on voluntary collective action in common property resources in five villages in Haryana and Punjab in India (Chopra and Kadekodi and Murty;1990), the studies on irrigation system in Nepal, southern India and Central Mexico (Bardhan and Johnson, 2001), the case studies in firewood collection in rural Nepal (Bardhan and Mukherjee;2001) strongly nullify the Olson's favourable inequality effect on collective action in common property resources. The most important source of failure of community approach to the management of common property resources has been found to lie in the large size and heterogeneity among members of the community. In a case study on CPR management in dry regions in India Jodha noted that failure of CPR institutions is less in the villages where access and benefits from CPR is equitable (Jodha; 1992).

Since common property resources is a de facto regulated institution of the community, the community in most of the cases can ensure the equal access right to the members of the community, not the equal bene¯t. The present paper proposes an analytical structure to handle with that kind of institutional problems of CPR management where withstanding the equal accessibility to CPR among the members of the community the potential of equitable bene¯t from CPR is tagged with distributional pattern of other private property resources in the community.

Although there exists a vast literature in empirical ¯eld, only a few attempt has been made so far to suggest a formal analytical structure to handle with the inequality e®ect on behavioural pattern of CPR users . Two seminal works in that context can be referred to, viz., one in (Hackett, Dudley and Walker; 1995) and the other in (Johnson and Bardhan; 1999). They share with some commonalities and the present paper proposes three steps of digression from them. Firstly they formalize the individual's action in common property ¯eld in such a way that it is not possible to separate out the e®ect of inequality on the individual's action itself. In their game theoretic decision setting in common property resources, the economic heterogeneity of the resource users have been completely captured either by the heterogeneity in e®ort endowment (Hackett, Dudley and Walker; 1995)[1]or by the heterogeneity in e®ort deployed (Dayton and Bardhan; 1999),[2] for appropriation of common property resources. In other words, in this kind of approach the substitutability of capital (where economic heterogeneity is traced out) by e®ort is not allowed. In this set up therefore the kind of remissive role of e®ort to be put in common property ¯eld on inequality (Dasgupta, 1987) cannot be handled. The homogeneity in e®ort endowment with heterogeneity in capital endowment is the most plausible assumption that the present paper has made.

The second digression of the current paper from the existing commonalities is the assumption of zero

[1]In Hackett, Dudley and Walker; 1995 the heterogeneity has been considered in terms of endowment of input, invested in common property resources without making any distinction between capital and labour inputs

[2]In Johnson and Bardhan; 1999, ¯shing \e®ort" included the number of boats and hours and intensity of labour

opportunity cost of e®ort of the resource users outside the common property ¯eld.[3] Our situation is familiar with that of some very poor rural community in the less developed economy where at the back drop of population explosion, and zero technological progress there exists almost no alternative option to put their e®ort outside the common property ¯eld. This assumption we made in our proposed model however is made for the sake of simplicity, which can be relaxed without a®ecting the results.

The third and the ¯nal important digression that the present analytical framework makes is treating common property resources as a source of inputs which, in conjunction with some private property inputs (which is unequally distributed among the resource users) produces some private goods.[4].Wealth inequality leads to unequal bene¯t from common property resources which may have further retrogressive e®ect on production of some private goods. In our proposed model the behavioural aspect of CPR users is assumed to hinge upon the inequality e®ect on production of private goods.

The main focus of this paper is addressed to the question: whether equal accessibility to common property resources in a private property regime with inequality will lead to equal bene¯t to the members of the community or not? In our proposed analytical framework the results from the theoretical exercises show that the answer depends on three important issues: viz., (1) the distribution of private property resources among the members of the community; (2) the degree of substitutability of private property resources by the e®ort used for appropriation of common property resources; (3) the degree of complementarity of private property resources to common property resources.

The paper has been organized as follows. Section 2 considers a general outline of the proposed two-stage

[3] In Johnson and Bardhan; 2002, the economic inequality has been re°ected in alternative income earning opportunities of the resource users outside the CPR ¯eld.

[4] Most of the theoretical exercises in the game theoretic setting, assume an uniform technology of appropriation of CPR units and de¯ne the payo® or bene¯t from CPR in terms of that appropriation function. In most of the cases however CPR units are used for intermediary consumption, not for ¯nal consumption. In that context therefore the payo® or bene¯t from CPR should be linked up with that production function in which CPR units are used for intermediary consumption. As a few exception to this kind of treatment of CPR , we can refer to Chopra and Kadekodi; 1991 and Murty; 1994, who although from a di®erent perspective incorporated the role of CPR in other private goods

backward induction CPR game with inequality in private property distribution. Section 3 gives a formal presentation of the model with derived propositions. Section 4 after compiling the results from section 3 reaches the conclusion.

2 A general outline of the model

The theme of this paper has been textured in terms of a two player-two stage (which is also considered here as two periods, `present' and `future') backward induction game for a simple model of a village community with production of a single commodity, milk. All members of the community are engaged in the production of milk and for that they depend on forest, the common property resources, which is the only source of fodder for feeding cattle. Cattle property (which is the private property resource for milk production) is unequally distributed among the members of the community. Forest is not merely a common pool resource here. By virtue of a well defined property right of the community on the forest (i.e., right to protection, right to regulation of usage and right to development)[5]. all non-members of the community are excluded from the use right and open grazing is not allowed. It is because of exclusionary principle and stall feeding ,the congestion externality becomes controllable. Forest has a finite stock of fodder which grows in these two periods at a constant rate of the initial stock.

Now the production of milk of each of the member of the society depends on the size of her cattle property, the effort she puts for each cattle for collection of fodder and some externality regarding how much fodder from the existing stock has been collected by other members in the society, given her own collection. Her production of milk (which is also considered as payoff in this model) is solely dependent on her own effort/ action i.e., independent of other's effort/ action in the community as long as the total effort deployed by the society for collection of fodder doesn't exceed the total stock. In other words, how far the per cattle

[5] Jodha (1993) in Beijer Discussion Paper Series, 41 identified those three sets of customary rights represented through the community's decision making, enforcing rules and practices with regard to CPRs

e®ort deployed in collection of fodder would be e®ective for production of milk depends upon the congestion externality. If forest is not congested, given the number of cattle she owns, given the production technology one may get the production of milk, as much as the e®ort she puts.

This problem of congestion externality in this model has been transcribed with a dynamic perspective in two periods game. The set of players and distribution of cattle property are assumed to remain the same between these two periods.[6] Given the backward induction strategy, the individual player being at the `present' (stage 1) anticipates the e®ect of her present period collection of fodder and that of her opponent, on her future period collection and the future period stock of fodder. On the basis of this anticipation she chooses the present period's subgame perfect Nash equilibrium strategy to maximize her total production of milk (payo®) in these two periods. Since our model deliberately assumes the non-existence of past before period 1 and non-existence of any future beyond period 2, given the ¯nite stock of resources, given the equal endowment of e®ort for each player in each period, this ¯nite two-stage game of perfect information shows how the individual players with di®erent size of cattle property in a non-cooperative way solve the problem of optimal allocation of per cattle e®ort (vis-a-vis, per cattle collection of fodder) between these two periods.

Regarding allocation of e®ort in our model two possible restrictions on per cattle e®ort have been taken into consideration. Firstly there is a restriction on individual's per cattle e®ort endowment which acts as a binding constraint to each individual in each period. The per cattle e®ort endowment restriction would vary from player to player according to the size of cattle property they hold. The second kind of restriction on per cattle e®ort comes out from technological complementarity of fodder (the CPR units) to the production of milk. Feeding the cattle more increases the milk production up to a certain point, not beyond that. This complementarity restriction in turn imposes restriction on substitutability of cattle by e®ort (cattle

[6]With the same kind of structure this story can be translated into the problem of intergenerational allocation of CPR assuming that within each type of player (say, father and son with the same number of cattle) the cattle property and payo® function remaining the same.

obviously is not perfectly substitutable by e®ort). Because of this complementarity restriction, one cannot compensate the loss of milk production (i.e., the loss of payo®) due to small size of cattle property just by increasing the per cattle e®ort for collection of fodder In our model we have focussed this complementarity restriction issue ¯rst by considering a situation where there is no upper bound complementarity restriction on per cattle e®ort. This step in our model has been used as an intermediary step to observe the e®ect of this kind of complementarity restriction on per cattle e®ort upon the sub-game perfect Nash equilibrium.

The general conclusion we derived from our theoretical exercises is that, in a private property regime with unequal distribution of private property (the cattle property in our model) equal e®ort per unit of PPR (vis-a-vis., equal level of per cattle collection of fodder) doesn't necessarily lead to equal bene¯t (i.e., equal level of production of milk) and vice versa.

3. A Simple Two-player-Two-stage Game with Common Property Resources

3.1 Model Speci¯cation

Consider a society of two players, N = f1; 2g, with a ¯nite time horizon of two periods, T = f1; 2g, the present and future, with no past. The life span of each player is assumed to cover these two periods:There is a common property resource, say, forest, with a ¯nite stock, which is S, at the begining of the game and grows by 4 between these two periods. 4 < S. The players collect fodder from the forest to feed the cattle since open grazing in this society is not allowed. The cattle are homogeneous in terms of productivity. There is inequality in the distribution of cattle (K) properties, so that the number of cattle owned by the ith player, $K_i > K_j$, the number of cattle owned by the jth player (assuming, i = 1and j = 2).For each of the ith player, K_i is assumed to be the same across the periods and so is K_j. Let $\frac{K_j}{K_i}$ = µ, the inequality index and µ < 1.

The strategic interaction of the players determines the amount of fodder that each player will collect from the forest in each period. The final outcome however is interdependent of players' decisions. Since this is a two period game, the move in the second period ("future") is conditioned by the outcome of the first, ('present') i.e. by the history of the game till the second stage is reached. This implies that each of the player's strategy is a complete plan of action for the whole game.

Let a_i^t; the effort per unit of cattle (expressed in terms of labour hours, intensity of labours etc.), deployed for collection of fodder, be the action variable of the ith player in period t. Given the number of cattle K_i fixed, and given the fixed endowment of total effort, E_i for each player in each period, $a_i^t \in [0; \frac{E_i}{K_i}] \to \Re_+$. It is assumed $E_i = E_j$ so that total effort endowment of each of the player in each period is the same although the per cattle effort endowment of ith player is less than that of jth player, i.e., $\frac{E_i}{K_i} < \frac{E_j}{K_j}$.

At the beginning of the period,1 say, 'present' since the stock of fodder is $S^1 = S$, and between these two periods, the stock grows by Δ, in period 2, say 'future' the maximum available fodder (if nothing is used in the 'present') is, $(S + \Delta)$.

The Specification of Production Function

The production function (also the payoff function) of milk in this model has two-parts; first part considers the total effort used for collection of fodder ($= K_i a_i^t$) and the second part constitutes the 'effectivity' of effort (ψ_i^t), which depends on congestional externality effect ($x^t - 1$).

$$\psi_i^t = e^{x^t - 1}$$

,

where,

$$x^t - 1 = \frac{S^t - (K_i a_i^t + K_j a_j^t)}{K_i a_i^t + K_j a_j^t}$$

$x^t - 1$ depends upon the existing stock S^t and the total effort deployed by the society (two players, here) for collection of fodder. For sake of convenience in this model, $(x^t - 1)$ will be denoted by $f_i^t((a_i^t; a_j^t); S^t)$.

In the second period since the maximum available stock $S^2 = (S + 4) - (K_i a_i^1 + K_j a_j^1)$

$$f_i^2((a_i^2; a_j^2); S^2) = x^2 - 1 = \frac{(S+4) - (K_i a_i^1 + K_j a_j^1) - (K_i a_i^2 + K_j a_j^2)}{K_i a_i^2 + K_j a_j^2}$$

According to this specification, the jth player's action enters into the ith player's payoff through the congestion externality effect. For those values of $(a_i^t; a_j^t) \in A^t$, the society's action profile, if there is no congestion in the forest (in the sense that society's total effort doesn't exceed the existing stock) the ith player's payoff/ production of milk can be defined in terms of ith player's action independently of jth player's action.

Since this model considers the two stage (two period) game with backward induction strategies, the effectivity of effort in the production of milk in the second period ψ_i^2 will depend on the previous period's action and stock i.e., on $(a_i^1; a_j^1; S)$. In the second period since, the maximum available stock

$S^2 = (S + 4) - (K_i a_i^1 + K_j a_j^1)$;and

$$f_i^2((a_i^2; a_j^2); S^2) = x^2 - 1 = \frac{(S+4) - (K_i a_i^1 + K_j a_j^1) - (K_i a_i^2 + K_j a_j^2)}{K_i a_i^2 + K_j a_j^2}$$

if the forest is congested in period 1, it will make the available stock less for period 2.

Now let us consider the following assumption on the value of the effectivity function, ψ_i^t

$$\begin{aligned} \psi_i^t &= e^{f_i^t((a_i^t; a_j^t); S^t)} \\ &= 1; \text{ for } f_i^t((a_i^t; a_j^t); S^t) \geq 0 \text{ i.e.; for; } (x^t - 1) \geq 0 \\ &< 1; \text{ for } -1 < f_i^t((a_i^t; a_j^t); S^t) < 0 \\ \text{i.e. for; } -1 &< (x^t - 1) < 0 \end{aligned}$$

The above assumption tries to capture the following intution in this model. The maximum fodder available in the forest at period t (= S^t) is expressed in terms of the total effort of the society which is just enough to exhaust the stock of fodder available in the forest. If $f_i^t((a_i^t; a_j^t); S^t) > 0$, i.e.,the maximum fodder available is greater than the total effort deployed by the society, $\psi_i^t = 1$, because by definition the effectivity can not exceed 1. If on the other hand, $-1 < f_i^t((a_i^t; a_j^t); S^t) < 0$, i.e., society's total effort for collection of fodder exceeds the available stock (i.e.,if forest is congested) effectivity will be less than 1.The effectivity of the total effort ($= K_i a_i^t$) of the ith player deployed for production of milk , therefore depends on her share of the total effort contributed in the society for collection of fodder , relative to the size of the stock of fodder available in the forest.

In terms of the above characterization now we can formally define the production function in the following way:

Definition D1. The production function (also the payoff function) of milk of the ith player at period t, Q_i^t is defined as a function of total effort, ($= K_i a_i^t$), and the effectivity function ψ_i^t, such that,

$$
\begin{aligned}
Q_i^t &= K_i a_i^t \psi_i^t (f_i^t((a_i^t; a_j^t); S^t)) = K_i a_i^t e^{x^t - 1}; \\
\text{for all } (a_i^t; a_j^t) &\in A^t \text{ such that; } -1 < f_i^t((a_i^t; a_j^t); S^t) < 0 \\
\text{i.e.; for; } -1 &< (x^t - 1) < 0 \\
&= K_i a_i^t; \text{ for all} (a_i^t; a_j^t) \in A^t \text{such that} \\
; f_i^t((a_i^t; a_j^t); S^t) &\geq 0 \text{i.e.; for; } (x^t - 1) \geq 0
\end{aligned}
$$

In this production function per cattle effort or action a_i^t is complementary to the production of milk. Other things remaining the same as per cattle effort vis-a-vis per cattle fodder collection increases, as a complementary effect of effort on output, milk production increases. There is no restriction on complementarity.

But suppose there is complementarity restriction on a_i^t so that, $a_i^t \quad a^*$, where a^* is exogeneously given, the production (or payo®) function as de¯ned in D1 now will satisfy either of the following two conditions:

$$Q_i^t \quad = \quad K_i a^{¤}; \text{ for all } (a^{¤}; a_j^t) \; 2 \; A_i^t \text{ such that; } f_i^t((a^{¤}; a_j^t); S) \; ¸ \; 0 \qquad \text{(C1)}$$

$$< \quad K_i a^{¤}; \text{ forall}(a^{¤}; a_j) \; 2 \; A_i^t \text{such that; } ¡1 \quad f_i^t((a^{¤}; a_j^t); S) < 0 \qquad \text{(C2)}$$

In other words the above complementarity restriction on production (payo®) function states that, given the number of cattle K_i, if the e®ectivity factor $Ã_i^t = 1$, i.e., forest is not congested, the milk production increases with a_i^t upto $a^{¤}$. Beyond $a^{¤}$with $Ã_i^t$ being equal to 1, milk production per cattle remains unchanged. In this way the complementarity restriction as speci¯ed here makes the production function discontinuous for $a_i^t > a^{¤}$, and other things remaining the same, the marginal productivity of $a_i > a^{¤}$ becomes zero. [7]

Incorporating the complementarity restriction into the production (also the payo®) function, in this way, we are now able to handle with two distinct possibilities; one, where, complementarity restriction acts as a binding constraint and the other, where the complementarity restriction does n't bind the individual's choice of per cattle action in the common property ¯eld.

Since this model assumes that the players play with backward induction strategies the game starts at the second stage with the history of possible per cattle (e®ort) action at the stage 1 $(a_i^1; a_j^1)$, given the stock S, . At the begining of the period 2 (i.e., the stage 2) the players know the history $h^1 = (a_i^1; a_j^1)$, which generates the simultaneous move game with the second stage action pro¯le, $A_i^2(h^1)$: The strategy at stage 2 thus maps `history' h^1 to the set of feasible action $A^2(h^1)$. Since the game ends at stage 2 the ¯nal history obtained is $h^2 = ((a_i^2(h^1); a_j^2(h^1)) = ((a_i^2(a_i^1; a_j^1); a_j^2(a_i^1; a_j^1))$, and correspondingly we get, $G(h^2)$ the subgame of the original game G with history h^2 and obtain the payo® $Q_i(h^2)$: Solving $G(h^2)$ we get, the subgame perfect

[7] This type of complementarity restriction may provide with an alternative explanation to why in a small society over extraction of natural resources doesn't take place. One of the possible reason is that size of the capital (which comes from private resources) in that society is so small that over extraction is not economically pro¯table.

Nash equilibrium strategy $(s_i^*(h^2); s_j^*(h^2))$, such that:

$$Q_i^2(s_i^*(h^2); s_j^*(h^2)) \geq Q_i^2(s_i(h^2); s_j^*(h^2)) \; \forall s_i(h^2) \in S_i(h^2)$$

$$Q_j^2(s_i^*(h^2); s_j^*(h^2)) \geq Q_j^2(s_i^*(h^2); s_j(h^2)) \forall s_j(h^2) \in S_j(h^2)$$

Then these payoffs are transferred into the first stage to solve the original game G.

This paper has considered the above two-player two-stage CPR game in two different versions:

Version V1. There is no restriction on a_i^t, other than the effort endowment restriction, i.e., $0 \leq a_i^t \leq \frac{E_i}{K_i}$

Version V2. In addition to the effort endowment restriction, there exists the complementarity restriction on a_i^t so that, $a_i^t \geq a^*$, where a^* is exogeneously given.

3.2 Two-player-Two-stage CPR Game with no Complementarity Restriction

As it has been specified in section 3.1, in the backward induction CPR game, the players in stage 1 anticipate their choice of per cattle effort in stage 2 (which is here, period, 2) for each possible choice they can make in stage 1(which is period 1). The production of milk of the ith player in period 2 is a function of her total effort deployed (= $K_i a_i^2$), and the effectivity factor, $\psi_i^2(f_i^t(:))$. The effectivity ψ_i^2 in period 2 depends not only on the collective effort of the society taken in period 2 to collect fodder but also on the total effort deployed in the previous period, relative to the stock available. This happens because the maximum available fodder in period 2 is determined by the growth of the stock Δ, and the stock left over from period 1.

Given the limited stock of fodder in the forest, given the fixed number of cattle she owns, in the non-cooperative game, each of the ith player makes a contingent choice of per cattle action (effort) for collecting fodder in period 1 ('present ') to support the action in period 2 (`future'). Each of the two players at stage 1 simultaneously choose, say, $\hat{a}_i^1; \hat{a}_j^1$ the per cattle actions for collecting fodder from the set of feasible actons belonging to A^1. The subgame perfect Nash equilibrium of the ith player with stage 2 actions is reached by solving $Q_i^2(a_i^2(\hat{a}_i^1; \hat{a}_j^1); a_j^2(\hat{a}_i^1; \hat{a}_j^1))$ with respect to a_i^2. After solving this, the first stage game is rolled back from the second stage subgame. Ultimately, the ith player chooses her optimal level of action $\hat{a}_i^1$ to maximize her present milk output contingent upon the anticipated action values for the future production, given the available stock, her optimization problem becomes,

$$\mathrm{Max}_{\hat{a}_i^1}[Q_i^1(\hat{a}_i^1; \hat{a}_j^1) + Q_i^2(a_i^2(\hat{a}_i^1; \hat{a}_j^1); a_j^2(\hat{a}_i^1; \hat{a}_j^1))]$$

(The solution to this backward induction CPR game is given in Appendix 1)

3.2.1 Results from CPR Game with no Complementarity Restriction

The Nash equilibrium results from solving the backward induction CPR game with no complementarity restriction are characterized as follows:

1)For period 1, the per cattle action of ith and jth players, respectively,

$$\hat{a}_i^1 = \frac{S}{4K_i} \quad (1)$$

$$\hat{a}_j^1 = \frac{S}{4K_j} \quad (2)$$

and milk output (payoff),

$$\hat{Q}_i^1 = \hat{Q}_j^1 = \frac{S}{4} \quad (3)$$

2)For period 2 the per cattle action of ith and jth players, respectively are,

$$\hat{a}_i^2(\hat{a}_i^1; \hat{a}_j^1) = \frac{S/2 + \Delta}{4K_i} \quad (4)$$

$$\hat{a}_j^2(\hat{a}_i^1; \hat{a}_j^1) = \frac{S/2 + \Delta}{4K_j} \quad (5)$$

and, milk output (payoff),

$$\hat{Q}_i^2 = \hat{Q}_j^2 = \frac{S/2 + \Delta}{4} \quad (6)$$

3)Taking into account two periods together, the total milk output of the ith and jth players respectively,

$$Q_i^{*} = Q_j^{*} = \frac{3S/2 + ¢}{4} \qquad (7)$$

$$Q^{*} = Q_i^{*} + Q_j^{*} = \frac{3S + 2¢}{4} < S + ¢ \text{(= Total available stock)} \qquad (8)$$

4)Finally all the subgame perfect Nash equilibria of this specific CPR game are characterized by congestion free situation of the forest.On the basis of the above characteristics, we derive the Proposition 1.

Proposition. P1. In the backward induction CPR game as specified by V1 (i.e., in the version of no complementariiy restriction on per cattle action of player other than the endowment restriction) with standing inequality in distribution of private property resources (the cattle property), and, unequal per cattle action, the non-cooperative Nash equilibrium strategy of the players generate equal levels of milk output (i.e., equal benefit from CPR) to the players.

The proposition P1.derived from the results 3.2.1 is illustrated in the diagram 1. For all combinations of $(a_i; K_i)$ and $(a_j; K_j)$ satisfying the endowment restriction $0 \le a_i \le \frac{E}{K_i}$ and $0 \le a_j \le \frac{E}{K_j}$, in the CPR game as specified by V1 the Nash equilibrium benefit from CPR (in terms of milk output) remains the same. The Nash equilibrium benefit curves ($B^1$for period 1 and B^2 for period 2) are the rectangular hyperbolic curves. With unequal size of cattle property and the unequal level of Nash per cattle action the total derived benefit (in terms of milk output) indicated by the area under the curve. In period 1 the size of the area under $B^1$is $OK_iP^ia_i^1 = OK_jP^ja_j^1 = \frac{S}{4}$ and the area B^2 curve is $OK_iQ^ia_i^1 = OK_j^jQ^ja_j^1 = \frac{S/2+¢}{4}$. Given that, $4 < S$;the Nash equilibrium benefit curve B^2 will lie below B^1, if $4 < \frac{S}{2}$, B^2 will coincide B^1 if $4 = \frac{S}{2}$;and B^2 will lie above B^1.

Insert Diagram 1 here

Note: The sub-game perfect equilibrium solution in the backward induction game shows that,with standing the inequality in distribution of private property resources (the cattle property) the optimal levels of milk output of the players will be equal. This has been possible because of the implicit assumption we made here, regarding the substitutability of cattle by e®ort. The `poor' owner of the cattle property with the given number of cattle and given endowment of e®ort is assumed to be able to wipe out the di®erence between her production and that of the rich owner just by increasing her e®ort i.e., just by using the complementary e®ect of per cattle e®ort vis-a-vis., per cattle fodder on production. How far the e®ort, vis-a-vis the fodder, i.e., the extracted units of CPR in a production process is substitutable to other inputs privately owned by the individual player? In other words, the degree of substitutability between the private property inputs and common property inputs most plausibly itself puts the constraint against the derivation of equal bene¯t (or, equal milk output, here) from common property resources. This particular issue will be taken into consideration in section 3.3 to show that with the complementarity restriction on per cattle action a_i^t will restrict the domain of production or payo® function to show that the results in two stage backward induction CPR game indicating the relation between inequality in cattle property and per cattle action and bene¯t derived will be di®erent.

3.3 Two-player two-stage CPR Game with Complementarity Restriction on Player's Action

In section 3.1 in V1 we have characterized the complementarity restriction on production (or payo®) function in terms of the restriction on per cattle action a_i^t, such that $a_i^t \quad a^*$, satisfying either of the two conditions C1 and C2, where

$a^* \in \Re^1_+$, i.e., a^* is any real number which is exogeneously given. Now we may consider the following four logical possibilities with di®erent ranges of values that a^* can take in relation to the previous ¯rst stage subgame values, $\hat{a}_i^1 = \frac{S}{4K_i}$, and $\hat{a}_j^1 = \frac{S}{4K_j}$, satisfying the endowment restriction, $\frac{S}{4K_i} \le \frac{E}{K_i}$ and $\frac{S}{4K_j} \le \frac{E}{K_j}$.

$$\text{Case I} \quad : \quad \{a^* \in \Re_+ \mid a^* \le \frac{S}{4K_i} < \frac{S}{4K_j}\} \tag{9}$$

$$\text{Case II} \quad : \quad \{a^* \in \Re_+ \mid \frac{S}{4K_i} < \frac{S}{4K_j} \le a^*\} \tag{10}$$

$$\text{Case III} \quad : \quad \{a^* \in \Re_+ \mid \frac{S}{4K_i} < a^* \le \frac{S}{4K_j}\} \tag{11}$$

Considering those three different ranges of values of a^*, we get the following lemmas:

Lemma L1. In case I $a_i^1 = a_j^1 = a^*$; $f_i^t((a^*, a^*); S) \ge 0$ and $Q_i^1 = K_i a^*$.

In other words Lemma1 states that if $a^* \le \frac{S}{4K_i}$ and $a_j^1 = a^* < \frac{S}{4K_j}$, satisfying each, the per cattle endowment restriction, forest would not be congested and ith player's milk output or payoff can be defined solely in terms of her own action.

(Proof is given in Appendix2).

Lemma L2. If $\frac{S}{4K_i} < \frac{S}{4K_j} \le a^*$, satisfying the endowment restriction $\frac{S}{4K_i} \le \frac{E}{K_i}$ and $\frac{S}{4K_j} \le \frac{E}{K_j}$;then, $a_i^1 = \frac{E}{K_i}$ and the forest would be congested (respectively not congested) for all $(\frac{E}{K_i}; a_j^1) \in A^1$; such that $0 < a_j^1 \le \frac{E}{K_j}$, iff $S - E \ge K_j a_j^1$ (respectively, $S - E < K_j a_j^1$).

Corollary Ci) If $a^* \ge \frac{E}{K_j} > \frac{S}{4K_j}$, $a_i^1 = \frac{E}{K_i}$ and $a_j^1 = \frac{E}{K_j}$, with $\frac{E}{K_i} < \frac{E}{K_i}$, i.e., both of them will fully utilize their per cattle effort endowment, then forest would not be congested (respectively congested) iff $E \le \frac{S}{2}$ (respectively, $E > \frac{S}{2}$)

Corollary Cii) If $\frac{E}{K_j} > a^* \ge \frac{S}{4K_j}$, $a_i^1 = \frac{E}{K_i}$ then for all values $(\frac{E}{K_i}; a_j^1) \in A^1$ such that $a_i^1 < a_j^1 \le a^*$ the forest would not be congested if $\frac{S}{2} < E + K_j a_j^1 \le S$

(Proof is given in Appendix 2)

Lemma L3. If $\frac{S}{4K_i} < a^* \le \frac{S}{4K_j}$; either, i) $\frac{S}{4K_i} < a^* \le \frac{E}{K_i} \le \frac{S}{4K_j}$; or, ii) $\frac{S}{4K_i} \le \frac{E}{K_i} < a^* \le \frac{S}{4K_j}$: In case i) $a_i^1 = a_j^1 = a^*$ the forest would not be congested, if $a^* \le \frac{S}{K_i + K_j}$. Where as in case ii) $a_i^1 = \frac{E}{K_i}$ and

$a_j^1 = a^*$ the * forest would not be congested, if $S \geq K_i a_i^1 + K_j a^*$

(Proof is given in Appendix 2).

Note: The above lemmas take into account the effect of two alternative restrictions on individual's action values, viz., complementarity restriction and endowment restriction. In some cases per cattle endowment restriction acts as a binding constraint in the sense that, $\max a_i^1 = \frac{E}{K_i}$, no matter what the complementarity restriction is. Complementarity restriction (a^*) acts as a binding constraint only when either it coincides with endowment restriction i.e., $a^* = \max a_i^1 = \frac{E}{K_i}$ or endowment restriction is inoperative in the sense that, $a^* < \max a_i^1 = \frac{E}{K_i}$.

Given those lemmas we can now characterize the Nash equilibrium of the CPR game with complementarity restriction in relation to our original CPR game in the following way:

Proposition P2. When complementarity restrictions are incorporated, the subgame perfect Nash equilibrium $((\hat{a}_i^1, \hat{a}_i^2(a_i^1, a_j^1)), (\hat{a}_j^1, \hat{a}_j^2(a_i^1, a_j^1)))$, must satisfy the following conditions:

i)

$$\hat{a}_i^1 = \frac{S}{4K_i} \quad a^*; \; \hat{a}_j^1 = \frac{S}{4K_j} \quad a^* \tag{12}$$

and, ii)

$$\hat{a}_i^2(\hat{a}_i^1, \hat{a}_j^1) \quad \frac{S/2 + \Delta}{4K_i} \quad a^*; \; \hat{a}_j^2(\hat{a}_i^1, \hat{a}_j^1) \quad \frac{S/2 + \Delta}{4K_j} \quad a^* \tag{13}$$

Proposition P3 The Nash equilibrium with complementarity restriction as shown in proposition P2 is perfectly consistent with Lemma2 indicating two alternative sets of action values ($a_i^1 = \frac{E}{K_i}$ and $a_j^1 = \frac{E}{K_j}$), and ($a_i^1 = \frac{E}{K_i}$; $a_j^1 = a^*$). with the following properties:

i) If $a^* \geq \frac{E}{K_j} > \frac{S}{4K_j}$ we get equal level of optimal milk output (or payoff) $Q_i^* = Q_j^*$ from unequal level of per cattle action.

ii) If $\frac{E}{K_j} > a^* \geq \frac{S}{4K_j}$, $a_i^1 = \frac{E}{K_i}$ unequal action values generate unequal levels of milk output (or payoff) $Q_i^* > Q_j^*$

iii) In case i) the society's output with complementarity restriction will be greater than the original Nash equilibrium solution.

iv) In case ii) the society's output with complementarity restriction will be greater than the original Nash equilibrium solution if $E + K_j a^* = S$: And we can not compare between them if $\frac{S}{2} < E + K_j a^* < S$

(Proof is given in Appendix 3)

The Nash equilibrium results of the CPR game with complementarity restriction compared to the Nash results of our original CPR game have been illustrated in diagram 2. B_1^* is the Nash benefit curve at stage 1 characterizing the properties of corollary i) of lemma 2. B_1^{**} is the Nash benefit curve at stage 1 with complementarity restriction. If for both the players the endowment restriction acts as the binding constraint not the complementarity restriction, so that, $a_i^1 = \frac{E}{K_i}$ and $a_j^1 = \frac{E}{K_j}$, the players Nash action will lie along the rectangular hyperbolic curve B_1^*.

Insert Diagram 2 here

In that case with proposition P3 satisfying lemma 2, corollary i), the optimal milk output (also, payoff in CPR game) for both the players will be the same. Where as in the second case with complementarity restriction, a^* is such that for the ith player with larger size of the cattle property endowment restriction acts as the binding constraint and for the jth player with smaller size of the cattle property, the complementarity restriction is operative, i.e., $a_j^1 = a^* < \frac{E}{K_j}$: In this case the Nash benefit curve B_1^{**} will be no more rectangular hyperbolic indicating unequal level of milk output (payoff) from unequal level of per cattle action. In this situation milk output which is also the benefit from CPR as specified in our model for the `poor' cattle owner is less than the `rich' cattle owner by the amount $E - K_j a^*$: Lesser and lesser is the size of of the cattle property of the `poor' owner compared to the `rich', the greater is the difference, $E - K_j a^*$: In other words in this specific context for the `poor' cattle owner, a part of her total effort endowment will remain unutilized, which indicates that, her opportunity cost of investing effort (which although is not explicitly considered in this model) in the forest, the CPR, is lower than the `rich' cattle owner. This leads to a set of far reaching implications. In this particular society now we can define two potential bands $(\overline{\omega}_i; \underline{\omega}_j)$ (Diagram 3) on the choice of individual's per cattle action, where $\overline{\omega}_i = K_i a^* - E$ and $\underline{\omega}_j = E - K_j a^*$, satisfying the `no congestion' properties as specified by lemma 2. Because of endowment restriction the ith individual is not in a position to reach $\overline{\omega}_i$, and for complementarity restriction the jth individual is unable to reach $\underline{\omega}_j$. This bands may play a crucial role in defining the `Nash bargaining' frontier for searching an alternative institution for better solution (Bardhan, 1999; p 15).

Insert Diagram 3 here

The non-cooperative Nash equilibrium solution to CPR game as specified in our model is sub-optimal in character. In this kind of solution with the non- cooperative behaviour since the stock of fodder in the forest (a CPR) is not totally exhausted, there is a possibility to improvise an institution for collective action. Instead of choosing independently the per cattle action, individual can now choose collectively the per cattle action without causing congestion in the forest that will improve their solution. One of the formidable barriers against evolving an institution for collective action lies into the unequal benefits derived by individuals before they join for collective action (Elster, 1989). People will be interested for participating in the collective action if by doing that they can shift outward their previous Nash bargaining frontier.

In the context of our present model one can think of a transfer programme (either the transfer of cattle property or the transfer of per cattlle action between the players) so that each of them can move closer to the the potential bands $(\top_i;\perp_j)$: Since the bands in our hypothetical society di®ers because of the di®erences in cattle property (i.e., inequality in distribution of private property resources) greater and greater is the inequality in cattle property, greater is the di®erence in bands $(\top_i;\perp_j)$; more it is di±cult to suggest a well de¯ned tranfer programme for the society as a whole. This happens because greater the di®erence in bands the greater will be the variation in marginal bene¯t from transfer (either in terms of cattle property or in terms of per cattle action) among the players.

4.Conclusion

With the help of a simple model of a village society producing milk with cattle (private property, which is unequally distributed) and fodder (which is collected from forest, a common property) this paper examines the e®ect of inequality in private property resources on individual's optimal allocation of e®ort in the common property ¯eld and individual's as well as society's bene¯t from the common property resources. In the generalized version of a two-player, two-stage backward induction game the inequality issue has been handled in two di®erent context. Firstly it has considered a situation where inequality itself is manifestated through unequal per cattle e®ort endowment, where the per cattle e®ort endowment for the `rich' cattle owner is less compared to the `poor' owner. Secondly, it has considered a situation where in conjunction with per cattle e®ort endowment there exists a limit on per cattle action (or e®ort) beyond which milk production can not be increased, which is called complementarity restriction in the model. In these context subgame perfect Nash solution to the problem of individual's allocation of e®ort has been derived and characterized in terms of the four parameters in the model, viz.,cattle property, e®ort endowment, stock of fodder in forest resource, complementarity restriction.

4.1 Summary of Results

In order to get the ¯nal results of the model, the theoretical exercises of this paper as an intermediary step has started with an oversimpli¯ed version of the two-stage CPR game, where on the choice of per cattle action for collecting fodder from forest, there is no restriction other than the per cattle endowment restriction. In this model the per cattle endowment restriction varies from individual to individual according to the size of cattle property. In the subgame perfect Nash equilibrium the per cattle action are found to be di®erent among the di®erent individuals according to the size of cattle property they hold, although milk output (or payo®) remains the same. In the second stage subgame the Nash equilibrium milk output which is contingent upon the ¯rst stage action is less than the ¯rst stage subgame output. In the present model at the second stage, forest is assumed to regenerate a fraction of its initial stock. In this simpli¯ed framework, although the congestion externality has been been taken into account in the de¯nition of payo® or milk output, the Nash solution satisfying the endowment restriction rules out the possibility of congestion.

Finally the oversimpli¯ed version of the two-stage CPR game has been extended to consider a situation where for the individual player along with endowment restriction and given size of cattle property it is not possible to increase production by increasing e®ort after a certain limit. (Since the complementary role of e®ort to increase milk production has been restricted by this assumption, this restriction has been called as complementarity restriction in this paper). In that situation to solve the problem of optimal allocation of e®ort in the common property ¯eld, if the value of complementarity restriction is such that the per cattle endowment restriction becomes the binding constraint for all the individuals concerned, the subgame perfect Nash equilibrium will result equal payo® or output from unequal e®ort. Alternatively, if for di®erent individuals di®erent kind of restrictions viz., endowment restriction and complementarity restriction become the binding constraints the subgame perfect Nash equilibrium will result unequal payo® or output from unequal e®ort. In a two-player-two-stage backward induction CPR game with inequality in private property resources the present paper has shown that it is for the `poor' cattle owner the complementarity restriction and for the `rich' the complementarity restriction and endowment restriction will be the binding constraint resulting unequal payo® or output from unequal per cattle action.

While considering an alternative institutional framework for collective action to reduce suboptimality of the non-cooperative Nash solution the above results from the theoretical exercises have a far reaching implication. Unequal bene¯ts from non-cooperative Nash solution create problems for evolving an institution of collective action. In this present paper it has been shown that unequal bene¯ts in Nash solution in a society with unequal distribution of private property resources may be rooted into di®erent kinds of binding restrictions (i.e., complementarity restriction and endowment restriction) for individual players.In that context this paper in a theoretical framework indicates how those binding restrictions play the crucial deterministic role in individual's voluntary participation in collective action in common property ¯eld.

Appendix 1.

The solution to the Two-player-Two-stage CPR game:

The game starts from solving the following second stage payo® funcion of the ith player, contingent upon the history of action from the ¯rst stage,

$$Q_i^2(a_i^2(\hat{b}_i^1;\hat{b}_j^1);a_j^2(\hat{b}_i^1;\hat{b}_j^1)) = (K - a_i^2)\Psi_i^2(f_i^t(.)) \quad (14)$$

$$= (K - a_i^2)e^{x^2 - 1} \text{ say,} \quad (15)$$

where, $x^2 - 1 = \frac{(S+\Delta) - (K - a_i^1 + K - a_j^1)}{K - a_i^2 + K - a_j^2} - 1$

The ¯rst order condition for maximization of this function with respect to a_i^2, requires that,

$$\frac{\partial}{\partial a_i^2}\left[(K - a_i^2)e^{x^2 - 1}\right] = 0 \quad (16)$$

for the given values $(\hat{b}_i^1;\hat{b}_j^1)$ this leads to the following two sets of equations, for ith and jth players respectively:

$$[(S+\Delta) - (K_i\hat{a}_i^1 + K_j\hat{a}_j^1)]K_i a_i^2 = (K_i a_i^2 + K_j a_j^2)^2 \tag{17}$$

$$[(S+\Delta) - (K_i\hat{a}_i^1 + K_j\hat{a}_j^1)]K_j a_j^2 = (K_i a_i^2 + K_j a_j^2)^2 \tag{18}$$

This is actually the best response of the ith player to choose a_i^2, the effort per unit of cattle to collect fodder, in period 2, given K_i, and given the choice of the opponent player a_j^2, and given the effort already put by the society in the previous period. Solving these two equations together, we get,

$$a_i^2 = \frac{K_j}{K_i} a_j^2 \tag{19}$$

$$\text{Or}, a_i^2 = \theta a_j^2 \tag{20}$$

Substituting these values into the response equations, we get the Nash-Cournot solution as:

$$\hat{a}_i^2 = \frac{(S+\Delta) - (K_i\hat{a}_i^1 + K_j\hat{a}_j^1)}{4K_i} \tag{21}$$

and,

$$\hat{a}_j^2 = \frac{(S+\Delta) - (K_i\hat{a}_i^1 + K_j\hat{a}_j^1)}{4K_j} \tag{22}$$

Now consider the first stage game rolled back from the second stage. The ith player will choose her optimal level of action $\hat{a}_i^1$ to maximize her present milk output contingent upon the anticipated action values for the future production, given the available stock. Her optimization problem is therefore,

$$Max_{\hat{a}_i^1}[Q_i^1(\hat{a}_i^1, \hat{a}_j^1) + Q_i^2(a_i^2(\hat{a}_i^1, \hat{a}_j^1), a_j^2(\hat{a}_i^1, \hat{a}_j^1))] \tag{23}$$

the first order condition of which requires that:

$$\frac{\partial}{\partial \hat{a}_i^1}[K_i a_i^1 e^{x^1 - 1} + K_i a_i^2 e^{x^2 - 1}] = 0 \tag{24}$$

$$\text{Or}, \frac{\partial}{\partial \hat{a}_i^1}[a_i^1 e^{x^1 - 1} + a_i^2 e^{x^2 - 1}] = 0 \tag{25}$$

Now,

$$x^2 - 1 = \frac{(S + \Delta) - (K_i \hat{a}_i^1 + K_j \hat{a}_j^1) - (K_i a_i^2 + K_j a_j^2)}{K_i a_i^2 + K_j a_j^2} \quad (26)$$

Substituting the value of a_i^2 from the Nash-Cournot solution into the above equation, we get, $x^2 - 1 = 1$. Or; $x^2 = 2$. Now by D1.1 the value of the effectivity function, $\psi_i^2 = 1$.

Therefore the first order condition for maximization of Q_i with respect to $\hat{a}_i^1$ is reduced to :

$$\frac{\partial}{\partial \hat{a}_i^1}[\hat{a}_i^1 e^{x^1 - 1} + a_i^2] = 0 \quad (27)$$

$$\text{Or; } \frac{\partial}{\partial \hat{a}_i^1}[\hat{a}_i^1 e^{x^1 - 1}] = 0 \quad (28)$$

$$\text{Or; } e^{x^1 - 1} + \hat{a}_i^1 \frac{\partial}{\partial a}(x^1 - 1) e^{x^1 - 1} = 0 \quad (29)$$

where,

$$x^1 - 1 = \frac{S - (K_i \hat{a}_i^1 + K_j \hat{a}_j^1)}{K_i \hat{a}_i^1 + K_j \hat{a}_j^1} \quad (30)$$

Now,

$$\frac{\partial}{\partial \hat{a}_i^1}(x^1 - 1) = \frac{-S K_i}{(K_i \hat{a}_i^1 + K_j \hat{a}_j^1)^2} \quad (31)$$

Plugging this value into the first order condition for optimization of the ith player, we get:

$$K_i \hat{a}_i^1 = \frac{(K_i \hat{a}_i^1 + K_j \hat{a}_j^1)^2}{S} \quad (32)$$

Similarly for the jth player solving the first order condition for maximization of Q_j with respect to $\hat{a}_j^1$, i.e., considering,

$$\frac{\partial}{\partial \hat{a}_j^1}[a_j^1 e^{x^1 - 1}] = 0 \quad (33)$$

we get:

$$K_i \hat{a}_j^1 = \frac{(K_i \hat{a}_i^1 + K_j \hat{a}_j^1)^2}{S} \quad (34)$$

Thus, $K_i \hat{a}_i^1 = K_j \hat{a}_j^1$, and if we substitute this into the optimal conditions for $K_i \hat{a}_i^1$ and $K_j \hat{a}_j^1$ respectively, finally we get the optimal values,

$$\hat{a}_i^1 = \frac{S}{4K_i} \tag{35}$$

$$\hat{a}_j^1 = \frac{S}{4K_j} \tag{36}$$

Now in order to get Nash-Cournot equilibrium values at stage 2, contingent upon stage 1, we substitute the values $(\hat{a}_i^1; \hat{a}_j^1)$ into, $\hat{a}_i^2 = \frac{(S+\Delta) - (K_i \hat{a}_i^1 + K_j \hat{a}_j^1)}{4K_i}$, and $\hat{a}_j^2 = \frac{(S+\Delta) - (K_i \hat{a}_i^1 + K_j \hat{a}_j^1)}{4K_j}$, and we get:

$$\hat{a}_i^2 = \frac{S/2 + \Delta}{4K_i} \tag{37}$$

$$\hat{a}_j^2 = \frac{S/2 + \Delta}{4K_j} \tag{38}$$

Note: $K_i \hat{a}_i^t = K_j \hat{a}_j^t$; but, $\hat{a}_i^t \neq \hat{a}_j^t$ for $t = 1; 2$.

Now it is possible to find out the value of effectivity ratio at stage 1. $A^1 = e^{x^1 - 1}$ at the subgame perfect equilibrium;

$$x^1 - 1 = \frac{S}{K_i \hat{a}_i^1 + K_j \hat{a}_j^1} - 1 \tag{39}$$

$$= \frac{S}{S/2} - 1 \tag{40}$$

Since $x^1 > 1$, by definition, $\psi^1 = 1$:

Now the optimal value of the milk-output of the ith player,

$$Q_i^* = \hat{Q}_i^1(\hat{a}_i^1; \hat{a}_j^1) + \hat{Q}_i^2(\hat{a}_i^2(\hat{a}_i^1; \hat{a}_j^1); \hat{a}_j^2(\hat{a}_i^1; \hat{a}_j^1)) \tag{41}$$

$$= \left(\frac{S}{4} + \frac{S/2 + \Delta}{4}\right) = \frac{3S/2 + \Delta}{4} \tag{42}$$

Similarly for the jth player,

$$Q_j^* = \frac{3S/2 + \Delta}{4} \tag{43}$$

The total milk output in the society:

$$Q^{*} = Q_i^{*} + Q_j^{*} = \frac{3S + 2\Delta}{4} < S + \Delta\,(= \text{Total available stock}) \tag{44}$$

Since $\frac{3S+2\Delta}{4} < S + \Delta$, it is obvious that the sub-game perfect equilibrium solution gives us a sub-optimal solution with underutilized stock of resources in the society.

Appendix 2.

The solution to the two-stage backward induction CPR game:

The game starts from solving the following second stage payoff funcion of the ith player, contingent upon the history of action from the first stage,

$$Q_i^2(a_i^2(\hat{a}_i^1;\hat{a}_j^1);a_j^2(\hat{a}_i^1;\hat{a}_j^1)) = (K_i a_i^2)\psi_i^2(f_i^t(.)) \tag{45}$$

$$= (K_i a_i^2)e^{x^2-1} \text{ say,} \tag{46}$$

where, $x^2 - 1 = \frac{(S+\Delta)-(K_i a_i^1 + K_j a_j^1)}{K_i a_i^2 + K_j a_j^2} - 1$

The first order condition for maximization of this function with respect to a_i^2, requires that,

$$\frac{\partial}{\partial a_i^2}\left[(K_i a_i^2)e^{x^2-1}\right] = 0 \tag{47}$$

for the given values $(\hat{a}_i^1;\hat{a}_j^1)$ this leads to the following two sets of equations, for ith and jth players respectively:

$$[(S+\Delta) - (K_i\hat{a}_i^1 + K_j\hat{a}_j^1)]K_i a_i^2 = (K_i a_i^2 + K_j a_j^2)^2 \tag{48}$$

$$[(S+\Delta) - (K_i\hat{a}_i^1 + K_j\hat{a}_j^1)]K_j a_j^2 = (K_i a_i^2 + K_j a_j^2)^2 \tag{49}$$

This is actually the best response of the ith player to choose a_i^2,the effort per unit of cattle to collect fodder, in period 2, given K_i, and given the choice of the opponent player a_j^2, and given the effort already put by the society in the previous period. Solving these two equations together, we get,

$$a_i^2 = \frac{K_j}{K_i} a_j^2 \tag{50}$$

$$\text{Or}, a_i^2 = \theta a_j^2 \tag{51}$$

Substituting these values into the response equations, we get the Nash-Cournot solution as:

$$\hat{a}_i^2 = \frac{(S + \Delta) - (K_i \hat{a}_i^1 + K_j \hat{a}_j^1)}{4K_i} \tag{52}$$

and,

$$\hat{a}_j^2 = \frac{(S + \Delta) - (K_i \hat{a}_i^1 + K_j \hat{a}_j^1)}{4K_j} \tag{53}$$

Now consider the first stage game rolled back from the second stage. The ith player will choose her optimal level of action $\hat{a}_i^1$ to maximize her present milk output contingent upon the anticipated action values for the future production, given the available stock. Her optimization problem is therefore,

$$\text{Max}_{\hat{a}_i^1}[Q_i^1(\hat{a}_i^1, \hat{a}_j^1) + Q_i^2(a_i^2(\hat{a}_i^1, \hat{a}_j^1), a_j^2(\hat{a}_i^1, \hat{a}_j^1))] \tag{54}$$

the first order condition of which requires that:

$$\frac{\partial}{\partial \hat{a}_i^1}[K_i a_i^1 e^{x^1 - 1} + K_i a_i^2 e^{x^2 - 1}] = 0 \tag{55}$$

$$\text{Or}, \frac{\partial}{\partial \hat{a}_i^1}[a_i^1 e^{x^1 - 1} + a_i^2 e^{x^2 - 1}] = 0 \tag{56}$$

Now,

$$x^2 - 1 = \frac{(S + \Delta) - (K_i \hat{a}_i^1 + K_j \hat{a}_j^1) - (K_i a_i^2 + K_j a_j^2)}{K_i a_i^2 + K_j a_j^2} \tag{57}$$

Substituting the value of a_i^2 from the Nash-Cournot solution into the above equation, we get, $x^2 - 1 = 1$. Or, $x^2 = 2$. Now by Ð1.1 the value of the effectivity function, $\psi_i^2 = 1$.

Therefore the first order condition for maximization of Q_i with respect to $\hat{a}_i^1$ is reduced to :

$$\frac{\partial}{\partial \hat{a}_i^1}[\hat{a}_i^1 e^{x^1 - 1} + a_i^2] = 0 \tag{58}$$

$$\text{Or;} \frac{\partial}{\partial \hat{a}_i^1}[\hat{a}_i^1 e^{x^1 - 1}] = 0 \tag{59}$$

$$\text{Or;} e^{x^1 - 1} + \hat{a}_i^1 \frac{\partial}{\partial a}(x^1 - 1)e^{x^1 - 1} = 0 \tag{60}$$

where,

$$x^1 - 1 = \frac{S - (K_i \hat{a}_i^1 + K_j \hat{a}_j^1)}{K_i \hat{a}_i^1 + K_j \hat{a}_j^1} \tag{61}$$

Now,

$$\frac{\partial}{\partial \hat{a}_i^1}(x^1 - 1) = \frac{-SK_i}{(K_i \hat{a}_i^1 + K_j \hat{a}_j^1)^2} \tag{62}$$

Plugging this value into the first order condition for optimization of the ith player, we get:

$$K_i \hat{a}_i^1 = \frac{(K_i \hat{a}_i^1 + K_j \hat{a}_j^1)^2}{S} \tag{63}$$

Similarly for the jth player solving the first order condition for maximization of Q_j with respect to $\hat{a}_j^1$, i.e., considering,

$$\frac{\partial}{\partial \hat{a}_j^1}[a_j^1 e^{x^1 - 1}] = 0 \tag{64}$$

we get:

$$K_i \hat{a}_j^1 = \frac{(K_i \hat{a}_i^1 + K_j \hat{a}_j^1)^2}{S} \tag{65}$$

Thus, $K_i \hat{a}_i^1 = K_j \hat{a}_j^1$, and if we substitute this into the optimal conditions for $K_i \hat{a}_i^1$ and $K_j \hat{a}_j^1$ respectively, finally we get the optimal values,

$$\hat{a}_i^1 = \frac{S}{4K_i} \tag{66}$$

$$\hat{a}_j^1 = \frac{S}{4K_j} \tag{67}$$

Now in order to get Nash-Cournot equilibrium values at stage 2, contingent upon stage 1, we substitute the values $(\hat{a}_i^1; \hat{a}_j^1)$ into, $\hat{a}_i^2 = \frac{(S+\Delta) - (K_i \hat{a}_i^1 + K_j \hat{a}_j^1)}{4K_i}$, and $\hat{a}_j^2 = \frac{(S+\Delta) - (K_i \hat{a}_i^1 + K_j \hat{a}_j^1)}{4K_j}$, and we get:

$$\hat{a}_i^2 = \frac{S/2 + \Delta}{4K_i} \tag{68}$$

$$\hat{a}_j^2 = \frac{S/2 + \Delta}{4K_j} \tag{69}$$

Note: $K_i \hat{a}_i^t = K_j \hat{a}_j^t$; but, $\hat{a}_i^t \neq \hat{a}_j^t$ for $t = 1; 2$.

Now it is possible to find out the value of effectivity ratio at stage 1. $A^1 = e^{x^1 - 1}$ at the subgame perfect equilibrium;

$$x^1 - 1 = \frac{S}{K_i \hat{a}_i^1 + K_j \hat{a}_j^1} - 1 \tag{70}$$

$$= \frac{S}{S/2} - 1 \tag{71}$$

Since $x^1 > 1$, by definition, $\psi^1 = 1$:

Now the optimal value of the milk-output of the ith player,

$$Q_i^* = \hat{Q}_i^1(\hat{a}_i^1; \hat{a}_j^1) + \hat{Q}_i^2(\hat{a}_i^2(\hat{a}_i^1; \hat{a}_j^1); \hat{a}_j^2(\hat{a}_i^1; \hat{a}_j^1)) \tag{72}$$

$$= \left(\frac{S}{4} + \frac{S/2 + \Delta}{4}\right) = \frac{3S/2 + \Delta}{4} \tag{73}$$

Similarly for the jth player,

$$Q_j^* = \frac{3S/2 + \Delta}{4} \tag{74}$$

The total milk output in the society:

$$Q^* = Q_i^* + Q_j^* = \frac{3S + 2\Delta}{4} < S + \Delta \text{ (= Total available stock)} \tag{75}$$

Since $\frac{3S+2\Delta}{4} < S + \Delta$, it is obvious that the sub-game perfect equilibrium solution gives us a sub-optimal solution with underutilized stock of resources in the society.

Appendix 2.

Proof: Lemma1.When $a^{*} \quad \frac{S}{4K_i} < \frac{S}{4K_j}$ i.e., $a^{*} \quad \frac{S}{4K_i}$ and $a_j^1 < \frac{S}{4K_j}$; then $K_i a^{*} \quad \frac{S}{4}$ and $K_j a_j^1 < \frac{S}{4}$

Then, $K_i a^{*} + K_j a_j^1 < \frac{S}{2}$

$$; f_i^t((a^{*}; a_j^t); S) \geq 0 \tag{76}$$

$$\text{Since } ; f_i^t((a^{*}; a_j^t); S) = x^1 - 1; \tag{77}$$

$$\text{where, } x_1 - 1 = \frac{S - (K_i a^{*} + K_j a_j^1)}{K_i a^{*} + K_j a_j^1} \tag{78}$$

$$> 0 \tag{79}$$

$$\text{since; } K_i a^{*} + K_j a_j^1 < \frac{S}{2} \tag{80}$$

Proof: Lemma 2. $a^{*} \geq \frac{S}{4K_j} > \frac{S}{4K_i}$, satisfying the endowment restriction $\frac{S}{4K_i} \quad \frac{E}{K_i}$ and $\frac{S}{4K_j} \quad \frac{E}{K_j}$

Now the forest would or would not be congested for all $(\frac{E}{K_i}; a_j^1) \in A^1$; for all $0 < a_j^1 \quad \frac{E}{K_j}$, depends upon whether $; f_i^t((a^{*}; a_j^t); S) \gtreqless 0$

We know,

$$; f_i^t((a^{*}; a_j^t); S) = x^1 - 1; \tag{81}$$

$$\text{where, } x_1 - 1 = \frac{S - (E + K_j a_j^1)}{E + K_j a_j^1} \tag{82}$$

Since $\max(a_i^1) = \frac{E}{K_i} < a^{*}; K_i a_i^1 = E$

$$x_1 - 1 \gtreqless 0 \tag{83}$$

$$\Rightarrow S - (E + K_j a_j^1) \gtreqless 0 \tag{84}$$

$$\text{i:e:; } S - E \gtreqless K_j a_j^1 \tag{85}$$

In other words, with the conditions satisfying lemma2, the forest would not be congested (respectively congested) iff $S - E \geq K_j a_j^1$ (respectively iff $S - E < K_j a_j^1$

Corollary i) When, $a^{*} \geq \frac{E}{K_j} > \frac{S}{4K_j}$, $a_i^1 = \frac{E}{K_i}$ and $a_j^1 = \frac{E}{K_j}$, with $\frac{E}{K_i} < \frac{E}{K_i}$; not only for the ith individual, but also for the j th individual the complementarity restriction are no more the binding constraints. Henceforth, $a_i^1 = \frac{E}{K_i}$ and $a_j^1 = \frac{E}{K_j}$, i.e., $K_i\, a_i^1 = K_j$, $a_j^1 = E$

In that case, congestion of forest depends upon whether,

$$x_1 - 1 \gtreqless 0 \tag{86}$$

$$\Rightarrow \quad S - (E + E) \gtreqless 0 \tag{87}$$

$$\Rightarrow \quad S \gtreqless 2E \tag{88}$$

$$\Rightarrow \quad E \lesseqgtr \frac{S}{2} \tag{89}$$

In other words, the forest will not be congested iff, $E \quad \frac{S}{2}$:

Corollary ii) When $\frac{E}{K_j} > a^{*} \geq \frac{S}{4K_j}$; $a_i^1 = \frac{E}{K_i}$; $K_j a^{*} \geq \frac{S}{4}$ and $a_i^1 = \frac{E}{K_i} > \frac{S}{4K_i}$, i.e., $E > \frac{S}{4}$, $E + K_j a^{*} > \frac{S}{2}$:

$$\text{If } S \geq E + K_j a^{*} > \frac{S}{2}: \tag{90}$$

$$; f_i^t((E; a^{*}); S) \quad = \quad x_1 - 1 \tag{91}$$

$$= \quad S - (E + K_j a^{*}) \geq 0 \tag{92}$$

i.e., forest would not get congested. But if $E + K_j a^{*} > S$, $; f_i^t((E; a^{*}); S) < 0$ so that forest would get congested.Appendix 2

Lemma 3. $\frac{S}{4K_i} < a^{*} \quad \frac{S}{4K_j}$: In case i) when $\frac{S}{4K_i} < a^{*} \quad \frac{E}{K_i} \quad \frac{S}{4K_j}$, and $\frac{S}{4K_j} \quad \frac{E}{K_j}$ the endowment restriction is not a binding constraint. Therefore $a_i^1 = a_j^1 = a^{*}$ In that case, the forest is not congested if

$$; f_i^t((a^{*}; a^{*}); S) \quad = \quad x_1 - 1 \geq 0 \tag{93}$$

$$= \quad S - (K_i + K_j) a^{*} \geq 0 \tag{94}$$

$$\Rightarrow \quad a^{*} \quad \frac{S}{K_i + K_j} \tag{95}$$

In case ii) when $\frac{S}{4K_i} \leq \frac{E}{K_i} < a^* \leq \frac{S}{4K_j}$ and $\frac{S}{4K_j} \leq \frac{E}{K_j}$, $\max a_i^1 = \frac{E}{K_i}$, and for the jth player, the endowment restriction is inoperative. Therefore, $a_i^1 = \frac{E}{K_i}$, and $a_j^1 = a^*$: the forest is not congested if

$$; f_i^t((a^*; a^*); S) = x_1 - 1 \geq 0 \tag{96}$$

$$= S - (E + K_j a^*) \geq 0 \tag{97}$$

$$\Rightarrow E + K_j a^* \leq S \tag{98}$$

Appendix 3

Proof: Proposition 3. When $a^* \geq \frac{E}{K_j} > \frac{S}{4K_j}$; the optimal action values in (33) in Appendix 1 will be:

$$\hat{a}_i^1 = \frac{E}{K_i}$$

$$\hat{a}_j^1 = \frac{E}{K_j}$$

Then by corollary C1 of Lemma 2 if $E \leq \frac{S}{2}$;i.e., if forest is not congested, from equation (39) in Appendix 1, we get

$$Q_i^* = E + \frac{(S + \Delta) - 2E}{4}$$

$$= \frac{2E + (S + \Delta)}{4}$$

$$= Q_j^*$$

If $E = \frac{S}{2}$;then substituting this value in the above equation, we get,

$$Q_i^* = \frac{2S + \Delta}{4}$$

$$= Q_j^*$$

Total milk output in the society:

$$\overline{Q} = Q_i^* + Q_j^*$$

$$= \quad S+\frac{¢}{2}$$

Since $S+\frac{¢}{2} > \frac{3S+2¢}{4}$, $\overline{Q} > Q^{¤}$, the original tota milk output we obtained in the Nash solution (equation 42 in Appendix1).

When $\frac{E}{K_j} > a^{¤}$ ¸ $\frac{S}{4K_j}$, $a_i^1 = \frac{E}{K_i}$, $a_j^1 = a^{¤}$. Then from equation 19) we get, $a_i^2 = \frac{(S+¢)\,¡\,(E+K_j a^{¤})}{4}$: If $E + K_j a^{¤} = S$, then substituting this value into the above equations, we get:

$$Q_i^{¤} = E + \frac{¢}{4}$$

$$Q_j^{¤} = K_j a^{¤} + \frac{¢}{4}$$

The society's total output

$$\overline{Q} = Q_i^{¤} + Q_j^{¤}$$

$$= E + K_j a^{¤} + \frac{¢}{2}$$

$\overline{Q} > Q^{¤}$ if $E + K_j a^{¤} = S$. If $E + K_j a^{¤} < S$; we can not compare between them immediately.

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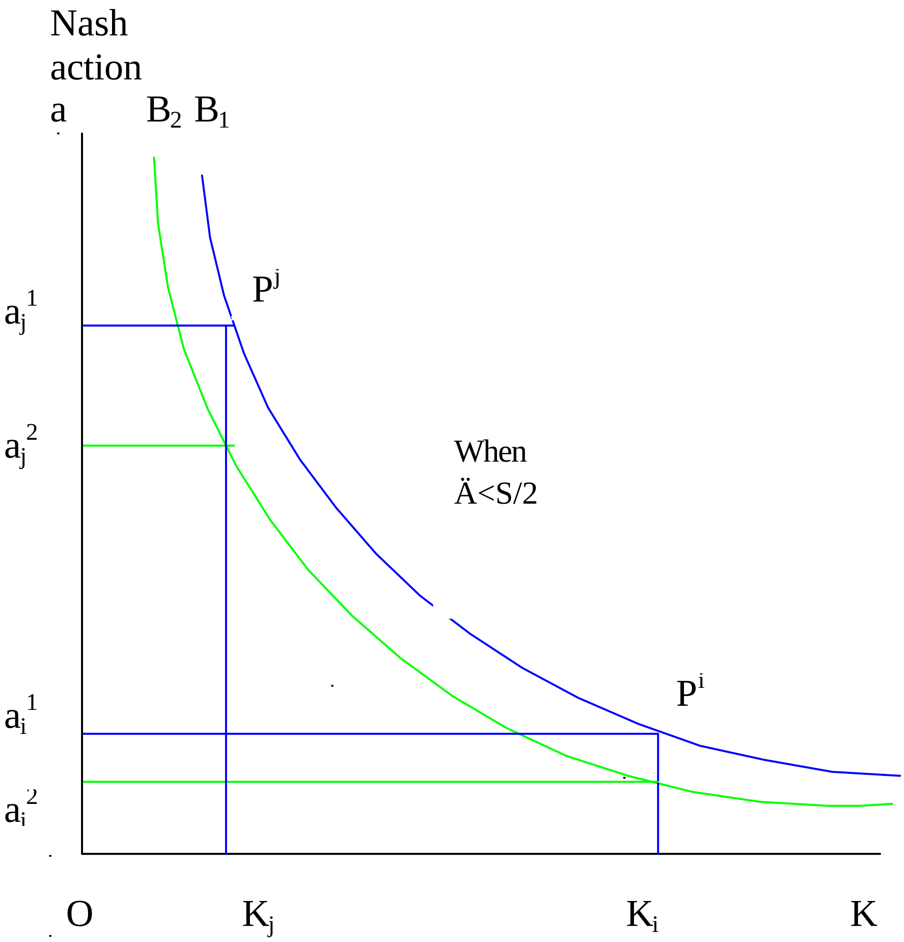
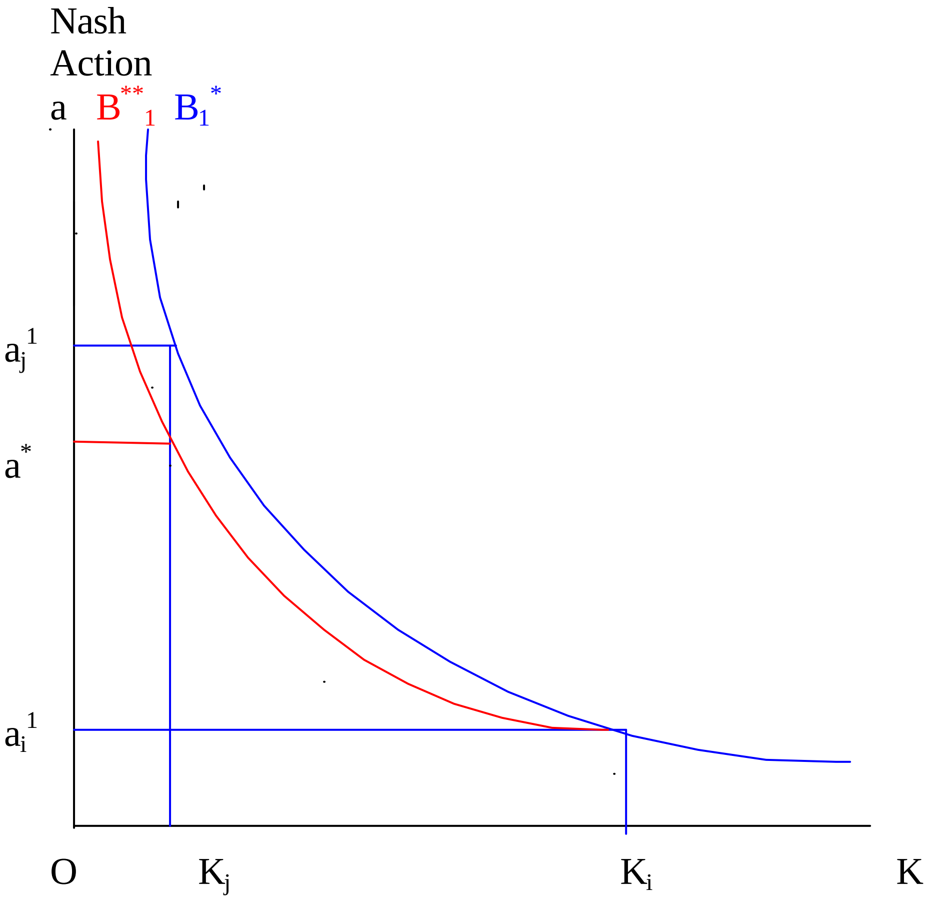


Diagram 1



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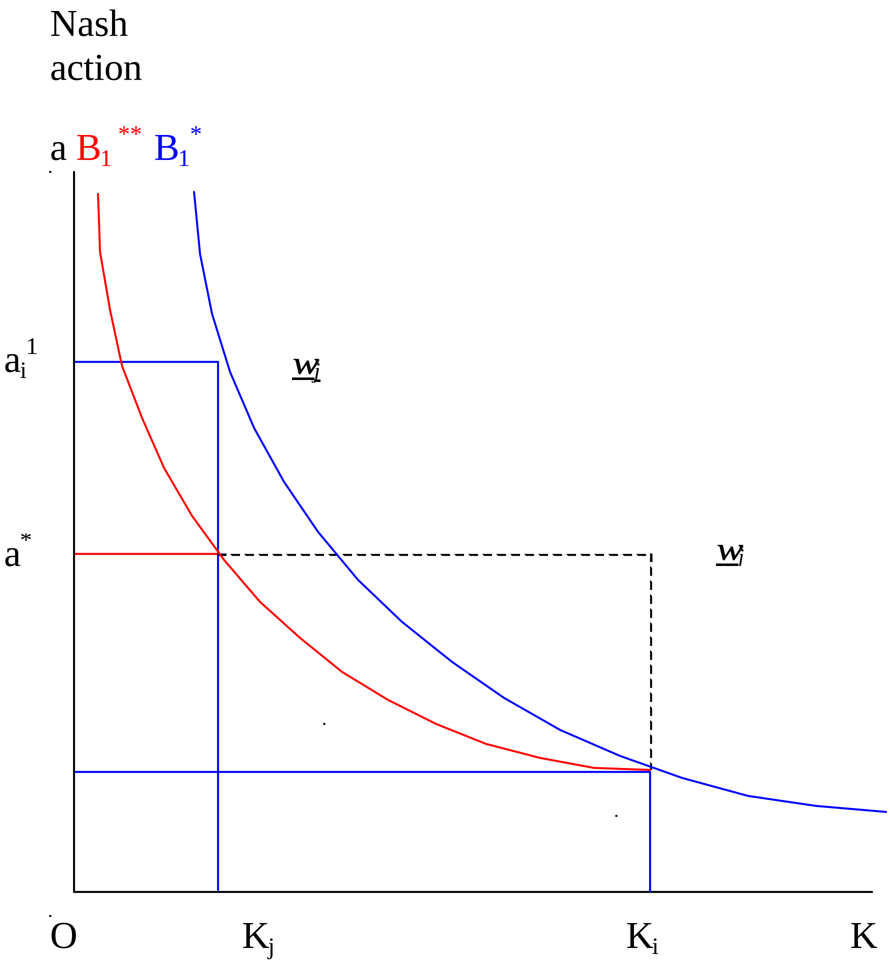

Diagram 3

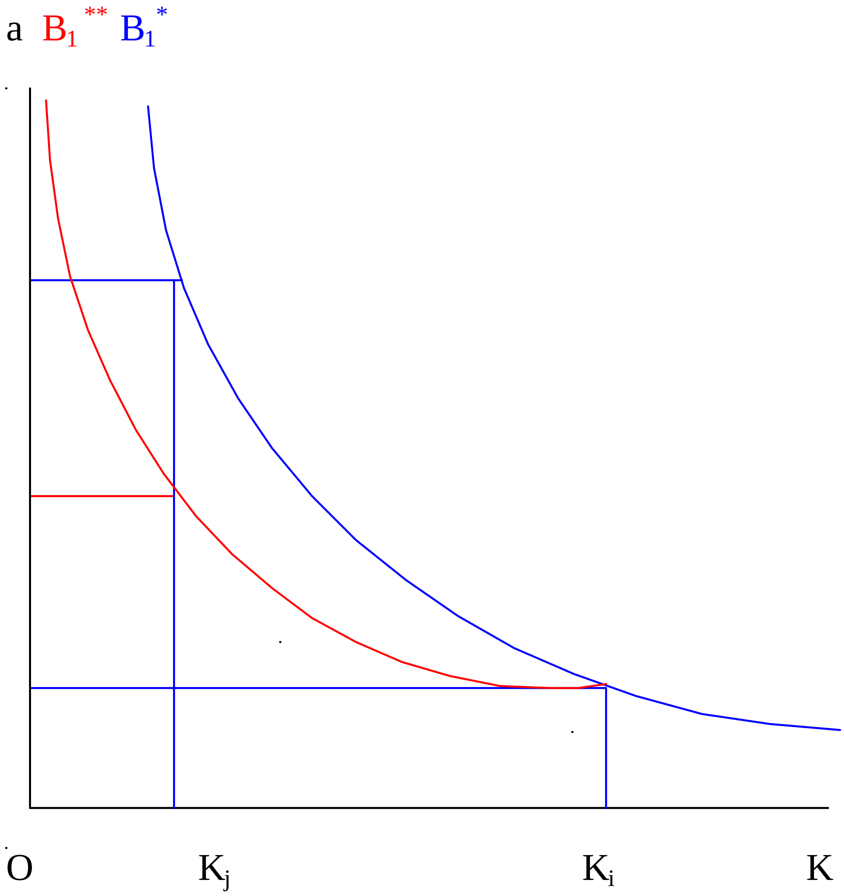

Diagram 2